Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.6-Exponentials-of-inverse-hyperbolic-tangent-functions

Nasser M. Abbasi

July 28, 2021 Compiled on July 28, 2021 at 7:17am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1378]. This is test number [196].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (1378)	% 0.00 (0)
Mathematica	% 98.19 (1353)	% 1.81 (25)
Maple	% 79.83 (1100)	% 20.17 (278)
Maxima	% 44.92 (619)	% 55.08 (759)
Fricas	% 81.13 (1118)	% 18.87 (260)
Sympy	% 32.95 (454)	% 67.05 (924)
Giac	% 48.98 (675)	% 51.02 (703)
Mupad	% 50.65 (698)	% 49.35 (680)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

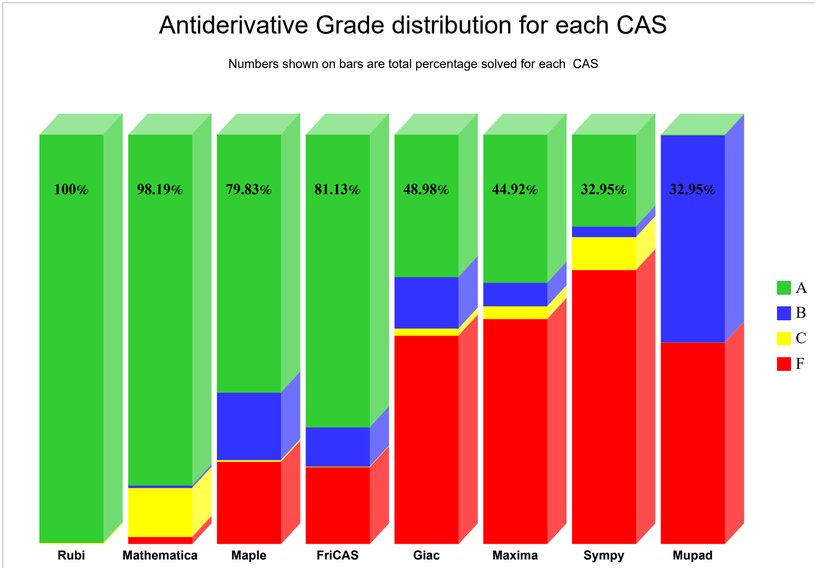
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

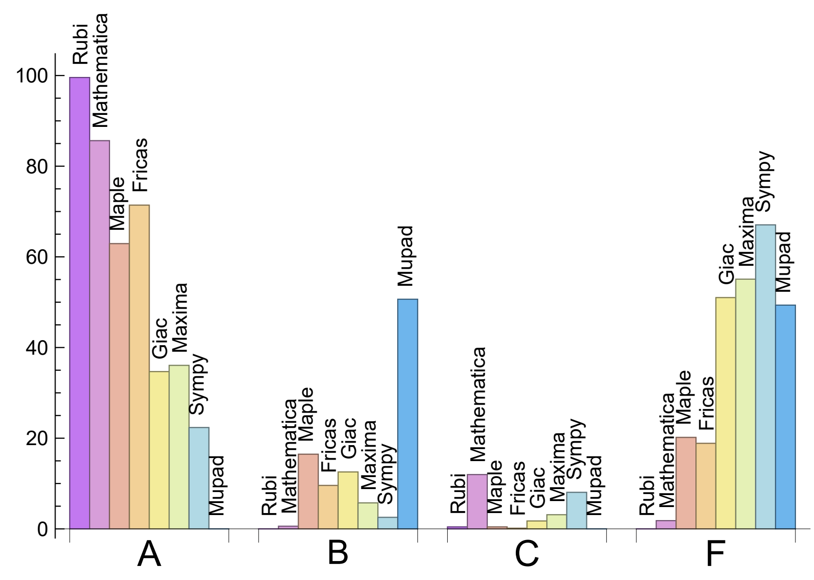
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.56	0.00	0.44	0.00
Mathematica	85.63	0.58	11.97	1.81
Maple	62.92	16.47	0.44	20.17
Maxima	36.07	5.73	3.12	55.08
Fricas	71.41	9.58	0.15	18.87
Sympy	22.35	2.54	8.06	67.05
Giac	34.69	12.55	1.74	51.02
Mupad	0.00	50.65	0.00	49.35

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	25	84.00 %	16.00 %	0.00 %
Maple	278	100.00 %	0.00 %	0.00 %
Maxima	759	96.84 %	0.13 %	3.03 %
Fricas	260	92.69 %	7.31 %	0.00 %
Sympy	924	90.37 %	8.98 %	0.65 %
Giac	703	69.84 %	1.28 %	28.88 %
Mupad	680	99.85 %	0.15 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

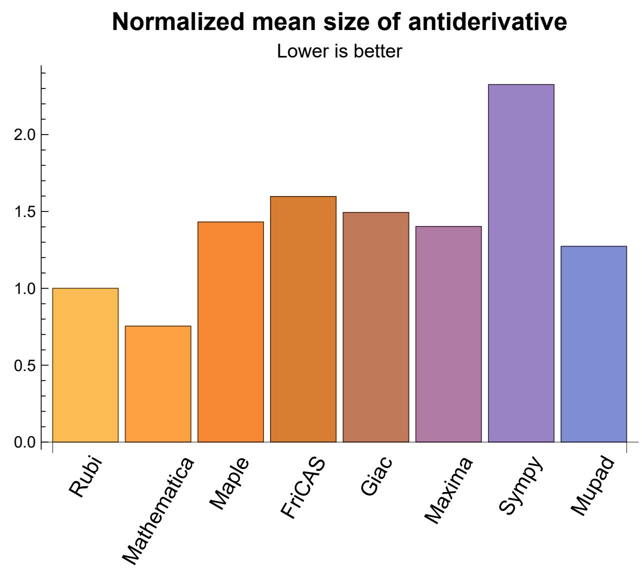
1.3 Performance

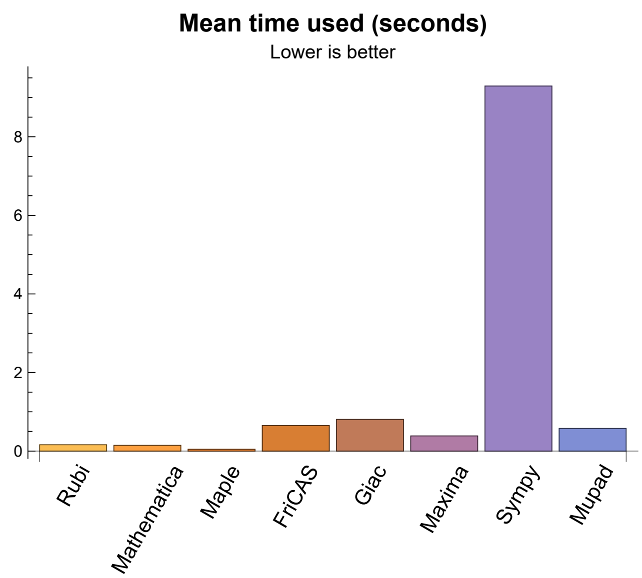
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	118.01	1.00	99.00	1.00
Mathematica	0.14	75.51	0.75	70.00	0.74
Maple	0.05	151.34	1.43	91.00	1.03
Maxima	0.39	116.14	1.40	73.00	1.04
Fricas	0.65	184.86	1.60	124.00	1.39
Sympy	9.29	209.70	2.33	85.00	1.22
Giac	0.81	131.62	1.49	82.00	1.14
Mupad	0.58	106.43	1.27	74.50	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {373, 620, 621, 791, 792, 795, 796, 1313, 1316, 1317, 1323, 1324, 1331, 1332, 1345, 1346, 1347, 1352, 1353, 1354}

Mathematica {6, 7, 8, 9, 10, 22, 23, 24, 25, 38, 39, 40, 41, 42, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 142, 144, 145, 147, 149, 152, 158, 159, 160, 162, 163, 164, 165, 166, 178, 180, 181, 182, 183, 184, 185, 186, 198, 199, 200, 202, 203, 204, 205, 206, 217, 218, 219, 220, 221, 226, 227, 228, 229, 231, 232, 233, 234, 243, 244, 245, 247, 248, 249, 250, 251, 258, 259, 260, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 291, 295, 296, 297, 298, 299, 300, 301, 306, 307, 308, 309, 310, 311, 312, 313, 316, 317, 318, 319, 320, 321, 322, 323, 324, 327, 328, 329, 330, 331, 336, 337, 338, 339, 340, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 387, 388, 389, 391, 392, 405, 411, 415, 416, 417, 418, 431, 432, 434, 435, 436, 448, 449, 450, 451, 452, 457, 458, 459, 466, 467, 468, 469, 470, 475, 477, 478, 485, 486, 487, 488, 494, 495, 496, 499, 502, 503, 504, 505, 510, 513, 514, 523, 524, 525, 526, 527, 528, 529, 530, 535, 539, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 573, 574, 575, 591, 592, 593, 594, 595, 596, 597, 598, 608, 609, 610, 611, 612, 620, 621, 623, 628, 629, 630, 645, 646, 662, 663, 664, 678, 679, 689, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 707, 717, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 791, 792, 796, 799, 801, 802, 803, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 822, 823, 824, 825, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 847, 848, 849, 850, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 873, 874, 876, 879, 883, 884, 885, 886, 891, 918, 919, 954, 1080, 1081, 1082, 1083, 1084, 1085, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1111, 1112, 1113, 1114, 1115, 1119, 1120, 1121, 1122, 1131, 1132, 1133, 1134, 1135, 1140, 1145, 1146, 1147, 1148, 1149, 1152, 1153, 1165, 1174, 1175, 1176, 1177, 1179, 1180, 1181, 1192, 1193, 1194, 1195, 1196, 1199, 1200, 1207, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1274, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301, 1302, 1304, 1311, 1313, 1315, 1316, 1317, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1331, 1332, 1342, 1343, 1344, 1345, 1346, 1347, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1358, 1367}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

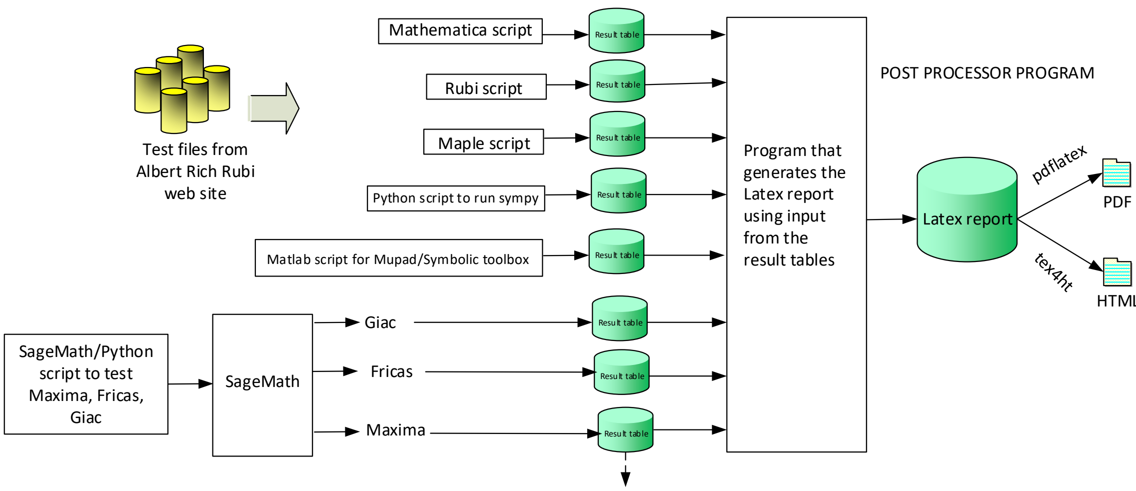
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550,

551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571,
572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592,
593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613,
614, 615, 616, 617, 618, 619, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635,
636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656,
657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677,
678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698,
699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719,
720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740,
741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761,
762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782,
783, 784, 785, 786, 787, 788, 789, 790, 793, 794, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806,
807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827,
828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848,
849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869,
870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890,
891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911,
912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932,
933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953,
954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974,
975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995,
996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013,
1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030,
1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047,
1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064,
1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081,
1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098,
1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115,
1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132,
1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149,
1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166,
1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183,
1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200,
1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217,
1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234,
1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251,
1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268,
1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285,
1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302,
1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319,
1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1333, 1334, 1335, 1336, 1337,
1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354,
1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371,
1372, 1373, 1374, 1375, 1376, 1377, 1378 }

B grade: { }

C grade: { 172, 620, 791, 792, 795, 1332 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 84, 104, 130, 141, 143, 146, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 270, 271, 272, 273, 274, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 346, 348, 349, 351, 352, 353, 354, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 433, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 556, 557, 558, 559, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 619, 621, 622, 623, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 802, 804, 805, 807, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 869, 870, 871, 873, 874, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1285, 1286, 1287, 1288, 1293, 1294, 1295, 1296, 1299, 1303, 1304, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378 }

B grade: { 291, 299, 301, 499, 620, 868, 1185, 1358 }

C grade: { 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 128, 129, 131, 132, 133, 135, 136, 137, 138, 139, 140, 142, 144, 145, 147, 172, 182, 217, 218, 219, 233, 234, 248, 250, 251, 260, 267, 268, 269, 275, 276, 277, 339, 345, 347, 350, 355, 360, 418, 432, 434, 435, 436, 470, 510, 523, 524, 525, 526, 527, 528, 529, 530, 551, 552, 553, 554, 555, 560, 561, 562, 573, 574, 628, 629, 630, 645, 646, 662, 663, 664, 678, 679, 801, 803, 806, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 872, 1053, 1133, 1134, 1135, 1145, 1146, 1232, 1252, 1281, 1282, 1283, 1284, 1289, 1290, 1291, 1292, 1297, 1298, 1300, 1301, 1302 }

F grade: { 60, 70, 80, 90, 100, 110, 120, 127, 134, 148, 447, 484, 617, 618, 624, 625, 626, 627, 800, 875, 1307, 1356, 1357, 1359, 1360 }

2.1.3 Maple

A grade: { 1, 2, 3, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 43, 44, 45, 46, 47, 48, 49, 50, 142, 144, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 179, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 217, 218, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 352, 354, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 381, 383, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 438, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 489, 494, 495, 496, 497, 498, 499, 500, 501, 507, 510, 511, 512, 513, 514, 515, 516, 518, 519, 520, 521, 527, 528, 529, 530, 531, 532, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 549, 550, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 608, 609, 610, 611, 612, 613, 614, 615, 616, 628, 629, 630, 631, 632, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 686, 687, 688, 689, 690, 691, 692, 693, 694, 699, 700, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 727, 728, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 780, 781, 782, 783, 784, 785, 786, 787, 788, 821, 827, 828, 829, 830, 831, 832, 833, 834, 853, 854, 855, 856, 857, 858, 859, 872, 885, 886, 887, 888, 889, 891, 892, 893, 894, 895, 896, 900, 901, 902, 904, 905, 906, 910, 911, 912, 913, 914, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 994, 998, 999, 1000, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1080, 1081, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1118, 1119, 1121, 1122, 1123, 1124, 1127, 1137, 1138, 1139, 1140, 1143, 1147, 1148, 1149, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1182, 1183, 1184, 1185, 1186, 1188, 1189, 1190, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1239, 1240, 1245, 1248, 1249, 1250, 1251, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1285, 1286, 1287, 1288, 1293, 1294, 1295, 1296, 1304, 1315, 1320, 1321, 1322, 1325, 1326, 1343, 1344, 1348, 1349, 1350, 1351, 1355, 1363, 1364, 1365, 1366, 1367, 1370, 1371, 1373, 1374, 1375, 1376, 1377, 1378 }

B grade: { 4, 6, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52, 53, 54, 55, 56, 57, 58, 59, 174, 178, 180, 182, 200, 201, 212, 219, 220, 341, 350, 351, 353, 360, 361, 362, 378, 380, 382, 384, 385, 386, 468, 486, 487, 488, 490, 491, 492, 502, 503, 504, 505, 506, 508, 509, 517, 522, 523, 524, 525, 526, 533, 534, 545, 546, 547, 548, 551, 552, 553, 575, 576, 577, 601, 602, 603, 604, 605, 606, 607, 633, 634, 635, 652, 666, 667, 668, 669, 680, 681, 682, 683, 684, 685, 695, 696, 697, 698, 701, 702, 703, 704, 723, 724, 725, 726, 729, 730, 731, 751, 752, 753, 754, 755, 775, 776, 777, 778, 779, 818, 819, 820, 822, 823, 824, 825, 826, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 873, 874, 883, 884, 890, 897, 898, 899, 903, 907, 908, 909, 915, 916, 917, 987, 988, 992, 993, 1078, 1079, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1117, 1120, 1125, 1126, 1141, 1142, 1144, 1145, 1146, 1150, 1151, 1187, 1238, 1241, 1242, 1243, 1244, 1246, 1247, 1368, 1369, 1372 }

C grade: { 141, 143, 146, 995, 996, 997 }

F grade: { 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 278, 279, 280, 281, 282, 283, 284, 285, 387, 411, 437, 439, 440, 441, 442, 443, 447, 456, 474, 484, 493, 564, 591, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 875, 876, 877, 878, 879, 880, 881, 882, 989, 990, 991, 1001, 1002, 1003, 1004, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1252, 1253, 1279, 1280, 1281, 1282, 1283, 1284, 1289, 1290, 1291, 1292, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1316, 1317, 1318, 1319, 1323, 1324, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1345, 1346, 1347, 1352, 1353, 1354, 1356, 1357, 1358, 1359, 1360, 1361, 1362 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 53, 54, 158, 159, 160, 161, 162, 167, 168, 169, 170, 171, 172, 173, 175, 176, 179, 181, 182, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 201, 207, 208, 209, 210, 211, 213, 214, 215, 219, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 286, 287, 288, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 336, 337, 338, 339, 345, 346, 347, 354, 355, 356, 363, 364, 365, 366, 368, 369, 370, 371, 372, 375, 376, 379, 388, 389, 390, 393, 394, 395, 396, 397, 398, 399, 400, 401, 412, 413, 414, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 438, 448, 449, 450, 451, 457, 458, 460, 461, 462, 463, 464, 465, 475, 476, 477, 478, 479, 480, 481, 482, 483, 488, 489, 494, 495, 496, 497, 498, 499, 500, 501, 507, 631, 636, 637, 638, 639, 640, 641, 642, 643, 644, 653, 654, 655, 656, 657, 658, 659, 660, 661, 670, 671, 672, 673, 674, 675, 676, 677, 821, 826, 827, 828, 829, 830, 831, 832, 833, 834, 844, 845, 846, 851, 852, 853, 854, 855, 856, 857, 858, 859, 863, 868, 883, 884, 885, 886, 893, 895, 904, 906, 916, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 954, 955, 956, 992, 993, 994, 998, 999, 1000, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1084, 1086, 1087, 1088, 1089, 1097, 1098, 1099, 1100, 1101, 1107, 1108, 1109, 1110, 1111, 1126, 1127, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1162, 1163, 1164, 1182, 1183, 1184, 1186, 1187, 1189, 1190, 1192, 1193, 1194, 1195, 1201, 1202, 1203, 1204, 1211, 1212, 1213, 1214, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1254, 1255, 1274, 1275, 1276, 1277, 1278, 1304, 1315, 1363, 1364, 1365, 1366, 1367, 1370, 1371, 1376, 1377 }

B grade: { 163, 164, 165, 166, 174, 178, 180, 183, 184, 185, 186, 194, 212, 243, 244, 245, 246, 289, 294, 340, 348, 349, 357, 358, 359, 373, 374, 377, 378, 466, 467, 468, 469, 628, 629, 630, 645, 646, 647, 648, 818, 819, 820, 835, 836, 837, 838, 843, 862, 869, 870, 871, 872, 887, 888, 889, 890, 891, 1035, 1095, 1116, 1117, 1118, 1119, 1123, 1124, 1125, 1150, 1151, 1152, 1166, 1167, 1168, 1185, 1188, 1368, 1369, 1372, 1378 }

C grade: { 51, 52, 217, 218, 367, 380, 690, 691, 709, 719, 737, 741, 742, 743, 744, 745, 746, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 860, 861, 1256, 1257 }

F grade: { 39, 40, 41, 42, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 202, 203, 204, 205, 206, 216, 220, 221, 222, 223, 224, 225, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 332, 333, 334, 335, 341, 342, 343, 344, 350, 351, 352, 353, 360, 361, 362, 381, 382, 383, 384, 385, 386, 387, 391, 392, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 415, 416, 417, 418, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 452, 453, 454, 455, 456, 459, 470, 471, 472, 473, 474, 484, 485, 486, 487, 490, 491, 492, 493, 502, 503, 504, 505, 506, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 632, 633, 634, 635, 649, 650, 651, 652, 662, 663, 664, 665, 666, 667, 668, 669, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 717, 718, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 738, 739, 740, 747, 748, 749, 750, 751, 752, 753, 754, 755, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 822, 823, 824, 825, 839, 840, 841, 842, 847, 848, 849, 850, 864, 865, 866, 867, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 892, 894, 896, 897, 898, 899, 900, 901, 902, 903, 905, 907, 908, 909, 910, 911, 912, 913, 914, 915, 917, 918, 919, 949, 950, 951, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 995, 996, 997, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1081, 1082, 1083, 1085, 1090, 1091, 1092, 1093, 1094, 1096, 1102, 1103, 1104, 1105, 1106, 1112, 1113, 1114, 1115, 1120, 1121, 1122, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1165, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1196, 1197, 1198, 1199, 1200, 1205, 1206, 1207, 1208, 1209, 1210, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1240, 1241, 1242, 1243, 1244, 1252, 1253, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1373, 1374, 1375 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 69, 77, 78, 79, 86, 87, 88, 89, 97, 98, 99, 107, 108, 109, 117, 118, 119, 121, 122, 123, 124, 125, 126, 128, 129, 131, 133, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 371, 372, 373, 375, 376, 379, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 438, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 714, 715, 716, 717, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 737, 738, 741, 742, 743, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 761, 762, 763, 764, 765, 766, 767, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 785, 786, 787, 788, 818, 819, 820, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 840, 841, 842, 843, 844, 845, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 865, 866, 867, 869, 870, 872, 873, 874, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 900, 901, 903, 904, 905, 906, 907, 908, 912, 916, 917, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 949, 950, 951, 954, 955, 956, 957, 958, 959, 962, 963, 964, 965, 970, 971, 979, 980, 981, 982, 986, 994, 998, 999, 1000, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1072, 1073, 1074, 1075, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1127, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1170, 1171, 1172, 1182, 1183, 1184, 1186, 1187, 1190, 1192, 1193, 1194, 1195, 1196, 1197, 1201, 1202, 1203, 1204, 1207, 1208, 1209, 1210, 1212, 1213, 1214, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1254, 1255, 1256, 1257, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1276, 1277, 1278, 1285, 1286, 1287, 1288, 1315, 1320, 1321, 1322, 1325, 1326, 1343, 1344, 1348, 1349, 1350, 1351, 1355, 1363, 1364, 1365, 1366, 1367, 1370, 1371 }

B grade: { 6, 61, 62, 63, 64, 65, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 91, 92, 93, 94, 95, 96, 101, 102, 103, 104, 105, 106, 111, 112, 113, 114, 115, 116, 130, 132, 135, 136, 137, 138, 139, 140, 163, 174, 183, 184, 194, 201, 212, 242, 340, 368, 369, 374, 377, 378, 381, 382, 383, 384, 385, 386, 690, 691, 718, 719, 744, 745, 768, 769, 821, 822, 839, 846, 847, 864, 868, 871, 899, 902, 909, 910, 911, 913, 914, 915, 918, 919, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 960, 961, 992, 993, 1035, 1071, 1076, 1125, 1126, 1169, 1185, 1188, 1189, 1198, 1199, 1200, 1205, 1206, 1211, 1235, 1258, 1275, 1281, 1282, 1283, 1284, 1368, 1369, 1372, 1373, 1374, 1377, 1378 }

C grade: { 367, 380 }

F grade: { 60, 70, 80, 90, 100, 110, 120, 127, 134, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 278, 279, 280, 281, 282, 283, 284, 285, 387, 411, 437, 439, 440, 441, 442, 443, 447, 456, 474, 484, 493, 564, 591, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 692, 693, 694, 709, 712, 713, 720, 721, 722, 736, 739, 740, 759, 760, 783, 784, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 875, 876, 877, 878, 879, 880, 881, 882, 966, 967, 968, 969, 972, 973, 974, 975, 976, 977, 978, 983, 984, 985, 987, 988, 989, 990, 991, 995, 996, 997, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1160, 1161, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1252, 1253, 1266, 1267, 1279, 1280, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1316, 1317, 1318, 1319, 1323, 1324, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1345, 1346, 1347, 1352, 1353, 1354, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1375, 1376 }

2.1.6 Sympy

A grade: { 1, 3, 5, 11, 12, 13, 14, 15, 16, 17, 18, 26, 27, 28, 29, 30, 31, 32, 33, 43, 44, 45, 46, 47, 48, 49, 50, 158, 159, 160, 161, 167, 169, 170, 171, 172, 173, 175, 178, 179, 180, 181, 187, 188, 189, 190, 191, 192, 193, 195, 196, 201, 207, 208, 209, 210, 211, 213, 214, 215, 235, 236, 237, 238, 239, 240, 241, 242, 261, 262, 263, 264, 265, 266, 267, 268, 269, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 306, 308, 309, 316, 318, 319, 363, 364, 365, 366, 367, 368, 393, 394, 395, 396, 397, 419, 420, 421, 422, 423, 448, 449, 450, 451, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 480, 481, 482, 483, 489, 494, 495, 496, 497, 498, 500, 501, 507, 577, 603, 628, 629, 630, 631, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 653, 654, 655, 656, 657, 658, 659, 660, 661, 670, 671, 672, 673, 674, 675, 676, 677, 826, 827, 828, 829, 830, 851, 852, 853, 854, 855, 883, 884, 885, 886, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 992, 993, 994, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1086, 1088, 1098, 1100, 1125, 1126, 1127, 1137, 1139, 1140, 1186, 1189, 1190, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1370, 1371 }

B grade: { 6, 143, 168, 174, 176, 194, 212, 398, 399, 400, 401, 424, 425, 426, 427, 499, 831, 832, 833, 834, 856, 857, 858, 859, 995, 1035, 1182, 1183, 1184, 1185, 1187, 1188, 1368, 1369, 1372 }

C grade: { 2, 4, 7, 8, 9, 10, 144, 146, 296, 299, 300, 301, 302, 303, 304, 305, 307, 310, 311, 312, 313, 314, 315, 317, 320, 321, 322, 323, 324, 325, 326, 456, 518, 519, 520, 521, 662, 663, 664, 665, 678, 679, 680, 723, 724, 725, 726, 803, 804, 806, 987, 988, 996, 997, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1087, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1099, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1131, 1132, 1136, 1138, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1192, 1193, 1194, 1195, 1245, 1246, 1247, 1253, 1254, 1255, 1256 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 162, 163, 164, 165, 166, 177, 182, 183, 184, 185, 186, 197, 198, 199, 200, 202, 203, 204, 205, 206, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 452, 453, 454, 455, 470, 471, 472, 473, 474, 484, 485, 486, 487, 488, 490, 491, 492, 493, 502, 503, 504, 505, 506, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568,

569, 570, 571, 572, 573, 574, 575, 576, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 632, 633, 634, 635, 649, 650, 651, 652, 666, 667, 668, 669, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 805, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 989, 990, 991, 998, 999, 1000, 1001, 1002, 1003, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1128, 1129, 1130, 1133, 1134, 1135, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1248, 1249, 1250, 1251, 1252, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1373, 1374, 1375, 1376, 1377, 1378
}

2.1.7 Giac

A grade: { 2, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 43, 44, 48, 49, 50, 52, 53, 54, 158, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 201, 203, 204, 207, 208, 209, 213, 214, 215, 217, 218, 219, 220, 222, 226, 228, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 245, 248, 249, 250, 251, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 273, 274, 276, 286, 287, 288, 289, 290, 291, 293, 295, 296, 297, 298, 299, 306, 307, 308, 309, 310, 316, 317, 318, 319, 320, 321, 328, 330, 331, 332, 337, 346, 354, 356, 358, 363, 364, 365, 366, 368, 369, 370, 373, 374, 377, 378, 380, 381, 382, 383, 385, 386, 389, 390, 391, 392, 395, 396, 397, 398, 399, 400, 401, 403, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 451, 452, 454, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 470, 471, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 487, 488, 489, 490, 492, 494, 495, 496, 499, 500, 501, 503, 504, 505, 506, 507, 524, 525, 526, 549, 550, 551, 552, 553, 573, 574, 575, 632, 636, 637, 638, 639, 640, 641, 642, 643, 644, 648, 650, 653, 654, 655, 656, 657, 658, 659, 660, 661, 666, 670, 671, 674, 675, 676, 677, 681, 683, 695, 697, 698, 723, 725, 726, 728, 729, 730, 731, 747, 748, 749, 751, 753, 771, 772, 773, 775, 777, 818, 819, 820, 821, 822, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 843, 844, 845, 846, 847, 860, 861, 862, 863, 864, 868, 869, 870, 871, 872, 873, 883, 884, 885, 886, 887, 889, 890, 891, 892, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 994, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1078, 1079, 1080, 1081, 1082, 1086, 1087, 1088, 1089, 1090, 1091, 1098, 1099, 1100, 1101, 1102, 1103, 1107, 1117, 1121, 1122, 1137, 1138, 1139, 1140, 1141, 1147, 1148, 1149, 1151, 1153, 1182, 1183, 1184, 1186, 1187, 1188, 1189, 1190, 1192, 1193, 1194, 1195, 1196, 1228, 1229, 1230, 1232, 1233, 1234, 1235, 1239, 1240, 1249, 1254, 1255, 1256, 1257, 1259, 1370 }

B grade: { 6, 7, 8, 9, 10, 22, 23, 24, 25, 38, 40, 42, 45, 46, 47, 55, 56, 57, 58, 59, 164, 165, 174, 184, 185, 194, 210, 211, 212, 235, 236, 237, 292, 294, 300, 301, 302, 303, 304, 305, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 333, 334, 335, 348, 349, 350, 351, 352, 353, 359, 360, 361, 362, 384, 393, 394, 448, 449, 450, 466, 468, 485, 486, 497, 498, 502, 522, 523, 563, 576, 605, 606, 607, 628, 629, 630, 631, 645, 646, 647, 662, 663, 664, 665, 672, 673, 678, 679, 680, 752, 754, 755, 776, 778, 779, 823, 824, 825, 840, 841, 842, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 865, 866, 867, 874, 893, 894, 895, 992, 993, 1035, 1083, 1084, 1085, 1092, 1093, 1094, 1095, 1096, 1097, 1104, 1105, 1106, 1118, 1119, 1142, 1143, 1144, 1145, 1146, 1152, 1185, 1231, 1237, 1238, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1250, 1251, 1258, 1368, 1369, 1371, 1372, 1378 }

C grade: { 163, 166, 183, 186, 202, 205, 221, 224, 340, 341, 342, 343, 344, 367, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817 }

F grade: { 1, 3, 34, 35, 39, 41, 51, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 167, 177, 187, 197, 206, 216, 223, 225, 227, 229, 244, 246, 247, 252, 253, 254, 255, 257, 259, 270, 271, 272, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 327, 329, 336, 338, 339, 345, 347, 355, 357, 371, 372, 375, 376, 379, 387, 388, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 453, 455, 456, 472, 474, 484, 491, 493, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 554, 555, 556, 557, 558, 559, 560, 561, 562, 564, 565, 566, 567, 568, 569, 570, 571, 572, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 633, 634, 635, 649, 651, 652, 667, 668, 669, 682, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 696, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 724, 727, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 750, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 774, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 875, 876, 877, 878, 879, 880, 881, 882, 888, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1077, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1120, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1150, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1236, 1252, 1253, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1373, 1374, 1375, 1376, 1377 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 388, 389, 390, 393, 394, 395, 396, 397, 398, 399, 400, 401, 412, 413, 414, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 438, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 563, 569, 570, 571, 572, 573, 574, 578, 579, 580, 581, 596, 597, 598, 604, 605, 606, 607, 613, 614, 615, 616, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 741, 742, 743, 746, 765, 766, 767, 770, 821, 822, 823, 826, 827, 828, 829, 830, 831, 832, 833, 834, 851, 852, 853, 854, 855, 856, 857, 858, 859, 868, 871, 872, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 954, 955, 956, 992, 993, 994, 998, 999, 1000, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1118, 1119, 1123, 1124, 1125, 1126, 1127, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1165, 1166, 1167, 1168, 1170, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1249, 1250, 1251, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1276, 1285, 1286, 1287, 1288, 1293, 1294, 1295, 1296, 1304, 1315, 1320, 1321, 1322, 1325, 1326, 1343, 1344, 1348, 1349, 1350, 1351, 1355, 1363, 1364, 1365, 1366, 1367, 1369, 1370, 1371, 1372, 1373, 1374, 1377, 1378 }

C grade: { }

F grade: { 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 381, 382, 383, 384, 385, 386, 387, 391, 392, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 415, 416, 417, 418, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 447, 456, 474, 484, 493, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 564, 565, 566, 567, 568, 575, 576, 577, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 608, 609, 610, 611, 612, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 744, 745, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 768, 769, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 824, 825, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 860, 861, 862, 863, 864, 865, 866, 867, 869, 870, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 952, 953, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 995, 996, 997, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1120, 1121, 1122, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1169, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1191, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1252, 1253, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1289, 1290, 1291, 1292, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1316, 1317, 1318, 1319, 1323, 1324, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1345, 1346, 1347, 1352, 1353, 1354, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1368, 1375, 1376 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	60	127	105	73	221	0	112
normalized size	1	1.00	0.54	1.14	0.95	0.66	1.99	0.00	1.01
time (sec)	N/A	0.105	0.050	0.043	0.420	0.587	4.754	0.000	0.076

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	52	107	85	65	199	59	97
normalized size	1	1.00	0.60	1.23	0.98	0.75	2.29	0.68	1.11
time (sec)	N/A	0.075	0.040	0.037	0.417	0.463	4.457	1.178	0.834

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	87	65	57	133	0	82
normalized size	1	1.00	0.59	1.18	0.88	0.77	1.80	0.00	1.11
time (sec)	N/A	0.054	0.034	0.037	0.412	0.824	3.289	0.000	0.051

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	67	45	48	110	41	58
normalized size	1	1.00	0.87	1.76	1.18	1.26	2.89	1.08	1.53
time (sec)	N/A	0.022	0.028	0.033	0.422	0.544	3.157	1.114	0.796

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	45	26	42	19	29	36
normalized size	1	1.00	0.89	1.61	0.93	1.50	0.68	1.04	1.29
time (sec)	N/A	0.011	0.010	0.032	0.412	0.658	1.616	0.215	0.802

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	26	44	33	44	70	51	35
normalized size	1	1.00	1.18	2.00	1.50	2.00	3.18	2.32	1.59
time (sec)	N/A	0.042	0.015	0.036	0.405	0.477	5.922	0.226	0.035

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	35	47	41	65	96	34
normalized size	1	1.00	1.16	0.92	1.24	1.08	1.71	2.53	0.89
time (sec)	N/A	0.044	0.032	0.035	0.409	0.525	2.370	0.415	0.035

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	58	55	67	52	136	158	54
normalized size	1	1.00	0.91	0.86	1.05	0.81	2.12	2.47	0.84
time (sec)	N/A	0.063	0.042	0.039	0.409	0.425	3.039	0.408	0.039

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	67	77	87	61	185	210	78
normalized size	1	1.00	0.74	0.86	0.97	0.68	2.06	2.33	0.87
time (sec)	N/A	0.085	0.058	0.037	0.402	0.511	3.206	0.213	0.033

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	75	100	107	69	258	273	98
normalized size	1	1.00	0.66	0.88	0.94	0.61	2.26	2.39	0.86
time (sec)	N/A	0.107	0.062	0.038	0.413	0.448	4.437	0.176	0.030

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	40	43	42	39	47	39
normalized size	1	1.00	1.00	0.91	0.98	0.95	0.89	1.07	0.89
time (sec)	N/A	0.042	0.019	0.027	0.309	0.401	0.105	0.935	0.045

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	32	34	33	29	38	31
normalized size	1	1.00	1.00	0.94	1.00	0.97	0.85	1.12	0.91
time (sec)	N/A	0.034	0.014	0.027	0.312	0.413	0.101	0.166	0.044

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	26	25	22	30	23
normalized size	1	1.00	1.00	0.92	1.00	0.96	0.85	1.15	0.88
time (sec)	N/A	0.029	0.012	0.027	0.307	0.612	0.095	0.156	0.803

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	15	17	12	16	15
normalized size	1	1.00	1.00	1.00	0.94	1.06	0.75	1.00	0.94
time (sec)	N/A	0.012	0.010	0.024	0.309	0.424	0.084	0.241	0.031

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	11	11	10	13	12
normalized size	1	1.00	1.00	1.00	0.92	0.92	0.83	1.08	1.00
time (sec)	N/A	0.025	0.007	0.033	0.315	0.683	0.121	0.342	0.818

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	20	23	17	22	16
normalized size	1	1.00	1.00	1.00	0.95	1.10	0.81	1.05	0.76
time (sec)	N/A	0.028	0.010	0.032	0.310	0.763	0.143	0.245	0.807

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	30	35	27	32	24
normalized size	1	1.00	1.00	0.94	0.91	1.06	0.82	0.97	0.73
time (sec)	N/A	0.032	0.012	0.034	0.309	0.507	0.163	0.246	0.814

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	39	38	43	36	40	31
normalized size	1	1.00	1.00	0.95	0.93	1.05	0.88	0.98	0.76
time (sec)	N/A	0.036	0.015	0.033	0.309	0.470	0.182	0.192	0.052

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	58	122	100	85	0	87	143
normalized size	1	1.00	0.63	1.33	1.09	0.92	0.00	0.95	1.55
time (sec)	N/A	0.649	0.060	0.051	0.409	0.907	0.000	0.235	0.071

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	53	102	80	76	0	78	102
normalized size	1	1.00	0.60	1.16	0.91	0.86	0.00	0.89	1.16
time (sec)	N/A	0.383	0.040	0.039	0.410	1.131	0.000	0.220	0.806

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	39	79	60	65	0	63	81
normalized size	1	1.00	0.71	1.44	1.09	1.18	0.00	1.15	1.47
time (sec)	N/A	0.049	0.032	0.035	0.414	0.437	0.000	0.231	0.807

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	75	65	82	0	87	82
normalized size	1	1.00	1.06	1.56	1.35	1.71	0.00	1.81	1.71
time (sec)	N/A	0.787	0.046	0.036	0.411	0.475	0.000	0.216	0.073

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	82	80	78	0	150	82
normalized size	1	1.00	0.90	1.30	1.27	1.24	0.00	2.38	1.30
time (sec)	N/A	0.697	0.057	0.039	0.311	0.429	0.000	0.215	0.799

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	108	102	97	0	213	106
normalized size	1	1.00	0.82	1.19	1.12	1.07	0.00	2.34	1.16
time (sec)	N/A	0.741	0.072	0.042	0.315	0.432	0.000	0.422	0.819

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	81	146	122	105	0	265	126
normalized size	1	1.00	0.69	1.25	1.04	0.90	0.00	2.26	1.08
time (sec)	N/A	0.738	0.086	0.045	0.318	0.564	0.000	0.402	0.813

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	58	66	49	61	57
normalized size	1	1.00	1.00	0.91	1.02	1.16	0.86	1.07	1.00
time (sec)	N/A	0.050	0.050	0.033	0.305	0.437	0.170	0.244	0.813

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	49	57	39	52	49
normalized size	1	1.00	1.00	0.94	1.04	1.21	0.83	1.11	1.04
time (sec)	N/A	0.044	0.039	0.031	0.316	0.410	0.159	0.197	0.795

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	41	49	31	44	38
normalized size	1	1.00	1.00	0.92	1.05	1.26	0.79	1.13	0.97
time (sec)	N/A	0.029	0.037	0.031	0.313	0.455	0.147	0.169	0.041

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	26	38	19	26	25
normalized size	1	1.00	0.96	0.96	0.96	1.41	0.70	0.96	0.93
time (sec)	N/A	0.014	0.018	0.033	0.309	0.434	0.131	0.172	0.037

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	18	8	13	12
normalized size	1	1.00	1.00	1.00	0.92	1.38	0.62	1.00	0.92
time (sec)	N/A	0.025	0.010	0.032	0.314	0.440	0.160	0.158	0.037

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	34	55	26	36	28
normalized size	1	1.00	1.00	0.97	1.06	1.72	0.81	1.12	0.88
time (sec)	N/A	0.033	0.026	0.034	0.317	0.403	0.225	1.094	0.808

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	43	48	73	41	47	41
normalized size	1	1.00	1.00	0.93	1.04	1.59	0.89	1.02	0.89
time (sec)	N/A	0.037	0.032	0.036	0.303	0.547	0.254	0.389	0.058

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	56	81	49	55	49
normalized size	1	1.00	1.00	0.94	1.04	1.50	0.91	1.02	0.91
time (sec)	N/A	0.045	0.046	0.034	0.308	0.448	0.275	0.175	0.815

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	51	154	80	65	0	0	97
normalized size	1	1.00	0.59	1.77	0.92	0.75	0.00	0.00	1.11
time (sec)	N/A	0.077	0.043	0.043	0.405	0.456	0.000	0.000	0.811

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	43	134	61	57	0	0	82
normalized size	1	1.00	0.59	1.84	0.84	0.78	0.00	0.00	1.12
time (sec)	N/A	0.055	0.035	0.039	0.415	0.433	0.000	0.000	0.827

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	34	119	45	48	0	41	58
normalized size	1	1.00	0.87	3.05	1.15	1.23	0.00	1.05	1.49
time (sec)	N/A	0.023	0.026	0.036	0.407	0.785	0.000	0.172	0.777

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	66	25	41	0	28	35
normalized size	1	1.00	0.85	2.44	0.93	1.52	0.00	1.04	1.30
time (sec)	N/A	0.011	0.013	0.030	0.416	0.573	0.000	0.593	0.034

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	28	93	42	44	0	52	36
normalized size	1	1.00	1.17	3.88	1.75	1.83	0.00	2.17	1.50
time (sec)	N/A	0.042	0.015	0.041	0.415	0.490	0.000	0.175	0.034

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	44	162	0	40	0	0	33
normalized size	1	1.00	1.19	4.38	0.00	1.08	0.00	0.00	0.89
time (sec)	N/A	0.042	0.028	0.044	0.000	0.468	0.000	0.000	0.034

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	186	0	51	0	159	53
normalized size	1	1.00	0.90	2.95	0.00	0.81	0.00	2.52	0.84
time (sec)	N/A	0.060	0.044	0.043	0.000	0.625	0.000	1.599	0.038

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	66	207	0	60	0	0	78
normalized size	1	1.00	0.73	2.30	0.00	0.67	0.00	0.00	0.87
time (sec)	N/A	0.083	0.055	0.050	0.000	0.515	0.000	0.000	0.033

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	74	226	0	68	0	273	98
normalized size	1	1.00	0.65	1.98	0.00	0.60	0.00	2.39	0.86
time (sec)	N/A	0.103	0.058	0.053	0.000	0.513	0.000	0.203	0.030

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	43	42	37	66	39
normalized size	1	1.00	1.00	0.93	1.00	0.98	0.86	1.53	0.91
time (sec)	N/A	0.039	0.020	0.026	0.313	0.724	0.106	0.388	0.039

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	34	33	27	57	30
normalized size	1	1.00	1.00	0.97	1.06	1.03	0.84	1.78	0.94
time (sec)	N/A	0.033	0.013	0.026	0.318	0.478	0.101	0.200	0.041

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	26	25	20	52	23
normalized size	1	1.00	1.00	0.96	1.04	1.00	0.80	2.08	0.92
time (sec)	N/A	0.024	0.011	0.025	0.313	0.463	0.093	0.178	0.793

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	17	10	64	15
normalized size	1	1.00	1.00	1.07	1.00	1.13	0.67	4.27	1.00
time (sec)	N/A	0.010	0.010	0.025	0.310	0.493	0.085	0.156	0.030

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	43	12
normalized size	1	1.00	1.00	1.09	1.00	1.00	0.91	3.91	1.09
time (sec)	N/A	0.024	0.006	0.030	0.310	0.416	0.119	0.360	0.794

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	22	17	30	16
normalized size	1	1.00	1.00	1.05	1.00	1.10	0.85	1.50	0.80
time (sec)	N/A	0.027	0.009	0.031	0.314	0.418	0.139	0.174	0.048

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	30	35	27	50	23
normalized size	1	1.00	1.00	0.97	0.94	1.09	0.84	1.56	0.72
time (sec)	N/A	0.030	0.010	0.032	0.311	0.420	0.157	0.183	0.796

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	38	43	36	62	32
normalized size	1	1.00	1.00	0.97	0.97	1.10	0.92	1.59	0.82
time (sec)	N/A	0.032	0.012	0.033	0.313	0.517	0.177	0.156	0.050

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	70	235	215	92	0	0	154
normalized size	1	1.00	0.53	1.79	1.64	0.70	0.00	0.00	1.18
time (sec)	N/A	0.689	0.057	0.052	0.430	0.490	0.000	0.000	0.067

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	58	170	177	83	0	87	141
normalized size	1	1.00	0.61	1.79	1.86	0.87	0.00	0.92	1.48
time (sec)	N/A	0.638	0.069	0.046	0.426	0.443	0.000	0.639	0.054

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	44	169	110	75	0	78	101
normalized size	1	1.00	0.51	1.97	1.28	0.87	0.00	0.91	1.17
time (sec)	N/A	0.364	0.052	0.042	0.441	0.558	0.000	0.222	0.794

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	39	164	63	64	0	64	81
normalized size	1	1.00	0.70	2.93	1.12	1.14	0.00	1.14	1.45
time (sec)	N/A	0.049	0.031	0.036	0.429	0.405	0.000	0.200	0.048

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	49	200	0	82	0	86	80
normalized size	1	1.00	1.09	4.44	0.00	1.82	0.00	1.91	1.78
time (sec)	N/A	0.708	0.036	0.047	0.000	0.558	0.000	0.219	0.801

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	261	0	75	0	150	81
normalized size	1	1.00	0.92	4.21	0.00	1.21	0.00	2.42	1.31
time (sec)	N/A	0.690	0.058	0.049	0.000	0.426	0.000	0.200	0.804

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	75	319	0	93	0	214	105
normalized size	1	1.00	0.83	3.54	0.00	1.03	0.00	2.38	1.17
time (sec)	N/A	0.735	0.063	0.053	0.000	0.475	0.000	0.218	0.805

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	82	338	0	101	0	265	125
normalized size	1	1.00	0.71	2.91	0.00	0.87	0.00	2.28	1.08
time (sec)	N/A	0.748	0.080	0.062	0.000	0.468	0.000	0.195	0.048

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	89	359	0	109	0	326	144
normalized size	1	1.00	0.66	2.66	0.00	0.81	0.00	2.41	1.07
time (sec)	N/A	0.822	0.080	0.066	0.000	0.510	0.000	0.187	0.052

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.377	0.039	0.000	0.623	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	69	0	0	544	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	1.93	0.00	0.00	-0.00
time (sec)	N/A	0.214	0.036	0.098	0.000	0.677	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	56	0	0	535	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	2.10	0.00	0.00	-0.00
time (sec)	N/A	0.173	0.024	0.037	0.000	0.499	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	149	0	0	511	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	2.30	0.00	0.00	-0.00
time (sec)	N/A	0.146	0.231	0.038	0.000	0.628	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	83	0	0	417	0	0	-1
normalized size	1	1.00	0.37	0.00	0.00	1.84	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.032	0.038	0.000	0.564	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	58	0	0	123	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	1.68	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.015	0.037	0.000	0.508	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	70	0	0	144	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.021	0.038	0.000	0.514	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	78	0	0	153	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.023	0.040	0.000	0.561	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	86	0	0	161	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.028	0.039	0.000	0.482	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	94	0	0	169	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.033	0.038	0.000	0.612	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.028	0.385	0.036	0.000	0.600	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	131	0	0	557	0	0	-1
normalized size	1	1.00	0.45	0.00	0.00	1.92	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.108	0.043	0.000	0.507	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	69	0	0	549	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.031	0.043	0.000	0.598	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	54	0	0	541	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.017	0.043	0.000	0.532	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	48	0	0	519	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	2.33	0.00	0.00	-0.00
time (sec)	N/A	0.134	0.064	0.040	0.000	0.469	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	83	0	0	417	0	0	-1
normalized size	1	1.00	0.37	0.00	0.00	1.84	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.031	0.040	0.000	0.521	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	0	0	131	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.014	0.040	0.000	0.537	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	70	0	0	149	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.018	0.043	0.000	0.641	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	78	0	0	157	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.024	0.043	0.000	0.677	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	86	0	0	165	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.029	0.044	0.000	0.570	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.439	0.038	0.000	0.578	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	74	0	0	553	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	1.74	0.00	0.00	-0.00
time (sec)	N/A	0.241	0.046	0.056	0.000	0.609	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	66	0	0	545	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	1.79	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.032	0.053	0.000	0.567	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	61	0	0	537	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	1.92	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.028	0.047	0.000	0.534	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	174	0	0	512	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	2.07	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.237	0.044	0.000	0.848	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	93	0	0	442	0	0	-1
normalized size	1	1.00	0.38	0.00	0.00	1.78	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.040	0.052	0.000	0.759	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	74	0	0	125	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.021	0.058	0.000	0.431	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	86	0	0	145	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.026	0.055	0.000	0.569	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	91	0	0	153	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.029	0.061	0.000	0.616	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	99	0	0	161	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.034	0.060	0.000	0.656	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.397	0.036	0.000	0.606	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	116	0	0	557	0	0	-1
normalized size	1	1.00	0.40	0.00	0.00	1.92	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.107	0.044	0.000	0.595	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	62	0	0	549	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.028	0.044	0.000	0.558	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	55	0	0	540	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.020	0.041	0.000	0.538	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	35	0	0	518	0	0	-1
normalized size	1	1.00	0.16	0.00	0.00	2.34	0.00	0.00	-0.00
time (sec)	N/A	0.138	0.037	0.042	0.000	1.103	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	83	0	0	417	0	0	-1
normalized size	1	1.00	0.37	0.00	0.00	1.84	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.026	0.042	0.000	0.661	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	0	0	130	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.014	0.041	0.000	0.507	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	69	0	0	148	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.022	0.042	0.000	0.516	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	78	0	0	157	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.024	0.043	0.000	0.501	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	86	0	0	165	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.031	0.043	0.000	0.508	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.398	0.035	0.000	0.548	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	116	0	0	553	0	0	-1
normalized size	1	1.00	0.40	0.00	0.00	1.91	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.108	0.055	0.000	0.503	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	62	0	0	545	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	1.93	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.031	0.052	0.000	0.597	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	56	0	0	537	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	2.11	0.00	0.00	-0.00
time (sec)	N/A	0.170	0.022	0.048	0.000	1.181	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	150	0	0	512	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	2.31	0.00	0.00	-0.00
time (sec)	N/A	0.140	0.226	0.046	0.000	0.709	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	83	0	0	417	0	0	-1
normalized size	1	1.00	0.37	0.00	0.00	1.84	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.028	0.050	0.000	0.478	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	0	0	124	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.015	0.054	0.000	0.515	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	70	0	0	145	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.024	0.058	0.000	0.517	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	78	0	0	153	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.024	0.060	0.000	0.484	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	86	0	0	161	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.032	0.062	0.000	0.500	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.678	0.039	0.000	0.484	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	79	0	0	597	0	0	-1
normalized size	1	1.00	0.25	0.00	0.00	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.040	0.044	0.000	0.539	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	70	0	0	589	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	1.93	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.037	0.044	0.000	0.514	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	64	0	0	581	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	2.08	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.026	0.044	0.000	0.648	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	33	0	0	554	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	2.24	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.059	0.037	0.000	0.491	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	90	0	0	499	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.068	0.045	0.000	2.443	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	55	0	0	170	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.018	0.043	0.000	0.575	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	70	0	0	192	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.025	0.044	0.000	0.935	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	78	0	0	200	0	0	-1
normalized size	1	1.00	0.47	0.00	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.026	0.043	0.000	0.592	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	86	0	0	208	0	0	-1
normalized size	1	1.00	0.44	0.00	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.033	0.044	0.000	0.635	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.484	0.032	0.000	0.585	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	59	0	0	308	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.385	0.028	0.076	0.000	0.580	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	49	0	0	301	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	1.34	0.00	0.00	-0.00
time (sec)	N/A	0.340	0.015	0.033	0.000	0.564	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	39	0	0	295	0	0	-1
normalized size	1	1.00	0.19	0.00	0.00	1.46	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.047	0.031	0.000	0.740	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	74	0	0	471	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	1.36	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.026	0.032	0.000	0.462	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	50	0	0	234	0	0	-1
normalized size	1	1.00	0.26	0.00	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.012	0.033	0.000	0.501	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	60	0	0	253	0	0	-1
normalized size	1	1.00	0.27	0.00	0.00	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.175	0.016	0.033	0.000	0.597	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.439	0.033	0.000	0.555	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	59	0	0	159	0	0	-1
normalized size	1	1.00	0.44	0.00	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.028	0.033	0.000	0.502	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	46	0	0	154	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.014	0.031	0.000	0.614	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	87	0	0	146	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.147	0.033	0.000	0.544	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	74	0	0	158	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.024	0.032	0.000	1.291	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	45	0	0	152	0	0	-1
normalized size	1	1.00	0.53	0.00	0.00	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.012	0.034	0.000	0.515	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	57	0	0	166	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.015	0.034	0.000	0.705	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.028	0.384	0.033	0.000	0.608	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	646	646	70	0	0	2486	0	0	-1
normalized size	1	1.00	0.11	0.00	0.00	3.85	0.00	0.00	-0.00
time (sec)	N/A	0.702	0.041	0.074	0.000	0.754	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	619	619	56	0	0	2476	0	0	-1
normalized size	1	1.00	0.09	0.00	0.00	4.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	0.024	0.036	0.000	0.609	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	48	0	0	2372	0	0	-1
normalized size	1	1.00	0.08	0.00	0.00	4.01	0.00	0.00	-0.00
time (sec)	N/A	0.437	0.053	0.035	0.000	1.004	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	759	759	83	0	0	2281	0	0	-1
normalized size	1	1.00	0.11	0.00	0.00	3.01	0.00	0.00	-0.00
time (sec)	N/A	0.527	0.033	0.035	0.000	0.543	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	58	0	0	544	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.129	0.016	0.035	0.000	1.236	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	73	0	0	585	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.023	0.034	0.000	0.590	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	461	0	0	0	0	-1
normalized size	1	1.00	1.04	10.24	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.024	0.394	0.000	0.545	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	92	139	0	0	0	0	-1
normalized size	1	1.00	0.61	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.881	0.084	0.326	0.000	0.491	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	26	184	0	0	99	0	-1
normalized size	1	1.00	0.72	5.11	0.00	0.00	2.75	0.00	-0.03
time (sec)	N/A	0.025	0.008	0.352	0.000	0.918	3.621	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	70	67	0	0	97	0	-1
normalized size	1	1.00	0.95	0.91	0.00	0.00	1.31	0.00	-0.01
time (sec)	N/A	0.042	0.032	0.239	0.000	0.544	3.219	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	31	0	0	0	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.029	0.312	0.000	2.421	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	126	0	0	119	0	-1
normalized size	1	1.00	0.73	3.41	0.00	0.00	3.22	0.00	-0.03
time (sec)	N/A	0.025	0.008	0.271	0.000	0.458	3.172	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	55	0	0	0	0	0	-1
normalized size	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.814	0.059	0.311	0.000	0.559	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.030	0.250	0.066	0.000	0.697	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	182	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.191	0.054	0.000	0.826	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	96	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.052	0.052	0.000	0.777	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	86	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.026	0.044	0.000	0.698	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.032	0.040	0.000	0.723	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.023	0.043	0.000	0.922	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	65	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.017	0.046	0.000	0.901	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.032	0.052	0.000	0.511	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	101	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.057	0.054	0.000	0.512	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.030	0.307	0.000	0.576	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	75	137	118	92	226	78	128
normalized size	1	1.00	0.61	1.11	0.96	0.75	1.84	0.63	1.04
time (sec)	N/A	0.080	0.106	0.046	0.450	0.749	8.979	0.215	0.794

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	114	95	82	134	66	105
normalized size	1	1.00	0.74	1.25	1.04	0.90	1.47	0.73	1.15
time (sec)	N/A	0.056	0.094	0.038	0.423	0.682	6.371	0.216	0.040

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	91	72	70	102	54	82
normalized size	1	1.00	0.97	1.49	1.18	1.15	1.67	0.89	1.34
time (sec)	N/A	0.037	0.082	0.038	0.424	1.172	5.384	0.173	0.043

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	46	27	47	37	30	37
normalized size	1	1.00	0.91	1.39	0.82	1.42	1.12	0.91	1.12
time (sec)	N/A	0.019	0.012	0.032	0.417	0.728	3.645	0.294	0.810

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	76	40	62	0	53	71
normalized size	1	1.00	1.07	1.77	0.93	1.44	0.00	1.23	1.65
time (sec)	N/A	0.041	0.024	0.035	0.436	0.787	0.000	0.207	0.821

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	35	73	60	0	66	32
normalized size	1	1.00	0.91	1.09	2.28	1.88	0.00	2.06	1.00
time (sec)	N/A	0.033	0.011	0.031	0.425	0.423	0.000	0.345	0.822

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	35	40	126	89	0	145	183
normalized size	1	1.00	0.54	0.62	1.94	1.37	0.00	2.23	2.82
time (sec)	N/A	0.050	0.019	0.028	0.436	0.551	0.000	0.359	0.832

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	189	116	0	199	49
normalized size	1	1.00	0.44	0.51	1.95	1.20	0.00	2.05	0.51
time (sec)	N/A	0.069	0.023	0.028	0.498	0.441	0.000	0.223	0.835

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	51	57	264	144	0	321	57
normalized size	1	1.00	0.40	0.44	2.05	1.12	0.00	2.49	0.44
time (sec)	N/A	0.091	0.028	0.031	0.434	0.762	0.000	0.261	0.866

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	30	35	29	126	0	29
normalized size	1	1.00	0.71	0.73	0.85	0.71	3.07	0.00	0.71
time (sec)	N/A	0.044	0.025	0.025	0.377	0.516	0.955	0.000	0.843

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	45	59	59	66	59	59
normalized size	1	1.00	0.62	1.22	1.59	1.59	1.78	1.59	1.59
time (sec)	N/A	0.034	0.020	0.026	0.345	0.622	0.089	0.162	0.044

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	29	37	37	36	37	37
normalized size	1	1.00	0.86	0.78	1.00	1.00	0.97	1.00	1.00
time (sec)	N/A	0.032	0.016	0.025	0.338	0.773	0.078	0.160	0.050

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	29	37	37	37	37	37
normalized size	1	1.00	0.81	0.78	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.035	0.014	0.023	0.315	0.637	0.077	0.169	0.048

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	15	17	17	15	17	15
normalized size	1	1.00	0.84	0.79	0.89	0.89	0.79	0.89	0.79
time (sec)	N/A	0.026	0.008	0.024	0.306	0.621	0.068	0.328	0.027

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	26	26	11	11	11	10	11	9
normalized size	1	2.00	2.00	0.85	0.85	0.85	0.77	0.85	0.69
time (sec)	N/A	0.012	0.008	0.025	0.317	0.498	0.063	1.272	0.023

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	29	28	20	30	28
normalized size	1	1.00	0.81	0.97	0.94	0.90	0.65	0.97	0.90
time (sec)	N/A	0.038	0.018	0.032	0.339	0.460	0.138	0.174	0.803

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	28	25	25	22	32	12
normalized size	1	1.00	1.92	2.15	1.92	1.92	1.69	2.46	0.92
time (sec)	N/A	0.026	0.008	0.031	0.342	0.492	0.181	0.243	0.808

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	47	47	48	21	21
normalized size	1	1.00	0.62	0.81	1.27	1.27	1.30	0.57	0.57
time (sec)	N/A	0.039	0.015	0.030	0.337	0.588	0.233	0.153	0.078

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	57	57	61	21	21
normalized size	1	1.00	0.62	0.81	1.54	1.54	1.65	0.57	0.57
time (sec)	N/A	0.039	0.014	0.031	0.328	0.457	0.290	0.178	0.099

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.025	0.391	0.000	0.589	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	183	164	93	459	78	128
normalized size	1	1.00	0.90	2.20	1.98	1.12	5.53	0.94	1.54
time (sec)	N/A	0.049	0.103	0.069	0.417	0.603	22.993	0.359	0.035

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	44	96	77	65	301	48	64
normalized size	1	1.00	0.75	1.63	1.31	1.10	5.10	0.81	1.08
time (sec)	N/A	0.036	0.040	0.044	0.415	0.506	8.537	0.288	0.053

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	137	118	71	221	54	82
normalized size	1	1.00	0.97	2.25	1.93	1.16	3.62	0.89	1.34
time (sec)	N/A	0.046	0.081	0.049	0.418	0.418	14.794	1.184	0.037

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	104	85	52	165	38	58
normalized size	1	1.00	0.74	1.60	1.31	0.80	2.54	0.58	0.89
time (sec)	N/A	0.048	0.037	0.039	0.418	0.614	10.289	0.193	0.033

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	45	146	89	94	0	79	114
normalized size	1	1.00	0.61	1.97	1.20	1.27	0.00	1.07	1.54
time (sec)	N/A	0.059	0.013	0.036	0.427	0.436	0.000	0.252	0.075

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	35	146	89	0	83	34
normalized size	1	1.00	0.91	1.09	4.56	2.78	0.00	2.59	1.06
time (sec)	N/A	0.037	0.011	0.030	0.337	0.528	0.000	0.313	0.081

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	34	40	216	116	0	199	299
normalized size	1	1.00	0.52	0.62	3.32	1.78	0.00	3.06	4.60
time (sec)	N/A	0.051	0.020	0.029	0.356	0.773	0.000	0.206	0.805

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	327	145	0	253	492
normalized size	1	1.00	0.44	0.51	3.37	1.49	0.00	2.61	5.07
time (sec)	N/A	0.069	0.024	0.029	0.352	0.648	0.000	0.233	0.849

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	51	57	462	171	0	509	604
normalized size	1	1.00	0.40	0.44	3.58	1.33	0.00	3.95	4.68
time (sec)	N/A	0.092	0.029	0.029	0.356	0.690	0.000	0.575	0.870

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	74	77	81	541	0	57
normalized size	1	1.00	0.76	1.12	1.17	1.23	8.20	0.00	0.86
time (sec)	N/A	0.058	0.093	0.031	0.402	0.695	3.053	0.000	0.940

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	31	45	59	59	63	59	59
normalized size	1	1.00	0.58	0.85	1.11	1.11	1.19	1.11	1.11
time (sec)	N/A	0.044	0.021	0.024	0.328	0.519	0.100	0.203	0.795

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	23	28	28	29	28	24
normalized size	1	1.00	0.81	0.72	0.88	0.88	0.91	0.88	0.75
time (sec)	N/A	0.033	0.015	0.024	0.332	0.514	0.082	0.187	0.798

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	29	37	37	37	37	37
normalized size	1	1.00	0.86	0.83	1.06	1.06	1.06	1.06	1.06
time (sec)	N/A	0.038	0.015	0.028	0.331	1.243	0.087	0.174	0.049

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	21	16	25	25	24	25	19
normalized size	1	1.00	1.24	0.94	1.47	1.47	1.41	1.47	1.12
time (sec)	N/A	0.025	0.012	0.023	0.317	0.421	0.080	0.159	0.036

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	25	24	28	26	35	26
normalized size	1	1.00	0.96	0.93	0.89	1.04	0.96	1.30	0.96
time (sec)	N/A	0.023	0.011	0.027	0.325	0.536	0.127	0.171	0.044

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	46	44	49	37	37	42
normalized size	1	1.00	0.75	0.96	0.92	1.02	0.77	0.77	0.88
time (sec)	N/A	0.045	0.020	0.031	0.319	0.555	0.223	0.176	0.065

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	42	51	51	51	50	25
normalized size	1	1.00	1.00	1.68	2.04	2.04	2.04	2.00	1.00
time (sec)	N/A	0.028	0.008	0.033	0.331	0.544	0.265	0.187	0.078

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	41	65	65	70	29	29
normalized size	1	1.00	0.60	0.79	1.25	1.25	1.35	0.56	0.56
time (sec)	N/A	0.043	0.018	0.031	0.328	0.696	0.325	0.159	0.853

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	31	42	77	77	80	29	29
normalized size	1	1.00	0.58	0.79	1.45	1.45	1.51	0.55	0.55
time (sec)	N/A	0.042	0.018	0.030	0.333	0.460	0.379	0.131	0.878

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.024	0.393	0.000	0.680	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	80	160	89	81	0	67	105
normalized size	1	1.00	0.60	1.20	0.67	0.61	0.00	0.50	0.79
time (sec)	N/A	0.096	0.072	0.041	0.432	0.745	0.000	0.782	0.033

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	72	142	71	71	0	54	82
normalized size	1	1.00	0.71	1.41	0.70	0.70	0.00	0.53	0.81
time (sec)	N/A	0.070	0.084	0.038	0.434	0.521	0.000	0.757	0.037

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	114	45	52	0	38	59
normalized size	1	1.00	0.97	1.81	0.71	0.83	0.00	0.60	0.94
time (sec)	N/A	0.043	0.053	0.036	0.463	1.430	0.000	0.182	0.800

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	154	11	30	44	14	21
normalized size	1	1.00	1.00	14.00	1.00	2.73	4.00	1.27	1.91
time (sec)	N/A	0.028	0.008	0.040	0.432	1.319	4.559	0.222	0.037

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	28	0	37	0	70	47
normalized size	1	1.00	0.90	0.97	0.00	1.28	0.00	2.41	1.62
time (sec)	N/A	0.036	0.010	0.029	0.000	0.506	0.000	0.241	0.045

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	34	33	0	62	0	91	32
normalized size	1	1.00	0.52	0.51	0.00	0.95	0.00	1.40	0.49
time (sec)	N/A	0.052	0.018	0.029	0.000	0.422	0.000	0.412	0.056

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	43	42	0	91	0	145	127
normalized size	1	1.00	0.44	0.43	0.00	0.94	0.00	1.49	1.31
time (sec)	N/A	0.068	0.022	0.031	0.000	0.590	0.000	0.247	0.060

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	51	50	0	117	0	164	167
normalized size	1	1.00	0.40	0.39	0.00	0.91	0.00	1.27	1.29
time (sec)	N/A	0.090	0.025	0.031	0.000	0.629	0.000	0.210	0.801

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.016	0.310	0.000	0.467	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	64	63	68	68	94	63
normalized size	1	1.00	0.62	0.70	0.69	0.75	0.75	1.03	0.69
time (sec)	N/A	0.049	0.020	0.026	0.327	0.575	0.172	0.212	0.052

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	53	52	57	56	82	52
normalized size	1	1.00	0.66	0.73	0.71	0.78	0.77	1.12	0.71
time (sec)	N/A	0.043	0.017	0.025	0.334	0.602	0.151	0.178	0.786

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	39	42	41	45	41	68	41
normalized size	1	1.00	0.72	0.78	0.76	0.83	0.76	1.26	0.76
time (sec)	N/A	0.036	0.014	0.029	0.349	0.442	0.132	0.191	0.049

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	25	24	28	24	50	26
normalized size	1	1.00	0.96	0.96	0.92	1.08	0.92	1.92	1.00
time (sec)	N/A	0.023	0.014	0.025	0.314	0.467	0.114	0.160	0.041

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	27	13
normalized size	1	1.00	1.00	1.08	1.00	1.00	0.77	2.08	1.00
time (sec)	N/A	0.026	0.006	0.026	0.309	0.503	0.068	0.756	0.033

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	30	29	23	22	25	11
normalized size	1	1.00	1.00	2.73	2.64	2.09	2.00	2.27	1.00
time (sec)	N/A	0.027	0.008	0.032	0.311	0.487	0.144	0.284	0.070

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	45	48	46	39	43	31
normalized size	1	1.00	0.94	1.36	1.45	1.39	1.18	1.30	0.94
time (sec)	N/A	0.040	0.019	0.033	0.343	0.461	0.222	0.186	0.067

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	63	76	56	58	47
normalized size	1	1.00	0.69	1.18	1.24	1.49	1.10	1.14	0.92
time (sec)	N/A	0.047	0.024	0.032	0.326	0.520	0.281	0.575	0.818

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	44	75	84	113	76	89	64
normalized size	1	1.00	0.64	1.09	1.22	1.64	1.10	1.29	0.93
time (sec)	N/A	0.053	0.031	0.036	0.347	0.515	0.372	0.201	0.826

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.030	0.401	0.000	0.563	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	45	245	233	118	0	103	166
normalized size	1	1.00	0.28	1.50	1.43	0.72	0.00	0.63	1.02
time (sec)	N/A	0.125	0.021	0.048	0.452	0.513	0.000	0.255	0.825

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	45	179	196	106	0	91	150
normalized size	1	1.00	0.34	1.37	1.50	0.81	0.00	0.69	1.15
time (sec)	N/A	0.094	0.018	0.044	0.436	0.500	0.000	0.189	0.057

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	43	169	109	81	0	73	96
normalized size	1	1.00	0.47	1.86	1.20	0.89	0.00	0.80	1.05
time (sec)	N/A	0.064	0.016	0.043	0.449	0.453	0.000	0.203	0.803

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	292	0	60	0	53	70
normalized size	1	1.00	1.44	7.12	0.00	1.46	0.00	1.29	1.71
time (sec)	N/A	0.041	0.054	0.048	0.000	0.461	0.000	0.274	0.050

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	34	0	35	0	69	46
normalized size	1	1.00	0.96	1.21	0.00	1.25	0.00	2.46	1.64
time (sec)	N/A	0.033	0.010	0.031	0.000	0.437	0.000	0.291	0.034

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	32	0	33	0	29	17
normalized size	1	1.00	1.00	1.68	0.00	1.74	0.00	1.53	0.89
time (sec)	N/A	0.028	0.012	0.026	0.000	0.435	0.000	0.301	0.071

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	45	49	0	89	0	0	48
normalized size	1	1.00	0.82	0.89	0.00	1.62	0.00	0.00	0.87
time (sec)	N/A	0.045	0.018	0.029	0.000	0.534	0.000	0.000	0.827

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	52	56	0	98	0	174	233
normalized size	1	1.00	0.60	0.64	0.00	1.13	0.00	2.00	2.68
time (sec)	N/A	0.062	0.022	0.029	0.000	0.452	0.000	0.337	0.828

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	61	65	0	144	0	0	347
normalized size	1	1.00	0.51	0.55	0.00	1.21	0.00	0.00	2.92
time (sec)	N/A	0.076	0.026	0.032	0.000	0.515	0.000	0.000	0.822

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	70	71	150	91	0	87	90
normalized size	1	1.00	0.40	0.40	0.85	0.52	0.00	0.49	0.51
time (sec)	N/A	0.129	0.053	0.031	0.359	0.478	0.000	0.271	1.117

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	62	63	128	80	0	0	79
normalized size	1	1.00	0.44	0.45	0.91	0.57	0.00	0.00	0.56
time (sec)	N/A	0.104	0.044	0.031	0.390	0.447	0.000	0.000	0.988

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	54	55	106	69	0	61	68
normalized size	1	1.00	0.51	0.52	1.00	0.65	0.00	0.58	0.64
time (sec)	N/A	0.082	0.037	0.030	0.351	0.626	0.000	0.199	0.951

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	47	82	52	0	0	49
normalized size	1	1.00	0.62	0.66	1.15	0.73	0.00	0.00	0.69
time (sec)	N/A	0.061	0.027	0.028	0.347	0.453	0.000	0.000	0.932

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	34	58	39	0	34	37
normalized size	1	1.00	1.06	0.97	1.66	1.11	0.00	0.97	1.06
time (sec)	N/A	0.041	0.014	0.026	0.347	0.728	0.000	0.174	0.866

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	84	0	215	0	88	-1
normalized size	1	1.00	0.75	1.01	0.00	2.59	0.00	1.06	-0.01
time (sec)	N/A	0.082	0.028	0.057	0.000	0.498	0.000	0.156	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	111	0	258	0	58	-1
normalized size	1	1.00	0.98	1.35	0.00	3.15	0.00	0.71	-0.01
time (sec)	N/A	0.080	0.058	0.049	0.000	0.675	0.000	0.335	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	57	156	0	308	0	76	-1
normalized size	1	1.00	0.47	1.28	0.00	2.52	0.00	0.62	-0.01
time (sec)	N/A	0.106	0.020	0.051	0.000	0.479	0.000	0.245	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	57	208	0	364	0	92	-1
normalized size	1	1.00	0.36	1.32	0.00	2.32	0.00	0.59	-0.01
time (sec)	N/A	0.130	0.024	0.050	0.000	0.707	0.000	0.312	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	32	60	172	205	32
normalized size	1	1.00	0.85	0.52	0.80	1.50	4.30	5.12	0.80
time (sec)	N/A	0.048	0.043	0.028	0.329	0.744	31.789	0.179	0.039

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	32	49	76	141	32
normalized size	1	1.00	0.85	0.52	0.80	1.22	1.90	3.52	0.80
time (sec)	N/A	0.047	0.039	0.028	0.314	0.586	14.059	0.178	0.769

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	21	32	32	58	71	32
normalized size	1	1.00	0.75	0.52	0.80	0.80	1.45	1.78	0.80
time (sec)	N/A	0.049	0.031	0.030	0.326	0.743	14.308	0.174	0.033

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	30	19	31	44	32
normalized size	1	1.00	0.61	0.53	0.79	0.50	0.82	1.16	0.84
time (sec)	N/A	0.043	0.026	0.029	0.318	0.475	4.684	0.296	0.030

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	20	30	29	48	32	20
normalized size	1	1.00	0.58	0.56	0.83	0.81	1.33	0.89	0.56
time (sec)	N/A	0.047	0.026	0.026	0.309	0.510	36.401	0.250	0.035

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	21	26	44	29	36	20
normalized size	1	1.00	0.89	0.55	0.68	1.16	0.76	0.95	0.53
time (sec)	N/A	0.048	0.046	0.028	0.320	0.561	41.880	0.232	0.778

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	24	56	31	34	20
normalized size	1	1.00	0.85	0.52	0.60	1.40	0.78	0.85	0.50
time (sec)	N/A	0.049	0.056	0.028	0.312	0.416	92.029	0.179	0.779

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	26	66	31	36	20
normalized size	1	1.00	0.85	0.52	0.65	1.65	0.78	0.90	0.50
time (sec)	N/A	0.052	0.057	0.029	0.312	0.417	61.977	0.216	0.775

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	62	63	254	91	0	73	90
normalized size	1	1.00	0.44	0.45	1.80	0.65	0.00	0.52	0.64
time (sec)	N/A	0.109	0.049	0.032	0.412	0.419	0.000	0.221	1.022

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	54	55	210	80	0	0	79
normalized size	1	1.00	0.51	0.52	1.98	0.75	0.00	0.00	0.75
time (sec)	N/A	0.088	0.037	0.029	0.384	0.475	0.000	0.000	0.976

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	46	47	164	68	0	47	68
normalized size	1	1.00	0.65	0.66	2.31	0.96	0.00	0.66	0.96
time (sec)	N/A	0.067	0.032	0.027	0.402	0.421	0.000	0.161	0.967

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	34	121	49	0	0	49
normalized size	1	1.00	1.06	0.97	3.46	1.40	0.00	0.00	1.40
time (sec)	N/A	0.050	0.022	0.024	0.423	0.546	0.000	0.000	0.920

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	67	95	0	220	0	0	-1
normalized size	1	1.00	0.56	0.80	0.00	1.85	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.035	0.042	0.000	0.562	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	57	127	0	259	0	70	-1
normalized size	1	1.00	0.50	1.10	0.00	2.25	0.00	0.61	-0.01
time (sec)	N/A	0.101	0.024	0.051	0.000	0.495	0.000	0.224	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	91	158	0	312	0	73	-1
normalized size	1	1.00	0.75	1.30	0.00	2.56	0.00	0.60	-0.01
time (sec)	N/A	0.105	0.078	0.049	0.000	0.485	0.000	0.414	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	57	208	0	364	0	92	-1
normalized size	1	1.00	0.36	1.32	0.00	2.32	0.00	0.59	-0.01
time (sec)	N/A	0.134	0.027	0.052	0.000	0.524	0.000	0.229	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	57	258	0	420	0	105	-1
normalized size	1	1.00	0.30	1.34	0.00	2.19	0.00	0.55	-0.01
time (sec)	N/A	0.159	0.026	0.055	0.000	0.530	0.000	0.319	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	73	72	82	91	0	0	96
normalized size	1	1.00	0.35	0.35	0.40	0.44	0.00	0.00	0.47
time (sec)	N/A	0.162	0.059	0.031	0.375	0.604	0.000	0.000	1.039

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	65	64	71	80	0	0	88
normalized size	1	1.00	0.38	0.37	0.42	0.47	0.00	0.00	0.51
time (sec)	N/A	0.134	0.041	0.031	0.366	0.408	0.000	0.000	1.015

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	57	56	60	69	0	0	80
normalized size	1	1.00	0.42	0.41	0.44	0.51	0.00	0.00	0.59
time (sec)	N/A	0.106	0.039	0.029	0.389	0.414	0.000	0.000	0.988

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	49	48	49	52	0	0	76
normalized size	1	1.00	0.49	0.48	0.49	0.51	0.00	0.00	0.75
time (sec)	N/A	0.087	0.030	0.030	0.350	0.512	0.000	0.000	0.945

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	39	37	39	0	50	56
normalized size	1	1.00	0.58	0.59	0.56	0.59	0.00	0.76	0.85
time (sec)	N/A	0.062	0.022	0.026	0.367	0.453	0.000	0.182	0.874

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	15	36	0	0	26
normalized size	1	1.00	1.00	0.90	0.50	1.20	0.00	0.00	0.87
time (sec)	N/A	0.047	0.016	0.026	0.366	0.591	0.000	0.000	0.960

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	68	0	133	0	62	-1
normalized size	1	1.00	0.86	1.33	0.00	2.61	0.00	1.22	-0.02
time (sec)	N/A	0.062	0.024	0.041	0.000	0.686	0.000	0.174	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	111	0	258	0	0	-1
normalized size	1	1.00	0.78	1.23	0.00	2.87	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.060	0.046	0.000	0.558	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	52	158	0	312	0	80	-1
normalized size	1	1.00	0.42	1.26	0.00	2.50	0.00	0.64	-0.01
time (sec)	N/A	0.105	0.027	0.049	0.000	0.445	0.000	0.638	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	88	101	123	203	128	161	112
normalized size	1	1.00	0.64	0.74	0.90	1.48	0.93	1.18	0.82
time (sec)	N/A	0.111	0.084	0.033	0.465	0.509	128.958	0.195	0.844

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	87	109	183	109	134	95
normalized size	1	1.00	0.69	0.75	0.94	1.58	0.94	1.16	0.82
time (sec)	N/A	0.098	0.062	0.033	0.420	0.472	76.554	0.186	0.065

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	71	73	95	145	90	107	78
normalized size	1	1.00	0.75	0.77	1.00	1.53	0.95	1.13	0.82
time (sec)	N/A	0.079	0.044	0.032	0.472	0.494	61.078	0.243	0.083

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	79	120	75	77	61
normalized size	1	1.00	0.80	0.78	1.04	1.58	0.99	1.01	0.80
time (sec)	N/A	0.066	0.044	0.031	0.436	0.559	8.099	0.192	0.068

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	45	67	118	58	51	47
normalized size	1	1.00	1.00	0.78	1.16	2.03	1.00	0.88	0.81
time (sec)	N/A	0.058	0.026	0.031	0.487	0.541	34.187	0.180	0.073

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	30	52	89	39	35	29
normalized size	1	1.00	1.00	0.79	1.37	2.34	1.03	0.92	0.76
time (sec)	N/A	0.051	0.018	0.030	0.449	0.460	53.996	0.748	0.823

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	36	50	71	146	60	53	46
normalized size	1	1.00	0.63	0.88	1.25	2.56	1.05	0.93	0.81
time (sec)	N/A	0.060	0.023	0.035	0.415	0.417	31.298	2.867	0.088

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	39	64	81	196	80	73	64
normalized size	1	1.00	0.47	0.77	0.98	2.36	0.96	0.88	0.77
time (sec)	N/A	0.070	0.026	0.038	0.498	0.625	66.364	0.140	0.835

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	39	78	101	252	99	93	78
normalized size	1	1.00	0.38	0.75	0.97	2.42	0.95	0.89	0.75
time (sec)	N/A	0.082	0.031	0.039	0.420	0.586	40.713	0.175	0.094

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	70	71	73	82	0	0	116
normalized size	1	1.00	0.41	0.42	0.43	0.48	0.00	0.00	0.68
time (sec)	N/A	0.137	0.042	0.030	0.345	0.448	0.000	0.000	1.098

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	61	62	61	62	0	0	99
normalized size	1	1.00	0.45	0.46	0.45	0.46	0.00	0.00	0.73
time (sec)	N/A	0.113	0.034	0.031	0.392	0.536	0.000	0.000	0.993

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	51	54	50	49	0	0	40
normalized size	1	1.00	0.50	0.52	0.49	0.48	0.00	0.00	0.39
time (sec)	N/A	0.084	0.030	0.028	0.346	0.474	0.000	0.000	0.969

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	40	46	30	43	0	50	64
normalized size	1	1.00	0.60	0.69	0.45	0.64	0.00	0.75	0.96
time (sec)	N/A	0.064	0.027	0.027	0.354	0.481	0.000	0.252	0.960

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	34	28	42	0	30	43
normalized size	1	1.00	1.06	1.03	0.85	1.27	0.00	0.91	1.30
time (sec)	N/A	0.050	0.025	0.025	0.334	0.485	0.000	0.199	1.017

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	55	82	0	234	0	0	-1
normalized size	1	1.00	0.65	0.96	0.00	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.027	0.048	0.000	0.574	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	57	124	0	310	0	82	-1
normalized size	1	1.00	0.46	0.99	0.00	2.48	0.00	0.66	-0.01
time (sec)	N/A	0.102	0.033	0.055	0.000	0.455	0.000	0.266	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	57	173	0	328	0	0	-1
normalized size	1	1.00	0.36	1.08	0.00	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.031	0.055	0.000	0.434	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.056	0.214	0.000	0.460	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.046	0.213	0.000	0.541	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.037	0.231	0.000	0.531	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.031	0.214	0.000	0.617	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.025	0.214	0.000	0.486	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.027	0.215	0.000	0.484	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.033	0.216	0.000	0.445	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.044	0.231	0.000	0.440	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	50	91	69	69	192	57	82
normalized size	1	1.00	0.60	1.10	0.83	0.83	2.31	0.69	0.99
time (sec)	N/A	0.071	0.060	0.040	0.401	0.506	6.525	0.224	0.781

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	32	43	58	39	66	47	36
normalized size	1	1.00	0.71	0.96	1.29	0.87	1.47	1.04	0.80
time (sec)	N/A	0.065	0.025	0.028	0.403	0.507	0.699	0.165	0.036

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	70	48	60	150	45	61
normalized size	1	1.00	0.69	1.21	0.83	1.03	2.59	0.78	1.05
time (sec)	N/A	0.057	0.045	0.036	0.460	0.425	4.879	0.189	0.784

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	33	37	29	42	18	18
normalized size	1	1.00	1.00	1.50	1.68	1.32	1.91	0.82	0.82
time (sec)	N/A	0.028	0.014	0.027	0.467	0.428	0.406	0.184	0.031

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	46	27	47	37	30	37
normalized size	1	1.00	0.91	1.39	0.82	1.42	1.12	0.91	1.12
time (sec)	N/A	0.020	0.010	0.033	0.401	0.410	3.594	0.198	0.002

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	79	32	44	36	66	53	31
normalized size	1	1.00	2.26	0.91	1.26	1.03	1.89	1.51	0.89
time (sec)	N/A	0.053	0.082	0.030	0.404	0.492	10.265	0.160	0.773

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	51	27	47	88	74	38
normalized size	1	1.00	0.97	1.76	0.93	1.62	3.03	2.55	1.31
time (sec)	N/A	0.043	0.031	0.038	0.466	0.480	3.425	0.199	0.050

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	67	40	51	47	73	70	38
normalized size	1	1.00	1.46	0.87	1.11	1.02	1.59	1.52	0.83
time (sec)	N/A	0.058	0.024	0.036	0.398	0.470	11.450	0.160	0.782

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	33	40	29	90	124	18
normalized size	1	1.00	1.00	1.50	1.82	1.32	4.09	5.64	0.82
time (sec)	N/A	0.039	0.014	0.026	0.444	0.414	3.234	0.188	0.036

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	89	163	141	104	486	92	154
normalized size	1	1.00	0.72	1.31	1.14	0.84	3.92	0.74	1.24
time (sec)	N/A	0.136	0.199	0.048	0.406	0.449	10.357	0.179	0.791

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	75	140	118	92	374	81	131
normalized size	1	1.00	0.66	1.24	1.04	0.81	3.31	0.72	1.16
time (sec)	N/A	0.121	0.085	0.043	0.525	0.436	7.524	0.253	0.033

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	117	95	82	330	69	108
normalized size	1	1.00	0.96	1.67	1.36	1.17	4.71	0.99	1.54
time (sec)	N/A	0.062	0.101	0.037	0.501	0.417	7.169	0.305	0.034

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	91	72	70	102	54	82
normalized size	1	1.00	0.97	1.49	1.18	1.15	1.67	0.89	1.34
time (sec)	N/A	0.037	0.066	0.034	0.428	0.486	5.298	0.199	0.002

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	125	86	76	76	201	84	77
normalized size	1	1.00	2.12	1.46	1.29	1.29	3.41	1.42	1.31
time (sec)	N/A	0.101	0.084	0.038	0.408	0.458	11.934	0.721	0.040

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	84	91	80	91	153	140	87
normalized size	1	1.00	1.45	1.57	1.38	1.57	2.64	2.41	1.50
time (sec)	N/A	0.103	0.187	0.038	0.404	0.521	4.597	0.228	0.043

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	147	95	86	95	226	192	90
normalized size	1	1.00	2.19	1.42	1.28	1.42	3.37	2.87	1.34
time (sec)	N/A	0.105	0.106	0.040	0.410	1.009	5.714	0.224	0.051

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	91	100	99	73	270	233	90
normalized size	1	1.00	1.21	1.33	1.32	0.97	3.60	3.11	1.20
time (sec)	N/A	0.093	0.030	0.040	0.399	0.477	6.260	0.535	0.801

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	99	125	122	84	415	240	113
normalized size	1	1.00	0.97	1.23	1.20	0.82	4.07	2.35	1.11
time (sec)	N/A	0.119	0.033	0.043	0.408	0.484	7.982	0.302	0.784

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	107	168	145	95	522	297	136
normalized size	1	1.00	0.83	1.30	1.12	0.74	4.05	2.30	1.05
time (sec)	N/A	0.140	0.039	0.043	0.407	0.436	8.614	0.297	0.789

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	115	193	168	106	644	424	159
normalized size	1	1.00	0.74	1.24	1.08	0.68	4.13	2.72	1.02
time (sec)	N/A	0.173	0.043	0.049	0.404	0.470	11.610	0.651	0.049

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	91	186	164	114	512	104	177
normalized size	1	1.00	0.61	1.26	1.11	0.77	3.46	0.70	1.20
time (sec)	N/A	0.238	0.131	0.052	0.403	0.415	11.221	0.212	0.044

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	83	163	141	104	423	92	154
normalized size	1	1.00	0.69	1.35	1.17	0.86	3.50	0.76	1.27
time (sec)	N/A	0.201	0.092	0.045	0.466	0.606	9.476	0.531	0.042

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	140	118	92	355	81	108
normalized size	1	1.00	0.80	1.49	1.26	0.98	3.78	0.86	1.15
time (sec)	N/A	0.128	0.112	0.043	0.411	0.483	7.712	0.370	0.064

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	114	95	82	134	66	105
normalized size	1	1.00	0.74	1.25	1.04	0.90	1.47	0.73	1.15
time (sec)	N/A	0.057	0.088	0.037	0.406	0.505	6.396	0.298	0.002

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	135	110	100	88	226	95	110
normalized size	1	1.00	1.80	1.47	1.33	1.17	3.01	1.27	1.47
time (sec)	N/A	0.173	0.104	0.039	0.486	0.554	12.535	0.411	0.062

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	143	113	102	104	199	152	108
normalized size	1	1.00	1.72	1.36	1.23	1.25	2.40	1.83	1.30
time (sec)	N/A	0.174	0.106	0.043	0.408	0.550	5.716	0.267	0.780

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	155	116	107	118	228	212	111
normalized size	1	1.00	1.68	1.26	1.16	1.28	2.48	2.30	1.21
time (sec)	N/A	0.179	0.124	0.040	0.408	0.526	5.006	0.225	0.778

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	156	119	110	105	279	250	114
normalized size	1	1.00	1.77	1.35	1.25	1.19	3.17	2.84	1.30
time (sec)	N/A	0.179	0.111	0.041	0.442	0.496	5.849	0.202	0.041

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	99	144	122	84	347	300	113
normalized size	1	1.00	0.97	1.41	1.20	0.82	3.40	2.94	1.11
time (sec)	N/A	0.163	0.034	0.042	0.434	0.448	7.337	0.423	0.043

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	107	190	145	95	476	297	136
normalized size	1	1.00	0.83	1.47	1.12	0.74	3.69	2.30	1.05
time (sec)	N/A	0.190	0.037	0.042	0.399	0.423	8.257	0.242	0.788

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	99	209	187	126	842	117	200
normalized size	1	1.00	0.57	1.21	1.08	0.73	4.87	0.68	1.16
time (sec)	N/A	0.333	0.231	0.071	0.410	0.444	18.306	0.215	0.056

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	91	186	164	114	683	104	177
normalized size	1	1.00	0.62	1.27	1.12	0.78	4.68	0.71	1.21
time (sec)	N/A	0.304	0.111	0.056	0.411	0.406	13.185	0.267	0.790

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	83	163	141	104	617	94	154
normalized size	1	1.00	0.53	1.03	0.89	0.66	3.91	0.59	0.97
time (sec)	N/A	0.141	0.133	0.049	0.424	0.482	12.485	0.181	0.036

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	75	137	118	92	226	78	128
normalized size	1	1.00	0.61	1.11	0.96	0.75	1.84	0.63	1.04
time (sec)	N/A	0.081	0.100	0.042	0.455	0.466	8.977	0.188	0.004

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	142	115	105	95	420	100	110
normalized size	1	1.00	1.41	1.14	1.04	0.94	4.16	0.99	1.09
time (sec)	N/A	0.246	0.098	0.040	0.405	0.480	19.057	0.291	0.778

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	152	136	125	116	306	164	131
normalized size	1	1.00	1.43	1.28	1.18	1.09	2.89	1.55	1.24
time (sec)	N/A	0.242	0.129	0.043	0.457	0.473	6.683	0.254	0.039

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	106	138	129	125	357	224	133
normalized size	1	1.00	0.91	1.19	1.11	1.08	3.08	1.93	1.15
time (sec)	N/A	0.247	0.227	0.043	0.524	0.486	7.721	0.227	0.780

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	164	140	131	129	359	270	135
normalized size	1	1.00	1.37	1.17	1.09	1.08	2.99	2.25	1.12
time (sec)	N/A	0.249	0.138	0.042	0.539	0.479	6.938	0.416	0.776

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	116	125	118	109	106	505	316	113
normalized size	1	1.05	1.14	1.07	0.99	0.96	4.59	2.87	1.03
time (sec)	N/A	0.248	0.229	0.044	0.408	0.512	9.356	0.197	0.046

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	107	207	145	95	607	354	136
normalized size	1	1.00	0.83	1.60	1.12	0.74	4.71	2.74	1.05
time (sec)	N/A	0.246	0.036	0.045	0.535	0.487	10.102	0.231	0.051

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	115	255	168	106	801	424	159
normalized size	1	1.00	0.74	1.63	1.08	0.68	5.13	2.72	1.02
time (sec)	N/A	0.264	0.039	0.048	0.396	0.537	13.508	0.239	0.813

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	81	166	129	95	0	0	163
normalized size	1	1.00	0.55	1.14	0.88	0.65	0.00	0.00	1.12
time (sec)	N/A	0.343	0.068	0.048	0.453	0.508	0.000	0.000	0.819

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	72	142	105	86	0	101	158
normalized size	1	1.00	0.63	1.25	0.92	0.75	0.00	0.89	1.39
time (sec)	N/A	0.286	0.051	0.040	0.477	0.601	0.000	0.208	0.817

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	120	83	78	0	0	129
normalized size	1	1.00	0.89	1.67	1.15	1.08	0.00	0.00	1.79
time (sec)	N/A	0.201	0.046	0.040	0.410	0.498	0.000	0.000	0.064

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	98	61	69	0	78	90
normalized size	1	1.00	0.83	1.53	0.95	1.08	0.00	1.22	1.41
time (sec)	N/A	0.078	0.038	0.036	0.454	0.434	0.000	0.244	0.058

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	76	40	62	0	53	71
normalized size	1	1.00	1.07	1.77	0.93	1.44	0.00	1.23	1.65
time (sec)	N/A	0.041	0.023	0.035	0.505	0.521	0.000	0.259	0.002

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	55	61	0	56	0	80	68
normalized size	1	1.00	1.22	1.36	0.00	1.24	0.00	1.78	1.51
time (sec)	N/A	0.168	0.025	0.039	0.000	0.494	0.000	0.187	0.053

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	77	0	80	0	159	91
normalized size	1	1.00	0.99	1.12	0.00	1.16	0.00	2.30	1.32
time (sec)	N/A	0.193	0.026	0.039	0.000	0.491	0.000	0.202	0.817

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	83	99	0	99	0	224	117
normalized size	1	1.00	0.83	0.99	0.00	0.99	0.00	2.24	1.17
time (sec)	N/A	0.258	0.041	0.042	0.000	0.427	0.000	0.204	0.831

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	91	142	0	107	0	283	140
normalized size	1	1.00	0.73	1.14	0.00	0.86	0.00	2.26	1.12
time (sec)	N/A	0.329	0.045	0.045	0.000	0.411	0.000	0.189	0.063

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	80	187	153	125	0	0	189
normalized size	1	1.00	0.50	1.18	0.96	0.79	0.00	0.00	1.19
time (sec)	N/A	0.517	0.104	0.049	0.441	0.451	0.000	0.000	0.812

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	72	164	130	117	0	176	166
normalized size	1	1.00	0.69	1.58	1.25	1.12	0.00	1.69	1.60
time (sec)	N/A	0.304	0.086	0.043	0.478	0.430	0.000	0.239	0.047

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	64	143	109	109	0	0	143
normalized size	1	1.00	0.62	1.38	1.05	1.05	0.00	0.00	1.38
time (sec)	N/A	0.188	0.076	0.045	0.435	0.437	0.000	0.000	0.811

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	122	88	102	0	0	108
normalized size	1	1.00	0.77	1.65	1.19	1.38	0.00	0.00	1.46
time (sec)	N/A	0.080	0.064	0.041	0.443	0.434	0.000	0.000	0.821

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	35	73	60	0	66	32
normalized size	1	1.00	0.91	1.09	2.28	1.88	0.00	2.06	1.00
time (sec)	N/A	0.034	0.010	0.029	0.399	0.475	0.000	0.241	0.002

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	147	0	93	0	245	119
normalized size	1	1.00	1.05	1.99	0.00	1.26	0.00	3.31	1.61
time (sec)	N/A	0.200	0.038	0.039	0.000	0.504	0.000	0.280	0.813

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	91	118	0	118	0	307	146
normalized size	1	1.00	0.92	1.19	0.00	1.19	0.00	3.10	1.47
time (sec)	N/A	0.267	0.044	0.044	0.000	0.487	0.000	0.240	0.831

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	103	181	0	136	0	332	169
normalized size	1	1.00	0.78	1.37	0.00	1.03	0.00	2.52	1.28
time (sec)	N/A	0.334	0.054	0.044	0.000	0.464	0.000	0.275	0.057

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	111	226	0	144	0	372	192
normalized size	1	1.00	0.69	1.40	0.00	0.89	0.00	2.31	1.19
time (sec)	N/A	0.423	0.059	0.046	0.000	0.448	0.000	0.297	0.830

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	122	208	185	153	0	0	302
normalized size	1	1.00	0.90	1.54	1.37	1.13	0.00	0.00	2.24
time (sec)	N/A	0.415	0.148	0.049	0.407	0.471	0.000	0.000	0.829

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	72	186	163	145	0	186	234
normalized size	1	1.00	0.53	1.36	1.19	1.06	0.00	1.36	1.71
time (sec)	N/A	0.333	0.076	0.047	0.463	0.487	0.000	0.195	0.054

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	77	167	144	138	0	0	259
normalized size	1	1.00	0.72	1.56	1.35	1.29	0.00	0.00	2.42
time (sec)	N/A	0.220	0.063	0.046	0.410	0.491	0.000	0.000	0.820

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	35	41	132	91	0	121	143
normalized size	1	1.00	0.54	0.63	2.03	1.40	0.00	1.86	2.20
time (sec)	N/A	0.078	0.020	0.028	0.404	0.616	0.000	0.203	0.851

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	35	40	126	89	0	145	183
normalized size	1	1.00	0.54	0.62	1.94	1.37	0.00	2.23	2.82
time (sec)	N/A	0.052	0.017	0.028	0.493	0.501	0.000	0.216	0.002

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	71	275	0	130	0	189	209
normalized size	1	1.00	0.73	2.84	0.00	1.34	0.00	1.95	2.15
time (sec)	N/A	0.274	0.093	0.041	0.000	0.592	0.000	0.222	0.075

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	101	248	0	155	0	269	234
normalized size	1	1.00	0.80	1.95	0.00	1.22	0.00	2.12	1.84
time (sec)	N/A	0.351	0.061	0.046	0.000	0.613	0.000	0.246	0.827

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	113	223	0	173	0	338	257
normalized size	1	1.00	0.70	1.38	0.00	1.07	0.00	2.09	1.59
time (sec)	N/A	0.430	0.063	0.049	0.000	0.447	0.000	0.254	0.832

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	121	355	0	181	0	393	328
normalized size	1	1.00	0.65	1.90	0.00	0.97	0.00	2.10	1.75
time (sec)	N/A	0.525	0.064	0.049	0.000	0.469	0.000	0.400	0.094

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	126	252	250	187	0	252	323
normalized size	1	1.00	0.76	1.52	1.51	1.13	0.00	1.52	1.95
time (sec)	N/A	0.533	0.341	0.053	0.411	0.481	0.000	0.169	0.833

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	95	231	229	179	0	0	350
normalized size	1	1.00	0.57	1.38	1.36	1.07	0.00	0.00	2.08
time (sec)	N/A	0.403	0.092	0.053	0.467	0.484	0.000	0.000	0.837

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	94	210	208	172	0	220	281
normalized size	1	1.00	0.68	1.52	1.51	1.25	0.00	1.59	2.04
time (sec)	N/A	0.272	0.247	0.046	0.645	0.603	0.000	0.204	0.069

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	197	118	0	0	347
normalized size	1	1.00	0.44	0.51	2.03	1.22	0.00	0.00	3.58
time (sec)	N/A	0.208	0.028	0.032	0.408	0.572	0.000	0.000	0.065

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	42	48	197	116	0	148	295
normalized size	1	1.00	0.43	0.49	2.03	1.20	0.00	1.53	3.04
time (sec)	N/A	0.101	0.021	0.030	0.431	0.672	0.000	0.506	0.815

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	189	116	0	199	49
normalized size	1	1.00	0.44	0.51	1.95	1.20	0.00	2.05	0.51
time (sec)	N/A	0.070	0.021	0.030	0.405	0.527	0.000	0.265	0.002

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	79	451	0	163	0	243	327
normalized size	1	1.00	0.62	3.52	0.00	1.27	0.00	1.90	2.55
time (sec)	N/A	0.311	0.183	0.047	0.000	0.463	0.000	0.261	0.826

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	109	423	0	188	0	323	352
normalized size	1	1.00	0.70	2.73	0.00	1.21	0.00	2.08	2.27
time (sec)	N/A	0.449	0.067	0.049	0.000	0.477	0.000	0.231	0.833

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	121	397	0	206	0	392	375
normalized size	1	1.00	0.63	2.07	0.00	1.07	0.00	2.04	1.95
time (sec)	N/A	0.524	0.070	0.049	0.000	0.470	0.000	0.524	0.841

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	41	40	38	37	21	22
normalized size	1	1.00	0.69	0.67	0.66	0.62	0.61	0.34	0.36
time (sec)	N/A	0.032	0.030	0.029	0.440	0.601	0.321	0.184	0.039

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	29	28	33	27	19	21
normalized size	1	1.00	0.79	0.62	0.60	0.70	0.57	0.40	0.45
time (sec)	N/A	0.017	0.013	0.027	0.473	0.443	0.184	0.185	0.029

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	51	57	56	45	54	28	29
normalized size	1	1.00	0.59	0.66	0.64	0.52	0.62	0.32	0.33
time (sec)	N/A	0.048	0.031	0.032	0.395	0.470	0.604	0.244	0.028

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	43	42	40	44	25	26
normalized size	1	1.00	0.66	0.64	0.63	0.60	0.66	0.37	0.39
time (sec)	N/A	0.029	0.024	0.031	0.491	0.426	0.331	0.181	0.026

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	13	17	11	11	8	11	11
normalized size	1	1.00	0.72	0.94	0.61	0.61	0.44	0.61	0.61
time (sec)	N/A	0.034	0.006	0.026	0.391	0.495	0.117	0.289	0.112

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	18	2	2	2
normalized size	1	1.00	1.00	1.50	1.00	9.00	1.00	1.00	1.00
time (sec)	N/A	0.019	0.003	0.030	0.399	0.449	0.118	0.181	0.006

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	24	18	44	0	24	18
normalized size	1	1.00	1.00	1.20	0.90	2.20	0.00	1.20	0.90
time (sec)	N/A	0.034	0.035	0.032	0.541	0.474	0.000	0.180	0.785

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	16	19	0	21	13
normalized size	1	1.00	1.00	0.78	0.89	1.06	0.00	1.17	0.72
time (sec)	N/A	0.018	0.004	0.030	0.407	0.425	0.000	0.158	0.026

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	35	29	31	0	0	52
normalized size	1	1.00	0.57	0.71	0.59	0.63	0.00	0.00	1.06
time (sec)	N/A	0.048	0.012	0.026	0.344	0.623	0.000	0.000	0.916

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	30	24	26	0	0	45
normalized size	1	1.00	0.61	0.79	0.63	0.68	0.00	0.00	1.18
time (sec)	N/A	0.029	0.009	0.028	0.310	0.677	0.000	0.000	0.880

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	47	21	34	48	38	0	27	33
normalized size	1	1.38	0.62	1.00	1.41	1.12	0.00	0.79	0.97
time (sec)	N/A	0.062	0.011	0.026	0.381	0.518	0.000	0.432	0.920

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	29	36	33	0	20	42
normalized size	1	1.00	0.70	1.26	1.57	1.43	0.00	0.87	1.83
time (sec)	N/A	0.034	0.008	0.028	0.330	0.424	0.000	0.221	0.866

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	28	22	24	0	0	45
normalized size	1	1.00	0.58	0.78	0.61	0.67	0.00	0.00	1.25
time (sec)	N/A	0.042	0.008	0.026	0.306	0.473	0.000	0.000	0.869

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	16	23	17	19	0	0	31
normalized size	1	1.00	0.64	0.92	0.68	0.76	0.00	0.00	1.24
time (sec)	N/A	0.023	0.006	0.026	0.300	0.454	0.000	0.000	0.854

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	29	38	31	0	20	42
normalized size	1	1.00	0.70	1.26	1.65	1.35	0.00	0.87	1.83
time (sec)	N/A	0.039	0.007	0.027	0.322	0.471	0.000	0.158	0.886

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	24	23	26	0	13	33
normalized size	1	1.00	1.00	2.18	2.09	2.36	0.00	1.18	3.00
time (sec)	N/A	0.020	0.005	0.024	0.375	0.564	0.000	0.404	0.861

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	16	23	15	19	0	0	19
normalized size	1	1.00	0.64	0.92	0.60	0.76	0.00	0.00	0.76
time (sec)	N/A	0.039	0.007	0.028	0.303	0.460	0.000	0.000	0.025

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	20	12	16	0	15	16
normalized size	1	1.00	1.00	1.82	1.09	1.45	0.00	1.36	1.45
time (sec)	N/A	0.019	0.004	0.025	0.371	0.414	0.000	0.180	0.925

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	61	0	79	0	44	-1
normalized size	1	1.00	0.86	1.45	0.00	1.88	0.00	1.05	-0.02
time (sec)	N/A	0.046	0.023	0.035	0.000	0.476	0.000	0.172	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	0	74	0	37	-1
normalized size	1	1.00	1.00	1.68	0.00	2.39	0.00	1.19	-0.03
time (sec)	N/A	0.034	0.006	0.035	0.000	0.476	0.000	0.234	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	50	0	71	0	43	-1
normalized size	1	1.00	1.00	1.47	0.00	2.09	0.00	1.26	-0.03
time (sec)	N/A	0.045	0.008	0.034	0.000	0.463	0.000	0.501	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	40	0	45	0	37	-1
normalized size	1	1.00	1.00	1.74	0.00	1.96	0.00	1.61	-0.04
time (sec)	N/A	0.028	0.004	0.033	0.000	0.600	0.000	0.311	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	78	0	91	0	49	-1
normalized size	1	1.00	0.78	1.53	0.00	1.78	0.00	0.96	-0.02
time (sec)	N/A	0.055	0.022	0.042	0.000	0.585	0.000	0.187	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	69	0	85	0	42	-1
normalized size	1	1.00	0.97	1.86	0.00	2.30	0.00	1.14	-0.03
time (sec)	N/A	0.037	0.026	0.044	0.000	0.437	0.000	0.193	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	46	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.124	0.018	0.325	0.000	0.572	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	51	48	106	58	0	0	53
normalized size	1	1.00	0.48	0.45	0.99	0.54	0.00	0.00	0.50
time (sec)	N/A	0.158	0.031	0.029	0.332	0.523	0.000	0.000	0.954

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	43	40	83	49	0	54	46
normalized size	1	1.00	0.62	0.58	1.20	0.71	0.00	0.78	0.67
time (sec)	N/A	0.090	0.023	0.026	0.327	0.572	0.000	0.197	0.920

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	34	58	39	0	34	37
normalized size	1	1.00	1.06	0.97	1.66	1.11	0.00	0.97	1.06
time (sec)	N/A	0.045	0.014	0.027	0.330	0.567	0.000	0.207	0.002

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	71	0	183	0	83	-1
normalized size	1	1.00	0.68	1.04	0.00	2.69	0.00	1.22	-0.01
time (sec)	N/A	0.152	0.021	0.042	0.000	0.493	0.000	0.157	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	57	78	0	207	0	97	-1
normalized size	1	1.00	0.79	1.08	0.00	2.88	0.00	1.35	-0.01
time (sec)	N/A	0.157	0.026	0.050	0.000	0.499	0.000	0.169	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	48	45	74	44	83	189	83
normalized size	1	1.00	0.48	0.45	0.73	0.44	0.82	1.87	0.82
time (sec)	N/A	0.132	0.067	0.031	0.306	0.649	10.430	0.482	0.044

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	40	37	60	36	68	142	66
normalized size	1	1.00	0.50	0.46	0.75	0.45	0.85	1.78	0.82
time (sec)	N/A	0.163	0.055	0.031	0.309	0.417	8.675	0.166	0.055

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	31	28	44	27	48	92	46
normalized size	1	1.00	0.54	0.49	0.77	0.47	0.84	1.61	0.81
time (sec)	N/A	0.086	0.042	0.028	0.300	0.651	7.407	0.156	0.049

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	30	19	31	44	32
normalized size	1	1.00	0.61	0.53	0.79	0.50	0.82	1.16	0.84
time (sec)	N/A	0.048	0.025	0.028	0.306	0.678	4.625	0.156	0.021

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	48	82	39	40	31
normalized size	1	1.00	1.00	0.82	1.23	2.10	1.00	1.03	0.79
time (sec)	N/A	0.105	0.032	0.033	0.447	0.709	7.282	0.200	0.809

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	45	62	96	119	47	35
normalized size	1	1.00	1.00	1.05	1.44	2.23	2.77	1.09	0.81
time (sec)	N/A	0.107	0.029	0.042	0.404	0.504	19.587	0.170	0.836

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	65	103	117	270	76	54
normalized size	1	1.00	0.81	0.96	1.51	1.72	3.97	1.12	0.79
time (sec)	N/A	0.116	0.050	0.038	0.401	0.557	36.999	0.185	0.075

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	63	80	134	133	439	104	74
normalized size	1	1.00	0.71	0.90	1.51	1.49	4.93	1.17	0.83
time (sec)	N/A	0.128	0.066	0.042	0.469	0.798	55.920	0.167	0.071

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	71	93	163	149	639	131	91
normalized size	1	1.00	0.65	0.85	1.48	1.35	5.81	1.19	0.83
time (sec)	N/A	0.145	0.069	0.041	0.458	0.748	78.973	0.389	0.104

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	92	146	0	274	0	0	-1
normalized size	1	1.00	0.37	0.59	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.180	0.084	0.046	0.000	0.536	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	84	129	0	258	0	130	-1
normalized size	1	1.00	0.50	0.76	0.00	1.53	0.00	0.77	-0.01
time (sec)	N/A	0.160	0.068	0.045	0.000	0.931	0.000	0.248	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	76	112	0	242	0	0	-1
normalized size	1	1.00	0.45	0.66	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.043	0.043	0.000	0.531	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	67	95	0	220	0	0	-1
normalized size	1	1.00	0.56	0.80	0.00	1.85	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.034	0.043	0.000	0.556	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	66	98	0	321	0	0	-1
normalized size	1	1.00	0.55	0.82	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.034	0.050	0.000	0.589	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	75	105	0	358	0	0	-1
normalized size	1	1.00	0.60	0.85	0.00	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.037	0.051	0.000	0.516	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	92	131	0	391	0	0	-1
normalized size	1	1.00	0.53	0.76	0.00	2.26	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.056	0.053	0.000	0.488	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	100	151	0	407	0	0	-1
normalized size	1	1.00	0.46	0.70	0.00	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.067	0.056	0.000	0.553	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	108	171	0	423	0	0	-1
normalized size	1	1.00	0.42	0.66	0.00	1.63	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.078	0.054	0.000	0.537	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	77	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.049	0.413	0.000	0.573	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	55	56	62	58	0	0	74
normalized size	1	1.00	0.41	0.41	0.46	0.43	0.00	0.00	0.55
time (sec)	N/A	0.213	0.037	0.030	0.326	0.449	0.000	0.000	0.951

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	46	47	50	49	0	65	75
normalized size	1	1.00	0.46	0.47	0.50	0.49	0.00	0.64	0.74
time (sec)	N/A	0.121	0.032	0.029	0.371	0.407	0.000	0.947	0.903

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	39	37	39	0	50	56
normalized size	1	1.00	0.58	0.59	0.56	0.59	0.00	0.76	0.85
time (sec)	N/A	0.065	0.022	0.027	0.383	0.456	0.000	0.377	0.002

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	44	69	0	184	0	82	-1
normalized size	1	1.00	0.65	1.01	0.00	2.71	0.00	1.21	-0.01
time (sec)	N/A	0.158	0.020	0.041	0.000	0.463	0.000	0.154	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	52	79	0	207	0	90	-1
normalized size	1	1.00	0.72	1.10	0.00	2.88	0.00	1.25	-0.01
time (sec)	N/A	0.161	0.030	0.049	0.000	0.491	0.000	1.017	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	64	101	0	232	0	117	-1
normalized size	1	1.00	0.57	0.90	0.00	2.07	0.00	1.04	-0.01
time (sec)	N/A	0.196	0.037	0.050	0.000	0.488	0.000	0.207	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	56	121	0	248	0	132	-1
normalized size	1	1.00	0.38	0.82	0.00	1.68	0.00	0.89	-0.01
time (sec)	N/A	0.237	0.023	0.053	0.000	0.602	0.000	0.260	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	87	101	123	169	126	159	114
normalized size	1	1.00	0.63	0.73	0.88	1.22	0.91	1.14	0.82
time (sec)	N/A	0.173	0.132	0.036	0.462	0.785	18.582	0.189	0.848

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	75	97	154	97	105	80
normalized size	1	1.00	0.80	0.77	1.00	1.59	1.00	1.08	0.82
time (sec)	N/A	0.158	0.091	0.036	0.424	0.553	14.871	0.166	0.097

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	70	73	95	137	94	105	80
normalized size	1	1.00	0.72	0.75	0.98	1.41	0.97	1.08	0.82
time (sec)	N/A	0.107	0.084	0.035	0.402	0.458	11.868	0.267	0.857

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	79	120	75	77	61
normalized size	1	1.00	0.80	0.78	1.04	1.58	0.99	1.01	0.80
time (sec)	N/A	0.066	0.043	0.031	0.404	0.524	8.084	0.173	0.002

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	58	97	157	80	67	57
normalized size	1	1.00	1.00	0.78	1.31	2.12	1.08	0.91	0.77
time (sec)	N/A	0.138	0.029	0.038	0.406	0.477	10.109	0.174	0.833

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	71	111	175	162	72	62
normalized size	1	1.00	1.00	0.90	1.41	2.22	2.05	0.91	0.78
time (sec)	N/A	0.140	0.045	0.042	0.437	0.632	14.770	0.185	0.860

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	93	95	152	204	352	106	88
normalized size	1	1.00	0.88	0.90	1.43	1.92	3.32	1.00	0.83
time (sec)	N/A	0.159	0.075	0.044	0.401	0.701	25.662	0.196	0.115

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	101	110	183	220	614	133	105
normalized size	1	1.00	0.80	0.87	1.44	1.73	4.83	1.05	0.83
time (sec)	N/A	0.182	0.098	0.041	0.413	0.501	29.743	0.151	0.127

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	109	123	212	236	991	160	122
normalized size	1	1.00	0.74	0.83	1.43	1.59	6.70	1.08	0.82
time (sec)	N/A	0.209	0.114	0.044	0.603	0.562	49.266	0.179	0.888

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	76	79	86	76	0	0	115
normalized size	1	1.00	0.32	0.34	0.37	0.32	0.00	0.00	0.49
time (sec)	N/A	0.154	0.050	0.030	0.338	0.480	0.000	0.000	1.069

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	68	71	75	68	0	0	107
normalized size	1	1.00	0.35	0.36	0.38	0.35	0.00	0.00	0.54
time (sec)	N/A	0.150	0.043	0.029	0.366	0.496	0.000	0.000	1.039

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	60	63	64	60	0	0	97
normalized size	1	1.00	0.38	0.40	0.41	0.38	0.00	0.00	0.62
time (sec)	N/A	0.105	0.038	0.028	0.405	0.449	0.000	0.000	1.004

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	51	54	50	49	0	0	40
normalized size	1	1.00	0.50	0.52	0.49	0.48	0.00	0.00	0.39
time (sec)	N/A	0.096	0.027	0.029	0.423	0.466	0.000	0.000	0.002

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	51	78	0	209	0	0	-1
normalized size	1	1.00	0.48	0.73	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.035	0.048	0.000	0.471	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	64	82	0	223	0	0	-1
normalized size	1	1.00	0.57	0.73	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.031	0.052	0.000	0.739	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	65	100	0	252	0	0	-1
normalized size	1	1.00	0.40	0.61	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.029	0.056	0.000	0.466	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	209	65	111	0	268	0	0	-1
normalized size	1	1.01	0.32	0.54	0.00	1.30	0.00	0.00	-0.00
time (sec)	N/A	0.138	0.031	0.053	0.000	0.498	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	252	65	122	0	284	0	0	-1
normalized size	1	1.01	0.26	0.49	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.029	0.059	0.000	0.475	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.026	0.250	0.000	0.444	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	32	30	37	0	0	37
normalized size	1	1.00	0.86	0.86	0.81	1.00	0.00	0.00	1.00
time (sec)	N/A	0.035	0.027	0.029	0.399	0.556	0.000	0.000	0.886

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	77	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.025	0.244	0.000	0.474	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	65	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.022	0.237	0.000	0.477	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	65	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.018	0.230	0.000	0.441	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.017	0.048	0.000	0.443	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.010	0.236	0.000	0.545	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	0	58	0	0	37
normalized size	1	1.00	1.00	0.85	0.00	1.49	0.00	0.00	0.95
time (sec)	N/A	0.039	0.020	0.029	0.000	0.601	0.000	0.000	1.105

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	51	46	0	128	0	0	100
normalized size	1	1.00	0.61	0.55	0.00	1.52	0.00	0.00	1.19
time (sec)	N/A	0.056	0.033	0.029	0.000	0.437	0.000	0.000	1.270

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	74	68	0	228	0	0	167
normalized size	1	1.00	0.55	0.50	0.00	1.69	0.00	0.00	1.24
time (sec)	N/A	0.081	0.046	0.033	0.000	0.730	0.000	0.000	1.336

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.102	1.397	0.244	0.000	0.552	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	86	142	190	132	357	263	137
normalized size	1	1.00	0.69	1.14	1.52	1.06	2.86	2.10	1.10
time (sec)	N/A	0.256	0.255	0.049	0.506	0.521	11.933	0.431	0.050

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	119	111	121	228	207	114
normalized size	1	1.00	0.77	1.23	1.14	1.25	2.35	2.13	1.18
time (sec)	N/A	0.191	0.199	0.040	0.572	0.603	7.375	0.379	0.795

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	78	95	89	94	151	139	91
normalized size	1	1.00	1.20	1.46	1.37	1.45	2.32	2.14	1.40
time (sec)	N/A	0.123	0.106	0.038	0.536	0.459	7.261	0.441	0.042

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	34	50	41	61	58	37
normalized size	1	1.00	1.02	0.83	1.22	1.00	1.49	1.41	0.90
time (sec)	N/A	0.068	0.044	0.030	0.446	0.425	20.408	0.216	0.818

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	52	96	0	68	0	73	91
normalized size	1	1.00	0.80	1.48	0.00	1.05	0.00	1.12	1.40
time (sec)	N/A	0.096	0.041	0.036	0.000	0.516	0.000	0.193	0.072

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	53	140	0	107	0	0	140
normalized size	1	1.00	0.51	1.35	0.00	1.03	0.00	0.00	1.35
time (sec)	N/A	0.185	0.136	0.044	0.000	0.486	0.000	0.000	0.818

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	61	184	0	143	0	181	225
normalized size	1	1.00	0.45	1.35	0.00	1.05	0.00	1.33	1.65
time (sec)	N/A	0.282	0.156	0.046	0.000	0.472	0.000	0.201	0.824

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	69	228	0	177	0	0	389
normalized size	1	1.00	0.41	1.36	0.00	1.05	0.00	0.00	2.32
time (sec)	N/A	0.352	0.191	0.049	0.000	0.476	0.000	0.000	0.833

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	46	0	0	0	274	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	4.64	0.00	-0.02
time (sec)	N/A	0.080	0.029	0.327	0.000	0.502	7.926	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	64	61	58	67	63	59	49
normalized size	1	1.00	1.03	0.98	0.94	1.08	1.02	0.95	0.79
time (sec)	N/A	0.113	0.225	0.032	0.475	0.548	0.281	0.158	0.855

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	39	38	43	39	39	35
normalized size	1	1.00	1.05	0.98	0.95	1.08	0.98	0.98	0.88
time (sec)	N/A	0.102	0.159	0.034	0.301	0.570	0.195	0.198	0.814

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	39	0	45	37	38	35
normalized size	1	1.00	1.05	0.98	0.00	1.12	0.92	0.95	0.88
time (sec)	N/A	0.098	0.126	0.033	0.000	0.411	0.170	0.204	0.051

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	18	22	17	18	20
normalized size	1	1.00	1.00	1.11	1.00	1.22	0.94	1.00	1.11
time (sec)	N/A	0.090	0.096	0.029	0.353	0.560	0.096	0.201	0.036

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	14	12	14	12
normalized size	1	1.00	1.00	1.08	1.00	1.08	0.92	1.08	0.92
time (sec)	N/A	0.054	0.037	0.026	0.408	0.512	0.090	0.175	0.034

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	37	36	41	26	37	35
normalized size	1	1.00	0.82	0.97	0.95	1.08	0.68	0.97	0.92
time (sec)	N/A	0.088	0.030	0.032	0.351	0.456	0.166	0.363	0.059

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	51	55	71	51	42	54
normalized size	1	1.00	0.96	0.96	1.04	1.34	0.96	0.79	1.02
time (sec)	N/A	0.114	0.098	0.033	0.307	0.421	0.253	0.159	0.072

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	67	76	100	75	51	73
normalized size	1	1.00	0.85	0.91	1.03	1.35	1.01	0.69	0.99
time (sec)	N/A	0.133	0.126	0.038	0.364	0.490	0.352	0.184	0.838

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	71	82	94	126	95	59	90
normalized size	1	1.00	0.81	0.93	1.07	1.43	1.08	0.67	1.02
time (sec)	N/A	0.139	0.154	0.035	0.357	0.424	0.450	0.169	0.848

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	85	180	349	132	354	262	135
normalized size	1	1.00	0.83	1.75	3.39	1.28	3.44	2.54	1.31
time (sec)	N/A	0.168	0.260	0.053	0.500	0.654	18.874	0.224	0.047

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	118	188	78	104	97	64
normalized size	1	1.00	1.00	1.53	2.44	1.01	1.35	1.26	0.83
time (sec)	N/A	0.121	0.136	0.041	0.412	0.492	35.547	0.360	0.059

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	82	133	209	93	151	139	90
normalized size	1	1.00	1.22	1.99	3.12	1.39	2.25	2.07	1.34
time (sec)	N/A	0.158	0.110	0.046	0.429	0.578	13.946	0.252	0.811

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	84	140	66	104	68	56
normalized size	1	1.00	0.94	1.68	2.80	1.32	2.08	1.36	1.12
time (sec)	N/A	0.164	0.055	0.038	0.474	0.428	10.248	2.096	0.053

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	62	168	0	101	0	126	129
normalized size	1	1.00	0.63	1.70	0.00	1.02	0.00	1.27	1.30
time (sec)	N/A	0.120	0.036	0.041	0.000	0.470	0.000	0.197	0.063

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	61	212	0	143	0	180	270
normalized size	1	1.00	0.48	1.66	0.00	1.12	0.00	1.41	2.11
time (sec)	N/A	0.257	0.135	0.047	0.000	0.493	0.000	0.208	0.817

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	69	256	0	177	0	0	340
normalized size	1	1.00	0.45	1.65	0.00	1.14	0.00	0.00	2.19
time (sec)	N/A	0.334	0.163	0.050	0.000	0.507	0.000	0.000	0.072

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	77	300	0	213	0	288	579
normalized size	1	1.00	0.41	1.58	0.00	1.12	0.00	1.52	3.05
time (sec)	N/A	0.443	0.195	0.055	0.000	0.792	0.000	0.670	0.869

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.052	0.327	0.000	0.507	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	61	59	67	63	60	51
normalized size	1	1.00	1.03	0.95	0.92	1.05	0.98	0.94	0.80
time (sec)	N/A	0.111	0.276	0.031	0.352	0.441	0.284	0.199	0.067

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	31	36	31	31	27
normalized size	1	1.00	1.00	0.90	1.03	1.20	1.03	1.03	0.90
time (sec)	N/A	0.098	0.189	0.032	0.375	0.491	0.158	0.234	0.807

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	37	34	43	37	35	31
normalized size	1	1.00	1.05	0.97	0.89	1.13	0.97	0.92	0.82
time (sec)	N/A	0.101	0.142	0.032	0.379	0.552	0.177	0.181	0.815

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	28	27	32	26	28	25
normalized size	1	1.00	1.07	1.04	1.00	1.19	0.96	1.04	0.93
time (sec)	N/A	0.096	0.100	0.033	0.352	0.408	0.130	0.200	0.808

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	24	23	17	26	24
normalized size	1	1.00	1.00	1.00	0.96	0.92	0.68	1.04	0.96
time (sec)	N/A	0.065	0.039	0.033	0.327	0.519	0.226	0.194	0.079

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	51	49	64	41	42	48
normalized size	1	1.00	0.96	0.96	0.92	1.21	0.77	0.79	0.91
time (sec)	N/A	0.102	0.033	0.035	0.335	0.504	0.245	0.182	0.065

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	66	75	100	73	50	71
normalized size	1	1.00	0.89	0.93	1.06	1.41	1.03	0.70	1.00
time (sec)	N/A	0.127	0.110	0.034	0.306	0.451	0.344	0.134	0.845

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	81	93	126	94	58	90
normalized size	1	1.00	0.80	0.91	1.04	1.42	1.06	0.65	1.01
time (sec)	N/A	0.145	0.142	0.033	0.342	0.514	0.440	0.330	0.081

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	79	96	113	154	114	66	109
normalized size	1	1.00	0.75	0.91	1.08	1.47	1.09	0.63	1.04
time (sec)	N/A	0.149	0.172	0.038	0.325	0.425	0.550	0.209	0.097

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.095	1.120	0.325	0.000	0.530	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	85	232	0	132	0	262	136
normalized size	1	1.00	0.61	1.66	0.00	0.94	0.00	1.87	0.97
time (sec)	N/A	0.420	0.253	0.050	0.000	0.433	0.000	0.204	0.847

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	77	209	0	119	0	206	113
normalized size	1	1.00	0.69	1.88	0.00	1.07	0.00	1.86	1.02
time (sec)	N/A	0.312	0.193	0.045	0.000	0.427	0.000	0.209	0.041

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	186	0	97	0	139	90
normalized size	1	1.00	1.01	2.27	0.00	1.18	0.00	1.70	1.10
time (sec)	N/A	0.238	0.126	0.041	0.000	0.484	0.000	0.431	0.816

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	106	70	67	0	68	56
normalized size	1	1.00	0.94	2.12	1.40	1.34	0.00	1.36	1.12
time (sec)	N/A	0.151	0.055	0.040	0.464	0.519	0.000	0.209	0.059

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	22	19	53	19	19
normalized size	1	1.00	1.00	0.95	1.05	0.90	2.52	0.90	0.90
time (sec)	N/A	0.074	0.014	0.027	0.330	0.553	9.018	0.160	0.040

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	47	198	0	71	0	72	89
normalized size	1	1.00	0.75	3.14	0.00	1.13	0.00	1.14	1.41
time (sec)	N/A	0.179	0.099	0.047	0.000	0.473	0.000	2.107	0.865

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	53	242	0	107	0	0	139
normalized size	1	1.00	0.56	2.57	0.00	1.14	0.00	0.00	1.48
time (sec)	N/A	0.260	0.136	0.049	0.000	0.643	0.000	0.000	0.063

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	61	286	0	143	0	180	272
normalized size	1	1.00	0.49	2.29	0.00	1.14	0.00	1.44	2.18
time (sec)	N/A	0.354	0.165	0.051	0.000	0.528	0.000	0.188	0.838

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.051	0.244	0.000	0.563	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	65	61	71	58	112	63
normalized size	1	1.00	1.03	0.98	0.92	1.08	0.88	1.70	0.95
time (sec)	N/A	0.118	0.176	0.038	0.310	0.451	0.472	0.167	0.105

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	57	54	52	62	44	100	51
normalized size	1	1.00	1.04	0.98	0.95	1.13	0.80	1.82	0.93
time (sec)	N/A	0.116	0.130	0.036	0.307	0.415	0.358	3.793	0.878

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	44	43	42	44	32	72	42
normalized size	1	1.00	1.05	1.02	1.00	1.05	0.76	1.71	1.00
time (sec)	N/A	0.111	0.100	0.034	0.313	0.434	0.285	1.985	0.872

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	23	19	56	25
normalized size	1	1.00	1.00	1.04	1.00	0.92	0.76	2.24	1.00
time (sec)	N/A	0.066	0.040	0.030	0.315	0.507	0.221	0.163	0.067

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	21	20	20	19	41	18
normalized size	1	1.00	0.90	1.05	1.00	1.00	0.95	2.05	0.90
time (sec)	N/A	0.088	0.016	0.028	0.346	0.553	0.111	1.767	0.041

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	40	36	35	27	36	35	16
normalized size	1	1.00	2.22	2.00	1.94	1.50	2.00	1.94	0.89
time (sec)	N/A	0.110	0.077	0.031	0.375	0.522	0.154	0.164	0.860

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	51	54	59	58	85	53
normalized size	1	1.00	0.98	0.88	0.93	1.02	1.00	1.47	0.91
time (sec)	N/A	0.125	0.112	0.034	0.340	0.399	0.308	0.202	0.096

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	69	66	70	93	75	95	68
normalized size	1	1.00	0.91	0.87	0.92	1.22	0.99	1.25	0.89
time (sec)	N/A	0.133	0.152	0.036	0.315	0.407	0.394	0.177	0.097

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	93	352	0	177	0	265	161
normalized size	1	1.00	0.66	2.51	0.00	1.26	0.00	1.89	1.15
time (sec)	N/A	0.368	0.302	0.056	0.000	0.473	0.000	0.198	0.062

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	81	329	0	149	0	197	138
normalized size	1	1.00	0.73	2.96	0.00	1.34	0.00	1.77	1.24
time (sec)	N/A	0.295	0.230	0.052	0.000	0.605	0.000	0.201	0.829

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	223	0	97	0	104	102
normalized size	1	1.00	0.79	2.90	0.00	1.26	0.00	1.35	1.32
time (sec)	N/A	0.206	0.145	0.050	0.000	0.434	0.000	4.041	0.837

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	292	0	67	0	73	90
normalized size	1	1.00	1.03	4.49	0.00	1.03	0.00	1.12	1.38
time (sec)	N/A	0.104	0.032	0.048	0.000	0.470	0.000	1.817	0.061

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	46	336	0	71	0	73	89
normalized size	1	1.00	0.73	5.33	0.00	1.13	0.00	1.16	1.41
time (sec)	N/A	0.125	0.147	0.054	0.000	0.545	0.000	0.181	0.058

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	43	45	53	34	38	116
normalized size	1	1.00	0.67	0.96	1.00	1.18	0.76	0.84	2.58
time (sec)	N/A	0.119	0.137	0.030	0.326	1.365	23.324	0.257	0.858

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	68	424	0	142	0	0	188
normalized size	1	1.00	0.71	4.42	0.00	1.48	0.00	0.00	1.96
time (sec)	N/A	0.165	0.190	0.064	0.000	0.854	0.000	0.000	0.070

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	76	468	0	151	0	0	275
normalized size	1	1.00	0.61	3.74	0.00	1.21	0.00	0.00	2.20
time (sec)	N/A	0.318	0.235	0.065	0.000	0.578	0.000	0.000	0.906

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	101	172	0	386	0	0	-1
normalized size	1	1.00	0.45	0.76	0.00	1.72	0.00	0.00	-0.00
time (sec)	N/A	0.209	0.082	0.103	0.000	0.552	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	95	154	0	364	0	0	-1
normalized size	1	1.00	0.52	0.85	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.075	0.060	0.000	0.702	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	87	136	0	330	0	0	-1
normalized size	1	1.00	0.51	0.80	0.00	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.064	0.058	0.000	1.678	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	70	108	0	266	0	0	-1
normalized size	1	1.00	0.55	0.84	0.00	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.052	0.054	0.000	0.596	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	85	69	89	0	249	0	0	-1
normalized size	1	1.01	0.82	1.06	0.00	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.042	0.047	0.000	0.586	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	105	168	0	447	0	0	-1
normalized size	1	1.00	0.67	1.07	0.00	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.068	0.053	0.000	0.570	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	126	276	0	526	0	0	-1
normalized size	1	1.00	0.64	1.39	0.00	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.158	0.059	0.000	0.568	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	139	390	0	600	0	0	-1
normalized size	1	1.00	0.56	1.57	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.181	0.066	0.000	0.641	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	91	163	0	234	2421	0	-1
normalized size	1	1.00	0.63	1.12	0.00	1.61	16.70	0.00	-0.01
time (sec)	N/A	0.185	0.148	0.046	0.000	0.457	31.186	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	83	144	0	212	777	0	-1
normalized size	1	1.00	0.69	1.20	0.00	1.77	6.48	0.00	-0.01
time (sec)	N/A	0.161	0.091	0.045	0.000	0.412	25.992	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	108	0	182	143	0	-1
normalized size	1	1.00	0.78	1.12	0.00	1.90	1.49	0.00	-0.01
time (sec)	N/A	0.143	0.064	0.043	0.000	0.537	6.735	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	57	103	0	136	163	0	-1
normalized size	1	1.00	0.80	1.45	0.00	1.92	2.30	0.00	-0.01
time (sec)	N/A	0.121	0.050	0.044	0.000	0.441	30.513	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	120	0	125	0	97	-1
normalized size	1	1.00	1.00	2.35	0.00	2.45	0.00	1.90	-0.02
time (sec)	N/A	0.101	0.031	0.043	0.000	0.447	0.000	4.940	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	194	0	175	0	123	-1
normalized size	1	1.00	0.62	2.73	0.00	2.46	0.00	1.73	-0.01
time (sec)	N/A	0.162	0.028	0.045	0.000	0.442	0.000	2.171	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	55	260	0	238	0	136	-1
normalized size	1	1.00	0.57	2.71	0.00	2.48	0.00	1.42	-0.01
time (sec)	N/A	0.150	0.031	0.046	0.000	0.655	0.000	0.195	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	59	328	0	294	0	165	-1
normalized size	1	1.00	0.50	2.76	0.00	2.47	0.00	1.39	-0.01
time (sec)	N/A	0.159	0.030	0.050	0.000	0.438	0.000	0.195	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	46	396	0	346	0	187	-1
normalized size	1	1.00	0.32	2.71	0.00	2.37	0.00	1.28	-0.01
time (sec)	N/A	0.190	0.042	0.053	0.000	0.442	0.000	0.227	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	85	172	0	386	0	0	-1
normalized size	1	1.00	0.38	0.77	0.00	1.73	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.068	0.056	0.000	0.601	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	77	154	0	364	0	0	-1
normalized size	1	1.00	0.35	0.71	0.00	1.68	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.056	0.058	0.000	0.466	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	66	136	0	330	0	0	-1
normalized size	1	1.00	0.38	0.77	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.042	0.052	0.000	0.534	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	42	109	0	268	0	0	-1
normalized size	1	1.00	0.33	0.86	0.00	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.026	0.053	0.000	0.538	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	105	165	0	442	0	0	-1
normalized size	1	1.00	0.68	1.06	0.00	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.073	0.059	0.000	0.535	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	120	276	0	513	0	0	-1
normalized size	1	1.00	0.62	1.42	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.106	0.061	0.000	0.835	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	139	390	0	600	0	0	-1
normalized size	1	1.00	0.56	1.57	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.142	0.066	0.000	0.604	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	147	504	0	668	0	0	-1
normalized size	1	1.00	0.50	1.72	0.00	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.168	0.071	0.000	0.530	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	108	172	0	386	0	0	-1
normalized size	1	1.00	0.48	0.76	0.00	1.72	0.00	0.00	-0.00
time (sec)	N/A	0.188	2.832	0.063	0.000	0.457	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	95	154	0	364	0	0	-1
normalized size	1	1.00	0.53	0.86	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.096	0.064	0.000	0.699	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	87	136	0	330	0	0	-1
normalized size	1	1.00	0.64	0.99	0.00	2.41	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.065	0.060	0.000	0.670	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	71	109	0	268	0	0	-1
normalized size	1	1.00	0.56	0.87	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.052	0.059	0.000	0.593	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	67	91	0	250	0	0	-1
normalized size	1	1.00	0.74	1.01	0.00	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.052	0.053	0.000	0.449	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	71	92	0	254	0	0	-1
normalized size	1	1.00	0.81	1.05	0.00	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.040	0.046	0.000	0.498	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	105	168	0	453	0	0	-1
normalized size	1	1.00	0.66	1.06	0.00	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.066	0.059	0.000	0.560	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	127	276	0	528	0	0	-1
normalized size	1	1.00	0.61	1.33	0.00	2.54	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.183	0.061	0.000	0.571	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	139	390	0	600	0	0	-1
normalized size	1	1.00	0.55	1.55	0.00	2.38	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.229	0.070	0.000	0.599	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	133	305	0	342	0	0	-1
normalized size	1	1.00	0.70	1.61	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.265	0.352	0.052	0.000	0.591	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	125	281	0	320	0	0	-1
normalized size	1	1.00	0.76	1.71	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.184	0.048	0.000	0.500	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	116	257	0	282	0	0	-1
normalized size	1	1.00	0.83	1.85	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.107	0.046	0.000	0.482	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	96	229	0	231	0	0	-1
normalized size	1	1.00	0.85	2.03	0.00	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.083	0.045	0.000	0.476	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	190	0	217	0	0	-1
normalized size	1	1.00	1.00	2.04	0.00	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.050	0.046	0.000	0.450	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	136	0	232	0	130	-1
normalized size	1	1.00	1.00	1.42	0.00	2.42	0.00	1.35	-0.01
time (sec)	N/A	0.161	0.050	0.036	0.000	0.438	0.000	0.219	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	134	0	231	0	129	-1
normalized size	1	1.00	1.00	1.41	0.00	2.43	0.00	1.36	-0.01
time (sec)	N/A	0.163	0.054	0.042	0.000	0.425	0.000	0.179	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	67	370	0	284	0	166	-1
normalized size	1	1.00	0.56	3.11	0.00	2.39	0.00	1.39	-0.01
time (sec)	N/A	0.190	0.054	0.048	0.000	0.437	0.000	0.197	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	77	497	0	356	0	187	-1
normalized size	1	1.00	0.52	3.36	0.00	2.41	0.00	1.26	-0.01
time (sec)	N/A	0.228	0.058	0.048	0.000	0.437	0.000	0.192	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	82	626	0	428	0	207	-1
normalized size	1	1.00	0.47	3.62	0.00	2.47	0.00	1.20	-0.01
time (sec)	N/A	0.257	0.062	0.053	0.000	0.415	0.000	0.193	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	234	227	0	408	0	0	-1
normalized size	1	1.00	0.88	0.85	0.00	1.53	0.00	0.00	-0.00
time (sec)	N/A	0.243	10.483	0.066	0.000	0.465	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	200	209	0	386	0	0	-1
normalized size	1	1.00	0.89	0.93	0.00	1.72	0.00	0.00	-0.00
time (sec)	N/A	0.207	3.756	0.065	0.000	0.553	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	97	191	0	352	0	0	-1
normalized size	1	1.00	0.54	1.06	0.00	1.94	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.079	0.066	0.000	0.547	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	80	164	0	298	0	0	-1
normalized size	1	1.00	0.60	1.23	0.00	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.055	0.062	0.000	0.556	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	80	140	0	282	0	0	-1
normalized size	1	1.00	0.65	1.14	0.00	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.053	0.056	0.000	0.505	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	86	143	0	286	0	0	-1
normalized size	1	1.00	0.68	1.13	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.065	0.056	0.000	0.535	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	44	143	0	294	0	0	-1
normalized size	1	1.00	0.34	1.09	0.00	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.042	0.053	0.000	0.448	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	162	279	0	492	0	0	-1
normalized size	1	1.00	0.81	1.40	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.164	0.065	0.000	0.680	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	234	315	0	596	0	0	-1
normalized size	1	1.00	0.93	1.25	0.00	2.37	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.649	0.068	0.000	0.593	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	92	165	0	120	0	217	138
normalized size	1	1.00	0.88	1.57	0.00	1.14	0.00	2.07	1.31
time (sec)	N/A	0.322	0.042	0.043	0.000	0.432	0.000	0.218	0.870

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.230	0.025	0.248	0.000	0.432	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	88	125	0	292	0	0	-1
normalized size	1	1.00	0.49	0.70	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.058	0.044	0.000	0.638	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	80	105	0	272	0	0	-1
normalized size	1	1.00	0.59	0.78	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.051	0.047	0.000	0.514	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	69	91	0	249	0	0	-1
normalized size	1	1.00	0.81	1.07	0.00	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.036	0.042	0.000	0.480	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	61	90	0	236	0	0	-1
normalized size	1	1.00	0.71	1.05	0.00	2.74	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.039	0.044	0.000	0.933	0.000	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	0	47	0	0	44
normalized size	1	1.00	1.00	0.98	0.00	1.15	0.00	0.00	1.07
time (sec)	N/A	0.227	0.022	0.032	0.000	0.497	0.000	0.000	1.122

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	47	46	0	58	0	0	52
normalized size	1	1.00	0.56	0.55	0.00	0.69	0.00	0.00	0.62
time (sec)	N/A	0.216	0.025	0.033	0.000	0.755	0.000	0.000	1.054

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	55	54	0	66	0	0	60
normalized size	1	1.00	0.43	0.42	0.00	0.52	0.00	0.00	0.47
time (sec)	N/A	0.231	0.029	0.029	0.000	1.122	0.000	0.000	1.137

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	63	62	0	74	0	0	68
normalized size	1	1.00	0.37	0.36	0.00	0.43	0.00	0.00	0.40
time (sec)	N/A	0.233	0.033	0.032	0.000	0.401	0.000	0.000	1.127

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	50	172	0	179	0	142	111
normalized size	1	1.00	0.38	1.32	0.00	1.38	0.00	1.09	0.85
time (sec)	N/A	0.245	0.034	0.042	0.000	0.449	0.000	0.209	1.349

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	50	155	0	163	0	127	90
normalized size	1	1.00	0.48	1.48	0.00	1.55	0.00	1.21	0.86
time (sec)	N/A	0.217	0.032	0.040	0.000	0.460	0.000	0.253	1.326

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	139	0	147	0	112	-1
normalized size	1	1.00	0.96	1.74	0.00	1.84	0.00	1.40	-0.01
time (sec)	N/A	0.143	0.075	0.036	0.000	0.426	0.000	0.225	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	120	0	125	0	97	-1
normalized size	1	1.00	1.00	2.35	0.00	2.45	0.00	1.90	-0.02
time (sec)	N/A	0.096	0.039	0.041	0.000	0.534	0.000	0.227	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	98	0	111	39	0	-1
normalized size	1	1.00	1.00	2.09	0.00	2.36	0.83	0.00	-0.02
time (sec)	N/A	0.191	0.033	0.044	0.000	0.514	9.647	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	28	27	0	28	0	0	24
normalized size	1	1.00	0.67	0.64	0.00	0.67	0.00	0.00	0.57
time (sec)	N/A	0.216	0.033	0.030	0.000	0.414	0.000	0.000	0.897

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	36	35	0	36	0	0	32
normalized size	1	1.00	0.52	0.51	0.00	0.52	0.00	0.00	0.46
time (sec)	N/A	0.222	0.042	0.031	0.000	0.436	0.000	0.000	0.908

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	44	43	0	44	0	0	77
normalized size	1	1.00	0.46	0.45	0.00	0.46	0.00	0.00	0.80
time (sec)	N/A	0.224	0.050	0.032	0.000	0.504	0.000	0.000	0.910

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	52	51	0	52	0	0	98
normalized size	1	1.00	0.43	0.42	0.00	0.43	0.00	0.00	0.81
time (sec)	N/A	0.238	0.058	0.032	0.000	0.617	0.000	0.000	0.900

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	130	247	0	500	0	0	-1
normalized size	1	1.00	0.45	0.85	0.00	1.71	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.101	0.059	0.000	0.546	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	122	219	0	484	0	0	-1
normalized size	1	1.00	0.49	0.88	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.292	0.086	0.057	0.000	0.652	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	114	191	0	468	0	0	-1
normalized size	1	1.00	0.56	0.94	0.00	2.29	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.073	0.050	0.000	0.607	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	105	165	0	442	0	0	-1
normalized size	1	1.00	0.68	1.06	0.00	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.068	0.056	0.000	0.555	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	103	166	0	429	0	0	-1
normalized size	1	1.00	0.67	1.08	0.00	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.051	0.059	0.000	0.914	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	93	150	0	309	0	0	-1
normalized size	1	1.00	0.63	1.02	0.00	2.10	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.053	0.061	0.000	0.554	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	101	181	0	337	0	0	-1
normalized size	1	1.00	0.53	0.95	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.062	0.064	0.000	0.529	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	109	209	0	353	0	0	-1
normalized size	1	1.00	0.46	0.88	0.00	1.49	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.068	0.064	0.000	0.585	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	117	237	0	369	0	0	-1
normalized size	1	1.00	0.42	0.84	0.00	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.307	0.079	0.064	0.000	0.476	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	87	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.293	0.060	0.328	0.000	0.498	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	88	125	0	292	0	0	-1
normalized size	1	1.00	0.48	0.69	0.00	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.308	0.089	0.052	0.000	0.491	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	80	107	0	276	0	0	-1
normalized size	1	1.00	0.58	0.78	0.00	2.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.050	0.048	0.000	0.539	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	67	91	0	250	0	0	-1
normalized size	1	1.00	0.74	1.01	0.00	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.055	0.053	0.000	0.555	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	60	91	0	235	0	0	-1
normalized size	1	1.00	0.67	1.02	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.051	0.053	0.000	0.627	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	47	46	0	48	0	0	60
normalized size	1	1.00	0.56	0.55	0.00	0.57	0.00	0.00	0.71
time (sec)	N/A	0.260	0.032	0.031	0.000	0.457	0.000	0.000	1.005

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	55	54	0	58	0	0	79
normalized size	1	1.00	0.43	0.42	0.00	0.45	0.00	0.00	0.62
time (sec)	N/A	0.267	0.032	0.029	0.000	0.428	0.000	0.000	1.050

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	63	62	0	66	0	0	86
normalized size	1	1.00	0.37	0.36	0.00	0.38	0.00	0.00	0.50
time (sec)	N/A	0.263	0.036	0.030	0.000	0.544	0.000	0.000	1.094

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	116	259	0	274	0	0	-1
normalized size	1	1.00	0.67	1.51	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.352	0.147	0.041	0.000	0.635	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	108	237	0	256	0	0	-1
normalized size	1	1.00	0.73	1.61	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.314	0.102	0.039	0.000	0.581	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	100	216	0	242	0	0	-1
normalized size	1	1.00	0.82	1.77	0.00	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.084	0.040	0.000	0.693	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	190	0	217	0	0	-1
normalized size	1	1.00	1.00	2.04	0.00	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.048	0.043	0.000	0.469	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	227	0	206	80	0	-1
normalized size	1	1.00	1.00	2.64	0.00	2.40	0.93	0.00	-0.01
time (sec)	N/A	0.237	0.037	0.048	0.000	0.574	12.331	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	254	0	157	0	0	67
normalized size	1	1.00	0.84	3.10	0.00	1.91	0.00	0.00	0.82
time (sec)	N/A	0.236	0.066	0.044	0.000	0.625	0.000	0.000	1.417

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	79	278	0	185	0	278	96
normalized size	1	1.00	0.70	2.46	0.00	1.64	0.00	2.46	0.85
time (sec)	N/A	0.253	0.084	0.047	0.000	0.570	0.000	1.181	1.740

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	87	302	0	197	0	356	96
normalized size	1	1.00	0.77	2.67	0.00	1.74	0.00	3.15	0.85
time (sec)	N/A	0.281	0.130	0.050	0.000	0.639	0.000	1.231	2.105

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	95	326	0	217	0	434	138
normalized size	1	1.00	0.58	2.00	0.00	1.33	0.00	2.66	0.85
time (sec)	N/A	0.303	0.138	0.051	0.000	0.513	0.000	2.350	2.620

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	108	194	0	336	0	0	-1
normalized size	1	1.00	0.41	0.74	0.00	1.28	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.065	0.062	0.000	0.691	0.000	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	100	176	0	320	0	0	-1
normalized size	1	1.00	0.46	0.81	0.00	1.47	0.00	0.00	-0.00
time (sec)	N/A	0.250	0.065	0.061	0.000	0.665	0.000	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	92	158	0	304	0	0	-1
normalized size	1	1.00	0.53	0.91	0.00	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.057	0.058	0.000	0.629	0.000	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	80	140	0	282	0	0	-1
normalized size	1	1.00	0.65	1.14	0.00	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.049	0.058	0.000	0.704	0.000	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	70	142	0	264	0	0	-1
normalized size	1	1.00	0.56	1.15	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.237	0.053	0.059	0.000	0.579	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	50	61	0	58	0	0	80
normalized size	1	1.00	0.41	0.50	0.00	0.47	0.00	0.00	0.65
time (sec)	N/A	0.223	0.032	0.030	0.000	0.613	0.000	0.000	1.124

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	58	69	0	68	0	0	99
normalized size	1	1.00	0.35	0.42	0.00	0.41	0.00	0.00	0.60
time (sec)	N/A	0.234	0.032	0.033	0.000	0.549	0.000	0.000	1.203

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	66	77	0	76	0	0	120
normalized size	1	1.00	0.31	0.36	0.00	0.36	0.00	0.00	0.56
time (sec)	N/A	0.244	0.034	0.031	0.000	0.650	0.000	0.000	1.208

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	74	85	0	84	0	0	139
normalized size	1	1.00	0.29	0.33	0.00	0.33	0.00	0.00	0.54
time (sec)	N/A	0.249	0.037	0.031	0.000	0.542	0.000	0.000	1.211

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.099	0.639	0.194	0.000	0.588	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.107	0.441	0.125	0.000	0.486	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.097	0.018	0.118	0.000	0.700	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	B	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	71	262	0	0	0	0	0	-1
normalized size	1	0.55	2.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.497	0.049	0.000	0.660	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	184	180	0	0	0	0	0	-1
normalized size	1	0.98	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.263	0.044	0.000	0.595	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.043	0.044	0.000	0.663	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	194	0	0	0	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.472	0.046	0.000	0.462	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.169	180.007	0.043	0.000	0.523	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.150	180.007	0.043	0.000	0.541	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.154	180.007	0.046	0.000	0.618	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.170	180.007	0.045	0.000	0.558	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	70	233	515	175	1119	505	228
normalized size	1	1.00	0.41	1.38	3.05	1.04	6.62	2.99	1.35
time (sec)	N/A	0.219	0.032	0.065	0.420	0.638	26.210	0.211	0.885

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	70	187	332	153	687	385	182
normalized size	1	1.00	0.51	1.38	2.44	1.12	5.05	2.83	1.34
time (sec)	N/A	0.187	0.029	0.053	0.416	0.625	16.349	0.273	0.054

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	141	189	131	354	263	136
normalized size	1	1.00	0.68	1.37	1.83	1.27	3.44	2.55	1.32
time (sec)	N/A	0.158	0.026	0.045	0.410	0.397	9.860	0.575	0.040

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	85	79	84	144	128	76
normalized size	1	1.00	0.95	1.47	1.36	1.45	2.48	2.21	1.31
time (sec)	N/A	0.101	0.043	0.039	0.403	0.421	5.603	0.256	0.831

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	94	0	68	0	72	90
normalized size	1	1.00	0.87	1.54	0.00	1.11	0.00	1.18	1.48
time (sec)	N/A	0.129	0.028	0.038	0.000	0.772	0.000	0.202	0.837

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	78	177	0	142	0	0	188
normalized size	1	1.00	0.81	1.84	0.00	1.48	0.00	0.00	1.96
time (sec)	N/A	0.150	0.051	0.047	0.000	0.423	0.000	0.000	0.070

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	108	259	0	211	0	0	367
normalized size	1	1.00	0.84	2.01	0.00	1.64	0.00	0.00	2.84
time (sec)	N/A	0.197	0.074	0.052	0.000	0.533	0.000	0.000	0.882

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	126	341	0	282	0	0	613
normalized size	1	1.00	0.78	2.10	0.00	1.74	0.00	0.00	3.78
time (sec)	N/A	0.223	0.099	0.056	0.000	0.634	0.000	0.000	0.974

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	128	117	115	122	126	116	90
normalized size	1	1.00	1.00	0.91	0.90	0.95	0.98	0.91	0.70
time (sec)	N/A	0.150	0.039	0.036	0.307	0.410	0.816	0.189	0.886

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	84	82	89	90	83	66
normalized size	1	1.00	1.00	0.92	0.90	0.98	0.99	0.91	0.73
time (sec)	N/A	0.132	0.026	0.033	0.314	0.613	0.488	0.202	0.066

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	71	78	78	72	57
normalized size	1	1.00	1.00	0.94	0.91	1.00	1.00	0.92	0.73
time (sec)	N/A	0.124	0.022	0.033	0.309	0.731	0.352	0.193	0.878

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	36	43	41	37	35
normalized size	1	1.00	1.00	0.98	0.88	1.05	1.00	0.90	0.85
time (sec)	N/A	0.112	0.015	0.031	0.312	0.680	0.202	0.190	0.050

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	27	20	22	24
normalized size	1	1.00	1.00	1.05	1.00	1.29	0.95	1.05	1.14
time (sec)	N/A	0.068	0.012	0.033	0.307	0.466	0.121	0.189	0.837

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	36	34	41	37	36	35
normalized size	1	1.00	1.00	0.95	0.89	1.08	0.97	0.95	0.92
time (sec)	N/A	0.124	0.021	0.033	0.304	0.485	0.166	0.183	0.050

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	69	66	70	93	75	58	68
normalized size	1	1.00	0.91	0.87	0.92	1.22	0.99	0.76	0.89
time (sec)	N/A	0.146	0.044	0.035	0.306	0.487	0.411	0.178	0.099

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	103	96	98	137	104	81	94
normalized size	1	1.00	0.93	0.86	0.88	1.23	0.94	0.73	0.85
time (sec)	N/A	0.169	0.059	0.037	0.307	0.574	0.624	0.174	0.116

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	123	126	146	233	158	97	144
normalized size	1	1.00	0.84	0.86	1.00	1.60	1.08	0.66	0.99
time (sec)	N/A	0.199	0.089	0.038	0.308	0.568	0.887	0.396	0.151

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	191	273	745	175	935	505	228
normalized size	1	1.00	1.00	1.43	3.90	0.92	4.90	2.64	1.19
time (sec)	N/A	0.366	0.211	0.088	0.421	1.312	30.596	0.243	0.892

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	186	227	443	153	687	385	182
normalized size	1	1.00	1.18	1.45	2.82	0.97	4.38	2.45	1.16
time (sec)	N/A	0.315	0.104	0.060	0.421	0.657	19.487	0.509	0.066

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	128	181	347	131	357	262	136
normalized size	1	1.00	1.03	1.46	2.80	1.06	2.88	2.11	1.10
time (sec)	N/A	0.271	0.067	0.052	0.416	0.583	16.834	0.309	0.824

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	121	200	82	150	129	81
normalized size	1	1.00	0.77	1.66	2.74	1.12	2.05	1.77	1.11
time (sec)	N/A	0.209	0.047	0.046	0.425	0.600	12.327	0.251	0.038

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	78	168	0	101	0	0	129
normalized size	1	1.00	0.82	1.77	0.00	1.06	0.00	0.00	1.36
time (sec)	N/A	0.226	0.080	0.042	0.000	0.561	0.000	0.000	0.071

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	88	212	0	143	0	180	272
normalized size	1	1.00	0.70	1.70	0.00	1.14	0.00	1.44	2.18
time (sec)	N/A	0.335	0.103	0.048	0.000	0.609	0.000	0.240	0.845

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	96	256	0	212	0	0	1548
normalized size	1	1.00	0.62	1.65	0.00	1.37	0.00	0.00	9.99
time (sec)	N/A	0.440	0.118	0.054	0.000	0.529	0.000	0.000	2.609

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	124	367	0	281	0	0	2165
normalized size	1	1.00	0.67	1.98	0.00	1.52	0.00	0.00	11.70
time (sec)	N/A	0.564	0.147	0.062	0.000	0.723	0.000	0.000	4.435

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	105	103	111	112	104	81
normalized size	1	1.00	1.00	0.91	0.89	0.96	0.97	0.90	0.70
time (sec)	N/A	0.143	0.039	0.036	0.312	0.562	0.663	0.196	0.086

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	93	92	100	100	93	72
normalized size	1	1.00	1.00	0.93	0.92	1.00	1.00	0.93	0.72
time (sec)	N/A	0.137	0.028	0.033	0.303	0.478	0.513	0.182	0.864

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	60	59	67	65	60	48
normalized size	1	1.00	1.00	0.95	0.94	1.06	1.03	0.95	0.76
time (sec)	N/A	0.123	0.021	0.035	0.308	0.637	0.320	0.140	0.056

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	50	46	56	53	47	43
normalized size	1	1.00	1.00	0.98	0.90	1.10	1.04	0.92	0.84
time (sec)	N/A	0.119	0.018	0.032	0.302	0.675	0.221	0.204	0.851

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	33	32	35	26	34	32
normalized size	1	1.00	1.00	1.00	0.97	1.06	0.79	1.03	0.97
time (sec)	N/A	0.077	0.019	0.033	0.304	0.599	0.286	0.153	0.074

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	51	49	64	41	42	48
normalized size	1	1.00	1.00	0.96	0.92	1.21	0.77	0.79	0.91
time (sec)	N/A	0.134	0.032	0.033	0.308	1.005	0.245	0.167	0.066

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	66	75	100	83	50	71
normalized size	1	1.00	0.89	0.93	1.06	1.41	1.17	0.70	1.00
time (sec)	N/A	0.136	0.037	0.033	0.311	0.546	0.333	0.166	0.074

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	95	107	163	114	73	104
normalized size	1	1.00	0.80	0.86	0.96	1.47	1.03	0.66	0.94
time (sec)	N/A	0.166	0.063	0.034	0.313	0.497	0.633	0.156	0.913

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	98	125	135	207	144	96	131
normalized size	1	1.00	0.67	0.86	0.92	1.42	0.99	0.66	0.90
time (sec)	N/A	0.192	0.099	0.048	0.318	0.656	0.900	0.183	0.943

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	70	249	0	176	1110	504	227
normalized size	1	1.00	0.41	1.47	0.00	1.04	6.57	2.98	1.34
time (sec)	N/A	0.232	0.029	0.084	0.000	0.839	16.184	0.172	0.900

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	70	203	0	154	692	384	181
normalized size	1	1.00	0.51	1.49	0.00	1.13	5.09	2.82	1.33
time (sec)	N/A	0.223	0.026	0.054	0.000	1.152	10.021	0.219	0.867

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	157	0	132	381	262	135
normalized size	1	1.00	0.68	1.52	0.00	1.28	3.70	2.54	1.31
time (sec)	N/A	0.170	0.027	0.043	0.000	0.553	6.342	0.239	0.049

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	100	0	85	177	128	76
normalized size	1	1.00	0.95	1.72	0.00	1.47	3.05	2.21	1.31
time (sec)	N/A	0.112	0.040	0.040	0.000	0.456	4.908	0.427	0.037

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	192	0	66	0	71	88
normalized size	1	1.00	0.90	3.20	0.00	1.10	0.00	1.18	1.47
time (sec)	N/A	0.141	0.029	0.044	0.000	0.420	0.000	0.203	0.060

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	274	0	141	0	0	188
normalized size	1	1.00	0.80	2.82	0.00	1.45	0.00	0.00	1.94
time (sec)	N/A	0.163	0.053	0.051	0.000	0.566	0.000	0.000	1.155

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	108	356	0	208	0	0	365
normalized size	1	1.00	0.83	2.74	0.00	1.60	0.00	0.00	2.81
time (sec)	N/A	0.199	0.078	0.061	0.000	0.503	0.000	0.000	1.283

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	126	438	0	281	0	0	614
normalized size	1	1.00	0.77	2.69	0.00	1.72	0.00	0.00	3.77
time (sec)	N/A	0.241	0.098	0.065	0.000	1.091	0.000	0.000	1.539

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	84	82	89	88	160	66
normalized size	1	1.00	1.00	0.92	0.90	0.98	0.97	1.76	0.73
time (sec)	N/A	0.139	0.027	0.035	0.306	0.474	0.461	0.177	0.076

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	71	78	76	136	57
normalized size	1	1.00	1.00	0.94	0.91	1.00	0.97	1.74	0.73
time (sec)	N/A	0.128	0.021	0.035	0.491	0.930	0.332	0.287	0.853

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	38	43	39	112	35
normalized size	1	1.00	1.00	0.98	0.95	1.08	0.98	2.80	0.88
time (sec)	N/A	0.116	0.016	0.033	0.306	0.521	0.190	0.216	0.890

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	27	20	88	24
normalized size	1	1.00	1.00	1.05	1.00	1.29	0.95	4.19	1.14
time (sec)	N/A	0.070	0.011	0.032	0.302	0.424	0.113	0.166	0.046

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	36	33	39	37	55	33
normalized size	1	1.00	0.80	1.03	0.94	1.11	1.06	1.57	0.94
time (sec)	N/A	0.132	0.028	0.032	0.304	0.703	0.153	0.183	0.054

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	66	70	92	76	101	68
normalized size	1	1.00	0.92	0.89	0.95	1.24	1.03	1.36	0.92
time (sec)	N/A	0.145	0.044	0.038	0.306	0.585	0.391	0.160	0.887

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	96	98	137	104	140	94
normalized size	1	1.00	0.95	0.88	0.90	1.26	0.95	1.28	0.86
time (sec)	N/A	0.166	0.064	0.037	0.310	0.637	0.626	0.172	0.114

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	121	126	146	233	158	164	142
normalized size	1	1.00	0.84	0.88	1.01	1.62	1.10	1.14	0.99
time (sec)	N/A	0.201	0.094	0.040	0.312	0.492	0.885	0.175	0.950

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	191	289	0	176	1110	506	229
normalized size	1	1.00	1.00	1.51	0.00	0.92	5.81	2.65	1.20
time (sec)	N/A	0.362	0.182	0.099	0.000	0.519	18.713	0.301	0.101

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	186	243	0	154	695	386	183
normalized size	1	1.00	1.18	1.55	0.00	0.98	4.43	2.46	1.17
time (sec)	N/A	0.321	0.104	0.059	0.000	0.615	11.652	0.206	0.849

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	128	299	0	132	384	263	137
normalized size	1	1.00	1.02	2.39	0.00	1.06	3.07	2.10	1.10
time (sec)	N/A	0.276	0.084	0.052	0.000	0.507	7.630	0.183	0.862

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	266	0	86	0	130	82
normalized size	1	1.00	0.77	3.59	0.00	1.16	0.00	1.76	1.11
time (sec)	N/A	0.208	0.064	0.048	0.000	0.714	0.000	0.235	0.041

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	330	0	101	0	0	129
normalized size	1	1.00	0.80	3.40	0.00	1.04	0.00	0.00	1.33
time (sec)	N/A	0.234	0.083	0.054	0.000	0.632	0.000	0.000	0.853

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	86	412	0	142	0	181	271
normalized size	1	1.00	0.67	3.19	0.00	1.10	0.00	1.40	2.10
time (sec)	N/A	0.363	0.106	0.064	0.000	0.875	0.000	0.238	0.058

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	94	494	0	213	0	0	436
normalized size	1	1.00	0.59	3.11	0.00	1.34	0.00	0.00	2.74
time (sec)	N/A	0.451	0.129	0.069	0.000	0.566	0.000	0.000	1.443

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	122	576	0	278	0	0	671
normalized size	1	1.00	0.65	3.05	0.00	1.47	0.00	0.00	3.55
time (sec)	N/A	0.636	0.157	0.077	0.000	0.488	0.000	0.000	1.794

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	114	118	0	606	0	0	-1
normalized size	1	1.00	0.30	0.32	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.104	0.086	0.000	0.700	0.000	0.000	0.000

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	98	102	0	542	0	0	-1
normalized size	1	1.00	0.33	0.34	0.00	1.81	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.060	0.051	0.000	0.787	0.000	0.000	0.000

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	82	86	0	478	0	0	-1
normalized size	1	1.00	0.37	0.39	0.00	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.047	0.052	0.000	1.711	0.000	0.000	0.000

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	72	70	0	373	0	0	-1
normalized size	1	1.00	0.49	0.48	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.036	0.048	0.000	1.533	0.000	0.000	0.000

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	37	52	17	320	0	0	-1
normalized size	1	1.00	0.54	0.76	0.25	4.71	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.022	0.039	0.390	0.689	0.000	0.000	0.000

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	48	50	21	366	0	0	-1
normalized size	1	1.00	0.61	0.63	0.27	4.63	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.030	0.040	0.433	1.413	0.000	0.000	0.000

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	91	93	0	0	0	0	-1
normalized size	1	1.00	0.52	0.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.054	0.052	0.000	0.707	0.000	0.000	0.000

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	87	167	0	0	0	0	-1
normalized size	1	1.00	0.33	0.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.135	0.056	0.000	0.651	0.000	0.000	0.000

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	147	239	0	0	0	0	-1
normalized size	1	1.00	0.41	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.122	0.059	0.000	0.785	0.000	0.000	0.000

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	166	965	0	482	0	707	-1
normalized size	1	1.00	0.37	2.14	0.00	1.07	0.00	1.57	-0.00
time (sec)	N/A	0.557	0.196	0.125	0.000	0.770	0.000	137.057	0.000

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	150	795	0	438	0	0	-1
normalized size	1	1.00	0.40	2.14	0.00	1.18	0.00	0.00	-0.00
time (sec)	N/A	0.497	0.161	0.070	0.000	0.635	0.000	0.000	0.000

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	134	625	0	394	0	416	-1
normalized size	1	1.00	0.46	2.13	0.00	1.34	0.00	1.41	-0.00
time (sec)	N/A	0.454	0.132	0.053	0.000	0.496	0.000	6.541	0.000

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	115	455	0	316	0	265	-1
normalized size	1	1.00	0.54	2.13	0.00	1.48	0.00	1.24	-0.00
time (sec)	N/A	0.395	0.104	0.049	0.000	1.394	0.000	0.450	0.000

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	81	198	0	270	0	0	-1
normalized size	1	1.00	0.69	1.68	0.00	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.305	0.085	0.046	0.000	0.561	0.000	0.000	0.000

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	69	178	0	216	0	0	-1
normalized size	1	1.00	0.63	1.62	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.068	0.042	0.000	1.550	0.000	0.000	0.000

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	326	0	281	0	0	-1
normalized size	1	1.00	0.77	2.65	0.00	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.373	0.084	0.047	0.000	0.982	0.000	0.000	0.000

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	105	462	0	353	0	0	-1
normalized size	1	1.00	0.52	2.28	0.00	1.74	0.00	0.00	-0.00
time (sec)	N/A	0.405	0.099	0.050	0.000	0.627	0.000	0.000	0.000

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	133	572	0	497	0	0	-1
normalized size	1	1.00	0.47	2.02	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.427	0.118	0.064	0.000	0.859	0.000	0.000	0.000

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	151	682	0	569	0	0	-1
normalized size	1	1.00	0.42	1.88	0.00	1.57	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.150	0.108	0.000	0.988	0.000	0.000	0.000

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	98	102	0	542	0	0	-1
normalized size	1	1.00	0.33	0.34	0.00	1.81	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.067	0.051	0.000	0.793	0.000	0.000	0.000

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	98	102	0	542	0	0	-1
normalized size	1	1.00	0.33	0.34	0.00	1.80	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.062	0.053	0.000	0.773	0.000	0.000	0.000

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	90	86	0	474	0	0	-1
normalized size	1	1.00	0.41	0.39	0.00	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.048	0.050	0.000	1.107	0.000	0.000	0.000

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	64	70	0	378	0	0	-1
normalized size	1	1.00	0.44	0.48	0.00	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.037	0.049	0.000	0.596	0.000	0.000	0.000

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	47	61	148	0	0	0	-1
normalized size	1	1.00	0.44	0.56	1.37	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.030	0.049	0.449	3.933	0.000	0.000	0.000

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	59	77	0	440	0	0	-1
normalized size	1	1.00	0.48	0.63	0.00	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.046	0.049	0.000	2.048	0.000	0.000	0.000

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	87	106	0	477	0	0	-1
normalized size	1	1.00	0.50	0.61	0.00	2.74	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.056	0.051	0.000	2.468	0.000	0.000	0.000

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	113	176	0	0	0	0	-1
normalized size	1	1.00	0.42	0.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.082	0.055	0.000	2.625	0.000	0.000	0.000

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	146	248	0	0	0	0	-1
normalized size	1	1.00	0.40	0.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.119	0.057	0.000	2.794	0.000	0.000	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	115	118	0	606	0	0	-1
normalized size	1	1.00	0.31	0.31	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.108	0.053	0.000	2.279	0.000	0.000	0.000

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	98	102	0	542	0	0	-1
normalized size	1	1.00	0.33	0.34	0.00	1.81	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.057	0.049	0.000	2.263	0.000	0.000	0.000

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	82	86	0	478	0	0	-1
normalized size	1	1.00	0.37	0.39	0.00	2.17	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.049	0.049	0.000	0.747	0.000	0.000	0.000

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	72	70	0	377	0	0	-1
normalized size	1	1.00	0.50	0.49	0.00	2.62	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.036	0.048	0.000	0.772	0.000	0.000	0.000

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	38	53	0	321	0	0	-1
normalized size	1	1.00	0.55	0.77	0.00	4.65	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.024	0.041	0.000	1.239	0.000	0.000	0.000

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	50	51	21	367	0	0	-1
normalized size	1	1.00	0.65	0.66	0.27	4.77	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.030	0.042	0.376	0.656	0.000	0.000	0.000

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	91	92	0	0	0	0	-1
normalized size	1	1.00	0.52	0.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.056	0.053	0.000	1.701	0.000	0.000	0.000

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	88	167	0	0	0	0	-1
normalized size	1	1.00	0.33	0.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.124	0.055	0.000	1.231	0.000	0.000	0.000

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	145	239	0	0	0	0	-1
normalized size	1	1.00	0.40	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.117	0.056	0.000	0.630	0.000	0.000	0.000

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	455	455	166	965	0	482	1408	707	-1
normalized size	1	1.00	0.36	2.12	0.00	1.06	3.09	1.55	-0.00
time (sec)	N/A	0.537	0.195	0.120	0.000	0.603	50.944	101.601	0.000

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	150	795	0	438	1059	0	-1
normalized size	1	1.00	0.40	2.12	0.00	1.17	2.82	0.00	-0.00
time (sec)	N/A	0.467	0.161	0.066	0.000	1.216	24.007	0.000	0.000

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	134	625	0	394	500	416	-1
normalized size	1	1.00	0.46	2.13	0.00	1.34	1.71	1.42	-0.00
time (sec)	N/A	0.427	0.141	0.054	0.000	1.654	13.612	13.718	0.000

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	115	454	0	317	376	265	-1
normalized size	1	1.00	0.54	2.14	0.00	1.50	1.77	1.25	-0.00
time (sec)	N/A	0.384	0.109	0.049	0.000	0.640	9.249	4.417	0.000

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	198	0	270	0	0	-1
normalized size	1	1.00	0.68	1.68	0.00	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.296	0.091	0.045	0.000	0.693	0.000	0.000	0.000

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	69	177	0	214	0	97	-1
normalized size	1	1.00	0.62	1.59	0.00	1.93	0.00	0.87	-0.01
time (sec)	N/A	0.241	0.069	0.041	0.000	0.597	0.000	0.207	0.000

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	95	326	0	280	0	127	-1
normalized size	1	1.00	0.77	2.63	0.00	2.26	0.00	1.02	-0.01
time (sec)	N/A	0.366	0.084	0.048	0.000	0.700	0.000	0.220	0.000

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	105	462	0	352	0	208	-1
normalized size	1	1.00	0.54	2.38	0.00	1.81	0.00	1.07	-0.01
time (sec)	N/A	0.398	0.097	0.051	0.000	0.570	0.000	0.254	0.000

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	131	572	0	496	0	251	-1
normalized size	1	1.00	0.49	2.12	0.00	1.84	0.00	0.93	-0.00
time (sec)	N/A	0.427	0.119	0.061	0.000	0.779	0.000	3.818	0.000

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	97	102	0	544	0	0	-1
normalized size	1	1.00	0.32	0.34	0.00	1.82	0.00	0.00	-0.00
time (sec)	N/A	0.201	0.095	0.051	0.000	0.787	0.000	0.000	0.000

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	98	102	0	544	0	0	-1
normalized size	1	1.00	0.33	0.34	0.00	1.81	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.060	0.047	0.000	0.714	0.000	0.000	0.000

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	90	86	0	480	0	0	-1
normalized size	1	1.00	0.41	0.39	0.00	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.049	0.049	0.000	0.611	0.000	0.000	0.000

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	64	70	0	376	0	0	-1
normalized size	1	1.00	0.44	0.48	0.00	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.039	0.047	0.000	0.772	0.000	0.000	0.000

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	45	60	0	0	0	0	-1
normalized size	1	1.00	0.42	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.028	0.050	0.000	0.600	0.000	0.000	0.000

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	58	78	53	439	0	0	-1
normalized size	1	1.00	0.48	0.64	0.43	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.047	0.049	0.344	1.734	0.000	0.000	0.000

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	86	106	0	477	0	0	-1
normalized size	1	1.00	0.51	0.62	0.00	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.056	0.052	0.000	0.660	0.000	0.000	0.000

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	110	176	0	0	0	0	-1
normalized size	1	1.00	0.41	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	0.093	0.055	0.000	0.859	0.000	0.000	0.000

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	146	248	0	0	0	0	-1
normalized size	1	1.00	0.41	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.124	0.061	0.000	1.726	0.000	0.000	0.000

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	51	53	30	78	0	0	45
normalized size	1	1.00	0.64	0.66	0.38	0.98	0.00	0.00	0.56
time (sec)	N/A	0.220	0.033	0.028	0.385	1.041	0.000	0.000	1.300

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	43	20	58	0	0	38
normalized size	1	1.00	0.57	0.58	0.27	0.78	0.00	0.00	0.51
time (sec)	N/A	0.212	0.021	0.027	0.392	0.605	0.000	0.000	0.997

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	41	42	18	57	0	0	36
normalized size	1	1.00	0.58	0.59	0.25	0.80	0.00	0.00	0.51
time (sec)	N/A	0.128	0.020	0.029	0.388	0.635	0.000	0.000	0.972

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	37	52	17	320	0	0	-1
normalized size	1	1.00	0.54	0.76	0.25	4.71	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.018	0.039	0.386	0.696	0.000	0.000	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	37	52	19	294	0	0	-1
normalized size	1	1.00	0.56	0.79	0.29	4.45	0.00	0.00	-0.02
time (sec)	N/A	0.223	0.021	0.043	0.381	0.882	0.000	0.000	0.000

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	43	20	63	0	0	35
normalized size	1	1.00	0.98	1.00	0.47	1.47	0.00	0.00	0.81
time (sec)	N/A	0.212	0.019	0.030	0.384	0.556	0.000	0.000	1.013

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	93	196	0	222	0	128	-1
normalized size	1	1.00	0.58	1.22	0.00	1.39	0.00	0.80	-0.01
time (sec)	N/A	0.408	0.091	0.045	0.000	0.543	0.000	3.087	0.000

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	84	174	0	204	0	116	-1
normalized size	1	1.00	0.68	1.41	0.00	1.66	0.00	0.94	-0.01
time (sec)	N/A	0.331	0.065	0.040	0.000	0.662	0.000	0.190	0.000

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	77	147	0	188	0	106	-1
normalized size	1	1.00	0.79	1.50	0.00	1.92	0.00	1.08	-0.01
time (sec)	N/A	0.210	0.063	0.038	0.000	0.584	0.000	0.216	0.000

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	81	198	0	270	0	0	-1
normalized size	1	1.00	0.69	1.68	0.00	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.288	0.076	0.043	0.000	0.634	0.000	0.000	0.000

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	85	305	0	255	0	128	-1
normalized size	1	1.00	0.72	2.58	0.00	2.16	0.00	1.08	-0.01
time (sec)	N/A	0.383	0.072	0.049	0.000	1.493	0.000	0.318	0.000

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	79	347	0	177	0	195	-1
normalized size	1	1.00	0.71	3.13	0.00	1.59	0.00	1.76	-0.01
time (sec)	N/A	0.382	0.069	0.050	0.000	0.599	0.000	3.008	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	71	378	0	201	0	231	-1
normalized size	1	1.00	0.51	2.72	0.00	1.45	0.00	1.66	-0.01
time (sec)	N/A	0.388	0.083	0.051	0.000	0.757	0.000	11.138	0.000

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	95	410	0	217	0	316	-1
normalized size	1	1.00	0.61	2.63	0.00	1.39	0.00	2.03	-0.01
time (sec)	N/A	0.411	0.082	0.052	0.000	0.758	0.000	6.417	0.000

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	103	447	0	233	0	362	-1
normalized size	1	1.00	0.57	2.47	0.00	1.29	0.00	2.00	-0.01
time (sec)	N/A	0.430	0.089	0.057	0.000	0.893	0.000	5.489	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	71	85	196	419	0	0	-1
normalized size	1	1.00	0.38	0.45	1.04	2.23	0.00	0.00	-0.01
time (sec)	N/A	0.265	0.050	0.046	0.464	0.972	0.000	0.000	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	65	78	172	405	0	0	-1
normalized size	1	1.00	0.42	0.51	1.12	2.65	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.041	0.045	0.461	1.936	0.000	0.000	0.000

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	55	69	149	387	0	0	-1
normalized size	1	1.00	0.48	0.61	1.31	3.39	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.026	0.042	0.467	1.423	0.000	0.000	0.000

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	47	61	148	0	0	0	-1
normalized size	1	1.00	0.44	0.56	1.37	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.029	0.044	0.474	0.478	0.000	0.000	0.000

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	49	63	144	0	0	0	-1
normalized size	1	1.00	0.46	0.59	1.35	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	0.033	0.052	0.515	1.466	0.000	0.000	0.000

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	64	78	159	492	0	0	-1
normalized size	1	1.00	0.43	0.53	1.07	3.32	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.044	0.052	0.453	1.182	0.000	0.000	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	74	86	184	530	0	0	-1
normalized size	1	1.00	0.40	0.46	0.98	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.050	0.054	0.458	3.919	0.000	0.000	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	80	94	209	556	0	0	-1
normalized size	1	1.00	0.36	0.42	0.94	2.50	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.058	0.052	0.457	0.966	0.000	0.000	0.000

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	92	102	240	582	0	0	-1
normalized size	1	1.00	0.35	0.39	0.91	2.21	0.00	0.00	-0.00
time (sec)	N/A	0.274	0.063	0.051	0.462	0.940	0.000	0.000	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	52	69	55	78	0	0	78
normalized size	1	1.00	0.64	0.85	0.68	0.96	0.00	0.00	0.96
time (sec)	N/A	0.234	0.038	0.029	0.390	0.832	0.000	0.000	1.155

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	57	43	58	0	0	67
normalized size	1	1.00	0.57	0.77	0.58	0.78	0.00	0.00	0.91
time (sec)	N/A	0.209	0.024	0.028	0.382	2.438	0.000	0.000	0.970

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	41	56	51	57	0	0	67
normalized size	1	1.00	0.58	0.79	0.72	0.80	0.00	0.00	0.94
time (sec)	N/A	0.131	0.020	0.028	0.387	1.160	0.000	0.000	0.930

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	38	53	0	321	0	0	-1
normalized size	1	1.00	0.55	0.77	0.00	4.65	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.021	0.035	0.000	0.847	0.000	0.000	0.000

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	38	51	0	295	0	0	-1
normalized size	1	1.00	0.57	0.76	0.00	4.40	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.023	0.046	0.000	1.108	0.000	0.000	0.000

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	57	0	63	0	0	64
normalized size	1	1.00	1.00	1.30	0.00	1.43	0.00	0.00	1.45
time (sec)	N/A	0.222	0.020	0.030	0.000	1.048	0.000	0.000	0.961

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	93	196	0	222	0	128	-1
normalized size	1	1.00	0.57	1.20	0.00	1.36	0.00	0.79	-0.01
time (sec)	N/A	0.403	0.086	0.043	0.000	3.201	0.000	0.267	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	84	173	0	204	0	117	-1
normalized size	1	1.00	0.67	1.37	0.00	1.62	0.00	0.93	-0.01
time (sec)	N/A	0.327	0.067	0.040	0.000	2.050	0.000	0.235	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	147	0	188	0	106	-1
normalized size	1	1.00	1.01	1.48	0.00	1.90	0.00	1.07	-0.01
time (sec)	N/A	0.213	0.059	0.038	0.000	0.745	0.000	0.182	0.000

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	198	0	270	0	0	-1
normalized size	1	1.00	0.68	1.68	0.00	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.085	0.042	0.000	0.742	0.000	0.000	0.000

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	83	307	0	255	0	126	-1
normalized size	1	1.00	0.70	2.60	0.00	2.16	0.00	1.07	-0.01
time (sec)	N/A	0.418	0.067	0.044	0.000	1.568	0.000	0.342	0.000

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	78	348	0	176	0	195	-1
normalized size	1	1.00	0.70	3.11	0.00	1.57	0.00	1.74	-0.01
time (sec)	N/A	0.391	0.065	0.047	0.000	0.514	0.000	0.381	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	86	378	0	200	0	231	-1
normalized size	1	1.00	0.61	2.70	0.00	1.43	0.00	1.65	-0.01
time (sec)	N/A	0.393	0.072	0.047	0.000	0.723	0.000	2.776	0.000

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	94	410	0	216	0	316	-1
normalized size	1	1.00	0.60	2.63	0.00	1.38	0.00	2.03	-0.01
time (sec)	N/A	0.415	0.080	0.051	0.000	1.576	0.000	6.169	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	447	0	232	0	362	-1
normalized size	1	1.00	0.56	2.47	0.00	1.28	0.00	2.00	-0.01
time (sec)	N/A	0.440	0.087	0.055	0.000	0.568	0.000	7.929	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	70	85	0	419	0	0	-1
normalized size	1	1.00	0.37	0.45	0.00	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.053	0.044	0.000	0.670	0.000	0.000	0.000

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	64	78	0	407	0	0	-1
normalized size	1	1.00	0.42	0.51	0.00	2.68	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.047	0.046	0.000	0.688	0.000	0.000	0.000

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	54	69	0	387	0	0	-1
normalized size	1	1.00	0.48	0.61	0.00	3.42	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.032	0.043	0.000	0.604	0.000	0.000	0.000

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	45	60	0	0	0	0	-1
normalized size	1	1.00	0.42	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.027	0.043	0.000	0.699	0.000	0.000	0.000

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	48	62	0	0	0	0	-1
normalized size	1	1.00	0.45	0.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.261	0.034	0.047	0.000	0.618	0.000	0.000	0.000

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	63	78	0	485	0	0	-1
normalized size	1	1.00	0.43	0.53	0.00	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.041	0.048	0.000	0.619	0.000	0.000	0.000

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	73	86	0	523	0	0	-1
normalized size	1	1.00	0.39	0.46	0.00	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.281	0.045	0.049	0.000	0.791	0.000	0.000	0.000

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	78	94	0	551	0	0	-1
normalized size	1	1.00	0.35	0.43	0.00	2.50	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.054	0.055	0.000	0.682	0.000	0.000	0.000

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	87	102	0	575	0	0	-1
normalized size	1	1.00	0.33	0.39	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.262	0.066	0.050	0.000	1.045	0.000	0.000	0.000

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.116	0.020	0.212	0.000	0.709	0.000	0.000	0.000

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.116	0.016	0.131	0.000	0.757	0.000	0.000	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	71	229	0	0	0	0	0	-1
normalized size	1	0.21	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.133	0.853	0.060	0.000	0.796	0.000	0.000	0.000

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	70	126	0	0	0	0	0	-1
normalized size	1	0.51	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.225	0.051	0.000	1.374	0.000	0.000	0.000

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	82	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.106	0.044	0.000	0.603	0.000	0.000	0.000

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	178	0	0	0	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.423	0.159	0.059	0.000	0.985	0.000	0.000	0.000

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	103	190	0	0	0	0	0	-1
normalized size	1	0.24	0.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	1.686	0.056	0.000	1.105	0.000	0.000	0.000

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	302	208	0	0	0	0	0	-1
normalized size	1	1.11	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	0.173	0.043	0.000	0.575	0.000	0.000	0.000

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	130	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.105	0.048	0.000	0.701	0.000	0.000	0.000

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	186	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	0.250	0.046	0.000	0.735	0.000	0.000	0.000

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1039	1039	227	0	0	0	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.770	6.323	0.046	0.000	0.593	0.000	0.000	0.000

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.387	0.151	0.000	0.526	0.000	0.000	0.000

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	217	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	0.210	0.335	0.000	0.706	0.000	0.000	0.000

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	175	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.330	0.163	0.337	0.000	1.525	0.000	0.000	0.000

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	142	0	0	0	697	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	3.21	0.00	-0.00
time (sec)	N/A	0.261	0.108	0.344	0.000	0.646	11.345	0.000	0.000

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	112	0	0	0	178	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	1.30	0.00	-0.01
time (sec)	N/A	0.133	0.043	0.258	0.000	0.991	14.933	0.000	0.000

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	112	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.030	0.342	0.000	0.497	0.000	0.000	0.000

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	142	0	0	0	695	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	3.19	0.00	-0.00
time (sec)	N/A	0.264	0.106	0.255	0.000	1.067	11.768	0.000	0.000

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	173	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	0.142	0.348	0.000	1.432	0.000	0.000	0.000

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	184	0	639	0	0	124	-1
normalized size	1	1.00	0.77	0.00	2.66	0.00	0.00	0.52	-0.00
time (sec)	N/A	0.397	9.269	0.362	0.438	0.649	0.000	0.195	0.000

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	134	0	349	0	0	74	-1
normalized size	1	1.00	0.95	0.00	2.48	0.00	0.00	0.52	-0.01
time (sec)	N/A	0.170	8.415	0.355	0.405	2.480	0.000	0.178	0.000

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	168	0	909	0	0	108	-1
normalized size	1	1.00	1.03	0.00	5.58	0.00	0.00	0.66	-0.01
time (sec)	N/A	0.298	8.676	0.367	0.496	0.454	0.000	0.190	0.000

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	498	0	0	66	-1
normalized size	1	1.00	1.07	0.00	6.92	0.00	0.00	0.92	-0.01
time (sec)	N/A	0.122	0.025	0.375	0.440	0.674	0.000	0.180	0.000

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	200	0	1010	0	0	202	-1
normalized size	1	1.00	0.60	0.00	3.01	0.00	0.00	0.60	-0.00
time (sec)	N/A	0.476	9.770	0.361	0.470	0.731	0.000	0.236	0.000

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	178	0	639	0	0	122	-1
normalized size	1	1.00	0.75	0.00	2.71	0.00	0.00	0.52	-0.00
time (sec)	N/A	0.263	8.690	0.356	0.441	0.541	0.000	0.220	0.000

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	193	0	1488	0	0	122	-1
normalized size	1	1.00	1.00	0.00	7.71	0.00	0.00	0.63	-0.01
time (sec)	N/A	0.401	8.595	0.375	0.573	0.624	0.000	0.216	0.000

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	176	0	912	0	0	86	-1
normalized size	1	1.00	1.12	0.00	5.81	0.00	0.00	0.55	-0.01
time (sec)	N/A	0.233	7.197	0.375	0.501	0.753	0.000	0.189	0.000

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	162	0	347	0	0	74	-1
normalized size	1	1.00	1.16	0.00	2.48	0.00	0.00	0.53	-0.01
time (sec)	N/A	0.188	8.175	0.370	0.409	0.722	0.000	0.521	0.000

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	C	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	70	0	168	0	0	48	-1
normalized size	1	1.00	1.13	0.00	2.71	0.00	0.00	0.77	-0.02
time (sec)	N/A	0.110	0.029	0.364	0.376	1.998	0.000	0.182	0.000

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	149	487	540	144	0	139	-1
normalized size	1	1.00	0.96	3.12	3.46	0.92	0.00	0.89	-0.01
time (sec)	N/A	0.160	0.394	0.042	0.426	0.595	0.000	0.232	0.000

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	159	315	355	116	0	97	-1
normalized size	1	1.00	1.22	2.42	2.73	0.89	0.00	0.75	-0.01
time (sec)	N/A	0.167	0.175	0.039	0.425	1.162	0.000	0.213	0.000

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	130	178	209	92	0	59	-1
normalized size	1	1.00	1.55	2.12	2.49	1.10	0.00	0.70	-0.01
time (sec)	N/A	0.064	0.093	0.036	0.406	0.478	0.000	0.393	0.000

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	28	71	65	76	0	36	101
normalized size	1	1.00	0.72	1.82	1.67	1.95	0.00	0.92	2.59
time (sec)	N/A	0.026	0.017	0.035	0.408	0.654	0.000	0.215	1.421

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	106	168	0	306	0	79	132
normalized size	1	1.00	1.66	2.62	0.00	4.78	0.00	1.23	2.06
time (sec)	N/A	0.075	0.061	0.035	0.000	0.698	0.000	0.230	1.392

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	265	0	282	0	190	241
normalized size	1	1.00	0.90	2.70	0.00	2.88	0.00	1.94	2.46
time (sec)	N/A	0.056	0.053	0.041	0.000	1.120	0.000	0.254	1.691

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	123	453	0	359	0	622	-1
normalized size	1	1.00	0.76	2.80	0.00	2.22	0.00	3.84	-0.01
time (sec)	N/A	0.108	0.131	0.041	0.000	0.722	0.000	0.386	0.000

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	193	683	0	488	0	1376	-1
normalized size	1	1.00	0.91	3.21	0.00	2.29	0.00	6.46	-0.00
time (sec)	N/A	0.183	0.266	0.044	0.000	1.760	0.000	1.055	0.000

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	161	94	93	102	122	142
normalized size	1	1.00	0.90	1.94	1.13	1.12	1.23	1.47	1.71
time (sec)	N/A	0.083	0.063	0.028	0.305	0.606	0.340	0.179	0.086

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	108	68	67	66	82	106
normalized size	1	1.00	1.00	1.64	1.03	1.02	1.00	1.24	1.61
time (sec)	N/A	0.060	0.042	0.030	0.305	0.704	0.258	0.151	0.888

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	68	46	45	42	52	74
normalized size	1	1.00	0.90	1.39	0.94	0.92	0.86	1.06	1.51
time (sec)	N/A	0.051	0.032	0.028	0.307	0.968	0.207	0.148	0.056

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	38	30	29	26	34	40
normalized size	1	1.00	1.00	1.12	0.88	0.85	0.76	1.00	1.18
time (sec)	N/A	0.031	0.018	0.029	0.300	0.566	0.155	0.152	0.870

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	18	14	17	16
normalized size	1	1.00	1.00	0.89	0.84	0.95	0.74	0.89	0.84
time (sec)	N/A	0.012	0.010	0.025	0.316	1.116	0.116	0.240	0.040

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	35	27	23	88	34	28
normalized size	1	1.00	0.79	1.06	0.82	0.70	2.67	1.03	0.85
time (sec)	N/A	0.038	0.017	0.034	0.310	0.459	0.461	0.188	0.116

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	34	46	48	39	144	57	47
normalized size	1	1.00	0.71	0.96	1.00	0.81	3.00	1.19	0.98
time (sec)	N/A	0.041	0.023	0.034	0.313	1.176	0.352	0.144	0.098

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	63	74	65	209	91	66
normalized size	1	1.00	0.81	0.94	1.10	0.97	3.12	1.36	0.99
time (sec)	N/A	0.051	0.034	0.033	0.309	1.130	0.448	0.179	0.931

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	75	76	108	88	260	120	84
normalized size	1	1.00	0.91	0.93	1.32	1.07	3.17	1.46	1.02
time (sec)	N/A	0.062	0.047	0.035	0.318	1.410	0.559	0.174	0.900

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	203	756	2349	192	0	193	-1
normalized size	1	1.00	1.09	4.04	12.56	1.03	0.00	1.03	-0.01
time (sec)	N/A	0.182	0.289	0.066	0.451	0.734	0.000	0.516	0.000

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	170	552	1645	159	0	148	-1
normalized size	1	1.00	1.01	3.29	9.79	0.95	0.00	0.88	-0.01
time (sec)	N/A	0.191	0.222	0.053	0.433	0.743	0.000	0.300	0.000

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	142	381	1137	131	0	109	-1
normalized size	1	1.00	1.17	3.15	9.40	1.08	0.00	0.90	-0.01
time (sec)	N/A	0.096	0.164	0.046	0.425	0.728	0.000	0.255	0.000

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	43	388	753	100	0	75	-1
normalized size	1	1.00	0.63	5.71	11.07	1.47	0.00	1.10	-0.01
time (sec)	N/A	0.034	0.041	0.037	0.416	0.582	0.000	0.312	0.000

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	153	1019	0	439	0	139	-1
normalized size	1	1.00	1.43	9.52	0.00	4.10	0.00	1.30	-0.01
time (sec)	N/A	0.091	0.590	0.041	0.000	0.702	0.000	0.319	0.000

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	106	1520	0	370	0	498	-1
normalized size	1	1.00	0.79	11.34	0.00	2.76	0.00	3.72	-0.01
time (sec)	N/A	0.075	0.079	0.046	0.000	0.804	0.000	0.226	0.000

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	141	2194	0	523	0	691	-1
normalized size	1	1.00	0.70	10.86	0.00	2.59	0.00	3.42	-0.00
time (sec)	N/A	0.122	0.168	0.046	0.000	1.551	0.000	0.452	0.000

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	199	2947	0	704	0	1521	-1
normalized size	1	1.00	0.77	11.33	0.00	2.71	0.00	5.85	-0.00
time (sec)	N/A	0.210	0.309	0.049	0.000	0.877	0.000	0.278	0.000

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	160	809	338	143	0	148	-1
normalized size	1	1.00	1.03	5.19	2.17	0.92	0.00	0.95	-0.01
time (sec)	N/A	0.166	0.436	0.046	0.422	0.678	0.000	0.220	0.000

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	126	535	174	117	0	106	-1
normalized size	1	1.00	0.97	4.12	1.34	0.90	0.00	0.82	-0.01
time (sec)	N/A	0.146	0.160	0.043	0.426	0.484	0.000	0.838	0.000

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	99	302	107	91	0	68	-1
normalized size	1	1.00	1.18	3.60	1.27	1.08	0.00	0.81	-0.01
time (sec)	N/A	0.063	0.114	0.036	0.414	0.646	0.000	0.176	0.000

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	26	95	37	77	0	44	-1
normalized size	1	1.00	0.68	2.50	0.97	2.03	0.00	1.16	-0.03
time (sec)	N/A	0.028	0.019	0.032	0.444	0.587	0.000	0.181	0.000

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	106	249	0	303	0	89	-1
normalized size	1	1.00	1.56	3.66	0.00	4.46	0.00	1.31	-0.01
time (sec)	N/A	0.060	0.039	0.043	0.000	0.610	0.000	0.407	0.000

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	89	565	0	277	0	223	-1
normalized size	1	1.00	0.95	6.01	0.00	2.95	0.00	2.37	-0.01
time (sec)	N/A	0.057	0.065	0.047	0.000	0.583	0.000	0.273	0.000

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	122	1116	0	356	0	750	-1
normalized size	1	1.00	0.75	6.89	0.00	2.20	0.00	4.63	-0.01
time (sec)	N/A	0.098	0.135	0.050	0.000	0.659	0.000	0.290	0.000

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	194	1711	0	483	0	1659	-1
normalized size	1	1.00	0.92	8.15	0.00	2.30	0.00	7.90	-0.00
time (sec)	N/A	0.168	0.242	0.053	0.000	0.716	0.000	0.253	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	159	94	93	102	202	141
normalized size	1	1.00	1.00	2.24	1.32	1.31	1.44	2.85	1.99
time (sec)	N/A	0.079	0.058	0.027	0.351	0.510	0.283	0.309	0.063

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	109	68	67	68	148	109
normalized size	1	1.00	1.00	1.91	1.19	1.18	1.19	2.60	1.91
time (sec)	N/A	0.056	0.042	0.027	0.306	2.260	0.228	0.261	0.049

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	67	46	45	41	102	73
normalized size	1	1.00	1.00	1.63	1.12	1.10	1.00	2.49	1.78
time (sec)	N/A	0.046	0.029	0.026	0.321	0.673	0.196	0.160	0.062

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	38	30	29	26	69	42
normalized size	1	1.00	1.00	1.31	1.03	1.00	0.90	2.38	1.45
time (sec)	N/A	0.032	0.018	0.026	0.318	0.481	0.144	0.169	0.892

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	18	12	38	16
normalized size	1	1.00	1.00	1.06	1.00	1.12	0.75	2.38	1.00
time (sec)	N/A	0.012	0.011	0.025	0.312	1.308	0.112	0.527	0.038

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	23	34	27	23	90	66	28
normalized size	1	1.00	0.82	1.21	0.96	0.82	3.21	2.36	1.00
time (sec)	N/A	0.035	0.016	0.031	0.306	0.589	0.456	0.321	0.103

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	47	48	36	143	80	47
normalized size	1	1.00	0.76	1.15	1.17	0.88	3.49	1.95	1.15
time (sec)	N/A	0.043	0.022	0.034	0.306	2.384	0.346	0.186	0.920

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	63	74	65	209	121	66
normalized size	1	1.00	0.88	1.09	1.28	1.12	3.60	2.09	1.14
time (sec)	N/A	0.050	0.036	0.036	0.308	0.602	0.445	0.181	0.915

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	75	108	86	262	159	83
normalized size	1	1.00	1.00	1.07	1.54	1.23	3.74	2.27	1.19
time (sec)	N/A	0.058	0.047	0.033	0.307	0.578	0.535	0.439	0.103

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	231	1271	985	191	0	211	-1
normalized size	1	1.00	1.24	6.80	5.27	1.02	0.00	1.13	-0.01
time (sec)	N/A	0.188	0.211	0.054	0.434	0.818	0.000	0.235	0.000

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	190	830	622	159	0	166	-1
normalized size	1	1.00	1.14	4.97	3.72	0.95	0.00	0.99	-0.01
time (sec)	N/A	0.201	0.170	0.050	0.421	0.665	0.000	0.210	0.000

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	157	543	299	130	0	127	-1
normalized size	1	1.00	1.32	4.56	2.51	1.09	0.00	1.07	-0.01
time (sec)	N/A	0.092	0.133	0.043	0.412	1.264	0.000	0.240	0.000

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	43	264	104	101	0	94	-1
normalized size	1	1.00	0.63	3.88	1.53	1.49	0.00	1.38	-0.01
time (sec)	N/A	0.034	0.044	0.036	0.414	0.628	0.000	0.208	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	137	1062	0	438	0	154	-1
normalized size	1	1.00	1.33	10.31	0.00	4.25	0.00	1.50	-0.01
time (sec)	N/A	0.085	0.443	0.051	0.000	0.457	0.000	0.219	0.000

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	106	1710	0	369	0	604	-1
normalized size	1	1.00	0.82	13.15	0.00	2.84	0.00	4.65	-0.01
time (sec)	N/A	0.077	0.096	0.056	0.000	0.702	0.000	0.213	0.000

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	139	2848	0	520	0	822	-1
normalized size	1	1.00	0.70	14.24	0.00	2.60	0.00	4.11	-0.00
time (sec)	N/A	0.129	0.159	0.064	0.000	0.664	0.000	0.435	0.000

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	194	4212	0	697	0	1839	-1
normalized size	1	1.00	0.75	16.39	0.00	2.71	0.00	7.16	-0.00
time (sec)	N/A	0.216	0.321	0.069	0.000	0.762	0.000	0.255	0.000

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	37	34	18	28	0	15	10
normalized size	1	1.00	3.70	3.40	1.80	2.80	0.00	1.50	1.00
time (sec)	N/A	0.035	0.013	0.028	0.395	0.636	0.000	0.192	0.966

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	90	499	576	150	0	125	-1
normalized size	1	1.00	0.83	4.58	5.28	1.38	0.00	1.15	-0.01
time (sec)	N/A	0.169	0.369	0.044	0.603	0.667	0.000	0.245	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	325	329	120	0	94	-1
normalized size	1	1.00	0.82	4.17	4.22	1.54	0.00	1.21	-0.01
time (sec)	N/A	0.137	0.212	0.042	0.522	0.828	0.000	0.249	0.000

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	160	120	98	0	66	229
normalized size	1	1.00	1.11	3.64	2.73	2.23	0.00	1.50	5.20
time (sec)	N/A	0.082	0.137	0.039	0.460	0.566	0.000	0.277	1.646

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	12	42	65	37	0	40	26
normalized size	1	1.00	0.44	1.56	2.41	1.37	0.00	1.48	0.96
time (sec)	N/A	0.037	0.081	0.029	0.426	0.530	0.000	0.396	0.180

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	118	391	0	316	0	112	-1
normalized size	1	1.00	1.27	4.20	0.00	3.40	0.00	1.20	-0.01
time (sec)	N/A	0.132	0.248	0.037	0.000	0.622	0.000	0.371	0.000

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	149	671	0	459	0	519	-1
normalized size	1	1.00	0.99	4.47	0.00	3.06	0.00	3.46	-0.01
time (sec)	N/A	0.159	0.320	0.043	0.000	0.717	0.000	0.202	0.000

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.840	0.072	0.000	0.633	0.000	0.000	0.000

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	220	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.183	0.251	0.056	0.000	0.702	0.000	0.000	0.000

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	127	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.114	0.052	0.000	0.740	0.000	0.000	0.000

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	96	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.043	0.046	0.000	0.676	0.000	0.000	0.000

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	50	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.022	0.040	0.000	0.585	0.000	0.000	0.000

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	111	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.037	0.047	0.000	0.676	0.000	0.000	0.000

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	83	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.023	0.049	0.000	0.599	0.000	0.000	0.000

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.062	0.057	0.000	0.590	0.000	0.000	0.000

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	107	229	210	136	452	127	118
normalized size	1	1.00	0.84	1.80	1.65	1.07	3.56	1.00	0.93
time (sec)	N/A	0.064	0.157	0.092	0.414	0.673	40.905	0.198	0.086

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	183	164	115	267	103	100
normalized size	1	1.00	0.87	1.74	1.56	1.10	2.54	0.98	0.95
time (sec)	N/A	0.056	0.130	0.059	0.411	0.646	24.981	0.190	0.901

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	137	118	92	143	78	82
normalized size	1	1.00	0.90	1.65	1.42	1.11	1.72	0.94	0.99
time (sec)	N/A	0.046	0.102	0.044	0.408	0.475	20.149	0.209	0.908

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	57	83	64	63	53	46	80
normalized size	1	1.00	1.04	1.51	1.16	1.15	0.96	0.84	1.45
time (sec)	N/A	0.029	0.075	0.035	0.403	0.543	4.822	0.207	0.881

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	74	143	204	86	0	102	140
normalized size	1	1.00	0.73	1.42	2.02	0.85	0.00	1.01	1.39
time (sec)	N/A	0.132	0.046	0.045	0.541	0.614	0.000	0.209	0.886

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	119	178	77	0	0	128
normalized size	1	1.00	0.88	1.61	2.41	1.04	0.00	0.00	1.73
time (sec)	N/A	0.108	0.040	0.038	0.490	0.500	0.000	0.000	0.930

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	98	157	69	0	78	93
normalized size	1	1.00	0.90	1.63	2.62	1.15	0.00	1.30	1.55
time (sec)	N/A	0.107	0.028	0.038	0.458	0.572	0.000	0.495	0.070

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	45	79	137	64	0	59	64
normalized size	1	1.00	1.15	2.03	3.51	1.64	0.00	1.51	1.64
time (sec)	N/A	0.064	0.030	0.041	0.443	0.746	0.000	0.220	0.897

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	26	25	125	33	0	37	47
normalized size	1	1.00	2.00	1.92	9.62	2.54	0.00	2.85	3.62
time (sec)	N/A	0.029	0.009	0.029	0.417	0.489	0.000	0.210	0.872

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	55	60	0	55	0	80	67
normalized size	1	1.00	1.25	1.36	0.00	1.25	0.00	1.82	1.52
time (sec)	N/A	0.099	0.022	0.040	0.000	0.700	0.000	0.193	0.895

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	75	104	78	0	159	90
normalized size	1	1.00	0.96	1.07	1.49	1.11	0.00	2.27	1.29
time (sec)	N/A	0.117	0.025	0.043	0.324	0.596	0.000	0.628	0.898

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	83	97	0	99	0	224	117
normalized size	1	1.00	0.84	0.98	0.00	1.00	0.00	2.26	1.18
time (sec)	N/A	0.139	0.040	0.045	0.000	0.571	0.000	0.216	0.069

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	91	140	148	107	0	283	140
normalized size	1	1.00	0.71	1.09	1.16	0.84	0.00	2.21	1.09
time (sec)	N/A	0.166	0.040	0.046	0.327	0.638	0.000	0.212	0.901

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	93	225	0	159	0	0	241
normalized size	1	1.00	0.68	1.64	0.00	1.16	0.00	0.00	1.76
time (sec)	N/A	0.154	0.060	0.054	0.000	0.562	0.000	0.000	0.077

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	202	0	152	0	0	218
normalized size	1	1.00	0.78	1.84	0.00	1.38	0.00	0.00	1.98
time (sec)	N/A	0.136	0.046	0.050	0.000	0.725	0.000	0.000	0.898

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	78	180	0	144	0	0	196
normalized size	1	1.00	0.79	1.82	0.00	1.45	0.00	0.00	1.98
time (sec)	N/A	0.122	0.048	0.046	0.000	0.678	0.000	0.000	0.068

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	69	160	0	137	0	0	176
normalized size	1	1.00	0.93	2.16	0.00	1.85	0.00	0.00	2.38
time (sec)	N/A	0.104	0.041	0.045	0.000	0.630	0.000	0.000	0.074

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	41	0	91	0	0	102
normalized size	1	1.00	0.79	0.72	0.00	1.60	0.00	0.00	1.79
time (sec)	N/A	0.094	0.020	0.031	0.000	0.495	0.000	0.000	0.927

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	44	41	0	89	0	0	63
normalized size	1	1.00	0.80	0.75	0.00	1.62	0.00	0.00	1.15
time (sec)	N/A	0.064	0.020	0.030	0.000	0.524	0.000	0.000	0.916

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	42	0	89	0	0	48
normalized size	1	1.00	0.87	0.81	0.00	1.71	0.00	0.00	0.92
time (sec)	N/A	0.040	0.014	0.030	0.000	0.548	0.000	0.000	0.079

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	77	182	0	127	0	0	173
normalized size	1	1.00	1.04	2.46	0.00	1.72	0.00	0.00	2.34
time (sec)	N/A	0.116	0.034	0.045	0.000	0.740	0.000	0.000	0.910

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	150	144	153	0	0	198
normalized size	1	1.00	0.87	1.43	1.37	1.46	0.00	0.00	1.89
time (sec)	N/A	0.142	0.042	0.048	0.340	0.636	0.000	0.000	0.900

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	103	214	0	171	0	0	221
normalized size	1	1.00	0.77	1.60	0.00	1.28	0.00	0.00	1.65
time (sec)	N/A	0.159	0.047	0.049	0.000	0.735	0.000	0.000	0.072

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	110	260	191	178	0	0	244
normalized size	1	1.00	0.68	1.61	1.19	1.11	0.00	0.00	1.52
time (sec)	N/A	0.189	0.051	0.049	0.339	0.843	0.000	0.000	0.913

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	116	283	0	221	0	0	352
normalized size	1	1.00	0.81	1.98	0.00	1.55	0.00	0.00	2.46
time (sec)	N/A	0.181	0.082	0.054	0.000	0.740	0.000	0.000	0.104

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	108	262	0	213	0	0	379
normalized size	1	1.00	0.81	1.97	0.00	1.60	0.00	0.00	2.85
time (sec)	N/A	0.154	0.075	0.052	0.000	0.606	0.000	0.000	0.921

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	100	243	0	206	0	0	312
normalized size	1	1.00	0.93	2.25	0.00	1.91	0.00	0.00	2.89
time (sec)	N/A	0.129	0.060	0.050	0.000	0.721	0.000	0.000	0.927

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	68	58	0	146	0	0	335
normalized size	1	1.00	0.84	0.72	0.00	1.80	0.00	0.00	4.14
time (sec)	N/A	0.116	0.026	0.032	0.000	0.747	0.000	0.000	0.920

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	88	68	58	0	145	0	0	287
normalized size	1	1.19	0.92	0.78	0.00	1.96	0.00	0.00	3.88
time (sec)	N/A	0.106	0.025	0.032	0.000	0.554	0.000	0.000	0.078

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	58	0	146	0	0	329
normalized size	1	1.00	0.77	0.66	0.00	1.66	0.00	0.00	3.74
time (sec)	N/A	0.108	0.026	0.032	0.000	0.805	0.000	0.000	0.910

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	58	0	145	0	0	271
normalized size	1	1.00	0.85	0.72	0.00	1.81	0.00	0.00	3.39
time (sec)	N/A	0.069	0.033	0.031	0.000	0.683	0.000	0.000	0.075

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	59	58	0	144	0	0	279
normalized size	1	1.00	0.80	0.78	0.00	1.95	0.00	0.00	3.77
time (sec)	N/A	0.045	0.024	0.033	0.000	0.556	0.000	0.000	0.897

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	108	384	0	198	0	0	306
normalized size	1	1.00	1.07	3.80	0.00	1.96	0.00	0.00	3.03
time (sec)	N/A	0.140	0.049	0.051	0.000	0.606	0.000	0.000	0.088

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	121	314	178	223	0	0	335
normalized size	1	1.00	0.90	2.33	1.32	1.65	0.00	0.00	2.48
time (sec)	N/A	0.171	0.059	0.051	0.361	0.652	0.000	0.000	0.094

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	133	326	0	241	0	0	357
normalized size	1	1.00	0.81	1.99	0.00	1.47	0.00	0.00	2.18
time (sec)	N/A	0.194	0.065	0.055	0.000	0.749	0.000	0.000	0.920

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	74	0	198	0	0	477
normalized size	1	1.00	0.78	0.77	0.00	2.06	0.00	0.00	4.97
time (sec)	N/A	0.053	0.033	0.033	0.000	0.736	0.000	0.000	0.933

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	91	90	0	252	0	0	783
normalized size	1	1.00	0.77	0.76	0.00	2.14	0.00	0.00	6.64
time (sec)	N/A	0.064	0.033	0.033	0.000	0.790	0.000	0.000	0.992

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	43	43	42	39	44	42
normalized size	1	1.00	1.00	0.88	0.88	0.86	0.80	0.90	0.86
time (sec)	N/A	0.100	0.026	0.026	0.314	0.487	18.386	0.259	0.037

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	35	35	34	31	36	34
normalized size	1	1.00	1.00	0.90	0.90	0.87	0.79	0.92	0.87
time (sec)	N/A	0.096	0.019	0.029	0.318	0.595	0.100	0.159	0.037

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	26	25	22	27	23
normalized size	1	1.00	1.00	0.93	0.90	0.86	0.76	0.93	0.79
time (sec)	N/A	0.091	0.017	0.026	0.321	0.540	0.089	0.199	0.872

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	15	14	19	15
normalized size	1	1.00	1.00	1.00	0.95	0.79	0.74	1.00	0.79
time (sec)	N/A	0.062	0.013	0.029	0.362	0.535	0.082	0.153	0.857

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	11	11	8	12	11
normalized size	1	1.00	1.00	1.00	0.92	0.92	0.67	1.00	0.92
time (sec)	N/A	0.034	0.005	0.026	0.311	0.593	0.059	0.201	0.023

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	11	11	8	13	9
normalized size	1	1.00	1.00	1.00	0.92	0.92	0.67	1.08	0.75
time (sec)	N/A	0.080	0.007	0.032	0.314	0.542	0.109	0.191	0.045

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	19	22	15	21	16
normalized size	1	1.00	1.00	1.00	0.95	1.10	0.75	1.05	0.80
time (sec)	N/A	0.087	0.013	0.031	0.306	0.583	0.138	0.191	0.868

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	30	29	35	26	31	23
normalized size	1	1.00	1.00	0.94	0.91	1.09	0.81	0.97	0.72
time (sec)	N/A	0.090	0.014	0.033	0.306	0.441	0.171	0.144	0.052

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	37	43	34	39	31
normalized size	1	1.00	1.00	0.90	0.88	1.02	0.81	0.93	0.74
time (sec)	N/A	0.091	0.016	0.032	0.310	0.758	0.172	0.174	0.054

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	45	49	52	62	48	56	54
normalized size	1	1.00	0.78	0.84	0.90	1.07	0.83	0.97	0.93
time (sec)	N/A	0.115	0.048	0.034	0.314	0.611	0.257	0.146	0.911

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	39	41	43	55	39	42	46
normalized size	1	1.00	0.81	0.85	0.90	1.15	0.81	0.88	0.96
time (sec)	N/A	0.113	0.032	0.033	0.305	0.568	0.253	0.192	0.065

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	36	38	42	34	37	39
normalized size	1	1.00	0.77	0.84	0.88	0.98	0.79	0.86	0.91
time (sec)	N/A	0.113	0.026	0.033	0.306	0.755	0.211	0.168	0.949

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	36	38	42	32	37	22
normalized size	1	1.00	0.81	1.33	1.41	1.56	1.19	1.37	0.81
time (sec)	N/A	0.078	0.018	0.033	0.339	0.507	0.175	0.173	0.049

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	36	36	40	29	37	22
normalized size	1	1.00	0.74	1.33	1.33	1.48	1.07	1.37	0.81
time (sec)	N/A	0.048	0.015	0.033	0.321	0.585	0.184	0.489	0.916

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	32	29	28	45	29	31	30
normalized size	1	1.00	0.89	0.81	0.78	1.25	0.81	0.86	0.83
time (sec)	N/A	0.109	0.031	0.033	0.335	0.667	0.258	0.192	0.052

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	42	75	42	44	40
normalized size	1	1.00	1.00	0.85	0.91	1.63	0.91	0.96	0.87
time (sec)	N/A	0.110	0.040	0.034	0.320	0.644	0.371	0.201	0.888

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	54	59	96	58	59	58
normalized size	1	1.00	0.94	0.86	0.94	1.52	0.92	0.94	0.92
time (sec)	N/A	0.118	0.051	0.037	0.314	0.623	0.405	0.188	0.905

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	67	62	67	105	66	67	66
normalized size	1	1.00	0.92	0.85	0.92	1.44	0.90	0.92	0.90
time (sec)	N/A	0.122	0.059	0.038	0.319	0.593	0.428	0.153	0.914

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	65	74	80	125	82	77	82
normalized size	1	1.00	0.74	0.84	0.91	1.42	0.93	0.88	0.93
time (sec)	N/A	0.140	0.094	0.038	0.320	0.656	0.436	0.189	0.076

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	55	65	72	117	71	64	73
normalized size	1	1.00	0.72	0.86	0.95	1.54	0.93	0.84	0.96
time (sec)	N/A	0.127	0.076	0.037	0.326	0.540	0.426	0.165	0.932

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	60	66	101	68	58	67
normalized size	1	1.00	0.76	0.83	0.92	1.40	0.94	0.81	0.93
time (sec)	N/A	0.125	0.066	0.037	0.319	0.500	0.378	0.273	0.178

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	60	66	101	66	58	53
normalized size	1	1.00	0.95	1.07	1.18	1.80	1.18	1.04	0.95
time (sec)	N/A	0.124	0.034	0.035	0.314	0.736	0.311	0.192	0.057

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	60	65	100	63	57	54
normalized size	1	1.00	0.93	1.07	1.16	1.79	1.12	1.02	0.96
time (sec)	N/A	0.123	0.032	0.036	0.321	0.719	0.292	0.162	0.057

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	48	65	100	61	57	51
normalized size	1	1.00	0.68	1.17	1.59	2.44	1.49	1.39	1.24
time (sec)	N/A	0.084	0.028	0.036	0.317	0.527	0.288	0.164	0.931

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	60	64	99	65	58	49
normalized size	1	1.00	0.95	1.07	1.14	1.77	1.16	1.04	0.88
time (sec)	N/A	0.057	0.026	0.034	0.313	0.602	0.305	0.326	0.061

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	47	57	122	60	51	57
normalized size	1	1.00	0.92	0.80	0.97	2.07	1.02	0.86	0.97
time (sec)	N/A	0.118	0.048	0.036	0.311	0.592	0.393	0.181	0.070

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	59	72	150	78	67	71
normalized size	1	1.00	0.92	0.83	1.01	2.11	1.10	0.94	1.00
time (sec)	N/A	0.127	0.062	0.037	0.310	0.653	0.516	0.163	0.941

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	83	78	89	172	95	82	89
normalized size	1	1.00	0.93	0.88	1.00	1.93	1.07	0.92	1.00
time (sec)	N/A	0.135	0.075	0.039	0.329	0.689	0.558	0.401	0.963

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	91	86	97	180	104	90	97
normalized size	1	1.00	0.89	0.84	0.95	1.76	1.02	0.88	0.95
time (sec)	N/A	0.145	0.087	0.040	0.307	0.689	0.585	0.256	0.939

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	37	0	50	0	0	38
normalized size	1	1.00	0.57	0.50	0.00	0.68	0.00	0.00	0.51
time (sec)	N/A	0.184	0.027	0.026	0.000	0.672	0.000	0.000	1.053

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	37	0	50	0	0	38
normalized size	1	1.00	0.57	0.50	0.00	0.68	0.00	0.00	0.51
time (sec)	N/A	0.128	0.020	0.027	0.000	0.556	0.000	0.000	0.981

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	40	34	0	47	0	0	34
normalized size	1	1.00	0.58	0.49	0.00	0.68	0.00	0.00	0.49
time (sec)	N/A	0.072	0.016	0.026	0.000	0.890	0.000	0.000	0.949

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	36	48	56	262	0	0	-1
normalized size	1	1.00	0.55	0.74	0.86	4.03	0.00	0.00	-0.02
time (sec)	N/A	0.176	0.020	0.047	0.338	0.869	0.000	0.000	0.000

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	40	50	60	264	0	0	-1
normalized size	1	1.00	0.59	0.74	0.88	3.88	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.020	0.039	0.348	0.745	0.000	0.000	0.000

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	65	66	68	0	0	55
normalized size	1	1.00	0.64	0.73	0.74	0.76	0.00	0.00	0.62
time (sec)	N/A	0.087	0.028	0.028	0.331	0.807	0.000	0.000	1.055

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	60	81	111	98	0	0	85
normalized size	1	1.00	0.44	0.60	0.82	0.72	0.00	0.00	0.62
time (sec)	N/A	0.095	0.039	0.032	0.338	0.873	0.000	0.000	1.051

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	68	97	154	120	0	0	107
normalized size	1	1.00	0.37	0.53	0.84	0.66	0.00	0.00	0.58
time (sec)	N/A	0.105	0.049	0.030	0.407	0.688	0.000	0.000	1.058

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	71	83	0	387	0	0	-1
normalized size	1	1.00	0.37	0.43	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.047	0.039	0.000	0.682	0.000	0.000	0.000

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	63	75	0	371	0	0	-1
normalized size	1	1.00	0.41	0.48	0.00	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.038	0.038	0.000	0.810	0.000	0.000	0.000

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	54	66	0	352	0	0	-1
normalized size	1	1.00	0.47	0.57	0.00	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.033	0.038	0.000	0.899	0.000	0.000	0.000

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	45	55	0	331	0	0	-1
normalized size	1	1.00	0.58	0.71	0.00	4.30	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.025	0.035	0.000	0.669	0.000	0.000	0.000

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	51	0	227	0	0	-1
normalized size	1	1.00	1.00	1.24	0.00	5.54	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.013	0.033	0.000	0.574	0.000	0.000	0.000

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	42	53	0	302	0	0	-1
normalized size	1	1.00	0.59	0.75	0.00	4.25	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.017	0.041	0.000	0.815	0.000	0.000	0.000

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	50	62	0	416	0	0	-1
normalized size	1	1.00	0.47	0.58	0.00	3.89	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.029	0.043	0.000	0.828	0.000	0.000	0.000

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	62	76	0	453	0	0	-1
normalized size	1	1.00	0.42	0.51	0.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.040	0.043	0.000	0.819	0.000	0.000	0.000

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	72	84	0	482	0	0	-1
normalized size	1	1.00	0.39	0.45	0.00	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.043	0.044	0.000	0.893	0.000	0.000	0.000

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	87	119	0	0	0	0	-1
normalized size	1	1.00	0.33	0.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	0.075	0.049	0.000	1.083	0.000	0.000	0.000

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	77	110	0	0	0	0	-1
normalized size	1	1.00	0.35	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.236	0.063	0.050	0.000	0.776	0.000	0.000	0.000

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	71	101	0	0	0	0	-1
normalized size	1	1.00	0.40	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.053	0.048	0.000	0.894	0.000	0.000	0.000

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	76	90	0	0	0	0	-1
normalized size	1	1.00	0.55	0.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.043	0.046	0.000	0.724	0.000	0.000	0.000

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	60	88	0	348	0	0	-1
normalized size	1	1.00	0.66	0.97	0.00	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.036	0.051	0.000	1.265	0.000	0.000	0.000

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	60	88	0	343	0	0	-1
normalized size	1	1.00	0.66	0.97	0.00	3.77	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.029	0.046	0.000	0.756	0.000	0.000	0.000

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	65	96	0	0	0	0	-1
normalized size	1	1.00	0.39	0.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.055	0.048	0.000	0.872	0.000	0.000	0.000

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	76	122	0	0	0	0	-1
normalized size	1	1.00	0.37	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.063	0.048	0.000	0.769	0.000	0.000	0.000

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	91	142	0	0	0	0	-1
normalized size	1	1.00	0.36	0.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.066	0.050	0.000	0.859	0.000	0.000	0.000

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	99	151	0	0	0	0	-1
normalized size	1	1.00	0.33	0.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	0.078	0.051	0.000	0.767	0.000	0.000	0.000

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	97	190	0	0	0	0	-1
normalized size	1	1.00	0.31	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.255	0.134	0.052	0.000	0.848	0.000	0.000	0.000

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	87	182	0	0	0	0	-1
normalized size	1	1.00	0.32	0.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	0.122	0.049	0.000	0.635	0.000	0.000	0.000

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	87	166	0	0	0	0	-1
normalized size	1	1.00	0.38	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.104	0.050	0.000	0.744	0.000	0.000	0.000

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	85	166	0	461	0	0	-1
normalized size	1	1.00	0.46	0.90	0.00	2.51	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.067	0.052	0.000	0.687	0.000	0.000	0.000

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	84	161	0	457	0	0	-1
normalized size	1	1.00	0.46	0.88	0.00	2.48	0.00	0.00	-0.01
time (sec)	N/A	0.237	0.060	0.049	0.000	0.543	0.000	0.000	0.000

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	60	161	0	459	0	0	-1
normalized size	1	1.00	0.44	1.18	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.069	0.051	0.000	0.675	0.000	0.000	0.000

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	85	166	0	455	0	0	-1
normalized size	1	1.00	0.46	0.90	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.050	0.049	0.000	0.709	0.000	0.000	0.000

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	86	193	0	0	0	0	-1
normalized size	1	1.00	0.34	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.071	0.050	0.000	0.799	0.000	0.000	0.000

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	97	222	0	0	0	0	-1
normalized size	1	1.00	0.33	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.093	0.051	0.000	0.762	0.000	0.000	0.000

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	115	242	0	0	0	0	-1
normalized size	1	1.00	0.33	0.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	0.122	0.053	0.000	0.850	0.000	0.000	0.000

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	103	238	0	567	0	0	-1
normalized size	1	1.00	0.37	0.86	0.00	2.05	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.083	0.052	0.000	1.201	0.000	0.000	0.000

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	227	0	0	223	0	-1
normalized size	1	1.00	1.02	2.84	0.00	0.00	2.79	0.00	-0.01
time (sec)	N/A	0.096	0.035	0.369	0.000	0.578	14.388	0.000	0.000

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	72	143	0	0	104	0	-1
normalized size	1	1.00	0.95	1.88	0.00	0.00	1.37	0.00	-0.01
time (sec)	N/A	0.079	0.045	0.260	0.000	0.757	4.503	0.000	0.000

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.032	0.327	0.000	0.721	0.000	0.000	0.000

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.032	0.325	0.000	0.720	0.000	0.000	0.000

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.034	0.327	0.000	0.741	0.000	0.000	0.000

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	338	82	285	1760	460	374
normalized size	1	1.00	0.85	4.12	1.00	3.48	21.46	5.61	4.56
time (sec)	N/A	0.118	0.049	0.032	0.317	0.670	1.381	0.221	1.163

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	142	54	128	585	197	160
normalized size	1	1.00	1.00	2.63	1.00	2.37	10.83	3.65	2.96
time (sec)	N/A	0.099	0.132	0.030	0.349	0.505	0.692	0.183	1.001

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	27	24	29	82	41	26
normalized size	1	1.00	0.83	1.12	1.00	1.21	3.42	1.71	1.08
time (sec)	N/A	0.080	0.018	0.025	0.304	0.608	0.253	0.236	1.101

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	100	0	0	44	0	-1
normalized size	1	1.00	1.00	4.55	0.00	0.00	2.00	0.00	-0.05
time (sec)	N/A	0.082	0.011	0.260	0.000	0.641	2.693	0.000	0.000

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	177	0	0	673	0	-1
normalized size	1	1.00	0.96	2.53	0.00	0.00	9.61	0.00	-0.01
time (sec)	N/A	0.104	0.035	0.256	0.000	0.667	29.317	0.000	0.000

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	224	0	0	2152	0	-1
normalized size	1	1.00	0.96	3.20	0.00	0.00	30.74	0.00	-0.01
time (sec)	N/A	0.116	0.034	0.285	0.000	0.708	50.308	0.000	0.000

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	102	377	144	476	0	0	468
normalized size	1	1.00	0.37	1.38	0.53	1.74	0.00	0.00	1.71
time (sec)	N/A	0.230	0.095	0.031	0.347	0.517	0.000	0.000	1.620

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	84	180	80	211	0	0	228
normalized size	1	1.00	0.48	1.03	0.46	1.21	0.00	0.00	1.31
time (sec)	N/A	0.203	0.096	0.030	0.370	0.742	0.000	0.000	1.255

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	49	52	30	80	0	0	51
normalized size	1	1.00	0.60	0.63	0.37	0.98	0.00	0.00	0.62
time (sec)	N/A	0.174	0.033	0.026	0.343	0.731	0.000	0.000	1.071

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.183	0.028	0.336	0.000	0.797	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	107	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.043	0.328	0.000	0.618	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	107	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.041	0.327	0.000	0.549	0.000	0.000	0.000

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	114	0	0	0	381	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	2.80	0.00	-0.01
time (sec)	N/A	0.149	0.043	0.339	0.000	0.571	69.566	0.000	0.000

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	47	0	0	258	0	-1
normalized size	1	1.00	0.91	0.55	0.00	0.00	3.04	0.00	-0.01
time (sec)	N/A	0.114	0.070	0.354	0.000	0.595	19.097	0.000	0.000

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	76	47	0	0	255	0	-1
normalized size	1	1.00	0.90	0.56	0.00	0.00	3.04	0.00	-0.01
time (sec)	N/A	0.115	0.049	0.340	0.000	0.745	13.509	0.000	0.000

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	47	0	0	301	0	-1
normalized size	1	1.00	1.03	0.81	0.00	0.00	5.19	0.00	-0.02
time (sec)	N/A	0.061	0.024	0.339	0.000	0.579	12.100	0.000	0.000

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	44	0	0	292	0	-1
normalized size	1	1.00	1.00	0.75	0.00	0.00	4.95	0.00	-0.02
time (sec)	N/A	0.039	0.015	0.344	0.000	0.633	11.473	0.000	0.000

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	74	90	0	0	286	0	-1
normalized size	1	1.00	1.03	1.25	0.00	0.00	3.97	0.00	-0.01
time (sec)	N/A	0.093	0.028	0.377	0.000	0.592	30.723	0.000	0.000

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	77	93	0	0	280	0	-1
normalized size	1	1.00	1.03	1.24	0.00	0.00	3.73	0.00	-0.01
time (sec)	N/A	0.100	0.028	0.362	0.000	0.659	12.387	0.000	0.000

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	112	0	0	287	0	-1
normalized size	1	1.00	1.03	1.44	0.00	0.00	3.68	0.00	-0.01
time (sec)	N/A	0.099	0.022	0.402	0.000	0.822	56.395	0.000	0.000

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	105	0	0	0	272	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	2.03	0.00	-0.01
time (sec)	N/A	0.181	0.093	0.333	0.000	0.565	54.206	0.000	0.000

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	102	0	0	0	269	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	2.02	0.00	-0.01
time (sec)	N/A	0.181	0.091	0.334	0.000	0.735	16.697	0.000	0.000

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	88	0	0	0	314	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	3.27	0.00	-0.01
time (sec)	N/A	0.109	0.037	0.329	0.000	0.766	17.587	0.000	0.000

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	306	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	3.56	0.00	-0.01
time (sec)	N/A	0.064	0.018	0.327	0.000	0.732	23.175	0.000	0.000

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	0	0	0	299	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	2.72	0.00	-0.01
time (sec)	N/A	0.160	0.029	0.326	0.000	0.677	17.332	0.000	0.000

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	105	0	0	0	294	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	2.60	0.00	-0.01
time (sec)	N/A	0.167	0.038	0.329	0.000	0.625	12.520	0.000	0.000

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	0	0	0	301	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	2.59	0.00	-0.01
time (sec)	N/A	0.161	0.023	0.330	0.000	0.611	13.758	0.000	0.000

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	23	23	23	24	23	20
normalized size	1	1.00	0.76	0.79	0.79	0.79	0.83	0.79	0.69
time (sec)	N/A	0.055	0.017	0.024	0.368	0.526	0.072	0.158	0.894

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	23	23	23	26	23	20
normalized size	1	1.00	0.76	0.79	0.79	0.79	0.90	0.79	0.69
time (sec)	N/A	0.055	0.018	0.025	0.334	0.627	0.074	0.189	0.036

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	23	23	23	24	23	20
normalized size	1	1.00	0.76	0.79	0.79	0.79	0.83	0.79	0.69
time (sec)	N/A	0.053	0.015	0.026	0.332	0.519	0.074	0.209	0.036

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	23	23	23	26	23	20
normalized size	1	1.00	0.76	0.79	0.79	0.79	0.90	0.79	0.69
time (sec)	N/A	0.040	0.015	0.025	0.332	0.566	0.071	0.211	0.034

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	19	14	19	19	19	19	17
normalized size	1	1.00	1.27	0.93	1.27	1.27	1.27	1.27	1.13
time (sec)	N/A	0.019	0.012	0.023	0.302	0.667	0.068	0.178	0.031

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	20	19	19	20	20	19
normalized size	1	1.00	0.90	0.95	0.90	0.90	0.95	0.95	0.90
time (sec)	N/A	0.049	0.015	0.027	0.320	1.023	0.099	0.288	0.035

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	19	23	17	20	20
normalized size	1	1.00	0.95	1.05	1.00	1.21	0.89	1.05	1.05
time (sec)	N/A	0.049	0.013	0.030	0.338	0.612	0.116	0.186	0.886

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	20	25	22	21	22
normalized size	1	1.00	0.96	0.96	0.87	1.09	0.96	0.91	0.96
time (sec)	N/A	0.052	0.018	0.030	0.329	1.005	0.149	0.198	0.050

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	23	21	21	24	21	21
normalized size	1	1.00	1.00	1.53	1.40	1.40	1.60	1.40	1.40
time (sec)	N/A	0.045	0.009	0.030	0.313	0.476	0.155	0.352	0.042

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	32	33	40	40	41	40	40
normalized size	1	1.00	0.67	0.69	0.83	0.83	0.85	0.83	0.83
time (sec)	N/A	0.088	0.025	0.025	0.351	0.523	0.085	0.167	0.061

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	33	40	40	44	40	40
normalized size	1	1.00	0.83	0.69	0.83	0.83	0.92	0.83	0.83
time (sec)	N/A	0.089	0.023	0.023	0.305	0.538	0.085	0.399	0.053

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	32	33	40	40	41	40	40
normalized size	1	1.00	0.67	0.69	0.83	0.83	0.85	0.83	0.83
time (sec)	N/A	0.090	0.021	0.030	0.320	0.660	0.086	0.165	0.048

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	32	33	40	40	44	40	40
normalized size	1	1.00	0.67	0.69	0.83	0.83	0.92	0.83	0.83
time (sec)	N/A	0.067	0.019	0.024	0.339	0.548	0.085	0.198	0.050

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	28	36	36	36	36	36
normalized size	1	1.00	0.66	0.80	1.03	1.03	1.03	1.03	1.03
time (sec)	N/A	0.037	0.015	0.025	0.318	0.710	0.083	1.406	0.050

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	33	37	36	36	39	37	36
normalized size	1	1.00	0.82	0.92	0.90	0.90	0.98	0.92	0.90
time (sec)	N/A	0.075	0.015	0.026	0.334	0.621	0.124	0.155	0.042

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	39	41	36	40	39
normalized size	1	1.00	1.00	0.98	0.95	1.00	0.88	0.98	0.95
time (sec)	N/A	0.082	0.012	0.033	0.345	0.664	0.137	0.219	0.042

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	31	37	37	39	37	29
normalized size	1	1.00	1.00	1.82	2.18	2.18	2.29	2.18	1.71
time (sec)	N/A	0.069	0.015	0.029	0.302	0.506	0.142	0.563	0.044

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	36	40	37	37	32
normalized size	1	1.00	1.00	0.97	0.92	1.03	0.95	0.95	0.82
time (sec)	N/A	0.080	0.015	0.032	0.310	0.880	0.189	0.448	0.044

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	40	40	42	41	41	32
normalized size	1	1.00	0.81	0.93	0.93	0.98	0.95	0.95	0.74
time (sec)	N/A	0.079	0.018	0.031	0.312	0.667	0.223	0.226	0.053

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	23	31	40	40	42	40	40
normalized size	1	1.00	0.55	0.74	0.95	0.95	1.00	0.95	0.95
time (sec)	N/A	0.081	0.010	0.030	0.298	0.542	0.249	0.177	0.036

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	70	57	73	73	76	73	73
normalized size	1	1.00	0.80	0.66	0.84	0.84	0.87	0.84	0.84
time (sec)	N/A	0.102	0.036	0.023	0.343	0.722	0.099	0.226	0.894

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	70	57	73	73	82	73	73
normalized size	1	1.00	0.80	0.66	0.84	0.84	0.94	0.84	0.84
time (sec)	N/A	0.099	0.034	0.024	0.360	0.585	0.097	0.186	0.034

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	57	73	73	78	73	73
normalized size	1	1.00	0.83	0.68	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.103	0.031	0.025	0.306	0.626	0.094	1.730	0.034

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	70	57	73	73	82	73	73
normalized size	1	1.00	1.01	0.83	1.06	1.06	1.19	1.06	1.06
time (sec)	N/A	0.080	0.029	0.026	0.315	0.485	0.096	0.173	0.034

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	52	69	69	70	69	69
normalized size	1	1.00	0.60	1.00	1.33	1.33	1.35	1.33	1.33
time (sec)	N/A	0.050	0.019	0.026	0.331	0.732	0.091	0.183	0.035

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	51	70	69	69	76	70	69
normalized size	1	1.00	0.65	0.89	0.87	0.87	0.96	0.89	0.87
time (sec)	N/A	0.086	0.028	0.026	0.304	1.358	0.153	0.150	0.041

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	59	71	70	75	70	71	70
normalized size	1	1.00	0.78	0.93	0.92	0.99	0.92	0.93	0.92
time (sec)	N/A	0.091	0.025	0.030	0.301	0.678	0.167	0.195	0.043

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	71	69	75	75	70	71
normalized size	1	1.00	0.74	0.91	0.88	0.96	0.96	0.90	0.91
time (sec)	N/A	0.095	0.024	0.034	0.308	0.671	0.200	1.037	0.040

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	69	70	74	70	71	69
normalized size	1	1.00	0.76	0.96	0.97	1.03	0.97	0.99	0.96
time (sec)	N/A	0.093	0.028	0.032	0.306	0.635	0.233	0.205	0.035

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	39	59	79	79	87	79	79
normalized size	1	1.00	0.57	0.86	1.14	1.14	1.26	1.14	1.14
time (sec)	N/A	0.056	0.025	0.024	0.326	0.585	0.100	0.307	0.039

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	61	57	59	53	70	64
normalized size	1	1.00	0.78	0.97	0.90	0.94	0.84	1.11	1.02
time (sec)	N/A	0.112	0.039	0.032	0.304	0.537	0.194	0.185	0.050

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	41	51	49	51	44	53	54
normalized size	1	1.00	0.77	0.96	0.92	0.96	0.83	1.00	1.02
time (sec)	N/A	0.104	0.030	0.033	0.309	0.583	0.175	0.235	0.891

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	28	39	40	42	32	39	38
normalized size	1	1.00	0.72	1.00	1.03	1.08	0.82	1.00	0.97
time (sec)	N/A	0.096	0.024	0.032	0.318	0.626	0.157	0.884	0.050

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	23	30	31	30	24	30	27
normalized size	1	1.00	0.77	1.00	1.03	1.00	0.80	1.00	0.90
time (sec)	N/A	0.065	0.019	0.031	0.317	0.640	0.132	0.231	0.042

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	16	15	15	12	15	13
normalized size	1	1.00	1.20	1.07	1.00	1.00	0.80	1.00	0.87
time (sec)	N/A	0.033	0.011	0.025	0.341	0.450	0.138	0.174	0.892

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	31	30	35	19	32	22
normalized size	1	1.00	0.77	1.00	0.97	1.13	0.61	1.03	0.71
time (sec)	N/A	0.087	0.024	0.033	0.313	0.645	0.220	0.210	0.906

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	43	42	57	31	45	34
normalized size	1	1.00	0.81	1.00	0.98	1.33	0.72	1.05	0.79
time (sec)	N/A	0.094	0.034	0.033	0.310	0.708	0.238	0.447	0.068

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	58	56	75	46	56	47
normalized size	1	1.00	0.87	0.97	0.93	1.25	0.77	0.93	0.78
time (sec)	N/A	0.101	0.051	0.035	0.313	0.543	0.269	0.425	0.076

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	69	64	83	54	64	55
normalized size	1	1.00	1.00	0.97	0.90	1.17	0.76	0.90	0.77
time (sec)	N/A	0.110	0.041	0.036	0.311	0.574	0.291	0.311	0.930

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	69	75	95	71	61	77
normalized size	1	1.00	0.86	0.87	0.95	1.20	0.90	0.77	0.97
time (sec)	N/A	0.119	0.045	0.035	0.344	0.613	0.459	0.156	0.100

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	60	66	79	63	52	65
normalized size	1	1.00	0.76	0.86	0.94	1.13	0.90	0.74	0.93
time (sec)	N/A	0.107	0.043	0.035	0.319	0.504	0.347	0.341	0.208

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	66	78	61	52	49
normalized size	1	1.00	0.69	1.18	1.29	1.53	1.20	1.02	0.96
time (sec)	N/A	0.105	0.029	0.033	0.316	0.583	0.296	0.301	0.068

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	60	59	77	53	47	42
normalized size	1	1.00	0.67	1.18	1.16	1.51	1.04	0.92	0.82
time (sec)	N/A	0.080	0.026	0.035	0.344	0.535	0.275	0.208	0.923

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	63	76	56	51	47
normalized size	1	1.00	0.69	1.18	1.24	1.49	1.10	1.00	0.92
time (sec)	N/A	0.053	0.020	0.036	0.346	0.513	0.297	0.341	0.062

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	54	60	89	58	50	60
normalized size	1	1.00	0.75	0.84	0.94	1.39	0.91	0.78	0.94
time (sec)	N/A	0.099	0.043	0.036	0.347	0.555	0.484	0.184	0.961

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	57	68	76	120	76	65	76
normalized size	1	1.00	0.73	0.87	0.97	1.54	0.97	0.83	0.97
time (sec)	N/A	0.112	0.054	0.036	0.309	0.650	0.579	0.199	0.111

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	72	87	92	141	92	79	91
normalized size	1	1.00	0.73	0.88	0.93	1.42	0.93	0.80	0.92
time (sec)	N/A	0.122	0.100	0.036	0.314	0.669	0.613	1.086	0.116

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	87	98	100	150	100	87	100
normalized size	1	1.00	0.79	0.89	0.91	1.36	0.91	0.79	0.91
time (sec)	N/A	0.134	0.088	0.036	0.312	0.628	0.652	0.160	0.985

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	87	90	94	123	92	75	94
normalized size	1	1.00	0.83	0.86	0.90	1.17	0.88	0.71	0.90
time (sec)	N/A	0.133	0.060	0.037	0.319	0.648	0.533	0.726	0.327

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	64	90	94	123	88	75	78
normalized size	1	1.00	0.74	1.05	1.09	1.43	1.02	0.87	0.91
time (sec)	N/A	0.138	0.042	0.035	0.330	0.546	0.454	0.188	0.074

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	64	90	94	123	88	75	77
normalized size	1	1.00	0.74	1.05	1.09	1.43	1.02	0.87	0.90
time (sec)	N/A	0.123	0.042	0.036	0.344	0.609	0.430	0.186	0.928

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	54	49	49	48	45	28
normalized size	1	1.00	0.97	1.74	1.58	1.58	1.55	1.45	0.90
time (sec)	N/A	0.089	0.030	0.032	0.319	0.564	0.362	0.193	0.073

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	75	93	123	87	74	73
normalized size	1	1.00	0.88	1.10	1.37	1.81	1.28	1.09	1.07
time (sec)	N/A	0.088	0.033	0.037	0.335	0.603	0.449	0.178	0.917

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	63	90	91	121	83	74	72
normalized size	1	1.00	0.73	1.05	1.06	1.41	0.97	0.86	0.84
time (sec)	N/A	0.068	0.032	0.038	0.324	0.653	0.433	0.208	0.071

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	66	78	89	143	87	73	88
normalized size	1	1.00	0.71	0.84	0.96	1.54	0.94	0.78	0.95
time (sec)	N/A	0.119	0.071	0.038	0.316	0.794	0.698	0.362	0.954

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	78	94	104	175	104	88	104
normalized size	1	1.00	0.72	0.86	0.95	1.61	0.95	0.81	0.95
time (sec)	N/A	0.135	0.087	0.039	0.312	1.090	0.744	0.190	0.992

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	98	117	120	197	121	102	119
normalized size	1	1.00	0.73	0.87	0.90	1.47	0.90	0.76	0.89
time (sec)	N/A	0.151	0.121	0.040	0.332	0.593	0.882	0.186	1.015

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	82	120	140	217	143	91	122
normalized size	1	1.00	0.68	0.99	1.16	1.79	1.18	0.75	1.01
time (sec)	N/A	0.091	0.060	0.037	0.339	0.589	0.608	1.119	0.137

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	96	210	120	184	0	0	-1
normalized size	1	1.00	0.70	1.53	0.88	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.329	0.144	0.045	0.600	0.688	0.000	0.000	0.000

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	186	96	168	0	84	-1
normalized size	1	1.00	0.79	1.66	0.86	1.50	0.00	0.75	-0.01
time (sec)	N/A	0.287	0.117	0.040	0.645	0.745	0.000	0.193	0.000

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	164	74	150	0	73	-1
normalized size	1	1.00	0.94	1.95	0.88	1.79	0.00	0.87	-0.01
time (sec)	N/A	0.182	0.095	0.040	0.517	0.836	0.000	0.233	0.000

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	134	52	134	0	62	-1
normalized size	1	1.00	0.88	1.56	0.60	1.56	0.00	0.72	-0.01
time (sec)	N/A	0.078	0.055	0.037	0.457	0.698	0.000	0.230	0.000

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	99	128	0	196	0	97	-1
normalized size	1	1.00	1.27	1.64	0.00	2.51	0.00	1.24	-0.01
time (sec)	N/A	0.245	0.079	0.041	0.000	0.656	0.000	0.187	0.000

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	106	211	0	212	0	133	-1
normalized size	1	1.00	1.29	2.57	0.00	2.59	0.00	1.62	-0.01
time (sec)	N/A	0.247	0.093	0.046	0.000	1.010	0.000	1.843	0.000

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	79	239	0	148	0	201	-1
normalized size	1	1.00	1.01	3.06	0.00	1.90	0.00	2.58	-0.01
time (sec)	N/A	0.246	0.104	0.045	0.000	1.078	0.000	0.188	0.000

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	82	262	140	164	0	250	-1
normalized size	1	1.00	0.81	2.59	1.39	1.62	0.00	2.48	-0.01
time (sec)	N/A	0.272	0.120	0.050	0.470	0.569	0.000	0.834	0.000

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	95	287	0	180	0	324	-1
normalized size	1	1.00	0.73	2.21	0.00	1.38	0.00	2.49	-0.01
time (sec)	N/A	0.309	0.144	0.053	0.000	0.681	0.000	0.199	0.000

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	113	268	213	234	420	117	-1
normalized size	1	1.00	0.70	1.66	1.32	1.45	2.61	0.73	-0.01
time (sec)	N/A	0.336	0.144	0.046	0.753	0.740	18.314	0.345	0.000

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	105	244	189	216	515	107	-1
normalized size	1	1.00	0.77	1.79	1.39	1.59	3.79	0.79	-0.01
time (sec)	N/A	0.308	0.126	0.041	0.646	0.640	29.497	0.208	0.000

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	97	222	167	198	306	98	-1
normalized size	1	1.00	0.87	2.00	1.50	1.78	2.76	0.88	-0.01
time (sec)	N/A	0.195	0.106	0.037	0.530	0.877	16.119	0.203	0.000

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	117	186	146	180	340	85	-1
normalized size	1	1.00	1.09	1.74	1.36	1.68	3.18	0.79	-0.01
time (sec)	N/A	0.086	0.094	0.036	0.523	0.676	18.344	0.762	0.000

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	115	179	0	233	267	116	-1
normalized size	1	1.00	1.14	1.77	0.00	2.31	2.64	1.15	-0.01
time (sec)	N/A	0.286	0.111	0.043	0.000	0.694	10.399	1.060	0.000

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	124	286	0	249	350	165	-1
normalized size	1	1.00	1.11	2.55	0.00	2.22	3.12	1.47	-0.01
time (sec)	N/A	0.285	0.153	0.045	0.000	0.702	19.257	0.999	0.000

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	129	316	0	267	366	274	-1
normalized size	1	1.00	1.07	2.61	0.00	2.21	3.02	2.26	-0.01
time (sec)	N/A	0.289	0.213	0.046	0.000	0.663	8.224	0.226	0.000

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	127	339	0	265	359	259	-1
normalized size	1	1.00	1.10	2.95	0.00	2.30	3.12	2.25	-0.01
time (sec)	N/A	0.282	0.152	0.051	0.000	0.689	18.177	0.549	0.000

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	96	364	0	191	447	371	-1
normalized size	1	1.00	0.91	3.43	0.00	1.80	4.22	3.50	-0.01
time (sec)	N/A	0.257	0.153	0.059	0.000	0.599	14.700	0.217	0.000

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	104	388	221	209	484	414	-1
normalized size	1	1.00	0.79	2.96	1.69	1.60	3.69	3.16	-0.01
time (sec)	N/A	0.293	0.168	0.067	0.457	0.654	24.934	0.213	0.000

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	109	412	0	227	636	443	-1
normalized size	1	1.00	0.70	2.64	0.00	1.46	4.08	2.84	-0.01
time (sec)	N/A	0.330	0.241	0.079	0.000	0.794	15.988	0.571	0.000

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	120	436	287	245	660	529	-1
normalized size	1	1.00	0.66	2.41	1.59	1.35	3.65	2.92	-0.01
time (sec)	N/A	0.371	0.190	0.099	0.451	0.739	39.568	0.270	0.000

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	131	330	239	307	763	155	-1
normalized size	1	1.00	0.70	1.76	1.28	1.64	4.08	0.83	-0.01
time (sec)	N/A	0.370	0.221	0.048	0.791	0.765	84.269	0.390	0.000

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	123	306	215	285	687	143	-1
normalized size	1	1.00	0.76	1.89	1.33	1.76	4.24	0.88	-0.01
time (sec)	N/A	0.325	0.175	0.042	0.672	0.880	21.276	0.278	0.000

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	115	284	193	263	586	131	-1
normalized size	1	1.00	0.84	2.07	1.41	1.92	4.28	0.96	-0.01
time (sec)	N/A	0.216	0.155	0.040	0.649	0.672	70.643	0.893	0.000

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	135	242	172	241	478	116	-1
normalized size	1	1.00	1.04	1.86	1.32	1.85	3.68	0.89	-0.01
time (sec)	N/A	0.098	0.144	0.037	0.547	0.702	15.691	0.252	0.000

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	136	235	0	295	508	150	-1
normalized size	1	1.00	1.00	1.73	0.00	2.17	3.74	1.10	-0.01
time (sec)	N/A	0.334	0.146	0.043	0.000	0.543	33.540	0.704	0.000

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	143	367	0	313	483	195	-1
normalized size	1	1.00	1.01	2.60	0.00	2.22	3.43	1.38	-0.01
time (sec)	N/A	0.335	0.236	0.045	0.000	0.751	69.466	0.980	0.000

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	151	399	0	329	401	302	-1
normalized size	1	1.00	1.00	2.64	0.00	2.18	2.66	2.00	-0.01
time (sec)	N/A	0.332	0.266	0.049	0.000	0.677	118.627	0.995	0.000

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	149	423	0	329	478	292	-1
normalized size	1	1.00	0.96	2.73	0.00	2.12	3.08	1.88	-0.01
time (sec)	N/A	0.342	0.276	0.054	0.000	0.735	26.912	0.232	0.000

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	151	447	0	329	575	440	-1
normalized size	1	1.00	0.97	2.88	0.00	2.12	3.71	2.84	-0.01
time (sec)	N/A	0.334	0.275	0.060	0.000	0.835	65.525	0.374	0.000

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	151	296	194	286	1091	141	-1
normalized size	1	1.00	0.99	1.93	1.27	1.87	7.13	0.92	-0.01
time (sec)	N/A	0.110	0.141	0.039	0.566	0.842	27.968	0.343	0.000

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	97	149	113	200	0	0	-1
normalized size	1	1.00	0.71	1.09	0.82	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.356	0.179	0.044	0.545	0.620	0.000	0.000	0.000

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	126	89	184	0	0	-1
normalized size	1	1.00	1.01	1.35	0.96	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.107	0.041	0.502	0.717	0.000	0.000	0.000

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	103	67	166	0	0	-1
normalized size	1	1.00	0.93	1.23	0.80	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.112	0.037	0.457	0.786	0.000	0.000	0.000

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	82	80	45	152	0	0	-1
normalized size	1	1.00	1.37	1.33	0.75	2.53	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.036	0.036	0.450	0.671	0.000	0.000	0.000

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	66	80	0	147	0	0	-1
normalized size	1	1.00	1.27	1.54	0.00	2.83	0.00	0.00	-0.02
time (sec)	N/A	0.228	0.124	0.038	0.000	0.925	0.000	0.000	0.000

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	99	0	178	0	0	-1
normalized size	1	1.00	1.01	1.29	0.00	2.31	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.137	0.043	0.000	0.702	0.000	0.000	0.000

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	124	0	208	0	0	-1
normalized size	1	1.00	0.86	1.14	0.00	1.91	0.00	0.00	-0.01
time (sec)	N/A	0.323	0.167	0.045	0.000	0.647	0.000	0.000	0.000

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	101	150	0	224	0	0	-1
normalized size	1	1.00	0.75	1.11	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.407	0.175	0.047	0.000	0.585	0.000	0.000	0.000

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	90	190	260	229	0	0	-1
normalized size	1	1.00	0.77	1.62	2.22	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.321	0.171	0.042	0.550	0.564	0.000	0.000	0.000

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	166	230	215	0	44	-1
normalized size	1	1.00	0.88	1.78	2.47	2.31	0.00	0.47	-0.01
time (sec)	N/A	0.263	0.121	0.043	0.511	0.651	0.000	0.643	0.000

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	38	32	218	50	0	117	34
normalized size	1	1.00	0.63	0.53	3.63	0.83	0.00	1.95	0.57
time (sec)	N/A	0.117	0.061	0.032	0.479	0.682	0.000	0.310	1.017

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	63	31	196	47	0	148	33
normalized size	1	1.00	1.24	0.61	3.84	0.92	0.00	2.90	0.65
time (sec)	N/A	0.064	0.032	0.029	0.380	0.786	0.000	0.352	0.951

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	75	152	0	202	0	0	-1
normalized size	1	1.00	0.91	1.85	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.118	0.040	0.000	0.505	0.000	0.000	0.000

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	89	178	0	238	0	100	-1
normalized size	1	1.00	0.82	1.65	0.00	2.20	0.00	0.93	-0.01
time (sec)	N/A	0.337	0.138	0.043	0.000	0.678	0.000	0.447	0.000

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	105	205	0	266	0	204	-1
normalized size	1	1.00	0.74	1.44	0.00	1.87	0.00	1.44	-0.01
time (sec)	N/A	0.420	0.146	0.048	0.000	0.868	0.000	0.410	0.000

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	53	47	218	75	0	0	56
normalized size	1	1.00	0.72	0.64	2.95	1.01	0.00	0.00	0.76
time (sec)	N/A	0.072	0.036	0.030	0.399	0.774	0.000	0.000	1.059

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	64	242	124	0	0	133
normalized size	1	1.00	0.99	0.66	2.49	1.28	0.00	0.00	1.37
time (sec)	N/A	0.080	0.051	0.033	0.448	0.975	0.000	0.000	1.098

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	88	476	304	540	3009	0	531
normalized size	1	1.00	0.73	3.97	2.53	4.50	25.08	0.00	4.42
time (sec)	N/A	0.113	0.068	0.031	0.386	0.589	3.472	0.000	1.235

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	146	114	179	706	0	173
normalized size	1	1.00	1.03	2.18	1.70	2.67	10.54	0.00	2.58
time (sec)	N/A	0.087	0.110	0.029	0.373	0.675	1.851	0.000	1.016

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	34	74	62	82	299	0	92
normalized size	1	1.00	0.81	1.76	1.48	1.95	7.12	0.00	2.19
time (sec)	N/A	0.063	0.058	0.028	0.378	0.547	1.104	0.000	0.941

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.081	0.015	0.416	0.000	0.725	0.000	0.000	0.000

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	92	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.071	0.416	0.000	0.729	0.000	0.000	0.000

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	194	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.157	0.415	0.000	0.657	0.000	0.000	0.000

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	180	0	0	0	226	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	1.28	0.00	-0.01
time (sec)	N/A	0.314	0.224	0.414	0.000	0.705	23.158	0.000	0.000

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	158	0	0	0	172	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	1.00	0.00	-0.01
time (sec)	N/A	0.300	0.150	0.418	0.000	0.553	10.262	0.000	0.000

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	130	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.291	0.227	0.411	0.000	0.652	0.000	0.000	0.000

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	66	0	0	0	0	0	-1
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.293	0.114	0.411	0.000	0.676	0.000	0.000	0.000

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	173	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.250	0.416	0.000	0.593	0.000	0.000	0.000

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	68	0	0	0	653	0	-1
normalized size	1	1.00	1.24	0.00	0.00	0.00	11.87	0.00	-0.02
time (sec)	N/A	0.063	0.018	0.469	0.000	0.636	12.138	0.000	0.000

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	70	191	169	89	483	78	140
normalized size	1	1.00	0.51	1.40	1.24	0.65	3.55	0.57	1.03
time (sec)	N/A	0.276	0.117	0.089	0.426	0.592	21.364	0.200	0.915

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	62	170	148	80	371	69	119
normalized size	1	1.00	0.56	1.53	1.33	0.72	3.34	0.62	1.07
time (sec)	N/A	0.247	0.090	0.069	0.422	0.613	18.454	0.232	0.030

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	54	148	126	71	326	58	98
normalized size	1	1.00	0.45	1.23	1.05	0.59	2.72	0.48	0.82
time (sec)	N/A	0.096	0.071	0.054	0.428	0.461	16.328	1.607	0.033

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	57	125	106	62	218	46	74
normalized size	1	1.00	0.63	1.37	1.16	0.68	2.40	0.51	0.81
time (sec)	N/A	0.057	0.052	0.048	0.427	0.635	14.314	0.287	0.037

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	49	121	111	69	197	76	70
normalized size	1	1.00	0.74	1.83	1.68	1.05	2.98	1.15	1.06
time (sec)	N/A	0.195	0.049	0.045	0.428	0.596	13.684	0.211	0.914

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	52	122	111	80	150	131	79
normalized size	1	1.00	0.79	1.85	1.68	1.21	2.27	1.98	1.20
time (sec)	N/A	0.191	0.069	0.046	0.425	0.673	13.977	0.232	0.041

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	60	125	116	85	223	179	83
normalized size	1	1.00	0.79	1.64	1.53	1.12	2.93	2.36	1.09
time (sec)	N/A	0.194	0.078	0.047	0.429	0.511	10.011	0.389	0.049

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	60	184	128	66	267	218	82
normalized size	1	1.00	0.64	1.96	1.36	0.70	2.84	2.32	0.87
time (sec)	N/A	0.200	0.063	0.043	0.331	0.645	28.084	0.230	0.894

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	97	231	149	75	411	280	103
normalized size	1	1.00	0.84	2.01	1.30	0.65	3.57	2.43	0.90
time (sec)	N/A	0.223	0.118	0.049	0.327	0.554	12.003	0.182	0.881

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	110	292	170	84	518	332	124
normalized size	1	1.00	0.76	2.03	1.18	0.58	3.60	2.31	0.86
time (sec)	N/A	0.250	0.133	0.052	0.330	0.642	49.285	0.572	0.040

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	75	183	164	93	340	78	128
normalized size	1	1.00	0.62	1.51	1.36	0.77	2.81	0.64	1.06
time (sec)	N/A	0.072	0.104	0.069	0.425	0.833	20.165	0.235	0.034

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	91	229	210	114	632	102	174
normalized size	1	1.00	0.64	1.60	1.47	0.80	4.42	0.71	1.22
time (sec)	N/A	0.082	0.123	0.112	0.436	0.635	25.475	0.225	0.048

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	107	275	256	137	996	125	220
normalized size	1	1.00	0.65	1.67	1.55	0.83	6.04	0.76	1.33
time (sec)	N/A	0.090	0.148	0.252	0.422	0.651	46.240	1.012	0.925

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	78	178	364	103	0	0	133
normalized size	1	1.00	0.82	1.87	3.83	1.08	0.00	0.00	1.40
time (sec)	N/A	0.187	0.086	0.040	0.569	0.623	0.000	0.000	0.940

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	155	241	96	0	112	108
normalized size	1	1.00	1.00	2.21	3.44	1.37	0.00	1.60	1.54
time (sec)	N/A	0.086	0.063	0.038	0.521	0.659	0.000	0.345	0.951

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	29	28	329	54	0	66	32
normalized size	1	1.00	1.61	1.56	18.28	3.00	0.00	3.67	1.78
time (sec)	N/A	0.031	0.012	0.029	0.488	0.530	0.000	0.340	0.065

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	43	49	0	91	0	145	127
normalized size	1	1.00	0.44	0.51	0.00	0.94	0.00	1.49	1.31
time (sec)	N/A	0.074	0.018	0.029	0.000	0.553	0.000	0.276	0.912

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	61	63	0	144	0	0	125
normalized size	1	1.00	0.51	0.53	0.00	1.21	0.00	0.00	1.05
time (sec)	N/A	0.089	0.021	0.032	0.000	0.549	0.000	0.000	1.196

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	75	74	0	198	0	0	156
normalized size	1	1.00	0.53	0.52	0.00	1.40	0.00	0.00	1.11
time (sec)	N/A	0.091	0.030	0.032	0.000	0.603	0.000	0.000	1.263

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	79	88	0	399	0	0	-1
normalized size	1	1.00	0.35	0.39	0.00	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.052	0.040	0.000	0.669	0.000	0.000	0.000

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	70	79	0	381	0	0	-1
normalized size	1	1.00	0.38	0.43	0.00	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.041	0.040	0.000	0.741	0.000	0.000	0.000

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	64	72	0	367	0	0	-1
normalized size	1	1.00	0.43	0.48	0.00	2.45	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.037	0.040	0.000	0.796	0.000	0.000	0.000

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	54	63	0	345	0	0	-1
normalized size	1	1.00	0.49	0.57	0.00	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.022	0.038	0.000	0.678	0.000	0.000	0.000

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	46	55	0	0	0	0	-1
normalized size	1	1.00	0.44	0.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.029	0.042	0.000	0.854	0.000	0.000	0.000

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	51	60	0	0	0	0	-1
normalized size	1	1.00	0.47	0.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.032	0.047	0.000	0.711	0.000	0.000	0.000

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	63	73	160	450	0	0	-1
normalized size	1	1.00	0.42	0.49	1.07	3.02	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.045	0.045	0.376	0.729	0.000	0.000	0.000

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	73	81	216	478	0	0	-1
normalized size	1	1.00	0.39	0.43	1.15	2.54	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.046	0.047	0.378	1.029	0.000	0.000	0.000

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	79	89	306	504	0	0	-1
normalized size	1	1.00	0.35	0.40	1.37	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.220	0.053	0.046	0.387	0.884	0.000	0.000	0.000

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	58	50	0	67	0	0	54
normalized size	1	1.00	1.32	1.14	0.00	1.52	0.00	0.00	1.23
time (sec)	N/A	0.082	0.030	0.030	0.000	0.638	0.000	0.000	1.051

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	52	81	237	98	0	0	85
normalized size	1	1.00	0.56	0.87	2.55	1.05	0.00	0.00	0.91
time (sec)	N/A	0.091	0.041	0.029	0.388	0.604	0.000	0.000	1.084

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	60	97	323	120	0	0	107
normalized size	1	1.00	0.43	0.70	2.32	0.86	0.00	0.00	0.77
time (sec)	N/A	0.104	0.049	0.030	0.431	0.990	0.000	0.000	1.089

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	68	97	409	120	0	0	106
normalized size	1	1.00	0.37	0.52	2.21	0.65	0.00	0.00	0.57
time (sec)	N/A	0.115	0.053	0.032	0.496	0.693	0.000	0.000	1.092

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	70	0	382	0	0	-1
normalized size	1	1.00	0.61	0.84	0.00	4.60	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.035	0.041	0.000	0.612	0.000	0.000	0.000

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	53	43	0	72	0	0	74
normalized size	1	1.00	1.13	0.91	0.00	1.53	0.00	0.00	1.57
time (sec)	N/A	0.084	0.041	0.027	0.000	0.693	0.000	0.000	1.247

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	73	159	0	459	0	0	-1
normalized size	1	1.00	0.39	0.86	0.00	2.48	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.054	0.046	0.000	0.781	0.000	0.000	0.000

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	101	238	0	565	0	0	-1
normalized size	1	1.00	0.36	0.86	0.00	2.03	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.078	0.051	0.000	0.743	0.000	0.000	0.000

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	74	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	0.050	0.417	0.000	0.483	0.000	0.000	0.000

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	186	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.416	0.168	0.413	0.000	0.673	0.000	0.000	0.000

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	176	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.355	0.205	0.415	0.000	0.650	0.000	0.000	0.000

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	179	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.339	0.240	0.413	0.000	0.986	0.000	0.000	0.000

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	134	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.382	0.416	0.000	0.834	0.000	0.000	0.000

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.032	0.414	0.000	0.671	0.000	0.000	0.000

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	159	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.225	0.483	0.000	1.258	0.000	0.000	0.000

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	133	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.331	0.257	0.423	0.000	0.772	0.000	0.000	0.000

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	154	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.367	0.298	0.414	0.000	0.929	0.000	0.000	0.000

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	39	75	101	101	109	101	101
normalized size	1	1.00	0.59	1.14	1.53	1.53	1.65	1.53	1.53
time (sec)	N/A	0.064	0.034	0.023	0.355	0.451	0.121	0.269	0.932

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	69	92	92	100	92	92
normalized size	1	1.00	0.60	1.33	1.77	1.77	1.92	1.77	1.77
time (sec)	N/A	0.057	0.026	0.025	0.318	0.649	0.112	0.345	0.051

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	45	59	59	63	59	59
normalized size	1	1.00	0.66	1.29	1.69	1.69	1.80	1.69	1.69
time (sec)	N/A	0.039	0.020	0.023	0.334	0.570	0.114	0.162	0.033

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	16	47	47	48	47	47
normalized size	1	1.00	2.18	0.94	2.76	2.76	2.82	2.76	2.76
time (sec)	N/A	0.030	0.019	0.024	0.319	0.592	0.114	0.159	0.029

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	46	36	34	33	37	36	44	33
normalized size	1	1.28	1.00	0.94	0.92	1.03	1.00	1.22	0.92
time (sec)	N/A	0.032	0.011	0.027	0.316	0.635	0.208	0.342	0.050

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	28	19	19	17	12	12
normalized size	1	1.00	1.92	2.15	1.46	1.46	1.31	0.92	0.92
time (sec)	N/A	0.034	0.009	0.030	0.316	0.542	0.236	0.157	0.919

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	16	41	41	42	15	40
normalized size	1	1.00	0.94	0.89	2.28	2.28	2.33	0.83	2.22
time (sec)	N/A	0.035	0.017	0.023	0.322	0.422	0.286	0.627	0.927

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	52	90	102	147	99	68	83
normalized size	1	1.00	0.60	1.03	1.17	1.69	1.14	0.78	0.95
time (sec)	N/A	0.068	0.036	0.034	0.318	0.593	0.600	0.189	0.105

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	80	120	130	191	129	91	111
normalized size	1	1.00	0.66	0.98	1.07	1.57	1.06	0.75	0.91
time (sec)	N/A	0.089	0.057	0.036	0.334	0.752	0.607	0.179	0.974

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	72	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.021	0.426	0.000	0.742	0.000	0.000	0.000

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	107	201	182	137	996	124	220
normalized size	1	1.00	0.84	1.58	1.43	1.08	7.84	0.98	1.73
time (sec)	N/A	0.068	0.150	0.073	0.430	0.655	20.604	0.198	0.929

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	155	136	114	629	101	174
normalized size	1	1.00	0.87	1.48	1.30	1.09	5.99	0.96	1.66
time (sec)	N/A	0.059	0.125	0.047	0.417	0.724	11.909	0.207	0.891

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	109	90	93	337	78	128
normalized size	1	1.00	0.90	1.31	1.08	1.12	4.06	0.94	1.54
time (sec)	N/A	0.051	0.099	0.039	0.446	0.661	6.892	0.202	0.039

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	57	64	45	62	109	46	80
normalized size	1	1.00	1.04	1.16	0.82	1.13	1.98	0.84	1.45
time (sec)	N/A	0.030	0.088	0.031	0.414	0.793	4.437	0.665	0.908

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	27	28	0	31	0	37	46
normalized size	1	1.00	1.69	1.75	0.00	1.94	0.00	2.31	2.88
time (sec)	N/A	0.031	0.010	0.029	0.000	0.593	0.000	0.220	0.045

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	42	0	87	0	0	48
normalized size	1	1.00	0.81	0.79	0.00	1.64	0.00	0.00	0.91
time (sec)	N/A	0.043	0.018	0.029	0.000	0.596	0.000	0.000	0.998

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	59	58	0	141	0	0	64
normalized size	1	1.00	0.79	0.77	0.00	1.88	0.00	0.00	0.85
time (sec)	N/A	0.050	0.028	0.031	0.000	0.836	0.000	0.000	1.084

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	74	0	197	0	0	145
normalized size	1	1.00	0.77	0.76	0.00	2.03	0.00	0.00	1.49
time (sec)	N/A	0.057	0.035	0.031	0.000	0.593	0.000	0.000	1.185

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	91	90	0	249	0	0	177
normalized size	1	1.00	0.76	0.76	0.00	2.09	0.00	0.00	1.49
time (sec)	N/A	0.066	0.041	0.033	0.000	0.754	0.000	0.000	1.330

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	50	68	63	80	0	0	52
normalized size	1	1.00	0.60	0.82	0.76	0.96	0.00	0.00	0.63
time (sec)	N/A	0.178	0.058	0.028	0.366	0.713	0.000	0.000	1.227

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	51	41	50	0	0	-1
normalized size	1	1.00	0.57	0.69	0.55	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.029	0.028	0.348	0.495	0.000	0.000	0.000

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	51	41	50	0	0	-1
normalized size	1	1.00	0.57	0.69	0.55	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.028	0.027	0.340	0.735	0.000	0.000	0.000

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	40	48	48	47	0	0	-1
normalized size	1	1.00	0.58	0.70	0.70	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.020	0.024	0.343	0.595	0.000	0.000	0.000

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	37	47	0	261	0	0	-1
normalized size	1	1.00	0.56	0.71	0.00	3.95	0.00	0.00	-0.02
time (sec)	N/A	0.177	0.027	0.033	0.000	0.766	0.000	0.000	0.000

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	41	49	0	263	0	0	-1
normalized size	1	1.00	0.59	0.71	0.00	3.81	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.030	0.039	0.000	0.790	0.000	0.000	0.000

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	57	65	0	68	0	0	-1
normalized size	1	1.00	0.63	0.71	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.036	0.028	0.000	0.576	0.000	0.000	0.000

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	60	81	0	98	0	0	-1
normalized size	1	1.00	0.43	0.58	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.052	0.034	0.000	0.614	0.000	0.000	0.000

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	68	97	0	120	0	0	-1
normalized size	1	1.00	0.36	0.52	0.00	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.072	0.030	0.000	0.607	0.000	0.000	0.000

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	76	113	0	142	0	0	-1
normalized size	1	1.00	0.32	0.48	0.00	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.119	0.083	0.031	0.000	0.603	0.000	0.000	0.000

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	40	13	227	0	0	-1
normalized size	1	1.00	1.00	1.03	0.33	5.82	0.00	0.00	-0.03
time (sec)	N/A	0.075	0.023	0.033	0.335	0.597	0.000	0.000	0.000

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	59	88	50	343	0	0	-1
normalized size	1	1.00	0.66	0.98	0.56	3.81	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.059	0.046	0.337	0.715	0.000	0.000	0.000

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	83	166	83	451	0	0	-1
normalized size	1	1.00	0.45	0.91	0.45	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.084	0.050	0.337	0.715	0.000	0.000	0.000

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	101	238	135	565	0	0	-1
normalized size	1	1.00	0.37	0.86	0.49	2.05	0.00	0.00	-0.00
time (sec)	N/A	0.138	0.121	0.050	0.347	0.658	0.000	0.000	0.000

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	115	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.061	0.413	0.000	0.832	0.000	0.000	0.000

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.084	0.423	0.000	0.607	0.000	0.000	0.000

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.074	0.426	0.000	0.555	0.000	0.000	0.000

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.030	0.422	0.000	0.646	0.000	0.000	0.000

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	56	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.024	0.420	0.000	0.721	0.000	0.000	0.000

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.034	0.426	0.000	0.654	0.000	0.000	0.000

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.035	0.421	0.000	0.583	0.000	0.000	0.000

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	119	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.076	0.415	0.000	0.568	0.000	0.000	0.000

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	102	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.096	0.418	0.000	0.617	0.000	0.000	0.000

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	88	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.037	0.419	0.000	0.608	0.000	0.000	0.000

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	83	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.036	0.418	0.000	0.622	0.000	0.000	0.000

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	103	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.038	0.420	0.000	0.688	0.000	0.000	0.000

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	102	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.040	0.417	0.000	0.676	0.000	0.000	0.000

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	39	61	81	81	87	78	81
normalized size	1	1.00	0.53	0.84	1.11	1.11	1.19	1.07	1.11
time (sec)	N/A	0.055	0.030	0.026	0.340	0.527	0.092	1.649	0.045

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	31	52	69	69	70	66	69
normalized size	1	1.00	0.56	0.95	1.25	1.25	1.27	1.20	1.25
time (sec)	N/A	0.047	0.026	0.026	0.337	0.592	0.085	0.167	0.038

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	29	37	37	36	54	37
normalized size	1	1.00	0.86	0.78	1.00	1.00	0.97	1.46	1.00
time (sec)	N/A	0.037	0.017	0.025	0.335	0.589	0.077	0.149	0.047

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	20	14	20	20	19	34	17
normalized size	1	1.00	1.25	0.88	1.25	1.25	1.19	2.12	1.06
time (sec)	N/A	0.019	0.013	0.024	0.375	0.674	0.067	0.182	0.040

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	16	14	14	12	15	13
normalized size	1	1.00	1.20	1.07	0.93	0.93	0.80	1.00	0.87
time (sec)	N/A	0.033	0.015	0.025	0.358	0.542	0.123	0.166	0.901

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	60	63	76	56	55	47
normalized size	1	1.00	0.67	1.22	1.29	1.55	1.14	1.12	0.96
time (sec)	N/A	0.050	0.027	0.034	0.360	1.089	0.282	0.280	0.067

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	61	90	91	121	83	97	72
normalized size	1	1.00	0.73	1.07	1.08	1.44	0.99	1.15	0.86
time (sec)	N/A	0.068	0.047	0.037	0.357	0.618	0.413	0.351	0.088

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	80	120	140	217	143	122	120
normalized size	1	1.00	0.67	1.01	1.18	1.82	1.20	1.03	1.01
time (sec)	N/A	0.087	0.066	0.037	0.340	0.891	0.578	0.178	0.986

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	96	202	117	184	0	0	-1
normalized size	1	1.00	0.70	1.47	0.85	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.327	0.154	0.049	0.444	0.599	0.000	0.000	0.000

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	178	93	168	0	221	-1
normalized size	1	1.00	0.79	1.59	0.83	1.50	0.00	1.97	-0.01
time (sec)	N/A	0.284	0.111	0.043	0.431	0.732	0.000	0.411	0.000

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	154	70	150	0	174	-1
normalized size	1	1.00	0.94	1.81	0.82	1.76	0.00	2.05	-0.01
time (sec)	N/A	0.183	0.092	0.039	0.431	0.608	0.000	0.413	0.000

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	99	126	47	134	0	126	-1
normalized size	1	1.00	1.14	1.45	0.54	1.54	0.00	1.45	-0.01
time (sec)	N/A	0.075	0.058	0.036	0.441	0.634	0.000	0.402	0.000

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	99	120	0	196	0	124	-1
normalized size	1	1.00	1.27	1.54	0.00	2.51	0.00	1.59	-0.01
time (sec)	N/A	0.245	0.104	0.041	0.000	0.553	0.000	0.331	0.000

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	106	203	0	211	0	168	-1
normalized size	1	1.00	1.29	2.48	0.00	2.57	0.00	2.05	-0.01
time (sec)	N/A	0.248	0.108	0.043	0.000	0.621	0.000	0.230	0.000

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	76	231	0	149	0	152	-1
normalized size	1	1.00	0.97	2.96	0.00	1.91	0.00	1.95	-0.01
time (sec)	N/A	0.237	0.149	0.047	0.000	0.644	0.000	3.094	0.000

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	253	0	165	0	210	-1
normalized size	1	1.00	0.83	2.56	0.00	1.67	0.00	2.12	-0.01
time (sec)	N/A	0.275	0.149	0.052	0.000	0.594	0.000	0.387	0.000

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	95	279	0	181	0	258	-1
normalized size	1	1.00	0.73	2.15	0.00	1.39	0.00	1.98	-0.01
time (sec)	N/A	0.308	0.163	0.054	0.000	0.700	0.000	0.234	0.000

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	117	174	130	180	340	224	-1
normalized size	1	1.00	1.08	1.61	1.20	1.67	3.15	2.07	-0.01
time (sec)	N/A	0.087	0.114	0.036	0.466	0.799	6.784	0.281	0.000

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	135	226	154	241	478	320	-1
normalized size	1	1.00	1.03	1.73	1.18	1.84	3.65	2.44	-0.01
time (sec)	N/A	0.097	0.172	0.037	0.423	0.602	10.274	0.283	0.000

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	151	276	173	286	1091	416	-1
normalized size	1	1.00	0.98	1.79	1.12	1.86	7.08	2.70	-0.01
time (sec)	N/A	0.116	0.181	0.037	0.421	1.353	20.109	0.436	0.000

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	100	74	40	150	0	107	-1
normalized size	1	1.00	1.64	1.21	0.66	2.46	0.00	1.75	-0.02
time (sec)	N/A	0.067	0.080	0.036	0.452	0.739	0.000	0.211	0.000

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	63	31	60	47	0	82	33
normalized size	1	1.00	1.21	0.60	1.15	0.90	0.00	1.58	0.63
time (sec)	N/A	0.064	0.043	0.027	0.319	0.584	0.000	0.978	0.976

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	47	79	75	0	220	56
normalized size	1	1.00	1.05	0.63	1.05	1.00	0.00	2.93	0.75
time (sec)	N/A	0.071	0.067	0.031	0.325	0.830	0.000	0.280	1.095

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	64	98	124	0	300	133
normalized size	1	1.00	0.98	0.65	1.00	1.27	0.00	3.06	1.36
time (sec)	N/A	0.081	0.085	0.032	0.329	0.910	0.000	0.619	1.116

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	111	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.196	0.337	0.000	1.296	0.000	0.000	0.000

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	74	0	0	0	651	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	12.06	0.00	-0.02
time (sec)	N/A	0.059	0.039	0.382	0.000	0.656	10.901	0.000	0.000

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	107	173	154	136	996	126	220
normalized size	1	1.00	0.64	1.04	0.92	0.81	5.96	0.75	1.32
time (sec)	N/A	0.094	0.179	0.054	0.445	0.583	20.377	0.323	0.070

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	91	127	108	115	632	102	174
normalized size	1	1.00	0.63	0.88	0.74	0.79	4.36	0.70	1.20
time (sec)	N/A	0.087	0.148	0.039	0.452	0.582	13.785	0.197	0.045

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	75	189	147	92	340	78	128
normalized size	1	1.00	0.61	1.54	1.20	0.75	2.76	0.63	1.04
time (sec)	N/A	0.075	0.121	0.036	0.459	0.544	8.103	0.209	0.037

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	70	133	122	63	0	46	74
normalized size	1	1.00	0.75	1.43	1.31	0.68	0.00	0.49	0.80
time (sec)	N/A	0.057	0.104	0.040	0.499	0.614	0.000	0.267	0.040

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	29	28	0	55	0	66	32
normalized size	1	1.00	1.61	1.56	0.00	3.06	0.00	3.67	1.78
time (sec)	N/A	0.033	0.015	0.030	0.000	0.651	0.000	0.383	0.956

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	43	42	0	89	0	145	125
normalized size	1	1.00	0.46	0.45	0.00	0.95	0.00	1.54	1.33
time (sec)	N/A	0.080	0.020	0.029	0.000	1.043	0.000	0.234	0.060

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	59	58	0	144	0	0	125
normalized size	1	1.00	0.51	0.50	0.00	1.24	0.00	0.00	1.08
time (sec)	N/A	0.085	0.028	0.031	0.000	0.575	0.000	0.000	1.155

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	75	74	0	195	0	0	156
normalized size	1	1.00	0.54	0.54	0.00	1.41	0.00	0.00	1.13
time (sec)	N/A	0.093	0.037	0.031	0.000	0.653	0.000	0.000	1.272

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	81	88	0	399	0	0	-1
normalized size	1	1.00	0.36	0.39	0.00	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.067	0.040	0.000	1.118	0.000	0.000	0.000

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	69	79	0	383	0	0	-1
normalized size	1	1.00	0.38	0.43	0.00	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.054	0.038	0.000	0.675	0.000	0.000	0.000

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	63	72	0	367	0	0	-1
normalized size	1	1.00	0.42	0.48	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.050	0.037	0.000	0.785	0.000	0.000	0.000

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	53	63	0	347	0	0	-1
normalized size	1	1.00	0.48	0.57	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.028	0.038	0.000	0.731	0.000	0.000	0.000

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	44	56	0	0	0	0	-1
normalized size	1	1.00	0.43	0.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.028	0.044	0.000	0.615	0.000	0.000	0.000

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	50	60	0	0	0	0	-1
normalized size	1	1.00	0.47	0.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.035	0.044	0.000	1.296	0.000	0.000	0.000

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	62	73	0	447	0	0	-1
normalized size	1	1.00	0.42	0.49	0.00	3.02	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.045	0.045	0.000	0.738	0.000	0.000	0.000

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	72	81	0	477	0	0	-1
normalized size	1	1.00	0.39	0.43	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.047	0.046	0.000	0.780	0.000	0.000	0.000

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	77	89	0	503	0	0	-1
normalized size	1	1.00	0.35	0.40	0.00	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.058	0.046	0.000	1.022	0.000	0.000	0.000

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	68	97	0	120	0	0	-1
normalized size	1	1.00	0.36	0.51	0.00	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.058	0.030	0.000	0.711	0.000	0.000	0.000

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	60	97	0	120	0	0	-1
normalized size	1	1.00	0.42	0.68	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.054	0.030	0.000	1.097	0.000	0.000	0.000

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	52	81	0	98	0	0	-1
normalized size	1	1.00	0.55	0.85	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.043	0.030	0.000	0.700	0.000	0.000	0.000

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	58	64	70	67	0	0	-1
normalized size	1	1.00	1.29	1.42	1.56	1.49	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.036	0.031	0.350	0.654	0.000	0.000	0.000

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	69	33	381	0	0	-1
normalized size	1	1.00	0.66	0.84	0.40	4.65	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.037	0.043	0.344	1.697	0.000	0.000	0.000

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	53	38	29	72	0	0	58
normalized size	1	1.00	1.15	0.83	0.63	1.57	0.00	0.00	1.26
time (sec)	N/A	0.081	0.050	0.027	0.432	0.561	0.000	0.000	1.173

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	73	159	93	461	0	0	-1
normalized size	1	1.00	0.40	0.87	0.51	2.53	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.062	0.047	0.353	0.654	0.000	0.000	0.000

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	101	238	122	559	0	0	-1
normalized size	1	1.00	0.37	0.87	0.44	2.03	0.00	0.00	-0.00
time (sec)	N/A	0.140	0.096	0.054	0.343	0.834	0.000	0.000	0.000

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	74	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.056	0.443	0.000	0.807	0.000	0.000	0.000

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.034	0.446	0.000	0.617	0.000	0.000	0.000

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	42	0	0	557	0	0	-1
normalized size	1	1.00	0.12	0.00	0.00	1.55	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.021	0.269	0.000	0.679	0.000	0.000	0.000

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	42	0	0	541	0	0	-1
normalized size	1	1.00	0.14	0.00	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.017	0.269	0.000	1.659	0.000	0.000	0.000

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	42	0	0	525	0	0	-1
normalized size	1	1.00	0.16	0.00	0.00	2.06	0.00	0.00	-0.00
time (sec)	N/A	0.188	0.014	0.266	0.000	0.990	0.000	0.000	0.000

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	40	0	0	473	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	2.45	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.011	0.276	0.000	0.660	0.000	0.000	0.000

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	54	0	56	0	0	69
normalized size	1	1.00	0.86	1.46	0.00	1.51	0.00	0.00	1.86
time (sec)	N/A	0.040	0.014	0.030	0.000	0.635	0.000	0.000	1.075

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	48	70	0	78	0	0	115
normalized size	1	1.00	0.64	0.93	0.00	1.04	0.00	0.00	1.53
time (sec)	N/A	0.079	0.023	0.032	0.000	0.692	0.000	0.000	1.149

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	64	86	0	104	0	0	165
normalized size	1	1.00	0.57	0.77	0.00	0.93	0.00	0.00	1.47
time (sec)	N/A	0.117	0.033	0.033	0.000	0.587	0.000	0.000	1.314

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	80	102	0	126	0	0	211
normalized size	1	1.00	0.54	0.68	0.00	0.85	0.00	0.00	1.42
time (sec)	N/A	0.168	0.035	0.032	0.000	1.670	0.000	0.000	1.460

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	74	0	0	0	0	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	0.043	0.276	0.000	0.000	0.000	0.000	0.000

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	72	0	0	0	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	0.033	0.268	0.000	0.000	0.000	0.000	0.000

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	71	0	0	0	0	0	-1
normalized size	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	0.027	0.279	0.000	0.000	0.000	0.000	0.000

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	69	0	0	0	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.023	0.281	0.000	0.000	0.000	0.000	0.000

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	64	55	0	0	0	0	49
normalized size	1	1.00	1.56	1.34	0.00	0.00	0.00	0.00	1.20
time (sec)	N/A	0.045	0.029	0.029	0.000	0.000	0.000	0.000	1.262

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	80	71	0	0	0	0	97
normalized size	1	1.00	0.96	0.86	0.00	0.00	0.00	0.00	1.17
time (sec)	N/A	0.094	0.043	0.031	0.000	0.000	0.000	0.000	1.332

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	96	87	0	0	0	0	139
normalized size	1	1.00	0.77	0.70	0.00	0.00	0.00	0.00	1.12
time (sec)	N/A	0.143	0.056	0.031	0.000	0.000	0.000	0.000	1.420

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	112	103	0	0	0	0	183
normalized size	1	1.00	0.68	0.62	0.00	0.00	0.00	0.00	1.11
time (sec)	N/A	0.197	0.058	0.033	0.000	0.000	0.000	0.000	1.494

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	84	0	0	0	0	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.063	0.274	0.000	0.000	0.000	0.000	0.000

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	70	0	0	0	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.046	0.271	0.000	0.000	0.000	0.000	0.000

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	74	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.060	0.281	0.000	0.000	0.000	0.000	0.000

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	64	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.020	0.274	0.000	0.000	0.000	0.000	0.000

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	79	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.037	0.275	0.000	0.000	0.000	0.000	0.000

Problem 1302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	86	0	0	0	0	0	-1
normalized size	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.040	0.277	0.000	0.000	0.000	0.000	0.000

Problem 1303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	107	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.254	0.072	0.275	0.000	0.000	0.000	0.000	0.000

Problem 1304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	63	54	28	0	0	0	52
normalized size	1	1.00	1.54	1.32	0.68	0.00	0.00	0.00	1.27
time (sec)	N/A	0.119	0.035	0.031	1.060	0.000	0.000	0.000	1.258

Problem 1305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	93	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.043	0.278	0.000	0.000	0.000	0.000	0.000

Problem 1306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	69	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.025	0.280	0.000	0.000	0.000	0.000	0.000

Problem 1307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.623	0.293	0.000	0.000	0.000	0.000	0.000

Problem 1308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	65	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.025	0.049	0.000	1.012	0.000	0.000	0.000

Problem 1309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.025	0.272	0.000	1.211	0.000	0.000	0.000

Problem 1310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.024	0.269	0.000	0.860	0.000	0.000	0.000

Problem 1311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	172	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	0.232	0.281	0.000	0.601	0.000	0.000	0.000

Problem 1312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	120	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.073	0.268	0.000	0.905	0.000	0.000	0.000

Problem 1313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	127	82	0	0	0	0	0	-1
normalized size	1	1.48	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.066	0.273	0.000	0.919	0.000	0.000	0.000

Problem 1314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	74	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.061	0.273	0.000	0.567	0.000	0.000	0.000

Problem 1315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	33	18	30	27	0	0	31
normalized size	1	1.00	1.83	1.00	1.67	1.50	0.00	0.00	1.72
time (sec)	N/A	0.034	0.008	0.029	0.331	0.816	0.000	0.000	1.106

Problem 1316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	100	85	0	0	0	0	0	-1
normalized size	1	1.11	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.039	0.264	0.000	0.699	0.000	0.000	0.000

Problem 1317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	137	103	0	0	0	0	0	-1
normalized size	1	1.11	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.058	0.272	0.000	0.783	0.000	0.000	0.000

Problem 1318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	178	0	0	0	0	0	-1
normalized size	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	0.172	0.273	0.000	0.747	0.000	0.000	0.000

Problem 1319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	145	0	0	0	0	0	-1
normalized size	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	5.399	0.270	0.000	0.597	0.000	0.000	0.000

Problem 1320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	65	62	0	91	0	0	93
normalized size	1	1.00	0.82	0.78	0.00	1.15	0.00	0.00	1.18
time (sec)	N/A	0.127	0.056	0.031	0.000	0.473	0.000	0.000	1.170

Problem 1321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	102	56	47	0	78	0	0	46
normalized size	1	1.48	0.81	0.68	0.00	1.13	0.00	0.00	0.67
time (sec)	N/A	0.149	0.036	0.033	0.000	0.457	0.000	0.000	1.420

Problem 1322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	68	55	0	79	0	0	93
normalized size	1	1.00	0.94	0.76	0.00	1.10	0.00	0.00	1.29
time (sec)	N/A	0.074	0.030	0.030	0.000	0.460	0.000	0.000	1.215

Problem 1323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	200	121	0	0	0	0	0	-1
normalized size	1	1.05	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.070	0.276	0.000	0.624	0.000	0.000	0.000

Problem 1324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	253	163	0	0	0	0	0	-1
normalized size	1	1.06	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.100	0.269	0.000	0.613	0.000	0.000	0.000

Problem 1325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	111	101	0	174	0	0	179
normalized size	1	1.00	0.87	0.80	0.00	1.37	0.00	0.00	1.41
time (sec)	N/A	0.134	0.076	0.032	0.000	0.549	0.000	0.000	1.389

Problem 1326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	170	167	0	309	0	0	301
normalized size	1	1.00	0.86	0.85	0.00	1.57	0.00	0.00	1.53
time (sec)	N/A	0.186	0.132	0.034	0.000	0.720	0.000	0.000	1.372

Problem 1327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	236	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	0.557	0.264	0.000	0.580	0.000	0.000	0.000

Problem 1328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	135	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.207	0.272	0.000	0.671	0.000	0.000	0.000

Problem 1329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	125	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.137	0.270	0.000	0.478	0.000	0.000	0.000

Problem 1330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.055	0.270	0.000	0.853	0.000	0.000	0.000

Problem 1331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	299	207	0	0	0	0	0	-1
normalized size	1	1.11	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.171	0.273	0.000	0.581	0.000	0.000	0.000

Problem 1332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	97	138	0	0	0	0	0	-1
normalized size	1	0.36	0.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	0.471	0.273	0.000	0.517	0.000	0.000	0.000

Problem 1333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.087	0.276	0.000	0.658	0.000	0.000	0.000

Problem 1334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	187	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	0.291	0.271	0.000	0.635	0.000	0.000	0.000

Problem 1335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	141	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.185	0.276	0.000	0.654	0.000	0.000	0.000

Problem 1336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	124	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.104	0.278	0.000	0.952	0.000	0.000	0.000

Problem 1337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.039	0.267	0.000	0.538	0.000	0.000	0.000

Problem 1338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	99	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.054	0.278	0.000	0.656	0.000	0.000	0.000

Problem 1339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	117	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.078	0.271	0.000	0.478	0.000	0.000	0.000

Problem 1340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	134	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	0.136	0.273	0.000	0.503	0.000	0.000	0.000

Problem 1341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	186	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	0.293	0.291	0.000	0.576	0.000	0.000	0.000

Problem 1342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	155	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.276	0.276	0.000	0.675	0.000	0.000	0.000

Problem 1343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	81	49	0	82	0	0	68
normalized size	1	1.00	1.76	1.07	0.00	1.78	0.00	0.00	1.48
time (sec)	N/A	0.093	0.063	0.032	0.000	0.586	0.000	0.000	1.076

Problem 1344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	81	49	0	80	0	0	65
normalized size	1	1.00	1.76	1.07	0.00	1.74	0.00	0.00	1.41
time (sec)	N/A	0.053	0.055	0.030	0.000	0.413	0.000	0.000	1.101

Problem 1345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	243	247	149	0	0	0	0	0	-1
normalized size	1	1.02	0.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	0.164	0.266	0.000	0.518	0.000	0.000	0.000

Problem 1346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	325	173	0	0	0	0	0	-1
normalized size	1	1.01	0.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	0.195	0.269	0.000	0.617	0.000	0.000	0.000

Problem 1347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	422	219	0	0	0	0	0	-1
normalized size	1	1.01	0.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.421	0.254	0.273	0.000	0.778	0.000	0.000	0.000

Problem 1348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	112	93	0	175	0	0	162
normalized size	1	1.00	0.28	0.23	0.00	0.43	0.00	0.00	0.40
time (sec)	N/A	0.486	0.218	0.033	0.000	0.557	0.000	0.000	1.283

Problem 1349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	125	96	0	180	0	0	162
normalized size	1	1.00	1.23	0.94	0.00	1.76	0.00	0.00	1.59
time (sec)	N/A	0.199	0.219	0.031	0.000	0.502	0.000	0.000	1.228

Problem 1350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	86	0	171	0	0	163
normalized size	1	1.00	0.86	0.65	0.00	1.29	0.00	0.00	1.23
time (sec)	N/A	0.214	0.193	0.032	0.000	0.859	0.000	0.000	1.188

Problem 1351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	123	84	0	165	0	0	160
normalized size	1	1.00	1.21	0.82	0.00	1.62	0.00	0.00	1.57
time (sec)	N/A	0.106	0.161	0.032	0.000	0.520	0.000	0.000	1.208

Problem 1352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	421	222	0	0	0	0	0	-1
normalized size	1	1.01	0.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.458	0.303	0.273	0.000	0.624	0.000	0.000	0.000

Problem 1353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	507	511	268	0	0	0	0	0	-1
normalized size	1	1.01	0.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.535	0.362	0.274	0.000	0.542	0.000	0.000	0.000

Problem 1354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	623	628	267	0	0	0	0	0	-1
normalized size	1	1.01	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.647	0.833	0.325	0.000	0.573	0.000	0.000	0.000

Problem 1355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	182	140	0	291	0	0	276
normalized size	1	1.00	1.10	0.84	0.00	1.75	0.00	0.00	1.66
time (sec)	N/A	0.186	0.260	0.040	0.000	0.483	0.000	0.000	1.464

Problem 1356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.091	0.364	0.313	0.000	0.549	0.000	0.000	0.000

Problem 1357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.317	0.082	0.000	1.978	0.000	0.000	0.000

Problem 1358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	106	0	0	0	0	0	-1
normalized size	1	1.00	2.52	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.102	0.238	0.297	0.000	0.494	0.000	0.000	0.000

Problem 1359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.099	0.534	0.273	0.000	0.526	0.000	0.000	0.000

Problem 1360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.576	0.277	0.000	0.998	0.000	0.000	0.000

Problem 1361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	136	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.124	0.272	0.000	0.427	0.000	0.000	0.000

Problem 1362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	102	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.036	0.304	0.000	0.478	0.000	0.000	0.000

Problem 1363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	59	40	80	0	0	84
normalized size	1	1.00	0.76	1.44	0.98	1.95	0.00	0.00	2.05
time (sec)	N/A	0.048	0.024	0.029	0.340	0.535	0.000	0.000	1.116

Problem 1364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	58	60	41	81	0	0	86
normalized size	1	1.00	0.61	0.63	0.43	0.85	0.00	0.00	0.91
time (sec)	N/A	0.090	0.028	0.028	0.356	0.477	0.000	0.000	1.015

Problem 1365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	36	38	34	42	0	0	43
normalized size	1	1.00	0.73	0.78	0.69	0.86	0.00	0.00	0.88
time (sec)	N/A	0.073	0.034	0.026	0.341	0.455	0.000	0.000	1.062

Problem 1366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	36	40	36	44	0	0	44
normalized size	1	1.00	0.71	0.78	0.71	0.86	0.00	0.00	0.86
time (sec)	N/A	0.071	0.037	0.028	0.350	0.545	0.000	0.000	1.016

Problem 1367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	92	58	75	77	0	0	101
normalized size	1	1.00	1.74	1.09	1.42	1.45	0.00	0.00	1.91
time (sec)	N/A	0.109	0.090	0.030	0.364	0.479	0.000	0.000	1.087

Problem 1368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	426	379	379	405	299	-1
normalized size	1	1.00	0.97	13.74	12.23	12.23	13.06	9.65	-0.03
time (sec)	N/A	0.091	1.271	0.061	0.568	1.480	3.151	0.252	0.000

Problem 1369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	186	169	169	180	139	28
normalized size	1	1.00	0.97	6.00	5.45	5.45	5.81	4.48	0.90
time (sec)	N/A	0.089	0.255	0.039	0.356	0.472	1.224	0.190	1.512

Problem 1370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	54	49	49	48	45	28
normalized size	1	1.00	0.97	1.74	1.58	1.58	1.55	1.45	0.90
time (sec)	N/A	0.088	0.036	0.033	0.317	0.648	0.349	0.224	0.097

Problem 1371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	54	49	49	49	76	28
normalized size	1	1.00	0.97	1.74	1.58	1.58	1.58	2.45	0.90
time (sec)	N/A	0.089	0.039	0.036	0.329	0.606	0.356	0.167	0.114

Problem 1372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	186	169	169	182	139	28
normalized size	1	1.00	0.97	6.00	5.45	5.45	5.87	4.48	0.90
time (sec)	N/A	0.087	0.213	0.042	0.358	0.634	1.215	0.196	2.541

Problem 1373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	49	0	496	0	0	583
normalized size	1	1.00	0.98	0.82	0.00	8.27	0.00	0.00	9.72
time (sec)	N/A	0.231	0.594	0.045	0.000	0.979	0.000	0.000	2.396

Problem 1374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	49	0	193	0	0	186
normalized size	1	1.00	0.98	0.82	0.00	3.22	0.00	0.00	3.10
time (sec)	N/A	0.240	0.116	0.034	0.000	0.540	0.000	0.000	1.578

Problem 1375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	76	90	0	0	0	0	-1
normalized size	1	1.00	0.55	0.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.075	0.048	0.000	0.614	0.000	0.000	0.000

Problem 1376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	72	88	52	0	0	0	-1
normalized size	1	1.00	0.52	0.64	0.38	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.060	0.049	0.345	0.758	0.000	0.000	0.000

Problem 1377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	49	93	192	0	0	187
normalized size	1	1.00	0.98	0.82	1.55	3.20	0.00	0.00	3.12
time (sec)	N/A	0.237	0.103	0.034	0.363	0.952	0.000	0.000	1.538

Problem 1378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	49	273	497	0	489	363
normalized size	1	1.00	0.98	0.82	4.55	8.28	0.00	8.15	6.05
time (sec)	N/A	0.238	0.522	0.044	0.962	0.951	0.000	5.602	2.216

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [138] had the largest ratio of [1.429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	10	0.400
2	A	5	4	1.00	10	0.400
3	A	7	5	1.00	10	0.500
4	A	3	3	1.00	8	0.375
5	A	3	3	1.00	6	0.500
6	A	6	6	1.00	10	0.600
7	A	5	5	1.00	10	0.500
8	A	6	6	1.00	10	0.600
9	A	7	6	1.00	10	0.600
10	A	8	6	1.00	10	0.600
11	A	3	2	1.00	12	0.167
12	A	3	2	1.00	12	0.167
13	A	3	2	1.00	10	0.200
14	A	3	2	1.00	8	0.250
15	A	3	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	2	1.00	12	0.167
17	A	3	2	1.00	12	0.167
18	A	3	2	1.00	12	0.167
19	A	10	9	1.00	12	0.750
20	A	9	7	1.00	10	0.700
21	A	5	5	1.00	8	0.625
22	A	8	7	1.00	12	0.583
23	A	8	7	1.00	12	0.583
24	A	12	8	1.00	12	0.667
25	A	14	9	1.00	12	0.750
26	A	3	2	1.00	12	0.167
27	A	3	2	1.00	12	0.167
28	A	3	2	1.00	10	0.200
29	A	3	2	1.00	8	0.250
30	A	3	2	1.00	12	0.167
31	A	3	2	1.00	12	0.167
32	A	3	2	1.00	12	0.167
33	A	3	2	1.00	12	0.167
34	A	5	4	1.00	12	0.333
35	A	7	5	1.00	12	0.417
36	A	3	3	1.00	10	0.300
37	A	3	3	1.00	8	0.375
38	A	6	6	1.00	12	0.500
39	A	5	5	1.00	12	0.417
40	A	6	6	1.00	12	0.500
41	A	7	6	1.00	12	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	8	6	1.00	12	0.500
43	A	3	2	1.00	12	0.167
44	A	3	2	1.00	12	0.167
45	A	3	2	1.00	10	0.200
46	A	3	2	1.00	8	0.250
47	A	3	2	1.00	12	0.167
48	A	3	2	1.00	12	0.167
49	A	3	2	1.00	12	0.167
50	A	3	2	1.00	12	0.167
51	A	14	11	1.00	12	0.917
52	A	10	9	1.00	12	0.750
53	A	9	7	1.00	10	0.700
54	A	5	5	1.00	8	0.625
55	A	8	7	1.00	12	0.583
56	A	8	7	1.00	12	0.583
57	A	12	8	1.00	12	0.667
58	A	14	9	1.00	12	0.750
59	A	19	9	1.00	12	0.750
60	A	2	2	1.00	14	0.143
61	A	15	12	1.00	14	0.857
62	A	14	11	1.00	12	0.917
63	A	13	10	1.00	10	1.000
64	A	17	14	1.00	14	1.000
65	A	6	6	1.00	14	0.429
66	A	7	7	1.00	14	0.500
67	A	9	8	1.00	14	0.571

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	10	8	1.00	14	0.571
69	A	11	8	1.00	14	0.571
70	A	2	2	1.00	14	0.143
71	A	15	12	1.00	14	0.857
72	A	15	12	1.00	14	0.857
73	A	14	11	1.00	12	0.917
74	A	13	10	1.00	10	1.000
75	A	17	14	1.00	14	1.000
76	A	6	6	1.00	14	0.429
77	A	7	7	1.00	14	0.500
78	A	9	8	1.00	14	0.571
79	A	10	8	1.00	14	0.571
80	A	2	2	1.00	14	0.143
81	A	16	13	1.00	14	0.929
82	A	16	12	1.00	14	0.857
83	A	15	11	1.00	12	0.917
84	A	14	11	1.00	10	1.100
85	A	19	16	1.00	14	1.143
86	A	7	6	1.00	14	0.429
87	A	8	7	1.00	14	0.500
88	A	10	9	1.00	14	0.643
89	A	11	9	1.00	14	0.643
90	A	2	2	1.00	14	0.143
91	A	15	12	1.00	14	0.857
92	A	15	12	1.00	14	0.857
93	A	14	11	1.00	12	0.917

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	13	10	1.00	10	1.000
95	A	17	14	1.00	14	1.000
96	A	6	6	1.00	14	0.429
97	A	7	7	1.00	14	0.500
98	A	9	8	1.00	14	0.571
99	A	10	8	1.00	14	0.571
100	A	2	2	1.00	14	0.143
101	A	15	12	1.00	14	0.857
102	A	15	12	1.00	14	0.857
103	A	14	11	1.00	12	0.917
104	A	13	10	1.00	10	1.000
105	A	17	14	1.00	14	1.000
106	A	6	6	1.00	14	0.429
107	A	7	7	1.00	14	0.500
108	A	9	8	1.00	14	0.571
109	A	10	8	1.00	14	0.571
110	A	2	2	1.00	14	0.143
111	A	16	13	1.00	14	0.929
112	A	16	12	1.00	14	0.857
113	A	15	11	1.00	12	0.917
114	A	14	11	1.00	10	1.100
115	A	19	16	1.00	14	1.143
116	A	7	6	1.00	14	0.429
117	A	8	7	1.00	14	0.500
118	A	10	9	1.00	14	0.643
119	A	11	9	1.00	14	0.643

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	2	2	1.00	12	0.167
121	A	16	12	1.00	12	1.000
122	A	15	11	1.00	10	1.100
123	A	14	10	1.00	8	1.250
124	A	25	13	1.00	12	1.083
125	A	13	9	1.00	12	0.750
126	A	14	10	1.00	12	0.833
127	A	2	2	1.00	12	0.167
128	A	5	5	1.00	12	0.417
129	A	4	4	1.00	10	0.400
130	A	3	3	1.00	8	0.375
131	A	4	4	1.00	12	0.333
132	A	3	3	1.00	12	0.250
133	A	4	4	1.00	12	0.333
134	A	2	2	1.00	14	0.143
135	A	27	13	1.00	14	0.929
136	A	26	12	1.00	12	1.000
137	A	25	11	1.00	10	1.100
138	A	39	20	1.00	14	1.429
139	A	16	13	1.00	14	0.929
140	A	17	14	1.00	14	1.000
141	A	4	4	1.00	12	0.333
142	A	9	5	1.00	12	0.417
143	A	3	3	1.00	12	0.250
144	A	4	3	1.00	10	0.300
145	A	4	3	1.00	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	3	3	1.00	12	0.250
147	A	9	5	1.00	12	0.417
148	A	2	2	1.00	12	0.167
149	A	4	4	1.00	12	0.333
150	A	4	4	1.00	12	0.333
151	A	3	3	1.00	10	0.300
152	A	2	2	1.00	8	0.250
153	A	4	4	1.00	12	0.333
154	A	2	2	1.00	12	0.167
155	A	3	3	1.00	12	0.250
156	A	5	5	1.00	12	0.417
157	A	3	3	1.00	16	0.188
158	A	6	5	1.00	16	0.312
159	A	5	5	1.00	16	0.312
160	A	4	4	1.00	16	0.250
161	A	3	3	1.00	14	0.214
162	A	3	3	1.00	16	0.188
163	A	2	2	1.00	16	0.125
164	A	3	3	1.00	16	0.188
165	A	4	3	1.00	16	0.188
166	A	5	3	1.00	16	0.188
167	A	4	3	1.00	18	0.167
168	A	3	2	1.00	18	0.111
169	A	3	2	1.00	18	0.111
170	A	3	2	1.00	18	0.111
171	A	3	2	1.00	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	C	1	1	2.00	16	0.062
173	A	3	2	1.00	18	0.111
174	A	2	2	1.00	18	0.111
175	A	3	2	1.00	18	0.111
176	A	3	2	1.00	18	0.111
177	A	3	3	1.00	18	0.167
178	A	5	4	1.00	18	0.222
179	A	4	3	1.00	18	0.167
180	A	4	4	1.00	18	0.222
181	A	4	3	1.00	16	0.188
182	A	4	3	1.00	18	0.167
183	A	2	2	1.00	18	0.111
184	A	3	3	1.00	18	0.167
185	A	4	3	1.00	18	0.167
186	A	5	3	1.00	18	0.167
187	A	4	3	1.00	18	0.167
188	A	3	2	1.00	18	0.111
189	A	4	3	1.00	18	0.167
190	A	3	2	1.00	18	0.111
191	A	2	2	1.00	18	0.111
192	A	3	2	1.00	16	0.125
193	A	3	2	1.00	18	0.111
194	A	2	2	1.00	18	0.111
195	A	3	2	1.00	18	0.111
196	A	3	2	1.00	18	0.111
197	A	3	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
198	A	6	4	1.00	18	0.222
199	A	5	4	1.00	18	0.222
200	A	4	4	1.00	16	0.250
201	A	2	2	1.00	18	0.111
202	A	2	2	1.00	18	0.111
203	A	3	3	1.00	18	0.167
204	A	4	3	1.00	18	0.167
205	A	5	3	1.00	18	0.167
206	A	3	3	1.00	18	0.167
207	A	3	2	1.00	18	0.111
208	A	3	2	1.00	18	0.111
209	A	3	2	1.00	18	0.111
210	A	3	2	1.00	16	0.125
211	A	2	2	1.00	18	0.111
212	A	3	3	1.00	18	0.167
213	A	4	3	1.00	18	0.167
214	A	4	3	1.00	18	0.167
215	A	4	3	1.00	18	0.167
216	A	3	3	1.00	18	0.167
217	A	7	5	1.00	18	0.278
218	A	6	5	1.00	18	0.278
219	A	5	5	1.00	16	0.312
220	A	3	3	1.00	18	0.167
221	A	2	2	1.00	18	0.111
222	A	2	2	1.00	18	0.111
223	A	3	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	4	3	1.00	18	0.167
225	A	5	3	1.00	18	0.167
226	A	6	3	1.00	18	0.167
227	A	5	3	1.00	18	0.167
228	A	4	3	1.00	18	0.167
229	A	3	3	1.00	18	0.167
230	A	2	2	1.00	18	0.111
231	A	4	4	1.00	18	0.222
232	A	4	4	1.00	18	0.222
233	A	5	5	1.00	18	0.278
234	A	6	5	1.00	18	0.278
235	A	4	3	1.00	20	0.150
236	A	4	3	1.00	20	0.150
237	A	4	3	1.00	20	0.150
238	A	4	3	1.00	20	0.150
239	A	4	3	1.00	20	0.150
240	A	4	3	1.00	20	0.150
241	A	4	3	1.00	20	0.150
242	A	4	3	1.00	20	0.150
243	A	5	3	1.00	20	0.150
244	A	4	3	1.00	20	0.150
245	A	3	3	1.00	20	0.150
246	A	2	2	1.00	20	0.100
247	A	5	4	1.00	20	0.200
248	A	5	5	1.00	20	0.250
249	A	5	4	1.00	20	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
250	A	6	5	1.00	20	0.250
251	A	7	5	1.00	20	0.250
252	A	7	3	1.00	20	0.150
253	A	6	3	1.00	20	0.150
254	A	5	3	1.00	20	0.150
255	A	4	3	1.00	20	0.150
256	A	3	3	1.00	20	0.150
257	A	2	2	1.00	20	0.100
258	A	3	3	1.00	20	0.150
259	A	4	4	1.00	20	0.200
260	A	5	4	1.00	20	0.200
261	A	9	5	1.00	20	0.250
262	A	8	5	1.00	20	0.250
263	A	7	5	1.00	20	0.250
264	A	6	5	1.00	20	0.250
265	A	5	5	1.00	20	0.250
266	A	4	4	1.00	20	0.200
267	A	5	5	1.00	20	0.250
268	A	6	5	1.00	20	0.250
269	A	7	5	1.00	20	0.250
270	A	6	3	1.00	20	0.150
271	A	5	3	1.00	20	0.150
272	A	4	3	1.00	20	0.150
273	A	3	3	1.00	20	0.150
274	A	2	2	1.00	20	0.100
275	A	4	4	1.00	20	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
276	A	5	5	1.00	20	0.250
277	A	6	5	1.00	20	0.250
278	A	3	3	1.00	20	0.150
279	A	3	3	1.00	20	0.150
280	A	3	3	1.00	20	0.150
281	A	3	3	1.00	20	0.150
282	A	3	3	1.00	20	0.150
283	A	3	3	1.00	20	0.150
284	A	3	3	1.00	20	0.150
285	A	3	3	1.00	20	0.150
286	A	5	4	1.00	17	0.235
287	A	4	3	1.00	17	0.176
288	A	4	4	1.00	17	0.235
289	A	2	2	1.00	15	0.133
290	A	3	3	1.00	14	0.214
291	A	5	5	1.00	17	0.294
292	A	3	3	1.00	17	0.176
293	A	5	5	1.00	17	0.294
294	A	2	2	1.00	17	0.118
295	A	6	5	1.00	19	0.263
296	A	9	5	1.00	19	0.263
297	A	4	4	1.00	17	0.235
298	A	4	4	1.00	16	0.250
299	A	7	7	1.00	19	0.368
300	A	7	7	1.00	19	0.368
301	A	7	7	1.00	19	0.368

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	A	6	6	1.00	19	0.316
303	A	7	7	1.00	19	0.368
304	A	8	7	1.00	19	0.368
305	A	9	7	1.00	19	0.368
306	A	7	6	1.00	19	0.316
307	A	6	6	1.00	19	0.316
308	A	5	5	1.00	17	0.294
309	A	5	5	1.00	16	0.312
310	A	8	8	1.00	19	0.421
311	A	8	8	1.00	19	0.421
312	A	8	8	1.00	19	0.421
313	A	8	8	1.00	19	0.421
314	A	7	7	1.00	19	0.368
315	A	8	8	1.00	19	0.421
316	A	8	6	1.00	19	0.316
317	A	7	6	1.00	19	0.316
318	A	7	6	1.00	17	0.353
319	A	6	5	1.00	16	0.312
320	A	9	8	1.00	19	0.421
321	A	9	9	1.00	19	0.474
322	A	9	8	1.00	19	0.421
323	A	9	8	1.00	19	0.421
324	A	9	8	1.05	19	0.421
325	A	8	7	1.00	19	0.368
326	A	9	8	1.00	19	0.421
327	A	8	6	1.00	19	0.316

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	7	6	1.00	19	0.316
329	A	5	5	1.00	19	0.263
330	A	4	4	1.00	17	0.235
331	A	3	3	1.00	16	0.188
332	A	7	7	1.00	19	0.368
333	A	7	7	1.00	19	0.368
334	A	8	8	1.00	19	0.421
335	A	9	8	1.00	19	0.421
336	A	10	9	1.00	19	0.474
337	A	6	5	1.00	19	0.263
338	A	5	5	1.00	19	0.263
339	A	4	4	1.00	17	0.235
340	A	2	2	1.00	16	0.125
341	A	8	8	1.00	19	0.421
342	A	8	7	1.00	19	0.368
343	A	9	8	1.00	19	0.421
344	A	10	8	1.00	19	0.421
345	A	7	5	1.00	19	0.263
346	A	9	7	1.00	19	0.368
347	A	8	6	1.00	19	0.316
348	A	3	3	1.00	17	0.176
349	A	3	3	1.00	16	0.188
350	A	9	8	1.00	19	0.421
351	A	9	7	1.00	19	0.368
352	A	10	8	1.00	19	0.421
353	A	11	8	1.00	19	0.421

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
354	A	8	5	1.00	19	0.263
355	A	12	7	1.00	19	0.368
356	A	11	6	1.00	19	0.316
357	A	5	5	1.00	19	0.263
358	A	4	4	1.00	17	0.235
359	A	4	3	1.00	16	0.188
360	A	10	8	1.00	19	0.421
361	A	10	7	1.00	19	0.368
362	A	11	8	1.00	19	0.421
363	A	6	5	1.00	9	0.556
364	A	5	4	1.00	8	0.500
365	A	7	5	1.00	11	0.454
366	A	6	4	1.00	10	0.400
367	A	2	2	1.00	11	0.182
368	A	3	3	1.00	10	0.300
369	A	4	4	1.00	11	0.364
370	A	2	2	1.00	10	0.200
371	A	3	2	1.00	13	0.154
372	A	3	2	1.00	12	0.167
373	A	5	4	1.38	15	0.267
374	A	4	3	1.00	14	0.214
375	A	3	2	1.00	13	0.154
376	A	3	2	1.00	12	0.167
377	A	4	3	1.00	15	0.200
378	A	3	3	1.00	14	0.214
379	A	3	2	1.00	13	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
380	A	2	2	1.00	12	0.167
381	A	5	5	1.00	15	0.333
382	A	5	5	1.00	14	0.357
383	A	4	4	1.00	13	0.308
384	A	3	3	1.00	12	0.250
385	A	5	5	1.00	15	0.333
386	A	5	5	1.00	14	0.357
387	A	4	4	1.00	21	0.190
388	A	4	4	1.00	21	0.190
389	A	3	3	1.00	19	0.158
390	A	2	2	1.00	18	0.111
391	A	4	4	1.00	21	0.190
392	A	4	4	1.00	21	0.190
393	A	4	3	1.00	23	0.130
394	A	4	3	1.00	23	0.130
395	A	4	3	1.00	21	0.143
396	A	4	3	1.00	20	0.150
397	A	5	5	1.00	23	0.217
398	A	5	5	1.00	23	0.217
399	A	6	6	1.00	23	0.261
400	A	7	6	1.00	23	0.261
401	A	8	6	1.00	23	0.261
402	A	8	6	1.00	23	0.261
403	A	8	6	1.00	23	0.261
404	A	7	6	1.00	21	0.286
405	A	5	4	1.00	20	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	8	6	1.00	23	0.261
407	A	8	6	1.00	23	0.261
408	A	9	7	1.00	23	0.304
409	A	10	7	1.00	23	0.304
410	A	11	7	1.00	23	0.304
411	A	5	5	1.00	23	0.217
412	A	5	5	1.00	23	0.217
413	A	4	4	1.00	21	0.190
414	A	3	3	1.00	20	0.150
415	A	4	4	1.00	23	0.174
416	A	4	4	1.00	23	0.174
417	A	5	5	1.00	23	0.217
418	A	6	5	1.00	23	0.217
419	A	8	6	1.00	23	0.261
420	A	8	6	1.00	23	0.261
421	A	7	6	1.00	21	0.286
422	A	6	5	1.00	20	0.250
423	A	8	7	1.00	23	0.304
424	A	8	7	1.00	23	0.304
425	A	9	8	1.00	23	0.348
426	A	10	8	1.00	23	0.348
427	A	11	8	1.00	23	0.348
428	A	4	3	1.00	23	0.130
429	A	4	3	1.00	23	0.130
430	A	4	3	1.00	21	0.143
431	A	4	3	1.00	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	6	5	1.00	23	0.217
433	A	6	6	1.00	23	0.261
434	A	7	7	1.00	23	0.304
435	A	8	7	1.01	23	0.304
436	A	9	7	1.01	23	0.304
437	A	3	3	1.00	19	0.158
438	A	3	3	1.00	19	0.158
439	A	3	3	1.00	18	0.167
440	A	2	2	1.00	18	0.111
441	A	2	2	1.00	18	0.111
442	A	2	2	1.00	16	0.125
443	A	2	2	1.00	18	0.111
444	A	2	2	1.00	18	0.111
445	A	3	3	1.00	18	0.167
446	A	4	3	1.00	18	0.167
447	A	3	3	1.00	20	0.150
448	A	10	9	1.00	20	0.450
449	A	9	9	1.00	20	0.450
450	A	8	8	1.00	20	0.400
451	A	6	6	1.00	18	0.333
452	A	5	5	1.00	20	0.250
453	A	6	6	1.00	20	0.300
454	A	10	8	1.00	20	0.400
455	A	13	8	1.00	20	0.400
456	A	6	6	1.00	22	0.273
457	A	4	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
458	A	4	3	1.00	22	0.136
459	A	4	3	1.00	22	0.136
460	A	5	4	1.00	22	0.182
461	A	4	3	1.00	20	0.150
462	A	4	3	1.00	22	0.136
463	A	4	3	1.00	22	0.136
464	A	4	3	1.00	22	0.136
465	A	4	3	1.00	22	0.136
466	A	9	9	1.00	22	0.409
467	A	7	7	1.00	22	0.318
468	A	9	9	1.00	22	0.409
469	A	9	9	1.00	20	0.450
470	A	6	6	1.00	22	0.273
471	A	9	8	1.00	22	0.364
472	A	9	8	1.00	22	0.364
473	A	10	8	1.00	22	0.364
474	A	7	7	1.00	22	0.318
475	A	4	3	1.00	22	0.136
476	A	5	4	1.00	22	0.182
477	A	4	3	1.00	22	0.136
478	A	4	3	1.00	22	0.136
479	A	4	3	1.00	20	0.150
480	A	4	3	1.00	22	0.136
481	A	4	3	1.00	22	0.136
482	A	4	3	1.00	22	0.136
483	A	4	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
484	A	3	3	1.00	22	0.136
485	A	11	9	1.00	22	0.409
486	A	10	9	1.00	22	0.409
487	A	9	9	1.00	22	0.409
488	A	8	8	1.00	20	0.400
489	A	3	3	1.00	22	0.136
490	A	6	6	1.00	22	0.273
491	A	7	6	1.00	22	0.273
492	A	8	6	1.00	22	0.273
493	A	8	8	1.00	22	0.364
494	A	4	3	1.00	22	0.136
495	A	4	3	1.00	22	0.136
496	A	4	3	1.00	22	0.136
497	A	4	3	1.00	20	0.150
498	A	4	3	1.00	22	0.136
499	A	5	4	1.00	22	0.182
500	A	4	3	1.00	22	0.136
501	A	4	3	1.00	22	0.136
502	A	11	10	1.00	22	0.454
503	A	10	10	1.00	22	0.454
504	A	9	9	1.00	20	0.450
505	A	5	5	1.00	22	0.227
506	A	6	6	1.00	22	0.273
507	A	5	4	1.00	22	0.182
508	A	7	6	1.00	22	0.273
509	A	8	7	1.00	22	0.318

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
510	A	8	7	1.00	22	0.318
511	A	7	7	1.00	22	0.318
512	A	7	7	1.00	22	0.318
513	A	7	7	1.00	22	0.318
514	A	6	6	1.01	22	0.273
515	A	8	8	1.00	22	0.364
516	A	9	9	1.00	22	0.409
517	A	10	10	1.00	22	0.454
518	A	11	8	1.00	24	0.333
519	A	10	8	1.00	24	0.333
520	A	9	8	1.00	24	0.333
521	A	8	8	1.00	24	0.333
522	A	7	7	1.00	24	0.292
523	A	8	8	1.00	24	0.333
524	A	9	8	1.00	24	0.333
525	A	10	8	1.00	24	0.333
526	A	11	8	1.00	24	0.333
527	A	8	8	1.00	24	0.333
528	A	8	8	1.00	24	0.333
529	A	8	8	1.00	24	0.333
530	A	7	7	1.00	24	0.292
531	A	8	8	1.00	24	0.333
532	A	9	9	1.00	24	0.375
533	A	10	10	1.00	24	0.417
534	A	11	10	1.00	24	0.417
535	A	8	7	1.00	24	0.292

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
536	A	7	7	1.00	24	0.292
537	A	6	6	1.00	24	0.250
538	A	6	6	1.00	24	0.250
539	A	6	6	1.00	24	0.250
540	A	6	6	1.00	24	0.250
541	A	9	9	1.00	24	0.375
542	A	9	9	1.00	24	0.375
543	A	10	10	1.00	24	0.417
544	A	14	9	1.00	24	0.375
545	A	13	9	1.00	24	0.375
546	A	12	9	1.00	24	0.375
547	A	11	9	1.00	24	0.375
548	A	10	8	1.00	24	0.333
549	A	10	8	1.00	24	0.333
550	A	11	9	1.00	24	0.375
551	A	11	9	1.00	24	0.375
552	A	12	9	1.00	24	0.375
553	A	13	9	1.00	24	0.375
554	A	9	7	1.00	24	0.292
555	A	8	7	1.00	24	0.292
556	A	7	7	1.00	24	0.292
557	A	6	6	1.00	24	0.250
558	A	6	6	1.00	24	0.250
559	A	7	7	1.00	24	0.292
560	A	7	7	1.00	24	0.292
561	A	9	9	1.00	24	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
562	A	10	10	1.00	24	0.417
563	A	9	8	1.00	25	0.320
564	A	4	4	1.00	25	0.160
565	A	8	6	1.00	25	0.240
566	A	7	6	1.00	23	0.261
567	A	6	6	1.00	22	0.273
568	A	6	6	1.00	25	0.240
569	A	4	4	1.00	25	0.160
570	A	5	5	1.00	25	0.200
571	A	6	5	1.00	25	0.200
572	A	7	5	1.00	25	0.200
573	A	10	8	1.00	27	0.296
574	A	9	8	1.00	27	0.296
575	A	8	8	1.00	25	0.320
576	A	7	7	1.00	24	0.292
577	A	7	7	1.00	27	0.259
578	A	6	5	1.00	27	0.185
579	A	6	5	1.00	27	0.185
580	A	6	5	1.00	27	0.185
581	A	6	5	1.00	27	0.185
582	A	11	9	1.00	27	0.333
583	A	10	9	1.00	27	0.333
584	A	9	9	1.00	25	0.360
585	A	8	8	1.00	24	0.333
586	A	8	8	1.00	27	0.296
587	A	6	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
588	A	7	6	1.00	27	0.222
589	A	9	7	1.00	27	0.259
590	A	10	7	1.00	27	0.259
591	A	5	5	1.00	27	0.185
592	A	8	7	1.00	27	0.259
593	A	7	7	1.00	25	0.280
594	A	6	6	1.00	24	0.250
595	A	6	6	1.00	27	0.222
596	A	5	5	1.00	27	0.185
597	A	6	6	1.00	27	0.222
598	A	7	6	1.00	27	0.222
599	A	13	9	1.00	27	0.333
600	A	12	9	1.00	27	0.333
601	A	11	9	1.00	25	0.360
602	A	10	8	1.00	24	0.333
603	A	10	8	1.00	27	0.296
604	A	8	7	1.00	27	0.259
605	A	9	8	1.00	27	0.296
606	A	10	8	1.00	27	0.296
607	A	10	8	1.00	27	0.296
608	A	9	7	1.00	27	0.259
609	A	8	7	1.00	27	0.259
610	A	7	7	1.00	25	0.280
611	A	6	6	1.00	24	0.250
612	A	6	6	1.00	27	0.222
613	A	5	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
614	A	6	6	1.00	27	0.222
615	A	7	6	1.00	27	0.222
616	A	8	6	1.00	27	0.222
617	A	3	3	1.00	22	0.136
618	A	3	3	1.00	23	0.130
619	A	3	3	1.00	23	0.130
620	C	3	3	0.55	22	0.136
621	A	7	5	0.98	20	0.250
622	A	4	4	1.00	22	0.182
623	A	5	5	1.00	22	0.227
624	A	3	3	1.00	24	0.125
625	A	3	3	1.00	24	0.125
626	A	3	3	1.00	24	0.125
627	A	3	3	1.00	24	0.125
628	A	11	9	1.00	20	0.450
629	A	10	9	1.00	20	0.450
630	A	9	9	1.00	20	0.450
631	A	8	8	1.00	18	0.444
632	A	6	6	1.00	20	0.300
633	A	6	5	1.00	20	0.250
634	A	7	5	1.00	20	0.250
635	A	8	5	1.00	20	0.250
636	A	4	3	1.00	22	0.136
637	A	4	3	1.00	22	0.136
638	A	4	3	1.00	22	0.136
639	A	4	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
640	A	4	3	1.00	20	0.150
641	A	4	3	1.00	22	0.136
642	A	4	3	1.00	22	0.136
643	A	4	3	1.00	22	0.136
644	A	4	3	1.00	22	0.136
645	A	12	10	1.00	22	0.454
646	A	11	10	1.00	22	0.454
647	A	10	9	1.00	22	0.409
648	A	9	9	1.00	20	0.450
649	A	7	7	1.00	22	0.318
650	A	7	5	1.00	22	0.227
651	A	8	6	1.00	22	0.273
652	A	9	6	1.00	22	0.273
653	A	4	3	1.00	22	0.136
654	A	4	3	1.00	22	0.136
655	A	4	3	1.00	22	0.136
656	A	4	3	1.00	22	0.136
657	A	4	3	1.00	20	0.150
658	A	4	3	1.00	22	0.136
659	A	4	3	1.00	22	0.136
660	A	4	3	1.00	22	0.136
661	A	4	3	1.00	22	0.136
662	A	11	9	1.00	22	0.409
663	A	10	9	1.00	22	0.409
664	A	9	9	1.00	22	0.409
665	A	8	8	1.00	20	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
666	A	6	6	1.00	22	0.273
667	A	6	5	1.00	22	0.227
668	A	7	5	1.00	22	0.227
669	A	8	5	1.00	22	0.227
670	A	4	3	1.00	22	0.136
671	A	4	3	1.00	22	0.136
672	A	4	3	1.00	22	0.136
673	A	4	3	1.00	20	0.150
674	A	4	3	1.00	22	0.136
675	A	4	3	1.00	22	0.136
676	A	4	3	1.00	22	0.136
677	A	4	3	1.00	22	0.136
678	A	12	10	1.00	22	0.454
679	A	11	10	1.00	22	0.454
680	A	10	9	1.00	22	0.409
681	A	9	9	1.00	20	0.450
682	A	7	7	1.00	22	0.318
683	A	7	5	1.00	22	0.227
684	A	8	6	1.00	22	0.273
685	A	9	6	1.00	22	0.273
686	A	4	3	1.00	22	0.136
687	A	4	3	1.00	22	0.136
688	A	4	3	1.00	22	0.136
689	A	4	3	1.00	22	0.136
690	A	4	3	1.00	22	0.136
691	A	4	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
692	A	4	3	1.00	22	0.136
693	A	4	3	1.00	22	0.136
694	A	4	3	1.00	22	0.136
695	A	16	10	1.00	24	0.417
696	A	14	10	1.00	24	0.417
697	A	12	10	1.00	24	0.417
698	A	10	10	1.00	24	0.417
699	A	8	8	1.00	24	0.333
700	A	6	6	1.00	24	0.250
701	A	6	6	1.00	24	0.250
702	A	8	7	1.00	24	0.292
703	A	10	7	1.00	24	0.292
704	A	12	7	1.00	24	0.292
705	A	4	3	1.00	24	0.125
706	A	4	3	1.00	24	0.125
707	A	4	3	1.00	24	0.125
708	A	4	3	1.00	24	0.125
709	A	4	3	1.00	24	0.125
710	A	4	3	1.00	24	0.125
711	A	4	3	1.00	24	0.125
712	A	4	3	1.00	24	0.125
713	A	4	3	1.00	24	0.125
714	A	4	3	1.00	24	0.125
715	A	4	3	1.00	24	0.125
716	A	4	3	1.00	24	0.125
717	A	4	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
718	A	4	3	1.00	24	0.125
719	A	4	3	1.00	24	0.125
720	A	4	3	1.00	24	0.125
721	A	4	3	1.00	24	0.125
722	A	4	3	1.00	24	0.125
723	A	16	10	1.00	24	0.417
724	A	14	10	1.00	24	0.417
725	A	12	10	1.00	24	0.417
726	A	10	10	1.00	24	0.417
727	A	8	8	1.00	24	0.333
728	A	6	6	1.00	24	0.250
729	A	6	6	1.00	24	0.250
730	A	8	7	1.00	24	0.292
731	A	10	7	1.00	24	0.292
732	A	4	3	1.00	24	0.125
733	A	4	3	1.00	24	0.125
734	A	4	3	1.00	24	0.125
735	A	4	3	1.00	24	0.125
736	A	4	3	1.00	24	0.125
737	A	4	3	1.00	24	0.125
738	A	4	3	1.00	24	0.125
739	A	4	3	1.00	24	0.125
740	A	4	3	1.00	24	0.125
741	A	4	3	1.00	25	0.120
742	A	4	3	1.00	25	0.120
743	A	3	2	1.00	23	0.087

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
744	A	4	3	1.00	22	0.136
745	A	4	3	1.00	25	0.120
746	A	3	3	1.00	25	0.120
747	A	8	7	1.00	27	0.259
748	A	7	6	1.00	27	0.222
749	A	6	5	1.00	25	0.200
750	A	8	8	1.00	24	0.333
751	A	8	8	1.00	27	0.296
752	A	6	5	1.00	27	0.185
753	A	7	6	1.00	27	0.222
754	A	9	7	1.00	27	0.259
755	A	10	7	1.00	27	0.259
756	A	4	3	1.00	27	0.111
757	A	4	3	1.00	27	0.111
758	A	4	3	1.00	25	0.120
759	A	4	3	1.00	24	0.125
760	A	4	3	1.00	27	0.111
761	A	4	3	1.00	27	0.111
762	A	4	3	1.00	27	0.111
763	A	4	3	1.00	27	0.111
764	A	4	3	1.00	27	0.111
765	A	4	3	1.00	27	0.111
766	A	4	3	1.00	27	0.111
767	A	3	2	1.00	25	0.080
768	A	4	3	1.00	24	0.125
769	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
770	A	3	3	1.00	27	0.111
771	A	8	7	1.00	27	0.259
772	A	7	6	1.00	27	0.222
773	A	6	5	1.00	25	0.200
774	A	8	8	1.00	24	0.333
775	A	8	8	1.00	27	0.296
776	A	6	5	1.00	27	0.185
777	A	7	6	1.00	27	0.222
778	A	9	7	1.00	27	0.259
779	A	10	7	1.00	27	0.259
780	A	4	3	1.00	27	0.111
781	A	4	3	1.00	27	0.111
782	A	4	3	1.00	25	0.120
783	A	4	3	1.00	24	0.125
784	A	4	3	1.00	27	0.111
785	A	4	3	1.00	27	0.111
786	A	4	3	1.00	27	0.111
787	A	4	3	1.00	27	0.111
788	A	4	3	1.00	27	0.111
789	A	3	3	1.00	23	0.130
790	A	3	3	1.00	23	0.130
791	C	3	3	0.21	22	0.136
792	C	3	3	0.51	20	0.150
793	A	5	5	1.00	22	0.227
794	A	11	10	1.00	22	0.454
795	C	3	3	0.24	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	7	5	1.11	24	0.208
797	A	4	4	1.00	24	0.167
798	A	5	5	1.00	24	0.208
799	A	18	11	1.00	24	0.458
800	A	3	3	1.00	22	0.136
801	A	13	6	1.00	22	0.273
802	A	7	6	1.00	22	0.273
803	A	10	5	1.00	22	0.227
804	A	5	4	1.00	20	0.200
805	A	5	4	1.00	22	0.182
806	A	10	5	1.00	22	0.227
807	A	7	6	1.00	22	0.273
808	A	16	8	1.00	15	0.533
809	A	11	7	1.00	14	0.500
810	A	13	8	1.00	17	0.471
811	A	7	7	1.00	16	0.438
812	A	22	8	1.00	15	0.533
813	A	16	8	1.00	14	0.571
814	A	19	8	1.00	17	0.471
815	A	13	8	1.00	16	0.500
816	A	11	7	1.00	15	0.467
817	A	6	6	1.00	14	0.429
818	A	7	7	1.00	12	0.583
819	A	7	7	1.00	12	0.583
820	A	6	6	1.00	10	0.600
821	A	5	5	1.00	8	0.625

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
822	A	7	7	1.00	12	0.583
823	A	4	4	1.00	12	0.333
824	A	5	5	1.00	12	0.417
825	A	7	6	1.00	12	0.500
826	A	3	2	1.00	14	0.143
827	A	3	2	1.00	14	0.143
828	A	3	2	1.00	14	0.143
829	A	3	2	1.00	12	0.167
830	A	3	2	1.00	10	0.200
831	A	3	2	1.00	14	0.143
832	A	3	2	1.00	14	0.143
833	A	3	2	1.00	14	0.143
834	A	3	2	1.00	14	0.143
835	A	8	8	1.00	14	0.571
836	A	8	7	1.00	14	0.500
837	A	7	6	1.00	12	0.500
838	A	6	6	1.00	10	0.600
839	A	8	8	1.00	14	0.571
840	A	5	4	1.00	14	0.286
841	A	6	5	1.00	14	0.357
842	A	8	7	1.00	14	0.500
843	A	7	7	1.00	14	0.500
844	A	7	7	1.00	14	0.500
845	A	6	6	1.00	12	0.500
846	A	5	5	1.00	10	0.500
847	A	7	7	1.00	14	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
848	A	4	4	1.00	14	0.286
849	A	5	5	1.00	14	0.357
850	A	7	6	1.00	14	0.429
851	A	3	2	1.00	14	0.143
852	A	3	2	1.00	14	0.143
853	A	3	2	1.00	14	0.143
854	A	3	2	1.00	12	0.167
855	A	3	2	1.00	10	0.200
856	A	3	2	1.00	14	0.143
857	A	3	2	1.00	14	0.143
858	A	3	2	1.00	14	0.143
859	A	3	2	1.00	14	0.143
860	A	8	8	1.00	14	0.571
861	A	8	7	1.00	14	0.500
862	A	7	6	1.00	12	0.500
863	A	6	6	1.00	10	0.600
864	A	8	8	1.00	14	0.571
865	A	5	4	1.00	14	0.286
866	A	6	5	1.00	14	0.357
867	A	8	7	1.00	14	0.500
868	A	4	4	1.00	16	0.250
869	A	6	6	1.00	34	0.176
870	A	6	6	1.00	34	0.176
871	A	5	5	1.00	32	0.156
872	A	2	2	1.00	31	0.065
873	A	4	4	1.00	34	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
874	A	6	6	1.00	34	0.176
875	A	4	3	1.00	14	0.214
876	A	4	4	1.00	14	0.286
877	A	4	4	1.00	14	0.286
878	A	3	3	1.00	12	0.250
879	A	2	2	1.00	10	0.200
880	A	4	4	1.00	14	0.286
881	A	2	2	1.00	14	0.143
882	A	3	3	1.00	14	0.214
883	A	7	4	1.00	20	0.200
884	A	6	4	1.00	20	0.200
885	A	5	4	1.00	20	0.200
886	A	4	4	1.00	18	0.222
887	A	5	5	1.00	23	0.217
888	A	4	4	1.00	23	0.174
889	A	5	5	1.00	23	0.217
890	A	3	3	1.00	21	0.143
891	A	1	1	1.00	20	0.050
892	A	6	6	1.00	23	0.261
893	A	6	6	1.00	23	0.261
894	A	7	7	1.00	23	0.304
895	A	8	7	1.00	23	0.304
896	A	6	5	1.00	23	0.217
897	A	5	4	1.00	23	0.174
898	A	5	4	1.00	23	0.174
899	A	4	4	1.00	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
900	A	4	4	1.00	23	0.174
901	A	3	3	1.00	21	0.143
902	A	3	3	1.00	20	0.150
903	A	7	6	1.00	23	0.261
904	A	7	6	1.00	23	0.261
905	A	8	7	1.00	23	0.304
906	A	9	7	1.00	23	0.304
907	A	6	4	1.00	23	0.174
908	A	6	4	1.00	23	0.174
909	A	5	4	1.00	23	0.174
910	A	5	4	1.00	23	0.174
911	A	4	4	1.19	23	0.174
912	A	4	4	1.00	23	0.174
913	A	4	4	1.00	21	0.190
914	A	4	4	1.00	20	0.200
915	A	8	6	1.00	23	0.261
916	A	8	6	1.00	23	0.261
917	A	9	7	1.00	23	0.304
918	A	5	4	1.00	20	0.200
919	A	6	4	1.00	20	0.200
920	A	3	2	1.00	24	0.083
921	A	3	2	1.00	24	0.083
922	A	3	2	1.00	24	0.083
923	A	3	2	1.00	22	0.091
924	A	2	2	1.00	21	0.095
925	A	4	4	1.00	24	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
926	A	3	2	1.00	24	0.083
927	A	3	2	1.00	24	0.083
928	A	3	2	1.00	24	0.083
929	A	3	2	1.00	24	0.083
930	A	3	2	1.00	24	0.083
931	A	3	2	1.00	24	0.083
932	A	4	3	1.00	22	0.136
933	A	4	3	1.00	21	0.143
934	A	3	2	1.00	24	0.083
935	A	3	2	1.00	24	0.083
936	A	3	2	1.00	24	0.083
937	A	3	2	1.00	24	0.083
938	A	3	2	1.00	24	0.083
939	A	3	2	1.00	24	0.083
940	A	3	2	1.00	24	0.083
941	A	4	3	1.00	24	0.125
942	A	4	3	1.00	24	0.125
943	A	4	3	1.00	22	0.136
944	A	4	3	1.00	21	0.143
945	A	3	2	1.00	24	0.083
946	A	3	2	1.00	24	0.083
947	A	3	2	1.00	24	0.083
948	A	3	2	1.00	24	0.083
949	A	4	3	1.00	25	0.120
950	A	4	3	1.00	23	0.130
951	A	3	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
952	A	4	3	1.00	25	0.120
953	A	4	3	1.00	25	0.120
954	A	4	3	1.00	22	0.136
955	A	4	3	1.00	22	0.136
956	A	4	3	1.00	22	0.136
957	A	4	3	1.00	25	0.120
958	A	4	3	1.00	25	0.120
959	A	4	3	1.00	25	0.120
960	A	4	3	1.00	23	0.130
961	A	3	3	1.00	22	0.136
962	A	5	5	1.00	25	0.200
963	A	4	3	1.00	25	0.120
964	A	4	3	1.00	25	0.120
965	A	4	3	1.00	25	0.120
966	A	4	3	1.00	25	0.120
967	A	4	3	1.00	25	0.120
968	A	4	3	1.00	25	0.120
969	A	4	3	1.00	25	0.120
970	A	5	4	1.00	23	0.174
971	A	5	4	1.00	22	0.182
972	A	4	3	1.00	25	0.120
973	A	4	3	1.00	25	0.120
974	A	4	3	1.00	25	0.120
975	A	4	3	1.00	25	0.120
976	A	4	3	1.00	25	0.120
977	A	4	3	1.00	25	0.120

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
978	A	4	3	1.00	25	0.120
979	A	5	4	1.00	25	0.160
980	A	5	4	1.00	25	0.160
981	A	5	4	1.00	23	0.174
982	A	5	4	1.00	22	0.182
983	A	4	3	1.00	25	0.120
984	A	4	3	1.00	25	0.120
985	A	4	3	1.00	25	0.120
986	A	5	4	1.00	22	0.182
987	A	4	3	1.00	23	0.130
988	A	4	3	1.00	21	0.143
989	A	4	3	1.00	23	0.130
990	A	4	3	1.00	23	0.130
991	A	4	3	1.00	23	0.130
992	A	3	2	1.00	24	0.083
993	A	3	2	1.00	24	0.083
994	A	3	2	1.00	24	0.083
995	A	2	2	1.00	24	0.083
996	A	6	4	1.00	24	0.167
997	A	6	4	1.00	24	0.167
998	A	4	3	1.00	25	0.120
999	A	4	3	1.00	25	0.120
1000	A	4	3	1.00	25	0.120
1001	A	3	3	1.00	25	0.120
1002	A	7	5	1.00	25	0.200
1003	A	7	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1004	A	5	4	1.00	23	0.174
1005	A	6	5	1.00	22	0.227
1006	A	6	5	1.00	22	0.227
1007	A	4	4	1.00	20	0.200
1008	A	2	2	1.00	19	0.105
1009	A	5	5	1.00	22	0.227
1010	A	5	5	1.00	22	0.227
1011	A	5	5	1.00	22	0.227
1012	A	7	6	1.00	23	0.261
1013	A	7	6	1.00	23	0.261
1014	A	5	5	1.00	21	0.238
1015	A	3	3	1.00	20	0.150
1016	A	6	6	1.00	23	0.261
1017	A	6	6	1.00	23	0.261
1018	A	6	6	1.00	23	0.261
1019	A	3	2	1.00	23	0.087
1020	A	3	2	1.00	23	0.087
1021	A	3	2	1.00	23	0.087
1022	A	3	2	1.00	21	0.095
1023	A	2	2	1.00	20	0.100
1024	A	3	2	1.00	23	0.087
1025	A	3	2	1.00	23	0.087
1026	A	3	2	1.00	23	0.087
1027	A	2	2	1.00	23	0.087
1028	A	3	2	1.00	25	0.080
1029	A	3	2	1.00	25	0.080

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1030	A	3	2	1.00	25	0.080
1031	A	3	2	1.00	23	0.087
1032	A	3	2	1.00	22	0.091
1033	A	3	2	1.00	25	0.080
1034	A	3	2	1.00	25	0.080
1035	A	2	2	1.00	25	0.080
1036	A	3	2	1.00	25	0.080
1037	A	3	2	1.00	25	0.080
1038	A	3	2	1.00	25	0.080
1039	A	3	2	1.00	25	0.080
1040	A	3	2	1.00	25	0.080
1041	A	3	2	1.00	25	0.080
1042	A	3	2	1.00	23	0.087
1043	A	3	2	1.00	22	0.091
1044	A	3	2	1.00	25	0.080
1045	A	3	2	1.00	25	0.080
1046	A	3	2	1.00	25	0.080
1047	A	3	2	1.00	25	0.080
1048	A	3	2	1.00	22	0.091
1049	A	3	2	1.00	25	0.080
1050	A	3	2	1.00	25	0.080
1051	A	3	2	1.00	25	0.080
1052	A	3	2	1.00	23	0.087
1053	A	2	2	1.00	22	0.091
1054	A	3	2	1.00	25	0.080
1055	A	3	2	1.00	25	0.080

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1056	A	3	2	1.00	25	0.080
1057	A	3	2	1.00	25	0.080
1058	A	3	2	1.00	25	0.080
1059	A	3	2	1.00	25	0.080
1060	A	4	3	1.00	25	0.120
1061	A	4	3	1.00	23	0.130
1062	A	4	3	1.00	22	0.136
1063	A	3	2	1.00	25	0.080
1064	A	3	2	1.00	25	0.080
1065	A	3	2	1.00	25	0.080
1066	A	3	2	1.00	25	0.080
1067	A	3	2	1.00	25	0.080
1068	A	4	3	1.00	25	0.120
1069	A	4	3	1.00	25	0.120
1070	A	2	2	1.00	25	0.080
1071	A	4	3	1.00	23	0.130
1072	A	4	3	1.00	22	0.136
1073	A	3	2	1.00	25	0.080
1074	A	3	2	1.00	25	0.080
1075	A	3	2	1.00	25	0.080
1076	A	4	3	1.00	22	0.136
1077	A	7	6	1.00	27	0.222
1078	A	6	6	1.00	27	0.222
1079	A	5	5	1.00	25	0.200
1080	A	5	5	1.00	24	0.208
1081	A	8	8	1.00	27	0.296

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1082	A	8	8	1.00	27	0.296
1083	A	6	6	1.00	27	0.222
1084	A	7	7	1.00	27	0.259
1085	A	8	7	1.00	27	0.259
1086	A	8	7	1.00	27	0.259
1087	A	7	7	1.00	27	0.259
1088	A	6	6	1.00	25	0.240
1089	A	6	6	1.00	24	0.250
1090	A	9	9	1.00	27	0.333
1091	A	9	9	1.00	27	0.333
1092	A	9	9	1.00	27	0.333
1093	A	9	9	1.00	27	0.333
1094	A	7	7	1.00	27	0.259
1095	A	8	8	1.00	27	0.296
1096	A	9	8	1.00	27	0.296
1097	A	10	8	1.00	27	0.296
1098	A	9	7	1.00	27	0.259
1099	A	8	7	1.00	27	0.259
1100	A	7	6	1.00	25	0.240
1101	A	7	6	1.00	24	0.250
1102	A	10	9	1.00	27	0.333
1103	A	10	9	1.00	27	0.333
1104	A	10	10	1.00	27	0.370
1105	A	10	9	1.00	27	0.333
1106	A	10	10	1.00	27	0.370
1107	A	8	6	1.00	24	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1108	A	7	6	1.00	27	0.222
1109	A	5	5	1.00	27	0.185
1110	A	5	5	1.00	25	0.200
1111	A	4	4	1.00	24	0.167
1112	A	5	5	1.00	27	0.185
1113	A	6	6	1.00	27	0.222
1114	A	7	7	1.00	27	0.259
1115	A	8	7	1.00	27	0.259
1116	A	6	5	1.00	27	0.185
1117	A	5	5	1.00	27	0.185
1118	A	3	3	1.00	25	0.120
1119	A	3	3	1.00	24	0.125
1120	A	7	7	1.00	27	0.259
1121	A	7	6	1.00	27	0.222
1122	A	8	7	1.00	27	0.259
1123	A	4	4	1.00	24	0.167
1124	A	5	4	1.00	24	0.167
1125	A	3	2	1.00	25	0.080
1126	A	3	2	1.00	25	0.080
1127	A	3	2	1.00	23	0.087
1128	A	2	2	1.00	25	0.080
1129	A	6	5	1.00	25	0.200
1130	A	8	5	1.00	25	0.200
1131	A	7	5	1.00	27	0.185
1132	A	7	5	1.00	27	0.185
1133	A	7	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1134	A	7	5	1.00	27	0.185
1135	A	7	5	1.00	27	0.185
1136	A	3	3	1.00	22	0.136
1137	A	7	5	1.00	23	0.217
1138	A	6	5	1.00	23	0.217
1139	A	6	5	1.00	21	0.238
1140	A	5	4	1.00	20	0.200
1141	A	8	7	1.00	23	0.304
1142	A	8	8	1.00	23	0.348
1143	A	8	7	1.00	23	0.304
1144	A	7	6	1.00	23	0.261
1145	A	8	7	1.00	23	0.304
1146	A	9	7	1.00	23	0.304
1147	A	6	5	1.00	22	0.227
1148	A	7	5	1.00	22	0.227
1149	A	8	5	1.00	22	0.227
1150	A	6	6	1.00	25	0.240
1151	A	4	4	1.00	23	0.174
1152	A	1	1	1.00	22	0.045
1153	A	5	4	1.00	22	0.182
1154	A	6	4	1.00	22	0.182
1155	A	7	5	1.00	22	0.227
1156	A	4	3	1.00	27	0.111
1157	A	4	3	1.00	27	0.111
1158	A	4	3	1.00	25	0.120
1159	A	4	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1160	A	4	3	1.00	27	0.111
1161	A	4	3	1.00	27	0.111
1162	A	4	3	1.00	27	0.111
1163	A	4	3	1.00	27	0.111
1164	A	4	3	1.00	27	0.111
1165	A	3	3	1.00	24	0.125
1166	A	4	3	1.00	24	0.125
1167	A	4	3	1.00	24	0.125
1168	A	4	3	1.00	24	0.125
1169	A	4	3	1.00	24	0.125
1170	A	3	3	1.00	24	0.125
1171	A	5	4	1.00	24	0.167
1172	A	5	4	1.00	24	0.167
1173	A	5	4	1.00	27	0.148
1174	A	7	5	1.00	25	0.200
1175	A	8	7	1.00	25	0.280
1176	A	8	7	1.00	25	0.280
1177	A	5	5	1.00	23	0.217
1178	A	3	3	1.00	22	0.136
1179	A	8	8	1.00	25	0.320
1180	A	9	9	1.00	25	0.360
1181	A	8	7	1.00	25	0.280
1182	A	3	2	1.00	22	0.091
1183	A	3	2	1.00	22	0.091
1184	A	3	2	1.00	22	0.091
1185	A	2	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1186	A	3	2	1.28	20	0.100
1187	A	2	2	1.00	22	0.091
1188	A	2	2	1.00	22	0.091
1189	A	4	3	1.00	22	0.136
1190	A	4	3	1.00	22	0.136
1191	A	3	3	1.00	22	0.136
1192	A	7	4	1.00	22	0.182
1193	A	6	4	1.00	22	0.182
1194	A	5	4	1.00	22	0.182
1195	A	4	4	1.00	20	0.200
1196	A	1	1	1.00	22	0.045
1197	A	3	3	1.00	22	0.136
1198	A	4	4	1.00	22	0.182
1199	A	5	4	1.00	22	0.182
1200	A	6	4	1.00	22	0.182
1201	A	4	3	1.00	27	0.111
1202	A	4	3	1.00	27	0.111
1203	A	4	3	1.00	25	0.120
1204	A	3	2	1.00	24	0.083
1205	A	4	3	1.00	27	0.111
1206	A	4	3	1.00	27	0.111
1207	A	4	3	1.00	24	0.125
1208	A	4	3	1.00	24	0.125
1209	A	4	3	1.00	24	0.125
1210	A	4	3	1.00	24	0.125
1211	A	3	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1212	A	5	4	1.00	24	0.167
1213	A	5	4	1.00	24	0.167
1214	A	5	4	1.00	24	0.167
1215	A	5	4	1.00	25	0.160
1216	A	6	5	1.00	24	0.208
1217	A	6	5	1.00	24	0.208
1218	A	4	4	1.00	22	0.182
1219	A	2	2	1.00	21	0.095
1220	A	5	5	1.00	24	0.208
1221	A	5	5	1.00	24	0.208
1222	A	7	6	1.00	25	0.240
1223	A	7	6	1.00	25	0.240
1224	A	5	5	1.00	23	0.217
1225	A	3	3	1.00	22	0.136
1226	A	6	6	1.00	25	0.240
1227	A	6	6	1.00	25	0.240
1228	A	3	2	1.00	22	0.091
1229	A	3	2	1.00	22	0.091
1230	A	3	2	1.00	22	0.091
1231	A	2	2	1.00	20	0.100
1232	A	2	2	1.00	22	0.091
1233	A	4	3	1.00	22	0.136
1234	A	4	3	1.00	22	0.136
1235	A	4	3	1.00	22	0.136
1236	A	7	6	1.00	27	0.222
1237	A	6	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1238	A	5	5	1.00	25	0.200
1239	A	5	5	1.00	24	0.208
1240	A	8	8	1.00	27	0.296
1241	A	8	8	1.00	27	0.296
1242	A	6	6	1.00	27	0.222
1243	A	7	7	1.00	27	0.259
1244	A	8	7	1.00	27	0.259
1245	A	6	6	1.00	24	0.250
1246	A	7	6	1.00	24	0.250
1247	A	8	6	1.00	24	0.250
1248	A	4	4	1.00	24	0.167
1249	A	3	3	1.00	24	0.125
1250	A	4	4	1.00	24	0.167
1251	A	5	4	1.00	24	0.167
1252	A	7	5	1.00	27	0.185
1253	A	3	3	1.00	22	0.136
1254	A	8	5	1.00	22	0.227
1255	A	7	5	1.00	22	0.227
1256	A	6	5	1.00	22	0.227
1257	A	5	4	1.00	20	0.200
1258	A	1	1	1.00	22	0.045
1259	A	5	4	1.00	22	0.182
1260	A	6	4	1.00	22	0.182
1261	A	7	5	1.00	22	0.227
1262	A	4	3	1.00	27	0.111
1263	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1264	A	4	3	1.00	25	0.120
1265	A	4	3	1.00	24	0.125
1266	A	4	3	1.00	27	0.111
1267	A	4	3	1.00	27	0.111
1268	A	4	3	1.00	27	0.111
1269	A	4	3	1.00	27	0.111
1270	A	4	3	1.00	27	0.111
1271	A	4	3	1.00	24	0.125
1272	A	4	3	1.00	24	0.125
1273	A	4	3	1.00	24	0.125
1274	A	3	3	1.00	24	0.125
1275	A	4	3	1.00	24	0.125
1276	A	3	3	1.00	24	0.125
1277	A	5	4	1.00	24	0.167
1278	A	5	4	1.00	24	0.167
1279	A	5	4	1.00	27	0.148
1280	A	3	3	1.00	22	0.136
1281	A	18	10	1.00	25	0.400
1282	A	16	10	1.00	25	0.400
1283	A	14	10	1.00	25	0.400
1284	A	12	9	1.00	25	0.360
1285	A	1	1	1.00	25	0.040
1286	A	2	2	1.00	25	0.080
1287	A	3	2	1.00	25	0.080
1288	A	4	2	1.00	25	0.080
1289	A	19	11	1.00	26	0.423

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1290	A	17	11	1.00	26	0.423
1291	A	15	11	1.00	26	0.423
1292	A	13	10	1.00	26	0.385
1293	A	1	1	1.00	26	0.038
1294	A	2	2	1.00	26	0.077
1295	A	3	2	1.00	26	0.077
1296	A	4	2	1.00	26	0.077
1297	A	10	8	1.00	29	0.276
1298	A	8	7	1.00	29	0.241
1299	A	5	5	1.00	27	0.185
1300	A	5	5	1.00	26	0.192
1301	A	8	7	1.00	29	0.241
1302	A	9	8	1.00	29	0.276
1303	A	5	5	1.00	29	0.172
1304	A	1	1	1.00	29	0.034
1305	A	4	4	1.00	27	0.148
1306	A	3	3	1.00	26	0.115
1307	A	3	3	1.00	29	0.103
1308	A	2	2	1.00	20	0.100
1309	A	2	2	1.00	22	0.091
1310	A	2	2	1.00	22	0.091
1311	A	5	5	1.00	25	0.200
1312	A	4	4	1.00	25	0.160
1313	A	4	4	1.48	25	0.160
1314	A	3	3	1.00	23	0.130
1315	A	1	1	1.00	22	0.045

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1316	A	3	3	1.11	25	0.120
1317	A	5	5	1.11	25	0.200
1318	A	10	9	1.00	25	0.360
1319	A	10	5	1.00	25	0.200
1320	A	2	2	1.00	25	0.080
1321	A	3	3	1.48	23	0.130
1322	A	2	2	1.00	22	0.091
1323	A	6	5	1.05	25	0.200
1324	A	7	5	1.06	25	0.200
1325	A	3	2	1.00	22	0.091
1326	A	4	2	1.00	22	0.091
1327	A	5	5	1.00	27	0.185
1328	A	5	5	1.00	27	0.185
1329	A	4	4	1.00	25	0.160
1330	A	3	3	1.00	24	0.125
1331	A	7	5	1.11	27	0.185
1332	C	3	3	0.36	27	0.111
1333	A	3	3	1.00	24	0.125
1334	A	5	5	1.00	27	0.185
1335	A	5	5	1.00	27	0.185
1336	A	4	4	1.00	25	0.160
1337	A	3	3	1.00	24	0.125
1338	A	3	3	1.00	27	0.111
1339	A	4	4	1.00	27	0.148
1340	A	6	6	1.00	27	0.222
1341	A	5	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1342	A	4	4	1.00	27	0.148
1343	A	1	1	1.00	25	0.040
1344	A	1	1	1.00	24	0.042
1345	A	6	6	1.02	27	0.222
1346	A	7	6	1.01	27	0.222
1347	A	8	7	1.01	27	0.259
1348	A	7	7	1.00	27	0.259
1349	A	2	2	1.00	27	0.074
1350	A	3	3	1.00	25	0.120
1351	A	2	2	1.00	24	0.083
1352	A	8	6	1.01	27	0.222
1353	A	9	6	1.01	27	0.222
1354	A	10	7	1.01	27	0.259
1355	A	3	2	1.00	24	0.083
1356	A	2	2	1.00	25	0.080
1357	A	2	2	1.00	23	0.087
1358	A	2	2	1.00	25	0.080
1359	A	2	2	1.00	25	0.080
1360	A	3	3	1.00	25	0.120
1361	A	4	4	1.00	23	0.174
1362	A	3	3	1.00	22	0.136
1363	A	3	2	1.00	26	0.077
1364	A	4	3	1.00	27	0.111
1365	A	3	3	1.00	23	0.130
1366	A	3	3	1.00	23	0.130
1367	A	1	1	1.00	33	0.030

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1368	A	2	2	1.00	25	0.080
1369	A	2	2	1.00	25	0.080
1370	A	2	2	1.00	25	0.080
1371	A	2	2	1.00	25	0.080
1372	A	2	2	1.00	25	0.080
1373	A	3	3	1.00	27	0.111
1374	A	3	3	1.00	27	0.111
1375	A	4	3	1.00	25	0.120
1376	A	4	3	1.00	27	0.111
1377	A	3	3	1.00	27	0.111
1378	A	3	3	1.00	27	0.111

Chapter 3

Listing of integrals

3.1 $\int e^{\tanh^{-1}(ax)} x^4 \, dx$

Optimal. Leaf size=111

$$\frac{3\sin^{-1}(ax)}{8a^5}-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{(45ax+64)\sqrt{1-a^2x^2}}{120a^5}-\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}$$

```
[Out] 3/8*arcsin(a*x)/a^5-4/15*x^2*(-a^2*x^2+1)^(1/2)/a^3-1/4*x^3*(-a^2*x^2+1)^(1
/2)/a^2-1/5*x^4*(-a^2*x^2+1)^(1/2)/a-1/120*(45*a*x+64)*(-a^2*x^2+1)^(1/2)/a
^5
```

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6124, 833, 780, 216}

$$-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}-\frac{(45ax+64)\sqrt{1-a^2x^2}}{120a^5}+\frac{3\sin^{-1}(ax)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^4,x]
```

```
[Out] (-4*x^2*Sqrt[1 - a^2*x^2])/(15*a^3) - (x^3*Sqrt[1 - a^2*x^2])/(4*a^2) - (x^
4*Sqrt[1 - a^2*x^2])/(5*a) - ((64 + 45*a*x)*Sqrt[1 - a^2*x^2])/(120*a^5) +
(3*ArcSin[a*x])/(8*a^5)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^4\,dx &= \int \frac{x^4(1+ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{\int \frac{x^3\left(-4a-5a^2x\right)}{\sqrt{1-a^2x^2}}\,dx}{5a^2}\\
&= -\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}+\frac{\int \frac{x^2\left(15a^2+16a^3x\right)}{\sqrt{1-a^2x^2}}\,dx}{20a^4}\\
&= -\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{\int \frac{x\left(-32a^3-45a^4x\right)}{\sqrt{1-a^2x^2}}\,dx}{60a^6}\\
&= -\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{(64+45ax)\sqrt{1-a^2x^2}}{120a^5}+\frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^4}\\
&= -\frac{4x^2\sqrt{1-a^2x^2}}{15a^3}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}-\frac{(64+45ax)\sqrt{1-a^2x^2}}{120a^5}+\frac{3\sin^{-1}(ax)}{8a^5}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.54

$$\frac{45\sin^{-1}(ax)-\sqrt{1-a^2x^2}\left(24a^4x^4+30a^3x^3+32a^2x^2+45ax+64\right)}{120a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^4,x]
```

```
[Out] (-(Sqrt[1 - a^2*x^2]*(64 + 45*a*x + 32*a^2*x^2 + 30*a^3*x^3 + 24*a^4*x^4))
+ 45*ArcSin[a*x])/(120*a^5)
```

fricas [A] time = 0.59, size = 73, normalized size = 0.66

$$-\frac{\left(24\,a^4x^4+30\,a^3x^3+32\,a^2x^2+45\,ax+64\right)\sqrt{-a^2x^2+1}\,+90\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{120\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4,x, algorithm="fricas")
```

```
[Out] -1/120*((24*a^4*x^4 + 30*a^3*x^3 + 32*a^2*x^2 + 45*a*x + 64)*sqrt(-a^2*x^2
+ 1) + 90*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 127, normalized size = 1.14

$$-\frac{x^4\sqrt{-a^2x^2+1}}{5a}-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^3}-\frac{8\sqrt{-a^2x^2+1}}{15a^5}-\frac{x^3\sqrt{-a^2x^2+1}}{4a^2}-\frac{3x\sqrt{-a^2x^2+1}}{8a^4}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a^4\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4,x)
```

```
[Out] -1/5*x^4*(-a^2*x^2+1)^(1/2)/a-4/15*x^2*(-a^2*x^2+1)^(1/2)/a^3-8/15*(-a^2*x^
2+1)^(1/2)/a^5-1/4*x^3*(-a^2*x^2+1)^(1/2)/a^2-3/8*x*(-a^2*x^2+1)^(1/2)/a^4+
3/8/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.42, size = 105, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+1}\,x^4}{5\,a}-\frac{\sqrt{-a^2x^2+1}\,x^3}{4\,a^2}-\frac{4\,\sqrt{-a^2x^2+1}\,x^2}{15\,a^3}-\frac{3\,\sqrt{-a^2x^2+1}\,x}{8\,a^4}+\frac{3\,\arcsin(ax)}{8\,a^5}-\frac{8\,\sqrt{-a^2x^2+1}}{15\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-a^2*x^2 + 1)*x^4/a - 1/4*sqrt(-a^2*x^2 + 1)*x^3/a^2 - 4/15*sqrt(
-a^2*x^2 + 1)*x^2/a^3 - 3/8*sqrt(-a^2*x^2 + 1)*x/a^4 + 3/8*arcsin(a*x)/a^5
- 8/15*sqrt(-a^2*x^2 + 1)/a^5
```

mupad [B] time = 0.08, size = 112, normalized size = 1.01

$$\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a^4\,\sqrt{-a^2}}+\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{8}{15\,a^3\,\sqrt{-a^2}}+\frac{a\,x^4}{5\,\sqrt{-a^2}}-\frac{3\,x\,\sqrt{-a^2}}{8\,a^4}+\frac{4\,x^2}{15\,a\,\sqrt{-a^2}}+\frac{x^3\,{\left(-a^2\right)}^{3/2}}{4\,a^4}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (3*asinh(x*(-a^2)^(1/2)))/(8*a^4*(-a^2)^(1/2)) + ((1 - a^2*x^2)^(1/2)*(8/(1
5*a^3*(-a^2)^(1/2)) + (a*x^4)/(5*(-a^2)^(1/2)) - (3*x*(-a^2)^(1/2))/(8*a^4)
 + (4*x^2)/(15*a*(-a^2)^(1/2)) + (x^3*(-a^2)^(3/2))/(4*a^4)))/(-a^2)^(1/2)
```

sympy [A] time = 4.75, size = 221, normalized size = 1.99

$$a\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^4}-\frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for}\ a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\,\mathrm{acosh}(ax)}{8a^5}\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\,\mathrm{asin}(ax)}{8a^5}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4,x)
```

```
[Out] a*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a**2*x**2 +
 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/6, True)
) + Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x
**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), A
```

```
bs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**
2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), Tr
ue))
```

3.2 $\int e^{\tanh^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=87

$$\frac{3\sin^{-1}(ax)}{8a^4} - \frac{x^2\sqrt{1-a^2x^2}}{3a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{(9ax+16)\sqrt{1-a^2x^2}}{24a^4}$$

```
[Out] 3/8*arcsin(a*x)/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)/a^2-1/4*x^3*(-a^2*x^2+1)^(1/
2)/a-1/24*(9*a*x+16)*(-a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6124, 833, 780, 216}

$$-\frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{x^2\sqrt{1-a^2x^2}}{3a^2} - \frac{(9ax+16)\sqrt{1-a^2x^2}}{24a^4} + \frac{3\sin^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3,x]
```

```
[Out] -(x^2*Sqrt[1 - a^2*x^2])/(3*a^2) - (x^3*Sqrt[1 - a^2*x^2])/(4*a) - ((16 + 9
*a*x)*Sqrt[1 - a^2*x^2])/(24*a^4) + (3*ArcSin[a*x])/(8*a^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1+ax)}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{\int \frac{x^2\left(-3a-4a^2x\right)}{\sqrt{1-a^2x^2}}\,dx}{4a^2} \\
&= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{\int \frac{x\left(8a^2+9a^3x\right)}{\sqrt{1-a^2x^2}}\,dx}{12a^4} \\
&= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{(16+9ax)\sqrt{1-a^2x^2}}{24a^4} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^3} \\
&= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{(16+9ax)\sqrt{1-a^2x^2}}{24a^4} + \frac{3\sin^{-1}(ax)}{8a^4}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.60

$$\frac{9\sin^{-1}(ax)-\sqrt{1-a^2x^2}\left(6a^3x^3+8a^2x^2+9ax+16\right)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^3,x]
```

```
[Out] (-(Sqrt[1 - a^2*x^2]*(16 + 9*a*x + 8*a^2*x^2 + 6*a^3*x^3)) + 9*ArcSin[a*x])
/(24*a^4)
```

fricas [A] time = 0.46, size = 65, normalized size = 0.75

$$-\frac{\left(6\,a^3x^3+8\,a^2x^2+9\,ax+16\right)\sqrt{-a^2x^2+1}\;+18\;\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/24*((6*a^3*x^3 + 8*a^2*x^2 + 9*a*x + 16)*sqrt(-a^2*x^2 + 1) + 18*arctan(
(sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

giac [A] time = 1.18, size = 59, normalized size = 0.68

$$-\frac{1}{24}\sqrt{-a^2x^2+1}\left(\left(2\,x\left(\frac{3\,x}{a}+\frac{4}{a^2}\right)+\frac{9}{a^3}\right)x+\frac{16}{a^4}\right)+\frac{3\,\arcsin\left(ax\right)\operatorname{sgn}(a)}{8\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-a^2*x^2 + 1)*((2*x*(3*x/a + 4/a^2) + 9/a^3)*x + 16/a^4) + 3/8*a
rcsin(a*x)*sgn(a)/(a^3*abs(a))
```

maple [A] time = 0.04, size = 107, normalized size = 1.23

$$-\frac{x^3\sqrt{-a^2x^2+1}}{4a}-\frac{3x\sqrt{-a^2x^2+1}}{8a^3}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a^3\sqrt{a^2}}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3,x)
```

```
[Out] -1/4*x^3*(-a^2*x^2+1)^(1/2)/a-3/8*x*(-a^2*x^2+1)^(1/2)/a^3+3/8/a^3/(a^2)^(1
/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/3*x^2*(-a^2*x^2+1)^(1/2)/a^2
-2/3*(-a^2*x^2+1)^(1/2)/a^4
```

maxima [A] time = 0.42, size = 85, normalized size = 0.98

$$-\frac{\sqrt{-a^2x^2+1}\,x^3}{4\,a}-\frac{\sqrt{-a^2x^2+1}\,x^2}{3\,a^2}-\frac{3\,\sqrt{-a^2x^2+1}\,x}{8\,a^3}+\frac{3\,\arcsin\left(ax\right)}{8\,a^4}-\frac{2\,\sqrt{-a^2x^2+1}}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + 1)*x^3/a - 1/3*sqrt(-a^2*x^2 + 1)*x^2/a^2 - 3/8*sqrt(-
a^2*x^2 + 1)*x/a^3 + 3/8*arcsin(a*x)/a^4 - 2/3*sqrt(-a^2*x^2 + 1)/a^4
```

mupad [B] time = 0.83, size = 97, normalized size = 1.11

$$\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a^3\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2}{3\,{\left(-a^2\right)}^{3/2}}+\frac{3\,x\,\sqrt{-a^2}}{8\,a^3}+\frac{a^2\,x^2}{3\,{\left(-a^2\right)}^{3/2}}-\frac{x^3\,{\left(-a^2\right)}^{3/2}}{4\,a^3}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (3*asinh(x*(-a^2)^(1/2)))/(8*a^3*(-a^2)^(1/2)) - ((1 - a^2*x^2)^(1/2)*(2/(3
*(-a^2)^(3/2)) + (3*x*(-a^2)^(1/2))/(8*a^3) + (a^2*x^2)/(3*(-a^2)^(3/2)) -
(x^3*(-a^2)^(3/2))/(4*a^3)))/(-a^2)^(1/2)
```

sympy [C] time = 4.46, size = 199, normalized size = 2.29

$$a\left(\begin{cases} -\frac{ix^5}{4\sqrt{a^2x^2-1}} - \frac{ix^3}{8a^2\sqrt{a^2x^2-1}} + \frac{3ix}{8a^4\sqrt{a^2x^2-1}} - \frac{3i\operatorname{acosh}(ax)}{8a^5} & \text{for } |a^2x^2| > 1 \\ \frac{x^5}{4\sqrt{-a^2x^2+1}} + \frac{x^3}{8a^2\sqrt{-a^2x^2+1}} - \frac{3x}{8a^4\sqrt{-a^2x^2+1}} + \frac{3\operatorname{asin}(ax)}{8a^5} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{x^2\sqrt{-a^2x^2+1}}{3a^2} - \frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a \neq 0 \\ \frac{x^4}{4} & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3,x)
```

```
[Out] a*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**
2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs
(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*
x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), True
)) + Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1
)/(3*a**4), Ne(a, 0)), (x**4/4, True))
```

3.3 $\int e^{\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=74

$$\frac{\sin^{-1}(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3}-\frac{\sqrt{1-a^2x^2}}{a^3}$$

```
[Out] 1/3*(-a^2*x^2+1)^(3/2)/a^3+1/2*arcsin(a*x)/a^3-(-a^2*x^2+1)^(1/2)/a^3-1/2*x
*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6124, 797, 641, 195, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}-\frac{\sqrt{1-a^2x^2}}{a^3}+\frac{\sin^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2,x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a^3) - (x*Sqrt[1 - a^2*x^2])/(2*a^2) + (1 - a^2*x^2)^(3
/2)/(3*a^3) + ArcSin[a*x]/(2*a^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{a^2}-\frac{\int (1+ax)\sqrt{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}-\frac{\int \sqrt{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3}+\frac{\sin^{-1}(ax)}{a^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3}+\frac{\sin^{-1}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.59

$$\frac{3\sin^{-1}(ax)-\sqrt{1-a^2x^2}\left(2a^2x^2+3ax+4\right)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2,x]
[Out] (-(Sqrt[1 - a^2*x^2]*(4 + 3*a*x + 2*a^2*x^2)) + 3*ArcSin[a*x])/(6*a^3)
```

fricas [A] time = 0.82, size = 57, normalized size = 0.77

$$-\frac{\left(2\,a^2x^2+3\,ax+4\right)\sqrt{-a^2x^2+1}+6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/6*((2*a^2*x^2 + 3*a*x + 4)*sqrt(-a^2*x^2 + 1) + 6*arctan((sqrt(-a^2*x^2
+ 1) - 1)/(a*x)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 87, normalized size = 1.18

$$-\frac{x^{2}\sqrt{-a^{2}x^{2}+1}}{3a}-\frac{2\sqrt{-a^{2}x^{2}+1}}{3a^{3}}-\frac{x\sqrt{-a^{2}x^{2}+1}}{2a^{2}}+\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{2a^{2}\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2,x)
```

```
[Out] -1/3*x^2/a*(-a^2*x^2+1)^(1/2)-2/3*(-a^2*x^2+1)^(1/2)/a^3-1/2*x*(-a^2*x^2+1)
^(1/2)/a^2+1/2/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 65, normalized size = 0.88

$$-\frac{\sqrt{-a^{2}x^{2}+1}\,x^{2}}{3\,a}-\frac{\sqrt{-a^{2}x^{2}+1}\,x}{2\,a^{2}}+\frac{\arcsin\left(ax\right)}{2\,a^{3}}-\frac{2\,\sqrt{-a^{2}x^{2}+1}}{3\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + 1)*x^2/a - 1/2*sqrt(-a^2*x^2 + 1)*x/a^2 + 1/2*arcsin(a
*x)/a^3 - 2/3*sqrt(-a^2*x^2 + 1)/a^3
```

mupad [B] time = 0.05, size = 82, normalized size = 1.11

$$\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^2\,\sqrt{-a^2}}+\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2}{3\,a\,\sqrt{-a^2}}+\frac{a\,x^2}{3\,\sqrt{-a^2}}-\frac{x\,\sqrt{-a^2}}{2\,a^2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] asinh(x*(-a^2)^(1/2))/(2*a^2*(-a^2)^(1/2)) + ((1 - a^2*x^2)^(1/2)*(2/(3*a*(
-a^2)^(1/2)) + (a*x^2)/(3*(-a^2)^(1/2)) - (x*(-a^2)^(1/2))/(2*a^2)))/(-a^2)
^(1/2)
```

sympy [A] time = 3.29, size = 133, normalized size = 1.80

$$a\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\,\mathrm{acosh}\,(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\mathrm{asin}\,(ax)}{2a^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2,x)
```

```
[Out] a*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(
3*a**4), Ne(a, 0)), (x**4/4, True)) + Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(
2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x
**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True))
```

3.4 $\int e^{\tanh^{-1}(ax)} x \, dx$

Optimal. Leaf size=38

$$\frac{\sin^{-1}(ax)}{2a^2} - \frac{(ax+2)\sqrt{1-a^2x^2}}{2a^2}$$

```
[Out] 1/2*arcsin(a*x)/a^2-1/2*(a*x+2)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6124, 780, 216}

$$\frac{\sin^{-1}(ax)}{2a^2} - \frac{(ax+2)\sqrt{1-a^2x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x,x]
```

```
[Out] -((2 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^2) + ArcSin[a*x]/(2*a^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x\,dx = \int \frac{x(1+ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{(2+ax)\sqrt{1-a^2x^2}}{2a^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a}$$
$$= -\frac{(2+ax)\sqrt{1-a^2x^2}}{2a^2} + \frac{\sin^{-1}(ax)}{2a^2}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.87

$$\frac{\sin^{-1}(ax) - (ax+2)\sqrt{1-a^2x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x,x]
[Out] (-((2 + a*x)*Sqrt[1 - a^2*x^2]) + ArcSin[a*x])/(2*a^2)
```

fricas [A] time = 0.54, size = 48, normalized size = 1.26

$$-\frac{\sqrt{-a^2x^2+1}\,(ax+2) + 2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x,x, algorithm="fricas")
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*(a*x + 2) + 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)))/a^2
```

giac [A] time = 1.11, size = 41, normalized size = 1.08

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a} + \frac{2}{a^2}\right) + \frac{\arcsin(ax)\,\mathrm{sgn}(a)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x,x, algorithm="giac")
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x/a + 2/a^2) + 1/2*arcsin(a*x)*sgn(a)/(a*abs(a))
```

maple [B] time = 0.03, size = 67, normalized size = 1.76

$$-\frac{x\sqrt{-a^{2}x^{2}+1}}{2a}+\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{2a\sqrt{a^{2}}}-\frac{\sqrt{-a^{2}x^{2}+1}}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x,x)
```

```
[Out] -1/2*x*(-a^2*x^2+1)^(1/2)/a+1/2/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/a^2
```

maxima [A] time = 0.42, size = 45, normalized size = 1.18

$$-\frac{\sqrt{-a^{2}x^{2}+1}\,x}{2\,a}+\frac{\arcsin\left(ax\right)}{2\,a^{2}}-\frac{\sqrt{-a^{2}x^{2}+1}}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x/a + 1/2*arcsin(a*x)/a^2 - sqrt(-a^2*x^2 + 1)/a^2
```

mupad [B] time = 0.80, size = 58, normalized size = 1.53

$$\frac{\sqrt{1-a^{2}\,x^{2}}\,\left(\frac{1}{\sqrt{-a^{2}}}-\frac{x\,\sqrt{-a^{2}}}{2\,a}\right)+\frac{\mathrm{asinh}\left(x\,\sqrt{-a^{2}}\right)}{2\,a}}{\sqrt{-a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(1/(-a^2)^(1/2) - (x*(-a^2)^(1/2))/(2*a)) + asinh(x*(-
a^2)^(1/2))/(2*a))/(-a^2)^(1/2)
```

sympy [C] time = 3.16, size = 110, normalized size = 2.89

$$a\left(\begin{cases}-\frac{ix\sqrt{a^{2}x^{2}-1}}{2a^{2}}-\frac{i\,\mathrm{acosh}\left(ax\right)}{2a^{3}} & \text{for}\ \left|a^{2}x^{2}\right|>1\\ \frac{x^{3}}{2\sqrt{-a^{2}x^{2}+1}}-\frac{x}{2a^{2}\sqrt{-a^{2}x^{2}+1}}+\frac{\mathrm{asin}\left(ax\right)}{2a^{3}} & \text{otherwise}\end{cases}\right)+\begin{cases}\frac{x^{2}}{2} & \text{for}\ a^{2}=0\\ -\frac{\sqrt{-a^{2}x^{2}+1}}{a^{2}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x,x)
```

```
[Out] a*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs
(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**
2 + 1)) + asin(a*x)/(2*a**3), True)) + Piecewise((x**2/2, Eq(a**2, 0)), (-s
qrt(-a**2*x**2 + 1)/a**2, True))
```

3.5 $\int e^{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\frac{\sin^{-1}(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}$$

```
[Out] arcsin(a*x)/a-(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6123, 641, 216}

$$\frac{\sin^{-1}(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a) + ArcSin[a*x]/a
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6123

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 + a*x)^((n + 1)/2)/(
(1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2]), x] /; FreeQ[a, x] && IntegerQ[(n
- 1)/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\,dx = \int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1-a^2x^2}}{a} + \int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1-a^2x^2}}{a} + \frac{\sin^{-1}(ax)}{a}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.89

$$\frac{\sin^{-1}(ax) - \sqrt{1-a^2x^2}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x],x]
[Out] (-Sqrt[1 - a^2*x^2] + ArcSin[a*x])/a
```

fricas [A] time = 0.66, size = 42, normalized size = 1.50

$$-\frac{\sqrt{-a^2x^2+1} + 2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(sqrt(-a^2*x^2 + 1) + 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a
```

giac [A] time = 0.22, size = 29, normalized size = 1.04

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} - \frac{\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] arcsin(a*x)*sgn(a)/abs(a) - sqrt(-a^2*x^2 + 1)/a
```

maple [A] time = 0.03, size = 45, normalized size = 1.61

$$-\frac{\sqrt{-a^{2}x^{2}+1}}{a}+\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2),x)
[Out] -(-a^2*x^2+1)^(1/2)/a+1/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)
)
```

maxima [A] time = 0.41, size = 26, normalized size = 0.93

$$\frac{\arcsin\left(ax\right)}{a}-\frac{\sqrt{-a^{2}x^{2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsin(a*x)/a - sqrt(-a^2*x^2 + 1)/a
```

mupad [B] time = 0.80, size = 36, normalized size = 1.29

$$\frac{\operatorname{asinh}\left(x\sqrt{-a^{2}}\right)}{\sqrt{-a^{2}}}-\frac{\sqrt{1-a^{2}\,x^{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(1 - a^2*x^2)^(1/2),x)
[Out] asinh(x*(-a^2)^(1/2))/(-a^2)^(1/2) - (1 - a^2*x^2)^(1/2)/a
```

sympy [A] time = 1.62, size = 19, normalized size = 0.68

$$\begin{cases}\frac{-\sqrt{-a^{2}x^{2}+1}+\operatorname{asin}\left(ax\right)}{a} & \text{for}\ a\neq 0\\ x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] Piecewise(((-sqrt(-a**2*x**2 + 1) + asin(a*x))/a, Ne(a, 0)), (x, True))
```

3.6 $\int \frac{e^{\tanh^{-1}(ax)}}{x} dx$

Optimal. Leaf size=22

$$\sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1 - a^2x^2}\right)$$

```
[Out] arcsin(a*x)-arctanh((-a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6124, 844, 216, 266, 63, 208}

$$\sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1 - a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x,x]
```

```
[Out] ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{1+ax}{x\sqrt{1-a^2x^2}}\,dx \\
&= a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx + \int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= \sin^{-1}(ax) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \sin^{-1}(ax) - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2} \\
&= \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.18

$$-\log\left(\sqrt{1-a^2x^2}+1\right)+\sin^{-1}(ax)+\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x,x]
```

```
[Out] ArcSin[a*x] + Log[x] - Log[1 + Sqrt[1 - a^2*x^2]]
```

fricas [B] time = 0.48, size = 44, normalized size = 2.00

$$-2\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + log((sqrt(-a^2*x^2 + 1) - 1)/x)
```

giac [B] time = 0.23, size = 51, normalized size = 2.32

$$\frac{a \arcsin\left(ax\right) \operatorname{sgn}(a)}{|a|} - \frac{a \log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
2*a)/(a^2*abs(x)))/abs(a)
```

maple [B] time = 0.04, size = 44, normalized size = 2.00

$$\frac{a \arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-arctanh(1/(-a^2*x^2+
1)^(1/2))
```

maxima [A] time = 0.41, size = 33, normalized size = 1.50

$$\arcsin\left(ax\right) - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] arcsin(a*x) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.03, size = 35, normalized size = 1.59

$$\frac{a \operatorname{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}} - \operatorname{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - atanh((1 - a^2*x^2)^(1/2))
```

sympy [B] time = 5.92, size = 70, normalized size = 3.18

$$a\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)+\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] a*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*as
inh(x*sqrt(-a**2)), a**2 < 0)) + Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**
2) > 1), (I*asin(1/(a*x)), True))
```

3.7 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{1-a^2x^2}}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -a*arctanh((-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6124, 807, 266, 63, 208}

$$-\frac{\sqrt{1-a^2x^2}}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x^2,x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{1+ax}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{x} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{x} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{x} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1-a^2x^2}}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 1.16

$$-\frac{\sqrt{1-a^2x^2}}{x} - a\log\left(\sqrt{1-a^2x^2}+1\right) + a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x^2,x]
[Out] -(Sqrt[1 - a^2*x^2]/x) + a*Log[x] - a*Log[1 + Sqrt[1 - a^2*x^2]]
```

fricas [A] time = 0.52, size = 41, normalized size = 1.08

$$\frac{ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (a*x*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1))/x
```

giac [B] time = 0.42, size = 96, normalized size = 2.53

$$\frac{a^4 x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/2*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(x*abs(a))
```

maple [A] time = 0.04, size = 35, normalized size = 0.92

$$-a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -a*arctanh(1/(-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/x
```

maxima [A] time = 0.41, size = 47, normalized size = 1.24

$$-a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)/x
```

mupad [B] time = 0.04, size = 34, normalized size = 0.89

$$-a\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)-\frac{\sqrt{1-a^2\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] - a*atanh((1 - a^2*x^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/x
```

sympy **[C]** time = 2.37, size = 65, normalized size = 1.71

$$a\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ |a^2x^2| > 1 \\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] a*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True
)) + Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2
*x**2 + 1)/x, True))
```

3.8 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=64

$$-\frac{a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/2*a^2*arctanh((-a^2*x^2+1)^(1/2))-1/2*(-a^2*x^2+1)^(1/2)/x^2-a*(-a^2*x^2
+1)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6124, 835, 807, 266, 63, 208}

$$-\frac{a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x^3,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) - (a*Sqrt[1 - a^2*x^2])/x - (a^2*ArcTanh[Sqrt[1
- a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{1+ax}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{1}{2}\int \frac{-2a-a^2x}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{a\sqrt{1-a^2x^2}}{x} + \frac{1}{2}a^2\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{a\sqrt{1-a^2x^2}}{x} + \frac{1}{4}a^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{a\sqrt{1-a^2x^2}}{x} - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{a\sqrt{1-a^2x^2}}{x} - \frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.91

$$\frac{1}{2}\left(-\frac{(2ax+1)\sqrt{1-a^2x^2}}{x^2}-a^2\log\left(\sqrt{1-a^2x^2}+1\right)+a^2\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x^3,x]
```

```
[Out] (-(((1 + 2*a*x)*Sqrt[1 - a^2*x^2])/x^2) + a^2*Log[x] - a^2*Log[1 + Sqrt[1 -
 a^2*x^2]])/2
```

fricas [A] time = 0.43, size = 52, normalized size = 0.81

$$\frac{a^2x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}\,(2\,ax+1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1)*(2*a*x +
1))/x^2
```

giac [B] time = 0.41, size = 158, normalized size = 2.47

$$\frac{\left(a^3+\frac{4\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}-\frac{a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{4\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3 + 4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x)*a^4*x^2/((sqrt(-a^2*x^2 +
 1)*abs(a) + a)^2*abs(a)) - 1/2*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a
) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*ab
s(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2))/a^2
```

maple [A] time = 0.04, size = 55, normalized size = 0.86

$$-\frac{a\sqrt{-a^2x^2+1}}{x}-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] -a*(-a^2*x^2+1)^(1/2)/x-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*
x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 67, normalized size = 1.05

$$-\frac{1}{2}\,a^2\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-a^2x^2+1}\,a}{x}-\frac{\sqrt{-a^2x^2+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*a
/x - 1/2*sqrt(-a^2*x^2 + 1)/x^2
```

mupad [B] time = 0.04, size = 54, normalized size = 0.84

$$-\frac{a^2\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{2}-\frac{\sqrt{1-a^2\,x^2}}{2\,x^2}-\frac{a\,\sqrt{1-a^2\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] - (a^2*atanh((1 - a^2*x^2)^(1/2)))/2 - (1 - a^2*x^2)^(1/2)/(2*x^2) - (a*(1
- a^2*x^2)^(1/2))/x
```

sympy [C] time = 3.04, size = 136, normalized size = 2.12

$$a\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{a^2\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for}\ \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\,\mathrm{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] a*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x*
*2 + 1)/x, True)) + Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2
*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sq
rt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

3.9 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=90

$$-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/2*a^3*arctanh((-a^2*x^2+1)^(1/2))-1/3*(-a^2*x^2+1)^(1/2)/x^3-1/2*a*(-a^2
*x^2+1)^(1/2)/x^2-2/3*a^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6124, 835, 807, 266, 63, 208}

$$-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x^4,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) - (a*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a^2*Sqrt[1
- a^2*x^2])/(3*x) - (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{x^4}\,dx &= \int \frac{1+ax}{x^4\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{1}{3}\int \frac{-3a-2a^2x}{x^3\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{a\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{6}\int \frac{4a^2+3a^3x}{x^2\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{2a^2\sqrt{1-a^2x^2}}{3x}+\frac{1}{2}a^3\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{2a^2\sqrt{1-a^2x^2}}{3x}+\frac{1}{4}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)\\ &= -\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.74

$$\frac{1}{6}\left(3a^3\log(x)-\frac{\sqrt{1-a^2x^2}\left(4a^2x^2+3ax+2\right)}{x^3}-3a^3\log\left(\sqrt{1-a^2x^2}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x^4,x]
```

```
[Out] (-((Sqrt[1 - a^2*x^2]*(2 + 3*a*x + 4*a^2*x^2))/x^3) + 3*a^3*Log[x] - 3*a^3*
Log[1 + Sqrt[1 - a^2*x^2]])/6
```

fricas [A] time = 0.51, size = 61, normalized size = 0.68

$$\frac{3\,a^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(4\,a^2x^2+3\,ax+2\right)\sqrt{-a^2x^2+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^3*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (4*a^2*x^2 + 3*a*x + 2)*sq
rt(-a^2*x^2 + 1))/x^3
```

giac [B] time = 0.21, size = 210, normalized size = 2.33

$$\frac{\left(a^4 + \frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2}{x} + \frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|} - \frac{a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{2|a|} - \frac{\frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^4}{x} + \frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{x^2}}{24a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 9*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*abs(a)) - 1/
2*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a)
- 1/24*(9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x + 3*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^2*a^2/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a))
```

maple [A] time = 0.04, size = 77, normalized size = 0.86

$$-\frac{\sqrt{-a^2x^2+1}}{3x^3} - \frac{2a^2\sqrt{-a^2x^2+1}}{3x} + a\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2} - \frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] -1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x+a*(-1/2*(-a^2*x^2+
1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.40, size = 87, normalized size = 0.97

$$-\frac{1}{2}a^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{2\sqrt{-a^2x^2+1}\,a^2}{3x} - \frac{\sqrt{-a^2x^2+1}\,a}{2x^2} - \frac{\sqrt{-a^2x^2+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*a^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/3*sqrt(-a^2*x^2 +
1)*a^2/x - 1/2*sqrt(-a^2*x^2 + 1)*a/x^2 - 1/3*sqrt(-a^2*x^2 + 1)/x^3
```

mupad [B] time = 0.03, size = 78, normalized size = 0.87

$$-\frac{\sqrt{1-a^2\,x^2}}{3\,x^3}-\frac{a\,\sqrt{1-a^2\,x^2}}{2\,x^2}-\frac{2\,a^2\,\sqrt{1-a^2\,x^2}}{3\,x}+\frac{a^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^3*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/2 - (1 - a^2*x^2)^(1/2)/(3*x^3) - (a*
(1 - a^2*x^2)^(1/2))/(2*x^2) - (2*a^2*(1 - a^2*x^2)^(1/2))/(3*x)
```

sympy [C] time = 3.21, size = 185, normalized size = 2.06

$$a\left(\begin{cases}-\frac{a^2\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\,\mathrm{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } |a^2x^2|>1\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] a*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/A
bs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**
2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) + Piecewise((-2*I*a**2*
sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2)
> 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3),
True))
```

3.10 $\int \frac{e^{\tanh^{-1}(ax)}}{x^5} \, dx$

Optimal. Leaf size=114

$$-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}-\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{2a^3\sqrt{1-a^2x^2}}{3x}$$

```
[Out] -3/8*a^4*arctanh((-a^2*x^2+1)^(1/2))-1/4*(-a^2*x^2+1)^(1/2)/x^4-1/3*a*(-a^2
*x^2+1)^(1/2)/x^3-3/8*a^2*(-a^2*x^2+1)^(1/2)/x^2-2/3*a^3*(-a^2*x^2+1)^(1/2)
/x
```

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6124, 835, 807, 266, 63, 208}

$$-\frac{2a^3\sqrt{1-a^2x^2}}{3x}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}-\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/x^5,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(4*x^4) - (a*Sqrt[1 - a^2*x^2])/(3*x^3) - (3*a^2*Sqrt[1
- a^2*x^2])/(8*x^2) - (2*a^3*Sqrt[1 - a^2*x^2])/(3*x) - (3*a^4*ArcTanh[Sqrt
[1 - a^2*x^2]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^5}\,dx &= \int \frac{1+ax}{x^5\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{1}{4}\int \frac{-4a-3a^2x}{x^4\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} + \frac{1}{12}\int \frac{9a^2+8a^3x}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{1}{24}\int \frac{-16a^3-9a^4x}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{2a^3\sqrt{1-a^2x^2}}{3x} + \frac{1}{8}\left(3a^4\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{2a^3\sqrt{1-a^2x^2}}{3x} + \frac{1}{16}\left(3a^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} \right. \\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{2a^3\sqrt{1-a^2x^2}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, \right. \\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{2a^3\sqrt{1-a^2x^2}}{3x} - \frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.06, size = 75, normalized size = 0.66

$$\frac{1}{24}\left(9a^4\log(x) - 9a^4\log\left(\sqrt{1-a^2x^2}+1\right) - \frac{\sqrt{1-a^2x^2}\left(16a^3x^3+9a^2x^2+8ax+6\right)}{x^4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/x^5,x]
```

```
[Out] (-((Sqrt[1 - a^2*x^2]*(6 + 8*a*x + 9*a^2*x^2 + 16*a^3*x^3))/x^4) + 9*a^4*Lo
g[x] - 9*a^4*Log[1 + Sqrt[1 - a^2*x^2]])/24
```

fricas [**A**] time = 0.45, size = 69, normalized size = 0.61

$$\frac{9\,a^4x^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(16\,a^3x^3 + 9\,a^2x^2 + 8\,ax + 6\right)\sqrt{-a^2x^2+1}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a^4*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (16*a^3*x^3 + 9*a^2*x^2 +
 8*a*x + 6)*sqrt(-a^2*x^2 + 1))/x^4
```

giac [B] time = 0.18, size = 273, normalized size = 2.39

$$\frac{\left(3\,a^5+\frac{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^3}{x}+\frac{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2a}{x^2}+\frac{72\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{ax^3}\right)a^8x^4}{192\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4|a|}-\frac{3\,a^5\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|}-\frac{72\left(\sqrt{-a^2x^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^5 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3/x + 24*(sqrt(-a^2*x^2
+ 1)*abs(a) + a)^2*a/x^2 + 72*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a*x^3))*a^
8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)) - 3/8*a^5*log(1/2*abs(-2*s
qrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/192*(72*(sqrt(-a^2
*x^2 + 1)*abs(a) + a)*a^5*abs(a)/x + 24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a
^3*abs(a)/x^2 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*abs(a)/x^3 + 3*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)/(a*x^4))/a^4
```

maple [A] time = 0.04, size = 100, normalized size = 0.88

$$a\left(-\frac{\sqrt{-a^2x^2+1}}{3x^3}-\frac{2a^2\sqrt{-a^2x^2+1}}{3x}\right)-\frac{\sqrt{-a^2x^2+1}}{4x^4}+\frac{3a^2\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^5,x)
```

```
[Out] a*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x)-1/4*(-a^2*x^2+
1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x
^2+1)^(1/2)))
```

maxima [A] time = 0.41, size = 107, normalized size = 0.94

$$-\frac{3}{8}\,a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{2\sqrt{-a^2x^2+1}\,a^3}{3\,x}-\frac{3\sqrt{-a^2x^2+1}\,a^2}{8\,x^2}-\frac{\sqrt{-a^2x^2+1}\,a}{3\,x^3}-\frac{\sqrt{-a^2x^2+1}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -3/8*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/3*sqrt(-a^2*x^2 +
1)*a^3/x - 3/8*sqrt(-a^2*x^2 + 1)*a^2/x^2 - 1/3*sqrt(-a^2*x^2 + 1)*a/x^3 -
1/4*sqrt(-a^2*x^2 + 1)/x^4
```

mupad [B] time = 0.03, size = 98, normalized size = 0.86

$$-\frac{\sqrt{1-a^2\,x^2}}{4\,x^4}-\frac{a\,\sqrt{1-a^2\,x^2}}{3\,x^3}-\frac{3\,a^2\,\sqrt{1-a^2\,x^2}}{8\,x^2}-\frac{2\,a^3\,\sqrt{1-a^2\,x^2}}{3\,x}+\frac{a^4\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,3\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^5*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^4*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/8 - (1 - a^2*x^2)^(1/2)/(4*x^4) - (a*
(1 - a^2*x^2)^(1/2))/(3*x^3) - (3*a^2*(1 - a^2*x^2)^(1/2))/(8*x^2) - (2*a^3
*(1 - a^2*x^2)^(1/2))/(3*x)
```

sympy [C] time = 4.44, size = 258, normalized size = 2.26

$$a\left(\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } \left|a^2x^2\right|>1\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{3a^4\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{8}+\frac{3a^3}{8x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{a}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{1}{4ax^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for} \\ \frac{3ia^4\,\mathrm{asin}\left(\frac{1}{ax}\right)}{8}-\frac{3ia^3}{8x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{ia}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{oth}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] a*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3
*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a*
*2*x**2 + 1)/(3*x**3), True)) + Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**
3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/
(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(
1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 -
1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

3.11 $\int e^{2\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=44

$$-\frac{2\log(1-ax)}{a^4}-\frac{2x}{a^3}-\frac{x^2}{a^2}-\frac{2x^3}{3a}-\frac{x^4}{4}$$

```
[Out] -2*x/a^3-x^2/a^2-2/3*x^3/a-1/4*x^4-2*ln(-a*x+1)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{x^2}{a^2}-\frac{2x}{a^3}-\frac{2\log(1-ax)}{a^4}-\frac{2x^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3,x]
```

```
[Out] (-2*x)/a^3 - x^2/a^2 - (2*x^3)/(3*a) - x^4/4 - (2*Log[1 - a*x])/a^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+ax)}{1-ax}\,dx$$
$$= \int \left(-\frac{2}{a^3} - \frac{2x}{a^2} - \frac{2x^2}{a} - x^3 - \frac{2}{a^3(-1+ax)}\right)dx$$
$$= -\frac{2x}{a^3} - \frac{x^2}{a^2} - \frac{2x^3}{3a} - \frac{x^4}{4} - \frac{2\log(1-ax)}{a^4}$$

Mathematica [**A**] time = 0.02, size = 44, normalized size = 1.00

$$-\frac{2\log(1-ax)}{a^4} - \frac{2x}{a^3} - \frac{x^2}{a^2} - \frac{2x^3}{3a} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3,x]
```

```
[Out] (-2*x)/a^3 - x^2/a^2 - (2*x^3)/(3*a) - x^4/4 - (2*Log[1 - a*x])/a^4
```

fricas [**A**] time = 0.40, size = 42, normalized size = 0.95

$$-\frac{3\,a^4x^4 + 8\,a^3x^3 + 12\,a^2x^2 + 24\,ax + 24\,\log(ax-1)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x + 24*log(a*x - 1))/a^4
```

giac [**A**] time = 0.93, size = 47, normalized size = 1.07

$$-\frac{3\,a^4x^4 + 8\,a^3x^3 + 12\,a^2x^2 + 24\,ax}{12\,a^4} - \frac{2\,\log(|ax-1|)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x)/a^4 - 2*log(abs(a*x - 1
))/a^4
```

maple [**A**] time = 0.03, size = 40, normalized size = 0.91

$$-\frac{x^4}{4} - \frac{2x^3}{3a} - \frac{x^2}{a^2} - \frac{2x}{a^3} - \frac{2\ln(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3,x)
```

```
[Out] -1/4*x^4-2/3*x^3/a-x^2/a^2-2*x/a^3-2/a^4*ln(a*x-1)
```

maxima [A] time = 0.31, size = 43, normalized size = 0.98

$$-\frac{3\,a^3x^4+8\,a^2x^3+12\,ax^2+24\,x}{12\,a^3}-\frac{2\,\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^4 + 8*a^2*x^3 + 12*a*x^2 + 24*x)/a^3 - 2*log(a*x - 1)/a^4
```

mupad [B] time = 0.05, size = 39, normalized size = 0.89

$$-\frac{2\,\ln\left(a\,x-1\right)}{a^4}-\frac{2\,x}{a^3}-\frac{x^4}{4}-\frac{2\,x^3}{3\,a}-\frac{x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] - (2*log(a*x - 1))/a^4 - (2*x)/a^3 - x^4/4 - (2*x^3)/(3*a) - x^2/a^2
```

sympy [A] time = 0.11, size = 39, normalized size = 0.89

$$-\frac{x^4}{4}-\frac{2x^3}{3a}-\frac{x^2}{a^2}-\frac{2x}{a^3}-\frac{2\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3,x)
```

```
[Out] -x**4/4 - 2*x**3/(3*a) - x**2/a**2 - 2*x/a**3 - 2*log(a*x - 1)/a**4
```

3.12 $\int e^{2\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=34

$$-\frac{2\log(1-ax)}{a^3}-\frac{2x}{a^2}-\frac{x^2}{a}-\frac{x^3}{3}$$

```
[Out] -2*x/a^2-x^2/a-1/3*x^3-2*ln(-a*x+1)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{2x}{a^2}-\frac{2\log(1-ax)}{a^3}-\frac{x^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2,x]
```

```
[Out] (-2*x)/a^2 - x^2/a - x^3/3 - (2*Log[1 - a*x])/a^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^2\,dx&=\int\frac{x^2(1+ax)}{1-ax}\,dx\\&=\int\left(-\frac{2}{a^2}-\frac{2x}{a}-x^2-\frac{2}{a^2(-1+ax)}\right)dx\\&=-\frac{2x}{a^2}-\frac{x^2}{a}-\frac{x^3}{3}-\frac{2\log(1-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{2\log(1-ax)}{a^3}-\frac{2x}{a^2}-\frac{x^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2,x]
[Out] (-2*x)/a^2 - x^2/a - x^3/3 - (2*Log[1 - a*x])/a^3
```

fricas [A] time = 0.41, size = 33, normalized size = 0.97

$$-\frac{a^3x^3+3\,a^2x^2+6\,ax+6\,\log\left(ax-1\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2,x, algorithm="fricas")
[Out] -1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^3
```

giac [A] time = 0.17, size = 38, normalized size = 1.12

$$-\frac{a^3x^3+3\,a^2x^2+6\,ax}{3\,a^3}-\frac{2\,\log\left(|ax-1|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2,x, algorithm="giac")
[Out] -1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x)/a^3 - 2*log(abs(a*x - 1))/a^3
```

maple [A] time = 0.03, size = 32, normalized size = 0.94

$$-\frac{x^3}{3}-\frac{x^2}{a}-\frac{2x}{a^2}-\frac{2\ln\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2,x)
[Out] -1/3*x^3-x^2/a-2*x/a^2-2/a^3*ln(a*x-1)
```

maxima [A] time = 0.31, size = 34, normalized size = 1.00

$$-\frac{a^2x^3+3\,ax^2+6\,x}{3\,a^2}-\frac{2\,\log\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2,x, algorithm="maxima")
[Out] -1/3*(a^2*x^3 + 3*a*x^2 + 6*x)/a^2 - 2*log(a*x - 1)/a^3
```

mupad [B] time = 0.04, size = 31, normalized size = 0.91

$$-\frac{2\,\ln\left(a\,x-1\right)}{a^3}-\frac{2\,x}{a^2}-\frac{x^3}{3}-\frac{x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] - (2*log(a*x - 1))/a^3 - (2*x)/a^2 - x^3/3 - x^2/a
```

sympy [A] time = 0.10, size = 29, normalized size = 0.85

$$-\frac{x^3}{3}-\frac{x^2}{a}-\frac{2x}{a^2}-\frac{2\log\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2,x)
[Out] -x**3/3 - x**2/a - 2*x/a**2 - 2*log(a*x - 1)/a**3
```

3.13 $\int e^{2\tanh^{-1}(ax)} x \, dx$

Optimal. Leaf size=26

$$-\frac{2\log(1-ax)}{a^2}-\frac{2x}{a}-\frac{x^2}{2}$$

```
[Out] -2*x/a-1/2*x^2-2*ln(-a*x+1)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6126, 77}

$$-\frac{2\log(1-ax)}{a^2}-\frac{2x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x,x]
```

```
[Out] (-2*x)/a - x^2/2 - (2*Log[1 - a*x])/a^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)} x \, dx &= \int \frac{x(1+ax)}{1-ax} \, dx \\
&= \int \left(-\frac{2}{a}-x-\frac{2}{a(-1+ax)}\right) dx \\
&= -\frac{2x}{a}-\frac{x^2}{2}-\frac{2\log(1-ax)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{2\log(1-ax)}{a^2}-\frac{2x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x,x]
[Out] (-2*x)/a - x^2/2 - (2*Log[1 - a*x])/a^2
```

fricas [A] time = 0.61, size = 25, normalized size = 0.96

$$-\frac{a^2x^2+4\,ax+4\,\log\left(ax-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x,x, algorithm="fricas")
[Out] -1/2*(a^2*x^2 + 4*a*x + 4*log(a*x - 1))/a^2
```

giac [A] time = 0.16, size = 30, normalized size = 1.15

$$-\frac{a^2x^2+4\,ax}{2\,a^2}-\frac{2\,\log\left(|ax-1|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x,x, algorithm="giac")
[Out] -1/2*(a^2*x^2 + 4*a*x)/a^2 - 2*log(abs(a*x - 1))/a^2
```

maple [A] time = 0.03, size = 24, normalized size = 0.92

$$-\frac{x^2}{2}-\frac{2x}{a}-\frac{2\ln\left(ax-1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x,x)
[Out] -1/2*x^2-2*x/a-2/a^2*ln(a*x-1)
```

maxima [A] time = 0.31, size = 26, normalized size = 1.00

$$-\frac{ax^2+4\,x}{2\,a}-\frac{2\,\log\left(ax-1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x,x, algorithm="maxima")
[Out] -1/2*(a*x^2 + 4*x)/a - 2*log(a*x - 1)/a^2
```

mupad [B] time = 0.80, size = 23, normalized size = 0.88

$$-\frac{2\ln\left(a\,x-1\right)}{a^2}-\frac{2\,x}{a}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] - (2*log(a*x - 1))/a^2 - (2*x)/a - x^2/2
```

sympy [A] time = 0.09, size = 22, normalized size = 0.85

$$-\frac{x^2}{2}-\frac{2x}{a}-\frac{2\log\left(ax-1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x,x)
[Out] -x**2/2 - 2*x/a - 2*log(a*x - 1)/a**2
```

3.14 $\int e^{2\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=16

$$-\frac{2\log(1-ax)}{a}-x$$

```
[Out] -x-2*ln(-a*x+1)/a
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6125, 43}

$$-\frac{2\log(1-ax)}{a}-x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x]),x]
```

```
[Out] -x - (2*Log[1 - a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\,dx&=\int\frac{1+ax}{1-ax}\,dx\\&=\int\left(-1-\frac{2}{-1+ax}\right)dx\\&=-x-\frac{2\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2\log(1-ax)}{a}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x]),x]
```

```
[Out] -x - (2*Log[1 - a*x])/a
```

fricas [A] time = 0.42, size = 17, normalized size = 1.06

$$-\frac{ax+2\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -(a*x + 2*log(a*x - 1))/a
```

giac [A] time = 0.24, size = 16, normalized size = 1.00

$$-x-\frac{2\log\left(|ax-1|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -x - 2*log(abs(a*x - 1))/a
```

maple [A] time = 0.02, size = 16, normalized size = 1.00

$$-x-\frac{2\ln\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1),x)
```

```
[Out] -x-2/a*ln(a*x-1)
```

maxima [A] time = 0.31, size = 15, normalized size = 0.94

$$-x-\frac{2\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x - 2*log(a*x - 1)/a
```

mupad [B] time = 0.03, size = 15, normalized size = 0.94

$$-x - \frac{2\ \ln(a\,x - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(a^2*x^2 - 1),x)
```

```
[Out] - x - (2*log(a*x - 1))/a
```

sympy [A] time = 0.08, size = 12, normalized size = 0.75

$$-x - \frac{2\log(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1),x)
```

```
[Out] -x - 2*log(a*x - 1)/a
```

3.15 $\int \frac{e^{2\tanh^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=12

$$\log(x) - 2\log(1 - ax)$$

```
[Out] ln(x)-2*ln(-a*x+1)
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 72}

$$\log(x) - 2\log(1 - ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/x,x]
```

```
[Out] Log[x] - 2*Log[1 - a*x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x} \, dx &= \int \frac{1 + ax}{x(1 - ax)} \, dx \\
&= \int \left(\frac{1}{x} - \frac{2a}{-1 + ax}\right) dx \\
&= \log(x) - 2\log(1 - ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\log(x) - 2\log(1 - ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/x,x]
```

```
[Out] Log[x] - 2*Log[1 - a*x]
```

fricas [A] time = 0.68, size = 11, normalized size = 0.92

$$-2 \log(ax-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] -2*log(a*x - 1) + log(x)
```

giac [A] time = 0.34, size = 13, normalized size = 1.08

$$-2 \log(|ax-1|)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] -2*log(abs(a*x - 1)) + log(abs(x))
```

maple [A] time = 0.03, size = 12, normalized size = 1.00

$$\ln(x)-2\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x,x)
```

```
[Out] ln(x)-2*ln(a*x-1)
```

maxima [A] time = 0.31, size = 11, normalized size = 0.92

$$-2 \log(ax-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] -2*log(a*x - 1) + log(x)
```

mupad [B] time = 0.82, size = 12, normalized size = 1.00

$$\ln(x)-2\ln(3\,a\,x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x*(a^2*x^2 - 1)),x)
```

```
[Out] log(x) - 2*log(3*a*x - 3)
```

sympy **[A]** time = 0.12, size = 10, normalized size = 0.83

$$\log(x) - 2\log\left(x - \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x,x)
```

```
[Out] log(x) - 2*log(x - 1/a)
```

3.16 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=21

$$2a\log(x) - 2a\log(1-ax) - \frac{1}{x}$$

```
[Out] -1/x+2*a*ln(x)-2*a*ln(-a*x+1)
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$2a\log(x) - 2a\log(1-ax) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/x^2,x]
```

```
[Out] -x^(-1) + 2*a*Log[x] - 2*a*Log[1 - a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^2} dx = \int \frac{1+ax}{x^2(1-ax)} dx$$
$$= \int \left(\frac{1}{x^2} + \frac{2a}{x} - \frac{2a^2}{-1+ax}\right) dx$$
$$= -\frac{1}{x} + 2a\log(x) - 2a\log(1-ax)$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$2a\log(x) - 2a\log(1-ax) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/x^2,x]
[Out] -x^(-1) + 2*a*Log[x] - 2*a*Log[1 - a*x]
```

fricas [A] time = 0.76, size = 23, normalized size = 1.10

$$-\frac{2\,ax\log(ax-1) - 2\,ax\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2,x, algorithm="fricas")
[Out] -(2*a*x*log(a*x - 1) - 2*a*x*log(x) + 1)/x
```

giac [A] time = 0.25, size = 22, normalized size = 1.05

$$-2\,a\log(|ax-1|) + 2\,a\log(|x|) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2,x, algorithm="giac")
[Out] -2*a*log(abs(a*x - 1)) + 2*a*log(abs(x)) - 1/x
```

maple [A] time = 0.03, size = 21, normalized size = 1.00

$$-\frac{1}{x} + 2a\ln(x) - 2a\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/x+2*a*ln(x)-2*a*ln(a*x-1)
```

maxima [A] time = 0.31, size = 20, normalized size = 0.95

$$-2\,a\log\left(ax-1\right)+2\,a\log(x)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] -2*a*log(a*x - 1) + 2*a*log(x) - 1/x
```

mupad [B] time = 0.81, size = 16, normalized size = 0.76

$$4\,a\,\mathrm{atanh}\left(2\,a\,x-1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] 4*a*atanh(2*a*x - 1) - 1/x
```

sympy [A] time = 0.14, size = 17, normalized size = 0.81

$$-2a\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2,x)
```

```
[Out] -2*a*(-log(x) + log(x - 1/a)) - 1/x
```

3.17 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=33

$$2a^2\log(x) - 2a^2\log(1-ax) - \frac{2a}{x} - \frac{1}{2x^2}$$

```
[Out] -1/2/x^2-2*a/x+2*a^2*ln(x)-2*a^2*ln(-a*x+1)
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$2a^2\log(x) - 2a^2\log(1-ax) - \frac{2a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 - a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{1+ax}{x^3(1-ax)}\,dx$$
$$= \int \left(\frac{1}{x^3} + \frac{2a}{x^2} + \frac{2a^2}{x} - \frac{2a^3}{-1+ax}\right)dx$$
$$= -\frac{1}{2x^2} - \frac{2a}{x} + 2a^2\log(x) - 2a^2\log(1-ax)$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$2a^2\log(x) - 2a^2\log(1-ax) - \frac{2a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/x^3,x]
[Out] -1/2*1/x^2 - (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 - a*x]
```

fricas [A] time = 0.51, size = 35, normalized size = 1.06

$$-\frac{4\,a^2x^2\log\left(ax-1\right) - 4\,a^2x^2\log(x) + 4\,ax + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3,x, algorithm="fricas")
[Out] -1/2*(4*a^2*x^2*log(a*x - 1) - 4*a^2*x^2*log(x) + 4*a*x + 1)/x^2
```

giac [A] time = 0.25, size = 32, normalized size = 0.97

$$-2\,a^2\log\left(|ax-1|\right) + 2\,a^2\log\left(|x|\right) - \frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3,x, algorithm="giac")
[Out] -2*a^2*log(abs(a*x - 1)) + 2*a^2*log(abs(x)) - 1/2*(4*a*x + 1)/x^2
```

maple [A] time = 0.03, size = 31, normalized size = 0.94

$$-\frac{1}{2x^2} - \frac{2a}{x} + 2a^2\ln(x) - 2a^2\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3,x)
```

```
[Out] -1/2/x^2-2*a/x+2*a^2*ln(x)-2*a^2*ln(a*x-1)
```

maxima [A] time = 0.31, size = 30, normalized size = 0.91

$$-2\,a^2\log\left(ax-1\right)+2\,a^2\log(x)-\frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] -2*a^2*log(a*x - 1) + 2*a^2*log(x) - 1/2*(4*a*x + 1)/x^2
```

mupad [B] time = 0.81, size = 24, normalized size = 0.73

$$4\,a^2\,\mathrm{atanh}\left(2\,a\,x-1\right)-\frac{2\,a\,x+\frac{1}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] 4*a^2*atanh(2*a*x - 1) - (2*a*x + 1/2)/x^2
```

sympy [A] time = 0.16, size = 27, normalized size = 0.82

$$-2a^2\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)-\frac{4ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3,x)
```

```
[Out] -2*a**2*(-log(x) + log(x - 1/a)) - (4*a*x + 1)/(2*x**2)
```

3.18 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=41

$$2a^3\log(x) - 2a^3\log(1-ax) - \frac{2a^2}{x} - \frac{a}{x^2} - \frac{1}{3x^3}$$

```
[Out] -1/3/x^3-a/x^2-2*a^2/x+2*a^3*ln(x)-2*a^3*ln(-a*x+1)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{2a^2}{x} + 2a^3\log(x) - 2a^3\log(1-ax) - \frac{a}{x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - a/x^2 - (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 - a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{1+ax}{x^4(1-ax)}\,dx$$
$$= \int \left(\frac{1}{x^4} + \frac{2a}{x^3} + \frac{2a^2}{x^2} + \frac{2a^3}{x} - \frac{2a^4}{-1+ax}\right) dx$$
$$= -\frac{1}{3x^3} - \frac{a}{x^2} - \frac{2a^2}{x} + 2a^3\log(x) - 2a^3\log(1-ax)$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$2a^3\log(x) - 2a^3\log(1-ax) - \frac{2a^2}{x} - \frac{a}{x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/x^4,x]
[Out] -1/3*1/x^3 - a/x^2 - (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 - a*x]
```

fricas [A] time = 0.47, size = 43, normalized size = 1.05

$$-\frac{6\,a^3x^3\log(ax-1) - 6\,a^3x^3\log(x) + 6\,a^2x^2 + 3\,ax + 1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4,x, algorithm="fricas")
[Out] -1/3*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 1)/x^
3
```

giac [A] time = 0.19, size = 40, normalized size = 0.98

$$-2\,a^3\log(|ax-1|) + 2\,a^3\log(|x|) - \frac{6\,a^2x^2 + 3\,ax + 1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4,x, algorithm="giac")
[Out] -2*a^3*log(abs(a*x - 1)) + 2*a^3*log(abs(x)) - 1/3*(6*a^2*x^2 + 3*a*x + 1)/
x^3
```

maple [A] time = 0.03, size = 39, normalized size = 0.95

$$-\frac{1}{3x^3} - \frac{a}{x^2} - \frac{2a^2}{x} + 2a^3\ln(x) - 2a^3\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^4,x)
[Out] -1/3/x^3-a/x^2-2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x-1)
```

maxima [A] time = 0.31, size = 38, normalized size = 0.93

$$-2\,a^3\log\left(ax-1\right)+2\,a^3\log(x)-\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4,x, algorithm="maxima")
[Out] -2*a^3*log(a*x - 1) + 2*a^3*log(x) - 1/3*(6*a^2*x^2 + 3*a*x + 1)/x^3
```

mupad [B] time = 0.05, size = 31, normalized size = 0.76

$$4\,a^3\,\mathrm{atanh}\left(2\,a\,x-1\right)-\frac{2\,a^2\,x^2+a\,x+\frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^4*(a^2*x^2 - 1)),x)
[Out] 4*a^3*atanh(2*a*x - 1) - (a*x + 2*a^2*x^2 + 1/3)/x^3
```

sympy [A] time = 0.18, size = 36, normalized size = 0.88

$$-2a^3\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)-\frac{6a^2x^2+3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**4,x)
[Out] -2*a**3*(-log(x) + log(x - 1/a)) - (6*a**2*x**2 + 3*a*x + 1)/(3*x**3)
```

3.19 $\int e^{3\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=92

$$-\frac{11\sin^{-1}(ax)}{2a^3}+\frac{(ax+1)^3}{a^3\sqrt{1-a^2x^2}}+\frac{(ax+3)^2\sqrt{1-a^2x^2}}{3a^3}+\frac{(3ax+28)\sqrt{1-a^2x^2}}{6a^3}$$

```
[Out] -11/2*arcsin(a*x)/a^3+(a*x+1)^3/a^3/(-a^2*x^2+1)^(1/2)+1/3*(a*x+3)^2*(-a^2*
x^2+1)^(1/2)/a^3+1/6*(3*a*x+28)*(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.65, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6124, 1633, 1593, 12, 852, 1635, 1654, 780, 216}

$$\frac{(ax+1)^3}{a^3\sqrt{1-a^2x^2}}+\frac{(ax+3)^2\sqrt{1-a^2x^2}}{3a^3}+\frac{(3ax+28)\sqrt{1-a^2x^2}}{6a^3}-\frac{11\sin^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2,x]
```

```
[Out] (1 + a*x)^3/(a^3*Sqrt[1 - a^2*x^2]) + ((3 + a*x)^2*Sqrt[1 - a^2*x^2])/(3*a^
3) + ((28 + 3*a*x)*Sqrt[1 - a^2*x^2])/(6*a^3) - (11*ArcSin[a*x])/(2*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
```

```
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1+ax)^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx \\
&= -\left(a\int \frac{\sqrt{1-a^2x^2}\left(-\frac{x^2}{a}-x^3\right)}{(1-ax)^2}\,dx\right) \\
&= -\left(a\int \frac{\left(-\frac{1}{a}-x\right)x^2\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx\right) \\
&= a^2\int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{a^2(1-ax)^3}\,dx \\
&= \int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}\,dx \\
&= \int \frac{x^2(1+ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx \\
&= \frac{(1+ax)^3}{a^3\sqrt{1-a^2x^2}} - \int \frac{\left(\frac{3}{a^2}+\frac{x}{a}\right)(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{(1+ax)^3}{a^3\sqrt{1-a^2x^2}} + \frac{(3+ax)^2\sqrt{1-a^2x^2}}{3a^3} + \frac{1}{3}\int \frac{\left(\frac{3}{a^2}+\frac{x}{a}\right)(-5-3ax)}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{(1+ax)^3}{a^3\sqrt{1-a^2x^2}} + \frac{(3+ax)^2\sqrt{1-a^2x^2}}{3a^3} + \frac{(28+3ax)\sqrt{1-a^2x^2}}{6a^3} - \frac{11\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2} \\
&= \frac{(1+ax)^3}{a^3\sqrt{1-a^2x^2}} + \frac{(3+ax)^2\sqrt{1-a^2x^2}}{3a^3} + \frac{(28+3ax)\sqrt{1-a^2x^2}}{6a^3} - \frac{11\sin^{-1}(ax)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.63

$$\frac{\frac{\sqrt{1-a^2x^2}\left(2a^3x^3+7a^2x^2+19ax-52\right)}{ax-1}-33\sin^{-1}(ax)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-52 + 19*a*x + 7*a^2*x^2 + 2*a^3*x^3))/(-1 + a*x) - 33
*ArcSin[a*x])/(6*a^3)
```

fricas [A] time = 0.91, size = 85, normalized size = 0.92

$$\frac{52\,ax + 66\,(ax - 1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(2\,a^3x^3 + 7\,a^2x^2 + 19\,ax - 52\right)\sqrt{-a^2x^2+1} - 52}{6\left(a^4x - a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(52*a*x + 66*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3*
x^3 + 7*a^2*x^2 + 19*a*x - 52)*sqrt(-a^2*x^2 + 1) - 52)/(a^4*x - a^3)
```

giac [A] time = 0.24, size = 87, normalized size = 0.95

$$\frac{1}{6}\sqrt{-a^2x^2+1}\left(x\left(\frac{2\,x}{a} + \frac{9}{a^2}\right) + \frac{28}{a^3}\right) - \frac{11\,\arcsin\left(ax\right)\mathrm{sgn}(a)}{2\,a^2|a|} + \frac{8}{a^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-a^2*x^2 + 1)*(x*(2*x/a + 9/a^2) + 28/a^3) - 11/2*arcsin(a*x)*sgn(
a)/(a^2*abs(a)) + 8/(a^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(
a))
```

maple [A] time = 0.05, size = 122, normalized size = 1.33

$$-\frac{a\,x^4}{3\sqrt{-a^2x^2+1}} - \frac{13x^2}{3a\sqrt{-a^2x^2+1}} + \frac{26}{3a^3\sqrt{-a^2x^2+1}} - \frac{3x^3}{2\sqrt{-a^2x^2+1}} + \frac{11x}{2a^2\sqrt{-a^2x^2+1}} - \frac{11\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a^2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2,x)
```

```
[Out] -1/3*a*x^4/(-a^2*x^2+1)^(1/2)-13/3/a*x^2/(-a^2*x^2+1)^(1/2)+26/3/a^3/(-a^2*
x^2+1)^(1/2)-3/2*x^3/(-a^2*x^2+1)^(1/2)+11/2*x/a^2/(-a^2*x^2+1)^(1/2)-11/2/
a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 100, normalized size = 1.09

$$-\frac{ax^4}{3\sqrt{-a^2x^2+1}}-\frac{3x^3}{2\sqrt{-a^2x^2+1}}-\frac{13x^2}{3\sqrt{-a^2x^2+1}\,a}+\frac{11x}{2\sqrt{-a^2x^2+1}\,a^2}-\frac{11\arcsin(ax)}{2a^3}+\frac{26}{3\sqrt{-a^2x^2+1}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*a*x^4/sqrt(-a^2*x^2 + 1) - 3/2*x^3/sqrt(-a^2*x^2 + 1) - 13/3*x^2/(sqrt
(-a^2*x^2 + 1)*a) + 11/2*x/(sqrt(-a^2*x^2 + 1)*a^2) - 11/2*arcsin(a*x)/a^3
+ 26/3/(sqrt(-a^2*x^2 + 1)*a^3)
```

mupad [B] time = 0.07, size = 143, normalized size = 1.55

$$\frac{4\sqrt{1-a^2x^2}}{a^2\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{\sqrt{1-a^2x^2}\left(\frac{2}{3a\sqrt{-a^2}}-\frac{4\sqrt{-a^2}}{a^3}+\frac{a\,x^2}{3\sqrt{-a^2}}-\frac{3x\sqrt{-a^2}}{2a^2}\right)}{\sqrt{-a^2}}-\frac{11\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{2a^2\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (4*(1 - a^2*x^2)^(1/2))/(a^2*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2)
) - ((1 - a^2*x^2)^(1/2)*(2/(3*a*(-a^2)^(1/2)) - (4*(-a^2)^(1/2))/a^3 + (a*
x^2)/(3*(-a^2)^(1/2)) - (3*x*(-a^2)^(1/2))/(2*a^2)))/(-a^2)^(1/2) - (11*asi
nh(x*(-a^2)^(1/2)))/(2*a^2*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2,x)
```

```
[Out] Integral(x**2*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.20 $\int e^{3\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=88

$$\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1-ax)}+\frac{9\sqrt{1-a^2x^2}}{2a^2}-\frac{9\sin^{-1}(ax)}{2a^2}$$

```
[Out] 3/2*(-a^2*x^2+1)^(3/2)/a^2/(-a*x+1)+(-a^2*x^2+1)^(5/2)/a^2/(-a*x+1)^3-9/2*a
rcsin(a*x)/a^2+9/2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.38, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6124, 1633, 1593, 12, 793, 665, 216}

$$\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1-ax)}+\frac{9\sqrt{1-a^2x^2}}{2a^2}-\frac{9\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x,x]
```

```
[Out] (9*Sqrt[1 - a^2*x^2])/(2*a^2) + (3*(1 - a^2*x^2)^(3/2))/(2*a^2*(1 - a*x)) +
 (1 - a^2*x^2)^(5/2)/(a^2*(1 - a*x)^3) - (9*ArcSin[a*x])/(2*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x\,dx = \int \frac{x(1+ax)^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx$$
$$= -\left(a\int \frac{\left(-\frac{x}{a}-x^2\right)\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx\right)$$
$$= -\left(a\int \frac{\left(-\frac{1}{a}-x\right)x\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx\right)$$
$$= a^2\int \frac{x\left(1-a^2x^2\right)^{3/2}}{a^2(1-ax)^3}\,dx$$
$$= \int \frac{x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}\,dx$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3} - \frac{3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(1-ax)^2}\,dx}{a}$$
$$= \frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1-ax)} + \frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3} - \frac{9\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx}{2a}$$
$$= \frac{9\sqrt{1-a^2x^2}}{2a^2} + \frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1-ax)} + \frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3} - \frac{9\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a}$$
$$= \frac{9\sqrt{1-a^2x^2}}{2a^2} + \frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1-ax)} + \frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1-ax)^3} - \frac{9\sin^{-1}(ax)}{2a^2}$$

Mathematica [**A**] time = 0.04, size = 53, normalized size = 0.60

$$\sqrt{1-a^2x^2}\left(-\frac{4}{a^2(ax-1)}+\frac{3}{a^2}+\frac{x}{2a}\right)-\frac{9\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x,x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(3/a^2 + x/(2*a) - 4/(a^2*(-1 + a*x))) - (9*ArcSin[a*x])/
(2*a^2)
```

fricas [A]   time = 1.13, size = 76, normalized size = 0.86

$$\frac{14\,ax+18\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(a^2x^2+5\,ax-14\right)\sqrt{-a^2x^2+1}-14}{2\,\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(14*a*x + 18*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a^2*x^
2 + 5*a*x - 14)*sqrt(-a^2*x^2 + 1) - 14)/(a^3*x - a^2)
```

giac [A]   time = 0.22, size = 78, normalized size = 0.89

$$\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a}+\frac{6}{a^2}\right)-\frac{9\,\arcsin\left(ax\right)\mathrm{sgn}(a)}{2\,a|a|}+\frac{8}{a\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(x/a + 6/a^2) - 9/2*arcsin(a*x)*sgn(a)/(a*abs(a)) +
8/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A]   time = 0.04, size = 102, normalized size = 1.16

$$-\frac{a\,x^3}{2\sqrt{-a^2x^2+1}}+\frac{9x}{2a\sqrt{-a^2x^2+1}}-\frac{9\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a\sqrt{a^2}}-\frac{3x^2}{\sqrt{-a^2x^2+1}}+\frac{7}{a^2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x,x)
```

```
[Out] -1/2*a*x^3/(-a^2*x^2+1)^(1/2)+9/2*x/a/(-a^2*x^2+1)^(1/2)-9/2/a/(a^2)^(1/2)*
arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-3*x^2/(-a^2*x^2+1)^(1/2)+7/a^2/(-a
^2*x^2+1)^(1/2)
```

maxima [A]   time = 0.41, size = 80, normalized size = 0.91

$$-\frac{ax^3}{2\sqrt{-a^2x^2+1}}-\frac{3\,x^2}{\sqrt{-a^2x^2+1}}+\frac{9\,x}{2\sqrt{-a^2x^2+1}\,a}-\frac{9\,\arcsin\left(ax\right)}{2\,a^2}+\frac{7}{\sqrt{-a^2x^2+1}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*a*x^3/sqrt(-a^2*x^2 + 1) - 3*x^2/sqrt(-a^2*x^2 + 1) + 9/2*x/(sqrt(-a^2
*x^2 + 1)*a) - 9/2*arcsin(a*x)/a^2 + 7/(sqrt(-a^2*x^2 + 1)*a^2)
```

mupad [B] time = 0.81, size = 102, normalized size = 1.16

$$-\frac{\left(\frac{3}{\sqrt{-a^2}}-\frac{x\,\sqrt{-a^2}}{2\,a}\right)\sqrt{1-a^2\,x^2}+\frac{9\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a}-\frac{4\,\sqrt{1-a^2\,x^2}}{a\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] -((3/(-a^2)^(1/2) - (x*(-a^2)^(1/2))/(2*a))*(1 - a^2*x^2)^(1/2) + (9*asinh(
x*(-a^2)^(1/2)))/(2*a) - (4*(1 - a^2*x^2)^(1/2))/(a*(x*(-a^2)^(1/2) - (-a^2
)^(1/2)/a)))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(ax+1)^3}{\left(-(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x,x)
```

```
[Out] Integral(x*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.21 $\int e^{3\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$\frac{2(ax+1)^2}{a\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{a}-\frac{3\sin^{-1}(ax)}{a}$$

```
[Out] -3*arcsin(a*x)/a+2*(a*x+1)^2/a/(-a^2*x^2+1)^(1/2)+3*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6123, 853, 669, 641, 216}

$$\frac{2(ax+1)^2}{a\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{a}-\frac{3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*(1 + a*x)^2)/(a*Sqrt[1 - a^2*x^2]) + (3*Sqrt[1 - a^2*x^2])/a - (3*ArcSin
[a*x])/a
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
```

```
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 6123

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 + a*x)^((n + 1)/2)/(
(1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2]), x] /; FreeQ[a, x] && IntegerQ[(n
- 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\,dx &= \int \frac{(1+ax)^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx\\
&= \int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx\\
&= \frac{2(1+ax)^2}{a\sqrt{1-a^2x^2}} - 3\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{2(1+ax)^2}{a\sqrt{1-a^2x^2}} + \frac{3\sqrt{1-a^2x^2}}{a} - 3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{2(1+ax)^2}{a\sqrt{1-a^2x^2}} + \frac{3\sqrt{1-a^2x^2}}{a} - \frac{3\sin^{-1}(ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.71

$$\frac{\sqrt{1-a^2x^2}\left(1-\frac{4}{ax-1}\right)}{a} - \frac{3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x]),x]
[Out] (Sqrt[1 - a^2*x^2]*(1 - 4/(-1 + a*x)))/a - (3*ArcSin[a*x])/a
```

fricas [A] time = 0.44, size = 65, normalized size = 1.18

$$\frac{5\,ax + 6\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(ax-5) - 5}{a^2x - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (5*a*x + 6*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(a*x - 5) - 5)/(a^2*x - a)
```

giac [A] time = 0.23, size = 63, normalized size = 1.15

$$-\frac{3 \arcsin(ax)\operatorname{sgn}(a)}{|a|}+\frac{\sqrt{-a^2x^2+1}}{a}+\frac{8}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/abs(a) + sqrt(-a^2*x^2 + 1)/a + 8/(((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 79, normalized size = 1.44

$$-\frac{a\,x^2}{\sqrt{-a^2x^2+1}}+\frac{5}{a\sqrt{-a^2x^2+1}}+\frac{4x}{\sqrt{-a^2x^2+1}}-\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -a*x^2/(-a^2*x^2+1)^(1/2)+5/a/(-a^2*x^2+1)^(1/2)+4*x/(-a^2*x^2+1)^(1/2)-3/(
a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 60, normalized size = 1.09

$$-\frac{ax^2}{\sqrt{-a^2x^2+1}}+\frac{4\,x}{\sqrt{-a^2x^2+1}}-\frac{3\,\arcsin(ax)}{a}+\frac{5}{\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*x^2/sqrt(-a^2*x^2 + 1) + 4*x/sqrt(-a^2*x^2 + 1) - 3*arcsin(a*x)/a + 5/(s
qrt(-a^2*x^2 + 1)*a)
```

mupad [B] time = 0.81, size = 81, normalized size = 1.47

$$\frac{\sqrt{1-a^2\,x^2}}{a}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{4\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/a - (3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) + (4*(1 - a^
2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.22 $\int \frac{e^{3\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=48

$$\frac{4\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

```
[Out] -arcsin(a*x)-arctanh((-a^2*x^2+1)^(1/2))+4*(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

Rubi [A] time = 0.79, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6124, 6742, 216, 266, 63, 208, 651}

$$\frac{4\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/x,x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 - a*x) - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{(1+ax)^2}{x(1-ax)\sqrt{1-a^2x^2}}\,dx\\
&= \int \left(-\frac{a}{\sqrt{1-a^2x^2}}+\frac{1}{x\sqrt{1-a^2x^2}}-\frac{4a}{(-1+ax)\sqrt{1-a^2x^2}}\right)dx\\
&= -\left(a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\right)-(4a)\int \frac{1}{(-1+ax)\sqrt{1-a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{4\sqrt{1-a^2x^2}}{1-ax}-\sin^{-1}(ax)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= \frac{4\sqrt{1-a^2x^2}}{1-ax}-\sin^{-1}(ax)-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\
&= \frac{4\sqrt{1-a^2x^2}}{1-ax}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 51, normalized size = 1.06

$$-\frac{4\sqrt{1-a^2x^2}}{ax-1}-\log\left(\sqrt{1-a^2x^2}+1\right)-\sin^{-1}(ax)+\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/x,x]
```

```
[Out] (-4*Sqrt[1 - a^2*x^2])/(-1 + a*x) - ArcSin[a*x] + Log[x] - Log[1 + Sqrt[1 -
 a^2*x^2]]
```

fricas [A] time = 0.48, size = 82, normalized size = 1.71

$$\frac{4\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-4\sqrt{-a^2x^2+1}-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x - 1)*log
((sqrt(-a^2*x^2 + 1) - 1)/x) - 4*sqrt(-a^2*x^2 + 1) - 4)/(a*x - 1)
```

giac [B] time = 0.22, size = 87, normalized size = 1.81

$$-\frac{a\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{8\,a}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x)
- 1)*abs(a))
```

maple [A] time = 0.04, size = 75, normalized size = 1.56

$$\frac{4ax}{\sqrt{-a^2x^2+1}}-\frac{a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{4}{\sqrt{-a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] 4*a*x/(-a^2*x^2+1)^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1
/2))+4/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 65, normalized size = 1.35

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}}+\frac{4}{\sqrt{-a^2x^2+1}}-\arcsin\left(ax\right)-\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(
-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.07, size = 82, normalized size = 1.71

$$\frac{4\,a\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{a\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (4*a*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - atanh((1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ax+1\right)^3}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.23 $\int \frac{e^{3\tanh^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=63

$$\frac{4a\sqrt{1-a^2x^2}}{1-ax} - \frac{\sqrt{1-a^2x^2}}{x} - 3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -3*a*arctanh((-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/x+4*a*(-a^2*x^2+1)^(1/2
)/(-a*x+1)
```

Rubi [A] time = 0.70, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6124, 6742, 264, 266, 63, 208, 651}

$$\frac{4a\sqrt{1-a^2x^2}}{1-ax} - \frac{\sqrt{1-a^2x^2}}{x} - 3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/x^2,x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) + (4*a*Sqrt[1 - a^2*x^2])/(1 - a*x) - 3*a*ArcTanh[Sq
rt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+ax)^2}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \left(\frac{1}{x^2\sqrt{1-a^2x^2}}+\frac{3a}{x\sqrt{1-a^2x^2}}-\frac{4a^2}{(-1+ax)\sqrt{1-a^2x^2}}\right)dx \\
&= (3a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx-\left(4a^2\right)\int \frac{1}{(-1+ax)\sqrt{1-a^2x^2}}\,dx+\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{x}+\frac{4a\sqrt{1-a^2x^2}}{1-ax}+\frac{1}{2}(3a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{x}+\frac{4a\sqrt{1-a^2x^2}}{1-ax}-\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1-a^2x^2}}{x}+\frac{4a\sqrt{1-a^2x^2}}{1-ax}-3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.90

$$\sqrt{1-a^2x^2}\left(-\frac{4a}{ax-1}-\frac{1}{x}\right)-3a\log\left(\sqrt{1-a^2x^2}+1\right)+3a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/x^2,x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(-x^(-1) - (4*a)/(-1 + a*x)) + 3*a*Log[x] - 3*a*Log[1 + S
qrt[1 - a^2*x^2]]
```

fricas [A] time = 0.43, size = 78, normalized size = 1.24

$$\frac{4a^2x^2-4ax+3\left(a^2x^2-ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}\,(5ax-1)}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] (4*a^2*x^2 - 4*a*x + 3*(a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sq
rt(-a^2*x^2 + 1)*(5*a*x - 1))/(a*x^2 - x)
```

giac [B] time = 0.21, size = 150, normalized size = 2.38

$$-\frac{3a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{|a|}-\frac{\left(a^2-\frac{17\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -3*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a)
 - 1/2*(a^2 - 17*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x)*a^2*x/((sqrt(-a^2*x^2 +
 1)*abs(a) + a)*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 1/2
*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a))
```

maple [A] time = 0.04, size = 82, normalized size = 1.30

$$\frac{a}{\sqrt{-a^2x^2+1}}+\frac{5a^2x}{\sqrt{-a^2x^2+1}}+3a\left(\frac{1}{\sqrt{-a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)-\frac{1}{x\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] a/(-a^2*x^2+1)^(1/2)+5*a^2*x/(-a^2*x^2+1)^(1/2)+3*a*(1/(-a^2*x^2+1)^(1/2)-a
rctanh(1/(-a^2*x^2+1)^(1/2)))-1/x/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.31, size = 80, normalized size = 1.27

$$\frac{5\,a^2x}{\sqrt{-a^2x^2+1}}-3\,a\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{4\,a}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5*a^2*x/sqrt(-a^2*x^2 + 1) - 3*a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)
) + 4*a/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x)
```

mupad [B] time = 0.80, size = 82, normalized size = 1.30

$$\frac{4\,a^2\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{x}-3\,a\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (4*a^2*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2)
) - (1 - a^2*x^2)^(1/2)/x - 3*a*atanh((1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^3}{x^2\,(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.24 $\int \frac{e^{3\tanh^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=91

$$\frac{4a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{3a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -9/2*a^2*arctanh((-a^2*x^2+1)^(1/2))-1/2*(-a^2*x^2+1)^(1/2)/x^2-3*a*(-a^2*x
^2+1)^(1/2)/x+4*a^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

Rubi [A] time = 0.74, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6124, 6742, 266, 51, 63, 208, 264, 651}

$$\frac{4a^2\sqrt{1-a^2x^2}}{1-ax}-\frac{3a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/x^3,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) - (3*a*Sqrt[1 - a^2*x^2])/x + (4*a^2*Sqrt[1 - a^
2*x^2])/(1 - a*x) - (9*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{(1+ax)^2}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx$$

$$= \int \left(\frac{1}{x^3\sqrt{1-a^2x^2}} + \frac{3a}{x^2\sqrt{1-a^2x^2}} + \frac{4a^2}{x\sqrt{1-a^2x^2}} - \frac{4a^3}{(-1+ax)\sqrt{1-a^2x^2}}\right)dx$$

$$= (3a)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx + \left(4a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(4a^3\right)\int \frac{1}{(-1+ax)\sqrt{1-a^2x^2}}\,dx + \int$$

$$= -\frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1-ax} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) + \left(2a^2\right)\text{Subst}\left(\int\right.$$

$$= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1-ax} - 4\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) + \frac{1}{4}a$$

$$= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1-ax} - 4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{1}{2}\,\text{Subst}\left(\int\right.$$

$$= -\frac{\sqrt{1-a^2x^2}}{2x^2} - \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1-ax} - \frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.82

$$\sqrt{1-a^2x^2}\left(-\frac{4a^2}{ax-1} - \frac{3a}{x} - \frac{1}{2x^2}\right) - \frac{9}{2}a^2\log\left(\sqrt{1-a^2x^2}+1\right) + \frac{9}{2}a^2\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/x^3,x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(-1/2*1/x^2 - (3*a)/x - (4*a^2)/(-1 + a*x)) + (9*a^2*Log[
x])/2 - (9*a^2*Log[1 + Sqrt[1 - a^2*x^2]])/2
```

fricas [A] time = 0.43, size = 97, normalized size = 1.07

$$\frac{8\,a^3x^3 - 8\,a^2x^2 + 9\left(a^3x^3 - a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(14\,a^2x^2 - 5\,ax - 1\right)\sqrt{-a^2x^2+1}}{2\left(ax^3 - x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

[Out] 1/2*(8*a^3*x^3 - 8*a^2*x^2 + 9*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (14*a^2*x^2 - 5*a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 - x^2)

giac [B] time = 0.42, size = 213, normalized size = 2.34

$$-\frac{\left(a^3+\frac{11\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x}-\frac{76\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{9\,a^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{12\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}\right.}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")

[Out] -1/8*(a^3 + 11*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 76*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 9/2*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(12*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2))/a^2

maple [A] time = 0.04, size = 108, normalized size = 1.19

$$\frac{a^3x}{\sqrt{-a^2x^2+1}}+\frac{9a^2\left(\frac{1}{\sqrt{-a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{2}+3a\left(-\frac{1}{x\sqrt{-a^2x^2+1}}+\frac{2a^2x}{\sqrt{-a^2x^2+1}}\right)-\frac{1}{2x^2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^3,x)

[Out] a^3*x/(-a^2*x^2+1)^(1/2)+9/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))+3*a*(-1/x/(-a^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2))-1/2/x^2/(-a^2*x^2+1)^(1/2)

maxima [A] time = 0.31, size = 102, normalized size = 1.12

$$\frac{7\,a^3x}{\sqrt{-a^2x^2+1}}-\frac{9}{2}\,a^2\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{9\,a^2}{2\,\sqrt{-a^2x^2+1}}-\frac{3\,a}{\sqrt{-a^2x^2+1}\,x}-\frac{1}{2\,\sqrt{-a^2x^2+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")

[Out] 7*a^3*x/sqrt(-a^2*x^2 + 1) - 9/2*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 9/2*a^2/sqrt(-a^2*x^2 + 1) - 3*a/(sqrt(-a^2*x^2 + 1)*x) - 1/2/(sqrt(-a^2*x^2 + 1)*x^2)

mupad [B] time = 0.82, size = 106, normalized size = 1.16

$$\frac{4\,a^3\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{3\,a\,\sqrt{1-a^2\,x^2}}{x}-\frac{\sqrt{1-a^2\,x^2}}{2\,x^2}+\frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,9\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*9i)/2 - (1 - a^2*x^2)^(1/2)/(2*x^2) - (3*
a*(1 - a^2*x^2)^(1/2))/x + (4*a^3*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (
-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{x^3\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral((a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.25 $\int \frac{e^{3\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=117

$$-\frac{14a^2\sqrt{1-a^2x^2}}{3x}-\frac{3a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{4a^3\sqrt{1-a^2x^2}}{1-ax}-\frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -11/2*a^3*arctanh((-a^2*x^2+1)^(1/2))-1/3*(-a^2*x^2+1)^(1/2)/x^3-3/2*a*(-a^
2*x^2+1)^(1/2)/x^2-14/3*a^2*(-a^2*x^2+1)^(1/2)/x+4*a^3*(-a^2*x^2+1)^(1/2)/(
-a*x+1)
```

Rubi [A] time = 0.74, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6124, 6742, 271, 264, 266, 51, 63, 208, 651}

$$\frac{4a^3\sqrt{1-a^2x^2}}{1-ax}-\frac{14a^2\sqrt{1-a^2x^2}}{3x}-\frac{3a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/x^4,x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) - (3*a*Sqrt[1 - a^2*x^2])/(2*x^2) - (14*a^2*Sqrt
[1 - a^2*x^2])/(3*x) + (4*a^3*Sqrt[1 - a^2*x^2])/(1 - a*x) - (11*a^3*ArcTan
h[Sqrt[1 - a^2*x^2]])/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+ax)^2}{x^4(1-ax)\sqrt{1-a^2x^2}}\,dx$$
$$= \int \left(\frac{1}{x^4\sqrt{1-a^2x^2}} + \frac{3a}{x^3\sqrt{1-a^2x^2}} + \frac{4a^2}{x^2\sqrt{1-a^2x^2}} + \frac{4a^3}{x\sqrt{1-a^2x^2}} - \frac{4a^4}{(-1+ax)\sqrt{1-a^2x^2}}\right) dx$$
$$= (3a)\int \frac{1}{x^3\sqrt{1-a^2x^2}}\,dx + \left(4a^2\right)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx + \left(4a^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(4a^4\right)\int \frac{}{(-1}$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{4a^2\sqrt{1-a^2x^2}}{x} + \frac{4a^3\sqrt{1-a^2x^2}}{1-ax} + \frac{1}{2}(3a)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) + \frac{1}{3}$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} + \frac{4a^3\sqrt{1-a^2x^2}}{1-ax} - (4a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,\right.$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} + \frac{4a^3\sqrt{1-a^2x^2}}{1-ax} - 4a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) -$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} + \frac{4a^3\sqrt{1-a^2x^2}}{1-ax} - \frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.09, size = 81, normalized size = 0.69

$$\frac{1}{6}\left(33a^3\log(x) - 33a^3\log\left(\sqrt{1-a^2x^2}+1\right) + \frac{\sqrt{1-a^2x^2}\left(-52a^3x^3+19a^2x^2+7ax+2\right)}{x^3(ax-1)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/x^4,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(2 + 7*a*x + 19*a^2*x^2 - 52*a^3*x^3))/(x^3*(-1 + a*x))
 + 33*a^3*Log[x] - 33*a^3*Log[1 + Sqrt[1 - a^2*x^2]])/6
```

fricas [A] time = 0.56, size = 105, normalized size = 0.90

$$\frac{24\,a^4x^4 - 24\,a^3x^3 + 33\left(a^4x^4 - a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(52\,a^3x^3 - 19\,a^2x^2 - 7\,ax - 2\right)\sqrt{-a^2x^2+1}}{6\left(ax^4 - x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(24*a^4*x^4 - 24*a^3*x^3 + 33*(a^4*x^4 - a^3*x^3)*log((sqrt(-a^2*x^2 +
1) - 1)/x) - (52*a^3*x^3 - 19*a^2*x^2 - 7*a*x - 2)*sqrt(-a^2*x^2 + 1))/(a*x
^4 - x^3)
```

giac [B] time = 0.40, size = 265, normalized size = 2.26

$$-\frac{\left(a^4+\frac{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2}{x}+\frac{48\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{x^2}-\frac{249\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^2x^3}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{11\,a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{57\left(\sqrt{-a^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*(a^4 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 48*(sqrt(-a^2*x^2 +
1)*abs(a) + a)^2/x^2 - 249*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^2*x^3))*a^6
*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a
^2*x) - 1)*abs(a)) - 11/2*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*
a)/(a^2*abs(x)))/abs(a) - 1/24*(57*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x +
9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3/x^3)/(a^2*abs(a))
```

maple [A] time = 0.04, size = 146, normalized size = 1.25

$$a^3\left(\frac{1}{\sqrt{-a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)+\frac{13a^2\left(-\frac{1}{x\sqrt{-a^2x^2+1}}+\frac{2a^2x}{\sqrt{-a^2x^2+1}}\right)}{3}-\frac{1}{3x^3\sqrt{-a^2x^2+1}}+3a\left(-\frac{1}{2x^2\sqrt{-a^2x\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] a^3*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))+13/3*a^2*(-1/x/(-a
^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2))-1/3/x^3/(-a^2*x^2+1)^(1/2)+3*a*
(-1/2/x^2/(-a^2*x^2+1)^(1/2)+3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*
x^2+1)^(1/2))))
```

maxima [A] time = 0.32, size = 122, normalized size = 1.04

$$\frac{26\,a^4x}{3\sqrt{-a^2x^2+1}}-\frac{11}{2}\,a^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{11\,a^3}{2\sqrt{-a^2x^2+1}}-\frac{13\,a^2}{3\sqrt{-a^2x^2+1}\,x}-\frac{3\,a}{2\sqrt{-a^2x^2+1}\,x^2}-\frac{1}{3\sqrt{-a^2x^2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 26/3*a^4*x/sqrt(-a^2*x^2 + 1) - 11/2*a^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) +
2/abs(x)) + 11/2*a^3/sqrt(-a^2*x^2 + 1) - 13/3*a^2/(sqrt(-a^2*x^2 + 1)*x) -
 3/2*a/(sqrt(-a^2*x^2 + 1)*x^2) - 1/3/(sqrt(-a^2*x^2 + 1)*x^3)
```

mupad [B] time = 0.81, size = 126, normalized size = 1.08

$$\frac{4\,a^4\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{3\,a\,\sqrt{1-a^2\,x^2}}{2\,x^2}-\frac{14\,a^2\,\sqrt{1-a^2\,x^2}}{3\,x}-\frac{\sqrt{1-a^2\,x^2}}{3\,x^3}+\frac{a^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,11\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/(x^4*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (a^3*atan((1 - a^2*x^2)^(1/2)*1i)*11i)/2 - (1 - a^2*x^2)^(1/2)/(3*x^3) - (3
*a*(1 - a^2*x^2)^(1/2))/(2*x^2) - (14*a^2*(1 - a^2*x^2)^(1/2))/(3*x) + (4*a
^4*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{x^4\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral((a*x + 1)**3/(x**4*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.26 $\int e^{4\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=57

$$\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

```
[Out] 12*x/a^3+4*x^2/a^2+4/3*x^3/a+1/4*x^4+4/a^4/(-a*x+1)+16*ln(-a*x+1)/a^4
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+ax)^2}{(1-ax)^2}\,dx$$
$$= \int \left(\frac{12}{a^3} + \frac{8x}{a^2} + \frac{4x^2}{a} + x^3 + \frac{4}{a^3(-1+ax)^2} + \frac{16}{a^3(-1+ax)}\right) dx$$
$$= \frac{12x}{a^3} + \frac{4x^2}{a^2} + \frac{4x^3}{3a} + \frac{x^4}{4} + \frac{4}{a^4(1-ax)} + \frac{16\log(1-ax)}{a^4}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 1.00

$$\frac{4}{a^4(1-ax)} + \frac{16\log(1-ax)}{a^4} + \frac{12x}{a^3} + \frac{4x^2}{a^2} + \frac{4x^3}{3a} + \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

fricas [A] time = 0.44, size = 66, normalized size = 1.16

$$\frac{3\,a^5x^5 + 13\,a^4x^4 + 32\,a^3x^3 + 96\,a^2x^2 - 144\,ax + 192\,(ax-1)\log(ax-1) - 48}{12\left(a^5x - a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^5*x^5 + 13*a^4*x^4 + 32*a^3*x^3 + 96*a^2*x^2 - 144*a*x + 192*(a*x
 - 1)*log(a*x - 1) - 48)/(a^5*x - a^4)
```

giac [A] time = 0.24, size = 61, normalized size = 1.07

$$\frac{16\,\log\left(|ax-1|\right)}{a^4} - \frac{4}{(ax-1)a^4} + \frac{3\,a^8x^4 + 16\,a^7x^3 + 48\,a^6x^2 + 144\,a^5x}{12\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^3,x, algorithm="giac")
```

```
[Out] 16*log(abs(a*x - 1))/a^4 - 4/((a*x - 1)*a^4) + 1/12*(3*a^8*x^4 + 16*a^7*x^3
 + 48*a^6*x^2 + 144*a^5*x)/a^8
```

maple [A] time = 0.03, size = 52, normalized size = 0.91

$$\frac{x^4}{4}+\frac{4x^3}{3a}+\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{16\ln(ax-1)}{a^4}-\frac{4}{a^4(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x^3,x)
```

```
[Out] 1/4*x^4+4/3*x^3/a+4*x^2/a^2+12*x/a^3+16/a^4*ln(a*x-1)-4/a^4/(a*x-1)
```

maxima [A] time = 0.31, size = 58, normalized size = 1.02

$$-\frac{4}{a^5x-a^4}+\frac{3\,a^3x^4+16\,a^2x^3+48\,ax^2+144\,x}{12\,a^3}+\frac{16\,\log(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -4/(a^5*x - a^4) + 1/12*(3*a^3*x^4 + 16*a^2*x^3 + 48*a*x^2 + 144*x)/a^3 + 1
6*log(a*x - 1)/a^4
```

mupad [B] time = 0.81, size = 57, normalized size = 1.00

$$\frac{16\,\ln(a\,x-1)}{a^4}-\frac{4}{a\,\left(a^4\,x-a^3\right)}+\frac{12\,x}{a^3}+\frac{x^4}{4}+\frac{4\,x^3}{3\,a}+\frac{4\,x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (16*log(a*x - 1))/a^4 - 4/(a*(a^4*x - a^3)) + (12*x)/a^3 + x^4/4 + (4*x^3)/
(3*a) + (4*x^2)/a^2
```

sympy [A] time = 0.17, size = 49, normalized size = 0.86

$$\frac{x^4}{4}-\frac{4}{a^5x-a^4}+\frac{4x^3}{3a}+\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{16\log(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x**3,x)
```

```
[Out] x**4/4 - 4/(a**5*x - a**4) + 4*x**3/(3*a) + 4*x**2/a**2 + 12*x/a**3 + 16*lo
g(a*x - 1)/a**4
```

3.27 $\int e^{4\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=47

$$\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}$$

```
[Out] 8*x/a^2+2*x^2/a+1/3*x^3+4/a^3/(-a*x+1)+12*ln(-a*x+1)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{8x}{a^2}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{2x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}x^2\,dx&=\int\frac{x^2(1+ax)^2}{(1-ax)^2}\,dx\\&=\int\left(\frac{8}{a^2}+\frac{4x}{a}+x^2+\frac{4}{a^2(-1+ax)^2}+\frac{12}{a^2(-1+ax)}\right)dx\\&=\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.00

$$\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

fricas [A] time = 0.41, size = 57, normalized size = 1.21

$$\frac{a^4x^4+5\,a^3x^3+18\,a^2x^2-24\,ax+36\,(ax-1)\log\left(ax-1\right)-12}{3\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 24*a*x + 36*(a*x - 1)*log(a*x - 1)
- 12)/(a^4*x - a^3)
```

giac [A] time = 0.20, size = 52, normalized size = 1.11

$$\frac{12\,\log\left(|ax-1|\right)}{a^3}-\frac{4}{(ax-1)a^3}+\frac{a^6x^3+6\,a^5x^2+24\,a^4x}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2,x, algorithm="giac")
```

```
[Out] 12*log(abs(a*x - 1))/a^3 - 4/((a*x - 1)*a^3) + 1/3*(a^6*x^3 + 6*a^5*x^2 + 2
4*a^4*x)/a^6
```

maple [A] time = 0.03, size = 44, normalized size = 0.94

$$\frac{x^3}{3}+\frac{2x^2}{a}+\frac{8x}{a^2}+\frac{12\ln\left(ax-1\right)}{a^3}-\frac{4}{a^3\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x^2,x)
```

```
[Out] 1/3*x^3+2*x^2/a+8*x/a^2+12/a^3*ln(a*x-1)-4/a^3/(a*x-1)
```

maxima [A] time = 0.32, size = 49, normalized size = 1.04

$$-\frac{4}{a^4x - a^3} + \frac{a^2x^3 + 6\,ax^2 + 24\,x}{3\,a^2} + \frac{12\,\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2,x, algorithm="maxima")
[Out] -4/(a^4*x - a^3) + 1/3*(a^2*x^3 + 6*a*x^2 + 24*x)/a^2 + 12*log(a*x - 1)/a^3
```

mupad [B] time = 0.79, size = 49, normalized size = 1.04

$$\frac{12\,\ln(a\,x-1)}{a^3} - \frac{4}{a\,\left(a^3\,x - a^2\right)} + \frac{8\,x}{a^2} + \frac{x^3}{3} + \frac{2\,x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] (12*log(a*x - 1))/a^3 - 4/(a*(a^3*x - a^2)) + (8*x)/a^2 + x^3/3 + (2*x^2)/a
```

sympy [A] time = 0.16, size = 39, normalized size = 0.83

$$\frac{x^3}{3} - \frac{4}{a^4x - a^3} + \frac{2x^2}{a} + \frac{8x}{a^2} + \frac{12\log(ax-1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x**2,x)
[Out] x**3/3 - 4/(a**4*x - a**3) + 2*x**2/a + 8*x/a**2 + 12*log(a*x - 1)/a**3
```

3.28 $\int e^{4\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=39

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

```
[Out] 4*x/a+1/2*x^2+4/a^2/(-a*x+1)+8*ln(-a*x+1)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6126, 77}

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*x,x]
```

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}x\,dx = \int \frac{x(1+ax)^2}{(1-ax)^2}\,dx$$
$$= \int \left(\frac{4}{a} + x + \frac{4}{a(-1+ax)^2} + \frac{8}{a(-1+ax)}\right)dx$$
$$= \frac{4x}{a} + \frac{x^2}{2} + \frac{4}{a^2(1-ax)} + \frac{8\log(1-ax)}{a^2}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 1.00

$$\frac{4}{a^2(1-ax)} + \frac{8\log(1-ax)}{a^2} + \frac{4x}{a} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*x,x]
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

fricas [A] time = 0.45, size = 49, normalized size = 1.26

$$\frac{a^3x^3 + 7\,a^2x^2 - 8\,ax + 16\,(ax-1)\log(ax-1) - 8}{2\left(a^3x - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x,x, algorithm="fricas")
[Out] 1/2*(a^3*x^3 + 7*a^2*x^2 - 8*a*x + 16*(a*x - 1)*log(a*x - 1) - 8)/(a^3*x -
a^2)
```

giac [A] time = 0.17, size = 44, normalized size = 1.13

$$\frac{8\,\log\left(|ax-1|\right)}{a^2} + \frac{a^4x^2 + 8\,a^3x}{2\,a^4} - \frac{4}{(ax-1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x,x, algorithm="giac")
[Out] 8*log(abs(a*x - 1))/a^2 + 1/2*(a^4*x^2 + 8*a^3*x)/a^4 - 4/((a*x - 1)*a^2)
```

maple [A] time = 0.03, size = 36, normalized size = 0.92

$$\frac{x^2}{2} + \frac{4x}{a} + \frac{8\ln(ax-1)}{a^2} - \frac{4}{a^2\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x,x)
```

```
[Out] 1/2*x^2+4*x/a+8/a^2*ln(a*x-1)-4/a^2/(a*x-1)
```

maxima [A] time = 0.31, size = 41, normalized size = 1.05

$$\frac{ax^2 + 8\,x}{2\,a} - \frac{4}{a^3x - a^2} + \frac{8\,\log\left(ax - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 + 8*x)/a - 4/(a^3*x - a^2) + 8*log(a*x - 1)/a^2
```

mupad [B] time = 0.04, size = 38, normalized size = 0.97

$$\frac{8\,\ln\left(a\,x - 1\right)}{a^2} + \frac{4\,x}{a} + \frac{x^2}{2} + \frac{4}{a\,\left(a - a^2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (8*log(a*x - 1))/a^2 + (4*x)/a + x^2/2 + 4/(a*(a - a^2*x))
```

sympy [A] time = 0.15, size = 31, normalized size = 0.79

$$\frac{x^2}{2} - \frac{4}{a^3x - a^2} + \frac{4x}{a} + \frac{8\log\left(ax - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x,x)
```

```
[Out] x**2/2 - 4/(a**3*x - a**2) + 4*x/a + 8*log(a*x - 1)/a**2
```

3.29 $\int e^{4\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}+x$$

[Out] x+4/a/(-a*x+1)+4*ln(-a*x+1)/a

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6125, 43}

$$\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcTanh[a*x]),x]

[Out] x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\,dx &= \int \frac{(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(1+\frac{4}{(-1+ax)^2}+\frac{4}{-1+ax}\right)dx\\ &= x+\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$-\frac{4}{a(ax-1)}+\frac{4\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x]),x]
[Out] x - 4/(a*(-1 + a*x)) + (4*Log[1 - a*x])/a
```

fricas [A] time = 0.43, size = 38, normalized size = 1.41

$$\frac{a^2x^2-ax+4\,(ax-1)\log\,(ax-1)-4}{a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2,x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 4*(a*x - 1)*log(a*x - 1) - 4)/(a^2*x - a)
```

giac [A] time = 0.17, size = 26, normalized size = 0.96

$$x+\frac{4\,\log\,(|ax-1|)}{a}-\frac{4}{(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2,x, algorithm="giac")
[Out] x + 4*log(abs(a*x - 1))/a - 4/((a*x - 1)*a)
```

maple [A] time = 0.03, size = 26, normalized size = 0.96

$$x+\frac{4\ln\,(ax-1)}{a}-\frac{4}{a\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2,x)
[Out] x+4/a*ln(a*x-1)-4/a/(a*x-1)
```

maxima [A] time = 0.31, size = 26, normalized size = 0.96

$$x+\frac{4\,\log\,(ax-1)}{a}-\frac{4}{a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2,x, algorithm="maxima")
[Out] x + 4*log(a*x - 1)/a - 4/(a^2*x - a)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.93

$$x - \frac{4}{a\,(a\,x-1)} + \frac{4\,\ln(a\,x-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/(a^2*x^2 - 1)^2,x)
[Out] x - 4/(a*(a*x - 1)) + (4*log(a*x - 1))/a
```

sympy [A] time = 0.13, size = 19, normalized size = 0.70

$$x - \frac{4}{a^2x - a} + \frac{4\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2,x)
[Out] x - 4/(a**2*x - a) + 4*log(a*x - 1)/a
```

3.30 $\int \frac{e^{4\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=13

$$\frac{4}{1-ax}+\log(x)$$

```
[Out] 4/(-a*x+1)+ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4}{1-ax}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/x,x]
```

```
[Out] 4/(1 - a*x) + Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{(1+ax)^2}{x(1-ax)^2}\,dx\\ &= \int \left(\frac{1}{x}+\frac{4a}{(-1+ax)^2}\right)dx\\ &= \frac{4}{1-ax}+\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{4}{1-ax}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/x,x]
[Out] 4/(1 - a*x) + Log[x]
```

fricas [A] time = 0.44, size = 18, normalized size = 1.38

$$\frac{(ax-1)\log(x)-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x,x, algorithm="fricas")
[Out] ((a*x - 1)*log(x) - 4)/(a*x - 1)
```

giac [A] time = 0.16, size = 13, normalized size = 1.00

$$-\frac{4}{ax-1}+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x,x, algorithm="giac")
[Out] -4/(a*x - 1) + log(abs(x))
```

maple [A] time = 0.03, size = 13, normalized size = 1.00

$$\ln(x)-\frac{4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/x,x)
[Out] ln(x)-4/(a*x-1)
```

maxima [A] time = 0.31, size = 12, normalized size = 0.92

$$-\frac{4}{ax-1}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x,x, algorithm="maxima")
[Out] -4/(a*x - 1) + log(x)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.92

$$\ln(x) - \frac{4}{a\,x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/(x*(a^2*x^2 - 1)^2),x)
[Out] log(x) - 4/(a*x - 1)
```

sympy [A] time = 0.16, size = 8, normalized size = 0.62

$$\log(x) - \frac{4}{ax - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/x,x)
[Out] log(x) - 4/(a*x - 1)
```

3.31 $\int \frac{e^{4\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=32

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

```
[Out] -1/x+4*a/(-a*x+1)+4*a*ln(x)-4*a*ln(-a*x+1)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+ax)^2}{x^2(1-ax)^2}\,dx\\ &= \int \left(\frac{1}{x^2}+\frac{4a}{x}+\frac{4a^2}{(-1+ax)^2}-\frac{4a^2}{-1+ax}\right)dx\\ &= -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.00

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

fricas [A] time = 0.40, size = 55, normalized size = 1.72

$$-\frac{5\,ax+4\left(a^2x^2-ax\right)\log\left(ax-1\right)-4\left(a^2x^2-ax\right)\log(x)-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a*x + 4*(a^2*x^2 - a*x)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 1)/(a
*x^2 - x)
```

giac [A] time = 1.09, size = 36, normalized size = 1.12

$$-4\,a\log\left(|ax-1|\right)+4\,a\log\left(|x|\right)-\frac{5\,ax-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^2,x, algorithm="giac")
```

```
[Out] -4*a*log(abs(a*x - 1)) + 4*a*log(abs(x)) - (5*a*x - 1)/(a*x^2 - x)
```

maple [A] time = 0.03, size = 31, normalized size = 0.97

$$-\frac{1}{x}+4a\ln(x)-\frac{4a}{ax-1}-4a\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/x^2,x)
```

```
[Out] -1/x+4*a*ln(x)-4*a/(a*x-1)-4*a*ln(a*x-1)
```

maxima [A] time = 0.32, size = 34, normalized size = 1.06

$$-4\,a\log\left(ax-1\right)+4\,a\log(x)-\frac{5\,ax-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4*a*log(a*x - 1) + 4*a*log(x) - (5*a*x - 1)/(a*x^2 - x)
```

mupad [B] time = 0.81, size = 28, normalized size = 0.88

$$8\,a\,\mathrm{atanh}\left(2\,a\,x-1\right)+\frac{5\,a\,x-1}{x-a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/(x^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] 8*a*atanh(2*a*x - 1) + (5*a*x - 1)/(x - a*x^2)
```

sympy [A] time = 0.23, size = 26, normalized size = 0.81

$$4a\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-5ax+1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/x**2,x)
```

```
[Out] 4*a*(log(x) - log(x - 1/a)) + (-5*a*x + 1)/(a*x**2 - x)
```

3.32 $\int \frac{e^{4\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

```
[Out] -1/2/x^2-4*a/x+4*a^2/(-a*x+1)+8*a^2*ln(x)-8*a^2*ln(-a*x+1)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{(1+ax)^2}{x^3(1-ax)^2}\,dx$$
$$= \int \left(\frac{1}{x^3} + \frac{4a}{x^2} + \frac{8a^2}{x} + \frac{4a^3}{(-1+ax)^2} - \frac{8a^3}{-1+ax}\right) dx$$
$$= -\frac{1}{2x^2} - \frac{4a}{x} + \frac{4a^2}{1-ax} + 8a^2\log(x) - 8a^2\log(1-ax)$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{4a^2}{1-ax} + 8a^2\log(x) - 8a^2\log(1-ax) - \frac{4a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/x^3,x]
```

```
[Out] -1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

fricas [A] time = 0.55, size = 73, normalized size = 1.59

$$-\frac{16\,a^2x^2 - 7\,ax + 16\left(a^3x^3 - a^2x^2\right)\log\left(ax-1\right) - 16\left(a^3x^3 - a^2x^2\right)\log(x) - 1}{2\left(ax^3 - x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(16*a^2*x^2 - 7*a*x + 16*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 16*(a^3*x^
3 - a^2*x^2)*log(x) - 1)/(a*x^3 - x^2)
```

giac [A] time = 0.39, size = 47, normalized size = 1.02

$$-8\,a^2\log\left(|ax-1|\right) + 8\,a^2\log\left(|x|\right) - \frac{16\,a^2x^2 - 7\,ax - 1}{2\,(ax-1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^3,x, algorithm="giac")
```

```
[Out] -8*a^2*log(abs(a*x - 1)) + 8*a^2*log(abs(x)) - 1/2*(16*a^2*x^2 - 7*a*x - 1)
/((a*x - 1)*x^2)
```

maple [A] time = 0.04, size = 43, normalized size = 0.93

$$-\frac{1}{2x^2}-\frac{4a}{x}+8a^2\ln(x)-\frac{4a^2}{ax-1}-8a^2\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/x^3,x)
[Out] -1/2/x^2-4*a/x+8*a^2*ln(x)-4*a^2/(a*x-1)-8*a^2*ln(a*x-1)
```

maxima [A] time = 0.30, size = 48, normalized size = 1.04

$$-8\,a^2\log(ax-1)+8\,a^2\log(x)-\frac{16\,a^2x^2-7\,ax-1}{2\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^3,x, algorithm="maxima")
[Out] -8*a^2*log(a*x - 1) + 8*a^2*log(x) - 1/2*(16*a^2*x^2 - 7*a*x - 1)/(a*x^3 -
x^2)
```

mupad [B] time = 0.06, size = 41, normalized size = 0.89

$$16\,a^2\,\mathrm{atanh}\left(2\,a\,x-1\right)+\frac{-8\,a^2\,x^2+\frac{7\,a\,x}{2}+\frac{1}{2}}{a\,x^3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/(x^3*(a^2*x^2 - 1)^2),x)
[Out] 16*a^2*atanh(2*a*x - 1) + ((7*a*x)/2 - 8*a^2*x^2 + 1/2)/(a*x^3 - x^2)
```

sympy [A] time = 0.25, size = 41, normalized size = 0.89

$$8a^2\left(\log(x)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-16a^2x^2+7ax+1}{2ax^3-2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/x**3,x)
[Out] 8*a**2*(log(x) - log(x - 1/a)) + (-16*a**2*x**2 + 7*a*x + 1)/(2*a*x**3 - 2*
x**2)
```

3.33 $\int \frac{e^{4\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=54

$$\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)-\frac{8a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+4*a^3/(-a*x+1)+12*a^3*ln(x)-12*a^3*ln(-a*x+1)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 88}

$$\frac{4a^3}{1-ax}-\frac{8a^2}{x}+12a^3\log(x)-12a^3\log(1-ax)-\frac{2a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+ax)^2}{x^4(1-ax)^2}\,dx$$
$$= \int \left(\frac{1}{x^4}+\frac{4a}{x^3}+\frac{8a^2}{x^2}+\frac{12a^3}{x}+\frac{4a^4}{(-1+ax)^2}-\frac{12a^4}{-1+ax}\right)dx$$
$$= -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.00

$$\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)-\frac{8a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/x^4,x]
```

```
[Out] -1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

fricas [A] time = 0.45, size = 81, normalized size = 1.50

$$-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax+36\left(a^4x^4-a^3x^3\right)\log\left(ax-1\right)-36\left(a^4x^4-a^3x^3\right)\log(x)-1}{3\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x + 36*(a^4*x^4 - a^3*x^3)*log(a*x - 1)
 - 36*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*x^4 - x^3)
```

giac [A] time = 0.18, size = 55, normalized size = 1.02

$$-12\,a^3\log\left(|ax-1|\right)+12\,a^3\log\left(|x|\right)-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax-1}{3\,(ax-1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^4,x, algorithm="giac")
```

```
[Out] -12*a^3*log(abs(a*x - 1)) + 12*a^3*log(abs(x)) - 1/3*(36*a^3*x^3 - 18*a^2*x
^2 - 5*a*x - 1)/((a*x - 1)*x^3)
```

maple [A] time = 0.03, size = 51, normalized size = 0.94

$$-\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+12a^3\ln(x)-\frac{4a^3}{ax-1}-12a^3\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/x^4,x)
```

```
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+12*a^3*ln(x)-4*a^3/(a*x-1)-12*a^3*ln(a*x-1)
```

maxima [A] time = 0.31, size = 56, normalized size = 1.04

$$-12\,a^3\log(ax-1)+12\,a^3\log(x)-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax-1}{3\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] -12*a^3*log(a*x - 1) + 12*a^3*log(x) - 1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x
 - 1)/(a*x^4 - x^3)
```

mupad [B] time = 0.82, size = 49, normalized size = 0.91

$$24\,a^3\,\mathrm{atanh}\left(2\,a\,x-1\right)+\frac{-12\,a^3\,x^3+6\,a^2\,x^2+\frac{5\,a\,x}{3}+\frac{1}{3}}{a\,x^4-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/(x^4*(a^2*x^2 - 1)^2),x)
```

```
[Out] 24*a^3*atanh(2*a*x - 1) + ((5*a*x)/3 + 6*a^2*x^2 - 12*a^3*x^3 + 1/3)/(a*x^4
 - x^3)
```

sympy [A] time = 0.27, size = 49, normalized size = 0.91

$$12a^3\left(\log(x)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-36a^3x^3+18a^2x^2+5ax+1}{3ax^4-3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/x**4,x)
```

```
[Out] 12*a**3*(log(x) - log(x - 1/a)) + (-36*a**3*x**3 + 18*a**2*x**2 + 5*a*x + 1
)/(3*a*x**4 - 3*x**3)
```

3.34 $\int e^{-\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=87

$$-\frac{3\sin^{-1}(ax)}{8a^4}-\frac{x^2\sqrt{1-a^2x^2}}{3a^2}+\frac{x^3\sqrt{1-a^2x^2}}{4a}-\frac{(16-9ax)\sqrt{1-a^2x^2}}{24a^4}$$

```
[Out] -3/8*arcsin(a*x)/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)/a^2+1/4*x^3*(-a^2*x^2+1)^(1
/2)/a-1/24*(-9*a*x+16)*(-a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6124, 833, 780, 216}

$$\frac{x^3\sqrt{1-a^2x^2}}{4a}-\frac{x^2\sqrt{1-a^2x^2}}{3a^2}-\frac{(16-9ax)\sqrt{1-a^2x^2}}{24a^4}-\frac{3\sin^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^ArcTanh[a*x],x]
```

```
[Out] -(x^2*Sqrt[1 - a^2*x^2])/(3*a^2) + (x^3*Sqrt[1 - a^2*x^2])/(4*a) - ((16 - 9
*a*x)*Sqrt[1 - a^2*x^2])/(24*a^4) - (3*ArcSin[a*x])/(8*a^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1-ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{\int \frac{x^2\left(3a-4a^2x\right)}{\sqrt{1-a^2x^2}}\,dx}{4a^2}\\
&= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} + \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{\int \frac{x\left(8a^2-9a^3x\right)}{\sqrt{1-a^2x^2}}\,dx}{12a^4}\\
&= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} + \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{(16-9ax)\sqrt{1-a^2x^2}}{24a^4} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^3}\\
&= -\frac{x^2\sqrt{1-a^2x^2}}{3a^2} + \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{(16-9ax)\sqrt{1-a^2x^2}}{24a^4} - \frac{3\sin^{-1}(ax)}{8a^4}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.59

$$\frac{\sqrt{1-a^2x^2}\,\left(6a^3x^3-8a^2x^2+9ax-16\right)-9\sin^{-1}(ax)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-16 + 9*a*x - 8*a^2*x^2 + 6*a^3*x^3) - 9*ArcSin[a*x])/(
24*a^4)
```

fricas [A] time = 0.46, size = 65, normalized size = 0.75

$$\frac{\left(6\,a^3x^3-8\,a^2x^2+9\,ax-16\right)\sqrt{-a^2x^2+1}\,+18\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((6*a^3*x^3 - 8*a^2*x^2 + 9*a*x - 16)*sqrt(-a^2*x^2 + 1) + 18*arctan((
sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.04, size = 154, normalized size = 1.77

$$-\frac{x\left(-a^2x^2+1\right)^{\frac{3}{2}}}{4a^3}+\frac{5x\sqrt{-a^2x^2+1}}{8a^3}+\frac{5\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a^3\sqrt{a^2}}+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3a^4}-\frac{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}{a^4}-\frac{\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4/a^3*x*(-a^2*x^2+1)^(3/2)+5/8*x*(-a^2*x^2+1)^(1/2)/a^3+5/8/a^3/(a^2)^(1
/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/3*(-a^2*x^2+1)^(3/2)/a^4-1/a
^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-1/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*
x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [A] time = 0.41, size = 80, normalized size = 0.92

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{4\,a^3}+\frac{5\,\sqrt{-a^2x^2+1}\,x}{8\,a^3}+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^4}-\frac{3\,\arcsin\left(ax\right)}{8\,a^4}-\frac{\sqrt{-a^2x^2+1}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-a^2*x^2 + 1)^(3/2)*x/a^3 + 5/8*sqrt(-a^2*x^2 + 1)*x/a^3 + 1/3*(-a^2*
x^2 + 1)^(3/2)/a^4 - 3/8*arcsin(a*x)/a^4 - sqrt(-a^2*x^2 + 1)/a^4
```

mupad [B] time = 0.81, size = 97, normalized size = 1.11

$$-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a^3\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2}{3\,{\left(-a^2\right)}^{3/2}}-\frac{3\,x\,\sqrt{-a^2}}{8\,a^3}+\frac{a^2\,x^2}{3\,{\left(-a^2\right)}^{3/2}}+\frac{x^3\,{\left(-a^2\right)}^{3/2}}{4\,a^3}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] - (3*asinh(x*(-a^2)^(1/2)))/(8*a^3*(-a^2)^(1/2)) - ((1 - a^2*x^2)^(1/2)*(2/
(3*(-a^2)^(3/2)) - (3*x*(-a^2)^(1/2))/(8*a^3) + (a^2*x^2)/(3*(-a^2)^(3/2))
+ (x^3*(-a^2)^(3/2))/(4*a^3)))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.35 $\int e^{-\tanh^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=73

$$\frac{\sin^{-1}(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} - \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\sqrt{1-a^2x^2}}{a^3}$$

```
[Out] -1/3*(-a^2*x^2+1)^(3/2)/a^3+1/2*arcsin(a*x)/a^3+(-a^2*x^2+1)^(1/2)/a^3-1/2*
x*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6124, 797, 641, 195, 216}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} + \frac{\sqrt{1-a^2x^2}}{a^3} + \frac{\sin^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/a^3 - (x*Sqrt[1 - a^2*x^2])/(2*a^2) - (1 - a^2*x^2)^(3/2)
/(3*a^3) + ArcSin[a*x]/(2*a^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1-ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{a^2} - \frac{\int (1-ax)\sqrt{1-a^2x^2}\,dx}{a^2}\\
&= \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2} - \frac{\int \sqrt{1-a^2x^2}\,dx}{a^2}\\
&= \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} - \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\sin^{-1}(ax)}{a^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} - \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3} + \frac{\sin^{-1}(ax)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.59

$$\frac{\sqrt{1-a^2x^2}\left(2a^2x^2-3ax+4\right)+3\sin^{-1}(ax)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^ArcTanh[a*x],x]
[Out] (Sqrt[1 - a^2*x^2]*(4 - 3*a*x + 2*a^2*x^2) + 3*ArcSin[a*x])/(6*a^3)
```

fricas [A] time = 0.43, size = 57, normalized size = 0.78

$$\frac{\left(2\,a^2x^2-3\,ax+4\right)\sqrt{-a^2x^2+1}-6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^2*x^2 - 3*a*x + 4)*sqrt(-a^2*x^2 + 1) - 6*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.04, size = 134, normalized size = 1.84

$$-\frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{3a^{3}}-\frac{x\sqrt{-a^{2}x^{2}+1}}{2a^{2}}-\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{2a^{2}\sqrt{a^{2}}}+\frac{\sqrt{-a^{2}\left(x+\frac{1}{a}\right)^{2}+2a\left(x+\frac{1}{a}\right)}}{a^{3}}+\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}\left(x+\frac{1}{a}\right)^{2}+2a\left(x+\frac{1}{a}\right)}}\right)}{a^{2}\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/3*(-a^2*x^2+1)^(3/2)/a^3-1/2*x*(-a^2*x^2+1)^(1/2)/a^2-1/2/a^2/(a^2)^(1/2
)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^3*(-a^2*(x+1/a)^2+2*a*(x+1/a
))^(1/2)+1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a)
)^(1/2))
```

maxima [A] time = 0.41, size = 61, normalized size = 0.84

$$-\frac{\sqrt{-a^{2}x^{2}+1}\,x}{2\,a^{2}}-\frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{3\,a^{3}}+\frac{\arcsin\left(ax\right)}{2\,a^{3}}+\frac{\sqrt{-a^{2}x^{2}+1}}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x/a^2 - 1/3*(-a^2*x^2 + 1)^(3/2)/a^3 + 1/2*arcsin(a
*x)/a^3 + sqrt(-a^2*x^2 + 1)/a^3
```

mupad [B] time = 0.83, size = 82, normalized size = 1.12

$$\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{2\,a^2\,\sqrt{-a^2}}+\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2\,a}{3\,{\left(-a^2\right)}^{3/2}}-\frac{x\,\sqrt{-a^2}}{2\,a^2}+\frac{a^3\,x^2}{3\,{\left(-a^2\right)}^{3/2}}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] asinh(x*(-a^2)^(1/2))/(2*a^2*(-a^2)^(1/2)) + ((1 - a^2*x^2)^(1/2)*((2*a)/(3
*(-a^2)^(3/2)) - (x*(-a^2)^(1/2))/(2*a^2) + (a^3*x^2)/(3*(-a^2)^(3/2))))/(-
a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.36 $\int e^{-\tanh^{-1}(ax)} x \, dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{1-a^2x^2}\,(2-ax)}{2a^2}-\frac{\sin^{-1}(ax)}{2a^2}$$

```
[Out] -1/2*arcsin(a*x)/a^2-1/2*(-a*x+2)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6124, 780, 216}

$$-\frac{\sqrt{1-a^2x^2}\,(2-ax)}{2a^2}-\frac{\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^ArcTanh[a*x],x]
```

```
[Out] -((2 - a*x)*Sqrt[1 - a^2*x^2])/(2*a^2) - ArcSin[a*x]/(2*a^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x\,dx = \int \frac{x(1-ax)}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{(2-ax)\sqrt{1-a^2x^2}}{2a^2} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a}$$
$$= -\frac{(2-ax)\sqrt{1-a^2x^2}}{2a^2} - \frac{\sin^{-1}(ax)}{2a^2}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.87

$$\frac{(ax-2)\sqrt{1-a^2x^2} - \sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^ArcTanh[a*x],x]
```

```
[Out] ((-2 + a*x)*Sqrt[1 - a^2*x^2] - ArcSin[a*x])/(2*a^2)
```

fricas [A] time = 0.78, size = 48, normalized size = 1.23

$$\frac{\sqrt{-a^2x^2+1}\,(ax-2) + 2\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-a^2*x^2 + 1)*(a*x - 2) + 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)
))/a^2
```

giac [A] time = 0.17, size = 41, normalized size = 1.05

$$\frac{1}{2}\,\sqrt{-a^2x^2+1}\left(\frac{x}{a} - \frac{2}{a^2}\right) - \frac{\arcsin(ax)\,\mathrm{sgn}(a)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(x/a - 2/a^2) - 1/2*arcsin(a*x)*sgn(a)/(a*abs(a))
```

maple [B]   time = 0.04, size = 119, normalized size = 3.05

$$\frac{x\sqrt{-a^2x^2+1}}{2a}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a\sqrt{a^2}}-\frac{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}{a^2}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}\right)}{a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*(-a^2*x^2+1)^(1/2)/a+1/2/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2
+1)^(1/2))-1/a^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-1/a/(a^2)^(1/2)*arctan(
(a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [A]   time = 0.41, size = 45, normalized size = 1.15

$$\frac{\sqrt{-a^2x^2+1}\,x}{2\,a}-\frac{\arcsin\left(ax\right)}{2\,a^2}-\frac{\sqrt{-a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*x/a - 1/2*arcsin(a*x)/a^2 - sqrt(-a^2*x^2 + 1)/a^2
```

mupad [B]   time = 0.78, size = 58, normalized size = 1.49

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{1}{\sqrt{-a^2}}+\frac{x\,\sqrt{-a^2}}{2\,a}\right)-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a}}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(1/(-a^2)^(1/2) + (x*(-a^2)^(1/2))/(2*a)) - asinh(x*(-
a^2)^(1/2))/(2*a))/(-a^2)^(1/2)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.37 $\int e^{-\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{1-a^2x^2}}{a}+\frac{\sin^{-1}(ax)}{a}$$

```
[Out] arcsin(a*x)/a+(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6123, 641, 216}

$$\frac{\sqrt{1-a^2x^2}}{a}+\frac{\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcTanh[a*x]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/a + ArcSin[a*x]/a
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6123

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 + a*x)^((n + 1)/2)/(
(1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2]), x] /; FreeQ[a, x] && IntegerQ[(n
- 1)/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\,dx = \int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\sqrt{1-a^2x^2}}{a} + \int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{\sqrt{1-a^2x^2}}{a} + \frac{\sin^{-1}(ax)}{a}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.85

$$\frac{\sqrt{1-a^2x^2}+\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-ArcTanh[a*x]),x]
[Out] (Sqrt[1 - a^2*x^2] + ArcSin[a*x])/a
```

fricas [A] time = 0.57, size = 41, normalized size = 1.52

$$\frac{\sqrt{-a^2x^2+1} - 2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (sqrt(-a^2*x^2 + 1) - 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a
```

giac [A] time = 0.59, size = 28, normalized size = 1.04

$$\frac{\arcsin(ax)\operatorname{sgn}(a)}{|a|} + \frac{\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] arcsin(a*x)*sgn(a)/abs(a) + sqrt(-a^2*x^2 + 1)/a
```

maple [B] time = 0.03, size = 66, normalized size = 2.44

$$\frac{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}{a}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(
-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [A] time = 0.42, size = 25, normalized size = 0.93

$$\frac{\arcsin(ax)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a
```

mupad [B] time = 0.03, size = 35, normalized size = 1.30

$$\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{\sqrt{1-a^2\,x^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(a*x + 1),x)
```

```
[Out] asinh(x*(-a^2)^(1/2))/(-a^2)^(1/2) + (1 - a^2*x^2)^(1/2)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.38 $\int \frac{e^{-\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=24

$$-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

```
[Out] -arcsin(a*x)-arctanh((-a^2*x^2+1)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6124, 844, 216, 266, 63, 208}

$$-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{1-ax}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\left(a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\right) + \int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\sin^{-1}(ax) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)\\
&= -\sin^{-1}(ax) - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2}\\
&= -\sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.17

$$-\log\left(\sqrt{1-a^2x^2}+1\right) - \sin^{-1}(ax) + \log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x),x]
[Out] -ArcSin[a*x] + Log[x] - Log[1 + Sqrt[1 - a^2*x^2]]
```

fricas [A] time = 0.49, size = 44, normalized size = 1.83

$$2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + log((sqrt(-a^2*x^2 + 1) - 1)/x)
```

giac [B] time = 0.17, size = 52, normalized size = 2.17

$$-\frac{a \arcsin\left(ax\right) \operatorname{sgn}(a)}{|a|} - \frac{a \log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
 2*a)/(a^2*abs(x)))/abs(a)
```

maple [B] time = 0.04, size = 93, normalized size = 3.88

$$\sqrt{-a^2x^2+1} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \sqrt{-a^2\left(x+\frac{1}{a}\right)^2 + 2a\left(x+\frac{1}{a}\right)} - \frac{a \arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))-(-a^2*(x+1/a)^2+2*a*(x+1/a
))^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1
/2))
```

maxima [A] time = 0.42, size = 42, normalized size = 1.75

$$-a\left(\frac{\arcsin\left(ax\right)}{a} + \frac{\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a*(arcsin(a*x)/a + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a)
```

mupad [B] time = 0.03, size = 36, normalized size = 1.50

$$-\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)-\frac{a\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x*(a*x + 1)),x)
[Out] - atanh((1 - a^2*x^2)^(1/2)) - (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{x\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x*(a*x + 1)), x)
```

3.39 $\int \frac{e^{-\tanh^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=37

$$a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}}{x}$$

```
[Out] a*arctanh((-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6124, 807, 266, 63, 208}

$$a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) + a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{1-ax}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{x} - a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{x} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{x} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{\sqrt{1-a^2x^2}}{x} + a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 1.19

$$-\frac{\sqrt{1-a^2x^2}}{x} + a\log\left(\sqrt{1-a^2x^2}+1\right) - a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x^2),x]
[Out] -(Sqrt[1 - a^2*x^2]/x) - a*Log[x] + a*Log[1 + Sqrt[1 - a^2*x^2]]
```

fricas [A] time = 0.47, size = 40, normalized size = 1.08

$$-\frac{ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + \sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*x*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.04, size = 162, normalized size = 4.38

$$-a\sqrt{-a^2x^2+1}+a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{x}-a^2x\sqrt{-a^2x^2+1}-\frac{a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+a\sqrt{-a^2\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -a*(-a^2*x^2+1)^(1/2)+a*arctanh(1/(-a^2*x^2+1)^(1/2))-1/x*(-a^2*x^2+1)^(3/2
)-a^2*x*(-a^2*x^2+1)^(1/2)-a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1
)^(1/2))+a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+a^2/(a^2)^(1/2)*arctan((a^2)^
(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^2), x)
```

mupad [B] time = 0.03, size = 33, normalized size = 0.89

$$a\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)-\frac{\sqrt{1-a^2\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^2*(a*x + 1)),x)
```

```
[Out] a*atanh((1 - a^2*x^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{x^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x**2*(a*x + 1)), x)
```

3.40 $\int \frac{e^{-\tanh^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=63

$$\frac{a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/2*a^2*arctanh((-a^2*x^2+1)^(1/2))-1/2*(-a^2*x^2+1)^(1/2)/x^2+a*(-a^2*x^2
+1)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6124, 835, 807, 266, 63, 208}

$$\frac{a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) + (a*Sqrt[1 - a^2*x^2])/x - (a^2*ArcTanh[Sqrt[1
- a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{1-ax}{x^3\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{2}\int \frac{2a-a^2x}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}+\frac{a\sqrt{1-a^2x^2}}{x}+\frac{1}{2}a^2\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}+\frac{a\sqrt{1-a^2x^2}}{x}+\frac{1}{4}a^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}+\frac{a\sqrt{1-a^2x^2}}{x}-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2}+\frac{a\sqrt{1-a^2x^2}}{x}-\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.90

$$\frac{1}{2}\left(\frac{(2ax-1)\sqrt{1-a^2x^2}}{x^2}-a^2\log\left(\sqrt{1-a^2x^2}+1\right)+a^2\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] (((-1 + 2*a*x)*Sqrt[1 - a^2*x^2])/x^2 + a^2*Log[x] - a^2*Log[1 + Sqrt[1 - a
^2*x^2]])/2
```

fricas [A] time = 0.63, size = 51, normalized size = 0.81

$$\frac{a^2x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\,(2\,ax-1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(2*a*x -
1))/x^2
```

giac [B] time = 1.60, size = 159, normalized size = 2.52

$$\frac{\left(a^3-\frac{4\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}-\frac{a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\frac{\frac{4\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a|a|}{x}-\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3 - 4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x)*a^4*x^2/((sqrt(-a^2*x^2 +
 1)*abs(a) + a)^2*abs(a)) - 1/2*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a
) - 2*a)/(a^2*abs(x)))/abs(a) + 1/8*(4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*ab
s(a)/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2))/a^2
```

maple [B] time = 0.04, size = 186, normalized size = 2.95

$$\frac{a^2\sqrt{-a^2x^2+1}}{2}-\frac{a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{a\left(-a^2x^2+1\right)^{\frac{3}{2}}}{x}+a^3x\sqrt{-a^2x^2+1}+\frac{a^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] 1/2*a^2*(-a^2*x^2+1)^(1/2)-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+a/x*(-a^2*
x^2+1)^(3/2)+a^3*x*(-a^2*x^2+1)^(1/2)+a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/
(-a^2*x^2+1)^(1/2))-1/2/x^2*(-a^2*x^2+1)^(3/2)-a^2*(-a^2*(x+1/a)^2+2*a*(x+1
/a))^(1/2)-a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a)
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^3), x)
```

mupad [B] time = 0.04, size = 53, normalized size = 0.84

$$\frac{a\sqrt{1-a^2\,x^2}}{x}-\frac{\sqrt{1-a^2\,x^2}}{2\,x^2}-\frac{a^2\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^3*(a*x + 1)),x)
```

```
[Out] (a*(1 - a^2*x^2)^(1/2))/x - (1 - a^2*x^2)^(1/2)/(2*x^2) - (a^2*atanh((1 - a
^2*x^2)^(1/2)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{x^3(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x**3*(a*x + 1)), x)
```

3.41 $\int \frac{e^{-\tanh^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=90

$$-\frac{2a^2\sqrt{1-a^2x^2}}{3x}+\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] 1/2*a^3*arctanh((-a^2*x^2+1)^(1/2))-1/3*(-a^2*x^2+1)^(1/2)/x^3+1/2*a*(-a^2*
x^2+1)^(1/2)/x^2-2/3*a^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6124, 835, 807, 266, 63, 208}

$$-\frac{2a^2\sqrt{1-a^2x^2}}{3x}+\frac{a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) + (a*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a^2*Sqrt[1
- a^2*x^2])/(3*x) + (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{x^4} dx &= \int \frac{1-ax}{x^4\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{1}{3}\int \frac{3a-2a^2x}{x^3\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{a\sqrt{1-a^2x^2}}{2x^2} + \frac{1}{6}\int \frac{4a^2-3a^3x}{x^2\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{a\sqrt{1-a^2x^2}}{2x^2} - \frac{2a^2\sqrt{1-a^2x^2}}{3x} - \frac{1}{2}a^3\int \frac{1}{x\sqrt{1-a^2x^2}} dx \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{a\sqrt{1-a^2x^2}}{2x^2} - \frac{2a^2\sqrt{1-a^2x^2}}{3x} - \frac{1}{4}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{a\sqrt{1-a^2x^2}}{2x^2} - \frac{2a^2\sqrt{1-a^2x^2}}{3x} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}} dx, x, \sqrt{1-a^2x^2}\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{a\sqrt{1-a^2x^2}}{2x^2} - \frac{2a^2\sqrt{1-a^2x^2}}{3x} + \frac{1}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.73

$$
\frac{1}{6}\left(-3a^3\log(x) + \frac{\left(-4a^2x^2+3ax-2\right)\sqrt{1-a^2x^2}}{x^3} + 3a^3\log\left(\sqrt{1-a^2x^2}+1\right)\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] (((-2 + 3*a*x - 4*a^2*x^2)*Sqrt[1 - a^2*x^2])/x^3 - 3*a^3*Log[x] + 3*a^3*Lo
g[1 + Sqrt[1 - a^2*x^2]])/6
```

fricas [A] time = 0.52, size = 60, normalized size = 0.67

$$
-\frac{3\,a^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + \left(4\,a^2x^2-3\,ax+2\right)\sqrt{-a^2x^2+1}}{6\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^3*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (4*a^2*x^2 - 3*a*x + 2)*s
qrt(-a^2*x^2 + 1))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 207, normalized size = 2.30

$$-\frac{a^{3}\sqrt{-a^{2}x^{2}+1}}{2}+\frac{a^{3}\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)}{2}-\frac{a^{2}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{x}-a^{4}x\sqrt{-a^{2}x^{2}+1}-\frac{a^{4}\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{\sqrt{a^{2}}}-\frac{\left(-a^{2}x^{2}+1\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] -1/2*a^3*(-a^2*x^2+1)^(1/2)+1/2*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))-a^2/x*(-a
^2*x^2+1)^(3/2)-a^4*x*(-a^2*x^2+1)^(1/2)-a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)
*x/(-a^2*x^2+1)^(1/2))-1/3/x^3*(-a^2*x^2+1)^(3/2)+1/2*a/x^2*(-a^2*x^2+1)^(3
/2)+a^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+a^4/(a^2)^(1/2)*arctan((a^2)^(1/
2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}}{(ax+1)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^4), x)
```

mupad [B] time = 0.03, size = 78, normalized size = 0.87

$$\frac{a\,\sqrt{1-a^2\,x^2}}{2\,x^2}-\frac{\sqrt{1-a^2\,x^2}}{3\,x^3}-\frac{2\,a^2\,\sqrt{1-a^2\,x^2}}{3\,x}-\frac{a^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^4*(a*x + 1)),x)
```

```
[Out] (a*(1 - a^2*x^2)^(1/2))/(2*x^2) - (1 - a^2*x^2)^(1/2)/(3*x^3) - (a^3*atan((
1 - a^2*x^2)^(1/2)*1i)*1i)/2 - (2*a^2*(1 - a^2*x^2)^(1/2))/(3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{x^4(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x**4*(a*x + 1)), x)
```

3.42 $\int \frac{e^{-\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=114

$$-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}-\frac{\sqrt{1-a^2x^2}}{4x^4}+\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{2a^3\sqrt{1-a^2x^2}}{3x}$$

```
[Out] -3/8*a^4*arctanh((-a^2*x^2+1)^(1/2))-1/4*(-a^2*x^2+1)^(1/2)/x^4+1/3*a*(-a^2
*x^2+1)^(1/2)/x^3-3/8*a^2*(-a^2*x^2+1)^(1/2)/x^2+2/3*a^3*(-a^2*x^2+1)^(1/2)
/x
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6124, 835, 807, 266, 63, 208}

$$\frac{2a^3\sqrt{1-a^2x^2}}{3x}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}+\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*x^5),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(4*x^4) + (a*Sqrt[1 - a^2*x^2])/(3*x^3) - (3*a^2*Sqrt[1
- a^2*x^2])/(8*x^2) + (2*a^3*Sqrt[1 - a^2*x^2])/(3*x) - (3*a^4*ArcTanh[Sqrt
[1 - a^2*x^2]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{x^5}\,dx &= \int \frac{1-ax}{x^5\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} - \frac{1}{4}\int \frac{4a-3a^2x}{x^4\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{3x^3} + \frac{1}{12}\int \frac{9a^2-8a^3x}{x^3\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} - \frac{1}{24}\int \frac{16a^3-9a^4x}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{2a^3\sqrt{1-a^2x^2}}{3x} + \frac{1}{8}\left(3a^4\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{2a^3\sqrt{1-a^2x^2}}{3x} + \frac{1}{16}\left(3a^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a}}\right.\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{2a^3\sqrt{1-a^2x^2}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx\right.\\
&= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{3x^3} - \frac{3a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{2a^3\sqrt{1-a^2x^2}}{3x} - \frac{3}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.65

$$
\frac{1}{24}\left(9a^4\log(x) - 9a^4\log\left(\sqrt{1-a^2x^2}+1\right) + \frac{\sqrt{1-a^2x^2}\left(16a^3x^3-9a^2x^2+8ax-6\right)}{x^4}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*x^5),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-6 + 8*a*x - 9*a^2*x^2 + 16*a^3*x^3))/x^4 + 9*a^4*Log[
x] - 9*a^4*Log[1 + Sqrt[1 - a^2*x^2]])/24
```

fricas [A] time = 0.51, size = 68, normalized size = 0.60

$$
\frac{9\,a^4x^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + \left(16\,a^3x^3 - 9\,a^2x^2 + 8\,ax - 6\right)\sqrt{-a^2x^2+1}}{24\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a^4*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (16*a^3*x^3 - 9*a^2*x^2 +
 8*a*x - 6)*sqrt(-a^2*x^2 + 1))/x^4
```

giac [B] time = 0.20, size = 273, normalized size = 2.39

$$\frac{\left(3\,a^5 - \frac{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^3}{x} + \frac{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2 a}{x^2} - \frac{72\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{ax^3}\right)a^8x^4}{192\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4|a|} - \frac{3\,a^5\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|} + \frac{72\left(\sqrt{-a^2x^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^5 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3/x + 24*(sqrt(-a^2*x^2
+ 1)*abs(a) + a)^2*a/x^2 - 72*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a*x^3))*a^
8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)) - 3/8*a^5*log(1/2*abs(-2*s
qrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/192*(72*(sqrt(-a^2
*x^2 + 1)*abs(a) + a)*a^5*abs(a)/x - 24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a
^3*abs(a)/x^2 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*abs(a)/x^3 - 3*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)/(a*x^4))/a^4
```

maple [B] time = 0.05, size = 226, normalized size = 1.98

$$\frac{3a^4\sqrt{-a^2x^2+1}}{8} - \frac{3a^4\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{8} + \frac{a^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{x} + a^5x\sqrt{-a^2x^2+1} + \frac{a^5\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}} + \frac{a\left(-a^2x^2\right.}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^5,x)
```

```
[Out] 3/8*a^4*(-a^2*x^2+1)^(1/2)-3/8*a^4*arctanh(1/(-a^2*x^2+1)^(1/2))+a^3/x*(-a^
2*x^2+1)^(3/2)+a^5*x*(-a^2*x^2+1)^(1/2)+a^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*
x/(-a^2*x^2+1)^(1/2))+1/3*a/x^3*(-a^2*x^2+1)^(3/2)-5/8*a^2/x^2*(-a^2*x^2+1)
^(3/2)-1/4/x^4*(-a^2*x^2+1)^(3/2)-a^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-a^
5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^5), x)
```

mupad [B] time = 0.03, size = 98, normalized size = 0.86

$$\frac{a\sqrt{1-a^2x^2}}{3x^3}-\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{3a^2\sqrt{1-a^2x^2}}{8x^2}+\frac{2a^3\sqrt{1-a^2x^2}}{3x}+\frac{a^4\,\mathrm{atan}\left(\sqrt{1-a^2x^2}\,1\mathrm{i}\right)3\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^5*(a*x + 1)),x)
```

```
[Out] (a^4*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/8 - (1 - a^2*x^2)^(1/2)/(4*x^4) + (a*
(1 - a^2*x^2)^(1/2))/(3*x^3) - (3*a^2*(1 - a^2*x^2)^(1/2))/(8*x^2) + (2*a^3
*(1 - a^2*x^2)^(1/2))/(3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(ax-1)(ax+1)}}{x^5(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(x**5*(a*x + 1)), x)
```

3.43 $\int e^{-2\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=43

$$-\frac{2\log(ax+1)}{a^4}+\frac{2x}{a^3}-\frac{x^2}{a^2}+\frac{2x^3}{3a}-\frac{x^4}{4}$$

```
[Out] 2*x/a^3-x^2/a^2+2/3*x^3/a-1/4*x^4-2*ln(a*x+1)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{x^2}{a^2}+\frac{2x}{a^3}-\frac{2\log(ax+1)}{a^4}+\frac{2x^3}{3a}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*x)/a^3 - x^2/a^2 + (2*x^3)/(3*a) - x^4/4 - (2*Log[1 + a*x])/a^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-ax)}{1+ax}\,dx$$
$$= \int \left(\frac{2}{a^3} - \frac{2x}{a^2} + \frac{2x^2}{a} - x^3 - \frac{2}{a^3(1+ax)}\right) dx$$
$$= \frac{2x}{a^3} - \frac{x^2}{a^2} + \frac{2x^3}{3a} - \frac{x^4}{4} - \frac{2\log(1+ax)}{a^4}$$

Mathematica [**A**] time = 0.02, size = 43, normalized size = 1.00

$$-\frac{2\log(ax+1)}{a^4} + \frac{2x}{a^3} - \frac{x^2}{a^2} + \frac{2x^3}{3a} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(2*ArcTanh[a*x]),x]
[Out] (2*x)/a^3 - x^2/a^2 + (2*x^3)/(3*a) - x^4/4 - (2*Log[1 + a*x])/a^4
```

fricas [**A**] time = 0.72, size = 42, normalized size = 0.98

$$-\frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax + 24\,\log\left(ax+1\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x + 24*log(a*x + 1))/a^4
```

giac [**A**] time = 0.39, size = 66, normalized size = 1.53

$$\frac{(ax+1)^4\left(\frac{20}{ax+1} - \frac{54}{(ax+1)^2} + \frac{84}{(ax+1)^3} - 3\right)}{12\,a^4} + \frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] 1/12*(a*x + 1)^4*(20/(a*x + 1) - 54/(a*x + 1)^2 + 84/(a*x + 1)^3 - 3)/a^4 +
 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a^4
```

maple [**A**] time = 0.03, size = 40, normalized size = 0.93

$$\frac{2x}{a^3} - \frac{x^2}{a^2} + \frac{2x^3}{3a} - \frac{x^4}{4} - \frac{2\ln\left(ax+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2*x/a^3-x^2/a^2+2/3*x^3/a-1/4*x^4-2*ln(a*x+1)/a^4
```

maxima [A] time = 0.31, size = 43, normalized size = 1.00

$$-\frac{3\,a^3x^4 - 8\,a^2x^3 + 12\,ax^2 - 24\,x}{12\,a^3} - \frac{2\,\log\left(ax+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^4 - 8*a^2*x^3 + 12*a*x^2 - 24*x)/a^3 - 2*log(a*x + 1)/a^4
```

mupad [B] time = 0.04, size = 39, normalized size = 0.91

$$\frac{2\,x}{a^3} - \frac{2\,\ln\left(a\,x+1\right)}{a^4} - \frac{x^4}{4} + \frac{2\,x^3}{3\,a} - \frac{x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] (2*x)/a^3 - (2*log(a*x + 1))/a^4 - x^4/4 + (2*x^3)/(3*a) - x^2/a^2
```

sympy [A] time = 0.11, size = 37, normalized size = 0.86

$$-\frac{x^4}{4} + \frac{2x^3}{3a} - \frac{x^2}{a^2} + \frac{2x}{a^3} - \frac{2\log\left(ax+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -x**4/4 + 2*x**3/(3*a) - x**2/a**2 + 2*x/a**3 - 2*log(a*x + 1)/a**4
```

3.44 $\int e^{-2\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=32

$$\frac{2\log(ax+1)}{a^3}-\frac{2x}{a^2}+\frac{x^2}{a}-\frac{x^3}{3}$$

[Out] -2*x/a^2+x^2/a-1/3*x^3+2*ln(a*x+1)/a^3

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{2x}{a^2}+\frac{2\log(ax+1)}{a^3}+\frac{x^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2/E^(2*ArcTanh[a*x]),x]

[Out] (-2*x)/a^2 + x^2/a - x^3/3 + (2*Log[1 + a*x])/a^3

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}x^2\,dx&=\int\frac{x^2(1-ax)}{1+ax}\,dx\\&=\int\left(-\frac{2}{a^2}+\frac{2x}{a}-x^2+\frac{2}{a^2(1+ax)}\right)dx\\&=-\frac{2x}{a^2}+\frac{x^2}{a}-\frac{x^3}{3}+\frac{2\log(1+ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{2\log(ax+1)}{a^3}-\frac{2x}{a^2}+\frac{x^2}{a}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-2*x)/a^2 + x^2/a - x^3/3 + (2*Log[1 + a*x])/a^3
```

fricas [A] time = 0.48, size = 33, normalized size = 1.03

$$-\frac{a^3x^3-3\,a^2x^2+6\,ax-6\,\log\left(ax+1\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x - 6*log(a*x + 1))/a^3
```

giac [A] time = 0.20, size = 57, normalized size = 1.78

$$\frac{(ax+1)^3\left(\frac{6}{ax+1}-\frac{15}{(ax+1)^2}-1\right)}{3\,a^3}-\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*(a*x + 1)^3*(6/(a*x + 1) - 15/(a*x + 1)^2 - 1)/a^3 - 2*log(abs(a*x + 1)
/((a*x + 1)^2*abs(a)))/a^3
```

maple [A] time = 0.03, size = 31, normalized size = 0.97

$$-\frac{2x}{a^2}+\frac{x^2}{a}-\frac{x^3}{3}+\frac{2\ln\left(ax+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -2*x/a^2+x^2/a-1/3*x^3+2/a^3*ln(a*x+1)
```

maxima [A] time = 0.32, size = 34, normalized size = 1.06

$$-\frac{a^2x^3-3\,ax^2+6\,x}{3\,a^2}+\frac{2\,\log\left(ax+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
[Out] -1/3*(a^2*x^3 - 3*a*x^2 + 6*x)/a^2 + 2*log(a*x + 1)/a^3
```

mupad [B] time = 0.04, size = 30, normalized size = 0.94

$$\frac{2\,\ln\left(a\,x+1\right)}{a^3}-\frac{2\,x}{a^2}-\frac{x^3}{3}+\frac{x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a^2*x^2 - 1))/(a*x + 1)^2,x)
[Out] (2*log(a*x + 1))/a^3 - (2*x)/a^2 - x^3/3 + x^2/a
```

sympy [A] time = 0.10, size = 27, normalized size = 0.84

$$-\frac{x^3}{3}+\frac{x^2}{a}-\frac{2x}{a^2}+\frac{2\log\left(ax+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**2*(-a**2*x**2+1),x)
[Out] -x**3/3 + x**2/a - 2*x/a**2 + 2*log(a*x + 1)/a**3
```

3.45 $\int e^{-2\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=25

$$-\frac{2\log(ax+1)}{a^2}+\frac{2x}{a}-\frac{x^2}{2}$$

[Out] 2*x/a-1/2*x^2-2*ln(a*x+1)/a^2

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6126, 77}

$$-\frac{2\log(ax+1)}{a^2}+\frac{2x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x/E^(2*ArcTanh[a*x]),x]

[Out] (2*x)/a - x^2/2 - (2*Log[1 + a*x])/a^2

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}x\,dx&=\int\frac{x(1-ax)}{1+ax}\,dx\\&=\int\left(\frac{2}{a}-x-\frac{2}{a(1+ax)}\right)dx\\&=\frac{2x}{a}-\frac{x^2}{2}-\frac{2\log(1+ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{2\log(ax+1)}{a^2}+\frac{2x}{a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(2*ArcTanh[a*x]),x]
[Out] (2*x)/a - x^2/2 - (2*Log[1 + a*x])/a^2
```

fricas [A] time = 0.46, size = 25, normalized size = 1.00

$$-\frac{a^2x^2-4\,ax+4\,\log\left(ax+1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -1/2*(a^2*x^2 - 4*a*x + 4*log(a*x + 1))/a^2
```

giac [B] time = 0.18, size = 52, normalized size = 2.08

$$\frac{\frac{(ax+1)^2\left(\frac{6}{ax+1}-1\right)}{a}+\frac{4\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] 1/2*((a*x + 1)^2*(6/(a*x + 1) - 1)/a + 4*log(abs(a*x + 1)/((a*x + 1)^2*abs(
a)))/a)/a
```

maple [A] time = 0.02, size = 24, normalized size = 0.96

$$\frac{2x}{a}-\frac{x^2}{2}-\frac{2\ln\left(ax+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)^2*(-a^2*x^2+1),x)
[Out] 2*x/a-1/2*x^2-2/a^2*ln(a*x+1)
```

maxima [A] time = 0.31, size = 26, normalized size = 1.04

$$-\frac{ax^2-4\,x}{2\,a}-\frac{2\,\log\left(ax+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2 - 4*x)/a - 2*log(a*x + 1)/a^2
```

mupad [B] time = 0.79, size = 23, normalized size = 0.92

$$\frac{2\,x}{a}-\frac{2\,\ln\left(a\,x+1\right)}{a^2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] (2*x)/a - (2*log(a*x + 1))/a^2 - x^2/2
```

sympy [A] time = 0.09, size = 20, normalized size = 0.80

$$-\frac{x^2}{2}+\frac{2x}{a}-\frac{2\log\left(ax+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -x**2/2 + 2*x/a - 2*log(a*x + 1)/a**2
```

3.46 $\int e^{-2\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=15

$$\frac{2\log(ax+1)}{a}-x$$

```
[Out] -x+2*ln(a*x+1)/a
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6125, 43}

$$\frac{2\log(ax+1)}{a}-x$$

Antiderivative was successfully verified.

```
[In] Int[E^(-2*ArcTanh[a*x]),x]
```

```
[Out] -x + (2*Log[1 + a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\,dx &= \int \frac{1-ax}{1+ax}\,dx\\ &= \int \left(-1+\frac{2}{1+ax}\right)dx\\ &= -x+\frac{2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{2\log(ax+1)}{a}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-2*ArcTanh[a*x]),x]
[Out] -x + (2*Log[1 + a*x])/a
```

fricas [A] time = 0.49, size = 17, normalized size = 1.13

$$-\frac{ax-2\,\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -(a*x - 2*log(a*x + 1))/a
```

giac [B] time = 0.16, size = 64, normalized size = 4.27

$$-a^2\left(\frac{ax+1}{a^3}+\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a^3}-\frac{1}{(ax+1)a^3}\right)-\frac{1}{(ax+1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] -a^2*((a*x + 1)/a^3 + 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a^3 - 1/((a*
x + 1)*a^3)) - 1/((a*x + 1)*a)
```

maple [A] time = 0.02, size = 16, normalized size = 1.07

$$-x+\frac{2\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1),x)
[Out] -x+2*ln(a*x+1)/a
```

maxima [A] time = 0.31, size = 15, normalized size = 1.00

$$-x+\frac{2\,\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x + 2*log(a*x + 1)/a
```

mupad [B] time = 0.03, size = 15, normalized size = 1.00

$$\frac{2\ln(a\,x+1)}{a}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/(a*x + 1)^2,x)
```

```
[Out] (2*log(a*x + 1))/a - x
```

sympy [A] time = 0.09, size = 10, normalized size = 0.67

$$-x+\frac{2\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -x + 2*log(a*x + 1)/a
```

3.47 $\int \frac{e^{-2\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=11

$$\log(x) - 2\log(ax+1)$$

```
[Out] ln(x)-2*ln(a*x+1)
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 72}

$$\log(x) - 2\log(ax+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] Log[x] - 2*Log[1 + a*x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{1-ax}{x(1+ax)}\,dx \\
&= \int \left(\frac{1}{x} - \frac{2a}{1+ax}\right)dx \\
&= \log(x) - 2\log(1+ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\log(x) - 2\log(ax+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] Log[x] - 2*Log[1 + a*x]
```

fricas [A] time = 0.42, size = 11, normalized size = 1.00

$$-2 \log(ax+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] -2*log(a*x + 1) + log(x)
```

giac [B] time = 0.36, size = 43, normalized size = 3.91

$$a\left(\frac{\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a} + \frac{\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] a*(log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + log(abs(-1/(a*x + 1) + 1))/a)
```

maple [A] time = 0.03, size = 12, normalized size = 1.09

$$\ln(x) - 2\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] ln(x)-2*ln(a*x+1)
```

maxima [A] time = 0.31, size = 11, normalized size = 1.00

$$-2 \log(ax+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] -2*log(a*x + 1) + log(x)
```

mupad [B] time = 0.79, size = 12, normalized size = 1.09

$$\ln(x) - 2\,\ln\left(3\,a\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/(x*(a*x + 1)^2),x)
[Out] log(x) - 2*log(3*a*x + 3)
```

sympy [A] time = 0.12, size = 10, normalized size = 0.91

$$\log\left(x\right) - 2\log\left(x + \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/x,x)
[Out] log(x) - 2*log(x + 1/a)
```

3.48 $\int \frac{e^{-2\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=20

$$-2a\log(x)+2a\log(ax+1)-\frac{1}{x}$$

```
[Out] -1/x-2*a*ln(x)+2*a*ln(a*x+1)
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-2a\log(x)+2a\log(ax+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -x^(-1) - 2*a*Log[x] + 2*a*Log[1 + a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{1-ax}{x^2(1+ax)}\,dx \\
&= \int \left(\frac{1}{x^2}-\frac{2a}{x}+\frac{2a^2}{1+ax}\right)dx \\
&= -\frac{1}{x}-2a\log(x)+2a\log(1+ax)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$-2a\log(x)+2a\log(ax+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*x^2),x]
[Out] -x^(-1) - 2*a*Log[x] + 2*a*Log[1 + a*x]
```

fricas [A] time = 0.42, size = 22, normalized size = 1.10

$$\frac{2\,ax\log\left(ax+1\right)-2\,ax\log(x)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fricas")
[Out] (2*a*x*log(a*x + 1) - 2*a*x*log(x) - 1)/x
```

giac [A] time = 0.17, size = 30, normalized size = 1.50

$$-2\,a\log\left(\left|-\frac{1}{ax+1}+1\right|\right)+\frac{a}{\frac{1}{ax+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="giac")
[Out] -2*a*log(abs(-1/(a*x + 1) + 1)) + a/(1/(a*x + 1) - 1)
```

maple [A] time = 0.03, size = 21, normalized size = 1.05

$$-\frac{1}{x}-2a\ln(x)+2a\ln\left(ax+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/x-2*a*ln(x)+2*a*ln(a*x+1)
```

maxima [A] time = 0.31, size = 20, normalized size = 1.00

$$2\,a\log\left(ax+1\right)-2\,a\log(x)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*log(a*x + 1) - 2*a*log(x) - 1/x
```

mupad [B] time = 0.05, size = 16, normalized size = 0.80

$$4\,a\,\mathrm{atanh}\left(2\,a\,x+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/(x^2*(a*x + 1)^2),x)
```

```
[Out] 4*a*atanh(2*a*x + 1) - 1/x
```

sympy [A] time = 0.14, size = 17, normalized size = 0.85

$$-2a\left(\log\left(x\right)-\log\left(x+\frac{1}{a}\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
```

```
[Out] -2*a*(log(x) - log(x + 1/a)) - 1/x
```

3.49 $\int \frac{e^{-2\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=32

$$2a^2\log(x) - 2a^2\log(ax+1) + \frac{2a}{x} - \frac{1}{2x^2}$$

```
[Out] -1/2/x^2+2*a/x+2*a^2*ln(x)-2*a^2*ln(a*x+1)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$2a^2\log(x) - 2a^2\log(ax+1) + \frac{2a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -1/(2*x^2) + (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 + a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{1-ax}{x^3(1+ax)}\,dx$$
$$= \int \left(\frac{1}{x^3} - \frac{2a}{x^2} + \frac{2a^2}{x} - \frac{2a^3}{1+ax}\right) dx$$
$$= -\frac{1}{2x^2} + \frac{2a}{x} + 2a^2\log(x) - 2a^2\log(1+ax)$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$2a^2\log(x) - 2a^2\log(ax+1) + \frac{2a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -1/2*1/x^2 + (2*a)/x + 2*a^2*Log[x] - 2*a^2*Log[1 + a*x]
```

fricas [A] time = 0.42, size = 35, normalized size = 1.09

$$-\frac{4\,a^2x^2\log\left(ax+1\right) - 4\,a^2x^2\log(x) - 4\,ax + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*x^2*log(a*x + 1) - 4*a^2*x^2*log(x) - 4*a*x + 1)/x^2
```

giac [A] time = 0.18, size = 50, normalized size = 1.56

$$2\,a^2\log\left(\left|-\frac{1}{ax+1}+1\right|\right) + \frac{5\,a^2 - \frac{6\,a^2}{ax+1}}{2\left(\frac{1}{ax+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] 2*a^2*log(abs(-1/(a*x + 1) + 1)) + 1/2*(5*a^2 - 6*a^2/(a*x + 1))/(1/(a*x +
1) - 1)^2
```

maple [A] time = 0.03, size = 31, normalized size = 0.97

$$-\frac{1}{2x^2}+\frac{2a}{x}+2a^2\ln(x)-2a^2\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)
```

```
[Out] -1/2/x^2+2*a/x+2*a^2*ln(x)-2*a^2*ln(a*x+1)
```

maxima [A] time = 0.31, size = 30, normalized size = 0.94

$$-2\,a^2\log(ax+1)+2\,a^2\log(x)+\frac{4\,ax-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] -2*a^2*log(a*x + 1) + 2*a^2*log(x) + 1/2*(4*a*x - 1)/x^2
```

mupad [B] time = 0.80, size = 23, normalized size = 0.72

$$\frac{2\,a\,x-\frac{1}{2}}{x^2}-4\,a^2\,\mathrm{atanh}\left(2\,a\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/(x^3*(a*x + 1)^2),x)
```

```
[Out] (2*a*x - 1/2)/x^2 - 4*a^2*atanh(2*a*x + 1)
```

sympy [A] time = 0.16, size = 27, normalized size = 0.84

$$-2a^2\left(-\log(x)+\log\left(x+\frac{1}{a}\right)\right)-\frac{-4ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -2*a**2*(-log(x) + log(x + 1/a)) - (-4*a*x + 1)/(2*x**2)
```

3.50 $\int \frac{e^{-2\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=39

$$-2a^3\log(x)+2a^3\log(ax+1)-\frac{2a^2}{x}+\frac{a}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3+a/x^2-2*a^2/x-2*a^3*ln(x)+2*a^3*ln(a*x+1)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 77}

$$-\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(ax+1)+\frac{a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] -1/(3*x^3) + a/x^2 - (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 + a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{1-ax}{x^4(1+ax)}\,dx$$
$$= \int \left(\frac{1}{x^4} - \frac{2a}{x^3} + \frac{2a^2}{x^2} - \frac{2a^3}{x} + \frac{2a^4}{1+ax}\right) dx$$
$$= -\frac{1}{3x^3} + \frac{a}{x^2} - \frac{2a^2}{x} - 2a^3\log(x) + 2a^3\log(1+ax)$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$-2a^3\log(x) + 2a^3\log(ax+1) - \frac{2a^2}{x} + \frac{a}{x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*x^4),x]
[Out] -1/3*1/x^3 + a/x^2 - (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 + a*x]
```

fricas [A] time = 0.52, size = 43, normalized size = 1.10

$$\frac{6\,a^3x^3\log\left(ax+1\right) - 6\,a^3x^3\log(x) - 6\,a^2x^2 + 3\,ax - 1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fricas")
[Out] 1/3*(6*a^3*x^3*log(a*x + 1) - 6*a^3*x^3*log(x) - 6*a^2*x^2 + 3*a*x - 1)/x^3
```

giac [A] time = 0.16, size = 62, normalized size = 1.59

$$-2\,a^3\log\left(\left|-\frac{1}{ax+1}+1\right|\right) + \frac{10\,a^3 - \frac{24\,a^3}{ax+1} + \frac{15\,a^3}{(ax+1)^2}}{3\left(\frac{1}{ax+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="giac")
[Out] -2*a^3*log(abs(-1/(a*x + 1) + 1)) + 1/3*(10*a^3 - 24*a^3/(a*x + 1) + 15*a^3
/(a*x + 1)^2)/(1/(a*x + 1) - 1)^3
```

maple [A] time = 0.03, size = 38, normalized size = 0.97

$$-\frac{1}{3x^3}+\frac{a}{x^2}-\frac{2a^2}{x}-2a^3\ln(x)+2a^3\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
[Out] -1/3/x^3+a/x^2-2*a^2/x-2*a^3*ln(x)+2*a^3*ln(a*x+1)
```

maxima [A] time = 0.31, size = 38, normalized size = 0.97

$$2\,a^3\log(ax+1)-2\,a^3\log(x)-\frac{6\,a^2x^2-3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="maxima")
[Out] 2*a^3*log(a*x + 1) - 2*a^3*log(x) - 1/3*(6*a^2*x^2 - 3*a*x + 1)/x^3
```

mupad [B] time = 0.05, size = 32, normalized size = 0.82

$$4\,a^3\,\mathrm{atanh}\left(2\,a\,x+1\right)-\frac{2\,a^2\,x^2-a\,x+\frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/(x^4*(a*x + 1)^2),x)
[Out] 4*a^3*atanh(2*a*x + 1) - (2*a^2*x^2 - a*x + 1/3)/x^3
```

sympy [A] time = 0.18, size = 36, normalized size = 0.92

$$-2a^3\left(\log(x)-\log\left(x+\frac{1}{a}\right)\right)-\frac{6a^2x^2-3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)
[Out] -2*a**3*(log(x) - log(x + 1/a)) - (6*a**2*x**2 - 3*a*x + 1)/(3*x**3)
```

3.51 $\int e^{-3\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=131

$$\frac{51\sin^{-1}(ax)}{8a^4}+\frac{x^2\sqrt{1-a^2x^2}}{a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a}+\frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4}+\frac{27\sqrt{1-a^2x^2}}{4a^4}+\frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}}$$

```
[Out] 51/8*arcsin(a*x)/a^4+(-a*x+1)^3/a^4/(-a^2*x^2+1)^(1/2)+27/4*(-a^2*x^2+1)^(1
/2)/a^4+x^2*(-a^2*x^2+1)^(1/2)/a^2-1/4*x^3*(-a^2*x^2+1)^(1/2)/a+9/8*(-3*a*x
+2)*(-a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.69, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6124, 1633, 1593, 12, 852, 1635, 1815, 27, 743, 641, 216}

$$-\frac{x^3\sqrt{1-a^2x^2}}{4a}+\frac{x^2\sqrt{1-a^2x^2}}{a^2}+\frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4}+\frac{27\sqrt{1-a^2x^2}}{4a^4}+\frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}}+\frac{51\sin^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (1 - a*x)^3/(a^4*Sqrt[1 - a^2*x^2]) + (27*Sqrt[1 - a^2*x^2])/(4*a^4) + (x^2
*Sqrt[1 - a^2*x^2])/a^2 - (x^3*Sqrt[1 - a^2*x^2])/(4*a) + (9*(2 - 3*a*x)*Sq
rt[1 - a^2*x^2])/(8*a^4) + (51*ArcSin[a*x])/(8*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}x^3\,dx &= \int \frac{x^3(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx \\
&= a\int \frac{\sqrt{1-a^2x^2}\left(\frac{x^3}{a}-x^4\right)}{(1+ax)^2}\,dx \\
&= a\int \frac{\left(\frac{1}{a}-x\right)x^3\sqrt{1-a^2x^2}}{(1+ax)^2}\,dx \\
&= a^2\int \frac{x^3\left(1-a^2x^2\right)^{3/2}}{a^2(1+ax)^3}\,dx \\
&= \int \frac{x^3\left(1-a^2x^2\right)^{3/2}}{(1+ax)^3}\,dx \\
&= \int \frac{x^3(1-ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx \\
&= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} - \int \frac{(1-ax)^2\left(-\frac{3}{a^3}+\frac{x}{a^2}-\frac{x^2}{a}\right)}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{\int \frac{\frac{12}{a}-28x+27ax^2-12a^2x^3}{\sqrt{1-a^2x^2}}\,dx}{4a^2} \\
&= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{\int \frac{-36a+108a^2x-81a^3x^2}{\sqrt{1-a^2x^2}}\,dx}{12a^4} \\
&= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} - \frac{\int -\frac{9a(-2+3ax)^2}{\sqrt{1-a^2x^2}}\,dx}{12a^4} \\
&= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{3\int \frac{(-2+3ax)^2}{\sqrt{1-a^2x^2}}\,dx}{4a^3} \\
&= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4} - \frac{3\int \frac{-17a^2+18a^3x}{\sqrt{1-a^2x^2}}\,dx}{8a^5} \\
&= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{27\sqrt{1-a^2x^2}}{4a^4} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4} + \frac{5\cdots}{\cdots} \\
&= \frac{(1-ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{27\sqrt{1-a^2x^2}}{4a^4} + \frac{x^2\sqrt{1-a^2x^2}}{a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a} + \frac{9(2-3ax)\sqrt{1-a^2x^2}}{8a^4} + \frac{5\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.53

$$\frac{51\sin^{-1}(ax)}{8a^4}+\sqrt{1-a^2x^2}\left(\frac{4}{a^4(ax+1)}+\frac{6}{a^4}-\frac{19x}{8a^3}+\frac{x^2}{a^2}-\frac{x^3}{4a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(6/a^4 - (19*x)/(8*a^3) + x^2/a^2 - x^3/(4*a) + 4/(a^4*(1
 + a*x))) + (51*ArcSin[a*x])/(8*a^4)
```

fricas [A] time = 0.49, size = 92, normalized size = 0.70

$$\frac{80\,ax-102\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^4x^4-6\,a^3x^3+11\,a^2x^2-29\,ax-80\right)\sqrt{-a^2x^2+1}+80}{8\left(a^5x+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(80*a*x - 102*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^4
*x^4 - 6*a^3*x^3 + 11*a^2*x^2 - 29*a*x - 80)*sqrt(-a^2*x^2 + 1) + 80)/(a^5*
x + a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 235, normalized size = 1.79

$$\frac{x\left(-a^2x^2+1\right)^{\frac{3}{2}}}{4a^3}+\frac{3x\sqrt{-a^2x^2+1}}{8a^3}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a^3\sqrt{a^2}}+\frac{\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^7\left(x+\frac{1}{a}\right)^3}+\frac{5\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x\right.\right.}{a^6\left(x+\frac{1}{a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/4/a^3*x*(-a^2*x^2+1)^(3/2)+3/8*x*(-a^2*x^2+1)^(1/2)/a^3+3/8/a^3/(a^2)^(1/
2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^7/(x+1/a)^3*(-a^2*(x+1/a)^2
+2*a*(x+1/a))^(5/2)+5/a^6/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+4/a^
4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+6/a^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/
2)*x+6/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1
/2))
```

maxima [C] time = 0.43, size = 215, normalized size = 1.64

$$-\frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{a^{6}x^{2}+2\,a^{5}x+a^{4}}+\frac{3\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{2\left(a^{5}x+a^{4}\right)}+\frac{6\,\sqrt{-a^{2}x^{2}+1}}{a^{5}x+a^{4}}+\frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x}{4\,a^{3}}-\frac{3\,\sqrt{a^{2}x^{2}+4\,ax+3}\,x}{2\,a^{3}}+\frac{3\,\sqrt{-a^{2}x^{2}+1}\,x}{8\,a^{3}}-\frac{(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-a^2*x^2 + 1)^(3/2)/(a^6*x^2 + 2*a^5*x + a^4) + 3/2*(-a^2*x^2 + 1)^(3/2)/
(a^5*x + a^4) + 6*sqrt(-a^2*x^2 + 1)/(a^5*x + a^4) + 1/4*(-a^2*x^2 + 1)^(3/
2)*x/a^3 - 3/2*sqrt(a^2*x^2 + 4*a*x + 3)*x/a^3 + 3/8*sqrt(-a^2*x^2 + 1)*x/a
^3 - (-a^2*x^2 + 1)^(3/2)/a^4 + 3/2*I*arcsin(a*x + 2)/a^4 + 63/8*arcsin(a*x
)/a^4 - 3*sqrt(a^2*x^2 + 4*a*x + 3)/a^4 + 9/2*sqrt(-a^2*x^2 + 1)/a^4
```

mupad [B] time = 0.07, size = 154, normalized size = 1.18

$$\frac{51\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a^3\,\sqrt{-a^2}}+\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2}{{\left(-a^2\right)}^{3/2}}-\frac{4}{a^2\,\sqrt{-a^2}}-\frac{19\,x\,\sqrt{-a^2}}{8\,a^3}+\frac{a^2\,x^2}{{\left(-a^2\right)}^{3/2}}+\frac{x^3\,{\left(-a^2\right)}^{3/2}}{4\,a^3}\right)}{\sqrt{-a^2}}-\frac{4\,\sqrt{1-a^2\,x^2}}{a^3\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (51*asinh(x*(-a^2)^(1/2)))/(8*a^3*(-a^2)^(1/2)) + ((1 - a^2*x^2)^(1/2)*(2/(
-a^2)^(3/2) - 4/(a^2*(-a^2)^(1/2)) - (19*x*(-a^2)^(1/2))/(8*a^3) + (a^2*x^2
)/(-a^2)^(3/2) + (x^3*(-a^2)^(3/2))/(4*a^3)))/(-a^2)^(1/2) - (4*(1 - a^2*x^
2)^(1/2))/(a^3*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
[Out] Integral(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

3.52 $\int e^{-3\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=95

$$-\frac{11\sin^{-1}(ax)}{2a^3}-\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}}-\frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3}-\frac{(28-3ax)\sqrt{1-a^2x^2}}{6a^3}$$

```
[Out] -11/2*arcsin(a*x)/a^3-(-a*x+1)^3/a^3/(-a^2*x^2+1)^(1/2)-1/6*(-3*a*x+28)*(-a
^2*x^2+1)^(1/2)/a^3-1/3*(-a*x+3)^2*(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.64, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6124, 1633, 1593, 12, 852, 1635, 1654, 780, 216}

$$-\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}}-\frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3}-\frac{(28-3ax)\sqrt{1-a^2x^2}}{6a^3}-\frac{11\sin^{-1}(ax)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((1 - a*x)^3/(a^3*Sqrt[1 - a^2*x^2])) - ((28 - 3*a*x)*Sqrt[1 - a^2*x^2])/(
6*a^3) - ((3 - a*x)^2*Sqrt[1 - a^2*x^2])/(3*a^3) - (11*ArcSin[a*x])/(2*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
```

```
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}x^2\,dx &= \int \frac{x^2(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx\\
&= a\int \frac{\sqrt{1-a^2x^2}\left(\frac{x^2}{a}-x^3\right)}{(1+ax)^2}\,dx\\
&= a\int \frac{\left(\frac{1}{a}-x\right)x^2\sqrt{1-a^2x^2}}{(1+ax)^2}\,dx\\
&= a^2\int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{a^2(1+ax)^3}\,dx\\
&= \int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{(1+ax)^3}\,dx\\
&= \int \frac{x^2(1-ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx\\
&= -\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}}-\int \frac{\left(\frac{3}{a^2}-\frac{x}{a}\right)(1-ax)^2}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}}-\frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3}+\frac{1}{3}\int \frac{\left(\frac{3}{a^2}-\frac{x}{a}\right)(-5+3ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}}-\frac{(28-3ax)\sqrt{1-a^2x^2}}{6a^3}-\frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3}-\frac{11\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= -\frac{(1-ax)^3}{a^3\sqrt{1-a^2x^2}}-\frac{(28-3ax)\sqrt{1-a^2x^2}}{6a^3}-\frac{(3-ax)^2\sqrt{1-a^2x^2}}{3a^3}-\frac{11\sin^{-1}(ax)}{2a^3}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 58, normalized size = 0.61

$$
-\frac{\frac{\sqrt{1-a^2x^2}\left(2a^3x^3-7a^2x^2+19ax+52\right)}{ax+1}+33\sin^{-1}(ax)}{6a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/6*((Sqrt[1 - a^2*x^2]*(52 + 19*a*x - 7*a^2*x^2 + 2*a^3*x^3))/(1 + a*x) +
 33*ArcSin[a*x])/a^3
```

fricas [A] time = 0.44, size = 83, normalized size = 0.87

$$-\frac{52\,ax - 66\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(2\,a^3x^3 - 7\,a^2x^2 + 19\,ax + 52\right)\sqrt{-a^2x^2+1} + 52}{6\left(a^4x + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(52*a*x - 66*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3
*x^3 - 7*a^2*x^2 + 19*a*x + 52)*sqrt(-a^2*x^2 + 1) + 52)/(a^4*x + a^3)
```

giac [A] time = 0.64, size = 87, normalized size = 0.92

$$-\frac{1}{6}\sqrt{-a^2x^2+1}\left(x\left(\frac{2\,x}{a} - \frac{9}{a^2}\right) + \frac{28}{a^3}\right) - \frac{11\,\arcsin\left(ax\right)\operatorname{sgn}(a)}{2\,a^2|a|} + \frac{8}{a^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a^2*x^2 + 1)*(x*(2*x/a - 9/a^2) + 28/a^3) - 11/2*arcsin(a*x)*sgn
(a)/(a^2*abs(a)) + 8/(a^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs
(a))
```

maple [B] time = 0.05, size = 170, normalized size = 1.79

$$-\frac{\left(-a^2\left(x+\frac{1}{a}\right)^2 + 2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^6\left(x+\frac{1}{a}\right)^3} - \frac{4\left(-a^2\left(x+\frac{1}{a}\right)^2 + 2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^5\left(x+\frac{1}{a}\right)^2} - \frac{11\left(-a^2\left(x+\frac{1}{a}\right)^2 + 2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3a^3} - \frac{11\sqrt{-a^2\left(x\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/a^6/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-4/a^5/(x+1/a)^2*(-a^2*(
x+1/a)^2+2*a*(x+1/a))^(5/2)-11/3/a^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-11/
```

```
2/a^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-11/2/a^2/(a^2)^(1/2)*arctan((a^2
)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [C] time = 0.43, size = 177, normalized size = 1.86

$$\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^5x^2+2\,a^4x+a^3}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^4x+a^3}-\frac{6\,\sqrt{-a^2x^2+1}}{a^4x+a^3}+\frac{\sqrt{a^2x^2+4\,ax+3}\,x}{2\,a^2}+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3\,a^3}-\frac{i\,\arcsin\left(ax+2\right)}{2\,a^3}-\frac{6\,\mathrm{arcsi}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] (-a^2*x^2 + 1)^(3/2)/(a^5*x^2 + 2*a^4*x + a^3) - (-a^2*x^2 + 1)^(3/2)/(a^4*
x + a^3) - 6*sqrt(-a^2*x^2 + 1)/(a^4*x + a^3) + 1/2*sqrt(a^2*x^2 + 4*a*x +
3)*x/a^2 + 1/3*(-a^2*x^2 + 1)^(3/2)/a^3 - 1/2*I*arcsin(a*x + 2)/a^3 - 6*arc
sin(a*x)/a^3 + sqrt(a^2*x^2 + 4*a*x + 3)/a^3 - 3*sqrt(-a^2*x^2 + 1)/a^3
```

mupad [B] time = 0.05, size = 141, normalized size = 1.48

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2}{3\,a\,\sqrt{-a^2}}-\frac{4\,\sqrt{-a^2}}{a^3}+\frac{a\,x^2}{3\,\sqrt{-a^2}}+\frac{3\,x\,\sqrt{-a^2}}{2\,a^2}\right)}{\sqrt{-a^2}}-\frac{11\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^2\,\sqrt{-a^2}}+\frac{4\,\sqrt{1-a^2\,x^2}}{a^2\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(2/(3*a*(-a^2)^(1/2)) - (4*(-a^2)^(1/2))/a^3 + (a*x^2)
/(3*(-a^2)^(1/2)) + (3*x*(-a^2)^(1/2))/(2*a^2)))/(-a^2)^(1/2) - (11*asinh(x
*(-a^2)^(1/2)))/(2*a^2*(-a^2)^(1/2)) + (4*(1 - a^2*x^2)^(1/2))/(a^2*(x*(-a^
2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

3.53 $\int e^{-3\tanh^{-1}(ax)} x \, dx$

Optimal. Leaf size=86

$$\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(ax+1)^3}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(ax+1)}+\frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{9\sin^{-1}(ax)}{2a^2}$$

```
[Out] 3/2*(-a^2*x^2+1)^(3/2)/a^2/(a*x+1)+(-a^2*x^2+1)^(5/2)/a^2/(a*x+1)^3+9/2*arc
sin(a*x)/a^2+9/2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.36, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6124, 1633, 1593, 12, 793, 665, 216}

$$\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(ax+1)^3}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(ax+1)}+\frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{9\sin^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (9*Sqrt[1 - a^2*x^2])/(2*a^2) + (3*(1 - a^2*x^2)^(3/2))/(2*a^2*(1 + a*x)) +
 (1 - a^2*x^2)^(5/2)/(a^2*(1 + a*x)^3) + (9*ArcSin[a*x])/(2*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}x\,dx &= \int \frac{x(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx \\
&= a\int \frac{\left(\frac{x}{a}-x^2\right)\sqrt{1-a^2x^2}}{(1+ax)^2}\,dx \\
&= a\int \frac{\left(\frac{1}{a}-x\right)x\sqrt{1-a^2x^2}}{(1+ax)^2}\,dx \\
&= a^2\int \frac{x\left(1-a^2x^2\right)^{3/2}}{a^2(1+ax)^3}\,dx \\
&= \int \frac{x\left(1-a^2x^2\right)^{3/2}}{(1+ax)^3}\,dx \\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1+ax)^3}+\frac{3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(1+ax)^2}\,dx}{a} \\
&= \frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1+ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1+ax)^3}+\frac{9\int \frac{\sqrt{1-a^2x^2}}{1+ax}\,dx}{2a} \\
&= \frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1+ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1+ax)^3}+\frac{9\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a} \\
&= \frac{9\sqrt{1-a^2x^2}}{2a^2}+\frac{3\left(1-a^2x^2\right)^{3/2}}{2a^2(1+ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^2(1+ax)^3}+\frac{9\sin^{-1}(ax)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.51

$$
\frac{\sqrt{1-a^2x^2}\left(-ax+\frac{8}{ax+1}+6\right)+9\sin^{-1}(ax)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(3*ArcTanh[a*x]),x]
[Out] (Sqrt[1 - a^2*x^2]*(6 - a*x + 8/(1 + a*x)) + 9*ArcSin[a*x])/(2*a^2)
```

fricas [A] time = 0.56, size = 75, normalized size = 0.87

$$
\frac{14\,ax-18\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(a^2x^2-5\,ax-14\right)\sqrt{-a^2x^2+1}+14}{2\left(a^3x+a^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(14*a*x - 18*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a^2*x^
2 - 5*a*x - 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^3*x + a^2)
```

giac [A] time = 0.22, size = 78, normalized size = 0.91

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a}-\frac{6}{a^2}\right)+\frac{9\arcsin(ax)\operatorname{sgn}(a)}{2a|a|}-\frac{8}{a\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x/a - 6/a^2) + 9/2*arcsin(a*x)*sgn(a)/(a*abs(a)) -
 8/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [B] time = 0.04, size = 169, normalized size = 1.97

$$\frac{\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^5\left(x+\frac{1}{a}\right)^3}+\frac{3\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^4\left(x+\frac{1}{a}\right)^2}+\frac{3\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{a^2}+\frac{9\sqrt{-a^2\left(x+\frac{1}{a}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/a^5/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+3/a^4/(x+1/a)^2*(-a^2*(x
+1/a)^2+2*a*(x+1/a))^(5/2)+3/a^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+9/2/a*(
-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+9/2/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/
(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [A] time = 0.44, size = 110, normalized size = 1.28

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^4x^2+2a^3x+a^2}+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{2\left(a^3x+a^2\right)}+\frac{6\sqrt{-a^2x^2+1}}{a^3x+a^2}+\frac{9\arcsin(ax)}{2a^2}+\frac{3\sqrt{-a^2x^2+1}}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-a^2*x^2 + 1)^(3/2)/(a^4*x^2 + 2*a^3*x + a^2) + 1/2*(-a^2*x^2 + 1)^(3/2)/
(a^3*x + a^2) + 6*sqrt(-a^2*x^2 + 1)/(a^3*x + a^2) + 9/2*arcsin(a*x)/a^2 +
3/2*sqrt(-a^2*x^2 + 1)/a^2
```

mupad [B] time = 0.79, size = 101, normalized size = 1.17

$$-\frac{\left(\frac{3}{\sqrt{-a^2}}+\frac{x\sqrt{-a^2}}{2a}\right)\sqrt{1-a^2x^2}-\frac{9\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{2a}+\frac{4\sqrt{1-a^2x^2}}{a\left(x\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)}}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] -((3/(-a^2)^(1/2) + (x*(-a^2)^(1/2))/(2*a))*(1 - a^2*x^2)^(1/2) - (9*asinh(
x*(-a^2)^(1/2)))/(2*a) + (4*(1 - a^2*x^2)^(1/2))/(a*(x*(-a^2)^(1/2) + (-a^2
)^(1/2)/a)))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

3.54 $\int e^{-3\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=56

$$-\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a}-\frac{3\sin^{-1}(ax)}{a}$$

```
[Out] -3*arcsin(a*x)/a-2*(-a*x+1)^2/a/(-a^2*x^2+1)^(1/2)-3*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6123, 853, 669, 641, 216}

$$-\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a}-\frac{3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-3*ArcTanh[a*x]),x]
```

```
[Out] (-2*(1 - a*x)^2)/(a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[1 - a^2*x^2])/a - (3*ArcSi
n[a*x])/a
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
```

```
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 6123

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 + a*x)^((n + 1)/2)/(
(1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2]), x] /; FreeQ[a, x] && IntegerQ[(n
- 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\tanh^{-1}(ax)}\,dx &= \int \frac{(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx\\ &= \int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx\\ &= -\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}}-3\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a}-3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{2(1-ax)^2}{a\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a}-\frac{3\sin^{-1}(ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.70

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{4}{ax+1}-1\right)}{a}-\frac{3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-3*ArcTanh[a*x]),x]
[Out] (Sqrt[1 - a^2*x^2]*(-1 - 4/(1 + a*x)))/a - (3*ArcSin[a*x])/a
```

fricas [A] time = 0.40, size = 64, normalized size = 1.14

$$-\frac{5\,ax-6\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,(ax+5)+5}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(5*a*x - 6*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 5) + 5)/(a^2*x + a)
```

giac [A] time = 0.20, size = 64, normalized size = 1.14

$$-\frac{3\arcsin(ax)\operatorname{sgn}(a)}{|a|}-\frac{\sqrt{-a^2x^2+1}}{a}+\frac{8}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/abs(a) - sqrt(-a^2*x^2 + 1)/a + 8/(((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [B] time = 0.04, size = 164, normalized size = 2.93

$$-\frac{\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^4\left(x+\frac{1}{a}\right)^3}-\frac{2\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^3\left(x+\frac{1}{a}\right)^2}-\frac{2\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{a}-3\sqrt{-a^2\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-2/a^3/(x+1/a)^2*(-a^2*(
x+1/a)^2+2*a*(x+1/a))^(5/2)-2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-3*(-a^2*
(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+
1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [A] time = 0.43, size = 63, normalized size = 1.12

$$\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^3x^2+2\,a^2x+a}-\frac{3\arcsin(ax)}{a}-\frac{6\sqrt{-a^2x^2+1}}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] (-a^2*x^2 + 1)^(3/2)/(a^3*x^2 + 2*a^2*x + a) - 3*arcsin(a*x)/a - 6*sqrt(-a^
2*x^2 + 1)/(a^2*x + a)
```

mupad [B] time = 0.05, size = 81, normalized size = 1.45

$$\frac{4\sqrt{1-a^2x^2}}{\left(x\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{\sqrt{1-a^2x^2}}{a}-\frac{3\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/(a*x + 1)^3,x)
```

```
[Out] (4*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2)) -
(1 - a^2*x^2)^(1/2)/a - (3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

3.55 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=45

$$\frac{4\sqrt{1-a^2x^2}}{ax+1}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\sin^{-1}(ax)$$

```
[Out] arcsin(a*x)-arctanh((-a^2*x^2+1)^(1/2))+4/(a*x+1)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6124, 6742, 216, 266, 63, 208, 651}

$$\frac{4\sqrt{1-a^2x^2}}{ax+1}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 + a*x) + ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{(1-ax)^2}{x(1+ax)\sqrt{1-a^2x^2}}\,dx\\
&= \int \left(\frac{a}{\sqrt{1-a^2x^2}}+\frac{1}{x\sqrt{1-a^2x^2}}-\frac{4a}{(1+ax)\sqrt{1-a^2x^2}}\right)dx\\
&= a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx-(4a)\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{4\sqrt{1-a^2x^2}}{1+ax}+\sin^{-1}(ax)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= \frac{4\sqrt{1-a^2x^2}}{1+ax}+\sin^{-1}(ax)-\frac{\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\
&= \frac{4\sqrt{1-a^2x^2}}{1+ax}+\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 1.09

$$\frac{4\sqrt{1-a^2x^2}}{ax+1}-\log\left(\sqrt{1-a^2x^2}+1\right)+\sin^{-1}(ax)+\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 + a*x) + ArcSin[a*x] + Log[x] - Log[1 + Sqrt[1 - a
^2*x^2]]
```

fricas [A] time = 0.56, size = 82, normalized size = 1.82

$$\frac{4\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(ax+1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+4\sqrt{-a^2x^2+1}+4}{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (4*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x + 1)*log
((sqrt(-a^2*x^2 + 1) - 1)/x) + 4*sqrt(-a^2*x^2 + 1) + 4)/(a*x + 1)
```

giac [B] time = 0.22, size = 86, normalized size = 1.91

$$\frac{a\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{8\,a}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
2*a)/(a^2*abs(x)))/abs(a) - 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) +
 1)*abs(a))
```

maple [B] time = 0.05, size = 200, normalized size = 4.44

$$\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}+\sqrt{-a^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^3\left(x+\frac{1}{a}\right)^3}+\frac{\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right.\right.}{a^2\left(x+\frac{1}{a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] 1/3*(-a^2*x^2+1)^(3/2)+(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))+1/a
^3/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+1/a^2/(x+1/a)^2*(-a^2*(x+1/
a)^2+2*a*(x+1/a))^(5/2)+2/3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+a*(-a^2*(x+1
/a)^2+2*a*(x+1/a))^(1/2)*x+a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)
^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x), x)
```

mupad [B] time = 0.80, size = 80, normalized size = 1.78

$$\frac{a\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)-\frac{4\,a\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/(x*(a*x + 1)^3),x)
```

```
[Out] (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - atanh((1 - a^2*x^2)^(1/2)) - (4*a*
(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x*(a*x + 1)**3), x)
```

3.56 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=62

$$-\frac{4a\sqrt{1-a^2x^2}}{ax+1}-\frac{\sqrt{1-a^2x^2}}{x}+3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] 3*a*arctanh((-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/x-4*a*(-a^2*x^2+1)^(1/2)
/(a*x+1)
```

Rubi [A] time = 0.69, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6124, 6742, 264, 266, 63, 208, 651}

$$-\frac{4a\sqrt{1-a^2x^2}}{ax+1}-\frac{\sqrt{1-a^2x^2}}{x}+3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/x) - (4*a*Sqrt[1 - a^2*x^2])/(1 + a*x) + 3*a*ArcTanh[Sq
rt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-ax)^2}{x^2(1+ax)\sqrt{1-a^2x^2}}\,dx\\
&= \int \left(\frac{1}{x^2\sqrt{1-a^2x^2}}-\frac{3a}{x\sqrt{1-a^2x^2}}+\frac{4a^2}{(1+ax)\sqrt{1-a^2x^2}}\right)dx\\
&= -\left((3a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\right)+\left(4a^2\right)\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx+\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{x}-\frac{4a\sqrt{1-a^2x^2}}{1+ax}-\frac{1}{2}(3a)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{x}-\frac{4a\sqrt{1-a^2x^2}}{1+ax}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a}\\
&= -\frac{\sqrt{1-a^2x^2}}{x}-\frac{4a\sqrt{1-a^2x^2}}{1+ax}+3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.92

$$\sqrt{1-a^2x^2}\left(-\frac{4a}{ax+1}-\frac{1}{x}\right)+3a\log\left(\sqrt{1-a^2x^2}+1\right)-3a\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(-x^(-1) - (4*a)/(1 + a*x)) - 3*a*Log[x] + 3*a*Log[1 + Sq
rt[1 - a^2*x^2]]
```

fricas [A] time = 0.43, size = 75, normalized size = 1.21

$$-\frac{4\,a^2x^2+4\,ax+3\left(a^2x^2+ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\,(5\,ax+1)}{ax^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(4*a^2*x^2 + 4*a*x + 3*(a^2*x^2 + a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) + s
qrt(-a^2*x^2 + 1)*(5*a*x + 1))/(a*x^2 + x)
```

giac [B] time = 0.20, size = 150, normalized size = 2.42

$$\frac{3\,a^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\left(a^2+\frac{17\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a)
+ 1/2*(a^2 + 17*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x)*a^2*x/((sqrt(-a^2*x^2 +
1)*abs(a) + a)*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a)) - 1/2*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a))
```

maple [B] time = 0.05, size = 261, normalized size = 4.21

$$-a\left(-a^2x^2+1\right)^{\frac{3}{2}}-3a\sqrt{-a^2x^2+1}+3a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}}{x}-a^2x\left(-a^2x^2+1\right)^{\frac{3}{2}}-\frac{3a^2x\sqrt{-a^2x^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] -a*(-a^2*x^2+1)^(3/2)-3*a*(-a^2*x^2+1)^(1/2)+3*a*arctanh(1/(-a^2*x^2+1)^(1/
2))-1/x*(-a^2*x^2+1)^(5/2)-a^2*x*(-a^2*x^2+1)^(3/2)-3/2*a^2*x*(-a^2*x^2+1)^
(1/2)-3/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/a^2/(x
+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(
3/2)+3/2*a^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+3/2*a^2/(a^2)^(1/2)*arcta
n((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^2), x)
```

mupad [B] time = 0.80, size = 81, normalized size = 1.31

$$3\,a\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)-\frac{\sqrt{1-a^2\,x^2}}{x}+\frac{4\,a^2\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/(x^2*(a*x + 1)^3),x)
```

```
[Out] 3*a*atanh((1 - a^2*x^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/x + (4*a^2*(1 - a^2*x^
2)^(1/2))/((x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{x^2(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x**2*(a*x + 1)**3), x)
```

3.57 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=90

$$\frac{4a^2\sqrt{1-a^2x^2}}{ax+1}+\frac{3a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -9/2*a^2*arctanh((-a^2*x^2+1)^(1/2))-1/2*(-a^2*x^2+1)^(1/2)/x^2+3*a*(-a^2*x
^2+1)^(1/2)/x+4*a^2*(-a^2*x^2+1)^(1/2)/(a*x+1)
```

Rubi [A] time = 0.74, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6124, 6742, 266, 51, 63, 208, 264, 651}

$$\frac{4a^2\sqrt{1-a^2x^2}}{ax+1}+\frac{3a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2x^2}-\frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2) + (3*a*Sqrt[1 - a^2*x^2])/x + (4*a^2*Sqrt[1 - a^
2*x^2])/(1 + a*x) - (9*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{(1-ax)^2}{x^3(1+ax)\sqrt{1-a^2x^2}}\,dx$$
$$= \int \left(\frac{1}{x^3\sqrt{1-a^2x^2}} - \frac{3a}{x^2\sqrt{1-a^2x^2}} + \frac{4a^2}{x\sqrt{1-a^2x^2}} - \frac{4a^3}{(1+ax)\sqrt{1-a^2x^2}}\right)dx$$
$$= -\left((3a)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx\right) + \left(4a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(4a^3\right)\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx + \int$$
$$= \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1+ax} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) + \left(2a^2\right)\text{Subst}\left(\int \frac{}{x\sqrt{1}}\right.$$
$$= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1+ax} - 4\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) + \frac{1}{4}a$$
$$= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1+ax} - 4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-}\right.$$
$$= -\frac{\sqrt{1-a^2x^2}}{2x^2} + \frac{3a\sqrt{1-a^2x^2}}{x} + \frac{4a^2\sqrt{1-a^2x^2}}{1+ax} - \frac{9}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.83

$$\sqrt{1-a^2x^2}\left(\frac{4a^2}{ax+1} + \frac{3a}{x} - \frac{1}{2x^2}\right) - \frac{9}{2}a^2\log\left(\sqrt{1-a^2x^2}+1\right) + \frac{9}{2}a^2\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] Sqrt[1 - a^2*x^2]*(-1/2*1/x^2 + (3*a)/x + (4*a^2)/(1 + a*x)) + (9*a^2*Log[x
])/2 - (9*a^2*Log[1 + Sqrt[1 - a^2*x^2]])/2
```

fricas [A] time = 0.47, size = 93, normalized size = 1.03

$$\frac{8\,a^3x^3 + 8\,a^2x^2 + 9\left(a^3x^3 + a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + \left(14\,a^2x^2 + 5\,ax - 1\right)\sqrt{-a^2x^2+1}}{2\left(ax^3 + x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(8*a^3*x^3 + 8*a^2*x^2 + 9*(a^3*x^3 + a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) + (14*a^2*x^2 + 5*a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 + x^2)
```

giac [B] time = 0.22, size = 214, normalized size = 2.38

$$\frac{\left(a^3 - \frac{11\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x} - \frac{76\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|} - \frac{9\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} + \frac{\frac{12\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a|a|}{x} - \frac{\left(\sqrt{-a^2x^2+1}\right.}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3 - 11*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 76*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a
^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a)) - 9/2*a^3*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/8*(12*(sqrt(-a^2*x^2 +
 1)*abs(a) + a)*a*abs(a)/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^
2))/a^2
```

maple [B] time = 0.05, size = 319, normalized size = 3.54

$$\frac{3a^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{2} + \frac{9a^2\sqrt{-a^2x^2+1}}{2} - \frac{9a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2} + \frac{3a\left(-a^2x^2+1\right)^{\frac{5}{2}}}{x} + 3a^3x\left(-a^2x^2+1\right)^{\frac{3}{2}} + \frac{9a^3x\sqrt{-a^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] 3/2*a^2*(-a^2*x^2+1)^(3/2)+9/2*a^2*(-a^2*x^2+1)^(1/2)-9/2*a^2*arctanh(1/(-a
^2*x^2+1)^(1/2))+3*a/x*(-a^2*x^2+1)^(5/2)+3*a^3*x*(-a^2*x^2+1)^(3/2)+9/2*a^
3*x*(-a^2*x^2+1)^(1/2)+9/2*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1
)^(1/2))-1/2/x^2*(-a^2*x^2+1)^(5/2)+1/a/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/
a))^(5/2)-1/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-3*a^2*(-a^2*(x+1/a
)^2+2*a*(x+1/a))^(3/2)-9/2*a^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-9/2*a^3
/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^3), x)
```

mupad [B] time = 0.81, size = 105, normalized size = 1.17

$$\frac{3\,a\,\sqrt{1-a^2\,x^2}}{x}-\frac{\sqrt{1-a^2\,x^2}}{2\,x^2}-\frac{4\,a^3\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}+\frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1{}\mathrm{i}\right)\,9{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/(x^3*(a*x + 1)^3),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*9i)/2 - (1 - a^2*x^2)^(1/2)/(2*x^2) + (3*
a*(1 - a^2*x^2)^(1/2))/x - (4*a^3*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) + (
-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x^3\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x**3*(a*x + 1)**3), x)
```

3.58 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=116

$$-\frac{14a^2\sqrt{1-a^2x^2}}{3x}+\frac{3a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}-\frac{4a^3\sqrt{1-a^2x^2}}{ax+1}+\frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] 11/2*a^3*arctanh((-a^2*x^2+1)^(1/2))-1/3*(-a^2*x^2+1)^(1/2)/x^3+3/2*a*(-a^2
*x^2+1)^(1/2)/x^2-14/3*a^2*(-a^2*x^2+1)^(1/2)/x-4*a^3*(-a^2*x^2+1)^(1/2)/(a
*x+1)
```

Rubi [A] time = 0.75, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6124, 6742, 271, 264, 266, 51, 63, 208, 651}

$$-\frac{4a^3\sqrt{1-a^2x^2}}{ax+1}-\frac{14a^2\sqrt{1-a^2x^2}}{3x}+\frac{3a\sqrt{1-a^2x^2}}{2x^2}-\frac{\sqrt{1-a^2x^2}}{3x^3}+\frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3) + (3*a*Sqrt[1 - a^2*x^2])/(2*x^2) - (14*a^2*Sqrt
[1 - a^2*x^2])/(3*x) - (4*a^3*Sqrt[1 - a^2*x^2])/(1 + a*x) + (11*a^3*ArcTan
h[Sqrt[1 - a^2*x^2]])/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-ax)^2}{x^4(1+ax)\sqrt{1-a^2x^2}}\,dx$$
$$= \int \left(\frac{1}{x^4\sqrt{1-a^2x^2}} - \frac{3a}{x^3\sqrt{1-a^2x^2}} + \frac{4a^2}{x^2\sqrt{1-a^2x^2}} - \frac{4a^3}{x\sqrt{1-a^2x^2}} + \frac{4a^4}{(1+ax)\sqrt{1-a^2x^2}}\right) dx$$
$$= -\left((3a)\int \frac{1}{x^3\sqrt{1-a^2x^2}}\,dx\right) + \left(4a^2\right)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx - \left(4a^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx + \left(4a^4\right)$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3} - \frac{4a^2\sqrt{1-a^2x^2}}{x} - \frac{4a^3\sqrt{1-a^2x^2}}{1+ax} - \frac{1}{2}(3a)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) +$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} - \frac{4a^3\sqrt{1-a^2x^2}}{1+ax} + (4a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\right.$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} - \frac{4a^3\sqrt{1-a^2x^2}}{1+ax} + 4a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3} + \frac{3a\sqrt{1-a^2x^2}}{2x^2} - \frac{14a^2\sqrt{1-a^2x^2}}{3x} - \frac{4a^3\sqrt{1-a^2x^2}}{1+ax} + \frac{11}{2}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right.$$

Mathematica [A] time = 0.08, size = 82, normalized size = 0.71

$$\frac{1}{6}\left(-33a^3\log(x) + 33a^3\log\left(\sqrt{1-a^2x^2}+1\right) - \frac{\sqrt{1-a^2x^2}\left(52a^3x^3+19a^2x^2-7ax+2\right)}{x^3(ax+1)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (-((Sqrt[1 - a^2*x^2]*(2 - 7*a*x + 19*a^2*x^2 + 52*a^3*x^3))/(x^3*(1 + a*x)
)) - 33*a^3*Log[x] + 33*a^3*Log[1 + Sqrt[1 - a^2*x^2]])/6
```

fricas [A] time = 0.47, size = 101, normalized size = 0.87

$$-\frac{24\,a^4x^4 + 24\,a^3x^3 + 33\left(a^4x^4+a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + \left(52\,a^3x^3+19\,a^2x^2-7\,ax+2\right)\sqrt{-a^2x^2+1}}{6\left(ax^4+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(24*a^4*x^4 + 24*a^3*x^3 + 33*(a^4*x^4 + a^3*x^3)*log((sqrt(-a^2*x^2 +
 1) - 1)/x) + (52*a^3*x^3 + 19*a^2*x^2 - 7*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a*
x^4 + x^3)
```

giac [B] time = 0.20, size = 265, normalized size = 2.28

$$\frac{\left(a^4 - \frac{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2}{x} + \frac{48\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{x^2} + \frac{249\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^2x^3}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|} + \frac{11\,a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|} - \frac{57\left(\sqrt{-a^2x^2+\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 48*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^2/x^2 + 249*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^2*x^3))*a^6*
x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^
2*x) + 1)*abs(a)) + 11/2*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a
)/(a^2*abs(x)))/abs(a) - 1/24*(57*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x - 9
*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) + a
)^3/x^3)/(a^2*abs(a))
```

maple [B] time = 0.06, size = 338, normalized size = 2.91

$$-\frac{11a^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{6} - \frac{11a^3\sqrt{-a^2x^2+1}}{2} + \frac{11a^3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2} - \frac{16a^2\left(-a^2x^2+1\right)^{\frac{5}{2}}}{3x} - \frac{16a^4x\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3} - 8a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] -11/6*a^3*(-a^2*x^2+1)^(3/2)-11/2*a^3*(-a^2*x^2+1)^(1/2)+11/2*a^3*arctanh(1
/(-a^2*x^2+1)^(1/2))-16/3*a^2/x*(-a^2*x^2+1)^(5/2)-16/3*a^4*x*(-a^2*x^2+1)^
(3/2)-8*a^4*x*(-a^2*x^2+1)^(1/2)-8*a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2))-1/3/x^3*(-a^2*x^2+1)^(5/2)+3/2*a/x^2*(-a^2*x^2+1)^(5/2)-1/
(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+2*a/(x+1/a)^2*(-a^2*(x+1/a)^2+
2*a*(x+1/a))^(5/2)+16/3*a^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+8*a^4*(-a^2*
(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+8*a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^4), x)
```

mupad [B] time = 0.05, size = 125, normalized size = 1.08

$$\frac{3\,a\,\sqrt{1-a^2\,x^2}}{2\,x^2}-\frac{\sqrt{1-a^2\,x^2}}{3\,x^3}-\frac{14\,a^2\,\sqrt{1-a^2\,x^2}}{3\,x}+\frac{4\,a^4\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{a^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,11\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/(x^4*(a*x + 1)^3),x)
```

```
[Out] (3*a*(1 - a^2*x^2)^(1/2))/(2*x^2) - (1 - a^2*x^2)^(1/2)/(3*x^3) - (a^3*atan
((1 - a^2*x^2)^(1/2)*1i)*11i)/2 - (14*a^2*(1 - a^2*x^2)^(1/2))/(3*x) + (4*a
^4*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x^4\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x**4*(a*x + 1)**3), x)
```

3.59 $\int \frac{e^{-3\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=135

$$-\frac{19a^2\sqrt{1-a^2x^2}}{8x^2}-\frac{\sqrt{1-a^2x^2}}{4x^4}+\frac{a\sqrt{1-a^2x^2}}{x^3}+\frac{4a^4\sqrt{1-a^2x^2}}{ax+1}-\frac{51}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{6a^3\sqrt{1-a^2x^2}}{x}$$

```
[Out] -51/8*a^4*arctanh((-a^2*x^2+1)^(1/2))-1/4*(-a^2*x^2+1)^(1/2)/x^4+a*(-a^2*x^
2+1)^(1/2)/x^3-19/8*a^2*(-a^2*x^2+1)^(1/2)/x^2+6*a^3*(-a^2*x^2+1)^(1/2)/x+4
*a^4*(-a^2*x^2+1)^(1/2)/(a*x+1)
```

Rubi [A] time = 0.82, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6124, 6742, 266, 51, 63, 208, 271, 264, 651}

$$\frac{4a^4\sqrt{1-a^2x^2}}{ax+1}+\frac{6a^3\sqrt{1-a^2x^2}}{x}-\frac{19a^2\sqrt{1-a^2x^2}}{8x^2}+\frac{a\sqrt{1-a^2x^2}}{x^3}-\frac{\sqrt{1-a^2x^2}}{4x^4}-\frac{51}{8}a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(4*x^4) + (a*Sqrt[1 - a^2*x^2])/x^3 - (19*a^2*Sqrt[1 - a
^2*x^2])/(8*x^2) + (6*a^3*Sqrt[1 - a^2*x^2])/x + (4*a^4*Sqrt[1 - a^2*x^2])/
(1 + a*x) - (51*a^4*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{(1-ax)^2}{x^5(1+ax)\sqrt{1-a^2x^2}}\,dx$$

$$= \int \left(\frac{1}{x^5\sqrt{1-a^2x^2}} - \frac{3a}{x^4\sqrt{1-a^2x^2}} + \frac{4a^2}{x^3\sqrt{1-a^2x^2}} - \frac{4a^3}{x^2\sqrt{1-a^2x^2}} + \frac{4a^4}{x\sqrt{1-a^2x^2}} - \frac{4}{(1+ax)}\right.$$

$$= -\left((3a)\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,dx\right) + \left(4a^2\right)\int \frac{1}{x^3\sqrt{1-a^2x^2}}\,dx - \left(4a^3\right)\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx + \left(4a^4\right)$$

$$= \frac{a\sqrt{1-a^2x^2}}{x^3} + \frac{4a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-a^2x}}\,dx, x, x^2\right) + \left(2a^2\right)$$

$$= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{x^3} - \frac{2a^2\sqrt{1-a^2x^2}}{x^2} + \frac{6a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} + \frac{1}{8}\left(3a^2\right)\text{Su}$$

$$= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{x^3} - \frac{19a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{6a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} - 4a^4\tanh$$

$$= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{x^3} - \frac{19a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{6a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} - 6a^4\tanh$$

$$= -\frac{\sqrt{1-a^2x^2}}{4x^4} + \frac{a\sqrt{1-a^2x^2}}{x^3} - \frac{19a^2\sqrt{1-a^2x^2}}{8x^2} + \frac{6a^3\sqrt{1-a^2x^2}}{x} + \frac{4a^4\sqrt{1-a^2x^2}}{1+ax} - \frac{51}{8}a^4\tan$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.66

$$\frac{1}{8}\left(51a^4\log(x) - 51a^4\log\left(\sqrt{1-a^2x^2}+1\right) + \frac{\sqrt{1-a^2x^2}\left(80a^4x^4+29a^3x^3-11a^2x^2+6ax-2\right)}{x^4(ax+1)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-2 + 6*a*x - 11*a^2*x^2 + 29*a^3*x^3 + 80*a^4*x^4))/(x
^4*(1 + a*x)) + 51*a^4*Log[x] - 51*a^4*Log[1 + Sqrt[1 - a^2*x^2]])/8
```

fricas [A] time = 0.51, size = 109, normalized size = 0.81

$$\frac{32\,a^5x^5 + 32\,a^4x^4 + 51\left(a^5x^5 + a^4x^4\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + \left(80\,a^4x^4 + 29\,a^3x^3 - 11\,a^2x^2 + 6\,ax - 2\right)\sqrt{-a^2x^2+1}}{8\left(ax^5 + x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(32*a^5*x^5 + 32*a^4*x^4 + 51*(a^5*x^5 + a^4*x^4)*log((sqrt(-a^2*x^2 +
1) - 1)/x) + (80*a^4*x^4 + 29*a^3*x^3 - 11*a^2*x^2 + 6*a*x - 2)*sqrt(-a^2*x
^2 + 1))/(a*x^5 + x^4)
```

giac [B] time = 0.19, size = 326, normalized size = 2.41

$$\frac{\left(a^5 - \frac{7\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^3}{x} + \frac{32\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2 a}{x^2} - \frac{160\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{ax^3} - \frac{712\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^3x^4}\right)a^8x^4}{64\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|} - \frac{51\,a^5\log\left(\frac{\left|-2\sqrt{-a^2x}\right.}{2}\right.}{8\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/64*(a^5 - 7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3/x + 32*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^2*a/x^2 - 160*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a*x^3) - 712
*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^3*x^4))*a^8*x^4/((sqrt(-a^2*x^2 + 1)*
abs(a) + a)^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a)) - 51/8*
a^5*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) +
1/64*(200*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*abs(a)/x - 40*(sqrt(-a^2*x^2
+ 1)*abs(a) + a)^2*a^3*abs(a)/x^2 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*a
bs(a)/x^3 - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)/(a*x^4))/a^4
```

maple [B] time = 0.07, size = 359, normalized size = 2.66

$$-\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}}{4x^4}+\frac{17a^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}{8}-8a^4\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}+\frac{51a^4\sqrt{-a^2x^2+1}}{8}-\frac{51a^4\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x}}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] -1/4/x^4*(-a^2*x^2+1)^(5/2)+17/8*a^4*(-a^2*x^2+1)^(3/2)-8*a^4*(-a^2*(x+1/a)
^2+2*a*(x+1/a))^(3/2)+51/8*a^4*(-a^2*x^2+1)^(1/2)-51/8*a^4*arctanh(1/(-a^2*
x^2+1)^(1/2))+a/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-3*a^2/(x+1/a)^
2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-12*a^5*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1
/2)*x+8*a^3/x*(-a^2*x^2+1)^(5/2)+8*a^5*x*(-a^2*x^2+1)^(3/2)+a/x^3*(-a^2*x^2
```

```
+1)^(5/2)-23/8*a^2/x^2*(-a^2*x^2+1)^(5/2)-12*a^5/(a^2)^(1/2)*arctan((a^2)^(
1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+12*a^5*x*(-a^2*x^2+1)^(1/2)+12*a
^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^5), x)
```

mupad [B] time = 0.05, size = 144, normalized size = 1.07

$$\frac{a\,\sqrt{1-a^2\,x^2}}{x^3}-\frac{\sqrt{1-a^2\,x^2}}{4\,x^4}-\frac{19\,a^2\,\sqrt{1-a^2\,x^2}}{8\,x^2}+\frac{6\,a^3\,\sqrt{1-a^2\,x^2}}{x}-\frac{4\,a^5\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}+\frac{a^4\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/(x^5*(a*x + 1)^3),x)
```

```
[Out] (a^4*atan((1 - a^2*x^2)^(1/2)*1i)*51i)/8 - (1 - a^2*x^2)^(1/2)/(4*x^4) + (a
*(1 - a^2*x^2)^(1/2))/x^3 - (19*a^2*(1 - a^2*x^2)^(1/2))/(8*x^2) + (6*a^3*(
1 - a^2*x^2)^(1/2))/x - (4*a^5*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) + (-a^
2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x^5\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(x**5*(a*x + 1)**3), x)
```

3.60 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{4},-\frac{1}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/4,-1/4,2+m,a*x,-a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{4},-\frac{1}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/4, -1/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{1}{4},-\frac{1}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.38, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

[In] Integrate[E^(ArcTanh[a*x]/2)*x^m,x]

[Out] Integrate[E^(ArcTanh[a*x]/2)*x^m, x]

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="fricas")

[Out] integral(x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="giac")

[Out] integrate(x^m*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x)

[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
[Out] int(x^m*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x***m,x)
[Out] Integral(x**m*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

3.61 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=282

$$-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{12a^3}-\frac{3(1-ax)^{3/4}\sqrt[4]{ax+1}}{8a^3}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}+\frac{3\text{ ta}}{}$$

```
[Out] -3/8*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a^3-1/12*(-a*x+1)^(3/4)*(a*x+1)^(5/4)/a^3
-1/3*x*(-a*x+1)^(3/4)*(a*x+1)^(5/4)/a^2-3/16*arctan(-1+(-a*x+1)^(1/4)*2^(1/
2)/(a*x+1)^(1/4))/a^3*2^(1/2)-3/16*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^
(1/4))/a^3*2^(1/2)-3/32*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^
(1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)+3/32*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(
1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x(1-ax)^{3/4}(ax+1)^{5/4}}{3a^2}-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{12a^3}-\frac{3(1-ax)^{3/4}\sqrt[4]{ax+1}}{8a^3}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}+\frac{3\log\left(\frac{\sqrt{1-}}{\sqrt{ax-}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*x^2,x]
```

```
[Out] (-3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(8*a^3) - ((1 - a*x)^(3/4)*(1 + a*x)^(
5/4))/(12*a^3) - (x*(1 - a*x)^(3/4)*(1 + a*x)^(5/4))/(3*a^2) + (3*ArcTan[1
- (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (3*ArcTan[1
 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (3*Log[1 +
 Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/
(16*Sqrt[2]*a^3) + (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a
*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx$$

$$= -\frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{\int \frac{\left(-1-\frac{ax}{2}\right)\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{3a^2}$$

$$= -\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \frac{3\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{8a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \frac{3\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{\cdots}}}{16a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{3\,\mathrm{Subst}\left(\int \frac{x}{(2-x\cdots}\right.}{\cdots}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{3\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\right.}{4a\cdots}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \frac{3\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\right.}{8a\cdots}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{3\,\mathrm{Subst}\left(\int \frac{}{1-\sqrt{\cdots}}\right.}{1\cdots}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} - \frac{3\log\left(1+\frac{\sqrt{1-a\cdots}}{\sqrt{1+a\cdots}}\right.}{16\sqrt{\cdots}}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\sqrt[4]{\cdots}}\right.}{8\sqrt{2}\,a^3}$$

Mathematica [C] time = 0.04, size = 69, normalized size = 0.24

$$-\frac{(1-ax)^{3/4}\left(\sqrt[4]{ax+1}\left(4a^2x^2+5ax+1\right)+6\sqrt[4]{2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right)\right)}{12a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*x^2,x]
```

[Out] -1/12*((1 - a*x)^(3/4)*((1 + a*x)^(1/4)*(1 + 5*a*x + 4*a^2*x^2) + 6*2^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (1 - a*x)/2]))/a^3

fricas [B] time = 0.68, size = 544, normalized size = 1.93

$$-\frac{36\sqrt{2}\,a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^9\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-\sqrt{2}\,a^9\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="fricas")

[Out] -1/96*(36*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt((sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)*a^9*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - 1) + 36*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)*a^9*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + 1) - 9*sqrt(2)*a^3*(a^(-12))^(1/4)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 9*sqrt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(8*a^3*x^3 + 2*a^2*x^2 + a*x - 11)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^3

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="giac")

[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**2,x)
```

```
[Out] Integral(x**2*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

3.62 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=255

$$-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{2a^2}-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{4a^2}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}+\frac{\tan^{-1}\left(\ldots\right)}{4\ldots}$$

```
[Out] -1/4*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a^2-1/2*(-a*x+1)^(3/4)*(a*x+1)^(5/4)/a^2-
1/8*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^2*2^(1/2)-1/8*arctan(
1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^2*2^(1/2)-1/16*ln(1-(-a*x+1)^(1/4
)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(1/2)+1/16*ln(1
+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{2a^2}-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{4a^2}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}+\frac{\tan^{-1}\left(\ldots\right)}{4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*x,x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*a^2) - ((1 - a*x)^(3/4)*(1 + a*x)^(5/
4))/(2*a^2) + ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt
[2]*a^2) - ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt[2]
*a^2) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/(8*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt
[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(8*Sqrt[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{4a}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx}{8a}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2} - \frac{\text{Subst}\left(\int \frac{1+\cdots}{1+\cdots}\right.}{\cdots}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a^2} - \frac{\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}a^2} + \frac{\log\left(1+\frac{\sqrt{1\cdots}}{\sqrt{1\cdots}}\right.}{8\cdots}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}a^2} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-a\cdots}}{\sqrt[4]{1+ax}}\right.}{4\sqrt{2}a^2}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.22

$$-\frac{(1-ax)^{3/4}\left(2\sqrt[4]{2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right)+3(ax+1)^{5/4}\right)}{6a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*x,x]
```

```
[Out] -1/6*((1 - a*x)^(3/4)*(3*(1 + a*x)^(5/4) + 2*2^(1/4)*Hypergeometric2F1[-1/4
, 3/4, 7/4, (1 - a*x)/2]))/a^2
```

fricas [B] time = 0.50, size = 535, normalized size = 2.10

$$-\frac{4\sqrt{2}\,a^2\left(\frac{1}{a^8}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^6\sqrt{\frac{\sqrt{2}\left(a^3x-a^2\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{1}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}-\sqrt{2}\,a^6\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}-1\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt((sqrt(2)*(a^3*x
 - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*x - a^4)*
sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - 1) + 4*sqrt(2)*a^2*(a^
(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^
2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*sqrt(-sqrt(-a^2*x^2 + 1
)/(a*x - 1))*(a^(-8))^(3/4) + 1) - sqrt(2)*a^2*(a^(-8))^(1/4)*log((sqrt(2)*
(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*x -
 a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + sqrt(2)*a^2*(a^(-8))^
(1/4)*log(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-
8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4
*(2*a^2*x^2 + a*x - 3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="maxima")
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
[Out] int(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x,x)
[Out] Integral(x*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

3.63 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=222

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}\,a}-\frac{\tan^{-1}\left(\right.}{}$$

```
[Out] -(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a-1/2*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1
)^(1/4))/a*2^(1/2)-1/2*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(
1/2)-1/4*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a*2^(1/2)+1/4*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2
)/(a*x+1)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6125, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}\,a}-\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2),x]
```

```
[Out] -(((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/a) + ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)]/(Sqrt[2]*a) - ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
 a*x)^(1/4)]/(Sqrt[2]*a) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(
1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a) + Log[1 + Sqrt[1 - a*x]/Sqrt
[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\,dx = \int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{1}{2}\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a} - \text{Subst}($$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}}{}\right.}{2\sqrt{2}\,a}$$

Mathematica [A] time = 0.23, size = 149, normalized size = 0.67

$$\frac{-\frac{8e^{\frac{1}{2}\tanh^{-1}(ax)}}{e^{2\tanh^{-1}(ax)}+1}-\sqrt{2}\log\left(-\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)}+e^{\tanh^{-1}(ax)}+1\right)+\sqrt{2}\log\left(\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)}+e^{\tanh^{-1}(ax)}+1\right)-2\sqrt{2}\tan\cdots}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2),x]
```

```
[Out] ((-8*E^(ArcTanh[a*x]/2))/(1 + E^(2*ArcTanh[a*x])) - 2*Sqrt[2]*ArcTan[1 - Sq
rt[2]*E^(ArcTanh[a*x]/2)] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcTanh[a*x]/2)
] - Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]] + Sqrt[2]*
Log[1 + Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]])/(4*a)
```

fricas [B] time = 0.63, size = 511, normalized size = 2.30

$$-\frac{4\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}a^3\sqrt{\frac{\sqrt{2}\left(a^2x-a\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-1\right)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^2*x -
a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(
a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - 1) + 4*sqrt(2)*a*(a^(-4))^(
1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1
))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
1))*(a^(-4))^(3/4) + 1) - sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^2*x - a)
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(a^
(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + sqrt(2)*a*(a^(-4))^(1/4)*log(-(sqr
t(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - (a^3*
x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(a*x - 1)*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

3.64 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=227

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)-$$

```
[Out] -2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-2*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1
/4))-1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(
1/2))*2^(1/2)+1/2*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/
(a*x+1)^(1/2))*2^(1/2)-arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1
/2)-arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6126, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x,x]
```

```
[Out] -2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
 - a*x)^(1/4))/(1 + a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/
4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Log[1 +
 Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/S
qrt[2] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
 + a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{\sqrt[4]{1+ax}}{x\sqrt[4]{1-ax}}\,dx \\
&= a\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx \\
&= -\left(4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)\right) + 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 4\,\text{Subst}\left(\int \frac{}{1}\right. \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\,\text{Subst} \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int\right.}{} \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.}{\sqrt{2}} \\
&= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\tanh
\end{aligned}
$$

<u>Mathematica</u> [C] time = 0.03, size = 83, normalized size = 0.37

$$
-\frac{2(1-ax)^{3/4}\left(\sqrt[4]{2}\,(ax+1)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right) + 2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)\right)}{3(ax+1)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x,x]
```

```
[Out] (-2*(1 - a*x)^(3/4)*(2^(1/4)*(1 + a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/
4, (1 - a*x)/2] + 2*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(
3*(1 + a*x)^(3/4))
```

fricas [B] time = 0.56, size = 417, normalized size = 1.84

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}\,(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\sqrt{2}\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(
-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2
)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(
a*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 1/2*sqrt(2)*
log(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-
a^2*x^2 + 1) - 1)/(a*x - 1)) - 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - 2
*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1)) + 1) + log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x,x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x,x)
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x, x)
```

3.65 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=73

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{x} - a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right) - a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] -(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x-a*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-a*ar
ctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 212, 206, 203}

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{x} - a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right) - a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^2,x]
```

```
[Out] -(((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/x) - a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)
^(1/4)] - a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{\sqrt[4]{1+ax}}{x^2\sqrt[4]{1-ax}}\,dx \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x}+\frac{1}{2}a\int \frac{1}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x}+(2a)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x}-a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[4]{1+a}}{\sqrt[4]{1-a}}\right. \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x}-a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.79

$$-\frac{(1-ax)^{3/4}\left(2ax\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)+3ax+3\right)}{3x(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^2,x]
```

```
[Out] -1/3*((1 - a*x)^(3/4)*(3 + 3*a*x + 2*a*x*Hypergeometric2F1[3/4, 1, 7/4, (1
- a*x)/(1 + a*x)]))/(x*(1 + a*x)^(3/4))
```

fricas [B] time = 0.51, size = 123, normalized size = 1.68

$$-\frac{2\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\,(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + a*x*log(sqrt(-sqr
t(-a^2*x^2 + 1)/(a*x - 1)) + 1) - a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1
)) - 1) - 2*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x**2, x)
```

3.66 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=110

$$-\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{2x^2}-\frac{a(1-ax)^{3/4}\sqrt[4]{ax+1}}{4x}$$

```
[Out] -1/4*a*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x-1/2*(-a*x+1)^(3/4)*(a*x+1)^(5/4)/x^2-
1/4*a^2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-1/4*a^2*arctanh((a*x+1)^(1/4)/
(-a*x+1)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6126, 96, 94, 93, 212, 206, 203}

$$-\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{3/4}(ax+1)^{5/4}}{2x^2}-\frac{a(1-ax)^{3/4}\sqrt[4]{ax+1}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^3,x]
```

```
[Out] -(a*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x) - ((1 - a*x)^(3/4)*(1 + a*x)^(5/
4))/(2*x^2) - (a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (a^2*ArcTan
h[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[4]{1+ax}}{x^3\sqrt[4]{1-ax}}\,dx$$
$$= -\frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2} + \frac{1}{4}a\int \frac{\sqrt[4]{1+ax}}{x^2\sqrt[4]{1-ax}}\,dx$$
$$= -\frac{a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2} + \frac{1}{8}a^2\int \frac{1}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$
$$= -\frac{a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= -\frac{a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{1}{4}$$
$$= -\frac{a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2x^2} - \frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.02, size = 70, normalized size = 0.64

$$-\frac{(1-ax)^{3/4}\left(2a^2x^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)+9a^2x^2+15ax+6\right)}{12x^2(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^3,x]
```

```
[Out] -1/12*((1 - a*x)^(3/4)*(6 + 15*a*x + 9*a^2*x^2 + 2*a^2*x^2*Hypergeometric2F
1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(x^2*(1 + a*x)^(3/4))
```

fricas [A] time = 0.51, size = 144, normalized size = 1.31

$$-\frac{2\,a^2x^2\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(3\,a^2x^2-ax-\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + a^2*x^2*log(s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - a^2*x^2*log(sqrt(-sqrt(-a^2*x^2 +
```

```
1)/(a*x - 1)) - 1) - 2*(3*a^2*x^2 - a*x - 2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1)))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^3,x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x**3, x)
```

3.67 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{3}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{24x}-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{3x^3}-\frac{5a(1-ax)^{3/4}\sqrt[4]{ax}}{12x^2}$$

```
[Out] -1/3*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x^3-5/12*a*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x
^2-11/24*a^2*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x-3/8*a^3*arctan((a*x+1)^(1/4)/(-
a*x+1)^(1/4))-3/8*a^3*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{24x}-\frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{3}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{5a(1-ax)^{3/4}\sqrt[4]{ax+1}}{12x^2}-\frac{(1-ax)^{3/4}\sqrt[4]{ax}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^4,x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(3*x^3) - (5*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(12*x^2) - (11*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(24*x) - (3*a^3
*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 - (3*a^3*ArcTanh[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{1+ax}}{x^4\sqrt[4]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{1}{3}\int \frac{\frac{5a}{2}+2a^2x}{x^3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} - \frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{1}{6}\int \frac{-\frac{11a^2}{4}-\frac{5a^3x}{2}}{x^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} - \frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} + \frac{1}{6}\int \frac{9a^3}{8x\sqrt[4]{1-ax}\,(1}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} - \frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} + \frac{1}{16}\left(3a^3\right)\int \frac{}{x\sqrt[4]{1-}}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} - \frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} + \frac{1}{4}\left(3a^3\right)\mathrm{Subst}\Bigg(\int$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} - \frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} - \frac{1}{8}\left(3a^3\right)\mathrm{Subst}\Bigg(\int$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} - \frac{5a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} - \frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.02, size = 78, normalized size = 0.56

$$-\frac{(1-ax)^{3/4}\left(6a^3x^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)+11a^3x^3+21a^2x^2+18ax+8\right)}{24x^3(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^4,x]
[Out] -1/24*((1 - a*x)^(3/4)*(8 + 18*a*x + 21*a^2*x^2 + 11*a^3*x^3 + 6*a^3*x^3*Hy
pergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(x^3*(1 + a*x)^(3/4))
```

fricas [A] time = 0.56, size = 153, normalized size = 1.10

$$-\frac{18\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+9\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-9\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(11\,a^3x^3-a\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(18*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 9*a^3*x^3*l
og(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 9*a^3*x^3*log(sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1)) - 1) - 2*(11*a^3*x^3 - a^2*x^2 - 2*a*x - 8)*sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^4, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x**4, x)
```

3.68 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=168

$$-\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{83a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{192x}-\frac{29a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{96x^2}-\frac{(1-ax)}{}$$

```
[Out] -1/4*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x^4-7/24*a*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x
^3-29/96*a^2*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x^2-83/192*a^3*(-a*x+1)^(3/4)*(a*
x+1)^(1/4)/x-11/64*a^4*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-11/64*a^4*arcta
nh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{29a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{96x^2}-\frac{83a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{192x}-\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{7a(1-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^5,x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x^4) - (7*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(24*x^3) - (29*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(96*x^2) - (83*
a^3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(192*x) - (11*a^4*ArcTan[(1 + a*x)^(1/
4)/(1 - a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{\sqrt[4]{1+ax}}{x^5\sqrt[4]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} + \frac{1}{4}\int \frac{\frac{7a}{2}+3a^2x}{x^4\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{1}{12}\int \frac{-\frac{29a^2}{4}-7a^3x}{x^3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} + \frac{1}{24}\int \frac{\frac{83a^3}{8}}{x^2\sqrt[4]{1-a}}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}}{192x}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}}{192x}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}}{192x}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}}{192x}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4} - \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x^3} - \frac{29a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{96x^2} - \frac{83a^3(1-ax)^{3/4}}{192x}$$

Mathematica [C] time = 0.03, size = 86, normalized size = 0.51

$$-\frac{(1-ax)^{3/4}\left(22a^4x^4\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)+83a^4x^4+141a^3x^3+114a^2x^2+104ax+48\right)}{192x^4(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^5,x]
```

```
[Out] -1/192*((1 - a*x)^(3/4)*(48 + 104*a*x + 114*a^2*x^2 + 141*a^3*x^3 + 83*a^4*
x^4 + 22*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(x^4
*(1 + a*x)^(3/4))
```

fricas [A] time = 0.48, size = 161, normalized size = 0.96

$$-\frac{66\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+33\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-33\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(83\,a^4x^4\right.}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(66*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 33*a^4*x^4
*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 33*a^4*x^4*log(sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(83*a^4*x^4 - 25*a^3*x^3 - 2*a^2*x^2 - 8*a
*x - 48)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^5, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x**5, x)
```

3.69 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^6}\,dx$

Optimal. Leaf size=197

$$-\frac{31}{128}a^5\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{31}{128}a^5\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{611a^4(1-ax)^{3/4}\sqrt[4]{ax+1}}{1920x}-\frac{269a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{960x^2}-\frac{11a^2(\cdots}{}$$

```
[Out] -1/5*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x^5-9/40*a*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x
^4-11/48*a^2*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x^3-269/960*a^3*(-a*x+1)^(3/4)*(a
*x+1)^(1/4)/x^2-611/1920*a^4*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x-31/128*a^5*arct
an((a*x+1)^(1/4)/(-a*x+1)^(1/4))-31/128*a^5*arctanh((a*x+1)^(1/4)/(-a*x+1)^
(1/4))
```

Rubi [A] time = 0.10, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{269a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{960x^2}-\frac{11a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{48x^3}-\frac{611a^4(1-ax)^{3/4}\sqrt[4]{ax+1}}{1920x}-\frac{31}{128}a^5\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{31}{128}a^5\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/x^6,x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(5*x^5) - (9*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(40*x^4) - (11*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(48*x^3) - (269
*a^3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(960*x^2) - (611*a^4*(1 - a*x)^(3/4)*
(1 + a*x)^(1/4))/(1920*x) - (31*a^5*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
)/128 - (31*a^5*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/128
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
```

]

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^6}\,dx = \int \frac{\sqrt[4]{1+ax}}{x^6\sqrt[4]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} + \frac{1}{5}\int \frac{\frac{9a}{2}+4a^2x}{x^5\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{1}{20}\int \frac{-\frac{55a^2}{4}-\frac{27a^3x}{2}}{x^4\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} + \frac{1}{60}\int \frac{\frac{269a^3}{8}+}{x^3\sqrt[4]{1-ax}\,(}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{}}{960x^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{}}{960x^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{}}{960x^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{}}{960x^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{}}{960x^2}$$

$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{5x^5} - \frac{9a(1-ax)^{3/4}\sqrt[4]{1+ax}}{40x^4} - \frac{11a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{48x^3} - \frac{269a^3(1-ax)^{3/4}\sqrt[4]{}}{960x^2}$$

Mathematica [C] time = 0.03, size = 94, normalized size = 0.48

$$-\frac{(1-ax)^{3/4}\left(310a^5x^5\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)+611a^5x^5+1149a^4x^4+978a^3x^3+872a^2x^2+816ax+384\right)}{1920x^5(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/x^6,x]
```

[Out] -1/1920*((1 - a*x)^(3/4)*(384 + 816*a*x + 872*a^2*x^2 + 978*a^3*x^3 + 1149*a^4*x^4 + 611*a^5*x^5 + 310*a^5*x^5*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(x^5*(1 + a*x)^(3/4))

fricas [A] time = 0.61, size = 169, normalized size = 0.86

$$-\frac{930\,a^5x^5\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+465\,a^5x^5\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-465\,a^5x^5\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(61\right.}{3840\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="fricas")

[Out] -1/3840*(930*a^5*x^5*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 465*a^5*x^5*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 465*a^5*x^5*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(611*a^5*x^5 - 73*a^4*x^4 - 98*a^3*x^3 - 8*a^2*x^2 - 48*a*x - 384)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^5

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="giac")

[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^6, x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x)

[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/x^6, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^6,x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/x^6, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**6,x)
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/x**6, x)
```

3.70 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;\frac{3}{4},-\frac{3}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,3/4,-3/4,2+m,a*x,-a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{3}{4},-\frac{3}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 3/4, -3/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{3}{4},-\frac{3}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.39, size = 0, normalized size = 0.00

$$\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)*x^m,x]
[Out] Integrate[E^((3*ArcTanh[a*x])/2)*x^m, x]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a*x -
 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="giac")
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
[Out] int(x^m*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)*x**m,x)
[Out] Timed out
```

3.71 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{7/4}(4ax+11)}{32a^4}-\frac{41\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}\,a^4}-\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a\cdots}}\right.}{128\sqrt{2}\,a^4}$$

```
[Out] -41/64*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a^4-1/4*x^2*(-a*x+1)^(1/4)*(a*x+1)^(7/4
)/a^2-1/32*(-a*x+1)^(1/4)*(a*x+1)^(7/4)*(4*a*x+11)/a^4-123/128*arctan(-1+(-
a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^4*2^(1/2)-123/128*arctan(1+(-a*x+1)^(
1/4)*2^(1/2)/(a*x+1)^(1/4))/a^4*2^(1/2)+123/256*ln(1-(-a*x+1)^(1/4)*2^(1/2)
/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4*2^(1/2)-123/256*ln(1+(-a*x
+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 100, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^2\sqrt[4]{1-ax}\,(ax+1)^{7/4}}{4a^2}-\frac{\sqrt[4]{1-ax}\,(ax+1)^{7/4}(4ax+11)}{32a^4}-\frac{41\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}\,a^4}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)*x^3,x]
```

```
[Out] (-41*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*a^4) - (x^2*(1 - a*x)^(1/4)*(1 +
a*x)^(7/4))/(4*a^2) - ((1 - a*x)^(1/4)*(1 + a*x)^(7/4)*(11 + 4*a*x))/(32*a^
4) + (123*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2
]*a^4) - (123*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sq
rt[2]*a^4) + (123*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^
(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (123*Log[1 + Sqrt[1 - a*x]/Sqr
t[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx$$

$$= -\frac{x^2\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^2} - \frac{\int \frac{x\left(-2-\frac{3ax}{2}\right)(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{4a^2}$$

$$= -\frac{x^2\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}(11+4ax)}{32a^4} + \frac{41\int \frac{(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{64a^3}$$

$$= -\frac{41\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}(11+4ax)}{32a^4} + \frac{123\int}{}$$

$$= -\frac{41\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}(11+4ax)}{32a^4} - \frac{123\,\mathrm{Su}}{}$$

$$= -\frac{41\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}(11+4ax)}{32a^4} - \frac{123\,\mathrm{Su}}{}$$

$$= -\frac{41\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}(11+4ax)}{32a^4} - \frac{123\,\mathrm{Su}}{}$$

$$= -\frac{41\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}(11+4ax)}{32a^4} - \frac{123\,\mathrm{Su}}{}$$

$$= -\frac{41\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}(11+4ax)}{32a^4} + \frac{123\,\mathrm{lo}}{}$$

$$= -\frac{41\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} - \frac{x^2\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}(11+4ax)}{32a^4} + \frac{123\,\mathrm{ta}}{}$$

Mathematica [C] time = 0.11, size = 131, normalized size = 0.45

$$-\frac{\sqrt[4]{1-ax}\left(a^3x^3(ax+1)^{3/4} + a^2x^2(ax+1)^{3/4} + 24\,2^{3/4}\,{}_2F_1\left(-\frac{11}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-ax)\right) - 8\,2^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-ax)\right)\right.}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)*x^3,x]
```

```
[Out] -1/4*((1 - a*x)^(1/4)*(a^2*x^2*(1 + a*x)^(3/4) + a^3*x^3*(1 + a*x)^(3/4) +
24*2^(3/4)*Hypergeometric2F1[-11/4, 1/4, 5/4, (1 - a*x)/2] - 8*2^(3/4)*Hype
rgeometric2F1[-7/4, 1/4, 5/4, (1 - a*x)/2] - 2*2^(3/4)*Hypergeometric2F1[-3
/4, 1/4, 5/4, (1 - a*x)/2]))/a^4
```

fricas [B] time = 0.51, size = 557, normalized size = 1.92

$$-\frac{492\sqrt{2}\,a^4\frac{1}{a^{16}}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^4\sqrt{\frac{\sqrt{2}\left(a^{13}x-a^{12}\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^{16}}^{\frac{3}{4}}+\left(a^9x-a^8\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^{16}}^{\frac{1}{4}}-\sqrt{2}\,a^4\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^{16}}^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/256*(492*sqrt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt((sqrt(2)*(a
^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + (a^9*x
- a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/4) - sqr
t(2)*a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - 1) + 492*sqr
t(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt(-(sqrt(2)*(a^13*x - a^12)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^
(-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/4) - sqrt(2)*a^4*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + 1) + 123*sqrt(2)*a^4*(a^(-
16))^(1/4)*log((sqrt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
*(a^(-16))^(3/4) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x -
 1)) - 123*sqrt(2)*a^4*(a^(-16))^(1/4)*log(-(sqrt(2)*(a^13*x - a^12)*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^(-16))
 + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(16*a^3*x^3 + 24*a^2*x^2 + 30*a*x + 6
3)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
```

```
[Out] int(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)*x**3,x)
```

```
[Out] Timed out
```

3.72 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=282

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{7/4}}{4a^3}-\frac{17\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{24a^3}+\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}-\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}+\cdots$$

```
[Out] -17/24*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a^3-1/4*(-a*x+1)^(1/4)*(a*x+1)^(7/4)/a^
3-1/3*x*(-a*x+1)^(1/4)*(a*x+1)^(7/4)/a^2-17/16*arctan(-1+(-a*x+1)^(1/4)*2^(
1/2)/(a*x+1)^(1/4))/a^3*2^(1/2)-17/16*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+
1)^(1/4))/a^3*2^(1/2)+17/32*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x
+1)^(1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)-17/32*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x
+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x\sqrt[4]{1-ax}\,(ax+1)^{7/4}}{3a^2}-\frac{\sqrt[4]{1-ax}\,(ax+1)^{7/4}}{4a^3}-\frac{17\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{24a^3}+\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)*x^2,x]
```

```
[Out] (-17*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(24*a^3) - ((1 - a*x)^(1/4)*(1 + a*x)
^(7/4))/(4*a^3) - (x*(1 - a*x)^(1/4)*(1 + a*x)^(7/4))/(3*a^2) + (17*ArcTan[
1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (17*ArcTa
n[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (17*Log
[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4
)])/(16*Sqrt[2]*a^3) - (17*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(
1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

n]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{3}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx$$

$$= -\frac{x\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{3a^2} - \frac{\int \frac{\left(-1-\frac{3ax}{2}\right)(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{3a^2}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{3a^2} + \frac{17\int \frac{(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{24a^2}$$

$$= -\frac{17\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{3a^2} + \frac{17\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}}{16a^2}$$

$$= -\frac{17\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\right.}{4a}$$

$$= -\frac{17\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{1}{1+x^4}\right.}{4a^3}$$

$$= -\frac{17\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\right.}{8a^3}$$

$$= -\frac{17\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{3a^2} - \frac{17\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}}\right.}{16}$$

$$= -\frac{17\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{3a^2} + \frac{17\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.}{16\sqrt{2}}$$

$$= -\frac{17\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24a^3} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{4a^3} - \frac{x\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{3a^2} + \frac{17\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{1}}\right.}{8\sqrt{2}\,a^3}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 0.24

$$-\frac{\sqrt[4]{1-ax}\left((ax+1)^{3/4}\left(4a^2x^2+7ax+3\right)+34\,2^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-ax)\right)\right)}{12a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)*x^2,x]
```

```
[Out] -1/12*((1 - a*x)^(1/4)*((1 + a*x)^(3/4)*(3 + 7*a*x + 4*a^2*x^2) + 34*2^(3/4
)*Hypergeometric2F1[-3/4, 1/4, 5/4, (1 - a*x)/2]))/a^3
```

fricas [B] time = 0.60, size = 549, normalized size = 1.95

$$-\frac{204\,\sqrt{2}\,a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^3\sqrt{\frac{\sqrt{2}\left(a^{10}x-a^9\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-\sqrt{2}\,a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^
10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + (a^7*x -
a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - sqrt(
2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - 1) + 204*sqrt(
2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^10*x - a^9)*sqr
t(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*x - a^6)*sqrt(a^(-1
2)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - sqrt(2)*a^3*sqrt(-sq
rt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + 1) + 51*sqrt(2)*a^3*(a^(-12))
^(1/4)*log((sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(
-12))^(3/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))
- 51*sqrt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt
(-a^2*x^2 + 1))/(a*x - 1)) + 4*(8*a^2*x^2 + 14*a*x + 23)*sqrt(-a^2*x^2 + 1)
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
```

```
[Out] int(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)*x**2,x)
```

```
[Out] Integral(x**2*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

3.73 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=255

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{7/4}}{2a^2}-\frac{3\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{4a^2}+\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}+\frac{9\tan}{}$$

```
[Out] -3/4*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a^2-1/2*(-a*x+1)^(1/4)*(a*x+1)^(7/4)/a^2-
9/8*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^2*2^(1/2)-9/8*arctan(
1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^2*2^(1/2)+9/16*ln(1-(-a*x+1)^(1/4
)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(1/2)-9/16*ln(1
+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{7/4}}{2a^2}-\frac{3\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{4a^2}+\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}+\frac{9\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)*x,x]
```

```
[Out] (-3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*a^2) - ((1 - a*x)^(1/4)*(1 + a*x)^(
7/4))/(2*a^2) + (9*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(
4*Sqrt[2]*a^2) - (9*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/
(4*Sqrt[2]*a^2) + (9*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*
x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - a*x]/Sqrt
[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{3}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2a^2} + \frac{3\int \frac{(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx}{4a}$$

$$= -\frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2a^2} + \frac{9\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{8a}$$

$$= -\frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a^2}$$

$$= -\frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$

$$= -\frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2} - \frac{9\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a^2} - \frac{9\,\text{Sub}}{}$$

$$= -\frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2a^2} + \frac{9\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}\,a^2} - \frac{9\log\left(1+\right.}{}$$

$$= -\frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2a^2} + \frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}\,a^2} - \frac{9\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1-}}\right.}{4\sqrt{2}\,a^2}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.21

$$-\frac{\sqrt[4]{1-ax}\left(6\,2^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-ax)\right)+(ax+1)^{7/4}\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)*x,x]
```

```
[Out] -1/2*((1 - a*x)^(1/4)*((1 + a*x)^(7/4) + 6*2^(3/4)*Hypergeometric2F1[-3/4,
1/4, 5/4, (1 - a*x)/2]))/a^2
```

fricas [B] time = 0.53, size = 541, normalized size = 2.12

$$-\frac{36\sqrt{2}\,a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^2\sqrt{\frac{\sqrt{2}\left(a^7x-a^6\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{3}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{1}{4}}-\sqrt{2}\,a^2\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{1}{4}}-1\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="fricas")
```

```
[Out] -1/16*(36*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt((sqrt(2)*(a^7*
x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a^5*x - a^4)
*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - 1) + 36*sqrt(2)*a^2*(
a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt(-(sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-
a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2*sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1))*(a^(-8))^(1/4) + 1) + 9*sqrt(2)*a^2*(a^(-8))^(1/4)*log((sqrt
(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a^5
*x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 9*sqrt(2)*a^2*(a^
(-8))^(1/4)*log(-(sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
*(a^(-8))^(3/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1
)) + 4*sqrt(-a^2*x^2 + 1)*(2*a*x + 5)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/
a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
```

```
[Out] int(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)*x,x)
```

```
[Out] Integral(x*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

3.74 $\int e^{\frac{3}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=223

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}\,a}-\frac{3\,\mathrm{t}}{}$$

```
[Out] -(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a-3/2*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1
)^(1/4))/a*2^(1/2)-3/2*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(
1/2)+3/4*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a*2^(1/2)-3/4*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2
)/(a*x+1)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6125, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}\,a}-\frac{3\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/a) + (3*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(
1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) - (3*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) + (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] -
 (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a) - (3*Log[1 + Sqr
t[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*S
qrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] && !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{\frac{3}{2}\tanh^{-1}(ax)}\,dx = \int \frac{(1+ax)^{3/4}}{(1-ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} + \frac{3}{2}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} - \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} - \frac{6\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} - \frac{3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{3\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-a}}{\sqrt[4]{1+a}}\right.}{2a}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} + \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a} - \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a} - \frac{3\,\text{Su}}{}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} - \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} + \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.}{2\sqrt{2}\,a}$$

Mathematica [C] time = 0.06, size = 48, normalized size = 0.22

$$\frac{8e^{\frac{3}{2}\tanh^{-1}(ax)}\left({}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\tanh^{-1}(ax)}\right)-\frac{1}{e^{2\tanh^{-1}(ax)}+1}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] (8*E^((3*ArcTanh[a*x])/2)*(-(1 + E^(2*ArcTanh[a*x]))^(-1) + Hypergeometric2
F1[3/4, 2, 7/4, -E^(2*ArcTanh[a*x])]))/a
```

fricas [B] time = 0.47, size = 519, normalized size = 2.33

$$-\frac{12\sqrt{2}\,a\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^4}^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^4}^{\frac{1}{4}}-\sqrt{2}\,a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^4}^{\frac{1}{4}}-1\right)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x - a
^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt
(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 12*sqrt(2)*a*(a^(-4))^(
1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1
))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)*(a^(-4))^(1/4) + 1) + 3*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*x - a^
3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(
a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 3*sqrt(2)*a*(a^(-4))^(1/4)*log(-
(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) -
 (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*sqrt(-a^2*
x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{3/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

3.75 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=227

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)-$$

```
[Out] 2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-2*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/
4))+1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1
/2))*2^(1/2)-1/2*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(
a*x+1)^(1/2))*2^(1/2)-arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/
2)-arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6126, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x,x]
```

```
[Out] 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
- a*x)^(1/4))/(1 + a*x)^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 +
Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sq
rt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

```
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x}\,dx = \int \frac{(1+ax)^{3/4}}{x(1-ax)^{3/4}}\,dx$$
$$= a\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx + \int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\left(4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 4\,\text{Subst}\Bigg(\int$$
$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\,\text{Subst}\Bigg($$
$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} + \frac{\text{Subst}\Big(\int}{}$$
$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\log\Big(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+}{\sqrt{2}}$$
$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\tanh^{-1}$$

Mathematica [C] time = 0.03, size = 83, normalized size = 0.37

$$-2\,2^{3/4}\sqrt[4]{1-ax}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-ax)\right) - \frac{4\sqrt[4]{1-ax}\;{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{1-ax}{-ax-1}\right)}{\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x,x]
```

```
[Out] -2*2^(3/4)*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - a*x)/2] -
(4*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - a*x)/(-1 - a*x))])
/(1 + a*x)^(1/4)
```

fricas [B] time = 0.52, size = 417, normalized size = 1.84

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\sqrt{2}\,\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(
-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2
)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(
a*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/2*sqrt(2)*
log(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-
a^2*x^2 + 1) - 1)/(a*x - 1)) + 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) + 2
*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1)) + 1) + log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x,x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x,x)
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)/x, x)
```

3.76 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{x}+3a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-3a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] -(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x+3*a*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-3*
a*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 298, 203, 206}

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{x}+3a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-3a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x^2,x]
```

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/x) + 3*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*
x)^(1/4)] - 3*a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+ax)^{3/4}}{x^2(1-ax)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{x} + \frac{1}{2}(3a)\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\\
&= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{x} + (6a)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{x} - (3a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + (3a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x,\right.\\
&= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{x} + 3a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 3a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.75

$$-\frac{\sqrt[4]{1-ax}\left(6ax\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)+ax+1\right)}{x\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x^2,x]
```

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x + 6*a*x*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x
)/(1 + a*x)]))/(x*(1 + a*x)^(1/4)))
```

fricas [B] time = 0.54, size = 131, normalized size = 1.79

$$\frac{6\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-3\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+3\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 3*a*x*log(sqrt(-sq
rt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 3*a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1)) - 1) - 2*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,x]=[0,0]Warning
, need to choose a branch for the root of a polynomial with parameters. Thi
s might be wrong.The choice was done assuming [a,x]=[0,0]sym2poly/r2sym(con
st gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x^2,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
```

```
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)/x**2, x)
```

3.77 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=110

$$\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{\sqrt[4]{1-ax}\,(ax+1)^{7/4}}{2x^2}-\frac{3a\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{4x}$$

```
[Out] -3/4*a*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x-1/2*(-a*x+1)^(1/4)*(a*x+1)^(7/4)/x^2+
9/4*a^2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-9/4*a^2*arctanh((a*x+1)^(1/4)/
(-a*x+1)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6126, 96, 94, 93, 298, 203, 206}

$$\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{\sqrt[4]{1-ax}\,(ax+1)^{7/4}}{2x^2}-\frac{3a\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x^3,x]
```

```
[Out] (-3*a*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x) - ((1 - a*x)^(1/4)*(1 + a*x)^(
7/4))/(2*x^2) + (9*a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (9*a^2*
ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{(1+ax)^{3/4}}{x^3(1-ax)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2x^2} + \frac{1}{4}(3a)\int \frac{(1+ax)^{3/4}}{x^2(1-ax)^{3/4}}\,dx$$
$$= -\frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2x^2} + \frac{1}{8}\left(9a^2\right)\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2x^2} + \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= -\frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2x^2} - \frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) +$$
$$= -\frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x} - \frac{\sqrt[4]{1-ax}\,(1+ax)^{7/4}}{2x^2} + \frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.02, size = 70, normalized size = 0.64

$$-\frac{\sqrt[4]{1-ax}\left(18a^2x^2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)+5a^2x^2+7ax+2\right)}{4x^2\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x^3,x]
```

```
[Out] -1/4*((1 - a*x)^(1/4)*(2 + 7*a*x + 5*a^2*x^2 + 18*a^2*x^2*Hypergeometric2F1
[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(x^2*(1 + a*x)^(1/4))
```

fricas [A] time = 0.64, size = 149, normalized size = 1.35

$$\frac{18\,a^2x^2\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-9\,a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+9\,a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\sqrt{-a^2x^2+1}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 9*a^2*x^2*log
(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 9*a^2*x^2*log(sqrt(-sqrt(-a^2*x
```

```
^2 + 1)/(a*x - 1)) - 1) - 2*sqrt(-a^2*x^2 + 1)*(5*a*x + 2)*sqrt(-sqrt(-a^2*
x^2 + 1)/(a*x - 1)))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)/x**3, x)
```

3.78 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=139

$$\frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{17}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{23a^2\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{24x}-\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{3x^3}-\frac{7a\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{12x^2}$$

```
[Out] -1/3*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x^3-7/12*a*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x
^2-23/24*a^2*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x+17/8*a^3*arctan((a*x+1)^(1/4)/(
-a*x+1)^(1/4))-17/8*a^3*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 298, 203, 206}

$$-\frac{23a^2\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{24x}+\frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{17}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{7a\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x^4,x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(3*x^3) - (7*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(12*x^2) - (23*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(24*x) + (17*a^
3*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 - (17*a^3*ArcTanh[(1 + a*x)^(1
/4)/(1 - a*x)^(1/4)])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+ax)^{3/4}}{x^4(1-ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} + \frac{1}{3}\int \frac{\frac{7a}{2}+2a^2x}{x^3(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{1}{6}\int \frac{-\frac{23a^2}{4}-\frac{7a^3x}{2}}{x^2(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} + \frac{1}{6}\int \frac{51a}{8x(1-ax)^{3/4}}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} + \frac{1}{16}\left(17a^3\right)\int \frac{}{x(1}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} + \frac{1}{4}\left(17a^3\right)\text{Subst}\Bigg(\int$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} - \frac{1}{8}\left(17a^3\right)\text{Subst}\Bigg(\int$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} - \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{23a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} + \frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.02, size = 78, normalized size = 0.56

$$-\frac{\sqrt[4]{1-ax}\left(102a^3x^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)+23a^3x^3+37a^2x^2+22ax+8\right)}{24x^3\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x^4,x]
```

```
[Out] -1/24*((1 - a*x)^(1/4)*(8 + 22*a*x + 37*a^2*x^2 + 23*a^3*x^3 + 102*a^3*x^3*
Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(x^3*(1 + a*x)^(1/4))
```

fricas [A] time = 0.68, size = 157, normalized size = 1.13

$$\frac{102\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-51\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+51\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(23\,a^2x^2\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(102*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 51*a^3*x^3*
log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 51*a^3*x^3*log(sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(23*a^2*x^2 + 14*a*x + 8)*sqrt(-a^2*x^2 + 1
)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,x]=[0,0]Warning
, need to choose a branch for the root of a polynomial with parameters. Thi
s might be wrong.The choice was done assuming [a,x]=[0,0]sym2poly/r2sym(con
st gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)/x**4, x)
```

3.79 $\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=168

$$\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{63a^3\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{64x}-\frac{15a^2\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{32x^2}-\frac{\sqrt[4]{1-a}}{}$$

```
[Out] -1/4*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x^4-3/8*a*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x^
3-15/32*a^2*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x^2-63/64*a^3*(-a*x+1)^(1/4)*(a*x+
1)^(3/4)/x+123/64*a^4*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-123/64*a^4*arcta
nh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 298, 203, 206}

$$-\frac{15a^2\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{32x^2}-\frac{63a^3\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{64x}+\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{3a\sqrt[4]{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcTanh[a*x])/2)/x^5,x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x^4) - (3*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(8*x^3) - (15*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(32*x^2) - (63*a
^3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*x) + (123*a^4*ArcTan[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{(1+ax)^{3/4}}{x^5(1-ax)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{\frac{9a}{2}+3a^2x}{x^4(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8x^3} - \frac{1}{12}\int \frac{-\frac{45a^2}{4}-9a^3x}{x^3(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{32x^2} + \frac{1}{24}\int \frac{\frac{189a^3}{8}}{x^2(1-ax)}$$
$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}\,(1}{64x}$$
$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}\,(1}{64x}$$
$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}\,(1}{64x}$$
$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}\,(1}{64x}$$
$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} - \frac{3a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8x^3} - \frac{15a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{32x^2} - \frac{63a^3\sqrt[4]{1-ax}\,(1}{64x}$$

Mathematica [C] time = 0.03, size = 86, normalized size = 0.51

$$-\frac{\sqrt[4]{1-ax}\left(246a^4x^4\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)+63a^4x^4+93a^3x^3+54a^2x^2+40ax+16\right)}{64x^4\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcTanh[a*x])/2)/x^5,x]
```

```
[Out] -1/64*((1 - a*x)^(1/4)*(16 + 40*a*x + 54*a^2*x^2 + 93*a^3*x^3 + 63*a^4*x^4
+ 246*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(x^4*(1
 + a*x)^(1/4))
```

fricas [A] time = 0.57, size = 165, normalized size = 0.98

$$\frac{246\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-123\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+123\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\left(63\,a^3x\right.}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/128*(246*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 123*a^4*x^
4*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 123*a^4*x^4*log(sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(63*a^3*x^3 + 30*a^2*x^2 + 24*a*x + 16)*
sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^5, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x^5,x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)/x**5, x)
```

3.80 $\int e^{\frac{5}{2}\tanh^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=31

$$\frac{x^{m+1} F_1\left(m+1; \frac{5}{4}, -\frac{5}{4}; m+2; ax, -ax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,5/4,-5/4,2+m,a*x,-a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1} F_1\left(m+1; \frac{5}{4}, -\frac{5}{4}; m+2; ax, -ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 5/4, -5/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned} \int e^{\frac{5}{2}\tanh^{-1}(ax)} x^m \, dx &= \int \frac{x^m (1+ax)^{5/4}}{(1-ax)^{5/4}} \, dx \\ &= \frac{x^{1+m} F_1\left(1+m; \frac{5}{4}, -\frac{5}{4}; 2+m; ax, -ax\right)}{1+m} \end{aligned}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)*x^m,x]
```

```
[Out] Integrate[E^((5*ArcTanh[a*x])/2)*x^m, x]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{(ax+1)x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-(a*x + 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a*x - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
[Out] int(x^m*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)*x**m,x)
[Out] Timed out
```

3.81 $\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=317

$$\frac{(1-ax)^{3/4}(ax+1)^{5/4}(452ax+521)}{96a^4}+\frac{475(1-ax)^{3/4}\sqrt[4]{ax+1}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}\,a^4}-\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right.}{128}$$

```
[Out] 475/64*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a^4+4*x^3*(a*x+1)^(5/4)/a/(-a*x+1)^(1/4
)+17/4*x^2*(-a*x+1)^(3/4)*(a*x+1)^(5/4)/a^2+1/96*(-a*x+1)^(3/4)*(a*x+1)^(5/
4)*(452*a*x+521)/a^4+475/128*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)
)/a^4*2^(1/2)+475/128*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^4*2^
(1/2)+475/256*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x
+1)^(1/2))/a^4*2^(1/2)-475/256*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-
a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6126, 97, 153, 147, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{17x^2(1-ax)^{3/4}(ax+1)^{5/4}}{4a^2}+\frac{(1-ax)^{3/4}(ax+1)^{5/4}(452ax+521)}{96a^4}+\frac{475(1-ax)^{3/4}\sqrt[4]{ax+1}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\right.}{128\sqrt{2}\,a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)*x^3,x]
```

```
[Out] (475*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(64*a^4) + (4*x^3*(1 + a*x)^(5/4))/(a
*(1 - a*x)^(1/4)) + (17*x^2*(1 - a*x)^(3/4)*(1 + a*x)^(5/4))/(4*a^2) + ((1
- a*x)^(3/4)*(1 + a*x)^(5/4)*(521 + 452*a*x))/(96*a^4) - (475*ArcTan[1 - (S
qrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (475*ArcTan[1
+ (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (475*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(128*Sqrt[2]*a^4) - (475*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(
1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1+ax)^{5/4}}{(1-ax)^{5/4}}\,dx$$

$$= \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - \frac{4\int \frac{x^2\sqrt[4]{1+ax}\left(3+\frac{17ax}{4}\right)}{\sqrt[4]{1-ax}}\,dx}{a}$$

$$= \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{\int \frac{x\sqrt[4]{1+ax}\left(-\frac{17a}{2}-\frac{113a^2x}{8}\right)}{\sqrt[4]{1-ax}}\,dx}{a^3}$$

$$= \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax)^{5/4}(521+452ax)}{96a^4} - \frac{475\int}{}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax}{96}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax}{96}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax}{96}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax}{96}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax}{96}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax}{96}$$

$$= \frac{475(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} + \frac{4x^3(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{17x^2(1-ax)^{3/4}(1+ax)^{5/4}}{4a^2} + \frac{(1-ax)^{3/4}(1+ax}{96}$$

Mathematica [C] time = 0.05, size = 74, normalized size = 0.23

$$\frac{(ax+1)^{9/4}\left(-6a^2x^2-5ax+59\right)-380\sqrt[4]{2}\,(ax-1)\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right)}{24a^4\sqrt[4]{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)*x^3,x]
```

```
[Out] ((1 + a*x)^(9/4)*(59 - 5*a*x - 6*a^2*x^2) - 380*2^(1/4)*(-1 + a*x)*Hypergeo
metric2F1[-5/4, 3/4, 7/4, (1 - a*x)/2])/(24*a^4*(1 - a*x)^(1/4))
```

fricas [B] time = 0.61, size = 553, normalized size = 1.74

$$\frac{5700\sqrt{2}\,a^4\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^{12}\sqrt{\frac{\sqrt{2}\left(a^5x-a^4\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}+\left(a^9x-a^8\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{3}{4}}-\sqrt{2}\,a^{12}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{3}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/768*(5700*sqrt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^12*sqrt((sqrt(2)*(
a^5*x - a^4)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + (a^9*x -
 a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(3/4) - sqrt
(2)*a^12*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - 1) + 5700*sq
rt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^12*sqrt(-(sqrt(2)*(a^5*x - a^4)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - (a^9*x - a^8)*sqrt(a^
(-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(3/4) - sqrt(2)*a^12*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + 1) - 1425*sqrt(2)*a^4*(a^
(-16))^(1/4)*log((sqrt(2)*(a^5*x - a^4)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
*(a^(-16))^(1/4) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x -
 1)) + 1425*sqrt(2)*a^4*(a^(-16))^(1/4)*log(-(sqrt(2)*(a^5*x - a^4)*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - (a^9*x - a^8)*sqrt(a^(-16))
+ sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(48*a^4*x^4 + 136*a^3*x^3 + 226*a^2*x^
2 + 521*a*x - 2467)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="maxima")
[Out] integrate(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
[Out] int(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)*x**3,x)
[Out] Timed out
```

3.82 $\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=305

$$\frac{(1-ax)^{3/4}(ax+1)^{9/4}}{3a^3}+\frac{2(ax+1)^{9/4}}{a^3\sqrt[4]{1-ax}}+\frac{11(1-ax)^{3/4}(ax+1)^{5/4}}{4a^3}+\frac{55(1-ax)^{3/4}\sqrt[4]{ax+1}}{8a^3}+\frac{55\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\right.}{16\sqrt{2}\,a^3}$$

```
[Out] 55/8*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a^3+11/4*(-a*x+1)^(3/4)*(a*x+1)^(5/4)/a^3
+2*(a*x+1)^(9/4)/a^3/(-a*x+1)^(1/4)+1/3*(-a*x+1)^(3/4)*(a*x+1)^(9/4)/a^3+55
/16*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^3*2^(1/2)+55/16*arcta
n(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^3*2^(1/2)+55/32*ln(1-(-a*x+1)^(
1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)-55/32*
ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3
*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 89, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{(1-ax)^{3/4}(ax+1)^{9/4}}{3a^3}+\frac{2(ax+1)^{9/4}}{a^3\sqrt[4]{1-ax}}+\frac{11(1-ax)^{3/4}(ax+1)^{5/4}}{4a^3}+\frac{55(1-ax)^{3/4}\sqrt[4]{ax+1}}{8a^3}+\frac{55\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\right.}{16\sqrt{2}\,a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)*x^2,x]
```

```
[Out] (55*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(8*a^3) + (11*(1 - a*x)^(3/4)*(1 + a*x
)^(5/4))/(4*a^3) + (2*(1 + a*x)^(9/4))/(a^3*(1 - a*x)^(1/4)) + ((1 - a*x)^(
3/4)*(1 + a*x)^(9/4))/(3*a^3) - (55*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1
 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (55*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/
(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (55*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x]
 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3) - (55*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(16*Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2(1+ax)^{5/4}}{(1-ax)^{5/4}}\,dx$$

$$= \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} - \frac{2\int \frac{(1+ax)^{5/4}\left(\frac{5a}{2}+\frac{a^2x}{2}\right)}{\sqrt[4]{1-ax}}\,dx}{a^3}$$

$$= \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} - \frac{11\int \frac{(1+ax)^{5/4}}{\sqrt[4]{1-ax}}\,dx}{2a^2}$$

$$= \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} - \frac{55\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{8a^2}$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} -$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} +$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} +$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} -$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} +$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} +$$

$$= \frac{55(1-ax)^{3/4}\sqrt[4]{1+ax}}{8a^3} + \frac{11(1-ax)^{3/4}(1+ax)^{5/4}}{4a^3} + \frac{2(1+ax)^{9/4}}{a^3\sqrt[4]{1-ax}} + \frac{(1-ax)^{3/4}(1+ax)^{9/4}}{3a^3} -$$

Mathematica [C] time = 0.03, size = 66, normalized size = 0.22

$$\frac{(7-ax)(ax+1)^{9/4}-44\sqrt[4]{2}\,(ax-1)\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right)}{3a^3\sqrt[4]{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)*x^2,x]
```

```
[Out] ((7 - a*x)*(1 + a*x)^(9/4) - 44*2^(1/4)*(-1 + a*x)*Hypergeometric2F1[-5/4,
3/4, 7/4, (1 - a*x)/2])/(3*a^3*(1 - a*x)^(1/4))
```

fricas [B] time = 0.57, size = 545, normalized size = 1.79

$$660\sqrt{2}\,a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^9\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-\sqrt{2}\,a^9\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt((sqrt(2)*(a^4
*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + (a^7*x - a^
6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)
*a^9*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - 1) + 660*sqrt(2)
*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12))
 + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)*a^9*sqrt(-sqrt(
-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + 1) - 165*sqrt(2)*a^3*(a^(-12))^(
1/4)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12
))^(1/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 1
65*sqrt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*
x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a
^2*x^2 + 1))/(a*x - 1)) - 4*(8*a^3*x^3 + 26*a^2*x^2 + 61*a*x - 287)*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
```

```
[Out] int(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)*x**2,x)
```

```
[Out] Timed out
```

3.83 $\int e^{\frac{5}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=279

$$\frac{2(ax+1)^{9/4}}{a^2\sqrt[4]{1-ax}}+\frac{5(1-ax)^{3/4}(ax+1)^{5/4}}{2a^2}+\frac{25(1-ax)^{3/4}\sqrt[4]{ax+1}}{4a^2}+\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\right.}{8\sqrt{2}\,a^2}$$

```
[Out] 25/4*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a^2+5/2*(-a*x+1)^(3/4)*(a*x+1)^(5/4)/a^2+
2*(a*x+1)^(9/4)/a^2/(-a*x+1)^(1/4)+25/8*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a
*x+1)^(1/4))/a^2*2^(1/2)+25/8*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)
)/a^2*2^(1/2)+25/16*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2
)/(a*x+1)^(1/2))/a^2*2^(1/2)-25/16*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4
)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 78, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{2(ax+1)^{9/4}}{a^2\sqrt[4]{1-ax}}+\frac{5(1-ax)^{3/4}(ax+1)^{5/4}}{2a^2}+\frac{25(1-ax)^{3/4}\sqrt[4]{ax+1}}{4a^2}+\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\right.}{8\sqrt{2}\,a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)*x,x]
```

```
[Out] (25*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*a^2) + (5*(1 - a*x)^(3/4)*(1 + a*x)
^(5/4))/(2*a^2) + (2*(1 + a*x)^(9/4))/(a^2*(1 - a*x)^(1/4)) - (25*ArcTan[1
- (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*ArcTan[
1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(8*Sqrt[2]*a^2) - (25*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 -
 a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x(1+ax)^{5/4}}{(1-ax)^{5/4}}\,dx$$

$$= \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{5\int \frac{(1+ax)^{5/4}}{\sqrt[4]{1-ax}}\,dx}{a}$$

$$= \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{25\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx}{4a}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{25\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx}{8a}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} + \frac{25\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x\right.}{2a^2}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} + \frac{25\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \right.}{2a^2}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{25\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \right.}{4a^2}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} + \frac{25\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,\right.}{8a^2}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} + \frac{25\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{1-}}\right.}{8\sqrt{2}\,a^2}$$

$$= \frac{25(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} + \frac{5(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{2(1+ax)^{9/4}}{a^2\sqrt[4]{1-ax}} - \frac{25\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}\,a^2}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.22

$$\frac{6(ax+1)^{9/4} - 40\sqrt[4]{2}\,(ax-1)\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right)}{3a^2\sqrt[4]{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)*x,x]
```

```
[Out] (6*(1 + a*x)^(9/4) - 40*2^(1/4)*(-1 + a*x)*Hypergeometric2F1[-5/4, 3/4, 7/4
, (1 - a*x)/2])/(3*a^2*(1 - a*x)^(1/4))
```

fricas [B] time = 0.53, size = 537, normalized size = 1.92

$$100\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}a^6\sqrt{\frac{\sqrt{2}\left(a^3x-a^2\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{1}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{3}{4}}-\sqrt{2}a^6\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^8}^{\frac{3}{4}}-1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="fricas")
```

```
[Out] 1/16*(100*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt((sqrt(2)*(a^3*
x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*x - a^4)
*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - 1) + 100*sqrt(2)*a^2*
(a^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(
-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1))*(a^(-8))^(3/4) + 1) - 25*sqrt(2)*a^2*(a^(-8))^(1/4)*log((sq
rt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a
^5*x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 25*sqrt(2)*a^2*
(a^(-8))^(1/4)*log(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
1))*(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x
- 1)) - 4*(2*a^2*x^2 + 9*a*x - 43)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
```

```
[Out] int(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)*x,x)
```

```
[Out] Timed out
```

3.84 $\int e^{\frac{5}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=247

$$\frac{4(ax+1)^{5/4}}{a\sqrt[4]{1-ax}}+\frac{5(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}+\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}-\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}-\frac{5\tan^{-1}\left(1-\right.}{\sqrt{2}\,a}$$

```
[Out] 5*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a+4*(a*x+1)^(5/4)/a/(-a*x+1)^(1/4)+5/2*arcta
n(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)+5/2*arctan(1+(-a*x+1)^
(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)+5/4*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x
+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*2^(1/2)-5/4*ln(1+(-a*x+1)^(1/4)*2
^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6125, 47, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{4(ax+1)^{5/4}}{a\sqrt[4]{1-ax}}+\frac{5(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}+\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}-\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}-\frac{5\tan^{-1}\left(1-\right.}{\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (5*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/a + (4*(1 + a*x)^(5/4))/(a*(1 - a*x)^(1
/4)) - (5*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a
) + (5*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) +
 (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*
x)^(1/4)])/(2*Sqrt[2]*a) - (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2
]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{\frac{5}{2}\tanh^{-1}(ax)}\,dx = \int \frac{(1+ax)^{5/4}}{(1-ax)^{5/4}}\,dx$$

$$= \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - 5\int \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\,dx$$

$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - \frac{5}{2}\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$

$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{10\,\mathrm{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{10\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - \frac{5\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} + \frac{5\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x\right.}{a}$$

$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} + \frac{5\,\mathrm{Subst}\left(\int \frac{}{1+\sqrt{}}\right.}{}$$

$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} + \frac{5\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a} - \frac{5\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \right.}{2\sqrt{2}\,a}$$

$$= \frac{5(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{4(1+ax)^{5/4}}{a\sqrt[4]{1-ax}} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} + \frac{5}{}$$

Mathematica [A] time = 0.24, size = 174, normalized size = 0.70

$$\frac{\frac{40e^{\frac{1}{2}\tanh^{-1}(ax)}}{e^{2\tanh^{-1}(ax)}+1} + \frac{32e^{\frac{5}{2}\tanh^{-1}(ax)}}{e^{2\tanh^{-1}(ax)}+1} + 5\sqrt{2}\,\log\left(-\sqrt{2}\,e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)} + 1\right) - 5\sqrt{2}\,\log\left(\sqrt{2}\,e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)}\right.}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] ((40*E^(ArcTanh[a*x]/2))/(1 + E^(2*ArcTanh[a*x])) + (32*E^((5*ArcTanh[a*x])
/2))/(1 + E^(2*ArcTanh[a*x])) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcTanh[a*
```

```
x]/2)] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcTanh[a*x]/2)] + 5*Sqrt[2]*Log[
1 - Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2
]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]])/(4*a)
```

fricas [B] time = 0.85, size = 512, normalized size = 2.07

$$\frac{20\sqrt{2}\,a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^3\sqrt{\frac{\sqrt{2}\left(a^2x-a\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}\,a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-1\right)+20}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^2*x -
a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(
a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - 1) + 20*sqrt(2)*a*(a^(-4))^
(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1
)/(a*x - 1))*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 +
1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))*(a^(-4))^(3/4) + 1) - 5*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^2*x -
 a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt
(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 5*sqrt(2)*a*(a^(-4))^(1/4)*log(
-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) -
(a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(a*x - 9)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

3.85 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=248

$$\frac{8\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{ax\ldots}}\right.$$

```
[Out] 8*(a*x+1)^(1/4)/(-a*x+1)^(1/4)-2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-2*arc
tanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))+1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^
(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*2^(1/2)-1/2*ln(1+(-a*x+1)^(1/4)*2^(1/2)
/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*2^(1/2)+arctan(-1+(-a*x+1)^(1/
4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/2)+arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(
1/4))*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6126, 98, 21, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{8\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{ax\ldots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x,x]
```

```
[Out] (8*(1 + a*x)^(1/4))/(1 - a*x)^(1/4) - 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1
/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] + Sqrt
[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 +
a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2
]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x}\,dx = \int \frac{(1+ax)^{5/4}}{x(1-ax)^{5/4}}\,dx$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - \frac{4\int \frac{-\frac{a}{4}+\frac{a^2x}{4}}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx}{a}$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} + \int \frac{(1-ax)^{3/4}}{x(1+ax)^{3/4}}\,dx$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - a\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx + \int \frac{1}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} + 4\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right) + 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 4\,\text{Su}$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}}$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\log}{}$$
$$= \frac{8\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.$$

Mathematica [C] time = 0.04, size = 93, normalized size = 0.38

$$\frac{4\left(3\sqrt[4]{2}\,(ax+1)^{3/4}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-ax)\right) + (ax-1)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right) + 3ax + 3\right)}{3\sqrt[4]{1-ax}\,(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x,x]
```

[Out] (4*(3 + 3*a*x + 3*2^(1/4)*(1 + a*x)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4
, (1 - a*x)/2] + (-1 + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a
x)]))/(3(1 - a*x)^(1/4)*(1 + a*x)^(3/4))

fricas [B] time = 0.76, size = 442, normalized size = 1.78

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\sqrt{2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")

[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1)) - 1) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2)
*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a
*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/2*sqrt(2)*l
og(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a
^2*x^2 + 1) - 1)/(a*x - 1)) + 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) + 8*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*
x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + log(sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)) - 1)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x,x)
```

```
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(5/2)/x, x)
```

3.86 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=95

$$-\frac{(ax+1)^{5/4}}{x\sqrt[4]{1-ax}}+\frac{10a\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-5a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-5a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] 10*a*(a*x+1)^(1/4)/(-a*x+1)^(1/4)-(a*x+1)^(5/4)/x/(-a*x+1)^(1/4)-5*a*arctan
((a*x+1)^(1/4)/(-a*x+1)^(1/4))-5*a*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 212, 206, 203}

$$-\frac{(ax+1)^{5/4}}{x\sqrt[4]{1-ax}}+\frac{10a\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-5a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-5a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x^2,x]
```

```
[Out] (10*a*(1 + a*x)^(1/4))/(1 - a*x)^(1/4) - (1 + a*x)^(5/4)/(x*(1 - a*x)^(1/4)
) - 5*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - 5*a*ArcTanh[(1 + a*x)^(1/
4)/(1 - a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1+ax)^{5/4}}{x^2(1-ax)^{5/4}}\,dx\\
&= -\frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}} + \frac{1}{2}(5a)\int \frac{\sqrt[4]{1+ax}}{x(1-ax)^{5/4}}\,dx\\
&= \frac{10a\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - \frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}} + \frac{1}{2}(5a)\int \frac{1}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx\\
&= \frac{10a\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - \frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}} + (10a)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= \frac{10a\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - \frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}} - (5a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - (5a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,d\right.\\
&= \frac{10a\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}} - \frac{(1+ax)^{5/4}}{x\sqrt[4]{1-ax}} - 5a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 5a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 74, normalized size = 0.78

$$\frac{3\left(9a^2x^2+8ax-1\right)+10ax(ax-1)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)}{3x\sqrt[4]{1-ax}\,(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x^2,x]
```

```
[Out] (3*(-1 + 8*a*x + 9*a^2*x^2) + 10*a*x*(-1 + a*x)*Hypergeometric2F1[3/4, 1, 7
/4, (1 - a*x)/(1 + a*x)])/(3*x*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))
```

fricas [A] time = 0.43, size = 125, normalized size = 1.32

$$-\frac{10\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+5\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-5\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\,(9\,ax-1)\sqrt{-\cdots}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(10*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 5*a*x*log(sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 5*a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*
x - 1)) - 1) - 2*(9*a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

3.87 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=136

$$\frac{25a^2\sqrt[4]{ax+1}}{2\sqrt[4]{1-ax}}-\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(ax+1)^{9/4}}{2x^2\sqrt[4]{1-ax}}-\frac{5a(ax+1)^{5/4}}{4x\sqrt[4]{1-ax}}$$

```
[Out] 25/2*a^2*(a*x+1)^(1/4)/(-a*x+1)^(1/4)-5/4*a*(a*x+1)^(5/4)/x/(-a*x+1)^(1/4)-
1/2*(a*x+1)^(9/4)/x^2/(-a*x+1)^(1/4)-25/4*a^2*arctan((a*x+1)^(1/4)/(-a*x+1)
^(1/4))-25/4*a^2*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6126, 96, 94, 93, 212, 206, 203}

$$\frac{25a^2\sqrt[4]{ax+1}}{2\sqrt[4]{1-ax}}-\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(ax+1)^{9/4}}{2x^2\sqrt[4]{1-ax}}-\frac{5a(ax+1)^{5/4}}{4x\sqrt[4]{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x^3,x]
```

```
[Out] (25*a^2*(1 + a*x)^(1/4))/(2*(1 - a*x)^(1/4)) - (5*a*(1 + a*x)^(5/4))/(4*x*(
1 - a*x)^(1/4)) - (1 + a*x)^(9/4)/(2*x^2*(1 - a*x)^(1/4)) - (25*a^2*ArcTan[
(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (25*a^2*ArcTanh[(1 + a*x)^(1/4)/(1 -
a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{(1+ax)^{5/4}}{x^3(1-ax)^{5/4}}\,dx$$
$$= -\frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} + \frac{1}{4}(5a)\int \frac{(1+ax)^{5/4}}{x^2(1-ax)^{5/4}}\,dx$$
$$= -\frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} + \frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1+ax}}{x(1-ax)^{5/4}}\,dx$$
$$= \frac{25a^2\sqrt[4]{1+ax}}{2\sqrt[4]{1-ax}} - \frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} + \frac{1}{8}\left(25a^2\right)\int \frac{1}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$
$$= \frac{25a^2\sqrt[4]{1+ax}}{2\sqrt[4]{1-ax}} - \frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} + \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right.$$
$$= \frac{25a^2\sqrt[4]{1+ax}}{2\sqrt[4]{1-ax}} - \frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} - \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= \frac{25a^2\sqrt[4]{1+ax}}{2\sqrt[4]{1-ax}} - \frac{5a(1+ax)^{5/4}}{4x\sqrt[4]{1-ax}} - \frac{(1+ax)^{9/4}}{2x^2\sqrt[4]{1-ax}} - \frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

Mathematica [C] time = 0.03, size = 86, normalized size = 0.63

$$\frac{50a^2x^2(ax-1)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right) + 3\left(43a^3x^3+34a^2x^2-11ax-2\right)}{12x^2\sqrt[4]{1-ax}\,(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x^3,x]
```

```
[Out] (3*(-2 - 11*a*x + 34*a^2*x^2 + 43*a^3*x^3) + 50*a^2*x^2*(-1 + a*x)*Hypergeo
metric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)])/(12*x^2*(1 - a*x)^(1/4)*(1 + a
*x)^(3/4))
```

fricas [A] time = 0.57, size = 145, normalized size = 1.07

$$-\frac{50\,a^2x^2\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right) + 25\,a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right) - 25\,a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right) - 2\left(43\,a^2x\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(50*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 25*a^2*x^2*l
og(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 25*a^2*x^2*log(sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1)) - 1) - 2*(43*a^2*x^2 - 9*a*x - 2)*sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1)))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{5/2}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

3.88 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=165

$$\frac{287a^3\sqrt[4]{ax+1}}{24\sqrt[4]{1-ax}}-\frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{55}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{61a^2\sqrt[4]{ax+1}}{24x\sqrt[4]{1-ax}}-\frac{\sqrt[4]{ax+1}}{3x^3\sqrt[4]{1-ax}}-\frac{13a\sqrt[4]{ax+1}}{12x^2\sqrt[4]{1-ax}}$$

```
[Out] 287/24*a^3*(a*x+1)^(1/4)/(-a*x+1)^(1/4)-1/3*(a*x+1)^(1/4)/x^3/(-a*x+1)^(1/4
)-13/12*a*(a*x+1)^(1/4)/x^2/(-a*x+1)^(1/4)-61/24*a^2*(a*x+1)^(1/4)/x/(-a*x+
1)^(1/4)-55/8*a^3*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-55/8*a^3*arctanh((a*
x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6126, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{287a^3\sqrt[4]{ax+1}}{24\sqrt[4]{1-ax}}-\frac{61a^2\sqrt[4]{ax+1}}{24x\sqrt[4]{1-ax}}-\frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{55}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{13a\sqrt[4]{ax+1}}{12x^2\sqrt[4]{1-ax}}-\frac{\sqrt[4]{ax+1}}{3x^3\sqrt[4]{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x^4,x]
```

```
[Out] (287*a^3*(1 + a*x)^(1/4))/(24*(1 - a*x)^(1/4)) - (1 + a*x)^(1/4)/(3*x^3*(1
- a*x)^(1/4)) - (13*a*(1 + a*x)^(1/4))/(12*x^2*(1 - a*x)^(1/4)) - (61*a^2*(
1 + a*x)^(1/4))/(24*x*(1 - a*x)^(1/4)) - (55*a^3*ArcTan[(1 + a*x)^(1/4)/(1
- a*x)^(1/4)])/8 - (55*a^3*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1+ax)^{5/4}}{x^4(1-ax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{1}{3}\int \frac{-\frac{13a}{2}-6a^2x}{x^3(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} + \frac{1}{6}\int \frac{\frac{61a^2}{4}+13a^3x}{x^2(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} - \frac{1}{6}\int \frac{-\frac{165a^3}{8}-\frac{61a^4x}{4}}{x(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$

$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} + \frac{\int \frac{165a^4}{16x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx}{3a}$$

$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} + \frac{1}{16}\left(55a^3\right)\int \frac{1}{x\sqrt[4]{1-ax}\,(1+ \cdots}$$

$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} + \frac{1}{4}\left(55a^3\right)\mathrm{Subst}\left(\int \frac{1}{-1+x \cdots}\right.$$

$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} - \frac{1}{8}\left(55a^3\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2} \cdots\right.$$

$$= \frac{287a^3\sqrt[4]{1+ax}}{24\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{3x^3\sqrt[4]{1-ax}} - \frac{13a\sqrt[4]{1+ax}}{12x^2\sqrt[4]{1-ax}} - \frac{61a^2\sqrt[4]{1+ax}}{24x\sqrt[4]{1-ax}} - \frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \cdots$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.55

$$\frac{287a^4x^4+110a^3x^3(ax-1)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)+226a^3x^3-87a^2x^2-34ax-8}{24x^3\sqrt[4]{1-ax}\,(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x^4,x]
```

```
[Out] (-8 - 34*a*x - 87*a^2*x^2 + 226*a^3*x^3 + 287*a^4*x^4 + 110*a^3*x^3*(-1 + a
*x)*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)])/(24*x^3*(1 - a*x)^
(1/4)*(1 + a*x)^(3/4))
```

fricas [A] time = 0.62, size = 153, normalized size = 0.93

$$-\frac{330\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+165\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-165\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2\,(287}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(330*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 165*a^3*x^
3*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 165*a^3*x^3*log(sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(287*a^3*x^3 - 61*a^2*x^2 - 26*a*x - 8)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

```
[Out] Timed out
```

3.89 $\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=194

$$\frac{2467a^4\sqrt[4]{ax+1}}{192\sqrt[4]{1-ax}}-\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{521a^3\sqrt[4]{ax+1}}{192x\sqrt[4]{1-ax}}-\frac{113a^2\sqrt[4]{ax+1}}{96x^2\sqrt[4]{1-ax}}-\frac{\sqrt[4]{ax+}}{4x^4\sqrt[4]{1}}$$

```
[Out] 2467/192*a^4*(a*x+1)^(1/4)/(-a*x+1)^(1/4)-1/4*(a*x+1)^(1/4)/x^4/(-a*x+1)^(1
/4)-17/24*a*(a*x+1)^(1/4)/x^3/(-a*x+1)^(1/4)-113/96*a^2*(a*x+1)^(1/4)/x^2/(
-a*x+1)^(1/4)-521/192*a^3*(a*x+1)^(1/4)/x/(-a*x+1)^(1/4)-475/64*a^4*arctan(
(a*x+1)^(1/4)/(-a*x+1)^(1/4))-475/64*a^4*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/
4))
```

Rubi [A] time = 0.09, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6126, 98, 151, 155, 12, 93, 212, 206, 203}

$$-\frac{113a^2\sqrt[4]{ax+1}}{96x^2\sqrt[4]{1-ax}}+\frac{2467a^4\sqrt[4]{ax+1}}{192\sqrt[4]{1-ax}}-\frac{521a^3\sqrt[4]{ax+1}}{192x\sqrt[4]{1-ax}}-\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{17a\sqrt[4]{}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcTanh[a*x])/2)/x^5,x]
```

```
[Out] (2467*a^4*(1 + a*x)^(1/4))/(192*(1 - a*x)^(1/4)) - (1 + a*x)^(1/4)/(4*x^4*(
1 - a*x)^(1/4)) - (17*a*(1 + a*x)^(1/4))/(24*x^3*(1 - a*x)^(1/4)) - (113*a^
2*(1 + a*x)^(1/4))/(96*x^2*(1 - a*x)^(1/4)) - (521*a^3*(1 + a*x)^(1/4))/(19
2*x*(1 - a*x)^(1/4)) - (475*a^4*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
 - (475*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{(1+ax)^{5/4}}{x^5(1-ax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{1}{4}\int \frac{-\frac{17a}{2}-8a^2x}{x^4(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} + \frac{1}{12}\int \frac{\frac{113a^2}{4}+\frac{51a^3x}{2}}{x^3(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{1}{24}\int \frac{-\frac{521a^3}{8}-\frac{113a^4x}{2}}{x^2(1-ax)^{5/4}(1+ax)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} + \frac{1}{24}\int \frac{\frac{1425a^4}{16}+\frac{521a^5x}{8}}{x(1-ax)^{5/4}(1+a$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} - \frac{\int -\frac{}{32x}}{}$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} + \frac{1}{128}\Big(4$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} + \frac{1}{32}\Big(47$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} - \frac{1}{64}\Big(47$$

$$= \frac{2467a^4\sqrt[4]{1+ax}}{192\sqrt[4]{1-ax}} - \frac{\sqrt[4]{1+ax}}{4x^4\sqrt[4]{1-ax}} - \frac{17a\sqrt[4]{1+ax}}{24x^3\sqrt[4]{1-ax}} - \frac{113a^2\sqrt[4]{1+ax}}{96x^2\sqrt[4]{1-ax}} - \frac{521a^3\sqrt[4]{1+ax}}{192x\sqrt[4]{1-ax}} - \frac{475}{64}a^4$$

Mathematica [C] time = 0.03, size = 99, normalized size = 0.51

$$\frac{2467a^5x^5+950a^4x^4(ax-1)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)+1946a^4x^4-747a^3x^3-362a^2x^2-184ax-48}{192x^4\sqrt[4]{1-ax}\,(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcTanh[a*x])/2)/x^5,x]
```

```
[Out] (-48 - 184*a*x - 362*a^2*x^2 - 747*a^3*x^3 + 1946*a^4*x^4 + 2467*a^5*x^5 +
950*a^4*x^4*(-1 + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)])
/(192*x^4*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))
```

fricas [A] time = 0.66, size = 161, normalized size = 0.83

$$-\frac{2850\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+1425\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-1425\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)-2}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/384*(2850*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 1425*a^4
*x^4*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1425*a^4*x^4*log(sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 2*(2467*a^4*x^4 - 521*a^3*x^3 - 226*a^
2*x^2 - 136*a*x - 48)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x^5,x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
[Out] Timed out
```

3.90 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{4},\frac{1}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,-1/4,1/4,2+m,a*x,-a*x)/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{4},\frac{1}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(ArcTanh[a*x]/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -1/4, 1/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{1}{2}\tanh^{-1}(ax)} x^m \, dx &= \int \frac{x^m \sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} \, dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{1}{4},\frac{1}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^(ArcTanh[a*x]/2),x]
[Out] Integrate[x^m/E^(ArcTanh[a*x]/2), x]
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a*x + 
1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(x^m/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x^m/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
[Out] int(x^m/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
[Out] Integral(x**m/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

3.91 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=290

$$-\frac{(25-4ax)(1-ax)^{5/4}(ax+1)^{3/4}}{96a^4}-\frac{11\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{64a^4}-\frac{11\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}\,a^4}+\frac{11\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{ax+}}\right.}{128\sqrt{2}\,a^4}$$

```
[Out] -11/64*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a^4-1/4*x^2*(-a*x+1)^(5/4)*(a*x+1)^(3/4
)/a^2-1/96*(-4*a*x+25)*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/a^4+11/128*arctan(-1+(-
a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^4*2^(1/2)+11/128*arctan(1+(-a*x+1)^(1
/4)*2^(1/2)/(a*x+1)^(1/4))/a^4*2^(1/2)-11/256*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(
a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4*2^(1/2)+11/256*ln(1+(-a*x+1)
^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 100, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^2(1-ax)^{5/4}(ax+1)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(ax+1)^{3/4}}{96a^4}-\frac{11\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{64a^4}-\frac{11\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}\,a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(ArcTanh[a*x]/2),x]
```

```
[Out] (-11*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*a^4) - (x^2*(1 - a*x)^(5/4)*(1 +
a*x)^(3/4))/(4*a^2) - ((25 - 4*a*x)*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(96*a^
4) - (11*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]
*a^4) + (11*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt
[2]*a^4) - (11*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/
4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4) + (11*Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^3\,dx=\int \frac{x^3\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$

$$=-\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2}-\frac{\int \frac{x\sqrt[4]{1-ax}\left(-2+\frac{ax}{2}\right)}{\sqrt[4]{1+ax}}\,dx}{4a^2}$$

$$=-\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4}-\frac{11\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{64a^3}$$

$$=-\frac{11\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4}-\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4}-\frac{11}{}$$

$$=-\frac{11\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4}-\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4}+\frac{11}{}$$

$$=-\frac{11\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4}-\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4}+\frac{11}{}$$

$$=-\frac{11\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4}-\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4}+\frac{11}{}$$

$$=-\frac{11\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4}-\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4}+\frac{11}{}$$

$$=-\frac{11\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4}-\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4}-\frac{11}{}$$

$$=-\frac{11\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4}-\frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2}-\frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4}-\frac{11}{}$$

<u>**Mathematica**</u> **[C]** time = 0.11, size = 116, normalized size = 0.40

$$\frac{(1-ax)^{5/4}\left(-5a^2x^2(ax+1)^{3/4}+4\,2^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-ax)\right)-12\,2^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-ax)\right)+5\,2^{3/4}\,{}_2F_1\left(\frac{1}{4}\right.\right.}{20a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(ArcTanh[a*x]/2),x]
```

```
[Out] ((1 - a*x)^(5/4)*(-5*a^2*x^2*(1 + a*x)^(3/4) + 4*2^(3/4)*Hypergeometric2F1[
-7/4, 5/4, 9/4, (1 - a*x)/2] - 12*2^(3/4)*Hypergeometric2F1[-3/4, 5/4, 9/4,
 (1 - a*x)/2] + 5*2^(3/4)*Hypergeometric2F1[1/4, 5/4, 9/4, (1 - a*x)/2]))/(
20*a^4)
```

fricas [B] time = 0.60, size = 557, normalized size = 1.92

$$132\sqrt{2}a^4\frac{1}{a^{16}}^{\frac{1}{4}}\arctan\left(\sqrt{2}a^4\sqrt{\frac{\sqrt{2}\left(a^{13}x-a^{12}\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^{16}}^{\frac{3}{4}}+\left(a^9x-a^8\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^{16}}^{\frac{1}{4}}-\sqrt{2}a^4\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\frac{1}{a^{16}}^{\frac{1}{4}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(132*sqrt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt((sqrt(2)*(a^
13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + (a^9*x -
 a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/4) - sqrt
(2)*a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - 1) + 132*sqrt
(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt(-(sqrt(2)*(a^13*x - a^12)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^(
-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/4) - sqrt(2)*a^4*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + 1) + 33*sqrt(2)*a^4*(a^(-16
))^(1/4)*log((sqrt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(
a^(-16))^(3/4) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1
)) - 33*sqrt(2)*a^4*(a^(-16))^(1/4)*log(-(sqrt(2)*(a^13*x - a^12)*sqrt(-sqr
t(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^(-16)) +
sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(48*a^3*x^3 - 56*a^2*x^2 + 58*a*x - 83)*
sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(x^3/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

3.92 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=282

$$\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{12a^3}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}}{8a^3}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}+\frac{3\tan}{}$$

```
[Out] 3/8*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a^3+1/12*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/a^3-
1/3*x*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/a^2-3/16*arctan(-1+(-a*x+1)^(1/4)*2^(1/2
)/(a*x+1)^(1/4))/a^3*2^(1/2)-3/16*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(
1/4))/a^3*2^(1/2)+3/32*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(
1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)-3/32*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1
/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x(ax+1)^{3/4}(1-ax)^{5/4}}{3a^2}+\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{12a^3}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}}{8a^3}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}-\frac{3\log\left(\frac{\sqrt{1-}}{\sqrt{ax}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(ArcTanh[a*x]/2),x]
```

```
[Out] (3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(8*a^3) + ((1 - a*x)^(5/4)*(1 + a*x)^(3
/4))/(12*a^3) - (x*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(3*a^2) + (3*ArcTan[1 -
 (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (3*ArcTan[1
+ (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (3*Log[1 +
Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(
16*Sqrt[2]*a^3) - (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*
x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

```
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-\frac{1}{2}\tanh^{-1}(ax)} x^2\,dx = \int \frac{x^2\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{\int \frac{\sqrt[4]{1-ax}\left(-1+\frac{ax}{2}\right)}{\sqrt[4]{1+ax}}\,dx}{3a^2}$$

$$= \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} + \frac{3\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{8a^2}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} + \frac{3\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+a\cdots}}}{16a^2}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-\cdots}}\cdots\right.}{4\cdots}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\cdots\right.}{4a\cdots}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{3\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\cdots\right.}{8a\cdots}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} - \frac{3\,\mathrm{Subst}\left(\int \frac{\cdots}{1-\sqrt{2}\cdots}\cdots\right.}{1\cdots}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} + \frac{3\log\left(1+\frac{\sqrt{1-a\cdots}}{\sqrt{1+a\cdots}}\cdots\right.}{16\sqrt{2}\cdots}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\sqrt[4]{\cdots}}\cdots\right.}{8\sqrt{2}\,a^3}$$

Mathematica [C] time = 0.03, size = 62, normalized size = 0.22

$$-\frac{(1-ax)^{5/4}\left(9\,2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-ax)\right)+5(ax+1)^{3/4}(4ax-1)\right)}{60a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(ArcTanh[a*x]/2),x]
```

```
[Out] -1/60*((1 - a*x)^(5/4)*(5*(1 + a*x)^(3/4)*(-1 + 4*a*x) + 9*2^(3/4)*Hypergeo
metric2F1[1/4, 5/4, 9/4, (1 - a*x)/2]))/a^3
```

fricas [B] time = 0.56, size = 549, normalized size = 1.95

$$-\frac{36\sqrt{2}\,a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^3\sqrt{\frac{\sqrt{2}\left(a^{10}x-a^9\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-\sqrt{2}\,a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/96*(36*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^1
0*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + (a^7*x - a
^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - sqrt(2
)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - 1) + 36*sqrt(2)
*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^10*x - a^9)*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*x - a^6)*sqrt(a^(-12)
) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - sqrt(2)*a^3*sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + 1) + 9*sqrt(2)*a^3*(a^(-12))^(1
/4)*log((sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12
))^(3/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 9
*sqrt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x
^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a^
2*x^2 + 1))/(a*x - 1)) - 4*(8*a^2*x^2 - 10*a*x + 11)*sqrt(-a^2*x^2 + 1)*sqr
t(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

3.93 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=255

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{2a^2}-\frac{(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a^2}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-\frac{\tan^{-1}\left(1\right.}{4}$$

```
[Out] -1/4*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a^2-1/2*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/a^2+
1/8*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^2*2^(1/2)+1/8*arctan(
1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^2*2^(1/2)-1/16*ln(1-(-a*x+1)^(1/4
)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(1/2)+1/16*ln(1
+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{2a^2}-\frac{(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a^2}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-\frac{\tan^{-1}\left(1\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(ArcTanh[a*x]/2),x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*a^2) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/
4))/(2*a^2) - ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt
[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(4*Sqrt[2]
*a^2) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/(8*Sqrt[2]*a^2) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt
[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(8*Sqrt[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-\frac{1}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{4a}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{8a}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a^2}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2} + \frac{\text{Subst}\left(\int \cdots\right.}{}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a^2} + \frac{\text{Subst}\cdots}{}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}\,a^2} + \frac{\log\left(1+\cdots\right.}{}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}\,a^2} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-\cdots}}{\sqrt[4]{1+a\cdots}}\right.}{4\sqrt{2}\,a^2}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.22

$$\frac{(1-ax)^{5/4}\left(2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-ax)\right)-5(ax+1)^{3/4}\right)}{10a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(ArcTanh[a*x]/2),x]
```

```
[Out] ((1 - a*x)^(5/4)*(-5*(1 + a*x)^(3/4) + 2^(3/4)*Hypergeometric2F1[1/4, 5/4,
9/4, (1 - a*x)/2]))/(10*a^2)
```

fricas [B] time = 0.54, size = 540, normalized size = 2.12

$$4\sqrt{2}\,a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^2\sqrt{\frac{\sqrt{2}\left(a^7x-a^6\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{3}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{1}{4}}-\sqrt{2}\,a^2\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{1}{4}}-1\right)+4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt((sqrt(2)*(a^7*x
- a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a^5*x - a^4)*s
qrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - 1) + 4*sqrt(2)*a^2*(a^(
-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt(-(sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*
x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2
*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-8))^(1/4) + 1) + sqrt(2)*a^2*(a^(-8))^(1/4)*log((sqrt(2)*(
a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a^5*x -
a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - sqrt(2)*a^2*(a^(-8))^(
1/4)*log(-(sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8
))^(3/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*
sqrt(-a^2*x^2 + 1)*(2*a*x - 3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

[Out] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)

[Out] int(x/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)

[Out] Integral(x/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)

3.94 $\int e^{-\frac{1}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=221

$$\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}\,a}-\frac{\tan^{-1}\left(\right.}{}$$

```
[Out] (-a*x+1)^(1/4)*(a*x+1)^(3/4)/a-1/2*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)
^(1/4))/a*2^(1/2)-1/2*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1
/2)+1/4*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a*2^(1/2)-1/4*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)
/(a*x+1)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6125, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}\,a}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}\,a}-\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcTanh[a*x]/2),x]
```

```
[Out] ((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/a + ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/
(1 + a*x)^(1/4)]/(Sqrt[2]*a) - ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*
x)^(1/4)]/(Sqrt[2]*a) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 -
 a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a) - Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(2*Sqrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-\frac{1}{2}\tanh^{-1}(ax)}\,dx &= \int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx\\
&= \frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a}+\frac{1}{2}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\\
&= \frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a}-\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx,x,\sqrt[4]{1-ax}\right)}{a}\\
&= \frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a}-\frac{2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}\\
&= \frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a}-\frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}-\frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}\\
&= \frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a}-\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a}-\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a}-\\
&= \frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a}+\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a}-\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a}-\frac{\text{Subst}\Big(}{}\\
&= \frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a}-\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a}+\frac{\log\Big(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}}{}}{2\sqrt{2}\,a}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 35, normalized size = 0.16

$$\frac{8e^{\frac{3}{2}\tanh^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\tanh^{-1}(ax)}\right)}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-1/2*ArcTanh[a*x]),x]
```

```
[Out] (8*E^((3*ArcTanh[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcTanh[a*x]
)])/(3*a)
```

fricas [B] time = 1.10, size = 518, normalized size = 2.34

$$-\frac{4\sqrt{2}\,a\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^4}^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^4}^{\frac{1}{4}}-\sqrt{2}\,a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^4}^{\frac{1}{4}}-1\right)+4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x - a^
3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(
a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 4*sqrt(2)*a*(a^(-4))^(1/
4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))
/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*
(a^(-4))^(1/4) + 1) + sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*x - a^3)*s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(a^(-
4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - sqrt(2)*a*(a^(-4))^(1/4)*log(-(sqrt(
2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*
x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*sqrt(-a^2*x^2 +
1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
[Out] int(1/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1)), x)
```

3.95 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=227

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)+$$

```
[Out] 2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-2*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/
4))-1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1
/2))*2^(1/2)+1/2*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(
a*x+1)^(1/2))*2^(1/2)+arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/
2)+arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6126, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x),x]
```

```
[Out] 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
- a*x)^(1/4))/(1 + a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Log[1 +
Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sq
rt[2] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

```
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[4]{1-ax}}{x\sqrt[4]{1+ax}}\,dx$$
$$= -\left(a\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\right) + \int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 4\,\text{Subst}\Bigg(\int$$
$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) + 2\,\text{Subst}\Bigg($$
$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\text{Subst}\Big(\int}{}$$
$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} + \frac{\log\Big(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}{\sqrt{2}}$$
$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\tanh^{-}$$

Mathematica [C] time = 0.03, size = 83, normalized size = 0.37

$$2\,2^{3/4}\sqrt[4]{1-ax}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-ax)\right) - \frac{4\sqrt[4]{1-ax}\;{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{1-ax}{-ax-1}\right)}{\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x),x]
```

```
[Out] 2*2^(3/4)*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - a*x)/2] - (
4*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - a*x)/(-1 - a*x))])/
(1 + a*x)^(1/4)
```

fricas [B] time = 0.66, size = 417, normalized size = 1.84

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}\,(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\sqrt{2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1)) - 1) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2)
*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a
*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 1/2*sqrt(2)*l
og(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a
^2*x^2 + 1) - 1)/(a*x - 1)) - 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) + 2*
arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)/
(a*x - 1)) + 1) + log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(1/(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)),x)
[Out] int(1/(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x,x)
[Out] Integral(1/(x*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))), x)
```

3.96 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{x}-a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] -(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x-a*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))+a*ar
ctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 298, 203, 206}

$$-\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{x}-a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x^2),x]
```

```
[Out] -(((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/x) - a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)
^(1/4)] + a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt[4]{1-ax}}{x^2\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{x} - \frac{1}{2}a\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{x} - (2a)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{x} + a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{x} - a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.76

$$\frac{\sqrt[4]{1-ax}\left(2ax\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)-ax-1\right)}{x\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x^2),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-1 - a*x + 2*a*x*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)
/(1 + a*x)]))/(x*(1 + a*x)^(1/4))
```

fricas [B] time = 0.51, size = 130, normalized size = 1.81

$$-\frac{2\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - a*x*log(sqrt(-sqr
t(-a^2*x^2 + 1)/(a*x - 1)) + 1) + a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1
)) - 1) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))), x)
```

3.97 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=110

$$\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{5/4}(ax+1)^{3/4}}{2x^2}+\frac{a\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{4x}$$

```
[Out] 1/4*a*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x-1/2*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/x^2+1
/4*a^2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-1/4*a^2*arctanh((a*x+1)^(1/4)/(
-a*x+1)^(1/4))
```

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6126, 96, 94, 93, 298, 203, 206}

$$\frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{5/4}(ax+1)^{3/4}}{2x^2}+\frac{a\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x^3),x]
```

```
[Out] (a*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/4
))/(2*x^2) + (a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (a^2*ArcTanh
[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[4]{1-ax}}{x^3\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} - \frac{1}{4}a\int \frac{\sqrt[4]{1-ax}}{x^2\sqrt[4]{1+ax}}\,dx$$
$$= \frac{a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} + \frac{1}{8}a^2\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= \frac{a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= \frac{a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{1}{4}a$$
$$= \frac{a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2x^2} + \frac{1}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{1}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.63

$$\frac{\sqrt[4]{1-ax}\left(-2a^2x^2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)+3a^2x^2+ax-2\right)}{4x^2\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x^3),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-2 + a*x + 3*a^2*x^2 - 2*a^2*x^2*Hypergeometric2F1[1/4, 1
, 5/4, (1 - a*x)/(1 + a*x)]))/(4*x^2*(1 + a*x)^(1/4))
```

fricas [A] time = 0.52, size = 148, normalized size = 1.35

$$\frac{2\,a^2x^2\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right) - a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right) + a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right) + 2\sqrt{-a^2x^2+1}\,(3\,a}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - a^2*x^2*log(sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + a^2*x^2*log(sqrt(-sqrt(-a^2*x^2 +
```

```
1)/(a*x - 1)) - 1) + 2*sqrt(-a^2*x^2 + 1)*(3*a*x - 2)*sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1)))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(1/(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(1/(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)),x)
[Out] int(1/(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
[Out] Integral(1/(x**3*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))), x)
```

3.98 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{3}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11a^2\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{24x}-\frac{\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{3x^3}+\frac{5a\sqrt[4]{1-ax}\,(ax+}{12x^2}$$

```
[Out] -1/3*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x^3+5/12*a*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x
^2-11/24*a^2*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x-3/8*a^3*arctan((a*x+1)^(1/4)/(-
a*x+1)^(1/4))+3/8*a^3*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 298, 203, 206}

$$-\frac{11a^2\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{24x}-\frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{3}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{5a\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{12x^2}-\frac{\sqrt[4]{1-ax}\,(ax+}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x^4),x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(3*x^3) + (5*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(12*x^2) - (11*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(24*x) - (3*a^3
*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 + (3*a^3*ArcTanh[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{\sqrt[4]{1-ax}}{x^4\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} + \frac{1}{3}\int \frac{-\frac{5a}{2}+2a^2x}{x^3(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{1}{6}\int \frac{-\frac{11a^2}{4}+\frac{5a^3x}{2}}{x^2(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} + \frac{1}{6}\int -\frac{9}{8x(1-ax)}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} - \frac{1}{16}\left(3a^3\right)\int \frac{}{x(1-}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} - \frac{1}{4}\left(3a^3\right)\text{Subst}\Bigg(\int$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} + \frac{1}{8}\left(3a^3\right)\text{Subst}\Bigg(\int$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{3x^3} + \frac{5a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{12x^2} - \frac{11a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x} - \frac{3}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.02, size = 78, normalized size = 0.56

$$\frac{\sqrt[4]{1-ax}\left(18a^3x^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)-11a^3x^3-a^2x^2+2ax-8\right)}{24x^3\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x^4),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-8 + 2*a*x - a^2*x^2 - 11*a^3*x^3 + 18*a^3*x^3*Hypergeome
tric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(24*x^3*(1 + a*x)^(1/4))
```

fricas [A] time = 0.50, size = 157, normalized size = 1.13

$$-\frac{18\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-9\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+9\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\left(11\,a^2x^2-1\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(18*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 9*a^3*x^3*l
og(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 9*a^3*x^3*log(sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1)) - 1) + 2*(11*a^2*x^2 - 10*a*x + 8)*sqrt(-a^2*x^2 + 1)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)),x)
[Out] int(1/(x^4*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
[Out] Integral(1/(x**4*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))), x)
```

3.99 $\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=168

$$\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{83a^3\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{192x}-\frac{29a^2\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{96x^2}-\frac{\sqrt[4]{1-ax}\,(}{4}$$

```
[Out] -1/4*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x^4+7/24*a*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x
^3-29/96*a^2*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/x^2+83/192*a^3*(-a*x+1)^(1/4)*(a*
x+1)^(3/4)/x+11/64*a^4*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-11/64*a^4*arcta
nh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 298, 203, 206}

$$-\frac{29a^2\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{96x^2}+\frac{83a^3\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{192x}+\frac{11}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{11}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{7a\sqrt[4]{1-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcTanh[a*x]/2)*x^5),x]
```

```
[Out] -((1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*x^4) + (7*a*(1 - a*x)^(1/4)*(1 + a*x)
^(3/4))/(24*x^3) - (29*a^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(96*x^2) + (83*
a^3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(192*x) + (11*a^4*ArcTan[(1 + a*x)^(1/
4)/(1 - a*x)^(1/4)])/64 - (11*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{\sqrt[4]{1-ax}}{x^5\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} + \frac{1}{4}\int \frac{-\frac{7a}{2}+3a^2x}{x^4(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x^3} - \frac{1}{12}\int \frac{-\frac{29a^2}{4}+7a^3x}{x^3(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{96x^2} + \frac{1}{24}\int \frac{-\frac{83a}{8}\cdots}{x^2(1-a\cdots}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}\,(\cdots}{192x\cdots}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}\,(\cdots}{192x\cdots}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}\,(\cdots}{192x\cdots}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}\,(\cdots}{192x\cdots}$$

$$= -\frac{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4x^4} + \frac{7a\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{24x^3} - \frac{29a^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{96x^2} + \frac{83a^3\sqrt[4]{1-ax}\,(\cdots}{192x\cdots}$$

Mathematica [C] time = 0.03, size = 86, normalized size = 0.51

$$\frac{\sqrt[4]{1-ax}\left(-66a^4x^4\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)+83a^4x^4+25a^3x^3-2a^2x^2+8ax-48\right)}{192x^4\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcTanh[a*x]/2)*x^5),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-48 + 8*a*x - 2*a^2*x^2 + 25*a^3*x^3 + 83*a^4*x^4 - 66*a^
4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(192*x^4*(1 + a
*x)^(1/4))
```

fricas [A] time = 0.51, size = 165, normalized size = 0.98

$$\frac{66\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-33\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)+33\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\left(83\,a^3x^3-\right.}{384\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/384*(66*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 33*a^4*x^4*
log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 33*a^4*x^4*log(sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)) - 1) + 2*(83*a^3*x^3 - 58*a^2*x^2 + 56*a*x - 48)*sqr
t(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/(x^5*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(1/(x^5*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)),x)
[Out] int(1/(x^5*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
[Out] Integral(1/(x**5*sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))), x)
```

3.100 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;-\frac{3}{4},\frac{3}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,-3/4,3/4,2+m,a*x,-a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{3}{4},\frac{3}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -3/4, 3/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{3}{4},\frac{3}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((3*ArcTanh[a*x])/2),x]
[Out] Integrate[x^m/E^((3*ArcTanh[a*x])/2), x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax-1)x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
[Out] integral(-(a*x - 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate(x^m/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
[Out] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate(x^m/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
[Out] int(x^m/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
[Out] Integral(x**m/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

3.101 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=290

$$-\frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{ax+1}}{32a^4}-\frac{41(1-ax)^{3/4}\sqrt[4]{ax+1}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}\,a^4}-\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{128\sqrt{2}\,a^4}$$

```
[Out] -41/64*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a^4-1/4*x^2*(-a*x+1)^(7/4)*(a*x+1)^(1/4
)/a^2-1/32*(-4*a*x+11)*(-a*x+1)^(7/4)*(a*x+1)^(1/4)/a^4+123/128*arctan(-1+(
-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^4*2^(1/2)+123/128*arctan(1+(-a*x+1)^
(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^4*2^(1/2)+123/256*ln(1-(-a*x+1)^(1/4)*2^(1/2
)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4*2^(1/2)-123/256*ln(1+(-a*
x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 100, 147, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^2(1-ax)^{7/4}\sqrt[4]{ax+1}}{4a^2}-\frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{ax+1}}{32a^4}-\frac{41(1-ax)^{3/4}\sqrt[4]{ax+1}}{64a^4}+\frac{123\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}\,a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] (-41*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(64*a^4) - (x^2*(1 - a*x)^(7/4)*(1 +
a*x)^(1/4))/(4*a^2) - ((11 - 4*a*x)*(1 - a*x)^(7/4)*(1 + a*x)^(1/4))/(32*a^
4) - (123*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2
]*a^4) + (123*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sq
rt[2]*a^4) + (123*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^
(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4) - (123*Log[1 + Sqrt[1 - a*x]/Sqr
t[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx$$
$$= -\frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{\int \frac{x(1-ax)^{3/4}\left(-2+\frac{3ax}{2}\right)}{(1+ax)^{3/4}}\,dx}{4a^2}$$
$$= -\frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} - \frac{41\int \frac{(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx}{64a^3}$$
$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} - \frac{123\int}{}$$
$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} + \frac{123\,\mathrm{Su}}{}$$
$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} + \frac{123\,\mathrm{Su}}{}$$
$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} - \frac{123\,\mathrm{Su}}{}$$
$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} + \frac{123\,\mathrm{Su}}{}$$
$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} + \frac{123\log}{}$$
$$= -\frac{41(1-ax)^{3/4}\sqrt[4]{1+ax}}{64a^4} - \frac{x^2(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(11-4ax)(1-ax)^{7/4}\sqrt[4]{1+ax}}{32a^4} - \frac{123\tan}{}$$

Mathematica [C] time = 0.11, size = 116, normalized size = 0.40

$$\frac{(1-ax)^{7/4}\left(-7a^2x^2\sqrt[4]{ax+1} + 12\sqrt[4]{2}\,{}_2F_1\left(-\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-ax)\right) - 20\sqrt[4]{2}\,{}_2F_1\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-ax)\right) + 7\sqrt[4]{2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4}\right.\right.}{28a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] ((1 - a*x)^(7/4)*(-7*a^2*x^2*(1 + a*x)^(1/4) + 12*2^(1/4)*Hypergeometric2F1
[-5/4, 7/4, 11/4, (1 - a*x)/2] - 20*2^(1/4)*Hypergeometric2F1[-1/4, 7/4, 11
/4, (1 - a*x)/2] + 7*2^(1/4)*Hypergeometric2F1[3/4, 7/4, 11/4, (1 - a*x)/2]
))/(28*a^4)
```

fricas [B] time = 0.50, size = 553, normalized size = 1.91

$$492\sqrt{2}\,a^4\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^{12}\sqrt{\frac{\sqrt{2}\left(a^5x-a^4\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{1}{4}}+\left(a^9x-a^8\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{3}{4}}-\sqrt{2}\,a^{12}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{16}}\right)^{\frac{3}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/256*(492*sqrt(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^12*sqrt((sqrt(2)*(a
^5*x - a^4)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + (a^9*x -
a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(3/4) - sqrt(
2)*a^12*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) - 1) + 492*sqrt
(2)*a^4*(a^(-16))^(1/4)*arctan(sqrt(2)*a^12*sqrt(-(sqrt(2)*(a^5*x - a^4)*sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - (a^9*x - a^8)*sqrt(a^(-
16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(3/4) - sqrt(2)*a^12*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + 1) - 123*sqrt(2)*a^4*(a^(-1
6))^(1/4)*log((sqrt(2)*(a^5*x - a^4)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a
^(-16))^(1/4) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)
) + 123*sqrt(2)*a^4*(a^(-16))^(1/4)*log(-(sqrt(2)*(a^5*x - a^4)*sqrt(-sqrt(
-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - (a^9*x - a^8)*sqrt(a^(-16)) + sq
rt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(16*a^4*x^4 - 40*a^3*x^3 + 54*a^2*x^2 - 93
*a*x + 63)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
```

```
[Out] int(x^3/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
```

```
[Out] Integral(x**3/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

3.102 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=282

$$\frac{\sqrt[4]{ax+1}\,(1-ax)^{7/4}}{4a^3}+\frac{17\sqrt[4]{ax+1}\,(1-ax)^{3/4}}{24a^3}-\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}+\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}+\cdots$$

```
[Out] 17/24*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a^3+1/4*(-a*x+1)^(7/4)*(a*x+1)^(1/4)/a^3
-1/3*x*(-a*x+1)^(7/4)*(a*x+1)^(1/4)/a^2-17/16*arctan(-1+(-a*x+1)^(1/4)*2^(1
/2)/(a*x+1)^(1/4))/a^3*2^(1/2)-17/16*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1
)^(1/4))/a^3*2^(1/2)-17/32*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+
1)^(1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)+17/32*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+
1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x\sqrt[4]{ax+1}\,(1-ax)^{7/4}}{3a^2}+\frac{\sqrt[4]{ax+1}\,(1-ax)^{7/4}}{4a^3}+\frac{17\sqrt[4]{ax+1}\,(1-ax)^{3/4}}{24a^3}-\frac{17\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{16\sqrt{2}\,a^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] (17*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(24*a^3) + ((1 - a*x)^(7/4)*(1 + a*x)^
(1/4))/(4*a^3) - (x*(1 - a*x)^(7/4)*(1 + a*x)^(1/4))/(3*a^2) + (17*ArcTan[1
 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (17*ArcTan
[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (17*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(16*Sqrt[2]*a^3) + (17*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1
 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx$$
$$= -\frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{\int \frac{(1-ax)^{3/4}\left(-1+\frac{3ax}{2}\right)}{(1+ax)^{3/4}}\,dx}{3a^2}$$
$$= \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} + \frac{17\int \frac{(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx}{24a^2}$$
$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} + \frac{17\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,d}{16a^2}$$
$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{17\,\mathrm{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\right.}{4a^3}$$
$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{17\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx\right.}{4a^3}$$
$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} + \frac{17\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx\right.}{8a^3}$$
$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{17\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+}\right.}{16a^3}$$
$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} - \frac{17\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\right.}{16\sqrt{2}\,a^3}$$
$$= \frac{17(1-ax)^{3/4}\sqrt[4]{1+ax}}{24a^3} + \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{4a^3} - \frac{x(1-ax)^{7/4}\sqrt[4]{1+ax}}{3a^2} + \frac{17\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{8\sqrt{2}\,a^3}$$

Mathematica [C]   time = 0.03, size = 62, normalized size = 0.22

$$-\frac{(1-ax)^{7/4}\left(17\sqrt[4]{2}\;{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-ax)\right)+7\sqrt[4]{ax+1}\,(4ax-3)\right)}{84a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] -1/84*((1 - a*x)^(7/4)*(7*(1 + a*x)^(1/4)*(-3 + 4*a*x) + 17*2^(1/4)*Hyperge
ometric2F1[3/4, 7/4, 11/4, (1 - a*x)/2]))/a^3
```

fricas [B] time = 0.60, size = 545, normalized size = 1.93

$$-\frac{204\sqrt{2}\,a^3\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^9\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}-\sqrt{2}\,a^9\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt((sqrt(2)*(a^
4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + (a^7*x - a
^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2
)*a^9*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - 1) + 204*sqrt(2
)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*a^9*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12)
) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(3/4) - sqrt(2)*a^9*sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + 1) - 51*sqrt(2)*a^3*(a^(-12))^(
1/4)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12
))^(1/4) + (a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 5
1*sqrt(2)*a^3*(a^(-12))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x
^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a^
2*x^2 + 1))/(a*x - 1)) + 4*(8*a^3*x^3 - 22*a^2*x^2 + 37*a*x - 23)*sqrt(-sqr
t(-a^2*x^2 + 1)/(a*x - 1)))/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
[Out] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
[Out] int(x^2/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
[Out] Integral(x**2/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

3.103 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=255

$$-\frac{\sqrt[4]{ax+1}\,(1-ax)^{7/4}}{2a^2}-\frac{3\sqrt[4]{ax+1}\,(1-ax)^{3/4}}{4a^2}+\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-$$

```
[Out] -3/4*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/a^2-1/2*(-a*x+1)^(7/4)*(a*x+1)^(1/4)/a^2+
9/8*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^2*2^(1/2)+9/8*arctan(
1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^2*2^(1/2)+9/16*ln(1-(-a*x+1)^(1/4
)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(1/2)-9/16*ln(1
+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6126, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{ax+1}\,(1-ax)^{7/4}}{2a^2}-\frac{3\sqrt[4]{ax+1}\,(1-ax)^{3/4}}{4a^2}+\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-\frac{9\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] (-3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*a^2) - ((1 - a*x)^(7/4)*(1 + a*x)^(
1/4))/(2*a^2) - (9*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(
4*Sqrt[2]*a^2) + (9*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/
(4*Sqrt[2]*a^2) + (9*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*
x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2) - (9*Log[1 + Sqrt[1 - a*x]/Sqrt
[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-\frac{3}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx$$

$$= -\frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} - \frac{3\int \frac{(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx}{4a}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} - \frac{9\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx}{8a}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} + \frac{9\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} + \frac{9\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a^2}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} - \frac{9\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a^2} + \frac{9\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} + \frac{9\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a^2} + \frac{9\,\text{Sub}}{}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} + \frac{9\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}\,a^2} - \frac{9\log\left(1+\right.}{}$$

$$= -\frac{3(1-ax)^{3/4}\sqrt[4]{1+ax}}{4a^2} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2a^2} - \frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}\,a^2} + \frac{9\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{4\sqrt{2}\,a^2}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.22

$$\frac{(1-ax)^{7/4}\left(3\sqrt[4]{2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-ax)\right)-7\sqrt[4]{ax+1}\right)}{14a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((3*ArcTanh[a*x])/2),x]
```

```
[Out] ((1 - a*x)^(7/4)*(-7*(1 + a*x)^(1/4) + 3*2^(1/4)*Hypergeometric2F1[3/4, 7/4
, 11/4, (1 - a*x)/2]))/(14*a^2)
```

fricas [B] time = 1.18, size = 537, normalized size = 2.11

$$\frac{36\sqrt{2}\,a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^6\sqrt{\frac{\sqrt{2}\left(a^3x-a^2\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{1}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{3}{4}}-\sqrt{2}\,a^6\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{3}{4}}-1\right)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(36*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt((sqrt(2)*(a^3*x
 - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*x - a^4)*
sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - 1) + 36*sqrt(2)*a^2*(a
^(-8))^(1/4)*arctan(sqrt(2)*a^6*sqrt(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a
^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(3/4) - sqrt(2)*a^6*sqrt(-sqrt(-a^2*x^2 +
1)/(a*x - 1))*(a^(-8))^(3/4) + 1) - 9*sqrt(2)*a^2*(a^(-8))^(1/4)*log((sqrt(
2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + (a^5*
x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 9*sqrt(2)*a^2*(a^(
-8))^(1/4)*log(-(sqrt(2)*(a^3*x - a^2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*
(a^(-8))^(1/4) - (a^5*x - a^4)*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)
) - 4*(2*a^2*x^2 - 7*a*x + 5)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,x]=[0,0]Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming [a
,x]=[0,0]Warning, integration of abs or sign assumes constant sign by inter
vals (correct if the argument is real):Check [abs(a*t_nostep-1)]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.The choice was done assuming [a,x]=[0,0]Warning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.The choice was done assuming [a,x]=[0,0]Warning, integration of abs or si
gn assumes constant sign by intervals (correct if the argument is real):Che
```

```
ck [abs(a*t_nostep-1)]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,t_nostep]=[0,0]Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.The choice was done assuming [a,t
_nostep]=[0,0]Evaluation time: 0.75sym2poly/r2sym(const gen & e,const index
_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
[Out] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate(x/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
[Out] int(x/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
[Out] Integral(x/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2), x)
```

3.104 $\int e^{-\frac{3}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=222

$$\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{3\text{ta}}{}$$

```
[Out] (-a*x+1)^(3/4)*(a*x+1)^(1/4)/a-3/2*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)
^(1/4))/a*2^(1/2)-3/2*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1
/2)-3/4*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a*2^(1/2)+3/4*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)
/(a*x+1)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6125, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{\sqrt{2}a}-\frac{3\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-3*ArcTanh[a*x])/2),x]
```

```
[Out] ((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/a + (3*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4
))/(1 + a*x)^(1/4)])/(Sqrt[2]*a) - (3*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/
(1 + a*x)^(1/4)])/(Sqrt[2]*a) - (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (S
qrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a) + (3*Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt
[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\frac{3}{2}\tanh^{-1}(ax)}\,dx &= \int \frac{(1-ax)^{3/4}}{(1+ax)^{3/4}}\,dx \\
&= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{3}{2}\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx \\
&= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{6\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a} \\
&= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{6\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} \\
&= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{3\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} \\
&= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} \\
&= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} - \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a} + \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a} - \frac{3\,\text{Su}}{} \\
&= \frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{a} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} - \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} - \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.}{2\sqrt{2}\,a}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 150, normalized size = 0.68

$$\frac{\frac{8e^{\frac{1}{2}\tanh^{-1}(ax)}}{e^{2\tanh^{-1}(ax)}+1} - 3\sqrt{2}\log\left(-\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)} + 1\right) + 3\sqrt{2}\log\left(\sqrt{2}e^{\frac{1}{2}\tanh^{-1}(ax)} + e^{\tanh^{-1}(ax)} + 1\right) - 6\sqrt{2}\ \mathrm{t}}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-3*ArcTanh[a*x])/2),x]
```

```
[Out] ((8*E^(ArcTanh[a*x]/2))/(1 + E^(2*ArcTanh[a*x])) - 6*Sqrt[2]*ArcTan[1 - Sqr
t[2]*E^(ArcTanh[a*x]/2)] + 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcTanh[a*x]/2)]
 - 3*Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]] + 3*Sqrt[
2]*Log[1 + Sqrt[2]*E^(ArcTanh[a*x]/2) + E^ArcTanh[a*x]])/(4*a)
```

fricas [B] time = 0.71, size = 512, normalized size = 2.31

$$-\frac{12\sqrt{2}\,a\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^3\sqrt{\frac{\sqrt{2}\left(a^2x-a\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^4}^{\frac{1}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^4}^{\frac{3}{4}}-\sqrt{2}\,a^3\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^4}^{\frac{3}{4}}-1\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sqrt(2)*(a^2*x -
 a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt
(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - 1) + 12*sqrt(2)*a*(a^(-4))
^(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 +
1)/(a*x - 1))*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 +
 1))/(a*x - 1))*(a^(-4))^(3/4) - sqrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1))*(a^(-4))^(3/4) + 1) - 3*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^2*x
- a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + (a^3*x - a^2)*sqr
t(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 3*sqrt(2)*a*(a^(-4))^(1/4)*log
(-(sqrt(2)*(a^2*x - a)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) -
 (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(a*x - 1)*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2),x)
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(-3/2), x)
```

3.105 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=227

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)+\sqrt{2}$$

```
[Out] -2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-2*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1
/4))+1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(
1/2))*2^(1/2)-1/2*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/
(a*x+1)^(1/2))*2^(1/2)+arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1
/2)+arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6126, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)+\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x),x]
```

```
[Out] -2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] - Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1
 - a*x)^(1/4))/(1 + a*x)^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/
4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 +
 Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/S
qrt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1
 + a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x}\,dx = \int \frac{(1-ax)^{3/4}}{x(1+ax)^{3/4}}\,dx$$
$$= -\left(a\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx\right) + \int \frac{1}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$
$$= 4\,\mathrm{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right) + 4\,\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\right) - 2\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 4\,\mathrm{Subst}\Big(\int$$
$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) + 2\,\mathrm{Sub}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} + \frac{\mathrm{Subst}}{}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\log\Big(1+\frac{\sqrt{1-}}{\sqrt{1+}}}{}$$
$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - 2\tan$$

Mathematica [C] time = 0.03, size = 83, normalized size = 0.37

$$\frac{2(1-ax)^{3/4}\left(\sqrt[4]{2}\,(ax+1)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right) - 2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)\right)}{3(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x),x]
```

```
[Out] (2*(1 - a*x)^(3/4)*(2^(1/4)*(1 + a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4
, (1 - a*x)/2] - 2*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(3
*(1 + a*x)^(3/4))
```

fricas [B] time = 0.48, size = 417, normalized size = 1.84

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\sqrt{2}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1)) - 1) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*x - sqrt(2)
*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a
*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/2*sqrt(2)*l
og(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a
^2*x^2 + 1) - 1)/(a*x - 1)) + 1/2*sqrt(2)*log(4*(a*x - sqrt(2)*(a*x - 1)*sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - 2*
arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - log(sqrt(-sqrt(-a^2*x^2 + 1)/
(a*x - 1)) + 1) + log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x,x)
```

```
[Out] Integral(1/(x*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)), x)
```

3.106 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=73

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{x}+3a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+3a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] -(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x+3*a*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))+3*
a*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 212, 206, 203}

$$-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{x}+3a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+3a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x^2),x]
```

```
[Out] -(((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/x) + 3*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*
x)^(1/4)] + 3*a*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-ax)^{3/4}}{x^2(1+ax)^{3/4}}\,dx \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} - \frac{1}{2}(3a)\int \frac{1}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} - (6a)\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} + (3a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + (3a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right. \\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{x} + 3a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 3a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.75

$$\frac{(1-ax)^{3/4}\left(2ax\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)-ax-1\right)}{x(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x^2),x]

[Out] ((1 - a*x)^(3/4)*(-1 - a*x + 2*a*x*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(x*(1 + a*x)^(3/4))

fricas [B] time = 0.51, size = 124, normalized size = 1.70

$$\frac{6\,ax\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+3\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-3\,ax\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\,(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")

[Out] 1/2*(6*a*x*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 3*a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 3*a*x*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1) + 2*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate(1/(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)),x)
[Out] int(1/(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
[Out] Integral(1/(x**2*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)), x)
```

3.107 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=110

$$-\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{7/4}\sqrt[4]{ax+1}}{2x^2}+\frac{3a(1-ax)^{3/4}\sqrt[4]{ax+1}}{4x}$$

```
[Out] 3/4*a*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x-1/2*(-a*x+1)^(7/4)*(a*x+1)^(1/4)/x^2-9
/4*a^2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-9/4*a^2*arctanh((a*x+1)^(1/4)/(
-a*x+1)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6126, 96, 94, 93, 212, 206, 203}

$$-\frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{7/4}\sqrt[4]{ax+1}}{2x^2}+\frac{3a(1-ax)^{3/4}\sqrt[4]{ax+1}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x^3),x]
```

```
[Out] (3*a*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x) - ((1 - a*x)^(7/4)*(1 + a*x)^(1
/4))/(2*x^2) - (9*a^2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (9*a^2*A
rcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{(1-ax)^{3/4}}{x^3(1+ax)^{3/4}}\,dx$$
$$= -\frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} - \frac{1}{4}(3a)\int \frac{(1-ax)^{3/4}}{x^2(1+ax)^{3/4}}\,dx$$
$$= \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} + \frac{1}{8}\left(9a^2\right)\int \frac{1}{x\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$
$$= \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} + \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} - \frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{1}{4}$$
$$= \frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x} - \frac{(1-ax)^{7/4}\sqrt[4]{1+ax}}{2x^2} - \frac{9}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{9}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+a}}{\sqrt[4]{1-a}}\right.$$

Mathematica [C] time = 0.02, size = 70, normalized size = 0.64

$$\frac{(1-ax)^{3/4}\left(-6a^2x^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)+5a^2x^2+3ax-2\right)}{4x^2(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x^3),x]
```

```
[Out] ((1 - a*x)^(3/4)*(-2 + 3*a*x + 5*a^2*x^2 - 6*a^2*x^2*Hypergeometric2F1[3/4,
 1, 7/4, (1 - a*x)/(1 + a*x)]))/(4*x^2*(1 + a*x)^(3/4))
```

fricas [A] time = 0.52, size = 145, normalized size = 1.32

$$-\frac{18\,a^2x^2\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+9\,a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-9\,a^2x^2\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\left(5\,a^2x^2-7\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(18*a^2*x^2*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 9*a^2*x^2*lo
g(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 9*a^2*x^2*log(sqrt(-sqrt(-a^2*
```

```
x^2 + 1)/(a*x - 1)) - 1) + 2*(5*a^2*x^2 - 7*a*x + 2)*sqrt(-sqrt(-a^2*x^2 +
1)/(a*x - 1)))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
[Out] integrate(1/(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(1/(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)),x)
[Out] int(1/(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
[Out] Integral(1/(x**3*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)), x)
```

3.108 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=139

$$\frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{17}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{23a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{24x}-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{3x^3}+\frac{7a(1-ax)^{3/4}\sqrt[4]{ax+1}}{12x^2}$$

```
[Out] -1/3*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x^3+7/12*a*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x
^2-23/24*a^2*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x+17/8*a^3*arctan((a*x+1)^(1/4)/(
-a*x+1)^(1/4))+17/8*a^3*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{23a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{24x}+\frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{17}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{7a(1-ax)^{3/4}\sqrt[4]{ax+1}}{12x^2}-\frac{(1-ax)^{3/4}\sqrt[4]{ax+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x^4),x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(3*x^3) + (7*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(12*x^2) - (23*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(24*x) + (17*a^
3*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8 + (17*a^3*ArcTanh[(1 + a*x)^(1
/4)/(1 - a*x)^(1/4)])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-ax)^{3/4}}{x^4(1+ax)^{3/4}}\,dx$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{1}{3}\int \frac{-\frac{7a}{2}+2a^2x}{x^3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{1}{6}\int \frac{-\frac{23a^2}{4}+\frac{7a^3x}{2}}{x^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} + \frac{1}{6}\int -\frac{}{8x\sqrt[4]{1-}}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} - \frac{1}{16}\left(17a^3\right)\int \frac{}{x}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} - \frac{1}{4}\left(17a^3\right)\text{Subst}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} + \frac{1}{8}\left(17a^3\right)\text{Subst}$$
$$= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{3x^3} + \frac{7a(1-ax)^{3/4}\sqrt[4]{1+ax}}{12x^2} - \frac{23a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{24x} + \frac{17}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.$$

Mathematica [C] time = 0.02, size = 78, normalized size = 0.56

$$\frac{(1-ax)^{3/4}\left(34a^3x^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)-23a^3x^3-9a^2x^2+6ax-8\right)}{24x^3(ax+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x^4),x]
```

```
[Out] ((1 - a*x)^(3/4)*(-8 + 6*a*x - 9*a^2*x^2 - 23*a^3*x^3 + 34*a^3*x^3*Hypergeo
metric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(24*x^3*(1 + a*x)^(3/4))
```

fricas [A] time = 0.48, size = 153, normalized size = 1.10

$$\frac{102\,a^3x^3\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+51\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-51\,a^3x^3\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\left(23\,a^3x^3\right.}{48\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/48*(102*a^3*x^3*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 51*a^3*x^3*
log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 51*a^3*x^3*log(sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)) - 1) + 2*(23*a^3*x^3 - 37*a^2*x^2 + 22*a*x - 8)*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{3}{2}} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{4} \left(\frac{a\,x+1}{\sqrt{1-a^{2}\,x^{2}}}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^4*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Integral(1/(x**4*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)), x)
```

3.109 $\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=168

$$-\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{63a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{64x}-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{32x^2}-\frac{(1-ax}{}$$

```
[Out] -1/4*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x^4+3/8*a*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x^
3-15/32*a^2*(-a*x+1)^(3/4)*(a*x+1)^(1/4)/x^2+63/64*a^3*(-a*x+1)^(3/4)*(a*x+
1)^(1/4)/x-123/64*a^4*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-123/64*a^4*arcta
nh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6126, 99, 151, 12, 93, 212, 206, 203}

$$-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{ax+1}}{32x^2}+\frac{63a^3(1-ax)^{3/4}\sqrt[4]{ax+1}}{64x}-\frac{123}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{123}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{3a(1-}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcTanh[a*x])/2)*x^5),x]
```

```
[Out] -((1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(4*x^4) + (3*a*(1 - a*x)^(3/4)*(1 + a*x)
^(1/4))/(8*x^3) - (15*a^2*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(32*x^2) + (63*a
^3*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))/(64*x) - (123*a^4*ArcTan[(1 + a*x)^(1/4
)/(1 - a*x)^(1/4)])/64 - (123*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])
/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\frac{3}{2}\tanh^{-1}(ax)}}{x^5}\,dx &= \int \frac{(1-ax)^{3/4}}{x^5(1+ax)^{3/4}}\,dx\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4}+\frac{1}{4}\int \frac{-\frac{9a}{2}+3a^2x}{x^4\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4}+\frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3}-\frac{1}{12}\int \frac{-\frac{45a^2}{4}+9a^3x}{x^3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4}+\frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3}-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2}+\frac{1}{24}\int \frac{-\frac{189a^3}{8}}{x^2\sqrt[4]{1-ax}}\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4}+\frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3}-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2}+\frac{63a^3(1-ax)^{3/4}}{64x}\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4}+\frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3}-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2}+\frac{63a^3(1-ax)^{3/4}}{64x}\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4}+\frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3}-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2}+\frac{63a^3(1-ax)^{3/4}}{64x}\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4}+\frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3}-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2}+\frac{63a^3(1-ax)^{3/4}}{64x}\\
&= -\frac{(1-ax)^{3/4}\sqrt[4]{1+ax}}{4x^4}+\frac{3a(1-ax)^{3/4}\sqrt[4]{1+ax}}{8x^3}-\frac{15a^2(1-ax)^{3/4}\sqrt[4]{1+ax}}{32x^2}+\frac{63a^3(1-ax)^{3/4}}{64x}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 86, normalized size = 0.51

$$
\frac{(1-ax)^{3/4}\left(-82a^4x^4\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1-ax}{ax+1}\right)+63a^4x^4+33a^3x^3-6a^2x^2+8ax-16\right)}{64x^4(ax+1)^{3/4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcTanh[a*x])/2)*x^5),x]
```

```
[Out] ((1 - a*x)^(3/4)*(-16 + 8*a*x - 6*a^2*x^2 + 33*a^3*x^3 + 63*a^4*x^4 - 82*a^
4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (1 - a*x)/(1 + a*x)]))/(64*x^4*(1 + a*
x)^(3/4))
```

fricas [A] time = 0.50, size = 161, normalized size = 0.96

$$-\frac{246\,a^4x^4\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)+123\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+1\right)-123\,a^4x^4\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+2\,(63}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/128*(246*a^4*x^4*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 123*a^4*x
^4*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 123*a^4*x^4*log(sqrt(-sqr
t(-a^2*x^2 + 1)/(a*x - 1)) - 1) + 2*(63*a^4*x^4 - 93*a^3*x^3 + 54*a^2*x^2 -
 40*a*x + 16)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/(x^5*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
[Out] integrate(1/(x^5*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5 \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)),x)
[Out] int(1/(x^5*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
[Out] Integral(1/(x**5*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(3/2)), x)
```

3.110 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=31

$$\frac{x^{m+1}F_1\left(m+1;-\frac{5}{4},\frac{5}{4};m+2;ax,-ax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,-5/4,5/4,2+m,a*x,-a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;-\frac{5}{4},\frac{5}{4};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, -5/4, 5/4, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;-\frac{5}{4},\frac{5}{4};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.68, size = 0, normalized size = 0.00

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] Integrate[x^m/E^((5*ArcTanh[a*x])/2), x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(ax-1)x^m\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^m*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
1))/(a^2*x^2 + 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(x^m/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
[Out] int(x^m/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Timed out
```

3.111 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=317

$$\frac{(521-452ax)(1-ax)^{5/4}(ax+1)^{3/4}}{96a^4}+\frac{475\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{128\sqrt{2}\,a^4}-\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\right.}{128\sqrt{}}$$

```
[Out] -4*x^3*(-a*x+1)^(5/4)/a/(a*x+1)^(1/4)+475/64*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a
^4+17/4*x^2*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/a^2+1/96*(-452*a*x+521)*(-a*x+1)^(
5/4)*(a*x+1)^(3/4)/a^4-475/128*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/
4))/a^4*2^(1/2)-475/128*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^4*
2^(1/2)+475/256*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a
*x+1)^(1/2))/a^4*2^(1/2)-475/256*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+
(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^4*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6126, 97, 153, 147, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{17x^2(1-ax)^{5/4}(ax+1)^{3/4}}{4a^2}+\frac{(521-452ax)(1-ax)^{5/4}(ax+1)^{3/4}}{96a^4}+\frac{475\sqrt[4]{1-ax}\,(ax+1)^{3/4}}{64a^4}+\frac{475\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{ax}}\right.}{128\sqrt{2}\,a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (-4*x^3*(1 - a*x)^(5/4))/(a*(1 + a*x)^(1/4)) + (475*(1 - a*x)^(1/4)*(1 + a*
x)^(3/4))/(64*a^4) + (17*x^2*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(4*a^2) + ((5
21 - 452*a*x)*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(96*a^4) + (475*ArcTan[1 - (
Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) - (475*ArcTan[1
 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a^4) + (475*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(128*Sqrt[2]*a^4) - (475*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*
(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^3\,dx = \int \frac{x^3(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{4\int \frac{x^2\left(3-\frac{17ax}{4}\right)\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{a}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{\int \frac{x\sqrt[4]{1-ax}\left(\frac{17a}{2}-\frac{113a^2x}{8}\right)}{\sqrt[4]{1+ax}}\,dx}{a^3}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} + \frac{475}{}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1}{9}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1}{9}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1}{9}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1}{9}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1}{9}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1}{9}$$

$$= -\frac{4x^3(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} + \frac{475\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a^4} + \frac{17x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} + \frac{(521-452ax)(1}{9}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.25

$$-\frac{(1-ax)^{9/4}\left(3\left(6a^2x^2-5ax-59\right)+95\,2^{3/4}\sqrt[4]{ax+1}\;{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-ax)\right)\right)}{72a^4\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] -1/72*((1 - a*x)^(9/4)*(3*(-59 - 5*a*x + 6*a^2*x^2) + 95*2^(3/4)*(1 + a*x)^
(1/4)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - a*x)/2]))/(a^4*(1 + a*x)^(1/4)
)
```

fricas [B] time = 0.54, size = 597, normalized size = 1.88

$$-\frac{5700\sqrt{2}\left(a^5x+a^4\right)\frac{1}{a^{16}}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^4\sqrt{\frac{\sqrt{2}\left(a^{13}x-a^{12}\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^{16}}^{\frac{3}{4}}+\left(a^9x-a^8\right)\sqrt{\frac{1}{a^{16}}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^{16}}^{\frac{1}{4}}-\sqrt{2}\,a^4\sqrt{-\frac{\sqrt{-}}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/768*(5700*sqrt(2)*(a^5*x + a^4)*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt(
(sqrt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4
) + (a^9*x - a^8)*sqrt(a^(-16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^
(1/4) - sqrt(2)*a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) - 1
) + 5700*sqrt(2)*(a^5*x + a^4)*(a^(-16))^(1/4)*arctan(sqrt(2)*a^4*sqrt(-(sq
rt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) -
 (a^9*x - a^8)*sqrt(a^(-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-16))^(1/
4) - sqrt(2)*a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(1/4) + 1) +
 1425*sqrt(2)*(a^5*x + a^4)*(a^(-16))^(1/4)*log((sqrt(2)*(a^13*x - a^12)*sq
rt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-16))^(3/4) + (a^9*x - a^8)*sqrt(a^(-
16)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 1425*sqrt(2)*(a^5*x + a^4)*(a^(-16)
)^(1/4)*log(-(sqrt(2)*(a^13*x - a^12)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(
a^(-16))^(3/4) - (a^9*x - a^8)*sqrt(a^(-16)) + sqrt(-a^2*x^2 + 1))/(a*x - 1
)) + 4*(48*a^4*x^4 - 136*a^3*x^3 + 226*a^2*x^2 - 521*a*x - 2467)*sqrt(-a^2*
x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/(a^5*x + a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
```

```
[Out] int(x^3/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Integral(x**3/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(5/2), x)
```

3.112 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=305

$$-\frac{(ax+1)^{3/4}(1-ax)^{9/4}}{3a^3}-\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{ax+1}}-\frac{11(ax+1)^{3/4}(1-ax)^{5/4}}{4a^3}-\frac{55(ax+1)^{3/4}\sqrt[4]{1-ax}}{8a^3}-\frac{55\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right.}{16\sqrt{2}\,a^3}$$

```
[Out] -2*(-a*x+1)^(9/4)/a^3/(a*x+1)^(1/4)-55/8*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a^3-1
1/4*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/a^3-1/3*(-a*x+1)^(9/4)*(a*x+1)^(3/4)/a^3+5
5/16*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^3*2^(1/2)+55/16*arct
an(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a^3*2^(1/2)-55/32*ln(1-(-a*x+1)^
(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^3*2^(1/2)+55/32
*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^
3*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6126, 89, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{3/4}(1-ax)^{9/4}}{3a^3}-\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{ax+1}}-\frac{11(ax+1)^{3/4}(1-ax)^{5/4}}{4a^3}-\frac{55(ax+1)^{3/4}\sqrt[4]{1-ax}}{8a^3}-\frac{55\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right.}{16\sqrt{2}\,a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (-2*(1 - a*x)^(9/4))/(a^3*(1 + a*x)^(1/4)) - (55*(1 - a*x)^(1/4)*(1 + a*x)^
(3/4))/(8*a^3) - (11*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(4*a^3) - ((1 - a*x)^
(9/4)*(1 + a*x)^(3/4))/(3*a^3) - (55*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(
1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) + (55*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))
/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^3) - (55*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x
] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(16*Sqrt[2]*a^3) + (55*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(16*Sqrt[2]*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x^2\,dx = \int \frac{x^2(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} + \frac{2\int \frac{(1-ax)^{5/4}\left(-\frac{5a}{2}+\frac{a^2x}{2}\right)}{\sqrt[4]{1+ax}}\,dx}{a^3}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} - \frac{11\int \frac{(1-ax)^{5/4}}{\sqrt[4]{1+ax}}\,dx}{2a^2}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} - \frac{55\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{8a^2}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3}$$

$$= -\frac{2(1-ax)^{9/4}}{a^3\sqrt[4]{1+ax}} - \frac{55\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.23

$$\frac{(1-ax)^{9/4}\left(11\ 2^{3/4}\sqrt[4]{ax+1}\ {}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-ax)\right)-3(ax+7)\right)}{9a^3\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] ((1 - a*x)^(9/4)*(-3*(7 + a*x) + 11*2^(3/4)*(1 + a*x)^(1/4)*Hypergeometric2
F1[1/4, 9/4, 13/4, (1 - a*x)/2]))/(9*a^3*(1 + a*x)^(1/4))
```

fricas [B] time = 0.51, size = 589, normalized size = 1.93

$$\frac{660\sqrt{2}\left(a^4x+a^3\right)\frac{1}{a^{12}}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^3\sqrt{\frac{\sqrt{2}\left(a^{10}x-a^9\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^{12}}^{\frac{3}{4}}+\left(a^7x-a^6\right)\sqrt{\frac{1}{a^{12}}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^{12}}^{\frac{1}{4}}-\sqrt{2}\,a^3\sqrt{-\frac{\sqrt{-a^2x^2+}}{ax-1}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*(a^4*x + a^3)*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt((sq
rt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) +
(a^7*x - a^6)*sqrt(a^(-12)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4
) - sqrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) - 1) +
660*sqrt(2)*(a^4*x + a^3)*(a^(-12))^(1/4)*arctan(sqrt(2)*a^3*sqrt(-(sqrt(2)
*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) - (a^7*
x - a^6)*sqrt(a^(-12)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-12))^(1/4) - s
qrt(2)*a^3*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(1/4) + 1) + 165*s
qrt(2)*(a^4*x + a^3)*(a^(-12))^(1/4)*log((sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(3/4) + (a^7*x - a^6)*sqrt(a^(-12)) - s
qrt(-a^2*x^2 + 1))/(a*x - 1)) - 165*sqrt(2)*(a^4*x + a^3)*(a^(-12))^(1/4)*l
og(-(sqrt(2)*(a^10*x - a^9)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-12))^(
3/4) - (a^7*x - a^6)*sqrt(a^(-12)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*(8*
a^3*x^3 - 26*a^2*x^2 + 61*a*x + 287)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1)))/(a^4*x + a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
[Out] int(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
[Out] int(x^2/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Integral(x**2/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(5/2), x)
```

3.113 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=279

$$\frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{ax+1}}+\frac{5(ax+1)^{3/4}(1-ax)^{5/4}}{2a^2}+\frac{25(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a^2}+\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\sqrt{\ }\right.}{8\sqrt{2}a^2}$$

```
[Out] 2*(-a*x+1)^(9/4)/a^2/(a*x+1)^(1/4)+25/4*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a^2+5/
2*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/a^2-25/8*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a
*x+1)^(1/4))/a^2*2^(1/2)-25/8*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)
)/a^2*2^(1/2)+25/16*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2
)/(a*x+1)^(1/2))/a^2*2^(1/2)-25/16*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4
)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a^2*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6126, 78, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{ax+1}}+\frac{5(ax+1)^{3/4}(1-ax)^{5/4}}{2a^2}+\frac{25(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a^2}+\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a^2}-\frac{25\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\sqrt{\ }\right.}{8\sqrt{2}a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (2*(1 - a*x)^(9/4))/(a^2*(1 + a*x)^(1/4)) + (25*(1 - a*x)^(1/4)*(1 + a*x)^(
3/4))/(4*a^2) + (5*(1 - a*x)^(5/4)*(1 + a*x)^(3/4))/(2*a^2) + (25*ArcTan[1
- (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) - (25*ArcTan[
1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a^2) + (25*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(8*Sqrt[2]*a^2) - (25*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 -
 a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}x\,dx = \int \frac{x(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx$$

$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{5\int \frac{(1-ax)^{5/4}}{\sqrt[4]{1+ax}}\,dx}{a}$$

$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{25\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{4a}$$

$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{25\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{8a}$$

$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{25\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \ldots\right.}{2a^2}$$

$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{25\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \ldots\right.}{2a^2}$$

$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{25\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \ldots\right.}{4a^2}$$

$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{25\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx \ldots\right.}{8a^2}$$

$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{25\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\ldots}{\sqrt[4]{\ldots}}\right.}{8\sqrt{2}\,a^2}$$

$$= \frac{2(1-ax)^{9/4}}{a^2\sqrt[4]{1+ax}} + \frac{25\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{25\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}\,a^2}$$

Mathematica [C] time = 0.03, size = 64, normalized size = 0.23

$$-\frac{2(1-ax)^{9/4}\left(5\,2^{3/4}\sqrt[4]{ax+1}\;{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-ax)\right)-9\right)}{9a^2\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((5*ArcTanh[a*x])/2),x]
```

```
[Out] (-2*(1 - a*x)^(9/4)*(-9 + 5*2^(3/4)*(1 + a*x)^(1/4)*Hypergeometric2F1[1/4,
9/4, 13/4, (1 - a*x)/2]))/(9*a^2*(1 + a*x)^(1/4))
```

fricas [B] time = 0.65, size = 581, normalized size = 2.08

$$-\frac{100\sqrt{2}\left(a^3x+a^2\right)\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a^2\sqrt{\frac{\sqrt{2}\left(a^7x-a^6\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{3}{4}}+\left(a^5x-a^4\right)\sqrt{\frac{1}{a^8}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^8}^{\frac{1}{4}}-\sqrt{2}\,a^2\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(100*sqrt(2)*(a^3*x + a^2)*(a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt((sq
rt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) + (a
^5*x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) -
sqrt(2)*a^2*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) - 1) + 100*s
qrt(2)*(a^3*x + a^2)*(a^(-8))^(1/4)*arctan(sqrt(2)*a^2*sqrt(-(sqrt(2)*(a^7*
x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - (a^5*x - a^4)
*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-8))^(1/4) - sqrt(2)*a^2
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(1/4) + 1) + 25*sqrt(2)*(a^3*
x + a^2)*(a^(-8))^(1/4)*log((sqrt(2)*(a^7*x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-8))^(3/4) + (a^5*x - a^4)*sqrt(a^(-8)) - sqrt(-a^2*x^2 + 1
))/(a*x - 1)) - 25*sqrt(2)*(a^3*x + a^2)*(a^(-8))^(1/4)*log(-(sqrt(2)*(a^7*
x - a^6)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-8))^(3/4) - (a^5*x - a^4)
*sqrt(a^(-8)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(2*a^2*x^2 - 9*a*x - 43)
*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/(a^3*x + a^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

```
[Out] int(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
```

```
[Out] int(x/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
```

```
[Out] Integral(x/((a*x + 1)/sqrt(-a**2*x**2 + 1))**(5/2), x)
```

3.114 $\int e^{-\frac{5}{2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=247

$$-\frac{4(1-ax)^{5/4}}{a\sqrt[4]{ax+1}}-\frac{5(ax+1)^{3/4}\sqrt[4]{1-ax}}{a}-\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{5\tan^{-1}\left(1-\right.}{\sqrt{2}}$$

```
[Out] -4*(-a*x+1)^(5/4)/a/(a*x+1)^(1/4)-5*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a+5/2*arct
an(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)+5/2*arctan(1+(-a*x+1)
^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)-5/4*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*
x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*2^(1/2)+5/4*ln(1+(-a*x+1)^(1/4)*
2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6125, 47, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{4(1-ax)^{5/4}}{a\sqrt[4]{ax+1}}-\frac{5(ax+1)^{3/4}\sqrt[4]{1-ax}}{a}-\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}+\frac{5\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{2\sqrt{2}a}-\frac{5\tan^{-1}\left(1-\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-5*ArcTanh[a*x])/2),x]
```

```
[Out] (-4*(1 - a*x)^(5/4))/(a*(1 + a*x)^(1/4)) - (5*(1 - a*x)^(1/4)*(1 + a*x)^(3/
4))/a - (5*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*
a) + (5*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(Sqrt[2]*a)
- (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a
*x)^(1/4)])/(2*Sqrt[2]*a) + (5*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[
2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(2*Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{-\frac{5}{2}\tanh^{-1}(ax)}\,dx = \int \frac{(1-ax)^{5/4}}{(1+ax)^{5/4}}\,dx$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - 5\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} - \frac{5}{2}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} + \frac{10\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} + \frac{10\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} + \frac{5\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} + \frac{5\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,d\right.}{a}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} + \frac{5\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a} + \frac{5\,\text{Subst}\left(\int \frac{}{1-}\right.}{}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} - \frac{5\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2\sqrt{2}\,a} + \frac{5\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.}{2\sqrt{2}\,a}$$
$$= -\frac{4(1-ax)^{5/4}}{a\sqrt[4]{1+ax}} - \frac{5\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{a} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} -$$

Mathematica [C] time = 0.06, size = 33, normalized size = 0.13

$$-\frac{8e^{-\frac{1}{2}\tanh^{-1}(ax)}\,{}_2F_1\left(-\frac{1}{4}, 2; \frac{3}{4}; -e^{2\tanh^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-5*ArcTanh[a*x])/2),x]
```

```
[Out] (-8*Hypergeometric2F1[-1/4, 2, 3/4, -E^(2*ArcTanh[a*x])])/(a*E^(ArcTanh[a*x
]/2))
```

fricas [B] time = 0.49, size = 554, normalized size = 2.24

$$\frac{20\sqrt{2}\left(a^2x+a\right)\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}\,a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*(a^2*x + a)*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(
a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x -
a^2)*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)
*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 20*sqrt(2)*(a^
2*x + a)*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4))
+ sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) + 1) + 5*sqrt(2)*(a^2*x + a)*(a^(-4))^(
1/4)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4)
)^(3/4) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 5*s
qrt(2)*(a^2*x + a)*(a^(-4))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-
a^2*x^2 + 1))/(a*x - 1)) - 4*sqrt(-a^2*x^2 + 1)*(a*x + 9)*sqrt(-sqrt(-a^2*x
^2 + 1)/(a*x - 1)))/(a^2*x + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(-5/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2),x)
[Out] int(1/((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2),x)
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(-5/2), x)
```

3.115 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=248

$$\frac{8\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a\cdots}}\right.$$

```
[Out] 8*(-a*x+1)^(1/4)/(a*x+1)^(1/4)+2*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))-2*arc
tanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))+1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^
(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*2^(1/2)-1/2*ln(1+(-a*x+1)^(1/4)*2^(1/2)
/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*2^(1/2)-arctan(-1+(-a*x+1)^(1/
4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/2)-arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(
1/4))*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6126, 98, 21, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{8\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}}+2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x),x]
```

```
[Out] (8*(1 - a*x)^(1/4))/(1 + a*x)^(1/4) + 2*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1
/4)] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] - Sqrt
[2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)] - 2*ArcTanh[(1 +
a*x)^(1/4)/(1 - a*x)^(1/4)] + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2
]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - a*x]/Sqrt[1
+ a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x}\,dx = \int \frac{(1-ax)^{5/4}}{x(1+ax)^{5/4}}\,dx$$

$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + \frac{4\int \frac{\frac{a}{4}+\frac{a^2x}{4}}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{a}$$

$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + \int \frac{(1+ax)^{3/4}}{x(1-ax)^{3/4}}\,dx$$

$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + a\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx + \int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} - 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right) + 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$

$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 4\,\text{S}$$

$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.$$

$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}}$$

$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}} - \frac{\text{lo}}{}$$

$$= \frac{8\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-a}}{\sqrt[4]{1+ax}}\right.$$

Mathematica [C] time = 0.07, size = 90, normalized size = 0.36

$$\frac{\sqrt[4]{1-ax}\left(-20\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)+2^{3/4}(1-ax)\sqrt[4]{ax+1}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-ax)\right)+20\right)}{5\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x),x]
```

```
[Out] ((1 - a*x)^(1/4)*(20 - 20*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x
)] + 2^(3/4)*(1 - a*x)*(1 + a*x)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, (1
- a*x)/2]))/(5*(1 + a*x)^(1/4))
```

fricas [B] time = 2.44, size = 499, normalized size = 2.01

$$-\frac{4\sqrt{2}(ax+1)\arctan\left(\sqrt{2}\sqrt{\frac{ax+\sqrt{2}(ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-\sqrt{-a^2x^2+1}-1}{ax-1}}-\sqrt{2}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}-1\right)+4\sqrt{2}(ax+1)\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(a*x + 1)*arctan(sqrt(2)*sqrt((a*x + sqrt(2)*(a*x - 1)*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt
(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1) + 4*sqrt(2)*(a*x + 1)*arctan(s
qrt(2)*sqrt((a*x - sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) -
sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1)) + 1) + sqrt(2)*(a*x + 1)*log(4*(a*x + sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)) - sqrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - sqrt(2)*(a*x
+ 1)*log(4*(a*x - sqrt(2)*(a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - s
qrt(-a^2*x^2 + 1) - 1)/(a*x - 1)) - 4*(a*x + 1)*arctan(sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1))) + 2*(a*x + 1)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1)
 - 2*(a*x + 1)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1) - 16*sqrt(-a^2*
x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/(a*x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x)
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
[Out] integrate(1/(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)),x)
[Out] int(1/(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x,x)
[Out] Integral(1/(x*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(5/2)), x)
```

3.116 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=95

$$-\frac{(1-ax)^{5/4}}{x\sqrt[4]{ax+1}}-\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-5a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+5a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

```
[Out] -10*a*(-a*x+1)^(1/4)/(a*x+1)^(1/4)-(-a*x+1)^(5/4)/x/(a*x+1)^(1/4)-5*a*arcta
n((a*x+1)^(1/4)/(-a*x+1)^(1/4))+5*a*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6126, 94, 93, 298, 203, 206}

$$-\frac{(1-ax)^{5/4}}{x\sqrt[4]{ax+1}}-\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-5a\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+5a\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x^2),x]
```

```
[Out] (-10*a*(1 - a*x)^(1/4))/(1 + a*x)^(1/4) - (1 - a*x)^(5/4)/(x*(1 + a*x)^(1/4
)) - 5*a*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)] + 5*a*ArcTanh[(1 + a*x)^(1
/4)/(1 - a*x)^(1/4)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^2}\,dx &= \int \frac{(1-ax)^{5/4}}{x^2(1+ax)^{5/4}}\,dx\\
&= -\frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}}-\frac{1}{2}(5a)\int \frac{\sqrt[4]{1-ax}}{x(1+ax)^{5/4}}\,dx\\
&= -\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}-\frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}}-\frac{1}{2}(5a)\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx\\
&= -\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}-\frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}}-(10a)\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)\\
&= -\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}-\frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}}+(5a)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)-(5a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\right.\\
&= -\frac{10a\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}-\frac{(1-ax)^{5/4}}{x\sqrt[4]{1+ax}}-5a\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)+5a\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.58

$$\frac{\sqrt[4]{1-ax}\left(10ax\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)-9ax-1\right)}{x\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x^2),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-1 - 9*a*x + 10*a*x*Hypergeometric2F1[1/4, 1, 5/4, (1 - a
*x)/(1 + a*x)]))/(x*(1 + a*x)^(1/4))
```

fricas [B] time = 0.57, size = 170, normalized size = 1.79

$$-\frac{2\sqrt{-a^2x^2+1}\,(9\,ax+1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+10\left(a^2x^2+ax\right)\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-5\left(a^2x^2+ax\right)\log\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right.}{2\left(ax^2+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(-a^2*x^2 + 1)*(9*a*x + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
+ 10*(a^2*x^2 + a*x)*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 5*(a^2*x
^2 + a*x)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 5*(a^2*x^2 + a*x)*
log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1))/(a*x^2 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)),x)
```

```
[Out] int(1/(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(5/2)), x)
```

3.117 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=136

$$\frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{ax+1}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{ax+1}}+\frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{ax+1}}$$

```
[Out] 25/2*a^2*(-a*x+1)^(1/4)/(a*x+1)^(1/4)+5/4*a*(-a*x+1)^(5/4)/x/(a*x+1)^(1/4)-
1/2*(-a*x+1)^(9/4)/x^2/(a*x+1)^(1/4)+25/4*a^2*arctan((a*x+1)^(1/4)/(-a*x+1)
^(1/4))-25/4*a^2*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6126, 96, 94, 93, 298, 203, 206}

$$\frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{ax+1}}+\frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{ax+1}}+\frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x^3),x]
```

```
[Out] (25*a^2*(1 - a*x)^(1/4))/(2*(1 + a*x)^(1/4)) + (5*a*(1 - a*x)^(5/4))/(4*x*(
1 + a*x)^(1/4)) - (1 - a*x)^(9/4)/(2*x^2*(1 + a*x)^(1/4)) + (25*a^2*ArcTan[
(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/4 - (25*a^2*ArcTanh[(1 + a*x)^(1/4)/(1 -
a*x)^(1/4)])/4
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{(1-ax)^{5/4}}{x^3(1+ax)^{5/4}}\,dx$$
$$= -\frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} - \frac{1}{4}(5a)\int \frac{(1-ax)^{5/4}}{x^2(1+ax)^{5/4}}\,dx$$
$$= \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} + \frac{1}{8}\left(25a^2\right)\int \frac{\sqrt[4]{1-ax}}{x(1+ax)^{5/4}}\,dx$$
$$= \frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{1+ax}} + \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} + \frac{1}{8}\left(25a^2\right)\int \frac{1}{x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= \frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{1+ax}} + \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} + \frac{1}{2}\left(25a^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= \frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{1+ax}} + \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} - \frac{1}{4}\left(25a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$
$$= \frac{25a^2\sqrt[4]{1-ax}}{2\sqrt[4]{1+ax}} + \frac{5a(1-ax)^{5/4}}{4x\sqrt[4]{1+ax}} - \frac{(1-ax)^{9/4}}{2x^2\sqrt[4]{1+ax}} + \frac{25}{4}a^2\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) - \frac{25}{4}a^2\tanh^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)$$

Mathematica [C] time = 0.02, size = 70, normalized size = 0.51

$$\frac{\sqrt[4]{1-ax}\left(-50a^2x^2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)+43a^2x^2+9ax-2\right)}{4x^2\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x^3),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-2 + 9*a*x + 43*a^2*x^2 - 50*a^2*x^2*Hypergeometric2F1[1/
4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(4*x^2*(1 + a*x)^(1/4))
```

fricas [A] time = 0.94, size = 192, normalized size = 1.41

$$\frac{2\left(43\,a^2x^2+9\,ax-2\right)\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}} + 50\left(a^3x^3+a^2x^2\right)\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right) - 25\left(a^3x^3+a^2x^2\right)\log\cdots}{8\left(ax^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(43*a^2*x^2 + 9*a*x - 2)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1)) + 50*(a^3*x^3 + a^2*x^2)*arctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))) - 25*(a^3*x^3 + a^2*x^2)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1)
 + 25*(a^3*x^3 + a^2*x^2)*log(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1))/(a*
x^3 + x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3} \left(\frac{a\,x+1}{\sqrt{1-a^{2}\,x^{2}}}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)),x)
```

```
[Out] int(1/(x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(5/2)), x)
```

3.118 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=165

$$-\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{ax+1}}-\frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{55}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{ax+1}}-\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{ax+1}}+\frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{ax+1}}$$

```
[Out] -287/24*a^3*(-a*x+1)^(1/4)/(a*x+1)^(1/4)-1/3*(-a*x+1)^(1/4)/x^3/(a*x+1)^(1/
4)+13/12*a*(-a*x+1)^(1/4)/x^2/(a*x+1)^(1/4)-61/24*a^2*(-a*x+1)^(1/4)/x/(a*x
+1)^(1/4)-55/8*a^3*arctan((a*x+1)^(1/4)/(-a*x+1)^(1/4))+55/8*a^3*arctanh((a
*x+1)^(1/4)/(-a*x+1)^(1/4))
```

Rubi [A] time = 0.08, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6126, 98, 151, 155, 12, 93, 298, 203, 206}

$$-\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{ax+1}}-\frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{ax+1}}-\frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{55}{8}a^3\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{ax+1}}-\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x^4),x]
```

```
[Out] (-287*a^3*(1 - a*x)^(1/4))/(24*(1 + a*x)^(1/4)) - (1 - a*x)^(1/4)/(3*x^3*(1
 + a*x)^(1/4)) + (13*a*(1 - a*x)^(1/4))/(12*x^2*(1 + a*x)^(1/4)) - (61*a^2*
(1 - a*x)^(1/4))/(24*x*(1 + a*x)^(1/4)) - (55*a^3*ArcTan[(1 + a*x)^(1/4)/(1
 - a*x)^(1/4)])/8 + (55*a^3*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-ax)^{5/4}}{x^4(1+ax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} - \frac{1}{3}\int \frac{\frac{13a}{2}-6a^2x}{x^3(1-ax)^{3/4}(1+ax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} + \frac{1}{6}\int \frac{\frac{61a^2}{4}-13a^3x}{x^2(1-ax)^{3/4}(1+ax)^{5/4}}\,dx$$

$$= -\frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{1}{6}\int \frac{\frac{165a^3}{8}-\frac{61a^4x}{4}}{x(1-ax)^{3/4}(1+ax)^{5/4}}\,dx$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{\int \frac{165a^4}{16x(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{3a}$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{1}{16}\left(55a^3\right)\int \frac{1}{x(1-ax)^{3/}}$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{1}{4}\left(55a^3\right)\text{Subst}\left(\int \frac{x^{}}{-1+}\right.$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} + \frac{1}{8}\left(55a^3\right)\text{Subst}\left(\int \frac{1}{1-}\right.$$

$$= -\frac{287a^3\sqrt[4]{1-ax}}{24\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{3x^3\sqrt[4]{1+ax}} + \frac{13a\sqrt[4]{1-ax}}{12x^2\sqrt[4]{1+ax}} - \frac{61a^2\sqrt[4]{1-ax}}{24x\sqrt[4]{1+ax}} - \frac{55}{8}a^3\tan^{-1}\left(\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right) -$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.47

$$\frac{\sqrt[4]{1-ax}\left(330a^3x^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)-287a^3x^3-61a^2x^2+26ax-8\right)}{24x^3\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x^4),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-8 + 26*a*x - 61*a^2*x^2 - 287*a^3*x^3 + 330*a^3*x^3*Hype
rgeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(24*x^3*(1 + a*x)^(1/4))
```

fricas [A] time = 0.59, size = 200, normalized size = 1.21

$$-\frac{2\left(287\,a^3x^3+61\,a^2x^2-26\,ax+8\right)\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}+330\left(a^4x^4+a^3x^3\right)\arctan\left(\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\right)-165}{48\left(ax^4+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(287*a^3*x^3 + 61*a^2*x^2 - 26*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1)) + 330*(a^4*x^4 + a^3*x^3)*arctan(sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))) - 165*(a^4*x^4 + a^3*x^3)*log(sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1)) + 1) + 165*(a^4*x^4 + a^3*x^3)*log(sqrt(-sqrt(-a^2*x^2 + 1)/
(a*x - 1)) - 1))/(a*x^4 + x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)}^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)),x)
```

```
[Out] int(1/(x^4*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

```
[Out] Timed out
```

3.119 $\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=194

$$\frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{ax+1}}+\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{ax+1}}-\frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{ax+1}}-\frac{\sqrt[4]{1-}}{4x^4\sqrt[4]{ax}}$$

```
[Out] 2467/192*a^4*(-a*x+1)^(1/4)/(a*x+1)^(1/4)-1/4*(-a*x+1)^(1/4)/x^4/(a*x+1)^(1
/4)+17/24*a*(-a*x+1)^(1/4)/x^3/(a*x+1)^(1/4)-113/96*a^2*(-a*x+1)^(1/4)/x^2/
(a*x+1)^(1/4)+521/192*a^3*(-a*x+1)^(1/4)/x/(a*x+1)^(1/4)+475/64*a^4*arctan(
(a*x+1)^(1/4)/(-a*x+1)^(1/4))-475/64*a^4*arctanh((a*x+1)^(1/4)/(-a*x+1)^(1/
4))
```

Rubi [A] time = 0.10, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6126, 98, 151, 155, 12, 93, 298, 203, 206}

$$-\frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{ax+1}}+\frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{ax+1}}+\frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{ax+1}}+\frac{475}{64}a^4\tan^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)-\frac{475}{64}a^4\tanh^{-1}\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}\right)+\frac{17a\sqrt[4]{}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcTanh[a*x])/2)*x^5),x]
```

```
[Out] (2467*a^4*(1 - a*x)^(1/4))/(192*(1 + a*x)^(1/4)) - (1 - a*x)^(1/4)/(4*x^4*(
1 + a*x)^(1/4)) + (17*a*(1 - a*x)^(1/4))/(24*x^3*(1 + a*x)^(1/4)) - (113*a^
2*(1 - a*x)^(1/4))/(96*x^2*(1 + a*x)^(1/4)) + (521*a^3*(1 - a*x)^(1/4))/(19
2*x*(1 + a*x)^(1/4)) + (475*a^4*ArcTan[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
 - (475*a^4*ArcTanh[(1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$
\int \frac{e^{-\frac{5}{2}\tanh^{-1}(ax)}}{x^5}\,dx = \int \frac{(1-ax)^{5/4}}{x^5(1+ax)^{5/4}}\,dx
$$

$$
= -\frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} - \frac{1}{4}\int \frac{\frac{17a}{2}-8a^2x}{x^4(1-ax)^{3/4}(1+ax)^{5/4}}\,dx
$$

$$
= -\frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} + \frac{1}{12}\int \frac{\frac{113a^2}{4}-\frac{51a^3x}{2}}{x^3(1-ax)^{3/4}(1+ax)^{5/4}}\,dx
$$

$$
= -\frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} - \frac{1}{24}\int \frac{\frac{521a^3}{8}-\frac{113a^4x}{2}}{x^2(1-ax)^{3/4}(1+ax)^{5/4}}\,dx
$$

$$
= -\frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{1}{24}\int \frac{\frac{1425a^4}{16}-\frac{521a}{8}}{x(1-ax)^{3/4}(1+}
$$

$$
= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{\int \frac{}{32x}}{}
$$

$$
= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{1}{128}(
$$

$$
= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{1}{32}(4
$$

$$
= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} - \frac{1}{64}(4
$$

$$
= \frac{2467a^4\sqrt[4]{1-ax}}{192\sqrt[4]{1+ax}} - \frac{\sqrt[4]{1-ax}}{4x^4\sqrt[4]{1+ax}} + \frac{17a\sqrt[4]{1-ax}}{24x^3\sqrt[4]{1+ax}} - \frac{113a^2\sqrt[4]{1-ax}}{96x^2\sqrt[4]{1+ax}} + \frac{521a^3\sqrt[4]{1-ax}}{192x\sqrt[4]{1+ax}} + \frac{475}{64}a
$$

Mathematica [C]   time = 0.03, size = 86, normalized size = 0.44

$$
\frac{\sqrt[4]{1-ax}\left(-2850a^4x^4\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1-ax}{ax+1}\right)+2467a^4x^4+521a^3x^3-226a^2x^2+136ax-48\right)}{192x^4\sqrt[4]{ax+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcTanh[a*x])/2)*x^5),x]
```

```
[Out] ((1 - a*x)^(1/4)*(-48 + 136*a*x - 226*a^2*x^2 + 521*a^3*x^3 + 2467*a^4*x^4
- 2850*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (1 - a*x)/(1 + a*x)]))/(192*x
^4*(1 + a*x)^(1/4))
```

fricas [A] time = 0.64, size = 208, normalized size = 1.07

$$\frac{2\left(2467\,a^{4}x^{4}+521\,a^{3}x^{3}-226\,a^{2}x^{2}+136\,ax-48\right)\sqrt{-a^{2}x^{2}+1}\sqrt{-\frac{\sqrt{-a^{2}x^{2}+1}}{ax-1}}+2850\left(a^{5}x^{5}+a^{4}x^{4}\right)\arctan\left(\sqrt{-\cdots}\right.}{384\left(ax^{5}+x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/384*(2*(2467*a^4*x^4 + 521*a^3*x^3 - 226*a^2*x^2 + 136*a*x - 48)*sqrt(-a^
2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 2850*(a^5*x^5 + a^4*x^4)*a
rctan(sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 1425*(a^5*x^5 + a^4*x^4)*log(s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + 1425*(a^5*x^5 + a^4*x^4)*log(sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - 1))/(a*x^5 + x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}} x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

```
[Out] int(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5}\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
[Out] integrate(1/(x^5*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5 \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)),x)
[Out] int(1/(x^5*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x+1)/(-a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
[Out] Timed out
```

3.120 $\int e^{\frac{1}{3}\tanh^{-1}(x)}x^m\,dx$

Optimal. Leaf size=28

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{6},-\frac{1}{6};m+2;x,-x\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/6,-1/6,2+m,x,-x)/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{6},-\frac{1}{6};m+2;x,-x\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/6, -1/6, 2 + m, x, -x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{3}\tanh^{-1}(x)}x^m\,dx &= \int \frac{x^m\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{1}{6},-\frac{1}{6};2+m;x,-x\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.48, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{3}\tanh^{-1}(x)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcTanh[x]/3)*x^m,x]
[Out] Integrate[E^(ArcTanh[x]/3)*x^m, x]
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x, algorithm="fricas")
[Out] integral(x^m*(-sqrt(-x^2 + 1)/(x - 1))^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x, algorithm="giac")
[Out] integrate(x^m*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x)
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \left(\frac{x+1}{\sqrt{1-x^2}}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((x + 1)/(1 - x^2)^(1/2))^(1/3),x)
[Out] int(x^m*((x + 1)/(1 - x^2)^(1/2))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)*x***m,x)
[Out] Timed out
```

3.121 $\int e^{\frac{1}{3}\tanh^{-1}(x)}x^2\,dx$

Optimal. Leaf size=245

$$-\frac{1}{3}(1-x)^{5/6}x(x+1)^{7/6}-\frac{1}{18}(1-x)^{5/6}(x+1)^{7/6}-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{x+1}-\frac{19\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{108\sqrt{3}}+\frac{19\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}}{\sqrt[6]{}}\right.}{108\sqrt{3}}$$

```
[Out] -19/54*(1-x)^(5/6)*(1+x)^(1/6)-1/18*(1-x)^(5/6)*(1+x)^(7/6)-1/3*(1-x)^(5/6)
*x*(1+x)^(7/6)-19/81*arctan((1-x)^(1/6)/(1+x)^(1/6))-19/162*arctan(2*(1-x)^
(1/6)/(1+x)^(1/6)-3^(1/2))-19/162*arctan(2*(1-x)^(1/6)/(1+x)^(1/6)+3^(1/2))
-19/324*ln(1+(1-x)^(1/3)/(1+x)^(1/3)-(1-x)^(1/6)*3^(1/2)/(1+x)^(1/6))*3^(1/
2)+19/324*ln(1+(1-x)^(1/3)/(1+x)^(1/3)+(1-x)^(1/6)*3^(1/2)/(1+x)^(1/6))*3^(
1/2)
```

Rubi [A] time = 0.39, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6126, 90, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$-\frac{1}{3}(1-x)^{5/6}x(x+1)^{7/6}-\frac{1}{18}(1-x)^{5/6}(x+1)^{7/6}-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{x+1}-\frac{19\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{108\sqrt{3}}+\frac{19\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}}{\sqrt[6]{}}\right.}{108\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)*x^2,x]
```

```
[Out] (-19*(1 - x)^(5/6)*(1 + x)^(1/6))/54 - ((1 - x)^(5/6)*(1 + x)^(7/6))/18 - (
(1 - x)^(5/6)*x*(1 + x)^(7/6))/3 - (19*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])
/81 + (19*ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)])/162 - (19*ArcT
an[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)])/162 - (19*Log[1 + (1 - x)^(1
/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)])/(108*Sqrt[3]) +
 (19*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^
(1/6)])/(108*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
```

```
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{1}{3}\tanh^{-1}(x)}x^2\,dx=\int\frac{x^2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$
$$=-\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6}-\frac{1}{3}\int\frac{\left(-1-\frac{x}{3}\right)\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$
$$=-\frac{1}{18}(1-x)^{5/6}(1+x)^{7/6}-\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6}+\frac{19}{54}\int\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$
$$=-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x}-\frac{1}{18}(1-x)^{5/6}(1+x)^{7/6}-\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6}+\frac{19}{162}\int\frac{1}{\sqrt[6]{1-x}\,(1+$$
$$=-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x}-\frac{1}{18}(1-x)^{5/6}(1+x)^{7/6}-\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6}-\frac{19}{27}\,\text{Subst}\Bigg(\int\frac{x}{(2-$$
$$=-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x}-\frac{1}{18}(1-x)^{5/6}(1+x)^{7/6}-\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6}-\frac{19}{27}\,\text{Subst}\Bigg(\int\frac{x^4}{1+x$$
$$=-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x}-\frac{1}{18}(1-x)^{5/6}(1+x)^{7/6}-\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6}-\frac{19}{81}\,\text{Subst}\Bigg(\int\frac{1}{1+x$$
$$=-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x}-\frac{1}{18}(1-x)^{5/6}(1+x)^{7/6}-\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6}-\frac{19}{81}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right.$$
$$=-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x}-\frac{1}{18}(1-x)^{5/6}(1+x)^{7/6}-\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6}-\frac{19}{81}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right.$$
$$=-\frac{19}{54}(1-x)^{5/6}\sqrt[6]{1+x}-\frac{1}{18}(1-x)^{5/6}(1+x)^{7/6}-\frac{1}{3}(1-x)^{5/6}x(1+x)^{7/6}-\frac{19}{81}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right.$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.24

$$-\frac{1}{90}(1-x)^{5/6}\left(38\sqrt[6]{2}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{1-x}{2}\right)+5\sqrt[6]{x+1}\left(6x^2+7x+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)*x^2,x]
```

```
[Out] -1/90*((1 - x)^(5/6)*(5*(1 + x)^(1/6)*(1 + 7*x + 6*x^2) + 38*2^(1/6)*Hyperg
eometric2F1[-1/6, 5/6, 11/6, (1 - x)/2]))
```

fricas [A] time = 0.58, size = 308, normalized size = 1.26

$$\frac{19}{324}\sqrt{3}\log\left(1444\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+1444\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+1444\right)-\frac{19}{324}\sqrt{3}\log\left(-1444\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="fricas")
```

```
[Out] 19/324*sqrt(3)*log(1444*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1444*(-sq
rt(-x^2 + 1)/(x - 1))^(2/3) + 1444) - 19/324*sqrt(3)*log(-1444*sqrt(3)*(-sq
rt(-x^2 + 1)/(x - 1))^(1/3) + 1444*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1444)
+ 1/54*(18*x^3 + 3*x^2 + x - 22)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 19/81*ar
ctan(sqrt(3) + 1/19*sqrt(-1444*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 14
44*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1444) - 2*(-sqrt(-x^2 + 1)/(x - 1))^(1
/3)) - 19/81*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/
3) + (-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1) - 2*(-sqrt(-x^2 + 1)/(x - 1))^(1/
3)) + 19/81*arctan((-sqrt(-x^2 + 1)/(x - 1))^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(\frac{x+1}{\sqrt{1-x^2}}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((x + 1)/(1 - x^2)^(1/2))^(1/3),x)
[Out] int(x^2*((x + 1)/(1 - x^2)^(1/2))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[3]{\frac{x+1}{\sqrt{1-x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)*x**2,x)
[Out] Integral(x**2*((x + 1)/sqrt(1 - x**2))**(1/3), x)
```

3.122 $\int e^{\frac{1}{3}\tanh^{-1}(x)} x \, dx$

Optimal. Leaf size=224

$$-\frac{1}{2}(1-x)^{5/6}(x+1)^{7/6}-\frac{1}{6}(1-x)^{5/6}\sqrt[6]{x+1}-\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{12\sqrt{3}}+\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{12\sqrt{3}}-\frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right.$$

```
[Out] -1/6*(1-x)^(5/6)*(1+x)^(1/6)-1/2*(1-x)^(5/6)*(1+x)^(7/6)-1/9*arctan((1-x)^(
1/6)/(1+x)^(1/6))-1/18*arctan(2*(1-x)^(1/6)/(1+x)^(1/6)-3^(1/2))-1/18*arcta
n(2*(1-x)^(1/6)/(1+x)^(1/6)+3^(1/2))-1/36*ln(1+(1-x)^(1/3)/(1+x)^(1/3)-(1-x
)^(1/6)*3^(1/2)/(1+x)^(1/6))*3^(1/2)+1/36*ln(1+(1-x)^(1/3)/(1+x)^(1/3)+(1-x
)^(1/6)*3^(1/2)/(1+x)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6126, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$-\frac{1}{2}(1-x)^{5/6}(x+1)^{7/6}-\frac{1}{6}(1-x)^{5/6}\sqrt[6]{x+1}-\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{12\sqrt{3}}+\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{12\sqrt{3}}-\frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)*x,x]
```

```
[Out] -((1 - x)^(5/6)*(1 + x)^(1/6))/6 - ((1 - x)^(5/6)*(1 + x)^(7/6))/2 - ArcTan
[(1 - x)^(1/6)/(1 + x)^(1/6)]/9 + ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x
)^(1/6)]/18 - ArcTan[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/18 - Log[1
+ (1 - x)^(1/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*
Sqrt[3]) + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1
 + x)^(1/6)]/(12*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{1}{3}\tanh^{-1}(x)}x\,dx = \int \frac{x\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$
$$= -\frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} + \frac{1}{6}\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} + \frac{1}{18}\int \frac{1}{\sqrt[6]{1-x}\,(1+x)^{5/6}}\,dx$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{1-x}\right)$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{1}{9}\,\text{Subst}$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{1}{36}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+\right.$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{\log\left(1+\frac{\sqrt[3]{1-x}}{\sqrt[3]{1+x}} - \frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right.}{12\sqrt{3}}$$
$$= -\frac{1}{6}(1-x)^{5/6}\sqrt[6]{1+x} - \frac{1}{2}(1-x)^{5/6}(1+x)^{7/6} - \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \frac{1}{18}\tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right.$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.22

$$-\frac{1}{10}(1-x)^{5/6}\left(2\sqrt[6]{2}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{1-x}{2}\right)+5(x+1)^{7/6}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)*x,x]
```

```
[Out] -1/10*((1 - x)^(5/6)*(5*(1 + x)^(7/6) + 2*2^(1/6)*Hypergeometric2F1[-1/6, 5
/6, 11/6, (1 - x)/2]))
```

fricas [A] time = 0.56, size = 301, normalized size = 1.34

$$\frac{1}{36}\sqrt{3}\log\left(4\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+4\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+4\right)-\frac{1}{36}\sqrt{3}\log\left(-4\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+4\left(-\frac{\sqrt{-x^2+1}}{x-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*log(4*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^2
+ 1)/(x - 1))^(2/3) + 4) - 1/36*sqrt(3)*log(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(x
- 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) + 1/6*(3*x^2 + x - 4)*
(-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1/9*arctan(sqrt(3) + sqrt(-4*sqrt(3)*(-sq
rt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) - 2*(-
sqrt(-x^2 + 1)/(x - 1))^(1/3)) - 1/9*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*(-sqr
t(-x^2 + 1)/(x - 1))^(1/3) + (-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1) - 2*(-sqr
t(-x^2 + 1)/(x - 1))^(1/3)) + 1/9*arctan((-sqrt(-x^2 + 1)/(x - 1))^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)*x,x, algorithm="maxima")
[Out] integrate(x*((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(\frac{x+1}{\sqrt{1-x^2}}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((x + 1)/(1 - x^2)^(1/2))^(1/3),x)
[Out] int(x*((x + 1)/(1 - x^2)^(1/2))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt[3]{\frac{x+1}{\sqrt{1-x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)*x,x)
[Out] Integral(x*((x + 1)/sqrt(1 - x**2))**(1/3), x)
```

3.123 $\int e^{\frac{1}{3}\tanh^{-1}(x)}\,dx$

Optimal. Leaf size=202

$$-(1-x)^{5/6}\sqrt[6]{x+1}-\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{2\sqrt{3}}+\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{2\sqrt{3}}-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)+\frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\cdots}{\sqrt[6]{\cdots}}\right.$$

```
[Out] -(1-x)^(5/6)*(1+x)^(1/6)-2/3*arctan((1-x)^(1/6)/(1+x)^(1/6))-1/3*arctan(2*(
1-x)^(1/6)/(1+x)^(1/6)-3^(1/2))-1/3*arctan(2*(1-x)^(1/6)/(1+x)^(1/6)+3^(1/2
))-1/6*ln(1+(1-x)^(1/3)/(1+x)^(1/3)-(1-x)^(1/6)*3^(1/2)/(1+x)^(1/6))*3^(1/2
)+1/6*ln(1+(1-x)^(1/3)/(1+x)^(1/3)+(1-x)^(1/6)*3^(1/2)/(1+x)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {6125, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$-(1-x)^{5/6}\sqrt[6]{x+1}-\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{2\sqrt{3}}+\frac{\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)}{2\sqrt{3}}-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)+\frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\cdots}{\sqrt[6]{\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3),x]
```

```
[Out] -((1 - x)^(5/6)*(1 + x)^(1/6)) - (2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])/3
+ ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - ArcTan[Sqrt[3] + (2
*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (S
qrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(2*Sqrt[3]) + Log[1 + (1 - x)^(1/3)/(1
 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(2*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{1}{3}\tanh^{-1}(x)}\,dx &= \int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx\\
&= -(1-x)^{5/6}\sqrt[6]{1+x} + \frac{1}{3}\int \frac{1}{\sqrt[6]{1-x}\,(1+x)^{5/6}}\,dx\\
&= -(1-x)^{5/6}\sqrt[6]{1+x} - 2\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{1-x}\right)\\
&= -(1-x)^{5/6}\sqrt[6]{1+x} - 2\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)\\
&= -(1-x)^{5/6}\sqrt[6]{1+x} - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1}}{\sqrt[6]{1}}\right.\\
&= -(1-x)^{5/6}\sqrt[6]{1+x} - \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{1}{6}\,\text{Subst}\\
&= -(1-x)^{5/6}\sqrt[6]{1+x} - \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{\log\left(1+\frac{\sqrt[3]{1-x}}{\sqrt[3]{1+x}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right)}{2\sqrt{3}} + \frac{\log\left(1+\frac{\sqrt[3]{1-x}}{\sqrt[3]{1+x}}+\frac{\sqrt{3}\sqrt[6]{1}}{\sqrt[6]{1+}}\right.}{2\sqrt{3}}\\
&= -(1-x)^{5/6}\sqrt[6]{1+x} - \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \frac{1}{3}\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right.
\end{aligned}$$

Mathematica [C] time = 0.05, size = 39, normalized size = 0.19

$$2e^{\frac{1}{3}\tanh^{-1}(x)}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-e^{2\tanh^{-1}(x)}\right)-\frac{1}{e^{2\tanh^{-1}(x)}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3),x]
```

```
[Out] 2*E^(ArcTanh[x]/3)*(-(1 + E^(2*ArcTanh[x]))^(-1) + Hypergeometric2F1[1/6, 1
, 7/6, -E^(2*ArcTanh[x])])
```

fricas [A] time = 0.74, size = 295, normalized size = 1.46

$$\frac{1}{6}\sqrt{3}\log\left(4\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+4\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+4\right)-\frac{1}{6}\sqrt{3}\log\left(-4\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+4\left(-\frac{\sqrt{-x^2+1}}{x-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(4*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^2 +
 1)/(x - 1))^(2/3) + 4) - 1/6*sqrt(3)*log(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(x -
1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) + (x - 1)*(-sqrt(-x^2 +
1)/(x - 1))^(1/3) - 2/3*arctan(sqrt(3) + sqrt(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(
x - 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) - 2*(-sqrt(-x^2 + 1)
/(x - 1))^(1/3)) - 2/3*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*(-sqrt(-x^2 + 1)/(x
 - 1))^(1/3) + (-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1) - 2*(-sqrt(-x^2 + 1)/(x
 - 1))^(1/3)) + 2/3*arctan((-sqrt(-x^2 + 1)/(x - 1))^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3),x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{x+1}{\sqrt{1-x^2}}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(1 - x^2)^(1/2))^(1/3),x)
```

```
[Out] int(((x + 1)/(1 - x^2)^(1/2))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\frac{x+1}{\sqrt{1-x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3),x)
```

```
[Out] Integral(((x + 1)/sqrt(1 - x**2))**(1/3), x)
```

3.124 $\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=346

$$-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)$$

```
[Out] -2*arctan((1-x)^(1/6)/(1+x)^(1/6))-arctan(2*(1-x)^(1/6)/(1+x)^(1/6)-3^(1/2)
)-arctan(2*(1-x)^(1/6)/(1+x)^(1/6)+3^(1/2))-2*arctanh((1+x)^(1/6)/(1-x)^(1/
6))+1/2*ln(1-(1+x)^(1/6)/(1-x)^(1/6)+(1+x)^(1/3)/(1-x)^(1/3))-1/2*ln(1+(1+x
)^(1/6)/(1-x)^(1/6)+(1+x)^(1/3)/(1-x)^(1/3))+arctan(1/3*(1-2*(1+x)^(1/6)/(1
-x)^(1/6))*3^(1/2))*3^(1/2)-arctan(1/3*(1+2*(1+x)^(1/6)/(1-x)^(1/6))*3^(1/2
))*3^(1/2)-1/2*ln(1+(1-x)^(1/3)/(1+x)^(1/3)-(1-x)^(1/6)*3^(1/2)/(1+x)^(1/6)
)*3^(1/2)+1/2*ln(1+(1-x)^(1/3)/(1+x)^(1/3)+(1-x)^(1/6)*3^(1/2)/(1+x)^(1/6))
*3^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6126, 105, 63, 331, 295, 634, 618, 204, 628, 203, 93, 210, 206}

$$-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}-\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+\frac{\sqrt{3}\sqrt[6]{1-x}}{\sqrt[6]{x+1}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)/x,x]
```

```
[Out] -2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)] + ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))
/(1 + x)^(1/6)] - ArcTan[Sqrt[3] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)] + Sqrt[
3]*ArcTan[(1 - (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]] - Sqrt[3]*ArcTan[(
1 + (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]] - 2*ArcTanh[(1 + x)^(1/6)/(1
- x)^(1/6)] - (Sqrt[3]*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (Sqrt[3]*(1 -
x)^(1/6))/(1 + x)^(1/6)])/2 + (Sqrt[3]*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)
+ (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)])/2 + Log[1 - (1 + x)^(1/6)/(1 - x)
^(1/6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/2 - Log[1 + (1 + x)^(1/6)/(1 - x)^(1/
6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
```

```
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x}\,dx = \int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x}\,dx$$
$$= \int \frac{1}{\sqrt[6]{1-x}\,(1+x)^{5/6}}\,dx + \int \frac{1}{\sqrt[6]{1-x}\,x(1+x)^{5/6}}\,dx$$
$$= -\left(6\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{1-x}\right)\right) + 6\,\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)\right) - 2\,\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) - 2\,\text{Subst}\Bigg(\int$$
$$= -2\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x,\right.$$
$$= -2\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{2}\log\left(1-\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}+\frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\right) - \frac{1}{2}\log\left(1+\frac{\sqrt[6]{1+}}{\sqrt[6]{1-}}\right.$$
$$= -2\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right) - \sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)$$
$$= -2\tan^{-1}\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) - \tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{1-x}}{\sqrt[6]{1+x}}\right) + \sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1}}{\sqrt[6]{1-}}}{\sqrt{3}}\right.$$

Mathematica [C] time = 0.03, size = 74, normalized size = 0.21

$$-\frac{3(1-x)^{5/6}\left(\sqrt[6]{2}\,(x+1)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{1-x}{2}\right)+2\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};\frac{1-x}{x+1}\right)\right)}{5(x+1)^{5/6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)/x,x]
```

```
[Out] (-3*(1 - x)^(5/6)*(2^(1/6)*(1 + x)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6,
(1 - x)/2] + 2*Hypergeometric2F1[5/6, 1, 11/6, (1 - x)/(1 + x)]))/(5*(1 + x
)^(5/6))
```

fricas [A] time = 0.46, size = 471, normalized size = 1.36

$$-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\log\left(4\sqrt{3}\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1/3*sqrt(3))
- sqrt(3)*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1/3*sqrt(3))
 + 1/2*sqrt(3)*log(4*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^
2 + 1)/(x - 1))^(2/3) + 4) - 1/2*sqrt(3)*log(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(x
 - 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 4) - 2*arctan(sqrt(3) +
sqrt(-4*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 4*(-sqrt(-x^2 + 1)/(x - 1
))^(2/3) + 4) - 2*(-sqrt(-x^2 + 1)/(x - 1))^(1/3)) - 2*arctan(-sqrt(3) + 2*
sqrt(sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + (-sqrt(-x^2 + 1)/(x - 1))^(2
/3) + 1) - 2*(-sqrt(-x^2 + 1)/(x - 1))^(1/3)) + 2*arctan((-sqrt(-x^2 + 1)/(
x - 1))^(1/3)) - 1/2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) + (-sqrt(-x^2 + 1)
/(x - 1))^(1/3) + 1) + 1/2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - (-sqrt(-x^
2 + 1)/(x - 1))^(1/3) + 1) - log((-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1) + log
((-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x,x, algorithm="maxima")
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{1-x^2}}\right)^{1/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(1 - x^2)^(1/2))^(1/3)/x,x)
[Out] int(((x + 1)/(1 - x^2)^(1/2))^(1/3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\frac{x+1}{\sqrt{1-x^2}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)/x,x)
[Out] Integral(((x + 1)/sqrt(1 - x**2))**(1/3)/x, x)
```

3.125 $\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=194

$$-\frac{(1-x)^{5/6}\sqrt[6]{x+1}}{x}+\frac{1}{6}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}+\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{x+1}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2 \cdots}{\cdots}\right)}{\sqrt{\cdots}}$$

```
[Out] -(1-x)^(5/6)*(1+x)^(1/6)/x-2/3*arctanh((1+x)^(1/6)/(1-x)^(1/6))+1/6*ln(1-(1
+x)^(1/6)/(1-x)^(1/6)+(1+x)^(1/3)/(1-x)^(1/3))-1/6*ln(1+(1+x)^(1/6)/(1-x)^(
1/6)+(1+x)^(1/3)/(1-x)^(1/3))+1/3*arctan(1/3*(1-2*(1+x)^(1/6)/(1-x)^(1/6))*
3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*(1+x)^(1/6)/(1-x)^(1/6))*3^(1/2))*3^(1
/2)
```

Rubi [A] time = 0.16, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6126, 94, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{(1-x)^{5/6}\sqrt[6]{x+1}}{x}+\frac{1}{6}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}+\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{x+1}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2 \cdots}{\cdots}\right)}{\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)/x^2,x]
```

```
[Out] -(((1 - x)^(5/6)*(1 + x)^(1/6))/x) + ArcTan[(1 - (2*(1 + x)^(1/6))/(1 - x)^
(1/6))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt
[3]]/Sqrt[3] - (2*ArcTanh[(1 + x)^(1/6)/(1 - x)^(1/6)])/3 + Log[1 - (1 + x)
^(1/6)/(1 - x)^(1/6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/6 - Log[1 + (1 + x)^(1/
6)/(1 - x)^(1/6) + (1 + x)^(1/3)/(1 - x)^(1/3)]/6
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x^2}\,dx &= \int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^2}\,dx \\
&= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x}+\frac{1}{3}\int \frac{1}{\sqrt[6]{1-x}\,x(1+x)^{5/6}}\,dx \\
&= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x}+2\,\mathrm{Subst}\left(\int \frac{1}{-1+x^6}\,dx,x,\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) \\
&= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x}-\frac{2}{3}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)-\frac{2}{3}\,\mathrm{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx,x,\frac{\sqrt[6]{1-}}{\sqrt[6]{1-}}\right. \\
&= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x}-\frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)+\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)-\frac{1}{6}\,\mathrm{Subs} \\
&= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x}-\frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)+\frac{1}{6}\log\left(1-\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}+\frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\right)-\frac{1}{6}\log\left(1+\frac{\sqrt[6]{1}}{\sqrt[6]{1}}\right. \\
&= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{x}-\frac{\tan^{-1}\left(\frac{-1+\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)+\frac{1}{6}\log\left(1\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.26

$$-\frac{(1-x)^{5/6}\left(2x\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};\frac{1-x}{x+1}\right)+5x+5\right)}{5x(x+1)^{5/6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)/x^2,x]
```

[Out] -1/5*((1 - x)^(5/6)*(5 + 5*x + 2*x*Hypergeometric2F1[5/6, 1, 11/6, (1 - x)/(1 + x)]))/(x*(1 + x)^(5/6))

fricas [A] time = 0.50, size = 234, normalized size = 1.21

$$-\frac{2\sqrt{3}\,x\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}\,x\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1/3*sqrt(3)) + 2*sqrt(3)*x*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1/3*sqrt(3)) + x*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) + (-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1) - x*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - (-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1) + 2*x*log((-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1) - 2*x*log((-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1) - 6*(x - 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3))/x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="giac")

[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x^2, x)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x)

[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="maxima")
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{x+1}{\sqrt{1-x^2}}\right)^{1/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(1 - x^2)^(1/2))^(1/3)/x^2,x)
[Out] int(((x + 1)/(1 - x^2)^(1/2))^(1/3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\frac{x+1}{\sqrt{1-x^2}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)/x**2,x)
[Out] Integral(((x + 1)/sqrt(1 - x**2))**(1/3)/x**2, x)
```

3.126 $\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=224

$$-\frac{(1-x)^{5/6}(x+1)^{7/6}}{2x^2}-\frac{(1-x)^{5/6}\sqrt[6]{x+1}}{6x}+\frac{1}{36}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{36}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}+\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)+\frac{\tan^{-}}{}$$

```
[Out] -1/6*(1-x)^(5/6)*(1+x)^(1/6)/x-1/2*(1-x)^(5/6)*(1+x)^(7/6)/x^2-1/9*arctanh(
(1+x)^(1/6)/(1-x)^(1/6))+1/36*ln(1-(1+x)^(1/6)/(1-x)^(1/6)+(1+x)^(1/3)/(1-x
)^(1/3))-1/36*ln(1+(1+x)^(1/6)/(1-x)^(1/6)+(1+x)^(1/3)/(1-x)^(1/3))+1/18*ar
ctan(1/3*(1-2*(1+x)^(1/6)/(1-x)^(1/6))*3^(1/2))*3^(1/2)-1/18*arctan(1/3*(1+
2*(1+x)^(1/6)/(1-x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6126, 96, 94, 93, 210, 634, 618, 204, 628, 206}

$$-\frac{(1-x)^{5/6}(x+1)^{7/6}}{2x^2}-\frac{(1-x)^{5/6}\sqrt[6]{x+1}}{6x}+\frac{1}{36}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}-\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)-\frac{1}{36}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{1-x}}+\frac{\sqrt[6]{x+1}}{\sqrt[6]{1-x}}+1\right)+\frac{\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[x]/3)/x^3,x]
```

```
[Out] -((1 - x)^(5/6)*(1 + x)^(1/6))/(6*x) - ((1 - x)^(5/6)*(1 + x)^(7/6))/(2*x^2
) + ArcTan[(1 - (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]]/(6*Sqrt[3]) - Arc
Tan[(1 + (2*(1 + x)^(1/6))/(1 - x)^(1/6))/Sqrt[3]]/(6*Sqrt[3]) - ArcTanh[(1
 + x)^(1/6)/(1 - x)^(1/6)]/9 + Log[1 - (1 + x)^(1/6)/(1 - x)^(1/6) + (1 + x
)^(1/3)/(1 - x)^(1/3)]/36 - Log[1 + (1 + x)^(1/6)/(1 - x)^(1/6) + (1 + x)^(
1/3)/(1 - x)^(1/3)]/36
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\tanh^{-1}(x)}}{x^3}\,dx = \int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^3}\,dx$$
$$= -\frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} + \frac{1}{6}\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^2}\,dx$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} + \frac{1}{18}\int \frac{1}{\sqrt[6]{1-x}\,x(1+x)^{5/6}}\,dx$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right)$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) - \frac{1}{9}\text{Subst}\Bigg(\int$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} - \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{36}\text{Subst}\Bigg(\int \frac{-1+2x}{1-x+x^2}\,dx$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} - \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\right) + \frac{1}{36}\log\Bigg(1 - \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}} + \frac{\sqrt[3]{1}}{\sqrt[3]{1}}$$
$$= -\frac{(1-x)^{5/6}\sqrt[6]{1+x}}{6x} - \frac{(1-x)^{5/6}(1+x)^{7/6}}{2x^2} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{1}{9}\tanh^{-1}\Bigg($$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.27

$$-\frac{(1-x)^{5/6}\left(2x^2\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};\frac{1-x}{x+1}\right)+5\left(4x^2+7x+3\right)\right)}{30x^2(x+1)^{5/6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[x]/3)/x^3,x]
```

```
[Out] -1/30*((1 - x)^(5/6)*(5*(3 + 7*x + 4*x^2) + 2*x^2*Hypergeometric2F1[5/6, 1,
 11/6, (1 - x)/(1 + x)]))/(x^2*(1 + x)^(5/6))
```

fricas [A] time = 0.60, size = 253, normalized size = 1.13

$$-\frac{2\sqrt{3}\,x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}\,x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x^2\log\Bigg(\Bigg(-\frac{\sqrt{-x^2+1}}{x-1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/36*(2*sqrt(3)*x^2*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1
/3*sqrt(3)) + 2*sqrt(3)*x^2*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(1
/3) - 1/3*sqrt(3)) + x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) + (-sqrt(-x^2
+ 1)/(x - 1))^(1/3) + 1) - x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - (-sqrt
(-x^2 + 1)/(x - 1))^(1/3) + 1) + 2*x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(1/3)
+ 1) - 2*x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1) - 6*(4*x^2 - x - 3)*(
-sqrt(-x^2 + 1)/(x - 1))^(1/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x^3, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(1/3)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{1-x^2}}\right)^{1/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(1 - x^2)^(1/2))^(1/3)/x^3,x)
```

```
[Out] int(((x + 1)/(1 - x^2)^(1/2))^(1/3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\frac{x+1}{\sqrt{1-x^2}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(1/3)/x**3,x)
```

```
[Out] Integral(((x + 1)/sqrt(1 - x**2))**(1/3)/x**3, x)
```

3.127 $\int e^{\frac{2}{3}\tanh^{-1}(x)}x^m\,dx$

Optimal. Leaf size=28

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{3},-\frac{1}{3};m+2;x,-x\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/3,-1/3,2+m,x,-x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{1}{3},-\frac{1}{3};m+2;x,-x\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, 1/3, -1/3, 2 + m, x, -x])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}\tanh^{-1}(x)}x^m\,dx &= \int \frac{x^m\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{1}{3},-\frac{1}{3};2+m;x,-x\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int e^{\frac{2}{3}\tanh^{-1}(x)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((2*ArcTanh[x])/3)*x^m,x]
[Out] Integrate[E^((2*ArcTanh[x])/3)*x^m, x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x, algorithm="fricas")
[Out] integral(x^m*(-sqrt(-x^2 + 1)/(x - 1))^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x, algorithm="giac")
[Out] integrate(x^m*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x)
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int x^m \left(\frac{x+1}{\sqrt{1-x^2}}\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((x + 1)/(1 - x^2)^(1/2))^(2/3),x)
[Out] int(x^m*((x + 1)/(1 - x^2)^(1/2))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)*x**m,x)
[Out] Timed out
```

3.128 $\int e^{\frac{2}{3}\tanh^{-1}(x)}x^2\,dx$

Optimal. Leaf size=133

$$-\frac{1}{3}(1-x)^{2/3}x(x+1)^{4/3}-\frac{1}{9}(1-x)^{2/3}(x+1)^{4/3}-\frac{11}{27}(1-x)^{2/3}\sqrt[3]{x+1}+\frac{11}{81}\log(x+1)+\frac{11}{27}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{22\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{27}$$

```
[Out] -11/27*(1-x)^(2/3)*(1+x)^(1/3)-1/9*(1-x)^(2/3)*(1+x)^(4/3)-1/3*(1-x)^(2/3)*
x*(1+x)^(4/3)+11/81*ln(1+x)+11/27*ln(1+(1-x)^(1/3)/(1+x)^(1/3))-22/81*arcta
n(-1/3*3^(1/2)+2/3*(1-x)^(1/3)/(1+x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6126, 90, 80, 50, 60}

$$-\frac{1}{3}(1-x)^{2/3}x(x+1)^{4/3}-\frac{1}{9}(1-x)^{2/3}(x+1)^{4/3}-\frac{11}{27}(1-x)^{2/3}\sqrt[3]{x+1}+\frac{11}{81}\log(x+1)+\frac{11}{27}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{22\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{27}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)*x^2,x]
```

```
[Out] (-11*(1 - x)^(2/3)*(1 + x)^(1/3))/27 - ((1 - x)^(2/3)*(1 + x)^(4/3))/9 - ((
1 - x)^(2/3)*x*(1 + x)^(4/3))/3 + (22*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/
(Sqrt[3]*(1 + x)^(1/3))])/(27*Sqrt[3]) + (11*Log[1 + x])/81 + (11*Log[1 + (
1 - x)^(1/3)/(1 + x)^(1/3)])/27
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
```

```
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{2}{3}\tanh^{-1}(x)}x^2\,dx=\int\frac{x^2\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx$$

$$=-\frac{1}{3}(1-x)^{2/3}x(1+x)^{4/3}-\frac{1}{3}\int\frac{\left(-1-\frac{2x}{3}\right)\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx$$

$$=-\frac{1}{9}(1-x)^{2/3}(1+x)^{4/3}-\frac{1}{3}(1-x)^{2/3}x(1+x)^{4/3}+\frac{11}{27}\int\frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx$$

$$=-\frac{11}{27}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{9}(1-x)^{2/3}(1+x)^{4/3}-\frac{1}{3}(1-x)^{2/3}x(1+x)^{4/3}+\frac{22}{81}\int\frac{1}{\sqrt[3]{1-x}\,(1+x)^{2}\cdots}$$

$$=-\frac{11}{27}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{9}(1-x)^{2/3}(1+x)^{4/3}-\frac{1}{3}(1-x)^{2/3}x(1+x)^{4/3}+\frac{22\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1\cdots}}{\sqrt{3}\sqrt[3]{\cdots}}\right.}{27\sqrt{3}}$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.44

$$-\frac{1}{18}(1-x)^{2/3}\left(11\sqrt[3]{2}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{1-x}{2}\right)+2\sqrt[3]{x+1}\left(3x^2+4x+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)*x^2,x]
```

```
[Out] -1/18*((1 - x)^(2/3)*(2*(1 + x)^(1/3)*(1 + 4*x + 3*x^2) + 11*2^(1/3)*Hyperg
eometric2F1[-1/3, 2/3, 5/3, (1 - x)/2]))
```

fricas [A] time = 0.50, size = 159, normalized size = 1.20

$$\frac{1}{27}\left(9\,x^3+3\,x^2+2\,x-14\right)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{22}{81}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{22}{81}\log\left(\left(-\frac{\sqrt{-x^2+}}{x-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^3 + 3*x^2 + 2*x - 14)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 22/81*sqr
t(3)*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1/3*sqrt(3)) + 22
/81*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1) - 11/81*log(-((x - 1)*(-sqrt(-
x^2 + 1)/(x - 1))^(2/3) - x + sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3
) + 1)/(x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x)
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(\frac{x+1}{\sqrt{1-x^2}}\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((x + 1)/(1 - x^2)^(1/2))^(2/3),x)
[Out] int(x^2*((x + 1)/(1 - x^2)^(1/2))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)*x**2,x)
[Out] Timed out
```

3.129 $\int e^{\frac{2}{3}\tanh^{-1}(x)}x\,dx$

Optimal. Leaf size=112

$$-\frac{1}{2}(1-x)^{2/3}(x+1)^{4/3}-\frac{1}{3}(1-x)^{2/3}\sqrt[3]{x+1}+\frac{1}{9}\log(x+1)+\frac{1}{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{3\sqrt{3}}$$

```
[Out] -1/3*(1-x)^(2/3)*(1+x)^(1/3)-1/2*(1-x)^(2/3)*(1+x)^(4/3)+1/9*ln(1+x)+1/3*ln
(1+(1-x)^(1/3)/(1+x)^(1/3))-2/9*arctan(-1/3*3^(1/2)+2/3*(1-x)^(1/3)/(1+x)^(
1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6126, 80, 50, 60}

$$-\frac{1}{2}(1-x)^{2/3}(x+1)^{4/3}-\frac{1}{3}(1-x)^{2/3}\sqrt[3]{x+1}+\frac{1}{9}\log(x+1)+\frac{1}{3}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)*x,x]
```

```
[Out] -((1 - x)^(2/3)*(1 + x)^(1/3))/3 - ((1 - x)^(2/3)*(1 + x)^(4/3))/2 + (2*Arc
Tan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sqrt[3]) + L
og[1 + x]/9 + Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)]/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}\tanh^{-1}(x)}x\,dx &= \int \frac{x\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx\\ &= -\frac{1}{2}(1-x)^{2/3}(1+x)^{4/3}+\frac{1}{3}\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx\\ &= -\frac{1}{3}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{2}(1-x)^{2/3}(1+x)^{4/3}+\frac{2}{9}\int \frac{1}{\sqrt[3]{1-x}\,(1+x)^{2/3}}\,dx\\ &= -\frac{1}{3}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{2}(1-x)^{2/3}(1+x)^{4/3}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{3\sqrt{3}}+\frac{1}{9}\log(1+x)+\frac{1}{3}\log\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.41

$$-\frac{1}{2}(1-x)^{2/3}\left(\sqrt[3]{2}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{1-x}{2}\right)+(x+1)^{4/3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)*x,x]
```

```
[Out] -1/2*((1 - x)^(2/3)*((1 + x)^(4/3) + 2^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5
/3, (1 - x)/2]))
```

fricas [A] time = 0.61, size = 154, normalized size = 1.38

$$\frac{1}{6}\left(3x^2+2x-5\right)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{2}{9}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{2}{9}\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 2*x - 5)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 2/9*sqrt(3)*arctan(
2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1/3*sqrt(3)) + 2/9*log((-sqrt
(-x^2 + 1)/(x - 1))^(2/3) + 1) - 1/9*log(-((x - 1)*(-sqrt(-x^2 + 1)/(x - 1)
)^(2/3) - x + sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1)/(x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x, algorithm="giac")
```

```
[Out] integrate(x*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)*x,x, algorithm="maxima")
[Out] integrate(x*((x + 1)/sqrt(-x^2 + 1))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(\frac{x+1}{\sqrt{1-x^2}}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((x + 1)/(1 - x^2)^(1/2))^(2/3),x)
[Out] int(x*((x + 1)/(1 - x^2)^(1/2))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{x+1}{\sqrt{1-x^2}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)*x,x)
[Out] Integral(x*((x + 1)/sqrt(1 - x**2))**(2/3), x)
```

3.130 $\int e^{\frac{2}{3}\tanh^{-1}(x)}\,dx$

Optimal. Leaf size=84

$$-(1-x)^{2/3}\sqrt[3]{x+1}+\frac{1}{3}\log(x+1)+\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{\sqrt{3}}$$

```
[Out] -(1-x)^(2/3)*(1+x)^(1/3)+1/3*ln(1+x)+ln(1+(1-x)^(1/3)/(1+x)^(1/3))-2/3*arct
an(-1/3*3^(1/2)+2/3*(1-x)^(1/3)/(1+x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6125, 50, 60}

$$-(1-x)^{2/3}\sqrt[3]{x+1}+\frac{1}{3}\log(x+1)+\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3),x]
```

```
[Out] -((1 - x)^(2/3)*(1 + x)^(1/3)) + (2*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(S
qrt[3]*(1 + x)^(1/3))])/Sqrt[3] + Log[1 + x]/3 + Log[1 + (1 - x)^(1/3)/(1 +
 x)^(1/3)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}\tanh^{-1}(x)}\,dx &= \int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx\\ &= -(1-x)^{2/3}\sqrt[3]{1+x}+\frac{2}{3}\int \frac{1}{\sqrt[3]{1-x}\,(1+x)^{2/3}}\,dx\\ &= -(1-x)^{2/3}\sqrt[3]{1+x}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1+x)+\log\left(1+\frac{\sqrt[3]{1-x}}{\sqrt[3]{1+x}}\right)\end{aligned}$$

Mathematica [A] time = 0.15, size = 87, normalized size = 1.04

$$-\frac{2e^{\frac{2}{3}\tanh^{-1}(x)}}{e^{2\tanh^{-1}(x)}+1}+\frac{2}{3}\log\left(e^{\frac{2}{3}\tanh^{-1}(x)}+1\right)-\frac{1}{3}\log\left(-e^{\frac{2}{3}\tanh^{-1}(x)}+e^{\frac{4}{3}\tanh^{-1}(x)}+1\right)+\frac{2\tan^{-1}\left(\frac{2e^{\frac{2}{3}\tanh^{-1}(x)}-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3),x]
```

```
[Out] (-2*E^((2*ArcTanh[x])/3))/(1 + E^(2*ArcTanh[x])) + (2*ArcTan[(-1 + 2*E^((2*
ArcTanh[x])/3))/Sqrt[3]])/Sqrt[3] + (2*Log[1 + E^((2*ArcTanh[x])/3)])/3 - L
og[1 - E^((2*ArcTanh[x])/3) + E^((4*ArcTanh[x])/3)]/3
```

fricas [B] time = 0.54, size = 146, normalized size = 1.74

$$(x-1)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{2}{3}\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+1\right)-\frac{1}{3}\log\left(-\frac{(x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3),x, algorithm="fricas")
```

[Out] (x - 1)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 2/3*sqrt(3)*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1/3*sqrt(3)) + 2/3*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1) - 1/3*log(-((x - 1)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) - x + sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1)/(x - 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3),x, algorithm="giac")

[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3), x)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3),x)

[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3),x, algorithm="maxima")

[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{x+1}{\sqrt{1-x^2}}\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(1 - x^2)^(1/2))^(2/3),x)
```

```
[Out] int(((x + 1)/(1 - x^2)^(1/2))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{x+1}{\sqrt{1-x^2}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3),x)
```

```
[Out] Integral(((x + 1)/sqrt(1 - x**2))**(2/3), x)
```

3.131 $\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=135

$$-\frac{\log(x)}{2}+\frac{1}{2}\log(x+1)+\frac{3}{2}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{3}{2}\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)+\sqrt{3}\tan$$

```
[Out] -1/2*ln(x)+1/2*ln(1+x)+3/2*ln(1+(1-x)^(1/3)/(1+x)^(1/3))+3/2*ln((1-x)^(1/3)
-(1+x)^(1/3))-arctan(-1/3*3^(1/2)+2/3*(1-x)^(1/3)/(1+x)^(1/3)*3^(1/2))*3^(1
/2)+arctan(1/3*3^(1/2)+2/3*(1-x)^(1/3)/(1+x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 105, 60, 91}

$$-\frac{\log(x)}{2}+\frac{1}{2}\log(x+1)+\frac{3}{2}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x+1}}+1\right)+\frac{3}{2}\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}\right)+\sqrt{3}\tan$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)/x,x]
```

```
[Out] Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))] + Sqr
t[3]*ArcTan[1/Sqrt[3] + (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))] - Log[x]
/2 + Log[1 + x]/2 + (3*Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3)])/2 + (3*Log[(1
- x)^(1/3) - (1 + x)^(1/3)])/2
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x}\,dx &= \int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x}\,dx\\ &= \int \frac{1}{\sqrt[3]{1-x}\,(1+x)^{2/3}}\,dx + \int \frac{1}{\sqrt[3]{1-x}\,x(1+x)^{2/3}}\,dx\\ &= \sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-x}}{\sqrt{3}\,\sqrt[3]{1+x}}\right) + \sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-x}}{\sqrt{3}\,\sqrt[3]{1+x}}\right) - \frac{\log(x)}{2} + \frac{1}{2}\log(1+x) + \frac{3}{2}\end{aligned}$$

Mathematica [C] time = 0.02, size = 74, normalized size = 0.55

$$-\frac{3(1-x)^{2/3}\left(\sqrt[3]{2}\,(x+1)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1-x}{2}\right) + 2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1-x}{x+1}\right)\right)}{4(x+1)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)/x,x]
```

```
[Out] (-3*(1 - x)^(2/3)*(2^(1/3)*(1 + x)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (
1 - x)/2] + 2*Hypergeometric2F1[2/3, 1, 5/3, (1 - x)/(1 + x)]))/(4*(1 + x)^
(2/3))
```

fricas [A] time = 1.29, size = 158, normalized size = 1.17

$$-\sqrt{3}\arctan\left(-\frac{\sqrt{3}(x-1)-2\sqrt{3}\sqrt{-x^2+1}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{1}{3}}}{3(x-1)}\right)-\frac{1}{2}\log\left(-\frac{(x+1)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-x+\sqrt{-x^2+1}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right.}{x-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-1/3*(sqrt(3)*(x - 1) - 2*sqrt(3)*sqrt(-x^2 + 1)*(-sqrt(-x^
2 + 1)/(x - 1))^(1/3))/(x - 1)) - 1/2*log(-((x + 1)*(-sqrt(-x^2 + 1)/(x - 1
))^(2/3) - x + sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) + 1)/(x - 1))
 + log(-(x + sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1))^(1/3) - 1)/(x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{x+1}{\sqrt{1-x^2}}\right)^{2/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(1 - x^2)^(1/2))^(2/3)/x,x)
```

```
[Out] int(((x + 1)/(1 - x^2)^(1/2))^(2/3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{1-x^2}}\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)/x,x)
```

```
[Out] Integral(((x + 1)/sqrt(1 - x**2))**(2/3)/x, x)
```

3.132 $\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=85

$$-\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{x}-\frac{\log(x)}{3}+\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(1-x)^(2/3)*(1+x)^(1/3)/x-1/3*ln(x)+ln((1-x)^(1/3)-(1+x)^(1/3))+2/3*arctan
(1/3*3^(1/2)+2/3*(1-x)^(1/3)/(1+x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6126, 94, 91}

$$-\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{x}-\frac{\log(x)}{3}+\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)/x^2,x]
```

```
[Out] -(((1 - x)^(2/3)*(1 + x)^(1/3))/x) + (2*ArcTan[1/Sqrt[3] + (2*(1 - x)^(1/3)
)/(Sqrt[3]*(1 + x)^(1/3))])/Sqrt[3] - Log[x]/3 + Log[(1 - x)^(1/3) - (1 + x
)^(1/3)]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x^2}\,dx &= \int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^2}\,dx\\ &= -\frac{(1-x)^{2/3}\sqrt[3]{1+x}}{x}+\frac{2}{3}\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)^{2/3}}\,dx\\ &= -\frac{(1-x)^{2/3}\sqrt[3]{1+x}}{x}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{\sqrt{3}}-\frac{\log(x)}{3}+\log\left(\sqrt[3]{1-x}-\sqrt[3]{1+x}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.53

$$-\frac{(1-x)^{2/3}\left(x\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1-x}{x+1}\right)+x+1\right)}{x(x+1)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)/x^2,x]
[Out] -(((1 - x)^(2/3)*(1 + x + x*Hypergeometric2F1[2/3, 1, 5/3, (1 - x)/(1 + x)]
))/(x*(1 + x)^(2/3)))
```

fricas [B] time = 0.52, size = 152, normalized size = 1.79

$$-\frac{2\sqrt{3}\,x\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)-2\,x\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-1\right)+x\log\left(\frac{(x-1)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+x-\sqrt{-x^2+1}\left(-\frac{\sqrt{-x^2}}{x-1}\right.}{x-1}\right.}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1/3*
sqrt(3)) - 2*x*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1) + x*log(((x - 1)*(-
sqrt(-x^2 + 1)/(x - 1))^(2/3) + x - sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1)/(x - 1)
)^(1/3) - 1)/(x - 1)) - 3*(x - 1)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{x+1}{\sqrt{1-x^2}}\right)}^{2/3}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(1 - x^2)^(1/2))^(2/3)/x^2,x)
```

```
[Out] int(((x + 1)/(1 - x^2)^(1/2))^(2/3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{1-x^2}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)/x**2,x)
```

```
[Out] Integral(((x + 1)/sqrt(1 - x**2))**(2/3)/x**2, x)
```

3.133 $\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=116

$$-\frac{(1-x)^{2/3}(x+1)^{4/3}}{2x^2}-\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{3x}-\frac{\log(x)}{9}+\frac{1}{3}\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/3*(1-x)^(2/3)*(1+x)^(1/3)/x-1/2*(1-x)^(2/3)*(1+x)^(4/3)/x^2-1/9*ln(x)+1/
3*ln((1-x)^(1/3)-(1+x)^(1/3))+2/9*arctan(1/3*3^(1/2)+2/3*(1-x)^(1/3)/(1+x)^
(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 96, 94, 91}

$$-\frac{(1-x)^{2/3}(x+1)^{4/3}}{2x^2}-\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{3x}-\frac{\log(x)}{9}+\frac{1}{3}\log\left(\sqrt[3]{1-x}-\sqrt[3]{x+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcTanh[x])/3)/x^3,x]
```

```
[Out] -((1 - x)^(2/3)*(1 + x)^(1/3))/(3*x) - ((1 - x)^(2/3)*(1 + x)^(4/3))/(2*x^2
) + (2*ArcTan[1/Sqrt[3] + (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sq
rt[3]) - Log[x]/9 + Log[(1 - x)^(1/3) - (1 + x)^(1/3)]/3
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\tanh^{-1}(x)}}{x^3}\,dx &= \int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^3}\,dx\\ &= -\frac{(1-x)^{2/3}(1+x)^{4/3}}{2x^2}+\frac{1}{3}\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^2}\,dx\\ &= -\frac{(1-x)^{2/3}\sqrt[3]{1+x}}{3x}-\frac{(1-x)^{2/3}(1+x)^{4/3}}{2x^2}+\frac{2}{9}\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)^{2/3}}\,dx\\ &= -\frac{(1-x)^{2/3}\sqrt[3]{1+x}}{3x}-\frac{(1-x)^{2/3}(1+x)^{4/3}}{2x^2}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{3\sqrt{3}}-\frac{\log(x)}{9}+\frac{1}{3}\log\left(\sqrt[3]{1-x\right.\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.49

$$-\frac{(1-x)^{2/3}\left(2x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1-x}{x+1}\right)+5x^2+8x+3\right)}{6x^2(x+1)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcTanh[x])/3)/x^3,x]
```

```
[Out] -1/6*((1 - x)^(2/3)*(3 + 8*x + 5*x^2 + 2*x^2*Hypergeometric2F1[2/3, 1, 5/3,
 (1 - x)/(1 + x)]))/(x^2*(1 + x)^(2/3))
```

fricas [A] time = 0.71, size = 166, normalized size = 1.43

$$-\frac{4\sqrt{3}\,x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)-4\,x^2\log\left(\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}-1\right)+2\,x^2\log\left(\frac{(x-1)\left(-\frac{\sqrt{-x^2+1}}{x-1}\right)^{\frac{2}{3}}+x-\sqrt{-x^2+1}\,\cdots}{x-1}\right.}{18\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(4*sqrt(3)*x^2*arctan(2/3*sqrt(3)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + 1
/3*sqrt(3)) - 4*x^2*log((-sqrt(-x^2 + 1)/(x - 1))^(2/3) - 1) + 2*x^2*log(((
x - 1)*(-sqrt(-x^2 + 1)/(x - 1))^(2/3) + x - sqrt(-x^2 + 1)*(-sqrt(-x^2 + 1
)/(x - 1))^(1/3) - 1)/(x - 1)) - 3*(5*x^2 - 2*x - 3)*(-sqrt(-x^2 + 1)/(x -
1))^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x^3, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1+x}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x)
```

```
[Out] int(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x+1}{\sqrt{-x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="maxima")
[Out] integrate(((x + 1)/sqrt(-x^2 + 1))^(2/3)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{x+1}{\sqrt{1-x^2}}\right)^{2/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(1 - x^2)^(1/2))^(2/3)/x^3,x)
[Out] int(((x + 1)/(1 - x^2)^(1/2))^(2/3)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(-x**2+1)**(1/2))**(2/3)/x**3,x)
[Out] Timed out
```

3.134 $\int e^{\frac{1}{4}\tanh^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=31

$$\frac{x^{m+1} F_1\left(m+1; \frac{1}{8}, -\frac{1}{8}; m+2; ax, -ax\right)}{m+1}$$

[Out] x^(1+m)*AppellF1(1+m,1/8,-1/8,2+m,a*x,-a*x)/(1+m)

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6126, 133}

$$\frac{x^{m+1} F_1\left(m+1; \frac{1}{8}, -\frac{1}{8}; m+2; ax, -ax\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[E^(ArcTanh[a*x]/4)*x^m,x]

[Out] (x^(1 + m)*AppellF1[1 + m, 1/8, -1/8, 2 + m, a*x, -(a*x)])/(1 + m)

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{1}{4}\tanh^{-1}(ax)} x^m \, dx &= \int \frac{x^m \sqrt[8]{1+ax}}{\sqrt[8]{1-ax}} \, dx \\
&= \frac{x^{1+m} F_1\left(1+m; \frac{1}{8}, -\frac{1}{8}; 2+m; ax, -ax\right)}{1+m}
\end{aligned}$$

Mathematica [F] time = 0.38, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{4}\tanh^{-1}(ax)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcTanh[a*x]/4)*x^m,x]
[Out] Integrate[E^(ArcTanh[a*x]/4)*x^m, x]
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="fricas")
[Out] integral(x^m*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4),x)
[Out] int(x^m*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)*x**m,x)
[Out] Timed out
```

3.135 $\int e^{\frac{1}{4}\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=646

$$-\frac{(1-ax)^{7/8}(ax+1)^{9/8}}{24a^3}-\frac{11(1-ax)^{7/8}\sqrt[8]{ax+1}}{32a^3}-\frac{11\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{256a^3}+\frac{11\sqrt{2-\sqrt{2}}\,\log\left(\right.}{}$$

```
[Out] -11/32*(-a*x+1)^(7/8)*(a*x+1)^(1/8)/a^3-1/24*(-a*x+1)^(7/8)*(a*x+1)^(9/8)/a
^3-1/3*x*(-a*x+1)^(7/8)*(a*x+1)^(9/8)/a^2+11/128*arctan((-2*(-a*x+1)^(1/8)/
(a*x+1)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a^3-1
1/128*arctan((2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))
^(1/2))*(2-2^(1/2))^(1/2)/a^3-11/256*ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)-(-a*
x+1)^(1/8)*(2-2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2-2^(1/2))^(1/2)/a^3+11/256*ln
(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)+(-a*x+1)^(1/8)*(2-2^(1/2))^(1/2)/(a*x+1)^(1
/8))*(2-2^(1/2))^(1/2)/a^3+11/128*arctan((-2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(
2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)/a^3-11/128*arctan((2
*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1
/2))^(1/2)/a^3-11/256*ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)-(-a*x+1)^(1/8)*(2+2
^(1/2))^(1/2)/(a*x+1)^(1/8))*(2+2^(1/2))^(1/2)/a^3+11/256*ln(1+(-a*x+1)^(1/
4)/(a*x+1)^(1/4)+(-a*x+1)^(1/8)*(2+2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2+2^(1/2)
)^(1/2)/a^3
```

Rubi [A] time = 0.70, antiderivative size = 646, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6126, 90, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$-\frac{x(1-ax)^{7/8}(ax+1)^{9/8}}{3a^2}-\frac{(1-ax)^{7/8}(ax+1)^{9/8}}{24a^3}-\frac{11(1-ax)^{7/8}\sqrt[8]{ax+1}}{32a^3}-\frac{11\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+\right.}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)*x^2,x]
```

```
[Out] (-11*(1 - a*x)^(7/8)*(1 + a*x)^(1/8))/(32*a^3) - ((1 - a*x)^(7/8)*(1 + a*x)
^(9/8))/(24*a^3) - (x*(1 - a*x)^(7/8)*(1 + a*x)^(9/8))/(3*a^2) + (11*Sqrt[2
 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)
)/Sqrt[2 + Sqrt[2]]])/(128*a^3) + (11*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sq
rt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(128*a^3)
 - (11*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1
```

```
 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(128*a^3) - (11*Sqrt[2 - Sqrt[2]]*ArcTan
[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - Sqrt[2]
]])/(128*a^3) - (11*Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/
4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(256*a^3) + (11*
Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (Sqrt[2 - Sqrt[
2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(256*a^3) - (11*Sqrt[2 + Sqrt[2]]*Lo
g[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))
/(1 + a*x)^(1/8)])/(256*a^3) + (11*Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4
)/(1 + a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(
256*a^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
```

```
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{1}{4}\tanh^{-1}(ax)} x^2\,dx = \int \frac{x^2\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{\int \frac{\left(-1-\frac{ax}{4}\right)\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx}{3a^2}$$

$$= -\frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} + \frac{11\int \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx}{32a^2}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} + \frac{11\int \frac{1}{\sqrt[8]{1-ax}\,(1+ax)\cdots}}{128a^2}\cdots$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\,\text{Subst}\left(\int \frac{x^6}{1+x\cdots}\right.}{16\cdots}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\,\text{Subst}\left(\int \frac{\cdots}{1-\cdots}\right.}{32\cdots}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} + \frac{11\,\text{Subst}\left(\int \frac{1-\cdots}{1-\cdots}\right.}{32\cdots}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\,\text{Subst}\left(\int \frac{\sqrt{2-\cdots}}{1-\cdots}\right.}{64\sqrt{\cdots}}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{\left(11\sqrt{\frac{1}{2}}\left(3-2\sqrt{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} - \frac{11\sqrt{2-\sqrt{2}}\,\log\cdots}{\cdots}$$

$$= -\frac{11(1-ax)^{7/8}\sqrt[8]{1+ax}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} + \frac{11\sqrt{2+\sqrt{2}}\,\tan\cdots}{1\cdots}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.11

$$-\frac{(1-ax)^{7/8}\left(7\sqrt[8]{ax+1}\,\left(8a^2x^2+9ax+1\right)+66\sqrt[8]{2}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{8};\frac{15}{8};\frac{1}{2}(1-ax)\right)\right)}{168a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)*x^2,x]
```

```
[Out] -1/168*((1 - a*x)^(7/8)*(7*(1 + a*x)^(1/8)*(1 + 9*a*x + 8*a^2*x^2) + 66*2^(
1/8)*Hypergeometric2F1[-1/8, 7/8, 15/8, (1 - a*x)/2]))/a^3
```

fricas [B] time = 0.75, size = 2486, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3072*(264*a^3*sqrt(sqrt(2) + 2)*(a^(-24))^(1/8)*arctan((2*sqrt(a^6*(a^(-
24))^(1/4) + a^3*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(
a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8)
- 2*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(7/8) - sqrt(-sqrt
(2) + 2))/sqrt(sqrt(2) + 2)) + 264*a^3*sqrt(sqrt(2) + 2)*(a^(-24))^(1/8)*ar
ctan((2*sqrt(a^6*(a^(-24))^(1/4) - a^3*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 +
 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))
*a^21*(a^(-24))^(7/8) - 2*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-2
4))^(7/8) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 264*a^3*sqrt(-sqrt(2)
+ 2)*(a^(-24))^(1/8)*arctan((2*sqrt(a^6*(a^(-24))^(1/4) + a^3*sqrt(sqrt(2)
+ 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8) - 2*a^21*(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))^(1/4)*(a^(-24))^(7/8) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) +
264*a^3*sqrt(-sqrt(2) + 2)*(a^(-24))^(1/8)*arctan((2*sqrt(a^6*(a^(-24))^(1/
4) - a^3*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^
(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8) - 2*a^21*
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(7/8) + sqrt(sqrt(2) + 2))/
sqrt(-sqrt(2) + 2)) - 66*a^3*sqrt(sqrt(2) + 2)*(a^(-24))^(1/8)*log(a^6*(a^(
-24))^(1/4) + a^3*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(
a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 66*a^3*sqrt(sqrt(2)
 + 2)*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4) - a^3*sqrt(sqrt(2) + 2)*(-sqr
t(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))) - 66*a^3*sqrt(-sqrt(2) + 2)*(a^(-24))^(1/8)*log(a^6*(a^(-24))^
(1/4) + a^3*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-2
4))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 66*a^3*sqrt(-sqrt(2) + 2
)*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4) - a^3*sqrt(-sqrt(2) + 2)*(-sqrt(-
```

```
a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a
*x - 1))) + 132*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sqrt(2)
+ 2))*(a^(-24))^(1/8)*arctan((2*sqrt(2)*sqrt(a^6*(a^(-24))^(1/4) + 1/2*(sqr
t(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^
2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1
)))*a^21*(a^(-24))^(7/8) - 2*sqrt(2)*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(
1/4)*(a^(-24))^(7/8) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2
) + 2) + sqrt(-sqrt(2) + 2))) + 132*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2
)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24))^(1/8)*arctan((2*sqrt(2)*sqrt(a^6*(a^(-2
4))^(1/4) - 1/2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2)
+ 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a
^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8) - 2*sqrt(2)*a^21*(-sqrt(-a^2*x
^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(7/8) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2
) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 132*(sqrt(2)*a^3*sqrt(s
qrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24))^(1/8)*arctan(-(2*sq
rt(2)*sqrt(a^6*(a^(-24))^(1/4) + 1/2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(
2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^
(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(-24))^(7/8) - 2*sqrt(
2)*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(7/8) - sqrt(sqrt(2
) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 13
2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24)
)^(1/8)*arctan(-(2*sqrt(2)*sqrt(a^6*(a^(-24))^(1/4) - 1/2*(sqrt(2)*a^3*sqrt
(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^21*(a^(
-24))^(7/8) - 2*sqrt(2)*a^21*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24)
)^(7/8) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt
(-sqrt(2) + 2))) - 33*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sq
rt(2) + 2))*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4) + 1/2*(sqrt(2)*a^3*sqrt
(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 33*(sqr
t(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24))^(1/8
)*log(a^6*(a^(-24))^(1/4) - 1/2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) + sqrt(2)*a^
3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8)
 + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 33*(sqrt(2)*a^3*sqrt(sqrt(2) + 2)
 - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4)
+ 1/2*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sq
rt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1
)/(a*x - 1))) + 33*(sqrt(2)*a^3*sqrt(sqrt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(
2) + 2))*(a^(-24))^(1/8)*log(a^6*(a^(-24))^(1/4) - 1/2*(sqrt(2)*a^3*sqrt(sq
rt(2) + 2) - sqrt(2)*a^3*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)^(1/4)*(a^(-24))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) - 32*(32*a^3
*x^3 + 4*a^2*x^2 + a*x - 37)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/a^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4),x)
[Out] int(x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)*x**2,x)
[Out] Integral(x**2*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4), x)
```

3.136 $\int e^{\frac{1}{4}\tanh^{-1}(ax)} x\,dx$

Optimal. Leaf size=619

$$-\frac{(1-ax)^{7/8}(ax+1)^{9/8}}{2a^2}-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{8a^2}-\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{64a^2}+\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right.}{64a^2}$$

```
[Out] -1/8*(-a*x+1)^(7/8)*(a*x+1)^(1/8)/a^2-1/2*(-a*x+1)^(7/8)*(a*x+1)^(9/8)/a^2+
1/32*arctan((-2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))
^(1/2))*(2-2^(1/2))^(1/2)/a^2-1/32*arctan((2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(
2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a^2-1/64*ln(1+(-a*x+
1)^(1/4)/(a*x+1)^(1/4)-(-a*x+1)^(1/8)*(2-2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2-2
^(1/2))^(1/2)/a^2+1/64*ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)+(-a*x+1)^(1/8)*(2-
2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2-2^(1/2))^(1/2)/a^2+1/32*arctan((-2*(-a*x+1
)^(1/8)/(a*x+1)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/
2)/a^2-1/32*arctan((2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2))*(2+2^(1/2))^(1/2)/a^2-1/64*ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)-
(-a*x+1)^(1/8)*(2+2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2+2^(1/2))^(1/2)/a^2+1/64*
ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)+(-a*x+1)^(1/8)*(2+2^(1/2))^(1/2)/(a*x+1)^
(1/8))*(2+2^(1/2))^(1/2)/a^2
```

Rubi [A] time = 0.48, antiderivative size = 619, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6126, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$-\frac{(1-ax)^{7/8}(ax+1)^{9/8}}{2a^2}-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{8a^2}-\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{64a^2}+\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right.}{64a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)*x,x]
```

```
[Out] -((1 - a*x)^(7/8)*(1 + a*x)^(1/8))/(8*a^2) - ((1 - a*x)^(7/8)*(1 + a*x)^(9/
8))/(2*a^2) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - a*x)^(
1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) + (Sqrt[2 - Sqrt[2]]*Ar
cTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - Sqr
t[2]]])/(32*a^2) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - a
*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(32*a^2) - (Sqrt[2 - Sqrt[2
]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2
```

```
- Sqrt[2]]])/(32*a^2) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x
)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(64*a^2) +
(Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (Sqrt[2 - Sqrt
[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(64*a^2) - (Sqrt[2 + Sqrt[2]]*Log[1
 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1
 + a*x)^(1/8)])/(64*a^2) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 +
a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(64*a^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
```

```
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{\frac{1}{4}\tanh^{-1}(ax)}x\,dx = \int \frac{x\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} + \frac{\int \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx}{8a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} + \frac{\int \frac{1}{\sqrt[8]{1-ax}\,(1+ax)^{7/8}}\,dx}{32a}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-ax}\right)}{4a^2}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{4a^2}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{8\sqrt{2}\,a^2} + \frac{\text{Subst}(\cdots}{\cdots}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} + \frac{\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{8\sqrt{2}\,a^2} - \frac{\text{Subst}(\cdots}{\cdots}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-(1-\sqrt{2})x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, \frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{16\sqrt{2\left(2-\sqrt{2}\right)}\,a^2} - \frac{\text{Su}\cdots}{\cdots}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx, x, \cdots\right.}{32a^2}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} - \frac{\sqrt{2-\sqrt{2}}\,\log\left(1+\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}} - \frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{64a^2} + \cdots$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{8a^2} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2a^2} + \frac{\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2-\cdots}}{\cdots}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.09

$$-\frac{(1-ax)^{7/8}\left(2\sqrt[8]{2}\,{}_2F_1\left(-\frac{1}{8},\frac{7}{8};\frac{15}{8};\frac{1}{2}(1-ax)\right)+7(ax+1)^{9/8}\right)}{14a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)*x,x]
```

```
[Out] -1/14*((1 - a*x)^(7/8)*(7*(1 + a*x)^(9/8) + 2*2^(1/8)*Hypergeometric2F1[-1/
8, 7/8, 15/8, (1 - a*x)/2]))/a^2
```

fricas [B] time = 0.61, size = 2476, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="fricas")
```

```
[Out] -1/256*(8*a^2*sqrt(sqrt(2) + 2)*(a^(-16))^(1/8)*arctan((2*sqrt(a^4*(a^(-16)
)^(1/4) + a^2*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(
-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2
*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(7/8) - sqrt(-sqrt(2)
 + 2))/sqrt(sqrt(2) + 2)) + 8*a^2*sqrt(sqrt(2) + 2)*(a^(-16))^(1/8)*arctan(
(2*sqrt(a^4*(a^(-16))^(1/4) - a^2*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14
*(a^(-16))^(7/8) - 2*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(
7/8) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 8*a^2*sqrt(-sqrt(2) + 2)*(a
^(-16))^(1/8)*arctan((2*sqrt(a^4*(a^(-16))^(1/4) + a^2*sqrt(sqrt(2) + 2)*(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 +
 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1
))^(1/4)*(a^(-16))^(7/8) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 8*a^2*s
qrt(-sqrt(2) + 2)*(a^(-16))^(1/8)*arctan((2*sqrt(a^4*(a^(-16))^(1/4) - a^2*
sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + s
qrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2*a^14*(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(7/8) + sqrt(sqrt(2) + 2))/sqrt(-sqr
t(2) + 2)) - 2*a^2*sqrt(sqrt(2) + 2)*(a^(-16))^(1/8)*log(a^4*(a^(-16))^(1/4
) + a^2*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(
1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 2*a^2*sqrt(sqrt(2) + 2)*(a^(-
16))^(1/8)*log(a^4*(a^(-16))^(1/4) - a^2*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2
+ 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
) - 2*a^2*sqrt(-sqrt(2) + 2)*(a^(-16))^(1/8)*log(a^4*(a^(-16))^(1/4) + a^2*
sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) +
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 2*a^2*sqrt(-sqrt(2) + 2)*(a^(-16))^(
1/8)*log(a^4*(a^(-16))^(1/4) - a^2*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/
```

```
(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 4
*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))
^(1/8)*arctan((2*sqrt(2)*sqrt(a^4*(a^(-16))^(1/4) + 1/2*(sqrt(2)*a^2*sqrt(s
qrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1
))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-1
6))^(7/8) - 2*sqrt(2)*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^
(7/8) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-
sqrt(2) + 2))) + 4*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(
2) + 2))*(a^(-16))^(1/8)*arctan((2*sqrt(2)*sqrt(a^4*(a^(-16))^(1/4) - 1/2*(
sqrt(2)*a^2*sqrt(sqrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1)))*a^14*(a^(-16))^(7/8) - 2*sqrt(2)*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)^(1/4)*(a^(-16))^(7/8) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqr
t(2) + 2) + sqrt(-sqrt(2) + 2))) - 4*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) - sqrt(
2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1/8)*arctan(-(2*sqrt(2)*sqrt(a^4*(a^(
-16))^(1/4) + 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(2
) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(
-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2*sqrt(2)*a^14*(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(7/8) - sqrt(sqrt(2) + 2) - sqrt(-sqrt
(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 4*(sqrt(2)*a^2*sqrt(s
qrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1/8)*arctan(-(2*sq
rt(2)*sqrt(a^4*(a^(-16))^(1/4) - 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(
2)*a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^
(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^14*(a^(-16))^(7/8) - 2*sqrt(
2)*a^14*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(7/8) + sqrt(sqrt(2
) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - (s
qrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1
/8)*log(a^4*(a^(-16))^(1/4) + 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*
a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/
8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + (sqrt(2)*a^2*sqrt(sqrt(2) + 2)
+ sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1/8)*log(a^4*(a^(-16))^(1/4) -
 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) + sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(-sqr
t(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))) - (sqrt(2)*a^2*sqrt(sqrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) +
 2))*(a^(-16))^(1/8)*log(a^4*(a^(-16))^(1/4) + 1/2*(sqrt(2)*a^2*sqrt(sqrt(2
) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1
/4)*(a^(-16))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + (sqrt(2)*a^2*s
qrt(sqrt(2) + 2) - sqrt(2)*a^2*sqrt(-sqrt(2) + 2))*(a^(-16))^(1/8)*log(a^4*
(a^(-16))^(1/4) - 1/2*(sqrt(2)*a^2*sqrt(sqrt(2) + 2) - sqrt(2)*a^2*sqrt(-sq
rt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-16))^(1/8) + sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1))) - 32*(4*a^2*x^2 + a*x - 5)*(-sqrt(-a^2*x^2 +
1)/(a*x - 1))^(1/4))/a^2
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x)
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="maxima")
[Out] integrate(x*((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4),x)
[Out] int(x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)*x,x)
[Out] Integral(x*((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4), x)
```

3.137 $\int e^{\frac{1}{4}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=591

$$-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{a}-\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{8a}+\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{8a}-\cdots$$

```
[Out] -(-a*x+1)^(7/8)*(a*x+1)^(1/8)/a+1/4*arctan((-2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)
+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)/a-1/4*arctan((2*(-
a*x+1)^(1/8)/(a*x+1)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2)
)^(1/2)/a-1/8*ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)-(-a*x+1)^(1/8)*(2-2^(1/2))^
(1/2)/(a*x+1)^(1/8))*(2-2^(1/2))^(1/2)/a+1/8*ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1
/4)+(-a*x+1)^(1/8)*(2-2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2-2^(1/2))^(1/2)/a+1/4
*arctan((-2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/
2))*(2+2^(1/2))^(1/2)/a-1/4*arctan((2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(2-2^(1/
2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)/a-1/8*ln(1+(-a*x+1)^(1/4)/(
a*x+1)^(1/4)-(-a*x+1)^(1/8)*(2+2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2+2^(1/2))^(1
/2)/a+1/8*ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)+(-a*x+1)^(1/8)*(2+2^(1/2))^(1/2
)/(a*x+1)^(1/8))*(2+2^(1/2))^(1/2)/a
```

Rubi [A] time = 0.44, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {6125, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{a}-\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{8a}+\frac{\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)}{8a}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4),x]
```

```
[Out] -(((1 - a*x)^(7/8)*(1 + a*x)^(1/8))/a) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2
- Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(4*a)
 + (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 +
a*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(4*a) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 -
 Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]])/(4*a)
- (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a
*x)^(1/8))/Sqrt[2 - Sqrt[2]]])/(4*a) - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)
^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8
)])/(8*a) + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (S
```

```
qrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(8*a) - (Sqrt[2 + Sqrt[
2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^
(1/8))/(1 + a*x)^(1/8)])/(8*a) + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)
/(1 + a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/(8
*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{\frac{1}{4}\tanh^{-1}(ax)}\,dx=\int\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\,dx$$

$$=-\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a}+\frac{1}{4}\int\frac{1}{\sqrt[8]{1-ax}\,(1+ax)^{7/8}}\,dx$$

$$=-\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a}-\frac{2\,\text{Subst}\left(\int\frac{x^6}{(2-x^8)^{7/8}}\,dx,x,\sqrt[8]{1-ax}\right)}{a}$$

$$=-\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a}-\frac{2\,\text{Subst}\left(\int\frac{x^6}{1+x^8}\,dx,x,\frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{a}$$

$$=-\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a}-\frac{\text{Subst}\left(\int\frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx,x,\frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{\sqrt{2}\,a}+\frac{\text{Subst}\left(\int\frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx,x,\frac{\sqrt[8]{1-a}}{\sqrt[8]{1+a}}\right.}{\sqrt{2}\,a}$$

$$=-\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a}+\frac{\text{Subst}\left(\int\frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx,x,\frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{\sqrt{2}\,a}-\frac{\text{Subst}\left(\int\frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx,x,\frac{\sqrt[8]{1-a}}{\sqrt[8]{1+a}}\right.}{\sqrt{2}\,a}$$

$$=-\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a}-\frac{\text{Subst}\left(\int\frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx,x,\frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}\,a}-\frac{\text{Subst}\left(\int\frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\right.}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

$$=-\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a}-\frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int\frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx,x,\frac{\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{4a}-\frac{\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right.}}{}$$

$$=-\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a}-\frac{\sqrt{2-\sqrt{2}}\,\log\left(1+\frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}\right)}{8a}+\frac{\sqrt{2-\sqrt{2}}\,\log\left(1+\frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{8a}$$

$$=-\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{a}+\frac{\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a}+\frac{\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}}{\sqrt{2-\sqrt{2}}}\right.}{4a}$$

Mathematica [C] time = 0.05, size = 48, normalized size = 0.08

$$\frac{2e^{\frac{1}{4}\tanh^{-1}(ax)}\left({}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-e^{2\tanh^{-1}(ax)}\right)-\frac{1}{e^{2\tanh^{-1}(ax)}+1}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4),x]
```

```
[Out] (2*E^(ArcTanh[a*x]/4)*(-(1 + E^(2*ArcTanh[a*x]))^(-1) + Hypergeometric2F1[1
/8, 1, 9/8, -E^(2*ArcTanh[a*x])]))/a
```

fricas [B] time = 1.00, size = 2372, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(8*a*sqrt(sqrt(2) + 2)*(a^(-8))^(1/8)*arctan((2*sqrt(a^2*(a^(-8))^(1/
4) + a*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1
/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(7/8) - 2*a^7*(-sqr
t(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) - sqrt(-sqrt(2) + 2))/sqrt(
sqrt(2) + 2)) + 8*a*sqrt(sqrt(2) + 2)*(a^(-8))^(1/8)*arctan((2*sqrt(a^2*(a^
(-8))^(1/4) - a*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a
^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(7/8) - 2*
a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) + sqrt(-sqrt(2) +
2))/sqrt(sqrt(2) + 2)) + 8*a*sqrt(-sqrt(2) + 2)*(a^(-8))^(1/8)*arctan((2*sq
rt(a^2*(a^(-8))^(1/4) + a*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(
7/8) - 2*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) - sqrt(sq
rt(2) + 2))/sqrt(-sqrt(2) + 2)) + 8*a*sqrt(-sqrt(2) + 2)*(a^(-8))^(1/8)*arc
tan((2*sqrt(a^2*(a^(-8))^(1/4) - a*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(
a^(-8))^(7/8) - 2*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8)
+ sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 2*a*sqrt(sqrt(2) + 2)*(a^(-8))^(
1/8)*log(a^2*(a^(-8))^(1/4) + a*sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 2*a*sq
rt(sqrt(2) + 2)*(a^(-8))^(1/8)*log(a^2*(a^(-8))^(1/4) - a*sqrt(sqrt(2) + 2)
*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2
 + 1)/(a*x - 1))) - 2*a*sqrt(-sqrt(2) + 2)*(a^(-8))^(1/8)*log(a^2*(a^(-8))^
(1/4) + a*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))
^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + 2*a*sqrt(-sqrt(2) + 2)*(a^(
-8))^(1/8)*log(a^2*(a^(-8))^(1/4) - a*sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 +
1)/(a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) +
 4*(sqrt(2)*a*sqrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-8))^(1
/8)*arctan((2*sqrt(2)*sqrt(a^2*(a^(-8))^(1/4) + 1/2*(sqrt(2)*a*sqrt(sqrt(2)
 + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)
*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(7/8) -
 2*sqrt(2)*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) - sqrt(
```

```
sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))
) + 4*(sqrt(2)*a*sqrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-8))
^(1/8)*arctan((2*sqrt(2)*sqrt(a^2*(a^(-8))^(1/4) - 1/2*(sqrt(2)*a*sqrt(sqrt
(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1
/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^(7/8
) - 2*sqrt(2)*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8) + sq
rt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) +
2))) - 4*(sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-
8))^(1/8)*arctan(-(2*sqrt(2)*sqrt(a^2*(a^(-8))^(1/4) + 1/2*(sqrt(2)*a*sqrt(
sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-8))^
(7/8) - 2*sqrt(2)*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7/8)
- sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2
) + 2))) - 4*(sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(
a^(-8))^(1/8)*arctan(-(2*sqrt(2)*sqrt(a^2*(a^(-8))^(1/4) - 1/2*(sqrt(2)*a*s
qrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x
- 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))*a^7*(a^(-
8))^(7/8) - 2*sqrt(2)*a^7*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(7
/8) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sq
rt(2) + 2))) - (sqrt(2)*a*sqrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))
*(a^(-8))^(1/8)*log(a^2*(a^(-8))^(1/4) + 1/2*(sqrt(2)*a*sqrt(sqrt(2) + 2) +
 sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8
))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + (sqrt(2)*a*sqrt(sqrt(2) +
 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-8))^(1/8)*log(a^2*(a^(-8))^(1/4) -
 1/2*(sqrt(2)*a*sqrt(sqrt(2) + 2) + sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a
^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1))) - (sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(
-8))^(1/8)*log(a^2*(a^(-8))^(1/4) + 1/2*(sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt
(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1
/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))) + (sqrt(2)*a*sqrt(sqrt(2) + 2) -
 sqrt(2)*a*sqrt(-sqrt(2) + 2))*(a^(-8))^(1/8)*log(a^2*(a^(-8))^(1/4) - 1/2*
(sqrt(2)*a*sqrt(sqrt(2) + 2) - sqrt(2)*a*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^
2 + 1)/(a*x - 1))^(1/4)*(a^(-8))^(1/8) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)) - 32*(a*x - 1)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/a
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x)
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4),x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4),x)
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4), x)
```

3.138 $\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=759

$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)-$$

```
[Out] -2*arctan((a*x+1)^(1/8)/(-a*x+1)^(1/8))-2*arctanh((a*x+1)^(1/8)/(-a*x+1)^(1
/8))+1/2*ln(1+(a*x+1)^(1/4)/(-a*x+1)^(1/4)-(a*x+1)^(1/8)*2^(1/2)/(-a*x+1)^(
1/8))*2^(1/2)-1/2*ln(1+(a*x+1)^(1/4)/(-a*x+1)^(1/4)+(a*x+1)^(1/8)*2^(1/2)/(
-a*x+1)^(1/8))*2^(1/2)+arctan(1-(a*x+1)^(1/8)*2^(1/2)/(-a*x+1)^(1/8))*2^(1/
2)-arctan(1+(a*x+1)^(1/8)*2^(1/2)/(-a*x+1)^(1/8))*2^(1/2)+arctan((-2*(-a*x+
1)^(1/8)/(a*x+1)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1
/2)-arctan((2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(
1/2))*(2-2^(1/2))^(1/2)-1/2*ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)-(-a*x+1)^(1/8
)*(2-2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2-2^(1/2))^(1/2)+1/2*ln(1+(-a*x+1)^(1/4
)/(a*x+1)^(1/4)+(-a*x+1)^(1/8)*(2-2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2-2^(1/2))
^(1/2)+arctan((-2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2
))^(1/2))*(2+2^(1/2))^(1/2)-arctan((2*(-a*x+1)^(1/8)/(a*x+1)^(1/8)+(2-2^(1/
2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/2*ln(1+(-a*x+1)^(1/4)/(a*
x+1)^(1/4)-(-a*x+1)^(1/8)*(2+2^(1/2))^(1/2)/(a*x+1)^(1/8))*(2+2^(1/2))^(1/2
)+1/2*ln(1+(-a*x+1)^(1/4)/(a*x+1)^(1/4)+(-a*x+1)^(1/8)*(2+2^(1/2))^(1/2)/(a
*x+1)^(1/8))*(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 759, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 20, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {6126, 105, 63, 331, 299, 1122, 1169, 634, 618, 204, 628, 93, 214, 212, 206, 203, 211, 1165, 1162, 617}

$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\,\log\left(\frac{\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-ax}}{\sqrt[8]{ax+1}}+1\right)-$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)/x,x]
```

```
[Out] -2*ArcTan[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)] + Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt
[2 - Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] + S
qrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - a*x)^(1/8))/(1 + a*x)^
```

```
(1/8))/Sqrt[2 - Sqrt[2]]] - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (
2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - Sqrt[2 - Sqrt[2]]*
ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - a*x)^(1/8))/(1 + a*x)^(1/8))/Sqrt[2 - S
qrt[2]]] + Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)] -
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)] - 2*ArcTanh[(
1 + a*x)^(1/8)/(1 - a*x)^(1/8)] - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4
)/(1 + a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/2
 + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4) + (Sqrt[2 - S
qrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 + (
1 - a*x)^(1/4)/(1 + a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a
*x)^(1/8)])/2 + (Sqrt[2 + Sqrt[2]]*Log[1 + (1 - a*x)^(1/4)/(1 + a*x)^(1/4)
+ (Sqrt[2 + Sqrt[2]]*(1 - a*x)^(1/8))/(1 + a*x)^(1/8)])/2 + Log[1 - (Sqrt[2
]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 - a*x)^(1/4)]/Sqrt[
2] - Log[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1
 - a*x)^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x}\,dx = \int \frac{\sqrt[8]{1+ax}}{x\sqrt[8]{1-ax}}\,dx$$

$$= a\int \frac{1}{\sqrt[8]{1-ax}\,(1+ax)^{7/8}}\,dx + \int \frac{1}{x\sqrt[8]{1-ax}\,(1+ax)^{7/8}}\,dx$$

$$= -\left(8\,\mathrm{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x, \sqrt[8]{1-ax}\right)\right) + 8\,\mathrm{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)$$

$$= -\left(4\,\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)\right) - 4\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 8\,\mathrm{Subst}\left(\int \right.$$

$$= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)\right) - 2\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\,\mathrm{Subst}\left(\int \right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)}{\sqrt{2}} + \frac{\mathrm{Subst}\left(\int \right.}{}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}+\frac{\sqrt[4]{1+ax}}{\sqrt[4]{1-ax}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt[8]{1-\ldots}}{\sqrt[8]{1-\ldots}}\right.}{\sqrt{\ldots}}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\tanh$$

$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - 2\tanh$$

$$= -2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-ax}}{\sqrt[8]{1+ax}}}{\sqrt{2+\sqrt{2}}}\right) + \sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\ldots}{\sqrt{2-\ldots}}\right.$$

Mathematica [C] time = 0.03, size = 83, normalized size = 0.11

$$-\frac{4(1-ax)^{7/8}\left(\sqrt[8]{2}\,(ax+1)^{7/8}\,{}_2F_1\left(\frac{7}{8},\frac{7}{8};\frac{15}{8};\frac{1}{2}(1-ax)\right)+2\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};\frac{1-ax}{ax+1}\right)\right)}{7(ax+1)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)/x,x]
```

```
[Out] (-4*(1 - a*x)^(7/8)*(2^(1/8)*(1 + a*x)^(7/8)*Hypergeometric2F1[7/8, 7/8, 15
/8, (1 - a*x)/2] + 2*Hypergeometric2F1[7/8, 1, 15/8, (1 - a*x)/(1 + a*x)]))
/(7*(1 + a*x)^(7/8))
```

fricas [B] time = 0.54, size = 2281, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((sqrt(
2)*sqrt(2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4)
- 2*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - sqrt(sqrt(2) + 2) + sqr
t(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/2*(sqrt(2)*s
qrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((sqrt(2)*sqrt(-2*(sqr
t(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(
a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - 2*sqrt(2)*(-
sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2
))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/2*(sqrt(2)*sqrt(sqrt(2) +
2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(sqrt(2)*sqrt(2*(sqrt(2)*sqrt(sqrt
(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4
) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - 2*sqrt(2)*(-sqrt(-a^2*x^2
+ 1)/(a*x - 1))^(1/4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(
2) + 2) - sqrt(-sqrt(2) + 2))) + 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*s
qrt(-sqrt(2) + 2))*arctan(-(sqrt(2)*sqrt(-2*(sqrt(2)*sqrt(sqrt(2) + 2) + sq
rt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-s
qrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - 2*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1
))^(1/4) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqr
t(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2)
+ 2))*log(2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt
(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4
) - 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(-2*(sq
rt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/
(a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) + 1/8*(sqrt(2
)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(sqrt(2)*sqrt(sqrt(
2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)
 + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - 1/8*(sqrt(2)*sqrt(sqrt(2) +
 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(-2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(
2)*sqrt(-sqrt(2) + 2))*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-sqrt
(-a^2*x^2 + 1)/(a*x - 1)) + 4) + 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(-sq
rt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1
```

```
) - sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - 1) + 2*sqrt(2)*arctan(1
/2*sqrt(2)*sqrt(-4*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-
sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) - sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1)
)^(1/4) + 1) - sqrt(-sqrt(2) + 2)*arctan((2*sqrt(sqrt(sqrt(2) + 2)*(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) -
sqrt(sqrt(2) + 2) - 2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/sqrt(-sqrt(2)
+ 2)) - sqrt(-sqrt(2) + 2)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^
2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + sqrt(s
qrt(2) + 2) - 2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/sqrt(-sqrt(2) + 2))
- sqrt(sqrt(2) + 2)*arctan((2*sqrt(sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/
(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - sqrt(-sqrt(2)
 + 2) - 2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/sqrt(sqrt(2) + 2)) - sqrt(
sqrt(2) + 2)*arctan((2*sqrt(-sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x -
 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) + sqrt(-sqrt(2) + 2)
- 2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/sqrt(sqrt(2) + 2)) - 1/2*sqrt(2)
*log(4*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^
2 + 1)/(a*x - 1)) + 4) + 1/2*sqrt(2)*log(-4*sqrt(2)*(-sqrt(-a^2*x^2 + 1)/(a
*x - 1))^(1/4) + 4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 4) + 1/4*sqrt(sqrt
(2) + 2)*log(sqrt(sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/4*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt
(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(
a*x - 1)) + 1) + 1/4*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*
x^2 + 1)/(a*x - 1))^(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 1/4*
sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^
(1/4) + sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + 1) - 2*arctan((-sqrt(-a^2*x^2
 + 1)/(a*x - 1))^(1/4)) - log((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 1) +
log((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{1/4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4)/x,x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)/x,x)
```

```
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4)/x, x)
```

3.139 $\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=271

$$-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{x}+\frac{a\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-\frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{4\sqrt{2}}-\frac{a\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{4\sqrt{2}}-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)+\frac{a\tan}{}$$

```
[Out] -(-a*x+1)^(7/8)*(a*x+1)^(1/8)/x-1/2*a*arctan((a*x+1)^(1/8)/(-a*x+1)^(1/8))-
1/2*a*arctanh((a*x+1)^(1/8)/(-a*x+1)^(1/8))+1/4*a*arctan(1-(a*x+1)^(1/8)*2^
(1/2)/(-a*x+1)^(1/8))*2^(1/2)-1/4*a*arctan(1+(a*x+1)^(1/8)*2^(1/2)/(-a*x+1)
^(1/8))*2^(1/2)+1/8*a*ln(1+(a*x+1)^(1/4)/(-a*x+1)^(1/4)-(a*x+1)^(1/8)*2^(1/
2)/(-a*x+1)^(1/8))*2^(1/2)-1/8*a*ln(1+(a*x+1)^(1/4)/(-a*x+1)^(1/4)+(a*x+1)^
(1/8)*2^(1/2)/(-a*x+1)^(1/8))*2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6126, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(1-ax)^{7/8}\sqrt[8]{ax+1}}{x}+\frac{a\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-\frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{4\sqrt{2}}-\frac{a\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\sqrt{2}\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{4\sqrt{2}}-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)+\frac{a\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)/x^2,x]
```

```
[Out] -(((1 - a*x)^(7/8)*(1 + a*x)^(1/8))/x) - (a*ArcTan[(1 + a*x)^(1/8)/(1 - a*x
)^(1/8)])/2 + (a*ArcTan[1 - (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(2*
Sqrt[2]) - (a*ArcTan[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(2*Sqr
t[2]) - (a*ArcTanh[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)])/2 + (a*Log[1 - (Sqrt[2
]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/(4*S
qrt[2]) - (a*Log[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^
(1/4)/(1 - a*x)^(1/4)])/(4*Sqrt[2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
```

```
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] && !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x^2}\,dx = \int \frac{\sqrt[8]{1+ax}}{x^2\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} + \frac{1}{4}a\int \frac{1}{x\sqrt[8]{1-ax}\,(1+ax)^{7/8}}\,dx$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} + (2a)\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - a\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - a\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+}}{\sqrt[8]{1-}}\right.$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \right.$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - \frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{4}a\,\text{Subst}\left(\int \frac{}{1-\sqrt{}}\right.$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - \frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{a\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right.}{4\sqrt{2}}$$

$$= -\frac{(1-ax)^{7/8}\sqrt[8]{1+ax}}{x} - \frac{1}{2}a\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)}{2\sqrt{2}} - \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1}}{\sqrt[8]{1-}}\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.21

$$-\frac{(1-ax)^{7/8}\left(2ax\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};\frac{1-ax}{ax+1}\right)+7ax+7\right)}{7x(ax+1)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)/x^2,x]
```

```
[Out] -1/7*((1 - a*x)^(7/8)*(7 + 7*a*x + 2*a*x*Hypergeometric2F1[7/8, 1, 15/8, (1
 - a*x)/(1 + a*x)]))/(x*(1 + a*x)^(7/8))
```

fricas [B] time = 1.24, size = 544, normalized size = 2.01

$$-\frac{4\,ax\arctan\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}\right)+2\,ax\log\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}+1\right)-2\,ax\log\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}-1\right)-4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*x*arctan((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)) + 2*a*x*log((-sqr
t(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 1) - 2*a*x*log((-sqrt(-a^2*x^2 + 1)/(a*x
 - 1))^(1/4) - 1) - 4*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3
/4)*a*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*
sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + sqrt(2)*(a^4)^(1/4)*a*(-sqrt(-a^2*x^2
 + 1)/(a*x - 1))^(1/4) + sqrt(a^4)))/a^4) - 4*sqrt(2)*(a^4)^(1/4)*x*arctan(
(a^4 - sqrt(2)*(a^4)^(3/4)*a*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(2
)*(a^4)^(3/4)*sqrt(a^2*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(2)*(a^4)^
(1/4)*a*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(a^4)))/a^4) + sqrt(2)*
(a^4)^(1/4)*x*log(a^2*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + sqrt(2)*(a^4)^(
1/4)*a*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(a^4)) - sqrt(2)*(a^4)^(
1/4)*x*log(a^2*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) - sqrt(2)*(a^4)^(1/4)*a*
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(a^4)) - 8*(a*x - 1)*(-sqrt(-a^
2*x^2 + 1)/(a*x - 1))^(1/4))/x
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{1/4}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4)/x^2,x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)/x**2,x)
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4)/x**2, x)
```

3.140 $\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=312

$$\frac{a^2\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-\frac{\sqrt{2}\,\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{32\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\sqrt{2}\,\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{32\sqrt{2}}-\frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)+\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)}{16\sqrt{2}}$$

```
[Out] -1/8*a*(-a*x+1)^(7/8)*(a*x+1)^(1/8)/x-1/2*(-a*x+1)^(7/8)*(a*x+1)^(9/8)/x^2-
1/16*a^2*arctan((a*x+1)^(1/8)/(-a*x+1)^(1/8))-1/16*a^2*arctanh((a*x+1)^(1/8
)/(-a*x+1)^(1/8))+1/32*a^2*arctan(1-(a*x+1)^(1/8)*2^(1/2)/(-a*x+1)^(1/8))*2
^(1/2)-1/32*a^2*arctan(1+(a*x+1)^(1/8)*2^(1/2)/(-a*x+1)^(1/8))*2^(1/2)+1/64
*a^2*ln(1+(a*x+1)^(1/4)/(-a*x+1)^(1/4)-(a*x+1)^(1/8)*2^(1/2)/(-a*x+1)^(1/8)
)*2^(1/2)-1/64*a^2*ln(1+(a*x+1)^(1/4)/(-a*x+1)^(1/4)+(a*x+1)^(1/8)*2^(1/2)/
(-a*x+1)^(1/8))*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6126, 96, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$\frac{a^2\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}-\frac{\sqrt{2}\,\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{32\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}}+\frac{\sqrt{2}\,\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}+1\right)}{32\sqrt{2}}-\frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)+\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{ax+1}}{\sqrt[8]{1-ax}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/4)/x^3,x]
```

```
[Out] -(a*(1 - a*x)^(7/8)*(1 + a*x)^(1/8))/(8*x) - ((1 - a*x)^(7/8)*(1 + a*x)^(9/
8))/(2*x^2) - (a^2*ArcTan[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)])/16 + (a^2*ArcTa
n[1 - (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(16*Sqrt[2]) - (a^2*ArcTa
n[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8)])/(16*Sqrt[2]) - (a^2*ArcTa
nh[(1 + a*x)^(1/8)/(1 - a*x)^(1/8)])/16 + (a^2*Log[1 - (Sqrt[2]*(1 + a*x)^(
1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 - a*x)^(1/4)])/(32*Sqrt[2]) - (a
^2*Log[1 + (Sqrt[2]*(1 + a*x)^(1/8))/(1 - a*x)^(1/8) + (1 + a*x)^(1/4)/(1 -
 a*x)^(1/4)])/(32*Sqrt[2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\tanh^{-1}(ax)}}{x^3}\,dx = \int \frac{\sqrt[8]{1+ax}}{x^3\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} + \frac{1}{8}a\int \frac{\sqrt[8]{1+ax}}{x^2\sqrt[8]{1-ax}}\,dx$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} + \frac{1}{32}a^2\int \frac{1}{x\sqrt[8]{1-ax}\,(1+ax)^{7/8}}\,dx$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} + \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right)$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{8}a^2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{8}$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{16}a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) -$$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{16}a^2\tanh^{-1}\Bigg($$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) - \frac{1}{16}a^2\tanh^{-1}\Bigg($$

$$= -\frac{a(1-ax)^{7/8}\sqrt[8]{1+ax}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{16}a^2\tan^{-1}\left(\frac{\sqrt[8]{1+ax}}{\sqrt[8]{1-ax}}\right) + \frac{a^2\tan^{-1}\Big(1-\sqrt{2}\cdots}{16\sqrt{2}}$$

Mathematica [C] time = 0.02, size = 73, normalized size = 0.23

$$-\frac{(1-ax)^{7/8}\left(2a^2x^2\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};\frac{1-ax}{ax+1}\right)+7\left(5a^2x^2+9ax+4\right)\right)}{56x^2(ax+1)^{7/8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/4)/x^3,x]
```

```
[Out] -1/56*((1 - a*x)^(7/8)*(7*(4 + 9*a*x + 5*a^2*x^2) + 2*a^2*x^2*Hypergeometri
c2F1[7/8, 1, 15/8, (1 - a*x)/(1 + a*x)]))/(x^2*(1 + a*x)^(7/8))
```

fricas [B] time = 0.59, size = 585, normalized size = 1.88

$$-\frac{4\,a^2x^2\arctan\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}\right)+2\,a^2x^2\log\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}+1\right)-2\,a^2x^2\log\left(\left(-\frac{\sqrt{-a^2x^2+1}}{ax-1}\right)^{\frac{1}{4}}-1\right)-4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*a^2*x^2*arctan((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4)) + 2*a^2*x^2*
log((-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + 1) - 2*a^2*x^2*log((-sqrt(-a^2*
x^2 + 1)/(a*x - 1))^(1/4) - 1) - 4*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + s
qrt(2)*(a^8)^(3/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) - sqrt(2)*(a^8
)^(3/4)*sqrt(a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)) + sqrt(2)*(a^8)^(1/4)*
a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sqrt(a^8)))/a^8) - 4*sqrt(2)*(a
^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a
*x - 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1)) - sqrt(2)*(a^8)^(1/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sq
rt(a^8)))/a^8) + sqrt(2)*(a^8)^(1/4)*x^2*log(a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(
a*x - 1)) + sqrt(2)*(a^8)^(1/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) +
 sqrt(a^8)) - sqrt(2)*(a^8)^(1/4)*x^2*log(a^4*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
 - 1)) - sqrt(2)*(a^8)^(1/4)*a^2*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4) + sq
rt(a^8)) - 8*(5*a^2*x^2 - a*x - 4)*(-sqrt(-a^2*x^2 + 1)/(a*x - 1))^(1/4))/x
^2
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/sqrt(-a^2*x^2 + 1))^(1/4)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}\right)^{1/4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4)/x^3,x)
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/4)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/4)/x**3,x)
[Out] Integral(((a*x + 1)/sqrt(-a**2*x**2 + 1))**(1/4)/x**3, x)
```

3.141 $\int e^{4\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=45

$$-4x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1+m)/(1+m)+4*x^(1+m)/(-a*x+1)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 89, 80, 64}

$$-4x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - a*x) - 4*x^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, a*x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
```

```
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}
\int e^{4\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+ax)^2}{(1-ax)^2}\,dx \\
&= \frac{4x^{1+m}}{1-ax} - \frac{\int \frac{x^m\left(a^2(3+4m)+a^3x\right)}{1-ax}\,dx}{a^2} \\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-ax} - (4(1+m))\int \frac{x^m}{1-ax}\,dx \\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-ax} - 4x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.04

$$\frac{x^{m+1}(-4(m+1)(ax-1)\,{}_2F_1(1,m+1;m+2;ax)+ax-4m-5)}{(m+1)(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(-5 - 4*m + a*x - 4*(1 + m)*(-1 + a*x)*Hypergeometric2F1[1, 1 +
m, 2 + m, a*x]))/((1 + m)*(-1 + a*x))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)x^m}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m/(a^2*x^2 - 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4 x^m}{(a^2x^2-1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^4*x^m/(a^2*x^2 - 1)^2, x)
```

maple [C] time = 0.39, size = 461, normalized size = 10.24

$$\frac{\left(-a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(-\frac{x^{1+m}\left(-a^2\right)^{\frac{5}{2}+\frac{m}{2}}\left(2a^2x^2-m-3\right)}{\left(-a^2x^2+1\right)a^4(1+m)}-\frac{x^{1+m}\left(-a^2\right)^{\frac{5}{2}+\frac{m}{2}}(3+m)\Phi\left(a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)}{2a^4}\right)}{2}+\frac{2\left(-a^2\right)^{-\frac{m}{2}}\left(-\frac{x^m\left(-a^2\right)^{\frac{m}{2}}\left(2a^2x^2-m-2\right)}{\left(-a^2x^2+1\right)m}-\frac{x^m\left(-a^2\right.}{}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x^m,x)
```

```
[Out] 1/2*(-a^2)^(-1/2-1/2*m)*(-x^(1+m)*(-a^2)^(5/2+1/2*m)*(2*a^2*x^2-m-3)/(-a^2*
x^2+1)/a^4/(1+m)-1/2*x^(1+m)*(-a^2)^(5/2+1/2*m)*(3+m)/a^4*LerchPhi(a^2*x^2,
1,1/2+1/2*m))+2/a*(-a^2)^(-1/2*m)*(-x^m*(-a^2)^(1/2*m)*(2*a^2*x^2-m-2)/(-a^
2*x^2+1)/m-1/2*x^m*(-a^2)^(1/2*m)*(2+m)*LerchPhi(a^2*x^2,1,1/2*m))-3*(-a^2)
^(-1/2-1/2*m)*(-1/(3+m)*x^(1+m)*(-a^2)^(3/2+1/2*m)*(-3-m)/a^2/(-a^2*x^2+1)-
1/2*x^(1+m)*(-a^2)^(3/2+1/2*m)*(1+m)/a^2*LerchPhi(a^2*x^2,1,1/2+1/2*m))-2/a
*(-a^2)^(-1/2*m)*(1/(2+m)*x^m*(-a^2)^(1/2*m)*(-m-2)/(-a^2*x^2+1)+1/2*x^m*(-
a^2)^(1/2*m)*m*LerchPhi(a^2*x^2,1,1/2*m))+1/2*(-a^2)^(-1/2-1/2*m)*(-2/(1+m)
*x^(1+m)*(-a^2)^(1/2+1/2*m)*(-1-m)/(-2*a^2*x^2+2)+2/(1+m)*x^(1+m)*(-a^2)^(1
/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(a^2*x^2,1,1/2+1/2*m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4 x^m}{(a^2x^2-1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^4*x^m/(a^2*x^2 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,{\left(a\,x+1\right)}^4}{{\left(a^2\,x^2-1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] int((x^m*(a*x + 1)^4)/(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(ax + 1\right)^{2}}{\left(ax - 1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x**m,x)
[Out] Integral(x**m*(a*x + 1)**2/(a*x - 1)**2, x)
```

3.142 $\int e^{3\tanh^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=151

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}+\frac{4ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\right.}{m+}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)-a*x^(2+m)*
hypergeom([1/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)+4*x^(1+m)*hypergeom([3/2,
 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+4*a*x^(2+m)*hypergeom([3/2, 1+1/2*m]
,[2+1/2*m],a^2*x^2)/(2+m)
```

Rubi [A] time = 0.88, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6124, 6742, 364, 850, 808}

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}+\frac{4ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\right.}{m+}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^m,x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m
) - (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2
+ m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/
(1 + m) + (4*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x
^2])/(2 + m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+ax)^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \left(-\frac{3x^m}{\sqrt{1-a^2x^2}} - \frac{ax^{1+m}}{\sqrt{1-a^2x^2}} + \frac{4x^m}{(1-ax)\sqrt{1-a^2x^2}}\right)dx \\
&= -\left(3\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx\right) + 4\int \frac{x^m}{(1-ax)\sqrt{1-a^2x^2}}\,dx - a\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m} - \frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m} + 4\int \frac{x^m(1+ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx \\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m} - \frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m} + 4\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}}\,dx + \\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m} - \frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m} + \frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+}{2}\right.}{1+m}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 92, normalized size = 0.61

$$\frac{\sqrt{-ax-1}\,\sqrt{1-ax}\,x^{m+1}\left(F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;-ax,ax\right)-2F_1\left(m+1;-\frac{1}{2},\frac{3}{2};m+2;-ax,ax\right)\right)}{(m+1)\sqrt{ax-1}\,\sqrt{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*Sqrt[-1 - a*x]*Sqrt[1 - a*x]*(AppellF1[1 + m, -1/2, 1/2, 2 + m,
-(a*x), a*x] - 2*AppellF1[1 + m, -1/2, 3/2, 2 + m, -(a*x), a*x]))/((1 + m)*
Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)x^m}{a^2x^2-2\,ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*x^m/(a^2*x^2 - 2*a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.33, size = 139, normalized size = 0.92

$$\frac{x^{1+m}\,\text{hypergeom}\left(\left[\frac{3}{2}, \frac{1}{2}+\frac{m}{2}\right], \left[\frac{3}{2}+\frac{m}{2}\right], a^2x^2\right)}{1+m}+\frac{a^3x^{4+m}\,\text{hypergeom}\left(\left[\frac{3}{2}, 2+\frac{m}{2}\right], \left[3+\frac{m}{2}\right], a^2x^2\right)}{4+m}+\frac{3a^2x^{3+m}\,\text{hype}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m,x)
```

```
[Out] x^(1+m)*hypergeom([3/2,1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a^3/(4+m)*x^(4
+m)*hypergeom([3/2,2+1/2*m],[3+1/2*m],a^2*x^2)+3*a^2/(3+m)*x^(3+m)*hypergeo
m([3/2,3/2+1/2*m],[5/2+1/2*m],a^2*x^2)+3*a*x^(2+m)*hypergeom([3/2,1+1/2*m],
[2+1/2*m],a^2*x^2)/(2+m)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 x^m}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m,x, algorithm="maxima")
[Out] integrate((a*x + 1)^3*x^m/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^m*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\,(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**m,x)
[Out] Integral(x**m*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.143 $\int e^{2\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{m+1}-\frac{x^{m+1}}{m+1}$$

```
[Out] -x^(1+m)/(1+m)+2*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6126, 80, 64}

$$\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{m+1}-\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m,x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]
)/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^m\,dx = \int \frac{x^m(1+ax)}{1-ax}\,dx$$
$$= -\frac{x^{1+m}}{1+m} + 2\int \frac{x^m}{1-ax}\,dx$$
$$= -\frac{x^{1+m}}{1+m} + \frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)}{1+m}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.72

$$\frac{x^{m+1}(2\,{}_2F_1(1,m+1;m+2;ax)-1)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m,x]
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(1 + m)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax+1)x^m}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m,x, algorithm="fricas")
[Out] integral(-(a*x + 1)*x^m/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2 x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m,x, algorithm="giac")
[Out] integrate(-(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

maple [C] time = 0.35, size = 184, normalized size = 5.11

$$-\frac{\left(-a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}\left(-a^2\right)^{\frac{3}{2}+\frac{m}{2}}(-3-m)}{(3+m)(1+m)a^2}+\frac{x^{1+m}\left(-a^2\right)^{\frac{3}{2}+\frac{m}{2}}\Phi\left(a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)}{a^2}\right)}{2}-\frac{\left(-a^2\right)^{-\frac{m}{2}}\left(-\frac{2x^m\left(-a^2\right)^{\frac{m}{2}}(-m-2)}{(2+m)m}-x^m\left(-a^2\right)^{\frac{m}{2}}\Phi\left(a^2x\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m,x)
```

```
[Out] -1/2*(-a^2)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a^2)^(3/2+1/2*m)*(-3-m)/(1+m)/a
^2+x^(1+m)*(-a^2)^(3/2+1/2*m)/a^2*LerchPhi(a^2*x^2,1,1/2+1/2*m))-1/a*(-a^2)
^(-1/2*m)*(-2/(2+m)*x^m*(-a^2)^(1/2*m)*(-m-2)/m-x^m*(-a^2)^(1/2*m)*LerchPhi
(a^2*x^2,1,1/2*m))+1/(1+m)*x^(1+m)*(1/2+1/2*m)*LerchPhi(a^2*x^2,1,1/2+1/2*m
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2 x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{x^m\,(a\,x+1)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^m*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [B] time = 3.62, size = 99, normalized size = 2.75

$$\frac{amx^2x^m\Phi(ax,1,m+2)\Gamma(m+2)}{\Gamma(m+3)}+\frac{2ax^2x^m\Phi(ax,1,m+2)\Gamma(m+2)}{\Gamma(m+3)}+\frac{mxx^m\Phi(ax,1,m+1)\Gamma(m+1)}{\Gamma(m+2)}+\frac{xx^m\Phi(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m,x)
```

```
[Out] a*m*x**2*x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) + 2*a*x**2*
x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) + m*x*x**m*lerchphi(
a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) + x*x**m*lerchphi(a*x, 1, m + 1)*g
amma(m + 1)/gamma(m + 2)
```

3.144 $\int e^{\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=74

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

```
[Out] x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a*x^(2+m)*hyp
ergeom([1/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6124, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) +
 (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m
)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1+ax)}{\sqrt{1-a^2x^2}}\,dx\\ &= a\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx + \int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m} + \frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}\end{aligned}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.95

$$-\frac{\sqrt{-ax-1}\,\sqrt{1-ax}\,x^{m+1}F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;-ax,ax\right)}{(m+1)\sqrt{ax-1}\,\sqrt{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^m,x]
```

```
[Out] -((x^(1 + m)*Sqrt[-1 - a*x]*Sqrt[1 - a*x]*AppellF1[1 + m, -1/2, 1/2, 2 + m,
 -(a*x), a*x])/((1 + m)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^m}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^m/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.24, size = 67, normalized size = 0.91

$$\frac{x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right], a^{2} x^{2}\right)}{1+m}+\frac{a\, x^{2+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right], a^{2} x^{2}\right)}{2+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m,x)
```

```
[Out] x^(1+m)*hypergeom([1/2,1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a*x^(2+m)*hype
rgeom([1/2,1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^{m}}{\sqrt{-a^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m}\,\left(a\,x+1\right)}{\sqrt{1-a^{2}\,x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^m*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 3.22, size = 97, normalized size = 1.31

$$\frac{ax^{2}x^{m}\Gamma\left(\frac{m}{2}+1\right) {}_{2}F_{1}\left(\left.\begin{matrix} \frac{1}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\right| a^{2}x^{2}e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{xx^{m}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_{2}F_{1}\left(\left.\begin{matrix} \frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\right| a^{2}x^{2}e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m,x)
```

```
[Out] a*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*exp_
polar(2*I*pi))/(2*gamma(m/2 + 2)) + x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2
 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3/2))
```

3.145 $\int e^{-\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=75

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

```
[Out] x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)-a*x^(2+m)*hyp
ergeom([1/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6124, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^ArcTanh[a*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) -
 (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m
)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}x^m\,dx&=\int\frac{x^m(1-ax)}{\sqrt{1-a^2x^2}}\,dx\\&=-\left(a\int\frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx\right)+\int\frac{x^m}{\sqrt{1-a^2x^2}}\,dx\\&=\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}-\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}\end{aligned}$$

Mathematica [C] time = 0.03, size = 31, normalized size = 0.41

$$\frac{x^{m+1}F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;ax,-ax\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^ArcTanh[a*x],x]
[Out] (x^(1 + m)*AppellF1[1 + m, -1/2, 1/2, 2 + m, a*x, -(a*x)])/(1 + m)
```

fricas [F] time = 2.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}\,x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^m/(a*x + 1), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \sqrt{-a^{2} x^{2}+1}}{a x +1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
[Out] int(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2} x^{2}+1}\, x^{m}}{a x +1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^m/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m}\, \sqrt{1-a^{2}\, x^{2}}}{a\, x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
[Out] int((x^m*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \sqrt{-\left(a x - 1\right) \left(a x + 1\right)}}{a x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**m*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.146 $\int e^{-2\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=37

$$\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{m+1}-\frac{x^{m+1}}{m+1}$$

```
[Out] -x^(1+m)/(1+m)+2*x^(1+m)*hypergeom([1, 1+m],[2+m],-a*x)/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6126, 80, 64}

$$\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{m+1}-\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*
x)])/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}x^m\,dx = \int \frac{x^m(1-ax)}{1+ax}\,dx$$
$$= -\frac{x^{1+m}}{1+m} + 2\int \frac{x^m}{1+ax}\,dx$$
$$= -\frac{x^{1+m}}{1+m} + \frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;-ax)}{1+m}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.73

$$\frac{x^{m+1}(2\,{}_2F_1(1,m+1;m+2;-ax)-1)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^(2*ArcTanh[a*x]),x]
[Out] (x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]))/(1 + m)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax-1)x^m}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-(a*x - 1)*x^m/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)x^m}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)*x^m/(a*x + 1)^2, x)
```

maple [C] time = 0.27, size = 126, normalized size = 3.41

$$-a^{-1-m}\left(\frac{x^m a^m\left(a^2 m\,x^2-amx-2ax-m^2-3m-2\right)}{(1+m)\,m\,(ax+1)} + x^m a^m\,(2+m)\,\Phi\,(-ax,1,m)\right)+a^{-1-m}\left(\frac{x^m a^m\,(-1-m)}{(1+m)\,(ax+1)} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -a^(-1-m)*(x^m*a^m*(a^2*m*x^2-a*m*x-2*a*x-m^2-3*m-2)/(1+m)/m/(a*x+1)+x^m*a^
m*(2+m)*LerchPhi(-a*x,1,m))+a^(-1-m)*(1/(1+m)*x^m*a^m*(-1-m)/(a*x+1)+x^m*a^
m*m*LerchPhi(-a*x,1,m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)x^m}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*x^m/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{x^m\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x^m*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [C] time = 3.17, size = 119, normalized size = 3.22

$$-\frac{amx^2x^m\Phi\left(axe^{i\pi},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}-\frac{2ax^2x^m\Phi\left(axe^{i\pi},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}+\frac{mxx^m\Phi\left(axe^{i\pi},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a*m*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/gamma(m
 + 3) - 2*a*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/
gamma(m + 3) + m*x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1
)/gamma(m + 2) + x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1
)/gamma(m + 2)
```

3.147 $\int e^{-3\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=150

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}-\frac{4ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\right.}{m}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a*x^(2+m)*
hypergeom([1/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)+4*x^(1+m)*hypergeom([3/2,
 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)-4*a*x^(2+m)*hypergeom([3/2, 1+1/2*m]
,[2+1/2*m],a^2*x^2)/(2+m)
```

Rubi [A] time = 0.81, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6124, 6742, 364, 850, 808}

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}-\frac{4ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\right.}{m}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m
) + (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2
+ m) + (4*x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/
(1 + m) - (4*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x
^2])/(2 + m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6124

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*
x)^((n + 1)/2)/((1 - a*x)^((n - 1)/2)*Sqrt[1 - a^2*x^2])), x] /; FreeQ[{a,
m}, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}x^m\,dx &= \int \frac{x^m(1-ax)^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx \\
&= \int \left(-\frac{3x^m}{\sqrt{1-a^2x^2}}+\frac{ax^{1+m}}{\sqrt{1-a^2x^2}}+\frac{4x^m}{(1+ax)\sqrt{1-a^2x^2}}\right)dx \\
&= -\left(3\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx\right)+4\int \frac{x^m}{(1+ax)\sqrt{1-a^2x^2}}\,dx+a\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}+4\int \frac{x^m(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx \\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}+4\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}}\,dx- \\
&= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}+\frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2}\right.}{1+m}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 55, normalized size = 0.37

$$-\frac{x^{m+1}\left(F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;ax,-ax\right)-2F_1\left(m+1;-\frac{1}{2},\frac{3}{2};m+2;ax,-ax\right)\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((x^(1 + m)*(AppellF1[1 + m, -1/2, 1/2, 2 + m, a*x, -(a*x)] - 2*AppellF1[1
 + m, -1/2, 3/2, 2 + m, a*x, -(a*x)]))/(1 + m))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(ax-1)x^m}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^m/(a^2*x^2 + 2*a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^m}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^m/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m \left(1-a^2 x^2\right)^{3/2}}{(a x+1)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int((x^m*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (-(ax-1)(ax+1))^{\frac{3}{2}}}{(ax+1)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**m*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

3.148 $\int e^{n\tanh^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=35

$$\frac{x^{m+1}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;ax,-ax\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(1+m,1/2*n,-1/2*n,2+m,a*x,-a*x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, n/2, -n/2, 2 + m, a*x, -(a*x)])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}x^m\,dx &= \int x^m(1-ax)^{-n/2}(1+ax)^{n/2}\,dx\\ &= \frac{x^{1+m}F_1\left(1+m;\frac{n}{2},-\frac{n}{2};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int e^{n\tanh^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^m,x]
```

```
[Out] Integrate[E^(n*ArcTanh[a*x])*x^m, x]
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \, e^{n \operatorname{atanh}(a x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*atanh(a*x)),x)
[Out] int(x^m*exp(n*atanh(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} e^{n \operatorname{atanh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m,x)
[Out] Integral(x**m*exp(n*atanh(a*x)), x)
```

3.149 $\int e^{n\tanh^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=155

$$-\frac{2^{\frac{n}{2}-2}n\left(n^2+8\right)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{3a^4(2-n)}-\frac{(ax+1)^{\frac{n+2}{2}}\left(2anx+n^2+6\right)(1-ax)^{1-\frac{n}{2}}}{24a^4}-\frac{x^2(ax+}{}$$

```
[Out] -1/4*x^2*(-a*x+1)^(1-1/2*n)*(a*x+1)^(1+1/2*n)/a^2-1/24*(-a*x+1)^(1-1/2*n)*(
a*x+1)^(1+1/2*n)*(2*a*n*x+n^2+6)/a^4-1/3*2^(-2+1/2*n)*n*(n^2+8)*(-a*x+1)^(1
-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],-1/2*a*x+1/2)/a^4/(2-n)
```

Rubi [A] time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 100, 147, 69}

$$-\frac{2^{\frac{n}{2}-2}n\left(n^2+8\right)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{3a^4(2-n)}-\frac{(ax+1)^{\frac{n+2}{2}}\left(2anx+n^2+6\right)(1-ax)^{1-\frac{n}{2}}}{24a^4}-\frac{x^2(ax+}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^3,x]
```

```
[Out] -(x^2*(1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(4*a^2) - ((1 - a*x)^(1 -
n/2)*(1 + a*x)^((2 + n)/2)*(6 + n^2 + 2*a*n*x))/(24*a^4) - (2^(-2 + n/2)*n*
(8 + n^2)*(1 - a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1
- a*x)/2])/(3*a^4*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x^3\,dx = \int x^3(1-ax)^{-n/2}(1+ax)^{n/2}\,dx$$
$$= -\frac{x^2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{4a^2} - \frac{\int x(1-ax)^{-n/2}(1+ax)^{n/2}(-2-anx)\,dx}{4a^2}$$
$$= -\frac{x^2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}\left(6+n^2+2anx\right)}{24a^4} + \frac{\left(n\left(8+n^2\right)\right)\int(1-$$
$$= -\frac{x^2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{4a^2} - \frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}\left(6+n^2+2anx\right)}{24a^4} - \frac{2^{-2+\frac{n}{2}}n\left(8+n^2\right)(}{}$$

Mathematica [A] time = 0.19, size = 182, normalized size = 1.17

$$-\frac{(1-ax)^{1-\frac{n}{2}}\left((n-2)\left(a^2x^2(ax+1)^{\frac{n}{2}+1}-2^{\frac{n}{2}+1}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)\right)-2^{\frac{n}{2}+3}n\,{}_2F_1\left(-\frac{n}{2}-2,1-\frac{n}{2};2-\right.\right.}{4a^4(n-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^3,x]
```

```
[Out] -1/4*((1 - a*x)^(1 - n/2)*(-(2^(3 + n/2)*n*Hypergeometric2F1[-2 - n/2, 1 -
n/2, 2 - n/2, (1 - a*x)/2]) + 2^(3 + n/2)*(-1 + n)*Hypergeometric2F1[-1 - n
```

```
/2, 1 - n/2, 2 - n/2, (1 - a*x)/2] + (-2 + n)*(a^2*x^2*(1 + a*x)^(1 + n/2)
- 2^(1 + n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - a*x)/2])))/(
a^4*(-2 + n))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(n*atanh(a*x)),x)
```

```
[Out] int(x^3*exp(n*atanh(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 e^{n\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3,x)
```

```
[Out] Integral(x**3*exp(n*atanh(a*x)), x)
```

3.150 $\int e^{n\tanh^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=141

$$-\frac{2^{n/2}\left(n^2+2\right)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{3a^3(2-n)}-\frac{n(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{6a^3}-\frac{x(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{3a^2}$$

```
[Out] -1/6*n*(-a*x+1)^(1-1/2*n)*(a*x+1)^(1+1/2*n)/a^3-1/3*x*(-a*x+1)^(1-1/2*n)*(a
*x+1)^(1+1/2*n)/a^2-1/3*2^(1/2*n)*(n^2+2)*(-a*x+1)^(1-1/2*n)*hypergeom([-1/
2*n, 1-1/2*n],[2-1/2*n],-1/2*a*x+1/2)/a^3/(2-n)
```

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 90, 80, 69}

$$-\frac{2^{n/2}\left(n^2+2\right)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{3a^3(2-n)}-\frac{n(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{6a^3}-\frac{x(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^2,x]
```

```
[Out] -(n*(1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(6*a^3) - (x*(1 - a*x)^(1 -
n/2)*(1 + a*x)^((2 + n)/2))/(3*a^2) - (2^(n/2)*(2 + n^2)*(1 - a*x)^(1 - n/2
)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(3*a^3*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}x^2\,dx &= \int x^2(1-ax)^{-n/2}(1+ax)^{n/2}\,dx\\ &= -\frac{x(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3a^2}-\frac{\int(1-ax)^{-n/2}(1+ax)^{n/2}(-1-anx)\,dx}{3a^2}\\ &= -\frac{n(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6a^3}-\frac{x(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3a^2}+\frac{\left(2+n^2\right)\int(1-ax)^{-n/2}(1+ax)^{n/2}\,d}{6a^2}\\ &= -\frac{n(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6a^3}-\frac{x(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3a^2}-\frac{2^{n/2}\left(2+n^2\right)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\right.}{3a^3(2-n)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 96, normalized size = 0.68

$$-\frac{(1-ax)^{1-\frac{n}{2}}\left((n-2)(ax+1)^{\frac{n}{2}+1}(2ax+n)-2^{\frac{n}{2}+1}\left(n^2+2\right)\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)\right)}{6a^3(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^2,x]
```

```
[Out] -1/6*((1 - a*x)^(1 - n/2)*((-2 + n)*(1 + a*x)^(1 + n/2)*(n + 2*a*x) - 2^(1
+ n/2)*(2 + n^2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - a*x)/2]))
/(a^3*(-2 + n))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2,x, algorithm="fricas")
[Out] integral(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2,x, algorithm="giac")
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2,x)
[Out] int(exp(n*arctanh(a*x))*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \, e^{n \operatorname{atanh}(a\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(n*atanh(a*x)),x)
```

```
[Out] int(x^2*exp(n*atanh(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{n \operatorname{atanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2,x)
```

```
[Out] Integral(x**2*exp(n*atanh(a*x)), x)
```

3.151 $\int e^{n\tanh^{-1}(ax)}x\,dx$

Optimal. Leaf size=99

$$-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2(2-n)}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{2a^2}$$

```
[Out] -1/2*(-a*x+1)^(1-1/2*n)*(a*x+1)^(1+1/2*n)/a^2-2^(1/2*n)*n*(-a*x+1)^(1-1/2*n
)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],-1/2*a*x+1/2)/a^2/(2-n)
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6126, 80, 69}

$$-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2(2-n)}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x,x]
```

```
[Out] -((1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(2*a^2) - (2^(n/2)*n*(1 - a*x)
^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a^2*(2
- n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
```

]

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x\,dx = \int x(1-ax)^{-n/2}(1+ax)^{n/2}\,dx$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{2a^2} + \frac{n\int(1-ax)^{-n/2}(1+ax)^{n/2}\,dx}{2a}$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{2a^2} - \frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2(2-n)}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 0.87

$$-\frac{(1-ax)^{1-\frac{n}{2}}\left((n-2)(ax+1)^{\frac{n}{2}+1}-2^{\frac{n}{2}+1}n\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)\right)}{2a^2(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x,x]
```

```
[Out] -1/2*((1 - a*x)^(1 - n/2)*((-2 + n)*(1 + a*x)^(1 + n/2) - 2^(1 + n/2)*n*Hyp
ergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - a*x)/2]))/(a^2*(-2 + n))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x,x, algorithm="fricas")
```

```
[Out] integral(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x,x, algorithm="giac")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x,x)
[Out] int(exp(n*arctanh(a*x))*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x,x, algorithm="maxima")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x e^{n \operatorname{atanh}(a x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(n*atanh(a*x)),x)
[Out] int(x*exp(n*atanh(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x e^{n \operatorname{atanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x,x)
[Out] Integral(x*exp(n*atanh(a*x)), x)
```

3.152 $\int e^{n \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=65

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)}$$

```
[Out] -2^(1+1/2*n)*(-a*x+1)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],-1/2*
a*x+1/2)/a/(2-n)
```

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6125, 69}

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x]),x]
```

```
[Out] -((2^(1 + n/2)*(1 - a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2
, (1 - a*x)/2])/(a*(2 - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\,dx = \int (1-ax)^{-n/2}(1+ax)^{n/2}\,dx$$
$$= -\frac{2^{1+\frac{n}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.71

$$\frac{4e^{(n+2)\tanh^{-1}(ax)}\,{}_2F_1\left(2,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\tanh^{-1}(ax)}\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x]),x]
```

```
[Out] (4*E^((2 + n)*ArcTanh[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(2*Ar
cTanh[a*x])])/(a*(2 + n))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x)),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x)),x)
```

```
[Out] int(exp(n*arctanh(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{n\,\mathrm{atanh}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x)),x)
```

```
[Out] int(exp(n*atanh(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x)),x)
```

```
[Out] Integral(exp(n*atanh(a*x)), x)
```

3.153 $\int \frac{e^{n \tanh^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=111

$$\frac{2(1-ax)^{-n/2}(ax+1)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{n}-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{n}$$

```
[Out] 2*(a*x+1)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(-a*x+1)/(a*x+1))/n/((-a*
x+1)^(1/2*n))-2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],-1/2*a*x+1/2
)/n/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6126, 105, 69, 131}

$$\frac{2(1-ax)^{-n/2}(ax+1)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{n}-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/x,x]
```

```
[Out] (2*(1 + a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (1 - a*x)/(1 + a*x)]
)/(n*(1 - a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2,
 (1 - a*x)/2])/(n*(1 - a*x)^(n/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{x}\,dx &= \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x}\,dx\\ &= -\left(a\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx\right)+\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}}{x}\,dx\\ &= \frac{2(1-ax)^{-n/2}(1+ax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{n}-\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-\right.}{n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 95, normalized size = 0.86

$$\frac{2(1-ax)^{-n/2}\left((ax+1)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{ax+1}\right)-2^{n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/x,x]
```

```
[Out] (2*((1 + a*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (1 - a*x)/(1 + a*
x)] - 2^(n/2)*Hypergeometric2F1[-1/2*n, -1/2*n, 1 - n/2, (1 - a*x)/2]))/(n*
(1 - a*x)^(n/2))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x,x)
[Out] int(exp(n*arctanh(a*x))/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/x,x)
[Out] int(exp(n*atanh(a*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x,x)
[Out] Integral(exp(n*atanh(a*x))/x, x)
```

3.154 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=67

$$-\frac{4a(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{2-n}$$

```
[Out] -4*a*(-a*x+1)^(1-1/2*n)*(a*x+1)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n]
,(-a*x+1)/(a*x+1))/(2-n)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6126, 131}

$$-\frac{4a(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/x^2,x]
```

```
[Out] (-4*a*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n
/2, 2 - n/2, (1 - a*x)/(1 + a*x)])/(2 - n)
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x^2}\,dx$$
$$= -\frac{4a(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{2-n}$$

<u>**Mathematica**</u> [A] time = 0.02, size = 65, normalized size = 0.97

$$\frac{4a(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n}{2}-1}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{n-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/x^2,x]
```

```
[Out] (4*a*(1 - a*x)^(1 - n/2)*(1 + a*x)^(-1 + n/2)*Hypergeometric2F1[2, 1 - n/2,
 2 - n/2, (1 - a*x)/(1 + a*x)])/(-2 + n)
```

<u>**fricas**</u> [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

<u>**giac**</u> [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n \operatorname{arctanh}(ax)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n \operatorname{atanh}(a\,x)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/x^2,x)
```

```
[Out] int(exp(n*atanh(a*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}{(ax)}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2,x)
```

```
[Out] Integral(exp(n*atanh(a*x))/x**2, x)
```

3.155 $\int \frac{e^{n \tanh^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=105

$$-\frac{2a^2n(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{2-n}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{2x^2}$$

```
[Out] -1/2*(-a*x+1)^(1-1/2*n)*(a*x+1)^(1+1/2*n)/x^2-2*a^2*n*(-a*x+1)^(1-1/2*n)*(a
*x+1)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(-a*x+1)/(a*x+1))/(2-n)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6126, 96, 131}

$$-\frac{2a^2n(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{2-n}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/x^3,x]
```

```
[Out] -((1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(2*x^2) - (2*a^2*n*(1 - a*x)^(
1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 -
 a*x)/(1 + a*x)])/(2 - n)
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{x^3}\,dx &= \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x^3}\,dx\\ &= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{2x^2}+\frac{1}{2}(an)\int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x^2}\,dx\\ &= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{2x^2}-\frac{2a^2n(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{2-n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 0.87

$$\frac{(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n}{2}-1}\left(4a^2nx^2\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)-(n-2)(ax+1)^2\right)}{2(n-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/x^3,x]
```

```
[Out] ((1 - a*x)^(1 - n/2)*(1 + a*x)^(-1 + n/2)*(-((-2 + n)*(1 + a*x)^2) + 4*a^2*
n*x^2*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)]))/(2*(-2
+ n)*x^2)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^3,x)
[Out] int(exp(n*arctanh(a*x))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{atanh}(a\,x)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/x^3,x)
[Out] int(exp(n*atanh(a*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}{(ax)}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**3,x)
```

```
[Out] Integral(exp(n*atanh(a*x))/x**3, x)
```

3.156 $\int \frac{e^{n \tanh^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=147

$$-\frac{2a^3(n^2+2)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{3(2-n)}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{3x^3}-\frac{an(ax+1)^{\frac{n+2}{2}}(1-ax)^{1}}{6x^2}$$

```
[Out] -1/3*(-a*x+1)^(1-1/2*n)*(a*x+1)^(1+1/2*n)/x^3-1/6*a*n*(-a*x+1)^(1-1/2*n)*(a
*x+1)^(1+1/2*n)/x^2-2/3*a^3*(n^2+2)*(-a*x+1)^(1-1/2*n)*(a*x+1)^(-1+1/2*n)*h
ypergeom([2, 1-1/2*n],[2-1/2*n],(-a*x+1)/(a*x+1))/(2-n)
```

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6126, 129, 151, 12, 131}

$$-\frac{2a^3(n^2+2)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{3(2-n)}-\frac{an(ax+1)^{\frac{n+2}{2}}(1-ax)^{1-\frac{n}{2}}}{6x^2}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/x^4,x]
```

```
[Out] -((1 - a*x)^(1 - n/2)*(1 + a*x)^((2 + n)/2))/(3*x^3) - (a*n*(1 - a*x)^(1 -
n/2)*(1 + a*x)^((2 + n)/2))/(6*x^2) - (2*a^3*(2 + n^2)*(1 - a*x)^(1 - n/2)*
(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 - a*x)/(1
+ a*x)])/(3*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^4}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x^4}\,dx$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3x^3} - \frac{1}{3}\int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(-an-a^2x\right)}{x^3}\,dx$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3x^3} - \frac{an(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6x^2} + \frac{1}{6}\int \frac{a^2\left(2+n^2\right)(1-ax)^{-n/2}(1+ax)^{n/2}}{x^2}$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3x^3} - \frac{an(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6x^2} + \frac{1}{6}\left(a^2\left(2+n^2\right)\right)\int \frac{(1-ax)^{-n/2}(1+ax}{x^2}$$
$$= -\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{3x^3} - \frac{an(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{6x^2} - \frac{2a^3\left(2+n^2\right)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+}}{3(2-n)}$$

Mathematica [A] time = 0.06, size = 101, normalized size = 0.69

$$\frac{(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n}{2}-1}\left(4a^3\left(n^2+2\right)x^3\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)-(n-2)(ax+1)^2(anx+2)\right)}{6(n-2)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/x^4,x]
```

```
[Out] ((1 - a*x)^(1 - n/2)*(1 + a*x)^(-1 + n/2)*(-((-2 + n)*(1 + a*x)^2*(2 + a*n*
x)) + 4*a^3*(2 + n^2)*x^3*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (1 - a*x)/
(1 + a*x)]))/(6*(-2 + n)*x^3)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^4,x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^4,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/x^4,x)
[Out] int(exp(n*atanh(a*x))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**4,x)
[Out] Integral(exp(n*atanh(a*x))/x**4, x)
```

3.157 $\int e^{\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=65

$$-\frac{2\sqrt{2}\,(c-acx)^{p+1}\,{}_2F_1\left(-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{ac(2p+1)\sqrt{1-ax}}$$

```
[Out] -2*(-a*c*x+c)^(1+p)*hypergeom([-1/2, 1/2+p],[3/2+p],-1/2*a*x+1/2)*2^(1/2)/a
/c/(1+2*p)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6130, 23, 69}

$$-\frac{2\sqrt{2}\,(c-acx)^{p+1}\,{}_2F_1\left(-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{ac(2p+1)\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (-2*Sqrt[2]*(c - a*c*x)^(1 + p)*Hypergeometric2F1[-1/2, 1/2 + p, 3/2 + p, (
1 - a*x)/2])/(a*c*(1 + 2*p)*Sqrt[1 - a*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^p\,dx = \int \frac{\sqrt{1+ax}\,(c-acx)^p}{\sqrt{1-ax}}\,dx$$
$$= \frac{\sqrt{c-acx}\int \sqrt{1+ax}\,(c-acx)^{-\frac{1}{2}+p}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{2}\,(c-acx)^{1+p}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}+p;\frac{3}{2}+p;\frac{1}{2}(1-ax)\right)}{ac(1+2p)\sqrt{1-ax}}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.82

$$-\frac{2\sqrt{2-2ax}\,(c-acx)^p\,{}_2F_1\left(-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}-\frac{ax}{2}\right)}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^p,x]
[Out] (-2*Sqrt[2 - 2*a*x]*(c - a*c*x)^p*Hypergeometric2F1[-1/2, 1/2 + p, 3/2 + p,
 1/2 - (a*x)/2])/(a + 2*a*p)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(-acx+c)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)(-acx+c)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(-a*c*x + c)^p/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,(-acx+c)^{p}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x)
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)(-acx+c)^{p}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)*(-a*c*x + c)^p/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c-a\,c\,x)^{p}\,\left(a\,x+1\right)}{\sqrt{1-a^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
[Out] int(((c - a*c*x)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\,(ax-1)\right)^{p}\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.158 $\int e^{\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=123

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\sin^{-1}(ax)}{8a}$$

```
[Out] 7/12*c^4*(-a^2*x^2+1)^(3/2)/a+7/20*c^4*(-a*x+1)*(-a^2*x^2+1)^(3/2)/a+1/5*c^
4*(-a*x+1)^2*(-a^2*x^2+1)^(3/2)/a+7/8*c^4*arcsin(a*x)/a+7/8*c^4*x*(-a^2*x^2
+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 671, 641, 195, 216}

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (7*c^4*x*Sqrt[1 - a^2*x^2])/8 + (7*c^4*(1 - a^2*x^2)^(3/2))/(12*a) + (7*c^4
*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(20*a) + (c^4*(1 - a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^4*ArcSin[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}(c-acx)^4\,dx &= c\int (c-acx)^3\sqrt{1-a^2x^2}\,dx\\
&= \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{5}\left(7c^2\right)\int (c-acx)^2\sqrt{1-a^2x^2}\,dx\\
&= \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^3\right)\int (c-acx)\sqrt{1-a^2x^2}\\
&= \frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^4\right)\int\\
&= \frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2\right.}{5a}\\
&= \frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2\right.}{5a}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 0.61

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^4,x]
```

```
[Out] -1/120*(c^4*(Sqrt[1 - a^2*x^2]*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
24*a^4*x^4) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/a
```

fricas [A] time = 0.75, size = 92, normalized size = 0.75

$$-\frac{210\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(24\,a^4c^4x^4-90\,a^3c^4x^3+112\,a^2c^4x^2-15\,ac^4x-136\,c^4\right)\sqrt{-a^2x^2+1}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(210*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c^4*x^4 -
90*a^3*c^4*x^3 + 112*a^2*c^4*x^2 - 15*a*c^4*x - 136*c^4)*sqrt(-a^2*x^2 + 1)
)/a
```

giac [A] time = 0.21, size = 78, normalized size = 0.63

$$\frac{7\,c^4\arcsin\left(ax\right)\mathrm{sgn}(a)}{8\,|a|}+\frac{1}{120}\,\sqrt{-a^2x^2+1}\left(\frac{136\,c^4}{a}+\left(15\,c^4-2\left(56\,ac^4+3\left(4\,a^3c^4x-15\,a^2c^4\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 7/8*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/120*sqrt(-a^2*x^2 + 1)*(136*c^4/a + (
15*c^4 - 2*(56*a*c^4 + 3*(4*a^3*c^4*x - 15*a^2*c^4)*x)*x)*x)
```

maple [A] time = 0.05, size = 137, normalized size = 1.11

$$-\frac{c^4a^3x^4\sqrt{-a^2x^2+1}}{5}-\frac{14c^4a\,x^2\sqrt{-a^2x^2+1}}{15}+\frac{17c^4\sqrt{-a^2x^2+1}}{15a}+\frac{3c^4a^2x^3\sqrt{-a^2x^2+1}}{4}+\frac{c^4x\sqrt{-a^2x^2+1}}{8}+\frac{7c^4\,\mathrm{arctan}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x)
```

```
[Out] -1/5*c^4*a^3*x^4*(-a^2*x^2+1)^(1/2)-14/15*c^4*a*x^2*(-a^2*x^2+1)^(1/2)+17/1
5*c^4*(-a^2*x^2+1)^(1/2)/a+3/4*c^4*a^2*x^3*(-a^2*x^2+1)^(1/2)+1/8*c^4*x*(-a
^2*x^2+1)^(1/2)+7/8*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)
)
```

maxima [A] time = 0.45, size = 118, normalized size = 0.96

$$-\frac{1}{5}\,\sqrt{-a^2x^2+1}\,a^3c^4x^4+\frac{3}{4}\,\sqrt{-a^2x^2+1}\,a^2c^4x^3-\frac{14}{15}\,\sqrt{-a^2x^2+1}\,ac^4x^2+\frac{1}{8}\,\sqrt{-a^2x^2+1}\,c^4x+\frac{7\,c^4\arcsin\left(ax\right)}{8\,a}+\frac{17\,\sqrt{-a}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^4 + 3/4*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^3 -
14/15*sqrt(-a^2*x^2 + 1)*a*c^4*x^2 + 1/8*sqrt(-a^2*x^2 + 1)*c^4*x + 7/8*c^4
*arcsin(a*x)/a + 17/15*sqrt(-a^2*x^2 + 1)*c^4/a
```

mupad [B] time = 0.79, size = 128, normalized size = 1.04

$$\frac{c^4\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{7\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}+\frac{17\,c^4\,\sqrt{1-a^2\,x^2}}{15\,a}-\frac{14\,a\,c^4\,x^2\,\sqrt{1-a^2\,x^2}}{15}+\frac{3\,a^2\,c^4\,x^3\,\sqrt{1-a^2\,x^2}}{4}-\frac{a^3\,c^4\,x^4\,\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^4*x*(1 - a^2*x^2)^(1/2))/8 + (7*c^4*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(1/
2)) + (17*c^4*(1 - a^2*x^2)^(1/2))/(15*a) - (14*a*c^4*x^2*(1 - a^2*x^2)^(1/
2))/15 + (3*a^2*c^4*x^3*(1 - a^2*x^2)^(1/2))/4 - (a^3*c^4*x^4*(1 - a^2*x^2)
^(1/2))/5
```

sympy [A] time = 8.98, size = 226, normalized size = 1.84

$$\begin{cases}\frac{3c^4\sqrt{-a^2x^2+1}+2c^4\left(\begin{cases}-\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\mathrm{asin}\,(ax)}{2} & \text{for } ax>-1 \wedge ax<1\end{cases}\right)+2c^4\left(\begin{cases}\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^2x^2+1} & \text{for } ax>-1 \wedge \cdots\end{cases}\right)\cdots}{\cdots} \\ c^4x\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4,x)
```

```
[Out] Piecewise(((3*c**4*sqrt(-a**2*x**2 + 1) + 2*c**4*Piecewise((-a*x*sqrt(-a**2
*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + 2*c**4*Piecewise(((-
a**2*x**2 + 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) -
3*c**4*Piecewise((a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 - a*x*sqrt(
-a**2*x**2 + 1)/2 + 3*asin(a*x)/8, (a*x > -1) & (a*x < 1))) + c**4*Piecewis
e((-(-a**2*x**2 + 1)**(5/2)/5 + 2*(-a**2*x**2 + 1)**(3/2)/3 - sqrt(-a**2*x*
*2 + 1), (a*x > -1) & (a*x < 1))) + c**4*asin(a*x))/a, Ne(a, 0)), (c**4*x,
True))
```

3.159 $\int e^{\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=91

$$\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{8a}$$

```
[Out] 5/12*c^3*(-a^2*x^2+1)^(3/2)/a+1/4*c^3*(-a*x+1)*(-a^2*x^2+1)^(3/2)/a+5/8*c^3
*arcsin(a*x)/a+5/8*c^3*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 671, 641, 195, 216}

$$\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/8 + (5*c^3*(1 - a^2*x^2)^(3/2))/(12*a) + (c^3*(
1 - a*x)*(1 - a^2*x^2)^(3/2))/(4*a) + (5*c^3*ArcSin[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}(c-acx)^3\,dx &= c\int (c-acx)^2\sqrt{1-a^2x^2}\,dx\\
&= \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{1}{4}\left(5c^2\right)\int (c-acx)\sqrt{1-a^2x^2}\,dx\\
&= \frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{1}{4}\left(5c^3\right)\int \sqrt{1-a^2x^2}\,dx\\
&= \frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{1}{8}\left(5c^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\sin^{-1}(ax)}{8a}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 67, normalized size = 0.74

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(6a^3x^3-16a^2x^2+9ax+16\right)-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 30*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(24*a)
```

fricas [A] time = 0.68, size = 82, normalized size = 0.90

$$-\frac{30\,c^{3}\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-\left(6\,a^{3}c^{3}x^{3}-16\,a^{2}c^{3}x^{2}+9\,ac^{3}x+16\,c^{3}\right)\sqrt{-a^{2}x^{2}+1}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(30*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (6*a^3*c^3*x^3 - 16*
a^2*c^3*x^2 + 9*a*c^3*x + 16*c^3)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.22, size = 66, normalized size = 0.73

$$\frac{5\,c^{3}\arcsin\left(ax\right)\mathrm{sgn}(a)}{8\,|a|}+\frac{1}{24}\,\sqrt{-a^{2}x^{2}+1}\left(\frac{16\,c^{3}}{a}+\left(9\,c^{3}+2\left(3\,a^{2}c^{3}x-8\,ac^{3}\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 5/8*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/24*sqrt(-a^2*x^2 + 1)*(16*c^3/a + (9*
c^3 + 2*(3*a^2*c^3*x - 8*a*c^3)*x)*x)
```

maple [A] time = 0.04, size = 114, normalized size = 1.25

$$\frac{c^{3}a^{2}x^{3}\sqrt{-a^{2}x^{2}+1}}{4}+\frac{3c^{3}x\sqrt{-a^{2}x^{2}+1}}{8}+\frac{5c^{3}\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{8\sqrt{a^{2}}}-\frac{2c^{3}a\,x^{2}\sqrt{-a^{2}x^{2}+1}}{3}+\frac{2c^{3}\sqrt{-a^{2}x^{2}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x)
```

```
[Out] 1/4*c^3*a^2*x^3*(-a^2*x^2+1)^(1/2)+3/8*c^3*x*(-a^2*x^2+1)^(1/2)+5/8*c^3/(a^
2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2/3*c^3*a*x^2*(-a^2*x^2+1
)^(1/2)+2/3*c^3*(-a^2*x^2+1)^(1/2)/a
```

maxima [A] time = 0.42, size = 95, normalized size = 1.04

$$\frac{1}{4}\,\sqrt{-a^{2}x^{2}+1}\,a^{2}c^{3}x^{3}-\frac{2}{3}\,\sqrt{-a^{2}x^{2}+1}\,ac^{3}x^{2}+\frac{3}{8}\,\sqrt{-a^{2}x^{2}+1}\,c^{3}x+\frac{5\,c^{3}\arcsin\left(ax\right)}{8\,a}+\frac{2\,\sqrt{-a^{2}x^{2}+1}\,c^{3}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^3 - 2/3*sqrt(-a^2*x^2 + 1)*a*c^3*x^2 + 3/8
*sqrt(-a^2*x^2 + 1)*c^3*x + 5/8*c^3*arcsin(a*x)/a + 2/3*sqrt(-a^2*x^2 + 1)*
c^3/a
```

mupad [B] time = 0.04, size = 105, normalized size = 1.15

$$\frac{3\,c^3\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{5\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}+\frac{2\,c^3\,\sqrt{1-a^2\,x^2}}{3\,a}-\frac{2\,a\,c^3\,x^2\,\sqrt{1-a^2\,x^2}}{3}+\frac{a^2\,c^3\,x^3\,\sqrt{1-a^2\,x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (3*c^3*x*(1 - a^2*x^2)^(1/2))/8 + (5*c^3*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(
1/2)) + (2*c^3*(1 - a^2*x^2)^(1/2))/(3*a) - (2*a*c^3*x^2*(1 - a^2*x^2)^(1/2
))/3 + (a^2*c^3*x^3*(1 - a^2*x^2)^(1/2))/4
```

sympy [A] time = 6.37, size = 134, normalized size = 1.47

$$\begin{cases} \frac{2c^3\sqrt{-a^2x^2+1}+2c^3\left(\begin{cases}\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^2x^2+1} & \text{for } ax>-1 \wedge ax<1\end{cases}\right)-c^3\left(\begin{cases}\frac{ax\left(-2a^2x^2+1\right)\sqrt{-a^2x^2+1}}{8}-\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{3\,\mathrm{asi}}{}\end{cases}\right.}{a} \\ c^3x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3,x)
```

```
[Out] Piecewise(((2*c**3*sqrt(-a**2*x**2 + 1) + 2*c**3*Piecewise(((-a**2*x**2 + 1
)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) - c**3*Piecewis
e((a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 - a*x*sqrt(-a**2*x**2 + 1)
/2 + 3*asin(a*x)/8, (a*x > -1) & (a*x < 1))) + c**3*asin(a*x))/a, Ne(a, 0))
, (c**3*x, True))
```

3.160 $\int e^{\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=61

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

```
[Out] 1/3*c^2*(-a^2*x^2+1)^(3/2)/a+1/2*c^2*arcsin(a*x)/a+1/2*c^2*x*(-a^2*x^2+1)^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6127, 641, 195, 216}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^2,x]
```

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/2 + (c^2*(1 - a^2*x^2)^(3/2))/(3*a) + (c^2*ArcSin
[a*x])/(2*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}(c-acx)^2\,dx &= c\int (c-acx)\sqrt{1-a^2x^2}\,dx\\ &= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+c^2\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{c^2\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 0.97

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(2a^2x^2-3ax-2\right)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^2,x]
```

```
[Out] -1/6*(c^2*(Sqrt[1 - a^2*x^2]*(-2 - 3*a*x + 2*a^2*x^2) + 6*ArcSin[Sqrt[1 - a
*x]/Sqrt[2]]))/a
```

fricas [A] time = 1.17, size = 70, normalized size = 1.15

$$-\frac{6\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^2c^2x^2-3\,ac^2x-2\,c^2\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^2*c^2*x^2 - 3*a*c
^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.17, size = 54, normalized size = 0.89

$$\frac{c^2\arcsin(ax)\,\mathrm{sgn}(a)}{2\,|a|} - \frac{1}{6}\sqrt{-a^2x^2+1}\left(\left(2\,ac^2x - 3\,c^2\right)x - \frac{2\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x - 3*
c^2)*x - 2*c^2/a)
```

maple [A] time = 0.04, size = 91, normalized size = 1.49

$$-\frac{c^2a\,x^2\sqrt{-a^2x^2+1}}{3} + \frac{c^2\sqrt{-a^2x^2+1}}{3a} + \frac{c^2x\sqrt{-a^2x^2+1}}{2} + \frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x)
```

```
[Out] -1/3*c^2*a*x^2*(-a^2*x^2+1)^(1/2)+1/3*c^2*(-a^2*x^2+1)^(1/2)/a+1/2*c^2*x*(-
a^2*x^2+1)^(1/2)+1/2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2
))
```

maxima [A] time = 0.42, size = 72, normalized size = 1.18

$$-\frac{1}{3}\sqrt{-a^2x^2+1}\,ac^2x^2 + \frac{1}{2}\sqrt{-a^2x^2+1}\,c^2x + \frac{c^2\arcsin(ax)}{2\,a} + \frac{\sqrt{-a^2x^2+1}\,c^2}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + 1)*a*c^2*x^2 + 1/2*sqrt(-a^2*x^2 + 1)*c^2*x + 1/2*c^2*
arcsin(a*x)/a + 1/3*sqrt(-a^2*x^2 + 1)*c^2/a
```

mupad [B] time = 0.04, size = 82, normalized size = 1.34

$$\frac{c^2\,x\,\sqrt{1-a^2\,x^2}}{2} + \frac{c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}} + \frac{c^2\,\sqrt{1-a^2\,x^2}}{3\,a} - \frac{a\,c^2\,x^2\,\sqrt{1-a^2\,x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^2*x*(1 - a^2*x^2)^(1/2))/2 + (c^2*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2)
) + (c^2*(1 - a^2*x^2)^(1/2))/(3*a) - (a*c^2*x^2*(1 - a^2*x^2)^(1/2))/3
```

sympy [A] time = 5.38, size = 102, normalized size = 1.67

$$\begin{cases} \frac{c^2\sqrt{-a^2x^2+1} - c^2\left(\begin{cases} -\frac{ax\sqrt{-a^2x^2+1}}{2} + \frac{\operatorname{asin}(ax)}{2} & \text{for}\ ax > -1 \wedge ax < 1 \end{cases}\right) + c^2\left(\begin{cases} \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3} - \sqrt{-a^2x^2+1} & \text{for}\ ax > -1 \wedge ax \end{cases}\right.}{a} \\ c^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2,x)
```

```
[Out] Piecewise(((c**2*sqrt(-a**2*x**2 + 1) - c**2*Piecewise((-a*x*sqrt(-a**2*x**
2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + c**2*Piecewise(((-a**2*x
**2 + 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) + c**2*a
sin(a*x))/a, Ne(a, 0)), (c**2*x, True))
```

3.161 $\int e^{\tanh^{-1}(ax)}(c - acx)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

[Out] 1/2*c*arcsin(a*x)/a+1/2*c*x*(-a^2*x^2+1)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6127, 195, 216}

$$\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[a*x]*(c - a*c*x),x]

[Out] (c*x*Sqrt[1 - a^2*x^2])/2 + (c*ArcSin[a*x])/(2*a)

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)\,dx = c\int\sqrt{1-a^2x^2}\,dx$$
$$= \frac{1}{2}cx\sqrt{1-a^2x^2} + \frac{1}{2}c\int\frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{1}{2}cx\sqrt{1-a^2x^2} + \frac{c\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{c\left(ax\sqrt{1-a^2x^2}+\sin^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x),x]
[Out] (c*(a*x*Sqrt[1 - a^2*x^2] + ArcSin[a*x]))/(2*a)
```

fricas [A] time = 0.73, size = 47, normalized size = 1.42

$$\frac{\sqrt{-a^2x^2+1}\,acx-2\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="fricas")
[Out] 1/2*(sqrt(-a^2*x^2 + 1)*a*c*x - 2*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))
/a
```

giac [A] time = 0.29, size = 30, normalized size = 0.91

$$\frac{1}{2}\sqrt{-a^2x^2+1}\,cx+\frac{c\arcsin(ax)\operatorname{sgn}(a)}{2\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="giac")
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(a*x)*sgn(a)/abs(a)
```

maple [A] time = 0.03, size = 46, normalized size = 1.39

$$\frac{cx\sqrt{-a^2x^2+1}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x)
```

```
[Out] 1/2*c*x*(-a^2*x^2+1)^(1/2)+1/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2
+1)^(1/2))
```

maxima [A] time = 0.42, size = 27, normalized size = 0.82

$$\frac{1}{2}\sqrt{-a^2x^2+1}\,cx+\frac{c\arcsin\left(ax\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(a*x)/a
```

mupad [B] time = 0.81, size = 37, normalized size = 1.12

$$\frac{c\,x\,\sqrt{1-a^2\,x^2}}{2}+\frac{c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c*x*(1 - a^2*x^2)^(1/2))/2 + (c*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2))
```

sympy [A] time = 3.64, size = 37, normalized size = 1.12

$$\begin{cases} \frac{c\left(\begin{cases} \frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\mathrm{asin}\,(ax)}{2} & \text{for}\ ax>-1\wedge ax<1 \end{cases}\right)}{a} & \text{for}\ a\neq 0 \\ cx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c),x)
```

```
[Out] Piecewise((c*Piecewise((a*x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1
) & (a*x < 1)))/a, Ne(a, 0)), (c*x, True))
```

3.162 $\int \frac{e^{\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}-\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] -arcsin(a*x)/a/c+2*(-a^2*x^2+1)^(1/2)/a/c/(-a*x+1)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 663, 216}

$$\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}-\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)) - ArcSin[a*x]/(a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{c-acx}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx$$
$$= \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} - \frac{\sin^{-1}(ax)}{ac}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.07

$$\frac{2\left(\frac{\sqrt{ax+1}}{\sqrt{1-ax}}+\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x),x]
[Out] (2*(Sqrt[1 + a*x]/Sqrt[1 - a*x] + ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*c)
```

fricas [A] time = 0.79, size = 62, normalized size = 1.44

$$\frac{2\left(ax+(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}-1\right)}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="fricas")
[Out] 2*(a*x + (a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 +
 1) - 1)/(a^2*c*x - a*c)
```

giac [A] time = 0.21, size = 53, normalized size = 1.23

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}+\frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c*abs(a)) + 4/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2
*x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 76, normalized size = 1.77

$$-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c\sqrt{a^2}}-\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{c\,a^2\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x)
```

```
[Out] -1/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2/c/a^2/(x-1/a)*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.44, size = 40, normalized size = 0.93

$$-\frac{2\sqrt{-a^2x^2+1}}{a^2cx-ac}-\frac{\arcsin(ax)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a^2*x^2 + 1)/(a^2*c*x - a*c) - arcsin(a*x)/(a*c)
```

mupad [B] time = 0.82, size = 71, normalized size = 1.65

$$\frac{2\sqrt{1-a^2\,x^2}}{c\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- asinh(x*(-a^2)^(1/2))/(c*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{ax}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{1}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Inte
gral(1/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.163 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^2} dx$

Optimal. Leaf size=32

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}$$

```
[Out] 1/3*(-a^2*x^2+1)^(3/2)/a/c^2/(-a*x+1)^3
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6127, 651}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^2,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(3*a*c^2*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^2} dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3} dx\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.91

$$\frac{(ax+1)^{3/2}}{3ac^2(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^2,x]

[Out] (1 + a*x)^(3/2)/(3*a*c^2*(1 - a*x)^(3/2))

fricas [B] time = 0.42, size = 60, normalized size = 1.88

$$\frac{a^2x^2 - 2\,ax + \sqrt{-a^2x^2 + 1}\,(ax + 1) + 1}{3\left(a^3c^2x^2 - 2\,a^2c^2x + ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")

[Out] 1/3*(a^2*x^2 - 2*a*x + sqrt(-a^2*x^2 + 1)*(a*x + 1) + 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)

giac [C] time = 0.35, size = 66, normalized size = 2.06

$$-\frac{i\,\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c) + \frac{\left(-\frac{2\,c}{acx-c}-1\right)^{\frac{3}{2}}}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}}{3\,c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")

[Out] -1/3*(I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c) + (-2*c/(a*c*x - c) - 1)^(3/2)/(sgn(1/(a*c*x - c))*sgn(a)*sgn(c)))/(c^2*abs(a))

maple [A] time = 0.03, size = 35, normalized size = 1.09

$$-\frac{(ax+1)^2}{3\,(ax-1)\,c^2\sqrt{-a^2x^2+1}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x)

```
[Out] -1/3*(a*x+1)^2/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a
```

maxima [B] time = 0.43, size = 73, normalized size = 2.28

$$\frac{2\sqrt{-a^2x^2+1}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}+\frac{\sqrt{-a^2x^2+1}}{3\left(a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(-a^2*x^2 + 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + 1/3*sqrt(-a^2*
x^2 + 1)/(a^2*c^2*x - a*c^2)
```

mupad [B] time = 0.82, size = 32, normalized size = 1.00

$$\frac{\sqrt{1-a^2\,x^2}\,\left(a\,x+1\right)}{3\,a\,c^2\,{\left(a\,x-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(a*x + 1))/(3*a*c^2*(a*x - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1)
+ sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**2*sqrt(-a**2*x**2 + 1) -
2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.164 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=65

$$\frac{(1-a^2x^2)^{3/2}}{15ac^3(1-ax)^3}+\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}$$

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)/a/c^3/(-a*x+1)^4+1/15*(-a^2*x^2+1)^(3/2)/a/c^3/(-a*x
+1)^3
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{(1-a^2x^2)^{3/2}}{15ac^3(1-ax)^3}+\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^3,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(5*a*c^3*(1 - a*x)^4) + (1 - a^2*x^2)^(3/2)/(15*a*c^3*(
1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^3}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4}+\frac{1}{5}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.54

$$\frac{(4-ax)(ax+1)^{3/2}}{15ac^3(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^3,x]
[Out] ((4 - a*x)*(1 + a*x)^(3/2))/(15*a*c^3*(1 - a*x)^(5/2))
```

fricas [A] time = 0.55, size = 89, normalized size = 1.37

$$\frac{4\,a^3x^3-12\,a^2x^2+12\,ax+\left(a^2x^2-3\,ax-4\right)\sqrt{-a^2x^2+1}-4}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] 1/15*(4*a^3*x^3 - 12*a^2*x^2 + 12*a*x + (a^2*x^2 - 3*a*x - 4)*sqrt(-a^2*x^2
 + 1) - 4)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

giac [B] time = 0.36, size = 145, normalized size = 2.23

$$-\frac{2\left(\frac{5\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{25\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}-4\right)}{15\,c^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/15*(5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 25*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2/(a^4*x^2) + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 1
5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 4)/(c^3*((sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

maple [A] time = 0.03, size = 40, normalized size = 0.62

$$-\frac{(ax-4)(ax+1)^2}{15(ax-1)^2 c^3\sqrt{-a^2x^2+1}\ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/15*(a*x-4)*(a*x+1)^2/(a*x-1)^2/c^3/(-a^2*x^2+1)^(1/2)/a
```

maxima [B] time = 0.44, size = 126, normalized size = 1.94

$$-\frac{2\sqrt{-a^2x^2+1}}{5\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}-\frac{\sqrt{-a^2x^2+1}}{15\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)}+\frac{\sqrt{-a^2x^2+1}}{15\left(a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-a^2*x^2 + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
 - 1/15*sqrt(-a^2*x^2 + 1)/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3) + 1/15*sqrt(
-a^2*x^2 + 1)/(a^2*c^3*x - a*c^3)
```

mupad [B] time = 0.83, size = 183, normalized size = 2.82

$$\frac{2\sqrt{1-a^2\,x^2}}{5\sqrt{-a^2}\,\left(3\,c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}+a^2\,c^3\,x^3\,\sqrt{-a^2}-3\,a\,c^3\,x^2\,\sqrt{-a^2}\right)}-\frac{\sqrt{1-a^2\,x^2}}{15\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}\right)}-\frac{a}{15\,\left(a^4\,c^3\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)
^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)^(1/2))) - (1 - a^
2*x^2)^(1/2)/(15*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a))
- (a*(1 - a^2*x^2)^(1/2))/(15*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**
2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(
1/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*
x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.165 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a/c^4/(-a*x+1)^5+2/35*(-a^2*x^2+1)^(3/2)/a/c^4/(-a*x
+1)^4+2/105*(-a^2*x^2+1)^(3/2)/a/c^4/(-a*x+1)^3
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a*c^4*(1 - a*x)^5) + (2*(1 - a^2*x^2)^(3/2))/(35*a*c
^4*(1 - a*x)^4) + (2*(1 - a^2*x^2)^(3/2))/(105*a*c^4*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^4}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2}{7}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{35c}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.44

$$-\frac{(ax+1)^{3/2}\left(-2a^2x^2+10ax-23\right)}{105ac^4(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -1/105*((1 + a*x)^(3/2)*(-23 + 10*a*x - 2*a^2*x^2))/(a*c^4*(1 - a*x)^(7/2))
```

fricas [A] time = 0.44, size = 116, normalized size = 1.20

$$\frac{23\,a^4x^4-92\,a^3x^3+138\,a^2x^2-92\,ax+\left(2\,a^3x^3-8\,a^2x^2+13\,ax+23\right)\sqrt{-a^2x^2+1}+23}{105\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(23*a^4*x^4 - 92*a^3*x^3 + 138*a^2*x^2 - 92*a*x + (2*a^3*x^3 - 8*a^2*
x^2 + 13*a*x + 23)*sqrt(-a^2*x^2 + 1) + 23)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 +
6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

giac [B] time = 0.22, size = 199, normalized size = 2.05

$$-\frac{2\left(\frac{56\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{273\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{350\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{455\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}+\frac{210\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5}{a^{10}x^5}-\frac{105\left(\sqrt{-\right.}}{}}{105\,c^4\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -2/105*(56*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 273*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^2/(a^4*x^2) + 350*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3)
 - 455*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 210*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^5/(a^10*x^5) - 105*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^
6) - 23)/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

maple [A] time = 0.03, size = 49, normalized size = 0.51

$$-\frac{\left(2a^2x^2-10ax+23\right)(ax+1)^2}{105\,(ax-1)^3\,c^4\sqrt{-a^2x^2+1}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)^2/(a*x-1)^3/c^4/(-a^2*x^2+1)^(1/2)/a
```

maxima [B] time = 0.50, size = 189, normalized size = 1.95

$$\frac{2\sqrt{-a^2x^2+1}}{7\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}+\frac{\sqrt{-a^2x^2+1}}{35\left(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4\right)}-\frac{2\sqrt{-a^2x^2+1}}{105\left(a^3c^4x^2-2\,a^2c^4x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2
*c^4*x + a*c^4) + 1/35*sqrt(-a^2*x^2 + 1)/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*
a^2*c^4*x - a*c^4) - 2/105*sqrt(-a^2*x^2 + 1)/(a^3*c^4*x^2 - 2*a^2*c^4*x +
a*c^4) + 2/105*sqrt(-a^2*x^2 + 1)/(a^2*c^4*x - a*c^4)
```

mupad [B] time = 0.83, size = 49, normalized size = 0.51

$$\frac{\sqrt{1-a^2\,x^2}\;\left(2\,a^3\,x^3-8\,a^2\,x^2+13\,a\,x+23\right)}{105\,a\,c^4\,(a\,x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(13*a*x - 8*a^2*x^2 + 2*a^3*x^3 + 23))/(105*a*c^4*(a*x
 - 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqr
t(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**
3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt
(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.166 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=129

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{315ac^5(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^5(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{21ac^5(1-ax)^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}$$

```
[Out] 1/9*(-a^2*x^2+1)^(3/2)/a/c^5/(-a*x+1)^6+1/21*(-a^2*x^2+1)^(3/2)/a/c^5/(-a*x
+1)^5+2/105*(-a^2*x^2+1)^(3/2)/a/c^5/(-a*x+1)^4+2/315*(-a^2*x^2+1)^(3/2)/a/
c^5/(-a*x+1)^3
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{315ac^5(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^5(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{21ac^5(1-ax)^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^5,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(9*a*c^5*(1 - a*x)^6) + (1 - a^2*x^2)^(3/2)/(21*a*c^5*(
1 - a*x)^5) + (2*(1 - a^2*x^2)^(3/2))/(105*a*c^5*(1 - a*x)^4) + (2*(1 - a^2
*x^2)^(3/2))/(315*a*c^5*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^5}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^6}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}+\frac{1}{3}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{3/2}}{21ac^5(1-ax)^5}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx}{21c}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{3/2}}{21ac^5(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^5(1-ax)^4}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{105c^2}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{9ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{3/2}}{21ac^5(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^5(1-ax)^4}+\frac{2\left(1-a^2x^2\right)^{3/2}}{315ac^5(1-ax)^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.40

$$\frac{(ax+1)^{3/2}\left(-2a^3x^3+12a^2x^2-33ax+58\right)}{315ac^5(1-ax)^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^5,x]
```

```
[Out] ((1 + a*x)^(3/2)*(58 - 33*a*x + 12*a^2*x^2 - 2*a^3*x^3))/(315*a*c^5*(1 - a*
x)^(9/2))
```

fricas [A] time = 0.76, size = 144, normalized size = 1.12

$$\frac{58\,a^5x^5-290\,a^4x^4+580\,a^3x^3-580\,a^2x^2+290\,ax+\left(2\,a^4x^4-10\,a^3x^3+21\,a^2x^2-25\,ax-58\right)\sqrt{-a^2x^2+1}\;-58}{315\left(a^6c^5x^5-5\,a^5c^5x^4+10\,a^4c^5x^3-10\,a^3c^5x^2+5\,a^2c^5x-ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/315*(58*a^5*x^5 - 290*a^4*x^4 + 580*a^3*x^3 - 580*a^2*x^2 + 290*a*x + (2*
a^4*x^4 - 10*a^3*x^3 + 21*a^2*x^2 - 25*a*x - 58)*sqrt(-a^2*x^2 + 1) - 58)/(
a^6*c^5*x^5 - 5*a^5*c^5*x^4 + 10*a^4*c^5*x^3 - 10*a^3*c^5*x^2 + 5*a^2*c^5*x
 - a*c^5)
```

giac [C] time = 0.26, size = 321, normalized size = 2.49

$$\frac{-\frac{16i\,\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}{c^3}-\frac{\frac{35\left(\frac{2c}{acx-c}+1\right)^4\sqrt{-\frac{2c}{acx-c}-1}-180\left(\frac{2c}{acx-c}+1\right)^3\sqrt{-\frac{2c}{acx-c}-1}+378\left(\frac{2c}{acx-c}+1\right)^2\sqrt{-\frac{2c}{acx-c}-1}+420\left(-\frac{2c}{acx-c}-1\right)^{\frac{3}{2}}+315\sqrt{-\frac{2c}{acx-c}-1}}{c^3}+\frac{9(\cdots}{\cdots}}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}}{2520\,c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/2520*(-16*I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/c^3 - ((35*(2*c/(a*c*x - c)
+ 1)^4*sqrt(-2*c/(a*c*x - c) - 1) - 180*(2*c/(a*c*x - c) + 1)^3*sqrt(-2*c/(
a*c*x - c) - 1) + 378*(2*c/(a*c*x - c) + 1)^2*sqrt(-2*c/(a*c*x - c) - 1) +
420*(-2*c/(a*c*x - c) - 1)^(3/2) + 315*sqrt(-2*c/(a*c*x - c) - 1))/c^3 + 9*
(5*(2*c/(a*c*x - c) + 1)^3*sqrt(-2*c/(a*c*x - c) - 1) - 21*(2*c/(a*c*x - c)
 + 1)^2*sqrt(-2*c/(a*c*x - c) - 1) - 35*(-2*c/(a*c*x - c) - 1)^(3/2) - 35*s
qrt(-2*c/(a*c*x - c) - 1))/c^3)/(sgn(1/(a*c*x - c))*sgn(a)*sgn(c)))/(c^2*ab
s(a))
```

maple [A] time = 0.03, size = 57, normalized size = 0.44

$$-\frac{\left(2x^3a^3-12a^2x^2+33ax-58\right)(ax+1)^2}{315\,(ax-1)^4\,c^5\sqrt{-a^2x^2+1}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/315*(2*a^3*x^3-12*a^2*x^2+33*a*x-58)*(a*x+1)^2/(a*x-1)^4/c^5/(-a^2*x^2+1
)^(1/2)/a
```

maxima [B] time = 0.43, size = 264, normalized size = 2.05

$$-\frac{2\sqrt{-a^2x^2+1}}{9\left(a^6c^5x^5-5\,a^5c^5x^4+10\,a^4c^5x^3-10\,a^3c^5x^2+5\,a^2c^5x-ac^5\right)}-\frac{\sqrt{-a^2x^2+1}}{63\left(a^5c^5x^4-4\,a^4c^5x^3+6\,a^3c^5x^2-4\,a^2c^5x+ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(-a^2*x^2 + 1)/(a^6*c^5*x^5 - 5*a^5*c^5*x^4 + 10*a^4*c^5*x^3 - 10*
a^3*c^5*x^2 + 5*a^2*c^5*x - a*c^5) - 1/63*sqrt(-a^2*x^2 + 1)/(a^5*c^5*x^4 -
 4*a^4*c^5*x^3 + 6*a^3*c^5*x^2 - 4*a^2*c^5*x + a*c^5) + 1/105*sqrt(-a^2*x^2
 + 1)/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5) - 2/315*sqrt(-a^2
*x^2 + 1)/(a^3*c^5*x^2 - 2*a^2*c^5*x + a*c^5) + 2/315*sqrt(-a^2*x^2 + 1)/(a
^2*c^5*x - a*c^5)
```

mupad [B] time = 0.87, size = 57, normalized size = 0.44

$$-\frac{\sqrt{1-a^2\,x^2}\,\left(-2\,a^4\,x^4+10\,a^3\,x^3-21\,a^2\,x^2+25\,a\,x+58\right)}{315\,a\,c^5\,{\left(a\,x-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^5),x)
```

```
[Out] -((1 - a^2*x^2)^(1/2)*(25*a*x - 21*a^2*x^2 + 10*a^3*x^3 - 2*a^4*x^4 + 58))/
(315*a*c^5*(a*x - 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{a^5x^5\sqrt{-a^2x^2+1}-5a^4x^4\sqrt{-a^2x^2+1}+10a^3x^3\sqrt{-a^2x^2+1}-10a^2x^2\sqrt{-a^2x^2+1}+5ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^5x^5\sqrt{-a^2x^2+1}-5a^4x^4\sqrt{-a^2}}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] -(Integral(a*x/(a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**4*x**4*sqrt(-a**2*x**
2 + 1) + 10*a**3*x**3*sqrt(-a**2*x**2 + 1) - 10*a**2*x**2*sqrt(-a**2*x**2 +
 1) + 5*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(1/(
a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 10*a**3
*x**3*sqrt(-a**2*x**2 + 1) - 10*a**2*x**2*sqrt(-a**2*x**2 + 1) + 5*a*x*sqrt
(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**5
```

3.167 $\int e^{2\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=41

$$\frac{(c-acx)^{p+1}}{ac(p+1)}-\frac{2(c-acx)^p}{ap}$$

```
[Out] -2*(-a*c*x+c)^p/a/p+(-a*c*x+c)^(1+p)/a/c/(1+p)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 21, 43}

$$\frac{(c-acx)^{p+1}}{ac(p+1)}-\frac{2(c-acx)^p}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (-2*(c - a*c*x)^p)/(a*p) + (c - a*c*x)^(1 + p)/(a*c*(1 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}(c-acx)^p\,dx = \int \frac{(1+ax)(c-acx)^p}{1-ax}\,dx$$
$$= c\int (1+ax)(c-acx)^{-1+p}\,dx$$
$$= c\int \left(2(c-acx)^{-1+p} - \frac{(c-acx)^p}{c}\right)dx$$
$$= -\frac{2(c-acx)^p}{ap} + \frac{(c-acx)^{1+p}}{ac(1+p)}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.71

$$-\frac{(apx+p+2)(c-acx)^p}{ap(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^p,x]
[Out] -(((c - a*c*x)^p*(2 + p + a*p*x))/(a*p*(1 + p)))
```

fricas [A] time = 0.52, size = 29, normalized size = 0.71

$$-\frac{\left(apx+p+2\right)(-acx+c)^p}{ap^2+ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^p,x, algorithm="fricas")
[Out] -(a*p*x + p + 2)*(-a*c*x + c)^p/(a*p^2 + a*p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2(-acx+c)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate(-(a*x + 1)^2*(-a*c*x + c)^p/(a^2*x^2 - 1), x)
```

maple [A] time = 0.02, size = 30, normalized size = 0.73

$$-\frac{(-acx+c)^{p}\left(apx+p+2\right)}{ap\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^p,x)
[Out] -(-a*c*x+c)^p*(a*p*x+p+2)/a/p/(1+p)
```

maxima [A] time = 0.38, size = 35, normalized size = 0.85

$$-\frac{\left(ac^{p}px+c^{p}\left(p+2\right)\right)(-ax+1)^{p}}{\left(p^{2}+p\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] -(a*c^p*p*x + c^p*(p + 2))*(-a*x + 1)^p/((p^2 + p)*a)
```

mupad [B] time = 0.84, size = 29, normalized size = 0.71

$$-\frac{(c-a\,c\,x)^{p}\,\left(p+a\,p\,x+2\right)}{a\,p\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^p*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] -((c - a*c*x)^p*(p + a*p*x + 2))/(a*p*(p + 1))
```

sympy [A] time = 0.96, size = 126, normalized size = 3.07

$$\begin{cases} c^{p}x & \text{for}\ a=0 \\ \frac{ax\log\left(x-\frac{1}{a}\right)}{a^{2}cx-ac}-\frac{\log\left(x-\frac{1}{a}\right)}{a^{2}cx-ac}-\frac{2}{a^{2}cx-ac} & \text{for}\ p=-1 \\ -x-\frac{2\log\left(x-\frac{1}{a}\right)}{a} & \text{for}\ p=0 \\ -\frac{apx(-acx+c)^{p}}{ap^{2}+ap}-\frac{p(-acx+c)^{p}}{ap^{2}+ap}-\frac{2(-acx+c)^{p}}{ap^{2}+ap} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((c**p*x, Eq(a, 0)), (a*x*log(x - 1/a)/(a**2*c*x - a*c) - log(x -
1/a)/(a**2*c*x - a*c) - 2/(a**2*c*x - a*c), Eq(p, -1)), (-x - 2*log(x - 1/a
)/a, Eq(p, 0)), (-a*p*x*(-a*c*x + c)**p/(a*p**2 + a*p) - p*(-a*c*x + c)**p/
(a*p**2 + a*p) - 2*(-a*c*x + c)**p/(a*p**2 + a*p), True))
```

3.168 $\int e^{2\tanh^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=37

$$\frac{c^5(1-ax)^6}{6a}-\frac{2c^5(1-ax)^5}{5a}$$

```
[Out] -2/5*c^5*(-a*x+1)^5/a+1/6*c^5*(-a*x+1)^6/a
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^5(1-ax)^6}{6a}-\frac{2c^5(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^5,x]
```

```
[Out] (-2*c^5*(1 - a*x)^5)/(5*a) + (c^5*(1 - a*x)^6)/(6*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^5\,dx&=c^5\int(1-ax)^4(1+ax)\,dx\\&=c^5\int\left(2(1-ax)^4-(1-ax)^5\right)dx\\&=-\frac{2c^5(1-ax)^5}{5a}+\frac{c^5(1-ax)^6}{6a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.62

$$\frac{c^5(ax-1)^5(5ax+7)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^5,x]
```

```
[Out] (c^5*(-1 + a*x)^5*(7 + 5*a*x))/(30*a)
```

fricas [A] time = 0.62, size = 59, normalized size = 1.59

$$\frac{1}{6}a^5c^5x^6 - \frac{3}{5}a^4c^5x^5 + \frac{1}{2}a^3c^5x^4 + \frac{2}{3}a^2c^5x^3 - \frac{3}{2}ac^5x^2 + c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*a^5*c^5*x^6 - 3/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 + 2/3*a^2*c^5*x^3 - 3/2
*a*c^5*x^2 + c^5*x
```

giac [A] time = 0.16, size = 59, normalized size = 1.59

$$\frac{1}{6}a^5c^5x^6 - \frac{3}{5}a^4c^5x^5 + \frac{1}{2}a^3c^5x^4 + \frac{2}{3}a^2c^5x^3 - \frac{3}{2}ac^5x^2 + c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/6*a^5*c^5*x^6 - 3/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 + 2/3*a^2*c^5*x^3 - 3/2
*a*c^5*x^2 + c^5*x
```

maple [A] time = 0.03, size = 45, normalized size = 1.22

$$c^5\left(\frac{1}{6}x^6a^5 - \frac{3}{5}a^4x^5 + \frac{1}{2}x^4a^3 + \frac{2}{3}x^3a^2 - \frac{3}{2}a\,x^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^5,x)
```

```
[Out] c^5*(1/6*x^6*a^5-3/5*a^4*x^5+1/2*x^4*a^3+2/3*x^3*a^2-3/2*a*x^2+x)
```

maxima [A] time = 0.35, size = 59, normalized size = 1.59

$$\frac{1}{6}a^5c^5x^6 - \frac{3}{5}a^4c^5x^5 + \frac{1}{2}a^3c^5x^4 + \frac{2}{3}a^2c^5x^3 - \frac{3}{2}ac^5x^2 + c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*a^5*c^5*x^6 - 3/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 + 2/3*a^2*c^5*x^3 - 3/2
*a*c^5*x^2 + c^5*x
```

mupad [B] time = 0.04, size = 59, normalized size = 1.59

$$\frac{a^5\,c^5\,x^6}{6}-\frac{3\,a^4\,c^5\,x^5}{5}+\frac{a^3\,c^5\,x^4}{2}+\frac{2\,a^2\,c^5\,x^3}{3}-\frac{3\,a\,c^5\,x^2}{2}+c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^5*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] c^5*x - (3*a*c^5*x^2)/2 + (2*a^2*c^5*x^3)/3 + (a^3*c^5*x^4)/2 - (3*a^4*c^5*
x^5)/5 + (a^5*c^5*x^6)/6
```

sympy [B] time = 0.09, size = 66, normalized size = 1.78

$$\frac{a^{5}c^{5}x^{6}}{6}-\frac{3a^{4}c^{5}x^{5}}{5}+\frac{a^{3}c^{5}x^{4}}{2}+\frac{2a^{2}c^{5}x^{3}}{3}-\frac{3ac^{5}x^{2}}{2}+c^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**5,x)
```

```
[Out] a**5*c**5*x**6/6 - 3*a**4*c**5*x**5/5 + a**3*c**5*x**4/2 + 2*a**2*c**5*x**3
/3 - 3*a*c**5*x**2/2 + c**5*x
```

3.169 $\int e^{2\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=37

$$\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}$$

```
[Out] -1/2*c^4*(-a*x+1)^4/a+1/5*c^4*(-a*x+1)^5/a
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] -(c^4*(1 - a*x)^4)/(2*a) + (c^4*(1 - a*x)^5)/(5*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^4\,dx &= c^4\int(1-ax)^3(1+ax)\,dx\\ &= c^4\int\left(2(1-ax)^3-(1-ax)^4\right)dx\\ &= -\frac{c^4(1-ax)^4}{2a}+\frac{c^4(1-ax)^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.86

$$c^4\left(-\frac{1}{5}a^4x^5+\frac{a^3x^4}{2}-ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^4,x]
[Out] c^4*(x - a*x^2 + (a^3*x^4)/2 - (a^4*x^5)/5)
```

fricas [A] time = 0.77, size = 37, normalized size = 1.00

$$-\frac{1}{5}\,a^4c^4x^5+\frac{1}{2}\,a^3c^4x^4-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^4,x, algorithm="fricas")
[Out] -1/5*a^4*c^4*x^5 + 1/2*a^3*c^4*x^4 - a*c^4*x^2 + c^4*x
```

giac [A] time = 0.16, size = 37, normalized size = 1.00

$$-\frac{1}{5}\,a^4c^4x^5+\frac{1}{2}\,a^3c^4x^4-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^4,x, algorithm="giac")
[Out] -1/5*a^4*c^4*x^5 + 1/2*a^3*c^4*x^4 - a*c^4*x^2 + c^4*x
```

maple [A] time = 0.02, size = 29, normalized size = 0.78

$$c^4\left(-\frac{1}{5}a^4x^5+\frac{1}{2}x^4a^3-a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^4,x)
[Out] c^4*(-1/5*a^4*x^5+1/2*x^4*a^3-a*x^2+x)
```

maxima [A] time = 0.34, size = 37, normalized size = 1.00

$$-\frac{1}{5}\,a^4c^4x^5+\frac{1}{2}\,a^3c^4x^4-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/5*a^4*c^4*x^5 + 1/2*a^3*c^4*x^4 - a*c^4*x^2 + c^4*x
```

mupad [B] time = 0.05, size = 37, normalized size = 1.00

$$-\frac{a^4\,c^4\,x^5}{5}+\frac{a^3\,c^4\,x^4}{2}-a\,c^4\,x^2+c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^4*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] c^4*x - a*c^4*x^2 + (a^3*c^4*x^4)/2 - (a^4*c^4*x^5)/5
```

sympy [A] time = 0.08, size = 36, normalized size = 0.97

$$-\frac{a^{4}c^{4}x^{5}}{5}+\frac{a^{3}c^{4}x^{4}}{2}-ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**4,x)
```

```
[Out] -a**4*c**4*x**5/5 + a**3*c**4*x**4/2 - a*c**4*x**2 + c**4*x
```

3.170 $\int e^{2\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=37

$$\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}$$

```
[Out] -2/3*c^3*(-a*x+1)^3/a+1/4*c^3*(-a*x+1)^4/a
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (-2*c^3*(1 - a*x)^3)/(3*a) + (c^3*(1 - a*x)^4)/(4*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^3\,dx &= c^3\int (1-ax)^2(1+ax)\,dx\\ &= c^3\int \left(2(1-ax)^2-(1-ax)^3\right)dx\\ &= -\frac{2c^3(1-ax)^3}{3a}+\frac{c^3(1-ax)^4}{4a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.81

$$\frac{1}{12}c^3x\left(3a^3x^3-4a^2x^2-6ax+12\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^3,x]
[Out] (c^3*x*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))/12
```

fricas [A] time = 0.64, size = 37, normalized size = 1.00

$$\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3-\frac{1}{2}\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^3,x, algorithm="fricas")
[Out] 1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 - 1/2*a*c^3*x^2 + c^3*x
```

giac [A] time = 0.17, size = 37, normalized size = 1.00

$$\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3-\frac{1}{2}\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^3,x, algorithm="giac")
[Out] 1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 - 1/2*a*c^3*x^2 + c^3*x
```

maple [A] time = 0.02, size = 29, normalized size = 0.78

$$c^3\left(\frac{1}{4}x^4a^3-\frac{1}{3}x^3a^2-\frac{1}{2}a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^3,x)
[Out] c^3*(1/4*x^4*a^3-1/3*x^3*a^2-1/2*a*x^2+x)
```

maxima [A] time = 0.32, size = 37, normalized size = 1.00

$$\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3-\frac{1}{2}\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 - 1/2*a*c^3*x^2 + c^3*x
```

mupad [B] time = 0.05, size = 37, normalized size = 1.00

$$\frac{a^3\,c^3\,x^4}{4}-\frac{a^2\,c^3\,x^3}{3}-\frac{a\,c^3\,x^2}{2}+c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] c^3*x - (a*c^3*x^2)/2 - (a^2*c^3*x^3)/3 + (a^3*c^3*x^4)/4
```

sympy [A] time = 0.08, size = 37, normalized size = 1.00

$$\frac{a^{3}c^{3}x^{4}}{4}-\frac{a^{2}c^{3}x^{3}}{3}-\frac{ac^{3}x^{2}}{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**3,x)
```

```
[Out] a**3*c**3*x**4/4 - a**2*c**3*x**3/3 - a*c**3*x**2/2 + c**3*x
```

3.171 $\int e^{2\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=19

$$c^2x-\frac{1}{3}a^2c^2x^3$$

```
[Out] c^2*x-1/3*a^2*c^2*x^3
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 41}

$$c^2x-\frac{1}{3}a^2c^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] c^2*x - (a^2*c^2*x^3)/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}(c-acx)^2\,dx &= c^2\int(1-ax)(1+ax)\,dx\\ &= c^2\int\left(1-a^2x^2\right)\,dx\\ &= c^2x-\frac{1}{3}a^2c^2x^3\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.84

$$c^2\left(x-\frac{a^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^2,x]
[Out] c^2*(x - (a^2*x^3)/3)
```

fricas [A] time = 0.62, size = 17, normalized size = 0.89

$$-\frac{1}{3}\,a^2c^2x^3+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/3*a^2*c^2*x^3 + c^2*x
```

giac [A] time = 0.33, size = 17, normalized size = 0.89

$$-\frac{1}{3}\,a^2c^2x^3+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^2,x, algorithm="giac")
[Out] -1/3*a^2*c^2*x^3 + c^2*x
```

maple [A] time = 0.02, size = 15, normalized size = 0.79

$$c^2\left(-\frac{1}{3}x^3a^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^2,x)
[Out] c^2*(-1/3*x^3*a^2+x)
```

maxima [A] time = 0.31, size = 17, normalized size = 0.89

$$-\frac{1}{3}\,a^2c^2x^3+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/3*a^2*c^2*x^3 + c^2*x
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{c^2\,x\,\left(a^2\,x^2-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] -(c^2*x*(a^2*x^2 - 3))/3
```

sympy [A] time = 0.07, size = 15, normalized size = 0.79

$$-\frac{a^{2}c^{2}x^{3}}{3}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**2,x)
[Out] -a**2*c**2*x**3/3 + c**2*x
```

3.172 $\int e^{2\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=13

$$\frac{1}{2}acx^2+cx$$

```
[Out] c*x+1/2*a*c*x^2
```

Rubi [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 2.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2288}

$$\frac{c\left(1-a^2x^2\right)e^{2\tanh^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] (c*E^(2*ArcTanh[a*x])*(1 - a^2*x^2))/(2*a)
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}(c-acx)\,dx=\frac{ce^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{2a}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 2.00

$$\frac{c\left(1-a^2x^2\right)e^{2\tanh^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] (c*E^(2*ArcTanh[a*x])*(1 - a^2*x^2))/(2*a)
```

fricas [A] time = 0.50, size = 11, normalized size = 0.85

$$\frac{1}{2}acx^2+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*a*c*x^2 + c*x
```

giac [A] time = 1.27, size = 11, normalized size = 0.85

$$\frac{1}{2}\,acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a*c*x^2 + c*x
```

maple [A] time = 0.02, size = 11, normalized size = 0.85

$$c\left(\frac{1}{2}a\,x^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c),x)
```

```
[Out] c*(1/2*a*x^2+x)
```

maxima [A] time = 0.32, size = 11, normalized size = 0.85

$$\frac{1}{2}\,acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*x^2 + c*x
```

mupad [B] time = 0.02, size = 9, normalized size = 0.69

$$\frac{c\,x\,\left(a\,x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c*x*(a*x + 2))/2
```

sympy [A] time = 0.06, size = 10, normalized size = 0.77

$$\frac{acx^2}{2} + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c),x)
```

```
[Out] a*c*x**2/2 + c*x
```

3.173 $\int \frac{e^{2\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=31

$$\frac{2}{ac(1-ax)}+\frac{\log(1-ax)}{ac}$$

[Out] 2/a/c/(-a*x+1)+ln(-a*x+1)/a/c

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{2}{ac(1-ax)}+\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x),x]

[Out] 2/(a*c*(1 - a*x)) + Log[1 - a*x]/(a*c)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{c-acx}\,dx = \frac{\int \frac{1+ax}{(1-ax)^2}\,dx}{c}$$
$$= \frac{\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{c}$$
$$= \frac{2}{ac(1-ax)}+\frac{\log(1-ax)}{ac}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.81

$$\frac{\frac{2}{1-ax}+\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x),x]
[Out] (2/(1 - a*x) + Log[1 - a*x])/(a*c)
```

fricas [A] time = 0.46, size = 28, normalized size = 0.90

$$\frac{(ax-1)\log(ax-1)-2}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="fricas")
[Out] ((a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

giac [A] time = 0.17, size = 30, normalized size = 0.97

$$\frac{\log(|ax-1|)}{ac}-\frac{2}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="giac")
[Out] log(abs(a*x - 1))/(a*c) - 2/((a*x - 1)*a*c)
```

maple [A] time = 0.03, size = 30, normalized size = 0.97

$$\frac{\ln(ax-1)}{ca}-\frac{2}{ca(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c),x)
```

```
[Out] 1/c/a*ln(a*x-1)-2/c/a/(a*x-1)
```

maxima [A] time = 0.34, size = 29, normalized size = 0.94

$$-\frac{2}{a^2cx-ac}+\frac{\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -2/(a^2*c*x - a*c) + log(a*x - 1)/(a*c)
```

mupad [B] time = 0.80, size = 28, normalized size = 0.90

$$\frac{2}{a\,\left(c-a\,c\,x\right)}+\frac{\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - a*c*x)),x)
```

```
[Out] 2/(a*(c - a*c*x)) + log(a*x - 1)/(a*c)
```

sympy [A] time = 0.14, size = 20, normalized size = 0.65

$$-\frac{2}{a^2cx-ac}+\frac{\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c),x)
```

```
[Out] -2/(a**2*c*x - a*c) + log(a*x - 1)/(a*c)
```

3.174 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=13

$$\frac{x}{c^2(1-ax)^2}$$

```
[Out] x/c^2/(-a*x+1)^2
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 34}

$$\frac{x}{c^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] x/(c^2*(1 - a*x)^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c^2}\\ &= \frac{x}{c^2(1-ax)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.92

$$\frac{(ax+1)^2}{4ac^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^2,x]
[Out] (1 + a*x)^2/(4*a*c^2*(1 - a*x)^2)
```

fricas [B] time = 0.49, size = 25, normalized size = 1.92

$$\frac{x}{a^2c^2x^2 - 2\,ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

giac [B] time = 0.24, size = 32, normalized size = 2.46

$$\frac{1}{(acx-c)^2a} + \frac{1}{(acx-c)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] 1/((a*c*x - c)^2*a) + 1/((a*c*x - c)*a*c)
```

maple [B] time = 0.03, size = 28, normalized size = 2.15

$$\frac{\frac{1}{a(ax-1)^2} + \frac{1}{a(ax-1)}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^2,x)
[Out] 1/c^2*(1/a/(a*x-1)^2+1/a/(a*x-1))
```

maxima [B] time = 0.34, size = 25, normalized size = 1.92

$$\frac{x}{a^2c^2x^2 - 2\,ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

mupad [B] time = 0.81, size = 12, normalized size = 0.92

$$\frac{x}{c^2\,{\left(a\,x-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - a*c*x)^2),x)
```

```
[Out] x/(c^2*(a*x - 1)^2)
```

sympy [B] time = 0.18, size = 22, normalized size = 1.69

$$\frac{x}{a^2c^2x^2 - 2ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**2,x)
```

```
[Out] x/(a**2*c**2*x**2 - 2*a*c**2*x + c**2)
```

3.175 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3ac^3(1-ax)^3}-\frac{1}{2ac^3(1-ax)^2}$$

```
[Out] 2/3/a/c^3/(-a*x+1)^3-1/2/a/c^3/(-a*x+1)^2
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{2}{3ac^3(1-ax)^3}-\frac{1}{2ac^3(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] 2/(3*a*c^3*(1 - a*x)^3) - 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\int \frac{1+ax}{(1-ax)^4}\,dx}{c^3}$$
$$= \frac{\int \left(\frac{2}{(-1+ax)^4}+\frac{1}{(-1+ax)^3}\right)dx}{c^3}$$
$$= \frac{2}{3ac^3(1-ax)^3} - \frac{1}{2ac^3(1-ax)^2}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.62

$$-\frac{3ax+1}{6ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^3,x]
[Out] -1/6*(1 + 3*a*x)/(a*c^3*(-1 + a*x)^3)
```

fricas [A] time = 0.59, size = 47, normalized size = 1.27

$$-\frac{3\,ax+1}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.15, size = 21, normalized size = 0.57

$$-\frac{3\,ax+1}{6\,(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="giac")
[Out] -1/6*(3*a*x + 1)/((a*x - 1)^3*a*c^3)
```

maple [A] time = 0.03, size = 30, normalized size = 0.81

$$\frac{-\frac{1}{2a(ax-1)^2}-\frac{2}{3a(ax-1)^3}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3*(-1/2/a/(a*x-1)^2-2/3/a/(a*x-1)^3)
```

maxima [A] time = 0.34, size = 47, normalized size = 1.27

$$-\frac{3\,ax+1}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

mupad [B] time = 0.08, size = 21, normalized size = 0.57

$$-\frac{3\,a\,x+1}{6\,a\,c^3\,{\left(a\,x-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - a*c*x)^3),x)
```

```
[Out] -(3*a*x + 1)/(6*a*c^3*(a*x - 1)^3)
```

sympy [A] time = 0.23, size = 48, normalized size = 1.30

$$\frac{-3ax-1}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**3,x)
```

```
[Out] (-3*a*x - 1)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a**2*c**3*x - 6*a*c
**3)
```

3.176 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2ac^4(1-ax)^4}-\frac{1}{3ac^4(1-ax)^3}$$

```
[Out] 1/2/a/c^4/(-a*x+1)^4-1/3/a/c^4/(-a*x+1)^3
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{1}{2ac^4(1-ax)^4}-\frac{1}{3ac^4(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] 1/(2*a*c^4*(1 - a*x)^4) - 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{1+ax}{(1-ax)^5}\,dx}{c^4}$$
$$= \frac{\int \left(-\frac{2}{(-1+ax)^5}-\frac{1}{(-1+ax)^4}\right)dx}{c^4}$$
$$= \frac{1}{2ac^4(1-ax)^4}-\frac{1}{3ac^4(1-ax)^3}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.62

$$\frac{2ax+1}{6ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] (1 + 2*a*x)/(6*a*c^4*(-1 + a*x)^4)
```

fricas [A] time = 0.46, size = 57, normalized size = 1.54

$$\frac{2\,ax+1}{6\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
+ a*c^4)
```

giac [A] time = 0.18, size = 21, normalized size = 0.57

$$\frac{2\,ax+1}{6\,(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*a*x + 1)/((a*x - 1)^4*a*c^4)
```

maple [A] time = 0.03, size = 30, normalized size = 0.81

$$\frac{\frac{1}{2a(ax-1)^4}+\frac{1}{3a(ax-1)^3}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(1/2/a/(a*x-1)^4+1/3/a/(a*x-1)^3)
```

maxima [A] time = 0.33, size = 57, normalized size = 1.54

$$\frac{2\,ax+1}{6\,\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
+ a*c^4)
```

mupad [B] time = 0.10, size = 21, normalized size = 0.57

$$\frac{2\,a\,x+1}{6\,a\,c^4\,{\left(a\,x-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - a*c*x)^4),x)
```

```
[Out] (2*a*x + 1)/(6*a*c^4*(a*x - 1)^4)
```

sympy [B] time = 0.29, size = 61, normalized size = 1.65

$$-\frac{-2ax-1}{6a^5c^4x^4-24a^4c^4x^3+36a^3c^4x^2-24a^2c^4x+6ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**4,x)
```

```
[Out] -(-2*a*x - 1)/(6*a**5*c**4*x**4 - 24*a**4*c**4*x**3 + 36*a**3*c**4*x**2 - 2
4*a**2*c**4*x + 6*a*c**4)
```

3.177 $\int e^{3\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=65

$$\frac{4\sqrt{2}\,(c-acx)^{p+1}\,{}_2F_1\left(-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{ac(1-2p)(1-ax)^{3/2}}$$

```
[Out] 4*(-a*c*x+c)^(1+p)*hypergeom([-3/2, -1/2+p],[1/2+p],-1/2*a*x+1/2)*2^(1/2)/a
/c/(1-2*p)/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 23, 69}

$$\frac{4\sqrt{2}\,(c-acx)^{p+1}\,{}_2F_1\left(-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{ac(1-2p)(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*Sqrt[2]*(c - a*c*x)^(1 + p)*Hypergeometric2F1[-3/2, -1/2 + p, 1/2 + p, (
1 - a*x)/2])/(a*c*(1 - 2*p)*(1 - a*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}(c-acx)^p\,dx &= \int \frac{(1+ax)^{3/2}(c-acx)^p}{(1-ax)^{3/2}}\,dx\\ &= \frac{(c-acx)^{3/2}\int (1+ax)^{3/2}(c-acx)^{-\frac{3}{2}+p}\,dx}{(1-ax)^{3/2}}\\ &= \frac{4\sqrt{2}\,(c-acx)^{1+p}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2}+p;\frac{1}{2}+p;\frac{1}{2}(1-ax)\right)}{ac(1-2p)(1-ax)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.89

$$\frac{4\sqrt{2}\,(c-acx)^p\,{}_2F_1\left(-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}-\frac{ax}{2}\right)}{(a-2ap)\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*Sqrt[2]*(c - a*c*x)^p*Hypergeometric2F1[-3/2, -1/2 + p, 1/2 + p, 1/2 - (
a*x)/2])/((a - 2*a*p)*Sqrt[1 - a*x])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)(-acx+c)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a*c*x + c)^p/(a^2*x^2 - 2*a*x + 1),
 x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 (-acx+c)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3(-acx+c)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3*(-a*c*x + c)^p/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c-a\,c\,x)^p\,(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - a*c*x)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1))^p\,(ax+1)^3}{(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.178 $\int e^{3\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=83

$$\frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^4x\sqrt{1-a^2x^2}+\frac{3c^4\sin^{-1}(ax)}{8a}$$

```
[Out] 1/4*c^4*x*(-a^2*x^2+1)^(3/2)+1/5*c^4*(-a^2*x^2+1)^(5/2)/a+3/8*c^4*arcsin(a*
x)/a+3/8*c^4*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 641, 195, 216}

$$\frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^4x\sqrt{1-a^2x^2}+\frac{3c^4\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (3*c^4*x*Sqrt[1 - a^2*x^2])/8 + (c^4*x*(1 - a^2*x^2)^(3/2))/4 + (c^4*(1 - a
^2*x^2)^(5/2))/(5*a) + (3*c^4*ArcSin[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}(c-acx)^4\,dx &= c^3\int(c-acx)\left(1-a^2x^2\right)^{3/2}\,dx\\ &= \frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a}+c^4\int\left(1-a^2x^2\right)^{3/2}\,dx\\ &= \frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}\left(3c^4\right)\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{3}{8}c^4x\sqrt{1-a^2x^2}+\frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{8}\left(3c^4\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{3}{8}c^4x\sqrt{1-a^2x^2}+\frac{1}{4}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{c^4\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{3c^4\sin^{-1}(ax)}{8a}\end{aligned}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.90

$$\frac{c^4\left(\sqrt{1-a^2x^2}\left(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8\right)-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(Sqrt[1 - a^2*x^2]*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a^4*x^4)
- 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(40*a)
```

fricas [A] time = 0.60, size = 93, normalized size = 1.12

$$-\frac{30\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(8\,a^4c^4x^4-10\,a^3c^4x^3-16\,a^2c^4x^2+25\,ac^4x+8\,c^4\right)\sqrt{-a^2x^2+1}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

[Out] -1/40*(30*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (8*a^4*c^4*x^4 - 10*a^3*c^4*x^3 - 16*a^2*c^4*x^2 + 25*a*c^4*x + 8*c^4)*sqrt(-a^2*x^2 + 1))/a

giac [A] time = 0.36, size = 78, normalized size = 0.94

$$\frac{3\,c^4\arcsin\left(ax\right)\mathrm{sgn}(a)}{8\,|a|}+\frac{1}{40}\,\sqrt{-a^2x^2+1}\left(\frac{8\,c^4}{a}+\left(25\,c^4-2\left(8\,ac^4-\left(4\,a^3c^4x-5\,a^2c^4\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^4,x, algorithm="giac")

[Out] 3/8*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/40*sqrt(-a^2*x^2 + 1)*(8*c^4/a + (25*c^4 - 2*(8*a*c^4 - (4*a^3*c^4*x - 5*a^2*c^4)*x)*x)*x)

maple [B] time = 0.07, size = 183, normalized size = 2.20

$$-\frac{c^4a^5x^6}{5\sqrt{-a^2x^2+1}}+\frac{3c^4a^3x^4}{5\sqrt{-a^2x^2+1}}-\frac{3c^4a\,x^2}{5\sqrt{-a^2x^2+1}}+\frac{c^4}{5a\sqrt{-a^2x^2+1}}+\frac{c^4a^4x^5}{4\sqrt{-a^2x^2+1}}-\frac{7c^4a^2x^3}{8\sqrt{-a^2x^2+1}}+\frac{5c^4x}{8\sqrt{-a^2x^2+1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^4,x)

[Out] -1/5*c^4*a^5*x^6/(-a^2*x^2+1)^(1/2)+3/5*c^4*a^3*x^4/(-a^2*x^2+1)^(1/2)-3/5*c^4*a*x^2/(-a^2*x^2+1)^(1/2)+1/5*c^4/a/(-a^2*x^2+1)^(1/2)+1/4*c^4*a^4*x^5/(-a^2*x^2+1)^(1/2)-7/8*c^4*a^2*x^3/(-a^2*x^2+1)^(1/2)+5/8*c^4*x/(-a^2*x^2+1)^(1/2)+3/8*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))

maxima [B] time = 0.42, size = 164, normalized size = 1.98

$$-\frac{a^5c^4x^6}{5\,\sqrt{-a^2x^2+1}}+\frac{a^4c^4x^5}{4\,\sqrt{-a^2x^2+1}}+\frac{3\,a^3c^4x^4}{5\,\sqrt{-a^2x^2+1}}-\frac{7\,a^2c^4x^3}{8\,\sqrt{-a^2x^2+1}}-\frac{3\,ac^4x^2}{5\,\sqrt{-a^2x^2+1}}+\frac{5\,c^4x}{8\,\sqrt{-a^2x^2+1}}+\frac{3\,c^4\arcsin\left(ax\right.}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^4,x, algorithm="maxima")

[Out] -1/5*a^5*c^4*x^6/sqrt(-a^2*x^2 + 1) + 1/4*a^4*c^4*x^5/sqrt(-a^2*x^2 + 1) + 3/5*a^3*c^4*x^4/sqrt(-a^2*x^2 + 1) - 7/8*a^2*c^4*x^3/sqrt(-a^2*x^2 + 1) - 3/5*a*c^4*x^2/sqrt(-a^2*x^2 + 1) + 5/8*c^4*x/sqrt(-a^2*x^2 + 1) + 3/8*c^4*arcsin(a*x)/a + 1/5*c^4/(sqrt(-a^2*x^2 + 1)*a)

mupad [B] time = 0.03, size = 128, normalized size = 1.54

$$\frac{5\,c^4\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{3\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}+\frac{c^4\,\sqrt{1-a^2\,x^2}}{5\,a}-\frac{2\,a\,c^4\,x^2\,\sqrt{1-a^2\,x^2}}{5}-\frac{a^2\,c^4\,x^3\,\sqrt{1-a^2\,x^2}}{4}+\frac{a^3\,c^4\,x^4\,\sqrt{1-}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (5*c^4*x*(1 - a^2*x^2)^(1/2))/8 + (3*c^4*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(
1/2)) + (c^4*(1 - a^2*x^2)^(1/2))/(5*a) - (2*a*c^4*x^2*(1 - a^2*x^2)^(1/2))
/5 - (a^2*c^4*x^3*(1 - a^2*x^2)^(1/2))/4 + (a^3*c^4*x^4*(1 - a^2*x^2)^(1/2)
)/5
```

sympy [A] time = 22.99, size = 459, normalized size = 5.53

$$-a^5c^4\left(\begin{cases} -\frac{x^4\sqrt{-a^2x^2+1}}{5a^2} - \frac{4x^2\sqrt{-a^2x^2+1}}{15a^4} - \frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for } a \neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases}\right) + a^4c^4\left(\begin{cases} -\frac{ix^5}{4\sqrt{a^2x^2-1}} - \frac{ix^3}{8a^2\sqrt{a^2x^2-1}} + \frac{3ix}{8a^4\sqrt{a^2x^2-1}} - \frac{3i\,a \\ \frac{x^5}{4\sqrt{-a^2x^2+1}} + \frac{x^3}{8a^2\sqrt{-a^2x^2+1}} - \frac{3x}{8a^4\sqrt{-a^2x^2+1}} + \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**4,x)
```

```
[Out] -a**5*c**4*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a*
*2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6
/6, True)) + a**4*c**4*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/
(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*aco
sh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**
3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asi
n(a*x)/(8*a**5), True)) + 2*a**3*c**4*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)
/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - 2
*a**2*c**4*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a
**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(
-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) - a*c**4*Piecewise((x**2/2, E
q(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) + c**4*Piecewise((sqrt(a**
(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a
**2 < 0))
```

3.179 $\int e^{3\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=59

$$\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^3x\sqrt{1-a^2x^2}+\frac{3c^3\sin^{-1}(ax)}{8a}$$

```
[Out] 1/4*c^3*x*(-a^2*x^2+1)^(3/2)+3/8*c^3*arcsin(a*x)/a+3/8*c^3*x*(-a^2*x^2+1)^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 195, 216}

$$\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^3x\sqrt{1-a^2x^2}+\frac{3c^3\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (3*c^3*x*Sqrt[1 - a^2*x^2])/8 + (c^3*x*(1 - a^2*x^2)^(3/2))/4 + (3*c^3*ArcS
in[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}(c-acx)^3\,dx = c^3\int\left(1-a^2x^2\right)^{3/2}\,dx$$
$$=\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{4}\left(3c^3\right)\int\sqrt{1-a^2x^2}\,dx$$
$$=\frac{3}{8}c^3x\sqrt{1-a^2x^2}+\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{8}\left(3c^3\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$=\frac{3}{8}c^3x\sqrt{1-a^2x^2}+\frac{1}{4}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{3c^3\sin^{-1}(ax)}{8a}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.75

$$\frac{c^3\left(ax\sqrt{1-a^2x^2}\left(5-2a^2x^2\right)+3\sin^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^3,x]
[Out] (c^3*(a*x*(5 - 2*a^2*x^2)*Sqrt[1 - a^2*x^2] + 3*ArcSin[a*x]))/(8*a)
```

fricas [A] time = 0.51, size = 65, normalized size = 1.10

$$-\frac{6\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3c^3x^3-5\,ac^3x\right)\sqrt{-a^2x^2+1}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/8*(6*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3*c^3*x^3 - 5*a*c
^3*x)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.29, size = 48, normalized size = 0.81

$$\frac{3\,c^3\arcsin(ax)\,\mathrm{sgn}(a)}{8\,|a|}-\frac{1}{8}\left(2\,a^2c^3x^2-5\,c^3\right)\sqrt{-a^2x^2+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 3/8*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/8*(2*a^2*c^3*x^2 - 5*c^3)*sqrt(-a^2*x
^2 + 1)*x
```

maple [A] time = 0.04, size = 96, normalized size = 1.63

$$\frac{c^3a^4x^5}{4\sqrt{-a^2x^2+1}}-\frac{7c^3a^2x^3}{8\sqrt{-a^2x^2+1}}+\frac{5c^3x}{8\sqrt{-a^2x^2+1}}+\frac{3c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^3,x)
```

```
[Out] 1/4*c^3*a^4*x^5/(-a^2*x^2+1)^(1/2)-7/8*c^3*a^2*x^3/(-a^2*x^2+1)^(1/2)+5/8*c
^3*x/(-a^2*x^2+1)^(1/2)+3/8*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))
```

maxima [A] time = 0.41, size = 77, normalized size = 1.31

$$\frac{a^4c^3x^5}{4\sqrt{-a^2x^2+1}}-\frac{7\,a^2c^3x^3}{8\sqrt{-a^2x^2+1}}+\frac{5\,c^3x}{8\sqrt{-a^2x^2+1}}+\frac{3\,c^3\arcsin\left(ax\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*c^3*x^5/sqrt(-a^2*x^2 + 1) - 7/8*a^2*c^3*x^3/sqrt(-a^2*x^2 + 1) + 5
/8*c^3*x/sqrt(-a^2*x^2 + 1) + 3/8*c^3*arcsin(a*x)/a
```

mupad [B] time = 0.05, size = 64, normalized size = 1.08

$$\frac{5\,c^3\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{3\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}-\frac{a^2\,c^3\,x^3\,\sqrt{1-a^2\,x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (5*c^3*x*(1 - a^2*x^2)^(1/2))/8 + (3*c^3*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(
1/2)) - (a^2*c^3*x^3*(1 - a^2*x^2)^(1/2))/4
```

sympy [A] time = 8.54, size = 301, normalized size = 5.10

$$a^4c^3\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\,\mathrm{acosh}\left(ax\right)}{8a^5} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\,\mathrm{asin}\left(ax\right)}{8a^5} & \text{otherwise}\end{cases}\right)-2a^2c^3\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\,\mathrm{acosh}\left(ax\right)}{2a^3}\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**3,x)
```

```
[Out] a**4*c**3*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(
a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a*
*5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqr
t(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**
5), True)) - 2*a**2*c**3*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*a
cosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x
/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + c**3*Piecewis
e((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt
(-a**2)), a**2 < 0))
```

3.180 $\int e^{3\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=61

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

```
[Out] -1/3*c^2*(-a^2*x^2+1)^(3/2)/a+1/2*c^2*arcsin(a*x)/a+1/2*c^2*x*(-a^2*x^2+1)^
(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 665, 195, 216}

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/2 - (c^2*(1 - a^2*x^2)^(3/2))/(3*a) + (c^2*ArcSin
[a*x])/(2*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}(c-acx)^2\,dx &= c^3\int\frac{\left(1-a^2x^2\right)^{3/2}}{c-acx}\,dx\\ &= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+c^2\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}c^2x\sqrt{1-a^2x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{1}{2}c^2x\sqrt{1-a^2x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{c^2\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 0.97

$$\frac{c^2\left(\sqrt{1-a^2x^2}\left(2a^2x^2+3ax-2\right)-6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(Sqrt[1 - a^2*x^2]*(-2 + 3*a*x + 2*a^2*x^2) - 6*ArcSin[Sqrt[1 - a*x]/S
qrt[2]]))/(6*a)
```

fricas [A] time = 0.42, size = 71, normalized size = 1.16

$$-\frac{6\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^2c^2x^2+3\,ac^2x-2\,c^2\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^2*c^2*x^2 + 3*a*c
^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 1.18, size = 54, normalized size = 0.89

$$\frac{c^2 \arcsin(ax)\operatorname{sgn}(a)}{2|a|} + \frac{1}{6}\sqrt{-a^2x^2+1}\left(\left(2\,ac^2x + 3\,c^2\right)x - \frac{2\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x + 3*
c^2)*x - 2*c^2/a)
```

maple [B] time = 0.05, size = 137, normalized size = 2.25

$$-\frac{c^2a^3x^4}{3\sqrt{-a^2x^2+1}}+\frac{2c^2a\,x^2}{3\sqrt{-a^2x^2+1}}-\frac{c^2}{3a\sqrt{-a^2x^2+1}}-\frac{c^2a^2x^3}{2\sqrt{-a^2x^2+1}}+\frac{c^2x}{2\sqrt{-a^2x^2+1}}+\frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^2,x)
```

```
[Out] -1/3*c^2*a^3*x^4/(-a^2*x^2+1)^(1/2)+2/3*c^2*a*x^2/(-a^2*x^2+1)^(1/2)-1/3*c^
2/a/(-a^2*x^2+1)^(1/2)-1/2*c^2*a^2*x^3/(-a^2*x^2+1)^(1/2)+1/2*c^2*x/(-a^2*x
^2+1)^(1/2)+1/2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [B] time = 0.42, size = 118, normalized size = 1.93

$$-\frac{a^3c^2x^4}{3\sqrt{-a^2x^2+1}}-\frac{a^2c^2x^3}{2\sqrt{-a^2x^2+1}}+\frac{2\,ac^2x^2}{3\sqrt{-a^2x^2+1}}+\frac{c^2x}{2\sqrt{-a^2x^2+1}}+\frac{c^2\arcsin(ax)}{2\,a}-\frac{c^2}{3\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a^3*c^2*x^4/sqrt(-a^2*x^2 + 1) - 1/2*a^2*c^2*x^3/sqrt(-a^2*x^2 + 1) +
2/3*a*c^2*x^2/sqrt(-a^2*x^2 + 1) + 1/2*c^2*x/sqrt(-a^2*x^2 + 1) + 1/2*c^2*a
rcsin(a*x)/a - 1/3*c^2/(sqrt(-a^2*x^2 + 1)*a)
```

mupad [B] time = 0.04, size = 82, normalized size = 1.34

$$\frac{c^2\,x\,\sqrt{1-a^2\,x^2}}{2}+\frac{c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{3\,a}+\frac{a\,c^2\,x^2\,\sqrt{1-a^2\,x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c^2*x*(1 - a^2*x^2)^(1/2))/2 + (c^2*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2)
) - (c^2*(1 - a^2*x^2)^(1/2))/(3*a) + (a*c^2*x^2*(1 - a^2*x^2)^(1/2))/3
```

sympy [A] time = 14.79, size = 221, normalized size = 3.62

$$-a^3c^2\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise}\end{cases}\right)+ac^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**2,x)
```

```
[Out] -a**3*c**2*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x*
*2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - a**2*c**2*Piecewise((-I*x*sq
rt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x
**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/
(2*a**3), True)) + a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**
2 + 1)/a**2, True)) + c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**
2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0))
```

3.181 $\int e^{3\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=65

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{3c\sin^{-1}(ax)}{2a}$$

```
[Out] -1/2*c*(-a^2*x^2+1)^(3/2)/a/(-a*x+1)+3/2*c*arcsin(a*x)/a-3/2*c*(-a^2*x^2+1)
^(1/2)/a
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 665, 216}

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{3c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] (-3*c*Sqrt[1 - a^2*x^2])/(2*a) - (c*(1 - a^2*x^2)^(3/2))/(2*a*(1 - a*x)) +
(3*c*ArcSin[a*x])/(2*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}(c-acx)\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^2}\,dx$$
$$= -\frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}+\frac{1}{2}\left(3c^2\right)\int \frac{\sqrt{1-a^2x^2}}{c-acx}\,dx$$
$$= -\frac{3c\sqrt{1-a^2x^2}}{2a}-\frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}+\frac{1}{2}(3c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{3c\sqrt{1-a^2x^2}}{2a}-\frac{c\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}+\frac{3c\sin^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.74

$$-\frac{c\left(\sqrt{1-a^2x^2}\,(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x),x]
[Out] -1/2*(c*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/a
```

fricas [A] time = 0.61, size = 52, normalized size = 0.80

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,(acx+4\,c)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c),x, algorithm="fricas")
[Out] -1/2*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*(a*c*
x + 4*c))/a
```

giac [A] time = 0.19, size = 38, normalized size = 0.58

$$\frac{3\,c\arcsin(ax)\,\mathrm{sgn}(a)}{2\,|a|}-\frac{1}{2}\,\sqrt{-a^2x^2+1}\left(cx+\frac{4\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 3/2*c*arcsin(a*x)*sgn(a)/abs(a) - 1/2*sqrt(-a^2*x^2 + 1)*(c*x + 4*c/a)
```

maple [A] time = 0.04, size = 104, normalized size = 1.60

$$\frac{c\,a^2x^3}{2\sqrt{-a^2x^2+1}}-\frac{cx}{2\sqrt{-a^2x^2+1}}+\frac{3c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}+\frac{2ca\,x^2}{\sqrt{-a^2x^2+1}}-\frac{2c}{a\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c),x)
```

```
[Out] 1/2*c*a^2*x^3/(-a^2*x^2+1)^(1/2)-1/2*c*x/(-a^2*x^2+1)^(1/2)+3/2*c/(a^2)^(1/
2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+2*c*a*x^2/(-a^2*x^2+1)^(1/2)-2*
c/a/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.42, size = 85, normalized size = 1.31

$$\frac{a^2cx^3}{2\sqrt{-a^2x^2+1}}+\frac{2\,acx^2}{\sqrt{-a^2x^2+1}}-\frac{cx}{2\sqrt{-a^2x^2+1}}+\frac{3\,c\arcsin\left(ax\right)}{2\,a}-\frac{2\,c}{\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c*x^3/sqrt(-a^2*x^2 + 1) + 2*a*c*x^2/sqrt(-a^2*x^2 + 1) - 1/2*c*x/s
qrt(-a^2*x^2 + 1) + 3/2*c*arcsin(a*x)/a - 2*c/(sqrt(-a^2*x^2 + 1)*a)
```

mupad [B] time = 0.03, size = 58, normalized size = 0.89

$$\frac{\frac{3\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2}+\sqrt{1-a^2\,x^2}\,\left(\frac{2\,a\,c}{\sqrt{-a^2}}-\frac{c\,x\,\sqrt{-a^2}}{2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] ((3*c*asinh(x*(-a^2)^(1/2)))/2 + (1 - a^2*x^2)^(1/2)*((2*a*c)/(-a^2)^(1/2)
- (c*x*(-a^2)^(1/2))/2))/(-a^2)^(1/2)
```

sympy [A] time = 10.29, size = 165, normalized size = 2.54

$$a^2c\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\,\mathrm{acosh}\left(ax\right)}{2a^3} & \text{for}\ \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\mathrm{asin}\left(ax\right)}{2a^3} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{asin}\left(x\sqrt{a\right. \\ \sqrt{-\frac{1}{a^2}}\ \mathrm{asinh}\left(x\right.\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c),x)
```

```
[Out] a**2*c*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3)
, Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**
2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + 2*a*c*Piecewise((x**2/2, Eq(a**
2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) + c*Piecewise((sqrt(a**(-2))*as
in(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)
)
```

3.182 $\int \frac{e^{3\tanh^{-1}(ax)}}{c-acx} dx$

Optimal. Leaf size=74

$$\frac{2(1-a^2x^2)^{3/2}}{3ac(1-ax)^3} - \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} + \frac{\sin^{-1}(ax)}{ac}$$

```
[Out] 2/3*(-a^2*x^2+1)^(3/2)/a/c/(-a*x+1)^3+arcsin(a*x)/a/c-2*(-a^2*x^2+1)^(1/2)/
a/c/(-a*x+1)
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 663, 216}

$$\frac{2(1-a^2x^2)^{3/2}}{3ac(1-ax)^3} - \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} + \frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)) + (2*(1 - a^2*x^2)^(3/2))/(3*a*c*(1
 - a*x)^3) + ArcSin[a*x]/(a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{c-acx}\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^4}\,dx$$
$$= \frac{2\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^3} - c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx$$
$$= -\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} + \frac{2\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^3} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= -\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} + \frac{2\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^3} + \frac{\sin^{-1}(ax)}{ac}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.61

$$\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - a*x)/2])/(3*a*c*(1 - a*
x)^(3/2))
```

fricas [A] time = 0.44, size = 94, normalized size = 1.27

$$-\frac{2\left(2\,a^2x^2-4\,ax+3\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-2\,\sqrt{-a^2x^2+1}\,(2\,ax-1)+2\right)}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -2/3*(2*a^2*x^2 - 4*a*x + 3*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1
) - 1)/(a*x)) - 2*sqrt(-a^2*x^2 + 1)*(2*a*x - 1) + 2)/(a^3*c*x^2 - 2*a^2*c*
x + a*c)
```

giac [A] time = 0.25, size = 79, normalized size = 1.07

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}+\frac{8\left(\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-1\right)}{3\,c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(c*abs(a)) + 8/3*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2
*x) - 1)/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a))
```

maple [B] time = 0.04, size = 146, normalized size = 1.97

$$-\frac{8x}{c\sqrt{-a^2x^2+1}}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c\sqrt{a^2}}-\frac{4}{ca\sqrt{-a^2x^2+1}}-\frac{8}{3c\,a^2\left(x-\frac{1}{a}\right)\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}+\frac{1}{3c\sqrt{-a^2\left(x-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x)
```

```
[Out] -8/c*x/(-a^2*x^2+1)^(1/2)+1/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-4/c/a/(-a^2*x^2+1)^(1/2)-8/3/c/a^2/(x-1/a)/(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(1/2)+16/3/c/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x
```

maxima [A] time = 0.43, size = 89, normalized size = 1.20

$$-\frac{8\,x}{3\,\sqrt{-a^2x^2+1}\,c}-\frac{8}{3\left(\sqrt{-a^2x^2+1}\,a^2cx-\sqrt{-a^2x^2+1}\,ac\right)}+\frac{\arcsin(ax)}{ac}-\frac{4}{\sqrt{-a^2x^2+1}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -8/3*x/(sqrt(-a^2*x^2 + 1)*c) - 8/3/(sqrt(-a^2*x^2 + 1)*a^2*c*x - sqrt(-a^2
*x^2 + 1)*a*c) + arcsin(a*x)/(a*c) - 4/(sqrt(-a^2*x^2 + 1)*a*c)
```

mupad [B] time = 0.07, size = 114, normalized size = 1.54

$$-\frac{4\,a\,\sqrt{1-a^2\,x^2}+3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}-8\,a^2\,x\,\sqrt{1-a^2\,x^2}+3\,a^2\,x^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}-6\,a\,x\,\mathrm{asinh}\left(x\right.}{3\,a^2\,c\,(a\,x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)),x)
```

```
[Out] -(4*a*(1 - a^2*x^2)^(1/2) + 3*asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2) - 8*a^2*x*
(1 - a^2*x^2)^(1/2) + 3*a^2*x^2*asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2) - 6*a*x*
asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2))/(3*a^2*c*(a*x - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{3ax}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^2}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(3*a*x/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x*
*2 + 1) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3
*a**2*x**2/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1
) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x*
*3/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*
sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-a**3*x**3*s
qrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sqrt(-a**2*x**2
+ 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.183 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=32

$$\frac{\left(1-a^2x^2\right)^{5/2}}{5ac^2(1-ax)^5}$$

```
[Out] 1/5*(-a^2*x^2+1)^(5/2)/a/c^2/(-a*x+1)^5
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 651}

$$\frac{\left(1-a^2x^2\right)^{5/2}}{5ac^2(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(5*a*c^2*(1 - a*x)^5)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^5}\,dx\\ &= \frac{\left(1-a^2x^2\right)^{5/2}}{5ac^2(1-ax)^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.91

$$\frac{(ax+1)^{5/2}}{5ac^2(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^(5/2)/(5*a*c^2*(1 - a*x)^(5/2))
```

fricas [B] time = 0.53, size = 89, normalized size = 2.78

$$\frac{a^3x^3 - 3\,a^2x^2 + 3\,ax - \left(a^2x^2 + 2\,ax + 1\right)\sqrt{-a^2x^2 + 1} - 1}{5\left(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - (a^2*x^2 + 2*a*x + 1)*sqrt(-a^2*x^2 + 1)
 - 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [C] time = 0.31, size = 83, normalized size = 2.59

$$-\frac{-i\,\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c) + \frac{\left(\frac{2\,c}{acx-c}+1\right)^2\sqrt{-\frac{2\,c}{acx-c}-1}}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}}{5\,c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/5*(-I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c) + (2*c/(a*c*x - c) + 1)^2*sqrt(-2
*c/(a*c*x - c) - 1)/(sgn(1/(a*c*x - c))*sgn(a)*sgn(c)))/(c^2*abs(a))
```

maple [A] time = 0.03, size = 35, normalized size = 1.09

$$-\frac{(ax+1)^4}{5\,(ax-1)\,c^2\left(-a^2x^2+1\right)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x)
```

[Out] -1/5*(a*x+1)^4/(a*x-1)/c^2/(-a^2*x^2+1)^(3/2)/a

maxima [B] time = 0.34, size = 146, normalized size = 4.56

$$\frac{8}{5\left(\sqrt{-a^2x^2+1}\,a^3c^2x^2-2\sqrt{-a^2x^2+1}\,a^2c^2x+\sqrt{-a^2x^2+1}\,ac^2\right)}+\frac{12}{5\left(\sqrt{-a^2x^2+1}\,a^2c^2x-\sqrt{-a^2x^2+1}\,ac^2\right)}+\frac{}{5\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 8/5/(sqrt(-a^2*x^2 + 1)*a^3*c^2*x^2 - 2*sqrt(-a^2*x^2 + 1)*a^2*c^2*x + sqrt
(-a^2*x^2 + 1)*a*c^2) + 12/5/(sqrt(-a^2*x^2 + 1)*a^2*c^2*x - sqrt(-a^2*x^2
+ 1)*a*c^2) + 1/5*x/(sqrt(-a^2*x^2 + 1)*c^2) + 1/(sqrt(-a^2*x^2 + 1)*a*c^2)
```

mupad [B] time = 0.08, size = 34, normalized size = 1.06

$$-\frac{\sqrt{1-a^2\,x^2}\,{\left(a\,x+1\right)}^2}{5\,a\,c^2\,{\left(a\,x-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^2),x)
```

```
[Out] -((1 - a^2*x^2)^(1/2)*(a*x + 1)^2)/(5*a*c^2*(a*x - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{3ax}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^2}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(3*a*x/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x
**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integra
l(3*a**2*x**2/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**
2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(
a**3*x**3/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2 +
1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-
a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a*x*s
qrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.184 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=65

$$\frac{\left(1-a^2x^2\right)^{5/2}}{35ac^3(1-ax)^5}+\frac{\left(1-a^2x^2\right)^{5/2}}{7ac^3(1-ax)^6}$$

```
[Out] 1/7*(-a^2*x^2+1)^(5/2)/a/c^3/(-a*x+1)^6+1/35*(-a^2*x^2+1)^(5/2)/a/c^3/(-a*x
+1)^5
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{\left(1-a^2x^2\right)^{5/2}}{35ac^3(1-ax)^5}+\frac{\left(1-a^2x^2\right)^{5/2}}{7ac^3(1-ax)^6}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(7*a*c^3*(1 - a*x)^6) + (1 - a^2*x^2)^(5/2)/(35*a*c^3*(
1 - a*x)^5)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^3}\,dx &= c^3 \int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^6}\,dx \\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{7ac^3(1-ax)^6} + \frac{1}{7}c^2 \int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^5}\,dx \\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{7ac^3(1-ax)^6} + \frac{\left(1-a^2x^2\right)^{5/2}}{35ac^3(1-ax)^5}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.52

$$-\frac{(ax-6)(ax+1)^{5/2}}{35ac^3(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -1/35*((-6 + a*x)*(1 + a*x)^(5/2))/(a*c^3*(1 - a*x)^(7/2))
```

fricas [B] time = 0.77, size = 116, normalized size = 1.78

$$\frac{6\,a^4x^4 - 24\,a^3x^3 + 36\,a^2x^2 - 24\,ax - \left(a^3x^3 - 4\,a^2x^2 - 11\,ax - 6\right)\sqrt{-a^2x^2+1} + 6}{35\left(a^5c^3x^4 - 4\,a^4c^3x^3 + 6\,a^3c^3x^2 - 4\,a^2c^3x + ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(6*a^4*x^4 - 24*a^3*x^3 + 36*a^2*x^2 - 24*a*x - (a^3*x^3 - 4*a^2*x^2 -
 11*a*x - 6)*sqrt(-a^2*x^2 + 1) + 6)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c
^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [B] time = 0.21, size = 199, normalized size = 3.06

$$-\frac{2\left(\frac{7\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x} - \frac{91\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2} + \frac{70\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3} - \frac{140\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4} + \frac{35\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5}{a^{10}x^5} - \frac{35\left(\sqrt{-a^2x\right.}}{a^{1}}\right.}{35\,c^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/35*(7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 91*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2/(a^4*x^2) + 70*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 1
40*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 35*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^5/(a^10*x^5) - 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^6) - 6
)/(c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

maple [A] time = 0.03, size = 40, normalized size = 0.62

$$-\frac{(ax-6)(ax+1)^4}{35(ax-1)^2 c^3 \left(-a^2x^2+1\right)^{\frac{3}{2}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/35*(a*x-6)*(a*x+1)^4/(a*x-1)^2/c^3/(-a^2*x^2+1)^(3/2)/a
```

maxima [B] time = 0.36, size = 216, normalized size = 3.32

$$-\frac{8}{7\left(\sqrt{-a^2x^2+1}\,a^4c^3x^3-3\sqrt{-a^2x^2+1}\,a^3c^3x^2+3\sqrt{-a^2x^2+1}\,a^2c^3x-\sqrt{-a^2x^2+1}\,ac^3\right)}-\frac{}{35\left(\sqrt{-a^2x^2+1}\,a^3c^3x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -8/7/(sqrt(-a^2*x^2 + 1)*a^4*c^3*x^3 - 3*sqrt(-a^2*x^2 + 1)*a^3*c^3*x^2 + 3
*sqrt(-a^2*x^2 + 1)*a^2*c^3*x - sqrt(-a^2*x^2 + 1)*a*c^3) - 52/35/(sqrt(-a^
2*x^2 + 1)*a^3*c^3*x^2 - 2*sqrt(-a^2*x^2 + 1)*a^2*c^3*x + sqrt(-a^2*x^2 + 1
)*a*c^3) - 18/35/(sqrt(-a^2*x^2 + 1)*a^2*c^3*x - sqrt(-a^2*x^2 + 1)*a*c^3)
+ 1/35*x/(sqrt(-a^2*x^2 + 1)*c^3)
```

mupad [B] time = 0.81, size = 299, normalized size = 4.60

$$\frac{8\,a^3\,\sqrt{1-a^2\,x^2}}{35\,\left(a^6\,c^3\,x^2-2\,a^5\,c^3\,x+a^4\,c^3\right)}-\frac{a\,\sqrt{1-a^2\,x^2}}{5\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{4\,a\,\sqrt{1-a^2\,x^2}}{7\,\left(a^6\,c^3\,x^4-4\,a^5\,c^3\,x^3+6\,a^4\,c^3\,x^2-4\,a^3\,c^3\,x+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^3),x)
```

```
[Out] (8*a^3*(1 - a^2*x^2)^(1/2))/(35*(a^4*c^3 - 2*a^5*c^3*x + a^6*c^3*x^2)) - (a
*(1 - a^2*x^2)^(1/2))/(5*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (4*a*(1 -
 a^2*x^2)^(1/2))/(7*(a^2*c^3 - 4*a^3*c^3*x + 6*a^4*c^3*x^2 - 4*a^5*c^3*x^3
+ a^6*c^3*x^4)) + (1 - a^2*x^2)^(1/2)/(35*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2)
- (c^3*(-a^2)^(1/2))/a)) - (16*(1 - a^2*x^2)^(1/2))/(35*(-a^2)^(1/2)*(3*c^3
*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3
*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{3ax}{-a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(3*a*x/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2
+ 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*
a**2*x**2/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 +
1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) +
3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**3
/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a*
*3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*x*sqr
t(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-a**5*x**5*sqrt
(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**3*x**3*sqrt(-a**
2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1)
 - sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.185 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$\frac{2\left(1-a^2x^2\right)^{5/2}}{315ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{5/2}}{63ac^4(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7}$$

```
[Out] 1/9*(-a^2*x^2+1)^(5/2)/a/c^4/(-a*x+1)^7+2/63*(-a^2*x^2+1)^(5/2)/a/c^4/(-a*x
+1)^6+2/315*(-a^2*x^2+1)^(5/2)/a/c^4/(-a*x+1)^5
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{2\left(1-a^2x^2\right)^{5/2}}{315ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{5/2}}{63ac^4(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(9*a*c^4*(1 - a*x)^7) + (2*(1 - a^2*x^2)^(5/2))/(63*a*c
^4*(1 - a*x)^6) + (2*(1 - a^2*x^2)^(5/2))/(315*a*c^4*(1 - a*x)^5)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^4}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^7}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7}+\frac{1}{9}\left(2c^2\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^6}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7}+\frac{2\left(1-a^2x^2\right)^{5/2}}{63ac^4(1-ax)^6}+\frac{1}{63}(2c)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^5}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{9ac^4(1-ax)^7}+\frac{2\left(1-a^2x^2\right)^{5/2}}{63ac^4(1-ax)^6}+\frac{2\left(1-a^2x^2\right)^{5/2}}{315ac^4(1-ax)^5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.44

$$\frac{(ax+1)^{5/2}\left(2a^2x^2-14ax+47\right)}{315ac^4(1-ax)^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] ((1 + a*x)^(5/2)*(47 - 14*a*x + 2*a^2*x^2))/(315*a*c^4*(1 - a*x)^(9/2))
```

fricas [A] time = 0.65, size = 145, normalized size = 1.49

$$\frac{47\,a^5x^5-235\,a^4x^4+470\,a^3x^3-470\,a^2x^2+235\,ax-\left(2\,a^4x^4-10\,a^3x^3+21\,a^2x^2+80\,ax+47\right)\sqrt{-a^2x^2+1}-47}{315\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(47*a^5*x^5 - 235*a^4*x^4 + 470*a^3*x^3 - 470*a^2*x^2 + 235*a*x - (2*
a^4*x^4 - 10*a^3*x^3 + 21*a^2*x^2 + 80*a*x + 47)*sqrt(-a^2*x^2 + 1) - 47)/(
a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x
 - a*c^4)
```

giac [B] time = 0.23, size = 253, normalized size = 2.61

$$-\frac{2\left(\frac{108\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{1062\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{1638\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{3402\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}+\frac{2520\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5}{a^{10}x^5}-\frac{23\cdots}{\cdots}\right.}{315\,c^4\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^9|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -2/315*(108*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1062*(sqrt(-a^2*x^2 +
 1)*abs(a) + a)^2/(a^4*x^2) + 1638*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x
^3) - 3402*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 2520*(sqrt(-a^2*x^
2 + 1)*abs(a) + a)^5/(a^10*x^5) - 2310*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a
^12*x^6) + 630*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/(a^14*x^7) - 315*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^8/(a^16*x^8) - 47)/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a)
+ a)/(a^2*x) - 1)^9*abs(a))
```

maple [A] time = 0.03, size = 49, normalized size = 0.51

$$-\frac{\left(2a^2x^2-14ax+47\right)(ax+1)^4}{315\,(ax-1)^3\,c^4\left(-a^2x^2+1\right)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/315*(2*a^2*x^2-14*a*x+47)*(a*x+1)^4/(a*x-1)^3/c^4/(-a^2*x^2+1)^(3/2)/a
```

maxima [B] time = 0.35, size = 327, normalized size = 3.37

$$\frac{8}{9\left(\sqrt{-a^2x^2+1}\,a^5c^4x^4-4\sqrt{-a^2x^2+1}\,a^4c^4x^3+6\sqrt{-a^2x^2+1}\,a^3c^4x^2-4\sqrt{-a^2x^2+1}\,a^2c^4x+\sqrt{-a^2x^2+1}\,ac^4\right)}+\frac{\cdots}{63\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 8/9/(sqrt(-a^2*x^2 + 1)*a^5*c^4*x^4 - 4*sqrt(-a^2*x^2 + 1)*a^4*c^4*x^3 + 6*
sqrt(-a^2*x^2 + 1)*a^3*c^4*x^2 - 4*sqrt(-a^2*x^2 + 1)*a^2*c^4*x + sqrt(-a^2
*x^2 + 1)*a*c^4) + 68/63/(sqrt(-a^2*x^2 + 1)*a^4*c^4*x^3 - 3*sqrt(-a^2*x^2
+ 1)*a^3*c^4*x^2 + 3*sqrt(-a^2*x^2 + 1)*a^2*c^4*x - sqrt(-a^2*x^2 + 1)*a*c^
```

```
4) + 106/315/(sqrt(-a^2*x^2 + 1)*a^3*c^4*x^2 - 2*sqrt(-a^2*x^2 + 1)*a^2*c^4
*x + sqrt(-a^2*x^2 + 1)*a*c^4) - 1/315/(sqrt(-a^2*x^2 + 1)*a^2*c^4*x - sqrt
(-a^2*x^2 + 1)*a*c^4) + 2/315*x/(sqrt(-a^2*x^2 + 1)*c^4)
```

mupad [B] time = 0.85, size = 492, normalized size = 5.07

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{12\,a^4}{35\,c^4\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^3}-\frac{8\,a^4}{35\,c^4\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}+\frac{4\,a^5}{7\,c^4\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^4\,\sqrt{-a^2}}+\frac{8\,a^7}{35\,c^4\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^2\,{\left(-a^2\right)}^{3/2}}\right)}{a^4\,\sqrt{-a^2}}+\frac{\sqrt{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^4),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((12*a^4)/(35*c^4*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^3)
 - (8*a^4)/(35*c^4*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)) + (4*a^5)/(7*c^4*(x*(
-a^2)^(1/2) - (-a^2)^(1/2)/a)^4*(-a^2)^(1/2)) + (8*a^7)/(35*c^4*(x*(-a^2)^(
1/2) - (-a^2)^(1/2)/a)^2*(-a^2)^(3/2))))/(a^4*(-a^2)^(1/2)) + ((1 - a^2*x^2
)^(1/2)*((32*a^5)/(315*c^4*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)) - (16*a^5)/(1
05*c^4*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^3) + (4*a^5)/(9*c^4*(x*(-a^2)^(1/2
) - (-a^2)^(1/2)/a)^5) - (16*a^2*(-a^2)^(3/2))/(63*c^4*(x*(-a^2)^(1/2) - (-
a^2)^(1/2)/a)^4) + (32*a^6)/(315*c^4*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^2*(-
a^2)^(1/2))))/(a^5*(-a^2)^(1/2)) + ((1 - a^2*x^2)^(1/2)*((2*a^3)/(15*c^4*(x
*(-a^2)^(1/2) - (-a^2)^(1/2)/a)) - a^3/(5*c^4*(x*(-a^2)^(1/2) - (-a^2)^(1/2
)/a)^3) + (2*a^4)/(15*c^4*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^2*(-a^2)^(1/2))
))/(a^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3ax}{-a^6x^6\sqrt{-a^2x^2+1}+4a^5x^5\sqrt{-a^2x^2+1}-5a^4x^4\sqrt{-a^2x^2+1}+5a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^6x^6\sqrt{-a^2x^2+1}+4a^5x^5\sqrt{-a^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(3*a*x/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x
**2 + 1) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 +
 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a
**2*x**2/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1
) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4
*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**3/
(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**
4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt
```

```
(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-a**6*x**6*sqrt(
-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**4*x**4*sqrt(-a**2
*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1)
+ sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.186 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=129

$$\frac{2\left(1-a^2x^2\right)^{5/2}}{1155ac^5(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{5/2}}{231ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7}+\frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8}$$

```
[Out] 1/11*(-a^2*x^2+1)^(5/2)/a/c^5/(-a*x+1)^8+1/33*(-a^2*x^2+1)^(5/2)/a/c^5/(-a*
x+1)^7+2/231*(-a^2*x^2+1)^(5/2)/a/c^5/(-a*x+1)^6+2/1155*(-a^2*x^2+1)^(5/2)/
a/c^5/(-a*x+1)^5
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{2\left(1-a^2x^2\right)^{5/2}}{1155ac^5(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{5/2}}{231ac^5(1-ax)^6}+\frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7}+\frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^5,x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(11*a*c^5*(1 - a*x)^8) + (1 - a^2*x^2)^(5/2)/(33*a*c^5*
(1 - a*x)^7) + (2*(1 - a^2*x^2)^(5/2))/(231*a*c^5*(1 - a*x)^6) + (2*(1 - a^
2*x^2)^(5/2))/(1155*a*c^5*(1 - a*x)^5)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^5}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^8}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8}+\frac{1}{11}\left(3c^2\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^7}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8}+\frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7}+\frac{1}{33}(2c)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^6}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8}+\frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7}+\frac{2\left(1-a^2x^2\right)^{5/2}}{231ac^5(1-ax)^6}+\frac{2}{231}\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^5}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{5/2}}{11ac^5(1-ax)^8}+\frac{\left(1-a^2x^2\right)^{5/2}}{33ac^5(1-ax)^7}+\frac{2\left(1-a^2x^2\right)^{5/2}}{231ac^5(1-ax)^6}+\frac{2\left(1-a^2x^2\right)^{5/2}}{1155ac^5(1-ax)^5}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.40

$$-\frac{(ax+1)^{5/2}\left(2a^3x^3-16a^2x^2+61ax-152\right)}{1155ac^5(1-ax)^{11/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^5,x]
```

```
[Out] -1/1155*((1 + a*x)^(5/2)*(-152 + 61*a*x - 16*a^2*x^2 + 2*a^3*x^3))/(a*c^5*(
1 - a*x)^(11/2))
```

fricas [A] time = 0.69, size = 171, normalized size = 1.33

$$\frac{152\,a^6x^6-912\,a^5x^5+2280\,a^4x^4-3040\,a^3x^3+2280\,a^2x^2-912\,ax-\left(2\,a^5x^5-12\,a^4x^4+31\,a^3x^3-46\,a^2x^2-243\right.}{1155\left(a^7c^5x^6-6\,a^6c^5x^5+15\,a^5c^5x^4-20\,a^4c^5x^3+15\,a^3c^5x^2-6\,a^2c^5x+ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/1155*(152*a^6*x^6 - 912*a^5*x^5 + 2280*a^4*x^4 - 3040*a^3*x^3 + 2280*a^2*
x^2 - 912*a*x - (2*a^5*x^5 - 12*a^4*x^4 + 31*a^3*x^3 - 46*a^2*x^2 - 243*a*x
 - 152)*sqrt(-a^2*x^2 + 1) + 152)/(a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 15*a^5*c^5
*x^4 - 20*a^4*c^5*x^3 + 15*a^3*c^5*x^2 - 6*a^2*c^5*x + a*c^5)
```

giac [C] time = 0.57, size = 509, normalized size = 3.95

$$\frac{\frac{48i\,\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}{c^3}+\frac{5\left(63\left(\frac{2c}{acx-c}+1\right)^5\sqrt{-\frac{2c}{acx-c}-1}-385\left(\frac{2c}{acx-c}+1\right)^4\sqrt{-\frac{2c}{acx-c}-1}+990\left(\frac{2c}{acx-c}+1\right)^3\sqrt{-\frac{2c}{acx-c}-1}-1386\left(\frac{2c}{acx-c}+1\right)^2\sqrt{-\frac{2c}{acx-c}-1}-1155\left(-\right.\right.}{c^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/27720*(48*I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/c^3 + (5*(63*(2*c/(a*c*x - c
) + 1)^5*sqrt(-2*c/(a*c*x - c) - 1) - 385*(2*c/(a*c*x - c) + 1)^4*sqrt(-2*c
/(a*c*x - c) - 1) + 990*(2*c/(a*c*x - c) + 1)^3*sqrt(-2*c/(a*c*x - c) - 1)
- 1386*(2*c/(a*c*x - c) + 1)^2*sqrt(-2*c/(a*c*x - c) - 1) - 1155*(-2*c/(a*c
*x - c) - 1)^(3/2) - 693*sqrt(-2*c/(a*c*x - c) - 1))/c^3 + 22*(35*(2*c/(a*c
*x - c) + 1)^4*sqrt(-2*c/(a*c*x - c) - 1) - 180*(2*c/(a*c*x - c) + 1)^3*sqr
t(-2*c/(a*c*x - c) - 1) + 378*(2*c/(a*c*x - c) + 1)^2*sqrt(-2*c/(a*c*x - c)
 - 1) + 420*(-2*c/(a*c*x - c) - 1)^(3/2) + 315*sqrt(-2*c/(a*c*x - c) - 1))/
c^3 + 99*(5*(2*c/(a*c*x - c) + 1)^3*sqrt(-2*c/(a*c*x - c) - 1) - 21*(2*c/(a
*c*x - c) + 1)^2*sqrt(-2*c/(a*c*x - c) - 1) - 35*(-2*c/(a*c*x - c) - 1)^(3/
2) - 35*sqrt(-2*c/(a*c*x - c) - 1))/c^3)/(sgn(1/(a*c*x - c))*sgn(a)*sgn(c))
)/(c^2*abs(a))
```

maple [A] time = 0.03, size = 57, normalized size = 0.44

$$-\frac{\left(2x^3a^3-16a^2x^2+61ax-152\right)(ax+1)^4}{1155\,(ax-1)^4\,c^5\left(-a^2x^2+1\right)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/1155*(2*a^3*x^3-16*a^2*x^2+61*a*x-152)*(a*x+1)^4/(a*x-1)^4/c^5/(-a^2*x^2
+1)^(3/2)/a
```

maxima [B] time = 0.36, size = 462, normalized size = 3.58

$$-\frac{8}{11\left(\sqrt{-a^2x^2+1}\,a^6c^5x^5-5\sqrt{-a^2x^2+1}\,a^5c^5x^4+10\sqrt{-a^2x^2+1}\,a^4c^5x^3-10\sqrt{-a^2x^2+1}\,a^3c^5x^2+5\sqrt{-a^2x^2+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -8/11/(sqrt(-a^2*x^2 + 1)*a^6*c^5*x^5 - 5*sqrt(-a^2*x^2 + 1)*a^5*c^5*x^4 +
10*sqrt(-a^2*x^2 + 1)*a^4*c^5*x^3 - 10*sqrt(-a^2*x^2 + 1)*a^3*c^5*x^2 + 5*s
qrt(-a^2*x^2 + 1)*a^2*c^5*x - sqrt(-a^2*x^2 + 1)*a*c^5) - 28/33/(sqrt(-a^2*
x^2 + 1)*a^5*c^5*x^4 - 4*sqrt(-a^2*x^2 + 1)*a^4*c^5*x^3 + 6*sqrt(-a^2*x^2 +
 1)*a^3*c^5*x^2 - 4*sqrt(-a^2*x^2 + 1)*a^2*c^5*x + sqrt(-a^2*x^2 + 1)*a*c^5
) - 58/231/(sqrt(-a^2*x^2 + 1)*a^4*c^5*x^3 - 3*sqrt(-a^2*x^2 + 1)*a^3*c^5*x
^2 + 3*sqrt(-a^2*x^2 + 1)*a^2*c^5*x - sqrt(-a^2*x^2 + 1)*a*c^5) + 1/1155/(s
qrt(-a^2*x^2 + 1)*a^3*c^5*x^2 - 2*sqrt(-a^2*x^2 + 1)*a^2*c^5*x + sqrt(-a^2*
x^2 + 1)*a*c^5) - 1/1155/(sqrt(-a^2*x^2 + 1)*a^2*c^5*x - sqrt(-a^2*x^2 + 1)
*a*c^5) + 2/1155*x/(sqrt(-a^2*x^2 + 1)*c^5)
```

mupad [B] time = 0.87, size = 604, normalized size = 4.68

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{32\,a^6}{693\,c^5\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}-\frac{16\,a^6}{231\,c^5\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^3}+\frac{20\,a^6}{99\,c^5\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^5}+\frac{4\,a^7}{11\,c^5\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^6\,\sqrt{-a^2}}+\frac{}{693\,c^5\,\left(x\,\sqrt{-\right.}}\right.}{a^6\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^5),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((32*a^6)/(693*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a))
- (16*a^6)/(231*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^3) + (20*a^6)/(99*c^5
*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^5) + (4*a^7)/(11*c^5*(x*(-a^2)^(1/2) - (
-a^2)^(1/2)/a)^6*(-a^2)^(1/2)) + (80*a^9)/(693*c^5*(x*(-a^2)^(1/2) - (-a^2)
^(1/2)/a)^4*(-a^2)^(3/2)) + (32*a^11)/(693*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/
2)/a)^2*(-a^2)^(5/2))))/(a^6*(-a^2)^(1/2)) - ((1 - a^2*x^2)^(1/2)*((32*a^5)
/(315*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)) - (16*a^5)/(105*c^5*(x*(-a^2)^
(1/2) - (-a^2)^(1/2)/a)^3) + (4*a^5)/(9*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/
a)^5) - (16*a^2*(-a^2)^(3/2))/(63*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^4)
+ (32*a^6)/(315*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^2*(-a^2)^(1/2))))/(a^
5*(-a^2)^(1/2)) - ((1 - a^2*x^2)^(1/2)*((3*a^4)/(35*c^5*(x*(-a^2)^(1/2) - (
-a^2)^(1/2)/a)^3) - (2*a^4)/(35*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)) + a^
5/(7*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^4*(-a^2)^(1/2)) + (2*a^7)/(35*c^
5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^2*(-a^2)^(3/2))))/(a^4*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{3ax}{-a^7x^7\sqrt{-a^2x^2+1}+5a^6x^6\sqrt{-a^2x^2+1}-9a^5x^5\sqrt{-a^2x^2+1}+5a^4x^4\sqrt{-a^2x^2+1}+5a^3x^3\sqrt{-a^2x^2+1}-9a^2x^2\sqrt{-a^2x^2+1}+5ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] -(Integral(3*a*x/(-a**7*x**7*sqrt(-a**2*x**2 + 1) + 5*a**6*x**6*sqrt(-a**2*
x**2 + 1) - 9*a**5*x**5*sqrt(-a**2*x**2 + 1) + 5*a**4*x**4*sqrt(-a**2*x**2
+ 1) + 5*a**3*x**3*sqrt(-a**2*x**2 + 1) - 9*a**2*x**2*sqrt(-a**2*x**2 + 1)
+ 5*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*
x**2/(-a**7*x**7*sqrt(-a**2*x**2 + 1) + 5*a**6*x**6*sqrt(-a**2*x**2 + 1) -
9*a**5*x**5*sqrt(-a**2*x**2 + 1) + 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**
3*x**3*sqrt(-a**2*x**2 + 1) - 9*a**2*x**2*sqrt(-a**2*x**2 + 1) + 5*a*x*sqrt
(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**3/(-a**7*x*
*7*sqrt(-a**2*x**2 + 1) + 5*a**6*x**6*sqrt(-a**2*x**2 + 1) - 9*a**5*x**5*sq
rt(-a**2*x**2 + 1) + 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**3*x**3*sqrt(-a
**2*x**2 + 1) - 9*a**2*x**2*sqrt(-a**2*x**2 + 1) + 5*a*x*sqrt(-a**2*x**2 +
1) - sqrt(-a**2*x**2 + 1)), x) + Integral(1/(-a**7*x**7*sqrt(-a**2*x**2 + 1
) + 5*a**6*x**6*sqrt(-a**2*x**2 + 1) - 9*a**5*x**5*sqrt(-a**2*x**2 + 1) + 5
*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**3*x**3*sqrt(-a**2*x**2 + 1) - 9*a**2
*x**2*sqrt(-a**2*x**2 + 1) + 5*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 +
 1)), x))/c**5
```

3.187 $\int e^{4\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=66

$$\frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

```
[Out] 4*c*(-a*c*x+c)^(-1+p)/a/(1-p)+4*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(1+p)/a/c/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 21, 43}

$$\frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*c*(c - a*c*x)^(-1 + p))/(a*(1 - p)) + (4*(c - a*c*x)^p)/(a*p) - (c - a*c
*x)^(1 + p)/(a*c*(1 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{4\tanh^{-1}(ax)}(c-acx)^p\,dx &= \int \frac{(1+ax)^2(c-acx)^p}{(1-ax)^2}\,dx \\
&= c^2\int (1+ax)^2(c-acx)^{-2+p}\,dx \\
&= c^2\int \left(4(c-acx)^{-2+p} - \frac{4(c-acx)^{-1+p}}{c} + \frac{(c-acx)^p}{c^2}\right)dx \\
&= \frac{4c(c-acx)^{-1+p}}{a(1-p)} + \frac{4(c-acx)^p}{ap} - \frac{(c-acx)^{1+p}}{ac(1+p)}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 50, normalized size = 0.76

$$
\frac{\left(\frac{ax}{p+1} + \frac{4}{(p-1)(ax-1)} + \frac{3p+4}{p(p+1)}\right)(c-acx)^p}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((c - a*c*x)^p*((4 + 3*p)/(p*(1 + p)) + (a*x)/(1 + p) + 4/((-1 + p)*(-1 + a
*x))))/a
```

fricas [A] time = 0.70, size = 81, normalized size = 1.23

$$
-\frac{\left(\left(a^2p^2 - a^2p\right)x^2 + p^2 + 2\left(ap^2 + ap - 2a\right)x + 3p + 4\right)(-acx+c)^p}{ap^3 - ap - \left(a^2p^3 - a^2p\right)x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] -((a^2*p^2 - a^2*p)*x^2 + p^2 + 2*(a*p^2 + a*p - 2*a)*x + 3*p + 4)*(-a*c*x
+ c)^p/(a*p^3 - a*p - (a^2*p^3 - a^2*p)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(ax+1)^4(-acx+c)^p}{\left(a^2x^2-1\right)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^4*(-a*c*x + c)^p/(a^2*x^2 - 1)^2, x)
```

maple [A] time = 0.03, size = 74, normalized size = 1.12

$$\frac{\left(a^{2} p^{2} x^{2}-a^{2} x^{2} p+2 a p^{2} x+2 a p x-4 a x+p^{2}+3 p+4\right) \left(-a c x+c\right)^{p}}{\left(a x-1\right) a p \left(p^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^p,x)
```

```
[Out] (a^2*p^2*x^2-a^2*p*x^2+2*a*p^2*x+2*a*p*x-4*a*x+p^2+3*p+4)*(-a*c*x+c)^p/(a*x
-1)/a/p/(p^2-1)
```

maxima [A] time = 0.40, size = 77, normalized size = 1.17

$$\frac{\left(\left(p^{2}-p\right) a^{2} c^{p} x^{2}+2 \left(p^{2}+p-2\right) a c^{p} x+\left(p^{2}+3 p+4\right) c^{p}\right) \left(-a x+1\right)^{p}}{\left(p^{3}-p\right) a^{2} x-\left(p^{3}-p\right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 - p)*a^2*c^p*x^2 + 2*(p^2 + p - 2)*a*c^p*x + (p^2 + 3*p + 4)*c^p)*(-a
*x + 1)^p/((p^3 - p)*a^2*x - (p^3 - p)*a)
```

mupad [B] time = 0.94, size = 57, normalized size = 0.86

$$\frac{4 \left(c-a c x\right)^{p}}{a \left(a x-1\right) \left(p-1\right)}+\frac{\left(c-a c x\right)^{p} \left(3 p+a p x+4\right)}{a p \left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^p*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (4*(c - a*c*x)^p)/(a*(a*x - 1)*(p - 1)) + ((c - a*c*x)^p*(3*p + a*p*x + 4))
/(a*p*(p + 1))
```

sympy [A] time = 3.05, size = 541, normalized size = 8.20

$$\begin{cases} c^p x \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac} + \frac{2ax\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac} + \frac{4ax}{a^3cx^2-2a^2cx+ac} - \frac{\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac} - \frac{2}{a^3cx^2-2a^2cx+ac} \\ \frac{a^2x^2}{a^2x-a} + \frac{4ax\log\left(x-\frac{1}{a}\right)}{a^2x-a} - \frac{ax}{a^2x-a} - \frac{4\log\left(x-\frac{1}{a}\right)}{a^2x-a} - \frac{4}{a^2x-a} \\ -\frac{acx^2}{2} - 3cx - \frac{4c\log\left(x-\frac{1}{a}\right)}{a} \\ \frac{a^2p^2x^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} - \frac{a^2px^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{2ap^2x(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{2apx(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} - \frac{4ax(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} + \frac{p^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((c**p*x, Eq(a, 0)), (-a**2*x**2*log(x - 1/a)/(a**3*c*x**2 - 2*a**
2*c*x + a*c) + 2*a*x*log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + 4*a*x/
(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x +
 a*c) - 2/(a**3*c*x**2 - 2*a**2*c*x + a*c), Eq(p, -1)), (a**2*x**2/(a**2*x
- a) + 4*a*x*log(x - 1/a)/(a**2*x - a) - a*x/(a**2*x - a) - 4*log(x - 1/a)/
(a**2*x - a) - 4/(a**2*x - a), Eq(p, 0)), (-a*c*x**2/2 - 3*c*x - 4*c*log(x
- 1/a)/a, Eq(p, 1)), (a**2*p**2*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*
x - a*p**3 + a*p) - a**2*p*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a
*p**3 + a*p) + 2*a*p**2*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3
+ a*p) + 2*a*p*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) -
4*a*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + p**2*(-a*c*
x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 3*p*(-a*c*x + c)**p/(a*
*2*p**3*x - a**2*p*x - a*p**3 + a*p) + 4*(-a*c*x + c)**p/(a**2*p**3*x - a**
2*p*x - a*p**3 + a*p), True))
```

3.188 $\int e^{4\tanh^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=53

$$-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

```
[Out] -c^5*(-a*x+1)^4/a+4/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -((c^5*(1 - a*x)^4)/a) + (4*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}(c-acx)^5\,dx &= c^5\int (1-ax)^3(1+ax)^2\,dx\\ &= c^5\int \left(4(1-ax)^3-4(1-ax)^4+(1-ax)^5\right)dx\\ &= -\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.58

$$-\frac{c^5(ax-1)^4\left(5a^2x^2+14ax+11\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -1/30*(c^5*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2))/a
```

fricas [A] time = 0.52, size = 59, normalized size = 1.11

$$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}a^2c^5x^3-\frac{1}{2}ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

giac [A] time = 0.20, size = 59, normalized size = 1.11

$$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}a^2c^5x^3-\frac{1}{2}ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

maple [A] time = 0.02, size = 45, normalized size = 0.85

$$c^5\left(-\frac{1}{6}x^6a^5+\frac{1}{5}a^4x^5+\frac{1}{2}x^4a^3-\frac{2}{3}x^3a^2-\frac{1}{2}a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^5,x)
```

```
[Out] c^5*(-1/6*x^6*a^5+1/5*a^4*x^5+1/2*x^4*a^3-2/3*x^3*a^2-1/2*a*x^2+x)
```

maxima [A] time = 0.33, size = 59, normalized size = 1.11

$$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}a^2c^5x^3-\frac{1}{2}ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

mupad [B] time = 0.79, size = 59, normalized size = 1.11

$$-\frac{a^5\,c^5\,x^6}{6}+\frac{a^4\,c^5\,x^5}{5}+\frac{a^3\,c^5\,x^4}{2}-\frac{2\,a^2\,c^5\,x^3}{3}-\frac{a\,c^5\,x^2}{2}+c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^5*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] c^5*x - (a*c^5*x^2)/2 - (2*a^2*c^5*x^3)/3 + (a^3*c^5*x^4)/2 + (a^4*c^5*x^5)
/5 - (a^5*c^5*x^6)/6
```

sympy [A] time = 0.10, size = 63, normalized size = 1.19

$$-\frac{a^5c^5x^6}{6}+\frac{a^4c^5x^5}{5}+\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}-\frac{ac^5x^2}{2}+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**5,x)
```

```
[Out] -a**5*c**5*x**6/6 + a**4*c**5*x**5/5 + a**3*c**5*x**4/2 - 2*a**2*c**5*x**3/
3 - a*c**5*x**2/2 + c**5*x
```

3.189 $\int e^{4\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=32

$$\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

```
[Out] c^4*x-2/3*a^2*c^4*x^3+1/5*a^4*c^4*x^5
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 41, 194}

$$\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^4,x]
```

```
[Out] c^4*x - (2*a^2*c^4*x^3)/3 + (a^4*c^4*x^5)/5
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{4\tanh^{-1}(ax)}(c-acx)^4\,dx &= c^4\int(1-ax)^2(1+ax)^2\,dx \\
&= c^4\int\left(1-a^2x^2\right)^2\,dx \\
&= c^4\int\left(1-2a^2x^2+a^4x^4\right)\,dx \\
&= c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.81

$$c^4\left(\frac{a^4x^5}{5}-\frac{2a^2x^3}{3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^4,x]
[Out] c^4*(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)
```

fricas [A] time = 0.51, size = 28, normalized size = 0.88

$$\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^4,x, algorithm="fricas")
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

giac [A] time = 0.19, size = 28, normalized size = 0.88

$$\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^4,x, algorithm="giac")
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

maple [A] time = 0.02, size = 23, normalized size = 0.72

$$c^4\left(\frac{1}{5}a^4x^5-\frac{2}{3}x^3a^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^4,x)
```

```
[Out] c^4*(1/5*a^4*x^5-2/3*x^3*a^2+x)
```

maxima [A] time = 0.33, size = 28, normalized size = 0.88

$$\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

mupad [B] time = 0.80, size = 24, normalized size = 0.75

$$\frac{c^4\,x\,\left(3\,a^4\,x^4-10\,a^2\,x^2+15\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (c^4*x*(3*a^4*x^4 - 10*a^2*x^2 + 15))/15
```

sympy [A] time = 0.08, size = 29, normalized size = 0.91

$$\frac{a^4c^4x^5}{5}-\frac{2a^2c^4x^3}{3}+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**4,x)
```

```
[Out] a**4*c**4*x**5/5 - 2*a**2*c**4*x**3/3 + c**4*x
```

3.190 $\int e^{4\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=35

$$\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

```
[Out] 2/3*c^3*(a*x+1)^3/a-1/4*c^3*(a*x+1)^4/a
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2*c^3*(1 + a*x)^3)/(3*a) - (c^3*(1 + a*x)^4)/(4*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}(c-acx)^3\,dx &= c^3\int(1-ax)(1+ax)^2\,dx\\ &= c^3\int\left(2(1+ax)^2-(1+ax)^3\right)dx\\ &= \frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.86

$$-\frac{1}{12}c^3x\left(3a^3x^3+4a^2x^2-6ax-12\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^3,x]
[Out] -1/12*(c^3*x*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))
```

fricas [A] time = 1.24, size = 37, normalized size = 1.06

$$-\frac{1}{4}a^3c^3x^4-\frac{1}{3}a^2c^3x^3+\frac{1}{2}ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

giac [A] time = 0.17, size = 37, normalized size = 1.06

$$-\frac{1}{4}a^3c^3x^4-\frac{1}{3}a^2c^3x^3+\frac{1}{2}ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^3,x, algorithm="giac")
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

maple [A] time = 0.03, size = 29, normalized size = 0.83

$$c^3\left(-\frac{1}{4}x^4a^3-\frac{1}{3}x^3a^2+\frac{1}{2}a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^3,x)
[Out] c^3*(-1/4*x^4*a^3-1/3*x^3*a^2+1/2*a*x^2+x)
```

maxima [A] time = 0.33, size = 37, normalized size = 1.06

$$-\frac{1}{4}a^3c^3x^4-\frac{1}{3}a^2c^3x^3+\frac{1}{2}ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

mupad [B] time = 0.05, size = 37, normalized size = 1.06

$$-\frac{a^3\,c^3\,x^4}{4}-\frac{a^2\,c^3\,x^3}{3}+\frac{a\,c^3\,x^2}{2}+c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] c^3*x + (a*c^3*x^2)/2 - (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

sympy [A] time = 0.09, size = 37, normalized size = 1.06

$$-\frac{a^{3}c^{3}x^{4}}{4}-\frac{a^{2}c^{3}x^{3}}{3}+\frac{ac^{3}x^{2}}{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**3,x)
[Out] -a**3*c**3*x**4/4 - a**2*c**3*x**3/3 + a*c**3*x**2/2 + c**3*x
```

3.191 $\int e^{4\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(ax+1)^3}{3a}$$

```
[Out] 1/3*c^2*(a*x+1)^3/a
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 32}

$$\frac{c^2(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(1 + a*x)^3)/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}(c-acx)^2\,dx &= c^2\int (1+ax)^2\,dx\\ &= \frac{c^2(1+ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.24

$$c^2\left(\frac{a^2x^3}{3}+ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] c^2*(x + a*x^2 + (a^2*x^3)/3)
```

fricas [A] time = 0.42, size = 25, normalized size = 1.47

$$\frac{1}{3}\,a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

giac [A] time = 0.16, size = 25, normalized size = 1.47

$$\frac{1}{3}\,a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

maple [A] time = 0.02, size = 16, normalized size = 0.94

$$\frac{c^2\left(ax+1\right)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^2,x)
```

```
[Out] 1/3*c^2*(a*x+1)^3/a
```

maxima [A] time = 0.32, size = 25, normalized size = 1.47

$$\frac{1}{3}\,a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

mupad [**B**] time = 0.04, size = 19, normalized size = 1.12

$$\frac{c^2\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (c^2*x*(3*a*x + a^2*x^2 + 3))/3
```

sympy [**A**] time = 0.08, size = 24, normalized size = 1.41

$$\frac{a^2c^2x^3}{3}+ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 + a*c**2*x**2 + c**2*x
```

3.192 $\int e^{4\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}-3cx$$

[Out] -3*c*x-1/2*a*c*x^2-4*c*ln(-a*x+1)/a

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6129, 43}

$$-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}-3cx$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcTanh[a*x])*(c - a*c*x),x]

[Out] -3*c*x - (a*c*x^2)/2 - (4*c*Log[1 - a*x])/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}(c-acx)\,dx&=c\int\frac{(1+ax)^2}{1-ax}\,dx\\&=c\int\left(-3-ax+\frac{4}{1-ax}\right)dx\\&=-3cx-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.96

$$c\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a*c*x),x]
[Out] c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

fricas [A] time = 0.54, size = 28, normalized size = 1.04

$$-\frac{a^2cx^2+6\,acx+8\,c\log\left(ax-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c),x, algorithm="fricas")
[Out] -1/2*(a^2*c*x^2 + 6*a*c*x + 8*c*log(a*x - 1))/a
```

giac [A] time = 0.17, size = 35, normalized size = 1.30

$$-\frac{4\,c\log\left(|ax-1|\right)}{a}-\frac{a^3cx^2+6\,a^2cx}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c),x, algorithm="giac")
[Out] -4*c*log(abs(a*x - 1))/a - 1/2*(a^3*c*x^2 + 6*a^2*c*x)/a^2
```

maple [A] time = 0.03, size = 25, normalized size = 0.93

$$-\frac{ac\,x^2}{2}-3cx-\frac{4c\ln\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c),x)
[Out] -1/2*a*c*x^2-3*c*x-4*c/a*ln(a*x-1)
```

maxima [A] time = 0.32, size = 24, normalized size = 0.89

$$-\frac{1}{2}\,acx^2-3\,cx-\frac{4\,c\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 - 3*c*x - 4*c*log(a*x - 1)/a
```

mupad [B] time = 0.04, size = 26, normalized size = 0.96

$$-\frac{c\,\left(8\,\ln\left(a\,x-1\right)+6\,a\,x+a^2\,x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] -(c*(8*log(a*x - 1) + 6*a*x + a^2*x^2))/(2*a)
```

sympy [A] time = 0.13, size = 26, normalized size = 0.96

$$-\frac{acx^2}{2} - 3cx - \frac{4c\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a*c*x+c),x)
```

```
[Out] -a*c*x**2/2 - 3*c*x - 4*c*log(a*x - 1)/a
```

3.193 $\int \frac{e^{4\tanh^{-1}(ax)}}{c-acx} dx$

Optimal. Leaf size=48

$$-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

```
[Out] 2/a/c/(-a*x+1)^2-4/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] 2/(a*c*(1 - a*x)^2) - 4/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{c-acx}\,dx = \frac{\int \frac{(1+ax)^2}{(1-ax)^3}\,dx}{c}$$
$$= \frac{\int \left(\frac{1}{1-ax} - \frac{4}{(-1+ax)^3} - \frac{4}{(-1+ax)^2}\right)dx}{c}$$
$$= \frac{2}{ac(1-ax)^2} - \frac{4}{ac(1-ax)} - \frac{\log(1-ax)}{ac}$$

Mathematica [**A**] time = 0.02, size = 36, normalized size = 0.75

$$\frac{4ax+(ax-1)^2(-\log(1-ax))-2}{ac(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a*c*x),x]
[Out] (-2 + 4*a*x - (-1 + a*x)^2*Log[1 - a*x])/(a*c*(-1 + a*x)^2)
```

fricas [**A**] time = 0.55, size = 49, normalized size = 1.02

$$\frac{4\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)-2}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c),x, algorithm="fricas")
[Out] (4*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x - 1) - 2)/(a^3*c*x^2 - 2*a^2*c*x + a
*c)
```

giac [**A**] time = 0.18, size = 37, normalized size = 0.77

$$-\frac{\log\left(|ax-1|\right)}{ac}+\frac{2\left(2\,ax-1\right)}{\left(ax-1\right)^2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c),x, algorithm="giac")
[Out] -log(abs(a*x - 1))/(a*c) + 2*(2*a*x - 1)/((a*x - 1)^2*a*c)
```

maple [A] time = 0.03, size = 46, normalized size = 0.96

$$\frac{2}{ca\,(ax-1)^2}-\frac{\ln(ax-1)}{ca}+\frac{4}{ca\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c),x)
[Out] 2/c/a/(a*x-1)^2-1/c/a*ln(a*x-1)+4/c/a/(a*x-1)
```

maxima [A] time = 0.32, size = 44, normalized size = 0.92

$$\frac{2\,(2\,ax-1)}{a^3cx^2-2\,a^2cx+ac}-\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c),x, algorithm="maxima")
[Out] 2*(2*a*x - 1)/(a^3*c*x^2 - 2*a^2*c*x + a*c) - log(a*x - 1)/(a*c)
```

mupad [B] time = 0.07, size = 42, normalized size = 0.88

$$\frac{4\,x-\frac{2}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}-\frac{\ln(a\,x-1)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((a^2*x^2 - 1)^2*(c - a*c*x)),x)
[Out] (4*x - 2/a)/(c + a^2*c*x^2 - 2*a*c*x) - log(a*x - 1)/(a*c)
```

sympy [A] time = 0.22, size = 37, normalized size = 0.77

$$-\frac{-4ax+2}{a^3cx^2-2a^2cx+ac}-\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a*c*x+c),x)
[Out] -(-4*a*x + 2)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(a*x - 1)/(a*c)
```

3.194 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=25

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

```
[Out] 1/6*(a*x+1)^3/a/c^2/(-a*x+1)^3
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 37}

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{(1+ax)^2}{(1-ax)^4}\,dx}{c^2}\\ &= \frac{(1+ax)^3}{6ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A]   time = 0.01, size = 25, normalized size = 1.00

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a*c*x)^2,x]
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

fricas [B]   time = 0.54, size = 51, normalized size = 2.04

$$-\frac{3\,a^2x^2+1}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [B]   time = 0.19, size = 50, normalized size = 2.00

$$-\frac{2}{(acx-c)^2a}-\frac{1}{(acx-c)ac}-\frac{4\,c}{3\,(acx-c)^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -2/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c) - 4/3*c/((a*c*x - c)^3*a)
```

maple [A]   time = 0.03, size = 42, normalized size = 1.68

$$\frac{-\frac{2}{a(ax-1)^2}-\frac{4}{3a(ax-1)^3}-\frac{1}{a(ax-1)}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^2,x)
[Out] 1/c^2*(-2/a/(a*x-1)^2-4/3/a/(a*x-1)^3-1/a/(a*x-1))
```

maxima [B]   time = 0.33, size = 51, normalized size = 2.04

$$-\frac{3\,a^2x^2+1}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

mupad [B] time = 0.08, size = 25, normalized size = 1.00

$$-\frac{3\,a^2\,x^2+1}{3\,a\,c^2\,{\left(a\,x-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((a^2*x^2 - 1)^2*(c - a*c*x)^2),x)
```

```
[Out] -(3*a^2*x^2 + 1)/(3*a*c^2*(a*x - 1)^3)
```

sympy [B] time = 0.26, size = 51, normalized size = 2.04

$$\frac{-3a^2x^2-1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a*c*x+c)**2,x)
```

```
[Out] (-3*a**2*x**2 - 1)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3
*a*c**2)
```

3.195 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=52

$$\frac{1}{2ac^3(1-ax)^2} - \frac{4}{3ac^3(1-ax)^3} + \frac{1}{ac^3(1-ax)^4}$$

```
[Out] 1/a/c^3/(-a*x+1)^4-4/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{1}{2ac^3(1-ax)^2} - \frac{4}{3ac^3(1-ax)^3} + \frac{1}{ac^3(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] 1/(a*c^3*(1 - a*x)^4) - 4/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\int \frac{(1+ax)^2}{(1-ax)^5}\,dx}{c^3}$$
$$= \frac{\int \left(-\frac{4}{(-1+ax)^5}-\frac{4}{(-1+ax)^4}-\frac{1}{(-1+ax)^3}\right)dx}{c^3}$$
$$= \frac{1}{ac^3(1-ax)^4}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.60

$$\frac{3a^2x^2+2ax+1}{6ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (1 + 2*a*x + 3*a^2*x^2)/(6*a*c^3*(-1 + a*x)^4)
```

fricas [A] time = 0.70, size = 65, normalized size = 1.25

$$\frac{3\,a^2x^2+2\,ax+1}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.16, size = 29, normalized size = 0.56

$$\frac{3\,a^2x^2+2\,ax+1}{6\,(ax-1)^4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/((a*x - 1)^4*a*c^3)
```

maple [A] time = 0.03, size = 41, normalized size = 0.79

$$\frac{\frac{1}{2a(ax-1)^{2}}+\frac{1}{a(ax-1)^{4}}+\frac{4}{3a(ax-1)^{3}}}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3*(1/2/a/(a*x-1)^2+1/a/(a*x-1)^4+4/3/a/(a*x-1)^3)
```

maxima [A] time = 0.33, size = 65, normalized size = 1.25

$$\frac{3\,a^{2}x^{2}+2\,ax+1}{6\,\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

mupad [B] time = 0.85, size = 29, normalized size = 0.56

$$\frac{3\,a^2\,x^2+2\,a\,x+1}{6\,a\,c^3\,{\left(a\,x-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((a^2*x^2 - 1)^2*(c - a*c*x)^3),x)
```

```
[Out] (2*a*x + 3*a^2*x^2 + 1)/(6*a*c^3*(a*x - 1)^4)
```

sympy [A] time = 0.33, size = 70, normalized size = 1.35

$$-\frac{-3a^{2}x^{2}-2ax-1}{6a^{5}c^{3}x^{4}-24a^{4}c^{3}x^{3}+36a^{3}c^{3}x^{2}-24a^{2}c^{3}x+6ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a*c*x+c)**3,x)
```

```
[Out] -(-3*a**2*x**2 - 2*a*x - 1)/(6*a**5*c**3*x**4 - 24*a**4*c**3*x**3 + 36*a**3
*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3)
```

3.196 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}$$

[Out] 4/5/a/c^4/(-a*x+1)^5-1/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcTanh[a*x])/(c - a*c*x)^4,x]

[Out] 4/(5*a*c^4*(1 - a*x)^5) - 1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{(1+ax)^2}{(1-ax)^6}\,dx}{c^4}$$
$$= \frac{\int \left(\frac{4}{(-1+ax)^6}+\frac{4}{(-1+ax)^5}+\frac{1}{(-1+ax)^4}\right)dx}{c^4}$$
$$= \frac{4}{5ac^4(1-ax)^5}-\frac{1}{ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.58

$$-\frac{5a^2x^2+5ax+2}{15ac^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/15*(2 + 5*a*x + 5*a^2*x^2)/(a*c^4*(-1 + a*x)^5)
```

fricas [A] time = 0.46, size = 77, normalized size = 1.45

$$-\frac{5\,a^2x^2+5\,ax+2}{15\,\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.13, size = 29, normalized size = 0.55

$$-\frac{5\,a^2x^2+5\,ax+2}{15\,(ax-1)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/((a*x - 1)^5*a*c^4)
```

maple [A] time = 0.03, size = 42, normalized size = 0.79

$$\frac{-\frac{1}{a(ax-1)^4}-\frac{1}{3a(ax-1)^3}-\frac{4}{5a(ax-1)^5}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-1/a/(a*x-1)^4-1/3/a/(a*x-1)^3-4/5/a/(a*x-1)^5)
```

maxima [A] time = 0.33, size = 77, normalized size = 1.45

$$-\frac{5\,a^2x^2+5\,ax+2}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

mupad [B] time = 0.88, size = 29, normalized size = 0.55

$$-\frac{5\,a^2\,x^2+5\,a\,x+2}{15\,a\,c^4\,{\left(a\,x-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((a^2*x^2 - 1)^2*(c - a*c*x)^4),x)
```

```
[Out] -(5*a*x + 5*a^2*x^2 + 2)/(15*a*c^4*(a*x - 1)^5)
```

sympy [A] time = 0.38, size = 80, normalized size = 1.51

$$\frac{-5a^2x^2-5ax-2}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a*c*x+c)**4,x)
```

```
[Out] (-5*a**2*x**2 - 5*a*x - 2)/(15*a**6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**
4*c**4*x**3 - 150*a**3*c**4*x**2 + 75*a**2*c**4*x - 15*a*c**4)
```

3.197 $\int e^{-\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{2}\sqrt{1-ax}\,(c-acx)^{p+1}\,{}_2F_1\left(\frac{1}{2},p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{ac(2p+3)}$$

```
[Out] -(-a*c*x+c)^(1+p)*hypergeom([1/2, 3/2+p],[5/2+p],-1/2*a*x+1/2)*2^(1/2)*(-a*
x+1)^(1/2)/a/c/(3+2*p)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 23, 69}

$$-\frac{\sqrt{2}\sqrt{1-ax}\,(c-acx)^{p+1}\,{}_2F_1\left(\frac{1}{2},p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{ac(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[2]*Sqrt[1 - a*x]*(c - a*c*x)^(1 + p)*Hypergeometric2F1[1/2, 3/2 + p
, 5/2 + p, (1 - a*x)/2])/(a*c*(3 + 2*p)))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^p\,dx = \int \frac{\sqrt{1-ax}\,(c-acx)^p}{\sqrt{1+ax}}\,dx$$
$$= \frac{\sqrt{1-ax}\int \frac{(c-acx)^{\frac{1}{2}+p}}{\sqrt{1+ax}}\,dx}{\sqrt{c-acx}}$$
$$= -\frac{\sqrt{2}\sqrt{1-ax}\,(c-acx)^{1+p}\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+p;\frac{5}{2}+p;\frac{1}{2}(1-ax)\right)}{ac(3+2p)}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.91

$$\frac{\sqrt{2-2ax}\,(ax-1)(c-acx)^p\,{}_2F_1\left(\frac{1}{2},p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}-\frac{ax}{2}\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^ArcTanh[a*x],x]
[Out] (Sqrt[2 - 2*a*x]*(-1 + a*x)*(c - a*c*x)^p*Hypergeometric2F1[1/2, 3/2 + p, 5
/2 + p, 1/2 - (a*x)/2])/(a*(3 + 2*p))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(-acx+c)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,(-acx+c)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(-acx+c)^{p} \sqrt{-a^{2}x^{2}+1}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
[Out] int((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}\,(-acx+c)^{p}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{1-a^{2}\,x^{2}}\,{\left(c-a\,c\,x\right)}^{p}}{a\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^p)/(a*x + 1),x)
[Out] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^p)/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^{p} \sqrt{-(ax-1)(ax+1)}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
[Out] Integral((-c*(a*x - 1))**p*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.198 $\int e^{-\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=133

$$\frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a}+\frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a}+\frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a}+\frac{35c^3\sqrt{1-a^2x^2}}{8a}+\frac{35c^3\sin^{-1}(ax)}{8a}$$

```
[Out] 35/8*c^3*arcsin(a*x)/a+35/8*c^3*(-a^2*x^2+1)^(1/2)/a+35/24*c^3*(-a*x+1)*(-a
^2*x^2+1)^(1/2)/a+7/12*c^3*(-a*x+1)^2*(-a^2*x^2+1)^(1/2)/a+1/4*c^3*(-a*x+1)
^3*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 671, 641, 216}

$$\frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a}+\frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a}+\frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a}+\frac{35c^3\sqrt{1-a^2x^2}}{8a}+\frac{35c^3\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^ArcTanh[a*x],x]
```

```
[Out] (35*c^3*Sqrt[1 - a^2*x^2])/(8*a) + (35*c^3*(1 - a*x)*Sqrt[1 - a^2*x^2])/(24
*a) + (7*c^3*(1 - a*x)^2*Sqrt[1 - a^2*x^2])/(12*a) + (c^3*(1 - a*x)^3*Sqrt[
1 - a^2*x^2])/(4*a) + (35*c^3*ArcSin[a*x])/(8*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^3\,dx=\frac{\int\frac{(c-acx)^4}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$=\frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a}+\frac{7}{4}\int\frac{(c-acx)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$=\frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a}+\frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a}+\frac{1}{12}(35c)\int\frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$=\frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a}+\frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a}+\frac{c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a}+\frac{1}{8}(35$$
$$=\frac{35c^3\sqrt{1-a^2x^2}}{8a}+\frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a}+\frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a}+\frac{c^3(1-ax)^3}{4a}$$
$$=\frac{35c^3\sqrt{1-a^2x^2}}{8a}+\frac{35c^3(1-ax)\sqrt{1-a^2x^2}}{24a}+\frac{7c^3(1-ax)^2\sqrt{1-a^2x^2}}{12a}+\frac{c^3(1-ax)^3}{4a}$$

Mathematica [A] time = 0.07, size = 80, normalized size = 0.60

$$\frac{c^3\left(\frac{\sqrt{ax+1}\left(6a^4x^4-38a^3x^3+113a^2x^2-241ax+160\right)}{\sqrt{1-ax}}-210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*((Sqrt[1 + a*x]*(160 - 241*a*x + 113*a^2*x^2 - 38*a^3*x^3 + 6*a^4*x^4)
)/Sqrt[1 - a*x] - 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a)
```

fricas [A] time = 0.75, size = 81, normalized size = 0.61

$$-\frac{210\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(6\,a^3c^3x^3-32\,a^2c^3x^2+81\,ac^3x-160\,c^3\right)\sqrt{-a^2x^2+1}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(210*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (6*a^3*c^3*x^3 - 32
*a^2*c^3*x^2 + 81*a*c^3*x - 160*c^3)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.78, size = 67, normalized size = 0.50

$$\frac{35\,c^3\arcsin(ax)\,\mathrm{sgn}(a)}{8\,|a|}+\frac{1}{24}\sqrt{-a^2x^2+1}\left(\frac{160\,c^3}{a}-\left(81\,c^3+2\left(3\,a^2c^3x-16\,ac^3\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 35/8*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/24*sqrt(-a^2*x^2 + 1)*(160*c^3/a - (
81*c^3 + 2*(3*a^2*c^3*x - 16*a*c^3)*x)*x)
```

maple [A] time = 0.04, size = 160, normalized size = 1.20

$$\frac{c^3x\left(-a^2x^2+1\right)^{\frac{3}{2}}}{4}-\frac{29c^3x\sqrt{-a^2x^2+1}}{8}-\frac{29c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8\sqrt{a^2}}-\frac{4c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3a}+\frac{8c^3\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\right.}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/4*c^3*x*(-a^2*x^2+1)^(3/2)-29/8*c^3*x*(-a^2*x^2+1)^(1/2)-29/8*c^3/(a^2)^(
1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-4/3*c^3*(-a^2*x^2+1)^(3/2)/a+
8*c^3/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+8*c^3/(a^2)^(1/2)*arctan((a^2)^(
1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [A] time = 0.43, size = 89, normalized size = 0.67

$$\frac{1}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3x-\frac{29}{8}\sqrt{-a^2x^2+1}\,c^3x-\frac{4\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3}{3\,a}+\frac{35\,c^3\arcsin(ax)}{8\,a}+\frac{8\sqrt{-a^2x^2+1}\,c^3}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-a^2*x^2 + 1)^(3/2)*c^3*x - 29/8*sqrt(-a^2*x^2 + 1)*c^3*x - 4/3*(-a^2*
x^2 + 1)^(3/2)*c^3/a + 35/8*c^3*arcsin(a*x)/a + 8*sqrt(-a^2*x^2 + 1)*c^3/a
```

mupad [B] time = 0.03, size = 105, normalized size = 0.79

$$\frac{35\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}-\frac{27\,c^3\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{20\,c^3\,\sqrt{1-a^2\,x^2}}{3\,a}+\frac{4\,a\,c^3\,x^2\,\sqrt{1-a^2\,x^2}}{3}-\frac{a^2\,c^3\,x^3\,\sqrt{1-a^2\,x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^3)/(a*x + 1),x)
```

```
[Out] (35*c^3*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(1/2)) - (27*c^3*x*(1 - a^2*x^2)^(
1/2))/8 + (20*c^3*(1 - a^2*x^2)^(1/2))/(3*a) + (4*a*c^3*x^2*(1 - a^2*x^2)^(
1/2))/3 - (a^2*c^3*x^3*(1 - a^2*x^2)^(1/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\left(-\frac{\sqrt{-a^2x^2+1}}{ax+1}\right)dx+\int\frac{3ax\sqrt{-a^2x^2+1}}{ax+1}\,dx+\int\left(-\frac{3a^2x^2\sqrt{-a^2x^2+1}}{ax+1}\right)dx+\int\frac{a^3x^3\sqrt{-a^2x^2+1}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -c**3*(Integral(-sqrt(-a**2*x**2 + 1)/(a*x + 1), x) + Integral(3*a*x*sqrt(-
a**2*x**2 + 1)/(a*x + 1), x) + Integral(-3*a**2*x**2*sqrt(-a**2*x**2 + 1)/(
a*x + 1), x) + Integral(a**3*x**3*sqrt(-a**2*x**2 + 1)/(a*x + 1), x))
```

3.199 $\int e^{-\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=101

$$\frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{5c^2\sqrt{1-a^2x^2}}{2a}+\frac{5c^2\sin^{-1}(ax)}{2a}$$

```
[Out] 5/2*c^2*arcsin(a*x)/a+5/2*c^2*(-a^2*x^2+1)^(1/2)/a+5/6*c^2*(-a*x+1)*(-a^2*x
^2+1)^(1/2)/a+1/3*c^2*(-a*x+1)^2*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 671, 641, 216}

$$\frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{5c^2\sqrt{1-a^2x^2}}{2a}+\frac{5c^2\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^ArcTanh[a*x],x]
```

```
[Out] (5*c^2*Sqrt[1 - a^2*x^2])/(2*a) + (5*c^2*(1 - a*x)*Sqrt[1 - a^2*x^2])/(6*a)
 + (c^2*(1 - a*x)^2*Sqrt[1 - a^2*x^2])/(3*a) + (5*c^2*ArcSin[a*x])/(2*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}(c-acx)^2\,dx &= \frac{\int \frac{(c-acx)^3}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= \frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5}{3}\int \frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}(5c)\int \frac{c-acx}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{5c^2\sqrt{1-a^2x^2}}{2a}+\frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}\left(5c^2\right)\int \frac{}{\sqrt{1-}}\\ &= \frac{5c^2\sqrt{1-a^2x^2}}{2a}+\frac{5c^2(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5c^2\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.71

$$\frac{c^2\left(\frac{\sqrt{ax+1}\left(-2a^3x^3+11a^2x^2-31ax+22\right)}{\sqrt{1-ax}}-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^ArcTanh[a*x],x]
```

```
[Out] (c^2*((Sqrt[1 + a*x]*(22 - 31*a*x + 11*a^2*x^2 - 2*a^3*x^3))/Sqrt[1 - a*x]
- 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(6*a)
```

fricas [A] time = 0.52, size = 71, normalized size = 0.70

$$-\frac{30\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^2c^2x^2-9\,ac^2x+22\,c^2\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^2*c^2*x^2 - 9*a*
c^2*x + 22*c^2)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.76, size = 54, normalized size = 0.53

$$\frac{5\,c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{2\,|a|}+\frac{1}{6}\,\sqrt{-a^2x^2+1}\left(\left(2\,ac^2x-9\,c^2\right)x+\frac{22\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 5/2*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x - 9*
c^2)*x + 22*c^2/a)
```

maple [A] time = 0.04, size = 142, normalized size = 1.41

$$-\frac{c^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3a}-\frac{3c^2x\sqrt{-a^2x^2+1}}{2}-\frac{3c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}+\frac{4c^2\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}{a}+\frac{4c^2\arctan\left(\frac{\cdots}{\sqrt{-a\cdots}}\right)}{\sqrt{a\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/3*c^2*(-a^2*x^2+1)^(3/2)/a-3/2*c^2*x*(-a^2*x^2+1)^(1/2)-3/2*c^2/(a^2)^(1
/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+4*c^2/a*(-a^2*(x+1/a)^2+2*a*(x
+1/a))^(1/2)+4*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+
1/a))^(1/2))
```

maxima [A] time = 0.43, size = 71, normalized size = 0.70

$$-\frac{3}{2}\,\sqrt{-a^2x^2+1}\,c^2x-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{3\,a}+\frac{5\,c^2\arcsin\left(ax\right)}{2\,a}+\frac{4\,\sqrt{-a^2x^2+1}\,c^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(-a^2*x^2 + 1)*c^2*x - 1/3*(-a^2*x^2 + 1)^(3/2)*c^2/a + 5/2*c^2*ar
csin(a*x)/a + 4*sqrt(-a^2*x^2 + 1)*c^2/a
```

mupad [B] time = 0.04, size = 82, normalized size = 0.81

$$\frac{5\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}-\frac{3\,c^2\,x\,\sqrt{1-a^2\,x^2}}{2}+\frac{11\,c^2\,\sqrt{1-a^2\,x^2}}{3\,a}+\frac{a\,c^2\,x^2\,\sqrt{1-a^2\,x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^2)/(a*x + 1),x)
```

```
[Out] (5*c^2*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2)) - (3*c^2*x*(1 - a^2*x^2)^(1/
2))/2 + (11*c^2*(1 - a^2*x^2)^(1/2))/(3*a) + (a*c^2*x^2*(1 - a^2*x^2)^(1/2)
)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\frac{\sqrt{-a^2x^2+1}}{ax+1}\,dx+\int\left(-\frac{2ax\sqrt{-a^2x^2+1}}{ax+1}\right)dx+\int\frac{a^2x^2\sqrt{-a^2x^2+1}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**2*(Integral(sqrt(-a**2*x**2 + 1)/(a*x + 1), x) + Integral(-2*a*x*sqrt(-a
**2*x**2 + 1)/(a*x + 1), x) + Integral(a**2*x**2*sqrt(-a**2*x**2 + 1)/(a*x
+ 1), x))
```

3.200 $\int e^{-\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=63

$$\frac{c(1-ax)\sqrt{1-a^2x^2}}{2a}+\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{3c\sin^{-1}(ax)}{2a}$$

```
[Out] 3/2*c*arcsin(a*x)/a+3/2*c*(-a^2*x^2+1)^(1/2)/a+1/2*c*(-a*x+1)*(-a^2*x^2+1)^
(1/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6127, 671, 641, 216}

$$\frac{c(1-ax)\sqrt{1-a^2x^2}}{2a}+\frac{3c\sqrt{1-a^2x^2}}{2a}+\frac{3c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^ArcTanh[a*x],x]
```

```
[Out] (3*c*Sqrt[1 - a^2*x^2])/(2*a) + (c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) + (3*
c*ArcSin[a*x])/(2*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}(c-acx)\,dx &= \frac{\int \frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{c(1-ax)\sqrt{1-a^2x^2}}{2a} + \frac{3}{2}\int \frac{c-acx}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{3c\sqrt{1-a^2x^2}}{2a} + \frac{c(1-ax)\sqrt{1-a^2x^2}}{2a} + \frac{1}{2}(3c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{3c\sqrt{1-a^2x^2}}{2a} + \frac{c(1-ax)\sqrt{1-a^2x^2}}{2a} + \frac{3c\sin^{-1}(ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.97

$$\frac{c\left(\frac{\sqrt{ax+1}\,(a^2x^2-5ax+4)}{\sqrt{1-ax}} - 6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^ArcTanh[a*x],x]
[Out] (c*((Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2))/Sqrt[1 - a*x] - 6*ArcSin[Sqrt[1 -
 a*x]/Sqrt[2]]))/(2*a)
```

fricas [A] time = 1.43, size = 52, normalized size = 0.83

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(acx-4\,c)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*(a*c*
x - 4*c))/a
```

giac [A] time = 0.18, size = 38, normalized size = 0.60

$$\frac{3\,c\arcsin\left(ax\right)\operatorname{sgn}(a)}{2\,|a|}-\frac{1}{2}\,\sqrt{-a^2x^2+1}\left(cx-\frac{4\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*c*arcsin(a*x)*sgn(a)/abs(a) - 1/2*sqrt(-a^2*x^2 + 1)*(c*x - 4*c/a)
```

maple [B] time = 0.04, size = 114, normalized size = 1.81

$$-\frac{cx\sqrt{-a^2x^2+1}}{2}-\frac{c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}+\frac{2c\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}{a}+\frac{2c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*c*x*(-a^2*x^2+1)^(1/2)-1/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))+2*c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+2*c/(a^2)^(1/2)*arctan
((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [A] time = 0.46, size = 45, normalized size = 0.71

$$-\frac{1}{2}\,\sqrt{-a^2x^2+1}\,cx+\frac{3\,c\arcsin\left(ax\right)}{2\,a}+\frac{2\,\sqrt{-a^2x^2+1}\,c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*c*x + 3/2*c*arcsin(a*x)/a + 2*sqrt(-a^2*x^2 + 1)*c/
a
```

mupad [B] time = 0.80, size = 59, normalized size = 0.94

$$\frac{\frac{3\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2}-\sqrt{1-a^2\,x^2}\,\left(\frac{2\,a\,c}{\sqrt{-a^2}}+\frac{c\,x\,\sqrt{-a^2}}{2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x))/(a*x + 1),x)
```

```
[Out] ((3*c*asinh(x*(-a^2)^(1/2)))/2 - (1 - a^2*x^2)^(1/2)*((2*a*c)/(-a^2)^(1/2)
+ (c*x*(-a^2)^(1/2))/2))/(-a^2)^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(-\frac{\sqrt{-a^2x^2+1}}{ax+1}\right)dx+\int\frac{ax\sqrt{-a^2x^2+1}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -c*(Integral(-sqrt(-a**2*x**2 + 1)/(a*x + 1), x) + Integral(a*x*sqrt(-a**2*
x**2 + 1)/(a*x + 1), x))
```

3.201 $\int \frac{e^{-\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] arcsin(a*x)/a/c
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 216}

$$\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)),x]
```

```
[Out] ArcSin[a*x]/(a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{c-acx}\,dx &= \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\ &= \frac{\sin^{-1}(ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)),x]
```

```
[Out] ArcSin[a*x]/(a*c)
```

fricas [B]   time = 1.32, size = 30, normalized size = 2.73

$$-\frac{2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x))/(a*c)
```

giac [A]   time = 0.22, size = 14, normalized size = 1.27

$$\frac{\arcsin(ax)\operatorname{sgn}(a)}{c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(c*abs(a))
```

maple [B]   time = 0.04, size = 154, normalized size = 14.00

$$-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{2ca}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{2c\sqrt{a^2}}+\frac{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}{2ca}+\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a^2\left(x+\frac{1}{a}\right.}}\right.}{2c\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x)
```

```
[Out] -1/2/c/a*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/2/c/(a^2)^(1/2)*arctan((a^2)^
(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/2/c/a*(-a^2*(x+1/a)^2+2*a*(x+
1/a))^(1/2)+1/2/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1
/a))^(1/2))
```

maxima [A]   time = 0.43, size = 11, normalized size = 1.00

$$\frac{\arcsin(ax)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] arcsin(a*x)/(a*c)
```

mupad [B] time = 0.04, size = 21, normalized size = 1.91

$$\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{c\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)*(a*x + 1)),x)
```

```
[Out] asinh(x*(-a^2)^(1/2))/(c*(-a^2)^(1/2))
```

sympy [A] time = 4.56, size = 44, normalized size = 4.00

$$\frac{\begin{cases} \sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2 > 0 \\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2 < 0 \end{cases}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c),x)
```

```
[Out] Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asin
h(x*sqrt(-a**2)), a**2 < 0))/c
```

3.202 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^2} dx$

Optimal. Leaf size=29

$$\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}$$

[Out] (-a^2*x^2+1)^(1/2)/a/c^2/(-a*x+1)

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 651}

$$\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}$$

Antiderivative was successfully verified.

[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^2),x]

[Out] Sqrt[1 - a^2*x^2]/(a*c^2*(1 - a*x))

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^2} dx &= \frac{\int \frac{1}{(c-acx)\sqrt{1-a^2x^2}} dx}{c}\\ &= \frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.90

$$\frac{\sqrt{ax+1}}{ac^2\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^2),x]
```

```
[Out] Sqrt[1 + a*x]/(a*c^2*Sqrt[1 - a*x])
```

fricas [A] time = 0.51, size = 37, normalized size = 1.28

$$\frac{ax-\sqrt{-a^2x^2+1}-1}{a^2c^2x-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*x - sqrt(-a^2*x^2 + 1) - 1)/(a^2*c^2*x - a*c^2)
```

giac [C] time = 0.24, size = 70, normalized size = 2.41

$$-c^2\left(\frac{\sqrt{-\frac{2c}{acx-c}-1}\,\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}{a^2c^4}-\frac{i\,\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}{a^2c^4}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -c^2*(sqrt(-2*c/(a*c*x - c) - 1)*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^4)
 - I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^4))*abs(a)
```

maple [A] time = 0.03, size = 28, normalized size = 0.97

$$-\frac{\sqrt{-a^2x^2+1}}{(ax-1)\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/(a*x-1)/a/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(acx-c)^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*c*x - c)^2*(a*x + 1)), x)
```

mupad [B] time = 0.04, size = 47, normalized size = 1.62

$$\frac{\sqrt{1-a^2\,x^2}}{c^2\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^2*(a*x + 1)),x)
[Out] (1 - a^2*x^2)^(1/2)/(c^2*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^3x^3-a^2x^2-ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**2,x)
[Out] Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**3 - a**2*x**2 - a*x + 1), x)/c**2
```

3.203 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=65

$$\frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)}+\frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)^2}$$

```
[Out] 1/3*(-a^2*x^2+1)^(1/2)/a/c^3/(-a*x+1)^2+1/3*(-a^2*x^2+1)^(1/2)/a/c^3/(-a*x+
1)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)}+\frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^3),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(3*a*c^3*(1 - a*x)^2) + Sqrt[1 - a^2*x^2]/(3*a*c^3*(1 - a
*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\int \frac{1}{(c-acx)^2\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)^2} + \frac{\int \frac{1}{(c-acx)\sqrt{1-a^2x^2}}\,dx}{3c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)^2} + \frac{\sqrt{1-a^2x^2}}{3ac^3(1-ax)}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.52

$$-\frac{(ax-2)\sqrt{ax+1}}{3ac^3(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^3),x]
[Out] -1/3*((-2 + a*x)*Sqrt[1 + a*x])/(a*c^3*(1 - a*x)^(3/2))
```

fricas [A] time = 0.42, size = 62, normalized size = 0.95

$$\frac{2\,a^2x^2 - 4\,ax - \sqrt{-a^2x^2+1}\,(ax-2) + 2}{3\left(a^3c^3x^2 - 2\,a^2c^3x + ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] 1/3*(2*a^2*x^2 - 4*a*x - sqrt(-a^2*x^2 + 1)*(a*x - 2) + 2)/(a^3*c^3*x^2 - 2
*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.41, size = 91, normalized size = 1.40

$$-\frac{2\left(\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x} - \frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2} - 2\right)}{3\,c^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} - 1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 3*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^2/(a^4*x^2) - 2)/(c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1
)^3*abs(a))
```

maple [A] time = 0.03, size = 33, normalized size = 0.51

$$-\frac{\sqrt{-a^2x^2+1}\,\left(ax-2\right)}{3\left(ax-1\right)^2c^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/3*(-a^2*x^2+1)^(1/2)*(a*x-2)/(a*x-1)^2/c^3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2x^2+1}}{\left(acx-c\right)^3\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a*c*x - c)^3*(a*x + 1)), x)
```

mupad [B] time = 0.06, size = 32, normalized size = 0.49

$$-\frac{\sqrt{1-a^2\,x^2}\,\left(a\,x-2\right)}{3\,a\,c^3\,{\left(a\,x-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^3*(a*x + 1)),x)
```

```
[Out] -((1 - a^2*x^2)^(1/2)*(a*x - 2))/(3*a*c^3*(a*x - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(sqrt(-a**2*x**2 + 1)/(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1), x)/c*
*3
```

3.204 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^4} dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)^2}+\frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3}$$

```
[Out] 1/5*(-a^2*x^2+1)^(1/2)/a/c^4/(-a*x+1)^3+2/15*(-a^2*x^2+1)^(1/2)/a/c^4/(-a*x
+1)^2+2/15*(-a^2*x^2+1)^(1/2)/a/c^4/(-a*x+1)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)^2}+\frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^4),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(5*a*c^4*(1 - a*x)^3) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^4*(
1 - a*x)^2) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^4*(1 - a*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{1}{(c-acx)^3\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3} + \frac{2\int \frac{1}{(c-acx)^2\sqrt{1-a^2x^2}}\,dx}{5c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3} + \frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)^2} + \frac{2\int \frac{1}{(c-acx)\sqrt{1-a^2x^2}}\,dx}{15c^3}$$
$$= \frac{\sqrt{1-a^2x^2}}{5ac^4(1-ax)^3} + \frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)^2} + \frac{2\sqrt{1-a^2x^2}}{15ac^4(1-ax)}$$

Mathematica [**A**] time = 0.02, size = 43, normalized size = 0.44

$$\frac{\sqrt{ax+1}\left(2a^2x^2-6ax+7\right)}{15ac^4(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^4),x]
```

```
[Out] (Sqrt[1 + a*x]*(7 - 6*a*x + 2*a^2*x^2))/(15*a*c^4*(1 - a*x)^(5/2))
```

fricas [**A**] time = 0.59, size = 91, normalized size = 0.94

$$\frac{7\,a^3x^3-21\,a^2x^2+21\,ax-\left(2\,a^2x^2-6\,ax+7\right)\sqrt{-a^2x^2+1}-7}{15\left(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(7*a^3*x^3 - 21*a^2*x^2 + 21*a*x - (2*a^2*x^2 - 6*a*x + 7)*sqrt(-a^2*x
^2 + 1) - 7)/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

giac [**A**] time = 0.25, size = 145, normalized size = 1.49

$$-\frac{2\left(\frac{20\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{40\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{30\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}-7\right)}{15\,c^4\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -2/15*(20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 40*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a^4*x^2) + 30*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) -
15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 7)/(c^4*((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

maple [A] time = 0.03, size = 42, normalized size = 0.43

$$-\frac{\sqrt{-a^{2} x^{2}+1}\, \left(2 a^{2} x^{2}-6 a x +7\right)}{15 \left(a x -1\right)^{3} c^{4} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/15*(-a^2*x^2+1)^(1/2)*(2*a^2*x^2-6*a*x+7)/(a*x-1)^3/c^4/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2} x^{2}+1}}{(acx-c)^{4}(ax+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*c*x - c)^4*(a*x + 1)), x)
```

mupad [B] time = 0.06, size = 127, normalized size = 1.31

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2\,a^3}{15\,c^4\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}-\frac{a^3}{5\,c^4\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^3}+\frac{2\,a^4}{15\,c^4\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^2\,\sqrt{-a^2}}\right)}{a^3\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^4*(a*x + 1)),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((2*a^3)/(15*c^4*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)) -
a^3/(5*c^4*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^3) + (2*a^4)/(15*c^4*(x*(-a^2)
^(1/2) - (-a^2)^(1/2)/a)^2*(-a^2)^(1/2))))/(a^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a*
*2*x**2 - 3*a*x + 1), x)/c**4
```

3.205 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^5} dx$

Optimal. Leaf size=129

$$\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)^2}+\frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3}+\frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4}$$

```
[Out] 1/7*(-a^2*x^2+1)^(1/2)/a/c^5/(-a*x+1)^4+3/35*(-a^2*x^2+1)^(1/2)/a/c^5/(-a*x
+1)^3+2/35*(-a^2*x^2+1)^(1/2)/a/c^5/(-a*x+1)^2+2/35*(-a^2*x^2+1)^(1/2)/a/c^
5/(-a*x+1)
```

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 651}

$$\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)^2}+\frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3}+\frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^5),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(7*a*c^5*(1 - a*x)^4) + (3*Sqrt[1 - a^2*x^2])/(35*a*c^5*(
1 - a*x)^3) + (2*Sqrt[1 - a^2*x^2])/(35*a*c^5*(1 - a*x)^2) + (2*Sqrt[1 - a^
2*x^2])/(35*a*c^5*(1 - a*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^5}\,dx &= \frac{\int \frac{1}{(c-acx)^4\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4}+\frac{3\int \frac{1}{(c-acx)^3\sqrt{1-a^2x^2}}\,dx}{7c^2}\\
&= \frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4}+\frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3}+\frac{6\int \frac{1}{(c-acx)^2\sqrt{1-a^2x^2}}\,dx}{35c^3}\\
&= \frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4}+\frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3}+\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)^2}+\frac{2\int \frac{1}{(c-acx)\sqrt{1-a^2x^2}}\,dx}{35c^4}\\
&= \frac{\sqrt{1-a^2x^2}}{7ac^5(1-ax)^4}+\frac{3\sqrt{1-a^2x^2}}{35ac^5(1-ax)^3}+\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)^2}+\frac{2\sqrt{1-a^2x^2}}{35ac^5(1-ax)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.40

$$-\frac{\sqrt{ax+1}\left(2a^3x^3-8a^2x^2+13ax-12\right)}{35ac^5(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^5),x]
```

```
[Out] -1/35*(Sqrt[1 + a*x]*(-12 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(a*c^5*(1 - a*
x)^(7/2))
```

fricas [A] time = 0.63, size = 117, normalized size = 0.91

$$\frac{12\,a^4x^4-48\,a^3x^3+72\,a^2x^2-48\,ax-\left(2\,a^3x^3-8\,a^2x^2+13\,ax-12\right)\sqrt{-a^2x^2+1}+12}{35\left(a^5c^5x^4-4\,a^4c^5x^3+6\,a^3c^5x^2-4\,a^2c^5x+ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/35*(12*a^4*x^4 - 48*a^3*x^3 + 72*a^2*x^2 - 48*a*x - (2*a^3*x^3 - 8*a^2*x^
2 + 13*a*x - 12)*sqrt(-a^2*x^2 + 1) + 12)/(a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 6*
a^3*c^5*x^2 - 4*a^2*c^5*x + a*c^5)
```

giac [C] time = 0.21, size = 164, normalized size = 1.27

$$\frac{1}{280}c^2\left(\frac{\left(5\left(\frac{2c}{acx-c}+1\right)^3\sqrt{-\frac{2c}{acx-c}-1}-21\left(\frac{2c}{acx-c}+1\right)^2\sqrt{-\frac{2c}{acx-c}-1}-35\left(-\frac{2c}{acx-c}-1\right)^{\frac{3}{2}}-35\sqrt{-\frac{2c}{acx-c}-1}\right)\operatorname{sgn}\left(\frac{1}{acx-c}\right)\cdots}{a^2c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/280*c^2*((5*(2*c/(a*c*x - c) + 1)^3*sqrt(-2*c/(a*c*x - c) - 1) - 21*(2*c/
(a*c*x - c) + 1)^2*sqrt(-2*c/(a*c*x - c) - 1) - 35*(-2*c/(a*c*x - c) - 1)^(
3/2) - 35*sqrt(-2*c/(a*c*x - c) - 1))*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2
*c^7) + 16*I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^7))*abs(a)
```

maple [A] time = 0.03, size = 50, normalized size = 0.39

$$-\frac{\sqrt{-a^2x^2+1}\,\left(2x^3a^3-8a^2x^2+13ax-12\right)}{35\,(ax-1)^4\,c^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/35*(-a^2*x^2+1)^(1/2)*(2*a^3*x^3-8*a^2*x^2+13*a*x-12)/(a*x-1)^4/c^5/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-a^2x^2+1}}{(acx-c)^5(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a*c*x - c)^5*(a*x + 1)), x)
```

mupad [B] time = 0.80, size = 167, normalized size = 1.29

$$-\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{3\,a^4}{35\,c^5\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^3}-\frac{2\,a^4}{35\,c^5\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}+\frac{a^5}{7\,c^5\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^4\,\sqrt{-a^2}}+\frac{2\,a^7}{35\,c^5\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^2\,{\left(-a^2\right)}^{3/2}}\right)}{a^4\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^5*(a*x + 1)),x)
```

```
[Out] -((1 - a^2*x^2)^(1/2)*((3*a^4)/(35*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^3)
 - (2*a^4)/(35*c^5*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)) + a^5/(7*c^5*(x*(-a^2
)^(1/2) - (-a^2)^(1/2)/a)^4*(-a^2)^(1/2)) + (2*a^7)/(35*c^5*(x*(-a^2)^(1/2)
 - (-a^2)^(1/2)/a)^2*(-a^2)^(3/2))))/(a^4*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] -Integral(sqrt(-a**2*x**2 + 1)/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a
**2*x**2 + 4*a*x - 1), x)/c**5
```

3.206 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=44

$$-\frac{(c-acx)^{p+2}\,{}_2F_1\left(1,p+2;p+3;\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

```
[Out] -1/2*(-a*c*x+c)^(2+p)*hypergeom([1, 2+p],[3+p],-1/2*a*x+1/2)/a/c^2/(2+p)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 21, 68}

$$-\frac{(c-acx)^{p+2}\,{}_2F_1\left(1,p+2;p+3;\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a
*c^2*(2 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}(c-acx)^p\,dx = \int \frac{(1-ax)(c-acx)^p}{1+ax}\,dx$$
$$= \frac{\int \frac{(c-acx)^{1+p}}{1+ax}\,dx}{c}$$
$$= -\frac{(c-acx)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.98

$$\frac{(ax-1)(c-acx)^p\left({}_2F_1\left(1,p+1;p+2;\frac{1}{2}(1-ax)\right)-1\right)}{a(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] ((-1 + a*x)*(c - a*c*x)^p*(-1 + Hypergeometric2F1[1, 1 + p, 2 + p, (1 - a*x
)/2]))/(a*(1 + p))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax-1)(-acx+c)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)(-acx+c)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*(-a*c*x + c)^p/(a*x + 1)^2, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(-acx+c)^{p} \left(-a^{2} x^{2}+1\right)}{(ax+1)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x)
[Out] int((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^{2} x^{2}-1\right) (-acx+c)^{p}}{(ax+1)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)*(-a*c*x + c)^p/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a^{2}\, x^{2}-1\right)\, (c-a\, c\, x)^{p}}{(a\, x+1)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^p)/(a*x + 1)^2,x)
[Out] -int(((a^2*x^2 - 1)*(c - a*c*x)^p)/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{(-acx+c)^{p}}{ax+1}\right) dx - \int \frac{ax\,(-acx+c)^{p}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p/(a*x+1)**2*(-a**2*x**2+1),x)
[Out] -Integral(-(-a*c*x + c)**p/(a*x + 1), x) - Integral(a*x*(-a*c*x + c)**p/(a*
x + 1), x)
```

3.207 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=91

$$\frac{c^4(1-ax)^5}{5a}+\frac{c^4(1-ax)^4}{2a}+\frac{4c^4(1-ax)^3}{3a}+\frac{4c^4(1-ax)^2}{a}+\frac{32c^4\log(ax+1)}{a}-16c^4x$$

```
[Out] -16*c^4*x+4*c^4*(-a*x+1)^2/a+4/3*c^4*(-a*x+1)^3/a+1/2*c^4*(-a*x+1)^4/a+1/5*
c^4*(-a*x+1)^5/a+32*c^4*ln(a*x+1)/a
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^4(1-ax)^5}{5a}+\frac{c^4(1-ax)^4}{2a}+\frac{4c^4(1-ax)^3}{3a}+\frac{4c^4(1-ax)^2}{a}+\frac{32c^4\log(ax+1)}{a}-16c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -16*c^4*x + (4*c^4*(1 - a*x)^2)/a + (4*c^4*(1 - a*x)^3)/(3*a) + (c^4*(1 - a
*x)^4)/(2*a) + (c^4*(1 - a*x)^5)/(5*a) + (32*c^4*Log[1 + a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}(c-acx)^4\,dx &= c^4\int\frac{(1-ax)^5}{1+ax}\,dx\\ &= c^4\int\left(-16-8(1-ax)-4(1-ax)^2-2(1-ax)^3-(1-ax)^4+\frac{32}{1+ax}\right)dx\\ &= -16c^4x+\frac{4c^4(1-ax)^2}{a}+\frac{4c^4(1-ax)^3}{3a}+\frac{c^4(1-ax)^4}{2a}+\frac{c^4(1-ax)^5}{5a}+\frac{32c^4\log(1+}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.62

$$-\frac{c^4\left(6a^5x^5-45a^4x^4+160a^3x^3-390a^2x^2+930ax-960\log(ax+1)-181\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/30*(c^4*(-181 + 930*a*x - 390*a^2*x^2 + 160*a^3*x^3 - 45*a^4*x^4 + 6*a^5
*x^5 - 960*Log[1 + a*x]))/a
```

fricas [A] time = 0.58, size = 68, normalized size = 0.75

$$-\frac{6\,a^5c^4x^5-45\,a^4c^4x^4+160\,a^3c^4x^3-390\,a^2c^4x^2+930\,ac^4x-960\,c^4\log\left(ax+1\right)}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/30*(6*a^5*c^4*x^5 - 45*a^4*c^4*x^4 + 160*a^3*c^4*x^3 - 390*a^2*c^4*x^2 +
 930*a*c^4*x - 960*c^4*log(a*x + 1))/a
```

giac [A] time = 0.21, size = 94, normalized size = 1.03

$$-\frac{\left(6\,c^4-\frac{75\,c^4}{ax+1}+\frac{400\,c^4}{(ax+1)^2}-\frac{1200\,c^4}{(ax+1)^3}+\frac{2400\,c^4}{(ax+1)^4}\right)(ax+1)^5}{30\,a}-\frac{32\,c^4\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/30*(6*c^4 - 75*c^4/(a*x + 1) + 400*c^4/(a*x + 1)^2 - 1200*c^4/(a*x + 1)^
3 + 2400*c^4/(a*x + 1)^4)*(a*x + 1)^5/a - 32*c^4*log(abs(a*x + 1)/((a*x + 1
)^2*abs(a)))/a
```

maple [A] time = 0.03, size = 64, normalized size = 0.70

$$-\frac{a^{4}c^{4}x^{5}}{5}+\frac{3c^{4}x^{4}a^{3}}{2}-\frac{16a^{2}c^{4}x^{3}}{3}+13c^{4}x^{2}a-31c^{4}x+\frac{32c^{4}\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^4/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/5*a^4*c^4*x^5+3/2*c^4*x^4*a^3-16/3*a^2*c^4*x^3+13*c^4*x^2*a-31*c^4*x+32*
c^4*ln(a*x+1)/a
```

maxima [A] time = 0.33, size = 63, normalized size = 0.69

$$-\frac{1}{5}\,a^{4}c^{4}x^{5}+\frac{3}{2}\,a^{3}c^{4}x^{4}-\frac{16}{3}\,a^{2}c^{4}x^{3}+13\,ac^{4}x^{2}-31\,c^{4}x+\frac{32\,c^{4}\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/5*a^4*c^4*x^5 + 3/2*a^3*c^4*x^4 - 16/3*a^2*c^4*x^3 + 13*a*c^4*x^2 - 31*c
^4*x + 32*c^4*log(a*x + 1)/a
```

mupad [B] time = 0.05, size = 63, normalized size = 0.69

$$13\,a\,c^{4}\,x^{2}-31\,c^{4}\,x-\frac{16\,a^{2}\,c^{4}\,x^{3}}{3}+\frac{3\,a^{3}\,c^{4}\,x^{4}}{2}-\frac{a^{4}\,c^{4}\,x^{5}}{5}+\frac{32\,c^{4}\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^4)/(a*x + 1)^2,x)
```

```
[Out] 13*a*c^4*x^2 - 31*c^4*x - (16*a^2*c^4*x^3)/3 + (3*a^3*c^4*x^4)/2 - (a^4*c^4
*x^5)/5 + (32*c^4*log(a*x + 1))/a
```

sympy [A] time = 0.17, size = 68, normalized size = 0.75

$$-\frac{a^{4}c^{4}x^{5}}{5}+\frac{3a^{3}c^{4}x^{4}}{2}-\frac{16a^{2}c^{4}x^{3}}{3}+13ac^{4}x^{2}-31c^{4}x+\frac{32c^{4}\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**4/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**4*c**4*x**5/5 + 3*a**3*c**4*x**4/2 - 16*a**2*c**4*x**3/3 + 13*a*c**4*x*
*2 - 31*c**4*x + 32*c**4*log(a*x + 1)/a
```

3.208 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=73

$$\frac{c^3(1-ax)^4}{4a}+\frac{2c^3(1-ax)^3}{3a}+\frac{2c^3(1-ax)^2}{a}+\frac{16c^3\log(ax+1)}{a}-8c^3x$$

```
[Out] -8*c^3*x+2*c^3*(-a*x+1)^2/a+2/3*c^3*(-a*x+1)^3/a+1/4*c^3*(-a*x+1)^4/a+16*c^
3*ln(a*x+1)/a
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^3(1-ax)^4}{4a}+\frac{2c^3(1-ax)^3}{3a}+\frac{2c^3(1-ax)^2}{a}+\frac{16c^3\log(ax+1)}{a}-8c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -8*c^3*x + (2*c^3*(1 - a*x)^2)/a + (2*c^3*(1 - a*x)^3)/(3*a) + (c^3*(1 - a*
x)^4)/(4*a) + (16*c^3*Log[1 + a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}(c-acx)^3\,dx = c^3\int\frac{(1-ax)^4}{1+ax}\,dx$$
$$= c^3\int\left(-8-4(1-ax)-2(1-ax)^2-(1-ax)^3+\frac{16}{1+ax}\right)dx$$
$$= -8c^3x+\frac{2c^3(1-ax)^2}{a}+\frac{2c^3(1-ax)^3}{3a}+\frac{c^3(1-ax)^4}{4a}+\frac{16c^3\log(1+ax)}{a}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.66

$$\frac{c^3\left(3a^4x^4-20a^3x^3+66a^2x^2-180ax+192\log(ax+1)+35\right)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(35 - 180*a*x + 66*a^2*x^2 - 20*a^3*x^3 + 3*a^4*x^4 + 192*Log[1 + a*x]
))/(12*a)
```

fricas [A] time = 0.60, size = 57, normalized size = 0.78

$$\frac{3\,a^4c^3x^4-20\,a^3c^3x^3+66\,a^2c^3x^2-180\,ac^3x+192\,c^3\log\left(ax+1\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*c^3*x^4 - 20*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 180*a*c^3*x + 192*c
^3*log(a*x + 1))/a
```

giac [A] time = 0.18, size = 82, normalized size = 1.12

$$\frac{\left(3\,c^3-\frac{32\,c^3}{ax+1}+\frac{144\,c^3}{(ax+1)^2}-\frac{384\,c^3}{(ax+1)^3}\right)(ax+1)^4}{12\,a}-\frac{16\,c^3\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3 - 32*c^3/(a*x + 1) + 144*c^3/(a*x + 1)^2 - 384*c^3/(a*x + 1)^3)
*(a*x + 1)^4/a - 16*c^3*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a
```

maple [A] time = 0.02, size = 53, normalized size = 0.73

$$\frac{c^3x^4a^3}{4} - \frac{5a^2c^3x^3}{3} + \frac{11c^3x^2a}{2} - 15c^3x + \frac{16c^3\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/4*c^3*x^4*a^3-5/3*a^2*c^3*x^3+11/2*c^3*x^2*a-15*c^3*x+16*c^3*ln(a*x+1)/a
```

maxima [A] time = 0.33, size = 52, normalized size = 0.71

$$\frac{1}{4}a^3c^3x^4 - \frac{5}{3}a^2c^3x^3 + \frac{11}{2}ac^3x^2 - 15c^3x + \frac{16c^3\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/4*a^3*c^3*x^4 - 5/3*a^2*c^3*x^3 + 11/2*a*c^3*x^2 - 15*c^3*x + 16*c^3*log(
a*x + 1)/a
```

mupad [B] time = 0.79, size = 52, normalized size = 0.71

$$\frac{11\,a\,c^3\,x^2}{2} - 15\,c^3\,x - \frac{5\,a^2\,c^3\,x^3}{3} + \frac{a^3\,c^3\,x^4}{4} + \frac{16\,c^3\,\ln(a\,x+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^3)/(a*x + 1)^2,x)
```

```
[Out] (11*a*c^3*x^2)/2 - 15*c^3*x - (5*a^2*c^3*x^3)/3 + (a^3*c^3*x^4)/4 + (16*c^3
*log(a*x + 1))/a
```

sympy [A] time = 0.15, size = 56, normalized size = 0.77

$$\frac{a^3c^3x^4}{4} - \frac{5a^2c^3x^3}{3} + \frac{11ac^3x^2}{2} - 15c^3x + \frac{16c^3\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] a**3*c**3*x**4/4 - 5*a**2*c**3*x**3/3 + 11*a*c**3*x**2/2 - 15*c**3*x + 16*c
**3*log(a*x + 1)/a
```

3.209 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=54

$$\frac{c^2(1-ax)^3}{3a}+\frac{c^2(1-ax)^2}{a}+\frac{8c^2\log(ax+1)}{a}-4c^2x$$

```
[Out] -4*c^2*x+c^2*(-a*x+1)^2/a+1/3*c^2*(-a*x+1)^3/a+8*c^2*ln(a*x+1)/a
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 43}

$$\frac{c^2(1-ax)^3}{3a}+\frac{c^2(1-ax)^2}{a}+\frac{8c^2\log(ax+1)}{a}-4c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -4*c^2*x + (c^2*(1 - a*x)^2)/a + (c^2*(1 - a*x)^3)/(3*a) + (8*c^2*Log[1 + a
*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}(c-acx)^2\,dx &= c^2\int \frac{(1-ax)^3}{1+ax}\,dx \\
&= c^2\int \left(-4-2(1-ax)-(1-ax)^2+\frac{8}{1+ax}\right)dx \\
&= -4c^2x+\frac{c^2(1-ax)^2}{a}+\frac{c^2(1-ax)^3}{3a}+\frac{8c^2\log(1+ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.72

$$
-\frac{c^2\left(a^3x^3-6a^2x^2+21ax-24\log(ax+1)-4\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^2/E^(2*ArcTanh[a*x]),x]
[Out] -1/3*(c^2*(-4 + 21*a*x - 6*a^2*x^2 + a^3*x^3 - 24*Log[1 + a*x]))/a
```

fricas [A] time = 0.44, size = 45, normalized size = 0.83

$$
-\frac{a^3c^2x^3-6\,a^2c^2x^2+21\,ac^2x-24\,c^2\log\left(ax+1\right)}{3\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -1/3*(a^3*c^2*x^3 - 6*a^2*c^2*x^2 + 21*a*c^2*x - 24*c^2*log(a*x + 1))/a
```

giac [A] time = 0.19, size = 68, normalized size = 1.26

$$
-\frac{\left(c^2-\frac{9\,c^2}{ax+1}+\frac{36\,c^2}{(ax+1)^2}\right)(ax+1)^3}{3\,a}-\frac{8\,c^2\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] -1/3*(c^2 - 9*c^2/(a*x + 1) + 36*c^2/(a*x + 1)^2)*(a*x + 1)^3/a - 8*c^2*log
(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a
```

maple [A] time = 0.03, size = 42, normalized size = 0.78

$$
-\frac{a^2c^2x^3}{3}+2c^2x^2a-7c^2x+\frac{8c^2\ln\left(ax+1\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/3*a^2*c^2*x^3+2*c^2*x^2*a-7*c^2*x+8*c^2*ln(a*x+1)/a
```

maxima [A] time = 0.35, size = 41, normalized size = 0.76

$$-\frac{1}{3}\,a^2c^2x^3 + 2\,ac^2x^2 - 7\,c^2x + \frac{8\,c^2\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/3*a^2*c^2*x^3 + 2*a*c^2*x^2 - 7*c^2*x + 8*c^2*log(a*x + 1)/a
```

mupad [B] time = 0.05, size = 41, normalized size = 0.76

$$2\,a\,c^2\,x^2 - 7\,c^2\,x - \frac{a^2\,c^2\,x^3}{3} + \frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^2)/(a*x + 1)^2,x)
```

```
[Out] 2*a*c^2*x^2 - 7*c^2*x - (a^2*c^2*x^3)/3 + (8*c^2*log(a*x + 1))/a
```

sympy [A] time = 0.13, size = 41, normalized size = 0.76

$$-\frac{a^2c^2x^3}{3} + 2ac^2x^2 - 7c^2x + \frac{8c^2\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**2*c**2*x**3/3 + 2*a*c**2*x**2 - 7*c**2*x + 8*c**2*log(a*x + 1)/a
```

3.210 $\int e^{-2\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}acx^2+\frac{4c\log(ax+1)}{a}-3cx$$

```
[Out] -3*c*x+1/2*a*c*x^2+4*c*ln(a*x+1)/a
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6129, 43}

$$\frac{1}{2}acx^2+\frac{4c\log(ax+1)}{a}-3cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -3*c*x + (a*c*x^2)/2 + (4*c*Log[1 + a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}(c-acx)\,dx &= c\int \frac{(1-ax)^2}{1+ax}\,dx\\ &= c\int \left(-3+ax+\frac{4}{1+ax}\right)dx\\ &= -3cx+\frac{1}{2}acx^2+\frac{4c\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$c\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a)
```

fricas [A] time = 0.47, size = 28, normalized size = 1.08

$$\frac{a^2cx^2-6\,acx+8\,c\log\left(ax+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*c*x^2 - 6*a*c*x + 8*c*log(a*x + 1))/a
```

giac [B] time = 0.16, size = 50, normalized size = 1.92

$$\frac{(ax+1)^2\left(c-\frac{8\,c}{ax+1}\right)}{2\,a}-\frac{4\,c\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*(a*x + 1)^2*(c - 8*c/(a*x + 1))/a - 4*c*log(abs(a*x + 1)/((a*x + 1)^2*a
bs(a)))/a
```

maple [A] time = 0.02, size = 25, normalized size = 0.96

$$-3cx+\frac{ac\,x^2}{2}+\frac{4c\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -3*c*x+1/2*a*c*x^2+4*c*ln(a*x+1)/a
```

maxima [A] time = 0.31, size = 24, normalized size = 0.92

$$\frac{1}{2}\,acx^2-3\,cx+\frac{4\,c\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
[Out] 1/2*a*c*x^2 - 3*c*x + 4*c*log(a*x + 1)/a
```

mupad [B] time = 0.04, size = 26, normalized size = 1.00

$$\frac{c\,\left(8\,\ln\left(a\,x+1\right)-6\,a\,x+a^2\,x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x))/(a*x + 1)^2,x)
[Out] (c*(8*log(a*x + 1) - 6*a*x + a^2*x^2))/(2*a)
```

sympy [A] time = 0.11, size = 24, normalized size = 0.92

$$\frac{acx^2}{2} - 3cx + \frac{4c\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)**2*(-a**2*x**2+1),x)
[Out] a*c*x**2/2 - 3*c*x + 4*c*log(a*x + 1)/a
```

3.211 $\int \frac{e^{-2\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=13

$$\frac{\log(ax+1)}{ac}$$

```
[Out] ln(a*x+1)/a/c
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 31}

$$\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)),x]
```

```
[Out] Log[1 + a*x]/(a*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{c-acx}\,dx &= \frac{\int \frac{1}{1+ax}\,dx}{c}\\ &= \frac{\log(1+ax)}{ac}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)),x]
[Out] Log[1 + a*x]/(a*c)
```

fricas [A] time = 0.50, size = 13, normalized size = 1.00

$$\frac{\log\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="fricas")
[Out] log(a*x + 1)/(a*c)
```

giac [B] time = 0.76, size = 27, normalized size = 2.08

$$-\frac{\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="giac")
[Out] -log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c)
```

maple [A] time = 0.03, size = 14, normalized size = 1.08

$$\frac{\ln\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c),x)
[Out] ln(a*x+1)/a/c
```

maxima [A] time = 0.31, size = 13, normalized size = 1.00

$$\frac{\log\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c),x, algorithm="maxima")
[Out] log(a*x + 1)/(a*c)
```

mupad [B] time = 0.03, size = 13, normalized size = 1.00

$$\frac{\ln\left(a\,x+1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a*c*x)*(a*x + 1)^2),x)
```

```
[Out] log(a*x + 1)/(a*c)
```

sympy [A] time = 0.07, size = 10, normalized size = 0.77

$$\frac{\log\left(acx+c\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c),x)
```

```
[Out] log(a*c*x + c)/(a*c)
```

3.212 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(ax)}{ac^2}$$

```
[Out] arctanh(a*x)/a/c^2
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 35, 206}

$$\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^2),x]
```

```
[Out] ArcTanh[a*x]/(a*c^2)
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c + b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{1}{(1-ax)(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \frac{1}{1-a^2x^2}\,dx}{c^2}$$
$$= \frac{\tanh^{-1}(ax)}{ac^2}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^2),x]
```

```
[Out] ArcTanh[a*x]/(a*c^2)
```

fricas [B] time = 0.49, size = 23, normalized size = 2.09

$$\frac{\log(ax+1)-\log(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(log(a*x + 1) - log(a*x - 1))/(a*c^2)
```

giac [B] time = 0.28, size = 25, normalized size = 2.27

$$\frac{\log\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^2)
```

maple [B] time = 0.03, size = 30, normalized size = 2.73

$$-\frac{\ln(ax-1)}{2c^2a}+\frac{\ln(ax+1)}{2a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^2,x)
```

```
[Out] -1/2/c^2/a*ln(a*x-1)+1/2*ln(a*x+1)/a/c^2
```

maxima [B] time = 0.31, size = 29, normalized size = 2.64

$$\frac{\log\left(ax+1\right)}{2\,ac^2}-\frac{\log\left(ax-1\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*log(a*x + 1)/(a*c^2) - 1/2*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.07, size = 11, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(a\,x\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a*c*x)^2*(a*x + 1)^2),x)
```

```
[Out] atanh(a*x)/(a*c^2)
```

sympy [B] time = 0.14, size = 22, normalized size = 2.00

$$-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**2,x)
```

```
[Out] -(log(x - 1/a)/2 - log(x + 1/a)/2)/(a*c**2)
```

3.213 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=33

$$\frac{1}{2ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{2ac^3}$$

```
[Out] 1/2/a/c^3/(-a*x+1)+1/2*arctanh(a*x)/a/c^3
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 44, 207}

$$\frac{1}{2ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{2ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^3),x]
```

```
[Out] 1/(2*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(2*a*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx = \frac{\int \frac{1}{(1-ax)^2(1+ax)}\,dx}{c^3}$$
$$= \frac{\int \left(\frac{1}{2(-1+ax)^2} - \frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{c^3}$$
$$= \frac{1}{2ac^3(1-ax)} - \frac{\int \frac{1}{-1+a^2x^2}\,dx}{2c^3}$$
$$= \frac{1}{2ac^3(1-ax)} + \frac{\tanh^{-1}(ax)}{2ac^3}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.94

$$\frac{\frac{1}{2a(1-ax)} + \frac{\tanh^{-1}(ax)}{2a}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^3),x]
```

```
[Out] (1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3
```

fricas [A] time = 0.46, size = 46, normalized size = 1.39

$$\frac{(ax-1)\log(ax+1) - (ax-1)\log(ax-1) - 2}{4\left(a^2c^3x - ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*c^3*x - a*c^
3)
```

giac [A] time = 0.19, size = 43, normalized size = 1.30

$$-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{4\,ac^3} + \frac{1}{4\,ac^3\left(\frac{2}{ax+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*log(abs(-2/(a*x + 1) + 1))/(a*c^3) + 1/4/(a*c^3*(2/(a*x + 1) - 1))
```

maple [A] time = 0.03, size = 45, normalized size = 1.36

$$-\frac{1}{2c^3a\,(ax-1)}-\frac{\ln\,(ax-1)}{4c^3a}+\frac{\ln\,(ax+1)}{4a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^3,x)
```

```
[Out] -1/2/c^3/a/(a*x-1)-1/4/c^3/a*ln(a*x-1)+1/4*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.34, size = 48, normalized size = 1.45

$$-\frac{1}{2\left(a^2c^3x-ac^3\right)}+\frac{\log\,(ax+1)}{4\,ac^3}-\frac{\log\,(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a^2*c^3*x - a*c^3) + 1/4*log(a*x + 1)/(a*c^3) - 1/4*log(a*x - 1)/(a*c
^3)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.94

$$\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)}+\frac{\mathrm{atanh}\,(a\,x)}{2\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a*c*x)^3*(a*x + 1)^2),x)
```

```
[Out] 1/(2*a*(c^3 - a*c^3*x)) + atanh(a*x)/(2*a*c^3)
```

sympy [A] time = 0.22, size = 39, normalized size = 1.18

$$-\frac{1}{2a^2c^3x-2ac^3}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**3,x)
```

```
[Out] -1/(2*a**2*c**3*x - 2*a*c**3) + (-log(x - 1/a)/4 + log(x + 1/a)/4)/(a*c**3)
```

3.214 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=51

$$\frac{1}{4ac^4(1-ax)}+\frac{1}{4ac^4(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4ac^4}$$

```
[Out] 1/4/a/c^4/(-a*x+1)^2+1/4/a/c^4/(-a*x+1)+1/4*arctanh(a*x)/a/c^4
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 44, 207}

$$\frac{1}{4ac^4(1-ax)}+\frac{1}{4ac^4(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^4),x]
```

```
[Out] 1/(4*a*c^4*(1 - a*x)^2) + 1/(4*a*c^4*(1 - a*x)) + ArcTanh[a*x]/(4*a*c^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^4}$$
$$= \frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^4}$$
$$= \frac{1}{4ac^4(1-ax)^2}+\frac{1}{4ac^4(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^4}$$
$$= \frac{1}{4ac^4(1-ax)^2}+\frac{1}{4ac^4(1-ax)}+\frac{\tanh^{-1}(ax)}{4ac^4}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.69

$$\frac{-ax+(ax-1)^2\tanh^{-1}(ax)+2}{4ac^4(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (2 - a*x + (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^4*(-1 + a*x)^2)
```

fricas [A] time = 0.52, size = 76, normalized size = 1.49

$$-\frac{2\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)-4}{8\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 4)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.58, size = 58, normalized size = 1.14

$$-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^4}-\frac{\frac{3}{a}-\frac{8}{(ax+1)a}}{16\,c^4\left(\frac{2}{ax+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(-2/(a*x + 1) + 1))/(a*c^4) - 1/16*(3/a - 8/((a*x + 1)*a))/(c^4
*(2/(a*x + 1) - 1)^2)
```

maple [A] time = 0.03, size = 60, normalized size = 1.18

$$\frac{1}{4c^4a\,(ax-1)^2}-\frac{1}{4c^4a\,(ax-1)}-\frac{\ln(ax-1)}{8c^4a}+\frac{\ln(ax+1)}{8a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^4,x)
```

```
[Out] 1/4/c^4/a/(a*x-1)^2-1/4/c^4/a/(a*x-1)-1/8/c^4/a*ln(a*x-1)+1/8*ln(a*x+1)/a/c
^4
```

maxima [A] time = 0.33, size = 63, normalized size = 1.24

$$-\frac{ax-2}{4\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}+\frac{\log(ax+1)}{8\,ac^4}-\frac{\log(ax-1)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x - 2)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) + 1/8*log(a*x + 1)/(a*c^
4) - 1/8*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.82, size = 47, normalized size = 0.92

$$\frac{\operatorname{atanh}(a\,x)}{4\,a\,c^4}-\frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a*c*x)^4*(a*x + 1)^2),x)
```

```
[Out] atanh(a*x)/(4*a*c^4) - (x/4 - 1/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x)
```

sympy [A] time = 0.28, size = 56, normalized size = 1.10

$$-\frac{ax-2}{4a^3c^4x^2-8a^2c^4x+4ac^4}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**4,x)
```

```
[Out] -(a*x - 2)/(4*a**3*c**4*x**2 - 8*a**2*c**4*x + 4*a*c**4) - (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**4)
```

3.215 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=69

$$\frac{1}{8ac^5(1-ax)}+\frac{1}{8ac^5(1-ax)^2}+\frac{1}{6ac^5(1-ax)^3}+\frac{\tanh^{-1}(ax)}{8ac^5}$$

```
[Out] 1/6/a/c^5/(-a*x+1)^3+1/8/a/c^5/(-a*x+1)^2+1/8/a/c^5/(-a*x+1)+1/8*arctanh(a*
x)/a/c^5
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 44, 207}

$$\frac{1}{8ac^5(1-ax)}+\frac{1}{8ac^5(1-ax)^2}+\frac{1}{6ac^5(1-ax)^3}+\frac{\tanh^{-1}(ax)}{8ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^5),x]
```

```
[Out] 1/(6*a*c^5*(1 - a*x)^3) + 1/(8*a*c^5*(1 - a*x)^2) + 1/(8*a*c^5*(1 - a*x)) +
 ArcTanh[a*x]/(8*a*c^5)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\int \frac{1}{(1-ax)^4(1+ax)}\,dx}{c^5}$$
$$= \frac{\int \left(\frac{1}{2(-1+ax)^4} - \frac{1}{4(-1+ax)^3} + \frac{1}{8(-1+ax)^2} - \frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{c^5}$$
$$= \frac{1}{6ac^5(1-ax)^3} + \frac{1}{8ac^5(1-ax)^2} + \frac{1}{8ac^5(1-ax)} - \frac{\int \frac{1}{-1+a^2x^2}\,dx}{8c^5}$$
$$= \frac{1}{6ac^5(1-ax)^3} + \frac{1}{8ac^5(1-ax)^2} + \frac{1}{8ac^5(1-ax)} + \frac{\tanh^{-1}(ax)}{8ac^5}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.64

$$\frac{-3a^2x^2 + 9ax + 3(ax-1)^3\tanh^{-1}(ax) - 10}{24ac^5(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh[a*x])/(24*a*c^5*(-1 + a*x
)^3)
```

fricas [A] time = 0.51, size = 113, normalized size = 1.64

$$-\frac{6\,a^2x^2 - 18\,ax - 3\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\log\left(ax+1\right) + 3\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\log\left(ax-1\right) + 20}{48\left(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(6*a^2*x^2 - 18*a*x - 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x + 1
) + 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x - 1) + 20)/(a^4*c^5*x^3 - 3
*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

giac [A] time = 0.20, size = 89, normalized size = 1.29

$$\frac{\log\left(\left|-\frac{2\,c}{acx-c} - 1\right|\right)}{16\,ac^5} - \frac{\frac{3\,a^2c^2}{acx-c} - \frac{3\,a^2c^3}{(acx-c)^2} + \frac{4\,a^2c^4}{(acx-c)^3}}{24\,a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^5,x, algorithm="giac")

[Out] 1/16*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^5) - 1/24*(3*a^2*c^2/(a*c*x - c) - 3*a^2*c^3/(a*c*x - c)^2 + 4*a^2*c^4/(a*c*x - c)^3)/(a^3*c^6)

maple [A] time = 0.04, size = 75, normalized size = 1.09

$$-\frac{1}{6c^5a\,(ax-1)^3}+\frac{1}{8c^5a\,(ax-1)^2}-\frac{1}{8c^5a\,(ax-1)}-\frac{\ln(ax-1)}{16c^5a}+\frac{\ln(ax+1)}{16c^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^5,x)

[Out] -1/6/c^5/a/(a*x-1)^3+1/8/c^5/a/(a*x-1)^2-1/8/c^5/a/(a*x-1)-1/16/c^5/a*ln(a*x-1)+1/16/c^5/a*ln(a*x+1)

maxima [A] time = 0.35, size = 84, normalized size = 1.22

$$-\frac{3\,a^2x^2-9\,ax+10}{24\left(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5\right)}+\frac{\log(ax+1)}{16\,ac^5}-\frac{\log(ax-1)}{16\,ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^5,x, algorithm="maxima")

[Out] -1/24*(3*a^2*x^2 - 9*a*x + 10)/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5) + 1/16*log(a*x + 1)/(a*c^5) - 1/16*log(a*x - 1)/(a*c^5)

mupad [B] time = 0.83, size = 64, normalized size = 0.93

$$\frac{\frac{a\,x^2}{8}-\frac{3\,x}{8}+\frac{5}{12\,a}}{-a^3\,c^5\,x^3+3\,a^2\,c^5\,x^2-3\,a\,c^5\,x+c^5}+\frac{\mathrm{atanh}\left(a\,x\right)}{8\,a\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a^2*x^2 - 1)/((c - a*c*x)^5*(a*x + 1)^2),x)

[Out] ((a*x^2)/8 - (3*x)/8 + 5/(12*a))/(c^5 + 3*a^2*c^5*x^2 - a^3*c^5*x^3 - 3*a*c^5*x) + atanh(a*x)/(8*a*c^5)

sympy [A] time = 0.37, size = 76, normalized size = 1.10

$$\frac{-3a^2x^2+9ax-10}{24a^4c^5x^3-72a^3c^5x^2+72a^2c^5x-24ac^5}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16}+\frac{\log\left(x+\frac{1}{a}\right)}{16}}{ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**5,x)
```

```
[Out] (-3*a**2*x**2 + 9*a*x - 10)/(24*a**4*c**5*x**3 - 72*a**3*c**5*x**2 + 72*a**
2*c**5*x - 24*a*c**5) + (-log(x - 1/a)/16 + log(x + 1/a)/16)/(a*c**5)
```

3.216 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=65

$$-\frac{(1-ax)^{3/2}(c-acx)^{p+1}\,{}_2F_1\left(\frac{3}{2},p+\frac{5}{2};p+\frac{7}{2};\frac{1}{2}(1-ax)\right)}{\sqrt{2}\,ac(2p+5)}$$

```
[Out] -1/2*(-a*x+1)^(3/2)*(-a*c*x+c)^(1+p)*hypergeom([3/2, 5/2+p],[7/2+p],-1/2*a*
x+1/2)/a/c/(5+2*p)*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 23, 69}

$$-\frac{(1-ax)^{3/2}(c-acx)^{p+1}\,{}_2F_1\left(\frac{3}{2},p+\frac{5}{2};p+\frac{7}{2};\frac{1}{2}(1-ax)\right)}{\sqrt{2}\,ac(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(((1 - a*x)^(3/2)*(c - a*c*x)^(1 + p)*Hypergeometric2F1[3/2, 5/2 + p, 7/2
+ p, (1 - a*x)/2])/(Sqrt[2]*a*c*(5 + 2*p)))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^p\,dx=\int\frac{(1-ax)^{3/2}(c-acx)^p}{(1+ax)^{3/2}}\,dx$$
$$=\frac{(1-ax)^{3/2}\int\frac{(c-acx)^{\frac{3}{2}+p}}{(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$=-\frac{(1-ax)^{3/2}(c-acx)^{1+p}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2}+p;\frac{7}{2}+p;\frac{1}{2}(1-ax)\right)}{\sqrt{2}\,ac(5+2p)}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.92

$$-\frac{(1-ax)^{5/2}(c-acx)^p\,{}_2F_1\left(\frac{3}{2},p+\frac{5}{2};p+\frac{7}{2};\frac{1}{2}-\frac{ax}{2}\right)}{\sqrt{2}\,a(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(((1 - a*x)^(5/2)*(c - a*c*x)^p*Hypergeometric2F1[3/2, 5/2 + p, 7/2 + p, 1
/2 - (a*x)/2])/(Sqrt[2]*a*(5 + 2*p)))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(ax-1)(-acx+c)^p}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*(-a*c*x + c)^p/(a^2*x^2 + 2*a*x + 1)
, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(-acx+c)^{p} \left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{(ax+1)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} (-acx+c)^{p}}{(ax+1)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(-a*c*x + c)^p/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(1-a^{2}\, x^{2}\right)^{3/2}\, (c-a\,c\,x)^{p}}{(a\,x+1)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^p)/(a*x + 1)^3,x)
```

```
[Out] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^p)/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1))^{p}\, (-(ax-1)\,(ax+1))^{\frac{3}{2}}}{(ax+1)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
[Out] Integral((-c*(a*x - 1))**p*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

3.217 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=163

$$-\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}}-\frac{9c^3\sqrt{1-a^2x^2}\,(1-ax)^3}{4a}-\frac{21c^3\sqrt{1-a^2x^2}\,(1-ax)^2}{4a}-\frac{105c^3\sqrt{1-a^2x^2}\,(1-ax)}{8a}-\frac{315c^3\sqrt{1-a^2x^2}}{8a}-3$$

```
[Out] -315/8*c^3*arcsin(a*x)/a-2*c^3*(-a*x+1)^5/a/(-a^2*x^2+1)^(1/2)-315/8*c^3*(-
a^2*x^2+1)^(1/2)/a-105/8*c^3*(-a*x+1)*(-a^2*x^2+1)^(1/2)/a-21/4*c^3*(-a*x+1
)^2*(-a^2*x^2+1)^(1/2)/a-9/4*c^3*(-a*x+1)^3*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6127, 669, 671, 641, 216}

$$-\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}}-\frac{9c^3\sqrt{1-a^2x^2}\,(1-ax)^3}{4a}-\frac{21c^3\sqrt{1-a^2x^2}\,(1-ax)^2}{4a}-\frac{105c^3\sqrt{1-a^2x^2}\,(1-ax)}{8a}-\frac{315c^3\sqrt{1-a^2x^2}}{8a}-3$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^3*(1 - a*x)^5)/(a*Sqrt[1 - a^2*x^2]) - (315*c^3*Sqrt[1 - a^2*x^2])/(8
*a) - (105*c^3*(1 - a*x)*Sqrt[1 - a^2*x^2])/(8*a) - (21*c^3*(1 - a*x)^2*Sqr
t[1 - a^2*x^2])/(4*a) - (9*c^3*(1 - a*x)^3*Sqrt[1 - a^2*x^2])/(4*a) - (315*
c^3*ArcSin[a*x])/(8*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
```

```
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^3\,dx = \frac{\int \frac{(c-acx)^6}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{9\int \frac{(c-acx)^4}{\sqrt{1-a^2x^2}}\,dx}{c}$$

$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{9c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a} - \frac{63}{4}\int \frac{(c-acx)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{21c^3(1-ax)^2\sqrt{1-a^2x^2}}{4a} - \frac{9c^3(1-ax)^3\sqrt{1-a^2x^2}}{4a} - \frac{1}{4}(105c)\int \frac{(c}{\sqrt{}}$$

$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{105c^3(1-ax)\sqrt{1-a^2x^2}}{8a} - \frac{21c^3(1-ax)^2\sqrt{1-a^2x^2}}{4a} - \frac{9c^3(1-ax)^3}{4a}$$

$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{315c^3\sqrt{1-a^2x^2}}{8a} - \frac{105c^3(1-ax)\sqrt{1-a^2x^2}}{8a} - \frac{21c^3(1-ax)^2\sqrt{1-}}{4a}$$

$$= -\frac{2c^3(1-ax)^5}{a\sqrt{1-a^2x^2}} - \frac{315c^3\sqrt{1-a^2x^2}}{8a} - \frac{105c^3(1-ax)\sqrt{1-a^2x^2}}{8a} - \frac{21c^3(1-ax)^2\sqrt{1-}}{4a}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.28

$$-\frac{c^3(1-ax)^{11/2}\,{}_2F_1\left(\frac{3}{2},\frac{11}{2};\frac{13}{2};\frac{1}{2}(1-ax)\right)}{11\sqrt{2}\,a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/11*(c^3*(1 - a*x)^(11/2)*Hypergeometric2F1[3/2, 11/2, 13/2, (1 - a*x)/2]
)/(Sqrt[2]*a)
```

fricas [A] time = 0.51, size = 118, normalized size = 0.72

$$-\frac{496\,ac^3x+496\,c^3-630\left(ac^3x+c^3\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^4c^3x^4-14\,a^3c^3x^3+51\,a^2c^3x^2-173\,ac^3x-496\right.}{8\left(a^2x+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(496*a*c^3*x + 496*c^3 - 630*(a*c^3*x + c^3)*arctan((sqrt(-a^2*x^2 + 1
) - 1)/(a*x)) - (2*a^4*c^3*x^4 - 14*a^3*c^3*x^3 + 51*a^2*c^3*x^2 - 173*a*c^
3*x - 496*c^3)*sqrt(-a^2*x^2 + 1))/(a^2*x + a)
```

giac [A] time = 0.25, size = 103, normalized size = 0.63

$$-\frac{315\,c^3\arcsin\left(ax\right)\mathrm{sgn}(a)}{8\,|a|}-\frac{1}{8}\sqrt{-a^2x^2+1}\left(\frac{240\,c^3}{a}-\left(67\,c^3+2\left(a^2c^3x-8\,ac^3\right)x\right)x\right)+\frac{64\,c^3}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -315/8*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/8*sqrt(-a^2*x^2 + 1)*(240*c^3/a -
(67*c^3 + 2*(a^2*c^3*x - 8*a*c^3)*x)*x) + 64*c^3/(((sqrt(-a^2*x^2 + 1)*abs(
a) + a)/(a^2*x) + 1)*abs(a))
```

maple [A] time = 0.05, size = 245, normalized size = 1.50

$$-\frac{c^3x\left(-a^2x^2+1\right)^{\frac{3}{2}}}{4}-\frac{3c^3x\sqrt{-a^2x^2+1}}{8}-\frac{3c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8\sqrt{a^2}}-\frac{28c^3\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^3\left(x+\frac{1}{a}\right)^2}-\frac{26c^3\left(-a^2\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/4*c^3*x*(-a^2*x^2+1)^(3/2)-3/8*c^3*x*(-a^2*x^2+1)^(1/2)-3/8*c^3/(a^2)^(1
/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-28*c^3/a^3/(x+1/a)^2*(-a^2*(x+
1/a)^2+2*a*(x+1/a))^(5/2)-26*c^3/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-39*c^
3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-39*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2
)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-8*c^3/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2
+2*a*(x+1/a))^(5/2)
```

maxima [C] time = 0.45, size = 233, normalized size = 1.43

$$-\frac{1}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3x+3\sqrt{a^2x^2+4\,ax+3}\,c^3x-\frac{3}{8}\sqrt{-a^2x^2+1}\,c^3x+\frac{8\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3}{a^3x^2+2\,a^2x+a}-\frac{6\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3}{a^2x+a}+\frac{2\left(-a^2x^2\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-a^2*x^2 + 1)^(3/2)*c^3*x + 3*sqrt(a^2*x^2 + 4*a*x + 3)*c^3*x - 3/8*s
qrt(-a^2*x^2 + 1)*c^3*x + 8*(-a^2*x^2 + 1)^(3/2)*c^3/(a^3*x^2 + 2*a^2*x + a
) - 6*(-a^2*x^2 + 1)^(3/2)*c^3/(a^2*x + a) + 2*(-a^2*x^2 + 1)^(3/2)*c^3/a -
 3*I*c^3*arcsin(a*x + 2)/a - 339/8*c^3*arcsin(a*x)/a - 48*sqrt(-a^2*x^2 + 1
)*c^3/(a^2*x + a) + 6*sqrt(a^2*x^2 + 4*a*x + 3)*c^3/a - 18*sqrt(-a^2*x^2 +
1)*c^3/a
```

mupad [B] time = 0.82, size = 166, normalized size = 1.02

$$\frac{32\,c^3\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{315\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{4\,a^3\,c^3}{{\left(-a^2\right)}^{3/2}}-\frac{67\,c^3\,x\,\sqrt{-a^2}}{8}-\frac{26\,a\,c^3}{\sqrt{-a^2}}+\frac{c^3\,x^3\,{\left(-a^2\right)}^{3/2}}{4}+\frac{2\,a^5\,c^3}{\left(-a^2\right)}\right.}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^3)/(a*x + 1)^3,x)
```

```
[Out] (32*c^3*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2
)) - (315*c^3*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(1/2)) - ((1 - a^2*x^2)^(1/2
)*((4*a^3*c^3)/(-a^2)^(3/2) - (67*c^3*x*(-a^2)^(1/2))/8 - (26*a*c^3)/(-a^2)
^(1/2) + (c^3*x^3*(-a^2)^(3/2))/4 + (2*a^5*c^3*x^2)/(-a^2)^(3/2)))/(-a^2)^(
1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\left(-\frac{\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\right)dx+\int\frac{3ax\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int\left(-\frac{2a^2x^2\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\right)dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -c**3*(Integral(-sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1)
, x) + Integral(3*a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x
 + 1), x) + Integral(-2*a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*
x**2 + 3*a*x + 1), x) + Integral(-2*a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x*
*3 + 3*a**2*x**2 + 3*a*x + 1), x) + Integral(3*a**4*x**4*sqrt(-a**2*x**2 +
1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x) + Integral(-a**5*x**5*sqrt(-a*
*2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x))
```

3.218 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=131

$$-\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}}-\frac{7c^2\sqrt{1-a^2x^2}\,(1-ax)^2}{3a}-\frac{35c^2\sqrt{1-a^2x^2}\,(1-ax)}{6a}-\frac{35c^2\sqrt{1-a^2x^2}}{2a}-\frac{35c^2\sin^{-1}(ax)}{2a}$$

```
[Out] -35/2*c^2*arcsin(a*x)/a-2*c^2*(-a*x+1)^4/a/(-a^2*x^2+1)^(1/2)-35/2*c^2*(-a^
2*x^2+1)^(1/2)/a-35/6*c^2*(-a*x+1)*(-a^2*x^2+1)^(1/2)/a-7/3*c^2*(-a*x+1)^2*
(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6127, 669, 671, 641, 216}

$$-\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}}-\frac{7c^2\sqrt{1-a^2x^2}\,(1-ax)^2}{3a}-\frac{35c^2\sqrt{1-a^2x^2}\,(1-ax)}{6a}-\frac{35c^2\sqrt{1-a^2x^2}}{2a}-\frac{35c^2\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^2*(1 - a*x)^4)/(a*Sqrt[1 - a^2*x^2]) - (35*c^2*Sqrt[1 - a^2*x^2])/(2*
a) - (35*c^2*(1 - a*x)*Sqrt[1 - a^2*x^2])/(6*a) - (7*c^2*(1 - a*x)^2*Sqrt[1
 - a^2*x^2])/(3*a) - (35*c^2*ArcSin[a*x])/(2*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^2\,dx=\frac{\int \frac{(c-acx)^5}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$
$$=-\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}}-\frac{7\int \frac{(c-acx)^3}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$=-\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}}-\frac{7c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}-\frac{35}{3}\int \frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$=-\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}}-\frac{35c^2(1-ax)\sqrt{1-a^2x^2}}{6a}-\frac{7c^2(1-ax)^2\sqrt{1-a^2x^2}}{3a}-\frac{1}{2}(35c)\int \frac{c}{\sqrt{}}$$
$$=-\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}}-\frac{35c^2\sqrt{1-a^2x^2}}{2a}-\frac{35c^2(1-ax)\sqrt{1-a^2x^2}}{6a}-\frac{7c^2(1-ax)^2\sqrt{1-a^2}}{3a}$$
$$=-\frac{2c^2(1-ax)^4}{a\sqrt{1-a^2x^2}}-\frac{35c^2\sqrt{1-a^2x^2}}{2a}-\frac{35c^2(1-ax)\sqrt{1-a^2x^2}}{6a}-\frac{7c^2(1-ax)^2\sqrt{1-a^2}}{3a}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.34

$$-\frac{c^2(1-ax)^{9/2}\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1}{2}(1-ax)\right)}{9\sqrt{2}\,a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/9*(c^2*(1 - a*x)^(9/2)*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - a*x)/2])/(
Sqrt[2]*a)
```

fricas [A] time = 0.50, size = 106, normalized size = 0.81

$$-\frac{166\,ac^2x+166\,c^2-210\left(ac^2x+c^2\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3c^2x^3-13\,a^2c^2x^2+55\,ac^2x+166\,c^2\right)\sqrt{-a^2x^2+1}}{6\left(a^2x+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(166*a*c^2*x + 166*c^2 - 210*(a*c^2*x + c^2)*arctan((sqrt(-a^2*x^2 + 1
) - 1)/(a*x)) + (2*a^3*c^2*x^3 - 13*a^2*c^2*x^2 + 55*a*c^2*x + 166*c^2)*sqr
t(-a^2*x^2 + 1))/(a^2*x + a)
```

giac [A] time = 0.19, size = 91, normalized size = 0.69

$$-\frac{35\,c^2\arcsin\left(ax\right)\mathrm{sgn}(a)}{2\,|a|}-\frac{1}{6}\sqrt{-a^2x^2+1}\left(\left(2\,ac^2x-15\,c^2\right)x+\frac{70\,c^2}{a}\right)+\frac{32\,c^2}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -35/2*c^2*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x -
15*c^2)*x + 70*c^2/a) + 32*c^2/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) +
1)*abs(a))
```

maple [A] time = 0.04, size = 179, normalized size = 1.37

$$-\frac{12c^2\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^3\left(x+\frac{1}{a}\right)^2}-\frac{35c^2\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3a}-\frac{35c^2\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}\,x}{2}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -12*c^2/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-35/3*c^2/a*(-a^2*(
x+1/a)^2+2*a*(x+1/a))^(3/2)-35/2*c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-3
```

```
5/2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
)-4*c^2/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

maxima [C] time = 0.44, size = 196, normalized size = 1.50

$$\frac{1}{2}\sqrt{a^2x^2+4\,ax+3}\,c^2x+\frac{4\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{a^3x^2+2\,a^2x+a}-\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{a^2x+a}+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{3\,a}-\frac{i\,c^2\arcsin\left(ax+2\right)}{2\,a}-\frac{18\,c^2\,\mathrm{arcsi}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a^2*x^2 + 4*a*x + 3)*c^2*x + 4*(-a^2*x^2 + 1)^(3/2)*c^2/(a^3*x^2 +
 2*a^2*x + a) - 2*(-a^2*x^2 + 1)^(3/2)*c^2/(a^2*x + a) + 1/3*(-a^2*x^2 + 1)
^(3/2)*c^2/a - 1/2*I*c^2*arcsin(a*x + 2)/a - 18*c^2*arcsin(a*x)/a - 24*sqrt
(-a^2*x^2 + 1)*c^2/(a^2*x + a) + sqrt(a^2*x^2 + 4*a*x + 3)*c^2/a - 6*sqrt(-
a^2*x^2 + 1)*c^2/a
```

mupad [B] time = 0.06, size = 150, normalized size = 1.15

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{11\,a\,c^2}{\sqrt{-a^2}}+\frac{5\,c^2\,x\,\sqrt{-a^2}}{2}-\frac{2\,a^3\,c^2}{3\,{\left(-a^2\right)}^{3/2}}-\frac{a^5\,c^2\,x^2}{3\,{\left(-a^2\right)}^{3/2}}\right)}{\sqrt{-a^2}}-\frac{35\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}+\frac{16\,c^2\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^2)/(a*x + 1)^3,x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((11*a*c^2)/(-a^2)^(1/2) + (5*c^2*x*(-a^2)^(1/2))/2 -
(2*a^3*c^2)/(3*(-a^2)^(3/2)) - (a^5*c^2*x^2)/(3*(-a^2)^(3/2))))/(-a^2)^(1/2
) - (35*c^2*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2)) + (16*c^2*(1 - a^2*x^2)
^(1/2))/((x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\frac{\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int\left(-\frac{2ax\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\right)dx+\int\frac{2a^3x^3\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int\left(-\frac{}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c**2*(Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1),
x) + Integral(-2*a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x
+ 1), x) + Integral(2*a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x*
*2 + 3*a*x + 1), x) + Integral(-a**4*x**4*sqrt(-a**2*x**2 + 1)/(a**3*x**3 +
 3*a**2*x**2 + 3*a*x + 1), x))
```

3.219 $\int e^{-3\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=91

$$-\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}}-\frac{5c\sqrt{1-a^2x^2}\,(1-ax)}{2a}-\frac{15c\sqrt{1-a^2x^2}}{2a}-\frac{15c\sin^{-1}(ax)}{2a}$$

```
[Out] -15/2*c*arcsin(a*x)/a-2*c*(-a*x+1)^3/a/(-a^2*x^2+1)^(1/2)-15/2*c*(-a^2*x^2+
1)^(1/2)/a-5/2*c*(-a*x+1)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 669, 671, 641, 216}

$$-\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}}-\frac{5c\sqrt{1-a^2x^2}\,(1-ax)}{2a}-\frac{15c\sqrt{1-a^2x^2}}{2a}-\frac{15c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c*(1 - a*x)^3)/(a*Sqrt[1 - a^2*x^2]) - (15*c*Sqrt[1 - a^2*x^2])/(2*a) -
 (5*c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) - (15*c*ArcSin[a*x])/(2*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}(c-acx)\,dx &= \frac{\int \frac{(c-acx)^4}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\
&= -\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}}-\frac{5\int \frac{(c-acx)^2}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}}-\frac{5c(1-ax)\sqrt{1-a^2x^2}}{2a}-\frac{15}{2}\int \frac{c-acx}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}}-\frac{15c\sqrt{1-a^2x^2}}{2a}-\frac{5c(1-ax)\sqrt{1-a^2x^2}}{2a}-\frac{1}{2}(15c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{2c(1-ax)^3}{a\sqrt{1-a^2x^2}}-\frac{15c\sqrt{1-a^2x^2}}{2a}-\frac{5c(1-ax)\sqrt{1-a^2x^2}}{2a}-\frac{15c\sin^{-1}(ax)}{2a}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.47

$$
-\frac{c(1-ax)^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{1}{2}(1-ax)\right)}{7\sqrt{2}\,a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/7*(c*(1 - a*x)^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (1 - a*x)/2])/(Sqr
t[2]*a)
```

fricas [A] time = 0.45, size = 81, normalized size = 0.89

$$-\frac{24\,acx - 30\,(acx + c)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - \left(a^2cx^2 - 7\,acx - 24\,c\right)\sqrt{-a^2x^2+1} + 24\,c}{2\,\left(a^2x + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(24*a*c*x - 30*(a*c*x + c)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a
^2*c*x^2 - 7*a*c*x - 24*c)*sqrt(-a^2*x^2 + 1) + 24*c)/(a^2*x + a)
```

giac [A] time = 0.20, size = 73, normalized size = 0.80

$$-\frac{15\,c\arcsin\left(ax\right)\operatorname{sgn}(a)}{2\,|a|} + \frac{1}{2}\,\sqrt{-a^2x^2+1}\left(cx - \frac{8\,c}{a}\right) + \frac{16\,c}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -15/2*c*arcsin(a*x)*sgn(a)/abs(a) + 1/2*sqrt(-a^2*x^2 + 1)*(c*x - 8*c/a) +
16*c/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [B] time = 0.04, size = 169, normalized size = 1.86

$$-\frac{5c\left(-a^2\left(x+\frac{1}{a}\right)^2 + 2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^3\left(x+\frac{1}{a}\right)^2} - \frac{5c\left(-a^2\left(x+\frac{1}{a}\right)^2 + 2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{a} - \frac{15c\sqrt{-a^2\left(x+\frac{1}{a}\right)^2 + 2a\left(x+\frac{1}{a}\right)}\;x}{2} - \frac{15c\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -5*c/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-5*c/a*(-a^2*(x+1/a)^2
+2*a*(x+1/a))^(3/2)-15/2*c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-15/2*c/(a^2
)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-2*c/a^4/(x
+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

maxima [A] time = 0.45, size = 109, normalized size = 1.20

$$\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{a^3x^2 + 2\,a^2x + a} - \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{2\left(a^2x + a\right)} - \frac{15\,c\arcsin\left(ax\right)}{2\,a} - \frac{12\,\sqrt{-a^2x^2+1}\,c}{a^2x + a} - \frac{3\,\sqrt{-a^2x^2+1}\,c}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(-a^2*x^2 + 1)^(3/2)*c/(a^3*x^2 + 2*a^2*x + a) - 1/2*(-a^2*x^2 + 1)^(3/2)
*c/(a^2*x + a) - 15/2*c*arcsin(a*x)/a - 12*sqrt(-a^2*x^2 + 1)*c/(a^2*x + a)
 - 3/2*sqrt(-a^2*x^2 + 1)*c/a
```

mupad [B] time = 0.80, size = 96, normalized size = 1.05

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{4\,a\,c}{\sqrt{-a^2}}+\frac{c\,x\,\sqrt{-a^2}}{2}\right)-\frac{15\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2}+\frac{8\,c\,\sqrt{1-a^2\,x^2}}{x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}}}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x))/(a*x + 1)^3,x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((4*a*c)/(-a^2)^(1/2) + (c*x*(-a^2)^(1/2))/2) - (15*c*
asinh(x*(-a^2)^(1/2)))/2 + (8*c*(1 - a^2*x^2)^(1/2))/(x*(-a^2)^(1/2) + (-a^
2)^(1/2)/a))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(-\frac{\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\right)dx+\int\frac{ax\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -c*(Integral(-sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x
) + Integral(a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1)
, x) + Integral(a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3
*a*x + 1), x) + Integral(-a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**
2*x**2 + 3*a*x + 1), x))
```

3.220 $\int \frac{e^{-3\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=41

$$-\frac{2(1-ax)}{ac\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] -arcsin(a*x)/a/c-2*(-a*x+1)/a/c/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 653, 216}

$$-\frac{2(1-ax)}{ac\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)),x]
```

```
[Out] (-2*(1 - a*x))/(a*c*Sqrt[1 - a^2*x^2]) - ArcSin[a*x]/(a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{c-acx}\,dx = \frac{\int \frac{(c-acx)^2}{(1-a^2x^2)^{3/2}}\,dx}{c^3}$$
$$= -\frac{2(1-ax)}{ac\sqrt{1-a^2x^2}} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= -\frac{2(1-ax)}{ac\sqrt{1-a^2x^2}} - \frac{\sin^{-1}(ax)}{ac}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 1.44

$$\frac{2\left(\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)+ax-1\right)}{ac\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)),x]
```

```
[Out] (2*(-1 + a*x + Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*c*Sqrt[
1 - a^2*x^2])
```

fricas [A] time = 0.46, size = 60, normalized size = 1.46

$$-\frac{2\left(ax-(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}+1\right)}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -2*(a*x - (a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
+ 1) + 1)/(a^2*c*x + a*c)
```

giac [A] time = 0.27, size = 53, normalized size = 1.29

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}+\frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c*abs(a)) + 4/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2
*x) + 1)*abs(a))
```

maple [B] time = 0.05, size = 292, normalized size = 7.12

$$-\frac{\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{24ca}+\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}\, x}{16c}+\frac{\arctan\left(\frac{\sqrt{a^{2}}\, x}{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}\right)}{16c\sqrt{a^{2}}}-\frac{3\left(-a^{2}\left(x+\frac{1}{a}\right)^{2}+\right.}{4c\, a^{3}\left(x-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x)
```

```
[Out] -1/24/c/a*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+1/16/c*(-a^2*(x-1/a)^2-2*a*(x-
1/a))^(1/2)*x+1/16/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(
x-1/a))^(1/2))-3/4/c/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-17/24
/c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-17/16/c*(-a^2*(x+1/a)^2+2*a*(x+1/a)
)^(1/2)*x-17/16/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1
/a))^(1/2))-1/2/c/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{(acx-c)(ax+1)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)*(a*x + 1)^3), x)
```

mupad [B] time = 0.05, size = 70, normalized size = 1.71

$$\frac{2\sqrt{1-a^{2}x^{2}}}{c\left(x\sqrt{-a^{2}}+\frac{\sqrt{-a^{2}}}{a}\right)\sqrt{-a^{2}}}-\frac{\operatorname{asinh}\left(x\sqrt{-a^{2}}\right)}{c\sqrt{-a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)*(a*x + 1)^3),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(c*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- asinh(x*(-a^2)^(1/2))/(c*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^4x^4+2a^3x^3-2ax-1}\,dx + \int \left(-\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^4x^4+2a^3x^3-2ax-1}\right)dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(sqrt(-a**2*x**2 + 1)/(a**4*x**4 + 2*a**3*x**3 - 2*a*x - 1), x) +
 Integral(-a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**4*x**4 + 2*a**3*x**3 - 2*a*x
- 1), x))/c
```

3.221 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=28

$$-\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}$$

```
[Out] (a*x-1)/a/c^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 637}

$$-\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^2),x]
```

```
[Out] -((1 - a*x)/(a*c^2*Sqrt[1 - a^2*x^2]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{c-acx}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\ &= -\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.96

$$-\frac{\sqrt{1-ax}}{ac^2\sqrt{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^2),x]
```

```
[Out] -(Sqrt[1 - a*x]/(a*c^2*Sqrt[1 + a*x]))
```

fricas [A] time = 0.44, size = 35, normalized size = 1.25

$$-\frac{ax + \sqrt{-a^2x^2 + 1} + 1}{a^2c^2x + ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*c^2*x + a*c^2)
```

giac [C] time = 0.29, size = 69, normalized size = 2.46

$$c^2\left(\frac{i\,\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}{a^2c^4} + \frac{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}{a^2c^4\sqrt{-\frac{2c}{acx-c} - 1}}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] c^2*(I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^4) + sgn(1/(a*c*x - c))*sgn(
a)*sgn(c)/(a^2*c^4*sqrt(-2*c/(a*c*x - c) - 1)))*abs(a)
```

maple [A] time = 0.03, size = 34, normalized size = 1.21

$$\frac{\left(-a^2x^2 + 1\right)^{\frac{3}{2}}}{(ax - 1)\,c^2a\,(ax + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x)
```

```
[Out] (-a^2*x^2+1)^(3/2)/(a*x-1)/c^2/a/(a*x+1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2 + 1\right)^{\frac{3}{2}}}{(acx - c)^2(ax + 1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^2*(a*x + 1)^3), x)
```

mupad [B] time = 0.03, size = 46, normalized size = 1.64

$$\frac{\sqrt{1-a^2\,x^2}}{c^2\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^2*(a*x + 1)^3),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(c^2*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx + \int \left(-\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(sqrt(-a**2*x**2 + 1)/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**
2*x**2 + a*x + 1), x) + Integral(-a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5
 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x))/c**2
```

3.222 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x}{c^3\sqrt{1-a^2x^2}}$$

```
[Out] x/c^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 191}

$$\frac{x}{c^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^3),x]
```

```
[Out] x/(c^3*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^3}\,dx &= \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\ &= \frac{x}{c^3\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{x}{c^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^3),x]
```

```
[Out] x/(c^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.44, size = 33, normalized size = 1.74

$$-\frac{\sqrt{-a^2x^2+1}\,x}{a^2c^3x^2-c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] -sqrt(-a^2*x^2 + 1)*x/(a^2*c^3*x^2 - c^3)
```

giac [A] time = 0.30, size = 29, normalized size = 1.53

$$-\frac{\sqrt{-a^2x^2+1}\,x}{\left(a^2x^2-1\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*c^3)
```

maple [A] time = 0.03, size = 32, normalized size = 1.68

$$\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{\left(ax-1\right)^2c^3\left(ax+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x)
```

```
[Out] (-a^2*x^2+1)^(3/2)*x/(a*x-1)^2/c^3/(a*x+1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(acx-c\right)^3\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^3*(a*x + 1)^3), x)
```

mupad [B] time = 0.07, size = 17, normalized size = 0.89

$$\frac{x}{c^3\,\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^3*(a*x + 1)^3),x)
```

```
[Out] x/(c^3*(1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx + \int \left(-\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^6x^6-3a^4x^4+3a^2x^2-1}\right)dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(sqrt(-a**2*x**2 + 1)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1)
, x) + Integral(-a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**6*x**6 - 3*a**4*x**4 +
3*a**2*x**2 - 1), x))/c**3
```

3.223 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=55

$$\frac{2x}{3c^4\sqrt{1-a^2x^2}}+\frac{1}{3ac^4(1-ax)\sqrt{1-a^2x^2}}$$

```
[Out] 2/3*x/c^4/(-a^2*x^2+1)^(1/2)+1/3/a/c^4/(-a*x+1)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 191}

$$\frac{2x}{3c^4\sqrt{1-a^2x^2}}+\frac{1}{3ac^4(1-ax)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (2*x)/(3*c^4*Sqrt[1 - a^2*x^2]) + 1/(3*a*c^4*(1 - a*x)*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{1}{(c-acx)\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= \frac{1}{3ac^4(1-ax)\sqrt{1-a^2x^2}} + \frac{2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}$$

$$= \frac{2x}{3c^4\sqrt{1-a^2x^2}} + \frac{1}{3ac^4(1-ax)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.82

$$\frac{2a^2x^2-2ax-1}{3ac^4(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (-1 - 2*a*x + 2*a^2*x^2)/(3*a*c^4*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.53, size = 89, normalized size = 1.62

$$\frac{a^3x^3 - a^2x^2 - ax - \left(2\,a^2x^2 - 2\,ax - 1\right)\sqrt{-a^2x^2+1} + 1}{3\left(a^4c^4x^3 - a^3c^4x^2 - a^2c^4x + ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/3*(a^3*x^3 - a^2*x^2 - a*x - (2*a^2*x^2 - 2*a*x - 1)*sqrt(-a^2*x^2 + 1) +
 1)/(a^4*c^4*x^3 - a^3*c^4*x^2 - a^2*c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^4(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^4*(a*x + 1)^3), x)
```

maple [A] time = 0.03, size = 49, normalized size = 0.89

$$\frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \left(2 a^{2} x^{2}-2 a x -1\right)}{3 \left(a x -1\right)^{3} c^{4} a \left(a x +1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x)
[Out] 1/3*(-a^2*x^2+1)^(3/2)*(2*a^2*x^2-2*a*x-1)/(a*x-1)^3/c^4/a/(a*x+1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{\left(a c x - c\right)^{4} \left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima"
)
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^4*(a*x + 1)^3), x)
```

mupad [B] time = 0.83, size = 48, normalized size = 0.87

$$\frac{\sqrt{1-a^2\,x^2}\,\left(-2\,a^2\,x^2+2\,a\,x+1\right)}{3\,a\,c^4\,{\left(a\,x-1\right)}^2\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^4*(a*x + 1)^3),x)
[Out] ((1 - a^2*x^2)^(1/2)*(2*a*x - 2*a^2*x^2 + 1))/(3*a*c^4*(a*x - 1)^2*(a*x + 1
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{- a^{2} x^{2} + 1}}{a^{7} x^{7} - a^{6} x^{6} - 3 a^{5} x^{5} + 3 a^{4} x^{4} + 3 a^{3} x^{3} - 3 a^{2} x^{2} - a x + 1}\, dx + \int \left(- \frac{a^{2} x^{2} \sqrt{- a^{2} x^{2} + 1}}{a^{7} x^{7} - a^{6} x^{6} - 3 a^{5} x^{5} + 3 a^{4} x^{4} + 3 a^{3} x^{3} - 3 a^{2} x^{2} - a x + 1}\right)\, dx}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(sqrt(-a**2*x**2 + 1)/(a**7*x**7 - a**6*x**6 - 3*a**5*x**5 + 3*a**
4*x**4 + 3*a**3*x**3 - 3*a**2*x**2 - a*x + 1), x) + Integral(-a**2*x**2*sqr
t(-a**2*x**2 + 1)/(a**7*x**7 - a**6*x**6 - 3*a**5*x**5 + 3*a**4*x**4 + 3*a*
*3*x**3 - 3*a**2*x**2 - a*x + 1), x))/c**4
```

3.224 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=87

$$\frac{2x}{5c^5\sqrt{1-a^2x^2}}+\frac{1}{5ac^5(1-ax)\sqrt{1-a^2x^2}}+\frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}}$$

```
[Out] 2/5*x/c^5/(-a^2*x^2+1)^(1/2)+1/5/a/c^5/(-a*x+1)^2/(-a^2*x^2+1)^(1/2)+1/5/a/
c^5/(-a*x+1)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 191}

$$\frac{2x}{5c^5\sqrt{1-a^2x^2}}+\frac{1}{5ac^5(1-ax)\sqrt{1-a^2x^2}}+\frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (2*x)/(5*c^5*Sqrt[1 - a^2*x^2]) + 1/(5*a*c^5*(1 - a*x)^2*Sqrt[1 - a^2*x^2])
 + 1/(5*a*c^5*(1 - a*x)*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^5}\,dx = \frac{\int \frac{1}{(c-acx)^2(1-a^2x^2)^{3/2}}\,dx}{c^3}$$
$$= \frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}} + \frac{3\int \frac{1}{(c-acx)(1-a^2x^2)^{3/2}}\,dx}{5c^4}$$
$$= \frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}} + \frac{1}{5ac^5(1-ax)\sqrt{1-a^2x^2}} + \frac{2\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{5c^5}$$
$$= \frac{2x}{5c^5\sqrt{1-a^2x^2}} + \frac{1}{5ac^5(1-ax)^2\sqrt{1-a^2x^2}} + \frac{1}{5ac^5(1-ax)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.60

$$\frac{2a^3x^3-4a^2x^2+ax+2}{5ac^5(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(5*a*c^5*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.45, size = 98, normalized size = 1.13

$$\frac{2\,a^4x^4-4\,a^3x^3+4\,ax-\left(2\,a^3x^3-4\,a^2x^2+ax+2\right)\sqrt{-a^2x^2+1}-2}{5\left(a^5c^5x^4-2\,a^4c^5x^3+2\,a^2c^5x-ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas"
)
```

```
[Out] 1/5*(2*a^4*x^4 - 4*a^3*x^3 + 4*a*x - (2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt
(-a^2*x^2 + 1) - 2)/(a^5*c^5*x^4 - 2*a^4*c^5*x^3 + 2*a^2*c^5*x - a*c^5)
```

giac [C] time = 0.34, size = 174, normalized size = 2.00

$$\frac{1}{40}\left(a\left(\frac{5}{a^3c^7\sqrt{-\frac{2\,c}{acx-c}-1}} - \frac{a^{12}c^{28}\left(\frac{2\,c}{acx-c}+1\right)^2\sqrt{-\frac{2\,c}{acx-c}-1}+5\,a^{12}c^{28}\left(-\frac{2\,c}{acx-c}-1\right)^{\frac{3}{2}}+15\,a^{12}c^{28}\sqrt{-\frac{2\,c}{acx-c}-1}}{a^{15}c^{35}}\right)\mathrm{sgn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/40*(a*(5/(a^3*c^7*sqrt(-2*c/(a*c*x - c) - 1)) - (a^12*c^28*(2*c/(a*c*x -
c) + 1)^2*sqrt(-2*c/(a*c*x - c) - 1) + 5*a^12*c^28*(-2*c/(a*c*x - c) - 1)^(
3/2) + 15*a^12*c^28*sqrt(-2*c/(a*c*x - c) - 1))/(a^15*c^35))*sgn(1/(a*c*x -
 c))*sgn(a)*sgn(c) + 16*I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/(a^2*c^7))*c^2*a
bs(a)
```

maple [A] time = 0.03, size = 56, normalized size = 0.64

$$\frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \left(2 x^{3} a^{3}-4 a^{2} x^{2}+a x +2\right)}{5 \left(a x -1\right)^{4} c^{5} a \left(a x +1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x)
```

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)*(2*a^3*x^3-4*a^2*x^2+a*x+2)/(a*x-1)^4/c^5/a/(a*x+1)^
2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{\left(a c x - c\right)^{5} \left(a x + 1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima"
)
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^5*(a*x + 1)^3), x)
```

mupad [B] time = 0.83, size = 233, normalized size = 2.68

$$\frac{3\,a\,\sqrt{1-a^2\,x^2}}{20\,\left(a^4\,c^5\,x^2-2\,a^3\,c^5\,x+a^2\,c^5\right)}+\frac{\sqrt{1-a^2\,x^2}}{8\,\sqrt{-a^2}\,\left(c^5\,x\,\sqrt{-a^2}+\frac{c^5\,\sqrt{-a^2}}{a}\right)}+\frac{11\,\sqrt{1-a^2\,x^2}}{40\,\sqrt{-a^2}\,\left(c^5\,x\,\sqrt{-a^2}-\frac{c^5\,\sqrt{-a^2}}{a}\right)}+\frac{}{10\,\sqrt{-a^2}\,\left(3\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^5*(a*x + 1)^3),x)
```

```
[Out] (3*a*(1 - a^2*x^2)^(1/2))/(20*(a^2*c^5 - 2*a^3*c^5*x + a^4*c^5*x^2)) + (1 -
 a^2*x^2)^(1/2)/(8*(-a^2)^(1/2)*(c^5*x*(-a^2)^(1/2) + (c^5*(-a^2)^(1/2))/a)
```

```
) + (11*(1 - a^2*x^2)^(1/2))/(40*(-a^2)^(1/2)*(c^5*x*(-a^2)^(1/2) - (c^5*(-
a^2)^(1/2))/a)) + (1 - a^2*x^2)^(1/2)/(10*(-a^2)^(1/2)*(3*c^5*x*(-a^2)^(1/2
) - (c^5*(-a^2)^(1/2))/a + a^2*c^5*x^3*(-a^2)^(1/2) - 3*a*c^5*x^2*(-a^2)^(1
/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1}\,dx+\int \left(-\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1}\right)dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] -(Integral(sqrt(-a**2*x**2 + 1)/(a**8*x**8 - 2*a**7*x**7 - 2*a**6*x**6 + 6*
a**5*x**5 - 6*a**3*x**3 + 2*a**2*x**2 + 2*a*x - 1), x) + Integral(-a**2*x**
2*sqrt(-a**2*x**2 + 1)/(a**8*x**8 - 2*a**7*x**7 - 2*a**6*x**6 + 6*a**5*x**5
 - 6*a**3*x**3 + 2*a**2*x**2 + 2*a*x - 1), x))/c**5
```

3.225 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^6}\,dx$

Optimal. Leaf size=119

$$\frac{8x}{35c^6\sqrt{1-a^2x^2}}+\frac{4}{35ac^6(1-ax)\sqrt{1-a^2x^2}}+\frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}}+\frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}}$$

```
[Out] 8/35*x/c^6/(-a^2*x^2+1)^(1/2)+1/7/a/c^6/(-a*x+1)^3/(-a^2*x^2+1)^(1/2)+4/35/
a/c^6/(-a*x+1)^2/(-a^2*x^2+1)^(1/2)+4/35/a/c^6/(-a*x+1)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 659, 191}

$$\frac{8x}{35c^6\sqrt{1-a^2x^2}}+\frac{4}{35ac^6(1-ax)\sqrt{1-a^2x^2}}+\frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}}+\frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (8*x)/(35*c^6*Sqrt[1 - a^2*x^2]) + 1/(7*a*c^6*(1 - a*x)^3*Sqrt[1 - a^2*x^2]
) + 4/(35*a*c^6*(1 - a*x)^2*Sqrt[1 - a^2*x^2]) + 4/(35*a*c^6*(1 - a*x)*Sqrt
[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{\int \frac{1}{(c-acx)^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= \frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4\int \frac{1}{(c-acx)^2\left(1-a^2x^2\right)^{3/2}}\,dx}{7c^4}$$

$$= \frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}} + \frac{12\int \frac{1}{(c-acx)\left(1-a^2x^2\right)^{3/2}}\,dx}{35c^5}$$

$$= \frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)\sqrt{1-a^2x^2}} + \frac{8\int \frac{}{(1-}}{3}$$

$$= \frac{8x}{35c^6\sqrt{1-a^2x^2}} + \frac{1}{7ac^6(1-ax)^3\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)^2\sqrt{1-a^2x^2}} + \frac{4}{35ac^6(1-ax)\sqrt{1}}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.51

$$\frac{8a^4x^4-24a^3x^3+20a^2x^2+4ax-13}{35ac^6(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4)/(35*a*c^6*(-1 + a*x)^3*
Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.52, size = 144, normalized size = 1.21

$$\frac{13\,a^5x^5-39\,a^4x^4+26\,a^3x^3+26\,a^2x^2-39\,ax-\left(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13\right)\sqrt{-a^2x^2+1}+13}{35\left(a^6c^6x^5-3\,a^5c^6x^4+2\,a^4c^6x^3+2\,a^3c^6x^2-3\,a^2c^6x+ac^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^6,x, algorithm="fricas"
)
```

```
[Out] 1/35*(13*a^5*x^5 - 39*a^4*x^4 + 26*a^3*x^3 + 26*a^2*x^2 - 39*a*x - (8*a^4*x
^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt(-a^2*x^2 + 1) + 13)/(a^6*c^
```

```
6*x^5 - 3*a^5*c^6*x^4 + 2*a^4*c^6*x^3 + 2*a^3*c^6*x^2 - 3*a^2*c^6*x + a*c^6
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^6(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^6,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^6*(a*x + 1)^3), x)
```

maple [A] time = 0.03, size = 65, normalized size = 0.55

$$\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(8x^4a^4-24x^3a^3+20a^2x^2+4ax-13\right)}{35\,(ax-1)^5\,c^6a\,(ax+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^6,x)
```

```
[Out] 1/35*(-a^2*x^2+1)^(3/2)*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/(a*x-1)^
5/c^6/a/(a*x+1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(acx-c)^6(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^6,x, algorithm="maxima"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*c*x - c)^6*(a*x + 1)^3), x)
```

mupad [B] time = 0.82, size = 347, normalized size = 2.92

$$\frac{3\,a\,\sqrt{1-a^2\,x^2}}{40\,\left(a^4\,c^6\,x^2-2\,a^3\,c^6\,x+a^2\,c^6\right)}+\frac{a^3\,\sqrt{1-a^2\,x^2}}{35\,\left(a^6\,c^6\,x^2-2\,a^5\,c^6\,x+a^4\,c^6\right)}+\frac{a\,\sqrt{1-a^2\,x^2}}{14\,\left(a^6\,c^6\,x^4-4\,a^5\,c^6\,x^3+6\,a^4\,c^6\,x^2-4\,a^3\,c^6\,x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^6*(a*x + 1)^3),x)
```

```
[Out] (3*a*(1 - a^2*x^2)^(1/2))/(40*(a^2*c^6 - 2*a^3*c^6*x + a^4*c^6*x^2)) + (a^3
*(1 - a^2*x^2)^(1/2))/(35*(a^4*c^6 - 2*a^5*c^6*x + a^6*c^6*x^2)) + (a*(1 -
a^2*x^2)^(1/2))/(14*(a^2*c^6 - 4*a^3*c^6*x + 6*a^4*c^6*x^2 - 4*a^5*c^6*x^3
+ a^6*c^6*x^4)) + (1 - a^2*x^2)^(1/2)/(16*(-a^2)^(1/2)*(c^6*x*(-a^2)^(1/2)
+ (c^6*(-a^2)^(1/2))/a)) + (93*(1 - a^2*x^2)^(1/2))/(560*(-a^2)^(1/2)*(c^6*
x*(-a^2)^(1/2) - (c^6*(-a^2)^(1/2))/a)) + (13*(1 - a^2*x^2)^(1/2))/(140*(-a
^2)^(1/2)*(3*c^6*x*(-a^2)^(1/2) - (c^6*(-a^2)^(1/2))/a + a^2*c^6*x^3*(-a^2)
^(1/2) - 3*a*c^6*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^9x^9-3a^8x^8+8a^6x^6-6a^5x^5-6a^4x^4+8a^3x^3-3ax+1}\,dx + \int \left(-\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^9x^9-3a^8x^8+8a^6x^6-6a^5x^5-6a^4x^4+8a^3x^3-3ax+1}\right)dx}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**6,x)
```

```
[Out] (Integral(sqrt(-a**2*x**2 + 1)/(a**9*x**9 - 3*a**8*x**8 + 8*a**6*x**6 - 6*a
**5*x**5 - 6*a**4*x**4 + 8*a**3*x**3 - 3*a*x + 1), x) + Integral(-a**2*x**2
*sqrt(-a**2*x**2 + 1)/(a**9*x**9 - 3*a**8*x**8 + 8*a**6*x**6 - 6*a**5*x**5
- 6*a**4*x**4 + 8*a**3*x**3 - 3*a*x + 1), x))/c**6
```

3.226 $\int e^{\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=176

$$\frac{4096c^6\left(1-a^2x^2\right)^{3/2}}{3465a(c-acx)^{3/2}}+\frac{1024c^5\left(1-a^2x^2\right)^{3/2}}{1155a\sqrt{c-acx}}+\frac{128c^4\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{231a}+\frac{32c^3\left(1-a^2x^2\right)^{3/2}(c-acx)^{3/2}}{99a}+\frac{2c^2\left(1\right.}{}$$

```
[Out] 4096/3465*c^6*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(3/2)+32/99*c^3*(-a*c*x+c)^(3
/2)*(-a^2*x^2+1)^(3/2)/a+2/11*c^2*(-a*c*x+c)^(5/2)*(-a^2*x^2+1)^(3/2)/a+102
4/1155*c^5*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(1/2)+128/231*c^4*(-a^2*x^2+1)^(
3/2)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 657, 649}

$$\frac{4096c^6\left(1-a^2x^2\right)^{3/2}}{3465a(c-acx)^{3/2}}+\frac{1024c^5\left(1-a^2x^2\right)^{3/2}}{1155a\sqrt{c-acx}}+\frac{128c^4\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{231a}+\frac{32c^3\left(1-a^2x^2\right)^{3/2}(c-acx)^{3/2}}{99a}+\frac{2c^2\left(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (4096*c^6*(1 - a^2*x^2)^(3/2))/(3465*a*(c - a*c*x)^(3/2)) + (1024*c^5*(1 -
a^2*x^2)^(3/2))/(1155*a*Sqrt[c - a*c*x]) + (128*c^4*Sqrt[c - a*c*x]*(1 - a^
2*x^2)^(3/2))/(231*a) + (32*c^3*(c - a*c*x)^(3/2)*(1 - a^2*x^2)^(3/2))/(99*
a) + (2*c^2*(c - a*c*x)^(5/2)*(1 - a^2*x^2)^(3/2))/(11*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx = c\int (c-acx)^{7/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{2c^2(c-acx)^{5/2}\left(1-a^2x^2\right)^{3/2}}{11a} + \frac{1}{11}\left(16c^2\right)\int (c-acx)^{5/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{32c^3(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{99a} + \frac{2c^2(c-acx)^{5/2}\left(1-a^2x^2\right)^{3/2}}{11a} + \frac{1}{33}\left(64c^3\right)\int (c-ac$$
$$= \frac{128c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{231a} + \frac{32c^3(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{99a} + \frac{2c^2(c-acx)^{5/2}(1-}{11a}$$
$$= \frac{1024c^5\left(1-a^2x^2\right)^{3/2}}{1155a\sqrt{c-acx}} + \frac{128c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{231a} + \frac{32c^3(c-acx)^{3/2}\left(1-a^2x^2\right)^{3}}{99a}$$
$$= \frac{4096c^6\left(1-a^2x^2\right)^{3/2}}{3465a(c-acx)^{3/2}} + \frac{1024c^5\left(1-a^2x^2\right)^{3/2}}{1155a\sqrt{c-acx}} + \frac{128c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{231a} + \frac{32c}{}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.40

$$\frac{2c^4(ax+1)^{3/2}\left(315a^4x^4-1820a^3x^3+4530a^2x^2-6396ax+5419\right)\sqrt{c-acx}}{3465a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (2*c^4*(1 + a*x)^(3/2)*Sqrt[c - a*c*x]*(5419 - 6396*a*x + 4530*a^2*x^2 - 18
20*a^3*x^3 + 315*a^4*x^4))/(3465*a*Sqrt[1 - a*x])
```

fricas [A] time = 0.48, size = 91, normalized size = 0.52

$$-\frac{2\left(315\,a^5c^4x^5-1505\,a^4c^4x^4+2710\,a^3c^4x^3-1866\,a^2c^4x^2-977\,ac^4x+5419\,c^4\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{3465\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas"
)
```

```
[Out] -2/3465*(315*a^5*c^4*x^5 - 1505*a^4*c^4*x^4 + 2710*a^3*c^4*x^3 - 1866*a^2*c
^4*x^2 - 977*a*c^4*x + 5419*c^4)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x
 - a)
```

giac [A] time = 0.27, size = 87, normalized size = 0.49

$$-\frac{2\left(4096\sqrt{2}\,c^{\frac{7}{2}}-\frac{315\,(acx+c)^{\frac{11}{2}}-3080\,(acx+c)^{\frac{9}{2}}c+11880\,(acx+c)^{\frac{7}{2}}c^{2}-22176\,(acx+c)^{\frac{5}{2}}c^{3}+18480\,(acx+c)^{\frac{3}{2}}c^{4}}{c^{2}}\right)c^{2}}{3465\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -2/3465*(4096*sqrt(2)*c^(7/2) - (315*(a*c*x + c)^(11/2) - 3080*(a*c*x + c)^
(9/2)*c + 11880*(a*c*x + c)^(7/2)*c^2 - 22176*(a*c*x + c)^(5/2)*c^3 + 18480
*(a*c*x + c)^(3/2)*c^4)/c^2)*c^2/(a*abs(c))
```

maple [A] time = 0.03, size = 71, normalized size = 0.40

$$\frac{2\,(ax+1)^{2}\left(315x^{4}a^{4}-1820x^{3}a^{3}+4530a^{2}x^{2}-6396ax+5419\right)(-acx+c)^{\frac{9}{2}}}{3465a\,(ax-1)^{4}\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2),x)
```

```
[Out] 2/3465*(a*x+1)^2*(315*a^4*x^4-1820*a^3*x^3+4530*a^2*x^2-6396*a*x+5419)*(-a*
c*x+c)^(9/2)/a/(a*x-1)^4/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.36, size = 150, normalized size = 0.85

$$\frac{2\left(35\,a^{6}c^{\frac{9}{2}}x^{6}-175\,a^{5}c^{\frac{9}{2}}x^{5}+360\,a^{4}c^{\frac{9}{2}}x^{4}-422\,a^{3}c^{\frac{9}{2}}x^{3}+459\,a^{2}c^{\frac{9}{2}}x^{2}-1451\,ac^{\frac{9}{2}}x-2902\,c^{\frac{9}{2}}\right)}{385\sqrt{ax+1}\,a}+\frac{2\left(35\,a^{5}c^{\frac{9}{2}}x^{5}-185\,\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima"
)
```

```
[Out] 2/385*(35*a^6*c^(9/2)*x^6 - 175*a^5*c^(9/2)*x^5 + 360*a^4*c^(9/2)*x^4 - 422
*a^3*c^(9/2)*x^3 + 459*a^2*c^(9/2)*x^2 - 1451*a*c^(9/2)*x - 2902*c^(9/2))/(
```

```
sqrt(a*x + 1)*a) + 2/315*(35*a^5*c^(9/2)*x^5 - 185*a^4*c^(9/2)*x^4 + 422*a^
3*c^(9/2)*x^3 - 634*a^2*c^(9/2)*x^2 + 1591*a*c^(9/2)*x + 2867*c^(9/2))/(sqr
t(a*x + 1)*a)
```

mupad [B] time = 1.12, size = 90, normalized size = 0.51

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{8884\,c^4\,x}{3465}+\frac{10838\,c^4}{3465\,a}-\frac{5686\,a\,c^4\,x^2}{3465}+\frac{1688\,a^2\,c^4\,x^3}{3465}+\frac{482\,a^3\,c^4\,x^4}{693}-\frac{68\,a^4\,c^4\,x^5}{99}+\frac{2\,a^5\,c^4\,x^6}{11}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(9/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((8884*c^4*x)/3465 + (10838*c^4)/(3465*a) - (5686*a*c^4*
x^2)/3465 + (1688*a^2*c^4*x^3)/3465 + (482*a^3*c^4*x^4)/693 - (68*a^4*c^4*x
^5)/99 + (2*a^5*c^4*x^6)/11))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1))^{\frac{9}{2}}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(9/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.227 $\int e^{\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=141

$$\frac{256c^5\left(1-a^2x^2\right)^{3/2}}{315a(c-acx)^{3/2}}+\frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a\sqrt{c-acx}}+\frac{8c^3\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{21a}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}(c-acx)^{3/2}}{9a}$$

```
[Out] 256/315*c^5*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(3/2)+2/9*c^2*(-a*c*x+c)^(3/2)*
(-a^2*x^2+1)^(3/2)/a+64/105*c^4*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(1/2)+8/21*
c^3*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 657, 649}

$$\frac{256c^5\left(1-a^2x^2\right)^{3/2}}{315a(c-acx)^{3/2}}+\frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a\sqrt{c-acx}}+\frac{8c^3\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{21a}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}(c-acx)^{3/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (256*c^5*(1 - a^2*x^2)^(3/2))/(315*a*(c - a*c*x)^(3/2)) + (64*c^4*(1 - a^2*
x^2)^(3/2))/(105*a*Sqrt[c - a*c*x]) + (8*c^3*Sqrt[c - a*c*x]*(1 - a^2*x^2)^
(3/2))/(21*a) + (2*c^2*(c - a*c*x)^(3/2)*(1 - a^2*x^2)^(3/2))/(9*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx = c\int (c-acx)^{5/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{2c^2(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{9a} + \frac{1}{3}\left(4c^2\right)\int (c-acx)^{3/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{8c^3\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{21a} + \frac{2c^2(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{9a} + \frac{1}{21}\left(32c^3\right)\int \sqrt{c-acx}$$
$$= \frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a\sqrt{c-acx}} + \frac{8c^3\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{21a} + \frac{2c^2(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}{9a} + \frac{}{1}$$
$$= \frac{256c^5\left(1-a^2x^2\right)^{3/2}}{315a(c-acx)^{3/2}} + \frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a\sqrt{c-acx}} + \frac{8c^3\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{21a} + \frac{2c^2(c-ac}{}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.44

$$-\frac{2c^3(ax+1)^{3/2}\left(35a^3x^3-165a^2x^2+321ax-319\right)\sqrt{c-acx}}{315a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*(1 + a*x)^(3/2)*Sqrt[c - a*c*x]*(-319 + 321*a*x - 165*a^2*x^2 + 35*
a^3*x^3))/(315*a*Sqrt[1 - a*x])
```

fricas [A] time = 0.45, size = 80, normalized size = 0.57

$$\frac{2\left(35\,a^4c^3x^4-130\,a^3c^3x^3+156\,a^2c^3x^2+2\,ac^3x-319\,c^3\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/315*(35*a^4*c^3*x^4 - 130*a^3*c^3*x^3 + 156*a^2*c^3*x^2 + 2*a*c^3*x - 319
*c^3)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.03, size = 63, normalized size = 0.45

$$\frac{2\left(ax+1\right)^{2}\left(35x^{3}a^{3}-165a^{2}x^{2}+321ax-319\right)\left(-acx+c\right)^{\frac{7}{2}}}{315a\left(ax-1\right)^{3}\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/315*(a*x+1)^2*(35*a^3*x^3-165*a^2*x^2+321*a*x-319)*(-a*c*x+c)^(7/2)/a/(a*
x-1)^3/(-a^2*x^2+1)^(1/2)
```

maxima [**A**] time = 0.39, size = 128, normalized size = 0.91

$$-\frac{2\left(5\,a^{5}c^{\frac{7}{2}}x^{5}-20\,a^{4}c^{\frac{7}{2}}x^{4}+32\,a^{3}c^{\frac{7}{2}}x^{3}-34\,a^{2}c^{\frac{7}{2}}x^{2}+91\,ac^{\frac{7}{2}}x+182\,c^{\frac{7}{2}}\right)}{45\,\sqrt{ax+1}\,a}-\frac{2\left(5\,a^{4}c^{\frac{7}{2}}x^{4}-22\,a^{3}c^{\frac{7}{2}}x^{3}+44\,a^{2}c^{\frac{7}{2}}x^{2}-1\right.}{35\,\sqrt{ax+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima"
)
```

```
[Out] -2/45*(5*a^5*c^(7/2)*x^5 - 20*a^4*c^(7/2)*x^4 + 32*a^3*c^(7/2)*x^3 - 34*a^2
*c^(7/2)*x^2 + 91*a*c^(7/2)*x + 182*c^(7/2))/(sqrt(a*x + 1)*a) - 2/35*(5*a^
4*c^(7/2)*x^4 - 22*a^3*c^(7/2)*x^3 + 44*a^2*c^(7/2)*x^2 - 106*a*c^(7/2)*x -
 177*c^(7/2))/(sqrt(a*x + 1)*a)
```

mupad [**B**] time = 0.99, size = 79, normalized size = 0.56

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{634\,c^3\,x}{315}+\frac{638\,c^3}{315\,a}-\frac{316\,a\,c^3\,x^2}{315}-\frac{52\,a^2\,c^3\,x^3}{315}+\frac{38\,a^3\,c^3\,x^4}{63}-\frac{2\,a^4\,c^3\,x^5}{9}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(7/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((634*c^3*x)/315 + (638*c^3)/(315*a) - (316*a*c^3*x^2)/3
15 - (52*a^2*c^3*x^3)/315 + (38*a^3*c^3*x^4)/63 - (2*a^4*c^3*x^5)/9))/(1 -
a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^{\frac{7}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(7/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.228 $\int e^{\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=106

$$\frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a(c-acx)^{3/2}}+\frac{16c^3\left(1-a^2x^2\right)^{3/2}}{35a\sqrt{c-acx}}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{7a}$$

```
[Out] 64/105*c^4*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(3/2)+16/35*c^3*(-a^2*x^2+1)^(3/
2)/a/(-a*c*x+c)^(1/2)+2/7*c^2*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 657, 649}

$$\frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a(c-acx)^{3/2}}+\frac{16c^3\left(1-a^2x^2\right)^{3/2}}{35a\sqrt{c-acx}}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}\sqrt{c-acx}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (64*c^4*(1 - a^2*x^2)^(3/2))/(105*a*(c - a*c*x)^(3/2)) + (16*c^3*(1 - a^2*x
^2)^(3/2))/(35*a*Sqrt[c - a*c*x]) + (2*c^2*Sqrt[c - a*c*x]*(1 - a^2*x^2)^(3
/2))/(7*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx = c\int (c-acx)^{3/2}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{2c^2\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{7a}+\frac{1}{7}\left(8c^2\right)\int \sqrt{c-acx}\sqrt{1-a^2x^2}\,dx$$
$$= \frac{16c^3\left(1-a^2x^2\right)^{3/2}}{35a\sqrt{c-acx}}+\frac{2c^2\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{7a}+\frac{1}{35}\left(32c^3\right)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx$$
$$= \frac{64c^4\left(1-a^2x^2\right)^{3/2}}{105a(c-acx)^{3/2}}+\frac{16c^3\left(1-a^2x^2\right)^{3/2}}{35a\sqrt{c-acx}}+\frac{2c^2\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}{7a}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.51

$$\frac{2c^2(ax+1)^{3/2}\left(15a^2x^2-54ax+71\right)\sqrt{c-acx}}{105a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*(1 + a*x)^(3/2)*Sqrt[c - a*c*x]*(71 - 54*a*x + 15*a^2*x^2))/(105*a*S
qrt[1 - a*x])
```

fricas [A] time = 0.63, size = 69, normalized size = 0.65

$$-\frac{2\left(15\,a^3c^2x^3-39\,a^2c^2x^2+17\,ac^2x+71\,c^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{105\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/105*(15*a^3*c^2*x^3 - 39*a^2*c^2*x^2 + 17*a*c^2*x + 71*c^2)*sqrt(-a^2*x^
2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

giac [A] time = 0.20, size = 61, normalized size = 0.58

$$-\frac{2\left(64\sqrt{2}\,c^{\frac{3}{2}}-\frac{15\,(acx+c)^{\frac{7}{2}}-84\,(acx+c)^{\frac{5}{2}}c+140\,(acx+c)^{\frac{3}{2}}c^2}{c^2}\right)c^2}{105\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/105*(64*sqrt(2)*c^(3/2) - (15*(a*c*x + c)^(7/2) - 84*(a*c*x + c)^(5/2)*c
 + 140*(a*c*x + c)^(3/2)*c^2)/c^2)*c^2/(a*abs(c))
```

maple [A] time = 0.03, size = 55, normalized size = 0.52

$$\frac{2\left(ax+1\right)^{2}\left(15a^{2}x^{2}-54ax+71\right)\left(-acx+c\right)^{\frac{5}{2}}}{105a\left(ax-1\right)^{2}\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/105*(a*x+1)^2*(15*a^2*x^2-54*a*x+71)*(-a*c*x+c)^(5/2)/a/(a*x-1)^2/(-a^2*x
^2+1)^(1/2)
```

maxima [A] time = 0.35, size = 106, normalized size = 1.00

$$\frac{2\left(3\,a^{4}c^{\frac{5}{2}}x^{4}-9\,a^{3}c^{\frac{5}{2}}x^{3}+11\,a^{2}c^{\frac{5}{2}}x^{2}-23\,ac^{\frac{5}{2}}x-46\,c^{\frac{5}{2}}\right)}{21\,\sqrt{ax+1}\,a}+\frac{2\left(3\,a^{3}c^{\frac{5}{2}}x^{3}-11\,a^{2}c^{\frac{5}{2}}x^{2}+29\,ac^{\frac{5}{2}}x+43\,c^{\frac{5}{2}}\right)}{15\,\sqrt{ax+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 2/21*(3*a^4*c^(5/2)*x^4 - 9*a^3*c^(5/2)*x^3 + 11*a^2*c^(5/2)*x^2 - 23*a*c^(
5/2)*x - 46*c^(5/2))/(sqrt(a*x + 1)*a) + 2/15*(3*a^3*c^(5/2)*x^3 - 11*a^2*c
^(5/2)*x^2 + 29*a*c^(5/2)*x + 43*c^(5/2))/(sqrt(a*x + 1)*a)
```

mupad [B] time = 0.95, size = 68, normalized size = 0.64

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{176\,c^2\,x}{105}+\frac{142\,c^2}{105\,a}-\frac{44\,a\,c^2\,x^2}{105}-\frac{16\,a^2\,c^2\,x^3}{35}+\frac{2\,a^3\,c^2\,x^4}{7}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(5/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((176*c^2*x)/105 + (142*c^2)/(105*a) - (44*a*c^2*x^2)/10
5 - (16*a^2*c^2*x^3)/35 + (2*a^3*c^2*x^4)/7))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^{\frac{5}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(5/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.229 $\int e^{\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=71

$$\frac{8c^3\left(1-a^2x^2\right)^{3/2}}{15a(c-acx)^{3/2}}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{5a\sqrt{c-acx}}$$

```
[Out] 8/15*c^3*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(3/2)+2/5*c^2*(-a^2*x^2+1)^(3/2)/a
/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6127, 657, 649}

$$\frac{8c^3\left(1-a^2x^2\right)^{3/2}}{15a(c-acx)^{3/2}}+\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{5a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (8*c^3*(1 - a^2*x^2)^(3/2))/(15*a*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a^2*x^2)
^(3/2))/(5*a*Sqrt[c - a*c*x])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx = c\int \sqrt{c-acx}\,\sqrt{1-a^2x^2}\,dx$$
$$= \frac{2c^2\left(1-a^2x^2\right)^{3/2}}{5a\sqrt{c-acx}} + \frac{1}{5}\left(4c^2\right)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx$$
$$= \frac{8c^3\left(1-a^2x^2\right)^{3/2}}{15a(c-acx)^{3/2}} + \frac{2c^2\left(1-a^2x^2\right)^{3/2}}{5a\sqrt{c-acx}}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.62

$$-\frac{2c(ax+1)^{3/2}(3ax-7)\sqrt{c-acx}}{15a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*(1 + a*x)^(3/2)*(-7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a*Sqrt[1 - a*x])
```

fricas [A] time = 0.45, size = 52, normalized size = 0.73

$$\frac{2\left(3\,a^2cx^2-4\,acx-7\,c\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{15\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2/15*(3*a^2*c*x^2 - 4*a*c*x - 7*c)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2
*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 47, normalized size = 0.66

$$\frac{2\left(ax+1\right)^{2}\left(3ax-7\right)\left(-acx+c\right)^{\frac{3}{2}}}{15a\left(ax-1\right)\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/15*(a*x+1)^2*(3*a*x-7)*(-a*c*x+c)^(3/2)/a/(a*x-1)/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.35, size = 82, normalized size = 1.15

$$-\frac{2\left(a^{3}c^{\frac{3}{2}}x^{3}-2\,a^{2}c^{\frac{3}{2}}x^{2}+3\,ac^{\frac{3}{2}}x+6\,c^{\frac{3}{2}}\right)}{5\sqrt{ax+1}\,a}-\frac{2\left(a^{2}c^{\frac{3}{2}}x^{2}-4\,ac^{\frac{3}{2}}x-5\,c^{\frac{3}{2}}\right)}{3\sqrt{ax+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -2/5*(a^3*c^(3/2)*x^3 - 2*a^2*c^(3/2)*x^2 + 3*a*c^(3/2)*x + 6*c^(3/2))/(sqr
t(a*x + 1)*a) - 2/3*(a^2*c^(3/2)*x^2 - 4*a*c^(3/2)*x - 5*c^(3/2))/(sqrt(a*x
 + 1)*a)
```

mupad [B] time = 0.93, size = 49, normalized size = 0.69

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{22\,c\,x}{15}+\frac{14\,c}{15\,a}-\frac{2\,a^{2}\,c\,x^{3}}{5}+\frac{2\,a\,c\,x^{2}}{15}\right)}{\sqrt{1-a^{2}\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(3/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((22*c*x)/15 + (14*c)/(15*a) - (2*a^2*c*x^3)/5 + (2*a*c*
x^2)/15))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-c\left(ax-1\right)\right)^{\frac{3}{2}}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.230 $\int e^{\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=35

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}$$

```
[Out] 2/3*c^2*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 649}

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*c^2*(1 - a^2*x^2)^(3/2))/(3*a*(c - a*c*x)^(3/2))
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= c\int\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx\\ &= \frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.06

$$\frac{2(ax+1)^{3/2}\sqrt{c-acx}}{3a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + a*x)^(3/2)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a*x])
```

fricas [A] time = 0.73, size = 39, normalized size = 1.11

$$-\frac{2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\,(ax+1)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 1)/(a^2*x - a)
```

giac [A] time = 0.17, size = 34, normalized size = 0.97

$$-\frac{2\,c^2\left(\frac{2\,\sqrt{2}}{\sqrt{c}}-\frac{(acx+c)^{\frac{3}{2}}}{c^2}\right)}{3\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*c^2*(2*sqrt(2)/sqrt(c) - (a*c*x + c)^(3/2)/c^2)/(a*abs(c))
```

maple [A] time = 0.03, size = 34, normalized size = 0.97

$$\frac{2\,(ax+1)^2\,\sqrt{-acx+c}}{3a\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/3*(a*x+1)^2*(-a*c*x+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.35, size = 58, normalized size = 1.66

$$\frac{2\left(a^2\sqrt{c}\,x^2 - a\sqrt{c}\,x - 2\sqrt{c}\right)}{3\sqrt{ax+1}\,a} + \frac{2\left(a\sqrt{c}\,x + \sqrt{c}\right)}{\sqrt{ax+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/3*(a^2*sqrt(c)*x^2 - a*sqrt(c)*x - 2*sqrt(c))/(sqrt(a*x + 1)*a) + 2*(a*sq
rt(c)*x + sqrt(c))/(sqrt(a*x + 1)*a)
```

mupad [B] time = 0.87, size = 37, normalized size = 1.06

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{4\,x}{3}+\frac{2\,a\,x^2}{3}+\frac{2}{3\,a}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((4*x)/3 + (2*a*x^2)/3 + 2/(3*a)))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.231 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}-\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

```
[Out] 2*arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))*2^(1/2)/
a/c^(1/2)-2*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 665, 661, 208}

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}-\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) + (2*Sqrt[2]*ArcTanh[(Sqrt[c]*Sq
rt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(a*Sqrt[c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
```

```
|| EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{3/2}}\,dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}} + 2\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}} - (4ac)\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right) \\
&= -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.75

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{ac\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x] - Sqrt[2]*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]
))/(a*c*Sqrt[1 - a*x])
```

fricas [A] time = 0.50, size = 215, normalized size = 2.59

$$\left[\frac{\frac{\sqrt{2}\,(acx-c)\log\left(-\frac{a^2x^2+2\,ax-\frac{2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{\sqrt{c}}-3}{a^2x^2-2\,ax+1}\right)}{\sqrt{c}}+2\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{a^2cx-ac},\ \frac{2\left(\sqrt{2}\,(acx-c)\sqrt{-\frac{1}{c}}\ \arctan\left(\frac{\sqrt{2}\,\sqrt{-a^2x^2+1}\ \cdots}{a^2x^2\cdots}\right)\cdots\right.}{a^2cx\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [(sqrt(2)*(a*c*x - c)*log(-(a^2*x^2 + 2*a*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*
sqrt(-a*c*x + c)/sqrt(c) - 3)/(a^2*x^2 - 2*a*x + 1))/sqrt(c) + 2*sqrt(-a^2*
x^2 + 1)*sqrt(-a*c*x + c))/(a^2*c*x - a*c), 2*(sqrt(2)*(a*c*x - c)*sqrt(-1/
c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-1/c)/(a^2*x^2 -
 1)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^2*c*x - a*c)]
```

giac [A] time = 0.16, size = 88, normalized size = 1.06

$$-\frac{2\,c\left(\frac{\frac{\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+\sqrt{acx+c}}{c}-\frac{\sqrt{2}\left(c\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+\sqrt{-c}\,\sqrt{c}\right)}{\sqrt{-c}\,c}\right)}{a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*c*((sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) + sq
rt(a*c*x + c))/c - sqrt(2)*(c*arctan(sqrt(c)/sqrt(-c)) + sqrt(-c)*sqrt(c))/
(sqrt(-c)*c))/(a*abs(c))
```

maple [A] time = 0.06, size = 84, normalized size = 1.01

$$-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-\sqrt{c\,(ax+1)}\right)}{(ax-1)\sqrt{c\,(ax+1)}\,ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] -2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*2^(1/2)*arctanh(1/2*(c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2))-(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/
c/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-c\,(ax-1)}\,\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.232 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}\,ac^{3/2}}$$

```
[Out] -1/2*arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^(
3/2)*2^(1/2)+(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6127, 663, 661, 208}

$$\frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}\,ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*(c - a*c*x)^(3/2)) - ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2
])/(Sqrt[2]*Sqrt[c - a*c*x])]/(Sqrt[2]*a*c^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{5/2}}\,dx \\
&= \frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}} - \frac{\int \frac{1}{\sqrt{c-acx}\,\sqrt{1-a^2x^2}}\,dx}{2c} \\
&= \frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}} + a\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right) \\
&= \frac{\sqrt{1-a^2x^2}}{a(c-acx)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{\sqrt{2}\,ac^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.98

$$-\frac{\sqrt{c-acx}\left(2ax+(ax-1)\sqrt{2ax+2}\,\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)+2\right)}{2ac^2(ax-1)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[c - a*c*x]*(2 + 2*a*x + (-1 + a*x)*Sqrt[2 + 2*a*x]*ArcTanh[Sqrt[
1 + a*x]/Sqrt[2]]))/(a*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.67, size = 258, normalized size = 3.15

$$\left[\frac{\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}, -\frac{\sqrt{2}\left(a^2x^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*s
qrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x
+ 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*c^2*x^2 - 2*a^2*c^2*x +
 a*c^2), -1/2*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-
a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

giac [A] time = 0.33, size = 58, normalized size = 0.71

$$\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2\sqrt{acx+c}}{acx-c}}{2a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*sqrt
(a*c*x + c)/(a*c*x - c))/(a*abs(c))
```

maple [A] time = 0.05, size = 111, normalized size = 1.35

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac - \sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c + 2\sqrt{c\,(ax+1)}\,\sqrt{c}\right)}{2\,(ax-1)^2\,\sqrt{c\,(ax+1)}\,c^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(2^(1/2)*arctanh(1/2*(c*(a*x+1))^
(1/2)*2^(1/2)/c^(1/2))*x*a*c-2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/
c^(1/2))*c+2*(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^2/(c*(a*x+1))^(1/2)/c^(5/2)
/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\sqrt{1-a^2\,x^2}\,\left(c-a\,c\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(3/2)),x)
[Out] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\right)^{\frac{3}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(3/2),x)
[Out] Integral((a*x + 1)/((-c*(a*x - 1))**(3/2)*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.233 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{8\sqrt{2}\,ac^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}+\frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}$$

```
[Out] -1/16*arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^
(5/2)*2^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(5/2)-1/8*(-a^2*x^2+1)^(1
/2)/a/c/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6127, 663, 673, 661, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{8\sqrt{2}\,ac^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}+\frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*(c - a*c*x)^(5/2)) - Sqrt[1 - a^2*x^2]/(8*a*c*(c - a
*c*x)^(3/2)) - ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x]
)]/(8*Sqrt[2]*a*c^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
```

```
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{7/2}}\,dx\\
&= \frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}-\frac{\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{4c}\\
&= \frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}-\frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{16c^2}\\
&= \frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}+\frac{a\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{8c}\\
&= \frac{\sqrt{1-a^2x^2}}{2a(c-acx)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{8ac(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{8\sqrt{2}\,ac^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.47

$$\frac{(ax+1)^{3/2}\sqrt{c-acx}\;{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{1}{2}(ax+1)\right)}{12ac^3\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] ((1 + a*x)^(3/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[3/2, 3, 5/2, (1 + a*x)/2
])/(12*a*c^3*Sqrt[1 - a*x])
```

fricas [A] time = 0.48, size = 308, normalized size = 2.52

$$\left[\frac{\sqrt{2}\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\sqrt{c}\,\log\left(-\frac{a^{2}cx^{2}+2\,acx+2\,\sqrt{2}\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)-4\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{-acx+c}\,(ax+3}{32\left(a^{4}c^{3}x^{3}-3\,a^{3}c^{3}x^{2}+3\,a^{2}c^{3}x-ac^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [1/32*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 +
2*a*c*x + 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) - 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 3))/(a^4*
c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3), -1/16*(sqrt(2)*(a^3*x^3 - 3
*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a
*x + 3))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

giac [A] time = 0.25, size = 76, normalized size = 0.62

$$\frac{\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}\,c}+\frac{2\left((acx+c)^{\frac{3}{2}}+2\,\sqrt{acx+c}\,c\right)}{(acx-c)^{2}c}}{16\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c) + 2
*((a*c*x + c)^(3/2) + 2*sqrt(a*c*x + c)*c)/((a*c*x - c)^2*c))/(a*abs(c))
```

maple [A] time = 0.05, size = 156, normalized size = 1.28

$$\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c\,(ax-1)}\,\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^{2}a^{2}c-2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac-2xa\sqrt{c\,(ax+1)}\right.}{16c^{\frac{7}{2}}\,(ax-1)^{3}\,\sqrt{c\,(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(7/2)*(2^(1/2)*arctanh(1/2*(c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-2*2^(1/2)*arctanh(1/2*(c*(a*x+1))
^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+2^(1/2)*arcta
nh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-6*(c*(a*x+1))^(1/2)*c^(1/2))/(a
*x-1)^3/(c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\sqrt{1-a^2\,x^2}\,{\left(c-a\,c\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(5/2)),x)
```

```
[Out] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\right)^{\frac{5}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1))**(5/2)*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.234 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}\,ac^{7/2}}-\frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}}$$

```
[Out] -1/64*arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^
(7/2)*2^(1/2)+1/3*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(7/2)-1/24*(-a^2*x^2+1)^(
1/2)/a/c/(-a*c*x+c)^(5/2)-1/32*(-a^2*x^2+1)^(1/2)/a/c^2/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6127, 663, 673, 661, 208}

$$-\frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}\,ac^{7/2}}-\frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(3*a*(c - a*c*x)^(7/2)) - Sqrt[1 - a^2*x^2]/(24*a*c*(c -
a*c*x)^(5/2)) - Sqrt[1 - a^2*x^2]/(32*a*c^2*(c - a*c*x)^(3/2)) - ArcTanh[(S
qrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(32*Sqrt[2]*a*c^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
```

```
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{9/2}}\,dx\\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\int \frac{1}{(c-acx)^{5/2}\sqrt{1-a^2x^2}}\,dx}{6c}\\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}} - \frac{\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{16c^2}\\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}} - \frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}} - \frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{64c^3}\\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}} - \frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{32c^2}\\
&= \frac{\sqrt{1-a^2x^2}}{3a(c-acx)^{7/2}} - \frac{\sqrt{1-a^2x^2}}{24ac(c-acx)^{5/2}} - \frac{\sqrt{1-a^2x^2}}{32ac^2(c-acx)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}\,ac^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.36

$$\frac{(ax+1)^{3/2}\sqrt{c-acx}\;{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{1}{2}(ax+1)\right)}{24ac^4\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] ((1 + a*x)^(3/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[3/2, 4, 5/2, (1 + a*x)/2
])/(24*a*c^4*Sqrt[1 - a*x])
```

fricas [A] time = 0.71, size = 364, normalized size = 2.32

$$\left[\frac{3\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right)-4\left(3a^2x^2-10ax-25\right.}{384\left(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2-4a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas"
)
```

```
[Out] [1/384*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log
(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt
(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^2*x^2 - 10*a*x - 25)*sqrt(-a^2*x
^2 + 1)*sqrt(-a*c*x + c))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*
a^2*c^4*x + a*c^4), -1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*
a*x + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-
c)/(a^2*c*x^2 - c)) + 2*(3*a^2*x^2 - 10*a*x - 25)*sqrt(-a^2*x^2 + 1)*sqrt(-
a*c*x + c))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*
c^4)]
```

giac [A] time = 0.31, size = 92, normalized size = 0.59

$$\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}\,c^2}+\frac{2\left(3(acx+c)^{\frac{5}{2}}-16(acx+c)^{\frac{3}{2}}c-12\sqrt{acx+c}\,c^2\right)}{(acx-c)^3c^2}}{192\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/192*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^2
) + 2*(3*(a*c*x + c)^(5/2) - 16*(a*c*x + c)^(3/2)*c - 12*sqrt(a*c*x + c)*c^
2)/((a*c*x - c)^3*c^2))/(a*abs(c))
```

maple [A] time = 0.05, size = 208, normalized size = 1.32

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c(ax-1)}\,\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^3a^3c-9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c-6x^2a^2\sqrt{c(ax\right.}{192c^{\frac{9}{2}}(ax-}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x)

[Out] 1/192*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-9*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*x^2*a^2*(c*(a*x+1))^(1/2)*c^(1/2)+9*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+20*x*a*(c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+50*(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^4/(c*(a*x+1))^(1/2)/a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^(7/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\sqrt{1-a^2\,x^2}\,{\left(c-a\,c\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(7/2)),x)

[Out] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(7/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\right)^{\frac{7}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(7/2),x)

[Out] Integral((a*x + 1)/((-c*(a*x - 1))**(7/2)*sqrt(-(a*x - 1)*(a*x + 1))), x)

3.235 $\int e^{2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=40

$$\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}$$

```
[Out] -4/7*(-a*c*x+c)^(7/2)/a+2/9*(-a*c*x+c)^(9/2)/a/c
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-4*(c - a*c*x)^(7/2))/(7*a) + (2*(c - a*c*x)^(9/2))/(9*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx = \int \frac{(1+ax)(c-acx)^{7/2}}{1-ax}\,dx$$
$$= c\int (1+ax)(c-acx)^{5/2}\,dx$$
$$= c\int \left(2(c-acx)^{5/2}-\frac{(c-acx)^{7/2}}{c}\right)dx$$
$$= -\frac{4(c-acx)^{7/2}}{7a}+\frac{2(c-acx)^{9/2}}{9ac}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 0.85

$$\frac{2c^3(ax-1)^3(7ax+11)\sqrt{c-acx}}{63a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (2*c^3*(-1 + a*x)^3*(11 + 7*a*x)*Sqrt[c - a*c*x])/(63*a)
```

fricas [A] time = 0.74, size = 60, normalized size = 1.50

$$\frac{2\left(7\,a^4c^3x^4-10\,a^3c^3x^3-12\,a^2c^3x^2+26\,ac^3x-11\,c^3\right)\sqrt{-acx+c}}{63\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 12*a^2*c^3*x^2 + 26*a*c^3*x - 11*c^3
)*sqrt(-a*c*x + c)/a
```

giac [B] time = 0.18, size = 205, normalized size = 5.12

$$-\frac{2\left(90\,(acx-c)^3\sqrt{-acx+c}+378\,(acx-c)^2\sqrt{-acx+c}\,c-630\,(-acx+c)^{\frac{3}{2}}c^2+945\,\sqrt{-acx+c}\,c^3+210\,\left((-acx\right.\right.}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -2/315*(90*(a*c*x - c)^3*sqrt(-a*c*x + c) + 378*(a*c*x - c)^2*sqrt(-a*c*x +
 c)*c - 630*(-a*c*x + c)^(3/2)*c^2 + 945*sqrt(-a*c*x + c)*c^3 + 210*((-a*c*
x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c^2 - (35*(a*c*x - c)^4*sqrt(-a*c*x +
c) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c + 378*(a*c*x - c)^2*sqrt(-a*c*x +
 c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*sqrt(-a*c*x + c)*c^4)/c)/a
```

maple [A] time = 0.03, size = 21, normalized size = 0.52

$$-\frac{2\left(-acx+c\right)^{\frac{7}{2}}\left(7ax+11\right)}{63a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(7/2),x)
```

```
[Out] -2/63*(-a*c*x+c)^(7/2)*(7*a*x+11)/a
```

maxima [A] time = 0.33, size = 32, normalized size = 0.80

$$\frac{2\left(7\left(-acx+c\right)^{\frac{9}{2}}-18\left(-acx+c\right)^{\frac{7}{2}}c\right)}{63\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*(-a*c*x + c)^(9/2) - 18*(-a*c*x + c)^(7/2)*c)/(a*c)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.80

$$\frac{2\left(c-a\,c\,x\right)^{9/2}}{9\,a\,c}-\frac{4\left(c-a\,c\,x\right)^{7/2}}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(7/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (2*(c - a*c*x)^(9/2))/(9*a*c) - (4*(c - a*c*x)^(7/2))/(7*a)
```

sympy [A] time = 31.79, size = 172, normalized size = 4.30

$$c^3\left(\begin{cases}\sqrt{c}\,x & \text{for } a=0\\ 0 & \text{for } c=0\\ -\frac{2(-acx+c)^{\frac{3}{2}}}{3ac} & \text{otherwise}\end{cases}\right)-\frac{2c\left(-\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a}+\frac{2\left(\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a}+\frac{2\left(-\frac{c^3(-acx\cdots}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(7/2),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*x, Eq(a, 0)), (0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2
)/(3*a*c), True)) - 2*c*(-c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/
a + 2*(c**2*(-a*c*x + c)**(3/2)/3 - 2*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x + c
)**(7/2)/7)/a + 2*(-c**3*(-a*c*x + c)**(3/2)/3 + 3*c**2*(-a*c*x + c)**(5/2)
/5 - 3*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a*c)
```

3.236 $\int e^{2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=40

$$\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}$$

```
[Out] -4/5*(-a*c*x+c)^(5/2)/a+2/7*(-a*c*x+c)^(7/2)/a/c
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (-4*(c - a*c*x)^(5/2))/(5*a) + (2*(c - a*c*x)^(7/2))/(7*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx = \int \frac{(1+ax)(c-acx)^{5/2}}{1-ax}\,dx$$
$$= c\int (1+ax)(c-acx)^{3/2}\,dx$$
$$= c\int \left(2(c-acx)^{3/2}-\frac{(c-acx)^{5/2}}{c}\right)dx$$
$$= -\frac{4(c-acx)^{5/2}}{5a}+\frac{2(c-acx)^{7/2}}{7ac}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 0.85

$$-\frac{2c^2(ax-1)^2(5ax+9)\sqrt{c-acx}}{35a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
[Out] (-2*c^2*(-1 + a*x)^2*(9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a)
```

fricas [A] time = 0.59, size = 49, normalized size = 1.22

$$-\frac{2\left(5\,a^3c^2x^3-a^2c^2x^2-13\,ac^2x+9\,c^2\right)\sqrt{-acx+c}}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
[Out] -2/35*(5*a^3*c^2*x^3 - a^2*c^2*x^2 - 13*a*c^2*x + 9*c^2)*sqrt(-a*c*x + c)/a
```

giac [B] time = 0.18, size = 141, normalized size = 3.52

$$\frac{2\left(21\,(acx-c)^2\sqrt{-acx+c}\;-70\,(-acx+c)^{\frac{3}{2}}c-35\left((-acx+c)^{\frac{3}{2}}-3\,\sqrt{-acx+c}\,c\right)c-\frac{3\left(5\,(acx-c)^3\,\sqrt{-acx+c}+21\,(acx-c)^2\right.}{}\right.}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(5/2),x, algorithm="giac")
[Out] 2/105*(21*(a*c*x - c)^2*sqrt(-a*c*x + c) - 70*(-a*c*x + c)^(3/2)*c - 35*((-
a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c - 3*(5*(a*c*x - c)^3*sqrt(-a*c*x
```

```
 + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 3
5*sqrt(-a*c*x + c)*c^3)/c)/a
```

maple [A] time = 0.03, size = 21, normalized size = 0.52

$$-\frac{2\left(-acx+c\right)^{\frac{5}{2}}\left(5ax+9\right)}{35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(5/2),x)
[Out] -2/35*(-a*c*x+c)^(5/2)*(5*a*x+9)/a
```

maxima [A] time = 0.31, size = 32, normalized size = 0.80

$$\frac{2\left(5\left(-acx+c\right)^{\frac{7}{2}}-14\left(-acx+c\right)^{\frac{5}{2}}c\right)}{35\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] 2/35*(5*(-a*c*x + c)^(7/2) - 14*(-a*c*x + c)^(5/2)*c)/(a*c)
```

mupad [B] time = 0.77, size = 32, normalized size = 0.80

$$\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a\,c}-\frac{4\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] (2*(c - a*c*x)^(7/2))/(7*a*c) - (4*(c - a*c*x)^(5/2))/(5*a)
```

sympy [A] time = 14.06, size = 76, normalized size = 1.90

$$c^{2}\left(\begin{cases}\sqrt{c}\,x & \text{for}\ a=0\\0 & \text{for}\ c=0\\-\frac{2\left(-acx+c\right)^{\frac{3}{2}}}{3ac} & \text{otherwise}\end{cases}\right)+\frac{2\left(\frac{c^{2}\left(-acx+c\right)^{\frac{3}{2}}}{3}-\frac{2c\left(-acx+c\right)^{\frac{5}{2}}}{5}+\frac{\left(-acx+c\right)^{\frac{7}{2}}}{7}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(5/2),x)
```

```
[Out] c**2*Piecewise((sqrt(c)*x, Eq(a, 0)), (0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2
)/(3*a*c), True)) + 2*(c**2*(-a*c*x + c)**(3/2)/3 - 2*c*(-a*c*x + c)**(5/2)
/5 + (-a*c*x + c)**(7/2)/7)/(a*c)
```

3.237 $\int e^{2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=40

$$\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}$$

```
[Out] -4/3*(-a*c*x+c)^(3/2)/a+2/5*(-a*c*x+c)^(5/2)/a/c
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (-4*(c - a*c*x)^(3/2))/(3*a) + (2*(c - a*c*x)^(5/2))/(5*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx &= \int \frac{(1+ax)(c-acx)^{3/2}}{1-ax}\,dx \\
&= c\int (1+ax)\sqrt{c-acx}\,dx \\
&= c\int \left(2\sqrt{c-acx}-\frac{(c-acx)^{3/2}}{c}\right)dx \\
&= -\frac{4(c-acx)^{3/2}}{3a}+\frac{2(c-acx)^{5/2}}{5ac}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 30, normalized size = 0.75

$$\frac{2c(ax-1)(3ax+7)\sqrt{c-acx}}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*c*(-1 + a*x)*(7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a)
```

fricas [A] time = 0.74, size = 32, normalized size = 0.80

$$\frac{2\left(3\,a^2cx^2+4\,acx-7\,c\right)\sqrt{-acx+c}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*c*x^2 + 4*a*c*x - 7*c)*sqrt(-a*c*x + c)/a
```

giac [B] time = 0.17, size = 71, normalized size = 1.78

$$-\frac{2\left(15\,\sqrt{-acx+c}\,c-\frac{3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}\,c^2}{c}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(-a*c*x + c)*c - (3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c
*x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/c)/a
```

maple [A] time = 0.03, size = 21, normalized size = 0.52

$$-\frac{2\left(-acx+c\right)^{\frac{3}{2}}\left(3ax+7\right)}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(3/2),x)
```

```
[Out] -2/15*(-a*c*x+c)^(3/2)*(3*a*x+7)/a
```

maxima [A] time = 0.33, size = 32, normalized size = 0.80

$$\frac{2\left(3\left(-acx+c\right)^{\frac{5}{2}}-10\left(-acx+c\right)^{\frac{3}{2}}c\right)}{15\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(-a*c*x + c)^(5/2) - 10*(-a*c*x + c)^(3/2)*c)/(a*c)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.80

$$\frac{2\left(c-a\,c\,x\right)^{5/2}}{5\,a\,c}-\frac{4\left(c-a\,c\,x\right)^{3/2}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (2*(c - a*c*x)^(5/2))/(5*a*c) - (4*(c - a*c*x)^(3/2))/(3*a)
```

sympy [A] time = 14.31, size = 58, normalized size = 1.45

$$c\left(\begin{cases}\sqrt{c}\,x & \text{for}\: a=0\\0 & \text{for}\: c=0\\-\frac{2\left(-acx+c\right)^{\frac{3}{2}}}{3ac} & \text{otherwise}\end{cases}\right)+\frac{2\left(-\frac{c\left(-acx+c\right)^{\frac{3}{2}}}{3}+\frac{\left(-acx+c\right)^{\frac{5}{2}}}{5}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(3/2),x)
```

```
[Out] c*Piecewise((sqrt(c)*x, Eq(a, 0)), (0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(
3*a*c), True)) + 2*(-c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/(a*c)
```

3.238 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a/c-4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx&=\int\frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\&=c\int\frac{1+ax}{\sqrt{c-acx}}\,dx\\&=c\int\left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\\&=-\frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.61

$$-\frac{2(ax+5)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
[Out] (-2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

fricas [A] time = 0.48, size = 19, normalized size = 0.50

$$-\frac{2\sqrt{-acx+c}\,(ax+5)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

giac [A] time = 0.30, size = 44, normalized size = 1.16

$$-\frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c}{c}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] -2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

maple [A] time = 0.03, size = 20, normalized size = 0.53

$$-\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x)
[Out] -2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

maxima [A] time = 0.32, size = 30, normalized size = 0.79

$$\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.84

$$\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c}-\frac{4\,\sqrt{c-a\,c\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

sympy [A] time = 4.68, size = 31, normalized size = 0.82

$$-\frac{2\left(2c\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2),x)
[Out] -2*(2*c*sqrt(-a*c*x + c) - (-a*c*x + c)**(3/2)/3)/(a*c)
```

3.239 $\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=36

$$\frac{2\sqrt{c-acx}}{ac}+\frac{4}{a\sqrt{c-acx}}$$

```
[Out] 4/a/(-a*c*x+c)^(1/2)+2*(-a*c*x+c)^(1/2)/a/c
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 21, 43}

$$\frac{2\sqrt{c-acx}}{ac}+\frac{4}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] 4/(a*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x])/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= \int \frac{1+ax}{(1-ax)\sqrt{c-acx}}\,dx\\ &= c\int \frac{1+ax}{(c-acx)^{3/2}}\,dx\\ &= c\int \left(\frac{2}{(c-acx)^{3/2}}-\frac{1}{c\sqrt{c-acx}}\right)dx\\ &= \frac{4}{a\sqrt{c-acx}}+\frac{2\sqrt{c-acx}}{ac}\end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 0.58

$$\frac{6-2ax}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
[Out] (6 - 2*a*x)/(a*Sqrt[c - a*c*x])
```

fricas [A] time = 0.51, size = 29, normalized size = 0.81

$$\frac{2\sqrt{-acx+c}\,(ax-3)}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(-a*c*x + c)*(a*x - 3)/(a^2*c*x - a*c)
```

giac [A] time = 0.25, size = 32, normalized size = 0.89

$$\frac{4}{\sqrt{-acx+c}\,a}+\frac{2\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 4/(sqrt(-a*c*x + c)*a) + 2*sqrt(-a*c*x + c)/(a*c)
```

maple [A] time = 0.03, size = 20, normalized size = 0.56

$$-\frac{2\,(ax-3)}{\sqrt{-acx+c}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x)
```

```
[Out] -2*(a*x-3)/(-a*c*x+c)^(1/2)/a
```

maxima [A] time = 0.31, size = 30, normalized size = 0.83

$$\frac{2\left(\sqrt{-acx+c}+\frac{2\,c}{\sqrt{-acx+c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(-a*c*x + c) + 2*c/sqrt(-a*c*x + c))/(a*c)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.56

$$-\frac{2\,a\,x-6}{a\,\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - a*c*x)^(1/2)),x)
```

```
[Out] -(2*a*x - 6)/(a*(c - a*c*x)^(1/2))
```

sympy [A] time = 36.40, size = 48, normalized size = 1.33

$$\begin{cases} \frac{\frac{2}{\sqrt{-acx+c}}-\frac{2\left(-\frac{c}{\sqrt{-acx+c}}-\sqrt{-acx+c}\right)}{c}}{a} & \text{for}\ a\neq 0 \\ \frac{x}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise(((2/sqrt(-a*c*x + c) - 2*(-c/sqrt(-a*c*x + c) - sqrt(-a*c*x + c))
/c)/a, Ne(a, 0)), (x/sqrt(c), True))
```

3.240 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{4}{3a(c-acx)^{3/2}}-\frac{2}{ac\sqrt{c-acx}}$$

[Out] 4/3/a/(-a*c*x+c)^(3/2)-2/a/c/(-a*c*x+c)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 21, 43}

$$\frac{4}{3a(c-acx)^{3/2}}-\frac{2}{ac\sqrt{c-acx}}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]

[Out] 4/(3*a*(c - a*c*x)^(3/2)) - 2/(a*c*Sqrt[c - a*c*x])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{1+ax}{(1-ax)(c-acx)^{3/2}}\,dx$$
$$= c\int \frac{1+ax}{(c-acx)^{5/2}}\,dx$$
$$= c\int \left(\frac{2}{(c-acx)^{5/2}} - \frac{1}{c(c-acx)^{3/2}}\right)dx$$
$$= \frac{4}{3a(c-acx)^{3/2}} - \frac{2}{ac\sqrt{c-acx}}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 0.89

$$\frac{2(3ax-1)\sqrt{c-acx}}{3ac^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
[Out] (2*(-1 + 3*a*x)*Sqrt[c - a*c*x])/(3*a*c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.56, size = 44, normalized size = 1.16

$$\frac{2\sqrt{-acx+c}\,(3\,ax-1)}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-a*c*x + c)*(3*a*x - 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.23, size = 36, normalized size = 0.95

$$-\frac{2\,(3\,acx-c)}{3\,(acx-c)\sqrt{-acx+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] -2/3*(3*a*c*x - c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c)
```

maple [A] time = 0.03, size = 21, normalized size = 0.55

$$\frac{2ax - \frac{2}{3}}{(-acx + c)^{\frac{3}{2}}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x)
[Out] 2/3*(3*a*x-1)/(-a*c*x+c)^(3/2)/a
```

maxima [A] time = 0.32, size = 26, normalized size = 0.68

$$\frac{2\,(3\,acx - c)}{3\,(-acx + c)^{\frac{3}{2}}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] 2/3*(3*a*c*x - c)/((-a*c*x + c)^(3/2)*a*c)
```

mupad [B] time = 0.78, size = 20, normalized size = 0.53

$$\frac{6\,a\,x - 2}{3\,a\,(c - a\,c\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - a*c*x)^(3/2)),x)
[Out] (6*a*x - 2)/(3*a*(c - a*c*x)^(3/2))
```

sympy [A] time = 41.88, size = 29, normalized size = 0.76

$$\frac{4}{3a\,(-acx + c)^{\frac{3}{2}}} - \frac{2}{ac\sqrt{-acx + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**(3/2),x)
[Out] 4/(3*a*(-a*c*x + c)**(3/2)) - 2/(a*c*sqrt(-a*c*x + c))
```

3.241 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=40

$$\frac{4}{5a(c-acx)^{5/2}}-\frac{2}{3ac(c-acx)^{3/2}}$$

```
[Out] 4/5/a/(-a*c*x+c)^(5/2)-2/3/a/c/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 21, 43}

$$\frac{4}{5a(c-acx)^{5/2}}-\frac{2}{3ac(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] 4/(5*a*(c - a*c*x)^(5/2)) - 2/(3*a*c*(c - a*c*x)^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \int \frac{1+ax}{(1-ax)(c-acx)^{5/2}}\,dx$$
$$= c\int \frac{1+ax}{(c-acx)^{7/2}}\,dx$$
$$= c\int \left(\frac{2}{(c-acx)^{7/2}} - \frac{1}{c(c-acx)^{5/2}}\right)dx$$
$$= \frac{4}{5a(c-acx)^{5/2}} - \frac{2}{3ac(c-acx)^{3/2}}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.85

$$-\frac{2(5ax+1)\sqrt{c-acx}}{15ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
[Out] (-2*(1 + 5*a*x)*Sqrt[c - a*c*x])/(15*a*c^3*(-1 + a*x)^3)
```

fricas [A] time = 0.42, size = 56, normalized size = 1.40

$$-\frac{2\sqrt{-acx+c}\,(5\,ax+1)}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
[Out] -2/15*sqrt(-a*c*x + c)*(5*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)
```

giac [A] time = 0.18, size = 34, normalized size = 0.85

$$\frac{2\,(5\,acx+c)}{15\,(acx-c)^2\sqrt{-acx+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
[Out] 2/15*(5*a*c*x + c)/((a*c*x - c)^2*sqrt(-a*c*x + c)*a*c)
```

maple [A] time = 0.03, size = 21, normalized size = 0.52

$$\frac{\frac{2ax}{3}+\frac{2}{15}}{(-acx+c)^{\frac{5}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/15*(5*a*x+1)/(-a*c*x+c)^(5/2)/a
```

maxima [A] time = 0.31, size = 24, normalized size = 0.60

$$\frac{2\,(5\,acx+c)}{15\,(-acx+c)^{\frac{5}{2}}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*a*c*x + c)/((-a*c*x + c)^(5/2)*a*c)
```

mupad [B] time = 0.78, size = 20, normalized size = 0.50

$$\frac{10\,a\,x+2}{15\,a\,{\left(c-a\,c\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - a*c*x)^(5/2)),x)
```

```
[Out] (10*a*x + 2)/(15*a*(c - a*c*x)^(5/2))
```

sympy [A] time = 92.03, size = 31, normalized size = 0.78

$$\frac{4}{5a\,(-acx+c)^{\frac{5}{2}}}-\frac{2}{3ac\,(-acx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] 4/(5*a*(-a*c*x + c)**(5/2)) - 2/(3*a*c*(-a*c*x + c)**(3/2))
```

3.242 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=40

$$\frac{4}{7a(c-acx)^{7/2}}-\frac{2}{5ac(c-acx)^{5/2}}$$

```
[Out] 4/7/a/(-a*c*x+c)^(7/2)-2/5/a/c/(-a*c*x+c)^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 21, 43}

$$\frac{4}{7a(c-acx)^{7/2}}-\frac{2}{5ac(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] 4/(7*a*(c - a*c*x)^(7/2)) - 2/(5*a*c*(c - a*c*x)^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{1+ax}{(1-ax)(c-acx)^{7/2}}\,dx$$
$$= c\int \frac{1+ax}{(c-acx)^{9/2}}\,dx$$
$$= c\int \left(\frac{2}{(c-acx)^{9/2}} - \frac{1}{c(c-acx)^{7/2}}\right)dx$$
$$= \frac{4}{7a(c-acx)^{7/2}} - \frac{2}{5ac(c-acx)^{5/2}}$$

Mathematica [**A**] time = 0.06, size = 34, normalized size = 0.85

$$\frac{2(7ax+3)\sqrt{c-acx}}{35ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (2*(3 + 7*a*x)*Sqrt[c - a*c*x])/(35*a*c^4*(-1 + a*x)^4)
```

fricas [**B**] time = 0.42, size = 66, normalized size = 1.65

$$\frac{2\sqrt{-acx+c}\,(7\,ax+3)}{35\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*sqrt(-a*c*x + c)*(7*a*x + 3)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*
x^2 - 4*a^2*c^4*x + a*c^4)
```

giac [**A**] time = 0.22, size = 36, normalized size = 0.90

$$-\frac{2\,(7\,acx+3\,c)}{35\,(acx-c)^3\sqrt{-acx+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -2/35*(7*a*c*x + 3*c)/((a*c*x - c)^3*sqrt(-a*c*x + c)*a*c)
```

maple [A] time = 0.03, size = 21, normalized size = 0.52

$$\frac{\frac{2ax}{5}+\frac{6}{35}}{\left(-acx+c\right)^{\frac{7}{2}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/35*(7*a*x+3)/(-a*c*x+c)^(7/2)/a
```

maxima [A] time = 0.31, size = 26, normalized size = 0.65

$$\frac{2\,(7\,acx+3\,c)}{35\,(-acx+c)^{\frac{7}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/35*(7*a*c*x + 3*c)/((-a*c*x + c)^(7/2)*a*c)
```

mupad [B] time = 0.78, size = 20, normalized size = 0.50

$$\frac{14\,a\,x+6}{35\,a\,(c-a\,c\,x)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - a*c*x)^(7/2)),x)
```

```
[Out] (14*a*x + 6)/(35*a*(c - a*c*x)^(7/2))
```

sympy [A] time = 61.98, size = 31, normalized size = 0.78

$$\frac{4}{7a\,(-acx+c)^{\frac{7}{2}}}-\frac{2}{5ac\,(-acx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] 4/(7*a*(-a*c*x + c)**(7/2)) - 2/(5*a*c*(-a*c*x + c)**(5/2))
```

3.243 $\int e^{3\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=141

$$\frac{256c^7\left(1-a^2x^2\right)^{5/2}}{1155a(c-acx)^{5/2}}+\frac{64c^6\left(1-a^2x^2\right)^{5/2}}{231a(c-acx)^{3/2}}+\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}\sqrt{c-acx}}{11a}$$

```
[Out] 256/1155*c^7*(-a^2*x^2+1)^(5/2)/a/(-a*c*x+c)^(5/2)+64/231*c^6*(-a^2*x^2+1)^
(5/2)/a/(-a*c*x+c)^(3/2)+8/33*c^5*(-a^2*x^2+1)^(5/2)/a/(-a*c*x+c)^(1/2)+2/1
1*c^4*(-a^2*x^2+1)^(5/2)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{256c^7\left(1-a^2x^2\right)^{5/2}}{1155a(c-acx)^{5/2}}+\frac{64c^6\left(1-a^2x^2\right)^{5/2}}{231a(c-acx)^{3/2}}+\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}\sqrt{c-acx}}{11a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^(9/2),x]
```

```
[Out] (256*c^7*(1 - a^2*x^2)^(5/2))/(1155*a*(c - a*c*x)^(5/2)) + (64*c^6*(1 - a^2
*x^2)^(5/2))/(231*a*(c - a*c*x)^(3/2)) + (8*c^5*(1 - a^2*x^2)^(5/2))/(33*a*
Sqrt[c - a*c*x]) + (2*c^4*Sqrt[c - a*c*x]*(1 - a^2*x^2)^(5/2))/(11*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx &= c^3\int (c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}\,dx\\
&= \frac{2c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{5/2}}{11a}+\frac{1}{11}\left(12c^4\right)\int \sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}\,dx\\
&= \frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}}+\frac{2c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{5/2}}{11a}+\frac{1}{33}\left(32c^5\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{\sqrt{c-acx}}\,dx\\
&= \frac{64c^6\left(1-a^2x^2\right)^{5/2}}{231a(c-acx)^{3/2}}+\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}}+\frac{2c^4\sqrt{c-acx}\left(1-a^2x^2\right)^{5/2}}{11a}+\frac{1}{231}\left(128c^6\right.\\
&= \frac{256c^7\left(1-a^2x^2\right)^{5/2}}{1155a(c-acx)^{5/2}}+\frac{64c^6\left(1-a^2x^2\right)^{5/2}}{231a(c-acx)^{3/2}}+\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{33a\sqrt{c-acx}}+\frac{2c^4\sqrt{c-acx}\left(1-\right.}{11a}\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.44

$$-\frac{2c^4(ax+1)^{5/2}\left(105a^3x^3-455a^2x^2+755ax-533\right)\sqrt{c-acx}}{1155a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^(9/2),x]
```

```
[Out] (-2*c^4*(1 + a*x)^(5/2)*Sqrt[c - a*c*x]*(-533 + 755*a*x - 455*a^2*x^2 + 105
*a^3*x^3))/(1155*a*Sqrt[1 - a*x])
```

fricas [A] time = 0.42, size = 91, normalized size = 0.65

$$\frac{2\left(105\,a^5c^4x^5-245\,a^4c^4x^4-50\,a^3c^4x^3+522\,a^2c^4x^2-311\,ac^4x-533\,c^4\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{1155\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="frica
s")
```

```
[Out] 2/1155*(105*a^5*c^4*x^5 - 245*a^4*c^4*x^4 - 50*a^3*c^4*x^3 + 522*a^2*c^4*x^
2 - 311*a*c^4*x - 533*c^4)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

giac [A] time = 0.22, size = 73, normalized size = 0.52

$$-\frac{2\left(512\,\sqrt{2}\,c^{\frac{7}{2}}+\frac{105\,(acx+c)^{\frac{11}{2}}-770\,(acx+c)^{\frac{9}{2}}c+1980\,(acx+c)^{\frac{7}{2}}c^{2}-1848\,(acx+c)^{\frac{5}{2}}c^{3}}{c^{2}}\right)c^{2}}{1155\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="giac"
)
```

```
[Out] -2/1155*(512*sqrt(2)*c^(7/2) + (105*(a*c*x + c)^(11/2) - 770*(a*c*x + c)^(9
/2)*c + 1980*(a*c*x + c)^(7/2)*c^2 - 1848*(a*c*x + c)^(5/2)*c^3)/c^2)*c^2/(
a*abs(c))
```

maple [A] time = 0.03, size = 63, normalized size = 0.45

$$\frac{2\,(ax+1)^{4}\left(105x^{3}a^{3}-455a^{2}x^{2}+755ax-533\right)(-acx+c)^{\frac{9}{2}}}{1155a\,(ax-1)^{3}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(9/2),x)
```

```
[Out] 2/1155*(a*x+1)^4*(105*a^3*x^3-455*a^2*x^2+755*a*x-533)*(-a*c*x+c)^(9/2)/a/(
a*x-1)^3/(-a^2*x^2+1)^(3/2)
```

maxima [B] time = 0.41, size = 254, normalized size = 1.80

$$-\frac{2\left(35\,a^{6}c^{\frac{9}{2}}x^{6}-175\,a^{5}c^{\frac{9}{2}}x^{5}+415\,a^{4}c^{\frac{9}{2}}x^{4}-741\,a^{3}c^{\frac{9}{2}}x^{3}+1482\,a^{2}c^{\frac{9}{2}}x^{2}-5928\,ac^{\frac{9}{2}}x-11856\,c^{\frac{9}{2}}\right)}{385\,\sqrt{ax+1}\,a}-\frac{2\left(7\,a^{5}c^{\frac{9}{2}}x^{5}-37\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="maxim
a")
```

```
[Out] -2/385*(35*a^6*c^(9/2)*x^6 - 175*a^5*c^(9/2)*x^5 + 415*a^4*c^(9/2)*x^4 - 74
1*a^3*c^(9/2)*x^3 + 1482*a^2*c^(9/2)*x^2 - 5928*a*c^(9/2)*x - 11856*c^(9/2)
)/(sqrt(a*x + 1)*a) - 2/21*(7*a^5*c^(9/2)*x^5 - 37*a^4*c^(9/2)*x^4 + 97*a^3
*c^(9/2)*x^3 - 215*a^2*c^(9/2)*x^2 + 860*a*c^(9/2)*x + 1720*c^(9/2))/(sqrt(
a*x + 1)*a) - 6/35*(5*a^4*c^(9/2)*x^4 - 29*a^3*c^(9/2)*x^3 + 93*a^2*c^(9/2)
*x^2 - 407*a*c^(9/2)*x - 814*c^(9/2))/(sqrt(a*x + 1)*a) - 2/5*(a^3*c^(9/2)*
x^3 - 7*a^2*c^(9/2)*x^2 + 43*a*c^(9/2)*x + 91*c^(9/2))/(sqrt(a*x + 1)*a)
```

mupad [B] time = 1.02, size = 90, normalized size = 0.64

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{1688\,c^4\,x}{1155}+\frac{1066\,c^4}{1155\,a}-\frac{422\,a\,c^4\,x^2}{1155}-\frac{944\,a^2\,c^4\,x^3}{1155}+\frac{118\,a^3\,c^4\,x^4}{231}+\frac{8\,a^4\,c^4\,x^5}{33}-\frac{2\,a^5\,c^4\,x^6}{11}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(9/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((1688*c^4*x)/1155 + (1066*c^4)/(1155*a) - (422*a*c^4*x^
2)/1155 - (944*a^2*c^4*x^3)/1155 + (118*a^3*c^4*x^4)/231 + (8*a^4*c^4*x^5)/
33 - (2*a^5*c^4*x^6)/11))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1))^{\frac{9}{2}}\,(ax+1)^3}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(9/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2),
x)
```

3.244 $\int e^{3\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=106

$$\frac{64c^6\left(1-a^2x^2\right)^{5/2}}{315a(c-acx)^{5/2}}+\frac{16c^5\left(1-a^2x^2\right)^{5/2}}{63a(c-acx)^{3/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}}$$

```
[Out] 64/315*c^6*(-a^2*x^2+1)^(5/2)/a/(-a*c*x+c)^(5/2)+16/63*c^5*(-a^2*x^2+1)^(5/
2)/a/(-a*c*x+c)^(3/2)+2/9*c^4*(-a^2*x^2+1)^(5/2)/a/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{64c^6\left(1-a^2x^2\right)^{5/2}}{315a(c-acx)^{5/2}}+\frac{16c^5\left(1-a^2x^2\right)^{5/2}}{63a(c-acx)^{3/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (64*c^6*(1 - a^2*x^2)^(5/2))/(315*a*(c - a*c*x)^(5/2)) + (16*c^5*(1 - a^2*x
^2)^(5/2))/(63*a*(c - a*c*x)^(3/2)) + (2*c^4*(1 - a^2*x^2)^(5/2))/(9*a*Sqrt
[c - a*c*x])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx &= c^3\int\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}\,dx\\ &= \frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}}+\frac{1}{9}\left(8c^4\right)\int\frac{\left(1-a^2x^2\right)^{3/2}}{\sqrt{c-acx}}\,dx\\ &= \frac{16c^5\left(1-a^2x^2\right)^{5/2}}{63a(c-acx)^{3/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}}+\frac{1}{63}\left(32c^5\right)\int\frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{3/2}}\,dx\\ &= \frac{64c^6\left(1-a^2x^2\right)^{5/2}}{315a(c-acx)^{5/2}}+\frac{16c^5\left(1-a^2x^2\right)^{5/2}}{63a(c-acx)^{3/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{9a\sqrt{c-acx}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.51

$$\frac{2c^3(ax+1)^{5/2}\left(35a^2x^2-110ax+107\right)\sqrt{c-acx}}{315a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (2*c^3*(1 + a*x)^(5/2)*Sqrt[c - a*c*x]*(107 - 110*a*x + 35*a^2*x^2))/(315*a
*Sqrt[1 - a*x])
```

fricas [A] time = 0.48, size = 80, normalized size = 0.75

$$-\frac{2\left(35\,a^4c^3x^4-40\,a^3c^3x^3-78\,a^2c^3x^2+104\,ac^3x+107\,c^3\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="frica
s")
```

```
[Out] -2/315*(35*a^4*c^3*x^4 - 40*a^3*c^3*x^3 - 78*a^2*c^3*x^2 + 104*a*c^3*x + 10
7*c^3)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 55, normalized size = 0.52

$$\frac{2\,(ax+1)^4\left(35a^2x^2-110ax+107\right)(-acx+c)^{\frac{7}{2}}}{315a\,(ax-1)^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/315*(a*x+1)^4*(35*a^2*x^2-110*a*x+107)*(-a*c*x+c)^(7/2)/a/(a*x-1)^2/(-a^2
*x^2+1)^(3/2)
```

maxima [B] time = 0.38, size = 210, normalized size = 1.98

$$\frac{2\left(5\,a^5c^{\frac{7}{2}}x^5-20\,a^4c^{\frac{7}{2}}x^4+41\,a^3c^{\frac{7}{2}}x^3-82\,a^2c^{\frac{7}{2}}x^2+328\,ac^{\frac{7}{2}}x+656\,c^{\frac{7}{2}}\right)}{45\,\sqrt{ax+1}\,a}+\frac{2\left(15\,a^4c^{\frac{7}{2}}x^4-66\,a^3c^{\frac{7}{2}}x^3+167\,a^2c^{\frac{7}{2}}x^2-\right.}{35\,\sqrt{ax+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="maxim
a")
```

```
[Out] 2/45*(5*a^5*c^(7/2)*x^5 - 20*a^4*c^(7/2)*x^4 + 41*a^3*c^(7/2)*x^3 - 82*a^2*
c^(7/2)*x^2 + 328*a*c^(7/2)*x + 656*c^(7/2))/(sqrt(a*x + 1)*a) + 2/35*(15*a
^4*c^(7/2)*x^4 - 66*a^3*c^(7/2)*x^3 + 167*a^2*c^(7/2)*x^2 - 668*a*c^(7/2)*x
 - 1336*c^(7/2))/(sqrt(a*x + 1)*a) + 2/5*(3*a^3*c^(7/2)*x^3 - 16*a^2*c^(7/2
)*x^2 + 79*a*c^(7/2)*x + 158*c^(7/2))/(sqrt(a*x + 1)*a) + 2/3*(a^2*c^(7/2)*
x^2 - 10*a*c^(7/2)*x - 23*c^(7/2))/(sqrt(a*x + 1)*a)
```

mupad [B] time = 0.98, size = 79, normalized size = 0.75

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{422\,c^3\,x}{315}+\frac{214\,c^3}{315\,a}+\frac{52\,a\,c^3\,x^2}{315}-\frac{236\,a^2\,c^3\,x^3}{315}-\frac{2\,a^3\,c^3\,x^4}{63}+\frac{2\,a^4\,c^3\,x^5}{9}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(7/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((422*c^3*x)/315 + (214*c^3)/(315*a) + (52*a*c^3*x^2)/31
5 - (236*a^2*c^3*x^3)/315 - (2*a^3*c^3*x^4)/63 + (2*a^4*c^3*x^5)/9))/(1 - a
^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^{\frac{7}{2}}(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(7/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2),
x)
```

3.245 $\int e^{3\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=71

$$\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{35a(c-acx)^{5/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{7a(c-acx)^{3/2}}$$

```
[Out] 8/35*c^5*(-a^2*x^2+1)^(5/2)/a/(-a*c*x+c)^(5/2)+2/7*c^4*(-a^2*x^2+1)^(5/2)/a
/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{8c^5\left(1-a^2x^2\right)^{5/2}}{35a(c-acx)^{5/2}}+\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{7a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (8*c^5*(1 - a^2*x^2)^(5/2))/(35*a*(c - a*c*x)^(5/2)) + (2*c^4*(1 - a^2*x^2)
^(5/2))/(7*a*(c - a*c*x)^(3/2))
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{\sqrt{c-acx}}\,dx$$
$$= \frac{2c^4\left(1-a^2x^2\right)^{5/2}}{7a(c-acx)^{3/2}} + \frac{1}{7}\left(4c^4\right)\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{3/2}}\,dx$$
$$= \frac{8c^5\left(1-a^2x^2\right)^{5/2}}{35a(c-acx)^{5/2}} + \frac{2c^4\left(1-a^2x^2\right)^{5/2}}{7a(c-acx)^{3/2}}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.65

$$-\frac{2c^2(ax+1)^{5/2}(5ax-9)\sqrt{c-acx}}{35a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (-2*c^2*(1 + a*x)^(5/2)*(-9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - a*x])
```

fricas [A] time = 0.42, size = 68, normalized size = 0.96

$$\frac{2\left(5\,a^3c^2x^3 + a^2c^2x^2 - 13\,ac^2x - 9\,c^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{35\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/35*(5*a^3*c^2*x^3 + a^2*c^2*x^2 - 13*a*c^2*x - 9*c^2)*sqrt(-a^2*x^2 + 1)*
sqrt(-a*c*x + c)/(a^2*x - a)
```

giac [A] time = 0.16, size = 47, normalized size = 0.66

$$-\frac{2\left(16\,\sqrt{2}\,c^{\frac{3}{2}} + \frac{5\,(acx+c)^{\frac{7}{2}} - 14\,(acx+c)^{\frac{5}{2}}c}{c^2}\right)c^2}{35\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="giac"
)
```

```
[Out] -2/35*(16*sqrt(2)*c^(3/2) + (5*(a*c*x + c)^(7/2) - 14*(a*c*x + c)^(5/2)*c)/
c^2)*c^2/(a*abs(c))
```

maple [A] time = 0.03, size = 47, normalized size = 0.66

$$\frac{2\left(ax+1\right)^{4}\left(5ax-9\right)\left(-acx+c\right)^{\frac{5}{2}}}{35a\left(ax-1\right)\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/35*(a*x+1)^4*(5*a*x-9)*(-a*c*x+c)^(5/2)/a/(a*x-1)/(-a^2*x^2+1)^(3/2)
```

maxima [B] time = 0.40, size = 164, normalized size = 2.31

$$-\frac{2\left(a^{4}c^{\frac{5}{2}}x^{4}-3\,a^{3}c^{\frac{5}{2}}x^{3}+6\,a^{2}c^{\frac{5}{2}}x^{2}-24\,ac^{\frac{5}{2}}x-48\,c^{\frac{5}{2}}\right)}{7\,\sqrt{ax+1}\,a}-\frac{2\left(3\,a^{3}c^{\frac{5}{2}}x^{3}-11\,a^{2}c^{\frac{5}{2}}x^{2}+44\,ac^{\frac{5}{2}}x+88\,c^{\frac{5}{2}}\right)}{5\,\sqrt{ax+1}\,a}-\frac{2\left(a^{2}c^{\frac{5}{2}}x^{2}-7\right.}{\sqrt{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/7*(a^4*c^(5/2)*x^4 - 3*a^3*c^(5/2)*x^3 + 6*a^2*c^(5/2)*x^2 - 24*a*c^(5/2
)*x - 48*c^(5/2))/(sqrt(a*x + 1)*a) - 2/5*(3*a^3*c^(5/2)*x^3 - 11*a^2*c^(5/
2)*x^2 + 44*a*c^(5/2)*x + 88*c^(5/2))/(sqrt(a*x + 1)*a) - 2*(a^2*c^(5/2)*x^
2 - 7*a*c^(5/2)*x - 14*c^(5/2))/(sqrt(a*x + 1)*a) - 2*(a*c^(5/2)*x + 3*c^(5
/2))/(sqrt(a*x + 1)*a)
```

mupad [B] time = 0.97, size = 68, normalized size = 0.96

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{44\,c^2\,x}{35}+\frac{18\,c^2}{35\,a}+\frac{24\,a\,c^2\,x^2}{35}-\frac{12\,a^2\,c^2\,x^3}{35}-\frac{2\,a^3\,c^2\,x^4}{7}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(5/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((44*c^2*x)/35 + (18*c^2)/(35*a) + (24*a*c^2*x^2)/35 - (
12*a^2*c^2*x^3)/35 - (2*a^3*c^2*x^4)/7))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1))^{\frac{5}{2}}\,(ax+1)^3}{(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(5/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2),
x)
```

3.246 $\int e^{3\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=35

$$\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{5a(c-acx)^{5/2}}$$

[Out] 2/5*c^4*(-a^2*x^2+1)^(5/2)/a/(-a*c*x+c)^(5/2)

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6127, 649}

$$\frac{2c^4\left(1-a^2x^2\right)^{5/2}}{5a(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[E^(3*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]

[Out] (2*c^4*(1 - a^2*x^2)^(5/2))/(5*a*(c - a*c*x)^(5/2))

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx &= c^3\int\frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{3/2}}\,dx\\ &= \frac{2c^4\left(1-a^2x^2\right)^{5/2}}{5a(c-acx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.06

$$\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*(1 + a*x)^(5/2)*(c - a*c*x)^(3/2))/(5*a*(1 - a*x)^(3/2))
```

fricas [A] time = 0.55, size = 49, normalized size = 1.40

$$-\frac{2\left(a^2cx^2+2\,acx+c\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{5\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] -2/5*(a^2*c*x^2 + 2*a*c*x + c)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x -
 a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 34, normalized size = 0.97

$$\frac{2\,(ax+1)^4\,(-acx+c)^{\frac{3}{2}}}{5a\left(-a^2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/5*(a*x+1)^4*(-a*c*x+c)^(3/2)/a/(-a^2*x^2+1)^(3/2)
```

maxima [B] time = 0.42, size = 121, normalized size = 3.46

$$-\frac{2\,c^{\frac{3}{2}}}{\sqrt{ax+1}\,a}+\frac{2\left(a^3c^{\frac{3}{2}}x^3-2\,a^2c^{\frac{3}{2}}x^2+8\,ac^{\frac{3}{2}}x+16\,c^{\frac{3}{2}}\right)}{5\,\sqrt{ax+1}\,a}+\frac{2\left(a^2c^{\frac{3}{2}}x^2-4\,ac^{\frac{3}{2}}x-8\,c^{\frac{3}{2}}\right)}{\sqrt{ax+1}\,a}+\frac{6\left(ac^{\frac{3}{2}}x+2\,c^{\frac{3}{2}}\right)}{\sqrt{ax+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2*c^(3/2)/(sqrt(a*x + 1)*a) + 2/5*(a^3*c^(3/2)*x^3 - 2*a^2*c^(3/2)*x^2 + 8
*a*c^(3/2)*x + 16*c^(3/2))/(sqrt(a*x + 1)*a) + 2*(a^2*c^(3/2)*x^2 - 4*a*c^(
3/2)*x - 8*c^(3/2))/(sqrt(a*x + 1)*a) + 6*(a*c^(3/2)*x + 2*c^(3/2))/(sqrt(a
*x + 1)*a)
```

mupad [B] time = 0.92, size = 49, normalized size = 1.40

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{6\,c\,x}{5}+\frac{2\,c}{5\,a}+\frac{2\,a^2\,c\,x^3}{5}+\frac{6\,a\,c\,x^2}{5}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(3/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((6*c*x)/5 + (2*c)/(5*a) + (2*a^2*c*x^3)/5 + (6*a*c*x^2)
/5))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-c\,(ax-1))^{\frac{3}{2}}\,(ax+1)^3}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2),
x)
```

3.247 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=119

$$-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}$$

```
[Out] -2/3*c^2*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(3/2)+4*arctanh(1/2*c^(1/2)*(-a^2*
x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))*2^(1/2)*c^(1/2)/a-4*c*(-a^2*x^2+1)^(
1/2)/a/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6127, 665, 661, 208}

$$-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*c*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) - (2*c^2*(1 - a^2*x^2)^(3/2))/
(3*a*(c - a*c*x)^(3/2)) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*
x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
```

```
|| EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{5/2}}\,dx\\
&= -\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+\left(2c^2\right)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{3/2}}\,dx\\
&= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+(4c)\int \frac{1}{\sqrt{c-acx}\,\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\left(8ac^2\right)\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\
&= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.56

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}\,(ax+7)-6\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{3a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(7 + a*x) - 6*Sqrt[2]*ArcTanh[Sqrt[1 + a
*x]/Sqrt[2]]))/(3*a*Sqrt[1 - a*x])
```

fricas [A] time = 0.56, size = 220, normalized size = 1.85

$$\left[\frac{2\left(3\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,(ax+7)\right)}{3\left(a^2x-a\right)}, \frac{2\left(6\sqrt{2}\,(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqr
t(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + sq
rt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 7))/(a^2*x - a), 2/3*(6*sqrt(2)*(a
*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c
)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 7))/(a^2*x
- a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 95, normalized size = 0.80

$$-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(6\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-xa\sqrt{c\,(ax+1)}-7\sqrt{c\,(ax+1)}\right)}{3\,(ax-1)\sqrt{c\,(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*2^(1/2)*arctanh(1/2*(
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-x*a*(c*(a*x+1))^(1/2)-7*(c*(a*x+1))^(1/2)
)/(a*x-1)/(c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^{3}}{\left(1-a^{2}\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.248 $\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=115

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}}+\frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{a\sqrt{c}}$$

```
[Out] c^2*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(5/2)-3*arctanh(1/2*c^(1/2)*(-a^2*x^2+1
)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))*2^(1/2)/a/c^(1/2)+3*(-a^2*x^2+1)^(1/2)/a/
(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6127, 663, 665, 661, 208}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}}+\frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (3*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) + (c^2*(1 - a^2*x^2)^(3/2))/(a*(c
 - a*c*x)^(5/2)) - (3*Sqrt[2]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*
Sqrt[c - a*c*x])])/(a*Sqrt[c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
```

```
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{7/2}}\,dx\\
&= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}}-\frac{1}{2}(3c)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{3/2}}\,dx\\
&= \frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}}-3\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx\\
&= \frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}}+(6ac)\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\
&= \frac{3\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{a(c-acx)^{5/2}}-\frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.50

$$\frac{(ax+1)^{5/2}(c-acx)^{3/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{1}{2}(ax+1)\right)}{10ac^2(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] ((1 + a*x)^(5/2)*(c - a*c*x)^(3/2)*Hypergeometric2F1[2, 5/2, 7/2, (1 + a*x)
/2])/(10*a*c^2*(1 - a*x)^(3/2))
```

fricas [A] time = 0.49, size = 259, normalized size = 2.25

$$\left[-\frac{4\sqrt{-a^2x^2+1}\sqrt{-acx+c}\,(ax-2)-\frac{3\sqrt{2}\left(a^2cx^2-2\,acx+c\right)\log\left(-\frac{a^2x^2+2\,ax+\frac{2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{\sqrt{c}}-3}{a^2x^2-2\,ax+1}\right)}{\sqrt{c}}}{2\left(a^3cx^2-2\,a^2cx+ac\right)},-\frac{3\sqrt{2}\left(a^2cx^2-2\,acx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x - 2) - 3*sqrt(2)*(a^2*c*x
^2 - 2*a*c*x + c)*log(-(a^2*x^2 + 2*a*x + 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c)/sqrt(c) - 3)/(a^2*x^2 - 2*a*x + 1))/sqrt(c))/(a^3*c*x^2 - 2*a^
2*c*x + a*c), -(3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*arctan(sqrt(
2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-1/c)/(a^2*x^2 - 1)) + 2*sqrt(-
a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x - 2))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

giac [A] time = 0.22, size = 70, normalized size = 0.61

$$\frac{\frac{3\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{acx+c}-\frac{2\sqrt{acx+c}\,c}{acx-c}}{a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] (3*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt
(a*c*x + c) - 2*sqrt(a*c*x + c)*c/(a*c*x - c))/(a*abs(c))
```

maple [A] time = 0.05, size = 127, normalized size = 1.10

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac-2xa\sqrt{c\,(ax+1)}\,\sqrt{c}-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\right.}{c^{\frac{3}{2}}\,(ax-1)^2\sqrt{c\,(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(3*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1
/2)*2^(1/2)/c^(1/2))*x*a*c-2*x*a*(c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctan
h(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+4*(c*(a*x+1))^(1/2)*c^(1/2))/c^(
3/2)/(a*x-1)^2/(c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}\,\sqrt{c-a\,c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2)),x)
```

```
[Out] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\sqrt{-c\,(ax-1)}\,\left(-(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**3/(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)), x
)
```

3.249 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{4\sqrt{2}\,ac^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}-\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}$$

```
[Out] 1/2*c^2*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(7/2)+3/8*arctanh(1/2*c^(1/2)*(-a^2
*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^(3/2)*2^(1/2)-3/4*(-a^2*x^2+1)^
(1/2)/a/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6127, 663, 661, 208}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{4\sqrt{2}\,ac^{3/2}}-\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(4*a*(c - a*c*x)^(3/2)) + (c^2*(1 - a^2*x^2)^(3/2))/
(2*a*(c - a*c*x)^(7/2)) + (3*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*S
qrt[c - a*c*x])])/(4*Sqrt[2]*a*c^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
```

```
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{9/2}}\,dx\\
&= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}-\frac{1}{4}(3c)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{5/2}}\,dx\\
&= -\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}+\frac{3\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{8c}\\
&= -\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}-\frac{1}{4}(3a)\,\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\
&= -\frac{3\sqrt{1-a^2x^2}}{4a(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{2a(c-acx)^{7/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{4\sqrt{2}\,ac^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 91, normalized size = 0.75

$$
\frac{\sqrt{c-acx}\left(10a^2x^2+8ax+3(ax-1)^2\sqrt{2ax+2}\,\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)-2\right)}{8ac^2(ax-1)^2\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-2 + 8*a*x + 10*a^2*x^2 + 3*(-1 + a*x)^2*Sqrt[2 + 2*a*x]*
ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(8*a*c^2*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.48, size = 312, normalized size = 2.56

$$\left[\frac{3\sqrt{2}\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)-4\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,(5\,a}{16\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/16*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(a^2*c*x^2
+ 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) - 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(5*a*x - 1))/(
a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2), 1/8*(3*sqrt(2)*(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-
a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c
)*(5*a*x - 1))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

giac [A] time = 0.41, size = 73, normalized size = 0.60

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\left(5\,(acx+c)^{\frac{3}{2}}-6\,\sqrt{acx+c}\,c\right)}{(acx-c)^2}}{8\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*(
5*(a*c*x + c)^(3/2) - 6*sqrt(a*c*x + c)*c)/(a*c*x - c)^2)/(a*abs(c))
```

maple [A] time = 0.05, size = 158, normalized size = 1.30

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c-6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac+10xa\sqrt{c\,(ax}{8c^{\frac{5}{2}}\,(ax-1)^3\,\sqrt{c\,(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(5/2)*(3*2^(1/2)*arctanh(1/2*(
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*2^(1/2)*arctanh(1/2*(c*(a*x+1
```

```
))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+10*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+3*2^(1/2)*
arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-2*(c*(a*x+1))^(1/2)*c^(1/2
))/(a*x-1)^3/(c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(-a*c*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}\,(c-a\,c\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{(-c\,(ax-1))^{\frac{3}{2}}\,(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-c*(a*x - 1))**(3/2)*(-(a*x - 1)*(a*x + 1))**(3/2))
, x)
```

3.250 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{16\sqrt{2}\,ac^{5/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}-\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}$$

```
[Out] 1/3*c^2*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(9/2)+1/32*arctanh(1/2*c^(1/2)*(-a^
2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^(5/2)*2^(1/2)-1/4*(-a^2*x^2+1)
^(1/2)/a/(-a*c*x+c)^(5/2)+1/16*(-a^2*x^2+1)^(1/2)/a/c/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6127, 663, 673, 661, 208}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{16\sqrt{2}\,ac^{5/2}}-\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(4*a*(c - a*c*x)^(5/2)) + Sqrt[1 - a^2*x^2]/(16*a*c*(c -
 a*c*x)^(3/2)) + (c^2*(1 - a^2*x^2)^(3/2))/(3*a*(c - a*c*x)^(9/2)) + ArcTan
h[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(16*Sqrt[2]*a*c^(5
/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{11/2}}\,dx\\
&= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}-\frac{1}{2}c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{7/2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}+\frac{\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{8c}\\
&= -\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}+\frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{32c^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}-\frac{a\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{16c}\\
&= -\frac{\sqrt{1-a^2x^2}}{4a(c-acx)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{16ac(c-acx)^{3/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{16\sqrt{2}\,ac^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.36

$$\frac{(ax+1)^{5/2}(c-acx)^{3/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{1}{2}(ax+1)\right)}{40ac^4(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] ((1 + a*x)^(5/2)*(c - a*c*x)^(3/2)*Hypergeometric2F1[5/2, 4, 7/2, (1 + a*x)
/2])/(40*a*c^4*(1 - a*x)^(3/2))
```

fricas [A] time = 0.52, size = 364, normalized size = 2.32

$$\left[\frac{3\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right)+4\left(3a^2x^2+22ax+\cdots\right.}{192\left(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log
(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt
(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(3*a^2*x^2 + 22*a*x + 7)*sqrt(-a^2*x^
2 + 1)*sqrt(-a*c*x + c))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a
^2*c^3*x + a*c^3), 1/96*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x
 + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/
(a^2*c*x^2 - c)) + 2*(3*a^2*x^2 + 22*a*x + 7)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
]
```

giac [A] time = 0.23, size = 92, normalized size = 0.59

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}\,c}+\frac{2\left(3(acx+c)^{\frac{5}{2}}+16(acx+c)^{\frac{3}{2}}c-12\sqrt{acx+c}\,c^2\right)}{(acx-c)^3c}}{96a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac"
)
```

[Out] -1/96*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c)
+ 2*(3*(a*c*x + c)^(5/2) + 16*(a*c*x + c)^(3/2)*c - 12*sqrt(a*c*x + c)*c^2)
/((a*c*x - c)^3*c))/(a*abs(c))

maple [A] time = 0.05, size = 208, normalized size = 1.32

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^3a^3c-9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c-6x^2a^2\sqrt{c\,(a}\right.}{96c^{\frac{7}{2}}\,(ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x)

[Out] -1/96*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(7/2)*(3*2^(1/2)*arctanh(1/2*
(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-9*2^(1/2)*arctanh(1/2*(c*(a*x+
1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*x^2*a^2*(c*(a*x+1))^(1/2)*c^(1/2)+9*
2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-44*x*a*(c*(a*x
+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
c-14(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^4/(c*(a*x+1))^(1/2)/a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxim
a")

[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(-a*c*x + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}\,(c-a\,c\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(5/2)),x)

[Out] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{(-c(ax-1))^{\frac{5}{2}}(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-c*(a*x - 1))**(5/2)*(-(a*x - 1)*(a*x + 1))**(3/2))
, x)
```

3.251 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=192

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{256\sqrt{2}\,ac^{7/2}}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}}+\frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}}+\frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}}$$

```
[Out] 1/4*c^2*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(11/2)+3/512*arctanh(1/2*c^(1/2)*(-
a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^(7/2)*2^(1/2)-1/8*(-a^2*x^2+
1)^(1/2)/a/(-a*c*x+c)^(7/2)+1/64*(-a^2*x^2+1)^(1/2)/a/c/(-a*c*x+c)^(5/2)+3/
256*(-a^2*x^2+1)^(1/2)/a/c^2/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6127, 663, 673, 661, 208}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}}+\frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{256\sqrt{2}\,ac^{7/2}}-\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}}+\frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(8*a*(c - a*c*x)^(7/2)) + Sqrt[1 - a^2*x^2]/(64*a*c*(c -
 a*c*x)^(5/2)) + (3*Sqrt[1 - a^2*x^2])/(256*a*c^2*(c - a*c*x)^(3/2)) + (c^2
*(1 - a^2*x^2)^(3/2))/(4*a*(c - a*c*x)^(11/2)) + (3*ArcTanh[(Sqrt[c]*Sqrt[1
 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(256*Sqrt[2]*a*c^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{13/2}}\,dx$$

$$= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} - \frac{1}{8}(3c)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{9/2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} + \frac{\int \frac{1}{(c-acx)^{5/2}\sqrt{1-a^2x^2}}\,dx}{16c}$$

$$= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} + \frac{3\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{128c^2}$$

$$= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} + \frac{3\int \frac{1}{\sqrt{c-acx}\,\sqrt{1-}}}{512c^3}$$

$$= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} - \frac{(3a)\,\mathrm{Subst}\Big(\int}{}$$

$$= -\frac{\sqrt{1-a^2x^2}}{8a(c-acx)^{7/2}} + \frac{\sqrt{1-a^2x^2}}{64ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{256ac^2(c-acx)^{3/2}} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4a(c-acx)^{11/2}} + \frac{3\tanh^{-1}\Big(\frac{\sqrt{c}}{\sqrt{2}}}{256\sqrt{2}\,a}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.30

$$\frac{(ax+1)^{5/2}(c-acx)^{3/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{1}{2}(ax+1)\right)}{80ac^5(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] ((1 + a*x)^(5/2)*(c - a*c*x)^(3/2)*Hypergeometric2F1[5/2, 5, 7/2, (1 + a*x)
/2])/(80*a*c^5*(1 - a*x)^(3/2))
```

fricas [A] time = 0.53, size = 420, normalized size = 2.19

$$\left[\frac{3\sqrt{2}\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+4\left(3\,a^3x^3-\right.}{1024\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="frica
s")
```

```
[Out] [1/1024*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x -
 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-
a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(3*a^3*x^3 - 13*a^2*x^
2 - 79*a*x - 39)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^6*c^4*x^5 - 5*a^5*
c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4), 1/512*(3*
sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*sqrt(-c
)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 -
c)) + 2*(3*a^3*x^3 - 13*a^2*x^2 - 79*a*x - 39)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5
*a^2*c^4*x - a*c^4)]
```

giac [A] time = 0.32, size = 105, normalized size = 0.55

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}\,c^2}+\frac{2\left(3\,(acx+c)^{\frac{7}{2}}-22\,(acx+c)^{\frac{5}{2}}c-44\,(acx+c)^{\frac{3}{2}}c^2+24\,\sqrt{acx+c}\,c^3\right)}{(acx-c)^4c^2}}{512\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/512*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^
2) + 2*(3*(a*c*x + c)^(7/2) - 22*(a*c*x + c)^(5/2)*c - 44*(a*c*x + c)^(3/2)
*c^2 + 24*sqrt(a*c*x + c)*c^3)/((a*c*x - c)^4*c^2))/(a*abs(c))
```

maple [A] time = 0.06, size = 258, normalized size = 1.34

$$-\frac{\sqrt{-a^{2}x^{2}+1}\ \sqrt{-c\left(ax-1\right)}\ \left(3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\ \sqrt{2}}{2\sqrt{c}}\right)x^{4}a^{4}c-12\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\ \sqrt{2}}{2\sqrt{c}}\right)x^{3}a^{3}c-6x^{3}a^{3}\sqrt{c}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/512*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1/2)*arctanh(1/2
*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^4*a^4*c-12*2^(1/2)*arctanh(1/2*(c*(a*
x+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-6*x^3*a^3*(c*(a*x+1))^(1/2)*c^(1/2)+
18*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c+26*x^2*
a^2*(c*(a*x+1))^(1/2)*c^(1/2)-12*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1
/2)/c^(1/2))*x*a*c+158*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+3*2^(1/2)*arctanh(1/2*
(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+78*(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^
5/(c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+1\right)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\left(-acx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(-a*c*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}\,{\left(c-a\,c\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(7/2)),x)
```

```
[Out] int((a*x + 1)^3/((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(7/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{(-c(ax-1))^{\frac{7}{2}}(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-c*(a*x - 1))**(7/2)*(-(a*x - 1)*(a*x + 1))**(3/2))
, x)
```

3.252 $\int e^{-\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=206

$$\frac{16384c^5\sqrt{1-a^2x^2}}{693a\sqrt{c-acx}}+\frac{4096c^4\sqrt{1-a^2x^2}\sqrt{c-acx}}{693a}+\frac{512c^3\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}{231a}+\frac{640c^2\sqrt{1-a^2x^2}\,(c-acx)^{5/2}}{693a}+\frac{40}{}$$

```
[Out] 512/231*c^3*(-a*c*x+c)^(3/2)*(-a^2*x^2+1)^(1/2)/a+640/693*c^2*(-a*c*x+c)^(5
/2)*(-a^2*x^2+1)^(1/2)/a+40/99*c*(-a*c*x+c)^(7/2)*(-a^2*x^2+1)^(1/2)/a+2/11
*(-a*c*x+c)^(9/2)*(-a^2*x^2+1)^(1/2)/a+16384/693*c^5*(-a^2*x^2+1)^(1/2)/a/(
-a*c*x+c)^(1/2)+4096/693*c^4*(-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.16, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{16384c^5\sqrt{1-a^2x^2}}{693a\sqrt{c-acx}}+\frac{4096c^4\sqrt{1-a^2x^2}\sqrt{c-acx}}{693a}+\frac{512c^3\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}{231a}+\frac{640c^2\sqrt{1-a^2x^2}\,(c-acx)^{5/2}}{693a}+\frac{40}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] (16384*c^5*Sqrt[1 - a^2*x^2])/(693*a*Sqrt[c - a*c*x]) + (4096*c^4*Sqrt[c -
a*c*x]*Sqrt[1 - a^2*x^2])/(693*a) + (512*c^3*(c - a*c*x)^(3/2)*Sqrt[1 - a^2
*x^2])/(231*a) + (640*c^2*(c - a*c*x)^(5/2)*Sqrt[1 - a^2*x^2])/(693*a) + (4
0*c*(c - a*c*x)^(7/2)*Sqrt[1 - a^2*x^2])/(99*a) + (2*(c - a*c*x)^(9/2)*Sqrt
[1 - a^2*x^2])/(11*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}(c-acx)^{9/2}\,dx &= \frac{\int \frac{(c-acx)^{11/2}}{\sqrt{1-a^2x^2}}\,dx}{c} \\
&= \frac{2(c-acx)^{9/2}\sqrt{1-a^2x^2}}{11a} + \frac{20}{11}\int \frac{(c-acx)^{9/2}}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{40c(c-acx)^{7/2}\sqrt{1-a^2x^2}}{99a} + \frac{2(c-acx)^{9/2}\sqrt{1-a^2x^2}}{11a} + \frac{1}{99}(320c)\int \frac{(c-acx)^{7/2}}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{640c^2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{693a} + \frac{40c(c-acx)^{7/2}\sqrt{1-a^2x^2}}{99a} + \frac{2(c-acx)^{9/2}\sqrt{1-a^2x^2}}{11a} \\
&= \frac{512c^3(c-acx)^{3/2}\sqrt{1-a^2x^2}}{231a} + \frac{640c^2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{693a} + \frac{40c(c-acx)^{7/2}\sqrt{1-}}{99a} \\
&= \frac{4096c^4\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{693a} + \frac{512c^3(c-acx)^{3/2}\sqrt{1-a^2x^2}}{231a} + \frac{640c^2(c-acx)^{5/2}\sqrt{1}}{693a} \\
&= \frac{16384c^5\sqrt{1-a^2x^2}}{693a\sqrt{c-acx}} + \frac{4096c^4\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{693a} + \frac{512c^3(c-acx)^{3/2}\sqrt{1-a^2x^2}}{231a} +
\end{aligned}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 0.35

$$-\frac{2c^5\sqrt{1-a^2x^2}\left(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531\right)}{693a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c^5*Sqrt[1 - a^2*x^2]*(-11531 + 5419*a*x - 3198*a^2*x^2 + 1510*a^3*x^3
- 455*a^4*x^4 + 63*a^5*x^5))/(693*a*Sqrt[c - a*c*x])
```

fricas [A] time = 0.60, size = 91, normalized size = 0.44

$$\frac{2\left(63\,a^5c^4x^5-455\,a^4c^4x^4+1510\,a^3c^4x^3-3198\,a^2c^4x^2+5419\,ac^4x-11531\,c^4\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{693\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/693*(63*a^5*c^4*x^5 - 455*a^4*c^4*x^4 + 1510*a^3*c^4*x^3 - 3198*a^2*c^4*x
^2 + 5419*a*c^4*x - 11531*c^4)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x -
 a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 72, normalized size = 0.35

$$\frac{2\sqrt{-a^{2}x^{2}+1}\,\left(-acx+c\right)^{\frac{9}{2}}\left(63x^{5}a^{5}-455x^{4}a^{4}+1510x^{3}a^{3}-3198a^{2}x^{2}+5419ax-11531\right)}{693\left(ax-1\right)^{5}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/693*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(9/2)*(63*a^5*x^5-455*a^4*x^4+1510*a^3*
x^3-3198*a^2*x^2+5419*a*x-11531)/(a*x-1)^5/a
```

maxima [A] time = 0.38, size = 82, normalized size = 0.40

$$-\frac{2\left(63\,a^{5}c^{\frac{9}{2}}x^{5}-455\,a^{4}c^{\frac{9}{2}}x^{4}+1510\,a^{3}c^{\frac{9}{2}}x^{3}-3198\,a^{2}c^{\frac{9}{2}}x^{2}+5419\,ac^{\frac{9}{2}}x-11531\,c^{\frac{9}{2}}\right)\sqrt{ax+1}\,(ax-1)}{693\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/693*(63*a^5*c^(9/2)*x^5 - 455*a^4*c^(9/2)*x^4 + 1510*a^3*c^(9/2)*x^3 - 3
198*a^2*c^(9/2)*x^2 + 5419*a*c^(9/2)*x - 11531*c^(9/2))*sqrt(a*x + 1)*(a*x
- 1)/(a^2*x - a)
```

mupad [B] time = 1.04, size = 96, normalized size = 0.47

$$\frac{2\,c^4\,\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}\,\left(63\,a^4\,x^4-392\,a^3\,x^3+1118\,a^2\,x^2-2080\,a\,x+3339\right)}{693\,a}-\frac{16384\,c^4\,\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{693\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(9/2))/(a*x + 1),x)
```

```
[Out] (2*c^4*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2)*(1118*a^2*x^2 - 2080*a*x - 392
*a^3*x^3 + 63*a^4*x^4 + 3339))/(693*a) - (16384*c^4*(1 - a^2*x^2)^(1/2)*(c
- a*c*x)^(1/2))/(693*a*(a*x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1))^{\frac{9}{2}}\,\sqrt{-(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(9/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(9/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.253 $\int e^{-\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=171

$$\frac{4096c^4\sqrt{1-a^2x^2}}{315a\sqrt{c-acx}}+\frac{1024c^3\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{315a}+\frac{128c^2\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}{105a}+\frac{32c\sqrt{1-a^2x^2}\,(c-acx)^{5/2}}{63a}+\frac{2\sqrt{1-}}{}$$

```
[Out] 128/105*c^2*(-a*c*x+c)^(3/2)*(-a^2*x^2+1)^(1/2)/a+32/63*c*(-a*c*x+c)^(5/2)*
(-a^2*x^2+1)^(1/2)/a+2/9*(-a*c*x+c)^(7/2)*(-a^2*x^2+1)^(1/2)/a+4096/315*c^4
*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(1/2)+1024/315*c^3*(-a*c*x+c)^(1/2)*(-a^2*
x^2+1)^(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{4096c^4\sqrt{1-a^2x^2}}{315a\sqrt{c-acx}}+\frac{1024c^3\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{315a}+\frac{128c^2\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}{105a}+\frac{32c\sqrt{1-a^2x^2}\,(c-acx)^{5/2}}{63a}+\frac{2\sqrt{1-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] (4096*c^4*Sqrt[1 - a^2*x^2])/(315*a*Sqrt[c - a*c*x]) + (1024*c^3*Sqrt[c - a
*c*x]*Sqrt[1 - a^2*x^2])/(315*a) + (128*c^2*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*
x^2])/(105*a) + (32*c*(c - a*c*x)^(5/2)*Sqrt[1 - a^2*x^2])/(63*a) + (2*(c -
 a*c*x)^(7/2)*Sqrt[1 - a^2*x^2])/(9*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx &= \frac{\int \frac{(c-acx)^{9/2}}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{2(c-acx)^{7/2}\sqrt{1-a^2x^2}}{9a}+\frac{16}{9}\int \frac{(c-acx)^{7/2}}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{32c(c-acx)^{5/2}\sqrt{1-a^2x^2}}{63a}+\frac{2(c-acx)^{7/2}\sqrt{1-a^2x^2}}{9a}+\frac{1}{21}(64c)\int \frac{(c-acx)^{5/2}}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{128c^2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{105a}+\frac{32c(c-acx)^{5/2}\sqrt{1-a^2x^2}}{63a}+\frac{2(c-acx)^{7/2}\sqrt{1-a^2x^2}}{9a}\\
&= \frac{1024c^3\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{315a}+\frac{128c^2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{105a}+\frac{32c(c-acx)^{5/2}\sqrt{1-a^2x^2}}{63a}\\
&= \frac{4096c^4\sqrt{1-a^2x^2}}{315a\sqrt{c-acx}}+\frac{1024c^3\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{315a}+\frac{128c^2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{105a}+
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.38

$$\frac{2c^4\sqrt{1-a^2x^2}\left(35a^4x^4-220a^3x^3+642a^2x^2-1276ax+2867\right)}{315a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^ArcTanh[a*x],x]
[Out] (2*c^4*Sqrt[1 - a^2*x^2]*(2867 - 1276*a*x + 642*a^2*x^2 - 220*a^3*x^3 + 35*
a^4*x^4))/(315*a*Sqrt[c - a*c*x])
```

fricas [A] time = 0.41, size = 80, normalized size = 0.47

$$-\frac{2\left(35\,a^4c^3x^4-220\,a^3c^3x^3+642\,a^2c^3x^2-1276\,ac^3x+2867\,c^3\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/315*(35*a^4*c^3*x^4 - 220*a^3*c^3*x^3 + 642*a^2*c^3*x^2 - 1276*a*c^3*x +
 2867*c^3)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 64, normalized size = 0.37

$$\frac{2\sqrt{-a^{2}x^{2}+1}\,\left(-acx+c\right)^{\frac{7}{2}}\left(35x^{4}a^{4}-220x^{3}a^{3}+642a^{2}x^{2}-1276ax+2867\right)}{315\left(ax-1\right)^{4}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/315*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(7/2)*(35*a^4*x^4-220*a^3*x^3+642*a^2*x
^2-1276*a*x+2867)/(a*x-1)^4/a
```

maxima [A] time = 0.37, size = 71, normalized size = 0.42

$$\frac{2\left(35\,a^{4}c^{\frac{7}{2}}x^{4}-220\,a^{3}c^{\frac{7}{2}}x^{3}+642\,a^{2}c^{\frac{7}{2}}x^{2}-1276\,ac^{\frac{7}{2}}x+2867\,c^{\frac{7}{2}}\right)\sqrt{ax+1}\,(ax-1)}{315\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/315*(35*a^4*c^(7/2)*x^4 - 220*a^3*c^(7/2)*x^3 + 642*a^2*c^(7/2)*x^2 - 127
6*a*c^(7/2)*x + 2867*c^(7/2))*sqrt(a*x + 1)*(a*x - 1)/(a^2*x - a)
```

mupad [B] time = 1.02, size = 88, normalized size = 0.51

$$-\frac{4096\,c^{3}\,\sqrt{1-a^{2}\,x^{2}}\,\sqrt{c-a\,c\,x}}{315\,a\,\left(a\,x-1\right)}-\frac{2\,c^{3}\,\sqrt{1-a^{2}\,x^{2}}\,\sqrt{c-a\,c\,x}\,\left(35\,a^{3}\,x^{3}-185\,a^{2}\,x^{2}+457\,a\,x-819\right)}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(7/2))/(a*x + 1),x)
```

```
[Out] - (4096*c^3*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(315*a*(a*x - 1)) - (2*c
^3*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2)*(457*a*x - 185*a^2*x^2 + 35*a^3*x^
3 - 819))/(315*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(ax-1\right)\right)^{\frac{7}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(7/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.254 $\int e^{-\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=136

$$\frac{256c^3\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}+\frac{64c^2\sqrt{1-a^2x^2}\sqrt{c-acx}}{35a}+\frac{24c\sqrt{1-a^2x^2}(c-acx)^{3/2}}{35a}+\frac{2\sqrt{1-a^2x^2}(c-acx)^{5/2}}{7a}$$

```
[Out] 24/35*c*(-a*c*x+c)^(3/2)*(-a^2*x^2+1)^(1/2)/a+2/7*(-a*c*x+c)^(5/2)*(-a^2*x^
2+1)^(1/2)/a+256/35*c^3*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(1/2)+64/35*c^2*(-a
*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{256c^3\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}+\frac{64c^2\sqrt{1-a^2x^2}\sqrt{c-acx}}{35a}+\frac{24c\sqrt{1-a^2x^2}(c-acx)^{3/2}}{35a}+\frac{2\sqrt{1-a^2x^2}(c-acx)^{5/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] (256*c^3*Sqrt[1 - a^2*x^2])/(35*a*Sqrt[c - a*c*x]) + (64*c^2*Sqrt[c - a*c*x
]*Sqrt[1 - a^2*x^2])/(35*a) + (24*c*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2])/(3
5*a) + (2*(c - a*c*x)^(5/2)*Sqrt[1 - a^2*x^2])/(7*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx &= \frac{\int \frac{(c-acx)^{7/2}}{\sqrt{1-a^2x^2}}\,dx}{c} \\
&= \frac{2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{7a} + \frac{12}{7}\int \frac{(c-acx)^{5/2}}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{24c(c-acx)^{3/2}\sqrt{1-a^2x^2}}{35a} + \frac{2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{7a} + \frac{1}{35}(96c)\int \frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{64c^2\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{35a} + \frac{24c(c-acx)^{3/2}\sqrt{1-a^2x^2}}{35a} + \frac{2(c-acx)^{5/2}\sqrt{1-a^2x^2}}{7a} + \\
&= \frac{256c^3\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}} + \frac{64c^2\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{35a} + \frac{24c(c-acx)^{3/2}\sqrt{1-a^2x^2}}{35a} + \frac{2(c-
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.42

$$
-\frac{2c^3\sqrt{1-a^2x^2}\,\left(5a^3x^3-27a^2x^2+71ax-177\right)}{35a\sqrt{c-acx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c^3*Sqrt[1 - a^2*x^2]*(-177 + 71*a*x - 27*a^2*x^2 + 5*a^3*x^3))/(35*a*S
qrt[c - a*c*x])
```

fricas [A] time = 0.41, size = 69, normalized size = 0.51

$$
\frac{2\left(5\,a^3c^2x^3-27\,a^2c^2x^2+71\,ac^2x-177\,c^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{35\left(a^2x-a\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/35*(5*a^3*c^2*x^3 - 27*a^2*c^2*x^2 + 71*a*c^2*x - 177*c^2)*sqrt(-a^2*x^2
 + 1)*sqrt(-a*c*x + c)/(a^2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 56, normalized size = 0.41

$$\frac{2\sqrt{-a^{2}x^{2}+1}\;(-acx+c)^{\frac{5}{2}}\left(5x^{3}a^{3}-27a^{2}x^{2}+71ax-177\right)}{35\,(ax-1)^{3}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/35*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(5/2)*(5*a^3*x^3-27*a^2*x^2+71*a*x-177)/
(a*x-1)^3/a
```

maxima [A] time = 0.39, size = 60, normalized size = 0.44

$$-\frac{2\left(5\,a^{3}c^{\frac{5}{2}}x^{3}-27\,a^{2}c^{\frac{5}{2}}x^{2}+71\,ac^{\frac{5}{2}}x-177\,c^{\frac{5}{2}}\right)\sqrt{ax+1}\,(ax-1)}{35\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/35*(5*a^3*c^(5/2)*x^3 - 27*a^2*c^(5/2)*x^2 + 71*a*c^(5/2)*x - 177*c^(5/2
))*sqrt(a*x + 1)*(a*x - 1)/(a^2*x - a)
```

mupad [B] time = 0.99, size = 80, normalized size = 0.59

$$\frac{2\,c^{2}\,\sqrt{1-a^{2}\,x^{2}}\;\sqrt{c-a\,c\,x}\;\left(5\,a^{2}\,x^{2}-22\,a\,x+49\right)}{35\,a}-\frac{256\,c^{2}\,\sqrt{1-a^{2}\,x^{2}}\;\sqrt{c-a\,c\,x}}{35\,a\;(a\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(5/2))/(a*x + 1),x)
```

```
[Out] (2*c^2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2)*(5*a^2*x^2 - 22*a*x + 49))/(35
*a) - (256*c^2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(35*a*(a*x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1))^{\frac{5}{2}}\,\sqrt{-(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(5/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.255 $\int e^{-\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{64c^2\sqrt{1-a^2x^2}}{15a\sqrt{c-acx}}+\frac{16c\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{15a}+\frac{2\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}{5a}$$

```
[Out] 2/5*(-a*c*x+c)^(3/2)*(-a^2*x^2+1)^(1/2)/a+64/15*c^2*(-a^2*x^2+1)^(1/2)/a/(-
a*c*x+c)^(1/2)+16/15*c*(-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{64c^2\sqrt{1-a^2x^2}}{15a\sqrt{c-acx}}+\frac{16c\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{15a}+\frac{2\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (64*c^2*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[c - a*c*x]) + (16*c*Sqrt[c - a*c*x]*S
qrt[1 - a^2*x^2])/(15*a) + (2*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2])/(5*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{\int \frac{(c-acx)^{5/2}}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a} + \frac{8}{5}\int \frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{16c\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{15a} + \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a} + \frac{1}{15}(32c)\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{64c^2\sqrt{1-a^2x^2}}{15a\sqrt{c-acx}} + \frac{16c\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{15a} + \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.49

$$\frac{2c^2\sqrt{1-a^2x^2}\,\left(3a^2x^2-14ax+43\right)}{15a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (2*c^2*Sqrt[1 - a^2*x^2]*(43 - 14*a*x + 3*a^2*x^2))/(15*a*Sqrt[c - a*c*x])
```

fricas [A] time = 0.51, size = 52, normalized size = 0.51

$$-\frac{2\left(3\,a^2cx^2-14\,acx+43\,c\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{15\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/15*(3*a^2*c*x^2 - 14*a*c*x + 43*c)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(
a^2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 48, normalized size = 0.48

$$\frac{2\sqrt{-a^{2}x^{2}+1}\,\left(-acx+c\right)^{\frac{3}{2}}\left(3a^{2}x^{2}-14ax+43\right)}{15\left(ax-1\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/15*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(3/2)*(3*a^2*x^2-14*a*x+43)/(a*x-1)^2/a
```

maxima [A] time = 0.35, size = 49, normalized size = 0.49

$$\frac{2\left(3\,a^{2}c^{\frac{3}{2}}x^{2}-14\,ac^{\frac{3}{2}}x+43\,c^{\frac{3}{2}}\right)\sqrt{ax+1}\,(ax-1)}{15\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/15*(3*a^2*c^(3/2)*x^2 - 14*a*c^(3/2)*x + 43*c^(3/2))*sqrt(a*x + 1)*(a*x -
 1)/(a^2*x - a)
```

mupad [B] time = 0.94, size = 76, normalized size = 0.75

$$-\frac{\sqrt{c-a\,c\,x}\,\left(\frac{86\,c\,\sqrt{1-a^{2}\,x^{2}}}{15\,a^{2}}+\frac{2\,c\,x^{2}\,\sqrt{1-a^{2}\,x^{2}}}{5}-\frac{28\,c\,x\,\sqrt{1-a^{2}\,x^{2}}}{15\,a}\right)}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(3/2))/(a*x + 1),x)
```

```
[Out] -((c - a*c*x)^(1/2)*((86*c*(1 - a^2*x^2)^(1/2))/(15*a^2) + (2*c*x^2*(1 - a^
2*x^2)^(1/2))/5 - (28*c*x*(1 - a^2*x^2)^(1/2))/(15*a)))/(x - 1/a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-c\left(ax-1\right)\right)^{\frac{3}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.256 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=66

$$\frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{3a}$$

```
[Out] 8/3*c*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(1/2)+2/3*(-a*c*x+c)^(1/2)*(-a^2*x^2+
1)^(1/2)/a
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcTanh[a*x],x]
```

```
[Out] (8*c*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x]*Sqrt[1 -
 a^2*x^2])/(3*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\int \frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{2\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{3a} + \frac{4}{3}\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}} + \frac{2\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{3a}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.58

$$-\frac{2c(ax-5)\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*(-5 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x])
```

fricas [A] time = 0.45, size = 39, normalized size = 0.59

$$\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,(ax-5)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x - 5)/(a^2*x - a)
```

giac [A] time = 0.18, size = 50, normalized size = 0.76

$$-\frac{8\sqrt{2}\,|c|}{3\,a\sqrt{c}} - \frac{2\,(acx+c)^{\frac{3}{2}}|c|}{3\,ac^2} + \frac{4\sqrt{acx+c}\,|c|}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -8/3*sqrt(2)*abs(c)/(a*sqrt(c)) - 2/3*(a*c*x + c)^(3/2)*abs(c)/(a*c^2) + 4*
sqrt(a*c*x + c)*abs(c)/(a*c)
```

maple [A] time = 0.03, size = 39, normalized size = 0.59

$$\frac{2\sqrt{-a^2x^2+1}\ \sqrt{-acx+c}\ (ax-5)}{3\,(ax-1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/3*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)*(a*x-5)/(a*x-1)/a
```

maxima [A] time = 0.37, size = 37, normalized size = 0.56

$$-\frac{2\left(a\sqrt{c}\,x-5\,\sqrt{c}\right)\sqrt{ax+1}\,(ax-1)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(a*sqrt(c)*x - 5*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/(a^2*x - a)
```

mupad [B] time = 0.87, size = 56, normalized size = 0.85

$$-\frac{\sqrt{c-a\,c\,x}\,\left(\frac{10\,\sqrt{1-a^2\,x^2}}{3\,a^2}-\frac{2\,x\,\sqrt{1-a^2\,x^2}}{3\,a}\right)}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a*x + 1),x)
```

```
[Out] -((c - a*c*x)^(1/2)*((10*(1 - a^2*x^2)^(1/2))/(3*a^2) - (2*x*(1 - a^2*x^2)^
(1/2))/(3*a)))/(x - 1/a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\ \sqrt{-\,(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.257 $\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-acx}} \, dx$

Optimal. Leaf size=30

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

```
[Out] 2*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6127, 649}

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-acx}} \, dx &= \frac{\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}} \, dx}{c} \\
&= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x])
```

fricas [A] time = 0.59, size = 36, normalized size = 1.20

$$-\frac{2\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^2*c*x - a*c)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 27, normalized size = 0.90

$$\frac{2\sqrt{-a^2x^2+1}}{a\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(1/2)
```

maxima [A] time = 0.37, size = 15, normalized size = 0.50

$$\frac{2\sqrt{ax+1}}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2*sqrt(a*x + 1)/(a*sqrt(c))
```

mupad [B] time = 0.96, size = 26, normalized size = 0.87

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^(1/2)*(a*x + 1)),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(a*(c - a*c*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{-c(ax-1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(-c*(a*x - 1))*(a*x + 1)), x)
```

3.258 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{ac^{3/2}}$$

```
[Out] arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^(3/2)*
2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 661, 208}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(a
*c^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{c-acx}\,\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= -\left((2a)\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\right)$$
$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{ac^{3/2}}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.86

$$\frac{\sqrt{2-2ax}\,\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{ac\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[2 - 2*a*x]*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(a*c*Sqrt[c - a*c*x])
```

fricas [A] time = 0.69, size = 133, normalized size = 2.61

$$\left[\frac{\sqrt{2}\log\left(-\frac{a^2x^2+2\,ax-\frac{2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{\sqrt{c}}-3}{a^2x^2-2\,ax+1}\right)}{2\,ac^{\frac{3}{2}}}, \frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{-\frac{1}{c}}}{a^2x^2-1}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/2*sqrt(2)*log(-(a^2*x^2 + 2*a*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c)/sqrt(c) - 3)/(a^2*x^2 - 2*a*x + 1))/(a*c^(3/2)), sqrt(2)*sqrt(-1/c)
*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-1/c)/(a^2*x^2 - 1
))/(a*c)]
```

giac [A] time = 0.17, size = 62, normalized size = 1.22

$$-\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}\right)|c|}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - sqrt(
2)*arctan(sqrt(c)/sqrt(-c))/(a*sqrt(-c)))*abs(c)/c^2
```

maple [A] time = 0.04, size = 68, normalized size = 1.33

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{(-ax+1)\sqrt{c\,(ax+1)}\,c^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] 1/(-a*x+1)/(c*(a*x+1))^(1/2)/c^(3/2)/a*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2
)*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(-acx+c)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a*c*x + c)^(3/2)*(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-a\,c\,x\right)}^{3/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^(3/2)*(a*x + 1)),x)
[Out] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^(3/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1))^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(3/2),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1))**(3/2)*(a*x + 1)), x)
```

3.259 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{2\sqrt{2}\,ac^{5/2}}+\frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}}$$

```
[Out] 1/4*arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^(5
/2)*2^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/a/c/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6127, 673, 661, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{2\sqrt{2}\,ac^{5/2}}+\frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*c*(c - a*c*x)^(3/2)) + ArcTanh[(Sqrt[c]*Sqrt[1 - a^2
*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(2*Sqrt[2]*a*c^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
```

```
+ 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= \frac{\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{c}\\ &= \frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}}+\frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{4c^2}\\ &= \frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}}-\frac{a\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{2c}\\ &= \frac{\sqrt{1-a^2x^2}}{2ac(c-acx)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{2\sqrt{2}\,ac^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.78

$$-\frac{\sqrt{2}\,(ax-1)\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)-2\sqrt{ax+1}}{4ac^2\sqrt{1-ax}\,\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] -1/4*(-2*Sqrt[1 + a*x] + Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])
/(a*c^2*Sqrt[1 - a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.56, size = 258, normalized size = 2.87

$$\left[\frac{\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{8\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)},\ \frac{\sqrt{2}\left(a^2x^2-2\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/8*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*s
qrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x
+ 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*c^3*x^2 - 2*a^2*c^3*x +
 a*c^3), 1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 +
1)*sqrt(-a*c*x + c))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 111, normalized size = 1.23

$$-\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c\left(ax-1\right)}\,\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c-2\sqrt{c\left(ax+1\right)}\,\sqrt{c}\right)}{4c^{\frac{7}{2}}\left(ax-1\right)^{2}\sqrt{c\left(ax+1\right)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(7/2)*(2^(1/2)*arctanh(1/2*(c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)
*2^(1/2)/c^(1/2))*c-2*(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^2/(c*(a*x+1))^(1/2
)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}}{\left(-acx+c\right)^{\frac{5}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a*c*x + c)^(5/2)*(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-a\,c\,x\right)}^{5/2}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^(5/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^(5/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\left(-c\left(ax-1\right)\right)^{\frac{5}{2}}\left(ax+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1))**(5/2)*(a*x + 1)), x)
```

3.260 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{7/2}} \, dx$

Optimal. Leaf size=125

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{16\sqrt{2}\,ac^{7/2}}+\frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}}+\frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}}$$

```
[Out] 3/32*arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^(
7/2)*2^(1/2)+1/4*(-a^2*x^2+1)^(1/2)/a/c/(-a*c*x+c)^(5/2)+3/16*(-a^2*x^2+1)^
(1/2)/a/c^2/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6127, 673, 661, 208}

$$\frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{16\sqrt{2}\,ac^{7/2}}+\frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(4*a*c*(c - a*c*x)^(5/2)) + (3*Sqrt[1 - a^2*x^2])/(16*a*c
^2*(c - a*c*x)^(3/2)) + (3*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqr
t[c - a*c*x])])/(16*Sqrt[2]*a*c^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
```

```
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= \frac{\int \frac{1}{(c-acx)^{5/2}\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}} + \frac{3\int \frac{1}{(c-acx)^{3/2}\sqrt{1-a^2x^2}}\,dx}{8c^2}\\
&= \frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}} + \frac{3\int \frac{1}{\sqrt{c-acx}\,\sqrt{1-a^2x^2}}\,dx}{32c^3}\\
&= \frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{16c^2}\\
&= \frac{\sqrt{1-a^2x^2}}{4ac(c-acx)^{5/2}} + \frac{3\sqrt{1-a^2x^2}}{16ac^2(c-acx)^{3/2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{16\sqrt{2}\,ac^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.42

$$\frac{\sqrt{1-a^2x^2}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{1}{2}(ax+1)\right)}{4ac^3\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, 3, 3/2, (1 + a*x)/2])/(4*a*c^3*Sq
rt[c - a*c*x])
```

fricas [A] time = 0.44, size = 312, normalized size = 2.50

$$\left[\frac{3\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right)+4\sqrt{-a^2x^2+1}\sqrt{-acx+c}(3a}{64\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="frica
s")
```

```
[Out] [1/64*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(a^2*c*x^2
+ 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(3*a*x - 7))/(
a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4), 1/32*(3*sqrt(2)*(a^3*x^
3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x +
c)*(3*a*x - 7))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

giac [A] time = 0.64, size = 80, normalized size = 0.64

$$-\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}\,c^2}+\frac{2\left(3(acx+c)^{\frac{3}{2}}-10\sqrt{acx+c}\,c\right)}{(acx-c)^2c^2}\right)|c|}{32\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac"
)
```

```
[Out] -1/32*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^2
) + 2*(3*(a*c*x + c)^(3/2) - 10*sqrt(a*c*x + c)*c)/((a*c*x - c)^2*c^2))*abs
(c)/(a*c^2)
```

maple [A] time = 0.05, size = 158, normalized size = 1.26

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c(ax-1)}\,\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c-6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac-6xa\sqrt{c(ax}\right.}{32c^{\frac{9}{2}}(ax-1)^3\sqrt{c(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/32*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1/2)*arctanh(1/2*
(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*2^(1/2)*arctanh(1/2*(c*(a*x+
1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-6*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+3*2^(1/2)*
arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+14*(c*(a*x+1))^(1/2)*c^(1/
2))/(a*x-1)^3/(c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(-acx+c)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a*c*x + c)^(7/2)*(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-a\,c\,x\right)}^{7/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^(7/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - a*c*x)^(7/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1))^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1))**(7/2)*(a*x + 1)), x)
```

3.261 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=137

$$-\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{32c^3\sqrt{c-acx}}{a}+\frac{16c^2(c-acx)^{3/2}}{3a}+\frac{2(c-acx)^{9/2}}{9ac}+\frac{4(c-acx)^{7/2}}{7a}+\frac{8c(c-acx)^{5/2}}{5a}$$

```
[Out] 16/3*c^2*(-a*c*x+c)^(3/2)/a+8/5*c*(-a*c*x+c)^(5/2)/a+4/7*(-a*c*x+c)^(7/2)/a
+2/9*(-a*c*x+c)^(9/2)/a/c-32*c^(7/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c
^(1/2))*2^(1/2)/a+32*c^3*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6130, 21, 50, 63, 206}

$$\frac{16c^2(c-acx)^{3/2}}{3a}+\frac{32c^3\sqrt{c-acx}}{a}-\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{9/2}}{9ac}+\frac{4(c-acx)^{7/2}}{7a}+\frac{8c(c-acx)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (32*c^3*Sqrt[c - a*c*x])/a + (16*c^2*(c - a*c*x)^(3/2))/(3*a) + (8*c*(c - a
*c*x)^(5/2))/(5*a) + (4*(c - a*c*x)^(7/2))/(7*a) + (2*(c - a*c*x)^(9/2))/(9
*a*c) - (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx = \int \frac{(1-ax)(c-acx)^{7/2}}{1+ax}\,dx$$
$$= \frac{\int \frac{(c-acx)^{9/2}}{1+ax}\,dx}{c}$$
$$= \frac{2(c-acx)^{9/2}}{9ac} + 2\int \frac{(c-acx)^{7/2}}{1+ax}\,dx$$
$$= \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9/2}}{9ac} + (4c)\int \frac{(c-acx)^{5/2}}{1+ax}\,dx$$
$$= \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9/2}}{9ac} + \left(8c^2\right)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx$$
$$= \frac{16c^2(c-acx)^{3/2}}{3a} + \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9/2}}{9ac} + \left(16c^3\right)\int \frac{\sqrt{c}}{1}$$
$$= \frac{32c^3\sqrt{c-acx}}{a} + \frac{16c^2(c-acx)^{3/2}}{3a} + \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9}}{9ac}$$

$$= \frac{32c^3\sqrt{c-acx}}{a} + \frac{16c^2(c-acx)^{3/2}}{3a} + \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9}}{9ac}$$

$$= \frac{32c^3\sqrt{c-acx}}{a} + \frac{16c^2(c-acx)^{3/2}}{3a} + \frac{8c(c-acx)^{5/2}}{5a} + \frac{4(c-acx)^{7/2}}{7a} + \frac{2(c-acx)^{9}}{9ac}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 0.64

$$\frac{2c^3\left(\left(35a^4x^4-230a^3x^3+732a^2x^2-1754ax+6257\right)\sqrt{c-acx}-5040\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*c^3*(Sqrt[c - a*c*x]*(6257 - 1754*a*x + 732*a^2*x^2 - 230*a^3*x^3 + 35*a
^4*x^4) - 5040*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))
/(315*a)
```

fricas [A] time = 0.51, size = 203, normalized size = 1.48

$$\left[\frac{2\left(2520\sqrt{2}\,c^{\frac{7}{2}}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(35a^4c^3x^4-230a^3c^3x^3+732a^2c^3x^2-1754ac^3x+6257c^3\right)\sqrt{-ac}\right.}{315a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/315*(2520*sqrt(2)*c^(7/2)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) + (35*a^4*c^3*x^4 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2 -
 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c))/a, 2/315*(5040*sqrt(2)*sqrt(-c)
*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*c^3*x^4 - 23
0*a^3*c^3*x^3 + 732*a^2*c^3*x^2 - 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c)
)/a]
```

giac [A] time = 0.20, size = 161, normalized size = 1.18

$$\frac{32\sqrt{2}c^4\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}\,a^8c^8-90\,(acx-c)^3\sqrt{-acx+c}\,a^8c^9+252\,(acx-c)^2\sqrt{-\right.}{315\,a^9c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 32*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) +
 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^8*c^8 - 90*(a*c*x - c)^3*sqrt(-
a*c*x + c)*a^8*c^9 + 252*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^10 + 840*(-a*
c*x + c)^(3/2)*a^8*c^11 + 5040*sqrt(-a*c*x + c)*a^8*c^12)/(a^9*c^9)
```

maple [A] time = 0.03, size = 101, normalized size = 0.74

$$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}+\frac{4(-acx+c)^{\frac{7}{2}}c}{7}+\frac{8(-acx+c)^{\frac{5}{2}}c^2}{5}+\frac{16c^3(-acx+c)^{\frac{3}{2}}}{3}+32\sqrt{-acx+c}\;c^4-32c^{\frac{9}{2}}\sqrt{2}\;\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\;\sqrt{2}}{2\sqrt{c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/9*(-a*c*x+c)^(9/2)+2/7*(-a*c*x+c)^(7/2)*c+4/5*(-a*c*x+c)^(5/2)*c^2
+8/3*c^3*(-a*c*x+c)^(3/2)+16*(-a*c*x+c)^(1/2)*c^4-16*c^(9/2)*2^(1/2)*arctan
h(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.46, size = 123, normalized size = 0.90

$$\frac{2\left(2520\sqrt{2}\,c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+35\,(-acx+c)^{\frac{9}{2}}+90\,(-acx+c)^{\frac{7}{2}}c+252\,(-acx+c)^{\frac{5}{2}}c^2+840\,(-acx+c)^{\frac{3}{2}}c\right.}{315\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 2/315*(2520*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt
(2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) + 90*(-a*c*x + c)^
(7/2)*c + 252*(-a*c*x + c)^(5/2)*c^2 + 840*(-a*c*x + c)^(3/2)*c^3 + 5040*sq
rt(-a*c*x + c)*c^4)/(a*c)
```

mupad [B] time = 0.84, size = 112, normalized size = 0.82

$$\frac{4\,(c-a\,c\,x)^{7/2}}{7\,a}+\frac{8\,c\,(c-a\,c\,x)^{5/2}}{5\,a}+\frac{32\,c^3\,\sqrt{c-a\,c\,x}}{a}+\frac{16\,c^2\,(c-a\,c\,x)^{3/2}}{3\,a}+\frac{2\,(c-a\,c\,x)^{9/2}}{9\,a\,c}+\frac{\sqrt{2}\,c^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1}{2\,\sqrt{c}}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(7/2))/(a*x + 1)^2,x)
```

```
[Out] (4*(c - a*c*x)^(7/2))/(7*a) + (8*c*(c - a*c*x)^(5/2))/(5*a) + (32*c^3*(c -
a*c*x)^(1/2))/a + (16*c^2*(c - a*c*x)^(3/2))/(3*a) + (2*(c - a*c*x)^(9/2))/
(9*a*c) + (2^(1/2)*c^(7/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))
*32i)/a
```

sympy [A] time = 128.96, size = 128, normalized size = 0.93

$$\frac{32\sqrt{2}\,c^4\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{32c^3\sqrt{-acx+c}}{a}+\frac{16c^2\,(-acx+c)^{\frac{3}{2}}}{3a}+\frac{8c\,(-acx+c)^{\frac{5}{2}}}{5a}+\frac{4\,(-acx+c)^{\frac{7}{2}}}{7a}+\frac{2\,(-acx+c)^{\frac{9}{2}}}{9ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] 32*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) +
32*c**3*sqrt(-a*c*x + c)/a + 16*c**2*(-a*c*x + c)**(3/2)/(3*a) + 8*c*(-a*c*
x + c)**(5/2)/(5*a) + 4*(-a*c*x + c)**(7/2)/(7*a) + 2*(-a*c*x + c)**(9/2)/(
9*a*c)
```

3.262 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=116

$$-\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{16c^2\sqrt{c-acx}}{a}+\frac{2(c-acx)^{7/2}}{7ac}+\frac{4(c-acx)^{5/2}}{5a}+\frac{8c(c-acx)^{3/2}}{3a}$$

```
[Out] 8/3*c*(-a*c*x+c)^(3/2)/a+4/5*(-a*c*x+c)^(5/2)/a+2/7*(-a*c*x+c)^(7/2)/a/c-16
*c^(5/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a+16*c^2*(-a
*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6130, 21, 50, 63, 206}

$$\frac{16c^2\sqrt{c-acx}}{a}-\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{7/2}}{7ac}+\frac{4(c-acx)^{5/2}}{5a}+\frac{8c(c-acx)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (16*c^2*Sqrt[c - a*c*x])/a + (8*c*(c - a*c*x)^(3/2))/(3*a) + (4*(c - a*c*x)
^(5/2))/(5*a) + (2*(c - a*c*x)^(7/2))/(7*a*c) - (16*Sqrt[2]*c^(5/2)*ArcTanh
[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx = \int \frac{(1-ax)(c-acx)^{5/2}}{1+ax}\,dx$$
$$= \frac{\int \frac{(c-acx)^{7/2}}{1+ax}\,dx}{c}$$
$$= \frac{2(c-acx)^{7/2}}{7ac} + 2\int \frac{(c-acx)^{5/2}}{1+ax}\,dx$$
$$= \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} + (4c)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx$$
$$= \frac{8c(c-acx)^{3/2}}{3a} + \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} + \left(8c^2\right)\int \frac{\sqrt{c-acx}}{1+ax}\,dx$$
$$= \frac{16c^2\sqrt{c-acx}}{a} + \frac{8c(c-acx)^{3/2}}{3a} + \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} + \left(16c^3\right)\int \frac{\quad}{(1+\ldots}$$
$$= \frac{16c^2\sqrt{c-acx}}{a} + \frac{8c(c-acx)^{3/2}}{3a} + \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} - \frac{\left(32c^2\right)\mathrm{Subst}\Big(\ldots}{\ldots}$$
$$= \frac{16c^2\sqrt{c-acx}}{a} + \frac{8c(c-acx)^{3/2}}{3a} + \frac{4(c-acx)^{5/2}}{5a} + \frac{2(c-acx)^{7/2}}{7ac} - \frac{16\sqrt{2}\,c^{5/2}\tanh\ldots}{a}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.69

$$-\frac{2c^2\left(\left(15a^3x^3-87a^2x^2+269ax-1037\right)\sqrt{c-acx}+840\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^2*(Sqrt[c - a*c*x]*(-1037 + 269*a*x - 87*a^2*x^2 + 15*a^3*x^3) + 840*
Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(105*a)
```

fricas [A] time = 0.47, size = 183, normalized size = 1.58

$$\left[\frac{2\left(420\sqrt{2}c^{\frac{5}{2}}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\left(15a^3c^2x^3-87a^2c^2x^2+269ac^2x-1037c^2\right)\sqrt{-acx+c}\right)}{105a}, \frac{2\left(840\sqrt{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/105*(420*sqrt(2)*c^(5/2)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) - (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c^2*x - 1037*
c^2)*sqrt(-a*c*x + c))/a, 2/105*(840*sqrt(2)*sqrt(-c)*c^2*arctan(1/2*sqrt(2
)*sqrt(-a*c*x + c)*sqrt(-c)/c) - (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c
^2*x - 1037*c^2)*sqrt(-a*c*x + c))/a]
```

giac [A] time = 0.19, size = 134, normalized size = 1.16

$$\frac{16\sqrt{2}c^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left(15(acx-c)^3\sqrt{-acx+c}\,a^6c^6-42(acx-c)^2\sqrt{-acx+c}\,a^6c^7-140(-acx+c)^{\frac{3}{2}}a^6\right.}{105a^7c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 16*sqrt(2)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
 2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^6*c^6 - 42*(a*c*x - c)^2*sqrt(-
a*c*x + c)*a^6*c^7 - 140*(-a*c*x + c)^(3/2)*a^6*c^8 - 840*sqrt(-a*c*x + c)*
a^6*c^9)/(a^7*c^7)
```

maple [A] time = 0.03, size = 87, normalized size = 0.75

$$\frac{\frac{2(-acx+c)^{\frac{7}{2}}}{7}+\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{8(-acx+c)^{\frac{3}{2}}c^2}{3}+16\sqrt{-acx+c}\,c^3-16c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/7*(-a*c*x+c)^(7/2)+2/5*c*(-a*c*x+c)^(5/2)+4/3*(-a*c*x+c)^(3/2)*c^2
+8*(-a*c*x+c)^(1/2)*c^3-8*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1
/2)/c^(1/2)))
```

maxima [A] time = 0.42, size = 109, normalized size = 0.94

$$\frac{2\left(420\,\sqrt{2}\,c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+15\,(-acx+c)^{\frac{7}{2}}+42\,(-acx+c)^{\frac{5}{2}}c+140\,(-acx+c)^{\frac{3}{2}}c^{2}+840\,\sqrt{-acx+c}\,c^{3}\right)}{105\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 2/105*(420*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(
2)*sqrt(c) + sqrt(-a*c*x + c))) + 15*(-a*c*x + c)^(7/2) + 42*(-a*c*x + c)^(
5/2)*c + 140*(-a*c*x + c)^(3/2)*c^2 + 840*sqrt(-a*c*x + c)*c^3)/(a*c)
```

mupad [B] time = 0.06, size = 95, normalized size = 0.82

$$\frac{4\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a}+\frac{8\,c\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a}+\frac{16\,c^2\,\sqrt{c-a\,c\,x}}{a}+\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a\,c}+\frac{\sqrt{2}\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,16{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(5/2))/(a*x + 1)^2,x)
```

```
[Out] (4*(c - a*c*x)^(5/2))/(5*a) + (8*c*(c - a*c*x)^(3/2))/(3*a) + (16*c^2*(c -
a*c*x)^(1/2))/a + (2*(c - a*c*x)^(7/2))/(7*a*c) + (2^(1/2)*c^(5/2)*atan((2^
(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*16i)/a
```

sympy [A] time = 76.55, size = 109, normalized size = 0.94

$$\frac{16\sqrt{2}\,c^{3}\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{16c^{2}\sqrt{-acx+c}}{a}+\frac{8c\,(-acx+c)^{\frac{3}{2}}}{3a}+\frac{4\,(-acx+c)^{\frac{5}{2}}}{5a}+\frac{2\,(-acx+c)^{\frac{7}{2}}}{7ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] 16*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) +
16*c**2*sqrt(-a*c*x + c)/a + 8*c*(-a*c*x + c)**(3/2)/(3*a) + 4*(-a*c*x + c)
**(5/2)/(5*a) + 2*(-a*c*x + c)**(7/2)/(7*a*c)
```

3.263 $\int e^{-2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{5/2}}{5ac}+\frac{4(c-acx)^{3/2}}{3a}+\frac{8c\sqrt{c-acx}}{a}$$

```
[Out] 4/3*(-a*c*x+c)^(3/2)/a+2/5*(-a*c*x+c)^(5/2)/a/c-8*c^(3/2)*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a+8*c*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6130, 21, 50, 63, 206}

$$-\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{2(c-acx)^{5/2}}{5ac}+\frac{4(c-acx)^{3/2}}{3a}+\frac{8c\sqrt{c-acx}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (8*c*Sqrt[c - a*c*x])/a + (4*(c - a*c*x)^(3/2))/(3*a) + (2*(c - a*c*x)^(5/2
))/(5*a*c) - (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])
/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx &= \int \frac{(1-ax)(c-acx)^{3/2}}{1+ax}\,dx \\
&= \frac{\int \frac{(c-acx)^{5/2}}{1+ax}\,dx}{c} \\
&= \frac{2(c-acx)^{5/2}}{5ac} + 2\int \frac{(c-acx)^{3/2}}{1+ax}\,dx \\
&= \frac{4(c-acx)^{3/2}}{3a} + \frac{2(c-acx)^{5/2}}{5ac} + (4c)\int \frac{\sqrt{c-acx}}{1+ax}\,dx \\
&= \frac{8c\sqrt{c-acx}}{a} + \frac{4(c-acx)^{3/2}}{3a} + \frac{2(c-acx)^{5/2}}{5ac} + \left(8c^2\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{8c\sqrt{c-acx}}{a} + \frac{4(c-acx)^{3/2}}{3a} + \frac{2(c-acx)^{5/2}}{5ac} - \frac{(16c)\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-ac\right.}{a} \\
&= \frac{8c\sqrt{c-acx}}{a} + \frac{4(c-acx)^{3/2}}{3a} + \frac{2(c-acx)^{5/2}}{5ac} - \frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.75

$$
\frac{2c\left(3a^2x^2-16ax+73\right)\sqrt{c-acx}-120\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{15a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[c - a*c*x]*(73 - 16*a*x + 3*a^2*x^2) - 120*Sqrt[2]*c^(3/2)*ArcTan
h[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a)
```

fricas [A] time = 0.49, size = 145, normalized size = 1.53

$$\left[\frac{2\left(30\sqrt{2}c^{\frac{3}{2}}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(3a^2cx^2-16acx+73c\right)\sqrt{-acx+c}\right)}{15a}, \frac{2\left(60\sqrt{2}\sqrt{-c}\,c\arctan\left(\frac{\sqrt{2}\sqrt{-ac}}{2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*c^(3/2)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a, 2/1
5*(60*sqrt(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) +
(3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a]
```

giac [A] time = 0.24, size = 107, normalized size = 1.13

$$\frac{8\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left(3\,(acx-c)^2\sqrt{-acx+c}\,a^4c^4+10\,(-acx+c)^{\frac{3}{2}}a^4c^5+60\sqrt{-acx+c}\,a^4c^6\right)}{15\,a^5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 8*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) +
2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^4 + 10*(-a*c*x + c)^(3/2)*a^4*
c^5 + 60*sqrt(-a*c*x + c)*a^4*c^6)/(a^5*c^5)
```

maple [A] time = 0.03, size = 73, normalized size = 0.77

$$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{4c(-acx+c)^{\frac{3}{2}}}{3}+8\sqrt{-acx+c}\;c^2-8c^{\frac{5}{2}}\sqrt{2}\;\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\;\sqrt{2}}{2\sqrt{c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/5*(-a*c*x+c)^(5/2)+2/3*c*(-a*c*x+c)^(3/2)+4*(-a*c*x+c)^(1/2)*c^2-4
*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.47, size = 95, normalized size = 1.00

$$\frac{2\left(30\sqrt{2}\,c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3\,(-acx+c)^{\frac{5}{2}}+10\,(-acx+c)^{\frac{3}{2}}c+60\sqrt{-acx+c}\,c^{2}\right)}{15\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 2/15*(30*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)
*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 10*(-a*c*x + c)^(3/2
)*c + 60*sqrt(-a*c*x + c)*c^2)/(a*c)
```

mupad [B] time = 0.08, size = 78, normalized size = 0.82

$$\frac{4\,(c-a\,c\,x)^{3/2}}{3\,a}+\frac{8\,c\,\sqrt{c-a\,c\,x}}{a}+\frac{2\,(c-a\,c\,x)^{5/2}}{5\,a\,c}+\frac{\sqrt{2}\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,8\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(3/2))/(a*x + 1)^2,x)
```

```
[Out] (4*(c - a*c*x)^(3/2))/(3*a) + (8*c*(c - a*c*x)^(1/2))/a + (2*(c - a*c*x)^(5
/2))/(5*a*c) + (2^(1/2)*c^(3/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1
/2)))*8i)/a
```

sympy [A] time = 61.08, size = 90, normalized size = 0.95

$$\frac{8\sqrt{2}\,c^{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{8c\sqrt{-acx+c}}{a}+\frac{4\,(-acx+c)^{\frac{3}{2}}}{3a}+\frac{2\,(-acx+c)^{\frac{5}{2}}}{5ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] 8*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) + 8
*c*sqrt(-a*c*x + c)/a + 4*(-a*c*x + c)**(3/2)/(3*a) + 2*(-a*c*x + c)**(5/2)
/(5*a*c)
```

3.264 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{c-acx}}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a/c-4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^
(1/2)*c^(1/2)/a+4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6130, 21, 50, 63, 206}

$$\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{c-acx}}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*Sqrt[2]*Sqrt[c]*
ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= \int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= \frac{2(c-acx)^{3/2}}{3ac}+2\int \frac{\sqrt{c-acx}}{1+ax}\,dx\\
&= \frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}+(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= \frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}-\frac{8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= \frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.80

$$
-\frac{2(ax-7)\sqrt{c-acx}+12\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/3*(2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.56, size = 120, normalized size = 1.58

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\sqrt{-acx+c}\,(ax-7)\right)}{3a}, \frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+\right.}{3a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) - sqrt(-a*c*x + c)*(a*x - 7))/a, 2/3*(6*sqrt(2)*sqrt(-c)*arc
tan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))/
a]
```

giac [A] time = 0.19, size = 77, normalized size = 1.01

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}\,a^2c^3\right)}{3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) + 2/
3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

maple [A] time = 0.03, size = 59, normalized size = 0.78

$$\frac{\frac{2(-acx+c)^{\frac{3}{2}}}{3}+4c\sqrt{-acx+c}-4c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh(
1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.44, size = 79, normalized size = 1.04

$$\frac{2\left(3\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*s
qrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)/(a
*c)
```

mupad [B] time = 0.07, size = 61, normalized size = 0.80

$$\frac{4\sqrt{c-a\,c\,x}}{a}+\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c}-\frac{4\sqrt{2}\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(a*x + 1)^2,x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a + (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*2^(1/2)*c^(1/2
)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a
```

sympy [A] time = 8.10, size = 75, normalized size = 0.99

$$-\frac{2\left(-\frac{2\sqrt{2}c^{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}-2c\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -2*(-2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) -
2*c*sqrt(-a*c*x + c) - (-a*c*x + c)**(3/2)/3)/(a*c)
```

3.265 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{c-acx}}{ac}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] -2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)+2*(-a*c*
x+c)^(1/2)/a/c
```

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6130, 21, 50, 63, 206}

$$\frac{2\sqrt{c-acx}}{ac}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x])/(a*c) - (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqr
t[c])])/(a*Sqrt[c])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= \int \frac{1-ax}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{c} \\
&= \frac{2\sqrt{c-acx}}{ac} + 2\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= \frac{2\sqrt{c-acx}}{ac} - \frac{4\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac} \\
&= \frac{2\sqrt{c-acx}}{ac} - \frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.00

$$\frac{2\sqrt{c-acx}}{ac} - \frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x])/(a*c) - (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqr
t[c])])/(a*Sqrt[c])
```

fricas [A] time = 0.54, size = 118, normalized size = 2.03

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{ax+\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)+2\sqrt{-acx+c}}{ac}, -\frac{2\left(\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)-\sqrt{-acx+c}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*sqrt(c)*log((a*x + 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x +
 1)) + 2*sqrt(-a*c*x + c))/(a*c), -2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(2)*s
qrt(-a*c*x + c)*sqrt(-1/c)/(a*x - 1)) - sqrt(-a*c*x + c))/(a*c)]
```

giac [A] time = 0.18, size = 51, normalized size = 0.88

$$\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) + 2*sq
rt(-a*c*x + c)/(a*c)
```

maple [A] time = 0.03, size = 45, normalized size = 0.78

$$\frac{2\sqrt{-acx+c}\;-2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\;\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\;\sqrt{c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/c/a*((-a*c*x+c)^(1/2)-arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/
2)*c^(1/2))
```

maxima [A] time = 0.49, size = 67, normalized size = 1.16

$$\frac{\sqrt{2}\,\sqrt{c}\,\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+2\,\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c)
 + sqrt(-a*c*x + c))) + 2*sqrt(-a*c*x + c))/(a*c)
```

mupad [B] time = 0.07, size = 47, normalized size = 0.81

$$\frac{2\,\sqrt{c-a\,c\,x}}{a\,c}-\frac{2\,\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a*c*x)^(1/2)*(a*x + 1)^2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2))/(a*c) - (2*2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/
(2*c^(1/2))))/(a*c^(1/2))
```

sympy [A] time = 34.19, size = 58, normalized size = 1.00

$$\frac{2\sqrt{-acx+c}}{ac}+\frac{2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{\sqrt{-\frac{1}{c}}\,\sqrt{-acx+c}}\right)}{ac\sqrt{-\frac{1}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*sqrt(-a*c*x + c)/(a*c) + 2*sqrt(2)*atan(sqrt(2)/(sqrt(-1/c)*sqrt(-a*c*x +
 c)))/(a*c*sqrt(-1/c))
```

3.266 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] -arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6130, 21, 63, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= \int \frac{1-ax}{(1+ax)(c-acx)^{3/2}}\,dx\\ &= \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{c}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^2}\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(3/2)),x]
[Out] -((Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2)))
```

fricas [A] time = 0.46, size = 89, normalized size = 2.34

$$\left[\frac{\sqrt{2}\log\left(\frac{ax+\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)}{2\,ac^{\frac{3}{2}}}, -\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log((a*x + 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x + 1))/
(a*c^(3/2)), -sqrt(2)*sqrt(-1/c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-1/c)
/(a*x - 1))/(a*c)]
```

giac [A] time = 0.75, size = 35, normalized size = 0.92

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

maple [A] time = 0.03, size = 30, normalized size = 0.79

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

maxima [A] time = 0.45, size = 52, normalized size = 1.37

$$\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{2\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + sq
rt(-a*c*x + c)))/(a*c^(3/2))
```

mupad [B] time = 0.82, size = 29, normalized size = 0.76

$$-\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a*c*x)^(3/2)*(a*x + 1)^2),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(3/2))
```

sympy [A] time = 54.00, size = 39, normalized size = 1.03

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(3/2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*c*sqrt(-c))
```

3.267 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{5/2}} \, dx$

Optimal. Leaf size=57

$$\frac{1}{ac^2\sqrt{c-acx}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}$$

```
[Out] -1/2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)+1/a/c^
2/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6130, 21, 51, 63, 206}

$$\frac{1}{ac^2\sqrt{c-acx}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] 1/(a*c^2*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(Sqr
t[2]*a*c^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= \int \frac{1-ax}{(1+ax)(c-acx)^{5/2}}\,dx \\
&= \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{c} \\
&= \frac{1}{ac^2\sqrt{c-acx}} + \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{2c^2} \\
&= \frac{1}{ac^2\sqrt{c-acx}} - \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^3} \\
&= \frac{1}{ac^2\sqrt{c-acx}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 36, normalized size = 0.63

$$
\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1-ax)\right)}{ac^2\sqrt{c-acx}}
$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(5/2)),x]

[Out] Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]/(a*c^2*Sqrt[c - a*c*x])

fricas [A] time = 0.42, size = 146, normalized size = 2.56

$$\left[\frac{\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(\frac{acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{ax+1}\right)-4\,\sqrt{-acx+c}}{4\,\left(a^2c^3x-ac^3\right)},\ \frac{\sqrt{2}\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-c}}{2\,c}\right)-2\,\sqrt{-acx+c}}{2\,\left(a^2c^3x-ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) - 4*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3), 1/2*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3)]

giac [A] time = 2.87, size = 53, normalized size = 0.93

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{2\,a\sqrt{-c}\,c^2}+\frac{1}{\sqrt{-acx+c}\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^2) + 1/(sqrt(-a*c*x + c)*a*c^2)

maple [A] time = 0.04, size = 50, normalized size = 0.88

$$\frac{\frac{1}{c\sqrt{-acx+c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{2c^{\frac{3}{2}}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x)

[Out] 2/c/a*(1/2/c/(-a*c*x+c)^(1/2)-1/4/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))

maxima [A] time = 0.42, size = 71, normalized size = 1.25

$$\frac{\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}+\frac{4}{\sqrt{-acx+c}\,c}}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + s
qrt(-a*c*x + c)))/c^(3/2) + 4/(sqrt(-a*c*x + c)*c))/(a*c)
```

mupad [B] time = 0.09, size = 46, normalized size = 0.81

$$\frac{1}{a\,c^2\,\sqrt{c-a\,c\,x}}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{2\,a\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a*c*x)^(5/2)*(a*x + 1)^2),x)
```

```
[Out] 1/(a*c^2*(c - a*c*x)^(1/2)) - (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2
*c^(1/2))))/(2*a*c^(5/2))
```

sympy [A] time = 31.30, size = 60, normalized size = 1.05

$$\frac{1}{ac^2\sqrt{-acx+c}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2ac^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] 1/(a*c**2*sqrt(-a*c*x + c)) + sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt
(-c)))/(2*a*c**2*sqrt(-c))
```

3.268 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}+\frac{1}{2ac^3\sqrt{c-acx}}+\frac{1}{3ac^2(c-acx)^{3/2}}$$

```
[Out] 1/3/a/c^2/(-a*c*x+c)^(3/2)-1/4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
)/a/c^(7/2)*2^(1/2)+1/2/a/c^3/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6130, 21, 51, 63, 206}

$$\frac{1}{2ac^3\sqrt{c-acx}}+\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] 1/(3*a*c^2*(c - a*c*x)^(3/2)) + 1/(2*a*c^3*Sqrt[c - a*c*x]) - ArcTanh[Sqrt[
c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= \int \frac{1-ax}{(1+ax)(c-acx)^{7/2}}\,dx \\
&= \frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{c} \\
&= \frac{1}{3ac^2(c-acx)^{3/2}} + \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{2c^2} \\
&= \frac{1}{3ac^2(c-acx)^{3/2}} + \frac{1}{2ac^3\sqrt{c-acx}} + \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{4c^3} \\
&= \frac{1}{3ac^2(c-acx)^{3/2}} + \frac{1}{2ac^3\sqrt{c-acx}} - \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{2ac^4} \\
&= \frac{1}{3ac^2(c-acx)^{3/2}} + \frac{1}{2ac^3\sqrt{c-acx}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.47

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(1-ax)\right)}{3ac^2(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, (1 - a*x)/2]/(3*a*c^2*(c - a*c*x)^(3/2))
```

fricas [A] time = 0.62, size = 196, normalized size = 2.36

$$\left[\frac{3\sqrt{2}\left(a^2x^2-2ax+1\right)\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-4\sqrt{-acx+c}\,(3ax-5)}{24\left(a^3c^4x^2-2a^2c^4x+ac^4\right)},\frac{3\sqrt{2}\left(a^2x^2-2ax+1\right)\sqrt{-c}\,\mathrm{arct}}{12\left(a^3c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) - 4*sqrt(-a*c*x + c)*(3*a*x - 5))/(a^
3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqr
t(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x + c)*
(3*a*x - 5))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

giac [A] time = 0.14, size = 73, normalized size = 0.88

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4a\sqrt{-c}\,c^3}+\frac{3acx-5c}{6(acx-c)\sqrt{-acx+c}\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^3)
+ 1/6*(3*a*c*x - 5*c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c^3)
```

maple [A] time = 0.04, size = 64, normalized size = 0.77

$$\frac{\frac{1}{2c^2\sqrt{-acx+c}}+\frac{1}{3c(-acx+c)^{\frac{3}{2}}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{5}{2}}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/c/a*(1/4/c^2/(-a*c*x+c)^(1/2)+1/6/c/(-a*c*x+c)^(3/2)-1/8/c^(5/2)*2^(1/2)*
arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.50, size = 81, normalized size = 0.98

$$\frac{\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}-\frac{4(3acx-5c)}{(-acx+c)^{\frac{3}{2}}c^{2}}}{24ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c)
+ sqrt(-a*c*x + c)))/c^(5/2) - 4*(3*a*c*x - 5*c)/((-a*c*x + c)^(3/2)*c^2))/
(a*c)
```

mupad [B] time = 0.84, size = 64, normalized size = 0.77

$$\frac{\frac{c-a\,c\,x}{2\,c^2}+\frac{1}{3\,c}}{a\,c\,{\left(c-a\,c\,x\right)}^{3/2}}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{4\,a\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a*c*x)^(7/2)*(a*x + 1)^2),x)
```

```
[Out] ((c - a*c*x)/(2*c^2) + 1/(3*c))/(a*c*(c - a*c*x)^(3/2)) - (2^(1/2)*atanh((2
^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(4*a*c^(7/2))
```

sympy [A] time = 66.36, size = 80, normalized size = 0.96

$$\frac{1}{3ac^{2}\left(-acx+c\right)^{\frac{3}{2}}}+\frac{1}{2ac^{3}\sqrt{-acx+c}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4ac^{3}\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] 1/(3*a*c**2*(-a*c*x + c)**(3/2)) + 1/(2*a*c**3*sqrt(-a*c*x + c)) + sqrt(2)*
atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(4*a*c**3*sqrt(-c))
```

3.269 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}+\frac{1}{4ac^4\sqrt{c-acx}}+\frac{1}{6ac^3(c-acx)^{3/2}}+\frac{1}{5ac^2(c-acx)^{5/2}}$$

```
[Out] 1/5/a/c^2/(-a*c*x+c)^(5/2)+1/6/a/c^3/(-a*c*x+c)^(3/2)-1/8*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)+1/4/a/c^4/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6130, 21, 51, 63, 206}

$$\frac{1}{4ac^4\sqrt{c-acx}}+\frac{1}{6ac^3(c-acx)^{3/2}}+\frac{1}{5ac^2(c-acx)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] 1/(5*a*c^2*(c - a*c*x)^(5/2)) + 1/(6*a*c^3*(c - a*c*x)^(3/2)) + 1/(4*a*c^4*
Sqrt[c - a*c*x]) - ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(4*Sqrt[2]*a*
c^(9/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx &= \int \frac{1-ax}{(1+ax)(c-acx)^{9/2}}\,dx\\
&= \frac{\int \frac{1}{(1+ax)(c-acx)^{7/2}}\,dx}{c}\\
&= \frac{1}{5ac^2(c-acx)^{5/2}}+\frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{2c^2}\\
&= \frac{1}{5ac^2(c-acx)^{5/2}}+\frac{1}{6ac^3(c-acx)^{3/2}}+\frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{4c^3}\\
&= \frac{1}{5ac^2(c-acx)^{5/2}}+\frac{1}{6ac^3(c-acx)^{3/2}}+\frac{1}{4ac^4\sqrt{c-acx}}+\frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{8c^4}\\
&= \frac{1}{5ac^2(c-acx)^{5/2}}+\frac{1}{6ac^3(c-acx)^{3/2}}+\frac{1}{4ac^4\sqrt{c-acx}}-\frac{\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{4ac^5}\\
&= \frac{1}{5ac^2(c-acx)^{5/2}}+\frac{1}{6ac^3(c-acx)^{3/2}}+\frac{1}{4ac^4\sqrt{c-acx}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.38

$$\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(1-ax)\right)}{5ac^2(c-acx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] Hypergeometric2F1[-5/2, 1, -3/2, (1 - a*x)/2]/(5*a*c^2*(c - a*c*x)^(5/2))
```

fricas [A] time = 0.59, size = 252, normalized size = 2.42

$$\left[\frac{15\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-4\left(15a^2x^2-40ax+37\right)\sqrt{-acx+c}}{240\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-ac^5\right)}, \frac{15\sqrt{2}\left(a^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((a*c*x + 2
*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) - 4*(15*a^2*x^2 - 40*a*
x + 37)*sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^
5), 1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/2
*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*(15*a^2*x^2 - 40*a*x + 37)*sqrt(-
a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

giac [A] time = 0.17, size = 93, normalized size = 0.89

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8a\sqrt{-c}\,c^4}+\frac{15(acx-c)^2-10(acx-c)c+12c^2}{60(acx-c)^2\sqrt{-acx+c}\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^4)
+ 1/60*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c + 12*c^2)/((a*c*x - c)^2*sqrt(-
a*c*x + c)*a*c^4)
```

maple [A] time = 0.04, size = 78, normalized size = 0.75

$$\frac{\frac{1}{4c^3\sqrt{-acx+c}}+\frac{1}{6c^2(-acx+c)^{\frac{3}{2}}}+\frac{1}{5c(-acx+c)^{\frac{5}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{7}{2}}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(9/2),x)
```

```
[Out] 2/c/a*(1/8/c^3/(-a*c*x+c)^(1/2)+1/12/c^2/(-a*c*x+c)^(3/2)+1/10/c/(-a*c*x+c)
^(5/2)-1/16/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.42, size = 101, normalized size = 0.97

$$\frac{\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{7}{2}}}+\frac{4\left(15\,(acx-c)^2-10\,(acx-c)c+12\,c^2\right)}{(-acx+c)^{\frac{5}{2}}c^3}}{240\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/240*(15*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c
) + sqrt(-a*c*x + c)))/c^(7/2) + 4*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c + 1
2*c^2)/((-a*c*x + c)^(5/2)*c^3))/(a*c)
```

mupad [B] time = 0.09, size = 78, normalized size = 0.75

$$\frac{\frac{c-a\,c\,x}{6\,c^2}+\frac{1}{5\,c}+\frac{(c-a\,c\,x)^2}{4\,c^3}}{a\,c\,(c-a\,c\,x)^{5/2}}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{8\,a\,c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a*c*x)^(9/2)*(a*x + 1)^2),x)
```

```
[Out] ((c - a*c*x)/(6*c^2) + 1/(5*c) + (c - a*c*x)^2/(4*c^3))/(a*c*(c - a*c*x)^(5
/2)) - (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(8*a*c^(9/2
))
```

sympy [A] time = 40.71, size = 99, normalized size = 0.95

$$\frac{1}{5ac^2\,(-acx+c)^{\frac{5}{2}}}+\frac{1}{6ac^3\,(-acx+c)^{\frac{3}{2}}}+\frac{1}{4ac^4\sqrt{-acx+c}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8ac^4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a*c*x+c)**(9/2),x)
```

```
[Out] 1/(5*a*c**2*(-a*c*x + c)**(5/2)) + 1/(6*a*c**3*(-a*c*x + c)**(3/2)) + 1/(4*
a*c**4*sqrt(-a*c*x + c)) + sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c
)))/(8*a*c**4*sqrt(-c))
```

3.270 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=171

$$\frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}}-\frac{4096c^2\sqrt{c-acx}}{35a\sqrt{1-a^2x^2}}+\frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}}+\frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}}+\frac{1024c(c-acx)^{3/2}}{35a\sqrt{1-a^2x^2}}$$

```
[Out] 1024/35*c*(-a*c*x+c)^(3/2)/a/(-a^2*x^2+1)^(1/2)+128/35*(-a*c*x+c)^(5/2)/a/(
-a^2*x^2+1)^(1/2)+32/35*(-a*c*x+c)^(7/2)/a/c/(-a^2*x^2+1)^(1/2)+2/7*(-a*c*x
+c)^(9/2)/a/c^2/(-a^2*x^2+1)^(1/2)-4096/35*c^2*(-a*c*x+c)^(1/2)/a/(-a^2*x^2
+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}}-\frac{4096c^2\sqrt{c-acx}}{35a\sqrt{1-a^2x^2}}+\frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}}+\frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}}+\frac{1024c(c-acx)^{3/2}}{35a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-4096*c^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - a^2*x^2]) + (1024*c*(c - a*c*x)^
(3/2))/(35*a*Sqrt[1 - a^2*x^2]) + (128*(c - a*c*x)^(5/2))/(35*a*Sqrt[1 - a^
2*x^2]) + (32*(c - a*c*x)^(7/2))/(35*a*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x
)^(9/2))/(7*a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{\int \frac{(c-acx)^{11/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= \frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}} + \frac{16\int \frac{(c-acx)^{9/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{7c^2}$$

$$= \frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}} + \frac{192\int \frac{(c-acx)^{7/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{35c}$$

$$= \frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}} + \frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}} + \frac{512}{35}\int \frac{(c-acx)^{5/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx$$

$$= \frac{1024c(c-acx)^{3/2}}{35a\sqrt{1-a^2x^2}} + \frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}} + \frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{9/2}}{7ac^2\sqrt{1-a^2x^2}} + \frac{1}{35}(2$$

$$= -\frac{4096c^2\sqrt{c-acx}}{35a\sqrt{1-a^2x^2}} + \frac{1024c(c-acx)^{3/2}}{35a\sqrt{1-a^2x^2}} + \frac{128(c-acx)^{5/2}}{35a\sqrt{1-a^2x^2}} + \frac{32(c-acx)^{7/2}}{35ac\sqrt{1-a^2x^2}} + \frac{}{7}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.41

$$\frac{2c^3\sqrt{1-ax}\left(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451\right)}{35a\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c^3*Sqrt[1 - a*x]*(-1451 - 708*a*x + 142*a^2*x^2 - 36*a^3*x^3 + 5*a^4*x^
4))/(35*a*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.45, size = 82, normalized size = 0.48

$$-\frac{2\left(5\,a^4c^2x^4-36\,a^3c^2x^3+142\,a^2c^2x^2-708\,ac^2x-1451\,c^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{35\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -2/35*(5*a^4*c^2*x^4 - 36*a^3*c^2*x^3 + 142*a^2*c^2*x^2 - 708*a*c^2*x - 145
1*c^2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x^2 - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 71, normalized size = 0.42

$$\frac{2 \left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \left(-a c x +c \right)^{\frac{5}{2}} \left(5 x^{4} a^{4}-36 x^{3} a^{3}+142 a^{2} x^{2}-708 a x -1451\right)}{35 \left(a x +1\right)^{2} \left(a x -1\right)^{4} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/35*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(5/2)*(5*a^4*x^4-36*a^3*x^3+142*a^2*x^2-
708*a*x-1451)/(a*x+1)^2/(a*x-1)^4/a
```

maxima [A] time = 0.34, size = 73, normalized size = 0.43

$$\frac{2 \left(5 \, a^{4} c^{\frac{5}{2}} x^{4} - 36 \, a^{3} c^{\frac{5}{2}} x^{3} + 142 \, a^{2} c^{\frac{5}{2}} x^{2} - 708 \, a c^{\frac{5}{2}} x - 1451 \, c^{\frac{5}{2}}\right)} \sqrt{a x + 1} {\left(a x - 1\right)}}{35 \, {\left(a^{3} x^{2} - a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/35*(5*a^4*c^(5/2)*x^4 - 36*a^3*c^(5/2)*x^3 + 142*a^2*c^(5/2)*x^2 - 708*a*
c^(5/2)*x - 1451*c^(5/2))*sqrt(a*x + 1)*(a*x - 1)/(a^3*x^2 - a)
```

mupad [B] time = 1.10, size = 116, normalized size = 0.68

$$\frac{2048\,c^2\,\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{35\,a\,\left(a\,x-1\right)}-\frac{16\,c^2\,\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{a\,\left(a\,x+1\right)}-\frac{2\,c^2\,\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}\,\left(5\,a^2\,x^2-36\,a\,x+147\right)}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(5/2))/(a*x + 1)^3,x)
```

```
[Out] (2048*c^2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(35*a*(a*x - 1)) - (16*c^2
*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a*(a*x + 1)) - (2*c^2*(1 - a^2*x^2
)^(1/2)*(c - a*c*x)^(1/2)*(5*a^2*x^2 - 36*a*x + 147))/(35*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^{\frac{5}{2}}(-(ax-1)(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(5/2)*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3,
x)
```

3.271 $\int e^{-3\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}}+\frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}}+\frac{64(c-acx)^{3/2}}{5a\sqrt{1-a^2x^2}}-\frac{256c\sqrt{c-acx}}{5a\sqrt{1-a^2x^2}}$$

```
[Out] 64/5*(-a*c*x+c)^(3/2)/a/(-a^2*x^2+1)^(1/2)+8/5*(-a*c*x+c)^(5/2)/a/c/(-a^2*x
^2+1)^(1/2)+2/5*(-a*c*x+c)^(7/2)/a/c^2/(-a^2*x^2+1)^(1/2)-256/5*c*(-a*c*x+c
)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}}+\frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}}+\frac{64(c-acx)^{3/2}}{5a\sqrt{1-a^2x^2}}-\frac{256c\sqrt{c-acx}}{5a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-256*c*Sqrt[c - a*c*x])/(5*a*Sqrt[1 - a^2*x^2]) + (64*(c - a*c*x)^(3/2))/(
5*a*Sqrt[1 - a^2*x^2]) + (8*(c - a*c*x)^(5/2))/(5*a*c*Sqrt[1 - a^2*x^2]) +
(2*(c - a*c*x)^(7/2))/(5*a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{\int \frac{(c-acx)^{9/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$
$$= \frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}} + \frac{12\int \frac{(c-acx)^{7/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{5c^2}$$
$$= \frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}} + \frac{32\int \frac{(c-acx)^{5/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{5c}$$
$$= \frac{64(c-acx)^{3/2}}{5a\sqrt{1-a^2x^2}} + \frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}} + \frac{128}{5}\int \frac{(c-acx)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx$$
$$= -\frac{256c\sqrt{c-acx}}{5a\sqrt{1-a^2x^2}} + \frac{64(c-acx)^{3/2}}{5a\sqrt{1-a^2x^2}} + \frac{8(c-acx)^{5/2}}{5ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{7/2}}{5ac^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.45

$$-\frac{2c^2\sqrt{1-ax}\left(a^3x^3-7a^2x^2+43ax+91\right)}{5a\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^2*Sqrt[1 - a*x]*(91 + 43*a*x - 7*a^2*x^2 + a^3*x^3))/(5*a*Sqrt[1 + a*
x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.54, size = 62, normalized size = 0.46

$$\frac{2\left(a^3cx^3-7\,a^2cx^2+43\,acx+91\,c\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{5\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/5*(a^3*c*x^3 - 7*a^2*c*x^2 + 43*a*c*x + 91*c)*sqrt(-a^2*x^2 + 1)*sqrt(-a*
c*x + c)/(a^3*x^2 - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 62, normalized size = 0.46

$$\frac{2\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \left(-acx+c\right)^{\frac{3}{2}} \left(x^{3} a^{3}-7 a^{2} x^{2}+43 a x+91\right)}{5 \left(a x+1\right)^{2} \left(a x-1\right)^{3} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/5*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(3/2)*(a^3*x^3-7*a^2*x^2+43*a*x+91)/(a*x+
1)^2/(a*x-1)^3/a
```

maxima [A] time = 0.39, size = 61, normalized size = 0.45

$$-\frac{2\left(a^{3} c^{\frac{3}{2}} x^{3}-7 \, a^{2} c^{\frac{3}{2}} x^{2}+43 \, a c^{\frac{3}{2}} x+91 \, c^{\frac{3}{2}}\right) \sqrt{a x+1} \left(a x-1\right)}{5 \left(a^{3} x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2/5*(a^3*c^(3/2)*x^3 - 7*a^2*c^(3/2)*x^2 + 43*a*c^(3/2)*x + 91*c^(3/2))*sq
rt(a*x + 1)*(a*x - 1)/(a^3*x^2 - a)
```

mupad [B] time = 0.99, size = 99, normalized size = 0.73

$$-\frac{\sqrt{c-a\,c\,x}\,\left(\frac{182\,c\,\sqrt{1-a^2\,x^2}}{5\,a^3}+\frac{2\,c\,x^3\,\sqrt{1-a^2\,x^2}}{5}-\frac{14\,c\,x^2\,\sqrt{1-a^2\,x^2}}{5\,a}+\frac{86\,c\,x\,\sqrt{1-a^2\,x^2}}{5\,a^2}\right)}{\frac{1}{a^2}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] -((c - a*c*x)^(1/2)*((182*c*(1 - a^2*x^2)^(1/2))/(5*a^3) + (2*c*x^3*(1 - a^
2*x^2)^(1/2))/5 - (14*c*x^2*(1 - a^2*x^2)^(1/2))/(5*a) + (86*c*x*(1 - a^2*x
^2)^(1/2))/(5*a^2)))/(1/a^2 - x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^{\frac{3}{2}}\,(-(ax-1)(ax+1))^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3,
x)
```

3.272 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=103

$$\frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}+\frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}}-\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] 16/3*(-a*c*x+c)^(3/2)/a/c/(-a^2*x^2+1)^(1/2)+2/3*(-a*c*x+c)^(5/2)/a/c^2/(-a
^2*x^2+1)^(1/2)-64/3*(-a*c*x+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}+\frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}}-\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-64*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a^2*x^2]) + (16*(c - a*c*x)^(3/2))/(3*a
*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x)^(5/2))/(3*a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\int \frac{(c-acx)^{7/2}}{\left(1-a^2x^2\right)^{3/2}}dx}{c^3}$$
$$= \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}} + \frac{8\int \frac{(c-acx)^{5/2}}{\left(1-a^2x^2\right)^{3/2}}dx}{3c^2}$$
$$= \frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}} + \frac{32\int \frac{(c-acx)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}dx}{3c}$$
$$= -\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}} + \frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.50

$$\frac{2c\sqrt{1-ax}\left(a^2x^2-10ax-23\right)}{3a\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(-23 - 10*a*x + a^2*x^2))/(3*a*Sqrt[1 + a*x]*Sqrt[c - a*
c*x])
```

fricas [A] time = 0.47, size = 49, normalized size = 0.48

$$-\frac{2\left(a^2x^2-10\,ax-23\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{3\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x^2 -
 a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 54, normalized size = 0.52

$$\frac{2\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{-acx+c}\,\left(a^{2}x^{2}-10ax-23\right)}{3\left(ax+1\right)^{2}\left(ax-1\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/3*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(a^2*x^2-10*a*x-23)/(a*x+1)^2/(a*x-
1)^2/a
```

maxima [A] time = 0.35, size = 50, normalized size = 0.49

$$\frac{2\left(a^{2}\sqrt{c}\,x^{2}-10\,a\sqrt{c}\,x-23\,\sqrt{c}\right)\sqrt{ax+1}\,(ax-1)}{3\left(a^{3}x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*(a^2*sqrt(c)*x^2 - 10*a*sqrt(c)*x - 23*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)
/(a^3*x^2 - a)
```

mupad [B] time = 0.97, size = 40, normalized size = 0.39

$$-\frac{2\sqrt{c-a\,c\,x}\,\left(-a^{2}\,x^{2}+10\,a\,x+23\right)}{3\,a\,\sqrt{1-a^{2}\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(a*x + 1)^3,x)
```

```
[Out] -(2*(c - a*c*x)^(1/2)*(10*a*x - a^2*x^2 + 23))/(3*a*(1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,\left(-\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{(ax+1)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

3.273 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=67

$$\frac{2(c-acx)^{3/2}}{ac^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{c-acx}}{ac\sqrt{1-a^2x^2}}$$

```
[Out] 2*(-a*c*x+c)^(3/2)/a/c^2/(-a^2*x^2+1)^(1/2)-8*(-a*c*x+c)^(1/2)/a/c/(-a^2*x^
2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{3/2}}{ac^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{c-acx}}{ac\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(a*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x)^(3/2))/(a*c^2
*Sqrt[1 - a^2*x^2])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\int \frac{(c-acx)^{5/2}}{(1-a^2x^2)^{3/2}}\,dx}{c^3}$$

$$= \frac{2(c-acx)^{3/2}}{ac^2\sqrt{1-a^2x^2}} + \frac{4\int \frac{(c-acx)^{3/2}}{(1-a^2x^2)^{3/2}}\,dx}{c^2}$$

$$= -\frac{8\sqrt{c-acx}}{ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{3/2}}{ac^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.60

$$-\frac{2\sqrt{1-ax}\,(ax+3)}{a\sqrt{ax+1}\,\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-2*Sqrt[1 - a*x]*(3 + a*x))/(a*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.48, size = 43, normalized size = 0.64

$$\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,(ax+3)}{a^3cx^2-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 3)/(a^3*c*x^2 - a*c)
```

giac [A] time = 0.25, size = 50, normalized size = 0.75

$$-2\left(\frac{\sqrt{acx+c}}{ac^2} + \frac{2}{\sqrt{acx+c}\,ac}\right)|c| + \frac{4\sqrt{2}\,|c|}{ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] -2*(sqrt(a*c*x + c)/(a*c^2) + 2/(sqrt(a*c*x + c)*a*c))*abs(c) + 4*sqrt(2)*a
bs(c)/(a*c^(3/2))
```

maple [A] time = 0.03, size = 46, normalized size = 0.69

$$\frac{2\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}(ax+3)}{\sqrt{-acx+c}\,(ax+1)^{2}(ax-1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2*(-a^2*x^2+1)^(3/2)*(a*x+3)/(-a*c*x+c)^(1/2)/(a*x+1)^2/(a*x-1)/a
```

maxima [A] time = 0.35, size = 30, normalized size = 0.45

$$-\frac{2\,(ax+3)\sqrt{ax+1}}{a^{2}\sqrt{c}\,x+a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -2*(a*x + 3)*sqrt(a*x + 1)/(a^2*sqrt(c)*x + a*sqrt(c))
```

mupad [B] time = 0.96, size = 64, normalized size = 0.96

$$-\frac{\left(\frac{6\,\sqrt{1-a^{2}\,x^{2}}}{a^{3}\,c}+\frac{2\,x\,\sqrt{1-a^{2}\,x^{2}}}{a^{2}\,c}\right)\sqrt{c-a\,c\,x}}{\frac{1}{a^{2}}-x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^(1/2)*(a*x + 1)^3),x)
```

```
[Out] -(((6*(1 - a^2*x^2)^(1/2))/(a^3*c) + (2*x*(1 - a^2*x^2)^(1/2))/(a^2*c))*(c
- a*c*x)^(1/2))/(1/a^2 - x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{\sqrt{-c\,(ax-1)}\,(ax+1)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(sqrt(-c*(a*x - 1))*(a*x + 1)**3), x
)
```

3.274 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2\sqrt{c-acx}}{ac^2\sqrt{1-a^2x^2}}$$

```
[Out] -2*(-a*c*x+c)^(1/2)/a/c^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6127, 649}

$$-\frac{2\sqrt{c-acx}}{ac^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= \frac{\int \frac{(c-acx)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\ &= -\frac{2\sqrt{c-acx}}{ac^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 1.06

$$-\frac{2(1-ax)^{3/2}}{a\sqrt{ax+1}\,(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (-2*(1 - a*x)^(3/2))/(a*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))
```

fricas [A] time = 0.48, size = 42, normalized size = 1.27

$$\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{a^3c^2x^2-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*c^2*x^2 - a*c^2)
```

giac [A] time = 0.20, size = 30, normalized size = 0.91

$$\frac{\left(\frac{\sqrt{2}}{a\sqrt{c}}-\frac{2}{\sqrt{acx+c}\,a}\right)|c|}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] (sqrt(2)/(a*sqrt(c)) - 2/(sqrt(a*c*x + c)*a))*abs(c)/c^2
```

maple [A] time = 0.02, size = 34, normalized size = 1.03

$$-\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{3}{2}}\,(ax+1)^2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -2*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2)/(a*x+1)^2/a
```

maxima [A] time = 0.33, size = 28, normalized size = 0.85

$$-\frac{2\sqrt{ax+1}\sqrt{c}}{a^2c^2x+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -2*sqrt(a*x + 1)*sqrt(c)/(a^2*c^2*x + a*c^2)
```

mupad [B] time = 1.02, size = 43, normalized size = 1.30

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-a\,c\,x}}{a\left(c^2-a^2c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^(3/2)*(a*x + 1)^3),x)
```

```
[Out] -(2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a*(c^2 - a^2*c^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{(-c(ax-1))^{\frac{3}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(a*x - 1))**(3/2)*(a*x + 1)**3)
, x)
```

3.275 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}\,ac^{5/2}}-\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}}$$

```
[Out] 1/2*arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^(5
/2)*2^(1/2)-(-a*c*x+c)^(1/2)/a/c^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6127, 667, 661, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}\,ac^{5/2}}-\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(Sqrt[c - a*c*x]/(a*c^3*Sqrt[1 - a^2*x^2])) + ArcTanh[(Sqrt[c]*Sqrt[1 - a^
2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])]/(Sqrt[2]*a*c^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(d*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(p + 1)), x] + Dist[(d*(m + 2*p
 + 2))/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && LtQ[0, m,
```

```
1] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= \frac{\int \frac{\sqrt{c-acx}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\ &= -\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}}+\frac{\int \frac{1}{\sqrt{c-acx}\sqrt{1-a^2x^2}}\,dx}{2c^2}\\ &= -\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}}-\frac{a\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{c}\\ &= -\frac{\sqrt{c-acx}}{ac^3\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{\sqrt{2}\,ac^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 0.65

$$-\frac{(1-ax)^{3/2}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(ax+1)\right)}{ac\sqrt{ax+1}\,(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(((1 - a*x)^(3/2)*Hypergeometric2F1[-1/2, 1, 1/2, (1 + a*x)/2])/(a*c*Sqrt[
1 + a*x]*(c - a*c*x)^(3/2)))
```

fricas [A] time = 0.57, size = 234, normalized size = 2.75

$$\left[\frac{\sqrt{2}\left(a^2x^2-1\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{4\left(a^3c^3x^2-ac^3\right)},\ \frac{\sqrt{2}\left(a^2x^2-1\right)\sqrt{-c}\ \text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(2)*(a^2*x^2 - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) +
4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*c^3*x^2 - a*c^3), 1/2*(sqrt(2)*
(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*s
qrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*c^3*
x^2 - a*c^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 82, normalized size = 0.96

$$-\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{-c \left(a x -1\right)}\, \left(\operatorname{arctanh}\left(\frac{\sqrt{c \left(a x +1\right)}\, \sqrt{2}}{2 \sqrt{c}}\right) \sqrt{2}\, \sqrt{c \left(a x +1\right)}-2 \sqrt{c}\right)}{2 c^{\frac{7}{2}} \left(a x -1\right) \left(a x +1\right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(7/2)*(arctanh(1/2*(c*(a*x+1))
^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(c*(a*x+1))^(1/2)-2*c^(1/2))/(a*x-1)/(a*x+1
)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{\left(-a c x+c\right)^{\frac{5}{2}} \left(a x+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a*c*x + c)^(5/2)*(a*x + 1)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{{\left(c-a\,c\,x\right)}^{5/2}\,{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^(5/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^(5/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(-c\left(ax-1\right)\right)^{\frac{5}{2}}\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(a*x - 1))**(5/2)*(a*x + 1)**3)
, x)
```

3.276 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=125

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{4\sqrt{2}\,ac^{7/2}}-\frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}}+\frac{1}{2ac^3\sqrt{1-a^2x^2}\sqrt{c-acx}}$$

```
[Out] 3/8*arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^(7
/2)*2^(1/2)+1/2/a/c^3/(-a*c*x+c)^(1/2)/(-a^2*x^2+1)^(1/2)-3/4*(-a*c*x+c)^(1
/2)/a/c^4/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6127, 673, 667, 661, 208}

$$-\frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}}+\frac{1}{2ac^3\sqrt{1-a^2x^2}\sqrt{c-acx}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{4\sqrt{2}\,ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] 1/(2*a*c^3*Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - a*c*x])/(4*a*c^
4*Sqrt[1 - a^2*x^2]) + (3*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt
[c - a*c*x])])/(4*Sqrt[2]*a*c^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(d*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(p + 1)), x] + Dist[(d*(m + 2*p
 + 2))/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /;
```

```
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && LtQ[0, m,
1] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\int \frac{1}{\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= \frac{1}{2ac^3\sqrt{c-acx}\,\sqrt{1-a^2x^2}} + \frac{3\int \frac{\sqrt{c-acx}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{4c^4}$$

$$= \frac{1}{2ac^3\sqrt{c-acx}\,\sqrt{1-a^2x^2}} - \frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}} + \frac{3\int \frac{1}{\sqrt{c-acx}\,\sqrt{1-a^2x^2}}\,dx}{8c^3}$$

$$= \frac{1}{2ac^3\sqrt{c-acx}\,\sqrt{1-a^2x^2}} - \frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)}{4c^2}$$

$$= \frac{1}{2ac^3\sqrt{c-acx}\,\sqrt{1-a^2x^2}} - \frac{3\sqrt{c-acx}}{4ac^4\sqrt{1-a^2x^2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{2}\,\sqrt{c-acx}}\right)}{4\sqrt{2}\,ac^{7/2}}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.46

$$-\frac{(1-ax)^{3/2}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{1}{2}(ax+1)\right)}{2ac^2\sqrt{ax+1}\,(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/2*((1 - a*x)^(3/2)*Hypergeometric2F1[-1/2, 2, 1/2, (1 + a*x)/2])/(a*c^2*
Sqrt[1 + a*x]*(c - a*c*x)^(3/2))
```

fricas [A] time = 0.46, size = 310, normalized size = 2.48

$$\left[\frac{3\sqrt{2}\left(a^3x^3 - a^2x^2 - ax + 1\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right) + 4\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,(3\,ax -}{16\left(a^4c^4x^3 - a^3c^4x^2 - a^2c^4x + ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fri
cas")
```

```
[Out] [1/16*(3*sqrt(2)*(a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*
a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(3*a*x - 1))/(a^4*
c^4*x^3 - a^3*c^4*x^2 - a^2*c^4*x + a*c^4), 1/8*(3*sqrt(2)*(a^3*x^3 - a^2*x
^2 - a*x + 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*s
qrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(3*a*x - 1
))/(a^4*c^4*x^3 - a^3*c^4*x^2 - a^2*c^4*x + a*c^4)]
```

giac [A] time = 0.27, size = 82, normalized size = 0.66

$$-\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}\,c^2} + \frac{2\,(3\,acx-c)}{\left((acx+c)^{\frac{3}{2}}-2\,\sqrt{acx+c}\,c\right)ac^2}\right)|c|}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="gia
c")
```

```
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^
2) + 2*(3*a*c*x - c)/(((a*c*x + c)^(3/2) - 2*sqrt(a*c*x + c)*c)*a*c^2))*abs
(c)/c^2
```

maple [A] time = 0.06, size = 124, normalized size = 0.99

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xa\sqrt{c\,(ax+1)} - 3\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c\,(ax+}\right.}{8c^{\frac{9}{2}}\,(ax-1)^2\,(ax+1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(9/2)*(3*2^(1/2)*arctanh(1/2*(
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*(c*(a*x+1))^(1/2)-3*arctanh(1/2*(c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(c*(a*x+1))^(1/2)-6*x*a*c^(1/2)+2*c^(
1/2))/(a*x-1)^2/(a*x+1)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(-acx+c\right)^{\frac{7}{2}}\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a*c*x + c)^(7/2)*(a*x + 1)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{\left(c-a\,c\,x\right)^{7/2}\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^(7/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^(7/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(-c\left(ax-1\right)\right)^{\frac{7}{2}}\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(a*x - 1))**(7/2)*(a*x + 1)**3)
, x)
```

3.277 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal. Leaf size=160

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}\,ac^{9/2}}-\frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}}+\frac{5}{16ac^4\sqrt{1-a^2x^2}\sqrt{c-acx}}+\frac{1}{4ac^3\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}$$

```
[Out] 15/64*arctanh(1/2*c^(1/2)*(-a^2*x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))/a/c^
(9/2)*2^(1/2)+1/4/a/c^3/(-a*c*x+c)^(3/2)/(-a^2*x^2+1)^(1/2)+5/16/a/c^4/(-a*
c*x+c)^(1/2)/(-a^2*x^2+1)^(1/2)-15/32*(-a*c*x+c)^(1/2)/a/c^5/(-a^2*x^2+1)^(
1/2)
```

Rubi [A] time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6127, 673, 667, 661, 208}

$$-\frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}}+\frac{5}{16ac^4\sqrt{1-a^2x^2}\sqrt{c-acx}}+\frac{1}{4ac^3\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}+\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{32\sqrt{2}\,ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] 1/(4*a*c^3*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2]) + 5/(16*a*c^4*Sqrt[c - a*c*
x]*Sqrt[1 - a^2*x^2]) - (15*Sqrt[c - a*c*x])/(32*a*c^5*Sqrt[1 - a^2*x^2]) +
 (15*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/(32*Sq
rt[2]*a*c^(9/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(d*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(p + 1)), x] + Dist[(d*(m + 2*p
 + 2))/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && LtQ[0, m,
1] && IntegerQ[2*p]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[(m + 2*
p + 2)/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p
+ 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx = \frac{\int \frac{1}{(c-acx)^{3/2}\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5\int \frac{1}{\sqrt{c-acx}\left(1-a^2x^2\right)^{3/2}}\,dx}{8c^4}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5}{16ac^4\sqrt{c-acx}\,\sqrt{1-a^2x^2}} + \frac{15\int \frac{\sqrt{c-acx}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{32c^5}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5}{16ac^4\sqrt{c-acx}\,\sqrt{1-a^2x^2}} - \frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}} + \frac{15\int \frac{1}{\sqrt{c-acx}\,\sqrt{}}}{64c^4}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5}{16ac^4\sqrt{c-acx}\,\sqrt{1-a^2x^2}} - \frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}} - \frac{(15a)\,\mathrm{Subst}\big(}{}$$

$$= \frac{1}{4ac^3(c-acx)^{3/2}\sqrt{1-a^2x^2}} + \frac{5}{16ac^4\sqrt{c-acx}\,\sqrt{1-a^2x^2}} - \frac{15\sqrt{c-acx}}{32ac^5\sqrt{1-a^2x^2}} + \frac{15\tanh^{-1}\big(}{32\sqrt{2}}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.36

$$-\frac{(1-ax)^{3/2}\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{1}{2}(ax+1)\right)}{4ac^3\sqrt{ax+1}\,(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] -1/4*((1 - a*x)^(3/2)*Hypergeometric2F1[-1/2, 3, 1/2, (1 + a*x)/2])/(a*c^3*
Sqrt[1 + a*x]*(c - a*c*x)^(3/2))
```

fricas [A] time = 0.43, size = 328, normalized size = 2.05

$$\left[\frac{15\sqrt{2}\left(a^4x^4-2a^3x^3+2ax-1\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right)+4\left(15a^2x^2-20ax-3\right)\sqrt{-a^2}}{128\left(a^5c^5x^4-2a^4c^5x^3+2a^2c^5x-ac^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(9/2),x, algorithm="fri
cas")
```

```
[Out] [1/128*(15*sqrt(2)*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(-(a^2*c*x^
2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/
(a^2*x^2 - 2*a*x + 1)) + 4*(15*a^2*x^2 - 20*a*x - 3)*sqrt(-a^2*x^2 + 1)*sqr
t(-a*c*x + c))/(a^5*c^5*x^4 - 2*a^4*c^5*x^3 + 2*a^2*c^5*x - a*c^5), 1/64*(1
5*sqrt(2)*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*(15*a^2*x^2 - 20
*a*x - 3)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^5*c^5*x^4 - 2*a^4*c^5*x^3
 + 2*a^2*c^5*x - a*c^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(9/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 173, normalized size = 1.08

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^2a^2\sqrt{c\,(ax+1)}-30\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xa\sqrt{c\,(}\right.}{64c^{\frac{11}{2}}\,(ax-1)^3\,(ax+1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(9/2),x)
```

```
[Out] -1/64*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)/c^(11/2)*(15*2^(1/2)*arctanh(1/
2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*(c*(a*x+1))^(1/2)-30*2^(1/2)*a
rctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*(c*(a*x+1))^(1/2)-30*x^2*
a^2*c^(1/2)+15*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(c*(a
*x+1))^(1/2)+40*x*a*c^(1/2)+6*c^(1/2))/(a*x-1)^3/(a*x+1)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{9}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a*c*x+c)^(9/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a*c*x + c)^(9/2)*(a*x + 1)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{{\left(c-a\,c\,x\right)}^{9/2}\,{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^(9/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - a*c*x)^(9/2)*(a*x + 1)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.278 $\int e^{n\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{9/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{9-n}{2},-\frac{n}{2};\frac{11-n}{2};\frac{1}{2}(1-ax)\right)}{ac(9-n)}$$

```
[Out] -2^(1+1/2*n)*(-a*c*x+c)^(9/2)*hypergeom([-1/2*n, 9/2-1/2*n],[11/2-1/2*n],-1
/2*a*x+1/2)/a/c/(9-n)/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{9/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{9-n}{2},-\frac{n}{2};\frac{11-n}{2};\frac{1}{2}(1-ax)\right)}{ac(9-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(9/2)*Hypergeometric2F1[(9 - n)/2, -n/2, (11 - n
)/2, (1 - a*x)/2])/(a*c*(9 - n)*(1 - a*x)^(n/2)))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx &= \int (1-ax)^{-n/2}(1+ax)^{n/2}(c-acx)^{7/2}\,dx\\ &= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{\frac{7}{2}-\frac{n}{2}}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{9/2}\,{}_2F_1\left(\frac{9-n}{2},-\frac{n}{2};\frac{11-n}{2};\frac{1}{2}(1-ax)\right)}{ac(9-n)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.99

$$\frac{c^3 2^{\frac{n}{2}+1}\sqrt{c-acx}\,(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(\frac{9}{2}-\frac{n}{2},-\frac{n}{2};\frac{11}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{a(n-9)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (2^(1 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[9/2
- n/2, -1/2*n, 11/2 - n/2, 1/2 - (a*x)/2])/(a*(-9 + n))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^3c^3x^3-3\,a^2c^3x^2+3\,ac^3x-c^3\right)\sqrt{-acx+c}\,\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*sqrt(-a*c*x + c)*
((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,8,1,0,0]%%%}+%%%{-4,[0,6,1,1,0]%%%}+%%%{6,[0,4,1,2,0]%%%}+%%%{-
4,[0,2,1,3,0]%%%}+%%%{1,[0,0,1,4,0]%%%} / %%%{1,[0,0,0,4,4]%%%} Error: Bad
Argument Value
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} (-acx + c)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx + c)^{\frac{7}{2}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(7/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{atanh}(a\,x)} \, (c - a\,c\,x)^{7/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a*c*x)^(7/2),x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - a*c*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.279 $\int e^{n \tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{7/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{7-n}{2},-\frac{n}{2};\frac{9-n}{2};\frac{1}{2}(1-ax)\right)}{ac(7-n)}$$

```
[Out] -2^(1+1/2*n)*(-a*c*x+c)^(7/2)*hypergeom([-1/2*n, 7/2-1/2*n],[9/2-1/2*n],-1/
2*a*x+1/2)/a/c/(7-n)/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{7/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{7-n}{2},-\frac{n}{2};\frac{9-n}{2};\frac{1}{2}(1-ax)\right)}{ac(7-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(7/2)*Hypergeometric2F1[(7 - n)/2, -n/2, (9 - n)
/2, (1 - a*x)/2])/(a*c*(7 - n)*(1 - a*x)^(n/2)))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx = \int (1-ax)^{-n/2}(1+ax)^{n/2}(c-acx)^{5/2}\,dx$$
$$= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{\frac{5}{2}-\frac{n}{2}}\,dx$$
$$= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{7/2}\,{}_2F_1\left(\frac{7-n}{2},-\frac{n}{2};\frac{9-n}{2};\frac{1}{2}(1-ax)\right)}{ac(7-n)}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.99

$$\frac{c^2 2^{\frac{n}{2}+1}\sqrt{c-acx}\,(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(\frac{7}{2}-\frac{n}{2},-\frac{n}{2};\frac{9}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{a(n-7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2^(1 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[7/2
- n/2, -1/2*n, 9/2 - n/2, 1/2 - (a*x)/2])/(a*(-7 + n))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^2x^2 - 2\,ac^2x + c^2\right)\sqrt{-acx+c}\,\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x -
 1))^(1/2*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-1,[0,6,1,0,0]%%%}+%%%{3,[0,4,1,1,0]%%%}+%%%{-3,[0,2,1,2,0]%%%}+%%%{
1,[0,0,1,3,0]%%%} / %%%{1,[0,0,0,3,3]%%%} Error: Bad Argument Value
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n \,\mathrm{arctanh}(ax)} (-acx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^{\frac{5}{2}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(5/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n \,\mathrm{atanh}(a\, x)} \left(c - a\, c\, x\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a*c*x)^(5/2),x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - a*c*x)^(5/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**(5/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.280 $\int e^{n\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{5/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{5-n}{2},-\frac{n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{ac(5-n)}$$

```
[Out] -2^(1+1/2*n)*(-a*c*x+c)^(5/2)*hypergeom([-1/2*n, 5/2-1/2*n],[7/2-1/2*n],-1/
2*a*x+1/2)/a/c/(5-n)/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{5/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{5-n}{2},-\frac{n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{ac(5-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(5/2)*Hypergeometric2F1[(5 - n)/2, -n/2, (7 - n)
/2, (1 - a*x)/2])/(a*c*(5 - n)*(1 - a*x)^(n/2)))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx &= \int (1-ax)^{-n/2}(1+ax)^{n/2}(c-acx)^{3/2}\,dx\\ &= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{\frac{3}{2}-\frac{n}{2}}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{5/2}\,{}_2F_1\left(\frac{5-n}{2},-\frac{n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{ac(5-n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.96

$$\frac{c2^{\frac{n}{2}+1}\sqrt{c-acx}\,(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(\frac{5}{2}-\frac{n}{2},-\frac{n}{2};\frac{7}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{a(n-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^(3/2),x]
[Out] (2^(1 + n/2)*c*(1 - a*x)^(2 - n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[5/2 -
n/2, -1/2*n, 7/2 - n/2, 1/2 - (a*x)/2])/(a*(-5 + n))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-(acx-c)\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] integral(-(a*c*x - c)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,4,1,0,0]%%%}+%%%{-2,[0,2,1,1,0]%%%}+%%%{1,[0,0,1,2,0]%%%} / %%%
{1,[0,0,0,2,2]%%%} Error: Bad Argument Value
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} (-acx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^{\frac{3}{2}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{atanh}(a\, x)} (c - a\, c\, x)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a*c*x)^(3/2),x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - a*c*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.281 $\int e^{n\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{3/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{3-n}{2},-\frac{n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{ac(3-n)}$$

```
[Out] -2^(1+1/2*n)*(-a*c*x+c)^(3/2)*hypergeom([-1/2*n, 3/2-1/2*n],[5/2-1/2*n],-1/
2*a*x+1/2)/a/c/(3-n)/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(c-acx)^{3/2}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{3-n}{2},-\frac{n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{ac(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(3/2)*Hypergeometric2F1[(3 - n)/2, -n/2, (5 - n)
/2, (1 - a*x)/2])/(a*c*(3 - n)*(1 - a*x)^(n/2)))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\sqrt{c-acx}\,dx=\int(1-ax)^{-n/2}(1+ax)^{n/2}\sqrt{c-acx}\,dx$$
$$=\left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int(1+ax)^{n/2}(c-acx)^{\frac{1}{2}-\frac{n}{2}}\,dx$$
$$=-\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{3/2}\,{}_2F_1\left(\frac{3-n}{2},-\frac{n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{ac(3-n)}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.95

$$\frac{2^{\frac{n}{2}+1}\sqrt{c-acx}\,(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(\frac{3}{2}-\frac{n}{2},-\frac{n}{2};\frac{5}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{a(n-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
[Out] (2^(1 + n/2)*(1 - a*x)^(1 - n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[3/2 - n/
2, -1/2*n, 5/2 - n/2, 1/2 - (a*x)/2])/(a*(-3 + n))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}\sqrt{-acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x)
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{atanh}(a\,x)}\,\sqrt{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a*c*x)^(1/2),x)
[Out] int(exp(n*atanh(a*x))*(c - a*c*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**(1/2),x)
[Out] Timed out
```

3.282 $\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{n}{2}+1}\sqrt{c-acx}\,(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1-n}{2},-\frac{n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{ac(1-n)}$$

```
[Out] -2^(1+1/2*n)*hypergeom([-1/2*n, 1/2-1/2*n],[3/2-1/2*n],-1/2*a*x+1/2)*(-a*c*
x+c)^(1/2)/a/c/(1-n)/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}\sqrt{c-acx}\,(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1-n}{2},-\frac{n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{ac(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] -((2^(1 + n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[(1 - n)/2, -n/2, (3 - n)/2
, (1 - a*x)/2])/(a*c*(1 - n)*(1 - a*x)^(n/2)))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{\sqrt{c-acx}}\,dx\\ &= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{1}{2}-\frac{n}{2}}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\sqrt{c-acx}\;{}_2F_1\left(\frac{1-n}{2},-\frac{n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{ac(1-n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.96

$$\frac{2^{\frac{n}{2}+1}\sqrt{c-acx}\,(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}-\frac{n}{2},-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{ac(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/Sqrt[c - a*c*x],x]
[Out] (2^(1 + n/2)*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2 - n/2, -1/2*n, 3/2 - n/2
, 1/2 - (a*x)/2])/(a*c*(-1 + n)*(1 - a*x)^(n/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x)
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{atanh}(a\,x)}}{\sqrt{c-a\,c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a*c*x)^(1/2),x)
[Out] int(exp(n*atanh(a*x))/(c - a*c*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}{(ax)}}}{\sqrt{-c\,(ax-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**(1/2),x)
[Out] Integral(exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)), x)
```

3.283 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-n-1),-\frac{n}{2};\frac{1-n}{2};\frac{1}{2}(1-ax)\right)}{ac(n+1)\sqrt{c-acx}}$$

```
[Out] 2^(1+1/2*n)*hypergeom([-1/2*n, -1/2-1/2*n],[1/2-1/2*n],-1/2*a*x+1/2)/a/c/(1
+n)/((-a*x+1)^(1/2*n))/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 23, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-n-1),-\frac{n}{2};\frac{1-n}{2};\frac{1}{2}(1-ax)\right)}{ac(n+1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[(-1 - n)/2, -n/2, (1 - n)/2, (1 - a*x)/2])/(
a*c*(1 + n)*(1 - a*x)^(n/2)*Sqrt[c - a*c*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{(c-acx)^{3/2}}\,dx$$
$$= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{3}{2}-\frac{n}{2}}\,dx$$
$$= \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-1-n),-\frac{n}{2};\frac{1-n}{2};\frac{1}{2}(1-ax)\right)}{ac(1+n)\sqrt{c-acx}}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.00

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2}-\frac{1}{2},-\frac{n}{2};\frac{1}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{ac(n+1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[-1/2 - n/2, -1/2*n, 1/2 - n/2, 1/2 - (a*x)/2
])/(a*c*(1 + n)*(1 - a*x)^(n/2)*Sqrt[c - a*c*x])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2x^2-2\,ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*
c^2*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n \operatorname{arctanh}(ax)}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x)
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{(c-a\,c\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a*c*x)^(3/2),x)
[Out] int(exp(n*atanh(a*x))/(c - a*c*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}{(ax)}}}{(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**(3/2),x)
[Out] Integral(exp(n*atanh(a*x))/(-c*(a*x - 1))**(3/2), x)
```

3.284 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-n-3),-\frac{n}{2};\frac{1}{2}(-n-1);\frac{1}{2}(1-ax)\right)}{ac(n+3)(c-acx)^{3/2}}$$

```
[Out] 2^(1+1/2*n)*hypergeom([-1/2*n, -3/2-1/2*n],[-1/2-1/2*n],-1/2*a*x+1/2)/a/c/(
3+n)/((-a*x+1)^(1/2*n))/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 23, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-n-3),-\frac{n}{2};\frac{1}{2}(-n-1);\frac{1}{2}(1-ax)\right)}{ac(n+3)(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[(-3 - n)/2, -n/2, (-1 - n)/2, (1 - a*x)/2])/
(a*c*(3 + n)*(1 - a*x)^(n/2)*(c - a*c*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{(c-acx)^{5/2}}\,dx\\ &= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{5}{2}-\frac{n}{2}}\,dx\\ &= \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-3-n),-\frac{n}{2};\frac{1}{2}(-1-n);\frac{1}{2}(1-ax)\right)}{ac(3+n)(c-acx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 80, normalized size = 1.03

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-\frac{n}{2}-1}\,{}_2F_1\left(-\frac{n}{2}-\frac{3}{2},-\frac{n}{2};-\frac{n}{2}-\frac{1}{2};\frac{1}{2}-\frac{ax}{2}\right)}{ac^2(n+3)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (2^(1 + n/2)*(1 - a*x)^(-1 - n/2)*Hypergeometric2F1[-3/2 - n/2, -1/2*n, -1/
2 - n/2, 1/2 - (a*x)/2])/(a*c^2*(3 + n)*Sqrt[c - a*c*x])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^3c^3x^3-3\,a^2c^3x^2+3\,ac^3x-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*x^3 - 3*a
^2*c^3*x^2 + 3*a*c^3*x - c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)}}{(-acx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x)
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{(-acx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{atanh}(a\,x)}}{(c-a\,c\,x)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a*c*x)^(5/2),x)
[Out] int(exp(n*atanh(a*x))/(c - a*c*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**(5/2),x)
[Out] Timed out
```

3.285 $\int \frac{e^{n \tanh^{-1}(ax)}}{(c-acx)^{7/2}} \, dx$

Optimal. Leaf size=78

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2} {}_2F_1\left(\frac{1}{2}(-n-5),-\frac{n}{2};\frac{1}{2}(-n-3);\frac{1}{2}(1-ax)\right)}{ac(n+5)(c-acx)^{5/2}}$$

```
[Out] 2^(1+1/2*n)*hypergeom([-1/2*n, -5/2-1/2*n],[-3/2-1/2*n],-1/2*a*x+1/2)/a/c/(
5+n)/((-a*x+1)^(1/2*n))/(-a*c*x+c)^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 23, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2} {}_2F_1\left(\frac{1}{2}(-n-5),-\frac{n}{2};\frac{1}{2}(-n-3);\frac{1}{2}(1-ax)\right)}{ac(n+5)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[(-5 - n)/2, -n/2, (-3 - n)/2, (1 - a*x)/2])/
(a*c*(5 + n)*(1 - a*x)^(n/2)*(c - a*c*x)^(5/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \int \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{(c-acx)^{7/2}}\,dx$$
$$= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{7}{2}-\frac{n}{2}}\,dx$$
$$= \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(\frac{1}{2}(-5-n),-\frac{n}{2};\frac{1}{2}(-3-n);\frac{1}{2}(1-ax)\right)}{ac(5+n)(c-acx)^{5/2}}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 1.03

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-\frac{n}{2}-2}\,{}_2F_1\left(-\frac{n}{2}-\frac{5}{2},-\frac{n}{2};-\frac{n}{2}-\frac{3}{2};\frac{1}{2}-\frac{ax}{2}\right)}{ac^3(n+5)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (2^(1 + n/2)*(1 - a*x)^(-2 - n/2)*Hypergeometric2F1[-5/2 - n/2, -1/2*n, -3/
2 - n/2, 1/2 - (a*x)/2])/(a*c^3*(5 + n)*Sqrt[c - a*c*x])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-acx+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^4x^4-4a^3c^4x^3+6a^2c^4x^2-4ac^4x+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^4*x^4 - 4*a^
3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{e^{n \,\mathrm{arctanh}(ax)}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x)
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{(c-a\,c\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a*c*x)^(7/2),x)
[Out] int(exp(n*atanh(a*x))/(c - a*c*x)^(7/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**(7/2),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.286 $\int e^{\tanh^{-1}(ax)}x^4(c-acx)\,dx$

Optimal. Leaf size=83

$$\frac{c\sin^{-1}(ax)}{16a^5}+\frac{1}{6}cx^5\sqrt{1-a^2x^2}-\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}-\frac{cx\sqrt{1-a^2x^2}}{16a^4}$$

```
[Out] 1/16*c*arcsin(a*x)/a^5-1/16*c*x*(-a^2*x^2+1)^(1/2)/a^4-1/24*c*x^3*(-a^2*x^2
+1)^(1/2)/a^2+1/6*c*x^5*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 279, 321, 216}

$$\frac{1}{6}cx^5\sqrt{1-a^2x^2}-\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}-\frac{cx\sqrt{1-a^2x^2}}{16a^4}+\frac{c\sin^{-1}(ax)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^4*(c - a*c*x),x]
```

```
[Out] -(c*x*Sqrt[1 - a^2*x^2])/(16*a^4) - (c*x^3*Sqrt[1 - a^2*x^2])/(24*a^2) + (c
*x^5*Sqrt[1 - a^2*x^2])/6 + (c*ArcSin[a*x])/(16*a^5)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^4(c-acx)\,dx &= c\int x^4\sqrt{1-a^2x^2}\,dx\\
&= \frac{1}{6}cx^5\sqrt{1-a^2x^2}+\frac{1}{6}c\int\frac{x^4}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}+\frac{1}{6}cx^5\sqrt{1-a^2x^2}+\frac{c\int\frac{x^2}{\sqrt{1-a^2x^2}}\,dx}{8a^2}\\
&= -\frac{cx\sqrt{1-a^2x^2}}{16a^4}-\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}+\frac{1}{6}cx^5\sqrt{1-a^2x^2}+\frac{c\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{16a^4}\\
&= -\frac{cx\sqrt{1-a^2x^2}}{16a^4}-\frac{cx^3\sqrt{1-a^2x^2}}{24a^2}+\frac{1}{6}cx^5\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{16a^5}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.60

$$\frac{c\left(ax\sqrt{1-a^2x^2}\left(8a^4x^4-2a^2x^2-3\right)+3\sin^{-1}(ax)\right)}{48a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^4*(c - a*c*x),x]
```

```
[Out] (c*(a*x*Sqrt[1 - a^2*x^2]*(-3 - 2*a^2*x^2 + 8*a^4*x^4) + 3*ArcSin[a*x]))/(4
8*a^5)
```

fricas [A] time = 0.51, size = 69, normalized size = 0.83

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(8\,a^5cx^5-2\,a^3cx^3-3\,acx\right)\sqrt{-a^2x^2+1}}{48\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/48*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (8*a^5*c*x^5 - 2*a^3*c*
x^3 - 3*a*c*x)*sqrt(-a^2*x^2 + 1))/a^5
```

giac [A] time = 0.22, size = 57, normalized size = 0.69

$$\frac{1}{48}\sqrt{-a^2x^2+1}\left(2\left(4\,cx^2-\frac{c}{a^2}\right)x^2-\frac{3\,c}{a^4}\right)x+\frac{c\arcsin\left(ax\right)\operatorname{sgn}(a)}{16\,a^4|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(-a^2*x^2 + 1)*(2*(4*c*x^2 - c/a^2)*x^2 - 3*c/a^4)*x + 1/16*c*arcs
in(a*x)*sgn(a)/(a^4*abs(a))
```

maple [A] time = 0.04, size = 91, normalized size = 1.10

$$\frac{c\,x^5\sqrt{-a^2x^2+1}}{6}-\frac{c\,x^3\sqrt{-a^2x^2+1}}{24a^2}-\frac{cx\sqrt{-a^2x^2+1}}{16a^4}+\frac{c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{16a^4\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4*(-a*c*x+c),x)
```

```
[Out] 1/6*c*x^5*(-a^2*x^2+1)^(1/2)-1/24*c*x^3*(-a^2*x^2+1)^(1/2)/a^2-1/16*c*x*(-a
^2*x^2+1)^(1/2)/a^4+1/16*c/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1
)^(1/2))
```

maxima [A] time = 0.40, size = 69, normalized size = 0.83

$$\frac{1}{6}\sqrt{-a^2x^2+1}\,cx^5-\frac{\sqrt{-a^2x^2+1}\,cx^3}{24\,a^2}-\frac{\sqrt{-a^2x^2+1}\,cx}{16\,a^4}+\frac{c\arcsin\left(ax\right)}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(-a^2*x^2 + 1)*c*x^5 - 1/24*sqrt(-a^2*x^2 + 1)*c*x^3/a^2 - 1/16*sqr
t(-a^2*x^2 + 1)*c*x/a^4 + 1/16*c*arcsin(a*x)/a^5
```

mupad [B] time = 0.78, size = 82, normalized size = 0.99

$$\frac{c\,x^5\,\sqrt{1-a^2\,x^2}}{6}-\frac{c\,x^3\,\sqrt{1-a^2\,x^2}}{24\,a^2}-\frac{c\,x\,\sqrt{1-a^2\,x^2}}{16\,a^4}+\frac{c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{16\,a^4\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c - a*c*x)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c*x^5*(1 - a^2*x^2)^(1/2))/6 - (c*x^3*(1 - a^2*x^2)^(1/2))/(24*a^2) - (c*x
*(1 - a^2*x^2)^(1/2))/(16*a^4) + (c*asinh(x*(-a^2)^(1/2)))/(16*a^4*(-a^2)^(
1/2))
```

sympy [A] time = 6.53, size = 192, normalized size = 2.31

$$c\left(\begin{cases}\frac{ia^2x^7}{6\sqrt{a^2x^2-1}}-\frac{5ix^5}{24\sqrt{a^2x^2-1}}-\frac{ix^3}{48a^2\sqrt{a^2x^2-1}}+\frac{ix}{16a^4\sqrt{a^2x^2-1}}-\frac{i\operatorname{acosh}(ax)}{16a^5} & \text{for } \left|a^2x^2\right|>1\\ -\frac{a^2x^7}{6\sqrt{-a^2x^2+1}}+\frac{5x^5}{24\sqrt{-a^2x^2+1}}+\frac{x^3}{48a^2\sqrt{-a^2x^2+1}}-\frac{x}{16a^4\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}(ax)}{16a^5} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4*(-a*c*x+c),x)
```

```
[Out] c*Piecewise((I*a**2*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*x**5/(24*sqrt(a**2*x
**2 - 1)) - I*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*x/(16*a**4*sqrt(a**2*x
**2 - 1)) - I*acosh(a*x)/(16*a**5), Abs(a**2*x**2) > 1), (-a**2*x**7/(6*sqr
t(-a**2*x**2 + 1)) + 5*x**5/(24*sqrt(-a**2*x**2 + 1)) + x**3/(48*a**2*sqrt(
-a**2*x**2 + 1)) - x/(16*a**4*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(16*a**5),
True))
```

3.287 $\int e^{\tanh^{-1}(ax)} x^3 (c - acx)\, dx$

Optimal. Leaf size=45

$$\frac{c\left(1-a^2x^2\right)^{5/2}}{5a^4}-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^4}$$

```
[Out] -1/3*c*(-a^2*x^2+1)^(3/2)/a^4+1/5*c*(-a^2*x^2+1)^(5/2)/a^4
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6128, 266, 43}

$$\frac{c\left(1-a^2x^2\right)^{5/2}}{5a^4}-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(c - a*c*x),x]
```

```
[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*a^4) + (c*(1 - a^2*x^2)^(5/2))/(5*a^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^3(c-acx)\,dx &= c\int x^3\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}c\,\text{Subst}\left(\int x\sqrt{1-a^2x}\,dx,x,x^2\right)\\ &= \frac{1}{2}c\,\text{Subst}\left(\int\left(\frac{\sqrt{1-a^2x}}{a^2}-\frac{(1-a^2x)^{3/2}}{a^2}\right)dx,x,x^2\right)\\ &= -\frac{c(1-a^2x^2)^{3/2}}{3a^4}+\frac{c(1-a^2x^2)^{5/2}}{5a^4}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 32, normalized size = 0.71

$$-\frac{c(1-a^2x^2)^{3/2}(3a^2x^2+2)}{15a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a*c*x),x]
[Out] -1/15*(c*(1 - a^2*x^2)^(3/2)*(2 + 3*a^2*x^2))/a^4
```

fricas [**A**] time = 0.51, size = 39, normalized size = 0.87

$$\frac{(3\,a^4cx^4-a^2cx^2-2\,c)\sqrt{-a^2x^2+1}}{15\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c),x, algorithm="fricas")
[Out] 1/15*(3*a^4*c*x^4 - a^2*c*x^2 - 2*c)*sqrt(-a^2*x^2 + 1)/a^4
```

giac [**A**] time = 0.16, size = 47, normalized size = 1.04

$$\frac{3\,(a^2x^2-1)^2\sqrt{-a^2x^2+1}\,c-5\,(-a^2x^2+1)^{\frac{3}{2}}c}{15\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/15*(3*(a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*c - 5*(-a^2*x^2 + 1)^(3/2)*c)/a^
4
```

maple [A] time = 0.03, size = 43, normalized size = 0.96

$$-\frac{(ax-1)^2 (ax+1)^2 \left(3a^2x^2+2\right) c}{15a^4\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c),x)
```

```
[Out] -1/15*(a*x-1)^2*(a*x+1)^2*(3*a^2*x^2+2)*c/a^4/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.40, size = 58, normalized size = 1.29

$$\frac{1}{5}\sqrt{-a^2x^2+1}\,cx^4-\frac{\sqrt{-a^2x^2+1}\,cx^2}{15\,a^2}-\frac{2\sqrt{-a^2x^2+1}\,c}{15\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-a^2*x^2 + 1)*c*x^4 - 1/15*sqrt(-a^2*x^2 + 1)*c*x^2/a^2 - 2/15*sqr
t(-a^2*x^2 + 1)*c/a^4
```

mupad [B] time = 0.04, size = 36, normalized size = 0.80

$$-\frac{5\,c\,\left(1-a^2\,x^2\right)^{3/2}-3\,c\,\left(1-a^2\,x^2\right)^{5/2}}{15\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] -(5*c*(1 - a^2*x^2)^(3/2) - 3*c*(1 - a^2*x^2)^(5/2))/(15*a^4)
```

sympy [A] time = 0.70, size = 66, normalized size = 1.47

$$\begin{cases} \frac{cx^4\sqrt{-a^2x^2+1}}{5}-\frac{cx^2\sqrt{-a^2x^2+1}}{15a^2}-\frac{2c\sqrt{-a^2x^2+1}}{15a^4} & \text{for}\ a\neq 0 \\ \frac{cx^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a*c*x+c),x)
```

```
[Out] Piecewise((c*x**4*sqrt(-a**2*x**2 + 1)/5 - c*x**2*sqrt(-a**2*x**2 + 1)/(15*
a**2) - 2*c*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (c*x**4/4, True))
```

3.288 $\int e^{\tanh^{-1}(ax)}x^2(c-acx)\,dx$

Optimal. Leaf size=58

$$\frac{c\sin^{-1}(ax)}{8a^3}-\frac{cx\sqrt{1-a^2x^2}}{8a^2}+\frac{1}{4}cx^3\sqrt{1-a^2x^2}$$

```
[Out] 1/8*c*arcsin(a*x)/a^3-1/8*c*x*(-a^2*x^2+1)^(1/2)/a^2+1/4*c*x^3*(-a^2*x^2+1)
^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 279, 321, 216}

$$\frac{1}{4}cx^3\sqrt{1-a^2x^2}-\frac{cx\sqrt{1-a^2x^2}}{8a^2}+\frac{c\sin^{-1}(ax)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a*c*x),x]
```

```
[Out] -(c*x*Sqrt[1 - a^2*x^2])/(8*a^2) + (c*x^3*Sqrt[1 - a^2*x^2])/4 + (c*ArcSin[
a*x])/(8*a^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^2(c-acx)\,dx &= c\int x^2\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{4}cx^3\sqrt{1-a^2x^2}+\frac{1}{4}c\int\frac{x^2}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{cx\sqrt{1-a^2x^2}}{8a^2}+\frac{1}{4}cx^3\sqrt{1-a^2x^2}+\frac{c\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^2}\\ &= -\frac{cx\sqrt{1-a^2x^2}}{8a^2}+\frac{1}{4}cx^3\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{8a^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.69

$$\frac{c\left(ax\sqrt{1-a^2x^2}\left(2a^2x^2-1\right)+\sin^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a*c*x),x]
[Out] (c*(a*x*Sqrt[1 - a^2*x^2]*(-1 + 2*a^2*x^2) + ArcSin[a*x]))/(8*a^3)
```

fricas [A] time = 0.42, size = 60, normalized size = 1.03

$$-\frac{2\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^3cx^3-acx\right)\sqrt{-a^2x^2+1}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c),x, algorithm="fricas")
[Out] -1/8*(2*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^3*c*x^3 - a*c*x)*sq
rt(-a^2*x^2 + 1))/a^3
```

giac [A] time = 0.19, size = 45, normalized size = 0.78

$$\frac{1}{8}\sqrt{-a^2x^2+1}\left(2\,cx^2-\frac{c}{a^2}\right)x+\frac{c\arcsin\left(ax\right)\mathrm{sgn}(a)}{8\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-a^2*x^2 + 1)*(2*c*x^2 - c/a^2)*x + 1/8*c*arcsin(a*x)*sgn(a)/(a^2*
abs(a))
```

maple [A] time = 0.04, size = 70, normalized size = 1.21

$$\frac{c\,x^3\sqrt{-a^2x^2+1}}{4}-\frac{cx\sqrt{-a^2x^2+1}}{8a^2}+\frac{c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a^2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c),x)
```

```
[Out] 1/4*c*x^3*(-a^2*x^2+1)^(1/2)-1/8*c*x*(-a^2*x^2+1)^(1/2)/a^2+1/8*c/a^2/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.46, size = 48, normalized size = 0.83

$$\frac{1}{4}\sqrt{-a^2x^2+1}\,cx^3-\frac{\sqrt{-a^2x^2+1}\,cx}{8\,a^2}+\frac{c\arcsin\left(ax\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*c*x^3 - 1/8*sqrt(-a^2*x^2 + 1)*c*x/a^2 + 1/8*c*arcsi
n(a*x)/a^3
```

mupad [B] time = 0.78, size = 61, normalized size = 1.05

$$\frac{c\,x^3\,\sqrt{1-a^2\,x^2}}{4}-\frac{c\,x\,\sqrt{1-a^2\,x^2}}{8\,a^2}+\frac{c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a^2\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c*x^3*(1 - a^2*x^2)^(1/2))/4 - (c*x*(1 - a^2*x^2)^(1/2))/(8*a^2) + (c*asin
h(x*(-a^2)^(1/2)))/(8*a^2*(-a^2)^(1/2))
```

sympy [A] time = 4.88, size = 150, normalized size = 2.59

$$c\left(\begin{cases} \frac{ia^2x^5}{4\sqrt{a^2x^2-1}} - \frac{3ix^3}{8\sqrt{a^2x^2-1}} + \frac{ix}{8a^2\sqrt{a^2x^2-1}} - \frac{i\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2| > 1 \\ -\frac{a^2x^5}{4\sqrt{-a^2x^2+1}} + \frac{3x^3}{8\sqrt{-a^2x^2+1}} - \frac{x}{8a^2\sqrt{-a^2x^2+1}} + \frac{\operatorname{asin}(ax)}{8a^3} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c),x)
```

```
[Out] c*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*x*
*2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(a*
*2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**2*
x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True))
```

3.289 $\int e^{\tanh^{-1}(ax)}x(c-acx)\,dx$

Optimal. Leaf size=22

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^2}$$

[Out] -1/3*c*(-a^2*x^2+1)^(3/2)/a^2

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6128, 261}

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^2}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[a*x]*x*(c - a*c*x),x]

[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*a^2)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x(c-acx)\,dx &= c\int x\sqrt{1-a^2x^2}\,dx\\ &= -\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a*c*x),x]
[Out] -1/3*(c*(1 - a^2*x^2)^(3/2))/a^2
```

fricas [A] time = 0.43, size = 29, normalized size = 1.32

$$\frac{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c),x, algorithm="fricas")
[Out] 1/3*(a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)/a^2
```

giac [A] time = 0.18, size = 18, normalized size = 0.82

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c),x, algorithm="giac")
[Out] -1/3*(-a^2*x^2 + 1)^(3/2)*c/a^2
```

maple [A] time = 0.03, size = 33, normalized size = 1.50

$$-\frac{(ax-1)^2\,(ax+1)^2\,c}{3a^2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c),x)
[Out] -1/3*(a*x-1)^2*(a*x+1)^2*c/a^2/(-a^2*x^2+1)^(1/2)
```

maxima [B] time = 0.47, size = 37, normalized size = 1.68

$$\frac{1}{3}\,\sqrt{-a^2x^2+1}\,cx^2-\frac{\sqrt{-a^2x^2+1}\,c}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-a^2*x^2 + 1)*c*x^2 - 1/3*sqrt(-a^2*x^2 + 1)*c/a^2
```

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$-\frac{c\left(1-a^2\,x^2\right)^{3/2}}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*a^2)
```

sympy [A] time = 0.41, size = 42, normalized size = 1.91

$$\begin{cases} \frac{cx^2\sqrt{-a^2x^2+1}}{3} - \frac{c\sqrt{-a^2x^2+1}}{3a^2} & \text{for}\ a \neq 0 \\ \frac{cx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c),x)
```

```
[Out] Piecewise((c*x**2*sqrt(-a**2*x**2 + 1)/3 - c*sqrt(-a**2*x**2 + 1)/(3*a**2),
 Ne(a, 0)), (c*x**2/2, True))
```

3.290 $\int e^{\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

```
[Out] 1/2*c*arcsin(a*x)/a+1/2*c*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6127, 195, 216}

$$\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x),x]
```

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 + (c*ArcSin[a*x])/(2*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}(c-acx)\,dx &= c\int \sqrt{1-a^2x^2}\,dx\\
&= \frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{1}{2}c\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.91

$$
\frac{c\left(ax\sqrt{1-a^2x^2}+\sin^{-1}(ax)\right)}{2a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x),x]
[Out] (c*(a*x*Sqrt[1 - a^2*x^2] + ArcSin[a*x]))/(2*a)
```

fricas [A] time = 0.41, size = 47, normalized size = 1.42

$$
\frac{\sqrt{-a^2x^2+1}\,acx-2\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="fricas")
[Out] 1/2*(sqrt(-a^2*x^2 + 1)*a*c*x - 2*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))
/a
```

giac [A] time = 0.20, size = 30, normalized size = 0.91

$$
\frac{1}{2}\,\sqrt{-a^2x^2+1}\,cx+\frac{c\arcsin\left(ax\right)\mathrm{sgn}(a)}{2\,|a|}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="giac")
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(a*x)*sgn(a)/abs(a)
```

maple [A] time = 0.03, size = 46, normalized size = 1.39

$$\frac{cx\sqrt{-a^2x^2+1}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x)
```

```
[Out] 1/2*c*x*(-a^2*x^2+1)^(1/2)+1/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2
+1)^(1/2))
```

maxima [A] time = 0.40, size = 27, normalized size = 0.82

$$\frac{1}{2}\sqrt{-a^2x^2+1}\,cx+\frac{c\arcsin(ax)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(a*x)/a
```

mupad [B] time = 0.00, size = 37, normalized size = 1.12

$$\frac{c\,x\,\sqrt{1-a^2\,x^2}}{2}+\frac{c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c*x*(1 - a^2*x^2)^(1/2))/2 + (c*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2))
```

sympy [A] time = 3.59, size = 37, normalized size = 1.12

$$\begin{cases} \frac{c\left(\begin{cases} \frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\mathrm{asin}(ax)}{2} & \text{for}\ ax>-1\wedge ax<1 \end{cases}\right)}{a} & \text{for}\ a\neq 0 \\ cx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c),x)
```

```
[Out] Piecewise((c*Piecewise((a*x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1
) & (a*x < 1)))/a, Ne(a, 0)), (c*x, True))
```

3.291 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x} dx$

Optimal. Leaf size=35

$$c\sqrt{1-a^2x^2} - c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -c*arctanh((-a^2*x^2+1)^(1/2))+c*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6128, 266, 50, 63, 208}

$$c\sqrt{1-a^2x^2} - c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x))/x,x]
```

```
[Out] c*Sqrt[1 - a^2*x^2] - c*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x}\,dx\\
&= \frac{1}{2}c\,\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x}\,dx, x, x^2\right)\\
&= c\sqrt{1-a^2x^2} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)\\
&= c\sqrt{1-a^2x^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2}\\
&= c\sqrt{1-a^2x^2} - c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [B] time = 0.08, size = 79, normalized size = 2.26

$$
c\left(-\frac{a^2x^2}{\sqrt{1-a^2x^2}}+\frac{1}{\sqrt{1-a^2x^2}}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\sin^{-1}(ax)+2\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x))/x,x]
```

```
[Out] c*(1/Sqrt[1 - a^2*x^2] - (a^2*x^2)/Sqrt[1 - a^2*x^2] + ArcSin[a*x] + 2*ArcS
in[Sqrt[1 - a*x]/Sqrt[2]] - ArcTanh[Sqrt[1 - a^2*x^2]])
```

fricas [A] time = 0.49, size = 36, normalized size = 1.03

$$c\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x,x, algorithm="fricas")
[Out] c*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*c
```

giac [A] time = 0.16, size = 53, normalized size = 1.51

$$-\frac{1}{2}\,c\log\left(\sqrt{-a^2x^2+1}+1\right)+\frac{1}{2}\,c\log\left(-\sqrt{-a^2x^2+1}+1\right)+\sqrt{-a^2x^2+1}\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x,x, algorithm="giac")
[Out] -1/2*c*log(sqrt(-a^2*x^2 + 1) + 1) + 1/2*c*log(-sqrt(-a^2*x^2 + 1) + 1) + s
qrt(-a^2*x^2 + 1)*c
```

maple [A] time = 0.03, size = 32, normalized size = 0.91

$$-c\left(-\sqrt{-a^2x^2+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x,x)
[Out] -c*(-(-a^2*x^2+1)^(1/2)+arctanh(1/(-a^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.40, size = 44, normalized size = 1.26

$$-c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^2x^2+1}\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x,x, algorithm="maxima")
[Out] -c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)*c
```

mupad [B] time = 0.77, size = 31, normalized size = 0.89

$$-c\,\left(\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)-\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] -c*(atanh((1 - a^2*x^2)^(1/2)) - (1 - a^2*x^2)^(1/2))
```

sympy [A] time = 10.27, size = 66, normalized size = 1.89

$$\frac{a^2c\left(\begin{cases}-x^2 & \text{for } a^2=0\\ \frac{2\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)}{2}-\frac{c\left(-\log\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)+\log\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)/x,x)
```

```
[Out] a**2*c*Piecewise((-x**2, Eq(a**2, 0)), (2*sqrt(-a**2*x**2 + 1)/a**2, True))
/2 - c*(-log(-1 + 1/sqrt(-a**2*x**2 + 1)) + log(1 + 1/sqrt(-a**2*x**2 + 1))
)/2
```

3.292 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^2} dx$

Optimal. Leaf size=29

$$-\frac{c\sqrt{1-a^2x^2}}{x} - ac\sin^{-1}(ax)$$

```
[Out] -a*c*arcsin(a*x)-c*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6128, 277, 216}

$$-\frac{c\sqrt{1-a^2x^2}}{x} - ac\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x))/x^2,x]
```

```
[Out] -((c*Sqrt[1 - a^2*x^2])/x) - a*c*ArcSin[a*x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^2}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^2}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{x} - \left(a^2c\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{x} - ac\sin^{-1}(ax)$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.97

$$-\frac{c\left(\sqrt{1-a^2x^2}+ax\sin^{-1}(ax)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x))/x^2,x]
```

```
[Out] -((c*(Sqrt[1 - a^2*x^2] + a*x*ArcSin[a*x]))/x)
```

fricas [A] time = 0.48, size = 47, normalized size = 1.62

$$\frac{2\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}\,c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*c)/x
```

giac [B] time = 0.20, size = 74, normalized size = 2.55

$$\frac{a^4cx}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{a^2c\arcsin(ax)\operatorname{sgn}(a)}{|a|}-\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*c*arcsin(a*x)*sg
n(a)/abs(a) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c/(x*abs(a))
```

maple [A] time = 0.04, size = 51, normalized size = 1.76

$$-\frac{c\,a^{2}\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{\sqrt{a^{2}}}-\frac{c\sqrt{-a^{2}x^{2}+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^2,x)
```

```
[Out] -c*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c*(-a^2*x^2+1)^
(1/2)/x
```

maxima [A] time = 0.47, size = 27, normalized size = 0.93

$$-ac\arcsin\left(ax\right)-\frac{\sqrt{-a^{2}x^{2}+1}\,c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -a*c*arcsin(a*x) - sqrt(-a^2*x^2 + 1)*c/x
```

mupad [B] time = 0.05, size = 38, normalized size = 1.31

$$c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}-\frac{c\,\sqrt{1-a^2\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] c*asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2) - (c*(1 - a^2*x^2)^(1/2))/x
```

sympy [A] time = 3.43, size = 88, normalized size = 3.03

$$c\left(\begin{cases}-\frac{ia^{2}x}{\sqrt{a^{2}x^{2}-1}}+ia\,\mathrm{acosh}\left(ax\right)+\frac{i}{x\sqrt{a^{2}x^{2}-1}} & \text{for}\ \left|a^{2}x^{2}\right|>1\\ \frac{a^{2}x}{\sqrt{-a^{2}x^{2}+1}}-a\,\mathrm{asin}\left(ax\right)-\frac{1}{x\sqrt{-a^{2}x^{2}+1}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)/x**2,x)
```

```
[Out] c*Piecewise((-I*a**2*x/sqrt(a**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**
2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*
x) - 1/(x*sqrt(-a**2*x**2 + 1)), True))
```

3.293 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{c\sqrt{1-a^2x^2}}{2x^2}$$

```
[Out] 1/2*a^2*c*arctanh((-a^2*x^2+1)^(1/2))-1/2*c*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6128, 266, 47, 63, 208}

$$\frac{1}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{c\sqrt{1-a^2x^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x))/x^3,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(2*x^2) + (a^2*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^3}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx\\
&= \frac{1}{2}c\,\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{1}{4}\left(a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.46

$$
\frac{c\left(a^2x^2+a^2x^2\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-1\right)}{2x^2\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x))/x^3,x]
```

```
[Out] (c*(-1 + a^2*x^2 + a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(
2*x^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.47, size = 47, normalized size = 1.02

$$-\frac{a^{2} c x^{2} \log \left(\frac{\sqrt{-a^{2} x^{2}+1}-1}{x}\right)+\sqrt{-a^{2} x^{2}+1}\, c}{2\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^3,x, algorithm="fricas")
[Out] -1/2*(a^2*c*x^2*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*c)/x^2
```

giac [A] time = 0.16, size = 70, normalized size = 1.52

$$\frac{a^{4} c \log \left(\sqrt{-a^{2} x^{2}+1}+1\right)-a^{4} c \log \left(-\sqrt{-a^{2} x^{2}+1}+1\right)-\frac{2 \sqrt{-a^{2} x^{2}+1}\, a^{2} c}{x^{2}}}{4\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^3,x, algorithm="giac")
[Out] 1/4*(a^4*c*log(sqrt(-a^2*x^2 + 1) + 1) - a^4*c*log(-sqrt(-a^2*x^2 + 1) + 1)
 - 2*sqrt(-a^2*x^2 + 1)*a^2*c/x^2)/a^2
```

maple [A] time = 0.04, size = 40, normalized size = 0.87

$$-c \left(-\frac{a^{2} \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2} x^{2}+1}}\right)}{2}+\frac{\sqrt{-a^{2} x^{2}+1}}{2 x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^3,x)
[Out] -c*(-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+1/2*(-a^2*x^2+1)^(1/2)/x^2)
```

maxima [A] time = 0.40, size = 51, normalized size = 1.11

$$\frac{1}{2}\, a^{2} c \log \left(\frac{2 \sqrt{-a^{2} x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-a^{2} x^{2}+1}\, c}{2\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 1/2*sqrt(-a^2*x^2 +
 1)*c/x^2
```

mupad [B] time = 0.78, size = 38, normalized size = 0.83

$$\frac{a^2\,c\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{2}-\frac{c\,\sqrt{1-a^2\,x^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1))/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^2*c*atanh((1 - a^2*x^2)^(1/2)))/2 - (c*(1 - a^2*x^2)^(1/2))/(2*x^2)
```

sympy [A] time = 11.45, size = 73, normalized size = 1.59

$$-a^2c\left(\frac{\log\left(\sqrt{-a^2x^2+1}-1\right)}{4}-\frac{\log\left(\sqrt{-a^2x^2+1}+1\right)}{4}-\frac{1}{4\left(\sqrt{-a^2x^2+1}+1\right)}-\frac{1}{4\left(\sqrt{-a^2x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)/x**3,x)
```

```
[Out] -a**2*c*(log(sqrt(-a**2*x**2 + 1) - 1)/4 - log(sqrt(-a**2*x**2 + 1) + 1)/4
- 1/(4*(sqrt(-a**2*x**2 + 1) + 1)) - 1/(4*(sqrt(-a**2*x**2 + 1) - 1)))
```

3.294 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^4}\,dx$

Optimal. Leaf size=22

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3x^3}$$

```
[Out] -1/3*c*(-a^2*x^2+1)^(3/2)/x^3
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6128, 264}

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x))/x^4,x]
```

```
[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}(c-acx)}{x^4}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^4}\,dx\\ &= -\frac{c\left(1-a^2x^2\right)^{3/2}}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x))/x^4,x]
```

```
[Out] -1/3*(c*(1 - a^2*x^2)^(3/2))/x^3
```

fricas [A] time = 0.41, size = 29, normalized size = 1.32

$$\frac{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)/x^3
```

giac [B] time = 0.19, size = 124, normalized size = 5.64

$$\frac{\left(a^4c-\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|}+\frac{\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^4c}{x}-\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c}{x^3}}{24\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*c - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c/x^2)*a^6*x^3/((sqrt(-a^
2*x^2 + 1)*abs(a) + a)^3*abs(a)) + 1/24*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*
a^4*c/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c/x^3)/(a^2*abs(a))
```

maple [A] time = 0.03, size = 33, normalized size = 1.50

$$-\frac{(ax-1)^2\,(ax+1)^2\,c}{3x^3\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^4,x)
```

[Out] -1/3*(a*x-1)^2*(a*x+1)^2*c/x^3/(-a^2*x^2+1)^(1/2)

maxima [B] time = 0.44, size = 40, normalized size = 1.82

$$\frac{\sqrt{-a^2x^2+1}\,a^2c}{3\,x}-\frac{\sqrt{-a^2x^2+1}\,c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)/x^4,x, algorithm="maxima")

[Out] 1/3*sqrt(-a^2*x^2 + 1)*a^2*c/x - 1/3*sqrt(-a^2*x^2 + 1)*c/x^3

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$-\frac{c\,\left(1-a^2\,x^2\right)^{3/2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - a*c*x)*(a*x + 1))/(x^4*(1 - a^2*x^2)^(1/2)),x)

[Out] -(c*(1 - a^2*x^2)^(3/2))/(3*x^3)

sympy [A] time = 3.23, size = 90, normalized size = 4.09

$$c\left(\begin{cases}\frac{a^3\sqrt{-1+\frac{1}{a^2x^2}}}{3}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{3x^2} & \text{for}\ \frac{1}{\left|a^2x^2\right|}>1\\ \frac{ia^3\sqrt{1-\frac{1}{a^2x^2}}}{3}-\frac{ia\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)/x**4,x)

[Out] c*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2))/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))

3.295 $\int e^{\tanh^{-1}(ax)}x^3(c-acx)^2\,dx$

Optimal. Leaf size=124

$$-\frac{c^2\sin^{-1}(ax)}{16a^4}-\frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2}+\frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a}-\frac{c^2(16-15ax)\left(1-a^2x^2\right)^{3/2}}{120a^4}-\frac{c^2x\sqrt{1-a^2x^2}}{16a^3}$$

```
[Out] -1/5*c^2*x^2*(-a^2*x^2+1)^(3/2)/a^2+1/6*c^2*x^3*(-a^2*x^2+1)^(3/2)/a-1/120*
c^2*(-15*a*x+16)*(-a^2*x^2+1)^(3/2)/a^4-1/16*c^2*arcsin(a*x)/a^4-1/16*c^2*x
*(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 833, 780, 195, 216}

$$\frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a}-\frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2}-\frac{c^2x\sqrt{1-a^2x^2}}{16a^3}-\frac{c^2(16-15ax)\left(1-a^2x^2\right)^{3/2}}{120a^4}-\frac{c^2\sin^{-1}(ax)}{16a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*x*Sqrt[1 - a^2*x^2])/(16*a^3) - (c^2*x^2*(1 - a^2*x^2)^(3/2))/(5*a^2)
 + (c^2*x^3*(1 - a^2*x^2)^(3/2))/(6*a) - (c^2*(16 - 15*a*x)*(1 - a^2*x^2)^(
3/2))/(120*a^4) - (c^2*ArcSin[a*x])/(16*a^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^3(c-acx)^2\,dx = c\int x^3(c-acx)\sqrt{1-a^2x^2}\,dx$$
$$= \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} - \frac{c\int x^2\left(3ac-6a^2cx\right)\sqrt{1-a^2x^2}\,dx}{6a^2}$$
$$= -\frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2} + \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} + \frac{c\int x\left(12a^2c-15a^3cx\right)\sqrt{1-a^2x^2}\,dx}{30a^4}$$
$$= -\frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2} + \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} - \frac{c^2(16-15ax)\left(1-a^2x^2\right)^{3/2}}{120a^4} - \frac{c^2\int\sqrt{1-}}{8a^3}$$
$$= -\frac{c^2x\sqrt{1-a^2x^2}}{16a^3} - \frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2} + \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} - \frac{c^2(16-15ax)\left(1-a^2x^2\right.}{120a^4}$$
$$= -\frac{c^2x\sqrt{1-a^2x^2}}{16a^3} - \frac{c^2x^2\left(1-a^2x^2\right)^{3/2}}{5a^2} + \frac{c^2x^3\left(1-a^2x^2\right)^{3/2}}{6a} - \frac{c^2(16-15ax)\left(1-a^2x^2\right.}{120a^4}$$

Mathematica [A] time = 0.20, size = 89, normalized size = 0.72

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(40a^5x^5-48a^4x^4-10a^3x^3+16a^2x^2-15ax+32\right)-60\sin^{-1}(ax)-150\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a*c*x)^2,x]
```

```
[Out] -1/240*(c^2*(Sqrt[1 - a^2*x^2]*(32 - 15*a*x + 16*a^2*x^2 - 10*a^3*x^3 - 48*
a^4*x^4 + 40*a^5*x^5) - 60*ArcSin[a*x] - 150*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])
)/a^4
```

fricas [A] time = 0.45, size = 104, normalized size = 0.84

$$\frac{30\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(40\,a^5c^2x^5-48\,a^4c^2x^4-10\,a^3c^2x^3+16\,a^2c^2x^2-15\,ac^2x+32\,c^2\right)\sqrt{-a^2x^2+1}}{240\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/240*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (40*a^5*c^2*x^5 - 48
*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 16*a^2*c^2*x^2 - 15*a*c^2*x + 32*c^2)*sqrt(
-a^2*x^2 + 1))/a^4
```

giac [A] time = 0.18, size = 92, normalized size = 0.74

$$-\frac{1}{240}\sqrt{-a^2x^2+1}\left(\left(2\left(\left(4\left(5\,ac^2x-6\,c^2\right)x-\frac{5\,c^2}{a}\right)x+\frac{8\,c^2}{a^2}\right)x-\frac{15\,c^2}{a^3}\right)x+\frac{32\,c^2}{a^4}\right)-\frac{c^2\arcsin\left(ax\right)\mathrm{sgn}(a)}{16\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/240*sqrt(-a^2*x^2 + 1)*((2*((4*(5*a*c^2*x - 6*c^2)*x - 5*c^2/a)*x + 8*c^
2/a^2)*x - 15*c^2/a^3)*x + 32*c^2/a^4) - 1/16*c^2*arcsin(a*x)*sgn(a)/(a^3*a
bs(a))
```

maple [A] time = 0.05, size = 163, normalized size = 1.31

$$-\frac{c^2a\,x^5\sqrt{-a^2x^2+1}}{6}+\frac{c^2x^3\sqrt{-a^2x^2+1}}{24a}+\frac{c^2x\sqrt{-a^2x^2+1}}{16a^3}-\frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{16a^3\sqrt{a^2}}+\frac{c^2x^4\sqrt{-a^2x^2+1}}{5}-\frac{c^2x^2\sqrt{-a^2x^2}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^2,x)
```

```
[Out] -1/6*c^2*a*x^5*(-a^2*x^2+1)^(1/2)+1/24*c^2/a*x^3*(-a^2*x^2+1)^(1/2)+1/16*c^
2*x*(-a^2*x^2+1)^(1/2)/a^3-1/16*c^2/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-
```

```
a^2*x^2+1)^(1/2))+1/5*c^2*x^4*(-a^2*x^2+1)^(1/2)-1/15*c^2*x^2/a^2*(-a^2*x^2
+1)^(1/2)-2/15*c^2/a^4*(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.41, size = 141, normalized size = 1.14

$$-\frac{1}{6}\sqrt{-a^2x^2+1}\,ac^2x^5+\frac{1}{5}\sqrt{-a^2x^2+1}\,c^2x^4+\frac{\sqrt{-a^2x^2+1}\,c^2x^3}{24\,a}-\frac{\sqrt{-a^2x^2+1}\,c^2x^2}{15\,a^2}+\frac{\sqrt{-a^2x^2+1}\,c^2x}{16\,a^3}-\frac{c^2\arcsin(ax)}{16\,a^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/6*sqrt(-a^2*x^2 + 1)*a*c^2*x^5 + 1/5*sqrt(-a^2*x^2 + 1)*c^2*x^4 + 1/24*s
qrt(-a^2*x^2 + 1)*c^2*x^3/a - 1/15*sqrt(-a^2*x^2 + 1)*c^2*x^2/a^2 + 1/16*sq
rt(-a^2*x^2 + 1)*c^2*x/a^3 - 1/16*c^2*arcsin(a*x)/a^4 - 2/15*sqrt(-a^2*x^2
+ 1)*c^2/a^4
```

mupad [B] time = 0.79, size = 154, normalized size = 1.24

$$\frac{c^2\,x^4\,\sqrt{1-a^2\,x^2}}{5}-\frac{2\,c^2\,\sqrt{1-a^2\,x^2}}{15\,a^4}+\frac{c^2\,x\,\sqrt{1-a^2\,x^2}}{16\,a^3}-\frac{a\,c^2\,x^5\,\sqrt{1-a^2\,x^2}}{6}-\frac{c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{16\,a^3\,\sqrt{-a^2}}+\frac{c^2\,x^3\,\sqrt{1-a^2\,x^2}}{24\,a}-\frac{c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^2*x^4*(1 - a^2*x^2)^(1/2))/5 - (2*c^2*(1 - a^2*x^2)^(1/2))/(15*a^4) + (c
^2*x*(1 - a^2*x^2)^(1/2))/(16*a^3) - (a*c^2*x^5*(1 - a^2*x^2)^(1/2))/6 - (c
^2*asinh(x*(-a^2)^(1/2)))/(16*a^3*(-a^2)^(1/2)) + (c^2*x^3*(1 - a^2*x^2)^(1
/2))/(24*a) - (c^2*x^2*(1 - a^2*x^2)^(1/2))/(15*a^2)
```

sympy [A] time = 10.36, size = 486, normalized size = 3.92

$$a^3c^2\left(\begin{cases}-\frac{ix^7}{6\sqrt{a^2x^2-1}}-\frac{ix^5}{24a^2\sqrt{a^2x^2-1}}-\frac{5ix^3}{48a^4\sqrt{a^2x^2-1}}+\frac{5ix}{16a^6\sqrt{a^2x^2-1}}-\frac{5i\,\mathrm{acosh}(ax)}{16a^7} & \text{for } |a^2x^2|>1\\ \frac{x^7}{6\sqrt{-a^2x^2+1}}+\frac{x^5}{24a^2\sqrt{-a^2x^2+1}}+\frac{5x^3}{48a^4\sqrt{-a^2x^2+1}}-\frac{5x}{16a^6\sqrt{-a^2x^2+1}}+\frac{5\,\mathrm{asin}(ax)}{16a^7} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}\\ \frac{x^6}{6}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a*c*x+c)**2,x)
```

```
[Out] a**3*c**2*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt
(a**2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*
```

```
sqrt(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7
/(6*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(4
8*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(
a*x)/(16*a**7), True)) - a**2*c**2*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5
*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15
*a**6), Ne(a, 0)), (x**6/6, True)) - a*c**2*Piecewise((-I*x**5/(4*sqrt(a**2
*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2
*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-
a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a*
*2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), True)) + c**2*Piecewise((-x**2*sqrt(-
a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4
/4, True))
```

3.296 $\int e^{\tanh^{-1}(ax)}x^2(c-acx)^2\,dx$

Optimal. Leaf size=113

$$\frac{c^2\sin^{-1}(ax)}{8a^3}-\frac{c^2x\left(1-a^2x^2\right)^{3/2}}{4a^2}+\frac{c^2x\sqrt{1-a^2x^2}}{8a^2}-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3}$$

```
[Out] 1/3*c^2*(-a^2*x^2+1)^(3/2)/a^3-1/4*c^2*x*(-a^2*x^2+1)^(3/2)/a^2-1/5*c^2*(-a
^2*x^2+1)^(5/2)/a^3+1/8*c^2*arcsin(a*x)/a^3+1/8*c^2*x*(-a^2*x^2+1)^(1/2)/a^
2
```

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 797, 641, 195, 216}

$$-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3}-\frac{c^2x\left(1-a^2x^2\right)^{3/2}}{4a^2}+\frac{c^2x\sqrt{1-a^2x^2}}{8a^2}+\frac{c^2\sin^{-1}(ax)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a*c*x)^2,x]
```

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/(8*a^2) + (c^2*(1 - a^2*x^2)^(3/2))/(3*a^3) - (c^
2*x*(1 - a^2*x^2)^(3/2))/(4*a^2) - (c^2*(1 - a^2*x^2)^(5/2))/(5*a^3) + (c^2
*ArcSin[a*x])/(8*a^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^2(c-acx)^2\,dx = c\int x^2(c-acx)\sqrt{1-a^2x^2}\,dx$$
$$= \frac{c\int(c-acx)\sqrt{1-a^2x^2}\,dx}{a^2} - \frac{c\int(c-acx)\left(1-a^2x^2\right)^{3/2}\,dx}{a^2}$$
$$= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3} - \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3} + \frac{c^2\int\sqrt{1-a^2x^2}\,dx}{a^2} - \frac{c^2\int\left(1-a^2x^2\right)^{3/2}\,dx}{a^2}$$
$$= \frac{c^2x\sqrt{1-a^2x^2}}{2a^2} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3} - \frac{c^2x\left(1-a^2x^2\right)^{3/2}}{4a^2} - \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3} + \frac{c^2\int\frac{}{\sqrt{1-}}}{2a}$$
$$= \frac{c^2x\sqrt{1-a^2x^2}}{8a^2} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3} - \frac{c^2x\left(1-a^2x^2\right)^{3/2}}{4a^2} - \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3} + \frac{c^2\sin^{-1}}{2a^3}$$
$$= \frac{c^2x\sqrt{1-a^2x^2}}{8a^2} + \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^3} - \frac{c^2x\left(1-a^2x^2\right)^{3/2}}{4a^2} - \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a^3} + \frac{c^2\sin^{-1}}{8a^3}$$

Mathematica [A] time = 0.08, size = 75, normalized size = 0.66

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(24a^4x^4-30a^3x^3-8a^2x^2+15ax-16\right)+30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a*c*x)^2,x]
```

```
[Out] -1/120*(c^2*(Sqrt[1 - a^2*x^2]*(-16 + 15*a*x - 8*a^2*x^2 - 30*a^3*x^3 + 24*
a^4*x^4) + 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/a^3
```

fricas [A] time = 0.44, size = 92, normalized size = 0.81

$$-\frac{30\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(24\,a^4c^2x^4-30\,a^3c^2x^3-8\,a^2c^2x^2+15\,ac^2x-16\,c^2\right)\sqrt{-a^2x^2+1}}{120\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/120*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c^2*x^4 - 3
0*a^3*c^2*x^3 - 8*a^2*c^2*x^2 + 15*a*c^2*x - 16*c^2)*sqrt(-a^2*x^2 + 1))/a^
3
```

giac [A] time = 0.25, size = 81, normalized size = 0.72

$$-\frac{1}{120}\sqrt{-a^2x^2+1}\left(\left(2\left(3\left(4\,ac^2x-5\,c^2\right)x-\frac{4\,c^2}{a}\right)x+\frac{15\,c^2}{a^2}\right)x-\frac{16\,c^2}{a^3}\right)+\frac{c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{8\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/120*sqrt(-a^2*x^2 + 1)*((2*(3*(4*a*c^2*x - 5*c^2)*x - 4*c^2/a)*x + 15*c^
2/a^2)*x - 16*c^2/a^3) + 1/8*c^2*arcsin(a*x)*sgn(a)/(a^2*abs(a))
```

maple [A] time = 0.04, size = 140, normalized size = 1.24

$$-\frac{c^2a\,x^4\sqrt{-a^2x^2+1}}{5}+\frac{c^2x^2\sqrt{-a^2x^2+1}}{15a}+\frac{2c^2\sqrt{-a^2x^2+1}}{15a^3}+\frac{c^2x^3\sqrt{-a^2x^2+1}}{4}-\frac{c^2x\sqrt{-a^2x^2+1}}{8a^2}+\frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a^2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^2,x)
```

```
[Out] -1/5*c^2*a*x^4*(-a^2*x^2+1)^(1/2)+1/15*c^2/a*x^2*(-a^2*x^2+1)^(1/2)+2/15*c^
2/a^3*(-a^2*x^2+1)^(1/2)+1/4*c^2*x^3*(-a^2*x^2+1)^(1/2)-1/8*c^2*x*(-a^2*x^2
+1)^(1/2)/a^2+1/8*c^2/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/
2))
```

maxima [A] time = 0.52, size = 118, normalized size = 1.04

$$-\frac{1}{5}\sqrt{-a^2x^2+1}\,ac^2x^4+\frac{1}{4}\sqrt{-a^2x^2+1}\,c^2x^3+\frac{\sqrt{-a^2x^2+1}\,c^2x^2}{15\,a}-\frac{\sqrt{-a^2x^2+1}\,c^2x}{8\,a^2}+\frac{c^2\arcsin\left(ax\right)}{8\,a^3}+\frac{2\,\sqrt{-a^2x^2+1}\,c^2}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/5*sqrt(-a^2*x^2 + 1)*a*c^2*x^4 + 1/4*sqrt(-a^2*x^2 + 1)*c^2*x^3 + 1/15*s
qrt(-a^2*x^2 + 1)*c^2*x^2/a - 1/8*sqrt(-a^2*x^2 + 1)*c^2*x/a^2 + 1/8*c^2*ar
csin(a*x)/a^3 + 2/15*sqrt(-a^2*x^2 + 1)*c^2/a^3
```

mupad [B] time = 0.03, size = 131, normalized size = 1.16

$$\frac{2\,c^2\,\sqrt{1-a^2\,x^2}}{15\,a^3}+\frac{c^2\,x^3\,\sqrt{1-a^2\,x^2}}{4}-\frac{c^2\,x\,\sqrt{1-a^2\,x^2}}{8\,a^2}-\frac{a\,c^2\,x^4\,\sqrt{1-a^2\,x^2}}{5}+\frac{c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a^2\,\sqrt{-a^2}}+\frac{c^2\,x^2\,\sqrt{1-a^2\,x^2}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (2*c^2*(1 - a^2*x^2)^(1/2))/(15*a^3) + (c^2*x^3*(1 - a^2*x^2)^(1/2))/4 - (c
^2*x*(1 - a^2*x^2)^(1/2))/(8*a^2) - (a*c^2*x^4*(1 - a^2*x^2)^(1/2))/5 + (c^
2*asinh(x*(-a^2)^(1/2)))/(8*a^2*(-a^2)^(1/2)) + (c^2*x^2*(1 - a^2*x^2)^(1/2
))/(15*a)
```

sympy [C] time = 7.52, size = 374, normalized size = 3.31

$$a^3c^2\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^4}-\frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for } a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\,a\cdots}{} \\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c)**2,x)
```

```
[Out] a**3*c**2*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a**
2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/
6, True)) - a**2*c**2*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(
8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acos
h(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3
/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin
(a*x)/(8*a**5), True)) - a*c**2*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a*
*2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + c**2*Pi
ecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**
2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 +
1)) + asin(a*x)/(2*a**3), True))
```

3.297 $\int e^{\tanh^{-1}(ax)} x(c - acx)^2 \, dx$

Optimal. Leaf size=70

$$-\frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2}-\frac{c^2x\sqrt{1-a^2x^2}}{8a}-\frac{c^2\sin^{-1}(ax)}{8a^2}$$

```
[Out] -1/12*c^2*(-3*a*x+4)*(-a^2*x^2+1)^(3/2)/a^2-1/8*c^2*arcsin(a*x)/a^2-1/8*c^2
*x*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 780, 195, 216}

$$-\frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2}-\frac{c^2x\sqrt{1-a^2x^2}}{8a}-\frac{c^2\sin^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*x*Sqrt[1 - a^2*x^2])/(8*a) - (c^2*(4 - 3*a*x)*(1 - a^2*x^2)^(3/2))/(1
2*a^2) - (c^2*ArcSin[a*x])/(8*a^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x(c-acx)^2\,dx &= c\int x(c-acx)\sqrt{1-a^2x^2}\,dx\\
&= -\frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2}-\frac{c^2\int\sqrt{1-a^2x^2}\,dx}{4a}\\
&= -\frac{c^2x\sqrt{1-a^2x^2}}{8a}-\frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2}-\frac{c^2\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a}\\
&= -\frac{c^2x\sqrt{1-a^2x^2}}{8a}-\frac{c^2(4-3ax)\left(1-a^2x^2\right)^{3/2}}{12a^2}-\frac{c^2\sin^{-1}(ax)}{8a^2}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 0.96

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(6a^3x^3-8a^2x^2-3ax+8\right)-6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a*c*x)^2,x]
```

```
[Out] -1/24*(c^2*(Sqrt[1 - a^2*x^2]*(8 - 3*a*x - 8*a^2*x^2 + 6*a^3*x^3) - 6*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/a^2
```

fricas [A] time = 0.42, size = 82, normalized size = 1.17

$$\frac{6\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(6\,a^3c^2x^3-8\,a^2c^2x^2-3\,ac^2x+8\,c^2\right)\sqrt{-a^2x^2+1}}{24\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(6*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (6*a^3*c^2*x^3 - 8*a^2
*c^2*x^2 - 3*a*c^2*x + 8*c^2)*sqrt(-a^2*x^2 + 1))/a^2
```

giac [A] time = 0.30, size = 69, normalized size = 0.99

$$-\frac{c^2 \arcsin(ax)\,\mathrm{sgn}(a)}{8\,a|a|} - \frac{1}{24}\,\sqrt{-a^2x^2+1}\left(\left(2\left(3\,ac^2x - 4\,c^2\right)x - \frac{3\,c^2}{a}\right)x + \frac{8\,c^2}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*c^2*arcsin(a*x)*sgn(a)/(a*abs(a)) - 1/24*sqrt(-a^2*x^2 + 1)*((2*(3*a*c
^2*x - 4*c^2)*x - 3*c^2/a)*x + 8*c^2/a^2)
```

maple [A] time = 0.04, size = 117, normalized size = 1.67

$$-\frac{c^2a\,x^3\sqrt{-a^2x^2+1}}{4}+\frac{c^2x\sqrt{-a^2x^2+1}}{8a}-\frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a\sqrt{a^2}}+\frac{c^2x^2\sqrt{-a^2x^2+1}}{3}-\frac{c^2\sqrt{-a^2x^2+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^2,x)
```

```
[Out] -1/4*c^2*a*x^3*(-a^2*x^2+1)^(1/2)+1/8*c^2*x*(-a^2*x^2+1)^(1/2)/a-1/8*c^2/a/
(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/3*c^2*x^2*(-a^2*x^2+
1)^(1/2)-1/3*c^2/a^2*(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.50, size = 95, normalized size = 1.36

$$-\frac{1}{4}\,\sqrt{-a^2x^2+1}\,ac^2x^3+\frac{1}{3}\,\sqrt{-a^2x^2+1}\,c^2x^2+\frac{\sqrt{-a^2x^2+1}\,c^2x}{8\,a}-\frac{c^2\arcsin(ax)}{8\,a^2}-\frac{\sqrt{-a^2x^2+1}\,c^2}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + 1)*a*c^2*x^3 + 1/3*sqrt(-a^2*x^2 + 1)*c^2*x^2 + 1/8*sq
rt(-a^2*x^2 + 1)*c^2*x/a - 1/8*c^2*arcsin(a*x)/a^2 - 1/3*sqrt(-a^2*x^2 + 1)
*c^2/a^2
```

mupad [B] time = 0.03, size = 108, normalized size = 1.54

$$\frac{c^2\,x^2\,\sqrt{1-a^2\,x^2}}{3}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{3\,a^2}+\frac{c^2\,x\,\sqrt{1-a^2\,x^2}}{8\,a}-\frac{a\,c^2\,x^3\,\sqrt{1-a^2\,x^2}}{4}-\frac{c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^2*x^2*(1 - a^2*x^2)^(1/2))/3 - (c^2*(1 - a^2*x^2)^(1/2))/(3*a^2) + (c^2*
x*(1 - a^2*x^2)^(1/2))/(8*a) - (a*c^2*x^3*(1 - a^2*x^2)^(1/2))/4 - (c^2*asi
nh(x*(-a^2)^(1/2)))/(8*a*(-a^2)^(1/2))
```

sympy [A] time = 7.17, size = 330, normalized size = 4.71

$$a^3c^2\left(\begin{cases} -\frac{ix^5}{4\sqrt{a^2x^2-1}} - \frac{ix^3}{8a^2\sqrt{a^2x^2-1}} + \frac{3ix}{8a^4\sqrt{a^2x^2-1}} - \frac{3i\operatorname{acosh}(ax)}{8a^5} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{x^5}{4\sqrt{-a^2x^2+1}} + \frac{x^3}{8a^2\sqrt{-a^2x^2+1}} - \frac{3x}{8a^4\sqrt{-a^2x^2+1}} + \frac{3\operatorname{asin}(ax)}{8a^5} & \text{otherwise} \end{cases}\right) - a^2c^2\left(\begin{cases} -\frac{x^2\sqrt{-a^2x^2+1}}{3a^2} - \frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{fo} \\ \frac{x^4}{4} & \text{o} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c)**2,x)
```

```
[Out] a**3*c**2*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(
a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a*
*5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqr
t(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**
5), True)) - a**2*c**2*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*s
qrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - a*c**2*Piecewise
((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2)
 > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + a
sin(a*x)/(2*a**3), True)) + c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a
**2*x**2 + 1)/a**2, True))
```

3.298 $\int e^{\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=61

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

```
[Out] 1/3*c^2*(-a^2*x^2+1)^(3/2)/a+1/2*c^2*arcsin(a*x)/a+1/2*c^2*x*(-a^2*x^2+1)^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6127, 641, 195, 216}

$$\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^2,x]
```

```
[Out] (c^2*x*Sqrt[1 - a^2*x^2])/2 + (c^2*(1 - a^2*x^2)^(3/2))/(3*a) + (c^2*ArcSin
[a*x])/(2*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}(c-acx)^2\,dx &= c\int (c-acx)\sqrt{1-a^2x^2}\,dx\\ &= \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+c^2\int \sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{1}{2}c^2x\sqrt{1-a^2x^2}+\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{c^2\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.97

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(2a^2x^2-3ax-2\right)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^2,x]
```

```
[Out] -1/6*(c^2*(Sqrt[1 - a^2*x^2]*(-2 - 3*a*x + 2*a^2*x^2) + 6*ArcSin[Sqrt[1 - a
*x]/Sqrt[2]]))/a
```

fricas [A] time = 0.49, size = 70, normalized size = 1.15

$$-\frac{6\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^2c^2x^2-3\,ac^2x-2\,c^2\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^2*c^2*x^2 - 3*a*c
^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.20, size = 54, normalized size = 0.89

$$\frac{c^2 \arcsin(ax)\,\mathrm{sgn}(a)}{2\,|a|} - \frac{1}{6}\sqrt{-a^2x^2+1}\left(\left(2\,ac^2x - 3\,c^2\right)x - \frac{2\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c^2*x - 3*
c^2)*x - 2*c^2/a)
```

maple [A] time = 0.03, size = 91, normalized size = 1.49

$$-\frac{c^2a\,x^2\sqrt{-a^2x^2+1}}{3} + \frac{c^2\sqrt{-a^2x^2+1}}{3a} + \frac{c^2x\sqrt{-a^2x^2+1}}{2} + \frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x)
```

```
[Out] -1/3*c^2*a*x^2*(-a^2*x^2+1)^(1/2)+1/3*c^2*(-a^2*x^2+1)^(1/2)/a+1/2*c^2*x*(-
a^2*x^2+1)^(1/2)+1/2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2
))
```

maxima [A] time = 0.43, size = 72, normalized size = 1.18

$$-\frac{1}{3}\sqrt{-a^2x^2+1}\,ac^2x^2 + \frac{1}{2}\sqrt{-a^2x^2+1}\,c^2x + \frac{c^2\arcsin(ax)}{2\,a} + \frac{\sqrt{-a^2x^2+1}\,c^2}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + 1)*a*c^2*x^2 + 1/2*sqrt(-a^2*x^2 + 1)*c^2*x + 1/2*c^2*
arcsin(a*x)/a + 1/3*sqrt(-a^2*x^2 + 1)*c^2/a
```

mupad [B] time = 0.00, size = 82, normalized size = 1.34

$$\frac{c^2\,x\,\sqrt{1-a^2\,x^2}}{2} + \frac{c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}} + \frac{c^2\,\sqrt{1-a^2\,x^2}}{3\,a} - \frac{a\,c^2\,x^2\,\sqrt{1-a^2\,x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^2*x*(1 - a^2*x^2)^(1/2))/2 + (c^2*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2)
) + (c^2*(1 - a^2*x^2)^(1/2))/(3*a) - (a*c^2*x^2*(1 - a^2*x^2)^(1/2))/3
```

sympy [A] time = 5.30, size = 102, normalized size = 1.67

$$\begin{cases} \frac{c^2\sqrt{-a^2x^2+1} - c^2\left(\begin{cases} -\frac{ax\sqrt{-a^2x^2+1}}{2} + \frac{\operatorname{asin}(ax)}{2} & \text{for } ax > -1 \wedge ax < 1 \end{cases}\right) + c^2\left(\begin{cases} \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3} - \sqrt{-a^2x^2+1} & \text{for } ax > -1 \wedge ax \end{cases}\right.}{a} \\ c^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2,x)
```

```
[Out] Piecewise(((c**2*sqrt(-a**2*x**2 + 1) - c**2*Piecewise((-a*x*sqrt(-a**2*x**
2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + c**2*Piecewise(((-a**2*x
**2 + 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) + c**2*a
sin(a*x))/a, Ne(a, 0)), (c**2*x, True))
```

3.299 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x} dx$

Optimal. Leaf size=59

$$\frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2}-c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{1}{2}c^2\sin^{-1}(ax)$$

```
[Out] -1/2*c^2*arcsin(a*x)-c^2*arctanh((-a^2*x^2+1)^(1/2))+1/2*c^2*(-a*x+2)*(-a^2
*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 815, 844, 216, 266, 63, 208}

$$\frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2}-c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{1}{2}c^2\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x,x]
```

```
[Out] (c^2*(2 - a*x)*Sqrt[1 - a^2*x^2])/2 - (c^2*ArcSin[a*x])/2 - c^2*ArcTanh[Sqr
t[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x}\,dx = c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x}\,dx$$
$$= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2} - \frac{c\int \frac{-2a^2c+a^3cx}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}$$
$$= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2} + c^2\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \frac{1}{2}\left(ac^2\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2} - \frac{1}{2}c^2\sin^{-1}(ax) + \frac{1}{2}c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2} - \frac{1}{2}c^2\sin^{-1}(ax) - \frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2}$$
$$= \frac{1}{2}c^2(2-ax)\sqrt{1-a^2x^2} - \frac{1}{2}c^2\sin^{-1}(ax) - c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [B] time = 0.08, size = 125, normalized size = 2.12

$$\frac{c^2\left(a^3x^3-2a^2x^2+\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+4\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-2\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-ax+2\right)}{2\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x,x]
```

```
[Out] (c^2*(2 - a*x - 2*a^2*x^2 + a^3*x^3 + Sqrt[1 - a^2*x^2]*ArcSin[a*x] + 4*Sqr
t[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] - 2*Sqrt[1 - a^2*x^2]*ArcTanh[
Sqrt[1 - a^2*x^2]]))/(2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.46, size = 76, normalized size = 1.29

$$c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+c^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\frac{1}{2}\left(ac^2x-2\,c^2\right)\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x,x, algorithm="fricas")
```

```
[Out] c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c^2*log((sqrt(-a^2*x^2 + 1) -
1)/x) - 1/2*(a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1)
```

giac [A] time = 0.72, size = 84, normalized size = 1.42

$$-\frac{ac^2 \arcsin(ax)\,\mathrm{sgn}(a)}{2\,|a|} - \frac{ac^2 \log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|} - \frac{1}{2}\left(ac^2x - 2\,c^2\right)\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x,x, algorithm="giac")
```

```
[Out] -1/2*a*c^2*arcsin(a*x)*sgn(a)/abs(a) - a*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 +
 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/2*(a*c^2*x - 2*c^2)*sqrt(-a^2*x^
2 + 1)
```

maple [A] time = 0.04, size = 86, normalized size = 1.46

$$-\frac{c^2ax\sqrt{-a^2x^2+1}}{2} - \frac{c^2a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}} + c^2\sqrt{-a^2x^2+1} - c^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x,x)
```

```
[Out] -1/2*c^2*a*x*(-a^2*x^2+1)^(1/2)-1/2*c^2*a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/
(-a^2*x^2+1)^(1/2))+c^2*(-a^2*x^2+1)^(1/2)-c^2*arctanh(1/(-a^2*x^2+1)^(1/2)
)
```

maxima [A] time = 0.41, size = 76, normalized size = 1.29

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\,ac^2x - \frac{1}{2}c^2\arcsin(ax) - c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \sqrt{-a^2x^2+1}\,c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*a*c^2*x - 1/2*c^2*arcsin(a*x) - c^2*log(2*sqrt(-a^2
*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)*c^2
```

mupad [B] time = 0.04, size = 77, normalized size = 1.31

$$c^2\sqrt{1-a^2\,x^2} - c^2\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right) - \frac{a\,c^2\,x\,\sqrt{1-a^2\,x^2}}{2} - \frac{a\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] c^2*(1 - a^2*x^2)^(1/2) - c^2*atanh((1 - a^2*x^2)^(1/2)) - (a*c^2*x*(1 - a^
2*x^2)^(1/2))/2 - (a*c^2*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2))
```

sympy [C] time = 11.93, size = 201, normalized size = 3.41

$$a^3c^2\left(\begin{cases} -\frac{ix\sqrt{a^2x^2-1}}{2a^2} - \frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } |a^2x^2| > 1 \\ \frac{x^3}{2\sqrt{-a^2x^2+1}} - \frac{x}{2a^2\sqrt{-a^2x^2+1}} + \frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise} \end{cases}\right) - a^2c^2\left(\begin{cases} \frac{x^2}{2} & \text{for } a^2 = 0 \\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise} \end{cases}\right) - ac^2\left(\begin{cases} \sqrt{\frac{1}{a^2}}\operatorname{asin}(x \\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x,x)
```

```
[Out] a**3*c**2*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a*
*3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-
a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) - a**2*c**2*Piecewise((x**2/2,
 Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) - a*c**2*Piecewise((sqrt
(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)
), a**2 < 0)) + c**2*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*
asin(1/(a*x)), True))
```

3.300 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{c^2(ax+1)\sqrt{1-a^2x^2}}{x}+ac^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-ac^2\sin^{-1}(ax)$$

```
[Out] -a*c^2*arcsin(a*x)+a*c^2*arctanh((-a^2*x^2+1)^(1/2))-c^2*(a*x+1)*(-a^2*x^2+
1)^(1/2)/x
```

Rubi [A] time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(ax+1)\sqrt{1-a^2x^2}}{x}+ac^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-ac^2\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^2,x]
```

```
[Out] -((c^2*(1 + a*x)*Sqrt[1 - a^2*x^2])/x) - a*c^2*ArcSin[a*x] + a*c^2*ArcTanh[
Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^2}\,dx = c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^2}\,dx$$
$$= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x} - \frac{1}{2}c\int \frac{2ac+2a^2cx}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x} - \left(ac^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(a^2c^2\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x} - ac^2\sin^{-1}(ax) - \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)$$
$$= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x} - ac^2\sin^{-1}(ax) + \frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a}$$
$$= -\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{x} - ac^2\sin^{-1}(ax) + ac^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.19, size = 84, normalized size = 1.45

$$\frac{1}{2}c^2\left(\frac{2(ax-1)(ax+1)^2}{x\sqrt{1-a^2x^2}} + 2a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - a\sin^{-1}(ax) + 2a\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^2,x]
```

```
[Out] (c^2*((2*(-1 + a*x)*(1 + a*x)^2)/(x*Sqrt[1 - a^2*x^2]) - a*ArcSin[a*x] + 2*
a*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] + 2*a*ArcTanh[Sqrt[1 - a^2*x^2]]))/2
```

fricas [A] time = 0.52, size = 91, normalized size = 1.57

$$\frac{2\,ac^2x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - ac^2x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - ac^2x - \left(ac^2x+c^2\right)\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^2,x, algorithm="fricas"
)
```

```
[Out] (2*a*c^2*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a*c^2*x*log((sqrt(-a^2*
x^2 + 1) - 1)/x) - a*c^2*x - (a*c^2*x + c^2)*sqrt(-a^2*x^2 + 1))/x
```

giac [B] time = 0.23, size = 140, normalized size = 2.41

$$\frac{a^4c^2x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{a^2c^2\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}+\frac{a^2c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{|a|}-\sqrt{-a^2x^2+1}\,ac^2-\frac{\left(\sqrt{-a^2x^2+1}\,|a\right.}{2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*c^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*c^2*arcsin(a*x
)*sgn(a)/abs(a) + a^2*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(
a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*a*c^2 - 1/2*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)*c^2/(x*abs(a))
```

maple [A] time = 0.04, size = 91, normalized size = 1.57

$$-c^2a\sqrt{-a^2x^2+1}-\frac{c^2a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+c^2a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{c^2\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^2,x)
```

```
[Out] -c^2*a*(-a^2*x^2+1)^(1/2)-c^2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))+c^2*a*arctanh(1/(-a^2*x^2+1)^(1/2))-c^2/x*(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.40, size = 80, normalized size = 1.38

$$-ac^2\arcsin(ax)+ac^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}\,ac^2-\frac{\sqrt{-a^2x^2+1}\,c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^2,x, algorithm="maxima"
)
```

```
[Out] -a*c^2*arcsin(a*x) + a*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sq
rt(-a^2*x^2 + 1)*a*c^2 - sqrt(-a^2*x^2 + 1)*c^2/x
```

mupad [B] time = 0.04, size = 87, normalized size = 1.50

$$-a\,c^2\sqrt{1-a^2\,x^2}-\frac{c^2\sqrt{1-a^2\,x^2}}{x}-\frac{a^2\,c^2\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}-a\,c^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] - a*c^2*(1 - a^2*x^2)^(1/2) - (c^2*(1 - a^2*x^2)^(1/2))/x - a*c^2*atan((1 -
 a^2*x^2)^(1/2)*1i)*1i - (a^2*c^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [C] time = 4.60, size = 153, normalized size = 2.64

$$a^3c^2\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)-ac^2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**2,x)
```

```
[Out] a**3*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, Tru
e)) - a**2*c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sq
rt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - a*c**2*Piecewise((-acosh(1/(
a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + c**2*Piecewise((-I
*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True
))
```

3.301 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^3} dx$

Optimal. Leaf size=67

$$-\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{2}a^2c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^2c^2\sin^{-1}(ax)$$

```
[Out] a^2*c^2*arcsin(a*x)+1/2*a^2*c^2*arctanh((-a^2*x^2+1)^(1/2))-1/2*c^2*(-2*a*x
+1)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 811, 844, 216, 266, 63, 208}

$$-\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2}+\frac{1}{2}a^2c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^2c^2\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^3,x]
```

```
[Out] -(c^2*(1 - 2*a*x)*Sqrt[1 - a^2*x^2])/(2*x^2) + a^2*c^2*ArcSin[a*x] + (a^2*c
^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^3}\,dx = c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^3}\,dx$$
$$= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2} - \frac{1}{4}c\int \frac{2a^2c-4a^3cx}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2} - \frac{1}{2}\left(a^2c^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx + \left(a^3c^2\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2} + a^2c^2\sin^{-1}(ax) - \frac{1}{4}\left(a^2c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)$$
$$= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2} + a^2c^2\sin^{-1}(ax) + \frac{1}{2}c^2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)$$
$$= -\frac{c^2(1-2ax)\sqrt{1-a^2x^2}}{2x^2} + a^2c^2\sin^{-1}(ax) + \frac{1}{2}a^2c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [B] time = 0.11, size = 147, normalized size = 2.19

$$-\frac{c^2\left(4a^3x^3-2a^2x^2+a^2x^2\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+10a^2x^2\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-2a^2x^2\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1\right.\right.}{4x^2\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^3,x]
```

```
[Out] -1/4*(c^2*(2 - 4*a*x - 2*a^2*x^2 + 4*a^3*x^3 + a^2*x^2*Sqrt[1 - a^2*x^2]*Ar
cSin[a*x] + 10*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] - 2*
a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(x^2*Sqrt[1 - a^2*x^
2])
```

fricas [A] time = 1.01, size = 95, normalized size = 1.42

$$-\frac{4\,a^2c^2x^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+a^2c^2x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(2\,ac^2x-c^2\right)\sqrt{-a^2x^2+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^3,x, algorithm="fricas"
)
```

```
[Out] -1/2*(4*a^2*c^2*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a^2*c^2*x^2*lo
g((sqrt(-a^2*x^2 + 1) - 1)/x) - (2*a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1))/x^2
```

giac [B] time = 0.22, size = 192, normalized size = 2.87

$$\frac{a^3c^2\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}+\frac{a^3c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{2|a|}+\frac{\left(a^3c^2-\frac{4\left(\sqrt{-a^2x^2+1}\,|a|+a\right)ac^2}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}+\frac{\frac{4\left(\sqrt{-a^2x^2+1}\,|a|+a\right)ac^2|a|}{x}-\cdots}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^3,x, algorithm="giac")
```

```
[Out] a^3*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/2*a^3*c^2*log(1/2*abs(-2*sqrt(-a^2*x^
2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/8*(a^3*c^2 - 4*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)*a*c^2/x)*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a
)) + 1/8*(4*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*c^2*abs(a)/x - (sqrt(-a^2*x^2
 + 1)*abs(a) + a)^2*c^2*abs(a)/(a*x^2))/a^2
```

maple [A] time = 0.04, size = 95, normalized size = 1.42

$$\frac{c^2a^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{c^2a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{c^2a\sqrt{-a^2x^2+1}}{x}-\frac{c^2\sqrt{-a^2x^2+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^3,x)
```

```
[Out] c^2*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/2*c^2*a^2*ar
ctanh(1/(-a^2*x^2+1)^(1/2))+c^2*a/x*(-a^2*x^2+1)^(1/2)-1/2*c^2/x^2*(-a^2*x^
2+1)^(1/2)
```

maxima [A] time = 0.41, size = 86, normalized size = 1.28

$$a^2c^2\arcsin(ax)+\frac{1}{2}a^2c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-a^2x^2+1}\,ac^2}{x}-\frac{\sqrt{-a^2x^2+1}\,c^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^3,x, algorithm="maxima"
)
```

```
[Out] a^2*c^2*arcsin(a*x) + 1/2*a^2*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x
)) + sqrt(-a^2*x^2 + 1)*a*c^2/x - 1/2*sqrt(-a^2*x^2 + 1)*c^2/x^2
```

mupad [B] time = 0.05, size = 90, normalized size = 1.34

$$\frac{a\,c^2\,\sqrt{1-a^2\,x^2}}{x}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{2\,x^2}+\frac{a^3\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{a^2\,c^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a*c^2*(1 - a^2*x^2)^(1/2))/x - (c^2*(1 - a^2*x^2)^(1/2))/(2*x^2) - (a^2*c^
2*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/2 + (a^3*c^2*asinh(x*(-a^2)^(1/2)))/(-a^
2)^(1/2)
```

sympy [C] time = 5.71, size = 226, normalized size = 3.37

$$a^3c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\ \mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)-a^2c^2\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)-ac^2\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } \left|a^2x^2\right|\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**3,x)
```

```
[Out] a**3*c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/
a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - a**2*c**2*Piecewise((-acosh(1/(a*x
)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) - a*c**2*Piecewise((-I*
sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)
) + c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*
x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a
**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

3.302 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^4} dx$

Optimal. Leaf size=75

$$\frac{ac^2\sqrt{1-a^2x^2}}{2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{2}a^3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/3*c^2*(-a^2*x^2+1)^(3/2)/x^3-1/2*a^3*c^2*arctanh((-a^2*x^2+1)^(1/2))+1/2
*a*c^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 807, 266, 47, 63, 208}

$$\frac{ac^2\sqrt{1-a^2x^2}}{2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{2}a^3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^4,x]
```

```
[Out] (a*c^2*Sqrt[1 - a^2*x^2])/(2*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(3*x^3) - (a^
3*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^4}\,dx &= c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^4}\,dx \\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \left(ac^2\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx \\
&= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx, x, x^2\right) \\
&= \frac{ac^2\sqrt{1-a^2x^2}}{2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{4}\left(a^3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{ac^2\sqrt{1-a^2x^2}}{2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&= \frac{ac^2\sqrt{1-a^2x^2}}{2x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{2}a^3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.21

$$
-\frac{c^2\left(2a^4x^4+3a^3x^3-4a^2x^2+3a^3x^3\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-3ax+2\right)}{6x^3\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^4,x]
```

```
[Out] -1/6*(c^2*(2 - 3*a*x - 4*a^2*x^2 + 3*a^3*x^3 + 2*a^4*x^4 + 3*a^3*x^3*Sqrt[1
 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.48, size = 73, normalized size = 0.97

$$
\frac{3\,a^3c^2x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(2\,a^2c^2x^2+3\,ac^2x-2\,c^2\right)\sqrt{-a^2x^2+1}}{6\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/6*(3*a^3*c^2*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (2*a^2*c^2*x^2 + 3*a*c
^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/x^3
```

giac [B] time = 0.54, size = 233, normalized size = 3.11

$$\frac{\left(a^4c^2 - \frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2c^2}{x} - \frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^2}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|} - \frac{a^4c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{2|a|} + \frac{\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^4c^2}{x} + \frac{3\left(\sqrt{-a^2\right.}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*c^2 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2*c^2/x - 3*(sqrt(-a^2*
x^2 + 1)*abs(a) + a)^2*c^2/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*
abs(a)) - 1/2*a^4*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*
abs(x)))/abs(a) + 1/24*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^2/x + 3*(sq
rt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^2/x^2 - (sqrt(-a^2*x^2 + 1)*abs(a) + a
)^3*c^2/x^3)/(a^2*abs(a))
```

maple [A] time = 0.04, size = 100, normalized size = 1.33

$$c^2\left(-a^3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + \frac{a^2\sqrt{-a^2x^2+1}}{3x} - \frac{\sqrt{-a^2x^2+1}}{3x^3} - a\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2} - \frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^4,x)
```

```
[Out] c^2*(-a^3*arctanh(1/(-a^2*x^2+1)^(1/2))+1/3*a^2*(-a^2*x^2+1)^(1/2)/x-1/3*(-
a^2*x^2+1)^(1/2)/x^3-a*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2
*x^2+1)^(1/2))))
```

maxima [A] time = 0.40, size = 99, normalized size = 1.32

$$-\frac{1}{2}a^3c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-a^2x^2+1}\,a^2c^2}{3\,x} + \frac{\sqrt{-a^2x^2+1}\,ac^2}{2\,x^2} - \frac{\sqrt{-a^2x^2+1}\,c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^4,x, algorithm="maxima"
)
```

```
[Out] -1/2*a^3*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 1/3*sqrt(-a^2*x^
2 + 1)*a^2*c^2/x + 1/2*sqrt(-a^2*x^2 + 1)*a*c^2/x^2 - 1/3*sqrt(-a^2*x^2 + 1
)*c^2/x^3
```

mupad [B] time = 0.80, size = 90, normalized size = 1.20

$$\frac{a\,c^2\,\sqrt{1-a^2\,x^2}}{2\,x^2}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{3\,x^3}+\frac{a^2\,c^2\,\sqrt{1-a^2\,x^2}}{3\,x}+\frac{a^3\,c^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(x^4*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^3*c^2*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/2 - (c^2*(1 - a^2*x^2)^(1/2))/(3*
x^3) + (a*c^2*(1 - a^2*x^2)^(1/2))/(2*x^2) + (a^2*c^2*(1 - a^2*x^2)^(1/2))/
(3*x)
```

sympy [C] time = 6.26, size = 270, normalized size = 3.60

$$a^3c^2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ |a^2x^2|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)-ac^2\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} \\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\ }{2ax^3}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**4,x)
```

```
[Out] a**3*c**2*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x
)), True)) - a**2*c**2*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2)
> 1), (-sqrt(-a**2*x**2 + 1)/x, True)) - a*c**2*Piecewise((-a**2*acosh(1/(a
*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*a
sin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1
/(a**2*x**2))), True)) + c**2*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x
) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**
2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True))
```

3.303 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^5} dx$

Optimal. Leaf size=102

$$-\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{1}{8}a^4c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/4*c^2*(-a^2*x^2+1)^(3/2)/x^4+1/3*a*c^2*(-a^2*x^2+1)^(3/2)/x^3+1/8*a^4*c^
2*arctanh((-a^2*x^2+1)^(1/2))-1/8*a^2*c^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 835, 807, 266, 47, 63, 208}

$$-\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{1}{8}a^4c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^5,x]
```

```
[Out] -(a^2*c^2*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(4*x^4) +
(a*c^2*(1 - a^2*x^2)^(3/2))/(3*x^3) + (a^4*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/
8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^5}\,dx = c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^5}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{1}{4}c\int \frac{\left(4ac-a^2cx\right)\sqrt{1-a^2x^2}}{x^4}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{4}\left(a^2c^2\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{8}\left(a^2c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{16}\left(a^4c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-}}\right.$$
$$= -\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{8}\left(a^2c^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\right.$$
$$= -\frac{a^2c^2\sqrt{1-a^2x^2}}{8x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{8}a^4c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.03, size = 99, normalized size = 0.97

$$\frac{c^2\left(8a^5x^5-3a^4x^4-16a^3x^3+9a^2x^2+3a^4x^4\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+8ax-6\right)}{24x^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^5,x]
```

```
[Out] (c^2*(-6 + 8*a*x + 9*a^2*x^2 - 16*a^3*x^3 - 3*a^4*x^4 + 8*a^5*x^5 + 3*a^4*x
^4*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(24*x^4*Sqrt[1 - a^2*x^2]
)
```

fricas [A] time = 0.48, size = 84, normalized size = 0.82

$$-\frac{3\,a^4c^2x^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(8\,a^3c^2x^3-3\,a^2c^2x^2-8\,ac^2x+6\,c^2\right)\sqrt{-a^2x^2+1}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^5,x, algorithm="fricas"
)
```

```
[Out] -1/24*(3*a^4*c^2*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (8*a^3*c^2*x^3 - 3*a
^2*c^2*x^2 - 8*a*c^2*x + 6*c^2)*sqrt(-a^2*x^2 + 1))/x^4
```

giac [B] time = 0.30, size = 240, normalized size = 2.35

$$\frac{\left(3\,a^5c^2 - \frac{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^3c^2}{x} + \frac{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^2}{ax^3}\right)a^8x^4}{192\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4|a|} + \frac{a^5c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|} - \frac{\frac{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^5c^2|a|}{x} - }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^5*c^2 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3*c^2/x + 24*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a*x^3))*a^8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^4*abs(a)) + 1/8*a^5*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*
a)/(a^2*abs(x)))/abs(a) - 1/192*(24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c^2
*abs(a)/x - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*c^2*abs(a)/x^3 + 3*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^4*c^2*abs(a)/(a*x^4))/a^4
```

maple [A] time = 0.04, size = 125, normalized size = 1.23

$$c^2\left(-\frac{a^3\sqrt{-a^2x^2+1}}{x} - a\left(-\frac{\sqrt{-a^2x^2+1}}{3x^3} - \frac{2a^2\sqrt{-a^2x^2+1}}{3x}\right) - \frac{a^2\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2} - \frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)}{4} - \frac{\sqrt{-a^2x^2+}}{4x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^5,x)
```

```
[Out] c^2*(-a^3*(-a^2*x^2+1)^(1/2)/x-a*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2
*x^2+1)^(1/2)/x)-1/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a
^2*x^2+1)^(1/2)))-1/4*(-a^2*x^2+1)^(1/2)/x^4)
```

maxima [A] time = 0.41, size = 122, normalized size = 1.20

$$\frac{1}{8}\,a^4c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\sqrt{-a^2x^2+1}\,a^3c^2}{3\,x} + \frac{\sqrt{-a^2x^2+1}\,a^2c^2}{8\,x^2} + \frac{\sqrt{-a^2x^2+1}\,ac^2}{3\,x^3} - \frac{\sqrt{-a^2x^2+1}\,c^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^5,x, algorithm="maxima"
)
```

```
[Out] 1/8*a^4*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 1/3*sqrt(-a^2*x^2
 + 1)*a^3*c^2/x + 1/8*sqrt(-a^2*x^2 + 1)*a^2*c^2/x^2 + 1/3*sqrt(-a^2*x^2 +
1)*a*c^2/x^3 - 1/4*sqrt(-a^2*x^2 + 1)*c^2/x^4
```

mupad [B] time = 0.78, size = 113, normalized size = 1.11

$$\frac{a\,c^2\,\sqrt{1-a^2\,x^2}}{3\,x^3}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{4\,x^4}+\frac{a^2\,c^2\,\sqrt{1-a^2\,x^2}}{8\,x^2}-\frac{a^3\,c^2\,\sqrt{1-a^2\,x^2}}{3\,x}-\frac{a^4\,c^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(x^5*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a*c^2*(1 - a^2*x^2)^(1/2))/(3*x^3) - (c^2*(1 - a^2*x^2)^(1/2))/(4*x^4) - (
a^4*c^2*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/8 + (a^2*c^2*(1 - a^2*x^2)^(1/2))/
(8*x^2) - (a^3*c^2*(1 - a^2*x^2)^(1/2))/(3*x)
```

sympy [C] time = 7.98, size = 415, normalized size = 4.07

$$a^3c^2\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{a^2\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{\left|a^2x^2\right|}>1\\ \frac{ia^2\,\mathrm{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)-ac^2\left(\begin{cases}-\frac{2ia^2\sqrt{a \cdots}}{3 \cdots} \\ -\frac{2a^2\sqrt{- \cdots}}{3 \cdots}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**5,x)
```

```
[Out] a**3*c**2*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(
-a**2*x**2 + 1)/x, True)) - a**2*c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a
*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x
))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**
2))), True)) - a*c**2*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sq
rt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 +
 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True)) + c**2*Piecewise((-3*a**4
*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(
-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x*
*2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))
) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**
2))), True))
```

3.304 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^6}\,dx$

Optimal. Leaf size=129

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{1}{8}a^5c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2}$$

```
[Out] -1/5*c^2*(-a^2*x^2+1)^(3/2)/x^5+1/4*a*c^2*(-a^2*x^2+1)^(3/2)/x^4-2/15*a^2*c
^2*(-a^2*x^2+1)^(3/2)/x^3-1/8*a^5*c^2*arctanh((-a^2*x^2+1)^(1/2))+1/8*a^3*c
^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 835, 807, 266, 47, 63, 208}

$$\frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2}-\frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{1}{8}a^5c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^6,x]
```

```
[Out] (a^3*c^2*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(5*x^5) + (
a*c^2*(1 - a^2*x^2)^(3/2))/(4*x^4) - (2*a^2*c^2*(1 - a^2*x^2)^(3/2))/(15*x^
3) - (a^5*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^6}\,dx = c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^6}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{1}{5}c\int \frac{\left(5ac-2a^2cx\right)\sqrt{1-a^2x^2}}{x^5}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{1}{20}c\int \frac{\left(8a^2c-5a^3cx\right)\sqrt{1-a^2x^2}}{x^4}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{4}\left(a^3c^2\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{8}\left(a^3c^2\right)\text{Subst}\left(\int \frac{\sqrt{1}}{}\right.$$
$$= \frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{16}\left(a\right.$$
$$= \frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{8}\left(a^3\right.$$
$$= \frac{a^3c^2\sqrt{1-a^2x^2}}{8x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{2a^2c^2\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{8}a^5c^2$$

Mathematica [A] time = 0.04, size = 107, normalized size = 0.83

$$-\frac{c^2\left(16a^6x^6-15a^5x^5-8a^4x^4+45a^3x^3-32a^2x^2+15a^5x^5\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-30ax+24\right)}{120x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^6,x]
```

```
[Out] -1/120*(c^2*(24 - 30*a*x - 32*a^2*x^2 + 45*a^3*x^3 - 8*a^4*x^4 - 15*a^5*x^5
 + 16*a^6*x^6 + 15*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(
x^5*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.44, size = 95, normalized size = 0.74

$$\frac{15\,a^5c^2x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(16\,a^4c^2x^4-15\,a^3c^2x^3+8\,a^2c^2x^2+30\,ac^2x-24\,c^2\right)\sqrt{-a^2x^2+1}}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/120*(15*a^5*c^2*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (16*a^4*c^2*x^4 - 1
5*a^3*c^2*x^3 + 8*a^2*c^2*x^2 + 30*a*c^2*x - 24*c^2)*sqrt(-a^2*x^2 + 1))/x^
5
```

giac [B] time = 0.30, size = 297, normalized size = 2.30

$$\frac{\left(6\,a^6c^2 - \frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^4c^2}{x} + \frac{10\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2a^2c^2}{x^2} - \frac{60\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^2}{a^2x^4}\right)a^{10}x^5}{960\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5|a|} - \frac{a^6c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^6*c^2 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^2/x + 10*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^2/x^2 - 60*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^4*c^2/(a^2*x^4))*a^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 1/
8*a^6*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs
(a) + 1/960*(60*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^8*c^2/x - 10*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^3*a^4*c^2/x^3 + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*a^
2*c^2/x^4 - 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^2/x^5)/(a^4*abs(a))
```

maple [A] time = 0.04, size = 168, normalized size = 1.30

$$c^2\left(-\frac{a^2\left(-\frac{\sqrt{-a^2x^2+1}}{3x^3} - \frac{2a^2\sqrt{-a^2x^2+1}}{3x}\right)}{5} + a^3\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2} - \frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right) - a\left(-\frac{\sqrt{-a^2x^2+1}}{4x^4} + \frac{3a^2\left(-\cdots\right.}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^6,x)
```

```
[Out] c^2*(-1/5*a^2*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x)+a^
3*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))-a*(-1
/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arct
anh(1/(-a^2*x^2+1)^(1/2))))-1/5/x^5*(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 145, normalized size = 1.12

$$-\frac{1}{8}a^5c^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{2\sqrt{-a^2x^2+1}\,a^4c^2}{15\,x}-\frac{\sqrt{-a^2x^2+1}\,a^3c^2}{8\,x^2}+\frac{\sqrt{-a^2x^2+1}\,a^2c^2}{15\,x^3}+\frac{\sqrt{-a^2x^2+1}\,ac^2}{4\,x^4}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/8*a^5*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 2/15*sqrt(-a^2*x
^2 + 1)*a^4*c^2/x - 1/8*sqrt(-a^2*x^2 + 1)*a^3*c^2/x^2 + 1/15*sqrt(-a^2*x^2
 + 1)*a^2*c^2/x^3 + 1/4*sqrt(-a^2*x^2 + 1)*a*c^2/x^4 - 1/5*sqrt(-a^2*x^2 +
1)*c^2/x^5
```

mupad [B] time = 0.79, size = 136, normalized size = 1.05

$$\frac{a\,c^2\,\sqrt{1-a^2\,x^2}}{4\,x^4}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{5\,x^5}+\frac{a^2\,c^2\,\sqrt{1-a^2\,x^2}}{15\,x^3}-\frac{a^3\,c^2\,\sqrt{1-a^2\,x^2}}{8\,x^2}+\frac{2\,a^4\,c^2\,\sqrt{1-a^2\,x^2}}{15\,x}+\frac{a^5\,c^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\cdots\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(x^6*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^5*c^2*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/8 - (c^2*(1 - a^2*x^2)^(1/2))/(5*
x^5) + (a*c^2*(1 - a^2*x^2)^(1/2))/(4*x^4) + (a^2*c^2*(1 - a^2*x^2)^(1/2))/
(15*x^3) - (a^3*c^2*(1 - a^2*x^2)^(1/2))/(8*x^2) + (2*a^4*c^2*(1 - a^2*x^2)
^(1/2))/(15*x)
```

sympy [C] time = 8.61, size = 522, normalized size = 4.05

$$a^3c^2\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for}\ \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for}\ \left|a^2x^2\right|>1\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**6,x)
```

```
[Out] a**3*c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2
*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(
```

```
a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) - a**2*c**2*Pie
cewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3
), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x*
*2 + 1)/(3*x**3), True)) - a*c**2*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a
**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) -
1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asi
n(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1
- 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True)) + c**2*Pie
cewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2
))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1),
(-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(1
5*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))
```

3.305 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^7}\,dx$

Optimal. Leaf size=156

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4}+\frac{1}{16}a^6c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2}+\frac{2a^3c^2\left(1\right.}{15}$$

```
[Out] -1/6*c^2*(-a^2*x^2+1)^(3/2)/x^6+1/5*a*c^2*(-a^2*x^2+1)^(3/2)/x^5-1/8*a^2*c^
2*(-a^2*x^2+1)^(3/2)/x^4+2/15*a^3*c^2*(-a^2*x^2+1)^(3/2)/x^3+1/16*a^6*c^2*a
rctanh((-a^2*x^2+1)^(1/2))-1/16*a^4*c^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.17, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 835, 807, 266, 47, 63, 208}

$$-\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2}+\frac{2a^3c^2\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4}+\frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6}+\frac{1}{16}a^6c^2\tanh^{-1}\left(\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^7,x]
```

```
[Out] -(a^4*c^2*Sqrt[1 - a^2*x^2])/(16*x^2) - (c^2*(1 - a^2*x^2)^(3/2))/(6*x^6) +
 (a*c^2*(1 - a^2*x^2)^(3/2))/(5*x^5) - (a^2*c^2*(1 - a^2*x^2)^(3/2))/(8*x^4
) + (2*a^3*c^2*(1 - a^2*x^2)^(3/2))/(15*x^3) + (a^6*c^2*ArcTanh[Sqrt[1 - a^
2*x^2]])/16
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^2}{x^7}\,dx = c\int \frac{(c-acx)\sqrt{1-a^2x^2}}{x^7}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} - \frac{1}{6}c\int \frac{\left(6ac-3a^2cx\right)\sqrt{1-a^2x^2}}{x^6}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{1}{30}c\int \frac{\left(15a^2c-12a^3cx\right)\sqrt{1-a^2x^2}}{x^5}\,dx$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} - \frac{1}{120}c\int \frac{\left(48a^3c-15a^4c\right.}{x^{\cdot}}$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c^2\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{8}$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c^2\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{1}$$
$$= -\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c^2}{}$$
$$= -\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c}{}$$
$$= -\frac{a^4c^2\sqrt{1-a^2x^2}}{16x^2} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{ac^2\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{a^2c^2\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{2a^3c}{}$$

Mathematica [A] time = 0.04, size = 115, normalized size = 0.74

$$\frac{c^2\left(32a^7x^7-15a^6x^6-16a^5x^5+5a^4x^4-64a^3x^3+50a^2x^2+15a^6x^6\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+48ax-40\right)}{240x^6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^2)/x^7,x]
```

```
[Out] (c^2*(-40 + 48*a*x + 50*a^2*x^2 - 64*a^3*x^3 + 5*a^4*x^4 - 16*a^5*x^5 - 15*
a^6*x^6 + 32*a^7*x^7 + 15*a^6*x^6*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^
2]]))/(240*x^6*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.47, size = 106, normalized size = 0.68

$$-\frac{15\,a^6c^2x^6\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(32\,a^5c^2x^5-15\,a^4c^2x^4+16\,a^3c^2x^3-10\,a^2c^2x^2-48\,ac^2x+40\,c^2\right)\sqrt{-a^2x^2+1}}{240\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^7,x, algorithm="fricas"
)
```

```
[Out] -1/240*(15*a^6*c^2*x^6*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (32*a^5*c^2*x^5 -
15*a^4*c^2*x^4 + 16*a^3*c^2*x^3 - 10*a^2*c^2*x^2 - 48*a*c^2*x + 40*c^2)*sqr
t(-a^2*x^2 + 1))/x^6
```

giac [B] time = 0.65, size = 424, normalized size = 2.72

$$\frac{\left(5\,a^7c^2 - \frac{12\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^5c^2}{x} + \frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2a^3c^2}{x^2} - \frac{20\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3ac^2}{x^3} - \frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^2}{ax^4} + \frac{120\left(\sqrt{-a^2x^2+1}\,|a\right.}{a^3x^5}\right.}{1920\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^6|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^7*c^2 - 12*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c^2/x + 15*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)^2*a^3*c^2/x^2 - 20*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*a*c^2/x^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^2/(a*x^4) + 120*(sq
rt(-a^2*x^2 + 1)*abs(a) + a)^5*c^2/(a^3*x^5))*a^12*x^6/((sqrt(-a^2*x^2 + 1)
*abs(a) + a)^6*abs(a)) + 1/16*a^7*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs
(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/1920*(120*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)*a^9*c^2*abs(a)/x - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^7*c^2*abs(a)/x
^2 - 20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a^5*c^2*abs(a)/x^3 + 15*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^4*a^3*c^2*abs(a)/x^4 - 12*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^5*a*c^2*abs(a)/x^5 + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6*c^2*abs(a)/(
a*x^6))/a^6
```

maple [A] time = 0.05, size = 193, normalized size = 1.24

$$c^2\left(a^3\left(-\frac{\sqrt{-a^2x^2+1}}{3x^3} - \frac{2a^2\sqrt{-a^2x^2+1}}{3x}\right) - \frac{a^2\left(-\frac{\sqrt{-a^2x^2+1}}{4x^4} + \frac{3a^2\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2} - \frac{a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)}{4}\right)}{6} - \frac{\sqrt{-a^2x^2+1}}{6x^6} - a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^7,x)
```

```
[Out] c^2*(a^3*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x)-1/6*a^2
*(-1/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*
arctanh(1/(-a^2*x^2+1)^(1/2))))-1/6/x^6*(-a^2*x^2+1)^(1/2)-a*(-1/5/x^5*(-a^
2*x^2+1)^(1/2)+4/5*a^2*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1
/2)/x)))
```

maxima [A] time = 0.40, size = 168, normalized size = 1.08

$$\frac{1}{16}\,a^6c^2\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{2\,\sqrt{-a^2x^2+1}\,a^5c^2}{15\,x}+\frac{\sqrt{-a^2x^2+1}\,a^4c^2}{16\,x^2}-\frac{\sqrt{-a^2x^2+1}\,a^3c^2}{15\,x^3}+\frac{\sqrt{-a^2x^2+1}\,a^2c^2}{24\,x^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^2/x^7,x, algorithm="maxima"
)
```

```
[Out] 1/16*a^6*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/15*sqrt(-a^2*x
^2 + 1)*a^5*c^2/x + 1/16*sqrt(-a^2*x^2 + 1)*a^4*c^2/x^2 - 1/15*sqrt(-a^2*x^
2 + 1)*a^3*c^2/x^3 + 1/24*sqrt(-a^2*x^2 + 1)*a^2*c^2/x^4 + 1/5*sqrt(-a^2*x^
2 + 1)*a*c^2/x^5 - 1/6*sqrt(-a^2*x^2 + 1)*c^2/x^6
```

mupad [B] time = 0.05, size = 159, normalized size = 1.02

$$\frac{a\,c^2\,\sqrt{1-a^2\,x^2}}{5\,x^5}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{6\,x^6}+\frac{a^2\,c^2\,\sqrt{1-a^2\,x^2}}{24\,x^4}-\frac{a^3\,c^2\,\sqrt{1-a^2\,x^2}}{15\,x^3}+\frac{a^4\,c^2\,\sqrt{1-a^2\,x^2}}{16\,x^2}-\frac{2\,a^5\,c^2\,\sqrt{1-a^2\,x^2}}{15\,x}-\frac{a^6\,c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(x^7*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a*c^2*(1 - a^2*x^2)^(1/2))/(5*x^5) - (c^2*(1 - a^2*x^2)^(1/2))/(6*x^6) - (
a^6*c^2*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/16 + (a^2*c^2*(1 - a^2*x^2)^(1/2))
/(24*x^4) - (a^3*c^2*(1 - a^2*x^2)^(1/2))/(15*x^3) + (a^4*c^2*(1 - a^2*x^2)
^(1/2))/(16*x^2) - (2*a^5*c^2*(1 - a^2*x^2)^(1/2))/(15*x)
```

sympy [C] time = 11.61, size = 644, normalized size = 4.13

$$a^3c^2\left(\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } \left|a^2x^2\right|>1\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}-\frac{3a^4\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}+\frac{3a^3}{8x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{a}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{1}{4ax^5\sqrt{-1+}} \\ \frac{3ia^4\operatorname{asin}\left(\frac{1}{ax}\right)}{8}-\frac{3ia^3}{8x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{ia}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**2/x**7,x)
```

```
[Out] a**3*c**2*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2
 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) -
sqrt(-a**2*x**2 + 1)/(3*x**3), True)) - a**2*c**2*Piecewise((-3*a**4*acosh(
1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/
(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1
), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a
/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), T
rue)) - a*c**2*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt
(-1 + 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs
(a**2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 -
 1/(a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True)) +
c**2*Piecewise((-5*a**6*acosh(1/(a*x))/16 + 5*a**5/(16*x*sqrt(-1 + 1/(a**2*
x**2))) - 5*a**3/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) - a/(24*x**5*sqrt(-1 +
1/(a**2*x**2))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) >
 1), (5*I*a**6*asin(1/(a*x))/16 - 5*I*a**5/(16*x*sqrt(1 - 1/(a**2*x**2))) +
 5*I*a**3/(48*x**3*sqrt(1 - 1/(a**2*x**2))) + I*a/(24*x**5*sqrt(1 - 1/(a**2
*x**2))) + I/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), True))
```

3.306 $\int e^{\tanh^{-1}(ax)}x^3(c-acx)^3\,dx$

Optimal. Leaf size=148

$$-\frac{c^3\sin^{-1}(ax)}{8a^4}-\frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2}-\frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2}+\frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a}-\frac{c^3(88-105ax)\left(1-a^2x^2\right)^{3/2}}{420a^4}-\frac{c^3x\sqrt{}$$

```
[Out] -11/35*c^3*x^2*(-a^2*x^2+1)^(3/2)/a^2+1/3*c^3*x^3*(-a^2*x^2+1)^(3/2)/a-1/7*
c^3*x^4*(-a^2*x^2+1)^(3/2)-1/420*c^3*(-105*a*x+88)*(-a^2*x^2+1)^(3/2)/a^4-1
/8*c^3*arcsin(a*x)/a^4-1/8*c^3*x*(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.24, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1809, 833, 780, 195, 216}

$$-\frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2}+\frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a}-\frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2}-\frac{c^3x\sqrt{1-a^2x^2}}{8a^3}-\frac{c^3(88-105ax)\left(1-a^2x^2\right)^{3/2}}{420a^4}-\frac{c^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(c - a*c*x)^3,x]
```

```
[Out] -(c^3*x*Sqrt[1 - a^2*x^2])/(8*a^3) - (11*c^3*x^2*(1 - a^2*x^2)^(3/2))/(35*a
^2) + (c^3*x^3*(1 - a^2*x^2)^(3/2))/(3*a) - (c^3*x^4*(1 - a^2*x^2)^(3/2))/7
 - (c^3*(88 - 105*a*x)*(1 - a^2*x^2)^(3/2))/(420*a^4) - (c^3*ArcSin[a*x])/(
8*a^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^3(c-acx)^3\,dx = c\int x^3(c-acx)^2\sqrt{1-a^2x^2}\,dx$$
$$= -\frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} - \frac{c\int x^3\left(-11a^2c^2+14a^3c^2x\right)\sqrt{1-a^2x^2}\,dx}{7a^2}$$
$$= \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} + \frac{c\int x^2\left(-42a^3c^2+66a^4c^2x\right)\sqrt{1-a^2x^2}}{42a^4}$$
$$= -\frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} - \frac{c\int x\left(-132a^4c\right.}{}$$
$$= -\frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2} - \frac{c^3(88-105ax}{420}$$
$$= -\frac{c^3x\sqrt{1-a^2x^2}}{8a^3} - \frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2}$$
$$= -\frac{c^3x\sqrt{1-a^2x^2}}{8a^3} - \frac{11c^3x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{c^3x^3\left(1-a^2x^2\right)^{3/2}}{3a} - \frac{1}{7}c^3x^4\left(1-a^2x^2\right)^{3/2}$$

Mathematica [A] time = 0.13, size = 91, normalized size = 0.61

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(120a^6x^6-280a^5x^5+144a^4x^4+70a^3x^3-88a^2x^2+105ax-176\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{840a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(-176 + 105*a*x - 88*a^2*x^2 + 70*a^3*x^3 + 144*a^4
*x^4 - 280*a^5*x^5 + 120*a^6*x^6) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(84
0*a^4)
```

fricas [A] time = 0.41, size = 114, normalized size = 0.77

$$\frac{210\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(120\,a^6c^3x^6-280\,a^5c^3x^5+144\,a^4c^3x^4+70\,a^3c^3x^3-88\,a^2c^3x^2+105\,ac^3x-176\,c\right.}{840\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/840*(210*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (120*a^6*c^3*x^6 -
280*a^5*c^3*x^5 + 144*a^4*c^3*x^4 + 70*a^3*c^3*x^3 - 88*a^2*c^3*x^2 + 105*a
*c^3*x - 176*c^3)*sqrt(-a^2*x^2 + 1))/a^4
```

giac [A] time = 0.21, size = 104, normalized size = 0.70

$$\frac{1}{840}\sqrt{-a^2x^2+1}\left(\left(2\left(\left(\frac{35\,c^3}{a}+4\left(18\,c^3+5\left(3\,a^2c^3x-7\,ac^3\right)x\right)x\right)x-\frac{44\,c^3}{a^2}\right)x+\frac{105\,c^3}{a^3}\right)x-\frac{176\,c^3}{a^4}\right)-\frac{c^3\arcsin\left(ax\right)}{8\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/840*sqrt(-a^2*x^2 + 1)*((2*((35*c^3/a + 4*(18*c^3 + 5*(3*a^2*c^3*x - 7*a*
c^3)*x)*x)*x - 44*c^3/a^2)*x + 105*c^3/a^3)*x - 176*c^3/a^4) - 1/8*c^3*arcs
in(a*x)*sgn(a)/(a^3*abs(a))
```

maple [A] time = 0.05, size = 186, normalized size = 1.26

$$\frac{c^3a^2x^6\sqrt{-a^2x^2+1}}{7}+\frac{6c^3x^4\sqrt{-a^2x^2+1}}{35}-\frac{11c^3x^2\sqrt{-a^2x^2+1}}{105a^2}-\frac{22c^3\sqrt{-a^2x^2+1}}{105a^4}-\frac{c^3a\,x^5\sqrt{-a^2x^2+1}}{3}+\frac{c^3x^3\sqrt{-a^2x^2+1}}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^3,x)
```

```
[Out] 1/7*c^3*a^2*x^6*(-a^2*x^2+1)^(1/2)+6/35*c^3*x^4*(-a^2*x^2+1)^(1/2)-11/105*c
^3*x^2/a^2*(-a^2*x^2+1)^(1/2)-22/105*c^3/a^4*(-a^2*x^2+1)^(1/2)-1/3*c^3*a*x
^5*(-a^2*x^2+1)^(1/2)+1/12*c^3/a*x^3*(-a^2*x^2+1)^(1/2)+1/8*c^3*x*(-a^2*x^2
+1)^(1/2)/a^3-1/8*c^3/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/
2))
```

maxima [A] time = 0.40, size = 164, normalized size = 1.11

$$\frac{1}{7}\sqrt{-a^2x^2+1}\,a^2c^3x^6-\frac{1}{3}\sqrt{-a^2x^2+1}\,ac^3x^5+\frac{6}{35}\sqrt{-a^2x^2+1}\,c^3x^4+\frac{\sqrt{-a^2x^2+1}\,c^3x^3}{12\,a}-\frac{11\sqrt{-a^2x^2+1}\,c^3x^2}{105\,a^2}+\frac{\sqrt{-a^2x^2+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/7*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^6 - 1/3*sqrt(-a^2*x^2 + 1)*a*c^3*x^5 + 6/3
5*sqrt(-a^2*x^2 + 1)*c^3*x^4 + 1/12*sqrt(-a^2*x^2 + 1)*c^3*x^3/a - 11/105*s
qrt(-a^2*x^2 + 1)*c^3*x^2/a^2 + 1/8*sqrt(-a^2*x^2 + 1)*c^3*x/a^3 - 1/8*c^3*
arcsin(a*x)/a^4 - 22/105*sqrt(-a^2*x^2 + 1)*c^3/a^4
```

mupad [B] time = 0.04, size = 177, normalized size = 1.20

$$\frac{6\,c^3\,x^4\,\sqrt{1-a^2\,x^2}}{35}-\frac{22\,c^3\,\sqrt{1-a^2\,x^2}}{105\,a^4}+\frac{c^3\,x\,\sqrt{1-a^2\,x^2}}{8\,a^3}-\frac{a\,c^3\,x^5\,\sqrt{1-a^2\,x^2}}{3}-\frac{c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a^3\,\sqrt{-a^2}}+\frac{c^3\,x^3\,\sqrt{1-a^2\,x^2}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^3*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (6*c^3*x^4*(1 - a^2*x^2)^(1/2))/35 - (22*c^3*(1 - a^2*x^2)^(1/2))/(105*a^4)
 + (c^3*x*(1 - a^2*x^2)^(1/2))/(8*a^3) - (a*c^3*x^5*(1 - a^2*x^2)^(1/2))/3
- (c^3*asinh(x*(-a^2)^(1/2)))/(8*a^3*(-a^2)^(1/2)) + (c^3*x^3*(1 - a^2*x^2)
^(1/2))/(12*a) - (11*c^3*x^2*(1 - a^2*x^2)^(1/2))/(105*a^2) + (a^2*c^3*x^6*
(1 - a^2*x^2)^(1/2))/7
```

sympy [A] time = 11.22, size = 512, normalized size = 3.46

$$-a^4c^3\left(\begin{cases}-\frac{x^6\sqrt{-a^2x^2+1}}{7a^2}-\frac{6x^4\sqrt{-a^2x^2+1}}{35a^4}-\frac{8x^2\sqrt{-a^2x^2+1}}{35a^6}-\frac{16\sqrt{-a^2x^2+1}}{35a^8} & \text{for } a\neq 0\\ \frac{x^8}{8} & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}-\frac{ix^7}{6\sqrt{a^2x^2-1}}-\frac{ix^5}{24a^2\sqrt{a^2x^2-1}}\\ \frac{x^7}{6\sqrt{-a^2x^2+1}}+\frac{x^5}{24a^2\sqrt{-a^2x^2+1}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a*c*x+c)**3,x)
```

```
[Out] -a**4*c**3*Piecewise((-x**6*sqrt(-a**2*x**2 + 1)/(7*a**2) - 6*x**4*sqrt(-a*
*2*x**2 + 1)/(35*a**4) - 8*x**2*sqrt(-a**2*x**2 + 1)/(35*a**6) - 16*sqrt(-a
**2*x**2 + 1)/(35*a**8), Ne(a, 0)), (x**8/8, True)) + 2*a**3*c**3*Piecewise
((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt(a**2*x**2 - 1)) -
5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*sqrt(a**2*x**2 - 1)
) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7/(6*sqrt(-a**2*x**2
 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(48*a**4*sqrt(-a**2*x
**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(a*x)/(16*a**7), Tru
e)) - 2*a*c**3*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*
sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/
(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**
2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(
8*a**5), True)) + c**3*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*s
qrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True))
```

3.307 $\int e^{\tanh^{-1}(ax)} x^2 (c - acx)^3 \, dx$

Optimal. Leaf size=121

$$\frac{3c^3 \sin^{-1}(ax)}{16a^3} + \frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a} + \frac{3c^3x\sqrt{1-a^2x^2}}{16a^2} - \frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2} + \frac{c^3(32-45ax)\left(1-a^2x^2\right)^{3/2}}{120a^3}$$

```
[Out] 2/5*c^3*x^2*(-a^2*x^2+1)^(3/2)/a-1/6*c^3*x^3*(-a^2*x^2+1)^(3/2)+1/120*c^3*(
-45*a*x+32)*(-a^2*x^2+1)^(3/2)/a^3+3/16*c^3*arcsin(a*x)/a^3+3/16*c^3*x*(-a^
2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1809, 833, 780, 195, 216}

$$-\frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2} + \frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a} + \frac{c^3(32-45ax)\left(1-a^2x^2\right)^{3/2}}{120a^3} + \frac{3c^3x\sqrt{1-a^2x^2}}{16a^2} + \frac{3c^3 \sin^{-1}(ax)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a*c*x)^3,x]
```

```
[Out] (3*c^3*x*Sqrt[1 - a^2*x^2])/(16*a^2) + (2*c^3*x^2*(1 - a^2*x^2)^(3/2))/(5*a
) - (c^3*x^3*(1 - a^2*x^2)^(3/2))/6 + (c^3*(32 - 45*a*x)*(1 - a^2*x^2)^(3/2
))/(120*a^3) + (3*c^3*ArcSin[a*x])/(16*a^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^2(c-acx)^3\,dx = c\int x^2(c-acx)^2\sqrt{1-a^2x^2}\,dx$$

$$= -\frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2} - \frac{c\int x^2\left(-9a^2c^2+12a^3c^2x\right)\sqrt{1-a^2x^2}\,dx}{6a^2}$$

$$= \frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a} - \frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2} + \frac{c\int x\left(-24a^3c^2+45a^4c^2x\right)\sqrt{1-a^2x^2}\,d}{30a^4}$$

$$= \frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a} - \frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2} + \frac{c^3(32-45ax)\left(1-a^2x^2\right)^{3/2}}{120a^3} + \frac{\left(3c^3\right)\int \sqrt{}}{}$$

$$= \frac{3c^3x\sqrt{1-a^2x^2}}{16a^2} + \frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a} - \frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2} + \frac{c^3(32-45ax)\left(1-a^2\right.}{120a^3}$$

$$= \frac{3c^3x\sqrt{1-a^2x^2}}{16a^2} + \frac{2c^3x^2\left(1-a^2x^2\right)^{3/2}}{5a} - \frac{1}{6}c^3x^3\left(1-a^2x^2\right)^{3/2} + \frac{c^3(32-45ax)\left(1-a^2\right.}{120a^3}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.69

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(40a^5x^5-96a^4x^4+50a^3x^3+32a^2x^2-45ax+64\right)-90\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(64 - 45*a*x + 32*a^2*x^2 + 50*a^3*x^3 - 96*a^4*x^4
 + 40*a^5*x^5) - 90*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a^3)
```

fricas [A] time = 0.61, size = 104, normalized size = 0.86

$$-\frac{90\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(40\,a^5c^3x^5-96\,a^4c^3x^4+50\,a^3c^3x^3+32\,a^2c^3x^2-45\,ac^3x+64\,c^3\right)\sqrt{-a^2x^2+1}}{240\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/240*(90*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (40*a^5*c^3*x^5 - 9
6*a^4*c^3*x^4 + 50*a^3*c^3*x^3 + 32*a^2*c^3*x^2 - 45*a*c^3*x + 64*c^3)*sqrt
(-a^2*x^2 + 1))/a^3
```

giac [A] time = 0.53, size = 92, normalized size = 0.76

$$\frac{3\,c^3\arcsin\left(ax\right)\mathrm{sgn}(a)}{16\,a^2|a|}+\frac{1}{240}\,\sqrt{-a^2x^2+1}\left(\left(2\left(\frac{16\,c^3}{a}+\left(25\,c^3+4\left(5\,a^2c^3x-12\,ac^3\right)x\right)x\right)x-\frac{45\,c^3}{a^2}\right)x+\frac{64\,c^3}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 3/16*c^3*arcsin(a*x)*sgn(a)/(a^2*abs(a)) + 1/240*sqrt(-a^2*x^2 + 1)*((2*(16
*c^3/a + (25*c^3 + 4*(5*a^2*c^3*x - 12*a*c^3)*x)*x)*x - 45*c^3/a^2)*x + 64*
c^3/a^3)
```

maple [A] time = 0.04, size = 163, normalized size = 1.35

$$\frac{c^3a^2x^5\sqrt{-a^2x^2+1}}{6}+\frac{5c^3x^3\sqrt{-a^2x^2+1}}{24}-\frac{3c^3x\sqrt{-a^2x^2+1}}{16a^2}+\frac{3c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{16a^2\sqrt{a^2}}-\frac{2c^3a\,x^4\sqrt{-a^2x^2+1}}{5}+\frac{2c^3x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^3,x)
```

```
[Out] 1/6*c^3*a^2*x^5*(-a^2*x^2+1)^(1/2)+5/24*c^3*x^3*(-a^2*x^2+1)^(1/2)-3/16*c^3
*x*(-a^2*x^2+1)^(1/2)/a^2+3/16*c^3/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2))-2/5*c^3*a*x^4*(-a^2*x^2+1)^(1/2)+2/15*c^3/a*x^2*(-a^2*x^2+
1)^(1/2)+4/15*c^3/a^3*(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.47, size = 141, normalized size = 1.17

$$\frac{1}{6}\,\sqrt{-a^2x^2+1}\,a^2c^3x^5-\frac{2}{5}\,\sqrt{-a^2x^2+1}\,ac^3x^4+\frac{5}{24}\,\sqrt{-a^2x^2+1}\,c^3x^3+\frac{2\,\sqrt{-a^2x^2+1}\,c^3x^2}{15\,a}-\frac{3\,\sqrt{-a^2x^2+1}\,c^3x}{16\,a^2}+\frac{3\,c^3\,\mathrm{ar}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/6*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^5 - 2/5*sqrt(-a^2*x^2 + 1)*a*c^3*x^4 + 5/2
4*sqrt(-a^2*x^2 + 1)*c^3*x^3 + 2/15*sqrt(-a^2*x^2 + 1)*c^3*x^2/a - 3/16*sqr
t(-a^2*x^2 + 1)*c^3*x/a^2 + 3/16*c^3*arcsin(a*x)/a^3 + 4/15*sqrt(-a^2*x^2 +
 1)*c^3/a^3
```

mupad [B] time = 0.04, size = 154, normalized size = 1.27

$$\frac{4\,c^3\,\sqrt{1-a^2\,x^2}}{15\,a^3}+\frac{5\,c^3\,x^3\,\sqrt{1-a^2\,x^2}}{24}-\frac{3\,c^3\,x\,\sqrt{1-a^2\,x^2}}{16\,a^2}-\frac{2\,a\,c^3\,x^4\,\sqrt{1-a^2\,x^2}}{5}+\frac{3\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{16\,a^2\,\sqrt{-a^2}}+\frac{2\,c^3\,x^2\,\sqrt{1-}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^3*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (4*c^3*(1 - a^2*x^2)^(1/2))/(15*a^3) + (5*c^3*x^3*(1 - a^2*x^2)^(1/2))/24 -
 (3*c^3*x*(1 - a^2*x^2)^(1/2))/(16*a^2) - (2*a*c^3*x^4*(1 - a^2*x^2)^(1/2))
/5 + (3*c^3*asinh(x*(-a^2)^(1/2)))/(16*a^2*(-a^2)^(1/2)) + (2*c^3*x^2*(1 -
a^2*x^2)^(1/2))/(15*a) + (a^2*c^3*x^5*(1 - a^2*x^2)^(1/2))/6
```

sympy [C] time = 9.48, size = 423, normalized size = 3.50

$$-a^4c^3\left(\begin{cases} -\frac{ix^7}{6\sqrt{a^2x^2-1}} - \frac{ix^5}{24a^2\sqrt{a^2x^2-1}} - \frac{5ix^3}{48a^4\sqrt{a^2x^2-1}} + \frac{5ix}{16a^6\sqrt{a^2x^2-1}} - \frac{5i\operatorname{acosh}(ax)}{16a^7} & \text{for } |a^2x^2| > 1 \\ \frac{x^7}{6\sqrt{-a^2x^2+1}} + \frac{x^5}{24a^2\sqrt{-a^2x^2+1}} + \frac{5x^3}{48a^4\sqrt{-a^2x^2+1}} - \frac{5x}{16a^6\sqrt{-a^2x^2+1}} + \frac{5\operatorname{asin}(ax)}{16a^7} & \text{otherwise} \end{cases}\right) + 2a^3c^3\left(\begin{cases} -\frac{x^4\sqrt{-a^2x^2 \dots}}{5a^2} \\ \frac{x^6}{6} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c)**3,x)
```

```
[Out] -a**4*c**3*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqr
t(a**2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6
*sqrt(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**
7/(6*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(
48*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin
(a*x)/(16*a**7), True)) + 2*a**3*c**3*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)
/(5*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/
(15*a**6), Ne(a, 0)), (x**6/6, True)) - 2*a*c**3*Piecewise((-x**2*sqrt(-a**
2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4,
 True)) + c**3*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/
(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*s
qrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True))
```

3.308 $\int e^{\tanh^{-1}(ax)}x(c-acx)^3\,dx$

Optimal. Leaf size=94

$$-\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2}-\frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2}-\frac{c^3x\sqrt{1-a^2x^2}}{4a}-\frac{c^3\sin^{-1}(ax)}{4a^2}$$

```
[Out] -1/5*c^3*x^2*(-a^2*x^2+1)^(3/2)-1/30*c^3*(-15*a*x+14)*(-a^2*x^2+1)^(3/2)/a^
2-1/4*c^3*arcsin(a*x)/a^2-1/4*c^3*x*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6128, 1809, 780, 195, 216}

$$-\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2}-\frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2}-\frac{c^3x\sqrt{1-a^2x^2}}{4a}-\frac{c^3\sin^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(c - a*c*x)^3,x]
```

```
[Out] -(c^3*x*Sqrt[1 - a^2*x^2])/(4*a) - (c^3*x^2*(1 - a^2*x^2)^(3/2))/5 - (c^3*(
14 - 15*a*x)*(1 - a^2*x^2)^(3/2))/(30*a^2) - (c^3*ArcSin[a*x])/(4*a^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x(c-acx)^3\,dx &= c\int x(c-acx)^2\sqrt{1-a^2x^2}\,dx\\
&= -\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2}-\frac{c\int x\left(-7a^2c^2+10a^3c^2x\right)\sqrt{1-a^2x^2}\,dx}{5a^2}\\
&= -\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2}-\frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2}-\frac{c^3\int\sqrt{1-a^2x^2}\,dx}{2a}\\
&= -\frac{c^3x\sqrt{1-a^2x^2}}{4a}-\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2}-\frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2}-\frac{c^3\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{4a}\\
&= -\frac{c^3x\sqrt{1-a^2x^2}}{4a}-\frac{1}{5}c^3x^2\left(1-a^2x^2\right)^{3/2}-\frac{c^3(14-15ax)\left(1-a^2x^2\right)^{3/2}}{30a^2}-\frac{c^3\sin^{-1}(ax)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 0.80

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(12a^4x^4-30a^3x^3+16a^2x^2+15ax-28\right)+30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{60a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(-28 + 15*a*x + 16*a^2*x^2 - 30*a^3*x^3 + 12*a^4*x^
4) + 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(60*a^2)
```

fricas [A] time = 0.48, size = 92, normalized size = 0.98

$$\frac{30\,c^3 \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(12\,a^4c^3x^4 - 30\,a^3c^3x^3 + 16\,a^2c^3x^2 + 15\,ac^3x - 28\,c^3\right)\sqrt{-a^2x^2+1}}{60\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(30*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (12*a^4*c^3*x^4 - 30*
a^3*c^3*x^3 + 16*a^2*c^3*x^2 + 15*a*c^3*x - 28*c^3)*sqrt(-a^2*x^2 + 1))/a^2
```

giac [A] time = 0.37, size = 81, normalized size = 0.86

$$-\frac{c^3 \arcsin(ax)\,\mathrm{sgn}(a)}{4\,a|a|} + \frac{1}{60}\sqrt{-a^2x^2+1}\left(\left(\frac{15\,c^3}{a} + 2\left(8\,c^3 + 3\left(2\,a^2c^3x - 5\,ac^3\right)x\right)x\right)x - \frac{28\,c^3}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*c^3*arcsin(a*x)*sgn(a)/(a*abs(a)) + 1/60*sqrt(-a^2*x^2 + 1)*((15*c^3/a
 + 2*(8*c^3 + 3*(2*a^2*c^3*x - 5*a*c^3)*x)*x)*x - 28*c^3/a^2)
```

maple [A] time = 0.04, size = 140, normalized size = 1.49

$$\frac{c^3a^2x^4\sqrt{-a^2x^2+1}}{5}+\frac{4c^3x^2\sqrt{-a^2x^2+1}}{15}-\frac{7c^3\sqrt{-a^2x^2+1}}{15a^2}-\frac{c^3a\,x^3\sqrt{-a^2x^2+1}}{2}+\frac{c^3x\sqrt{-a^2x^2+1}}{4a}-\frac{c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{4a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^3,x)
```

```
[Out] 1/5*c^3*a^2*x^4*(-a^2*x^2+1)^(1/2)+4/15*c^3*x^2*(-a^2*x^2+1)^(1/2)-7/15*c^3
/a^2*(-a^2*x^2+1)^(1/2)-1/2*c^3*a*x^3*(-a^2*x^2+1)^(1/2)+1/4*c^3*x*(-a^2*x^
2+1)^(1/2)/a-1/4*c^3/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 118, normalized size = 1.26

$$\frac{1}{5}\sqrt{-a^2x^2+1}\,a^2c^3x^4-\frac{1}{2}\sqrt{-a^2x^2+1}\,ac^3x^3+\frac{4}{15}\sqrt{-a^2x^2+1}\,c^3x^2+\frac{\sqrt{-a^2x^2+1}\,c^3x}{4\,a}-\frac{c^3\arcsin(ax)}{4\,a^2}-\frac{7\sqrt{-a^2x^2+1}\ldots}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^4 - 1/2*sqrt(-a^2*x^2 + 1)*a*c^3*x^3 + 4/1
5*sqrt(-a^2*x^2 + 1)*c^3*x^2 + 1/4*sqrt(-a^2*x^2 + 1)*c^3*x/a - 1/4*c^3*arc
sin(a*x)/a^2 - 7/15*sqrt(-a^2*x^2 + 1)*c^3/a^2
```

mupad [B] time = 0.06, size = 108, normalized size = 1.15

$$\frac{c^3 \operatorname{asinh}\left(x\sqrt{-a^2}\right)\sqrt{-a^2}}{4a^3} - \frac{\frac{2c^3\left(1-a^2x^2\right)^{3/2}}{3} - \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5}}{a^2} - \frac{\frac{c^3x\sqrt{1-a^2x^2}}{4} - \frac{c^3x\left(1-a^2x^2\right)^{3/2}}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^3*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^3*asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2))/(4*a^3) - ((2*c^3*(1 - a^2*x^2)^(3
/2))/3 - (c^3*(1 - a^2*x^2)^(5/2))/5)/a^2 - ((c^3*x*(1 - a^2*x^2)^(1/2))/4
- (c^3*x*(1 - a^2*x^2)^(3/2))/2)/a
```

sympy [A] time = 7.71, size = 355, normalized size = 3.78

$$-a^4c^3\left(\begin{cases} -\frac{x^4\sqrt{-a^2x^2+1}}{5a^2} - \frac{4x^2\sqrt{-a^2x^2+1}}{15a^4} - \frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for } a \neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases}\right) + 2a^3c^3\left(\begin{cases} -\frac{ix^5}{4\sqrt{a^2x^2-1}} - \frac{ix^3}{8a^2\sqrt{a^2x^2-1}} + \frac{3ix}{8a^4\sqrt{a^2x^2-1}} - \frac{3i \\ \frac{x^5}{4\sqrt{-a^2x^2+1}} + \frac{x^3}{8a^2\sqrt{-a^2x^2+1}} - \frac{3x}{8a^4\sqrt{-a^2x^2+1}} + \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c)**3,x)
```

```
[Out] -a**4*c**3*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a*
*2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6
/6, True)) + 2*a**3*c**3*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**
3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*a
cosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x
**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*a
sin(a*x)/(8*a**5), True)) - 2*a*c**3*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2
*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x*
*2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + c
**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
```

3.309 $\int e^{\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=91

$$\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{8a}$$

```
[Out] 5/12*c^3*(-a^2*x^2+1)^(3/2)/a+1/4*c^3*(-a*x+1)*(-a^2*x^2+1)^(3/2)/a+5/8*c^3
*arcsin(a*x)/a+5/8*c^3*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 671, 641, 195, 216}

$$\frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a}+\frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{5}{8}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/8 + (5*c^3*(1 - a^2*x^2)^(3/2))/(12*a) + (c^3*(
1 - a*x)*(1 - a^2*x^2)^(3/2))/(4*a) + (5*c^3*ArcSin[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}(c-acx)^3\,dx = c\int (c-acx)^2\sqrt{1-a^2x^2}\,dx$$
$$= \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a} + \frac{1}{4}\left(5c^2\right)\int (c-acx)\sqrt{1-a^2x^2}\,dx$$
$$= \frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a} + \frac{1}{4}\left(5c^3\right)\int \sqrt{1-a^2x^2}\,dx$$
$$= \frac{5}{8}c^3x\sqrt{1-a^2x^2} + \frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a} + \frac{1}{8}\left(5c^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}$$
$$= \frac{5}{8}c^3x\sqrt{1-a^2x^2} + \frac{5c^3\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{c^3(1-ax)\left(1-a^2x^2\right)^{3/2}}{4a} + \frac{5c^3\sin^{-1}(ax)}{8a}$$

Mathematica [A] time = 0.09, size = 67, normalized size = 0.74

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(6a^3x^3-16a^2x^2+9ax+16\right)-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 30*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(24*a)
```

fricas [A] time = 0.51, size = 82, normalized size = 0.90

$$-\frac{30\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(6\,a^3c^3x^3-16\,a^2c^3x^2+9\,ac^3x+16\,c^3\right)\sqrt{-a^2x^2+1}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(30*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (6*a^3*c^3*x^3 - 16*
a^2*c^3*x^2 + 9*a*c^3*x + 16*c^3)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.30, size = 66, normalized size = 0.73

$$\frac{5\,c^3\arcsin\left(ax\right)\mathrm{sgn}(a)}{8\,|a|}+\frac{1}{24}\,\sqrt{-a^2x^2+1}\left(\frac{16\,c^3}{a}+\left(9\,c^3+2\left(3\,a^2c^3x-8\,ac^3\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 5/8*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/24*sqrt(-a^2*x^2 + 1)*(16*c^3/a + (9*
c^3 + 2*(3*a^2*c^3*x - 8*a*c^3)*x)*x)
```

maple [A] time = 0.04, size = 114, normalized size = 1.25

$$\frac{c^3a^2x^3\sqrt{-a^2x^2+1}}{4}+\frac{3c^3x\sqrt{-a^2x^2+1}}{8}+\frac{5c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8\sqrt{a^2}}-\frac{2c^3a\,x^2\sqrt{-a^2x^2+1}}{3}+\frac{2c^3\sqrt{-a^2x^2+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x)
```

```
[Out] 1/4*c^3*a^2*x^3*(-a^2*x^2+1)^(1/2)+3/8*c^3*x*(-a^2*x^2+1)^(1/2)+5/8*c^3/(a^
2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2/3*c^3*a*x^2*(-a^2*x^2+1
)^(1/2)+2/3*c^3*(-a^2*x^2+1)^(1/2)/a
```

maxima [A] time = 0.41, size = 95, normalized size = 1.04

$$\frac{1}{4}\,\sqrt{-a^2x^2+1}\,a^2c^3x^3-\frac{2}{3}\,\sqrt{-a^2x^2+1}\,ac^3x^2+\frac{3}{8}\,\sqrt{-a^2x^2+1}\,c^3x+\frac{5\,c^3\arcsin\left(ax\right)}{8\,a}+\frac{2\,\sqrt{-a^2x^2+1}\,c^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^3 - 2/3*sqrt(-a^2*x^2 + 1)*a*c^3*x^2 + 3/8
*sqrt(-a^2*x^2 + 1)*c^3*x + 5/8*c^3*arcsin(a*x)/a + 2/3*sqrt(-a^2*x^2 + 1)*
c^3/a
```

mupad [B] time = 0.00, size = 105, normalized size = 1.15

$$\frac{3\,c^3\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{5\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}+\frac{2\,c^3\,\sqrt{1-a^2\,x^2}}{3\,a}-\frac{2\,a\,c^3\,x^2\,\sqrt{1-a^2\,x^2}}{3}+\frac{a^2\,c^3\,x^3\,\sqrt{1-a^2\,x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (3*c^3*x*(1 - a^2*x^2)^(1/2))/8 + (5*c^3*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(
1/2)) + (2*c^3*(1 - a^2*x^2)^(1/2))/(3*a) - (2*a*c^3*x^2*(1 - a^2*x^2)^(1/2
))/3 + (a^2*c^3*x^3*(1 - a^2*x^2)^(1/2))/4
```

sympy [A] time = 6.40, size = 134, normalized size = 1.47

$$\begin{cases} \frac{2c^3\sqrt{-a^2x^2+1}+2c^3\left(\begin{cases}\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^2x^2+1} & \text{for } ax>-1\wedge ax<1\end{cases}\right)-c^3\left(\begin{cases}\frac{ax\left(-2a^2x^2+1\right)\sqrt{-a^2x^2+1}}{8}-\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{3\,\mathrm{asin}(}{8}\end{cases}\right.}{a} \\ c^3x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3,x)
```

```
[Out] Piecewise((((2*c**3*sqrt(-a**2*x**2 + 1) + 2*c**3*Piecewise(((-a**2*x**2 + 1
)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) - c**3*Piecewis
e((a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 - a*x*sqrt(-a**2*x**2 + 1)
/2 + 3*asin(a*x)/8, (a*x > -1) & (a*x < 1))) + c**3*asin(a*x))/a, Ne(a, 0))
, (c**3*x, True))
```

3.310 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x}\,dx$

Optimal. Leaf size=75

$$-\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}+c^3(1-ax)\sqrt{1-a^2x^2}-c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-c^3\sin^{-1}(ax)$$

```
[Out] -1/3*c^3*(-a^2*x^2+1)^(3/2)-c^3*arcsin(a*x)-c^3*arctanh((-a^2*x^2+1)^(1/2))
+c^3*(-a*x+1)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1809, 815, 844, 216, 266, 63, 208}

$$-\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2}+c^3(1-ax)\sqrt{1-a^2x^2}-c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-c^3\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x,x]
```

```
[Out] c^3*(1 - a*x)*Sqrt[1 - a^2*x^2] - (c^3*(1 - a^2*x^2)^(3/2))/3 - c^3*ArcSin[
a*x] - c^3*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x}\,dx = c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x}\,dx$$
$$= -\frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2} - \frac{c\int \frac{\left(-3a^2c^2+6a^3c^2x\right)\sqrt{1-a^2x^2}}{x}\,dx}{3a^2}$$
$$= c^3(1-ax)\sqrt{1-a^2x^2} - \frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2} + \frac{c\int \frac{6a^4c^2-6a^5c^2x}{x\sqrt{1-a^2x^2}}\,dx}{6a^4}$$
$$= c^3(1-ax)\sqrt{1-a^2x^2} - \frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2} + c^3\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(ac^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}$$
$$= c^3(1-ax)\sqrt{1-a^2x^2} - \frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2} - c^3\sin^{-1}(ax) + \frac{1}{2}c^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\right.$$
$$= c^3(1-ax)\sqrt{1-a^2x^2} - \frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2} - c^3\sin^{-1}(ax) - \frac{c^3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{\ }\right.}{a^2}$$
$$= c^3(1-ax)\sqrt{1-a^2x^2} - \frac{1}{3}c^3\left(1-a^2x^2\right)^{3/2} - c^3\sin^{-1}(ax) - c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.10, size = 135, normalized size = 1.80

$$\frac{c^3\left(-2a^4x^4+6a^3x^3-2a^2x^2+3\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+18\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-6\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-\right.\right.}{6\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x,x]
```

```
[Out] (c^3*(4 - 6*a*x - 2*a^2*x^2 + 6*a^3*x^3 - 2*a^4*x^4 + 3*Sqrt[1 - a^2*x^2]*A
rcSin[a*x] + 18*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] - 6*Sqrt[1
- a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(6*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.55, size = 88, normalized size = 1.17

$$2\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}\,-1}{ax}\right)+c^3\log\left(\frac{\sqrt{-a^2x^2+1}\,-1}{x}\right)+\frac{1}{3}\left(a^2c^3x^2-3\,ac^3x+2\,c^3\right)\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x,x, algorithm="fricas")
```

```
[Out] 2*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c^3*log((sqrt(-a^2*x^2 + 1)
- 1)/x) + 1/3*(a^2*c^3*x^2 - 3*a*c^3*x + 2*c^3)*sqrt(-a^2*x^2 + 1)
```

giac [A] time = 0.41, size = 95, normalized size = 1.27

$$-\frac{ac^3 \arcsin(ax)\,\mathrm{sgn}(a)}{|a|}-\frac{ac^3 \log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{1}{3}\sqrt{-a^2x^2+1}\left(2\,c^3+\left(a^2c^3x-3\,ac^3\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x,x, algorithm="giac")
```

```
[Out] -a*c^3*arcsin(a*x)*sgn(a)/abs(a) - a*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*
abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/3*sqrt(-a^2*x^2 + 1)*(2*c^3 + (a^2*c
^3*x - 3*a*c^3)*x)
```

maple [A] time = 0.04, size = 110, normalized size = 1.47

$$\frac{c^3a^2x^2\sqrt{-a^2x^2+1}}{3}+\frac{2c^3\sqrt{-a^2x^2+1}}{3}-c^3ax\sqrt{-a^2x^2+1}-\frac{c^3a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-c^3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x,x)
```

```
[Out] 1/3*c^3*a^2*x^2*(-a^2*x^2+1)^(1/2)+2/3*c^3*(-a^2*x^2+1)^(1/2)-c^3*a*x*(-a^2
*x^2+1)^(1/2)-c^3*a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c^
3*arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.49, size = 100, normalized size = 1.33

$$\frac{1}{3}\sqrt{-a^2x^2+1}\,a^2c^3x^2-\sqrt{-a^2x^2+1}\,ac^3x-c^3\arcsin(ax)-c^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{2}{3}\sqrt{-a^2x^2+1}\,c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^2 - sqrt(-a^2*x^2 + 1)*a*c^3*x - c^3*arcsi
n(a*x) - c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 2/3*sqrt(-a^2*x^
2 + 1)*c^3
```

mupad [B] time = 0.06, size = 110, normalized size = 1.47

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2\,a^4\,c^3}{3\,{\left(-a^2\right)}^{3/2}}+\frac{a^6\,c^3\,x^2}{3\,{\left(-a^2\right)}^{3/2}}-a\,c^3\,x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-c^3\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)-\frac{a\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - a*c*x)^3*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)

[Out] ((1 - a^2*x^2)^(1/2)*((2*a^4*c^3)/(3*(-a^2)^(3/2)) + (a^6*c^3*x^2)/(3*(-a^2)^(3/2)) - a*c^3*x*(-a^2)^(1/2)))/(-a^2)^(1/2) - c^3*atanh((1 - a^2*x^2)^(1/2)) - (a*c^3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)

sympy [C] time = 12.54, size = 226, normalized size = 3.01

$$-a^4c^3\left(\begin{cases} -\frac{x^2\sqrt{-a^2x^2+1}}{3a^2} - \frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}\right) + 2a^3c^3\left(\begin{cases} -\frac{ix\sqrt{a^2x^2-1}}{2a^2} - \frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{x^3}{2\sqrt{-a^2x^2+1}} - \frac{x}{2a^2\sqrt{-a^2x^2+1}} + \frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise} \end{cases}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x,x)

[Out] -a**4*c**3*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + 2*a**3*c**3*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) - 2*a*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + c**3*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))

3.311 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^2}\,dx$

Optimal. Leaf size=83

$$-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{x}-\frac{1}{2}ac^3(ax+4)\sqrt{1-a^2x^2}+2ac^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{1}{2}ac^3\sin^{-1}(ax)$$

```
[Out] -c^3*(-a^2*x^2+1)^(3/2)/x-1/2*a*c^3*arcsin(a*x)+2*a*c^3*arctanh((-a^2*x^2+1
)^(1/2))-1/2*a*c^3*(a*x+4)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 815, 844, 216, 266, 63, 208}

$$-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{x}-\frac{1}{2}ac^3(ax+4)\sqrt{1-a^2x^2}+2ac^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{1}{2}ac^3\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^2,x]
```

```
[Out] -(a*c^3*(4 + a*x)*Sqrt[1 - a^2*x^2])/2 - (c^3*(1 - a^2*x^2)^(3/2))/x - (a*c
^3*ArcSin[a*x])/2 + 2*a*c^3*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^2}\,dx = c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^2}\,dx$$

$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - c\int \frac{\left(2ac^2+a^2c^2x\right)\sqrt{1-a^2x^2}}{x}\,dx$$

$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} + \frac{c\int \frac{-4a^3c^2-a^4c^2x}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}$$

$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - \left(2ac^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \frac{1}{2}\left(a^2c^3\right)\int$$

$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - \frac{1}{2}ac^3\sin^{-1}(ax) - \left(ac^3\right)\text{Subst}\left(\int \frac{}{x\sqrt{1}}\right.$$

$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - \frac{1}{2}ac^3\sin^{-1}(ax) + \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\right.}{a}$$

$$= -\frac{1}{2}ac^3(4+ax)\sqrt{1-a^2x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{x} - \frac{1}{2}ac^3\sin^{-1}(ax) + 2ac^3\tanh^{-1}\left(\sqrt{1-a^2}\right.$$

Mathematica [A] time = 0.11, size = 143, normalized size = 1.72

$$-\frac{c^3\left(a^4x^4-4a^3x^3-3a^2x^2+2ax\sqrt{1-a^2x^2}\sin^{-1}(ax)+2ax\sqrt{1-a^2x^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-4ax\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1}\right.\right.}{2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^2,x]
```

```
[Out] -1/2*(c^3*(2 + 4*a*x - 3*a^2*x^2 - 4*a^3*x^3 + a^4*x^4 + 2*a*x*Sqrt[1 - a^2
*x^2]*ArcSin[a*x] + 2*a*x*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] -
 4*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(x*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.55, size = 104, normalized size = 1.25

$$\frac{2\,ac^3x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-4\,ac^3x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-4\,ac^3x+\left(a^2c^3x^2-4\,ac^3x-2\,c^3\right)\sqrt{-a^2x^2+1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^2,x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a*c^3*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 4*a*c^3*x*log((sqrt
(-a^2*x^2 + 1) - 1)/x) - 4*a*c^3*x + (a^2*c^3*x^2 - 4*a*c^3*x - 2*c^3)*sqrt
(-a^2*x^2 + 1))/x
```

giac [B] time = 0.27, size = 152, normalized size = 1.83

$$\frac{a^4c^3x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{a^2c^3\arcsin\left(ax\right)\operatorname{sgn}(a)}{2\,|a|}+\frac{2\,a^2c^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^3}{2\,x|a|}+\frac{1}{2}\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*c^3*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 1/2*a^2*c^3*arcsin
(a*x)*sgn(a)/abs(a) + 2*a^2*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
2*a)/(a^2*abs(x)))/abs(a) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(x*abs(
a)) + 1/2*(a^2*c^3*x - 4*a*c^3)*sqrt(-a^2*x^2 + 1)
```

maple [A] time = 0.04, size = 113, normalized size = 1.36

$$\frac{c^3a^2x\sqrt{-a^2x^2+1}}{2}-\frac{c^3a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}-2c^3a\sqrt{-a^2x^2+1}+2c^3a\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{c^3\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^2,x)
```

```
[Out] 1/2*c^3*a^2*x*(-a^2*x^2+1)^(1/2)-1/2*c^3*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)
*x/(-a^2*x^2+1)^(1/2))-2*c^3*a*(-a^2*x^2+1)^(1/2)+2*c^3*a*arctanh(1/(-a^2*x
^2+1)^(1/2))-c^3/x*(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.41, size = 102, normalized size = 1.23

$$\frac{1}{2}\,\sqrt{-a^2x^2+1}\,a^2c^3x-\frac{1}{2}\,ac^3\arcsin\left(ax\right)+2\,ac^3\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-2\,\sqrt{-a^2x^2+1}\,ac^3-\frac{\sqrt{-a^2x^2+1}\,c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*a^2*c^3*x - 1/2*a*c^3*arcsin(a*x) + 2*a*c^3*log(2*sq
rt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2*sqrt(-a^2*x^2 + 1)*a*c^3 - sqrt(-a^
2*x^2 + 1)*c^3/x
```

mupad [B] time = 0.78, size = 108, normalized size = 1.30

$$\frac{a^2\,c^3\,x\,\sqrt{1-a^2\,x^2}}{2}-\frac{c^3\,\sqrt{1-a^2\,x^2}}{x}-2\,a\,c^3\,\sqrt{1-a^2\,x^2}-\frac{a^2\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}-a\,c^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^2*c^3*x*(1 - a^2*x^2)^(1/2))/2 - (c^3*(1 - a^2*x^2)^(1/2))/x - a*c^3*ata
n((1 - a^2*x^2)^(1/2)*1i)*2i - 2*a*c^3*(1 - a^2*x^2)^(1/2) - (a^2*c^3*asinh
(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2))
```

sympy [C] time = 5.72, size = 199, normalized size = 2.40

$$-a^4c^3\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\,\mathrm{acosh}\,(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\mathrm{asin}\,(ax)}{2a^3} & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-2ac^3\left(\begin{cases}-\mathrm{acosh} \\ i\,\mathrm{asin}\left(\frac{1}{a}\right.\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**2,x)
```

```
[Out] -a**4*c**3*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a
**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(
-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + 2*a**3*c**3*Piecewise((x**2
/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) - 2*a*c**3*Piecewise(
(-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + c**3*Pi
ecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 +
 1)/x, True))
```

3.312 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^3}\,dx$

Optimal. Leaf size=92

$$-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{ac^3(ax+4)\sqrt{1-a^2x^2}}{2x}-\frac{1}{2}a^2c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2a^2c^3\sin^{-1}(ax)$$

```
[Out] -1/2*c^3*(-a^2*x^2+1)^(3/2)/x^2+2*a^2*c^3*arcsin(a*x)-1/2*a^2*c^3*arctanh((
-a^2*x^2+1)^(1/2))+1/2*a*c^3*(a*x+4)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.18, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 813, 844, 216, 266, 63, 208}

$$-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{ac^3(ax+4)\sqrt{1-a^2x^2}}{2x}-\frac{1}{2}a^2c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2a^2c^3\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^3,x]
```

```
[Out] (a*c^3*(4 + a*x)*Sqrt[1 - a^2*x^2])/(2*x) - (c^3*(1 - a^2*x^2)^(3/2))/(2*x^
2) + 2*a^2*c^3*ArcSin[a*x] - (a^2*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^3}\,dx = c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^3}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}c\int \frac{\left(4ac^2-a^2c^2x\right)\sqrt{1-a^2x^2}}{x^2}\,dx$$
$$= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{1}{4}c\int \frac{2a^2c^2+8a^3c^2x}{x\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{1}{2}\left(a^2c^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx + \left(2a^3c^3\right)\int$$
$$= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2} + 2a^2c^3\sin^{-1}(ax) + \frac{1}{4}\left(a^2c^3\right)\text{Subst}\left(\int \frac{}{x}\right.$$
$$= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2} + 2a^2c^3\sin^{-1}(ax) - \frac{1}{2}c^3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\right.$$
$$= \frac{ac^3(4+ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{2x^2} + 2a^2c^3\sin^{-1}(ax) - \frac{1}{2}a^2c^3\tanh^{-1}\left(\sqrt{1-a\right.$$

Mathematica [A] time = 0.12, size = 155, normalized size = 1.68

$$\frac{c^3\left(-4a^4x^4-8a^3x^3+6a^2x^2+a^2x^2\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-14a^2x^2\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-2a^2x^2\sqrt{1-a^2x^2}\,\tan\right.}{4x^2\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^3,x]
```

```
[Out] (c^3*(-2 + 8*a*x + 6*a^2*x^2 - 8*a^3*x^3 - 4*a^4*x^4 + a^2*x^2*Sqrt[1 - a^2
*x^2]*ArcSin[a*x] - 14*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[
2]] - 2*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(4*x^2*Sqrt[
1 - a^2*x^2])
```

fricas [A] time = 0.53, size = 118, normalized size = 1.28

$$-\frac{8\,a^2c^3x^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-a^2c^3x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-2\,a^2c^3x^2-\left(2\,a^2c^3x^2+4\,ac^3x-c^3\right)\sqrt{-a^2x^2+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^3,x, algorithm="fricas"
)
```

```
[Out] -1/2*(8*a^2*c^3*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a^2*c^3*x^2*lo
g((sqrt(-a^2*x^2 + 1) - 1)/x) - 2*a^2*c^3*x^2 - (2*a^2*c^3*x^2 + 4*a*c^3*x
- c^3)*sqrt(-a^2*x^2 + 1))/x^2
```

giac [B] time = 0.22, size = 212, normalized size = 2.30

$$\frac{2\,a^3c^3\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}-\frac{a^3c^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\sqrt{-a^2x^2+1}\,a^2c^3+\frac{\left(a^3c^3-\frac{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)ac^3}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}+\frac{8\left(\sqrt{-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^3,x, algorithm="giac")
```

```
[Out] 2*a^3*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/2*a^3*c^3*log(1/2*abs(-2*sqrt(-a^2*
x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*a^2*c^3 +
1/8*(a^3*c^3 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*c^3/x)*a^4*x^2/((sqrt(-a
^2*x^2 + 1)*abs(a) + a)^2*abs(a)) + 1/8*(8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*
a*c^3*abs(a)/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*abs(a)/(a*x^2))/a^2
```

maple [A] time = 0.04, size = 116, normalized size = 1.26

$$c^3a^2\sqrt{-a^2x^2+1}+\frac{2c^3a^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{2c^3a\sqrt{-a^2x^2+1}}{x}-\frac{c^3\sqrt{-a^2x^2+1}}{2x^2}-\frac{c^3a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^3,x)
```

```
[Out] c^3*a^2*(-a^2*x^2+1)^(1/2)+2*c^3*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*x^2+1)^(1/2))+2*c^3*a/x*(-a^2*x^2+1)^(1/2)-1/2*c^3/x^2*(-a^2*x^2+1)^(1/2)-
1/2*c^3*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 107, normalized size = 1.16

$$2\,a^2c^3\arcsin\left(ax\right)-\frac{1}{2}\,a^2c^3\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^2x^2+1}\,a^2c^3+\frac{2\,\sqrt{-a^2x^2+1}\,ac^3}{x}-\frac{\sqrt{-a^2x^2+1}\,c^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^3,x, algorithm="maxima"
)
```

```
[Out] 2*a^2*c^3*arcsin(a*x) - 1/2*a^2*c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs
(x)) + sqrt(-a^2*x^2 + 1)*a^2*c^3 + 2*sqrt(-a^2*x^2 + 1)*a*c^3/x - 1/2*sqrt
(-a^2*x^2 + 1)*c^3/x^2
```

mupad [B] time = 0.78, size = 111, normalized size = 1.21

$$a^2\,c^3\,\sqrt{1-a^2\,x^2}-\frac{c^3\,\sqrt{1-a^2\,x^2}}{2\,x^2}+\frac{2\,a\,c^3\,\sqrt{1-a^2\,x^2}}{x}+\frac{2\,a^3\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{a^2\,c^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^2*c^3*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/2 + a^2*c^3*(1 - a^2*x^2)^(1/2) -
 (c^3*(1 - a^2*x^2)^(1/2))/(2*x^2) + (2*a*c^3*(1 - a^2*x^2)^(1/2))/x + (2*a
^3*c^3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [C] time = 5.01, size = 228, normalized size = 2.48

$$-a^4c^3\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)-2ac^3\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } \left|a^2x^2\right.\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwis}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**3,x)
```

```
[Out] -a**4*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, Tr
ue)) + 2*a**3*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0),
(sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 2*a*c**3*Piecewise((-I*sq
rt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))
+ c**3*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x)
, 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**
2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

3.313 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^4}\,dx$

Optimal. Leaf size=88

$$-a^3c^3\sin^{-1}(ax)+\frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}-a^3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/3*c^3*(-a^2*x^2+1)^(3/2)/x^3-a^3*c^3*arcsin(a*x)-a^3*c^3*arctanh((-a^2*x
^2+1)^(1/2))+a*c^3*(-a*x+1)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.18, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 811, 844, 216, 266, 63, 208}

$$-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2}-a^3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-a^3c^3\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^4,x]
```

```
[Out] (a*c^3*(1 - a*x)*Sqrt[1 - a^2*x^2])/x^2 - (c^3*(1 - a^2*x^2)^(3/2))/(3*x^3)
 - a^3*c^3*ArcSin[a*x] - a^3*c^3*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^4}\,dx = c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^4}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{3}c\int \frac{\left(6ac^2-3a^2c^2x\right)\sqrt{1-a^2x^2}}{x^3}\,dx$$
$$= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{12}c\int \frac{12a^3c^2-12a^4c^2x}{x\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3} + \left(a^3c^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - \left(a^4c^3\right)\int \frac{}{\sqrt{1-}}$$
$$= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3} - a^3c^3\sin^{-1}(ax) + \frac{1}{2}\left(a^3c^3\right)\text{Subst}\left(\int \frac{}{x\sqrt{1}}\right.$$
$$= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3} - a^3c^3\sin^{-1}(ax) - \left(ac^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\right.$$
$$= \frac{ac^3(1-ax)\sqrt{1-a^2x^2}}{x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{3x^3} - a^3c^3\sin^{-1}(ax) - a^3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.11, size = 156, normalized size = 1.77

$$\frac{c^3\left(4a^4x^4-6a^3x^3-2a^2x^2+3a^3x^3\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+18a^3x^3\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-6a^3x^3\sqrt{1-a^2x^2}\,\tanh\right.}{6x^3\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^4,x]
```

```
[Out] (c^3*(-2 + 6*a*x - 2*a^2*x^2 - 6*a^3*x^3 + 4*a^4*x^4 + 3*a^3*x^3*Sqrt[1 - a
^2*x^2]*ArcSin[a*x] + 18*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqr
t[2]] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(6*x^3*Sqr
t[1 - a^2*x^2])
```

fricas [A] time = 0.50, size = 105, normalized size = 1.19

$$\frac{6\,a^3c^3x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,a^3c^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(2\,a^2c^3x^2-3\,ac^3x+c^3\right)\sqrt{-a^2x^2+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/3*(6*a^3*c^3*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a^3*c^3*x^3*l
og((sqrt(-a^2*x^2 + 1) - 1)/x) - (2*a^2*c^3*x^2 - 3*a*c^3*x + c^3)*sqrt(-a^
2*x^2 + 1))/x^3
```

giac [B] time = 0.20, size = 250, normalized size = 2.84

$$-\frac{a^4c^3\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}-\frac{a^4c^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{|a|}+\frac{\left(a^4c^3-\frac{6\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2c^3}{x}+\frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^3}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^4,x, algorithm="giac")
```

```
[Out] -a^4*c^3*arcsin(a*x)*sgn(a)/abs(a) - a^4*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 +
 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/24*(a^4*c^3 - 6*(sqrt(-a^2*x^2 +
 1)*abs(a) + a)*a^2*c^3/x + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2)*a^
6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*abs(a)) - 1/24*(9*(sqrt(-a^2*x^2 +
 1)*abs(a) + a)*a^4*c^3/x - 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^3/x^2
 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/x^3)/(a^2*abs(a))
```

maple [A] time = 0.04, size = 119, normalized size = 1.35

$$-\frac{c^3a^4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-c^3a^3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{c^3\sqrt{-a^2x^2+1}}{3x^3}-\frac{2c^3a^2\sqrt{-a^2x^2+1}}{3x}+\frac{c^3a\sqrt{-a^2x^2+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^4,x)
```

```
[Out] -c^3*a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c^3*a^3*arcta
nh(1/(-a^2*x^2+1)^(1/2))-1/3*c^3/x^3*(-a^2*x^2+1)^(1/2)-2/3*c^3*a^2/x*(-a^2
*x^2+1)^(1/2)+c^3*a/x^2*(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.44, size = 110, normalized size = 1.25

$$-a^3c^3\arcsin(ax)-a^3c^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{2\sqrt{-a^2x^2+1}\,a^2c^3}{3\,x}+\frac{\sqrt{-a^2x^2+1}\,ac^3}{x^2}-\frac{\sqrt{-a^2x^2+1}\,c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^4,x, algorithm="maxima"
)
```

```
[Out] -a^3*c^3*arcsin(a*x) - a^3*c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))
- 2/3*sqrt(-a^2*x^2 + 1)*a^2*c^3/x + sqrt(-a^2*x^2 + 1)*a*c^3/x^2 - 1/3*sqr
t(-a^2*x^2 + 1)*c^3/x^3
```

mupad [B] time = 0.04, size = 114, normalized size = 1.30

$$\frac{a\,c^3\,\sqrt{1-a^2\,x^2}}{x^2}-\frac{c^3\,\sqrt{1-a^2\,x^2}}{3\,x^3}-\frac{a^4\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{2\,a^2\,c^3\,\sqrt{1-a^2\,x^2}}{3\,x}+a^3\,c^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(x^4*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] a^3*c^3*atan((1 - a^2*x^2)^(1/2)*1i)*1i - (c^3*(1 - a^2*x^2)^(1/2))/(3*x^3)
 + (a*c^3*(1 - a^2*x^2)^(1/2))/x^2 - (a^4*c^3*asinh(x*(-a^2)^(1/2)))/(-a^2)
^(1/2) - (2*a^2*c^3*(1 - a^2*x^2)^(1/2))/(3*x)
```

sympy [C] time = 5.85, size = 279, normalized size = 3.17

$$-a^4c^3\left(\begin{cases}\sqrt{\frac{1}{a^2}}\,\mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\,\mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)+2a^3c^3\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)-2ac^3\left(\begin{cases}-\frac{a^2\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-\ldots}}{\ldots} & \\ \frac{ia^2\,\mathrm{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\ldots}} & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**4,x)
```

```
[Out] -a**4*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1
/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*a**3*c**3*Piecewise((-acosh(1/(
a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) - 2*a*c**3*Piecewise
((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**
2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2
*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) + c**3*Piecewise((-2*I*a**2*sqrt(a
**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1),
(-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True))
```

3.314 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^5} dx$

Optimal. Leaf size=102

$$-\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3}+\frac{5}{8}a^4c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/4*c^3*(-a^2*x^2+1)^(3/2)/x^4+2/3*a*c^3*(-a^2*x^2+1)^(3/2)/x^3+5/8*a^4*c^
3*arctanh((-a^2*x^2+1)^(1/2))-5/8*a^2*c^3*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 1807, 807, 266, 47, 63, 208}

$$-\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2}+\frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4}+\frac{5}{8}a^4c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^5,x]
```

```
[Out] (-5*a^2*c^3*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^3*(1 - a^2*x^2)^(3/2))/(4*x^4)
+ (2*a*c^3*(1 - a^2*x^2)^(3/2))/(3*x^3) + (5*a^4*c^3*ArcTanh[Sqrt[1 - a^2*x
^2]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^5}\,dx = c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^5}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{1}{4}c\int \frac{\left(8ac^2-5a^2c^2x\right)\sqrt{1-a^2x^2}}{x^4}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{4}\left(5a^2c^3\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{8}\left(5a^2c^3\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{16}\left(5a^4c^3\right)\text{Subst}\left(\int \frac{}{x}\right.$$
$$= -\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{1}{8}\left(5a^2c^3\right)\text{Subst}\left(\int \frac{}{\frac{1}{a^2}}\right.$$
$$= -\frac{5a^2c^3\sqrt{1-a^2x^2}}{8x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{2ac^3\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{5}{8}a^4c^3\tanh^{-1}\left(\sqrt{1-a^2}\right.$$

Mathematica [A] time = 0.03, size = 99, normalized size = 0.97

$$\frac{c^3\left(16a^5x^5+9a^4x^4-32a^3x^3-3a^2x^2+15a^4x^4\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+16ax-6\right)}{24x^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^5,x]
```

```
[Out] (c^3*(-6 + 16*a*x - 3*a^2*x^2 - 32*a^3*x^3 + 9*a^4*x^4 + 16*a^5*x^5 + 15*a^
4*x^4*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(24*x^4*Sqrt[1 - a^2*x
^2])
```

fricas [A] time = 0.45, size = 84, normalized size = 0.82

$$-\frac{15\,a^4c^3x^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(16\,a^3c^3x^3+9\,a^2c^3x^2-16\,ac^3x+6\,c^3\right)\sqrt{-a^2x^2+1}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^5,x, algorithm="fricas"
)
```

```
[Out] -1/24*(15*a^4*c^3*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (16*a^3*c^3*x^3 + 9
*a^2*c^3*x^2 - 16*a*c^3*x + 6*c^3)*sqrt(-a^2*x^2 + 1))/x^4
```

giac [B] time = 0.42, size = 300, normalized size = 2.94

$$\frac{\left(3\,a^5c^3 - \frac{16\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^3c^3}{x} + \frac{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2ac^3}{x^2} + \frac{48\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^3}{ax^3}\right)a^8x^4}{192\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4|a|} + \frac{5\,a^5c^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^5*c^3 - 16*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3*c^3/x + 24*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^2*a*c^3/x^2 + 48*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^
3*c^3/(a*x^3))*a^8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)) + 5/8*a^5
*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) -
 1/192*(48*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c^3*abs(a)/x + 24*(sqrt(-a^2
*x^2 + 1)*abs(a) + a)^2*a^3*c^3*abs(a)/x^2 - 16*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^3*a*c^3*abs(a)/x^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3*abs(a)/(a
*x^4))/a^4
```

maple [A] time = 0.04, size = 144, normalized size = 1.41

$$-c^3\left(-a^4\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + \frac{2a^3\sqrt{-a^2x^2+1}}{x} + 2a\left(-\frac{\sqrt{-a^2x^2+1}}{3x^3} - \frac{2a^2\sqrt{-a^2x^2+1}}{3x}\right) + \frac{\sqrt{-a^2x^2+1}}{4x^4} - \frac{3a}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^5,x)
```

```
[Out] -c^3*(-a^4*arctanh(1/(-a^2*x^2+1)^(1/2))+2*a^3*(-a^2*x^2+1)^(1/2)/x+2*a*(-1
/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x)+1/4*(-a^2*x^2+1)^(1
/2)/x^4-3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)
^(1/2))))
```

maxima [A] time = 0.43, size = 122, normalized size = 1.20

$$\frac{5}{8}\,a^4c^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{2\sqrt{-a^2x^2+1}\,a^3c^3}{3\,x} - \frac{3\sqrt{-a^2x^2+1}\,a^2c^3}{8\,x^2} + \frac{2\sqrt{-a^2x^2+1}\,ac^3}{3\,x^3} - \frac{\sqrt{-a^2x^2+1}\,c^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^5,x, algorithm="maxima"
)
```

```
[Out] 5/8*a^4*c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/3*sqrt(-a^2*x^2
 + 1)*a^3*c^3/x - 3/8*sqrt(-a^2*x^2 + 1)*a^2*c^3/x^2 + 2/3*sqrt(-a^2*x^2 +
1)*a*c^3/x^3 - 1/4*sqrt(-a^2*x^2 + 1)*c^3/x^4
```

mupad [B] time = 0.04, size = 113, normalized size = 1.11

$$\frac{2\,a\,c^3\,\sqrt{1-a^2\,x^2}}{3\,x^3}-\frac{c^3\,\sqrt{1-a^2\,x^2}}{4\,x^4}-\frac{3\,a^2\,c^3\,\sqrt{1-a^2\,x^2}}{8\,x^2}-\frac{2\,a^3\,c^3\,\sqrt{1-a^2\,x^2}}{3\,x}-\frac{a^4\,c^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,5\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(x^5*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (2*a*c^3*(1 - a^2*x^2)^(1/2))/(3*x^3) - (c^3*(1 - a^2*x^2)^(1/2))/(4*x^4) -
 (a^4*c^3*atan((1 - a^2*x^2)^(1/2)*1i)*5i)/8 - (3*a^2*c^3*(1 - a^2*x^2)^(1/
2))/(8*x^2) - (2*a^3*c^3*(1 - a^2*x^2)^(1/2))/(3*x)
```

sympy [C] time = 7.34, size = 347, normalized size = 3.40

$$-a^4c^3\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)-2ac^3\left(\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3}\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**5,x)
```

```
[Out] -a**4*c**3*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*
x)), True)) + 2*a**3*c**3*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**
2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)) - 2*a*c**3*Piecewise((-2*I*a**2*s
qrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) >
 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), T
rue)) + c**3*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/
(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 +
 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a*
*3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I
/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

3.315 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^6}\,dx$

Optimal. Leaf size=129

$$-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5}+\frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4}-\frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{1}{4}a^5c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2}$$

```
[Out] -1/5*c^3*(-a^2*x^2+1)^(3/2)/x^5+1/2*a*c^3*(-a^2*x^2+1)^(3/2)/x^4-7/15*a^2*c
^3*(-a^2*x^2+1)^(3/2)/x^3-1/4*a^5*c^3*arctanh((-a^2*x^2+1)^(1/2))+1/4*a^3*c
^3*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.19, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2}-\frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4}-\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{1}{4}a^5c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^6,x]
```

```
[Out] (a^3*c^3*Sqrt[1 - a^2*x^2])/(4*x^2) - (c^3*(1 - a^2*x^2)^(3/2))/(5*x^5) + (
a*c^3*(1 - a^2*x^2)^(3/2))/(2*x^4) - (7*a^2*c^3*(1 - a^2*x^2)^(3/2))/(15*x^
3) - (a^5*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^3}{x^6}\,dx = c\int \frac{(c-acx)^2\sqrt{1-a^2x^2}}{x^6}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{1}{5}c\int \frac{\left(10ac^2-7a^2c^2x\right)\sqrt{1-a^2x^2}}{x^5}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} + \frac{1}{20}c\int \frac{\left(28a^2c^2-10a^3c^2x\right)\sqrt{1-a^2x^2}}{x^4}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{2}\left(a^3c^3\right)\int \frac{\sqrt{1-a^2x^2}}{x^3}$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{4}\left(a^3c^3\right)\text{Subst}\Bigg(\int \frac{\sqrt{1}}{}$$
$$= \frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{8}\left(a^5c^3\right.$$
$$= \frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{4}\left(a^3c^3\right.$$
$$= \frac{a^3c^3\sqrt{1-a^2x^2}}{4x^2} - \frac{c^3\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{ac^3\left(1-a^2x^2\right)^{3/2}}{2x^4} - \frac{7a^2c^3\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{4}a^5c^3\,\mathrm{t}$$

Mathematica [A] time = 0.04, size = 107, normalized size = 0.83

$$-\frac{c^3\left(28a^6x^6-15a^5x^5-44a^4x^4+45a^3x^3+4a^2x^2+15a^5x^5\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-30ax+12\right)}{60x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^3)/x^6,x]
```

```
[Out] -1/60*(c^3*(12 - 30*a*x + 4*a^2*x^2 + 45*a^3*x^3 - 44*a^4*x^4 - 15*a^5*x^5
+ 28*a^6*x^6 + 15*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(x
^5*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.42, size = 95, normalized size = 0.74

$$\frac{15\,a^5c^3x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(28\,a^4c^3x^4-15\,a^3c^3x^3-16\,a^2c^3x^2+30\,ac^3x-12\,c^3\right)\sqrt{-a^2x^2+1}}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/60*(15*a^5*c^3*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (28*a^4*c^3*x^4 - 15
*a^3*c^3*x^3 - 16*a^2*c^3*x^2 + 30*a*c^3*x - 12*c^3)*sqrt(-a^2*x^2 + 1))/x^
5
```

giac [B] time = 0.24, size = 297, normalized size = 2.30

$$\frac{\left(3\,a^6c^3-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^4c^3}{x}+\frac{25\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2a^2c^3}{x^2}-\frac{90\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^3}{a^2x^4}\right)a^{10}x^5}{480\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5|a|}-\frac{a^6c^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{4\,|a|}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^6*c^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^3/x + 25*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^3/x^2 - 90*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^4*c^3/(a^2*x^4))*a^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 1/
4*a^6*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs
(a) + 1/480*(90*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^8*c^3/x - 25*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^3*a^4*c^3/x^3 + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*a^
2*c^3/x^4 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/x^5)/(a^4*abs(a))
```

maple [A] time = 0.04, size = 190, normalized size = 1.47

$$-c^3\left(-\frac{a^4\sqrt{-a^2x^2+1}}{x}-2a^3\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)+2a\left(-\frac{\sqrt{-a^2x^2+1}}{4x^4}+\frac{3a^2\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^6,x)
```

```
[Out] -c^3*(-a^4/x*(-a^2*x^2+1)^(1/2)-2*a^3*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*
arctanh(1/(-a^2*x^2+1)^(1/2)))+2*a*(-1/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1
/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))+1/5/x^5*(
-a^2*x^2+1)^(1/2)-4/5*a^2*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)
^(1/2)/x))
```

maxima [A] time = 0.40, size = 145, normalized size = 1.12

$$-\frac{1}{4}a^5c^3\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{7\sqrt{-a^2x^2+1}\,a^4c^3}{15\,x}-\frac{\sqrt{-a^2x^2+1}\,a^3c^3}{4\,x^2}-\frac{4\sqrt{-a^2x^2+1}\,a^2c^3}{15\,x^3}+\frac{\sqrt{-a^2x^2+1}\,ac^3}{2\,x^4}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^3/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/4*a^5*c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 7/15*sqrt(-a^2*x
^2 + 1)*a^4*c^3/x - 1/4*sqrt(-a^2*x^2 + 1)*a^3*c^3/x^2 - 4/15*sqrt(-a^2*x^2
 + 1)*a^2*c^3/x^3 + 1/2*sqrt(-a^2*x^2 + 1)*a*c^3/x^4 - 1/5*sqrt(-a^2*x^2 +
1)*c^3/x^5
```

mupad [B] time = 0.79, size = 136, normalized size = 1.05

$$\frac{a\,c^3\sqrt{1-a^2\,x^2}}{2\,x^4}-\frac{c^3\sqrt{1-a^2\,x^2}}{5\,x^5}-\frac{4\,a^2\,c^3\sqrt{1-a^2\,x^2}}{15\,x^3}-\frac{a^3\,c^3\sqrt{1-a^2\,x^2}}{4\,x^2}+\frac{7\,a^4\,c^3\sqrt{1-a^2\,x^2}}{15\,x}+\frac{a^5\,c^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\cdots\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(x^6*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^5*c^3*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/4 - (c^3*(1 - a^2*x^2)^(1/2))/(5*
x^5) + (a*c^3*(1 - a^2*x^2)^(1/2))/(2*x^4) - (4*a^2*c^3*(1 - a^2*x^2)^(1/2)
)/(15*x^3) - (a^3*c^3*(1 - a^2*x^2)^(1/2))/(4*x^2) + (7*a^4*c^3*(1 - a^2*x^
2)^(1/2))/(15*x)
```

sympy [C] time = 8.26, size = 476, normalized size = 3.69

$$-a^4c^3\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+2a^3c^3\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{\left|a^2x^2\right|}>1\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)-2ac^3\left(\cdots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**3/x**6,x)
```

```
[Out] -a**4*c**3*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt
(-a**2*x**2 + 1)/x, True)) + 2*a**3*c**3*Piecewise((-a**2*acosh(1/(a*x))/2
```

```
- a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(
a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*
x**2))), True)) - 2*a*c**3*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*
x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*
x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*
x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a*
*2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True)) + c**3*Piecewise(
(-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2))/(15*
x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1), (-8*I*a
**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(15*x**2)
 - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))
```

3.316 $\int e^{\tanh^{-1}(ax)} x^3 (c - acx)^4 \, dx$

Optimal. Leaf size=173

$$-\frac{29c^4\sin^{-1}(ax)}{128a^4}-\frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2}+\frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2}-\frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2}+\frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a}-\frac{c^4(2432}{}$$

```
[Out] -19/35*c^4*x^2*(-a^2*x^2+1)^(3/2)/a^2+29/48*c^4*x^3*(-a^2*x^2+1)^(3/2)/a-3/
7*c^4*x^4*(-a^2*x^2+1)^(3/2)+1/8*a*c^4*x^5*(-a^2*x^2+1)^(3/2)-1/6720*c^4*(-
3045*a*x+2432)*(-a^2*x^2+1)^(3/2)/a^4-29/128*c^4*arcsin(a*x)/a^4-29/128*c^4
*x*(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.33, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1809, 833, 780, 195, 216}

$$\frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2}-\frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2}+\frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a}-\frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2}-\frac{29c^4x\sqrt{1-a^2x^2}}{128a^3}-\frac{c^4(2432}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(c - a*c*x)^4,x]
```

```
[Out] (-29*c^4*x*Sqrt[1 - a^2*x^2])/(128*a^3) - (19*c^4*x^2*(1 - a^2*x^2)^(3/2))/
(35*a^2) + (29*c^4*x^3*(1 - a^2*x^2)^(3/2))/(48*a) - (3*c^4*x^4*(1 - a^2*x^
2)^(3/2))/7 + (a*c^4*x^5*(1 - a^2*x^2)^(3/2))/8 - (c^4*(2432 - 3045*a*x)*(1
 - a^2*x^2)^(3/2))/(6720*a^4) - (29*c^4*ArcSin[a*x])/(128*a^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^3(c-acx)^4\,dx = c\int x^3(c-acx)^3\sqrt{1-a^2x^2}\,dx$$
$$= \frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2} - \frac{c\int x^3\sqrt{1-a^2x^2}\,\left(-8a^2c^3+29a^3c^3x-24a^4c^3x^2\right)dx}{8a^2}$$
$$= -\frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2} + \frac{c\int x^3\left(152a^4c^3-203a^5c^3x\right)\sqrt{1-a^2}}{56a^4}$$
$$= \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{8}ac^4x^5\left(1-a^2x^2\right)^{3/2} - \frac{c\int x^2\left(609a^5c\right.}{}$$
$$= -\frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{8}ac^4x^5\left(1-a\right.$$
$$= -\frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{8}ac^4x^5\left(1-a\right.$$
$$= -\frac{29c^4x\sqrt{1-a^2x^2}}{128a^3} - \frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right.$$
$$= -\frac{29c^4x\sqrt{1-a^2x^2}}{128a^3} - \frac{19c^4x^2\left(1-a^2x^2\right)^{3/2}}{35a^2} + \frac{29c^4x^3\left(1-a^2x^2\right)^{3/2}}{48a} - \frac{3}{7}c^4x^4\left(1-a^2x^2\right.$$

Mathematica [A] time = 0.23, size = 99, normalized size = 0.57

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\,\left(1680a^7x^7-5760a^6x^6+6440a^5x^5-1536a^4x^4-2030a^3x^3+2432a^2x^2-3045ax+4864\right)-6090\right.}{13440a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a*c*x)^4,x]
```

```
[Out] -1/13440*(c^4*(Sqrt[1 - a^2*x^2]*(4864 - 3045*a*x + 2432*a^2*x^2 - 2030*a^3
*x^3 - 1536*a^4*x^4 + 6440*a^5*x^5 - 5760*a^6*x^6 + 1680*a^7*x^7) - 6090*Ar
cSin[Sqrt[1 - a*x]/Sqrt[2]]))/a^4
```

fricas [A] time = 0.44, size = 126, normalized size = 0.73

$$\frac{6090\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(1680\,a^7c^4x^7-5760\,a^6c^4x^6+6440\,a^5c^4x^5-1536\,a^4c^4x^4-2030\,a^3c^4x^3+2432\,a^2c\right.}{13440\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/13440*(6090*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (1680*a^7*c^4*x^
7 - 5760*a^6*c^4*x^6 + 6440*a^5*c^4*x^5 - 1536*a^4*c^4*x^4 - 2030*a^3*c^4*x
^3 + 2432*a^2*c^4*x^2 - 3045*a*c^4*x + 4864*c^4)*sqrt(-a^2*x^2 + 1))/a^4
```

giac [A] time = 0.21, size = 117, normalized size = 0.68

$$-\frac{29\,c^4\arcsin(ax)\,\mathrm{sgn}(a)}{128\,a^3|a|}-\frac{1}{13440}\sqrt{-a^2x^2+1}\left(\left(2\left(\frac{1216\,c^4}{a^2}-\left(\frac{1015\,c^4}{a}+4\left(192\,c^4-5\left(161\,ac^4+6\left(7\,a^3c^4x-24\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -29/128*c^4*arcsin(a*x)*sgn(a)/(a^3*abs(a)) - 1/13440*sqrt(-a^2*x^2 + 1)*((
2*(1216*c^4/a^2 - (1015*c^4/a + 4*(192*c^4 - 5*(161*a*c^4 + 6*(7*a^3*c^4*x
- 24*a^2*c^4)*x)*x)*x)*x)*x - 3045*c^4/a^3)*x + 4864*c^4/a^4)
```

maple [A] time = 0.07, size = 209, normalized size = 1.21

$$-\frac{c^4a^3x^7\sqrt{-a^2x^2+1}}{8}-\frac{23c^4a\,x^5\sqrt{-a^2x^2+1}}{48}+\frac{29c^4x^3\sqrt{-a^2x^2+1}}{192a}+\frac{29c^4x\sqrt{-a^2x^2+1}}{128a^3}-\frac{29c^4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{128a^3\sqrt{a^2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^4,x)
```

```
[Out] -1/8*c^4*a^3*x^7*(-a^2*x^2+1)^(1/2)-23/48*c^4*a*x^5*(-a^2*x^2+1)^(1/2)+29/1
92*c^4/a*x^3*(-a^2*x^2+1)^(1/2)+29/128*c^4*x*(-a^2*x^2+1)^(1/2)/a^3-29/128*
c^4/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3/7*c^4*a^2*x^
6*(-a^2*x^2+1)^(1/2)+4/35*c^4*x^4*(-a^2*x^2+1)^(1/2)-19/105*c^4*x^2/a^2*(-a
^2*x^2+1)^(1/2)-38/105*c^4/a^4*(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.41, size = 187, normalized size = 1.08

$$-\frac{1}{8}\sqrt{-a^2x^2+1}\,a^3c^4x^7+\frac{3}{7}\sqrt{-a^2x^2+1}\,a^2c^4x^6-\frac{23}{48}\sqrt{-a^2x^2+1}\,ac^4x^5+\frac{4}{35}\sqrt{-a^2x^2+1}\,c^4x^4+\frac{29\sqrt{-a^2x^2+1}\,c^4x^3}{192\,a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] -1/8*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^7 + 3/7*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^6 -
23/48*sqrt(-a^2*x^2 + 1)*a*c^4*x^5 + 4/35*sqrt(-a^2*x^2 + 1)*c^4*x^4 + 29/1
```

```
92*sqrt(-a^2*x^2 + 1)*c^4*x^3/a - 19/105*sqrt(-a^2*x^2 + 1)*c^4*x^2/a^2 + 2
9/128*sqrt(-a^2*x^2 + 1)*c^4*x/a^3 - 29/128*c^4*arcsin(a*x)/a^4 - 38/105*sq
rt(-a^2*x^2 + 1)*c^4/a^4
```

mupad [B] time = 0.06, size = 200, normalized size = 1.16

$$\frac{4\,c^4\,x^4\,\sqrt{1-a^2\,x^2}}{35}-\frac{38\,c^4\,\sqrt{1-a^2\,x^2}}{105\,a^4}+\frac{29\,c^4\,x\,\sqrt{1-a^2\,x^2}}{128\,a^3}-\frac{23\,a\,c^4\,x^5\,\sqrt{1-a^2\,x^2}}{48}-\frac{29\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{128\,a^3\,\sqrt{-a^2}}+\frac{29\,c^4\,x^3\,\cdots}{19\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^4*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (4*c^4*x^4*(1 - a^2*x^2)^(1/2))/35 - (38*c^4*(1 - a^2*x^2)^(1/2))/(105*a^4)
 + (29*c^4*x*(1 - a^2*x^2)^(1/2))/(128*a^3) - (23*a*c^4*x^5*(1 - a^2*x^2)^(
1/2))/48 - (29*c^4*asinh(x*(-a^2)^(1/2)))/(128*a^3*(-a^2)^(1/2)) + (29*c^4*
x^3*(1 - a^2*x^2)^(1/2))/(192*a) - (19*c^4*x^2*(1 - a^2*x^2)^(1/2))/(105*a^
2) + (3*a^2*c^4*x^6*(1 - a^2*x^2)^(1/2))/7 - (a^3*c^4*x^7*(1 - a^2*x^2)^(1/
2))/8
```

sympy [A] time = 18.31, size = 842, normalized size = 4.87

$$a^5c^4\left(\begin{cases} -\frac{ix^9}{8\sqrt{a^2x^2-1}}-\frac{ix^7}{48a^2\sqrt{a^2x^2-1}}-\frac{7ix^5}{192a^4\sqrt{a^2x^2-1}}-\frac{35ix^3}{384a^6\sqrt{a^2x^2-1}}+\frac{35ix}{128a^8\sqrt{a^2x^2-1}}-\frac{35i\,\mathrm{acosh}\,(ax)}{128a^9} & \text{for } \left|a^2x^2\right|>1 \\ \frac{x^9}{8\sqrt{-a^2x^2+1}}+\frac{x^7}{48a^2\sqrt{-a^2x^2+1}}+\frac{7x^5}{192a^4\sqrt{-a^2x^2+1}}+\frac{35x^3}{384a^6\sqrt{-a^2x^2+1}}-\frac{35x}{128a^8\sqrt{-a^2x^2+1}}+\frac{35\,\mathrm{asin}\,(ax)}{128a^9} & \text{otherwise} \end{cases}\right)-3\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a*c*x+c)**4,x)
```

```
[Out] a**5*c**4*Piecewise((-I*x**9/(8*sqrt(a**2*x**2 - 1)) - I*x**7/(48*a**2*sqrt
(a**2*x**2 - 1)) - 7*I*x**5/(192*a**4*sqrt(a**2*x**2 - 1)) - 35*I*x**3/(384
*a**6*sqrt(a**2*x**2 - 1)) + 35*I*x/(128*a**8*sqrt(a**2*x**2 - 1)) - 35*I*a
cosh(a*x)/(128*a**9), Abs(a**2*x**2) > 1), (x**9/(8*sqrt(-a**2*x**2 + 1)) +
 x**7/(48*a**2*sqrt(-a**2*x**2 + 1)) + 7*x**5/(192*a**4*sqrt(-a**2*x**2 + 1
)) + 35*x**3/(384*a**6*sqrt(-a**2*x**2 + 1)) - 35*x/(128*a**8*sqrt(-a**2*x*
*2 + 1)) + 35*asin(a*x)/(128*a**9), True)) - 3*a**4*c**4*Piecewise((-x**6*s
qrt(-a**2*x**2 + 1)/(7*a**2) - 6*x**4*sqrt(-a**2*x**2 + 1)/(35*a**4) - 8*x*
*2*sqrt(-a**2*x**2 + 1)/(35*a**6) - 16*sqrt(-a**2*x**2 + 1)/(35*a**8), Ne(a
, 0)), (x**8/8, True)) + 2*a**3*c**4*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 -
 1)) - I*x**5/(24*a**2*sqrt(a**2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x
**2 - 1)) + 5*I*x/(16*a**6*sqrt(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7),
 Abs(a**2*x**2) > 1), (x**7/(6*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-
a**2*x**2 + 1)) + 5*x**3/(48*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt
```

```
(-a**2*x**2 + 1)) + 5*asin(a*x)/(16*a**7), True)) + 2*a**2*c**4*Piecewise((
-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4)
 - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/6, True)) - 3*a*c**4*
Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**2
- 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs(a
**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*x*
*2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), True))
 + c**4*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2
+ 1)/(3*a**4), Ne(a, 0)), (x**4/4, True))
```

3.317 $\int e^{\tanh^{-1}(ax)}x^2(c-acx)^4\,dx$

Optimal. Leaf size=146

$$\frac{5c^4\sin^{-1}(ax)}{16a^3}+\frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a}+\frac{5c^4x\sqrt{1-a^2x^2}}{16a^2}+\frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2}-\frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2}+\frac{5c^4(16-21ax)\left(1\right.}{168a^3}$$

```
[Out] 5/7*c^4*x^2*(-a^2*x^2+1)^(3/2)/a-1/2*c^4*x^3*(-a^2*x^2+1)^(3/2)+1/7*a*c^4*x
^4*(-a^2*x^2+1)^(3/2)+5/168*c^4*(-21*a*x+16)*(-a^2*x^2+1)^(3/2)/a^3+5/16*c^
4*arcsin(a*x)/a^3+5/16*c^4*x*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.30, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1809, 833, 780, 195, 216}

$$\frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2}-\frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2}+\frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a}+\frac{5c^4(16-21ax)\left(1-a^2x^2\right)^{3/2}}{168a^3}+\frac{5c^4x\sqrt{1-a^2x^2}}{16a^2}+\frac{5c}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a*c*x)^4,x]
```

```
[Out] (5*c^4*x*Sqrt[1 - a^2*x^2])/(16*a^2) + (5*c^4*x^2*(1 - a^2*x^2)^(3/2))/(7*a
) - (c^4*x^3*(1 - a^2*x^2)^(3/2))/2 + (a*c^4*x^4*(1 - a^2*x^2)^(3/2))/7 + (
5*c^4*(16 - 21*a*x)*(1 - a^2*x^2)^(3/2))/(168*a^3) + (5*c^4*ArcSin[a*x])/(1
6*a^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^2(c-acx)^4\,dx = c\int x^2(c-acx)^3\sqrt{1-a^2x^2}\,dx$$
$$= \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} - \frac{c\int x^2\sqrt{1-a^2x^2}\left(-7a^2c^3+25a^3c^3x-21a^4c^3x^2\right)dx}{7a^2}$$
$$= -\frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{c\int x^2\left(105a^4c^3-150a^5c^3x\right)\sqrt{1-a^2}}{42a^4}$$
$$= \frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a} - \frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} - \frac{c\int x\left(300a^5c^3 -\right.}{}$$
$$= \frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a} - \frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2} + \frac{5c^4(16-21ax)\left(\right.}{168a}$$
$$= \frac{5c^4x\sqrt{1-a^2x^2}}{16a^2} + \frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a} - \frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2}$$
$$= \frac{5c^4x\sqrt{1-a^2x^2}}{16a^2} + \frac{5c^4x^2\left(1-a^2x^2\right)^{3/2}}{7a} - \frac{1}{2}c^4x^3\left(1-a^2x^2\right)^{3/2} + \frac{1}{7}ac^4x^4\left(1-a^2x^2\right)^{3/2}$$

Mathematica [A] time = 0.11, size = 91, normalized size = 0.62

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(48a^6x^6-168a^5x^5+192a^4x^4-42a^3x^3-80a^2x^2+105ax-160\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{336a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a*c*x)^4,x]
```

```
[Out] -1/336*(c^4*(Sqrt[1 - a^2*x^2]*(-160 + 105*a*x - 80*a^2*x^2 - 42*a^3*x^3 +
192*a^4*x^4 - 168*a^5*x^5 + 48*a^6*x^6) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]
))/a^3
```

fricas [A] time = 0.41, size = 114, normalized size = 0.78

$$-\frac{210\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(48\,a^6c^4x^6-168\,a^5c^4x^5+192\,a^4c^4x^4-42\,a^3c^4x^3-80\,a^2c^4x^2+105\,ac^4x-160\,c^4\right)}{336\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/336*(210*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (48*a^6*c^4*x^6 -
168*a^5*c^4*x^5 + 192*a^4*c^4*x^4 - 42*a^3*c^4*x^3 - 80*a^2*c^4*x^2 + 105*a
*c^4*x - 160*c^4)*sqrt(-a^2*x^2 + 1))/a^3
```

giac [A] time = 0.27, size = 104, normalized size = 0.71

$$\frac{5\,c^4\arcsin\left(ax\right)\mathrm{sgn}(a)}{16\,a^2|a|}-\frac{1}{336}\,\sqrt{-a^2x^2+1}\left(\left(\left(\frac{105\,c^4}{a^2}-2\left(\frac{40\,c^4}{a}+3\left(7\,c^4-4\left(8\,ac^4+\left(2\,a^3c^4x-7\,a^2c^4\right)x\right)x\right)x\right)x\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 5/16*c^4*arcsin(a*x)*sgn(a)/(a^2*abs(a)) - 1/336*sqrt(-a^2*x^2 + 1)*((105*c
^4/a^2 - 2*(40*c^4/a + 3*(7*c^4 - 4*(8*a*c^4 + (2*a^3*c^4*x - 7*a^2*c^4)*x)
*x)*x)*x)*x - 160*c^4/a^3)
```

maple [A] time = 0.06, size = 186, normalized size = 1.27

$$-\frac{c^4a^3x^6\sqrt{-a^2x^2+1}}{7}-\frac{4c^4a\,x^4\sqrt{-a^2x^2+1}}{7}+\frac{5c^4x^2\sqrt{-a^2x^2+1}}{21a}+\frac{10c^4\sqrt{-a^2x^2+1}}{21a^3}+\frac{c^4a^2x^5\sqrt{-a^2x^2+1}}{2}+\frac{c^4x^3\sqrt{-}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^4,x)
```

```
[Out] -1/7*c^4*a^3*x^6*(-a^2*x^2+1)^(1/2)-4/7*c^4*a*x^4*(-a^2*x^2+1)^(1/2)+5/21*c
^4/a*x^2*(-a^2*x^2+1)^(1/2)+10/21*c^4/a^3*(-a^2*x^2+1)^(1/2)+1/2*c^4*a^2*x^
5*(-a^2*x^2+1)^(1/2)+1/8*c^4*x^3*(-a^2*x^2+1)^(1/2)-5/16*c^4*x*(-a^2*x^2+1)
^(1/2)/a^2+5/16*c^4/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)
)
```

maxima [A] time = 0.41, size = 164, normalized size = 1.12

$$-\frac{1}{7}\,\sqrt{-a^2x^2+1}\,a^3c^4x^6+\frac{1}{2}\,\sqrt{-a^2x^2+1}\,a^2c^4x^5-\frac{4}{7}\,\sqrt{-a^2x^2+1}\,ac^4x^4+\frac{1}{8}\,\sqrt{-a^2x^2+1}\,c^4x^3+\frac{5\,\sqrt{-a^2x^2+1}\,c^4x^2}{21\,a}-\frac{5\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] -1/7*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^6 + 1/2*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^5 -
4/7*sqrt(-a^2*x^2 + 1)*a*c^4*x^4 + 1/8*sqrt(-a^2*x^2 + 1)*c^4*x^3 + 5/21*sq
rt(-a^2*x^2 + 1)*c^4*x^2/a - 5/16*sqrt(-a^2*x^2 + 1)*c^4*x/a^2 + 5/16*c^4*a
rcsin(a*x)/a^3 + 10/21*sqrt(-a^2*x^2 + 1)*c^4/a^3
```

mupad [B] time = 0.79, size = 177, normalized size = 1.21

$$\frac{10\,c^4\,\sqrt{1-a^2\,x^2}}{21\,a^3}+\frac{c^4\,x^3\,\sqrt{1-a^2\,x^2}}{8}-\frac{5\,c^4\,x\,\sqrt{1-a^2\,x^2}}{16\,a^2}-\frac{4\,a\,c^4\,x^4\,\sqrt{1-a^2\,x^2}}{7}+\frac{5\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{16\,a^2\,\sqrt{-a^2}}+\frac{5\,c^4\,x^2\,\sqrt{1-a}}{21\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^4*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (10*c^4*(1 - a^2*x^2)^(1/2))/(21*a^3) + (c^4*x^3*(1 - a^2*x^2)^(1/2))/8 - (
5*c^4*x*(1 - a^2*x^2)^(1/2))/(16*a^2) - (4*a*c^4*x^4*(1 - a^2*x^2)^(1/2))/7
 + (5*c^4*asinh(x*(-a^2)^(1/2)))/(16*a^2*(-a^2)^(1/2)) + (5*c^4*x^2*(1 - a^
2*x^2)^(1/2))/(21*a) + (a^2*c^4*x^5*(1 - a^2*x^2)^(1/2))/2 - (a^3*c^4*x^6*(
1 - a^2*x^2)^(1/2))/7
```

sympy [C] time = 13.19, size = 683, normalized size = 4.68

$$a^5c^4\left(\begin{cases}-\frac{x^6\sqrt{-a^2x^2+1}}{7a^2}-\frac{6x^4\sqrt{-a^2x^2+1}}{35a^4}-\frac{8x^2\sqrt{-a^2x^2+1}}{35a^6}-\frac{16\sqrt{-a^2x^2+1}}{35a^8} & \text{for } a\neq 0\\ \frac{x^8}{8} & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}-\frac{ix^7}{6\sqrt{a^2x^2-1}}-\frac{ix^5}{24a^2\sqrt{a^2x^2-1}}-\\ \frac{x^7}{6\sqrt{-a^2x^2+1}}+\frac{x^5}{24a^2\sqrt{-a^2x^2+1}}+\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c)**4,x)
```

```
[Out] a**5*c**4*Piecewise((-x**6*sqrt(-a**2*x**2 + 1)/(7*a**2) - 6*x**4*sqrt(-a**
2*x**2 + 1)/(35*a**4) - 8*x**2*sqrt(-a**2*x**2 + 1)/(35*a**6) - 16*sqrt(-a*
*2*x**2 + 1)/(35*a**8), Ne(a, 0)), (x**8/8, True)) - 3*a**4*c**4*Piecewise(
(-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt(a**2*x**2 - 1)) - 5
*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*sqrt(a**2*x**2 - 1))
 - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7/(6*sqrt(-a**2*x**2
+ 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(48*a**4*sqrt(-a**2*x*
*2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(a*x)/(16*a**7), True
)) + 2*a**3*c**4*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sq
rt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)),
 (x**6/6, True)) + 2*a**2*c**4*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) -
 I*x**3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) -
 3*I*acosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1
)) + x**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1))
 + 3*asin(a*x)/(8*a**5), True)) - 3*a*c**4*Piecewise((-x**2*sqrt(-a**2*x**2
 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)
) + c**4*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**
3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a
**2*x**2 + 1)) + asin(a*x)/(2*a**3), True))
```

3.318 $\int e^{\tanh^{-1}(ax)}x(c-acx)^4\,dx$

Optimal. Leaf size=158

$$-\frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2}-\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2}-\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2}-\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2}-\frac{7c^4x\sqrt{1-a^2x}}{16a}$$

```
[Out] -7/24*c^4*(-a^2*x^2+1)^(3/2)/a^2-7/40*c^4*(-a*x+1)*(-a^2*x^2+1)^(3/2)/a^2-1
/10*c^4*(-a*x+1)^2*(-a^2*x^2+1)^(3/2)/a^2-1/6*c^4*(-a*x+1)^3*(-a^2*x^2+1)^(
3/2)/a^2-7/16*c^4*arcsin(a*x)/a^2-7/16*c^4*x*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6128, 795, 671, 641, 195, 216}

$$-\frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2}-\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2}-\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2}-\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2}-\frac{7c^4x\sqrt{1-a^2x}}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(c - a*c*x)^4,x]
```

```
[Out] (-7*c^4*x*Sqrt[1 - a^2*x^2])/(16*a) - (7*c^4*(1 - a^2*x^2)^(3/2))/(24*a^2)
- (7*c^4*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(40*a^2) - (c^4*(1 - a*x)^2*(1 - a^
2*x^2)^(3/2))/(10*a^2) - (c^4*(1 - a*x)^3*(1 - a^2*x^2)^(3/2))/(6*a^2) - (7
*c^4*ArcSin[a*x])/(16*a^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x(c-acx)^4\,dx = c\int x(c-acx)^3\sqrt{1-a^2x^2}\,dx$$
$$= -\frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2} - \frac{c\int(c-acx)^3\sqrt{1-a^2x^2}\,dx}{2a}$$
$$= -\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2} - \frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2} - \frac{\left(7c^2\right)\int(c-acx)^2\sqrt{1-a^2x}}{10a}$$
$$= -\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2} - \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2} - \frac{c^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{6a^2} -$$
$$= -\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2} - \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2} - \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{10a^2} - \frac{c^4(1-}{}$$
$$= -\frac{7c^4x\sqrt{1-a^2x^2}}{16a} - \frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2} - \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2} - \frac{c^4(1-ax)^2\left(1-\right.}{10a^2}$$
$$= -\frac{7c^4x\sqrt{1-a^2x^2}}{16a} - \frac{7c^4\left(1-a^2x^2\right)^{3/2}}{24a^2} - \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{40a^2} - \frac{c^4(1-ax)^2\left(1-\right.}{10a^2}$$

Mathematica [A] time = 0.13, size = 83, normalized size = 0.53

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(40a^5x^5-144a^4x^4+170a^3x^3-32a^2x^2-105ax+176\right)-210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a*c*x)^4,x]
```

```
[Out] -1/240*(c^4*(Sqrt[1 - a^2*x^2]*(176 - 105*a*x - 32*a^2*x^2 + 170*a^3*x^3 -
144*a^4*x^4 + 40*a^5*x^5) - 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/a^2
```

fricas [A] time = 0.48, size = 104, normalized size = 0.66

$$\frac{210\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(40\,a^5c^4x^5-144\,a^4c^4x^4+170\,a^3c^4x^3-32\,a^2c^4x^2-105\,ac^4x+176\,c^4\right)\sqrt{-a^2x^2+1}}{240\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/240*(210*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (40*a^5*c^4*x^5 - 1
44*a^4*c^4*x^4 + 170*a^3*c^4*x^3 - 32*a^2*c^4*x^2 - 105*a*c^4*x + 176*c^4)*
sqrt(-a^2*x^2 + 1))/a^2
```

giac [A] time = 0.18, size = 94, normalized size = 0.59

$$-\frac{7\,c^4\arcsin\left(ax\right)\operatorname{sgn}(a)}{16\,a|a|}-\frac{1}{240}\,\sqrt{-a^2x^2+1}\left(\frac{176\,c^4}{a^2}-\left(\frac{105\,c^4}{a}+2\left(16\,c^4-\left(85\,ac^4+4\left(5\,a^3c^4x-18\,a^2c^4\right)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -7/16*c^4*arcsin(a*x)*sgn(a)/(a*abs(a)) - 1/240*sqrt(-a^2*x^2 + 1)*(176*c^4
/a^2 - (105*c^4/a + 2*(16*c^4 - (85*a*c^4 + 4*(5*a^3*c^4*x - 18*a^2*c^4)*x)
*x)*x)*x)
```

maple [A] time = 0.05, size = 163, normalized size = 1.03

$$-\frac{c^4a^3x^5\sqrt{-a^2x^2+1}}{6}-\frac{17c^4a\,x^3\sqrt{-a^2x^2+1}}{24}+\frac{7c^4x\sqrt{-a^2x^2+1}}{16a}-\frac{7c^4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{16a\sqrt{a^2}}+\frac{3c^4a^2x^4\sqrt{-a^2x^2+1}}{5}+\frac{2c^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^4,x)
```

```
[Out] -1/6*c^4*a^3*x^5*(-a^2*x^2+1)^(1/2)-17/24*c^4*a*x^3*(-a^2*x^2+1)^(1/2)+7/16
*c^4*x*(-a^2*x^2+1)^(1/2)/a-7/16*c^4/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2))+3/5*c^4*a^2*x^4*(-a^2*x^2+1)^(1/2)+2/15*c^4*x^2*(-a^2*x^2+
1)^(1/2)-11/15*c^4/a^2*(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.42, size = 141, normalized size = 0.89

$$-\frac{1}{6}\sqrt{-a^2x^2+1}\,a^3c^4x^5+\frac{3}{5}\sqrt{-a^2x^2+1}\,a^2c^4x^4-\frac{17}{24}\sqrt{-a^2x^2+1}\,ac^4x^3+\frac{2}{15}\sqrt{-a^2x^2+1}\,c^4x^2+\frac{7\sqrt{-a^2x^2+1}\,c^4x}{16\,a}-\frac{7\,c^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^5 + 3/5*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^4 -
17/24*sqrt(-a^2*x^2 + 1)*a*c^4*x^3 + 2/15*sqrt(-a^2*x^2 + 1)*c^4*x^2 + 7/16
*sqrt(-a^2*x^2 + 1)*c^4*x/a - 7/16*c^4*arcsin(a*x)/a^2 - 11/15*sqrt(-a^2*x^
2 + 1)*c^4/a^2
```

mupad [B] time = 0.04, size = 154, normalized size = 0.97

$$\frac{2\,c^4\,x^2\,\sqrt{1-a^2\,x^2}}{15}-\frac{11\,c^4\,\sqrt{1-a^2\,x^2}}{15\,a^2}+\frac{7\,c^4\,x\,\sqrt{1-a^2\,x^2}}{16\,a}-\frac{17\,a\,c^4\,x^3\,\sqrt{1-a^2\,x^2}}{24}-\frac{7\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{16\,a\,\sqrt{-a^2}}+\frac{3\,a^2\,c^4\,x^4\,\sqrt{}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^4*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (2*c^4*x^2*(1 - a^2*x^2)^(1/2))/15 - (11*c^4*(1 - a^2*x^2)^(1/2))/(15*a^2)
+ (7*c^4*x*(1 - a^2*x^2)^(1/2))/(16*a) - (17*a*c^4*x^3*(1 - a^2*x^2)^(1/2))
/24 - (7*c^4*asinh(x*(-a^2)^(1/2)))/(16*a*(-a^2)^(1/2)) + (3*a^2*c^4*x^4*(1
 - a^2*x^2)^(1/2))/5 - (a^3*c^4*x^5*(1 - a^2*x^2)^(1/2))/6
```

sympy [A] time = 12.49, size = 617, normalized size = 3.91

$$a^5c^4\left(\begin{cases} -\frac{ix^7}{6\sqrt{a^2x^2-1}} - \frac{ix^5}{24a^2\sqrt{a^2x^2-1}} - \frac{5ix^3}{48a^4\sqrt{a^2x^2-1}} + \frac{5ix}{16a^6\sqrt{a^2x^2-1}} - \frac{5i\operatorname{acosh}(ax)}{16a^7} & \text{for } |a^2x^2| > 1 \\ \frac{x^7}{6\sqrt{-a^2x^2+1}} + \frac{x^5}{24a^2\sqrt{-a^2x^2+1}} + \frac{5x^3}{48a^4\sqrt{-a^2x^2+1}} - \frac{5x}{16a^6\sqrt{-a^2x^2+1}} + \frac{5\operatorname{asin}(ax)}{16a^7} & \text{otherwise} \end{cases}\right) - 3a^4c^4\left(\begin{cases} -\frac{x^4\sqrt{-a^2x^2-}}{5a^2} \\ \frac{x^6}{6} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c)**4,x)
```

```
[Out] a**5*c**4*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt
(a**2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*
sqrt(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7
/(6*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(4
8*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(
a*x)/(16*a**7), True)) - 3*a**4*c**4*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/
(5*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(
15*a**6), Ne(a, 0)), (x**6/6, True)) + 2*a**3*c**4*Piecewise((-I*x**5/(4*sq
rt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sq
rt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4
*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*s
qrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5), True)) + 2*a**2*c**4*Piecewise
((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne
(a, 0)), (x**4/4, True)) - 3*a*c**4*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*
a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**
2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + c*
*4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
```

3.319 $\int e^{\tanh^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=123

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\sin^{-1}(ax)}{8a}$$

```
[Out] 7/12*c^4*(-a^2*x^2+1)^(3/2)/a+7/20*c^4*(-a*x+1)*(-a^2*x^2+1)^(3/2)/a+1/5*c^
4*(-a*x+1)^2*(-a^2*x^2+1)^(3/2)/a+7/8*c^4*arcsin(a*x)/a+7/8*c^4*x*(-a^2*x^2
+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6127, 671, 641, 195, 216}

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (7*c^4*x*Sqrt[1 - a^2*x^2])/8 + (7*c^4*(1 - a^2*x^2)^(3/2))/(12*a) + (7*c^4
*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(20*a) + (c^4*(1 - a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^4*ArcSin[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}(c-acx)^4\,dx &= c\int (c-acx)^3\sqrt{1-a^2x^2}\,dx\\
&= \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{5}\left(7c^2\right)\int (c-acx)^2\sqrt{1-a^2x^2}\,dx\\
&= \frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^3\right)\int (c-acx)\sqrt{1-a^2x^2}\\
&= \frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^4\right)\int\\
&= \frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2\right.}{5a}\\
&= \frac{7}{8}c^4x\sqrt{1-a^2x^2}+\frac{7c^4\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7c^4(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{c^4(1-ax)^2\left(1-a^2\right.}{5a}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.61

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a*c*x)^4,x]
```

```
[Out] -1/120*(c^4*(Sqrt[1 - a^2*x^2]*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
24*a^4*x^4) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/a
```

fricas [A] time = 0.47, size = 92, normalized size = 0.75

$$-\frac{210\, c^4 \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(24\, a^4c^4x^4 - 90\, a^3c^4x^3 + 112\, a^2c^4x^2 - 15\, ac^4x - 136\, c^4\right)\sqrt{-a^2x^2+1}}{120\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(210*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c^4*x^4 -
90*a^3*c^4*x^3 + 112*a^2*c^4*x^2 - 15*a*c^4*x - 136*c^4)*sqrt(-a^2*x^2 + 1)
)/a
```

giac [A] time = 0.19, size = 78, normalized size = 0.63

$$\frac{7\, c^4 \arcsin(ax)\,\mathrm{sgn}(a)}{8\, |a|} + \frac{1}{120}\sqrt{-a^2x^2+1}\left(\frac{136\, c^4}{a} + \left(15\, c^4 - 2\left(56\, ac^4 + 3\left(4\, a^3c^4x - 15\, a^2c^4\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 7/8*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/120*sqrt(-a^2*x^2 + 1)*(136*c^4/a + (
15*c^4 - 2*(56*a*c^4 + 3*(4*a^3*c^4*x - 15*a^2*c^4)*x)*x)*x)
```

maple [A] time = 0.04, size = 137, normalized size = 1.11

$$-\frac{c^4a^3x^4\sqrt{-a^2x^2+1}}{5} - \frac{14c^4a\,x^2\sqrt{-a^2x^2+1}}{15} + \frac{17c^4\sqrt{-a^2x^2+1}}{15a} + \frac{3c^4a^2x^3\sqrt{-a^2x^2+1}}{4} + \frac{c^4x\sqrt{-a^2x^2+1}}{8} + \frac{7c^4 \arctan}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x)
```

```
[Out] -1/5*c^4*a^3*x^4*(-a^2*x^2+1)^(1/2)-14/15*c^4*a*x^2*(-a^2*x^2+1)^(1/2)+17/1
5*c^4*(-a^2*x^2+1)^(1/2)/a+3/4*c^4*a^2*x^3*(-a^2*x^2+1)^(1/2)+1/8*c^4*x*(-a
^2*x^2+1)^(1/2)+7/8*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)
)
```

maxima [A] time = 0.46, size = 118, normalized size = 0.96

$$-\frac{1}{5}\sqrt{-a^2x^2+1}\,a^3c^4x^4 + \frac{3}{4}\sqrt{-a^2x^2+1}\,a^2c^4x^3 - \frac{14}{15}\sqrt{-a^2x^2+1}\,ac^4x^2 + \frac{1}{8}\sqrt{-a^2x^2+1}\,c^4x + \frac{7\,c^4\arcsin(ax)}{8\,a} + \frac{17\sqrt{-a}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^4 + 3/4*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^3 -
14/15*sqrt(-a^2*x^2 + 1)*a*c^4*x^2 + 1/8*sqrt(-a^2*x^2 + 1)*c^4*x + 7/8*c^4
*arcsin(a*x)/a + 17/15*sqrt(-a^2*x^2 + 1)*c^4/a
```

mupad [B] time = 0.00, size = 128, normalized size = 1.04

$$\frac{c^4\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{7\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}+\frac{17\,c^4\,\sqrt{1-a^2\,x^2}}{15\,a}-\frac{14\,a\,c^4\,x^2\,\sqrt{1-a^2\,x^2}}{15}+\frac{3\,a^2\,c^4\,x^3\,\sqrt{1-a^2\,x^2}}{4}-\frac{a^3\,c^4\,x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^4*x*(1 - a^2*x^2)^(1/2))/8 + (7*c^4*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(1/
2)) + (17*c^4*(1 - a^2*x^2)^(1/2))/(15*a) - (14*a*c^4*x^2*(1 - a^2*x^2)^(1/
2))/15 + (3*a^2*c^4*x^3*(1 - a^2*x^2)^(1/2))/4 - (a^3*c^4*x^4*(1 - a^2*x^2)
^(1/2))/5
```

sympy [A] time = 8.98, size = 226, normalized size = 1.84

$$\begin{cases} \frac{3c^4\sqrt{-a^2x^2+1}+2c^4\left(\begin{cases} -\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\mathrm{asin}\,(ax)}{2} & \text{for } ax>-1\wedge ax<1 \end{cases}\right)+2c^4\left(\begin{cases} \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\sqrt{-a^2x^2+1} & \text{for } ax>-1\wedge \end{cases}\right.}{} \\ c^4x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4,x)
```

```
[Out] Piecewise(((3*c**4*sqrt(-a**2*x**2 + 1) + 2*c**4*Piecewise((-a*x*sqrt(-a**2
*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) + 2*c**4*Piecewise(((-
a**2*x**2 + 1)**(3/2)/3 - sqrt(-a**2*x**2 + 1), (a*x > -1) & (a*x < 1))) -
3*c**4*Piecewise((a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 - a*x*sqrt(
-a**2*x**2 + 1)/2 + 3*asin(a*x)/8, (a*x > -1) & (a*x < 1))) + c**4*Piecewis
e((-(-a**2*x**2 + 1)**(5/2)/5 + 2*(-a**2*x**2 + 1)**(3/2)/3 - sqrt(-a**2*x*
*2 + 1), (a*x > -1) & (a*x < 1))) + c**4*asin(a*x))/a, Ne(a, 0)), (c**4*x,
True))
```

3.320 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x} \, dx$

Optimal. Leaf size=101

$$\frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2}-c^4\left(1-a^2x^2\right)^{3/2}+\frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2}-c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{13}{8}c^4\sin^{-1}(ax)$$

```
[Out] -c^4*(-a^2*x^2+1)^(3/2)+1/4*a*c^4*x*(-a^2*x^2+1)^(3/2)-13/8*c^4*arcsin(a*x)
-c^4*arctanh((-a^2*x^2+1)^(1/2))+1/8*c^4*(-13*a*x+8)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1809, 815, 844, 216, 266, 63, 208}

$$\frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2}-c^4\left(1-a^2x^2\right)^{3/2}+\frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2}-c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{13}{8}c^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x,x]
```

```
[Out] (c^4*(8 - 13*a*x)*Sqrt[1 - a^2*x^2])/8 - c^4*(1 - a^2*x^2)^(3/2) + (a*c^4*x
*(1 - a^2*x^2)^(3/2))/4 - (13*c^4*ArcSin[a*x])/8 - c^4*ArcTanh[Sqrt[1 - a^2
*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x}\,dx$$

$$= \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{c\int \frac{\sqrt{1-a^2x^2}\left(-4a^2c^3+13a^3c^3x-12a^4c^3x^2\right)}{x}\,dx}{4a^2}$$

$$= -c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} + \frac{c\int \frac{\left(12a^4c^3-39a^5c^3x\right)\sqrt{1-a^2x^2}}{x}\,dx}{12a^4}$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{c\int \frac{-24a^6c^3+39a^7c^3x}{x\sqrt{1-a^2x^2}}}{24a^6}$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} + c^4\int \frac{1}{x\sqrt{1-a^2x^2}}$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{13}{8}c^4\sin^{-1}(ax) +$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{13}{8}c^4\sin^{-1}(ax) -$$

$$= \frac{1}{8}c^4(8-13ax)\sqrt{1-a^2x^2} - c^4\left(1-a^2x^2\right)^{3/2} + \frac{1}{4}ac^4x\left(1-a^2x^2\right)^{3/2} - \frac{13}{8}c^4\sin^{-1}(ax) -$$

Mathematica [A] time = 0.10, size = 142, normalized size = 1.41

$$\frac{c^4\left(2a^5x^5-8a^4x^4+9a^3x^3+8a^2x^2+4\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+34\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-8\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\right.\right.}{8\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x,x]
```

```
[Out] (c^4*(-11*a*x + 8*a^2*x^2 + 9*a^3*x^3 - 8*a^4*x^4 + 2*a^5*x^5 + 4*Sqrt[1 -
a^2*x^2]*ArcSin[a*x] + 34*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] -
 8*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(8*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.48, size = 95, normalized size = 0.94

$$\frac{13}{4}\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}\,-1}{ax}\right)+c^4\log\left(\frac{\sqrt{-a^2x^2+1}\,-1}{x}\right)-\frac{1}{8}\left(2\,a^3c^4x^3-8\,a^2c^4x^2+11\,ac^4x\right)\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x,x, algorithm="fricas")
```

```
[Out] 13/4*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c^4*log((sqrt(-a^2*x^2 +
1) - 1)/x) - 1/8*(2*a^3*c^4*x^3 - 8*a^2*c^4*x^2 + 11*a*c^4*x)*sqrt(-a^2*x^2
 + 1)
```

giac [A] time = 0.29, size = 100, normalized size = 0.99

$$-\frac{13\,ac^4\arcsin\left(ax\right)\mathrm{sgn}(a)}{8\,|a|}-\frac{ac^4\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{1}{8}\left(11\,ac^4+2\left(a^3c^4x-4\,a^2c^4\right)x\right)\sqrt{-a^2x^2+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x,x, algorithm="giac")
```

```
[Out] -13/8*a*c^4*arcsin(a*x)*sgn(a)/abs(a) - a*c^4*log(1/2*abs(-2*sqrt(-a^2*x^2
+ 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(11*a*c^4 + 2*(a^3*c^4*x - 4*
a^2*c^4)*x)*sqrt(-a^2*x^2 + 1)*x
```

maple [A] time = 0.04, size = 115, normalized size = 1.14

$$-\frac{c^4a^3x^3\sqrt{-a^2x^2+1}}{4}-\frac{11c^4ax\sqrt{-a^2x^2+1}}{8}-\frac{13c^4a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8\sqrt{a^2}}+c^4a^2x^2\sqrt{-a^2x^2+1}-c^4\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x,x)
```

```
[Out] -1/4*c^4*a^3*x^3*(-a^2*x^2+1)^(1/2)-11/8*c^4*a*x*(-a^2*x^2+1)^(1/2)-13/8*c^
4*a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+c^4*a^2*x^2*(-a^2*
x^2+1)^(1/2)-c^4*arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.40, size = 105, normalized size = 1.04

$$-\frac{1}{4}\sqrt{-a^2x^2+1}\,a^3c^4x^3+\sqrt{-a^2x^2+1}\,a^2c^4x^2-\frac{11}{8}\sqrt{-a^2x^2+1}\,ac^4x-\frac{13}{8}c^4\arcsin\left(ax\right)-c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^3 + sqrt(-a^2*x^2 + 1)*a^2*c^4*x^2 - 11/8
*sqrt(-a^2*x^2 + 1)*a*c^4*x - 13/8*c^4*arcsin(a*x) - c^4*log(2*sqrt(-a^2*x^
2 + 1)/abs(x) + 2/abs(x))
```

mupad [B] time = 0.78, size = 110, normalized size = 1.09

$$a^2\,c^4\,x^2\,\sqrt{1-a^2\,x^2}-\frac{a^3\,c^4\,x^3\,\sqrt{1-a^2\,x^2}}{4}-\frac{11\,a\,c^4\,x\,\sqrt{1-a^2\,x^2}}{8}-\frac{13\,a\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}+c^4\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] c^4*atan((1 - a^2*x^2)^(1/2)*1i)*1i + a^2*c^4*x^2*(1 - a^2*x^2)^(1/2) - (a^
3*c^4*x^3*(1 - a^2*x^2)^(1/2))/4 - (11*a*c^4*x*(1 - a^2*x^2)^(1/2))/8 - (13
*a*c^4*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(1/2))
```

sympy [C] time = 19.06, size = 420, normalized size = 4.16

$$a^5c^4\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\,\mathrm{acosh}\,(ax)}{8a^5} & \text{for}\ \left|a^2x^2\right|>1\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\,\mathrm{asin}\,(ax)}{8a^5} & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{fo}\\ \frac{x^4}{4} & \text{o}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x,x)
```

```
[Out] a**5*c**4*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(
a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a*
*5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqr
t(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**
5), True)) - 3*a**4*c**4*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2
*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + 2*a**3*c**4*Pi
ecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**
2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 +
1)) + asin(a*x)/(2*a**3), True)) + 2*a**2*c**4*Piecewise((x**2/2, Eq(a**2,
0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) - 3*a*c**4*Piecewise((sqrt(a**(-2)
)*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2
< 0)) + c**4*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(
a*x)), True))
```

3.321 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^2} dx$

Optimal. Leaf size=106

$$\frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{x}-\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2}+3ac^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{1}{2}ac^4\sin^{-1}(ax)$$

```
[Out] 1/3*a*c^4*(-a^2*x^2+1)^(3/2)-c^4*(-a^2*x^2+1)^(3/2)/x+1/2*a*c^4*arcsin(a*x)
+3*a*c^4*arctanh((-a^2*x^2+1)^(1/2))-1/2*a*c^4*(-a*x+6)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6128, 1807, 1809, 815, 844, 216, 266, 63, 208}

$$\frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{x}-\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2}+3ac^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{1}{2}ac^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^2,x]
```

```
[Out] -(a*c^4*(6 - a*x)*Sqrt[1 - a^2*x^2])/2 + (a*c^4*(1 - a^2*x^2)^(3/2))/3 - (c
^4*(1 - a^2*x^2)^(3/2))/x + (a*c^4*ArcSin[a*x])/2 + 3*a*c^4*ArcTanh[Sqrt[1
- a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
```

```
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^2}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^2}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} - c\int \frac{\sqrt{1-a^2x^2}\left(3ac^3-a^2c^3x+a^3c^3x^2\right)}{x}\,dx$$
$$= \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{c\int \frac{\left(-9a^3c^3+3a^4c^3x\right)\sqrt{1-a^2x^2}}{x}\,dx}{3a^2}$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} - \frac{c\int \frac{18a^5c^3-3a^6c^3x}{x\sqrt{1-a^2x^2}}}{6a^4}$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} - \left(3ac^4\right)\int \frac{1}{x\sqrt{1-}}$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}ac^4\sin^{-1}(ax) -$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}ac^4\sin^{-1}(ax) -$$
$$= -\frac{1}{2}ac^4(6-ax)\sqrt{1-a^2x^2} + \frac{1}{3}ac^4\left(1-a^2x^2\right)^{3/2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}ac^4\sin^{-1}(ax) -$$

Mathematica [A] time = 0.13, size = 152, normalized size = 1.43

$$-\frac{c^4\left(-2a^5x^5+9a^4x^4-14a^3x^3-15a^2x^2+9ax\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+24ax\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-18ax\sqrt{1-}\right.}{6x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^2,x]
```

```
[Out] -1/6*(c^4*(6 + 16*a*x - 15*a^2*x^2 - 14*a^3*x^3 + 9*a^4*x^4 - 2*a^5*x^5 + 9
*a*x*Sqrt[1 - a^2*x^2]*ArcSin[a*x] + 24*a*x*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1
```

```
 - a*x]/Sqrt[2]] - 18*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(x
*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.47, size = 116, normalized size = 1.09

$$-\frac{6\,ac^4x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+18\,ac^4x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+16\,ac^4x+\left(2\,a^3c^4x^3-9\,a^2c^4x^2+16\,ac^4x+6\,c^4\right)\sqrt{-a^2x}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^2,x, algorithm="fricas"
)
```

```
[Out] -1/6*(6*a*c^4*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 18*a*c^4*x*log((sq
rt(-a^2*x^2 + 1) - 1)/x) + 16*a*c^4*x + (2*a^3*c^4*x^3 - 9*a^2*c^4*x^2 + 16
*a*c^4*x + 6*c^4)*sqrt(-a^2*x^2 + 1))/x
```

giac [A] time = 0.25, size = 164, normalized size = 1.55

$$\frac{a^4c^4x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}+\frac{a^2c^4\arcsin\left(ax\right)\mathrm{sgn}(a)}{2\,|a|}+\frac{3\,a^2c^4\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^4}{2\,x|a|}-\frac{1}{6}\left(16\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a^4*c^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + 1/2*a^2*c^4*arcsin
(a*x)*sgn(a)/abs(a) + 3*a^2*c^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
2*a)/(a^2*abs(x)))/abs(a) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(x*abs(
a)) - 1/6*(16*a*c^4 + (2*a^3*c^4*x - 9*a^2*c^4)*x)*sqrt(-a^2*x^2 + 1)
```

maple [A] time = 0.04, size = 136, normalized size = 1.28

$$-\frac{c^4a^3x^2\sqrt{-a^2x^2+1}}{3}-\frac{8c^4a\sqrt{-a^2x^2+1}}{3}+\frac{3c^4a^2x\sqrt{-a^2x^2+1}}{2}+\frac{c^4a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}+3c^4a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^2,x)
```

```
[Out] -1/3*c^4*a^3*x^2*(-a^2*x^2+1)^(1/2)-8/3*c^4*a*(-a^2*x^2+1)^(1/2)+3/2*c^4*a^
2*x*(-a^2*x^2+1)^(1/2)+1/2*c^4*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x
^2+1)^(1/2))+3*c^4*a*arctanh(1/(-a^2*x^2+1)^(1/2))-c^4/x*(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.46, size = 125, normalized size = 1.18

$$-\frac{1}{3}\sqrt{-a^2x^2+1}\,a^3c^4x^2+\frac{3}{2}\sqrt{-a^2x^2+1}\,a^2c^4x+\frac{1}{2}\,ac^4\arcsin(ax)+3\,ac^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{8}{3}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^2,x, algorithm="maxima"
)
```

```
[Out] -1/3*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^2 + 3/2*sqrt(-a^2*x^2 + 1)*a^2*c^4*x + 1/
2*a*c^4*arcsin(a*x) + 3*a*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) -
 8/3*sqrt(-a^2*x^2 + 1)*a*c^4 - sqrt(-a^2*x^2 + 1)*c^4/x
```

mupad [B] time = 0.04, size = 131, normalized size = 1.24

$$\frac{3\,a^2\,c^4\,x\,\sqrt{1-a^2\,x^2}}{2}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{x}-\frac{8\,a\,c^4\,\sqrt{1-a^2\,x^2}}{3}+\frac{a^2\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}-\frac{a^3\,c^4\,x^2\,\sqrt{1-a^2\,x^2}}{3}-a\,c^4\,\mathrm{atan}\left(\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (3*a^2*c^4*x*(1 - a^2*x^2)^(1/2))/2 - (c^4*(1 - a^2*x^2)^(1/2))/x - a*c^4*a
tan((1 - a^2*x^2)^(1/2)*1i)*3i - (8*a*c^4*(1 - a^2*x^2)^(1/2))/3 + (a^2*c^4
*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2)) - (a^3*c^4*x^2*(1 - a^2*x^2)^(1/2)
)/3
```

sympy [C] time = 6.68, size = 306, normalized size = 2.89

$$a^5c^4\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\,\mathrm{acosh}(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\mathrm{asin}(ax)}{2a^3} & \text{otherwise}\end{cases}\right)+2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**2,x)
```

```
[Out] a**5*c**4*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**
2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) - 3*a**4*c**4*Piecewise((-I*x*s
qrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (
x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)
/(2*a**3), True)) + 2*a**3*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a*
```

```
*2*x**2 + 1)/a**2, True)) + 2*a**2*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqr
t(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 3*a*
c**4*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), T
rue)) + c**4*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sq
rt(-a**2*x**2 + 1)/x, True))
```

3.322 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^3}\,dx$

Optimal. Leaf size=116

$$\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{5}{2}a^2c^4(ax+1)\sqrt{1-a^2x^2}-\frac{5}{2}a^2c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{5}{2}a^2c^4\sin^{-1}(ax)$$

```
[Out] -1/2*c^4*(-a^2*x^2+1)^(3/2)/x^2+3*a*c^4*(-a^2*x^2+1)^(3/2)/x+5/2*a^2*c^4*ar
csin(a*x)-5/2*a^2*c^4*arctanh((-a^2*x^2+1)^(1/2))+5/2*a^2*c^4*(a*x+1)*(-a^2
*x^2+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 815, 844, 216, 266, 63, 208}

$$\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2}+\frac{5}{2}a^2c^4(ax+1)\sqrt{1-a^2x^2}-\frac{5}{2}a^2c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{5}{2}a^2c^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^3,x]
```

```
[Out] (5*a^2*c^4*(1 + a*x)*Sqrt[1 - a^2*x^2])/2 - (c^4*(1 - a^2*x^2)^(3/2))/(2*x^
2) + (3*a*c^4*(1 - a^2*x^2)^(3/2))/x + (5*a^2*c^4*ArcSin[a*x])/2 - (5*a^2*c
^4*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^3}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^3}\,dx$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}c\int \frac{\sqrt{1-a^2x^2}\,\left(6ac^3-5a^2c^3x+2a^3c^3x^2\right)}{x^2}\,dx$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}c\int \frac{\left(5a^2c^3+10a^3c^3x\right)\sqrt{1-a^2x^2}}{x}\,dx$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} - \frac{c\int \frac{-10a^4c^3-10a^5c^3x}{x\sqrt{1-a^2x^2}}}{4a^2}$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{1}{2}\left(5a^2c^4\right)\int \frac{}{x\sqrt{}}$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{5}{2}a^2c^4\sin^{-1}(ax)$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{5}{2}a^2c^4\sin^{-1}(ax)$$

$$= \frac{5}{2}a^2c^4(1+ax)\sqrt{1-a^2x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{x} + \frac{5}{2}a^2c^4\sin^{-1}(ax)$$

Mathematica [A] time = 0.23, size = 106, normalized size = 0.91

$$\frac{1}{4}c^4\left(-10a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+5a^2\sin^{-1}(ax)-10a^2\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)+\frac{2(ax+1)^2\left(a^3x^3-8a^2x^2+8ax-1\right)}{x^2\sqrt{1-a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^3,x]
[Out] (c^4*((2*(1 + a*x)^2*(-1 + 8*a*x - 8*a^2*x^2 + a^3*x^3))/(x^2*Sqrt[1 - a^2*
x^2]) + 5*a^2*ArcSin[a*x] - 10*a^2*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] - 10*a^2*A
rcTanh[Sqrt[1 - a^2*x^2]]))/4
```

fricas [A] time = 0.49, size = 125, normalized size = 1.08

$$-\frac{10\,a^2c^4x^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-5\,a^2c^4x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^2c^4x^2+\left(a^3c^4x^3-6\,a^2c^4x^2-6\,ac^4x+c^4\right)\sqrt{-a^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^3,x, algorithm="fricas"
)
```

```
[Out] -1/2*(10*a^2*c^4*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 5*a^2*c^4*x^2
*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^2*c^4*x^2 + (a^3*c^4*x^3 - 6*a^2*c^4
*x^2 - 6*a*c^4*x + c^4)*sqrt(-a^2*x^2 + 1))/x^2
```

giac [B] time = 0.23, size = 224, normalized size = 1.93

$$\frac{5\,a^3c^4\arcsin\left(ax\right)\mathrm{sgn}(a)}{2\,|a|}-\frac{5\,a^3c^4\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\frac{\left(a^3c^4-\frac{12\left(\sqrt{-a^2x^2+1}\,|a|+a\right)ac^4}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}-\frac{1}{2}\left(a^3c^4x-6\,a^2c^4\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*c^4*arcsin(a*x)*sgn(a)/abs(a) - 5/2*a^3*c^4*log(1/2*abs(-2*sqrt(-a^
2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/8*(a^3*c^4 - 12*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)*a*c^4/x)*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*
abs(a)) - 1/2*(a^3*c^4*x - 6*a^2*c^4)*sqrt(-a^2*x^2 + 1) + 1/8*(12*(sqrt(-a
^2*x^2 + 1)*abs(a) + a)*a*c^4*abs(a)/x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*
c^4*abs(a)/(a*x^2))/a^2
```

maple [A] time = 0.04, size = 138, normalized size = 1.19

$$-\frac{c^4a^3x\sqrt{-a^2x^2+1}}{2}+\frac{5c^4a^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}+3c^4a^2\sqrt{-a^2x^2+1}-\frac{5c^4a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{3c^4a\sqrt{-a^2x^2+1}}{x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^3,x)
```

```
[Out] -1/2*c^4*a^3*x*(-a^2*x^2+1)^(1/2)+5/2*c^4*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2
)*x/(-a^2*x^2+1)^(1/2))+3*c^4*a^2*(-a^2*x^2+1)^(1/2)-5/2*c^4*a^2*arctanh(1/
(-a^2*x^2+1)^(1/2))+3*c^4*a/x*(-a^2*x^2+1)^(1/2)-1/2*c^4/x^2*(-a^2*x^2+1)^(
1/2)
```

maxima [A] time = 0.52, size = 129, normalized size = 1.11

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\,a^3c^4x+\frac{5}{2}\,a^2c^4\arcsin\left(ax\right)-\frac{5}{2}\,a^2c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+3\sqrt{-a^2x^2+1}\,a^2c^4+\frac{3\sqrt{-a^2x^2+1}\,ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^3,x, algorithm="maxima"
)
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*a^3*c^4*x + 5/2*a^2*c^4*arcsin(a*x) - 5/2*a^2*c^4*l
og(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 3*sqrt(-a^2*x^2 + 1)*a^2*c^4 +
 3*sqrt(-a^2*x^2 + 1)*a*c^4/x - 1/2*sqrt(-a^2*x^2 + 1)*c^4/x^2
```

mupad [B] time = 0.78, size = 133, normalized size = 1.15

$$3\,a^2\,c^4\,\sqrt{1-a^2\,x^2}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{2\,x^2}+\frac{3\,a\,c^4\,\sqrt{1-a^2\,x^2}}{x}-\frac{a^3\,c^4\,x\,\sqrt{1-a^2\,x^2}}{2}+\frac{5\,a^3\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}+\frac{a^2\,c^4\,\mathrm{atan}\left(\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^2*c^4*atan((1 - a^2*x^2)^(1/2)*1i)*5i)/2 + 3*a^2*c^4*(1 - a^2*x^2)^(1/2)
 - (c^4*(1 - a^2*x^2)^(1/2))/(2*x^2) + (3*a*c^4*(1 - a^2*x^2)^(1/2))/x - (a
^3*c^4*x*(1 - a^2*x^2)^(1/2))/2 + (5*a^3*c^4*asinh(x*(-a^2)^(1/2)))/(2*(-a^
2)^(1/2))
```

sympy [C] time = 7.72, size = 357, normalized size = 3.08

$$a^5c^4\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\,\mathrm{acosh}\,(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\mathrm{asin}\,(ax)}{2a^3} & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+2a^3c^4\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{as} \\ \sqrt{-\frac{1}{a^2}}\ \mathrm{a}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**3,x)
```

```
[Out] a**5*c**4*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a*
*3), Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-
a**2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) - 3*a**4*c**4*Piecewise((x**2/
2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) + 2*a**3*c**4*Piecewis
e((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt
(-a**2)), a**2 < 0)) + 2*a**2*c**4*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x
**2) > 1), (I*asin(1/(a*x)), True)) - 3*a*c**4*Piecewise((-I*sqrt(a**2*x**2
 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)) + c**4*Piece
wise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2
```

```
*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) +
I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

3.323 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^4}\,dx$

Optimal. Leaf size=120

$$-3a^3c^4\sin^{-1}(ax)+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2}-\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{1}{2}a^3c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/3*c^4*(-a^2*x^2+1)^(3/2)/x^3+3/2*a*c^4*(-a^2*x^2+1)^(3/2)/x^2-3*a^3*c^4*
arcsin(a*x)-1/2*a^3*c^4*arctanh((-a^2*x^2+1)^(1/2))-1/2*a^2*c^4*(-a*x+6)*(-
a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.25, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 813, 844, 216, 266, 63, 208}

$$\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3}-\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x}-\frac{1}{2}a^3c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-3a^3c^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^4,x]
```

```
[Out] -(a^2*c^4*(6 - a*x)*Sqrt[1 - a^2*x^2])/(2*x) - (c^4*(1 - a^2*x^2)^(3/2))/(3
*x^3) + (3*a*c^4*(1 - a^2*x^2)^(3/2))/(2*x^2) - 3*a^3*c^4*ArcSin[a*x] - (a^
3*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^4}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^4}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3} - \frac{1}{3}c\int \frac{\sqrt{1-a^2x^2}\,\left(9ac^3-9a^2c^3x+3a^3c^3x^2\right)}{x^3}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{1}{6}c\int \frac{\left(18a^2c^3+3a^3c^3x\right)\sqrt{1-a^2x^2}}{x^2}\,dx$$
$$= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2} - \frac{1}{12}c\int \frac{-6a^3c^3 +}{x\sqrt{1-}}$$
$$= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2} + \frac{1}{2}\left(a^3c^4\right)\int \frac{}{x\sqrt{1}}$$
$$= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2} - 3a^3c^4\sin^{-1}(ax) -$$
$$= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2} - 3a^3c^4\sin^{-1}(ax) -$$
$$= -\frac{a^2c^4(6-ax)\sqrt{1-a^2x^2}}{2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3x^3} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{2x^2} - 3a^3c^4\sin^{-1}(ax) -$$

Mathematica [A] time = 0.14, size = 164, normalized size = 1.37

$$\frac{c^4\left(12a^5x^5+32a^4x^4-30a^3x^3-28a^2x^2+3a^3x^3\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+78a^3x^3\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-6a^3x^3\right.}{12x^3\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^4,x]
```

```
[Out] (c^4*(-4 + 18*a*x - 28*a^2*x^2 - 30*a^3*x^3 + 32*a^4*x^4 + 12*a^5*x^5 + 3*a
^3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x] + 78*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[
Sqrt[1 - a*x]/Sqrt[2]] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x
^2]]))/(12*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.48, size = 129, normalized size = 1.08

$$\frac{36\,a^3c^4x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,a^3c^4x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^4x^3-\left(6\,a^3c^4x^3+16\,a^2c^4x^2-9\,ac^4x+2\,c^4\right)\sqrt{}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/6*(36*a^3*c^4*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a^3*c^4*x^3*
log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^4*x^3 - (6*a^3*c^4*x^3 + 16*a^2*c
^4*x^2 - 9*a*c^4*x + 2*c^4)*sqrt(-a^2*x^2 + 1))/x^3
```

giac [B] time = 0.42, size = 270, normalized size = 2.25

$$-\frac{3\,a^4c^4\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}-\frac{a^4c^4\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\sqrt{-a^2x^2+1}\,a^3c^4+\frac{\left(a^4c^4-\frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2c^4}{x}+\frac{33\left(\sqrt{-a^2\right.}}{}\right.}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^4,x, algorithm="giac")
```

```
[Out] -3*a^4*c^4*arcsin(a*x)*sgn(a)/abs(a) - 1/2*a^4*c^4*log(1/2*abs(-2*sqrt(-a^2
*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*a^3*c^4 +
 1/24*(a^4*c^4 - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2*c^4/x + 33*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^2*c^4/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^
3*abs(a)) - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x - 9*(sqrt(-a
^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c
^4/x^3)/(a^2*abs(a))
```

maple [A] time = 0.04, size = 140, normalized size = 1.17

$$-c^4a^3\sqrt{-a^2x^2+1}-\frac{3c^4a^4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{c^4a^3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{8c^4a^2\sqrt{-a^2x^2+1}}{3x}-\frac{c^4\sqrt{-a^2x^2+1}}{3x^3}+\frac{3c^4a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^4,x)
```

```
[Out] -c^4*a^3*(-a^2*x^2+1)^(1/2)-3*c^4*a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^
2*x^2+1)^(1/2))-1/2*c^4*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))-8/3*c^4*a^2/x*(-a
^2*x^2+1)^(1/2)-1/3*c^4/x^3*(-a^2*x^2+1)^(1/2)+3/2*c^4*a/x^2*(-a^2*x^2+1)^(
1/2)
```

maxima [A] time = 0.54, size = 131, normalized size = 1.09

$$-3\,a^3c^4\arcsin\left(ax\right)-\frac{1}{2}\,a^3c^4\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}\,a^3c^4-\frac{8\,\sqrt{-a^2x^2+1}\,a^2c^4}{3\,x}+\frac{3\,\sqrt{-a^2x^2+1}\,ac^4}{2\,x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^4,x, algorithm="maxima"
)
```

```
[Out] -3*a^3*c^4*arcsin(a*x) - 1/2*a^3*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/ab
s(x)) - sqrt(-a^2*x^2 + 1)*a^3*c^4 - 8/3*sqrt(-a^2*x^2 + 1)*a^2*c^4/x + 3/2
*sqrt(-a^2*x^2 + 1)*a*c^4/x^2 - 1/3*sqrt(-a^2*x^2 + 1)*c^4/x^3
```

mupad [B] time = 0.78, size = 135, normalized size = 1.12

$$\frac{3\,a\,c^4\,\sqrt{1-a^2\,x^2}}{2\,x^2}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{3\,x^3}-a^3\,c^4\,\sqrt{1-a^2\,x^2}-\frac{3\,a^4\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{8\,a^2\,c^4\,\sqrt{1-a^2\,x^2}}{3\,x}+\frac{a^3\,c^4\,\mathrm{atan}\left(\sqrt{1}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(x^4*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^3*c^4*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/2 - a^3*c^4*(1 - a^2*x^2)^(1/2) -
 (c^4*(1 - a^2*x^2)^(1/2))/(3*x^3) + (3*a*c^4*(1 - a^2*x^2)^(1/2))/(2*x^2)
- (3*a^4*c^4*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (8*a^2*c^4*(1 - a^2*x^2)
^(1/2))/(3*x)
```

sympy [C] time = 6.94, size = 359, normalized size = 2.99

$$a^5c^4\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\ \mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+2a^3c^4\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x}\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherw}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**4,x)
```

```
[Out] a**5*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, Tru
e)) - 3*a**4*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (
sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*a**3*c**4*Piecewise((-ac
osh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + 2*a**2*c**4
*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**
2 + 1)/x, True)) - 3*a*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 +
 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a
/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True
)) + c**4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2
```

```
 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) -
sqrt(-a**2*x**2 + 1)/(3*x**3), True))
```

3.324 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^5} dx$

Optimal. Leaf size=110

$$a^4c^4\sin^{-1}(ax)-\frac{11a^2c^4\sqrt{1-a^2x^2}}{8x^2}-\frac{c^4\sqrt{1-a^2x^2}}{4x^4}+\frac{ac^4\sqrt{1-a^2x^2}}{x^3}+\frac{13}{8}a^4c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] a^4*c^4*arcsin(a*x)+13/8*a^4*c^4*arctanh((-a^2*x^2+1)^(1/2))-1/4*c^4*(-a^2*
x^2+1)^(1/2)/x^4+a*c^4*(-a^2*x^2+1)^(1/2)/x^3-11/8*a^2*c^4*(-a^2*x^2+1)^(1/
2)/x^2
```

Rubi [A] time = 0.25, antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 811, 844, 216, 266, 63, 208}

$$\frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2}+\frac{13}{8}a^4c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^4c^4\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^5,x]
```

```
[Out] -(a^2*c^4*(13 - 8*a*x)*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^4*(1 - a^2*x^2)^(3/2
))/(4*x^4) + (a*c^4*(1 - a^2*x^2)^(3/2))/x^3 + a^4*c^4*ArcSin[a*x] + (13*a^
4*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^5}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^5}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{1}{4}c\int \frac{\sqrt{1-a^2x^2}\,\left(12ac^3-13a^2c^3x+4a^3c^3x^2\right)}{x^4}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} + \frac{1}{12}c\int \frac{\left(39a^2c^3-12a^3c^3x\right)\sqrt{1-a^2x^2}}{x^3}\,dx$$
$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} - \frac{1}{48}c\int \frac{78a^4c^3 \cdots}{x\sqrt{1 \cdots}}$$
$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} - \frac{1}{8}\left(13a^4c^4\right)\int \cdots$$
$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} + a^4c^4\sin^{-1}(ax) \cdots$$
$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} + a^4c^4\sin^{-1}(ax) \cdots$$
$$= -\frac{a^2c^4(13-8ax)\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{ac^4\left(1-a^2x^2\right)^{3/2}}{x^3} + a^4c^4\sin^{-1}(ax) \cdots$$

Mathematica [A] time = 0.23, size = 125, normalized size = 1.14

$$\frac{1}{16}c^4\left(-13a^4\sin^{-1}(ax)+26a^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{2\left(-11a^4x^4+8a^3x^3+9a^2x^2+29a^4x^4\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1 \cdots}}{\sqrt{\cdots}}\right) \cdots\right.}{x^4\sqrt{1-a^2x^2}} \cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^5,x]
```

```
[Out] (c^4*(-13*a^4*ArcSin[a*x] - (2*(2 - 8*a*x + 9*a^2*x^2 + 8*a^3*x^3 - 11*a^4*
x^4 + 29*a^4*x^4*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(x^4*Sqr
t[1 - a^2*x^2]) + 26*a^4*ArcTanh[Sqrt[1 - a^2*x^2]]))/16
```

fricas [A] time = 0.51, size = 106, normalized size = 0.96

$$-\frac{16\,a^4c^4x^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+13\,a^4c^4x^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(11\,a^2c^4x^2-8\,ac^4x+2\,c^4\right)\sqrt{-a^2x^2+1}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^5,x, algorithm="fricas"
)
```

```
[Out] -1/8*(16*a^4*c^4*x^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 13*a^4*c^4*x^
4*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (11*a^2*c^4*x^2 - 8*a*c^4*x + 2*c^4)*sq
rt(-a^2*x^2 + 1))/x^4
```

giac [B] time = 0.20, size = 316, normalized size = 2.87

$$\frac{a^5c^4\arcsin(ax)\operatorname{sgn}(a)}{|a|}+\frac{13\,a^5c^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|}+\frac{\left(a^5c^4-\frac{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^3c^4}{x}+\frac{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2ac^4}{x^2}-\frac{8\left(\sqrt{\cdots}\right.}{}\right.}{64\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^5,x, algorithm="giac")
```

```
[Out] a^5*c^4*arcsin(a*x)*sgn(a)/abs(a) + 13/8*a^5*c^4*log(1/2*abs(-2*sqrt(-a^2*x
^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/64*(a^5*c^4 - 8*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)*a^3*c^4/x + 24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a*c^4/
x^2 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^4/(a*x^3))*a^8*x^4/((sqrt(-a^2*
x^2 + 1)*abs(a) + a)^4*abs(a)) + 1/64*(8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^
5*c^4*abs(a)/x - 24*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^3*c^4*abs(a)/x^2 +
8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*c^4*abs(a)/x^3 - (sqrt(-a^2*x^2 + 1)*
abs(a) + a)^4*c^4*abs(a)/(a*x^4))/a^4
```

maple [A] time = 0.04, size = 118, normalized size = 1.07

$$\frac{c^4a^5\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{13c^4a^4\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{8}+\frac{a\,c^4\sqrt{-a^2x^2+1}}{x^3}-\frac{11a^2c^4\sqrt{-a^2x^2+1}}{8x^2}-\frac{c^4\sqrt{-a^2x^2+1}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^5,x)
```

```
[Out] c^4*a^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+13/8*c^4*a^4*a
rctanh(1/(-a^2*x^2+1)^(1/2))+a*c^4*(-a^2*x^2+1)^(1/2)/x^3-11/8*a^2*c^4*(-a^
2*x^2+1)^(1/2)/x^2-1/4*c^4*(-a^2*x^2+1)^(1/2)/x^4
```

maxima [A] time = 0.41, size = 109, normalized size = 0.99

$$a^4c^4\arcsin(ax)+\frac{13}{8}\,a^4c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{11\sqrt{-a^2x^2+1}\,a^2c^4}{8\,x^2}+\frac{\sqrt{-a^2x^2+1}\,ac^4}{x^3}-\frac{\sqrt{-a^2x^2+1}\,c^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^5,x, algorithm="maxima"
)
```

```
[Out] a^4*c^4*arcsin(a*x) + 13/8*a^4*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(
x)) - 11/8*sqrt(-a^2*x^2 + 1)*a^2*c^4/x^2 + sqrt(-a^2*x^2 + 1)*a*c^4/x^3 -
1/4*sqrt(-a^2*x^2 + 1)*c^4/x^4
```

mupad [B] time = 0.05, size = 113, normalized size = 1.03

$$\frac{a\,c^4\,\sqrt{1-a^2\,x^2}}{x^3}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{4\,x^4}+\frac{a^5\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{11\,a^2\,c^4\,\sqrt{1-a^2\,x^2}}{8\,x^2}-\frac{a^4\,c^4\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1{}\mathrm{i}\right)\,13{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(x^5*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a*c^4*(1 - a^2*x^2)^(1/2))/x^3 - (c^4*(1 - a^2*x^2)^(1/2))/(4*x^4) - (a^4*
c^4*atan((1 - a^2*x^2)^(1/2)*1i)*13i)/8 + (a^5*c^4*asinh(x*(-a^2)^(1/2)))/(
-a^2)^(1/2) - (11*a^2*c^4*(1 - a^2*x^2)^(1/2))/(8*x^2)
```

sympy [C] time = 9.36, size = 505, normalized size = 4.59

$$a^5c^4\left(\begin{cases}\sqrt{\frac{1}{a^2}}\,\mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\,\mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)-3a^4c^4\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)+2a^3c^4\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ |a\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{other}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**5,x)
```

```
[Out] a**5*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/
a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 3*a**4*c**4*Piecewise((-acosh(1/(a
*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + 2*a**3*c**4*Piecewi
se((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x
, True)) + 2*a**2*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a
**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x
*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) -
3*a*c**4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2
- 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - s
```

```
qrt(-a**2*x**2 + 1)/(3*x**3), True)) + c**4*Piecewise((-3*a**4*acosh(1/(a*x
))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*
x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*
I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x*
*3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

3.325 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^6}\,dx$

Optimal. Leaf size=129

$$-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{7}{8}a^5c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2}$$

```
[Out] -1/5*c^4*(-a^2*x^2+1)^(3/2)/x^5+3/4*a*c^4*(-a^2*x^2+1)^(3/2)/x^4-17/15*a^2*
c^4*(-a^2*x^2+1)^(3/2)/x^3-7/8*a^5*c^4*arctanh((-a^2*x^2+1)^(1/2))+7/8*a^3*
c^4*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.25, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 1807, 807, 266, 47, 63, 208}

$$\frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2}-\frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{7}{8}a^5c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^6,x]
```

```
[Out] (7*a^3*c^4*Sqrt[1 - a^2*x^2])/(8*x^2) - (c^4*(1 - a^2*x^2)^(3/2))/(5*x^5) +
 (3*a*c^4*(1 - a^2*x^2)^(3/2))/(4*x^4) - (17*a^2*c^4*(1 - a^2*x^2)^(3/2))/(
15*x^3) - (7*a^5*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^6}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^6}\,dx$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{1}{5}c\int \frac{\sqrt{1-a^2x^2}\left(15ac^3-17a^2c^3x+5a^3c^3x^2\right)}{x^5}\,dx$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} + \frac{1}{20}c\int \frac{\left(68a^2c^3-35a^3c^3x\right)\sqrt{1-a^2x^2}}{x^4}\,dx$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{4}\left(7a^3c^4\right)\int \frac{\sqrt{1-a}}{x^3}$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{8}\left(7a^3c^4\right)\text{Subst}\Bigg(\int$$

$$= \frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} + \frac{1}{16}$$

$$= \frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{1}{8}$$

$$= \frac{7a^3c^4\sqrt{1-a^2x^2}}{8x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{4x^4} - \frac{17a^2c^4\left(1-a^2x^2\right)^{3/2}}{15x^3} - \frac{7}{8}$$

Mathematica [A] time = 0.04, size = 107, normalized size = 0.83

$$-\frac{c^4\left(136a^6x^6+15a^5x^5-248a^4x^4+75a^3x^3+88a^2x^2+105a^5x^5\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-90ax+24\right)}{120x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^6,x]
```

```
[Out] -1/120*(c^4*(24 - 90*a*x + 88*a^2*x^2 + 75*a^3*x^3 - 248*a^4*x^4 + 15*a^5*x
^5 + 136*a^6*x^6 + 105*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]]
))/(x^5*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.49, size = 95, normalized size = 0.74

$$\frac{105\,a^5c^4x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(136\,a^4c^4x^4+15\,a^3c^4x^3-112\,a^2c^4x^2+90\,ac^4x-24\,c^4\right)\sqrt{-a^2x^2+1}}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/120*(105*a^5*c^4*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (136*a^4*c^4*x^4 +
 15*a^3*c^4*x^3 - 112*a^2*c^4*x^2 + 90*a*c^4*x - 24*c^4)*sqrt(-a^2*x^2 + 1)
)/x^5
```

giac [B] time = 0.23, size = 354, normalized size = 2.74

$$\frac{\left(6\,a^6c^4 - \frac{45\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^4c^4}{x} + \frac{130\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2a^2c^4}{x^2} - \frac{120\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^4}{x^3} - \frac{420\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^4}{a^2x^4}\right)a^{10}x^5}{960\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5|a|} - \frac{7\,a^6c^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^6,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^6*c^4 - 45*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x + 130*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 120*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^4/x^3 - 420*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4))*a^10*x
^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 7/8*a^6*c^4*log(1/2*abs(-2*
sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 1/960*(420*(sqrt(-a
^2*x^2 + 1)*abs(a) + a)*a^8*c^4/x + 120*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a
^6*c^4/x^2 - 130*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a^4*c^4/x^3 + 45*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^4*a^2*c^4/x^4 - 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^
5*c^4/x^5)/(a^4*abs(a))
```

maple [A] time = 0.04, size = 207, normalized size = 1.60

$$c^4\left(-a^5\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + \frac{3a^4\sqrt{-a^2x^2+1}}{x} + \frac{14a^2\left(-\frac{\sqrt{-a^2x^2+1}}{3x^3} - \frac{2a^2\sqrt{-a^2x^2+1}}{3x}\right)}{5} + 2a^3\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2} - \frac{a^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^6,x)
```

```
[Out] c^4*(-a^5*arctanh(1/(-a^2*x^2+1)^(1/2))+3*a^4/x*(-a^2*x^2+1)^(1/2)+14/5*a^2
*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)^(1/2)/x)+2*a^3*(-1/2*(-a
^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2)))-3*a*(-1/4*(-a^2*
```

```
x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a
^2*x^2+1)^(1/2))))-1/5/x^5*(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.54, size = 145, normalized size = 1.12

$$-\frac{7}{8}a^5c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{17\sqrt{-a^2x^2+1}\,a^4c^4}{15\,x}+\frac{\sqrt{-a^2x^2+1}\,a^3c^4}{8\,x^2}-\frac{14\sqrt{-a^2x^2+1}\,a^2c^4}{15\,x^3}+\frac{3\sqrt{-a^2x^2+1}\,\cdots}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^6,x, algorithm="maxima"
)
```

```
[Out] -7/8*a^5*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 17/15*sqrt(-a^2*
x^2 + 1)*a^4*c^4/x + 1/8*sqrt(-a^2*x^2 + 1)*a^3*c^4/x^2 - 14/15*sqrt(-a^2*x
^2 + 1)*a^2*c^4/x^3 + 3/4*sqrt(-a^2*x^2 + 1)*a*c^4/x^4 - 1/5*sqrt(-a^2*x^2
+ 1)*c^4/x^5
```

mupad [B] time = 0.05, size = 136, normalized size = 1.05

$$\frac{3\,a\,c^4\sqrt{1-a^2\,x^2}}{4\,x^4}-\frac{c^4\sqrt{1-a^2\,x^2}}{5\,x^5}-\frac{14\,a^2\,c^4\sqrt{1-a^2\,x^2}}{15\,x^3}+\frac{a^3\,c^4\sqrt{1-a^2\,x^2}}{8\,x^2}+\frac{17\,a^4\,c^4\sqrt{1-a^2\,x^2}}{15\,x}+\frac{a^5\,c^4\,\mathrm{atan}\left(\sqrt{1-\cdots}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(x^6*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^5*c^4*atan((1 - a^2*x^2)^(1/2)*1i)*7i)/8 - (c^4*(1 - a^2*x^2)^(1/2))/(5*
x^5) + (3*a*c^4*(1 - a^2*x^2)^(1/2))/(4*x^4) - (14*a^2*c^4*(1 - a^2*x^2)^(1
/2))/(15*x^3) + (a^3*c^4*(1 - a^2*x^2)^(1/2))/(8*x^2) + (17*a^4*c^4*(1 - a^
2*x^2)^(1/2))/(15*x)
```

sympy [C] time = 10.10, size = 607, normalized size = 4.71

$$a^5c^4\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } |a^2x^2|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+2a^3c^4\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x}\cdots\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**6,x)
```

```
[Out] a**5*c**4*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x
)), True)) - 3*a**4*c**4*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2
) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)) + 2*a**3*c**4*Piecewise((-a**2*aco
sh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I
*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqr
t(1 - 1/(a**2*x**2))), True)) + 2*a**2*c**4*Piecewise((-2*I*a**2*sqrt(a**2*
x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*
a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True)) - 3
*a*c**4*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2
*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a
**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8
*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a
*x**5*sqrt(1 - 1/(a**2*x**2))), True)) + c**4*Piecewise((-8*a**5*sqrt(-1 +
1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 +
 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**
2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(
a**2*x**2))/(5*x**4), True))
```

3.326 $\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^7}\,dx$

Optimal. Leaf size=156

$$-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4}+\frac{7}{16}a^6c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2}+\frac{11a^3c}{}$$

```
[Out] -1/6*c^4*(-a^2*x^2+1)^(3/2)/x^6+3/5*a*c^4*(-a^2*x^2+1)^(3/2)/x^5-7/8*a^2*c^
4*(-a^2*x^2+1)^(3/2)/x^4+11/15*a^3*c^4*(-a^2*x^2+1)^(3/2)/x^3+7/16*a^6*c^4*
arctanh((-a^2*x^2+1)^(1/2))-7/16*a^4*c^4*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.26, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 1807, 835, 807, 266, 47, 63, 208}

$$-\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2}+\frac{11a^3c^4\left(1-a^2x^2\right)^{3/2}}{15x^3}-\frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4}+\frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6}+\frac{7}{16}a^6c^4\tanh$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^7,x]
```

```
[Out] (-7*a^4*c^4*Sqrt[1 - a^2*x^2])/(16*x^2) - (c^4*(1 - a^2*x^2)^(3/2))/(6*x^6)
 + (3*a*c^4*(1 - a^2*x^2)^(3/2))/(5*x^5) - (7*a^2*c^4*(1 - a^2*x^2)^(3/2))/
(8*x^4) + (11*a^3*c^4*(1 - a^2*x^2)^(3/2))/(15*x^3) + (7*a^6*c^4*ArcTanh[Sq
rt[1 - a^2*x^2]])/16
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
```

```
0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}(c-acx)^4}{x^7}\,dx = c\int \frac{(c-acx)^3\sqrt{1-a^2x^2}}{x^7}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} - \frac{1}{6}c\int \frac{\sqrt{1-a^2x^2}\,\left(18ac^3-21a^2c^3x+6a^3c^3x^2\right)}{x^6}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} + \frac{1}{30}c\int \frac{\left(105a^2c^3-66a^3c^3x\right)\sqrt{1-a^2x^2}}{x^5}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} - \frac{1}{120}c\int \frac{\left(264a^3c^3-\right.}{}$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{11a^3c^4\left(1-a^2x^2\right)^{3/2}}{15x^3}$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{11a^3c^4\left(1-a^2x^2\right)^{3/2}}{15x^3}$$
$$= -\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{1}{}$$
$$= -\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{1}{}$$
$$= -\frac{7a^4c^4\sqrt{1-a^2x^2}}{16x^2} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{6x^6} + \frac{3ac^4\left(1-a^2x^2\right)^{3/2}}{5x^5} - \frac{7a^2c^4\left(1-a^2x^2\right)^{3/2}}{8x^4} + \frac{1}{}$$

Mathematica [A] time = 0.04, size = 115, normalized size = 0.74

$$\frac{c^4\left(176a^7x^7-105a^6x^6-208a^5x^5+275a^4x^4-112a^3x^3-130a^2x^2+105a^6x^6\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+1\right.}{240x^6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a*c*x)^4)/x^7,x]

[Out] (c^4*(-40 + 144*a*x - 130*a^2*x^2 - 112*a^3*x^3 + 275*a^4*x^4 - 208*a^5*x^5
 - 105*a^6*x^6 + 176*a^7*x^7 + 105*a^6*x^6*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1
 - a^2*x^2]]))/(240*x^6*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.54, size = 106, normalized size = 0.68

$$-\frac{105\,a^6c^4x^6\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(176\,a^5c^4x^5-105\,a^4c^4x^4-32\,a^3c^4x^3+170\,a^2c^4x^2-144\,ac^4x+40\,c^4\right)\sqrt{-a^2x^2+}}{240\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^7,x, algorithm="fricas"
)
```

```
[Out] -1/240*(105*a^6*c^4*x^6*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (176*a^5*c^4*x^5
- 105*a^4*c^4*x^4 - 32*a^3*c^4*x^3 + 170*a^2*c^4*x^2 - 144*a*c^4*x + 40*c^4
)*sqrt(-a^2*x^2 + 1))/x^6
```

giac [B] time = 0.24, size = 424, normalized size = 2.72

$$\frac{\left(5\,a^7c^4-\frac{36\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^5c^4}{x}+\frac{105\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2a^3c^4}{x^2}-\frac{140\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3ac^4}{x^3}-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^4}{ax^4}+\frac{600\left(\sqrt{-a^2x^2+1}\right.}{a^3x^5}\right.}{1920\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^6|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^7,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^7*c^4 - 36*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c^4/x + 105*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^2*a^3*c^4/x^2 - 140*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^3*a*c^4/x^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a*x^4) + 600*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^3*x^5))*a^12*x^6/((sqrt(-a^2*x^2 +
1)*abs(a) + a)^6*abs(a)) + 7/16*a^7*c^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*a
bs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/1920*(600*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)*a^9*c^4*abs(a)/x - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^7*c^4*abs(a)
/x^2 - 140*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a^5*c^4*abs(a)/x^3 + 105*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)^4*a^3*c^4*abs(a)/x^4 - 36*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^5*a*c^4*abs(a)/x^5 + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6*c^4*abs(
a)/(a*x^6))/a^6
```

maple [A] time = 0.05, size = 255, normalized size = 1.63

$$c^4\left(-\frac{a^5\sqrt{-a^2x^2+1}}{x}+2a^3\left(-\frac{\sqrt{-a^2x^2+1}}{3x^3}-\frac{2a^2\sqrt{-a^2x^2+1}}{3x}\right)-3a^4\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^7,x)
```

```
[Out] c^4*(-a^5*(-a^2*x^2+1)^(1/2)/x+2*a^3*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(
-a^2*x^2+1)^(1/2)/x)-3*a^4*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(
-a^2*x^2+1)^(1/2)))+17/6*a^2*(-1/4*(-a^2*x^2+1)^(1/2)/x^4+3/4*a^2*(-1/2*(-a
^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))-3*a*(-1/5/x^5*(
-a^2*x^2+1)^(1/2)+4/5*a^2*(-1/3*(-a^2*x^2+1)^(1/2)/x^3-2/3*a^2*(-a^2*x^2+1)
^(1/2)/x))-1/6/x^6*(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.40, size = 168, normalized size = 1.08

$$\frac{7}{16}a^6c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{11\sqrt{-a^2x^2+1}\,a^5c^4}{15\,x}+\frac{7\sqrt{-a^2x^2+1}\,a^4c^4}{16\,x^2}+\frac{2\sqrt{-a^2x^2+1}\,a^3c^4}{15\,x^3}-\frac{17\sqrt{-a^2x^2+1}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^4/x^7,x, algorithm="maxima"
)
```

```
[Out] 7/16*a^6*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 11/15*sqrt(-a^2*
x^2 + 1)*a^5*c^4/x + 7/16*sqrt(-a^2*x^2 + 1)*a^4*c^4/x^2 + 2/15*sqrt(-a^2*x
^2 + 1)*a^3*c^4/x^3 - 17/24*sqrt(-a^2*x^2 + 1)*a^2*c^4/x^4 + 3/5*sqrt(-a^2*
x^2 + 1)*a*c^4/x^5 - 1/6*sqrt(-a^2*x^2 + 1)*c^4/x^6
```

mupad [B] time = 0.81, size = 159, normalized size = 1.02

$$\frac{3\,a\,c^4\,\sqrt{1-a^2\,x^2}}{5\,x^5}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{6\,x^6}-\frac{17\,a^2\,c^4\,\sqrt{1-a^2\,x^2}}{24\,x^4}+\frac{2\,a^3\,c^4\,\sqrt{1-a^2\,x^2}}{15\,x^3}+\frac{7\,a^4\,c^4\,\sqrt{1-a^2\,x^2}}{16\,x^2}-\frac{11\,a^5\,c^4\,\sqrt{1-a^2\,x}}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(x^7*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (3*a*c^4*(1 - a^2*x^2)^(1/2))/(5*x^5) - (c^4*(1 - a^2*x^2)^(1/2))/(6*x^6) -
 (a^6*c^4*atan((1 - a^2*x^2)^(1/2)*1i)*7i)/16 - (17*a^2*c^4*(1 - a^2*x^2)^(
1/2))/(24*x^4) + (2*a^3*c^4*(1 - a^2*x^2)^(1/2))/(15*x^3) + (7*a^4*c^4*(1 -
 a^2*x^2)^(1/2))/(16*x^2) - (11*a^5*c^4*(1 - a^2*x^2)^(1/2))/(15*x)
```

sympy [C] time = 13.51, size = 801, normalized size = 5.13

$$a^5c^4\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)-3a^4c^4\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{\left|a^2x^2\right|}>1\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)+2a^3c^4\left(\begin{cases}-\frac{2i\cdots}{} \\ -\frac{2a\cdots}{}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**4/x**7,x)
```

```
[Out] a**5*c**4*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(
-a**2*x**2 + 1)/x, True)) - 3*a**4*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 -
 a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a
*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x
**2))), True)) + 2*a**3*c**4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x)
 - I*sqrt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2
*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True)) + 2*a**2*c**4*Piec
ewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a
/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))),
 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 -
1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1
 - 1/(a**2*x**2))), True)) - 3*a*c**4*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*
x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2
*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))
/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**
2))/(5*x**4), True)) + c**4*Piecewise((-5*a**6*acosh(1/(a*x))/16 + 5*a**5/(
16*x*sqrt(-1 + 1/(a**2*x**2))) - 5*a**3/(48*x**3*sqrt(-1 + 1/(a**2*x**2)))
- a/(24*x**5*sqrt(-1 + 1/(a**2*x**2))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2
))), 1/Abs(a**2*x**2) > 1), (5*I*a**6*asin(1/(a*x))/16 - 5*I*a**5/(16*x*sqr
t(1 - 1/(a**2*x**2))) + 5*I*a**3/(48*x**3*sqrt(1 - 1/(a**2*x**2))) + I*a/(2
4*x**5*sqrt(1 - 1/(a**2*x**2))) + I/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), Tru
e))
```

3.327 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{c-acx}\,dx$

Optimal. Leaf size=146

$$-\frac{27\sin^{-1}(ax)}{8a^5c}+\frac{x^3\sqrt{1-a^2x^2}}{4a^2c}+\frac{13\sqrt{1-a^2x^2}}{3a^5c}+\frac{(ax+1)^2}{a^5c\sqrt{1-a^2x^2}}+\frac{11x\sqrt{1-a^2x^2}}{8a^4c}+\frac{2x^2\sqrt{1-a^2x^2}}{3a^3c}$$

```
[Out] -27/8*arcsin(a*x)/a^5/c+(a*x+1)^2/a^5/c/(-a^2*x^2+1)^(1/2)+13/3*(-a^2*x^2+1
)^(1/2)/a^5/c+11/8*x*(-a^2*x^2+1)^(1/2)/a^4/c+2/3*x^2*(-a^2*x^2+1)^(1/2)/a^
3/c+1/4*x^3*(-a^2*x^2+1)^(1/2)/a^2/c
```

Rubi [A] time = 0.34, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 852, 1635, 1815, 641, 216}

$$\frac{x^3\sqrt{1-a^2x^2}}{4a^2c}+\frac{2x^2\sqrt{1-a^2x^2}}{3a^3c}+\frac{11x\sqrt{1-a^2x^2}}{8a^4c}+\frac{13\sqrt{1-a^2x^2}}{3a^5c}+\frac{(ax+1)^2}{a^5c\sqrt{1-a^2x^2}}-\frac{27\sin^{-1}(ax)}{8a^5c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a*c*x),x]
```

```
[Out] (1 + a*x)^2/(a^5*c*Sqrt[1 - a^2*x^2]) + (13*Sqrt[1 - a^2*x^2])/(3*a^5*c) +
(11*x*Sqrt[1 - a^2*x^2])/(8*a^4*c) + (2*x^2*Sqrt[1 - a^2*x^2])/(3*a^3*c) +
(x^3*Sqrt[1 - a^2*x^2])/(4*a^2*c) - (27*ArcSin[a*x])/(8*a^5*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{c-acx}\,dx = c\int \frac{x^4\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx$$
$$= \frac{\int \frac{x^4(c+acx)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$
$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} - \frac{\int \frac{(c+acx)\left(\frac{2}{a^4}+\frac{x}{a^3}+\frac{x^2}{a^2}+\frac{x^3}{a}\right)}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$
$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} + \frac{\int \frac{-\frac{8c}{a^2}-\frac{12cx}{a}-11cx^2-8acx^3}{\sqrt{1-a^2x^2}}\,dx}{4a^2c^2}$$
$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{2x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} - \frac{\int \frac{24c+52acx+33a^2cx^2}{\sqrt{1-a^2x^2}}\,dx}{12a^4c^2}$$
$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{11x\sqrt{1-a^2x^2}}{8a^4c} + \frac{2x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} + \frac{\int \frac{-81a^2c-104a^3cx}{\sqrt{1-a^2x^2}}\,dx}{24a^6c^2}$$
$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{13\sqrt{1-a^2x^2}}{3a^5c} + \frac{11x\sqrt{1-a^2x^2}}{8a^4c} + \frac{2x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} - \frac{27\int \frac{}{\sqrt{}}}{8}$$
$$= \frac{(1+ax)^2}{a^5c\sqrt{1-a^2x^2}} + \frac{13\sqrt{1-a^2x^2}}{3a^5c} + \frac{11x\sqrt{1-a^2x^2}}{8a^4c} + \frac{2x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{x^3\sqrt{1-a^2x^2}}{4a^2c} - \frac{27\sin}{8a}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 0.55

$$\frac{162\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right) - \frac{\sqrt{ax+1}\left(6a^4x^4+10a^3x^3+17a^2x^2+47ax-128\right)}{\sqrt{1-ax}}}{24a^5c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a*c*x),x]
```

```
[Out] (-((Sqrt[1 + a*x]*(-128 + 47*a*x + 17*a^2*x^2 + 10*a^3*x^3 + 6*a^4*x^4))/Sq
rt[1 - a*x]) + 162*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(24*a^5*c)
```

fricas [A] time = 0.51, size = 95, normalized size = 0.65

$$\frac{128\,ax + 162\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(6\,a^4x^4 + 10\,a^3x^3 + 17\,a^2x^2 + 47\,ax - 128\right)\sqrt{-a^2x^2+1} - 128}{24\left(a^6cx - a^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/24*(128*a*x + 162*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (6*a
^4*x^4 + 10*a^3*x^3 + 17*a^2*x^2 + 47*a*x - 128)*sqrt(-a^2*x^2 + 1) - 128)/
(a^6*c*x - a^5*c)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 166, normalized size = 1.14

$$\frac{x^{3}\sqrt{-a^{2}x^{2}+1}}{4a^{2}c}+\frac{11x\sqrt{-a^{2}x^{2}+1}}{8a^{4}c}-\frac{27\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{8c\,a^{4}\sqrt{a^{2}}}+\frac{2x^{2}\sqrt{-a^{2}x^{2}+1}}{3a^{3}c}+\frac{10\sqrt{-a^{2}x^{2}+1}}{3a^{5}c}-\frac{2\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{c\,a^{6}\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c),x)
```

```
[Out] 1/4*x^3*(-a^2*x^2+1)^(1/2)/a^2/c+11/8*x*(-a^2*x^2+1)^(1/2)/a^4/c-27/8/c/a^4
/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+2/3*x^2*(-a^2*x^2+1)^
(1/2)/a^3/c+10/3*(-a^2*x^2+1)^(1/2)/a^5/c-2/c/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2
*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.45, size = 129, normalized size = 0.88

$$\frac{\sqrt{-a^{2}x^{2}+1}\,x^{3}}{4\,a^{2}c}-\frac{2\,\sqrt{-a^{2}x^{2}+1}}{a^{6}cx-a^{5}c}+\frac{2\,\sqrt{-a^{2}x^{2}+1}\,x^{2}}{3\,a^{3}c}+\frac{11\,\sqrt{-a^{2}x^{2}+1}\,x}{8\,a^{4}c}-\frac{27\,\arcsin\left(ax\right)}{8\,a^{5}c}+\frac{10\,\sqrt{-a^{2}x^{2}+1}}{3\,a^{5}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*x^3/(a^2*c) - 2*sqrt(-a^2*x^2 + 1)/(a^6*c*x - a^5*c)
 + 2/3*sqrt(-a^2*x^2 + 1)*x^2/(a^3*c) + 11/8*sqrt(-a^2*x^2 + 1)*x/(a^4*c) -
 27/8*arcsin(a*x)/(a^5*c) + 10/3*sqrt(-a^2*x^2 + 1)/(a^5*c)
```

mupad [B] time = 0.82, size = 163, normalized size = 1.12

$$\frac{10\sqrt{1-a^2x^2}}{3a^5c}-\frac{2\sqrt{1-a^2x^2}}{\sqrt{-a^2}\left(a^3c\sqrt{-a^2}-a^4cx\sqrt{-a^2}\right)}+\frac{11x\sqrt{1-a^2x^2}}{8a^4c}-\frac{27\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{8a^4c\sqrt{-a^2}}+\frac{x^3\sqrt{1-a^2x^2}}{4a^2c}+\frac{2x^2\sqrt{1-a^2x^2}}{3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] (10*(1 - a^2*x^2)^(1/2))/(3*a^5*c) - (2*(1 - a^2*x^2)^(1/2))/((-a^2)^(1/2)*
(a^3*c*(-a^2)^(1/2) - a^4*c*x*(-a^2)^(1/2))) + (11*x*(1 - a^2*x^2)^(1/2))/(
8*a^4*c) - (27*asinh(x*(-a^2)^(1/2)))/(8*a^4*c*(-a^2)^(1/2)) + (x^3*(1 - a^
2*x^2)^(1/2))/(4*a^2*c) + (2*x^2*(1 - a^2*x^2)^(1/2))/(3*a^3*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{x^4}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{ax^5}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a*c*x+c),x)
```

```
[Out] -(Integral(x**4/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Int
egral(a*x**5/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.328 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{c-acx}\,dx$

Optimal. Leaf size=114

$$-\frac{3\sin^{-1}(ax)}{a^4c}+\frac{x^2\sqrt{1-a^2x^2}}{3a^2c}+\frac{11\sqrt{1-a^2x^2}}{3a^4c}+\frac{(ax+1)^2}{a^4c\sqrt{1-a^2x^2}}+\frac{x\sqrt{1-a^2x^2}}{a^3c}$$

```
[Out] -3*arcsin(a*x)/a^4/c+(a*x+1)^2/a^4/c/(-a^2*x^2+1)^(1/2)+11/3*(-a^2*x^2+1)^(
1/2)/a^4/c+x*(-a^2*x^2+1)^(1/2)/a^3/c+1/3*x^2*(-a^2*x^2+1)^(1/2)/a^2/c
```

Rubi [A] time = 0.29, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 852, 1635, 1815, 641, 216}

$$\frac{x^2\sqrt{1-a^2x^2}}{3a^2c}+\frac{x\sqrt{1-a^2x^2}}{a^3c}+\frac{11\sqrt{1-a^2x^2}}{3a^4c}+\frac{(ax+1)^2}{a^4c\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a*c*x),x]
```

```
[Out] (1 + a*x)^2/(a^4*c*Sqrt[1 - a^2*x^2]) + (11*Sqrt[1 - a^2*x^2])/(3*a^4*c) +
(x*Sqrt[1 - a^2*x^2])/(a^3*c) + (x^2*Sqrt[1 - a^2*x^2])/(3*a^2*c) - (3*ArcS
in[a*x])/(a^4*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{c-acx}\,dx = c\int \frac{x^3\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx$$

$$= \frac{\int \frac{x^3(c+acx)^2}{(1-a^2x^2)^{3/2}}\,dx}{c^3}$$

$$= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} - \frac{\int \frac{(c+acx)\left(\frac{2}{a^3}+\frac{x}{a^2}+\frac{x^2}{a}\right)}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} + \frac{x^2\sqrt{1-a^2x^2}}{3a^2c} + \frac{\int \frac{-\frac{6c}{a}-11cx-6acx^2}{\sqrt{1-a^2x^2}}\,dx}{3a^2c^2}$$

$$= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} + \frac{x\sqrt{1-a^2x^2}}{a^3c} + \frac{x^2\sqrt{1-a^2x^2}}{3a^2c} - \frac{\int \frac{18ac+22a^2cx}{\sqrt{1-a^2x^2}}\,dx}{6a^4c^2}$$

$$= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} + \frac{11\sqrt{1-a^2x^2}}{3a^4c} + \frac{x\sqrt{1-a^2x^2}}{a^3c} + \frac{x^2\sqrt{1-a^2x^2}}{3a^2c} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3c}$$

$$= \frac{(1+ax)^2}{a^4c\sqrt{1-a^2x^2}} + \frac{11\sqrt{1-a^2x^2}}{3a^4c} + \frac{x\sqrt{1-a^2x^2}}{a^3c} + \frac{x^2\sqrt{1-a^2x^2}}{3a^2c} - \frac{3\sin^{-1}(ax)}{a^4c}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.63

$$\frac{18\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right) - \frac{\sqrt{ax+1}\left(a^3x^3+2a^2x^2+5ax-14\right)}{\sqrt{1-ax}}}{3a^4c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a*c*x),x]
```

```
[Out] (-((Sqrt[1 + a*x]*(-14 + 5*a*x + 2*a^2*x^2 + a^3*x^3))/Sqrt[1 - a*x]) + 18*
ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(3*a^4*c)
```

fricas [A] time = 0.60, size = 86, normalized size = 0.75

$$\frac{14\,ax + 18\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(a^3x^3+2\,a^2x^2+5\,ax-14\right)\sqrt{-a^2x^2+1} - 14}{3\left(a^5cx - a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/3*(14*a*x + 18*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a^3*x^
3 + 2*a^2*x^2 + 5*a*x - 14)*sqrt(-a^2*x^2 + 1) - 14)/(a^5*c*x - a^4*c)
```

giac [A] time = 0.21, size = 101, normalized size = 0.89

$$\frac{1}{3}\sqrt{-a^2x^2+1}\left(x\left(\frac{x}{a^2c}+\frac{3}{a^3c}\right)+\frac{8}{a^4c}\right)-\frac{3\arcsin(ax)\operatorname{sgn}(a)}{a^3c|a|}+\frac{4}{a^3c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-a^2*x^2 + 1)*(x*(x/(a^2*c) + 3/(a^3*c)) + 8/(a^4*c)) - 3*arcsin(a
*x)*sgn(a)/(a^3*c*abs(a)) + 4/(a^3*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*
x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 142, normalized size = 1.25

$$\frac{x^2\sqrt{-a^2x^2+1}}{3a^2c}+\frac{8\sqrt{-a^2x^2+1}}{3a^4c}+\frac{x\sqrt{-a^2x^2+1}}{a^3c}-\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c\,a^3\sqrt{a^2}}-\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{c\,a^5\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c),x)
```

```
[Out] 1/3*x^2*(-a^2*x^2+1)^(1/2)/a^2/c+8/3*(-a^2*x^2+1)^(1/2)/a^4/c+x*(-a^2*x^2+1
)^(1/2)/a^3/c-3/c/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-
2/c/a^5/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.48, size = 105, normalized size = 0.92

$$-\frac{2\sqrt{-a^2x^2+1}}{a^5cx-a^4c}+\frac{\sqrt{-a^2x^2+1}\,x^2}{3\,a^2c}+\frac{\sqrt{-a^2x^2+1}\,x}{a^3c}-\frac{3\arcsin(ax)}{a^4c}+\frac{8\sqrt{-a^2x^2+1}}{3\,a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a^2*x^2 + 1)/(a^5*c*x - a^4*c) + 1/3*sqrt(-a^2*x^2 + 1)*x^2/(a^2*c
) + sqrt(-a^2*x^2 + 1)*x/(a^3*c) - 3*arcsin(a*x)/(a^4*c) + 8/3*sqrt(-a^2*x^
2 + 1)/(a^4*c)
```

mupad [B] time = 0.82, size = 158, normalized size = 1.39

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2}{3\,c\,{\left(-a^2\right)}^{3/2}}-\frac{2}{a^2\,c\,\sqrt{-a^2}}+\frac{a^2\,x^2}{3\,c\,{\left(-a^2\right)}^{3/2}}+\frac{x\,\sqrt{-a^2}}{a^3\,c}\right)}{\sqrt{-a^2}}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^3\,c\,\sqrt{-a^2}}+\frac{2\,\sqrt{1-a^2\,x^2}}{a^3\,c\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(2/(3*c*(-a^2)^(3/2)) - 2/(a^2*c*(-a^2)^(1/2)) + (a^2*
x^2)/(3*c*(-a^2)^(3/2)) + (x*(-a^2)^(1/2))/(a^3*c)))/(-a^2)^(1/2) - (3*asin
h(x*(-a^2)^(1/2)))/(a^3*c*(-a^2)^(1/2)) + (2*(1 - a^2*x^2)^(1/2))/(a^3*c*(x
*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^3}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a*c*x+c),x)
```

```
[Out] -(Integral(x**3/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Int
egral(a*x**4/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.329 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{c-acx}\,dx$

Optimal. Leaf size=72

$$-\frac{5\sin^{-1}(ax)}{2a^3c}+\frac{(ax+1)^2}{a^3c\sqrt{1-a^2x^2}}+\frac{(ax+6)\sqrt{1-a^2x^2}}{2a^3c}$$

```
[Out] -5/2*arcsin(a*x)/a^3/c+(a*x+1)^2/a^3/c/(-a^2*x^2+1)^(1/2)+1/2*(a*x+6)*(-a^2
*x^2+1)^(1/2)/a^3/c
```

Rubi [A] time = 0.20, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 852, 1635, 780, 216}

$$\frac{(ax+1)^2}{a^3c\sqrt{1-a^2x^2}}+\frac{(ax+6)\sqrt{1-a^2x^2}}{2a^3c}-\frac{5\sin^{-1}(ax)}{2a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a*c*x),x]
```

```
[Out] (1 + a*x)^2/(a^3*c*Sqrt[1 - a^2*x^2]) + ((6 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^
3*c) - (5*ArcSin[a*x])/(2*a^3*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{c-acx}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx\\
&= \frac{\int \frac{x^2(c+acx)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\
&= \frac{(1+ax)^2}{a^3c\sqrt{1-a^2x^2}}-\frac{\int \frac{\left(\frac{2}{a^2}+\frac{x}{a}\right)(c+acx)}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= \frac{(1+ax)^2}{a^3c\sqrt{1-a^2x^2}}+\frac{(6+ax)\sqrt{1-a^2x^2}}{2a^3c}-\frac{5\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^2c}\\
&= \frac{(1+ax)^2}{a^3c\sqrt{1-a^2x^2}}+\frac{(6+ax)\sqrt{1-a^2x^2}}{2a^3c}-\frac{5\sin^{-1}(ax)}{2a^3c}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.89

$$
\frac{10\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\frac{\sqrt{ax+1}\left(a^2x^2+3ax-8\right)}{\sqrt{1-ax}}}{2a^3c}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a*c*x),x]
```

```
[Out] (-((Sqrt[1 + a*x]*(-8 + 3*a*x + a^2*x^2))/Sqrt[1 - a*x]) + 10*ArcSin[Sqrt[1
 - a*x]/Sqrt[2]])/(2*a^3*c)
```

fricas [A] time = 0.50, size = 78, normalized size = 1.08

$$\frac{8\,ax + 10\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(a^2x^2 + 3\,ax - 8\right)\sqrt{-a^2x^2+1} - 8}{2\left(a^4cx - a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(8*a*x + 10*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a^2*x^2
 + 3*a*x - 8)*sqrt(-a^2*x^2 + 1) - 8)/(a^4*c*x - a^3*c)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 120, normalized size = 1.67

$$\frac{x\sqrt{-a^2x^2+1}}{2c\,a^2} - \frac{5\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2c\,a^2\sqrt{a^2}} + \frac{2\sqrt{-a^2x^2+1}}{a^3c} - \frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2 - 2a\left(x-\frac{1}{a}\right)}}{c\,a^4\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c),x)
```

```
[Out] 1/2/c*x/a^2*(-a^2*x^2+1)^(1/2)-5/2/c/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(
-a^2*x^2+1)^(1/2))+2*(-a^2*x^2+1)^(1/2)/a^3/c-2/c/a^4/(x-1/a)*(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.41, size = 83, normalized size = 1.15

$$-\frac{2\sqrt{-a^2x^2+1}}{a^4cx - a^3c} + \frac{\sqrt{-a^2x^2+1}\,x}{2\,a^2c} - \frac{5\,\arcsin\left(ax\right)}{2\,a^3c} + \frac{2\sqrt{-a^2x^2+1}}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a^2*x^2 + 1)/(a^4*c*x - a^3*c) + 1/2*sqrt(-a^2*x^2 + 1)*x/(a^2*c)
- 5/2*arcsin(a*x)/(a^3*c) + 2*sqrt(-a^2*x^2 + 1)/(a^3*c)
```

mupad [B] time = 0.06, size = 129, normalized size = 1.79

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2\,\sqrt{-a^2}}{a^3\,c}+\frac{x\,\sqrt{-a^2}}{2\,a^2\,c}\right)}{\sqrt{-a^2}}-\frac{5\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^2\,c\,\sqrt{-a^2}}+\frac{2\,\sqrt{1-a^2\,x^2}}{a^2\,c\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((2*(-a^2)^(1/2))/(a^3*c) + (x*(-a^2)^(1/2))/(2*a^2*c)
))/(-a^2)^(1/2) - (5*asinh(x*(-a^2)^(1/2)))/(2*a^2*c*(-a^2)^(1/2)) + (2*(1
- a^2*x^2)^(1/2))/(a^2*c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a*c*x+c),x)
```

```
[Out] -(Integral(x**2/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Int
egral(a*x**3/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.330 $\int \frac{e^{\tanh^{-1}(ax)}x}{c-acx}\,dx$

Optimal. Leaf size=64

$$\frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2}+\frac{2\sqrt{1-a^2x^2}}{a^2c}-\frac{2\sin^{-1}(ax)}{a^2c}$$

```
[Out] (-a^2*x^2+1)^(3/2)/a^2/c/(-a*x+1)^2-2*arcsin(a*x)/a^2/c+2*(-a^2*x^2+1)^(1/2
)/a^2/c
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 793, 665, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2}+\frac{2\sqrt{1-a^2x^2}}{a^2c}-\frac{2\sin^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a*c*x),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^2*c) + (1 - a^2*x^2)^(3/2)/(a^2*c*(1 - a*x)^2) - (
2*ArcSin[a*x])/(a^2*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
```

```
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{c-acx}\,dx &= c\int \frac{x\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx \\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2} - \frac{2\int \frac{\sqrt{1-a^2x^2}}{c-acx}\,dx}{a} \\
&= \frac{2\sqrt{1-a^2x^2}}{a^2c} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2} - \frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{ac} \\
&= \frac{2\sqrt{1-a^2x^2}}{a^2c} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^2c(1-ax)^2} - \frac{2\sin^{-1}(ax)}{a^2c}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.83

$$\frac{\frac{\sqrt{ax+1}\,(3-ax)}{\sqrt{1-ax}}+4\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a^2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a*c*x),x]
```

```
[Out] (((3 - a*x)*Sqrt[1 + a*x])/Sqrt[1 - a*x] + 4*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])
/(a^2*c)
```

fricas [A] time = 0.43, size = 69, normalized size = 1.08

$$\frac{3\,ax+4\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,(ax-3)-3}{a^3cx-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (3*a*x + 4*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(a*x - 3) - 3)/(a^3*c*x - a^2*c)
```

giac [A] time = 0.24, size = 78, normalized size = 1.22

$$-\frac{2 \arcsin\left(ax\right) \operatorname{sgn}\left(a\right)}{ac|a|} + \frac{\sqrt{-a^2x^2+1}}{a^2c} + \frac{4}{ac\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -2*arcsin(a*x)*sgn(a)/(a*c*abs(a)) + sqrt(-a^2*x^2 + 1)/(a^2*c) + 4/(a*c*((
sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 98, normalized size = 1.53

$$\frac{\sqrt{-a^2x^2+1}}{a^2c} - \frac{2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{ca\sqrt{a^2}} - \frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2 - 2a\left(x-\frac{1}{a}\right)}}{c\,a^3\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)/a^2/c-2/c/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1
)^(1/2))-2/c/a^3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.45, size = 61, normalized size = 0.95

$$-\frac{2\sqrt{-a^2x^2+1}}{a^3cx-a^2c} - \frac{2 \arcsin\left(ax\right)}{a^2c} + \frac{\sqrt{-a^2x^2+1}}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a^2*x^2 + 1)/(a^3*c*x - a^2*c) - 2*arcsin(a*x)/(a^2*c) + sqrt(-a^2
*x^2 + 1)/(a^2*c)
```

mupad [B] time = 0.06, size = 90, normalized size = 1.41

$$\frac{\sqrt{1-a^2\,x^2}}{a^2\,c}-\frac{2\,\sqrt{1-a^2\,x^2}}{\left(c\,\sqrt{-a^2}-a\,c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a\,c\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(a^2*c) - (2*(1 - a^2*x^2)^(1/2))/((c*(-a^2)^(1/2) - a*
c*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (2*asinh(x*(-a^2)^(1/2)))/(a*c*(-a^2)^(1/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^2}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a*c*x+c),x)
```

```
[Out] -(Integral(x/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integr
al(a*x**2/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.331 $\int \frac{e^{\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}-\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] -arcsin(a*x)/a/c+2*(-a^2*x^2+1)^(1/2)/a/c/(-a*x+1)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 663, 216}

$$\frac{2\sqrt{1-a^2x^2}}{ac(1-ax)}-\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)) - ArcSin[a*x]/(a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{c-acx}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx$$
$$= \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{2\sqrt{1-a^2x^2}}{ac(1-ax)} - \frac{\sin^{-1}(ax)}{ac}$$

Mathematica [**A**] time = 0.02, size = 46, normalized size = 1.07

$$\frac{2\left(\frac{\sqrt{ax+1}}{\sqrt{1-ax}}+\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x),x]
```

```
[Out] (2*(Sqrt[1 + a*x]/Sqrt[1 - a*x] + ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*c)
```

fricas [**A**] time = 0.52, size = 62, normalized size = 1.44

$$\frac{2\left(ax+(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}-1\right)}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 2*(a*x + (a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 +
 1) - 1)/(a^2*c*x - a*c)
```

giac [**A**] time = 0.26, size = 53, normalized size = 1.23

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}+\frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c*abs(a)) + 4/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2
*x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 76, normalized size = 1.77

$$-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c\sqrt{a^2}}-\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{c\,a^2\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x)
```

```
[Out] -1/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2/c/a^2/(x-1/a)*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.51, size = 40, normalized size = 0.93

$$-\frac{2\sqrt{-a^2x^2+1}}{a^2cx-ac}-\frac{\arcsin(ax)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a^2*x^2 + 1)/(a^2*c*x - a*c) - arcsin(a*x)/(a*c)
```

mupad [B] time = 0.00, size = 71, normalized size = 1.65

$$\frac{2\sqrt{1-a^2\,x^2}}{c\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{c\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- asinh(x*(-a^2)^(1/2))/(c*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{ax}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{1}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Inte
gral(1/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.332 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)}\,dx$

Optimal. Leaf size=45

$$\frac{2(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] -arctanh((-a^2*x^2+1)^(1/2))/c+2*(a*x+1)/c/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 12, 266, 63, 208}

$$\frac{2(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a*c*x)),x]
```

```
[Out] (2*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x(c-acx)^2}\,dx \\
&= \frac{\int \frac{(c+acx)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3} \\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}} + \frac{\int \frac{c^2}{x\sqrt{1-a^2x^2}}\,dx}{c^3} \\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c} \\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c} \\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2c} \\
&= \frac{2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.22

$$
\frac{-\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2ax+2}{c\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a*c*x)),x]
```

```
[Out] (2 + 2*a*x - Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(c*Sqrt[1 - a^2*
x^2])
```

fricas [A] time = 0.49, size = 56, normalized size = 1.24

$$
\frac{2\,ax+(ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-2\,\sqrt{-a^2x^2+1}-2}{acx-c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (2*a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 2*sqrt(-a^2*x^2 + 1) -
 2)/(a*c*x - c)
```

giac [A] time = 0.19, size = 80, normalized size = 1.78

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c|a|}+\frac{4\,a}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a))
 + 4*a/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 61, normalized size = 1.36

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c),x)
```

```
[Out] -1/c*(arctanh(1/(-a^2*x^2+1)^(1/2))+2/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a)
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)*x), x)
```

mupad [B] time = 0.05, size = 68, normalized size = 1.51

$$\frac{2\,a\,\sqrt{1-a^2\,x^2}}{c\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] (2*a*(1 - a^2*x^2)^(1/2))/(c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2)
) - atanh((1 - a^2*x^2)^(1/2))/c
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{ax^2\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{ax^2\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x) +
 Integral(1/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x))/c
```

3.333 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)}\,dx$

Optimal. Leaf size=69

$$\frac{2a(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{cx}-\frac{2a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] -2*a*arctanh((-a^2*x^2+1)^(1/2))/c+2*a*(a*x+1)/c/(-a^2*x^2+1)^(1/2)-(-a^2*x
^2+1)^(1/2)/c/x
```

Rubi [A] time = 0.19, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 807, 266, 63, 208}

$$\frac{2a(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{cx}-\frac{2a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a*c*x)),x]
```

```
[Out] (2*a*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(c*x) - (2*a*ArcT
anh[Sqrt[1 - a^2*x^2]])/c
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^2(c-acx)^2}\,dx$$

$$= \frac{\int \frac{(c+acx)^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\int \frac{-c^2-2ac^2x}{x^2\sqrt{1-a^2x^2}}\,dx}{c^3}$$

$$= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{cx} + \frac{(2a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c}$$

$$= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{cx} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{c}$$

$$= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{cx} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{ac}$$

$$= \frac{2a(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{cx} - \frac{2a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Mathematica [**A**] time = 0.03, size = 68, normalized size = 0.99

$$\frac{3a^2x^2 - 2ax\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) + 2ax - 1}{cx\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a*c*x)),x]
```

```
[Out] (-1 + 2*a*x + 3*a^2*x^2 - 2*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]
])/(c*x*Sqrt[1 - a^2*x^2])
```

fricas [**A**] time = 0.49, size = 80, normalized size = 1.16

$$\frac{2\,a^2x^2 - 2\,ax + 2\left(a^2x^2 - ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}\,(3\,ax-1)}{acx^2 - cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (2*a^2*x^2 - 2*a*x + 2*(a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sq
rt(-a^2*x^2 + 1)*(3*a*x - 1))/(a*c*x^2 - c*x)
```

giac [B] time = 0.20, size = 159, normalized size = 2.30

$$-\frac{2\,a^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c|a|}-\frac{\left(a^2-\frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{2\,cx|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -2*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs
(a)) - 1/2*(a^2 - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x)*a^2*x/((sqrt(-a^2*x^
2 + 1)*abs(a) + a)*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
- 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(c*x*abs(a))
```

maple [A] time = 0.04, size = 77, normalized size = 1.12

$$-\frac{2a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\sqrt{-a^2x^2+1}}{x}+\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c),x)
```

```
[Out] -1/c*(2*a*arctanh(1/(-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/x+2/(x-1/a)*(-a^
2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)*x^2), x)
```

mupad [B] time = 0.82, size = 91, normalized size = 1.32

$$\frac{2\,a^2\,\sqrt{1-a^2\,x^2}}{c\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{c\,x}-\frac{2\,a\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] (2*a^2*(1 - a^2*x^2)^(1/2))/(c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/
2)) - (1 - a^2*x^2)^(1/2)/(c*x) - (2*a*atanh((1 - a^2*x^2)^(1/2)))/c
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x
) + Integral(1/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x
))/c
```

3.334 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)}\,dx$

Optimal. Leaf size=100

$$\frac{2a^2(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{\sqrt{1-a^2x^2}}{2cx^2}-\frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

```
[Out] -5/2*a^2*arctanh((-a^2*x^2+1)^(1/2))/c+2*a^2*(a*x+1)/c/(-a^2*x^2+1)^(1/2)-1
/2*(-a^2*x^2+1)^(1/2)/c/x^2-2*a*(-a^2*x^2+1)^(1/2)/c/x
```

Rubi [A] time = 0.26, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{2a^2(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{\sqrt{1-a^2x^2}}{2cx^2}-\frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a*c*x)),x]
```

```
[Out] (2*a^2*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(2*c*x^2) - (2*
a*Sqrt[1 - a^2*x^2])/(c*x) - (5*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^3(c-acx)^2}\,dx \\
&= \frac{\int \frac{(c+acx)^2}{x^3(1-a^2x^2)^{3/2}}\,dx}{c^3} \\
&= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\int \frac{-c^2-2ac^2x-2a^2c^2x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{c^3} \\
&= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} + \frac{\int \frac{4ac^2+5a^2c^2x}{x^2\sqrt{1-a^2x^2}}\,dx}{2c^3} \\
&= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} + \frac{(5a^2)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{2c} \\
&= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} + \frac{(5a^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{4c} \\
&= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{2c} \\
&= \frac{2a^2(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} - \frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.83

$$-\frac{-8a^3x^3-5a^2x^2+5a^2x^2\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+4ax+1}{2cx^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a*c*x)),x]
```

```
[Out] -1/2*(1 + 4*a*x - 5*a^2*x^2 - 8*a^3*x^3 + 5*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTa
nh[Sqrt[1 - a^2*x^2]])/(c*x^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.43, size = 99, normalized size = 0.99

$$\frac{4\,a^3x^3-4\,a^2x^2+5\left(a^3x^3-a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(8\,a^2x^2-3\,ax-1\right)\sqrt{-a^2x^2+1}}{2\left(acx^3-cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^3*x^3 - 4*a^2*x^2 + 5*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (8*a^2*x^2 - 3*a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*c*x^3 - c*x^2)
```

giac [B] time = 0.20, size = 224, normalized size = 2.24

$$-\frac{\left(a^{3}+\frac{7\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)a}{x}-\frac{40\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{2}}{ax^{2}}\right)a^{4}x^{2}}{8\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{2}c\left(\frac{\sqrt{-a^{2}x^{2}+1}\,|a|+a}{a^{2}x}-1\right)|a|}-\frac{5\,a^{3}\log\left(\frac{\left|-2\,\sqrt{-a^{2}x^{2}+1}\,|a|-2\,a\right|}{2\,a^{2}|x|}\right)}{2\,c|a|}-\frac{\frac{8\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)ac|a|}{x}+\frac{\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{2}c|a|}{ax^{2}}}{8\,a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(a^3 + 7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 40*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c*((sqrt(
-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 5/2*a^3*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a)) - 1/8*(8*(sqrt(-a^2*
x^2 + 1)*abs(a) + a)*a*c*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c*abs
(a)/(a*x^2))/(a^2*c^2)
```

maple [A] time = 0.04, size = 99, normalized size = 0.99

$$-\frac{\frac{5a^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)}{2}+\frac{2a\sqrt{-a^{2}x^{2}+1}}{x}+\frac{2a\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}+\frac{\sqrt{-a^{2}x^{2}+1}}{2x^{2}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c),x)
```

```
[Out] -1/c*(5/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+2*a*(-a^2*x^2+1)^(1/2)/x+2*a/(x
-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}\,(acx-c)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)*x^3), x)
```

mupad [B] time = 0.83, size = 117, normalized size = 1.17

$$-\frac{\sqrt{1-a^2\,x^2}}{2\,c\,x^2}-\frac{2\,a\,\sqrt{1-a^2\,x^2}}{c\,x}-\frac{2\,a^3\,\sqrt{1-a^2\,x^2}}{\left(\frac{c\,\sqrt{-a^2}}{a}-c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+\frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,5\mathrm{i}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*5i)/(2*c) - (1 - a^2*x^2)^(1/2)/(2*c*x^2)
 - (2*a*(1 - a^2*x^2)^(1/2))/(c*x) - (2*a^3*(1 - a^2*x^2)^(1/2))/(((c*(-a^2
)^(1/2))/a - c*x*(-a^2)^(1/2))*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x
) + Integral(1/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x
))/c
```

3.335 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)}\,dx$

Optimal. Leaf size=125

$$-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{a\sqrt{1-a^2x^2}}{cx^2}-\frac{\sqrt{1-a^2x^2}}{3cx^3}+\frac{2a^3(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] -3*a^3*arctanh((-a^2*x^2+1)^(1/2))/c+2*a^3*(a*x+1)/c/(-a^2*x^2+1)^(1/2)-1/3
*(-a^2*x^2+1)^(1/2)/c/x^3-a*(-a^2*x^2+1)^(1/2)/c/x^2-8/3*a^2*(-a^2*x^2+1)^(
1/2)/c/x
```

Rubi [A] time = 0.33, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{2a^3(ax+1)}{c\sqrt{1-a^2x^2}}-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{a\sqrt{1-a^2x^2}}{cx^2}-\frac{\sqrt{1-a^2x^2}}{3cx^3}-\frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a*c*x)),x]
```

```
[Out] (2*a^3*(1 + a*x))/(c*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(3*c*x^3) - (a*
Sqrt[1 - a^2*x^2])/(c*x^2) - (8*a^2*Sqrt[1 - a^2*x^2])/(3*c*x) - (3*a^3*Arc
Tanh[Sqrt[1 - a^2*x^2]])/c
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^4(c-acx)^2}\,dx \\
&= \frac{\int \frac{(c+acx)^2}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\int \frac{-c^2-2ac^2x-2a^2c^2x^2-2a^3c^2x^3}{x^4\sqrt{1-a^2x^2}}\,dx}{c^3} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} + \frac{\int \frac{6ac^2+8a^2c^2x+6a^3c^2x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{3c^3} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{\int \frac{-16a^2c^2-18a^3c^2x}{x^2\sqrt{1-a^2x^2}}\,dx}{6c^3} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} + \frac{\left(3a^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} + \frac{\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right.}{2c} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-\right.}{c} \\
&= \frac{2a^3(1+ax)}{c\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3cx^3} - \frac{a\sqrt{1-a^2x^2}}{cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} - \frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.73

$$
-\frac{-14a^4x^4-9a^3x^3+7a^2x^2+9a^3x^3\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ax+1}{3cx^3\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a*c*x)),x]
```

```
[Out] -1/3*(1 + 3*a*x + 7*a^2*x^2 - 9*a^3*x^3 - 14*a^4*x^4 + 9*a^3*x^3*Sqrt[1 - a
^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(c*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.41, size = 107, normalized size = 0.86

$$\frac{6\,a^4x^4-6\,a^3x^3+9\left(a^4x^4-a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(14\,a^3x^3-5\,a^2x^2-2\,ax-1\right)\sqrt{-a^2x^2+1}}{3\left(acx^4-cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^4*x^4 - 6*a^3*x^3 + 9*(a^4*x^4 - a^3*x^3)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (14*a^3*x^3 - 5*a^2*x^2 - 2*a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*c*x^4
 - c*x^3)
```

giac [B] time = 0.19, size = 283, normalized size = 2.26

$$-\frac{\left(a^4+\frac{5\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2}{x}+\frac{27\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{x^2}-\frac{129\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^2x^3}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{3\,a^4\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c|a|}-\frac{33\left(\sqrt{-a^2x^2\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/24*(a^4 + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 27*(sqrt(-a^2*x^2 +
1)*abs(a) + a)^2/x^2 - 129*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^2*x^3))*a^6
*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/
(a^2*x) - 1)*abs(a)) - 3*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a
)/(a^2*abs(x)))/(c*abs(a)) - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c
^2/x + 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^2/x^2 + (sqrt(-a^2*x^2 + 1
)*abs(a) + a)^3*c^2/x^3)/(a^2*c^3*abs(a))
```

maple [A] time = 0.04, size = 142, normalized size = 1.14

$$-\frac{2a^3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{8a^2\sqrt{-a^2x^2+1}}{3x}+\frac{\sqrt{-a^2x^2+1}}{3x^3}+\frac{2a^2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}-2a\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\ldots\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c),x)
```

```
[Out] -1/c*(2*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))+8/3*a^2*(-a^2*x^2+1)^(1/2)/x+1/3*
(-a^2*x^2+1)^(1/2)/x^3+2*a^2/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2*a
*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)*x^4), x)
```

mupad [B] time = 0.06, size = 140, normalized size = 1.12

$$-\frac{\sqrt{1-a^2\,x^2}}{3\,c\,x^3}-\frac{a\,\sqrt{1-a^2\,x^2}}{c\,x^2}-\frac{8\,a^2\,\sqrt{1-a^2\,x^2}}{3\,c\,x}-\frac{2\,a^4\,\sqrt{1-a^2\,x^2}}{\left(\frac{c\,\sqrt{-a^2}}{a}-c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+\frac{a^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,3\mathrm{i}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(1 - a^2*x^2)^(1/2)*(c - a*c*x)),x)
```

```
[Out] (a^3*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/c - (1 - a^2*x^2)^(1/2)/(3*c*x^3) - (
a*(1 - a^2*x^2)^(1/2))/(c*x^2) - (8*a^2*(1 - a^2*x^2)^(1/2))/(3*c*x) - (2*a
^4*(1 - a^2*x^2)^(1/2))/(((c*(-a^2)^(1/2))/a - c*x*(-a^2)^(1/2))*(-a^2)^(1/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{ax^5\sqrt{-a^2x^2+1}-x^4\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{ax^5\sqrt{-a^2x^2+1}-x^4\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a*c*x+c),x)
```

```
[Out] -(Integral(a*x/(a*x**5*sqrt(-a**2*x**2 + 1) - x**4*sqrt(-a**2*x**2 + 1)), x
) + Integral(1/(a*x**5*sqrt(-a**2*x**2 + 1) - x**4*sqrt(-a**2*x**2 + 1)), x
))/c
```

3.336 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^2}\,dx$

Optimal. Leaf size=159

$$\frac{17\sin^{-1}(ax)}{2a^5c^2}-\frac{2(ax+1)^3}{a^5c^2\sqrt{1-a^2x^2}}+\frac{(ax+1)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}}-\frac{(ax+5)^2\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{(ax+5)\sqrt{1-a^2x^2}}{6a^5c^2}-\frac{5\sqrt{1-a^2x^2}}{2a^5c^2}$$

```
[Out] 1/3*(a*x+1)^3/a^5/c^2/(-a^2*x^2+1)^(3/2)+17/2*arcsin(a*x)/a^5/c^2-2*(a*x+1)
^3/a^5/c^2/(-a^2*x^2+1)^(1/2)-5/2*(-a^2*x^2+1)^(1/2)/a^5/c^2-1/6*(a*x+5)*(-
a^2*x^2+1)^(1/2)/a^5/c^2-1/3*(a*x+5)^2*(-a^2*x^2+1)^(1/2)/a^5/c^2
```

Rubi [A] time = 0.52, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {6128, 852, 1635, 1625, 1654, 21, 743, 641, 216}

$$-\frac{2(ax+1)^3}{a^5c^2\sqrt{1-a^2x^2}}+\frac{(ax+1)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}}-\frac{(ax+5)^2\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{(ax+5)\sqrt{1-a^2x^2}}{6a^5c^2}-\frac{5\sqrt{1-a^2x^2}}{2a^5c^2}+\frac{17\sin^{-1}(ax)}{2a^5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(3*a^5*c^2*(1 - a^2*x^2)^(3/2)) - (2*(1 + a*x)^3)/(a^5*c^2*Sqrt
[1 - a^2*x^2]) - (5*Sqrt[1 - a^2*x^2])/(2*a^5*c^2) - ((5 + a*x)*Sqrt[1 - a^
2*x^2])/(6*a^5*c^2) - ((5 + a*x)^2*Sqrt[1 - a^2*x^2])/(3*a^5*c^2) + (17*Arc
Sin[a*x])/(2*a^5*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1625

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*(a + c*x^2)^p,
x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemaind
er[Pq, d + e*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
```

```
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^2}\,dx = c\int \frac{x^4\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx$$

$$= \frac{\int \frac{x^4(c+acx)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^5}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{(c+acx)^2\left(\frac{3}{a^4}+\frac{3x}{a^3}+\frac{3x^2}{a^2}+\frac{3x^3}{a}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{(c+acx)^3\left(\frac{3}{a^4c}+\frac{3x^2}{a^2c}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{\left(\frac{15}{a^4c}+\frac{3x}{a^3c}\right)(c+acx)^2}{\sqrt{1-a^2x^2}}\,dx}{3c^3}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{a^4\int \frac{\left(-\frac{45}{a^4}-\frac{9x}{a^3}\right)\left(\frac{15}{a^4c}+\frac{3x}{a^3c}\right)}{\sqrt{1-a^2x^2}}\,dx}{81c}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} + \frac{a^4\int \frac{\left(-\frac{45}{a^4}-\frac{9x}{a^3}\right)^2}{\sqrt{1-a^2x^2}}\,dx}{243c^2}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{(5+ax)\sqrt{1-a^2x^2}}{6a^5c^2} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{a^2\int \cdots}{4\cdots}$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2a^5c^2} - \frac{(5+ax)\sqrt{1-a^2x^2}}{6a^5c^2} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} \cdots$$

$$= \frac{(1+ax)^3}{3a^5c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^3}{a^5c^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2a^5c^2} - \frac{(5+ax)\sqrt{1-a^2x^2}}{6a^5c^2} - \frac{(5+ax)^2\sqrt{1-a^2x^2}}{3a^5c^2} \cdots$$

Mathematica [A] time = 0.10, size = 80, normalized size = 0.50

$$-\frac{\frac{\sqrt{ax+1}\left(2a^4x^4+5a^3x^3+18a^2x^2-109ax+80\right)}{(1-ax)^{3/2}}+102\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{6a^5c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^2,x]
```

```
[Out] -1/6*((Sqrt[1 + a*x]*(80 - 109*a*x + 18*a^2*x^2 + 5*a^3*x^3 + 2*a^4*x^4))/(
1 - a*x)^(3/2) + 102*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(a^5*c^2)
```

fricas [A] time = 0.45, size = 125, normalized size = 0.79

$$-\frac{80\,a^2x^2-160\,ax+102\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^4x^4+5\,a^3x^3+18\,a^2x^2-109\,ax+80\right)\sqrt{-a}}{6\left(a^7c^2x^2-2\,a^6c^2x+a^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/6*(80*a^2*x^2 - 160*a*x + 102*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^
2 + 1) - 1)/(a*x)) + (2*a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 109*a*x + 80)*sq
rt(-a^2*x^2 + 1) + 80)/(a^7*c^2*x^2 - 2*a^6*c^2*x + a^5*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 187, normalized size = 1.18

$$-\frac{x^2\sqrt{-a^2x^2+1}}{3c^2a^3}-\frac{17\sqrt{-a^2x^2+1}}{3a^5c^2}-\frac{3x\sqrt{-a^2x^2+1}}{2c^2a^4}+\frac{17\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2c^2a^4\sqrt{a^2}}+\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3c^2a^7\left(x-\frac{1}{a}\right)^2}+\frac{25\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^2,x)
```

```
[Out] -1/3/c^2/a^3*x^2*(-a^2*x^2+1)^(1/2)-17/3*(-a^2*x^2+1)^(1/2)/a^5/c^2-3/2/c^2
/a^4*x*(-a^2*x^2+1)^(1/2)+17/2/c^2/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2))+2/3/c^2/a^7/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2
5/3/c^2/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.44, size = 153, normalized size = 0.96

$$\frac{2\sqrt{-a^2x^2+1}}{3\left(a^7c^2x^2-2a^6c^2x+a^5c^2\right)}+\frac{25\sqrt{-a^2x^2+1}}{3\left(a^6c^2x-a^5c^2\right)}-\frac{\sqrt{-a^2x^2+1}\,x^2}{3a^3c^2}-\frac{3\sqrt{-a^2x^2+1}\,x}{2a^4c^2}+\frac{17\arcsin(ax)}{2a^5c^2}-\frac{17\sqrt{-a^2x^2+1}}{3a^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] 2/3*sqrt(-a^2*x^2 + 1)/(a^7*c^2*x^2 - 2*a^6*c^2*x + a^5*c^2) + 25/3*sqrt(-a
^2*x^2 + 1)/(a^6*c^2*x - a^5*c^2) - 1/3*sqrt(-a^2*x^2 + 1)*x^2/(a^3*c^2) -
3/2*sqrt(-a^2*x^2 + 1)*x/(a^4*c^2) + 17/2*arcsin(a*x)/(a^5*c^2) - 17/3*sqrt
(-a^2*x^2 + 1)/(a^5*c^2)
```

mupad [B] time = 0.81, size = 189, normalized size = 1.19

$$\frac{2\sqrt{1-a^2x^2}}{3\left(a^7c^2x^2-2a^6c^2x+a^5c^2\right)}+\frac{25\sqrt{1-a^2x^2}}{3\left(a^3c^2\sqrt{-a^2}-a^4c^2x\sqrt{-a^2}\right)\sqrt{-a^2}}-\frac{17\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{3x\sqrt{1-a^2x^2}}{2a^4c^2}+\frac{17\,\mathrm{asinh}}{2a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(3*(a^5*c^2 - 2*a^6*c^2*x + a^7*c^2*x^2)) + (25*(1
- a^2*x^2)^(1/2))/(3*(a^3*c^2*(-a^2)^(1/2) - a^4*c^2*x*(-a^2)^(1/2))*(-a^2)
^(1/2)) - (17*(1 - a^2*x^2)^(1/2))/(3*a^5*c^2) - (3*x*(1 - a^2*x^2)^(1/2))/
(2*a^4*c^2) + (17*asinh(x*(-a^2)^(1/2)))/(2*a^4*c^2*(-a^2)^(1/2)) - (x^2*(1
 - a^2*x^2)^(1/2))/(3*a^3*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(x**4/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**5/(a**2*x**2*sqrt(-a**2*x**2 +
 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.337 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^2}\,dx$

Optimal. Leaf size=104

$$\frac{11\sin^{-1}(ax)}{2a^4c^2}+\frac{(ax+1)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3(ax+1)^2}{a^4c^2\sqrt{1-a^2x^2}}-\frac{(ax+12)\sqrt{1-a^2x^2}}{2a^4c^2}$$

```
[Out] 1/3*(a*x+1)^3/a^4/c^2/(-a^2*x^2+1)^(3/2)+11/2*arcsin(a*x)/a^4/c^2-3*(a*x+1)
^2/a^4/c^2/(-a^2*x^2+1)^(1/2)-1/2*(a*x+12)*(-a^2*x^2+1)^(1/2)/a^4/c^2
```

Rubi [A] time = 0.30, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 852, 1635, 780, 216}

$$\frac{(ax+1)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3(ax+1)^2}{a^4c^2\sqrt{1-a^2x^2}}-\frac{(ax+12)\sqrt{1-a^2x^2}}{2a^4c^2}+\frac{11\sin^{-1}(ax)}{2a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(3*a^4*c^2*(1 - a^2*x^2)^(3/2)) - (3*(1 + a*x)^2)/(a^4*c^2*Sqrt
[1 - a^2*x^2]) - ((12 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^4*c^2) + (11*ArcSin[a*
x])/(2*a^4*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^2}\,dx = c\int \frac{x^3\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx$$

$$= \frac{\int \frac{x^3(c+acx)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^5}$$

$$= \frac{(1+ax)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{(c+acx)^2\left(\frac{3}{a^3}+\frac{3x}{a^2}+\frac{3x^2}{a}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}$$

$$= \frac{(1+ax)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}} - \frac{3(1+ax)^2}{a^4c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{\left(\frac{15}{a^3}+\frac{3x}{a^2}\right)(c+acx)}{\sqrt{1-a^2x^2}}\,dx}{3c^3}$$

$$= \frac{(1+ax)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}} - \frac{3(1+ax)^2}{a^4c^2\sqrt{1-a^2x^2}} - \frac{(12+ax)\sqrt{1-a^2x^2}}{2a^4c^2} + \frac{11\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^3c^2}$$

$$= \frac{(1+ax)^3}{3a^4c^2\left(1-a^2x^2\right)^{3/2}} - \frac{3(1+ax)^2}{a^4c^2\sqrt{1-a^2x^2}} - \frac{(12+ax)\sqrt{1-a^2x^2}}{2a^4c^2} + \frac{11\sin^{-1}(ax)}{2a^4c^2}$$

Mathematica [**A**] time = 0.09, size = 72, normalized size = 0.69

$$-\frac{\frac{\sqrt{ax+1}\left(3a^3x^3+12a^2x^2-71ax+52\right)}{(1-ax)^{3/2}} + 66\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{6a^4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^2,x]
```

```
[Out] -1/6*((Sqrt[1 + a*x]*(52 - 71*a*x + 12*a^2*x^2 + 3*a^3*x^3))/(1 - a*x)^(3/2
) + 66*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(a^4*c^2)
```

fricas [**A**] time = 0.43, size = 117, normalized size = 1.12

$$-\frac{52\,a^2x^2 - 104\,ax + 66\left(a^2x^2 - 2\,ax + 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(3\,a^3x^3 + 12\,a^2x^2 - 71\,ax + 52\right)\sqrt{-a^2x^2+1} + 52}{6\left(a^6c^2x^2 - 2\,a^5c^2x + a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/6*(52*a^2*x^2 - 104*a*x + 66*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2
 + 1) - 1)/(a*x)) + (3*a^3*x^3 + 12*a^2*x^2 - 71*a*x + 52)*sqrt(-a^2*x^2 +
1) + 52)/(a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2)
```

giac [A] time = 0.24, size = 176, normalized size = 1.69

$$-\frac{4\,a^3c^6\left(-\frac{2\,c}{acx-c}-1\right)^{\frac{3}{2}}+132\,a^3c^6\arctan\left(\sqrt{-\frac{2\,c}{acx-c}-1}\right)-72\,a^3c^6\sqrt{-\frac{2\,c}{acx-c}-1}-\frac{3\left(7\,a^3c^6\left(-\frac{2\,c}{acx-c}-1\right)^{\frac{3}{2}}+5\,a^3c^6\sqrt{-\frac{2\,c}{acx-c}-1}\right)\left(acx-c\right)^2}{c^2}}{12\,a^6c^8|a|\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/12*(4*a^3*c^6*(-2*c/(a*c*x - c) - 1)^(3/2) + 132*a^3*c^6*arctan(sqrt(-2*
c/(a*c*x - c) - 1)) - 72*a^3*c^6*sqrt(-2*c/(a*c*x - c) - 1) - 3*(7*a^3*c^6*
(-2*c/(a*c*x - c) - 1)^(3/2) + 5*a^3*c^6*sqrt(-2*c/(a*c*x - c) - 1))*(a*c*x
 - c)^2/c^2)/(a^6*c^8*abs(a)*sgn(1/(a*c*x - c))*sgn(a)*sgn(c))
```

maple [A] time = 0.04, size = 164, normalized size = 1.58

$$-\frac{x\sqrt{-a^2x^2+1}}{2c^2a^3}+\frac{11\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2c^2a^3\sqrt{a^2}}-\frac{3\sqrt{-a^2x^2+1}}{c^2a^4}+\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3c^2a^6\left(x-\frac{1}{a}\right)^2}+\frac{19\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3c^2a^5\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^2,x)
```

```
[Out] -1/2/c^2/a^3*x*(-a^2*x^2+1)^(1/2)+11/2/c^2/a^3/(a^2)^(1/2)*arctan((a^2)^(1/
2)*x/(-a^2*x^2+1)^(1/2))-3/c^2/a^4*(-a^2*x^2+1)^(1/2)+2/3/c^2/a^6/(x-1/a)^2
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+19/3/c^2/a^5/(x-1/a)*(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(1/2)
```

maxima [A] time = 0.48, size = 130, normalized size = 1.25

$$\frac{2\sqrt{-a^2x^2+1}}{3\left(a^6c^2x^2-2\,a^5c^2x+a^4c^2\right)}+\frac{19\sqrt{-a^2x^2+1}}{3\left(a^5c^2x-a^4c^2\right)}-\frac{\sqrt{-a^2x^2+1}\,x}{2\,a^3c^2}+\frac{11\arcsin\left(ax\right)}{2\,a^4c^2}-\frac{3\sqrt{-a^2x^2+1}}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] 2/3*sqrt(-a^2*x^2 + 1)/(a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2) + 19/3*sqrt(-a
^2*x^2 + 1)/(a^5*c^2*x - a^4*c^2) - 1/2*sqrt(-a^2*x^2 + 1)*x/(a^3*c^2) + 11
/2*arcsin(a*x)/(a^4*c^2) - 3*sqrt(-a^2*x^2 + 1)/(a^4*c^2)
```

mupad [B] time = 0.05, size = 166, normalized size = 1.60

$$\frac{2\sqrt{1-a^2x^2}}{3\left(a^6c^2x^2-2a^5c^2x+a^4c^2\right)}+\frac{19\sqrt{1-a^2x^2}}{3\left(a^2c^2\sqrt{-a^2}-a^3c^2x\sqrt{-a^2}\right)\sqrt{-a^2}}-\frac{3\sqrt{1-a^2x^2}}{a^4c^2}-\frac{x\sqrt{1-a^2x^2}}{2a^3c^2}+\frac{11\,\mathrm{asinh}\left(x\right.}{2a^3c^2\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(3*(a^4*c^2 - 2*a^5*c^2*x + a^6*c^2*x^2)) + (19*(1
- a^2*x^2)^(1/2))/(3*(a^2*c^2*(-a^2)^(1/2) - a^3*c^2*x*(-a^2)^(1/2))*(-a^2)
^(1/2)) - (3*(1 - a^2*x^2)^(1/2))/(a^4*c^2) - (x*(1 - a^2*x^2)^(1/2))/(2*a^
3*c^2) + (11*asinh(x*(-a^2)^(1/2)))/(2*a^3*c^2*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(x**3/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**4/(a**2*x**2*sqrt(-a**2*x**2 +
 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.338 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^2}\,dx$

Optimal. Leaf size=104

$$\frac{3\sin^{-1}(ax)}{a^3c^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3}-\frac{6\sqrt{1-a^2x^2}}{a^3c^2(1-ax)}$$

```
[Out] 1/3*(-a^2*x^2+1)^(3/2)/a^3/c^2/(-a*x+1)^3+(-a^2*x^2+1)^(3/2)/a^3/c^2/(-a*x+
1)^2+3*arcsin(a*x)/a^3/c^2-6*(-a^2*x^2+1)^(1/2)/a^3/c^2/(-a*x+1)
```

Rubi [A] time = 0.19, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 1639, 793, 663, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3}-\frac{6\sqrt{1-a^2x^2}}{a^3c^2(1-ax)}+\frac{3\sin^{-1}(ax)}{a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^2,x]
```

```
[Out] (-6*Sqrt[1 - a^2*x^2])/(a^3*c^2*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(3*a^3*c^2
*(1 - a*x)^3) + (1 - a^2*x^2)^(3/2)/(a^3*c^2*(1 - a*x)^2) + (3*ArcSin[a*x])
/(a^3*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
```

```
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^2}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2}-\frac{\int \frac{\left(2a^2c^2-3a^3c^2x\right)\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{a^4c}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2}-\frac{3\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx}{a^2}\\
&= -\frac{6\sqrt{1-a^2x^2}}{a^3c^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2}+\frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2c^2}\\
&= -\frac{6\sqrt{1-a^2x^2}}{a^3c^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^3c^2(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^2(1-ax)^2}+\frac{3\sin^{-1}(ax)}{a^3c^2}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 64, normalized size = 0.62

$$\frac{\frac{\sqrt{ax+1}\left(-3a^2x^2+19ax-14\right)}{(1-ax)^{3/2}}-18\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{3a^3c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^2,x]
```

```
[Out] ((Sqrt[1 + a*x]*(-14 + 19*a*x - 3*a^2*x^2))/(1 - a*x)^(3/2) - 18*ArcSin[Sqr
t[1 - a*x]/Sqrt[2]])/(3*a^3*c^2)
```

fricas [A] time = 0.44, size = 109, normalized size = 1.05

$$-\frac{14\,a^2x^2-28\,ax+18\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^2x^2-19\,ax+14\right)\sqrt{-a^2x^2+1}+14}{3\left(a^5c^2x^2-2\,a^4c^2x+a^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/3*(14*a^2*x^2 - 28*a*x + 18*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2
+ 1) - 1)/(a*x)) + (3*a^2*x^2 - 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^5*
c^2*x^2 - 2*a^4*c^2*x + a^3*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 143, normalized size = 1.38

$$-\frac{\sqrt{-a^2x^2+1}}{c^2a^3}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^2a^2\sqrt{a^2}}+\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3c^2a^5\left(x-\frac{1}{a}\right)^2}+\frac{13\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3c^2a^4\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^2,x)
```

```
[Out] -1/c^2/a^3*(-a^2*x^2+1)^(1/2)+3/c^2/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-
a^2*x^2+1)^(1/2))+2/3/c^2/a^5/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+
13/3/c^2/a^4/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.43, size = 109, normalized size = 1.05

$$\frac{2\sqrt{-a^2x^2+1}}{3\left(a^5c^2x^2-2\,a^4c^2x+a^3c^2\right)}+\frac{13\sqrt{-a^2x^2+1}}{3\left(a^4c^2x-a^3c^2\right)}+\frac{3\,\arcsin\left(ax\right)}{a^3c^2}-\frac{\sqrt{-a^2x^2+1}}{a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] 2/3*sqrt(-a^2*x^2 + 1)/(a^5*c^2*x^2 - 2*a^4*c^2*x + a^3*c^2) + 13/3*sqrt(-a
^2*x^2 + 1)/(a^4*c^2*x - a^3*c^2) + 3*arcsin(a*x)/(a^3*c^2) - sqrt(-a^2*x^2
 + 1)/(a^3*c^2)
```

mupad [B] time = 0.81, size = 143, normalized size = 1.38

$$\frac{2\sqrt{1-a^2\,x^2}}{3\,\left(a^5\,c^2\,x^2-2\,a^4\,c^2\,x+a^3\,c^2\right)}+\frac{13\sqrt{1-a^2\,x^2}}{3\,\left(a\,c^2\,\sqrt{-a^2}-a^2\,c^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3\,c^2}+\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,c^2\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(3*(a^3*c^2 - 2*a^4*c^2*x + a^5*c^2*x^2)) + (13*(1
- a^2*x^2)^(1/2))/(3*(a*c^2*(-a^2)^(1/2) - a^2*c^2*x*(-a^2)^(1/2))*(-a^2)^(
1/2)) - (1 - a^2*x^2)^(1/2)/(a^3*c^2) + (3*asinh(x*(-a^2)^(1/2)))/(a^2*c^2*
(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(x**2/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**3/(a**2*x**2*sqrt(-a**2*x**2 +
 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.339 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^2}\,dx$

Optimal. Leaf size=74

$$\frac{(1-a^2x^2)^{3/2}}{3a^2c^2(1-ax)^3}-\frac{2\sqrt{1-a^2x^2}}{a^2c^2(1-ax)}+\frac{\sin^{-1}(ax)}{a^2c^2}$$

```
[Out] 1/3*(-a^2*x^2+1)^(3/2)/a^2/c^2/(-a*x+1)^3+arcsin(a*x)/a^2/c^2-2*(-a^2*x^2+1
)^(1/2)/a^2/c^2/(-a*x+1)
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 793, 663, 216}

$$\frac{(1-a^2x^2)^{3/2}}{3a^2c^2(1-ax)^3}-\frac{2\sqrt{1-a^2x^2}}{a^2c^2(1-ax)}+\frac{\sin^{-1}(ax)}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a*c*x)^2,x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a^2*c^2*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(3*a^2*c^2
*(1 - a*x)^3) + ArcSin[a*x]/(a^2*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
```

```
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^2}\,dx &= c\int \frac{x\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{3a^2c^2(1-ax)^3}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^2}\,dx}{a}\\
&= -\frac{2\sqrt{1-a^2x^2}}{a^2c^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^2c^2(1-ax)^3}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{ac^2}\\
&= -\frac{2\sqrt{1-a^2x^2}}{a^2c^2(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{3a^2c^2(1-ax)^3}+\frac{\sin^{-1}(ax)}{a^2c^2}
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 57, normalized size = 0.77

$$
-\frac{(ax+1)^{3/2}-4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-ax)\right)}{3a^2c^2(1-ax)^{3/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a*c*x)^2,x]
```

```
[Out] -1/3*((1 + a*x)^(3/2) - 4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 -
a*x)/2])/(a^2*c^2*(1 - a*x)^(3/2))
```

fricas [A] time = 0.43, size = 102, normalized size = 1.38

$$
-\frac{5\,a^2x^2-10\,ax+6\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}\,(7\,ax-5)+5}{3\left(a^4c^2x^2-2\,a^3c^2x+a^2c^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(5*a^2*x^2 - 10*a*x + 6*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*(7*a*x - 5) + 5)/(a^4*c^2*x^2 - 2*a^3*c
^2*x + a^2*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/abs
(a)/c^2/a/c*(-(-6*c*atan(i)-(-7*i)*c)/3*sign((a*c*x-c)^-1)*sign(a)*sign(c)-
2*a*c^2*(1/8*(4/3*sqrt(-2*a*c^2*(a*c*x-c)^-1/a/c-1)*(-2*a*c^2*(a*c*x-c)^-1/
a/c-1)-8*sqrt(-2*a*c^2*(a*c*x-c)^-1/a/c-1))+atan(sqrt(-2*a*c^2*(a*c*x-c)^-1
/a/c-1)))/a/c/sign((a*c*x-c)^-1)/sign(a)/sign(c))
```

maple [A] time = 0.04, size = 122, normalized size = 1.65

$$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^2a\sqrt{a^2}}+\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3c^2a^4\left(x-\frac{1}{a}\right)^2}+\frac{7\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3c^2a^3\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+2/3/c^2/a^4/(x
-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+7/3/c^2/a^3/(x-1/a)*(-a^2*(x-1/a
)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.44, size = 88, normalized size = 1.19

$$\frac{2\sqrt{-a^2x^2+1}}{3\left(a^4c^2x^2-2\,a^3c^2x+a^2c^2\right)}+\frac{7\sqrt{-a^2x^2+1}}{3\left(a^3c^2x-a^2c^2\right)}+\frac{\arcsin\left(ax\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(-a^2*x^2 + 1)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2) + 7/3*sqrt(-a^
2*x^2 + 1)/(a^3*c^2*x - a^2*c^2) + arcsin(a*x)/(a^2*c^2)
```

mupad [B] time = 0.82, size = 108, normalized size = 1.46

$$\frac{4}{3\,a^2\,c^2\,{\left(1-a^2\,x^2\right)}^{3/2}}-\frac{3}{a^2\,c^2\,\sqrt{1-a^2\,x^2}}-\frac{7\,x}{3\,a\,c^2\,\sqrt{1-a^2\,x^2}}+\frac{4\,x}{3\,a\,c^2\,{\left(1-a^2\,x^2\right)}^{3/2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}{a^3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)
```

```
[Out] 4/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - 3/(a^2*c^2*(1 - a^2*x^2)^(1/2)) - (7*x)
/(3*a*c^2*(1 - a^2*x^2)^(1/2)) + (4*x)/(3*a*c^2*(1 - a^2*x^2)^(3/2)) - (asi
nh(x*(-a^2)^(1/2))*(-a^2)^(1/2))/(a^3*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^{2} x^{2} \sqrt{- a^{2} x^{2} + 1} - 2 a x \sqrt{- a^{2} x^{2} + 1} + \sqrt{- a^{2} x^{2} + 1}}\, dx + \int \frac{a x^{2}}{a^{2} x^{2} \sqrt{- a^{2} x^{2} + 1} - 2 a x \sqrt{- a^{2} x^{2} + 1} + \sqrt{- a^{2} x^{2} + 1}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(x/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) +
sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**2/(a**2*x**2*sqrt(-a**2*x**2 + 1)
 - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.340 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^2} dx$

Optimal. Leaf size=32

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}$$

```
[Out] 1/3*(-a^2*x^2+1)^(3/2)/a/c^2/(-a*x+1)^3
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6127, 651}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^2,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(3*a*c^2*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^2} dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3} dx \\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.91

$$\frac{(ax+1)^{3/2}}{3ac^2(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^(3/2)/(3*a*c^2*(1 - a*x)^(3/2))
```

fricas [B] time = 0.48, size = 60, normalized size = 1.88

$$\frac{a^2x^2-2\,ax+\sqrt{-a^2x^2+1}\,(ax+1)+1}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*x^2 - 2*a*x + sqrt(-a^2*x^2 + 1)*(a*x + 1) + 1)/(a^3*c^2*x^2 - 2*a
^2*c^2*x + a*c^2)
```

giac [C] time = 0.24, size = 66, normalized size = 2.06

$$-\frac{i\,\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)+\frac{\left(-\frac{2\,c}{acx-c}-1\right)^{\frac{3}{2}}}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}}{3\,c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(I*sgn(1/(a*c*x - c))*sgn(a)*sgn(c) + (-2*c/(a*c*x - c) - 1)^(3/2)/(sg
n(1/(a*c*x - c))*sgn(a)*sgn(c)))/(c^2*abs(a))
```

maple [A] time = 0.03, size = 35, normalized size = 1.09

$$-\frac{(ax+1)^2}{3\,(ax-1)\,c^2\sqrt{-a^2x^2+1}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x)
```

[Out] -1/3*(a*x+1)^2/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a

maxima [B] time = 0.40, size = 73, normalized size = 2.28

$$\frac{2\sqrt{-a^2x^2+1}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}+\frac{\sqrt{-a^2x^2+1}}{3\left(a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")

[Out] 2/3*sqrt(-a^2*x^2 + 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + 1/3*sqrt(-a^2*x^2 + 1)/(a^2*c^2*x - a*c^2)

mupad [B] time = 0.00, size = 32, normalized size = 1.00

$$\frac{\sqrt{1-a^2\,x^2}\,\left(a\,x+1\right)}{3\,a\,c^2\,{\left(a\,x-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)

[Out] ((1 - a^2*x^2)^(1/2)*(a*x + 1))/(3*a*c^2*(a*x - 1)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**2,x)

[Out] (Integral(a*x/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2

3.341 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^2}\,dx$

Optimal. Leaf size=74

$$\frac{4(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{5ax+3}{3c^2\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

```
[Out] 4/3*(a*x+1)/c^2/(-a^2*x^2+1)^(3/2)-arctanh((-a^2*x^2+1)^(1/2))/c^2+1/3*(5*a
*x+3)/c^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 823, 12, 266, 63, 208}

$$\frac{4(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{5ax+3}{3c^2\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a*c*x)^2),x]
```

```
[Out] (4*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (3 + 5*a*x)/(3*c^2*Sqrt[1 - a^2
*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^2}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x(c-acx)^3}\,dx$$

$$= \frac{\int \frac{(c+acx)^3}{x(1-a^2x^2)^{5/2}}\,dx}{c^5}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-3c^3-5ac^3x}{x(1-a^2x^2)^{3/2}}\,dx}{3c^5}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} - \frac{\int -\frac{3a^2c^3}{x\sqrt{1-a^2x^2}}\,dx}{3a^2c^5}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c^2}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2c^2}$$

$$= \frac{4(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+5ax}{3c^2\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 1.05

$$\frac{5a^2x^2-3(ax-1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-2ax-7}{3c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a*c*x)^2),x]
```

```
[Out] (-7 - 2*a*x + 5*a^2*x^2 - 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a
^2*x^2]])/(3*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.50, size = 93, normalized size = 1.26

$$\frac{7\,a^2x^2 - 14\,ax + 3\left(a^2x^2 - 2\,ax + 1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}\,(5\,ax - 7) + 7}{3\left(a^2c^2x^2 - 2\,ac^2x + c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(7*a^2*x^2 - 14*a*x + 3*(a^2*x^2 - 2*a*x + 1)*log((sqrt(-a^2*x^2 + 1) -
 1)/x) - sqrt(-a^2*x^2 + 1)*(5*a*x - 7) + 7)/(a^2*c^2*x^2 - 2*a*c^2*x + c^2
)
```

giac [C] time = 0.28, size = 245, normalized size = 3.31

$$\frac{\left(\frac{(3\log(2)-6\log(i+1)+10i)\operatorname{sgn}\left(\frac{1}{acx-c}\right)\operatorname{sgn}(a)\operatorname{sgn}(c)}{c} + \frac{6\log\left(\sqrt{-\frac{2c}{acx-c}-1}+1\right)}{c\operatorname{sgn}\left(\frac{1}{acx-c}\right)\operatorname{sgn}(a)\operatorname{sgn}(c)} - \frac{6\log\left(\left|\sqrt{-\frac{2c}{acx-c}-1}-1\right|\right)}{c\operatorname{sgn}\left(\frac{1}{acx-c}\right)\operatorname{sgn}(a)\operatorname{sgn}(c)} - \frac{2\left(c^2\left(-\frac{2c}{acx-c}-1\right)^{\frac{3}{2}}\operatorname{sgn}\left(\frac{1}{acx-c}\right)^2\operatorname{sgn}\right.}{c^3\operatorname{s}}\right.}{6\,c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*((3*log(2) - 6*log(I + 1) + 10*I)*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)/c +
6*log(sqrt(-2*c/(a*c*x - c) - 1) + 1)/(c*sgn(1/(a*c*x - c))*sgn(a)*sgn(c))
- 6*log(abs(sqrt(-2*c/(a*c*x - c) - 1) - 1))/(c*sgn(1/(a*c*x - c))*sgn(a)*s
gn(c)) - 2*(c^2*(-2*c/(a*c*x - c) - 1)^(3/2)*sgn(1/(a*c*x - c))^2*sgn(a)^2*
sgn(c)^2 + 6*c^2*sqrt(-2*c/(a*c*x - c) - 1)*sgn(1/(a*c*x - c))^2*sgn(a)^2*s
gn(c)^2)/(c^3*sgn(1/(a*c*x - c))^3*sgn(a)^3*sgn(c)^3))*a/(c*abs(a))
```

maple [B] time = 0.04, size = 147, normalized size = 1.99

$$\frac{-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + \frac{\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2} - \frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}}{a} - \frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^2,x)
```

[Out] 1/c^2*(-arctanh(1/(-a^2*x^2+1)^(1/2))+2/a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-1/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^2,x, algorithm="maxima")

[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x), x)

mupad [B] time = 0.81, size = 119, normalized size = 1.61

$$\frac{2\,a^2\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^2\,x^2-2\,a^3\,c^2\,x+a^2\,c^2\right)}-\frac{\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{c^2}+\frac{5\,a\,\sqrt{1-a^2\,x^2}}{3\,\sqrt{-a^2}\,\left(c^2\,x\,\sqrt{-a^2}-\frac{c^2\,\sqrt{-a^2}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + 1)/(x*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)

[Out] (2*a^2*(1 - a^2*x^2)^(1/2))/(3*(a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) - atanh((1 - a^2*x^2)^(1/2))/c^2 + (5*a*(1 - a^2*x^2)^(1/2))/(3*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^(1/2))/a))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^2x^3\sqrt{-a^2x^2+1}-2ax^2\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^3\sqrt{-a^2x^2+1}-2ax^2\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a*c*x+c)**2,x)

[Out] (Integral(a*x/(a**2*x**3*sqrt(-a**2*x**2 + 1) - 2*a*x**2*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**3*sqrt(-a**2*x**2 + 1) - 2*a*x**2*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/c**2

3.342 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^2}\,dx$

Optimal. Leaf size=99

$$\frac{4a(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{c^2x}+\frac{a(11ax+9)}{3c^2\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

```
[Out] 4/3*a*(a*x+1)/c^2/(-a^2*x^2+1)^(3/2)-3*a*arctanh((-a^2*x^2+1)^(1/2))/c^2+1/
3*a*(11*a*x+9)/c^2/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/c^2/x
```

Rubi [A] time = 0.27, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 807, 266, 63, 208}

$$\frac{4a(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{c^2x}+\frac{a(11ax+9)}{3c^2\sqrt{1-a^2x^2}}-\frac{3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^2),x]
```

```
[Out] (4*a*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (a*(9 + 11*a*x))/(3*c^2*Sqrt[
1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(c^2*x) - (3*a*ArcTanh[Sqrt[1 - a^2*x^2]]
)/c^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^2}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^2(c-acx)^3}\,dx\\
&= \frac{\int \frac{(c+acx)^3}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c^5}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-3c^3-9ac^3x-8a^2c^3x^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^5}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3c^3+9ac^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{3c^5}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^2x} + \frac{(3a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^2x} + \frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c^2}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^2x} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{ac^2}\\
&= \frac{4a(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a(9+11ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^2x} - \frac{3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.92

$$
\frac{14a^3x^3-5a^2x^2-9ax(ax-1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-16ax+3}{3c^2x(ax-1)\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^2),x]
```

```
[Out] (3 - 16*a*x - 5*a^2*x^2 + 14*a^3*x^3 - 9*a*x*(-1 + a*x)*Sqrt[1 - a^2*x^2]*A
rcTanh[Sqrt[1 - a^2*x^2]])/(3*c^2*x*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.49, size = 118, normalized size = 1.19

$$\frac{13\,a^3x^3 - 26\,a^2x^2 + 13\,ax + 9\left(a^3x^3 - 2\,a^2x^2 + ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(14\,a^2x^2 - 19\,ax + 3\right)\sqrt{-a^2x^2+1}}{3\left(a^2c^2x^3 - 2\,ac^2x^2 + c^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/3*(13*a^3*x^3 - 26*a^2*x^2 + 13*a*x + 9*(a^3*x^3 - 2*a^2*x^2 + a*x)*log((
sqrt(-a^2*x^2 + 1) - 1)/x) - (14*a^2*x^2 - 19*a*x + 3)*sqrt(-a^2*x^2 + 1))/
(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x)
```

giac [C] time = 0.24, size = 307, normalized size = 3.10

$$\frac{\left(9\,a^2\log(2) - 18\,a^2\log\left(i+1\right) + 28i\,a^2\right)\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c) + \frac{18\,a^2\log\left(\sqrt{-\frac{2\,c}{acx-c}-1}+1\right)}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)} - \frac{18\,a^2\log\left(\left|\sqrt{-\frac{2\,c}{acx-c}-1}-1\right|\right)}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}}{6\,c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*((9*a^2*log(2) - 18*a^2*log(I + 1) + 28*I*a^2)*sgn(1/(a*c*x - c))*sgn(a
)*sgn(c) + 18*a^2*log(sqrt(-2*c/(a*c*x - c) - 1) + 1)/(sgn(1/(a*c*x - c))*s
gn(a)*sgn(c)) - 18*a^2*log(abs(sqrt(-2*c/(a*c*x - c) - 1) - 1))/(sgn(1/(a*c
*x - c))*sgn(a)*sgn(c)) - 6*a^2*sqrt(-2*c/(a*c*x - c) - 1)/((c/(a*c*x - c)
+ 1)*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)) - 2*(a^2*(-2*c/(a*c*x - c) - 1)^(3/2
)*sgn(1/(a*c*x - c))^2*sgn(a)^2*sgn(c)^2 + 12*a^2*sqrt(-2*c/(a*c*x - c) - 1
)*sgn(1/(a*c*x - c))^2*sgn(a)^2*sgn(c)^2)/(sgn(1/(a*c*x - c))^3*sgn(a)^3*sg
n(c)^3))/(c^2*abs(a))
```

maple [A] time = 0.04, size = 118, normalized size = 1.19

$$\frac{-3a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{\sqrt{-a^2x^2+1}}{x} + \frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2} - \frac{11\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-3*a*arctanh(1/(-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/x+2/3/a/(x-1/a
)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-11/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-
1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x^2), x)
```

mupad [B] time = 0.83, size = 146, normalized size = 1.47

$$\frac{2\,a^3\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^2\,x^2-2\,a^3\,c^2\,x+a^2\,c^2\right)}-\frac{\sqrt{1-a^2\,x^2}}{c^2\,x}+\frac{11\,a^2\,\sqrt{1-a^2\,x^2}}{3\,\sqrt{-a^2}\,\left(c^2\,x\,\sqrt{-a^2}-\frac{c^2\,\sqrt{-a^2}}{a}\right)}+\frac{a\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,3\mathrm{i}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)
```

```
[Out] (2*a^3*(1 - a^2*x^2)^(1/2))/(3*(a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) - (1
- a^2*x^2)^(1/2)/(c^2*x) + (a*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/c^2 + (11*a^
2*(1 - a^2*x^2)^(1/2))/(3*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^(1
/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^2x^4\sqrt{-a^2x^2+1}-2ax^3\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^4\sqrt{-a^2x^2+1}-2ax^3\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**4*sqrt(-a**2*x**2 + 1) - 2*a*x**3*sqrt(-a**2*x**2 +
1) + x**2*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**4*sqrt(-a**2*x**2
 + 1) - 2*a*x**3*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x))/c**
2
```

3.343 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^2}\,dx$

Optimal. Leaf size=132

$$\frac{a^2(17ax+15)}{3c^2\sqrt{1-a^2x^2}}+\frac{4a^2(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3a\sqrt{1-a^2x^2}}{c^2x}-\frac{\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{11a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

```
[Out] 4/3*a^2*(a*x+1)/c^2/(-a^2*x^2+1)^(3/2)-11/2*a^2*arctanh((-a^2*x^2+1)^(1/2))
/c^2+1/3*a^2*(17*a*x+15)/c^2/(-a^2*x^2+1)^(1/2)-1/2*(-a^2*x^2+1)^(1/2)/c^2/
x^2-3*a*(-a^2*x^2+1)^(1/2)/c^2/x
```

Rubi [A] time = 0.33, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^2(17ax+15)}{3c^2\sqrt{1-a^2x^2}}+\frac{4a^2(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3a\sqrt{1-a^2x^2}}{c^2x}-\frac{\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{11a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^2),x]
```

```
[Out] (4*a^2*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (a^2*(15 + 17*a*x))/(3*c^2*
Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(2*c^2*x^2) - (3*a*Sqrt[1 - a^2*x^2]
)/(c^2*x) - (11*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^2}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^3(c-acx)^3}\,dx$$

$$= \frac{\int \frac{(c+acx)^3}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^5}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-3c^3-9ac^3x-12a^2c^3x^2-8a^3c^3x^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^5}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3c^3+9ac^3x+15a^2c^3x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{3c^5}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{\int \frac{-18ac^3-33a^2c^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{6c^5}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{3a\sqrt{1-a^2x^2}}{c^2x} + \frac{\left(11a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{2c^2}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{3a\sqrt{1-a^2x^2}}{c^2x} + \frac{\left(11a^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\right.}{4c^2}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{3a\sqrt{1-a^2x^2}}{c^2x} - \frac{11\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x,\right.}{2c^2}$$

$$= \frac{4a^2(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(15+17ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{3a\sqrt{1-a^2x^2}}{c^2x} - \frac{11a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right.}{2c^2}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 0.78

$$\frac{52a^4x^4-19a^3x^3-59a^2x^2-33a^2x^2(ax-1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+15ax+3}{6c^2x^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^2),x]

[Out] (3 + 15*a*x - 59*a^2*x^2 - 19*a^3*x^3 + 52*a^4*x^4 - 33*a^2*x^2*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(6*c^2*x^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])

fricas [A] time = 0.46, size = 136, normalized size = 1.03

$$\frac{38\,a^{4}x^{4}-76\,a^{3}x^{3}+38\,a^{2}x^{2}+33\left(a^{4}x^{4}-2\,a^{3}x^{3}+a^{2}x^{2}\right)\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)-\left(52\,a^{3}x^{3}-71\,a^{2}x^{2}+12\,ax+3\right)\sqrt{-a^{2}}}{6\left(a^{2}c^{2}x^{4}-2\,ac^{2}x^{3}+c^{2}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^2,x, algorithm="fricas")

[Out] 1/6*(38*a^4*x^4 - 76*a^3*x^3 + 38*a^2*x^2 + 33*(a^4*x^4 - 2*a^3*x^3 + a^2*x^2)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (52*a^3*x^3 - 71*a^2*x^2 + 12*a*x + 3)*sqrt(-a^2*x^2 + 1))/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*x^2)

giac [C] time = 0.27, size = 332, normalized size = 2.52

$$\frac{\left(33\,a^{3}\log(2)-66\,a^{3}\log\left(i+1\right)+104i\,a^{3}\right)\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)+\frac{66\,a^{3}\log\left(\sqrt{-\frac{2\,c}{acx-c}-1}+1\right)}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}-\frac{66\,a^{3}\log\left(\left|\sqrt{-\frac{2\,c}{acx-c}-1}-1\right|\right)}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}}{12\,c^{2}|a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^2,x, algorithm="giac")

[Out] 1/12*((33*a^3*log(2) - 66*a^3*log(I + 1) + 104*I*a^3)*sgn(1/(a*c*x - c))*sgn(a)*sgn(c) + 66*a^3*log(sqrt(-2*c/(a*c*x - c) - 1) + 1)/(sgn(1/(a*c*x - c))*sgn(a)*sgn(c)) - 66*a^3*log(abs(sqrt(-2*c/(a*c*x - c) - 1) - 1))/(sgn(1/(a*c*x - c))*sgn(a)*sgn(c)) + 3*(7*a^3*(-2*c/(a*c*x - c) - 1)^(3/2) - 5*a^3*sqrt(-2*c/(a*c*x - c) - 1))/((c/(a*c*x - c) + 1)^2*sgn(1/(a*c*x - c))*sgn(a)*sgn(c)) - 4*(a^3*(-2*c/(a*c*x - c) - 1)^(3/2)*sgn(1/(a*c*x - c))^2*sgn(a)^2*sgn(c)^2 + 18*a^3*sqrt(-2*c/(a*c*x - c) - 1)*sgn(1/(a*c*x - c))^2*sgn(a)^2*sgn(c)^2)/(sgn(1/(a*c*x - c))^3*sgn(a)^3*sgn(c)^3))/(c^2*abs(a))

maple [A] time = 0.04, size = 181, normalized size = 1.37

$$\frac{-\frac{11a^{2}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)}{2}-\frac{3a\sqrt{-a^{2}x^{2}+1}}{x}+2a\left(\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^{2}}-\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}\right)-\frac{5a\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}-\frac{\sqrt{-a^{2}}}{2}}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-11/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-3*a*(-a^2*x^2+1)^(1/2)/x+2*a
*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1
/a)^2-2*a*(x-1/a))^(1/2))-5*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/
2*(-a^2*x^2+1)^(1/2)/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x^3), x)
```

mupad [B] time = 0.06, size = 169, normalized size = 1.28

$$\frac{2\,a^4\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^2\,x^2-2\,a^3\,c^2\,x+a^2\,c^2\right)}-\frac{\sqrt{1-a^2\,x^2}}{2\,c^2\,x^2}-\frac{3\,a\,\sqrt{1-a^2\,x^2}}{c^2\,x}+\frac{17\,a^3\,\sqrt{1-a^2\,x^2}}{3\,\sqrt{-a^2}\,\left(c^2\,x\,\sqrt{-a^2}-\frac{c^2\,\sqrt{-a^2}}{a}\right)}+\frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x}\right.}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*11i)/(2*c^2) + (2*a^4*(1 - a^2*x^2)^(1/2)
)/(3*(a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) - (1 - a^2*x^2)^(1/2)/(2*c^2*x^
2) - (3*a*(1 - a^2*x^2)^(1/2))/(c^2*x) + (17*a^3*(1 - a^2*x^2)^(1/2))/(3*(-
a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^2x^5\sqrt{-a^2x^2+1}-2ax^4\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^5\sqrt{-a^2x^2+1}-2ax^4\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**5*sqrt(-a**2*x**2 + 1) - 2*a*x**4*sqrt(-a**2*x**2 +
1) + x**3*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**5*sqrt(-a**2*x**2
```

```
 + 1) - 2*a*x**4*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)), x))/c**
2
```

3.344 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{17a^2\sqrt{1-a^2x^2}}{3c^2x}-\frac{3a\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{\sqrt{1-a^2x^2}}{3c^2x^3}+\frac{a^3(23ax+21)}{3c^2\sqrt{1-a^2x^2}}+\frac{4a^3(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{17a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

```
[Out] 4/3*a^3*(a*x+1)/c^2/(-a^2*x^2+1)^(3/2)-17/2*a^3*arctanh((-a^2*x^2+1)^(1/2))
/c^2+1/3*a^3*(23*a*x+21)/c^2/(-a^2*x^2+1)^(1/2)-1/3*(-a^2*x^2+1)^(1/2)/c^2/
x^3-3/2*a*(-a^2*x^2+1)^(1/2)/c^2/x^2-17/3*a^2*(-a^2*x^2+1)^(1/2)/c^2/x
```

Rubi [A] time = 0.42, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^3(23ax+21)}{3c^2\sqrt{1-a^2x^2}}+\frac{4a^3(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{17a^2\sqrt{1-a^2x^2}}{3c^2x}-\frac{3a\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{\sqrt{1-a^2x^2}}{3c^2x^3}-\frac{17a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a*c*x)^2),x]
```

```
[Out] (4*a^3*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) + (a^3*(21 + 23*a*x))/(3*c^2*
Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(3*c^2*x^3) - (3*a*Sqrt[1 - a^2*x^2]
)/(2*c^2*x^2) - (17*a^2*Sqrt[1 - a^2*x^2])/(3*c^2*x) - (17*a^3*ArcTanh[Sqrt
[1 - a^2*x^2]])/(2*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
```

```
0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)^2}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^4(c-acx)^3}\,dx$$

$$= \frac{\int \frac{(c+acx)^3}{x^4\left(1-a^2x^2\right)^{5/2}}\,dx}{c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-3c^3-9ac^3x-12a^2c^3x^2-12a^3c^3x^3-8a^4c^3x^4}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3c^3+9ac^3x+15a^2c^3x^2+21a^3c^3x^3}{x^4\sqrt{1-a^2x^2}}\,dx}{3c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{\int \frac{-27ac^3-51a^2c^3x-63a^3c^3x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{9c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} + \frac{\int \frac{102a^2c^3+153a^3c^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{18c^5}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{17a^2\sqrt{1-a^2x^2}}{3c^2x} + \frac{(17a^3}{}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{17a^2\sqrt{1-a^2x^2}}{3c^2x} + \frac{(17a^3}{}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{17a^2\sqrt{1-a^2x^2}}{3c^2x} - \frac{(17a)}{}$$

$$= \frac{4a^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(21+23ax)}{3c^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{3a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{17a^2\sqrt{1-a^2x^2}}{3c^2x} - \frac{17a^3}{}$$

Mathematica [A] time = 0.06, size = 111, normalized size = 0.69

$$\frac{80a^5x^5-29a^4x^4-91a^3x^3+23a^2x^2-51a^3x^3(ax-1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+7ax+2}{6c^2x^3(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a*c*x)^2),x]
```

```
[Out] (2 + 7*a*x + 23*a^2*x^2 - 91*a^3*x^3 - 29*a^4*x^4 + 80*a^5*x^5 - 51*a^3*x^3
*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(6*c^2*x^3*(-1 +
a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.45, size = 144, normalized size = 0.89

$$\frac{50\,a^5x^5-100\,a^4x^4+50\,a^3x^3+51\left(a^5x^5-2\,a^4x^4+a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(80\,a^4x^4-109\,a^3x^3+18\,a^2x^2+5\,ax\right.}{6\left(a^2c^2x^5-2\,ac^2x^4+c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/6*(50*a^5*x^5 - 100*a^4*x^4 + 50*a^3*x^3 + 51*(a^5*x^5 - 2*a^4*x^4 + a^3*
x^3)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (80*a^4*x^4 - 109*a^3*x^3 + 18*a^2*x
^2 + 5*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*c^2*x^5 - 2*a*c^2*x^4 + c^2*x^3)
```

giac [C] time = 0.30, size = 372, normalized size = 2.31

$$2\left(51\,a^4\log(2)-102\,a^4\log\left(i+1\right)+160i\,a^4\right)\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)+\frac{204\,a^4\log\left(\sqrt{-\frac{2c}{acx-c}-1}+1\right)}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sgn}(c)}-\frac{204\,a^4\log\left(\left|\sqrt{-\frac{2c}{acx-c}-\right.\right.}{\mathrm{sgn}\left(\frac{1}{acx-c}\right)\mathrm{sgn}(a)\mathrm{sg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/24*(2*(51*a^4*log(2) - 102*a^4*log(I + 1) + 160*I*a^4)*sgn(1/(a*c*x - c))
*sgn(a)*sgn(c) + 204*a^4*log(sqrt(-2*c/(a*c*x - c) - 1) + 1)/(sgn(1/(a*c*x
- c))*sgn(a)*sgn(c)) - 204*a^4*log(abs(sqrt(-2*c/(a*c*x - c) - 1) - 1))/(sg
n(1/(a*c*x - c))*sgn(a)*sgn(c)) - (45*a^4*(2*c/(a*c*x - c) + 1)^2*sqrt(-2*c
/(a*c*x - c) - 1) - 64*a^4*(-2*c/(a*c*x - c) - 1)^(3/2) + 27*a^4*sqrt(-2*c/
```

```
(a*c*x - c) - 1))/((c/(a*c*x - c) + 1)^3*sgn(1/(a*c*x - c))*sgn(a)*sgn(c))
- 8*(a^4*(-2*c/(a*c*x - c) - 1)^(3/2)*sgn(1/(a*c*x - c))^2*sgn(a)^2*sgn(c)^
2 + 24*a^4*sqrt(-2*c/(a*c*x - c) - 1)*sgn(1/(a*c*x - c))^2*sgn(a)^2*sgn(c)^
2)/(sgn(1/(a*c*x - c))^3*sgn(a)^3*sgn(c)^3))/(c^2*abs(a))
```

maple [A] time = 0.05, size = 226, normalized size = 1.40

$$\frac{-7a^3 \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{17a^2\sqrt{-a^2x^2+1}}{3x} + 2a^2\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2} - \frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}\right) - \frac{\sqrt{-a^2x^2+1}}{3x^3} - \frac{7a^2\sqrt{-a^2}}{}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^2,x)
```

```
[Out] 1/c^2*(-7*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))-17/3*a^2*(-a^2*x^2+1)^(1/2)/x+2
*a^2*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(1/2))-1/3*(-a^2*x^2+1)^(1/2)/x^3-7*a^2/(x-1/a)*(-a^
2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+3*a*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arc
tanh(1/(-a^2*x^2+1)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^2*x^4), x)
```

mupad [B] time = 0.83, size = 192, normalized size = 1.19

$$\frac{2\,a^5\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^2\,x^2-2\,a^3\,c^2\,x+a^2\,c^2\right)}-\frac{\sqrt{1-a^2\,x^2}}{3\,c^2\,x^3}-\frac{3\,a\,\sqrt{1-a^2\,x^2}}{2\,c^2\,x^2}-\frac{17\,a^2\,\sqrt{1-a^2\,x^2}}{3\,c^2\,x}+\frac{23\,a^4\,\sqrt{1-a^2\,x^2}}{3\,\sqrt{-a^2}\,\left(c^2\,x\,\sqrt{-a^2}-\frac{c^2\,\sqrt{-a^2}}{a}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^2),x)
```

```
[Out] (a^3*atan((1 - a^2*x^2)^(1/2)*1i)*17i)/(2*c^2) + (2*a^5*(1 - a^2*x^2)^(1/2)
)/(3*(a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) - (1 - a^2*x^2)^(1/2)/(3*c^2*x^
```

```
3) - (3*a*(1 - a^2*x^2)^(1/2))/(2*c^2*x^2) - (17*a^2*(1 - a^2*x^2)^(1/2))/(
3*c^2*x) + (23*a^4*(1 - a^2*x^2)^(1/2))/(3*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2)
 - (c^2*(-a^2)^(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^2x^6\sqrt{-a^2x^2+1}-2ax^5\sqrt{-a^2x^2+1}+x^4\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^2x^6\sqrt{-a^2x^2+1}-2ax^5\sqrt{-a^2x^2+1}+x^4\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(a*x/(a**2*x**6*sqrt(-a**2*x**2 + 1) - 2*a*x**5*sqrt(-a**2*x**2 +
1) + x**4*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**2*x**6*sqrt(-a**2*x**2
 + 1) - 2*a*x**5*sqrt(-a**2*x**2 + 1) + x**4*sqrt(-a**2*x**2 + 1)), x))/c**
2
```

3.345 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^3}\,dx$

Optimal. Leaf size=135

$$-\frac{19\sin^{-1}(ax)}{2a^5c^3}+\frac{(ax+1)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{19(ax+1)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{6(ax+1)^2}{a^5c^3\sqrt{1-a^2x^2}}+\frac{(ax+20)\sqrt{1-a^2x^2}}{2a^5c^3}$$

```
[Out] 1/5*(a*x+1)^4/a^5/c^3/(-a^2*x^2+1)^(5/2)-19/15*(a*x+1)^3/a^5/c^3/(-a^2*x^2+
1)^(3/2)-19/2*arcsin(a*x)/a^5/c^3+6*(a*x+1)^2/a^5/c^3/(-a^2*x^2+1)^(1/2)+1/
2*(a*x+20)*(-a^2*x^2+1)^(1/2)/a^5/c^3
```

Rubi [A] time = 0.42, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 852, 1635, 780, 216}

$$\frac{(ax+1)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{19(ax+1)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{6(ax+1)^2}{a^5c^3\sqrt{1-a^2x^2}}+\frac{(ax+20)\sqrt{1-a^2x^2}}{2a^5c^3}-\frac{19\sin^{-1}(ax)}{2a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^3,x]
```

```
[Out] (1 + a*x)^4/(5*a^5*c^3*(1 - a^2*x^2)^(5/2)) - (19*(1 + a*x)^3)/(15*a^5*c^3*
(1 - a^2*x^2)^(3/2)) + (6*(1 + a*x)^2)/(a^5*c^3*Sqrt[1 - a^2*x^2]) + ((20 +
 a*x)*Sqrt[1 - a^2*x^2])/(2*a^5*c^3) - (19*ArcSin[a*x])/(2*a^5*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^3}\,dx = c\int \frac{x^4\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx$$

$$= \frac{\int \frac{x^4(c+acx)^4}{(1-a^2x^2)^{7/2}}\,dx}{c^7}$$

$$= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{(c+acx)^3\left(\frac{4}{a^4}+\frac{5x}{a^3}+\frac{5x^2}{a^2}+\frac{5x^3}{a}\right)}{(1-a^2x^2)^{5/2}}\,dx}{5c^6}$$

$$= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{19(1+ax)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{(c+acx)^2\left(\frac{45}{a^4}+\frac{30x}{a^3}+\frac{15x^2}{a^2}\right)}{(1-a^2x^2)^{3/2}}\,dx}{15c^5}$$

$$= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{19(1+ax)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{6(1+ax)^2}{a^5c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{\left(\frac{135}{a^4}+\frac{15x}{a^3}\right)(c+acx)}{\sqrt{1-a^2x^2}}\,dx}{15c^4}$$

$$= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{19(1+ax)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{6(1+ax)^2}{a^5c^3\sqrt{1-a^2x^2}} + \frac{(20+ax)\sqrt{1-a^2x^2}}{2a^5c^3} - \frac{19\int}{}$$

$$= \frac{(1+ax)^4}{5a^5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{19(1+ax)^3}{15a^5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{6(1+ax)^2}{a^5c^3\sqrt{1-a^2x^2}} + \frac{(20+ax)\sqrt{1-a^2x^2}}{2a^5c^3} - \frac{19\,\mathrm{si}}{2}$$

Mathematica [C] time = 0.15, size = 122, normalized size = 0.90

$$\frac{\sqrt{ax+1}\left(-15a^4x^4-75a^3x^3+433a^2x^2-639ax+308\right)+140\sqrt{2}\,(ax-1)\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-ax)\right)+360(1-}{30a^5c^3(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^3,x]
```

```
[Out] (Sqrt[1 + a*x]*(308 - 639*a*x + 433*a^2*x^2 - 75*a^3*x^3 - 15*a^4*x^4) + 36
0*(1 - a*x)^(5/2)*ArcSin[Sqrt[1 - a*x]/Sqrt[2]] + 140*Sqrt[2]*(-1 + a*x)*Hy
pergeometric2F1[-3/2, -3/2, -1/2, (1 - a*x)/2])/(30*a^5*c^3*(1 - a*x)^(5/2)
)
```

fricas [A] time = 0.47, size = 153, normalized size = 1.13

$$\frac{448\,a^3x^3 - 1344\,a^2x^2 + 1344\,ax + 570\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(15\,a^4x^4 + 75\,a^3x^3 - 713\right.}{30\left(a^8c^3x^3 - 3\,a^7c^3x^2 + 3\,a^6c^3x - a^5c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/30*(448*a^3*x^3 - 1344*a^2*x^2 + 1344*a*x + 570*(a^3*x^3 - 3*a^2*x^2 + 3*
a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^4*x^4 + 75*a^3*x^3
- 713*a^2*x^2 + 1059*a*x - 448)*sqrt(-a^2*x^2 + 1) - 448)/(a^8*c^3*x^3 - 3*
a^7*c^3*x^2 + 3*a^6*c^3*x - a^5*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 208, normalized size = 1.54

$$\frac{x\sqrt{-a^2x^2+1}}{2c^3a^4} - \frac{19\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2c^3a^4\sqrt{a^2}} + \frac{4\sqrt{-a^2x^2+1}}{c^3a^5} - \frac{41\sqrt{-a^2\left(x-\frac{1}{a}\right)^2 - 2a\left(x-\frac{1}{a}\right)}}{15c^3a^7\left(x-\frac{1}{a}\right)^2} - \frac{199\sqrt{-a^2\left(x-\frac{1}{a}\right)^2 - 2a\left(\right.}}{15c^3a^6\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^3,x)
```

```
[Out] 1/2/c^3/a^4*x*(-a^2*x^2+1)^(1/2)-19/2/c^3/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2
)*x/(-a^2*x^2+1)^(1/2))+4/c^3/a^5*(-a^2*x^2+1)^(1/2)-41/15/c^3/a^7/(x-1/a)^
2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-199/15/c^3/a^6/(x-1/a)*(-a^2*(x-1/a)^2
-2*a*(x-1/a))^(1/2)-2/5/c^3/a^8/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2
)
```

maxima [A] time = 0.41, size = 185, normalized size = 1.37

$$-\frac{2\sqrt{-a^2x^2+1}}{5\left(a^8c^3x^3-3\,a^7c^3x^2+3\,a^6c^3x-a^5c^3\right)}-\frac{41\sqrt{-a^2x^2+1}}{15\left(a^7c^3x^2-2\,a^6c^3x+a^5c^3\right)}-\frac{199\sqrt{-a^2x^2+1}}{15\left(a^6c^3x-a^5c^3\right)}+\frac{\sqrt{-a^2x^2+1}\,x}{2\,a^4c^3}-\frac{19\,\mathrm{ar}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -2/5*sqrt(-a^2*x^2 + 1)/(a^8*c^3*x^3 - 3*a^7*c^3*x^2 + 3*a^6*c^3*x - a^5*c^
3) - 41/15*sqrt(-a^2*x^2 + 1)/(a^7*c^3*x^2 - 2*a^6*c^3*x + a^5*c^3) - 199/1
5*sqrt(-a^2*x^2 + 1)/(a^6*c^3*x - a^5*c^3) + 1/2*sqrt(-a^2*x^2 + 1)*x/(a^4*
c^3) - 19/2*arcsin(a*x)/(a^5*c^3) + 4*sqrt(-a^2*x^2 + 1)/(a^5*c^3)
```

mupad [B] time = 0.83, size = 302, normalized size = 2.24

$$\frac{4\,a^4\,\sqrt{1-a^2\,x^2}}{15\,\left(a^{11}\,c^3\,x^2-2\,a^{10}\,c^3\,x+a^9\,c^3\right)}-\frac{2\,\sqrt{1-a^2\,x^2}}{5\,\sqrt{-a^2}\,\left(a^3\,c^3\,\sqrt{-a^2}+3\,a^5\,c^3\,x^2\,\sqrt{-a^2}-a^6\,c^3\,x^3\,\sqrt{-a^2}-3\,a^4\,c^3\,x\,\sqrt{-a^2}\right)}-\frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] (4*a^4*(1 - a^2*x^2)^(1/2))/(15*(a^9*c^3 - 2*a^10*c^3*x + a^11*c^3*x^2)) -
(2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(a^3*c^3*(-a^2)^(1/2) + 3*a^5*c^3*x
^2*(-a^2)^(1/2) - a^6*c^3*x^3*(-a^2)^(1/2) - 3*a^4*c^3*x*(-a^2)^(1/2))) - (
3*(1 - a^2*x^2)^(1/2))/(a^5*c^3 - 2*a^6*c^3*x + a^7*c^3*x^2) - (199*(1 - a^
2*x^2)^(1/2))/(15*(a^3*c^3*(-a^2)^(1/2) - a^4*c^3*x*(-a^2)^(1/2))*(-a^2)^(1
/2)) + (4*(1 - a^2*x^2)^(1/2))/(a^5*c^3) + (x*(1 - a^2*x^2)^(1/2))/(2*a^4*c
^3) - (19*asinh(x*(-a^2)^(1/2)))/(2*a^4*c^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^4}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(x**4/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x*
*2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral
(a*x**5/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1)
+ 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.346 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^3} dx$

Optimal. Leaf size=137

$$-\frac{4\sin^{-1}(ax)}{a^4c^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2}-\frac{14\left(1-a^2x^2\right)^{3/2}}{15a^4c^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^4c^3(1-ax)^4}+\frac{8\sqrt{1-a^2x^2}}{a^4c^3(1-ax)}$$

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)/a^4/c^3/(-a*x+1)^4-14/15*(-a^2*x^2+1)^(3/2)/a^4/c^3/
(-a*x+1)^3-(-a^2*x^2+1)^(3/2)/a^4/c^3/(-a*x+1)^2-4*arcsin(a*x)/a^4/c^3+8*(-
a^2*x^2+1)^(1/2)/a^4/c^3/(-a*x+1)
```

Rubi [A] time = 0.33, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 1639, 1637, 659, 651, 663, 216}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2}-\frac{14\left(1-a^2x^2\right)^{3/2}}{15a^4c^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^4c^3(1-ax)^4}+\frac{8\sqrt{1-a^2x^2}}{a^4c^3(1-ax)}-\frac{4\sin^{-1}(ax)}{a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^3,x]
```

```
[Out] (8*Sqrt[1 - a^2*x^2])/(a^4*c^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^4*c^3*
(1 - a*x)^4) - (14*(1 - a^2*x^2)^(3/2))/(15*a^4*c^3*(1 - a*x)^3) - (1 - a^2
*x^2)^(3/2)/(a^4*c^3*(1 - a*x)^2) - (4*ArcSin[a*x])/(a^4*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
```

```
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^3}\,dx = c\int \frac{x^3\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx$$
$$= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2} + \frac{\int \frac{\sqrt{1-a^2x^2}\left(2a^2c^3-5a^3c^3x+4a^4c^3x^2\right)}{(c-acx)^4}\,dx}{a^5c^2}$$
$$= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2} + \frac{\int \left(\frac{a^2\sqrt{1-a^2x^2}}{c(-1+ax)^4} + \frac{3a^2\sqrt{1-a^2x^2}}{c(-1+ax)^3} + \frac{4a^2\sqrt{1-a^2x^2}}{c(-1+ax)^2}\right)dx}{a^5c^2}$$
$$= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2} + \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^3c^3} + \frac{3\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^3c^3} + \frac{4\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^3c^3}$$
$$= \frac{8\sqrt{1-a^2x^2}}{a^4c^3(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{5a^4c^3(1-ax)^4} - \frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^3} - \frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2} - \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5a^3c^3} - \frac{4\int \frac{1}{\sqrt{1-a^2x^2}}}{a^3c^3}$$
$$= \frac{8\sqrt{1-a^2x^2}}{a^4c^3(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{5a^4c^3(1-ax)^4} - \frac{14\left(1-a^2x^2\right)^{3/2}}{15a^4c^3(1-ax)^3} - \frac{\left(1-a^2x^2\right)^{3/2}}{a^4c^3(1-ax)^2} - \frac{4\sin^{-1}(ax)}{a^4c^3}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.53

$$\frac{\frac{\sqrt{ax+1}\left(-15a^3x^3+149a^2x^2-222ax+94\right)}{(1-ax)^{5/2}} + 120\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{15a^4c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^3,x]
```

```
[Out] ((Sqrt[1 + a*x]*(94 - 222*a*x + 149*a^2*x^2 - 15*a^3*x^3))/(1 - a*x)^(5/2)
+ 120*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(15*a^4*c^3)
```

fricas [A] time = 0.49, size = 145, normalized size = 1.06

$$\frac{94\,a^3x^3 - 282\,a^2x^2 + 282\,ax + 120\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(15\,a^3x^3 - 149\,a^2x^2 + 222\,ax\right.}{15\left(a^7c^3x^3 - 3\,a^6c^3x^2 + 3\,a^5c^3x - a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/15*(94*a^3*x^3 - 282*a^2*x^2 + 282*a*x + 120*(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^3*x^3 - 149*a^2*x^2 +
222*a*x - 94)*sqrt(-a^2*x^2 + 1) - 94)/(a^7*c^3*x^3 - 3*a^6*c^3*x^2 + 3*a^5
*c^3*x - a^4*c^3)
```

giac [A] time = 0.19, size = 186, normalized size = 1.36

$$-\frac{4 \arcsin(ax)\,\mathrm{sgn}(a)}{a^3c^3|a|}+\frac{\sqrt{-a^2x^2+1}}{a^4c^3}-\frac{2\left(\frac{335\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{505\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{285\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{60\left(\sqrt{-a^2x^2\right.}}{a^8}\right.}{15\,a^3c^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -4*arcsin(a*x)*sgn(a)/(a^3*c^3*abs(a)) + sqrt(-a^2*x^2 + 1)/(a^4*c^3) - 2/1
5*(335*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 505*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^2/(a^4*x^2) + 285*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 6
0*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 79)/(a^3*c^3*((sqrt(-a^2*x^
2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

maple [A] time = 0.05, size = 186, normalized size = 1.36

$$\frac{\sqrt{-a^2x^2+1}}{c^3a^4}-\frac{4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^3a^3\sqrt{a^2}}-\frac{31\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{15c^3a^6\left(x-\frac{1}{a}\right)^2}-\frac{104\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{15c^3a^5\left(x-\frac{1}{a}\right)}-\frac{2\sqrt{-a^2\left(\right.}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3/a^4*(-a^2*x^2+1)^(1/2)-4/c^3/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2))-31/15/c^3/a^6/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
-104/15/c^3/a^5/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5/c^3/a^7/(x-1
/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.46, size = 163, normalized size = 1.19

$$-\frac{2\sqrt{-a^2x^2+1}}{5\left(a^7c^3x^3-3\,a^6c^3x^2+3\,a^5c^3x-a^4c^3\right)}-\frac{31\sqrt{-a^2x^2+1}}{15\left(a^6c^3x^2-2\,a^5c^3x+a^4c^3\right)}-\frac{104\sqrt{-a^2x^2+1}}{15\left(a^5c^3x-a^4c^3\right)}-\frac{4\arcsin(ax)}{a^4c^3}+\frac{\sqrt{-a^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -2/5*sqrt(-a^2*x^2 + 1)/(a^7*c^3*x^3 - 3*a^6*c^3*x^2 + 3*a^5*c^3*x - a^4*c^
3) - 31/15*sqrt(-a^2*x^2 + 1)/(a^6*c^3*x^2 - 2*a^5*c^3*x + a^4*c^3) - 104/1
5*sqrt(-a^2*x^2 + 1)/(a^5*c^3*x - a^4*c^3) - 4*arcsin(a*x)/(a^4*c^3) + sqrt
(-a^2*x^2 + 1)/(a^4*c^3)
```

mupad [B] time = 0.05, size = 234, normalized size = 1.71

$$\frac{\sqrt{1-a^2\,x^2}}{a^4\,c^3}-\frac{2\,\sqrt{1-a^2\,x^2}}{5\,\sqrt{-a^2}\,\left(a^2\,c^3\,\sqrt{-a^2}+3\,a^4\,c^3\,x^2\,\sqrt{-a^2}-a^5\,c^3\,x^3\,\sqrt{-a^2}-3\,a^3\,c^3\,x\,\sqrt{-a^2}\right)}-\frac{104\,\sqrt{1-a^2\,}}{15\,\left(a^2\,c^3\,\sqrt{-a^2}-a^3\,c^3\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(a^4*c^3) - (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(a^
2*c^3*(-a^2)^(1/2) + 3*a^4*c^3*x^2*(-a^2)^(1/2) - a^5*c^3*x^3*(-a^2)^(1/2)
- 3*a^3*c^3*x*(-a^2)^(1/2))) - (104*(1 - a^2*x^2)^(1/2))/(15*(a^2*c^3*(-a^2
)^(1/2) - a^3*c^3*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (31*(1 - a^2*x^2)^(1/2))/
(15*(a^4*c^3 - 2*a^5*c^3*x + a^6*c^3*x^2)) - (4*asinh(x*(-a^2)^(1/2)))/(a^3
*c^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^3}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(x**3/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x*
*2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral
(a*x**4/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1)
+ 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.347 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^3} dx$

Optimal. Leaf size=107

$$-\frac{\sin^{-1}(ax)}{a^3c^3}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3c^3(1-ax)}$$

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)/a^3/c^3/(-a*x+1)^4-3/5*(-a^2*x^2+1)^(3/2)/a^3/c^3/(-
a*x+1)^3-arcsin(a*x)/a^3/c^3+2*(-a^2*x^2+1)^(1/2)/a^3/c^3/(-a*x+1)
```

Rubi [A] time = 0.22, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1637, 659, 651, 663, 216}

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3c^3(1-ax)}-\frac{\sin^{-1}(ax)}{a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^3,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^3*c^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*c^3*
(1 - a*x)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*c^3*(1 - a*x)^3) - ArcSin[a*x
]/(a^3*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
```

```
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^3}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\
&= c\int \left(\frac{\sqrt{1-a^2x^2}}{a^2c^4(-1+ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^2c^4(-1+ax)^3}+\frac{\sqrt{1-a^2x^2}}{a^2c^4(-1+ax)^2}\right)dx\\
&= \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^2c^3}+\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^2c^3}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^2c^3}\\
&= \frac{2\sqrt{1-a^2x^2}}{a^3c^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^4}-\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^3c^3(1-ax)^3}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5a^2c^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2c^3}\\
&= \frac{2\sqrt{1-a^2x^2}}{a^3c^3(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^4}-\frac{3\left(1-a^2x^2\right)^{3/2}}{5a^3c^3(1-ax)^3}-\frac{\sin^{-1}(ax)}{a^3c^3}
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 77, normalized size = 0.72

$$\frac{\sqrt{ax+1}\left(-a^2x^2+3ax+4\right)+20\sqrt{2}(ax-1)\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-ax)\right)}{15a^3c^3(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^3,x]
```

```
[Out] (Sqrt[1 + a*x]*(4 + 3*a*x - a^2*x^2) + 20*Sqrt[2]*(-1 + a*x)*Hypergeometric
2F1[-3/2, -3/2, -1/2, (1 - a*x)/2])/(15*a^3*c^3*(1 - a*x)^(5/2))
```

fricas [A] time = 0.49, size = 138, normalized size = 1.29

$$\frac{8\,a^3x^3-24\,a^2x^2+24\,ax+10\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(13\,a^2x^2-19\,ax+8\right)\sqrt{-a^2x^2+1}}{5\left(a^6c^3x^3-3\,a^5c^3x^2+3\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/5*(8*a^3*x^3 - 24*a^2*x^2 + 24*a*x + 10*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (13*a^2*x^2 - 19*a*x + 8)*sqrt(-a
^2*x^2 + 1) - 8)/(a^6*c^3*x^3 - 3*a^5*c^3*x^2 + 3*a^4*c^3*x - a^3*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 167, normalized size = 1.56

$$-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^3a^2\sqrt{a^2}}-\frac{7\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{5c^3a^5\left(x-\frac{1}{a}\right)^2}-\frac{13\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{5c^3a^4\left(x-\frac{1}{a}\right)}-\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{5c^3a^6\left(x-\frac{1}{a}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-7/5/c^3/a^5
/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-13/5/c^3/a^4/(x-1/a)*(-a^2*(x
-1/a)^2-2*a*(x-1/a))^(1/2)-2/5/c^3/a^6/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a
))^(1/2)
```

maxima [A] time = 0.41, size = 144, normalized size = 1.35

$$-\frac{2\sqrt{-a^2x^2+1}}{5\left(a^6c^3x^3-3\,a^5c^3x^2+3\,a^4c^3x-a^3c^3\right)}-\frac{7\sqrt{-a^2x^2+1}}{5\left(a^5c^3x^2-2\,a^4c^3x+a^3c^3\right)}-\frac{13\sqrt{-a^2x^2+1}}{5\left(a^4c^3x-a^3c^3\right)}-\frac{\arcsin\left(ax\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -2/5*sqrt(-a^2*x^2 + 1)/(a^6*c^3*x^3 - 3*a^5*c^3*x^2 + 3*a^4*c^3*x - a^3*c^
3) - 7/5*sqrt(-a^2*x^2 + 1)/(a^5*c^3*x^2 - 2*a^4*c^3*x + a^3*c^3) - 13/5*sq
rt(-a^2*x^2 + 1)/(a^4*c^3*x - a^3*c^3) - arcsin(a*x)/(a^3*c^3)
```

mupad [B] time = 0.82, size = 259, normalized size = 2.42

$$\frac{4\,a^2\,\sqrt{1-a^2\,x^2}}{15\,\left(a^7\,c^3\,x^2-2\,a^6\,c^3\,x+a^5\,c^3\right)}-\frac{13\,\sqrt{1-a^2\,x^2}}{5\,\left(a\,c^3\,\sqrt{-a^2}-a^2\,c^3\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{5\,\sqrt{1-a^2\,x^2}}{3\,\left(a^5\,c^3\,x^2-2\,a^4\,c^3\,x+a^3\,c^3\right)}-\frac{}{5\,\sqrt{-a^2}\,\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] (4*a^2*(1 - a^2*x^2)^(1/2))/(15*(a^5*c^3 - 2*a^6*c^3*x + a^7*c^3*x^2)) - (1
3*(1 - a^2*x^2)^(1/2))/(5*(a*c^3*(-a^2)^(1/2) - a^2*c^3*x*(-a^2)^(1/2))*(-a
^2)^(1/2)) - (5*(1 - a^2*x^2)^(1/2))/(3*(a^3*c^3 - 2*a^4*c^3*x + a^5*c^3*x^
2)) - (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(a*c^3*(-a^2)^(1/2) + 3*a^3*c
^3*x^2*(-a^2)^(1/2) - a^4*c^3*x^3*(-a^2)^(1/2) - 3*a^2*c^3*x*(-a^2)^(1/2)))
 - asinh(x*(-a^2)^(1/2))/(a^2*c^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(x**2/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x*
*2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral
(a*x**3/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1)
+ 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.348 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^3}\,dx$

Optimal. Leaf size=65

$$\frac{(1-a^2x^2)^{3/2}}{5a^2c^3(1-ax)^4}-\frac{4(1-a^2x^2)^{3/2}}{15a^2c^3(1-ax)^3}$$

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)/a^2/c^3/(-a*x+1)^4-4/15*(-a^2*x^2+1)^(3/2)/a^2/c^3/(
-a*x+1)^3
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6128, 793, 651}

$$\frac{(1-a^2x^2)^{3/2}}{5a^2c^3(1-ax)^4}-\frac{4(1-a^2x^2)^{3/2}}{15a^2c^3(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a*c*x)^3,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(5*a^2*c^3*(1 - a*x)^4) - (4*(1 - a^2*x^2)^(3/2))/(15*a
^2*c^3*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
```

```
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^3}\,dx &= c\int \frac{x\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{5a^2c^3(1-ax)^4}-\frac{4\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{5a}\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{5a^2c^3(1-ax)^4}-\frac{4\left(1-a^2x^2\right)^{3/2}}{15a^2c^3(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.54

$$\frac{(ax+1)^{3/2}(4ax-1)}{15a^2c^3(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a*c*x)^3,x]
[Out] ((1 + a*x)^(3/2)*(-1 + 4*a*x))/(15*a^2*c^3*(1 - a*x)^(5/2))
```

fricas [A] time = 0.62, size = 91, normalized size = 1.40

$$-\frac{a^3x^3-3\,a^2x^2+3\,ax+\left(4\,a^2x^2+3\,ax-1\right)\sqrt{-a^2x^2+1}-1}{15\left(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x + (4*a^2*x^2 + 3*a*x - 1)*sqrt(-a^2*x^2
+ 1) - 1)/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2*c^3)
```

giac [B] time = 0.20, size = 121, normalized size = 1.86

$$\frac{2\left(\frac{5\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}+\frac{5\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-1\right)}{15\,ac^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 2/15*(5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 5*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^2/(a^4*x^2) + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 1)/
(a*c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

maple [A] time = 0.03, size = 41, normalized size = 0.63

$$\frac{(4ax-1)(ax+1)^2}{15(ax-1)^2 c^3 \sqrt{-a^2x^2+1}\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^3,x)
```

```
[Out] 1/15*(4*a*x-1)*(a*x+1)^2/(a*x-1)^2/c^3/(-a^2*x^2+1)^(1/2)/a^2
```

maxima [B] time = 0.40, size = 132, normalized size = 2.03

$$-\frac{2\sqrt{-a^2x^2+1}}{5\left(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-a^2c^3\right)}-\frac{11\sqrt{-a^2x^2+1}}{15\left(a^4c^3x^2-2\,a^3c^3x+a^2c^3\right)}-\frac{4\sqrt{-a^2x^2+1}}{15\left(a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-a^2*x^2 + 1)/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2*c^
3) - 11/15*sqrt(-a^2*x^2 + 1)/(a^4*c^3*x^2 - 2*a^3*c^3*x + a^2*c^3) - 4/15*
sqrt(-a^2*x^2 + 1)/(a^3*c^3*x - a^2*c^3)
```

mupad [B] time = 0.85, size = 143, normalized size = 2.20

$$-\frac{360\,a^6\,c^3\,\sqrt{1-a^2\,x^2}-600\,a^6\,c^3\,{\left(1-a^2\,x^2\right)}^{3/2}+225\,a^6\,c^3\,{\left(1-a^2\,x^2\right)}^{5/2}+360\,a^7\,c^3\,x\,\sqrt{1-a^2\,x^2}-420\,a^7\,c^3\,x\,\left(1\right.}{225\,a^8\,c^6\,{\left(a^2\,x^2-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] -(360*a^6*c^3*(1 - a^2*x^2)^(1/2) - 600*a^6*c^3*(1 - a^2*x^2)^(3/2) + 225*a
^6*c^3*(1 - a^2*x^2)^(5/2) + 360*a^7*c^3*x*(1 - a^2*x^2)^(1/2) - 420*a^7*c^
3*x*(1 - a^2*x^2)^(3/2) + 60*a^7*c^3*x*(1 - a^2*x^2)^(5/2))/(225*a^8*c^6*(a
^2*x^2 - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x}{a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}-3a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+3ax\sqrt{-a^{2}x^{2}+1}-\sqrt{-a^{2}x^{2}+1}}\,dx+\int \frac{ax^{2}}{a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}-3a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+3ax\sqrt{-a^{2}x^{2}+1}-\sqrt{-a^{2}x^{2}+1}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(x/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2
+ 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a*
x**2/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3
*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.349 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=65

$$\frac{(1-a^2x^2)^{3/2}}{15ac^3(1-ax)^3}+\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}$$

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)/a/c^3/(-a*x+1)^4+1/15*(-a^2*x^2+1)^(3/2)/a/c^3/(-a*x
+1)^3
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{(1-a^2x^2)^{3/2}}{15ac^3(1-ax)^3}+\frac{(1-a^2x^2)^{3/2}}{5ac^3(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^3,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(5*a*c^3*(1 - a*x)^4) + (1 - a^2*x^2)^(3/2)/(15*a*c^3*(
1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^3}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4}+\frac{1}{5}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.54

$$
\frac{(4-ax)(ax+1)^{3/2}}{15ac^3(1-ax)^{5/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^3,x]
[Out] ((4 - a*x)*(1 + a*x)^(3/2))/(15*a*c^3*(1 - a*x)^(5/2))
```

fricas [A] time = 0.50, size = 89, normalized size = 1.37

$$
\frac{4\,a^3x^3-12\,a^2x^2+12\,ax+\left(a^2x^2-3\,ax-4\right)\sqrt{-a^2x^2+1}-4}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] 1/15*(4*a^3*x^3 - 12*a^2*x^2 + 12*a*x + (a^2*x^2 - 3*a*x - 4)*sqrt(-a^2*x^2
 + 1) - 4)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

giac [B] time = 0.22, size = 145, normalized size = 2.23

$$
-\frac{2\left(\frac{5\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{25\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}-4\right)}{15\,c^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/15*(5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 25*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2/(a^4*x^2) + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 1
5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 4)/(c^3*((sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

maple [A] time = 0.03, size = 40, normalized size = 0.62

$$-\frac{(ax-4)(ax+1)^2}{15(ax-1)^2 c^3\sqrt{-a^2x^2+1}\ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/15*(a*x-4)*(a*x+1)^2/(a*x-1)^2/c^3/(-a^2*x^2+1)^(1/2)/a
```

maxima [B] time = 0.49, size = 126, normalized size = 1.94

$$-\frac{2\sqrt{-a^2x^2+1}}{5\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}-\frac{\sqrt{-a^2x^2+1}}{15\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)}+\frac{\sqrt{-a^2x^2+1}}{15\left(a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-a^2*x^2 + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
 - 1/15*sqrt(-a^2*x^2 + 1)/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3) + 1/15*sqrt(
-a^2*x^2 + 1)/(a^2*c^3*x - a*c^3)
```

mupad [B] time = 0.00, size = 183, normalized size = 2.82

$$\frac{2\sqrt{1-a^2\,x^2}}{5\sqrt{-a^2}\,\left(3\,c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}+a^2\,c^3\,x^3\,\sqrt{-a^2}-3\,a\,c^3\,x^2\,\sqrt{-a^2}\right)}-\frac{\sqrt{1-a^2\,x^2}}{15\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}\right)}-\frac{a}{15\,\left(a^4\,c^3\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)
^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)^(1/2))) - (1 - a^
2*x^2)^(1/2)/(15*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a))
- (a*(1 - a^2*x^2)^(1/2))/(15*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**
2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(
1/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*
x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.350 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^3}\,dx$

Optimal. Leaf size=97

$$\frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8ax+5}{5c^3\sqrt{1-a^2x^2}}+\frac{8(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

```
[Out] 8/5*(a*x+1)/c^3/(-a^2*x^2+1)^(5/2)+4/5*a*x/c^3/(-a^2*x^2+1)^(3/2)-arctanh((
-a^2*x^2+1)^(1/2))/c^3+1/5*(8*a*x+5)/c^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 823, 12, 266, 63, 208}

$$\frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8ax+5}{5c^3\sqrt{1-a^2x^2}}+\frac{8(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a*c*x)^3),x]
```

```
[Out] (8*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a*x)/(5*c^3*(1 - a^2*x^2)^(3
/2)) + (5 + 8*a*x)/(5*c^3*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x(c-acx)^4}\,dx$$

$$= \frac{\int \frac{(c+acx)^4}{x\left(1-a^2x^2\right)^{7/2}}\,dx}{c^7}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{-5c^4-12ac^4x+5a^2c^4x^2}{x\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^7}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{15c^4+24ac^4x}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^7}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} + \frac{\int \frac{15a^2c^4}{x\sqrt{1-a^2x^2}}\,dx}{15a^2c^7}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^3}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c^3}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2c^3}$$

$$= \frac{8(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4ax}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{5+8ax}{5c^3\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Mathematica [C] time = 0.09, size = 71, normalized size = 0.73

$$\frac{24a^5x^5 - 60a^3x^3 + 3\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-a^2x^2\right) + 5a^2x^2 + 60ax + 16}{15c^3\left(1-a^2x^2\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a*c*x)^3),x]
```

```
[Out] (16 + 60*a*x + 5*a^2*x^2 - 60*a^3*x^3 + 24*a^5*x^5 + 3*Hypergeometric2F1[-5
/2, 1, -3/2, 1 - a^2*x^2])/(15*c^3*(1 - a^2*x^2)^(5/2))
```

fricas [A] time = 0.59, size = 130, normalized size = 1.34

$$\frac{13\,a^3x^3-39\,a^2x^2+39\,ax+5\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(8\,a^2x^2-19\,ax+13\right)\sqrt{-a^2x^2+1}-}{5\left(a^3c^3x^3-3\,a^2c^3x^2+3\,ac^3x-c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/5*(13*a^3*x^3 - 39*a^2*x^2 + 39*a*x + 5*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (8*a^2*x^2 - 19*a*x + 13)*sqrt(-a^2*x^2
+ 1) - 13)/(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)
```

giac [B] time = 0.22, size = 189, normalized size = 1.95

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c^3|a|}+\frac{2\left(13\,a-\frac{45\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{ax}+\frac{75\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^3x^2}-\frac{55\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^5x^3}+\frac{20\left(\sqrt{-a^2x^2+1}\,|a|+\right.}{a^7x^4}\right.}{5\,c^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^3*abs(a
)) + 2/5*(13*a - 45*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a*x) + 75*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2/(a^3*x^2) - 55*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^5
*x^3) + 20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^7*x^4))/(c^3*((sqrt(-a^2*x^
2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

maple [B] time = 0.04, size = 275, normalized size = 2.84

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}}{a}+\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}+\frac{\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{5a\left(x-\frac{1}{a}\right)^3}-\frac{4a\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right.}}{3a\left(x\right.}\right.}{}}{a^2}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3*(arctanh(1/(-a^2*x^2+1)^(1/2))-1/a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/a/(x-1
/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2/a^2*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(
1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^3*x), x)
```

mupad [B] time = 0.07, size = 209, normalized size = 2.15

$$\frac{3\,a^2\,\sqrt{1-a^2\,x^2}}{5\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{8\,a\,\sqrt{1-a^2\,x^2}}{5\,\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}\right)}+\frac{2\,a\,\sqrt{1-a^2\,x^2}}{5\,\sqrt{-a^2}\,\left(3\,c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}+a^2\,c^3\,x^3\,\sqrt{-a^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] (atan((1 - a^2*x^2)^(1/2)*1i)*1i)/c^3 + (3*a^2*(1 - a^2*x^2)^(1/2))/(5*(a^2
*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (8*a*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1
/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a)) + (2*a*(1 - a^2*x^2)^(1/2)
)/(5*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a + a^2*c^3*x^
3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{a^3x^4\sqrt{-a^2x^2+1}-3a^2x^3\sqrt{-a^2x^2+1}+3ax^2\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^4\sqrt{-a^2x^2+1}-3a^2x^3\sqrt{-a^2x^2+1}+3ax^2\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}}\,d}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**4*sqrt(-a**2*x**2 + 1) - 3*a**2*x**3*sqrt(-a**2*x**
2 + 1) + 3*a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x) + Inte
gral(1/(a**3*x**4*sqrt(-a**2*x**2 + 1) - 3*a**2*x**3*sqrt(-a**2*x**2 + 1) +
 3*a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.351 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^3}\,dx$

Optimal. Leaf size=127

$$\frac{8a(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{c^3x}+\frac{a(79ax+60)}{15c^3\sqrt{1-a^2x^2}}+\frac{4a(8ax+5)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{4a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

```
[Out] 8/5*a*(a*x+1)/c^3/(-a^2*x^2+1)^(5/2)+4/15*a*(8*a*x+5)/c^3/(-a^2*x^2+1)^(3/2
)-4*a*arctanh((-a^2*x^2+1)^(1/2))/c^3+1/15*a*(79*a*x+60)/c^3/(-a^2*x^2+1)^(
1/2)-(-a^2*x^2+1)^(1/2)/c^3/x
```

Rubi [A] time = 0.35, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 807, 266, 63, 208}

$$\frac{8a(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{c^3x}+\frac{a(79ax+60)}{15c^3\sqrt{1-a^2x^2}}+\frac{4a(8ax+5)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{4a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^3),x]
```

```
[Out] (8*a*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a*(5 + 8*a*x))/(15*c^3*(1
- a^2*x^2)^(3/2)) + (a*(60 + 79*a*x))/(15*c^3*Sqrt[1 - a^2*x^2]) - Sqrt[1 -
 a^2*x^2]/(c^3*x) - (4*a*ArcTanh[Sqrt[1 - a^2*x^2]])/c^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^2(c-acx)^4}\,dx$$

$$= \frac{\int \frac{(c+acx)^4}{x^2\left(1-a^2x^2\right)^{7/2}}\,dx}{c^7}$$

$$= \frac{8a(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{-5c^4-20ac^4x-27a^2c^4x^2}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^7}$$

$$= \frac{8a(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a(5+8ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{15c^4+60ac^4x+64a^2c^4x^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^7}$$

$$= \frac{8a(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a(5+8ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{-15c^4-60ac^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{15c^7}$$

$$= \frac{8a(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a(5+8ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^3x} + \frac{(4a)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^3}$$

$$= \frac{8a(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a(5+8ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^3x} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-}\cdots}\right.}{c^3}$$

$$= \frac{8a(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a(5+8ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^3x} - \frac{4\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx\right.}{ac^3}$$

$$= \frac{8a(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a(5+8ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a(60+79ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^3x} - \frac{4a\tanh^{-1}\left(\sqrt{1-a^2}\right.}{c^3}$$

Mathematica [A] time = 0.06, size = 101, normalized size = 0.80

$$\frac{94a^4x^4-128a^3x^3-73a^2x^2-60ax(ax-1)^2\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+134ax-15}{15c^3x(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^3),x]
```

```
[Out] (-15 + 134*a*x - 73*a^2*x^2 - 128*a^3*x^3 + 94*a^4*x^4 - 60*a*x*(-1 + a*x)^
2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(15*c^3*x*(-1 + a*x)^2*Sqrt
[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 155, normalized size = 1.22

$$\frac{104\,a^4x^4 - 312\,a^3x^3 + 312\,a^2x^2 - 104\,ax + 60\left(a^4x^4 - 3\,a^3x^3 + 3\,a^2x^2 - ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(94\,a^3x^3 - 222\,a\right.}{15\left(a^3c^3x^4 - 3\,a^2c^3x^3 + 3\,ac^3x^2 - c^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/15*(104*a^4*x^4 - 312*a^3*x^3 + 312*a^2*x^2 - 104*a*x + 60*(a^4*x^4 - 3*a
^3*x^3 + 3*a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (94*a^3*x^3 - 2
22*a^2*x^2 + 149*a*x - 15)*sqrt(-a^2*x^2 + 1))/(a^3*c^3*x^4 - 3*a^2*c^3*x^3
 + 3*a*c^3*x^2 - c^3*x)
```

giac [B] time = 0.25, size = 269, normalized size = 2.12

$$-\frac{4\,a^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c^3|a|} - \frac{\sqrt{-a^2x^2+1}\,|a|+a}{2\,c^3x|a|} - \frac{\left(15\,a^2 - \frac{491\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{x} + \frac{1690\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^2x^2} - \frac{2570\left(\sqrt{-a^2\right.}{a^4}\right.}{30\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -4*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^3*a
bs(a)) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(c^3*x*abs(a)) - 1/30*(15*a^2
- 491*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x + 1690*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^2/(a^2*x^2) - 2570*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^4*x^3) + 1815*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^6*x^4) - 555*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^5/(a^8*x^5))*a^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3*((sqrt(-a^2*x
^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

maple [B] time = 0.05, size = 248, normalized size = 1.95

$$-\frac{4a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\sqrt{-a^2x^2+1}}{x}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)^2}+\frac{5\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}+\frac{\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{5a\left(x-\frac{1}{a}\right)^3}-\frac{4a\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2}}{3a\left(x-\right.}\right.}{}}{a}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3*(4*a*arctanh(1/(-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/x-1/a/(x-1/a)^
2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+5/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))
^(1/2)+2/a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/a
/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2
*a*(x-1/a))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^3*x^2), x)
```

mupad [B] time = 0.83, size = 234, normalized size = 1.84

$$\frac{19\,a^3\,\sqrt{1-a^2\,x^2}}{15\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{c^3\,x}+\frac{79\,a^2\,\sqrt{1-a^2\,x^2}}{15\,\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}\right)}+\frac{2\,a^2\,\sqrt{1}}{5\,\sqrt{-a^2}\,\left(3\,c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] (19*a^3*(1 - a^2*x^2)^(1/2))/(15*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) - (
1 - a^2*x^2)^(1/2)/(c^3*x) + (a*atan((1 - a^2*x^2)^(1/2)*1i)*4i)/c^3 + (79*
a^2*(1 - a^2*x^2)^(1/2))/(15*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)
```

```
^(1/2))/a)) + (2*a^2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(
1/2) - (c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)
^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{a^3x^5\sqrt{-a^2x^2+1}-3a^2x^4\sqrt{-a^2x^2+1}+3ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^5\sqrt{-a^2x^2+1}-3a^2x^4\sqrt{-a^2x^2+1}+3ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}$$

$$c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**5*sqrt(-a**2*x**2 + 1) - 3*a**2*x**4*sqrt(-a**2*x**
2 + 1) + 3*a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x) + I
ntegral(1/(a**3*x**5*sqrt(-a**2*x**2 + 1) - 3*a**2*x**4*sqrt(-a**2*x**2 + 1
) + 3*a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.352 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^3}\,dx$

Optimal. Leaf size=162

$$\frac{a^2(164ax+135)}{15c^3\sqrt{1-a^2x^2}}+\frac{4a^2(13ax+10)}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8a^2(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{4a\sqrt{1-a^2x^2}}{c^3x}-\frac{\sqrt{1-a^2x^2}}{2c^3x^2}-\frac{19a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^3}$$

```
[Out] 8/5*a^2*(a*x+1)/c^3/(-a^2*x^2+1)^(5/2)+4/15*a^2*(13*a*x+10)/c^3/(-a^2*x^2+1
)^(3/2)-19/2*a^2*arctanh((-a^2*x^2+1)^(1/2))/c^3+1/15*a^2*(164*a*x+135)/c^3
/(-a^2*x^2+1)^(1/2)-1/2*(-a^2*x^2+1)^(1/2)/c^3/x^2-4*a*(-a^2*x^2+1)^(1/2)/c
^3/x
```

Rubi [A] time = 0.43, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^2(164ax+135)}{15c^3\sqrt{1-a^2x^2}}+\frac{4a^2(13ax+10)}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8a^2(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{4a\sqrt{1-a^2x^2}}{c^3x}-\frac{\sqrt{1-a^2x^2}}{2c^3x^2}-\frac{19a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^3),x]
```

```
[Out] (8*a^2*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a^2*(10 + 13*a*x))/(15*c
^3*(1 - a^2*x^2)^(3/2)) + (a^2*(135 + 164*a*x))/(15*c^3*Sqrt[1 - a^2*x^2])
- Sqrt[1 - a^2*x^2]/(2*c^3*x^2) - (4*a*Sqrt[1 - a^2*x^2])/(c^3*x) - (19*a^2
*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
```

```
0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^3(c-acx)^4}\,dx$$

$$= \frac{\int \frac{(c+acx)^4}{x^3\left(1-a^2x^2\right)^{7/2}}\,dx}{c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{-5c^4-20ac^4x-35a^2c^4x^2-32a^3c^4x^3}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{15c^4+60ac^4x+120a^2c^4x^2+104a^3c^4x^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{-15c^4-60ac^4x-135a^2c^4x^2}{x^3\sqrt{1-a^2x^2}}\,dx}{15c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} + \frac{\int \frac{120ac^4+285a^2c^4x}{x^2\sqrt{1-a^2x^2}}\,d}{30c^7}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{4a\sqrt{1-a^2x^2}}{c^3x} + \frac{(1}{}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{4a\sqrt{1-a^2x^2}}{c^3x} + \frac{(1}{}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{4a\sqrt{1-a^2x^2}}{c^3x} - \frac{19}{}$$

$$= \frac{8a^2(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^2(10+13ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(135+164ax)}{15c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{4a\sqrt{1-a^2x^2}}{c^3x} - \frac{19}{}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 0.70

$$\frac{448a^5x^5-611a^4x^4-346a^3x^3+638a^2x^2-285a^2x^2(ax-1)^2\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-90ax-15}{30c^3x^2(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^3),x]
```

```
[Out] (-15 - 90*a*x + 638*a^2*x^2 - 346*a^3*x^3 - 611*a^4*x^4 + 448*a^5*x^5 - 285
*a^2*x^2*(-1 + a*x)^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(30*c^3
*x^2*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.45, size = 173, normalized size = 1.07

$$\frac{398\,a^5x^5-1194\,a^4x^4+1194\,a^3x^3-398\,a^2x^2+285\left(a^5x^5-3\,a^4x^4+3\,a^3x^3-a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(448\,a^4x\right.}{30\left(a^3c^3x^5-3\,a^2c^3x^4+3\,ac^3x^3-c^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/30*(398*a^5*x^5 - 1194*a^4*x^4 + 1194*a^3*x^3 - 398*a^2*x^2 + 285*(a^5*x^
5 - 3*a^4*x^4 + 3*a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (448
*a^4*x^4 - 1059*a^3*x^3 + 713*a^2*x^2 - 75*a*x - 15)*sqrt(-a^2*x^2 + 1))/(a
^3*c^3*x^5 - 3*a^2*c^3*x^4 + 3*a*c^3*x^3 - c^3*x^2)
```

giac [B] time = 0.25, size = 338, normalized size = 2.09

$$-\frac{19\,a^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,c^3|a|}-\frac{\left(15\,a^3+\frac{165\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x}-\frac{4234\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{ax^2}+\frac{14330\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^3x^3}-\frac{20965}{\cdots}\right.}{120\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^3\left(\frac{\sqrt{-\cdots}}{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -19/2*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^
3*abs(a)) - 1/120*(15*a^3 + 165*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 4234*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a*x^2) + 14330*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)^3/(a^3*x^3) - 20965*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^5*x^4) + 14
```

```
385*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^7*x^5) - 4080*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^6/(a^9*x^6))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*((s
qrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a)) - 1/8*(16*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)*a*c^3*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*
abs(a)/(a*x^2))/(a^2*c^6)
```

maple [A] time = 0.05, size = 223, normalized size = 1.38

$$-\frac{\frac{19a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{4a\sqrt{-a^2x^2+1}}{x}-\frac{29a\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}\right)}{5}+\frac{9a\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}+\frac{\sqrt{-a^2x^2+1}}{2x^2}+}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3*(19/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+4*a*(-a^2*x^2+1)^(1/2)/x-29/
5*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(
x-1/a)^2-2*a*(x-1/a))^(1/2))+9*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
+1/2*(-a^2*x^2+1)^(1/2)/x^2+2/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^3*x^3), x)
```

mupad [B] time = 0.83, size = 257, normalized size = 1.59

$$\frac{29\,a^4\,\sqrt{1-a^2\,x^2}}{15\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{2\,c^3\,x^2}-\frac{4\,a\,\sqrt{1-a^2\,x^2}}{c^3\,x}+\frac{164\,a^3\,\sqrt{1-a^2\,x^2}}{15\,\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}\right)}+\frac{}{5\,\sqrt{-a^2}\,\left(3\,c^3\,x\,\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*19i)/(2*c^3) + (29*a^4*(1 - a^2*x^2)^(1/2
))/(15*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) - (1 - a^2*x^2)^(1/2)/(2*c^3*
x^2) - (4*a*(1 - a^2*x^2)^(1/2))/(c^3*x) + (164*a^3*(1 - a^2*x^2)^(1/2))/(1
5*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a)) + (2*a^3*(1 - a
^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a
 + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{a^3x^6\sqrt{-a^2x^2+1}-3a^2x^5\sqrt{-a^2x^2+1}+3ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^3x^6\sqrt{-a^2x^2+1}-3a^2x^5\sqrt{-a^2x^2+1}+3ax^4\sqrt{-a^2x^2+1}-x^3\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**6*sqrt(-a**2*x**2 + 1) - 3*a**2*x**5*sqrt(-a**2*x**
2 + 1) + 3*a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x) + I
ntegral(1/(a**3*x**6*sqrt(-a**2*x**2 + 1) - 3*a**2*x**5*sqrt(-a**2*x**2 + 1
) + 3*a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.353 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)^3} \, dx$

Optimal. Leaf size=187

$$-\frac{29a^2\sqrt{1-a^2x^2}}{3c^3x}-\frac{2a\sqrt{1-a^2x^2}}{c^3x^2}-\frac{\sqrt{1-a^2x^2}}{3c^3x^3}+\frac{a^3(93ax+80)}{5c^3\sqrt{1-a^2x^2}}+\frac{4a^3(6ax+5)}{5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8a^3(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{18a^3\tanh^{-1}\left(\right.}{c^3}$$

```
[Out] 8/5*a^3*(a*x+1)/c^3/(-a^2*x^2+1)^(5/2)+4/5*a^3*(6*a*x+5)/c^3/(-a^2*x^2+1)^(
3/2)-18*a^3*arctanh((-a^2*x^2+1)^(1/2))/c^3+1/5*a^3*(93*a*x+80)/c^3/(-a^2*x
^2+1)^(1/2)-1/3*(-a^2*x^2+1)^(1/2)/c^3/x^3-2*a*(-a^2*x^2+1)^(1/2)/c^3/x^2-2
9/3*a^2*(-a^2*x^2+1)^(1/2)/c^3/x
```

Rubi [A] time = 0.52, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^3(93ax+80)}{5c^3\sqrt{1-a^2x^2}}+\frac{4a^3(6ax+5)}{5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8a^3(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{29a^2\sqrt{1-a^2x^2}}{3c^3x}-\frac{2a\sqrt{1-a^2x^2}}{c^3x^2}-\frac{\sqrt{1-a^2x^2}}{3c^3x^3}-\frac{18a^3\tanh^{-1}\left(\sqrt{\right.}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a*c*x)^3),x]
```

```
[Out] (8*a^3*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (4*a^3*(5 + 6*a*x))/(5*c^3*
(1 - a^2*x^2)^(3/2)) + (a^3*(80 + 93*a*x))/(5*c^3*Sqrt[1 - a^2*x^2]) - Sqrt
[1 - a^2*x^2]/(3*c^3*x^3) - (2*a*Sqrt[1 - a^2*x^2])/(c^3*x^2) - (29*a^2*Sqr
t[1 - a^2*x^2])/(3*c^3*x) - (18*a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/c^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
```

```
0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-acx)^3}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^4(c-acx)^4}\,dx$$

$$= \frac{\int \frac{(c+acx)^4}{x^4\left(1-a^2x^2\right)^{7/2}}\,dx}{c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{-5c^4-20ac^4x-35a^2c^4x^2-40a^3c^4x^3-32a^4c^4x^4}{x^4\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{15c^4+60ac^4x+120a^2c^4x^2+180a^3c^4x^3+144a^4c^4x^4}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{-15c^4-60ac^4x-135a^2c^4x^2-240a^3c^4x^3}{x^4\sqrt{1-a^2x^2}}\,dx}{15c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} + \frac{\int \frac{180ac^4+435a^2c^4x+720\cdots}{x^3\sqrt{1-a^2x^2}}\cdots}{45c^7}$$

$$= \frac{8a^3(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{\int \cdots}{\cdots}$$

$$= \frac{8a^3(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{29a\cdots}{\cdots}$$

$$= \frac{8a^3(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{29a\cdots}{\cdots}$$

$$= \frac{8a^3(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{29a\cdots}{\cdots}$$

$$= \frac{8a^3(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{4a^3(5+6ax)}{5c^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^3(80+93ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{3c^3x^3} - \frac{2a\sqrt{1-a^2x^2}}{c^3x^2} - \frac{29a\cdots}{\cdots}$$

Mathematica [A] time = 0.06, size = 121, normalized size = 0.65

$$\frac{424a^6x^6-578a^5x^5-328a^4x^4+604a^3x^3-85a^2x^2-270a^3x^3(ax-1)^2\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-20ax-5}{15c^3x^3(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a*c*x)^3),x]
```

```
[Out] (-5 - 20*a*x - 85*a^2*x^2 + 604*a^3*x^3 - 328*a^4*x^4 - 578*a^5*x^5 + 424*a
^6*x^6 - 270*a^3*x^3*(-1 + a*x)^2*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^
2]])/(15*c^3*x^3*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.47, size = 181, normalized size = 0.97

$$\frac{324\,a^6x^6-972\,a^5x^5+972\,a^4x^4-324\,a^3x^3+270\left(a^6x^6-3\,a^5x^5+3\,a^4x^4-a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(424\,a^5x^5-\cdots\right.}{15\left(a^3c^3x^6-3\,a^2c^3x^5+3\,ac^3x^4-c^3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/15*(324*a^6*x^6 - 972*a^5*x^5 + 972*a^4*x^4 - 324*a^3*x^3 + 270*(a^6*x^6
- 3*a^5*x^5 + 3*a^4*x^4 - a^3*x^3)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (424*a
^5*x^5 - 1002*a^4*x^4 + 674*a^3*x^3 - 70*a^2*x^2 - 15*a*x - 5)*sqrt(-a^2*x^
2 + 1))/(a^3*c^3*x^6 - 3*a^2*c^3*x^5 + 3*a*c^3*x^4 - c^3*x^3)
```

giac [B] time = 0.40, size = 393, normalized size = 2.10

$$-\frac{18\,a^4\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c^3|a|}-\frac{\left(5\,a^4+\frac{35\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2}{x}+\frac{335\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{x^2}-\frac{7559\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^2x^3}+\frac{25195\left(\sqrt{-a\cdots}\right.}{a^4\cdots}\cdots\right.}{120\left(\sqrt{-a^2x^2+1}\,|a|+a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -18*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^3*
abs(a)) - 1/120*(5*a^4 + 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 335*(sq
rt(-a^2*x^2 + 1)*abs(a) + a)^2/x^2 - 7559*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3
/(a^2*x^3) + 25195*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^4*x^4) - 36035*(sqr
```

```
t(-a^2*x^2 + 1)*abs(a) + a)^5/(a^6*x^5) + 24225*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^6/(a^8*x^6) - 6585*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/(a^10*x^7))*a^6*x
^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/
(a^2*x) - 1)^5*abs(a)) - 1/24*(117*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^6/
x + 12*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^6/x^2 + (sqrt(-a^2*x^2 + 1)*
abs(a) + a)^3*c^6/x^3)/(a^2*c^9*abs(a))
```

maple [B] time = 0.05, size = 355, normalized size = 1.90

$$-\frac{16a^3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{29a^2\sqrt{-a^2x^2+1}}{3x}-7a^2\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}\right)+\frac{\sqrt{-a^2x^2+1}}{3x^3}+\frac{16a^2\sqrt{\cdots}}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^3,x)
```

```
[Out] -1/c^3*(16*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))+29/3*a^2*(-a^2*x^2+1)^(1/2)/x-
7*a^2*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/3*(-a^2*x^2+1)^(1/2)/x^3+16*a^2/(x-1/a)*(-
a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-4*a*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*a
rctanh(1/(-a^2*x^2+1)^(1/2)))+2*a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(1/2)-2/5*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x
-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a*c*x+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^3*x^4), x)
```

mupad [B] time = 0.09, size = 328, normalized size = 1.75

$$\frac{7\,a^5\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{4\,a^7\,\sqrt{1-a^2\,x^2}}{15\,\left(a^6\,c^3\,x^2-2\,a^5\,c^3\,x+a^4\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{3\,c^3\,x^3}-\frac{2\,a\,\sqrt{1-a^2\,x^2}}{c^3\,x^2}-\frac{29\,a^2\,\sqrt{1-a^2\,x^2}}{3\,c^3\,x}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^3),x)
```

```
[Out] (a^3*atan((1 - a^2*x^2)^(1/2)*1i)*18i)/c^3 + (7*a^5*(1 - a^2*x^2)^(1/2))/(3
*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (4*a^7*(1 - a^2*x^2)^(1/2))/(15*(
a^4*c^3 - 2*a^5*c^3*x + a^6*c^3*x^2)) - (1 - a^2*x^2)^(1/2)/(3*c^3*x^3) - (
2*a*(1 - a^2*x^2)^(1/2))/(c^3*x^2) - (29*a^2*(1 - a^2*x^2)^(1/2))/(3*c^3*x)
 + (93*a^4*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*
(-a^2)^(1/2))/a)) + (2*a^4*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(3*c^3*x*(-
a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*
(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{a^{3}x^{7}\sqrt{-a^{2}x^{2}+1}-3a^{2}x^{6}\sqrt{-a^{2}x^{2}+1}+3ax^{5}\sqrt{-a^{2}x^{2}+1}-x^{4}\sqrt{-a^{2}x^{2}+1}}\,dx+\int \frac{1}{a^{3}x^{7}\sqrt{-a^{2}x^{2}+1}-3a^{2}x^{6}\sqrt{-a^{2}x^{2}+1}+3ax^{5}\sqrt{-a^{2}x^{2}+1}-x^{4}\sqrt{-a^{2}x^{2}+1}}}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(a*x/(a**3*x**7*sqrt(-a**2*x**2 + 1) - 3*a**2*x**6*sqrt(-a**2*x**
2 + 1) + 3*a*x**5*sqrt(-a**2*x**2 + 1) - x**4*sqrt(-a**2*x**2 + 1)), x) + I
ntegral(1/(a**3*x**7*sqrt(-a**2*x**2 + 1) - 3*a**2*x**6*sqrt(-a**2*x**2 + 1
) + 3*a*x**5*sqrt(-a**2*x**2 + 1) - x**4*sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.354 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-acx)^4}\,dx$

Optimal. Leaf size=166

$$\frac{29\sin^{-1}(ax)}{2a^6c^4}+\frac{(ax+1)^5}{7a^6c^4\left(1-a^2x^2\right)^{7/2}}-\frac{33(ax+1)^4}{35a^6c^4\left(1-a^2x^2\right)^{5/2}}+\frac{317(ax+1)^3}{105a^6c^4\left(1-a^2x^2\right)^{3/2}}-\frac{10(ax+1)^2}{a^6c^4\sqrt{1-a^2x^2}}-\frac{(ax+30)\sqrt{1-}}{2a^6c^4}$$

```
[Out] 1/7*(a*x+1)^5/a^6/c^4/(-a^2*x^2+1)^(7/2)-33/35*(a*x+1)^4/a^6/c^4/(-a^2*x^2+
1)^(5/2)+317/105*(a*x+1)^3/a^6/c^4/(-a^2*x^2+1)^(3/2)+29/2*arcsin(a*x)/a^6/
c^4-10*(a*x+1)^2/a^6/c^4/(-a^2*x^2+1)^(1/2)-1/2*(a*x+30)*(-a^2*x^2+1)^(1/2)
/a^6/c^4
```

Rubi [A] time = 0.53, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 852, 1635, 780, 216}

$$\frac{(ax+1)^5}{7a^6c^4\left(1-a^2x^2\right)^{7/2}}-\frac{33(ax+1)^4}{35a^6c^4\left(1-a^2x^2\right)^{5/2}}+\frac{317(ax+1)^3}{105a^6c^4\left(1-a^2x^2\right)^{3/2}}-\frac{10(ax+1)^2}{a^6c^4\sqrt{1-a^2x^2}}-\frac{(ax+30)\sqrt{1-a^2x^2}}{2a^6c^4}+\frac{29\sin^{-}}{2a^6}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a*c*x)^4,x]
```

```
[Out] (1 + a*x)^5/(7*a^6*c^4*(1 - a^2*x^2)^(7/2)) - (33*(1 + a*x)^4)/(35*a^6*c^4*
(1 - a^2*x^2)^(5/2)) + (317*(1 + a*x)^3)/(105*a^6*c^4*(1 - a^2*x^2)^(3/2))
- (10*(1 + a*x)^2)/(a^6*c^4*Sqrt[1 - a^2*x^2]) - ((30 + a*x)*Sqrt[1 - a^2*x
^2])/(2*a^6*c^4) + (29*ArcSin[a*x])/(2*a^6*c^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-acx)^4}\,dx = c\int \frac{x^5\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx$$
$$= \frac{\int \frac{x^5(c+acx)^5}{(1-a^2x^2)^{9/2}}dx}{c^9}$$
$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{\int \frac{(c+acx)^4\left(\frac{5}{a^5}+\frac{7x}{a^4}+\frac{7x^2}{a^3}+\frac{7x^3}{a^2}+\frac{7x^4}{a}\right)}{(1-a^2x^2)^{7/2}}dx}{7c^8}$$
$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{\int \frac{(c+acx)^3\left(\frac{107}{a^5}+\frac{105x}{a^4}+\frac{70x^2}{a^3}+\frac{35x^3}{a^2}\right)}{(1-a^2x^2)^{5/2}}dx}{35c^7}$$
$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{317(1+ax)^3}{105a^6c^4(1-a^2x^2)^{3/2}} - \frac{\int \frac{(c+acx)^2\left(\frac{630}{a^5}+\frac{315x}{a^4}+\frac{105x}{a^3}\right.}{(1-a^2x^2)^{3/2}}}{105c^6}$$
$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{317(1+ax)^3}{105a^6c^4(1-a^2x^2)^{3/2}} - \frac{10(1+ax)^2}{a^6c^4\sqrt{1-a^2x^2}} + \frac{\int (}{}$$
$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{317(1+ax)^3}{105a^6c^4(1-a^2x^2)^{3/2}} - \frac{10(1+ax)^2}{a^6c^4\sqrt{1-a^2x^2}} - \frac{(30}{}$$
$$= \frac{(1+ax)^5}{7a^6c^4(1-a^2x^2)^{7/2}} - \frac{33(1+ax)^4}{35a^6c^4(1-a^2x^2)^{5/2}} + \frac{317(1+ax)^3}{105a^6c^4(1-a^2x^2)^{3/2}} - \frac{10(1+ax)^2}{a^6c^4\sqrt{1-a^2x^2}} - \frac{(30}{}$$

Mathematica [A] time = 0.34, size = 126, normalized size = 0.76

$$-\frac{(ax+1)\left(\sqrt{1-a^2x^2}\left(105a^5x^5+630a^4x^4-8404a^3x^3+18916a^2x^2-16091ax+4784\right)-945(ax-1)^4\sin^{-1}(ax\right.}{210a^6c^4(ax-1)^3\left(a^2x^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a*c*x)^4,x]
```

```
[Out] -1/210*((1 + a*x)*(Sqrt[1 - a^2*x^2]*(4784 - 16091*a*x + 18916*a^2*x^2 - 84
04*a^3*x^3 + 630*a^4*x^4 + 105*a^5*x^5) - 945*(-1 + a*x)^4*ArcSin[a*x] + 42
00*(-1 + a*x)^4*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a^6*c^4*(-1 + a*x)^3*(-1 +
 a^2*x^2))
```

fricas [A] time = 0.48, size = 187, normalized size = 1.13

$$-\frac{4784\,a^4x^4 - 19136\,a^3x^3 + 28704\,a^2x^2 - 19136\,ax + 6090\left(a^4x^4 - 4\,a^3x^3 + 6\,a^2x^2 - 4\,ax + 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right.}{210\left(a^{10}c^4x^4 - 4\,a^9c^4x^3 + 6\,a^8c^4x^2 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/210*(4784*a^4*x^4 - 19136*a^3*x^3 + 28704*a^2*x^2 - 19136*a*x + 6090*(a^
4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/
(a*x)) + (105*a^5*x^5 + 630*a^4*x^4 - 8404*a^3*x^3 + 18916*a^2*x^2 - 16091*
a*x + 4784)*sqrt(-a^2*x^2 + 1) + 4784)/(a^10*c^4*x^4 - 4*a^9*c^4*x^3 + 6*a^
8*c^4*x^2 - 4*a^7*c^4*x + a^6*c^4)
```

giac [A] time = 0.17, size = 252, normalized size = 1.52

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(\frac{x}{a^5c^4}+\frac{10}{a^6c^4}\right)+\frac{29\,\arcsin\left(ax\right)\operatorname{sgn}\left(a\right)}{2\,a^5c^4|a|}+\frac{2\left(\frac{11599\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{29442\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{38500\left(\sqrt{-a^2\right.}{a^6}\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x/(a^5*c^4) + 10/(a^6*c^4)) + 29/2*arcsin(a*x)*sgn
(a)/(a^5*c^4*abs(a)) + 2/105*(11599*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x)
 - 29442*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 38500*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^3/(a^6*x^3) - 26845*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^
8*x^4) + 9765*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^10*x^5) - 1470*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^6/(a^12*x^6) - 1867)/(a^5*c^4*((sqrt(-a^2*x^2 + 1)*a
bs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

maple [A] time = 0.05, size = 252, normalized size = 1.52

$$-\frac{x\sqrt{-a^2x^2+1}}{2c^4a^5}+\frac{29\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2c^4a^5\sqrt{a^2}}-\frac{5\sqrt{-a^2x^2+1}}{c^4a^6}+\frac{733\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{105c^4a^8\left(x-\frac{1}{a}\right)^2}+\frac{2417\sqrt{-a^2\left(x-\frac{1}{a}\right)^2 \cdots}}{105c^4a^7\left(x-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a*c*x+c)^4,x)
```

```
[Out] -1/2/c^4/a^5*x*(-a^2*x^2+1)^(1/2)+29/2/c^4/a^5/(a^2)^(1/2)*arctan((a^2)^(1/
2)*x/(-a^2*x^2+1)^(1/2))-5/c^4/a^6*(-a^2*x^2+1)^(1/2)+733/105/c^4/a^8/(x-1/
a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2417/105/c^4/a^7/(x-1/a)*(-a^2*(x-1
/a)^2-2*a*(x-1/a))^(1/2)+2/7/c^4/a^10/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a)
)^(1/2)+71/35/c^4/a^9/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.41, size = 250, normalized size = 1.51

$$\frac{2\sqrt{-a^2x^2+1}}{7\left(a^{10}c^4x^4-4\,a^9c^4x^3+6\,a^8c^4x^2-4\,a^7c^4x+a^6c^4\right)}+\frac{71\sqrt{-a^2x^2+1}}{35\left(a^9c^4x^3-3\,a^8c^4x^2+3\,a^7c^4x-a^6c^4\right)}+\frac{733\sqrt{-a^2x^2 \cdots}}{105\left(a^8c^4x^2-2\,a^7 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^10*c^4*x^4 - 4*a^9*c^4*x^3 + 6*a^8*c^4*x^2 - 4*a^
7*c^4*x + a^6*c^4) + 71/35*sqrt(-a^2*x^2 + 1)/(a^9*c^4*x^3 - 3*a^8*c^4*x^2
+ 3*a^7*c^4*x - a^6*c^4) + 733/105*sqrt(-a^2*x^2 + 1)/(a^8*c^4*x^2 - 2*a^7*
c^4*x + a^6*c^4) + 2417/105*sqrt(-a^2*x^2 + 1)/(a^7*c^4*x - a^6*c^4) - 1/2*
sqrt(-a^2*x^2 + 1)*x/(a^5*c^4) + 29/2*arcsin(a*x)/(a^6*c^4) - 5*sqrt(-a^2*x
^2 + 1)/(a^6*c^4)
```

mupad [B] time = 0.83, size = 323, normalized size = 1.95

$$\frac{2\sqrt{1-a^2\,x^2}}{7\left(a^{10}\,c^4\,x^4-4\,a^9\,c^4\,x^3+6\,a^8\,c^4\,x^2-4\,a^7\,c^4\,x+a^6\,c^4\right)}+\frac{733\sqrt{1-a^2\,x^2}}{105\left(a^8\,c^4\,x^2-2\,a^7\,c^4\,x+a^6\,c^4\right)}+\frac{\cdots}{35\sqrt{-a^2}\,\left(a^4\,c^4\sqrt{-a^2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(7*(a^6*c^4 - 4*a^7*c^4*x + 6*a^8*c^4*x^2 - 4*a^9*c
^4*x^3 + a^10*c^4*x^4)) + (733*(1 - a^2*x^2)^(1/2))/(105*(a^6*c^4 - 2*a^7*c
```

```
^4*x + a^8*c^4*x^2)) + (71*(1 - a^2*x^2)^(1/2))/(35*(-a^2)^(1/2)*(a^4*c^4*(
-a^2)^(1/2) + 3*a^6*c^4*x^2*(-a^2)^(1/2) - a^7*c^4*x^3*(-a^2)^(1/2) - 3*a^5
*c^4*x*(-a^2)^(1/2))) + (2417*(1 - a^2*x^2)^(1/2))/(105*(a^4*c^4*(-a^2)^(1/
2) - a^5*c^4*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (5*(1 - a^2*x^2)^(1/2))/(a^6*c
^4) - (x*(1 - a^2*x^2)^(1/2))/(2*a^5*c^4) + (29*asinh(x*(-a^2)^(1/2)))/(2*a
^5*c^4*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^6}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x**5/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**
2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sq
rt(-a**2*x**2 + 1)), x) + Integral(a*x**6/(a**4*x**4*sqrt(-a**2*x**2 + 1) -
 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.355 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^4}\,dx$

Optimal. Leaf size=168

$$\frac{5\sin^{-1}(ax)}{a^5c^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2}+\frac{184\left(1-a^2x^2\right)^{3/2}}{105a^5c^4(1-ax)^3}-\frac{26\left(1-a^2x^2\right)^{3/2}}{35a^5c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^5c^4(1-ax)^5}-\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a^5/c^4/(-a*x+1)^5-26/35*(-a^2*x^2+1)^(3/2)/a^5/c^4/
(-a*x+1)^4+184/105*(-a^2*x^2+1)^(3/2)/a^5/c^4/(-a*x+1)^3+(-a^2*x^2+1)^(3/2)
/a^5/c^4/(-a*x+1)^2+5*arcsin(a*x)/a^5/c^4-10*(-a^2*x^2+1)^(1/2)/a^5/c^4/(-a
*x+1)
```

Rubi [A] time = 0.40, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 1639, 1637, 659, 651, 663, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2}+\frac{184\left(1-a^2x^2\right)^{3/2}}{105a^5c^4(1-ax)^3}-\frac{26\left(1-a^2x^2\right)^{3/2}}{35a^5c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^5c^4(1-ax)^5}-\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)}+\frac{5\sin^{-1}(ax)}{a^5c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^4,x]
```

```
[Out] (-10*Sqrt[1 - a^2*x^2])/(a^5*c^4*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(7*a^5*c^
4*(1 - a*x)^5) - (26*(1 - a^2*x^2)^(3/2))/(35*a^5*c^4*(1 - a*x)^4) + (184*(
1 - a^2*x^2)^(3/2))/(105*a^5*c^4*(1 - a*x)^3) + (1 - a^2*x^2)^(3/2)/(a^5*c^
4*(1 - a*x)^2) + (5*ArcSin[a*x])/(a^5*c^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
```

```
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-acx)^4}\,dx = c\int \frac{x^4\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} - \frac{\int \frac{\sqrt{1-a^2x^2}\left(2a^2c^4-7a^3c^4x+9a^4c^4x^2-5a^5c^4x^3\right)}{(c-acx)^5}dx}{a^6c^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} - \frac{\int \left(\frac{a^2\sqrt{1-a^2x^2}}{c(-1+ax)^5} + \frac{4a^2\sqrt{1-a^2x^2}}{c(-1+ax)^4} + \frac{6a^2\sqrt{1-a^2x^2}}{c(-1+ax)^3} + \frac{5a^2\sqrt{1-a^2x^2}}{c(-1+ax)^2}\right)dx}{a^6c^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} - \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^5}dx}{a^4c^4} - \frac{4\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}dx}{a^4c^4} - \frac{5\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}dx}{a^4c^4} - \frac{6\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}dx}{a^4c^4}$$

$$= -\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{7a^5c^4(1-ax)^5} - \frac{4\left(1-a^2x^2\right)^{3/2}}{5a^5c^4(1-ax)^4} + \frac{2\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} + \frac{2\int}{}$$

$$= -\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{7a^5c^4(1-ax)^5} - \frac{26\left(1-a^2x^2\right)^{3/2}}{35a^5c^4(1-ax)^4} + \frac{26\left(1-a^2x^2\right)^{3/2}}{15a^5c^4(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} + \frac{5}{}$$

$$= -\frac{10\sqrt{1-a^2x^2}}{a^5c^4(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{7a^5c^4(1-ax)^5} - \frac{26\left(1-a^2x^2\right)^{3/2}}{35a^5c^4(1-ax)^4} + \frac{184\left(1-a^2x^2\right)^{3/2}}{105a^5c^4(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{a^5c^4(1-ax)^2} +$$

Mathematica [C] time = 0.09, size = 95, normalized size = 0.57

$$-\frac{\sqrt{ax+1}\left(105a^4x^4-44a^3x^3-244a^2x^2+29ax+124\right)-700\sqrt{2}\,(ax-1)^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-ax)\right)}{105a^5c^4(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a*c*x)^4,x]
```

```
[Out] -1/105*(Sqrt[1 + a*x]*(124 + 29*a*x - 244*a^2*x^2 - 44*a^3*x^3 + 105*a^4*x^
4) - 700*Sqrt[2]*(-1 + a*x)^2*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - a*x)
/2])/(a^5*c^4*(1 - a*x)^(7/2))
```

fricas [A] time = 0.48, size = 179, normalized size = 1.07

$$-\frac{824\,a^4x^4-3296\,a^3x^3+4944\,a^2x^2-3296\,ax+1050\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+}{105\left(a^9c^4x^4-4\,a^8c^4x^3+6\,a^7c^4x^2-4\,a^6c^4x-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/105*(824*a^4*x^4 - 3296*a^3*x^3 + 4944*a^2*x^2 - 3296*a*x + 1050*(a^4*x^
4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)) + (105*a^4*x^4 - 1444*a^3*x^3 + 3256*a^2*x^2 - 2771*a*x + 824)*sqrt(-a^2
*x^2 + 1) + 824)/(a^9*c^4*x^4 - 4*a^8*c^4*x^3 + 6*a^7*c^4*x^2 - 4*a^6*c^4*x
 + a^5*c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 231, normalized size = 1.38

$$-\frac{\sqrt{-a^2x^2+1}}{c^4a^5}+\frac{5\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^4a^4\sqrt{a^2}}+\frac{446\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{105c^4a^7\left(x-\frac{1}{a}\right)^2}+\frac{1024\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{105c^4a^6\left(x-\frac{1}{a}\right)}+\frac{2\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^4,x)
```

```
[Out] -1/c^4/a^5*(-a^2*x^2+1)^(1/2)+5/c^4/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-
a^2*x^2+1)^(1/2))+446/105/c^4/a^7/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2)+1024/105/c^4/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2/7/c^4/a^9
/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+57/35/c^4/a^8/(x-1/a)^3*(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.47, size = 229, normalized size = 1.36

$$\frac{2\sqrt{-a^2x^2+1}}{7\left(a^9c^4x^4-4\,a^8c^4x^3+6\,a^7c^4x^2-4\,a^6c^4x+a^5c^4\right)}+\frac{57\sqrt{-a^2x^2+1}}{35\left(a^8c^4x^3-3\,a^7c^4x^2+3\,a^6c^4x-a^5c^4\right)}+\frac{446\sqrt{-a^2x^2+}}{105\left(a^7c^4x^2-2\,a^6c^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^9*c^4*x^4 - 4*a^8*c^4*x^3 + 6*a^7*c^4*x^2 - 4*a^6
*c^4*x + a^5*c^4) + 57/35*sqrt(-a^2*x^2 + 1)/(a^8*c^4*x^3 - 3*a^7*c^4*x^2 +
 3*a^6*c^4*x - a^5*c^4) + 446/105*sqrt(-a^2*x^2 + 1)/(a^7*c^4*x^2 - 2*a^6*c
^4*x + a^5*c^4) + 1024/105*sqrt(-a^2*x^2 + 1)/(a^6*c^4*x - a^5*c^4) + 5*arc
sin(a*x)/(a^5*c^4) - sqrt(-a^2*x^2 + 1)/(a^5*c^4)
```

mupad [B] time = 0.84, size = 350, normalized size = 2.08

$$\frac{2\sqrt{1-a^2x^2}}{7\left(a^9c^4x^4-4a^8c^4x^3+6a^7c^4x^2-4a^6c^4x+a^5c^4\right)}+\frac{572\sqrt{1-a^2x^2}}{105\left(a^7c^4x^2-2a^6c^4x+a^5c^4\right)}+\frac{}{35\sqrt{-a^2}\left(a^3c^4\sqrt{-a^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(7*(a^5*c^4 - 4*a^6*c^4*x + 6*a^7*c^4*x^2 - 4*a^8*c
^4*x^3 + a^9*c^4*x^4)) + (572*(1 - a^2*x^2)^(1/2))/(105*(a^5*c^4 - 2*a^6*c^
4*x + a^7*c^4*x^2)) + (57*(1 - a^2*x^2)^(1/2))/(35*(-a^2)^(1/2)*(a^3*c^4*(-
a^2)^(1/2) + 3*a^5*c^4*x^2*(-a^2)^(1/2) - a^6*c^4*x^3*(-a^2)^(1/2) - 3*a^4*
c^4*x*(-a^2)^(1/2))) - (6*a^4*(1 - a^2*x^2)^(1/2))/(5*(a^9*c^4 - 2*a^10*c^4
*x + a^11*c^4*x^2)) + (1024*(1 - a^2*x^2)^(1/2))/(105*(a^3*c^4*(-a^2)^(1/2)
 - a^4*c^4*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a^5*c^4) +
(5*asinh(x*(-a^2)^(1/2)))/(a^4*c^4*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**
2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sq
rt(-a**2*x**2 + 1)), x) + Integral(a*x**5/(a**4*x**4*sqrt(-a**2*x**2 + 1) -
 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.356 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^4}\,dx$

Optimal. Leaf size=138

$$\frac{\sin^{-1}(ax)}{a^4c^4}+\frac{86\left(1-a^2x^2\right)^{3/2}}{105a^4c^4(1-ax)^3}-\frac{19\left(1-a^2x^2\right)^{3/2}}{35a^4c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^4c^4(1-ax)^5}-\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a^4/c^4/(-a*x+1)^5-19/35*(-a^2*x^2+1)^(3/2)/a^4/c^4/
(-a*x+1)^4+86/105*(-a^2*x^2+1)^(3/2)/a^4/c^4/(-a*x+1)^3+arcsin(a*x)/a^4/c^4
-2*(-a^2*x^2+1)^(1/2)/a^4/c^4/(-a*x+1)
```

Rubi [A] time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6128, 1637, 659, 651, 663, 216}

$$\frac{86\left(1-a^2x^2\right)^{3/2}}{105a^4c^4(1-ax)^3}-\frac{19\left(1-a^2x^2\right)^{3/2}}{35a^4c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^4c^4(1-ax)^5}-\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)}+\frac{\sin^{-1}(ax)}{a^4c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^4,x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a^4*c^4*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(7*a^4*c^4
*(1 - a*x)^5) - (19*(1 - a^2*x^2)^(3/2))/(35*a^4*c^4*(1 - a*x)^4) + (86*(1
- a^2*x^2)^(3/2))/(105*a^4*c^4*(1 - a*x)^3) + ArcSin[a*x]/(a^4*c^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
```

```
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-acx)^4}\,dx = c\int \frac{x^3\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx$$
$$= c\int \left(-\frac{\sqrt{1-a^2x^2}}{a^3c^5(-1+ax)^5} - \frac{3\sqrt{1-a^2x^2}}{a^3c^5(-1+ax)^4} - \frac{3\sqrt{1-a^2x^2}}{a^3c^5(-1+ax)^3} - \frac{\sqrt{1-a^2x^2}}{a^3c^5(-1+ax)^2}\right)dx$$
$$= -\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^5}\,dx}{a^3c^4} - \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^3c^4} - \frac{3\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^3c^4} - \frac{3\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^3c^4}$$
$$= -\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)} + \frac{(1-a^2x^2)^{3/2}}{7a^4c^4(1-ax)^5} - \frac{3(1-a^2x^2)^{3/2}}{5a^4c^4(1-ax)^4} + \frac{(1-a^2x^2)^{3/2}}{a^4c^4(1-ax)^3} + \frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{7a^3c^4} + \frac{3\int \frac{\sqrt{}}{(-}}{5}$$
$$= -\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)} + \frac{(1-a^2x^2)^{3/2}}{7a^4c^4(1-ax)^5} - \frac{19(1-a^2x^2)^{3/2}}{35a^4c^4(1-ax)^4} + \frac{4(1-a^2x^2)^{3/2}}{5a^4c^4(1-ax)^3} + \frac{\sin^{-1}(ax)}{a^4c^4} - \frac{2\int \frac{\sqrt{1-a}}{(-1+a}}{35a^3}$$
$$= -\frac{2\sqrt{1-a^2x^2}}{a^4c^4(1-ax)} + \frac{(1-a^2x^2)^{3/2}}{7a^4c^4(1-ax)^5} - \frac{19(1-a^2x^2)^{3/2}}{35a^4c^4(1-ax)^4} + \frac{86(1-a^2x^2)^{3/2}}{105a^4c^4(1-ax)^3} + \frac{\sin^{-1}(ax)}{a^4c^4}$$

Mathematica [A] time = 0.25, size = 94, normalized size = 0.68

$$\frac{\sqrt{ax+1}\left(\sqrt{1-a^2x^2}\left(296a^3x^3-659a^2x^2+559ax-166\right)+105(ax-1)^4\sin^{-1}(ax)\right)}{105a^4c^4(1-ax)^{7/2}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a*c*x)^4,x]
```

```
[Out] (Sqrt[1 + a*x]*(Sqrt[1 - a^2*x^2]*(-166 + 559*a*x - 659*a^2*x^2 + 296*a^3*x
^3) + 105*(-1 + a*x)^4*ArcSin[a*x]))/(105*a^4*c^4*(1 - a*x)^(7/2)*Sqrt[1 -
a^2*x^2])
```

fricas [A] time = 0.60, size = 172, normalized size = 1.25

$$-\frac{166\,a^4x^4 - 664\,a^3x^3 + 996\,a^2x^2 - 664\,ax + 210\left(a^4x^4 - 4\,a^3x^3 + 6\,a^2x^2 - 4\,ax + 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - \left(296\,a\right.}{105\left(a^8c^4x^4 - 4\,a^7c^4x^3 + 6\,a^6c^4x^2 - 4\,a^5c^4x + a^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/105*(166*a^4*x^4 - 664*a^3*x^3 + 996*a^2*x^2 - 664*a*x + 210*(a^4*x^4 -
4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) -
 (296*a^3*x^3 - 659*a^2*x^2 + 559*a*x - 166)*sqrt(-a^2*x^2 + 1) + 166)/(a^8
*c^4*x^4 - 4*a^7*c^4*x^3 + 6*a^6*c^4*x^2 - 4*a^5*c^4*x + a^4*c^4)
```

giac [A] time = 0.20, size = 220, normalized size = 1.59

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^3c^4|a|}+\frac{2\left(\frac{1057\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{2751\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{3640\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{2170\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}+\frac{73}{}\right.}{105\,a^3c^4\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(a^3*c^4*abs(a)) + 2/105*(1057*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)/(a^2*x) - 2751*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 3640*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 2170*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)^4/(a^8*x^4) + 735*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^10*x^5) - 105*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^6) - 166)/(a^3*c^4*((sqrt(-a^2*x^
2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

maple [A] time = 0.05, size = 210, normalized size = 1.52

$$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^4a^3\sqrt{a^2}}+\frac{229\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{105c^4a^6\left(x-\frac{1}{a}\right)^2}+\frac{296\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{105c^4a^5\left(x-\frac{1}{a}\right)}+\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a}}{7c^4a^8\left(x-\frac{1}{a}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+229/105/c^4/
a^6/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+296/105/c^4/a^5/(x-1/a)*(-
a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2/7/c^4/a^8/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*
(x-1/a))^(1/2)+43/35/c^4/a^7/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [A] time = 0.64, size = 208, normalized size = 1.51

$$\frac{2\sqrt{-a^2x^2+1}}{7\left(a^8c^4x^4-4\,a^7c^4x^3+6\,a^6c^4x^2-4\,a^5c^4x+a^4c^4\right)}+\frac{43\sqrt{-a^2x^2+1}}{35\left(a^7c^4x^3-3\,a^6c^4x^2+3\,a^5c^4x-a^4c^4\right)}+\frac{229\sqrt{-a^2x^2}}{105\left(a^6c^4x^2-2\,a^5c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^8*c^4*x^4 - 4*a^7*c^4*x^3 + 6*a^6*c^4*x^2 - 4*a^5
*c^4*x + a^4*c^4) + 43/35*sqrt(-a^2*x^2 + 1)/(a^7*c^4*x^3 - 3*a^6*c^4*x^2 +
 3*a^5*c^4*x - a^4*c^4) + 229/105*sqrt(-a^2*x^2 + 1)/(a^6*c^4*x^2 - 2*a^5*c
^4*x + a^4*c^4) + 296/105*sqrt(-a^2*x^2 + 1)/(a^5*c^4*x - a^4*c^4) + arcsin
(a*x)/(a^4*c^4)
```

mupad [B] time = 0.07, size = 281, normalized size = 2.04

$$\frac{2\sqrt{1-a^2x^2}}{7\left(a^8c^4x^4-4a^7c^4x^3+6a^6c^4x^2-4a^5c^4x+a^4c^4\right)}+\frac{229\sqrt{1-a^2x^2}}{105\left(a^6c^4x^2-2a^5c^4x+a^4c^4\right)}+\frac{}{35\sqrt{-a^2}\left(a^2c^4\sqrt{-a^2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(7*(a^4*c^4 - 4*a^5*c^4*x + 6*a^6*c^4*x^2 - 4*a^7*c
^4*x^3 + a^8*c^4*x^4)) + (229*(1 - a^2*x^2)^(1/2))/(105*(a^4*c^4 - 2*a^5*c^
4*x + a^6*c^4*x^2)) + (43*(1 - a^2*x^2)^(1/2))/(35*(-a^2)^(1/2)*(a^2*c^4*(-
a^2)^(1/2) + 3*a^4*c^4*x^2*(-a^2)^(1/2) - a^5*c^4*x^3*(-a^2)^(1/2) - 3*a^3*
c^4*x*(-a^2)^(1/2))) + (296*(1 - a^2*x^2)^(1/2))/(105*(a^2*c^4*(-a^2)^(1/2)
 - a^3*c^4*x*(-a^2)^(1/2))*(-a^2)^(1/2)) + asinh(x*(-a^2)^(1/2))/(a^3*c^4*(
-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{x^3}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int\frac{ax^4}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}+}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**
2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sq
rt(-a**2*x**2 + 1)), x) + Integral(a*x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) -
 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.357 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3c^4(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a^3/c^4/(-a*x+1)^5-12/35*(-a^2*x^2+1)^(3/2)/a^3/c^4/
(-a*x+1)^4+23/105*(-a^2*x^2+1)^(3/2)/a^3/c^4/(-a*x+1)^3
```

Rubi [A] time = 0.21, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6128, 1639, 793, 659, 651}

$$\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3c^4(1-ax)^3}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^3*c^4*(1 - a*x)^5) - (12*(1 - a^2*x^2)^(3/2))/(35*
a^3*c^4*(1 - a*x)^4) + (23*(1 - a^2*x^2)^(3/2))/(105*a^3*c^4*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-acx)^4}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx\\
&= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^4(1-ax)^4}+\frac{\int \frac{\left(4a^2c^2-3a^3c^2x\right)\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx}{a^4c}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3c^4(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx}{7a^2}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3c^4(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{35a^2c}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3c^4(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3c^4(1-ax)^4}+\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3c^4(1-ax)^3}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.44

$$-\frac{(ax+1)^{3/2}\left(-23a^2x^2+10ax-2\right)}{105a^3c^4(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a*c*x)^4,x]
```

```
[Out] -1/105*((1 + a*x)^(3/2)*(-2 + 10*a*x - 23*a^2*x^2))/(a^3*c^4*(1 - a*x)^(7/2
))
```

fricas [A] time = 0.57, size = 118, normalized size = 1.22

$$\frac{2\,a^4x^4-8\,a^3x^3+12\,a^2x^2-8\,ax+\left(23\,a^3x^3+13\,a^2x^2-8\,ax+2\right)\sqrt{-a^2x^2+1}+2}{105\left(a^7c^4x^4-4\,a^6c^4x^3+6\,a^5c^4x^2-4\,a^4c^4x+a^3c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/105*(2*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 8*a*x + (23*a^3*x^3 + 13*a^2*x^
2 - 8*a*x + 2)*sqrt(-a^2*x^2 + 1) + 2)/(a^7*c^4*x^4 - 4*a^6*c^4*x^3 + 6*a^5
*c^4*x^2 - 4*a^4*c^4*x + a^3*c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 49, normalized size = 0.51

$$-\frac{\left(23a^2x^2-10ax+2\right)(ax+1)^2}{105\,(ax-1)^3\,c^4\sqrt{-a^2x^2+1}\;a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^4,x)
```

[Out] -1/105*(23*a^2*x^2-10*a*x+2)*(a*x+1)^2/(a*x-1)^3/c^4/(-a^2*x^2+1)^(1/2)/a^3

maxima [B] time = 0.41, size = 197, normalized size = 2.03

$$\frac{2\sqrt{-a^2x^2+1}}{7\left(a^7c^4x^4-4a^6c^4x^3+6a^5c^4x^2-4a^4c^4x+a^3c^4\right)}+\frac{29\sqrt{-a^2x^2+1}}{35\left(a^6c^4x^3-3a^5c^4x^2+3a^4c^4x-a^3c^4\right)}+\frac{82\sqrt{-a^2x^2+1}}{105\left(a^5c^4x^2-2a^4c^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^7*c^4*x^4 - 4*a^6*c^4*x^3 + 6*a^5*c^4*x^2 - 4*a^4
*c^4*x + a^3*c^4) + 29/35*sqrt(-a^2*x^2 + 1)/(a^6*c^4*x^3 - 3*a^5*c^4*x^2 +
 3*a^4*c^4*x - a^3*c^4) + 82/105*sqrt(-a^2*x^2 + 1)/(a^5*c^4*x^2 - 2*a^4*c^
4*x + a^3*c^4) + 23/105*sqrt(-a^2*x^2 + 1)/(a^4*c^4*x - a^3*c^4)
```

mupad [B] time = 0.06, size = 347, normalized size = 3.58

$$\frac{2\sqrt{1-a^2x^2}}{7\left(a^7c^4x^4-4a^6c^4x^3+6a^5c^4x^2-4a^4c^4x+a^3c^4\right)}+\frac{4\sqrt{1-a^2x^2}}{3\left(a^5c^4x^2-2a^4c^4x+a^3c^4\right)}+\frac{4a\sqrt{1-a^2x^2}}{35\left(a^6c^4x^2-2a^5c^4x+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(7*(a^3*c^4 - 4*a^4*c^4*x + 6*a^5*c^4*x^2 - 4*a^6*c
^4*x^3 + a^7*c^4*x^4)) + (4*(1 - a^2*x^2)^(1/2))/(3*(a^3*c^4 - 2*a^4*c^4*x
+ a^5*c^4*x^2)) + (4*a*(1 - a^2*x^2)^(1/2))/(35*(a^4*c^4 - 2*a^5*c^4*x + a^
6*c^4*x^2)) + (23*(1 - a^2*x^2)^(1/2))/(105*(a*c^4*(-a^2)^(1/2) - a^2*c^4*x
*(-a^2)^(1/2))*(-a^2)^(1/2)) - (2*a^2*(1 - a^2*x^2)^(1/2))/(3*(a^5*c^4 - 2*
a^6*c^4*x + a^7*c^4*x^2)) + (29*(1 - a^2*x^2)^(1/2))/(35*(-a^2)^(1/2)*(a*c^
4*(-a^2)^(1/2) + 3*a^3*c^4*x^2*(-a^2)^(1/2) - a^4*c^4*x^3*(-a^2)^(1/2) - 3*
a^2*c^4*x*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}\cdots}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x**2/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**
2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sq
```

```
rt(-a**2*x**2 + 1)), x) + Integral(a*x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) -
 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.358 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{21a^2c^4(1-ax)^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a^2/c^4/(-a*x+1)^5-1/7*(-a^2*x^2+1)^(3/2)/a^2/c^4/(-
a*x+1)^4-1/21*(-a^2*x^2+1)^(3/2)/a^2/c^4/(-a*x+1)^3
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6128, 793, 659, 651}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{21a^2c^4(1-ax)^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^2*c^4*(1 - a*x)^5) - (1 - a^2*x^2)^(3/2)/(7*a^2*c^
4*(1 - a*x)^4) - (1 - a^2*x^2)^(3/2)/(21*a^2*c^4*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{(c-acx)^4}\,dx &= c\int \frac{x\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5} - \frac{5\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx}{7a}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5} - \frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^4} - \frac{\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{7ac}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^5} - \frac{\left(1-a^2x^2\right)^{3/2}}{7a^2c^4(1-ax)^4} - \frac{\left(1-a^2x^2\right)^{3/2}}{21a^2c^4(1-ax)^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.43

$$-\frac{(ax+1)^{3/2}\left(a^2x^2-5ax+1\right)}{21a^2c^4(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a*c*x)^4,x]
[Out] -1/21*((1 + a*x)^(3/2)*(1 - 5*a*x + a^2*x^2))/(a^2*c^4*(1 - a*x)^(7/2))
```

fricas [A] time = 0.67, size = 116, normalized size = 1.20

$$-\frac{a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+\left(a^3x^3-4\,a^2x^2-4\,ax+1\right)\sqrt{-a^2x^2+1}+1}{21\left(a^6c^4x^4-4\,a^5c^4x^3+6\,a^4c^4x^2-4\,a^3c^4x+a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/21*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + (a^3*x^3 - 4*a^2*x^2 - 4*a
*x + 1)*sqrt(-a^2*x^2 + 1) + 1)/(a^6*c^4*x^4 - 4*a^5*c^4*x^3 + 6*a^4*c^4*x^
2 - 4*a^3*c^4*x + a^2*c^4)
```

giac [A] time = 0.51, size = 148, normalized size = 1.53

$$\frac{2\left(\frac{7\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}+\frac{28\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{7\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}+\frac{21\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5}{a^{10}x^5}-1\right)}{21\,ac^4\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 2/21*(7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 28*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^3/(a^6*x^3) - 7*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 21*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^10*x^5) - 1)/(a*c^4*((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

maple [A] time = 0.03, size = 48, normalized size = 0.49

$$\frac{\left(a^2x^2-5ax+1\right)(ax+1)^2}{21\,(ax-1)^3\,c^4\sqrt{-a^2x^2+1}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^4,x)
```

```
[Out] 1/21*(a^2*x^2-5*a*x+1)*(a*x+1)^2/(a*x-1)^3/c^4/(-a^2*x^2+1)^(1/2)/a^2
```

maxima [B] time = 0.43, size = 197, normalized size = 2.03

$$\frac{2\sqrt{-a^2x^2+1}}{7\left(a^6c^4x^4-4\,a^5c^4x^3+6\,a^4c^4x^2-4\,a^3c^4x+a^2c^4\right)}+\frac{3\sqrt{-a^2x^2+1}}{7\left(a^5c^4x^3-3\,a^4c^4x^2+3\,a^3c^4x-a^2c^4\right)}+\frac{\sqrt{-a^2x^2+1}}{21\left(a^4c^4x^2-2\,a^3c^4x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^6*c^4*x^4 - 4*a^5*c^4*x^3 + 6*a^4*c^4*x^2 - 4*a^3
*c^4*x + a^2*c^4) + 3/7*sqrt(-a^2*x^2 + 1)/(a^5*c^4*x^3 - 3*a^4*c^4*x^2 + 3
```

```
*a^3*c^4*x - a^2*c^4) + 1/21*sqrt(-a^2*x^2 + 1)/(a^4*c^4*x^2 - 2*a^3*c^4*x
+ a^2*c^4) - 1/21*sqrt(-a^2*x^2 + 1)/(a^3*c^4*x - a^2*c^4)
```

mupad [B] time = 0.81, size = 295, normalized size = 3.04

$$\frac{2\sqrt{1-a^2x^2}}{7\left(a^6c^4x^4-4a^5c^4x^3+6a^4c^4x^2-4a^3c^4x+a^2c^4\right)}-\frac{\sqrt{1-a^2x^2}}{15\left(a^4c^4x^2-2a^3c^4x+a^2c^4\right)}-\frac{\sqrt{1-a^2x^2}}{21\left(c^4\sqrt{-a^2}-ac^4x\sqrt{-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(7*(a^2*c^4 - 4*a^3*c^4*x + 6*a^4*c^4*x^2 - 4*a^5*c
^4*x^3 + a^6*c^4*x^4)) - (1 - a^2*x^2)^(1/2)/(15*(a^2*c^4 - 2*a^3*c^4*x + a
^4*c^4*x^2)) - (1 - a^2*x^2)^(1/2)/(21*(c^4*(-a^2)^(1/2) - a*c^4*x*(-a^2)^(
1/2))*(-a^2)^(1/2)) + (4*a^2*(1 - a^2*x^2)^(1/2))/(35*(a^4*c^4 - 2*a^5*c^4*
x + a^6*c^4*x^2)) + (3*(1 - a^2*x^2)^(1/2))/(7*(-a^2)^(1/2)*(c^4*(-a^2)^(1/
2) + 3*a^2*c^4*x^2*(-a^2)^(1/2) - a^3*c^4*x^3*(-a^2)^(1/2) - 3*a*c^4*x*(-a^
2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^2}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**2 +
 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(
-a**2*x**2 + 1)), x) + Integral(a*x**2/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*
a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*s
qrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.359 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=97

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a/c^4/(-a*x+1)^5+2/35*(-a^2*x^2+1)^(3/2)/a/c^4/(-a*x
+1)^4+2/105*(-a^2*x^2+1)^(3/2)/a/c^4/(-a*x+1)^3
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6127, 659, 651}

$$\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a*c*x)^4,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a*c^4*(1 - a*x)^5) + (2*(1 - a^2*x^2)^(3/2))/(35*a*c
^4*(1 - a*x)^4) + (2*(1 - a^2*x^2)^(3/2))/(105*a*c^4*(1 - a*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{(c-acx)^4}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^5}\,dx\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2}{7}\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^4}\,dx\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^3}\,dx}{35c}\\ &= \frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}+\frac{2\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{2\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.44

$$-\frac{(ax+1)^{3/2}\left(-2a^2x^2+10ax-23\right)}{105ac^4(1-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a*c*x)^4,x]
[Out] -1/105*((1 + a*x)^(3/2)*(-23 + 10*a*x - 2*a^2*x^2))/(a*c^4*(1 - a*x)^(7/2))
```

fricas [A] time = 0.53, size = 116, normalized size = 1.20

$$\frac{23\,a^4x^4-92\,a^3x^3+138\,a^2x^2-92\,ax+\left(2\,a^3x^3-8\,a^2x^2+13\,ax+23\right)\sqrt{-a^2x^2+1}+23}{105\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
[Out] 1/105*(23*a^4*x^4 - 92*a^3*x^3 + 138*a^2*x^2 - 92*a*x + (2*a^3*x^3 - 8*a^2*
x^2 + 13*a*x + 23)*sqrt(-a^2*x^2 + 1) + 23)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 +
6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

giac [B] time = 0.26, size = 199, normalized size = 2.05

$$-\frac{2\left(\frac{56\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{273\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{350\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{455\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}+\frac{210\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5}{a^{10}x^5}-\frac{105\left(\sqrt{-\right.}}{}\right.}{105\,c^4\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -2/105*(56*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 273*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^2/(a^4*x^2) + 350*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3)
 - 455*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 210*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^5/(a^10*x^5) - 105*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^12*x^
6) - 23)/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs(a))
```

maple [A] time = 0.03, size = 49, normalized size = 0.51

$$-\frac{\left(2a^2x^2-10ax+23\right)(ax+1)^2}{105\,(ax-1)^3\,c^4\sqrt{-a^2x^2+1}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)^2/(a*x-1)^3/c^4/(-a^2*x^2+1)^(1/2)/a
```

maxima [B] time = 0.41, size = 189, normalized size = 1.95

$$\frac{2\sqrt{-a^2x^2+1}}{7\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}+\frac{\sqrt{-a^2x^2+1}}{35\left(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4\right)}-\frac{2\sqrt{-a^2x^2+1}}{105\left(a^3c^4x^2-2\,a^2c^4x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2
*c^4*x + a*c^4) + 1/35*sqrt(-a^2*x^2 + 1)/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*
a^2*c^4*x - a*c^4) - 2/105*sqrt(-a^2*x^2 + 1)/(a^3*c^4*x^2 - 2*a^2*c^4*x +
a*c^4) + 2/105*sqrt(-a^2*x^2 + 1)/(a^2*c^4*x - a*c^4)
```

mupad [B] time = 0.00, size = 49, normalized size = 0.51

$$\frac{\sqrt{1-a^2\,x^2}\,\left(2\,a^3\,x^3-8\,a^2\,x^2+13\,a\,x+23\right)}{105\,a\,c^4\,(a\,x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(13*a*x - 8*a^2*x^2 + 2*a^3*x^3 + 23))/(105*a*c^4*(a*x
 - 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqr
t(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**
3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt
(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.360 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^4}\,dx$

Optimal. Leaf size=128

$$-\frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{166ax+105}{105c^4\sqrt{1-a^2x^2}}+\frac{83ax+35}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{16(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^4}$$

```
[Out] 16/7*(a*x+1)/c^4/(-a^2*x^2+1)^(7/2)-4/35*(-3*a*x+7)/c^4/(-a^2*x^2+1)^(5/2)+
1/105*(83*a*x+35)/c^4/(-a^2*x^2+1)^(3/2)-arctanh((-a^2*x^2+1)^(1/2))/c^4+1/
105*(166*a*x+105)/c^4/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 823, 12, 266, 63, 208}

$$-\frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{166ax+105}{105c^4\sqrt{1-a^2x^2}}+\frac{83ax+35}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{16(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a*c*x)^4),x]
```

```
[Out] (16*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) - (4*(7 - 3*a*x))/(35*c^4*(1 - a
^2*x^2)^(5/2)) + (35 + 83*a*x)/(105*c^4*(1 - a^2*x^2)^(3/2)) + (105 + 166*a
*x)/(105*c^4*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
```

```
0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x(c-acx)^4}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x(c-acx)^5}\,dx$$

$$= \frac{\int \frac{(c+acx)^5}{x\left(1-a^2x^2\right)^{9/2}}\,dx}{c^9}$$

$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{\int \frac{-7c^5-19ac^5x+35a^2c^5x^2+7a^3c^5x^3}{x\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^9}$$

$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{35c^5+83ac^5x}{x\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^9}$$

$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{105a^2c^5+166a^3c^5x}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{105a^2c^9}$$

$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{105a^4c^5}{x\sqrt{1-a^2x^2}}}{105a^4c^9}$$

$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}}{c^4}$$

$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{16(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{4(7-3ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{35+83ax}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{105+166ax}{105c^4\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{\right.}{c^4}$$

Mathematica [C] time = 0.18, size = 79, normalized size = 0.62

$$\frac{-166a^7x^7+581a^5x^5-700a^3x^3+15\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};1-a^2x^2\right)+105a^2x^2+525ax+120}{105c^4\left(1-a^2x^2\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a*c*x)^4),x]
```

```
[Out] (120 + 525*a*x + 105*a^2*x^2 - 700*a^3*x^3 + 581*a^5*x^5 - 166*a^7*x^7 + 15
*Hypergeometric2F1[-7/2, 1, -5/2, 1 - a^2*x^2])/(105*c^4*(1 - a^2*x^2)^(7/2
))
```

fricas [A] time = 0.46, size = 163, normalized size = 1.27

$$\frac{296\,a^4x^4-1184\,a^3x^3+1776\,a^2x^2-1184\,ax+105\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(166\,a\right.}{105\left(a^4c^4x^4-4\,a^3c^4x^3+6\,a^2c^4x^2-4\,ac^4x+c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(296*a^4*x^4 - 1184*a^3*x^3 + 1776*a^2*x^2 - 1184*a*x + 105*(a^4*x^4
- 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (166
*a^3*x^3 - 559*a^2*x^2 + 659*a*x - 296)*sqrt(-a^2*x^2 + 1) + 296)/(a^4*c^4*
x^4 - 4*a^3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4)
```

giac [B] time = 0.26, size = 243, normalized size = 1.90

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c^4|a|}+\frac{2\left(296\,a-\frac{1547\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{ax}+\frac{4011\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^3x^2}-\frac{5600\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^5x^3}+\frac{4760\left(\sqrt{-}\right.}{}\right.}{105\,c^4\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^4*abs(a
)) + 2/105*(296*a - 1547*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a*x) + 4011*(sqrt
(-a^2*x^2 + 1)*abs(a) + a)^2/(a^3*x^2) - 5600*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3/(a^5*x^3) + 4760*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^7*x^4) - 2205*(s
```

```
qrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^9*x^5) + 525*(sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^6/(a^11*x^6))/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*abs
(a))
```

maple [B] time = 0.05, size = 451, normalized size = 3.52

$$\frac{-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}}{a}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}+\frac{\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{7a\left(x-\frac{1}{a}\right)^4}-\frac{6a\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2}}{5a\left(x-\right.}\right.}{}}{}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-arctanh(1/(-a^2*x^2+1)^(1/2))+1/a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-1/a/(x-1
/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2/a^3*(1/7/a/(x-1/a)^4*(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(1/2)-3/7*a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(
1/2)-2/5*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*
(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))))-1/a^2*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^
2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^4*x), x)
```

mupad [B] time = 0.83, size = 327, normalized size = 2.55

$$\frac{7\,a^2\,\sqrt{1-a^2\,x^2}}{15\,\left(a^4\,c^4\,x^2-2\,a^3\,c^4\,x+a^2\,c^4\right)}+\frac{4\,a^4\,\sqrt{1-a^2\,x^2}}{35\,\left(a^6\,c^4\,x^2-2\,a^5\,c^4\,x+a^4\,c^4\right)}+\frac{2\,a^2\,\sqrt{1-a^2\,x^2}}{7\,\left(a^6\,c^4\,x^4-4\,a^5\,c^4\,x^3+6\,a^4\,c^4\,x^2-4\,a^3\,c^4\,x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] (atan((1 - a^2*x^2)^(1/2)*1i)*1i)/c^4 + (7*a^2*(1 - a^2*x^2)^(1/2))/(15*(a^
2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) + (4*a^4*(1 - a^2*x^2)^(1/2))/(35*(a^4*
c^4 - 2*a^5*c^4*x + a^6*c^4*x^2)) + (2*a^2*(1 - a^2*x^2)^(1/2))/(7*(a^2*c^4
 - 4*a^3*c^4*x + 6*a^4*c^4*x^2 - 4*a^5*c^4*x^3 + a^6*c^4*x^4)) + (166*a*(1
- a^2*x^2)^(1/2))/(105*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2)
)/a)) + (13*a*(1 - a^2*x^2)^(1/2))/(35*(-a^2)^(1/2)*(3*c^4*x*(-a^2)^(1/2) -
 (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1/2) - 3*a*c^4*x^2*(-a^2)^(1/2)
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^4x^5\sqrt{-a^2x^2+1}-4a^3x^4\sqrt{-a^2x^2+1}+6a^2x^3\sqrt{-a^2x^2+1}-4ax^2\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^5\sqrt{-a^2x^2+1}-4a^3x^4\sqrt{-a^2x^2+1}+6a^2x^3\sqrt{-a^2x^2}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**5*sqrt(-a**2*x**2 + 1) - 4*a**3*x**4*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**3*sqrt(-a**2*x**2 + 1) - 4*a*x**2*sqrt(-a**2*x**2 + 1) +
x*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**5*sqrt(-a**2*x**2 + 1) -
4*a**3*x**4*sqrt(-a**2*x**2 + 1) + 6*a**2*x**3*sqrt(-a**2*x**2 + 1) - 4*a*x
**2*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.361 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^4}\,dx$

Optimal. Leaf size=155

$$\frac{16a(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{\sqrt{1-a^2x^2}}{c^4x}+\frac{a(719ax+525)}{105c^4\sqrt{1-a^2x^2}}+\frac{a(307ax+175)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{4a(17ax+7)}{35c^4\left(1-a^2x^2\right)^{5/2}}-\frac{5a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^4}$$

```
[Out] 16/7*a*(a*x+1)/c^4/(-a^2*x^2+1)^(7/2)+4/35*a*(17*a*x+7)/c^4/(-a^2*x^2+1)^(5
/2)+1/105*a*(307*a*x+175)/c^4/(-a^2*x^2+1)^(3/2)-5*a*arctanh((-a^2*x^2+1)^(
1/2))/c^4+1/105*a*(719*a*x+525)/c^4/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/c
^4/x
```

Rubi [A] time = 0.45, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6128, 852, 1805, 807, 266, 63, 208}

$$\frac{16a(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{\sqrt{1-a^2x^2}}{c^4x}+\frac{a(719ax+525)}{105c^4\sqrt{1-a^2x^2}}+\frac{a(307ax+175)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{4a(17ax+7)}{35c^4\left(1-a^2x^2\right)^{5/2}}-\frac{5a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^4),x]
```

```
[Out] (16*a*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) + (4*a*(7 + 17*a*x))/(35*c^4*(
1 - a^2*x^2)^(5/2)) + (a*(175 + 307*a*x))/(105*c^4*(1 - a^2*x^2)^(3/2)) + (
a*(525 + 719*a*x))/(105*c^4*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(c^4*x)
- (5*a*ArcTanh[Sqrt[1 - a^2*x^2]])/c^4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-acx)^4}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^2(c-acx)^5}\,dx$$

$$= \frac{\int \frac{(c+acx)^5}{x^2\left(1-a^2x^2\right)^{9/2}}\,dx}{c^9}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{\int \frac{-7c^5-35ac^5x-61a^2c^5x^2+7a^3c^5x^3}{x^2\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^9}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{35c^5+175ac^5x+272a^2c^5x^2}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^9}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-105c^5-525ac^5x-614a^2c^5x^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{105c^9}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{105c^5+52}{x^2\sqrt{1-a}}}{105}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^4x}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^4x}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^4x}$$

$$= \frac{16a(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a(7+17ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a(175+307ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a(525+719ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{c^4x}$$

Mathematica [A] time = 0.07, size = 109, normalized size = 0.70

$$\frac{824a^5x^5-1947a^4x^4+485a^3x^3+1812a^2x^2-525ax(ax-1)^3\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-1339ax+105}{105c^4x(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a*c*x)^4),x]
```

```
[Out] (105 - 1339*a*x + 1812*a^2*x^2 + 485*a^3*x^3 - 1947*a^4*x^4 + 824*a^5*x^5 -
 525*a*x*(-1 + a*x)^3*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(105*c^
4*x*(-1 + a*x)^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.48, size = 188, normalized size = 1.21

$$\frac{1024\,a^5x^5 - 4096\,a^4x^4 + 6144\,a^3x^3 - 4096\,a^2x^2 + 1024\,ax + 525\left(a^5x^5 - 4\,a^4x^4 + 6\,a^3x^3 - 4\,a^2x^2 + ax\right)\log\left(\frac{\sqrt{-}}{}\right.}{105\left(a^4c^4x^5 - 4\,a^3c^4x^4 + 6\,a^2c^4x^3 - 4\,ac^4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/105*(1024*a^5*x^5 - 4096*a^4*x^4 + 6144*a^3*x^3 - 4096*a^2*x^2 + 1024*a*x
 + 525*(a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2 + a*x)*log((sqrt(-a^2*x
^2 + 1) - 1)/x) - (824*a^4*x^4 - 2771*a^3*x^3 + 3256*a^2*x^2 - 1444*a*x + 1
05)*sqrt(-a^2*x^2 + 1))/(a^4*c^4*x^5 - 4*a^3*c^4*x^4 + 6*a^2*c^4*x^3 - 4*a*
c^4*x^2 + c^4*x)
```

giac [B] time = 0.23, size = 323, normalized size = 2.08

$$-\frac{5\,a^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c^4|a|}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{2\,c^4x|a|}-\frac{\left(105\,a^2-\frac{4831\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{x}+\frac{24997\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^2x^2}-\frac{61131\,(}{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -5*a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^4*a
bs(a)) - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(c^4*x*abs(a)) - 1/210*(105*a^
2 - 4831*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x + 24997*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^2/(a^2*x^2) - 61131*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^4*x^3) + 8
2915*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^6*x^4) - 66325*(sqrt(-a^2*x^2 + 1
)*abs(a) + a)^5/(a^8*x^5) + 29295*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a^10*x
^6) - 5985*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/(a^12*x^7))*a^2*x/((sqrt(-a^2*
x^2 + 1)*abs(a) + a)*c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^7*ab
s(a))
```

maple [B] time = 0.05, size = 423, normalized size = 2.73

$$\frac{-5a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{\sqrt{-a^2x^2+1}}{x} + \frac{4\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2} - \frac{19\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)} + \frac{\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{7a\left(x-\frac{1}{a}\right)^4} - \frac{6a\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2 \cdots}}{5a\left(x \cdots\right.} \cdots\right.}{\cdots}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-5*a*arctanh(1/(-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/x+4/3/a/(x-1/a
)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-19/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-
1/a))^(1/2)+2/a^2*(1/7/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-3/7*a
*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/a/(x-1/a)^2
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a)
)^(1/2))))-3/a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1
/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^4x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^4*x^2), x)
```

mupad [B] time = 0.83, size = 352, normalized size = 2.27

$$\frac{26\,a^3\,\sqrt{1-a^2\,x^2}}{15\,\left(a^4\,c^4\,x^2-2\,a^3\,c^4\,x+a^2\,c^4\right)}+\frac{4\,a^5\,\sqrt{1-a^2\,x^2}}{35\,\left(a^6\,c^4\,x^2-2\,a^5\,c^4\,x+a^4\,c^4\right)}-\frac{\sqrt{1-a^2\,x^2}}{c^4\,x}+\frac{2\,a^3\,\sqrt{1-a^2\,x^2}}{7\,\left(a^6\,c^4\,x^4-4\,a^5\,c^4\,x^3+6\,a^4\,c^4\,x^2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] (26*a^3*(1 - a^2*x^2)^(1/2))/(15*(a^2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) + (
4*a^5*(1 - a^2*x^2)^(1/2))/(35*(a^4*c^4 - 2*a^5*c^4*x + a^6*c^4*x^2)) - (1
- a^2*x^2)^(1/2)/(c^4*x) + (a*atan((1 - a^2*x^2)^(1/2)*1i)*5i)/c^4 + (2*a^3
*(1 - a^2*x^2)^(1/2))/(7*(a^2*c^4 - 4*a^3*c^4*x + 6*a^4*c^4*x^2 - 4*a^5*c^4
*x^3 + a^6*c^4*x^4)) + (719*a^2*(1 - a^2*x^2)^(1/2))/(105*(-a^2)^(1/2)*(c^4
*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a)) + (27*a^2*(1 - a^2*x^2)^(1/2))/(35
*(-a^2)^(1/2)*(3*c^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-
a^2)^(1/2) - 3*a*c^4*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^4x^6\sqrt{-a^2x^2+1}-4a^3x^5\sqrt{-a^2x^2+1}+6a^2x^4\sqrt{-a^2x^2+1}-4ax^3\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^6\sqrt{-a^2x^2+1}-4a^3x^5\sqrt{-a^2x^2+1}+6a^2x^4\sqrt{-a^2x}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**6*sqrt(-a**2*x**2 + 1) - 4*a**3*x**5*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**4*sqrt(-a**2*x**2 + 1) - 4*a*x**3*sqrt(-a**2*x**2 + 1) +
x**2*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**6*sqrt(-a**2*x**2 + 1)
 - 4*a**3*x**5*sqrt(-a**2*x**2 + 1) + 6*a**2*x**4*sqrt(-a**2*x**2 + 1) - 4*
a*x**3*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.362 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^4}\,dx$

Optimal. Leaf size=192

$$\frac{a^2(1867ax+1470)}{105c^4\sqrt{1-a^2x^2}}+\frac{a^2(671ax+455)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{4a^2(31ax+21)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{16a^2(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{5a\sqrt{1-a^2x^2}}{c^4x}-\frac{\sqrt{1-a^2x^2}}{2c^4x^2}-\frac{29a^2}{}$$

```
[Out] 16/7*a^2*(a*x+1)/c^4/(-a^2*x^2+1)^(7/2)+4/35*a^2*(31*a*x+21)/c^4/(-a^2*x^2+
1)^(5/2)+1/105*a^2*(671*a*x+455)/c^4/(-a^2*x^2+1)^(3/2)-29/2*a^2*arctanh((-
a^2*x^2+1)^(1/2))/c^4+1/105*a^2*(1867*a*x+1470)/c^4/(-a^2*x^2+1)^(1/2)-1/2*
(-a^2*x^2+1)^(1/2)/c^4/x^2-5*a*(-a^2*x^2+1)^(1/2)/c^4/x
```

Rubi [A] time = 0.52, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6128, 852, 1805, 1807, 807, 266, 63, 208}

$$\frac{a^2(1867ax+1470)}{105c^4\sqrt{1-a^2x^2}}+\frac{a^2(671ax+455)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{4a^2(31ax+21)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{16a^2(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{5a\sqrt{1-a^2x^2}}{c^4x}-\frac{\sqrt{1-a^2x^2}}{2c^4x^2}-\frac{29a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^4),x]
```

```
[Out] (16*a^2*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) + (4*a^2*(21 + 31*a*x))/(35*
c^4*(1 - a^2*x^2)^(5/2)) + (a^2*(455 + 671*a*x))/(105*c^4*(1 - a^2*x^2)^(3/
2)) + (a^2*(1470 + 1867*a*x))/(105*c^4*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^
2]/(2*c^4*x^2) - (5*a*Sqrt[1 - a^2*x^2])/(c^4*x) - (29*a^2*ArcTanh[Sqrt[1 -
 a^2*x^2]])/(2*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
```

```
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-acx)^4}\,dx = c\int \frac{\sqrt{1-a^2x^2}}{x^3(c-acx)^5}\,dx$$

$$= \frac{\int \frac{(c+acx)^5}{x^3\left(1-a^2x^2\right)^{9/2}}\,dx}{c^9}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{\int \frac{-7c^5-35ac^5x-77a^2c^5x^2-89a^3c^5x^3}{x^3\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^9}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{35c^5+175ac^5x+420a^2c^5x^2+496a^3c^5x^3}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^9}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{-105c^5-525ac^5x-1365a^2c^5x^2-13}{x^3\left(1-a^2x^2\right)^{3/2}}}{105c^9}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} + \int \frac{105}{}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-}}{2c^4}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-}}{2c^4}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-}}{2c^4}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-}}{2c^4}$$

$$= \frac{16a^2(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} + \frac{4a^2(21+31ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2(455+671ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^2(1470+1867ax)}{105c^4\sqrt{1-a^2x^2}} - \frac{\sqrt{1-}}{2c^4}$$

Mathematica [A] time = 0.07, size = 121, normalized size = 0.63

$$\frac{4784a^6x^6-11307a^5x^5+2825a^4x^4+10512a^3x^3-7774a^2x^2-3045a^2x^2(ax-1)^3\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{210c^4x^2(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a*c*x)^4),x]
```

```
[Out] (105 + 735*a*x - 7774*a^2*x^2 + 10512*a^3*x^3 + 2825*a^4*x^4 - 11307*a^5*x^
5 + 4784*a^6*x^6 - 3045*a^2*x^2*(-1 + a*x)^3*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt
[1 - a^2*x^2]])/(210*c^4*x^2*(-1 + a*x)^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.47, size = 206, normalized size = 1.07

$$\frac{4834\,a^6x^6-19336\,a^5x^5+29004\,a^4x^4-19336\,a^3x^3+4834\,a^2x^2+3045\left(a^6x^6-4\,a^5x^5+6\,a^4x^4-4\,a^3x^3+a^2x^2\right)\cdots}{210\left(a^4c^4x^6-4\,a^3c^4x^5+6\,a^2c^4x^4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/210*(4834*a^6*x^6 - 19336*a^5*x^5 + 29004*a^4*x^4 - 19336*a^3*x^3 + 4834*
a^2*x^2 + 3045*(a^6*x^6 - 4*a^5*x^5 + 6*a^4*x^4 - 4*a^3*x^3 + a^2*x^2)*log(
(sqrt(-a^2*x^2 + 1) - 1)/x) - (4784*a^5*x^5 - 16091*a^4*x^4 + 18916*a^3*x^3
 - 8404*a^2*x^2 + 630*a*x + 105)*sqrt(-a^2*x^2 + 1))/(a^4*c^4*x^6 - 4*a^3*c
^4*x^5 + 6*a^2*c^4*x^4 - 4*a*c^4*x^3 + c^4*x^2)
```

giac [B] time = 0.52, size = 392, normalized size = 2.04

$$-\frac{29\,a^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,c^4|a|}-\frac{\left(105\,a^3+\frac{1365\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x}-\frac{51167\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{ax^2}+\frac{260729\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^3x^3}-\cdots\right.}{840\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -29/2*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c^
4*abs(a)) - 1/840*(105*a^3 + 1365*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 511
67*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a*x^2) + 260729*(sqrt(-a^2*x^2 + 1)*a
```

```
bs(a) + a)^3/(a^3*x^3) - 621537*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^5*x^4)
 + 826175*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/(a^7*x^5) - 642005*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^6/(a^9*x^6) + 274995*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/
(a^11*x^7) - 52500*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^8/(a^13*x^8))*a^4*x^2/((
sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^4*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*
x) - 1)^7*abs(a)) - 1/8*(20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*c^4*abs(a)/x
+ (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^4*abs(a)/(a*x^2))/(a^2*c^8)
```

maple [B] time = 0.05, size = 397, normalized size = 2.07

$$\frac{-\frac{29a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{5a\sqrt{-a^2x^2+1}}{x}+11a\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}\right)-\frac{14a\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}-}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-29/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-5*a*(-a^2*x^2+1)^(1/2)/x+11*
a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-
1/a)^2-2*a*(x-1/a))^(1/2))-14*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-
1/2*(-a^2*x^2+1)^(1/2)/x^2+2/a*(1/7/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a)
)^(1/2)-3/7*a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/
3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^
2-2*a*(x-1/a))^(1/2))))-1/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,(acx-c)^4x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^4,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(a*c*x - c)^4*x^3), x)
```

mupad [B] time = 0.84, size = 375, normalized size = 1.95

$$\frac{11\,a^4\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^4\,x^2-2\,a^3\,c^4\,x+a^2\,c^4\right)}+\frac{4\,a^6\,\sqrt{1-a^2\,x^2}}{35\,\left(a^6\,c^4\,x^2-2\,a^5\,c^4\,x+a^4\,c^4\right)}-\frac{\sqrt{1-a^2\,x^2}}{2\,c^4\,x^2}+\frac{2\,a^4\,\sqrt{1-a^2\,x^2}}{7\,\left(a^6\,c^4\,x^4-4\,a^5\,c^4\,x^3+6\,a^4\,c^4\,x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^4),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*29i)/(2*c^4) + (11*a^4*(1 - a^2*x^2)^(1/2
))/(3*(a^2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) + (4*a^6*(1 - a^2*x^2)^(1/2))/
(35*(a^4*c^4 - 2*a^5*c^4*x + a^6*c^4*x^2)) - (1 - a^2*x^2)^(1/2)/(2*c^4*x^2
) + (2*a^4*(1 - a^2*x^2)^(1/2))/(7*(a^2*c^4 - 4*a^3*c^4*x + 6*a^4*c^4*x^2 -
 4*a^5*c^4*x^3 + a^6*c^4*x^4)) - (5*a*(1 - a^2*x^2)^(1/2))/(c^4*x) + (1867*
a^3*(1 - a^2*x^2)^(1/2))/(105*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2) - (c^4*(-a^2
)^(1/2))/a)) + (41*a^3*(1 - a^2*x^2)^(1/2))/(35*(-a^2)^(1/2)*(3*c^4*x*(-a^2
)^(1/2) - (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1/2) - 3*a*c^4*x^2*(-a
^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^4x^7\sqrt{-a^2x^2+1}-4a^3x^6\sqrt{-a^2x^2+1}+6a^2x^5\sqrt{-a^2x^2+1}-4ax^4\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^7\sqrt{-a^2x^2+1}-4a^3x^6\sqrt{-a^2x^2+1}+6a^2x^5\sqrt{-a^2x^2}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(a*x/(a**4*x**7*sqrt(-a**2*x**2 + 1) - 4*a**3*x**6*sqrt(-a**2*x**2
 + 1) + 6*a**2*x**5*sqrt(-a**2*x**2 + 1) - 4*a*x**4*sqrt(-a**2*x**2 + 1) +
x**3*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**7*sqrt(-a**2*x**2 + 1)
 - 4*a**3*x**6*sqrt(-a**2*x**2 + 1) + 6*a**2*x**5*sqrt(-a**2*x**2 + 1) - 4*
a*x**4*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.363 $\int e^{\tanh^{-1}(x)}x(1+x)\,dx$

Optimal. Leaf size=61

$$-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{1}{3}\sqrt{1-x}\,(x+1)^{3/2}-\sqrt{1-x}\,\sqrt{x+1}\,+\sin^{-1}(x)$$

```
[Out] arcsin(x)-1/3*(1+x)^(3/2)*(1-x)^(1/2)-1/3*(1+x)^(5/2)*(1-x)^(1/2)-(1-x)^(1/
2)*(1+x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6129, 80, 50, 41, 216}

$$-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{1}{3}\sqrt{1-x}\,(x+1)^{3/2}-\sqrt{1-x}\,\sqrt{x+1}\,+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*(1 + x),x]
```

```
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) - (Sqrt[1 - x]*(1 + x)^(3/2))/3 - (Sqrt[1 - x]*(
1 + x)^(5/2))/3 + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(x)}x(1+x)\,dx &= \int \frac{x(1+x)^{3/2}}{\sqrt{1-x}}\,dx\\ &= -\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{2}{3}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx\\ &= -\frac{1}{3}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.69

$$-\frac{1}{3}\sqrt{1-x^2}\,\left(x^2+3x+5\right)-2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*x*(1 + x),x]
[Out] -1/3*(Sqrt[1 - x^2]*(5 + 3*x + x^2)) - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

fricas [A] time = 0.60, size = 38, normalized size = 0.62

$$-\frac{1}{3}\left(x^2+3\,x+5\right)\sqrt{-x^2+1}\,-2\,\arctan\left(\frac{\sqrt{-x^2+1}\,-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -1/3*(x^2 + 3*x + 5)*sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.18, size = 21, normalized size = 0.34

$$-\frac{1}{3}((x+3)x+5)\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/3*((x + 3)*x + 5)*sqrt(-x^2 + 1) + arcsin(x)
```

maple [A] time = 0.03, size = 41, normalized size = 0.67

$$-\frac{x^2\sqrt{-x^2+1}}{3}-\frac{5\sqrt{-x^2+1}}{3}-x\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^2+1)^(1/2)*x,x)
```

```
[Out] -1/3*x^2*(-x^2+1)^(1/2)-5/3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)+arcsin(x)
```

maxima [A] time = 0.44, size = 40, normalized size = 0.66

$$-\frac{1}{3}\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}\,x-\frac{5}{3}\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - sqrt(-x^2 + 1)*x - 5/3*sqrt(-x^2 + 1) + arcsin(x)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.36

$$\operatorname{asin}(x)-\sqrt{1-x^2}\left(\frac{x^2}{3}+x+\frac{5}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

```
[Out] asin(x) - (1 - x^2)^(1/2)*(x + x^2/3 + 5/3)
```

sympy [A] time = 0.32, size = 37, normalized size = 0.61

$$-\frac{x^2\sqrt{1-x^2}}{3}-x\sqrt{1-x^2}-\frac{5\sqrt{1-x^2}}{3}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**2+1)**(1/2)*x,x)
[Out] -x**2*sqrt(1 - x**2)/3 - x*sqrt(1 - x**2) - 5*sqrt(1 - x**2)/3 + asin(x)
```

3.364 $\int e^{\tanh^{-1}(x)}(1+x)\,dx$

Optimal. Leaf size=47

$$-\frac{1}{2}\sqrt{1-x}\,(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\,\sqrt{x+1}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] 3/2*arcsin(x)-1/2*(1+x)^(3/2)*(1-x)^(1/2)-3/2*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6129, 50, 41, 216}

$$-\frac{1}{2}\sqrt{1-x}\,(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\,\sqrt{x+1}+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 + x),x]
```

```
[Out] (-3*Sqrt[1 - x]*Sqrt[1 + x])/2 - (Sqrt[1 - x]*(1 + x)^(3/2))/2 + (3*ArcSin[
x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(x)}(1+x)\,dx &= \int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\
&= -\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.79

$$-\frac{1}{2}\sqrt{1-x^2}\,(x+4)-3\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 + x),x]
[Out] -1/2*((4 + x)*Sqrt[1 - x^2]) - 3*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

fricas [A] time = 0.44, size = 33, normalized size = 0.70

$$-\frac{1}{2}\sqrt{-x^2+1}\,(x+4)-3\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) - 3*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.19, size = 19, normalized size = 0.40

$$-\frac{1}{2}\sqrt{-x^2+1}\,(x+4)+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) + 3/2*arcsin(x)
```

maple [A] time = 0.03, size = 29, normalized size = 0.62

$$-\frac{x\sqrt{-x^2+1}}{2}+\frac{3\arcsin(x)}{2}-2\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*x*(-x^2+1)^(1/2)+3/2*arcsin(x)-2*(-x^2+1)^(1/2)
```

maxima [A] time = 0.47, size = 28, normalized size = 0.60

$$-\frac{1}{2}\sqrt{-x^2+1}\,x-2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.45

$$\frac{3\,\mathrm{asin}(x)}{2}-\left(\frac{x}{2}+2\right)\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(1 - x^2)^(1/2),x)
```

```
[Out] (3*asin(x))/2 - (x/2 + 2)*(1 - x^2)^(1/2)
```

sympy [A] time = 0.18, size = 27, normalized size = 0.57

$$-\frac{x\sqrt{1-x^2}}{2}-2\sqrt{1-x^2}+\frac{3\,\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x*sqrt(1 - x**2)/2 - 2*sqrt(1 - x**2) + 3*asin(x)/2
```

3.365 $\int e^{\tanh^{-1}(x)} x(1+x)^2 \, dx$

Optimal. Leaf size=87

$$-\frac{1}{4}\sqrt{1-x}\,(x+1)^{7/2}-\frac{1}{4}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{8}\sqrt{1-x}\,(x+1)^{3/2}-\frac{15}{8}\sqrt{1-x}\,\sqrt{x+1}+\frac{15}{8}\sin^{-1}(x)$$

```
[Out] 15/8*arcsin(x)-5/8*(1+x)^(3/2)*(1-x)^(1/2)-1/4*(1+x)^(5/2)*(1-x)^(1/2)-1/4*
(1+x)^(7/2)*(1-x)^(1/2)-15/8*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6129, 80, 50, 41, 216}

$$-\frac{1}{4}\sqrt{1-x}\,(x+1)^{7/2}-\frac{1}{4}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{8}\sqrt{1-x}\,(x+1)^{3/2}-\frac{15}{8}\sqrt{1-x}\,\sqrt{x+1}+\frac{15}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*(1 + x)^2,x]
```

```
[Out] (-15*Sqrt[1 - x]*Sqrt[1 + x])/8 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/8 - (Sqrt[1
 - x]*(1 + x)^(5/2))/4 - (Sqrt[1 - x]*(1 + x)^(7/2))/4 + (15*ArcSin[x])/8
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(x)}x(1+x)^2\,dx &= \int \frac{x(1+x)^{5/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{4}\sqrt{1-x}\,(1+x)^{7/2}+\frac{3}{4}\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{4}\sqrt{1-x}\,(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}\,(1+x)^{7/2}+\frac{5}{4}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{5}{8}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{4}\sqrt{1-x}\,(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}\,(1+x)^{7/2}+\frac{15}{8}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx\\
&= -\frac{15}{8}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{8}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{4}\sqrt{1-x}\,(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}\,(1+x)^{7/2}+\\
&= -\frac{15}{8}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{8}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{4}\sqrt{1-x}\,(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}\,(1+x)^{7/2}+\\
&= -\frac{15}{8}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{8}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{4}\sqrt{1-x}\,(1+x)^{5/2}-\frac{1}{4}\sqrt{1-x}\,(1+x)^{7/2}+
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.59

$$\frac{1}{8}\left(-\sqrt{1-x^2}\,\left(2x^3+8x^2+15x+24\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*x*(1 + x)^2,x]
```

```
[Out] (-(Sqrt[1 - x^2]*(24 + 15*x + 8*x^2 + 2*x^3)) - 30*ArcSin[Sqrt[1 - x]/Sqrt[
2]])/8
```

fricas [A] time = 0.47, size = 45, normalized size = 0.52

$$-\frac{1}{8}\left(2\,x^{3}+8\,x^{2}+15\,x+24\right)\sqrt{-x^{2}+1}-\frac{15}{4}\,\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^3 + 8*x^2 + 15*x + 24)*sqrt(-x^2 + 1) - 15/4*arctan((sqrt(-x^2 +
1) - 1)/x)
```

giac [A] time = 0.24, size = 28, normalized size = 0.32

$$-\frac{1}{8}\,((2\,(x+4)x+15)x+24)\sqrt{-x^{2}+1}+\frac{15}{8}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/8*((2*(x + 4)*x + 15)*x + 24)*sqrt(-x^2 + 1) + 15/8*arcsin(x)
```

maple [A] time = 0.03, size = 57, normalized size = 0.66

$$-\frac{x^{3}\sqrt{-x^{2}+1}}{4}-\frac{15x\sqrt{-x^{2}+1}}{8}+\frac{15\arcsin(x)}{8}-x^{2}\sqrt{-x^{2}+1}-3\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^3/(-x^2+1)^(1/2)*x,x)
```

```
[Out] -1/4*x^3*(-x^2+1)^(1/2)-15/8*x*(-x^2+1)^(1/2)+15/8*arcsin(x)-x^2*(-x^2+1)^(
1/2)-3*(-x^2+1)^(1/2)
```

maxima [A] time = 0.40, size = 56, normalized size = 0.64

$$-\frac{1}{4}\,\sqrt{-x^{2}+1}\,x^{3}-\sqrt{-x^{2}+1}\,x^{2}-\frac{15}{8}\,\sqrt{-x^{2}+1}\,x-3\,\sqrt{-x^{2}+1}+\frac{15}{8}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 - sqrt(-x^2 + 1)*x^2 - 15/8*sqrt(-x^2 + 1)*x - 3*sq
rt(-x^2 + 1) + 15/8*arcsin(x)
```

mupad [B] time = 0.03, size = 29, normalized size = 0.33

$$\frac{15\,\mathrm{asin}(x)}{8}-\sqrt{1-x^2}\,\left(\frac{x^3}{4}+x^2+\frac{15\,x}{8}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^3)/(1 - x^2)^(1/2),x)
```

```
[Out] (15*asin(x))/8 - (1 - x^2)^(1/2)*((15*x)/8 + x^2 + x^3/4 + 3)
```

sympy [A] time = 0.60, size = 54, normalized size = 0.62

$$-\frac{x^3\sqrt{1-x^2}}{4}-x^2\sqrt{1-x^2}-\frac{15x\sqrt{1-x^2}}{8}-3\sqrt{1-x^2}+\frac{15\,\mathrm{asin}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/(-x**2+1)**(1/2)*x,x)
```

```
[Out] -x**3*sqrt(1 - x**2)/4 - x**2*sqrt(1 - x**2) - 15*x*sqrt(1 - x**2)/8 - 3*sq
rt(1 - x**2) + 15*asin(x)/8
```

3.366 $\int e^{\tanh^{-1}(x)}(1+x)^2\,dx$

Optimal. Leaf size=67

$$-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}\,(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{5}{2}\sin^{-1}(x)$$

```
[Out] 5/2*arcsin(x)-5/6*(1+x)^(3/2)*(1-x)^(1/2)-1/3*(1+x)^(5/2)*(1-x)^(1/2)-5/2*(
1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6129, 50, 41, 216}

$$-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}\,(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 + x)^2,x]
```

```
[Out] (-5*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/6 - (Sqrt[1
- x]*(1 + x)^(5/2))/3 + (5*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(x)}(1+x)^2\,dx &= \int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{3}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx\\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.66

$$-\frac{1}{6}\sqrt{1-x^2}\,\left(2x^2+9x+22\right)-5\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 + x)^2,x]
[Out] -1/6*(Sqrt[1 - x^2]*(22 + 9*x + 2*x^2)) - 5*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

fricas [A] time = 0.43, size = 40, normalized size = 0.60

$$-\frac{1}{6}\left(2\,x^2+9\,x+22\right)\sqrt{-x^2+1}\,-5\,\arctan\left(\frac{\sqrt{-x^2+1}\,-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 + 9*x + 22)*sqrt(-x^2 + 1) - 5*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.18, size = 25, normalized size = 0.37

$$-\frac{1}{6}\left((2x+9)x+22\right)\sqrt{-x^2+1}+\frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/6*((2*x + 9)*x + 22)*sqrt(-x^2 + 1) + 5/2*arcsin(x)
```

maple [A] time = 0.03, size = 43, normalized size = 0.64

$$-\frac{x^2\sqrt{-x^2+1}}{3}-\frac{11\sqrt{-x^2+1}}{3}-\frac{3x\sqrt{-x^2+1}}{2}+\frac{5\arcsin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^3/(-x^2+1)^(1/2),x)
[Out] -1/3*x^2*(-x^2+1)^(1/2)-11/3*(-x^2+1)^(1/2)-3/2*x*(-x^2+1)^(1/2)+5/2*arcsin
(x)
```

maxima [A] time = 0.49, size = 42, normalized size = 0.63

$$-\frac{1}{3}\sqrt{-x^2+1}\,x^2-\frac{3}{2}\sqrt{-x^2+1}\,x-\frac{11}{3}\sqrt{-x^2+1}+\frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x - 11/3*sqrt(-x^2 + 1) + 5/2*
arcsin(x)
```

mupad [B] time = 0.03, size = 26, normalized size = 0.39

$$\frac{5\,\mathrm{asin}(x)}{2}-\sqrt{1-x^2}\,\left(\frac{x^2}{3}+\frac{3\,x}{2}+\frac{11}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^3/(1 - x^2)^(1/2),x)
[Out] (5*asin(x))/2 - (1 - x^2)^(1/2)*((3*x)/2 + x^2/3 + 11/3)
```

sympy [A] time = 0.33, size = 44, normalized size = 0.66

$$-\frac{x^2\sqrt{1-x^2}}{3}-\frac{3x\sqrt{1-x^2}}{2}-\frac{11\sqrt{1-x^2}}{3}+\frac{5\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/(-x**2+1)**(1/2),x)
```

```
[Out] -x**2*sqrt(1 - x**2)/3 - 3*x*sqrt(1 - x**2)/2 - 11*sqrt(1 - x**2)/3 + 5*asi
n(x)/2
```

3.367 $\int \frac{e^{\tanh^{-1}(x)}x}{1+x}\,dx$

Optimal. Leaf size=18

$$-\sqrt{1-x}\,\sqrt{x+1}$$

```
[Out] -(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6129, 74}

$$-\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/(1 + x),x]
```

```
[Out] -(Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}x}{1+x}\,dx &= \int \frac{x}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{1+x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.72

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/(1 + x),x]
```

```
[Out] -Sqrt[1 - x^2]
```

fricas [C] time = 0.49, size = 11, normalized size = 0.61

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)
```

giac [C] time = 0.29, size = 11, normalized size = 0.61

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)
```

maple [A] time = 0.03, size = 17, normalized size = 0.94

$$\frac{(1+x)(-1+x)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)*x,x)
```

```
[Out] (1+x)*(-1+x)/(-x^2+1)^(1/2)
```

maxima [C] time = 0.39, size = 11, normalized size = 0.61

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)
```

mupad [B] time = 0.11, size = 11, normalized size = 0.61

$$-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^2)^(1/2),x)
```

```
[Out] -(1 - x^2)^(1/2)
```

sympy [A] time = 0.12, size = 8, normalized size = 0.44

$$-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**2)
```

3.368 $\int \frac{e^{\tanh^{-1}(x)}}{1+x} dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

[Out] arcsin(x)

Rubi [A] time = 0.02, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6129, 41, 216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[x]/(1 + x),x]

[Out] ArcSin[x]

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(x)}}{1+x}\,dx = \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \sin^{-1}(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/(1 + x),x]
[Out] ArcSin[x]
```

fricas [B] time = 0.45, size = 18, normalized size = 9.00

$$-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.18, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] arcsin(x)
```

maple [A] time = 0.03, size = 3, normalized size = 1.50

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2),x)
[Out] arcsin(x)
```

maxima [A] time = 0.40, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] arcsin(x)
```

mupad [B] time = 0.01, size = 2, normalized size = 1.00

$$\mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^2)^(1/2),x)
[Out] asin(x)
```

sympy [A] time = 0.12, size = 2, normalized size = 1.00

$$\mathrm{asin}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2),x)
[Out] asin(x)
```

3.369 $\int \frac{e^{\tanh^{-1}(x)}x}{(1+x)^2}\,dx$

Optimal. Leaf size=20

$$\frac{\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

```
[Out] arcsin(x)+(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6129, 78, 41, 216}

$$\frac{\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/(1 + x)^2,x]
```

```
[Out] Sqrt[1 - x]/Sqrt[1 + x] + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(x)}x}{(1+x)^2}\,dx = \int \frac{x}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx$$
$$= \frac{\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{\sqrt{1-x}}{\sqrt{1+x}} + \sin^{-1}(x)$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$\frac{\sqrt{1-x}}{\sqrt{x+1}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/(1 + x)^2,x]
```

```
[Out] Sqrt[1 - x]/Sqrt[1 + x] + ArcSin[x]
```

fricas [B] time = 0.47, size = 44, normalized size = 2.20

$$-\frac{2\,(x+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - x - \sqrt{-x^2+1} - 1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -(2*(x + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - x - sqrt(-x^2 + 1) - 1)/(x + 1
)
```

giac [A] time = 0.18, size = 24, normalized size = 1.20

$$-\frac{2}{\frac{\sqrt{-x^2+1}-1}{x} - 1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -2/((sqrt(-x^2 + 1) - 1)/x - 1) + arcsin(x)
```

maple [A] time = 0.03, size = 24, normalized size = 1.20

$$\arcsin(x) + \frac{\sqrt{-(1+x)^2 + 2x + 2}}{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-x^2+1)^(1/2)*x,x)
```

```
[Out] arcsin(x)+1/(1+x)*(-(1+x)^2+2*x+2)^(1/2)
```

maxima [A] time = 0.54, size = 18, normalized size = 0.90

$$\frac{\sqrt{-x^2+1}}{x+1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1)/(x + 1) + arcsin(x)
```

mupad [B] time = 0.79, size = 18, normalized size = 0.90

$$\operatorname{asin}(x) + \frac{\sqrt{1-x^2}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^2)^(1/2)*(x + 1)),x)
```

```
[Out] asin(x) + (1 - x^2)^(1/2)/(x + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**2+1)**(1/2)*x,x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*(x + 1)), x)
```

3.370 $\int \frac{e^{\tanh^{-1}(x)}}{(1+x)^2} dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

```
[Out] -(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6129, 37}

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}}{(1+x)^2} dx &= \int \frac{1}{\sqrt{1-x}(1+x)^{3/2}} dx \\ &= -\frac{\sqrt{1-x}}{\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/(1 + x)^2,x]
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

fricas [A] time = 0.42, size = 19, normalized size = 1.06

$$-\frac{x+\sqrt{-x^2+1}+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(x + sqrt(-x^2 + 1) + 1)/(x + 1)
```

giac [A] time = 0.16, size = 21, normalized size = 1.17

$$\frac{2}{\frac{\sqrt{-x^2+1}-1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 2/((sqrt(-x^2 + 1) - 1)/x - 1)
```

maple [A] time = 0.03, size = 14, normalized size = 0.78

$$\frac{-1+x}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-x^2+1)^(1/2),x)
[Out] (-1+x)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.41, size = 16, normalized size = 0.89

$$-\frac{\sqrt{-x^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/(x + 1)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.72

$$\frac{x-1}{\sqrt{1-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(x + 1)),x)
```

```
[Out] (x - 1)/(1 - x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x + 1)), x)
```

3.371 $\int e^{\tanh^{-1}(x)}x(1+x)^{3/2}\,dx$

Optimal. Leaf size=49

$$\frac{2}{7}(1-x)^{7/2}-2(1-x)^{5/2}+\frac{16}{3}(1-x)^{3/2}-8\sqrt{1-x}$$

```
[Out] 16/3*(1-x)^(3/2)-2*(1-x)^(5/2)+2/7*(1-x)^(7/2)-8*(1-x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6129, 77}

$$\frac{2}{7}(1-x)^{7/2}-2(1-x)^{5/2}+\frac{16}{3}(1-x)^{3/2}-8\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*(1 + x)^(3/2),x]
```

```
[Out] -8*Sqrt[1 - x] + (16*(1 - x)^(3/2))/3 - 2*(1 - x)^(5/2) + (2*(1 - x)^(7/2))
/7
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}x(1+x)^{3/2}\,dx = \int \frac{x(1+x)^2}{\sqrt{1-x}}\,dx$$
$$= \int \left(\frac{4}{\sqrt{1-x}} - 8\sqrt{1-x} + 5(1-x)^{3/2} - (1-x)^{5/2}\right)dx$$
$$= -8\sqrt{1-x} + \frac{16}{3}(1-x)^{3/2} - 2(1-x)^{5/2} + \frac{2}{7}(1-x)^{7/2}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.57

$$-\frac{2}{21}\sqrt{1-x}\left(3x^3+12x^2+23x+46\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*x*(1 + x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - x]*(46 + 23*x + 12*x^2 + 3*x^3))/21
```

fricas [A] time = 0.62, size = 31, normalized size = 0.63

$$-\frac{2\left(3x^3+12x^2+23x+46\right)\sqrt{-x^2+1}}{21\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] -2/21*(3*x^3 + 12*x^2 + 23*x + 46)*sqrt(-x^2 + 1)/sqrt(x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.03, size = 35, normalized size = 0.71

$$\frac{2(-1+x)\left(3x^3+12x^2+23x+46\right)\sqrt{1+x}}{21\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(-x^2+1)^(1/2)*x,x)
```

```
[Out] 2/21*(-1+x)*(3*x^3+12*x^2+23*x+46)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.34, size = 29, normalized size = 0.59

$$\frac{2\left(3\,x^4+9\,x^3+11\,x^2+23\,x-46\right)}{21\,\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^4 + 9*x^3 + 11*x^2 + 23*x - 46)/sqrt(-x + 1)
```

mupad [B] time = 0.92, size = 52, normalized size = 1.06

$$-\frac{\sqrt{1-x^2}\,\left(\frac{46\,x\,\sqrt{x+1}}{21}+\frac{92\,\sqrt{x+1}}{21}+\frac{8\,x^2\,\sqrt{x+1}}{7}+\frac{2\,x^3\,\sqrt{x+1}}{7}\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(5/2))/(1 - x^2)^(1/2),x)
```

```
[Out] -((1 - x^2)^(1/2)*((46*x*(x + 1)^(1/2))/21 + (92*(x + 1)^(1/2))/21 + (8*x^2
*(x + 1)^(1/2))/7 + (2*x^3*(x + 1)^(1/2))/7))/(x + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(x+1)^{\frac{5}{2}}}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(-x**2+1)**(1/2)*x,x)
```

```
[Out] Integral(x*(x + 1)**(5/2)/sqrt(-(x - 1)*(x + 1)), x)
```

3.372 $\int e^{\tanh^{-1}(x)}(1+x)^{3/2}\,dx$

Optimal. Leaf size=38

$$-\frac{2}{5}(1-x)^{5/2}+\frac{8}{3}(1-x)^{3/2}-8\sqrt{1-x}$$

```
[Out] 8/3*(1-x)^(3/2)-2/5*(1-x)^(5/2)-8*(1-x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 43}

$$-\frac{2}{5}(1-x)^{5/2}+\frac{8}{3}(1-x)^{3/2}-8\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 + x)^(3/2),x]
```

```
[Out] -8*Sqrt[1 - x] + (8*(1 - x)^(3/2))/3 - (2*(1 - x)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(x)}(1+x)^{3/2}\,dx &= \int \frac{(1+x)^2}{\sqrt{1-x}}\,dx\\ &= \int \left(\frac{4}{\sqrt{1-x}}-4\sqrt{1-x}+(1-x)^{3/2}\right)dx\\ &= -8\sqrt{1-x}+\frac{8}{3}(1-x)^{3/2}-\frac{2}{5}(1-x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.61

$$-\frac{2}{15}\sqrt{1-x}\left(3x^2+14x+43\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*(1 + x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - x]*(43 + 14*x + 3*x^2))/15
```

fricas [A] time = 0.68, size = 26, normalized size = 0.68

$$-\frac{2\left(3x^2+14x+43\right)\sqrt{-x^2+1}}{15\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*x^2 + 14*x + 43)*sqrt(-x^2 + 1)/sqrt(x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.03, size = 30, normalized size = 0.79

$$\frac{2(-1+x)\left(3x^2+14x+43\right)\sqrt{1+x}}{15\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 2/15*(-1+x)*(3*x^2+14*x+43)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.31, size = 24, normalized size = 0.63

$$\frac{2\left(3\,x^{3}+11\,x^{2}+29\,x-43\right)}{15\,\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^3 + 11*x^2 + 29*x - 43)/sqrt(-x + 1)
```

mupad [B] time = 0.88, size = 45, normalized size = 1.18

$$-\frac{6\,x^{2}\,\sqrt{1-x^{2}}+28\,x\,\sqrt{1-x^{2}}+86\,\sqrt{1-x^{2}}}{15\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x^2)^(1/2),x)
```

```
[Out] -(6*x^2*(1 - x^2)^(1/2) + 28*x*(1 - x^2)^(1/2) + 86*(1 - x^2)^(1/2))/(15*(x
 + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{5}{2}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral((x + 1)**(5/2)/sqrt(-(x - 1)*(x + 1)), x)
```

3.373 $\int e^{\tanh^{-1}(x)}(1-x)^{3/2}x\,dx$

Optimal. Leaf size=34

$$-\frac{2}{7}(x+1)^{7/2}+\frac{6}{5}(x+1)^{5/2}-\frac{4}{3}(x+1)^{3/2}$$

```
[Out] -4/3*(1+x)^(3/2)+6/5*(1+x)^(5/2)-2/7*(1+x)^(7/2)
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6128, 795, 627, 43}

$$-\frac{2}{7}\sqrt{1-x}\left(1-x^2\right)^{3/2}+\frac{2}{35}(x+1)^{5/2}-\frac{4}{21}(x+1)^{3/2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcTanh[x]*(1 - x)^(3/2)*x,x]
```

```
[Out] (-4*(1 + x)^(3/2))/21 + (2*(1 + x)^(5/2))/35 - (2*Sqrt[1 - x]*(1 - x^2)^(3/
2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(x)}(1-x)^{3/2}x\,dx &= \int \sqrt{1-x}\,x\sqrt{1-x^2}\,dx\\
&= -\frac{2}{7}\sqrt{1-x}\,\left(1-x^2\right)^{3/2}-\frac{1}{7}\int \sqrt{1-x}\,\sqrt{1-x^2}\,dx\\
&= -\frac{2}{7}\sqrt{1-x}\,\left(1-x^2\right)^{3/2}-\frac{1}{7}\int (1-x)\sqrt{1+x}\,dx\\
&= -\frac{2}{7}\sqrt{1-x}\,\left(1-x^2\right)^{3/2}-\frac{1}{7}\int \left(2\sqrt{1+x}-(1+x)^{3/2}\right)dx\\
&= -\frac{4}{21}(1+x)^{3/2}+\frac{2}{35}(1+x)^{5/2}-\frac{2}{7}\sqrt{1-x}\,\left(1-x^2\right)^{3/2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.62

$$-\frac{2}{105}(x+1)^{3/2}\left(15x^2-33x+22\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*(1 - x)^(3/2)*x,x]
[Out] (-2*(1 + x)^(3/2)*(22 - 33*x + 15*x^2))/105
```

fricas [A] time = 0.52, size = 38, normalized size = 1.12

$$\frac{2\left(15\,x^3-18\,x^2-11\,x+22\right)\sqrt{-x^2+1}\,\sqrt{-x+1}}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x,x, algorithm="fricas")
[Out] 2/105*(15*x^3 - 18*x^2 - 11*x + 22)*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

giac [A] time = 0.43, size = 27, normalized size = 0.79

$$-\frac{2}{7}\,(x+1)^{\frac{7}{2}}+\frac{6}{5}\,(x+1)^{\frac{5}{2}}-\frac{4}{3}\,(x+1)^{\frac{3}{2}}+\frac{16}{105}\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x,x, algorithm="giac")
```

```
[Out] -2/7*(x + 1)^(7/2) + 6/5*(x + 1)^(5/2) - 4/3*(x + 1)^(3/2) + 16/105*sqrt(2)
```

maple [A] time = 0.03, size = 34, normalized size = 1.00

$$-\frac{2\left(1+x\right)^{2}\left(15x^{2}-33x+22\right)\sqrt{1-x}}{105\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x,x)
```

```
[Out] -2/105*(1+x)^2*(15*x^2-33*x+22)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [B] time = 0.38, size = 48, normalized size = 1.41

$$-\frac{2\left(15\,x^{4}-24\,x^{3}+13\,x^{2}-52\,x-104\right)}{105\,\sqrt{x+1}}-\frac{2\left(x^{3}-2\,x^{2}+3\,x+6\right)}{5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] -2/105*(15*x^4 - 24*x^3 + 13*x^2 - 52*x - 104)/sqrt(x + 1) - 2/5*(x^3 - 2*x
^2 + 3*x + 6)/sqrt(x + 1)
```

mupad [B] time = 0.92, size = 33, normalized size = 0.97

$$\frac{2\,\sqrt{1-x^{2}}\,\left(-15\,x^{3}+18\,x^{2}+11\,x-22\right)}{105\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x)^(3/2)*(x + 1))/(1 - x^2)^(1/2),x)
```

```
[Out] (2*(1 - x^2)^(1/2)*(11*x + 18*x^2 - 15*x^3 - 22))/(105*(1 - x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(1-x\right)^{\frac{3}{2}}\left(x+1\right)}{\sqrt{-\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(3/2)*x,x)
```

```
[Out] Integral(x*(1 - x)**(3/2)*(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

3.374 $\int e^{\tanh^{-1}(x)}(1-x)^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{4}{3}(x+1)^{3/2}-\frac{2}{5}(x+1)^{5/2}$$

```
[Out] 4/3*(1+x)^(3/2)-2/5*(1+x)^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6127, 627, 43}

$$\frac{4}{3}(x+1)^{3/2}-\frac{2}{5}(x+1)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 - x)^(3/2),x]
```

```
[Out] (4*(1 + x)^(3/2))/3 - (2*(1 + x)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}(1-x)^{3/2}\,dx = \int \sqrt{1-x}\,\sqrt{1-x^2}\,dx$$
$$= \int (1-x)\sqrt{1+x}\,dx$$
$$= \int \left(2\sqrt{1+x} - (1+x)^{3/2}\right)\,dx$$
$$= \frac{4}{3}(1+x)^{3/2} - \frac{2}{5}(1+x)^{5/2}$$

Mathematica [**A**] time = 0.01, size = 16, normalized size = 0.70

$$-\frac{2}{15}(x+1)^{3/2}(3x-7)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*(1 - x)^(3/2),x]
[Out] (-2*(1 + x)^(3/2)*(-7 + 3*x))/15
```

fricas [**B**] time = 0.42, size = 33, normalized size = 1.43

$$\frac{2\left(3x^2-4x-7\right)\sqrt{-x^2+1}\,\sqrt{-x+1}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 - 4*x - 7)*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

giac [**A**] time = 0.22, size = 20, normalized size = 0.87

$$-\frac{2}{5}\,(x+1)^{\frac{5}{2}} + \frac{4}{3}\,(x+1)^{\frac{3}{2}} - \frac{16}{15}\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2),x, algorithm="giac")
[Out] -2/5*(x + 1)^(5/2) + 4/3*(x + 1)^(3/2) - 16/15*sqrt(2)
```

maple [**A**] time = 0.03, size = 29, normalized size = 1.26

$$-\frac{2\,(1+x)^2\,(3x-7)\,\sqrt{1-x}}{15\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2),x)
```

```
[Out] -2/15*(1+x)^2*(3*x-7)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [B] time = 0.33, size = 36, normalized size = 1.57

$$-\frac{2\left(x^3-2x^2+3x+6\right)}{5\sqrt{x+1}}-\frac{2\left(x^2-4x-5\right)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(x^3 - 2*x^2 + 3*x + 6)/sqrt(x + 1) - 2/3*(x^2 - 4*x - 5)/sqrt(x + 1)
```

mupad [B] time = 0.87, size = 42, normalized size = 1.83

$$\frac{16\sqrt{1-x^2}}{15\sqrt{1-x}}+\frac{2\,(3x-1)\,\sqrt{1-x^2}\,\sqrt{1-x}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x)^(3/2)*(x + 1))/(1 - x^2)^(1/2),x)
```

```
[Out] (16*(1 - x^2)^(1/2))/(15*(1 - x)^(1/2)) + (2*(3*x - 1)*(1 - x^2)^(1/2)*(1 -
x)^(1/2))/15
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(1-x)^{\frac{3}{2}}\,(x+1)}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(3/2),x)
```

```
[Out] Integral((1 - x)**(3/2)*(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

3.375 $\int e^{\tanh^{-1}(x)} x\sqrt{1+x}\, dx$

Optimal. Leaf size=36

$$-\frac{2}{5}(1-x)^{5/2}+2(1-x)^{3/2}-4\sqrt{1-x}$$

[Out] 2*(1-x)^(3/2)-2/5*(1-x)^(5/2)-4*(1-x)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6129, 77}

$$-\frac{2}{5}(1-x)^{5/2}+2(1-x)^{3/2}-4\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[x]*x*Sqrt[1 + x],x]

[Out] -4*Sqrt[1 - x] + 2*(1 - x)^(3/2) - (2*(1 - x)^(5/2))/5

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}x\sqrt{1+x}\,dx = \int \frac{x(1+x)}{\sqrt{1-x}}\,dx$$
$$= \int \left(\frac{2}{\sqrt{1-x}} - 3\sqrt{1-x} + (1-x)^{3/2}\right)dx$$
$$= -4\sqrt{1-x} + 2(1-x)^{3/2} - \frac{2}{5}(1-x)^{5/2}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.58

$$-\frac{2}{5}\sqrt{1-x}\left(x^2+3x+6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*x*Sqrt[1 + x],x]
[Out] (-2*Sqrt[1 - x]*(6 + 3*x + x^2))/5
```

fricas [A] time = 0.47, size = 24, normalized size = 0.67

$$-\frac{2\left(x^2+3x+6\right)\sqrt{-x^2+1}}{5\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
[Out] -2/5*(x^2 + 3*x + 6)*sqrt(-x^2 + 1)/sqrt(x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.03, size = 28, normalized size = 0.78

$$\frac{2\,(-1+x)\left(x^2+3x+6\right)\sqrt{1+x}}{5\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(-x^2+1)^(1/2)*x,x)
```

```
[Out] 2/5*(-1+x)*(x^2+3*x+6)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.31, size = 22, normalized size = 0.61

$$\frac{2\left(x^3+2\,x^2+3\,x-6\right)}{5\,\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 2/5*(x^3 + 2*x^2 + 3*x - 6)/sqrt(-x + 1)
```

mupad [B] time = 0.87, size = 45, normalized size = 1.25

$$-\frac{2\,x^2\,\sqrt{1-x^2}\,+6\,x\,\sqrt{1-x^2}\,+12\,\sqrt{1-x^2}}{5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(3/2))/(1 - x^2)^(1/2),x)
```

```
[Out] -(2*x^2*(1 - x^2)^(1/2) + 6*x*(1 - x^2)^(1/2) + 12*(1 - x^2)^(1/2))/(5*(x +
 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(x+1)^{\frac{3}{2}}}{\sqrt{-\,(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(-x**2+1)**(1/2)*x,x)
```

```
[Out] Integral(x*(x + 1)**(3/2)/sqrt(-(x - 1)*(x + 1)), x)
```

3.376 $\int e^{\tanh^{-1}(x)}\sqrt{1+x}\,dx$

Optimal. Leaf size=25

$$\frac{2}{3}(1-x)^{3/2}-4\sqrt{1-x}$$

[Out] 2/3*(1-x)^(3/2)-4*(1-x)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 43}

$$\frac{2}{3}(1-x)^{3/2}-4\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[x]*Sqrt[1 + x],x]

[Out] -4*Sqrt[1 - x] + (2*(1 - x)^(3/2))/3

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(x)}\sqrt{1+x}\,dx &= \int \frac{1+x}{\sqrt{1-x}}\,dx\\
&= \int \left(\frac{2}{\sqrt{1-x}}-\sqrt{1-x}\right)dx\\
&= -4\sqrt{1-x}+\frac{2}{3}(1-x)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.64

$$-\frac{2}{3}\sqrt{1-x}\,(x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 + x],x]
[Out] (-2*Sqrt[1 - x]*(5 + x))/3
```

fricas [A] time = 0.45, size = 19, normalized size = 0.76

$$-\frac{2\sqrt{-x^2+1}\,(x+5)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-x^2 + 1)*(x + 5)/sqrt(x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.03, size = 23, normalized size = 0.92

$$\frac{2\,(-1+x)\,(x+5)\,\sqrt{1+x}}{3\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(-x^2+1)^(1/2),x)
[Out] 2/3*(-1+x)*(x+5)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.30, size = 17, normalized size = 0.68

$$\frac{2\left(x^2+4x-5\right)}{3\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^2 + 4*x - 5)/sqrt(-x + 1)
```

mupad [B] time = 0.85, size = 31, normalized size = 1.24

$$-\frac{2x\sqrt{1-x^2}+10\sqrt{1-x^2}}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x^2)^(1/2),x)
```

```
[Out] -(2*x*(1 - x^2)^(1/2) + 10*(1 - x^2)^(1/2))/(3*(x + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral((x + 1)**(3/2)/sqrt(-(x - 1)*(x + 1)), x)
```

3.377 $\int e^{\tanh^{-1}(x)}\sqrt{1-x}\,x\,dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6128, 26, 43}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 - x]*x,x]
```

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}\sqrt{1-x}\,x\,dx = \int \frac{x\sqrt{1-x^2}}{\sqrt{1-x}}\,dx$$
$$= \int x\sqrt{1+x}\,dx$$
$$= \int \left(-\sqrt{1+x} + (1+x)^{3/2}\right)dx$$
$$= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.70

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 - x]*x,x]
[Out] (2*(1 + x)^(3/2)*(-2 + 3*x))/15
```

fricas [B] time = 0.47, size = 31, normalized size = 1.35

$$-\frac{2\left(3x^2+x-2\right)\sqrt{-x^2+1}\sqrt{-x+1}}{15(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x,x, algorithm="fricas")
[Out] -2/15*(3*x^2 + x - 2)*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

giac [A] time = 0.16, size = 20, normalized size = 0.87

$$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{4}{15}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x,x, algorithm="giac")
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2) - 4/15*sqrt(2)
```

maple [A] time = 0.03, size = 29, normalized size = 1.26

$$\frac{2(1+x)^2(3x-2)\sqrt{1-x}}{15\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x,x)
```

```
[Out] 2/15*(1+x)^2*(3*x-2)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [B] time = 0.32, size = 38, normalized size = 1.65

$$\frac{2\left(3x^3 - x^2 + 4x + 8\right)}{15\sqrt{x+1}} + \frac{2\left(x^2 - x - 2\right)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^3 - x^2 + 4*x + 8)/sqrt(x + 1) + 2/3*(x^2 - x - 2)/sqrt(x + 1)
```

mupad [B] time = 0.89, size = 42, normalized size = 1.83

$$\frac{4\sqrt{1-x^2}}{15\sqrt{1-x}} - \frac{2\ (3x+4)\ \sqrt{1-x^2}\ \sqrt{1-x}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x)^(1/2)*(x + 1))/(1 - x^2)^(1/2),x)
```

```
[Out] (4*(1 - x^2)^(1/2))/(15*(1 - x)^(1/2)) - (2*(3*x + 4)*(1 - x^2)^(1/2)*(1 -
x)^(1/2))/15
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{1-x}\ (x+1)}{\sqrt{-(x-1)(x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(1/2)*x,x)
```

```
[Out] Integral(x*sqrt(1 - x)*(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

3.378 $\int e^{\tanh^{-1}(x)}\sqrt{1-x}\,dx$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

```
[Out] 2/3*(1+x)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6127, 26, 32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 - x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}\sqrt{1-x}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx$$
$$= \int \sqrt{1+x}\,dx$$
$$= \frac{2}{3}(1+x)^{3/2}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 - x],x]
[Out] (2*(1 + x)^(3/2))/3
```

fricas [B] time = 0.56, size = 26, normalized size = 2.36

$$-\frac{2\sqrt{-x^2+1}\,(x+1)\sqrt{-x+1}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-x^2 + 1)*(x + 1)*sqrt(-x + 1)/(x - 1)
```

giac [A] time = 0.40, size = 13, normalized size = 1.18

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{4}{3}\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2),x, algorithm="giac")
[Out] 2/3*(x + 1)^(3/2) - 4/3*sqrt(2)
```

maple [B] time = 0.02, size = 24, normalized size = 2.18

$$\frac{2\,(1+x)^2\,\sqrt{1-x}}{3\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2),x)
```

```
[Out] 2/3*(1+x)^2*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [B] time = 0.38, size = 23, normalized size = 2.09

$$\frac{2\left(x^{2}-x-2\right)}{3\sqrt{x+1}}+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^2 - x - 2)/sqrt(x + 1) + 2*sqrt(x + 1)
```

mupad [B] time = 0.86, size = 33, normalized size = 3.00

$$\frac{2\,x\,\sqrt{1-x^{2}}+2\,\sqrt{1-x^{2}}}{3\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x)^(1/2)*(x + 1))/(1 - x^2)^(1/2),x)
```

```
[Out] (2*x*(1 - x^2)^(1/2) + 2*(1 - x^2)^(1/2))/(3*(1 - x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{1-x}\,(x+1)}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)*(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

3.379 $\int \frac{e^{\tanh^{-1}(x)}x}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=25

$$\frac{2}{3}(1-x)^{3/2}-2\sqrt{1-x}$$

```
[Out] 2/3*(1-x)^(3/2)-2*(1-x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6129, 43}

$$\frac{2}{3}(1-x)^{3/2}-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/Sqrt[1 + x],x]
```

```
[Out] -2*Sqrt[1 - x] + (2*(1 - x)^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(x)}x}{\sqrt{1+x}}\,dx = \int \frac{x}{\sqrt{1-x}}\,dx$$
$$= \int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x}\right) dx$$
$$= -2\sqrt{1-x} + \frac{2}{3}(1-x)^{3/2}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.64

$$-\frac{2}{3}\sqrt{1-x}\,(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/Sqrt[1 + x],x]
[Out] (-2*Sqrt[1 - x]*(2 + x))/3
```

fricas [A] time = 0.46, size = 19, normalized size = 0.76

$$-\frac{2\sqrt{-x^2+1}\,(x+2)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
[Out] -2/3*sqrt(-x^2 + 1)*(x + 2)/sqrt(x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.03, size = 23, normalized size = 0.92

$$\frac{2\left(-1+x\right)\left(x+2\right)\sqrt{1+x}}{3\sqrt{-x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2)*x,x)
[Out] 2/3*(-1+x)*(x+2)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [A] time = 0.30, size = 15, normalized size = 0.60

$$\frac{2\left(x^{2}+x-2\right)}{3\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
[Out] 2/3*(x^2 + x - 2)/sqrt(-x + 1)
```

mupad [B] time = 0.02, size = 19, normalized size = 0.76

$$-\frac{2\sqrt{1-x^{2}}\ \left(x+2\right)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(1/2))/(1 - x^2)^(1/2),x)
[Out] -(2*(1 - x^2)^(1/2)*(x + 2))/(3*(x + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{x+1}}{\sqrt{-\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2)*x,x)
[Out] Integral(x*sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

3.380 $\int \frac{e^{\tanh^{-1}(x)}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

```
[Out] -2*(1-x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/Sqrt[1 + x],x]
```

```
[Out] -2*Sqrt[1 - x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}}{\sqrt{1+x}}\,dx &= \int \frac{1}{\sqrt{1-x}}\,dx\\ &= -2\sqrt{1-x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/Sqrt[1 + x],x]
[Out] -2*Sqrt[1 - x]
```

fricas [C] time = 0.41, size = 16, normalized size = 1.45

$$-\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-x^2 + 1)/sqrt(x + 1)
```

giac [A] time = 0.18, size = 15, normalized size = 1.36

$$2\sqrt{2} - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2) - 2*sqrt(-x + 1)
```

maple [B] time = 0.02, size = 20, normalized size = 1.82

$$\frac{2(-1+x)\sqrt{1+x}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2),x)
[Out] 2*(-1+x)*(1+x)^(1/2)/(-x^2+1)^(1/2)
```

maxima [C] time = 0.37, size = 12, normalized size = 1.09

$$\frac{2(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 2*(x - 1)/sqrt(-x + 1)
```

mupad [B] time = 0.93, size = 16, normalized size = 1.45

$$-\frac{2\,\sqrt{1-x^2}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x^2)^(1/2),x)
[Out] -(2*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}}{\sqrt{-(x-1)(x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

3.381 $\int \frac{e^{\tanh^{-1}(x)}x}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=42

$$-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(1+x)^(3/2)+2*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)-2*(1+x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6129, 80, 50, 63, 206}

$$-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2
]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}x}{\sqrt{1-x}}\,dx &= \int \frac{x\sqrt{1+x}}{1-x}\,dx \\
&= -\frac{2}{3}(1+x)^{3/2} + \int \frac{\sqrt{1+x}}{1-x}\,dx \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + 2\int \frac{1}{(1-x)\sqrt{1+x}}\,dx \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + 4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+x}\right) \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.86

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right) - \frac{2}{3}\sqrt{x+1}\,(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/Sqrt[1 - x],x]
[Out] (-2*Sqrt[1 + x]*(4 + x))/3 + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2]]
```

fricas [B] time = 0.48, size = 79, normalized size = 1.88

$$\frac{3\sqrt{2}(x-1)\log\left(-\frac{x^2-2\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2x-3}{x^2-2x+1}\right)+2\sqrt{-x^2+1}(x+4)\sqrt{-x+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*(x - 1)*log(-(x^2 - 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x + 1) +
2*x - 3)/(x^2 - 2*x + 1)) + 2*sqrt(-x^2 + 1)*(x + 4)*sqrt(-x + 1))/(x - 1)
```

giac [A] time = 0.17, size = 44, normalized size = 1.05

$$-\frac{2}{3}(x+1)^{\frac{3}{2}}-\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(x + 1)^(3/2) - sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x
+ 1))) - 2*sqrt(x + 1)
```

maple [A] time = 0.04, size = 61, normalized size = 1.45

$$-\frac{2\sqrt{-x^2+1}\,\sqrt{1-x}\,\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{1+x}\,\sqrt{2}}{2}\right)\sqrt{2}-\sqrt{1+x}\,x-4\sqrt{1+x}\right)}{3(-1+x)\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(1/2),x)
```

```
[Out] -2/3*(-x^2+1)^(1/2)*(1-x)^(1/2)*(3*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)
-(1+x)^(1/2)*x-4*(1+x)^(1/2))/(-1+x)/(1+x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x+1)x}{\sqrt{-x^2+1}\,\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)*x/(sqrt(-x^2 + 1)*sqrt(-x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,(x+1)}{\sqrt{1-x^2}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1))/((1 - x^2)^(1/2)*(1 - x)^(1/2)),x)
[Out] int((x*(x + 1))/((1 - x^2)^(1/2)*(1 - x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(x+1)}{\sqrt{-(x-1)\,(x+1)}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*x/(1-x)**(1/2),x)
[Out] Integral(x*(x + 1)/(sqrt(-(x - 1)*(x + 1))*sqrt(1 - x)), x)
```

3.382 $\int \frac{e^{\tanh^{-1}(x)}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=31

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)-2\sqrt{x+1}$$

```
[Out] 2*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)-2*(1+x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6127, 627, 50, 63, 206}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)-2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 + x] + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}}{\sqrt{1-x}}\,dx &= \int \frac{\sqrt{1-x^2}}{(1-x)^{3/2}}\,dx \\
&= \int \frac{\sqrt{1+x}}{1-x}\,dx \\
&= -2\sqrt{1+x}+2\int \frac{1}{(1-x)\sqrt{1+x}}\,dx \\
&= -2\sqrt{1+x}+4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+x}\right) \\
&= -2\sqrt{1+x}+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)-2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/Sqrt[1 - x],x]
[Out] -2*Sqrt[1 + x] + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x]/Sqrt[2]]
```

fricas [B] time = 0.48, size = 74, normalized size = 2.39

$$\frac{\sqrt{2}\,(x-1)\log\left(-\frac{x^2-2\,\sqrt{2}\,\sqrt{-x^2+1}\,\sqrt{-x+1}+2\,x-3}{x^2-2\,x+1}\right)+2\,\sqrt{-x^2+1}\,\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(x - 1)*log(-(x^2 - 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x + 1) + 2*x -
3)/(x^2 - 2*x + 1)) + 2*sqrt(-x^2 + 1)*sqrt(-x + 1))/(x - 1)
```

giac [A] time = 0.23, size = 37, normalized size = 1.19

$$-\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) - 2*sqrt(x +
1)
```

maple [B] time = 0.04, size = 52, normalized size = 1.68

$$-\frac{2\sqrt{-x^2+1}\,\sqrt{1-x}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{1+x}\,\sqrt{2}}{2}\right)\sqrt{2}-\sqrt{1+x}\right)}{(-1+x)\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)/(1-x)^(1/2),x)
```

```
[Out] -2*(-x^2+1)^(1/2)*(1-x)^(1/2)*(arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)-(1+
x)^(1/2))/(-1+x)/(1+x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{-x^2+1}\sqrt{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(-x^2 + 1)*sqrt(-x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{x+1}{\sqrt{1-x^2}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((1 - x^2)^(1/2)*(1 - x)^(1/2)),x)
```

```
[Out] int((x + 1)/((1 - x^2)^(1/2)*(1 - x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{-(x-1)(x+1)}\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt(-(x - 1)*(x + 1))*sqrt(1 - x)), x)
```

3.383 $\int \frac{e^{\tanh^{-1}(x)}x}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-2\sqrt{1-x}$$

```
[Out] arctanh(1/2*(1-x)^(1/2)*2^(1/2))*2^(1/2)-2*(1-x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6129, 80, 63, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/(1 + x)^(3/2),x]
```

```
[Out] -2*Sqrt[1 - x] + Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}x}{(1+x)^{3/2}}\,dx &= \int \frac{x}{\sqrt{1-x}\,(1+x)}\,dx\\ &= -2\sqrt{1-x}-\int \frac{1}{\sqrt{1-x}\,(1+x)}\,dx\\ &= -2\sqrt{1-x}+2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1-x}\right)\\ &= -2\sqrt{1-x}+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/(1 + x)^(3/2),x]
[Out] -2*Sqrt[1 - x] + Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]]
```

fricas [B] time = 0.46, size = 71, normalized size = 2.09

$$\frac{\sqrt{2}\,(x+1)\log\left(-\frac{x^2-2\,\sqrt{2}\,\sqrt{-x^2+1}\,\sqrt{x+1}-2\,x-3}{x^2+2\,x+1}\right)-4\,\sqrt{-x^2+1}\,\sqrt{x+1}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*(x + 1)*log(-(x^2 - 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(x + 1) - 2*x
 - 3)/(x^2 + 2*x + 1)) - 4*sqrt(-x^2 + 1)*sqrt(x + 1))/(x + 1)
```

giac [A] time = 0.50, size = 43, normalized size = 1.26

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{2}+\sqrt{-x+1}}\right)-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log((sqrt(2) - sqrt(-x + 1))/(sqrt(2) + sqrt(-x + 1))) - 2*sqr
t(-x + 1)
```

maple [A] time = 0.03, size = 50, normalized size = 1.47

$$\frac{\sqrt{-x^2+1}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{1-x}\,\sqrt{2}}{2}\right)\sqrt{2}\,-2\sqrt{1-x}\right)}{\sqrt{1+x}\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2)/(-x^2+1)^(1/2)*x,x)
```

```
[Out] (-x^2+1)^(1/2)*(arctanh(1/2*(1-x)^(1/2)*2^(1/2))*2^(1/2)-2*(1-x)^(1/2))/(1+
x)^(1/2)/(1-x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^2+1}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^2 + 1)*sqrt(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{1-x^2}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^2)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int(x/((1 - x^2)^(1/2)*(x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-x**2+1)**(1/2)*x,x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(x + 1)), x)
```

3.384 $\int \frac{e^{\tanh^{-1}(x)}}{(1+x)^{3/2}} \, dx$

Optimal. Leaf size=23

$$-\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

```
[Out] -arctanh(1/2*(1-x)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6129, 63, 206}

$$-\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/(1 + x)^(3/2),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(x)}}{(1+x)^{3/2}}\,dx = \int \frac{1}{\sqrt{1-x}\,(1+x)}\,dx$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1-x}\right)\right)$$
$$= -\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/(1 + x)^(3/2),x]
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

fricas [B] time = 0.60, size = 45, normalized size = 1.96

$$\frac{1}{2}\,\sqrt{2}\,\log\left(-\frac{x^2+2\,\sqrt{2}\,\sqrt{-x^2+1}\,\sqrt{x+1}\,-2\,x-3}{x^2+2\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(x + 1) - 2*x - 3)/(x^
2 + 2*x + 1))
```

giac [B] time = 0.31, size = 37, normalized size = 1.61

$$-\frac{1}{2}\,\sqrt{2}\,\log\left(\sqrt{2}\,+\sqrt{-x+1}\right)+\frac{1}{2}\,\sqrt{2}\,\log\left(\sqrt{2}\,-\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(sqrt(2) + sqrt(-x + 1)) + 1/2*sqrt(2)*log(sqrt(2) - sqrt(-
x + 1))
```

maple [B] time = 0.03, size = 40, normalized size = 1.74

$$-\frac{\sqrt{-x^2+1}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1-x}\,\sqrt{2}}{2}\right)}{\sqrt{1+x}\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/(1+x)^(1/2)*(-x^2+1)^(1/2)/(1-x)^(1/2)*2^(1/2)*arctanh(1/2*(1-x)^(1/2)*2
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+1}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{1-x^2}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int(1/((1 - x^2)^(1/2)*(x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(x + 1)), x)
```

3.385 $\int \frac{e^{\tanh^{-1}(x)}x}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{(x+1)^{3/2}}{2(1-x)}+\frac{5\sqrt{x+1}}{2}-\frac{5\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*(1+x)^(3/2)/(1-x)-5/2*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)+5/2*(1+x
)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6129, 78, 50, 63, 206}

$$\frac{(x+1)^{3/2}}{2(1-x)}+\frac{5\sqrt{x+1}}{2}-\frac{5\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] (5*Sqrt[1 + x])/2 + (1 + x)^(3/2)/(2*(1 - x)) - (5*ArcTanh[Sqrt[1 + x]/Sqrt
[2]])/Sqrt[2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}x}{(1-x)^{3/2}}\,dx &= \int \frac{x\sqrt{1+x}}{(1-x)^2}\,dx \\
&= \frac{(1+x)^{3/2}}{2(1-x)} - \frac{5}{4}\int \frac{\sqrt{1+x}}{1-x}\,dx \\
&= \frac{5\sqrt{1+x}}{2} + \frac{(1+x)^{3/2}}{2(1-x)} - \frac{5}{2}\int \frac{1}{(1-x)\sqrt{1+x}}\,dx \\
&= \frac{5\sqrt{1+x}}{2} + \frac{(1+x)^{3/2}}{2(1-x)} - 5\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{5\sqrt{1+x}}{2} + \frac{(1+x)^{3/2}}{2(1-x)} - \frac{5\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.78

$$
\frac{\sqrt{x+1}\,(2x-3)}{x-1} - \frac{5\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] (Sqrt[1 + x]*(-3 + 2*x))/(-1 + x) - (5*ArcTanh[Sqrt[1 + x]/Sqrt[2]])/Sqrt[2
]
```

fricas [B] time = 0.59, size = 91, normalized size = 1.78

$$\frac{5\sqrt{2}\left(x^2-2x+1\right)\log\left(-\frac{x^2+2\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2x-3}{x^2-2x+1}\right)-4\sqrt{-x^2+1}\,(2x-3)\sqrt{-x+1}}{4\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x
 + 1) + 2*x - 3)/(x^2 - 2*x + 1)) - 4*sqrt(-x^2 + 1)*(2*x - 3)*sqrt(-x + 1)
)/(x^2 - 2*x + 1)
```

giac [A] time = 0.19, size = 49, normalized size = 0.96

$$\frac{5}{4}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)+2\sqrt{x+1}-\frac{\sqrt{x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 5/4*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) + 2*sqrt(x
 + 1) - sqrt(x + 1)/(x - 1)
```

maple [B] time = 0.04, size = 78, normalized size = 1.53

$$\frac{\sqrt{-x^2+1}\,\sqrt{1-x}\,\left(5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+x}\,\sqrt{2}}{2}\right)x-5\operatorname{arctanh}\left(\frac{\sqrt{1+x}\,\sqrt{2}}{2}\right)\sqrt{2}-4\sqrt{1+x}\,x+6\sqrt{1+x}\right)}{2\left(-1+x\right)^2\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(3/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(1-x)^(1/2)*(5*2^(1/2)*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*
x-5*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)-4*(1+x)^(1/2)*x+6*(1+x)^(1/2))
/(-1+x)^2/(1+x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)x}{\sqrt{-x^2+1}\,(-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*x/(1-x)^(3/2),x, algorithm="maxima")
[Out] integrate((x + 1)*x/(sqrt(-x^2 + 1)*(-x + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,(x+1)}{\sqrt{1-x^2}\,(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1))/((1 - x^2)^(1/2)*(1 - x)^(3/2)),x)
[Out] int((x*(x + 1))/((1 - x^2)^(1/2)*(1 - x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(x+1)}{\sqrt{-(x-1)\,(x+1)}\,(1-x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*x/(1-x)**(3/2),x)
[Out] Integral(x*(x + 1)/(sqrt(-(x - 1)*(x + 1))*(1 - x)**(3/2)), x)
```

3.386 $\int \frac{e^{\tanh^{-1}(x)}}{(1-x)^{3/2}} \, dx$

Optimal. Leaf size=37

$$\frac{\sqrt{x+1}}{1-x}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)+(1+x)^(1/2)/(1-x)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6127, 627, 47, 63, 206}

$$\frac{\sqrt{x+1}}{1-x}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]/(1 - x)^(3/2),x]
```

```
[Out] Sqrt[1 + x]/(1 - x) - ArcTanh[Sqrt[1 + x]/Sqrt[2]]/Sqrt[2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(x)}}{(1-x)^{3/2}}\,dx &= \int \frac{\sqrt{1-x^2}}{(1-x)^{5/2}}\,dx \\
&= \int \frac{\sqrt{1+x}}{(1-x)^2}\,dx \\
&= \frac{\sqrt{1+x}}{1-x} - \frac{1}{2}\int \frac{1}{(1-x)\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{1-x} - \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{\sqrt{1+x}}{1-x} - \frac{\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.97

$$
-\frac{\sqrt{x+1}}{x-1} - \frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[x]/(1 - x)^(3/2),x]
```

```
[Out] -(Sqrt[1 + x]/(-1 + x)) - ArcTanh[Sqrt[1 + x]/Sqrt[2]]/Sqrt[2]
```

fricas [B] time = 0.44, size = 85, normalized size = 2.30

$$\frac{\sqrt{2}\left(x^2-2\,x+1\right)\log\left(-\frac{x^2+2\,\sqrt{2}\,\sqrt{-x^2+1}\,\sqrt{-x+1}+2\,x-3}{x^2-2\,x+1}\right)+4\,\sqrt{-x^2+1}\,\sqrt{-x+1}}{4\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x +
 1) + 2*x - 3)/(x^2 - 2*x + 1)) + 4*sqrt(-x^2 + 1)*sqrt(-x + 1))/(x^2 - 2*x
 + 1)
```

giac [A] time = 0.19, size = 42, normalized size = 1.14

$$\frac{1}{4}\,\sqrt{2}\,\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)-\frac{\sqrt{x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) - sqrt(x +
 1)/(x - 1)
```

maple [B] time = 0.04, size = 69, normalized size = 1.86

$$\frac{\sqrt{-x^2+1}\,\sqrt{1-x}\,\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+x}\,\sqrt{2}}{2}\right)x-\operatorname{arctanh}\left(\frac{\sqrt{1+x}\,\sqrt{2}}{2}\right)\sqrt{2}+2\sqrt{1+x}\right)}{2\,(-1+x)^2\,\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)/(1-x)^(3/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(1-x)^(1/2)*(2^(1/2)*arctanh(1/2*(1+x)^(1/2)*2^(1/2))*x-
arctanh(1/2*(1+x)^(1/2)*2^(1/2))*2^(1/2)+2*(1+x)^(1/2))/(-1+x)^2/(1+x)^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{-x^2+1}\,(-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(-x^2 + 1)*(-x + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x+1}{\sqrt{1-x^2}\,(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((1 - x^2)^(1/2)*(1 - x)^(3/2)),x)
```

```
[Out] int((x + 1)/((1 - x^2)^(1/2)*(1 - x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{-(x-1)(x+1)}\,(1-x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)/(1-x)**(3/2),x)
```

```
[Out] Integral((x + 1)/(sqrt(-(x - 1)*(x + 1))*(1 - x)**(3/2)), x)
```

3.387 $\int e^{\tanh^{-1}(ax)}x^m\sqrt{c-acx}\,dx$

Optimal. Leaf size=64

$$\frac{2c(ax+1)\sqrt{1-a^2x^2}\,x^m(-ax)^{-m}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};ax+1\right)}{3a\sqrt{c-acx}}$$

```
[Out] 2/3*c*x^m*(a*x+1)*hypergeom([3/2, -m],[5/2],a*x+1)*(-a^2*x^2+1)^(1/2)/a/((-
a*x)^m)/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6128, 892, 67, 65}

$$\frac{2c(ax+1)\sqrt{1-a^2x^2}\,x^m(-ax)^{-m}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};ax+1\right)}{3a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*Sqrt[c - a*c*x],x]
```

```
[Out] (2*c*x^m*(1 + a*x)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[3/2, -m, 5/2, 1 + a*
x])/(3*a*(-(a*x))^m*Sqrt[c - a*c*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
```

```
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^m\sqrt{c-acx}\,dx &= c\int \frac{x^m\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx\\
&= \frac{\left(c\sqrt{1-a^2x^2}\right)\int x^m\sqrt{\frac{1}{c}+\frac{ax}{c}}\,dx}{\sqrt{\frac{1}{c}+\frac{ax}{c}}\sqrt{c-acx}}\\
&= \frac{\left(cx^m(-ax)^{-m}\sqrt{1-a^2x^2}\right)\int(-ax)^m\sqrt{\frac{1}{c}+\frac{ax}{c}}\,dx}{\sqrt{\frac{1}{c}+\frac{ax}{c}}\sqrt{c-acx}}\\
&= \frac{2cx^m(-ax)^{-m}(1+ax)\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};1+ax\right)}{3a\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.72

$$\frac{x^{m+1}\sqrt{c-acx}\,{}_2F_1\left(-\frac{1}{2},m+1;m+2;-ax\right)}{(m+1)\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*Sqrt[c - a*c*x],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, 1 + m, 2 + m, -(a*x)])/(
(1 + m)*Sqrt[1 - a*x])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}\,x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*x^m/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^m\sqrt{-acx+c}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a*c*x)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^m*(c - a*c*x)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{-c(ax-1)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.388 $\int e^{\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=107

$$\frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}}-\frac{8c^2\left(1-a^2x^2\right)^{3/2}}{105a^3(c-acx)^{3/2}}+\frac{8c\left(1-a^2x^2\right)^{3/2}}{35a^3\sqrt{c-acx}}$$

```
[Out] -8/105*c^2*(-a^2*x^2+1)^(3/2)/a^3/(-a*c*x+c)^(3/2)+2/7*c^2*x^2*(-a^2*x^2+1)
^(3/2)/a/(-a*c*x+c)^(3/2)+8/35*c*(-a^2*x^2+1)^(3/2)/a^3/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6128, 871, 795, 649}

$$\frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}}-\frac{8c^2\left(1-a^2x^2\right)^{3/2}}{105a^3(c-acx)^{3/2}}+\frac{8c\left(1-a^2x^2\right)^{3/2}}{35a^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (-8*c^2*(1 - a^2*x^2)^(3/2))/(105*a^3*(c - a*c*x)^(3/2)) + (2*c^2*x^2*(1 -
a^2*x^2)^(3/2))/(7*a*(c - a*c*x)^(3/2)) + (8*c*(1 - a^2*x^2)^(3/2))/(35*a^3
*Sqrt[c - a*c*x])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
```

```
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= c\int \frac{x^2\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx\\
&= \frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}} - \frac{(4c)\int \frac{x\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx}{7a}\\
&= \frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}} + \frac{8c\left(1-a^2x^2\right)^{3/2}}{35a^3\sqrt{c-acx}} - \frac{(4c)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx}{35a^2}\\
&= -\frac{8c^2\left(1-a^2x^2\right)^{3/2}}{105a^3(c-acx)^{3/2}} + \frac{2c^2x^2\left(1-a^2x^2\right)^{3/2}}{7a(c-acx)^{3/2}} + \frac{8c\left(1-a^2x^2\right)^{3/2}}{35a^3\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.48

$$\frac{2(ax+1)^{3/2}\left(15a^2x^2-12ax+8\right)\sqrt{c-acx}}{105a^3\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + a*x)^(3/2)*Sqrt[c - a*c*x]*(8 - 12*a*x + 15*a^2*x^2))/(105*a^3*Sqrt
[1 - a*x])
```

fricas [A] time = 0.52, size = 58, normalized size = 0.54

$$-\frac{2\left(15\,a^3x^3+3\,a^2x^2-4\,ax+8\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -2/105*(15*a^3*x^3 + 3*a^2*x^2 - 4*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x
+ c)/(a^4*x - a^3)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.03, size = 48, normalized size = 0.45

$$\frac{2\left(ax+1\right)^{2}\left(15a^{2}x^{2}-12ax+8\right)\sqrt{-acx+c}}{105a^{3}\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/105*(a*x+1)^2*(15*a^2*x^2-12*a*x+8)*(-a*c*x+c)^(1/2)/a^3/(-a^2*x^2+1)^(1/
2)
```

maxima [**A**] time = 0.33, size = 106, normalized size = 0.99

$$\frac{2\left(5\,a^{4}\sqrt{c}\,x^{4}-a^{3}\sqrt{c}\,x^{3}+2\,a^{2}\sqrt{c}\,x^{2}-8\,a\sqrt{c}\,x-16\,\sqrt{c}\right)}{35\,\sqrt{ax+1}\,a^{3}}+\frac{2\left(3\,a^{3}\sqrt{c}\,x^{3}-a^{2}\sqrt{c}\,x^{2}+4\,a\sqrt{c}\,x+8\,\sqrt{c}\right)}{15\,\sqrt{ax+1}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] 2/35*(5*a^4*sqrt(c)*x^4 - a^3*sqrt(c)*x^3 + 2*a^2*sqrt(c)*x^2 - 8*a*sqrt(c)
*x - 16*sqrt(c))/(sqrt(a*x + 1)*a^3) + 2/15*(3*a^3*sqrt(c)*x^3 - a^2*sqrt(c
)*x^2 + 4*a*sqrt(c)*x + 8*sqrt(c))/(sqrt(a*x + 1)*a^3)
```

mupad [B] time = 0.95, size = 53, normalized size = 0.50

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{8\,x}{105\,a^2}+\frac{2\,a\,x^4}{7}+\frac{16}{105\,a^3}+\frac{12\,x^3}{35}-\frac{2\,x^2}{105\,a}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((8*x)/(105*a^2) + (2*a*x^4)/7 + 16/(105*a^3) + (12*x^3)
/35 - (2*x^2)/(105*a)))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\,(ax-1)}\,\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.389 $\int e^{\tanh^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal. Leaf size=69

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{15a^2(c-acx)^{3/2}}-\frac{2c\left(1-a^2x^2\right)^{3/2}}{5a^2\sqrt{c-acx}}$$

```
[Out] 2/15*c^2*(-a^2*x^2+1)^(3/2)/a^2/(-a*c*x+c)^(3/2)-2/5*c*(-a^2*x^2+1)^(3/2)/a
^2/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6128, 795, 649}

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{15a^2(c-acx)^{3/2}}-\frac{2c\left(1-a^2x^2\right)^{3/2}}{5a^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*c^2*(1 - a^2*x^2)^(3/2))/(15*a^2*(c - a*c*x)^(3/2)) - (2*c*(1 - a^2*x^2)
^(3/2))/(5*a^2*Sqrt[c - a*c*x])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx = c\int \frac{x\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx$$
$$= -\frac{2c\left(1-a^2x^2\right)^{3/2}}{5a^2\sqrt{c-acx}} + \frac{c\int \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx}{5a}$$
$$= \frac{2c^2\left(1-a^2x^2\right)^{3/2}}{15a^2(c-acx)^{3/2}} - \frac{2c\left(1-a^2x^2\right)^{3/2}}{5a^2\sqrt{c-acx}}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.62

$$\frac{2(ax+1)^{3/2}(3ax-2)\sqrt{c-acx}}{15a^2\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + a*x)^(3/2)*(-2 + 3*a*x)*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - a*x])
```

fricas [A] time = 0.57, size = 49, normalized size = 0.71

$$-\frac{2\left(3\,a^2x^2+ax-2\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{15\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/15*(3*a^2*x^2 + a*x - 2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x - a^
2)
```

giac [A] time = 0.20, size = 54, normalized size = 0.78

$$-\frac{2\,c^2\left(\frac{2\,\sqrt{2}}{a\,\sqrt{c}} - \frac{3\,(acx+c)^{\frac{5}{2}}-5\,(acx+c)^{\frac{3}{2}}\,c}{ac^3}\right)}{15\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -2/15*c^2*(2*sqrt(2)/(a*sqrt(c)) - (3*(a*c*x + c)^(5/2) - 5*(a*c*x + c)^(3/
2)*c)/(a*c^3))/(a*abs(c))
```

maple [A] time = 0.03, size = 40, normalized size = 0.58

$$\frac{2 (ax+1)^2 (3ax-2) \sqrt{-acx+c}}{15a^2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/15*(a*x+1)^2*(3*a*x-2)*(-a*c*x+c)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.33, size = 83, normalized size = 1.20

$$\frac{2 \left(3 a^3 \sqrt{c} x^3 - a^2 \sqrt{c} x^2 + 4 a \sqrt{c} x + 8 \sqrt{c}\right)}{15 \sqrt{ax+1}\, a^2} + \frac{2 \left(a^2 \sqrt{c} x^2 - a \sqrt{c} x - 2 \sqrt{c}\right)}{3 \sqrt{ax+1}\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/15*(3*a^3*sqrt(c)*x^3 - a^2*sqrt(c)*x^2 + 4*a*sqrt(c)*x + 8*sqrt(c))/(sqr
t(a*x + 1)*a^2) + 2/3*(a^2*sqrt(c)*x^2 - a*sqrt(c)*x - 2*sqrt(c))/(sqrt(a*x
 + 1)*a^2)
```

mupad [B] time = 0.92, size = 46, normalized size = 0.67

$$-\frac{\sqrt{c - a\, c\, x} \left(\frac{2\, x}{15\, a} - \frac{2\, a\, x^3}{5} + \frac{4}{15\, a^2} - \frac{8\, x^2}{15}\right)}{\sqrt{1 - a^2\, x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] -((c - a*c*x)^(1/2)*((2*x)/(15*a) - (2*a*x^3)/5 + 4/(15*a^2) - (8*x^2)/15))
/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\,(ax-1)}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a*c*x+c)**(1/2),x)
[Out] Integral(x*sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.390 $\int e^{\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=35

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}$$

```
[Out] 2/3*c^2*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6127, 649}

$$\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*c^2*(1 - a^2*x^2)^(3/2))/(3*a*(c - a*c*x)^(3/2))
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= c\int\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\,dx\\ &= \frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.06

$$\frac{2(ax+1)^{3/2}\sqrt{c-acx}}{3a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + a*x)^(3/2)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a*x])
```

fricas [A] time = 0.57, size = 39, normalized size = 1.11

$$-\frac{2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\,(ax+1)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 1)/(a^2*x - a)
```

giac [A] time = 0.21, size = 34, normalized size = 0.97

$$-\frac{2\,c^2\left(\frac{2\sqrt{2}}{\sqrt{c}}-\frac{(acx+c)^{\frac{3}{2}}}{c^2}\right)}{3\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*c^2*(2*sqrt(2)/sqrt(c) - (a*c*x + c)^(3/2)/c^2)/(a*abs(c))
```

maple [A] time = 0.03, size = 34, normalized size = 0.97

$$\frac{2\,(ax+1)^2\sqrt{-acx+c}}{3a\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/3*(a*x+1)^2*(-a*c*x+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.33, size = 58, normalized size = 1.66

$$\frac{2\left(a^{2}\sqrt{c}\,x^{2}-a\sqrt{c}\,x-2\,\sqrt{c}\right)}{3\,\sqrt{ax+1}\,a}+\frac{2\left(a\sqrt{c}\,x+\sqrt{c}\right)}{\sqrt{ax+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/3*(a^2*sqrt(c)*x^2 - a*sqrt(c)*x - 2*sqrt(c))/(sqrt(a*x + 1)*a) + 2*(a*sq
rt(c)*x + sqrt(c))/(sqrt(a*x + 1)*a)
```

mupad [B] time = 0.00, size = 37, normalized size = 1.06

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{4\,x}{3}+\frac{2\,a\,x^2}{3}+\frac{2}{3\,a}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((4*x)/3 + (2*a*x^2)/3 + 2/(3*a)))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.391 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=68

$$\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -2*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)+2*c*(-a^2*x
^2+1)^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6128, 865, 875, 208}

$$\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1
 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}\,dx\\ &= \frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx\\ &= \frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+\left(2a^2c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\ &= \frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.68

$$\frac{\sqrt{c-acx}\left(2\sqrt{ax+1}-2\tanh^{-1}\left(\sqrt{ax+1}\right)\right)}{\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (Sqrt[c - a*c*x]*(2*Sqrt[1 + a*x] - 2*ArcTanh[Sqrt[1 + a*x]]))/Sqrt[1 - a*x
]
```

fricas [A] time = 0.49, size = 183, normalized size = 2.69

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{ax-1},-\frac{2\left((ax-1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-a}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*
c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c
))/(a*x - 1), -2*((a*x - 1)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x
+ c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x
- 1)]
```

giac [A] time = 0.16, size = 83, normalized size = 1.22

$$\frac{2\,c^3\left(\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}-\frac{\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{c}}{\sqrt{-c}}\right)+\sqrt{2}\,\sqrt{-c}}{\sqrt{-c}\,c^{\frac{3}{2}}}+\frac{\sqrt{acx+c}}{c^2}\right)}{|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*c^3*(arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c) - (sqrt(c)*arctan(sqrt
(2)*sqrt(c)/sqrt(-c)) + sqrt(2)*sqrt(-c))/(sqrt(-c)*c^(3/2)) + sqrt(a*c*x +
 c)/c^2)/abs(c)
```

maple [A] time = 0.04, size = 71, normalized size = 1.04

$$\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)-\sqrt{c\,(ax+1)}\right)}{(ax-1)\,\sqrt{c\,(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] 2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctanh((c*(a*x+1))^(1/2)/
c^(1/2))-(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{c}\int\frac{1}{\sqrt{ax+1}\,x}\,dx+\frac{2\left(a\sqrt{c}\,x+\sqrt{c}\right)}{\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] sqrt(c)*integrate(1/(sqrt(a*x + 1)*x), x) + 2*(a*sqrt(c)*x + sqrt(c))/sqrt(
a*x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(ax-1)}\,(ax+1)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.392 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}-a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -a*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)-c*(-a^2*x^2
+1)^(1/2)/x/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6128, 863, 875, 208}

$$-\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}-a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((c*Sqrt[1 - a^2*x^2])/(x*Sqrt[c - a*c*x])) - a*Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= c\int \frac{\sqrt{1-a^2x^2}}{x^2\sqrt{c-acx}}\,dx\\ &= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}+\frac{1}{2}a\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}+\left(a^3c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\ &= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}-a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.79

$$-\frac{\sqrt{c-acx}\left(ax+ax\sqrt{ax+1}\,\tanh^{-1}\left(\sqrt{ax+1}\right)+1\right)}{x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[c - a*c*x]*(1 + a*x + a*x*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 + a*x]]))/(x
*Sqrt[1 - a^2*x^2]))
```

fricas [A] time = 0.50, size = 207, normalized size = 2.88

$$\left[\frac{\left(a^2x^2-ax\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{ax^2-x}\right)+2\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{2\left(ax^2-x\right)},-\frac{\left(a^2x^2-ax\right)\sqrt{-c}\,\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c))/(a*x^2 - x), -((a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*x^2 + 1)*sqrt(-a
*c*x + c))/(a*x^2 - x)]
```

giac [A] time = 0.17, size = 97, normalized size = 1.35

$$\frac{\left(\frac{a^{2}\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)-\sqrt{2}a^{2}\sqrt{-c}}{\sqrt{-c}\sqrt{c}}-\frac{\sqrt{acx+c}\,a}{cx}\right)c^{2}}{a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] (a^2*arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) - (a^2*sqrt(c)*arctan(sqrt(2
)*sqrt(c)/sqrt(-c)) - sqrt(2)*a^2*sqrt(-c))/(sqrt(-c)*sqrt(c)) - sqrt(a*c*x
 + c)*a/(c*x))*c^2/(a*abs(c))
```

maple [A] time = 0.05, size = 78, normalized size = 1.08

$$\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c\,(ax-1)}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)xac+\sqrt{c\,(ax+1)}\,\sqrt{c}\right)}{(ax-1)\sqrt{c\,(ax+1)}\,x\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(arctanh((c*(a*x+1))^(1/2)/c^(1/2))*x
*a*c+(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/x/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}\,(ax+1)}{\sqrt{-a^{2}x^{2}+1}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\ \left(a\,x+1\right)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,\left(ax+1\right)}{x^{2}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.393 $\int e^{2\tanh^{-1}(ax)} x^3 \sqrt{c-acx}\, dx$

Optimal. Leaf size=101

$$-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{10(c-acx)^{7/2}}{7a^4c^3}-\frac{18(c-acx)^{5/2}}{5a^4c^2}+\frac{14(c-acx)^{3/2}}{3a^4c}-\frac{4\sqrt{c-acx}}{a^4}$$

```
[Out] 14/3*(-a*c*x+c)^(3/2)/a^4/c-18/5*(-a*c*x+c)^(5/2)/a^4/c^2+10/7*(-a*c*x+c)^(
7/2)/a^4/c^3-2/9*(-a*c*x+c)^(9/2)/a^4/c^4-4*(-a*c*x+c)^(1/2)/a^4
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6130, 21, 77}

$$-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{10(c-acx)^{7/2}}{7a^4c^3}-\frac{18(c-acx)^{5/2}}{5a^4c^2}+\frac{14(c-acx)^{3/2}}{3a^4c}-\frac{4\sqrt{c-acx}}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^4 + (14*(c - a*c*x)^(3/2))/(3*a^4*c) - (18*(c - a*c*
x)^(5/2))/(5*a^4*c^2) + (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) - (2*(c - a*c*x)
^(9/2))/(9*a^4*c^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx &= \int \frac{x^3(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= c\int \frac{x^3(1+ax)}{\sqrt{c-acx}}\,dx\\ &= c\int \left(\frac{2}{a^3\sqrt{c-acx}}-\frac{7\sqrt{c-acx}}{a^3c}+\frac{9(c-acx)^{3/2}}{a^3c^2}-\frac{5(c-acx)^{5/2}}{a^3c^3}+\frac{(c-acx)^{7/2}}{a^3c^4}\right) d\\ &= -\frac{4\sqrt{c-acx}}{a^4}+\frac{14(c-acx)^{3/2}}{3a^4c}-\frac{18(c-acx)^{5/2}}{5a^4c^2}+\frac{10(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{9/2}}{9a^4c^4}\end{aligned}$$

Mathematica [A] time = 0.07, size = 48, normalized size = 0.48

$$-\frac{2\left(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272\right)\sqrt{c-acx}}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(272 + 136*a*x + 102*a^2*x^2 + 85*a^3*x^3 + 35*a^4*x^4)
)/(315*a^4)
```

fricas [A] time = 0.65, size = 44, normalized size = 0.44

$$-\frac{2\left(35\,a^4x^4+85\,a^3x^3+102\,a^2x^2+136\,ax+272\right)\sqrt{-acx+c}}{315\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/315*(35*a^4*x^4 + 85*a^3*x^3 + 102*a^2*x^2 + 136*a*x + 272)*sqrt(-a*c*x
+ c)/a^4
```

giac [B] time = 0.48, size = 189, normalized size = 1.87

$$-\frac{2\left(\frac{9\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}\,c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\,\sqrt{-acx+c}\,c^3\right)}{a^3c^3}+\frac{35\,(acx-c)^4\sqrt{-acx+c}+180\,(acx-c)^3\sqrt{-acx+c}\,c+378\,(acx-}{a^3c^4}\right.}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/315*(9*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x
+ c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*x + c)*c^3)/(a^3*c^3) + (
35*(a*c*x - c)^4*sqrt(-a*c*x + c) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c +
378*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*s
qrt(-a*c*x + c)*c^4)/(a^3*c^4))/a
```

maple [A] time = 0.03, size = 45, normalized size = 0.45

$$-\frac{2\sqrt{-acx+c}\,\left(35x^4a^4+85x^3a^3+102a^2x^2+136ax+272\right)}{315a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/315*(-a*c*x+c)^(1/2)*(35*a^4*x^4+85*a^3*x^3+102*a^2*x^2+136*a*x+272)/a^4
```

maxima [A] time = 0.31, size = 74, normalized size = 0.73

$$-\frac{2\left(35\,(-acx+c)^{\frac{9}{2}}-225\,(-acx+c)^{\frac{7}{2}}c+567\,(-acx+c)^{\frac{5}{2}}c^2-735\,(-acx+c)^{\frac{3}{2}}c^3+630\,\sqrt{-acx+c}\,c^4\right)}{315\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/315*(35*(-a*c*x + c)^(9/2) - 225*(-a*c*x + c)^(7/2)*c + 567*(-a*c*x + c)
^(5/2)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(-a*c*x + c)*c^4)/(a^4*c^
4)
```

mupad [B] time = 0.04, size = 83, normalized size = 0.82

$$\frac{14\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^4\,c}-\frac{4\,\sqrt{c-a\,c\,x}}{a^4}-\frac{18\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^4\,c^2}+\frac{10\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^4\,c^3}-\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a^4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - a*c*x)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (14*(c - a*c*x)^(3/2))/(3*a^4*c) - (4*(c - a*c*x)^(1/2))/a^4 - (18*(c - a*c
*x)^(5/2))/(5*a^4*c^2) + (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) - (2*(c - a*c*x
)^(9/2))/(9*a^4*c^4)
```

sympy [A] time = 10.43, size = 83, normalized size = 0.82

$$\frac{2\left(-2c^4\sqrt{-acx+c}+\frac{7c^3(-acx+c)^{\frac{3}{2}}}{3}-\frac{9c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{5c(-acx+c)^{\frac{7}{2}}}{7}-\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(-2*c**4*sqrt(-a*c*x + c) + 7*c**3*(-a*c*x + c)**(3/2)/3 - 9*c**2*(-a*c*x
 + c)**(5/2)/5 + 5*c*(-a*c*x + c)**(7/2)/7 - (-a*c*x + c)**(9/2)/9)/(a**4*c
**4)
```

3.394 $\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=80

$$\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{8(c-acx)^{5/2}}{5a^3c^2}+\frac{10(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}$$

```
[Out] 10/3*(-a*c*x+c)^(3/2)/a^3/c-8/5*(-a*c*x+c)^(5/2)/a^3/c^2+2/7*(-a*c*x+c)^(7/
2)/a^3/c^3-4*(-a*c*x+c)^(1/2)/a^3
```

Rubi [A] time = 0.16, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6130, 21, 77}

$$\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{8(c-acx)^{5/2}}{5a^3c^2}+\frac{10(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^3 + (10*(c - a*c*x)^(3/2))/(3*a^3*c) - (8*(c - a*c*x
)^(5/2))/(5*a^3*c^2) + (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= \int \frac{x^2(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\
&= c\int \frac{x^2(1+ax)}{\sqrt{c-acx}}\,dx\\
&= c\int \left(\frac{2}{a^2\sqrt{c-acx}}-\frac{5\sqrt{c-acx}}{a^2c}+\frac{4(c-acx)^{3/2}}{a^2c^2}-\frac{(c-acx)^{5/2}}{a^2c^3}\right)dx\\
&= -\frac{4\sqrt{c-acx}}{a^3}+\frac{10(c-acx)^{3/2}}{3a^3c}-\frac{8(c-acx)^{5/2}}{5a^3c^2}+\frac{2(c-acx)^{7/2}}{7a^3c^3}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.50

$$
-\frac{2\left(15a^3x^3+39a^2x^2+52ax+104\right)\sqrt{c-acx}}{105a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(104 + 52*a*x + 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3)
```

fricas [A] time = 0.42, size = 36, normalized size = 0.45

$$
-\frac{2\left(15\,a^3x^3+39\,a^2x^2+52\,ax+104\right)\sqrt{-acx+c}}{105\,a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/105*(15*a^3*x^3 + 39*a^2*x^2 + 52*a*x + 104)*sqrt(-a*c*x + c)/a^3
```

giac [B] time = 0.17, size = 142, normalized size = 1.78

$$
-\frac{2\left(\frac{7\left(3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}\,c^2\right)}{a^2c^2}+\frac{3\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}\,c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\,\sqrt{-acx+c}\,c^3\right)}{a^2c^3}\right)}{105\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*(7*(3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*
sqrt(-a*c*x + c)*c^2)/(a^2*c^2) + 3*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*
(a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c
*x + c)*c^3)/(a^2*c^3))/a
```

maple [A] time = 0.03, size = 37, normalized size = 0.46

$$-\frac{2\sqrt{-acx+c}\,\left(15x^3a^3+39a^2x^2+52ax+104\right)}{105a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/105*(-a*c*x+c)^(1/2)*(15*a^3*x^3+39*a^2*x^2+52*a*x+104)/a^3
```

maxima [A] time = 0.31, size = 60, normalized size = 0.75

$$\frac{2\left(15\,(-acx+c)^{\frac{7}{2}}-84\,(-acx+c)^{\frac{5}{2}}c+175\,(-acx+c)^{\frac{3}{2}}c^2-210\,\sqrt{-acx+c}\,c^3\right)}{105\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/105*(15*(-a*c*x + c)^(7/2) - 84*(-a*c*x + c)^(5/2)*c + 175*(-a*c*x + c)^(
3/2)*c^2 - 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

mupad [B] time = 0.06, size = 66, normalized size = 0.82

$$\frac{10\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^3\,c}-\frac{4\,\sqrt{c-a\,c\,x}}{a^3}-\frac{8\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^3\,c^2}+\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - a*c*x)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (10*(c - a*c*x)^(3/2))/(3*a^3*c) - (4*(c - a*c*x)^(1/2))/a^3 - (8*(c - a*c*
x)^(5/2))/(5*a^3*c^2) + (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

sympy [A] time = 8.68, size = 68, normalized size = 0.85

$$-\frac{2\left(2c^3\sqrt{-acx+c}-\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{4c(-acx+c)^{\frac{5}{2}}}{5}-\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*(2*c**3*sqrt(-a*c*x + c) - 5*c**2*(-a*c*x + c)**(3/2)/3 + 4*c*(-a*c*x +
c)**(5/2)/5 - (-a*c*x + c)**(7/2)/7)/(a**3*c**3)
```

3.395 $\int e^{2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=57

$$-\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2(c-acx)^{3/2}}{a^2c}-\frac{4\sqrt{c-acx}}{a^2}$$

```
[Out] 2*(-a*c*x+c)^(3/2)/a^2/c-2/5*(-a*c*x+c)^(5/2)/a^2/c^2-4*(-a*c*x+c)^(1/2)/a^
2
```

Rubi [A] time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6130, 21, 77}

$$-\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2(c-acx)^{3/2}}{a^2c}-\frac{4\sqrt{c-acx}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^2 + (2*(c - a*c*x)^(3/2))/(a^2*c) - (2*(c - a*c*x)^(
5/2))/(5*a^2*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx&=\int\frac{x(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\&=c\int\frac{x(1+ax)}{\sqrt{c-acx}}\,dx\\&=c\int\left(\frac{2}{a\sqrt{c-acx}}-\frac{3\sqrt{c-acx}}{ac}+\frac{(c-acx)^{3/2}}{ac^2}\right)dx\\&=-\frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{a^2c}-\frac{2(c-acx)^{5/2}}{5a^2c^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 0.54

$$-\frac{2\left(a^2x^2+3ax+6\right)\sqrt{c-acx}}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*Sqrt[c - a*c*x],x]
[Out] (-2*Sqrt[c - a*c*x]*(6 + 3*a*x + a^2*x^2))/(5*a^2)
```

fricas [A] time = 0.65, size = 27, normalized size = 0.47

$$-\frac{2\left(a^2x^2+3\,ax+6\right)\sqrt{-acx+c}}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2/5*(a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)/a^2
```

giac [A] time = 0.16, size = 92, normalized size = 1.61

$$\frac{2\left(\frac{5\left((-acx+c)^{\frac{3}{2}}-3\,\sqrt{-acx+c}\,c\right)}{ac}-\frac{3\,(acx-c)^2\,\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}\,c^2}{ac^2}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/(a*c) - (3*(a*c*x - c)^
2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/(a*
c^2))/a
```

maple [A] time = 0.03, size = 28, normalized size = 0.49

$$-\frac{2\sqrt{-acx+c}\ \left(a^{2}x^{2}+3ax+6\right)}{5a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/5*(-a*c*x+c)^(1/2)*(a^2*x^2+3*a*x+6)/a^2
```

maxima [A] time = 0.30, size = 44, normalized size = 0.77

$$-\frac{2\left((-acx+c)^{\frac{5}{2}}-5\,(-acx+c)^{\frac{3}{2}}c+10\,\sqrt{-acx+c}\,c^{2}\right)}{5\,a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*((-a*c*x + c)^(5/2) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*c*x + c)*c^2
)/(a^2*c^2)
```

mupad [B] time = 0.05, size = 46, normalized size = 0.81

$$-\frac{2\,(c-a\,c\,x)^{5/2}-10\,c\,(c-a\,c\,x)^{3/2}+20\,c^2\,\sqrt{c-a\,c\,x}}{5\,a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - a*c*x)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -(2*(c - a*c*x)^(5/2) - 10*c*(c - a*c*x)^(3/2) + 20*c^2*(c - a*c*x)^(1/2))/
(5*a^2*c^2)
```

sympy [A] time = 7.41, size = 48, normalized size = 0.84

$$\frac{2\left(-2c^{2}\sqrt{-acx+c}\ +c\,(-acx+c)^{\frac{3}{2}}-\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(-2*c**2*sqrt(-a*c*x + c) + c*(-a*c*x + c)**(3/2) - (-a*c*x + c)**(5/2)/5
)/(a**2*c**2)
```

3.396 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a/c-4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6130, 21, 43}

$$\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\\ &= c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\\ &= -\frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.61

$$-\frac{2(ax+5)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
[Out] (-2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

fricas [A] time = 0.68, size = 19, normalized size = 0.50

$$-\frac{2\sqrt{-acx+c}\,(ax+5)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

giac [A] time = 0.16, size = 44, normalized size = 1.16

$$-\frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c}{c}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] -2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

maple [A] time = 0.03, size = 20, normalized size = 0.53

$$-\frac{2\sqrt{-acx+c}\,\left(ax+5\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x)
[Out] -2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

maxima [A] time = 0.31, size = 30, normalized size = 0.79

$$\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

mupad [B] time = 0.02, size = 32, normalized size = 0.84

$$\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a\,c}-\frac{4\,\sqrt{c-a\,c\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

sympy [A] time = 4.62, size = 31, normalized size = 0.82

$$-\frac{2\left(2c\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2),x)
[Out] -2*(2*c*sqrt(-a*c*x + c) - (-a*c*x + c)**(3/2)/3)/(a*c)
```

3.397 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=39

$$-2\sqrt{c-acx}-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] -2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-2*(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6130, 21, 80, 63, 208}

$$-2\sqrt{c-acx}-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{x(1-ax)}\,dx\\
&= c\int \frac{1+ax}{x\sqrt{c-acx}}\,dx\\
&= -2\sqrt{c-acx}+c\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= -2\sqrt{c-acx}-\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= -2\sqrt{c-acx}-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.00

$$-2\sqrt{c-acx}-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x,x]
[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

fricas [A] time = 0.71, size = 82, normalized size = 2.10

$$\left[\sqrt{c}\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)-2\sqrt{-acx+c}\,,2\sqrt{-c}\,\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-2\sqrt{-acx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas")

[Out] [sqrt(c)*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) - 2*sqrt(-a*c*x + c), 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x + c)]

giac [A] time = 0.20, size = 40, normalized size = 1.03

$$2\,c\left(\frac{\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")

[Out] 2*c*(arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/c)

maple [A] time = 0.03, size = 32, normalized size = 0.82

$$-2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}\;-2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x,x)

[Out] -2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-2*(-a*c*x+c)^(1/2)

maxima [A] time = 0.45, size = 48, normalized size = 1.23

$$\sqrt{c}\,\log\left(\frac{\sqrt{-acx+c}\;-\sqrt{c}}{\sqrt{-acx+c}\;+\sqrt{c}}\right)-2\,\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima")

[Out] sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c))) - 2*sqrt(-a*c*x + c)

mupad [B] time = 0.81, size = 31, normalized size = 0.79

$$-2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)-2\,\sqrt{c-a\,c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(1/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)),x)
[Out] - 2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) - 2*(c - a*c*x)^(1/2)
```

sympy [A] time = 7.28, size = 39, normalized size = 1.00

$$\frac{2c\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - 2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x,x)
[Out] 2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*sqrt(-a*c*x + c)
```

3.398 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{\sqrt{c-acx}}{x}-3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] -3*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.11, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6130, 21, 78, 63, 208}

$$-\frac{\sqrt{c-acx}}{x}-3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -(Sqrt[c - a*c*x]/x) - 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{x^2(1-ax)}\,dx \\
&= c\int \frac{1+ax}{x^2\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{x}+\frac{1}{2}(3ac)\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= -\frac{\sqrt{c-acx}}{x}-3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right) \\
&= -\frac{\sqrt{c-acx}}{x}-3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 1.00

$$-\frac{\sqrt{c-acx}}{x}-3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -(Sqrt[c - a*c*x]/x) - 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

fricas [A] time = 0.50, size = 96, normalized size = 2.23

$$\left[\frac{3\,a\sqrt{c}\,x\log\left(\frac{acx+2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)-2\,\sqrt{-acx+c}}{2\,x},\ \frac{3\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(3*a*sqrt(c)*x*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) - 2*s
qrt(-a*c*x + c))/x, (3*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - s
qrt(-a*c*x + c))/x]
```

giac [A] time = 0.17, size = 47, normalized size = 1.09

$$\frac{\frac{3\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}\,a}{x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)*a/x)
/a
```

maple [A] time = 0.04, size = 45, normalized size = 1.05

$$2ac\left(-\frac{\sqrt{-acx+c}}{2xac}-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] 2*a*c*(-1/2*(-a*c*x+c)^(1/2)/x/a/c-3/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(
1/2)))
```

maxima [A] time = 0.40, size = 62, normalized size = 1.44

$$\frac{1}{2}\,ac\left(\frac{3\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\,\sqrt{-acx+c}}{acx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*a*c*(3*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/s
qrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

mupad [B] time = 0.84, size = 35, normalized size = 0.81

$$-\frac{\sqrt{c-a\,c\,x}}{x}-3\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(1/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] - (c - a*c*x)^(1/2)/x - 3*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2))
```

sympy [B] time = 19.59, size = 119, normalized size = 2.77

$$\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{2ac\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 - a*c**2
*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + 2*a*c*atan(sq
rt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/x
```

3.399 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=68

$$-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}$$

```
[Out] -7/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-1/2*(-a*c*x+c)^(1/2)/x^2
-7/4*a*(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 78, 51, 63, 208}

$$-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] -Sqrt[c - a*c*x]/(2*x^2) - (7*a*Sqrt[c - a*c*x])/(4*x) - (7*a^2*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= \int \frac{(1+ax)\sqrt{c-acx}}{x^3(1-ax)}\,dx\\
&= c\int \frac{1+ax}{x^3\sqrt{c-acx}}\,dx\\
&= -\frac{\sqrt{c-acx}}{2x^2}+\frac{1}{4}(7ac)\int \frac{1}{x^2\sqrt{c-acx}}\,dx\\
&= -\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}+\frac{1}{8}\left(7a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= -\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}-\frac{1}{4}(7a)\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right)\\
&= -\frac{\sqrt{c-acx}}{2x^2}-\frac{7a\sqrt{c-acx}}{4x}-\frac{7}{4}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.81

$$-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{(7ax+2)\sqrt{c-acx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] -1/4*((2 + 7*a*x)*Sqrt[c - a*c*x])/x^2 - (7*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/Sqrt[c]])/4
```

fricas [A] time = 0.56, size = 117, normalized size = 1.72

$$\left[\frac{7\,a^2\sqrt{c}\,x^2\log\left(\frac{acx+2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)-2\,\sqrt{-acx+c}\,(7\,ax+2)}{8\,x^2},\frac{7\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-\sqrt{-acx+c}\,(7\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] [1/8*(7*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) -
 2*sqrt(-a*c*x + c)*(7*a*x + 2))/x^2, 1/4*(7*a^2*sqrt(-c)*x^2*arctan(sqrt(-
a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(7*a*x + 2))/x^2]
```

giac [A] time = 0.18, size = 76, normalized size = 1.12

$$\frac{\frac{7\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{7\,(-acx+c)^{\frac{3}{2}}a^3c-9\,\sqrt{-acx+c}\,a^3c^2}{a^2c^2x^2}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(7*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + (7*(-a*c*x + c)^(
3/2)*a^3*c - 9*sqrt(-a*c*x + c)*a^3*c^2)/(a^2*c^2*x^2))/a
```

maple [A] time = 0.04, size = 65, normalized size = 0.96

$$-2a^2c^2\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{x^2a^2c^2}+\frac{7\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^3,x)
```

```
[Out] -2*a^2*c^2*((-7/8/c*(-a*c*x+c)^(3/2)+9/8*(-a*c*x+c)^(1/2))/x^2/a^2/c^2+7/8/
c^(3/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.40, size = 103, normalized size = 1.51

$$\frac{1}{8}\,a^2c^2\left(\frac{2\left(7\,(-acx+c)^{\frac{3}{2}}-9\,\sqrt{-acx+c}\,c\right)}{(acx-c)^2c+2\,(acx-c)c^2+c^3}+\frac{7\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/8*a^2*c^2*(2*(7*(-a*c*x + c)^(3/2) - 9*sqrt(-a*c*x + c)*c)/((a*c*x - c)^2
*c + 2*(a*c*x - c)*c^2 + c^3) + 7*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a
*c*x + c) + sqrt(c)))/c^(3/2))
```

mupad [B] time = 0.08, size = 54, normalized size = 0.79

$$\frac{7\,{\left(c-a\,c\,x\right)}^{3/2}}{4\,c\,x^2}-\frac{7\,a^2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)}{4}-\frac{9\,\sqrt{c-a\,c\,x}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(1/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] (7*(c - a*c*x)^(3/2))/(4*c*x^2) - (7*a^2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^
(1/2)))/4 - (9*(c - a*c*x)^(1/2))/(4*x^2)
```

sympy [B] time = 37.00, size = 270, normalized size = 3.97

$$-\frac{10a^2c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2\,(-acx+c)^2}+\frac{6a^2c^3\,(-acx+c)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2\,(-acx+c)^2}+\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\,\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}-\frac{3a^2c^3}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] -10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)*
*2) + 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*
```

```
x + c)**2) + 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*
x + c))/8 - 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x
+ c))/8 + a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c
))/2 - a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2
 - a*sqrt(-a*c*x + c)/x
```

3.400 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=89

$$-\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{11a^2\sqrt{c-acx}}{8x}-\frac{\sqrt{c-acx}}{3x^3}-\frac{11a\sqrt{c-acx}}{12x^2}$$

```
[Out] -11/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-1/3*(-a*c*x+c)^(1/2)/x^
3-11/12*a*(-a*c*x+c)^(1/2)/x^2-11/8*a^2*(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 78, 51, 63, 208}

$$-\frac{11a^2\sqrt{c-acx}}{8x}-\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{11a\sqrt{c-acx}}{12x^2}-\frac{\sqrt{c-acx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] -Sqrt[c - a*c*x]/(3*x^3) - (11*a*Sqrt[c - a*c*x])/(12*x^2) - (11*a^2*Sqrt[c
 - a*c*x])/(8*x) - (11*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{(1+ax)\sqrt{c-acx}}{x^4(1-ax)}\,dx$$
$$= c\int \frac{1+ax}{x^4\sqrt{c-acx}}\,dx$$
$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{1}{6}(11ac)\int \frac{1}{x^3\sqrt{c-acx}}\,dx$$
$$= -\frac{\sqrt{c-acx}}{3x^3} - \frac{11a\sqrt{c-acx}}{12x^2} + \frac{1}{8}\left(11a^2c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx$$
$$= -\frac{\sqrt{c-acx}}{3x^3} - \frac{11a\sqrt{c-acx}}{12x^2} - \frac{11a^2\sqrt{c-acx}}{8x} + \frac{1}{16}\left(11a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx$$
$$= -\frac{\sqrt{c-acx}}{3x^3} - \frac{11a\sqrt{c-acx}}{12x^2} - \frac{11a^2\sqrt{c-acx}}{8x} - \frac{1}{8}\left(11a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-\right.$$
$$= -\frac{\sqrt{c-acx}}{3x^3} - \frac{11a\sqrt{c-acx}}{12x^2} - \frac{11a^2\sqrt{c-acx}}{8x} - \frac{11}{8}a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.71

$$-\frac{11}{8}a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - \frac{\left(33a^2x^2+22ax+8\right)\sqrt{c-acx}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] -1/24*(Sqrt[c - a*c*x]*(8 + 22*a*x + 33*a^2*x^2))/x^3 - (11*a^3*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

fricas [A] time = 0.80, size = 133, normalized size = 1.49

$$\left[\frac{33\,a^3\sqrt{c}\,x^3\log\left(\frac{acx+2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right) - 2\left(33\,a^2x^2+22\,ax+8\right)\sqrt{-acx+c}}{48\,x^3}, \frac{33\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right) - (3}{24\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/48*(33*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x)
 - 2*(33*a^2*x^2 + 22*a*x + 8)*sqrt(-a*c*x + c))/x^3, 1/24*(33*a^3*sqrt(-c)
*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (33*a^2*x^2 + 22*a*x + 8)*sqrt(-
a*c*x + c))/x^3]
```

giac [A] time = 0.17, size = 104, normalized size = 1.17

$$\frac{\frac{33\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{33\,(acx-c)^2\sqrt{-acx+c}\,a^4c-88\,(-acx+c)^{\frac{3}{2}}a^4c^2+63\,\sqrt{-acx+c}\,a^4c^3}{a^3c^3x^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(33*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (33*(a*c*x - c)
^2*sqrt(-a*c*x + c)*a^4*c - 88*(-a*c*x + c)^(3/2)*a^4*c^2 + 63*sqrt(-a*c*x
+ c)*a^4*c^3)/(a^3*c^3*x^3))/a
```

maple [A] time = 0.04, size = 80, normalized size = 0.90

$$2a^3c^3\left(-\frac{\frac{11(-acx+c)^{\frac{5}{2}}}{16c^2}-\frac{11(-acx+c)^{\frac{3}{2}}}{6c}+\frac{21\sqrt{-acx+c}}{16}}{x^3a^3c^3}-\frac{11\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^4,x)
```

```
[Out] 2*a^3*c^3*(-(11/16/c^2*(-a*c*x+c)^(5/2)-11/6/c*(-a*c*x+c)^(3/2)+21/16*(-a*c
*x+c)^(1/2))/x^3/a^3/c^3-11/16/c^(5/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.47, size = 134, normalized size = 1.51

$$-\frac{1}{48}\,a^3c^3\left(\frac{2\left(33\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+63\,\sqrt{-acx+c}\,c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{33\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] -1/48*a^3*c^3*(2*(33*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 63*sqrt
(-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)*
```

```
c^4 + c^5) - 33*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c
)))/c^(5/2))
```

mupad [B] time = 0.07, size = 74, normalized size = 0.83

$$\frac{11\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,c\,x^3}-\frac{21\,\sqrt{c-a\,c\,x}}{8\,x^3}-\frac{11\,{\left(c-a\,c\,x\right)}^{5/2}}{8\,c^2\,x^3}+\frac{a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,11{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(1/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/c^(1/2))*11i)/8 - (21*(c - a*c*x)^
(1/2))/(8*x^3) + (11*(c - a*c*x)^(3/2))/(3*c*x^3) - (11*(c - a*c*x)^(5/2))/
(8*c^2*x^3)
```

sympy [B] time = 55.92, size = 439, normalized size = 4.93

$$\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}-\frac{80a^3c^5\left(-acx+c\right)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] 66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x +
 c)**2 + 48*c**3*(-a*c*x + c)**3) - 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144*
a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) +
30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*
x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 10*a**3*c**4*sqrt(-a*c*x + c)/(16*a*
c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 5*a**3*c**4*sqrt(c**(-7))*log(-
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 - 5*a**3*c**4*sqrt(c**(-7))*log(c
**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 6*a**3*c**3*(-a*c*x + c)**(3/2)/
(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 3*a**3*c**3*sqrt(c**(-5))
*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 - 3*a**3*c**3*sqrt(c**(-5))*
log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8
```

3.401 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=110

$$-\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{75a^3\sqrt{c-acx}}{64x}-\frac{25a^2\sqrt{c-acx}}{32x^2}-\frac{\sqrt{c-acx}}{4x^4}-\frac{5a\sqrt{c-acx}}{8x^3}$$

```
[Out] -75/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-1/4*(-a*c*x+c)^(1/2)/x
^4-5/8*a*(-a*c*x+c)^(1/2)/x^3-25/32*a^2*(-a*c*x+c)^(1/2)/x^2-75/64*a^3*(-a*
c*x+c)^(1/2)/x
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 78, 51, 63, 208}

$$-\frac{25a^2\sqrt{c-acx}}{32x^2}-\frac{75a^3\sqrt{c-acx}}{64x}-\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-\frac{5a\sqrt{c-acx}}{8x^3}-\frac{\sqrt{c-acx}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] -Sqrt[c - a*c*x]/(4*x^4) - (5*a*Sqrt[c - a*c*x])/(8*x^3) - (25*a^2*Sqrt[c -
 a*c*x])/(32*x^2) - (75*a^3*Sqrt[c - a*c*x])/(64*x) - (75*a^4*Sqrt[c]*ArcTa
nh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{(1+ax)\sqrt{c-acx}}{x^5(1-ax)}\,dx$$
$$= c\int \frac{1+ax}{x^5\sqrt{c-acx}}\,dx$$
$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{1}{8}(15ac)\int \frac{1}{x^4\sqrt{c-acx}}\,dx$$
$$= -\frac{\sqrt{c-acx}}{4x^4} - \frac{5a\sqrt{c-acx}}{8x^3} + \frac{1}{16}\left(25a^2c\right)\int \frac{1}{x^3\sqrt{c-acx}}\,dx$$
$$= -\frac{\sqrt{c-acx}}{4x^4} - \frac{5a\sqrt{c-acx}}{8x^3} - \frac{25a^2\sqrt{c-acx}}{32x^2} + \frac{1}{64}\left(75a^3c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx$$
$$= -\frac{\sqrt{c-acx}}{4x^4} - \frac{5a\sqrt{c-acx}}{8x^3} - \frac{25a^2\sqrt{c-acx}}{32x^2} - \frac{75a^3\sqrt{c-acx}}{64x} + \frac{1}{128}\left(75a^4c\right)\int \frac{}{x\sqrt{}}$$
$$= -\frac{\sqrt{c-acx}}{4x^4} - \frac{5a\sqrt{c-acx}}{8x^3} - \frac{25a^2\sqrt{c-acx}}{32x^2} - \frac{75a^3\sqrt{c-acx}}{64x} - \frac{1}{64}\left(75a^3\right)\text{Subst}\Bigg($$
$$= -\frac{\sqrt{c-acx}}{4x^4} - \frac{5a\sqrt{c-acx}}{8x^3} - \frac{25a^2\sqrt{c-acx}}{32x^2} - \frac{75a^3\sqrt{c-acx}}{64x} - \frac{75}{64}a^4\sqrt{c}\tanh^{-1}\Bigg($$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.65

$$-\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - \frac{\left(75a^3x^3+50a^2x^2+40ax+16\right)\sqrt{c-acx}}{64x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^5,x]
[Out] -1/64*(Sqrt[c - a*c*x]*(16 + 40*a*x + 50*a^2*x^2 + 75*a^3*x^3))/x^4 - (75*a
^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

fricas [A] time = 0.75, size = 149, normalized size = 1.35

$$\left[\frac{75\,a^4\sqrt{c}\,x^4\log\left(\frac{acx+2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right) - 2\left(75\,a^3x^3+50\,a^2x^2+40\,ax+16\right)\sqrt{-acx+c}}{128\,x^4}, \frac{75\,a^4\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{-a}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fricas"
)
```

```
[Out] [1/128*(75*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x
) - 2*(75*a^3*x^3 + 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4, 1/64*(
75*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (75*a^3*x^3 + 50*
a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4]
```

giac [A] time = 0.39, size = 131, normalized size = 1.19

$$\frac{\frac{75\,a^5c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{75\,(acx-c)^3\sqrt{-acx+c}\,a^5c+275\,(acx-c)^2\sqrt{-acx+c}\,a^5c^2-365\,(-acx+c)^{\frac{3}{2}}a^5c^3+181\,\sqrt{-acx+c}\,a^5c^4}{a^4c^4x^4}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/64*(75*a^5*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (75*(a*c*x - c)
^3*sqrt(-a*c*x + c)*a^5*c + 275*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^5*c^2 - 36
5*(-a*c*x + c)^(3/2)*a^5*c^3 + 181*sqrt(-a*c*x + c)*a^5*c^4)/(a^4*c^4*x^4))
/a
```

maple [A] time = 0.04, size = 93, normalized size = 0.85

$$-2a^4c^4\left(\frac{-\frac{75(-acx+c)^{\frac{7}{2}}}{128c^3}+\frac{275(-acx+c)^{\frac{5}{2}}}{128c^2}-\frac{365(-acx+c)^{\frac{3}{2}}}{128c}+\frac{181\sqrt{-acx+c}}{128}}{x^4a^4c^4}+\frac{75\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{128c^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^5,x)
```

```
[Out] -2*a^4*c^4*((-75/128/c^3*(-a*c*x+c)^(7/2)+275/128/c^2*(-a*c*x+c)^(5/2)-365/
128/c*(-a*c*x+c)^(3/2)+181/128*(-a*c*x+c)^(1/2))/x^4/a^4/c^4+75/128/c^(7/2)
*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.46, size = 163, normalized size = 1.48

$$\frac{1}{128}\,a^4c^4\left(\frac{2\left(75\,(-acx+c)^{\frac{7}{2}}-275\,(-acx+c)^{\frac{5}{2}}c+365\,(-acx+c)^{\frac{3}{2}}c^2-181\,\sqrt{-acx+c}\,c^3\right)}{(acx-c)^4c^3+4\,(acx-c)^3c^4+6\,(acx-c)^2c^5+4\,(acx-c)c^6+c^7}+\frac{75\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxima"
)
```

```
[Out] 1/128*a^4*c^4*(2*(75*(-a*c*x + c)^(7/2) - 275*(-a*c*x + c)^(5/2)*c + 365*(-
a*c*x + c)^(3/2)*c^2 - 181*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4*(a*
c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 75*log((s
qrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(7/2))
```

mupad [B] time = 0.10, size = 91, normalized size = 0.83

$$\frac{365\,{\left(c-a\,c\,x\right)}^{3/2}}{64\,c\,x^4}-\frac{181\,\sqrt{c-a\,c\,x}}{64\,x^4}-\frac{275\,{\left(c-a\,c\,x\right)}^{5/2}}{64\,c^2\,x^4}+\frac{75\,{\left(c-a\,c\,x\right)}^{7/2}}{64\,c^3\,x^4}+\frac{a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,75{}\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a*c*x)^(1/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)),x)
```

```
[Out] (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/c^(1/2))*75i)/64 - (181*(c - a*c*x
)^(1/2))/(64*x^4) + (365*(c - a*c*x)^(3/2))/(64*c*x^4) - (275*(c - a*c*x)^(
5/2))/(64*c^2*x^4) + (75*(c - a*c*x)^(7/2))/(64*c^3*x^4)
```

sympy [B] time = 78.97, size = 639, normalized size = 5.81

$$-\frac{558a^4c^8\sqrt{-acx+c}}{1536ac^8x-1152c^8+2304c^6\left(-acx+c\right)^2-1536c^5\left(-acx+c\right)^3+384c^4\left(-acx+c\right)^4}+\frac{}{1536ac^8x-1152c^8+230}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] -558*a**4*c**8*sqrt(-a*c*x + c)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*
c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 1022*
a**4*c**7*(-a*c*x + c)**(3/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*
x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 770*a**
4*c**6*(-a*c*x + c)**(5/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x +
 c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 66*a**4*c*
*6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 4
8*c**3*(-a*c*x + c)**3) + 210*a**4*c**5*(-a*c*x + c)**(7/2)/(1536*a*c**8*x
- 1152*c**8 + 2304*c**6*(-a*c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c
**4*(-a*c*x + c)**4) - 80*a**4*c**5*(-a*c*x + c)**(3/2)/(-144*a*c**6*x + 96
*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) + 35*a**4*c**5*
sqrt(c**(-9))*log(-c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 - 35*a**4*c**
5*sqrt(c**(-9))*log(c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 30*a**4*c*
*4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2
+ 48*c**3*(-a*c*x + c)**3) + 5*a**4*c**4*sqrt(c**(-7))*log(-c**4*sqrt(c**(-
7)) + sqrt(-a*c*x + c))/16 - 5*a**4*c**4*sqrt(c**(-7))*log(c**4*sqrt(c**(-7
)) + sqrt(-a*c*x + c))/16
```

3.402 $\int e^{3\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=249

$$-\frac{2(ax+1)^{9/2}(c-acx)^{3/2}}{9a^4c(1-ax)^{3/2}}+\frac{2(ax+1)^{7/2}(c-acx)^{3/2}}{7a^4c(1-ax)^{3/2}}-\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}\,(c}{a^4c(1-}$$

```
[Out] -2/3*(a*x+1)^(3/2)*(-a*c*x+c)^(3/2)/a^4/c/(-a*x+1)^(3/2)-2/5*(a*x+1)^(5/2)*
(-a*c*x+c)^(3/2)/a^4/c/(-a*x+1)^(3/2)+2/7*(a*x+1)^(7/2)*(-a*c*x+c)^(3/2)/a^
4/c/(-a*x+1)^(3/2)-2/9*(a*x+1)^(9/2)*(-a*c*x+c)^(3/2)/a^4/c/(-a*x+1)^(3/2)+
4*(-a*c*x+c)^(3/2)*arctanh(1/2*(a*x+1)^(1/2)*2^(1/2))*2^(1/2)/a^4/c/(-a*x+1
)^(3/2)-4*(-a*c*x+c)^(3/2)*(a*x+1)^(1/2)/a^4/c/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 88, 50, 63, 208}

$$-\frac{2(ax+1)^{9/2}(c-acx)^{3/2}}{9a^4c(1-ax)^{3/2}}+\frac{2(ax+1)^{7/2}(c-acx)^{3/2}}{7a^4c(1-ax)^{3/2}}-\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}\,(c}{a^4c(1-}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(a^4*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)
^(3/2)*(c - a*c*x)^(3/2))/(3*a^4*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(5/2)*(c
 - a*c*x)^(3/2))/(5*a^4*c*(1 - a*x)^(3/2)) + (2*(1 + a*x)^(7/2)*(c - a*c*x)
^(3/2))/(7*a^4*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(9/2)*(c - a*c*x)^(3/2))/(
9*a^4*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(3/2)*ArcTanh[Sqrt[1 + a*
x]/Sqrt[2]])/(a^4*c*(1 - a*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \int \frac{x^3(1+ax)^{3/2}\sqrt{c-acx}}{(1-ax)^{3/2}}\,dx$$

$$= \frac{(c-acx)^{3/2}\int \frac{x^3(1+ax)^{3/2}}{c-acx}\,dx}{(1-ax)^{3/2}}$$

$$= \frac{(c-acx)^{3/2}\int \left(-\frac{(1+ax)^{3/2}}{a^3c}+\frac{(1+ax)^{5/2}}{a^3c}-\frac{(1+ax)^{7/2}}{a^3c}+\frac{(1+ax)^{3/2}}{a^3(c-acx)}\right)dx}{(1-ax)^{3/2}}$$

$$= -\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{9/2}(c-acx)^{3/2}}{9a^4c(1-ax)^{3/2}}+\frac{(c}{}$$

$$= -\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^4c(1-ax)^{3/2}}-2($$

$$= -\frac{4\sqrt{1+ax}\,(c-acx)^{3/2}}{a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1}{}$$

$$= -\frac{4\sqrt{1+ax}\,(c-acx)^{3/2}}{a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1}{}$$

$$= -\frac{4\sqrt{1+ax}\,(c-acx)^{3/2}}{a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^4c(1-ax)^{3/2}}-\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^4c(1-ax)^{3/2}}+\frac{2(1}{}$$

Mathematica [A] time = 0.08, size = 92, normalized size = 0.37

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}\left(35a^4x^4+95a^3x^3+138a^2x^2+236ax+788\right)-630\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{315a^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^3*Sqrt[c - a*c*x],x]

[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(788 + 236*a*x + 138*a^2*x^2 + 95*a^3*x^
3 + 35*a^4*x^4) - 630*Sqrt[2]*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(315*a^4*Sqr
t[1 - a*x])
```

fricas [A] time = 0.54, size = 274, normalized size = 1.10

$$\left[\frac{2\left(315\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(35\,a^4x^4+95\,a^3x^3+138\,a^2x^2+236\,ax+7\right.\right.}{315\left(a^5x-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [2/315*(315*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)
*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1))
+ (35*a^4*x^4 + 95*a^3*x^3 + 138*a^2*x^2 + 236*a*x + 788)*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c))/(a^5*x - a^4), 2/315*(630*sqrt(2)*(a*x - 1)*sqrt(-c)*ar
ctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c))
+ (35*a^4*x^4 + 95*a^3*x^3 + 138*a^2*x^2 + 236*a*x + 788)*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c))/(a^5*x - a^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 146, normalized size = 0.59

$$-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(-35x^4a^4\sqrt{c\,(ax+1)}-95x^3a^3\sqrt{c\,(ax+1)}-138x^2a^2\sqrt{c\,(ax+1)}+630\sqrt{c}\,\sqrt{2}\,\mathrm{ar}\right.}{315\,(ax-1)\sqrt{c\,(ax+1)}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/315*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(-35*x^4*a^4*(c*(a*x+1))^(1/2)
-95*x^3*a^3*(c*(a*x+1))^(1/2)-138*x^2*a^2*(c*(a*x+1))^(1/2)+630*c^(1/2)*2^(
1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-236*x*a*(c*(a*x+1))^(1/
2)-788*(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)^3x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*(c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{- c \left(a x - 1\right)} \left(a x + 1\right)^{3}}{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

3.403 $\int e^{3\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=169

$$-\frac{2(ax+1)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}\,(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{a^3c(1-ax)^{3/2}}$$

```
[Out] -2/3*(a*x+1)^(3/2)*(-a*c*x+c)^(3/2)/a^3/c/(-a*x+1)^(3/2)-2/7*(a*x+1)^(7/2)*
(-a*c*x+c)^(3/2)/a^3/c/(-a*x+1)^(3/2)+4*(-a*c*x+c)^(3/2)*arctanh(1/2*(a*x+1
)^(1/2)*2^(1/2))*2^(1/2)/a^3/c/(-a*x+1)^(3/2)-4*(-a*c*x+c)^(3/2)*(a*x+1)^(1
/2)/a^3/c/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 88, 50, 63, 208}

$$-\frac{2(ax+1)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}\,(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{a^3c(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(a^3*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)
^(3/2)*(c - a*c*x)^(3/2))/(3*a^3*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(7/2)*(c
 - a*c*x)^(3/2))/(7*a^3*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(3/2)*A
rcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(a^3*c*(1 - a*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \int \frac{x^2(1+ax)^{3/2}\sqrt{c-acx}}{(1-ax)^{3/2}}\,dx$$
$$= \frac{(c-acx)^{3/2}\int \frac{x^2(1+ax)^{3/2}}{c-acx}\,dx}{(1-ax)^{3/2}}$$
$$= \frac{(c-acx)^{3/2}\int \left(-\frac{(1+ax)^{5/2}}{a^2c}+\frac{(1+ax)^{3/2}}{a^2(c-acx)}\right)dx}{(1-ax)^{3/2}}$$
$$= -\frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{c-acx}\,dx}{a^2(1-ax)^{3/2}}$$
$$= -\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}} - \frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}} + \frac{\left(2(c-acx)^{3/2}\right)\int \frac{\sqrt{1+ax}}{c-acx}\,dx}{a^2(1-ax)^{3/2}}$$
$$= -\frac{4\sqrt{1+ax}\,(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}} - \frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}} - \frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}} + \frac{(4}{}$$
$$= -\frac{4\sqrt{1+ax}\,(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}} - \frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}} - \frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}} + \frac{(8}{}$$
$$= -\frac{4\sqrt{1+ax}\,(c-acx)^{3/2}}{a^3c(1-ax)^{3/2}} - \frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^3c(1-ax)^{3/2}} - \frac{2(1+ax)^{7/2}(c-acx)^{3/2}}{7a^3c(1-ax)^{3/2}} + \frac{4}{}$$

Mathematica [A] time = 0.07, size = 84, normalized size = 0.50

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}\left(3a^3x^3+9a^2x^2+16ax+52\right)-42\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{21a^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(52 + 16*a*x + 9*a^2*x^2 + 3*a^3*x^3) -
42*Sqrt[2]*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(21*a^3*Sqrt[1 - a*x])
```

fricas [A] time = 0.93, size = 258, normalized size = 1.53

$$\left[\frac{2\left(21\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(3\,a^3x^3+9\,a^2x^2+16\,ax+52\right)\sqrt{-a^2x^2+1}\right.}{21\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [2/21*(21*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) +
(3*a^3*x^3 + 9*a^2*x^2 + 16*a*x + 52)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/
(a^4*x - a^3), 2/21*(42*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2
*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 9*a^2*x
^2 + 16*a*x + 52)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^4*x - a^3)]
```

giac [A] time = 0.25, size = 130, normalized size = 0.77

$$\frac{2\,c^2\left(\frac{2\,\sqrt{2}\left(21\,c\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+40\,\sqrt{-c}\,\sqrt{c}\right)}{a^2\sqrt{-c}\,c}-\frac{\frac{42\,\sqrt{2}\,c^4\arctan\left(\frac{\sqrt{2}\,\sqrt{acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+3\,(acx+c)^{\frac{7}{2}}+7\,(acx+c)^{\frac{3}{2}}c^2+42\,\sqrt{acx+c}\,c^3}{a^2c^4}\right)}{21\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] 2/21*c^2*(2*sqrt(2)*(21*c*arctan(sqrt(c)/sqrt(-c)) + 40*sqrt(-c)*sqrt(c))/(
a^2*sqrt(-c)*c) - (42*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(a*c*x + c)/sqrt(-
c))/sqrt(-c) + 3*(a*c*x + c)^(7/2) + 7*(a*c*x + c)^(3/2)*c^2 + 42*sqrt(a*c*
x + c)*c^3)/(a^2*c^4))/(a*abs(c))
```

maple [A] time = 0.04, size = 129, normalized size = 0.76

$$-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(-3x^3a^3\sqrt{c\,(ax+1)}-9x^2a^2\sqrt{c\,(ax+1)}+42\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-16xa\cdots\right.}{21\,(ax-1)\sqrt{c\,(ax+1)}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/21*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(-3*x^3*a^3*(c*(a*x+1))^(1/2)-9
*x^2*a^2*(c*(a*x+1))^(1/2)+42*c^(1/2)*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)
*2^(1/2)/c^(1/2))-16*x*a*(c*(a*x+1))^(1/2)-52*(c*(a*x+1))^(1/2))/(a*x-1)/(c
*(a*x+1))^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)^3x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*(c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\,(ax-1)}\,\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

3.404 $\int e^{3\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=169

$$-\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}\,(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{a^2c(1-ax)^{3/2}}$$

```
[Out] -2/3*(a*x+1)^(3/2)*(-a*c*x+c)^(3/2)/a^2/c/(-a*x+1)^(3/2)-2/5*(a*x+1)^(5/2)*
(-a*c*x+c)^(3/2)/a^2/c/(-a*x+1)^(3/2)+4*(-a*c*x+c)^(3/2)*arctanh(1/2*(a*x+1
)^(1/2)*2^(1/2))*2^(1/2)/a^2/c/(-a*x+1)^(3/2)-4*(-a*c*x+c)^(3/2)*(a*x+1)^(1
/2)/a^2/c/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6130, 23, 80, 50, 63, 208}

$$-\frac{2(ax+1)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}}-\frac{2(ax+1)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}}-\frac{4\sqrt{ax+1}\,(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{a^2c(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(a^2*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)
^(3/2)*(c - a*c*x)^(3/2))/(3*a^2*c*(1 - a*x)^(3/2)) - (2*(1 + a*x)^(5/2)*(c
 - a*c*x)^(3/2))/(5*a^2*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(3/2)*A
rcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(a^2*c*(1 - a*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx = \int \frac{x(1+ax)^{3/2}\sqrt{c-acx}}{(1-ax)^{3/2}}\,dx$$
$$= \frac{(c-acx)^{3/2}\int \frac{x(1+ax)^{3/2}}{c-acx}\,dx}{(1-ax)^{3/2}}$$
$$= -\frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{c-acx}\,dx}{a(1-ax)^{3/2}}$$
$$= -\frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}} - \frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}} + \frac{\left(2(c-acx)^{3/2}\right)\int \frac{\sqrt{1+ax}}{c-acx}\,dx}{a(1-ax)^{3/2}}$$
$$= -\frac{4\sqrt{1+ax}\,(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}} - \frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}} - \frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}} + \frac{(4(c}{}$$
$$= -\frac{4\sqrt{1+ax}\,(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}} - \frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}} - \frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}} + \frac{(8(c}{}$$
$$= -\frac{4\sqrt{1+ax}\,(c-acx)^{3/2}}{a^2c(1-ax)^{3/2}} - \frac{2(1+ax)^{3/2}(c-acx)^{3/2}}{3a^2c(1-ax)^{3/2}} - \frac{2(1+ax)^{5/2}(c-acx)^{3/2}}{5a^2c(1-ax)^{3/2}} + \frac{4\sqrt{2}}{}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.45

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}\left(3a^2x^2+11ax+38\right)-30\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{15a^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(38 + 11*a*x + 3*a^2*x^2) - 30*Sqrt[2]*A
rcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(15*a^2*Sqrt[1 - a*x])
```

fricas [A] time = 0.53, size = 242, normalized size = 1.43

$$\left[\frac{2\left(15\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(3\,a^2x^2+11\,ax+38\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{15\left(a^3x-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [2/15*(15*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) +
(3*a^2*x^2 + 11*a*x + 38)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*x - a^2
), 2/15*(30*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sq
rt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (3*a^2*x^2 + 11*a*x + 38)*sqrt(-
a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^3*x - a^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 112, normalized size = 0.66

$$-\frac{2\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c\left(ax-1\right)}\,\left(-3x^{2}a^{2}\sqrt{c\left(ax+1\right)}+30\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-11xa\sqrt{c\left(ax+1\right)}-38\sqrt{c}}{15\left(ax-1\right)\sqrt{c\left(ax+1\right)}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/15*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(-3*x^2*a^2*(c*(a*x+1))^(1/2)+3
0*c^(1/2)*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-11*x*a*(c*
(a*x+1))^(1/2)-38*(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\left(ax+1\right)^{3}x}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3*x/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
[Out] int((x*(c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(ax-1\right)}\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(-a*c*x+c)**(1/2),x)
[Out] Integral(x*sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x
)
```

3.405 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=119

$$-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}$$

```
[Out] -2/3*c^2*(-a^2*x^2+1)^(3/2)/a/(-a*c*x+c)^(3/2)+4*arctanh(1/2*c^(1/2)*(-a^2*
x^2+1)^(1/2)*2^(1/2)/(-a*c*x+c)^(1/2))*2^(1/2)*c^(1/2)/a-4*c*(-a^2*x^2+1)^(
1/2)/a/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6127, 665, 661, 208}

$$-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*c*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a*c*x]) - (2*c^2*(1 - a^2*x^2)^(3/2))/
(3*a*(c - a*c*x)^(3/2)) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*
x^2])/(Sqrt[2]*Sqrt[c - a*c*x])])/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
```

```
|| EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= c^3\int \frac{\left(1-a^2x^2\right)^{3/2}}{(c-acx)^{5/2}}\,dx\\ &= -\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+\left(2c^2\right)\int \frac{\sqrt{1-a^2x^2}}{(c-acx)^{3/2}}\,dx\\ &= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+(4c)\int \frac{1}{\sqrt{c-acx}\,\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}-\left(8ac^2\right)\mathrm{Subst}\left(\int \frac{1}{-2a^2c+a^2c^2x^2}\,dx,x,\frac{\sqrt{1-a^2}}{\sqrt{c-ac}}\right.\\ &= -\frac{4c\sqrt{1-a^2x^2}}{a\sqrt{c-acx}}-\frac{2c^2\left(1-a^2x^2\right)^{3/2}}{3a(c-acx)^{3/2}}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{2}\sqrt{c-acx}}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.56

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}\,(ax+7)-6\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{3a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(7 + a*x) - 6*Sqrt[2]*ArcTanh[Sqrt[1 + a
*x]/Sqrt[2]]))/(3*a*Sqrt[1 - a*x])
```

fricas [A] time = 0.56, size = 220, normalized size = 1.85

$$\left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right)+\sqrt{-a^2x^2+1}\sqrt{-acx+c}(ax+7)\right)}{3(a^2x-a)}, \frac{2\left(6\sqrt{2}(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqr
t(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + sq
rt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 7))/(a^2*x - a), 2/3*(6*sqrt(2)*(a
*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c
)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x + 7))/(a^2*x
- a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 95, normalized size = 0.80

$$-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-c(ax-1)}\,\left(6\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-xa\sqrt{c(ax+1)}-7\sqrt{c(ax+1)}\right)}{3(ax-1)\sqrt{c(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*2^(1/2)*arctanh(1/2*(
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-x*a*(c*(a*x+1))^(1/2)-7*(c*(a*x+1))^(1/2)
)/(a*x-1)/(c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.406 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=119

$$-\frac{2\sqrt{ax+1}\,(c-acx)^{3/2}}{c(1-ax)^{3/2}}-\frac{2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}$$

```
[Out] -2*(-a*c*x+c)^(3/2)*arctanh((a*x+1)^(1/2))/c/(-a*x+1)^(3/2)+4*(-a*c*x+c)^(3
/2)*arctanh(1/2*(a*x+1)^(1/2)*2^(1/2))*2^(1/2)/c/(-a*x+1)^(3/2)-2*(-a*c*x+c
)^(3/2)*(a*x+1)^(1/2)/c/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 84, 156, 63, 208}

$$-\frac{2\sqrt{ax+1}\,(c-acx)^{3/2}}{c(1-ax)^{3/2}}-\frac{2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (-2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(c*(1 - a*x)^(3/2)) - (2*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]])/(c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x(1-ax)^{3/2}}\,dx$$
$$= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x(c-acx)}\,dx}{(1-ax)^{3/2}}$$
$$= -\frac{2\sqrt{1+ax}\,(c-acx)^{3/2}}{c(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-ac-3a^2cx}{x\sqrt{1+ax}\,(c-acx)}\,dx}{ac(1-ax)^{3/2}}$$
$$= -\frac{2\sqrt{1+ax}\,(c-acx)^{3/2}}{c(1-ax)^{3/2}} + \frac{\left(4a(c-acx)^{3/2}\right)\int \frac{1}{\sqrt{1+ax}\,(c-acx)}\,dx}{(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{1}{x\sqrt{1+a}}}{c(1-ax)^{3/2}}$$
$$= -\frac{2\sqrt{1+ax}\,(c-acx)^{3/2}}{c(1-ax)^{3/2}} + \frac{\left(8(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{2c-cx^2}\,dx, x, \sqrt{1+ax}\right)}{(1-ax)^{3/2}} + \frac{(2(c-}{}$$
$$= -\frac{2\sqrt{1+ax}\,(c-acx)^{3/2}}{c(1-ax)^{3/2}} - \frac{2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{1+ax}\right)}{c(1-ax)^{3/2}} + \frac{4\sqrt{2}\,(c-acx)^{3/2}\tanh}{c(1-ax)^{3/2}}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.55

$$-\frac{2\sqrt{c-acx}\left(\sqrt{ax+1}+\tanh^{-1}\left(\sqrt{ax+1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(Sqrt[1 + a*x] + ArcTanh[Sqrt[1 + a*x]] - 2*Sqrt[2]*Arc
Tanh[Sqrt[1 + a*x]/Sqrt[2]]))/Sqrt[1 - a*x]
```

fricas [A] time = 0.59, size = 321, normalized size = 2.70

$$\left[\frac{2\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right)+(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}}{ax^2-x}\right.}{ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] [(2*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + (a*x -
 1)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)
*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x
- 1), 2*(2*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqr
t(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - (a*x - 1)*sqrt(-c)*arctan(sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)
*sqrt(-a*c*x + c))/(a*x - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 98, normalized size = 0.82

$$-\frac{2\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c\,(ax-1)}\,\left(2\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)-\sqrt{c\,(ax+1)}\right)}{(ax-1)\sqrt{c\,(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] -2*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(2*c^(1/2)*2^(1/2)*arctanh(1/2*(c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-c^(1/2)*arctanh((c*(a*x+1))^(1/2)/c^(1/2))-
(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^3}{x\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}\left(ax+1\right)^{3}}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)),
 x)
```

3.407 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{\sqrt{ax+1}\,(c-acx)^{3/2}}{cx(1-ax)^{3/2}}-\frac{5a(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,a(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}$$

```
[Out] -5*a*(-a*c*x+c)^(3/2)*arctanh((a*x+1)^(1/2))/c/(-a*x+1)^(3/2)+4*a*(-a*c*x+c
)^(3/2)*arctanh(1/2*(a*x+1)^(1/2)*2^(1/2))*2^(1/2)/c/(-a*x+1)^(3/2)-(-a*c*x
+c)^(3/2)*(a*x+1)^(1/2)/c/x/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 98, 156, 63, 208}

$$-\frac{\sqrt{ax+1}\,(c-acx)^{3/2}}{cx(1-ax)^{3/2}}-\frac{5a(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,a(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(c*x*(1 - a*x)^(3/2))) - (5*a*(c - a*c*
x)^(3/2)*ArcTanh[Sqrt[1 + a*x]])/(c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a*(c - a*
c*x)^(3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x^2(1-ax)^{3/2}}\,dx$$
$$= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x^2(c-acx)}\,dx}{(1-ax)^{3/2}}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{cx(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-\frac{5ac}{2}-\frac{3}{2}a^2cx}{x\sqrt{1+ax}\,(c-acx)}\,dx}{c(1-ax)^{3/2}}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{cx(1-ax)^{3/2}} + \frac{\left(4a^2(c-acx)^{3/2}\right)\int \frac{1}{\sqrt{1+ax}\,(c-acx)}\,dx}{(1-ax)^{3/2}} + \frac{\left(5a(c-acx)^{3/2}\right)\int \frac{}{x\sqrt{1}}}{2c(1-ax)^{3/2}}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{cx(1-ax)^{3/2}} + \frac{\left(8a(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{2c-cx^2}\,dx, x, \sqrt{1+ax}\right)}{(1-ax)^{3/2}} + \frac{\left(5(c-a\right.}{}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{cx(1-ax)^{3/2}} - \frac{5a(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{1+ax}\right)}{c(1-ax)^{3/2}} + \frac{4\sqrt{2}\,a(c-acx)^{3/2}\tanh^{}}{c(1-ax)^{3/2}}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 0.60

$$-\frac{\sqrt{c-acx}\left(\sqrt{ax+1}+5ax\tanh^{-1}\left(\sqrt{ax+1}\right)-4\sqrt{2}\,ax\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)\right)}{x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[c - a*c*x]*(Sqrt[1 + a*x] + 5*a*x*ArcTanh[Sqrt[1 + a*x]] - 4*Sqrt[2
]*a*x*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(x*Sqrt[1 - a*x]))
```

fricas [A] time = 0.52, size = 358, normalized size = 2.89

$$\left[\frac{4\sqrt{2}\left(a^2x^2-ax\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+5\left(a^2x^2-ax\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-a^2x^2+1}\,\sqrt{-}}{ax^2-x}\right.}{2\left(ax^2-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] [1/2*(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(
2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x + 1)
) + 5*(a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c))/(a*x^2 - x), (4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt(2
)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 5*(a^2*x^
2 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*
c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x^2 - x)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.05, size = 105, normalized size = 0.85

$$\frac{\left(-4\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\ \sqrt{2}}{2\sqrt{c}}\right)xac+5\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)xac+\sqrt{c\,(ax+1)}\ \sqrt{c}\right)\sqrt{-a^{2}x^{2}+1}\ \sqrt{-c\,(ax-1)}}{(ax-1)\sqrt{c\,(ax+1)}\ \sqrt{c}\ x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] (-4*2^(1/2)*arctanh(1/2*(c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+5*arctanh(
(c*(a*x+1))^(1/2)/c^(1/2))*x*a*c+(c*(a*x+1))^(1/2)*c^(1/2))*(-a^2*x^2+1)^(1
/2)*(-c*(a*x-1))^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)/c^(1/2)/x
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^3}{x^2\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}\left(ax+1\right)^{3}}{x^{2}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2
)), x)
```

3.408 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=173

$$-\frac{23a^2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{4c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,a^2(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}-\frac{\sqrt{ax+1}\,(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}}-\frac{9a\sqrt{ax+1}\,(c-}{4cx(1-ax}$$

```
[Out] -23/4*a^2*(-a*c*x+c)^(3/2)*arctanh((a*x+1)^(1/2))/c/(-a*x+1)^(3/2)+4*a^2*(-
a*c*x+c)^(3/2)*arctanh(1/2*(a*x+1)^(1/2)*2^(1/2))*2^(1/2)/c/(-a*x+1)^(3/2)-
1/2*(-a*c*x+c)^(3/2)*(a*x+1)^(1/2)/c/x^2/(-a*x+1)^(3/2)-9/4*a*(-a*c*x+c)^(3
/2)*(a*x+1)^(1/2)/c/x/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 98, 151, 156, 63, 208}

$$-\frac{23a^2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{4c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,a^2(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}-\frac{\sqrt{ax+1}\,(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}}-\frac{9a\sqrt{ax+1}\,(c-}{4cx(1-ax}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] -(Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(2*c*x^2*(1 - a*x)^(3/2)) - (9*a*Sqrt[1
+ a*x]*(c - a*c*x)^(3/2))/(4*c*x*(1 - a*x)^(3/2)) - (23*a^2*(c - a*c*x)^(3/
2)*ArcTanh[Sqrt[1 + a*x]])/(4*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a^2*(c - a*c*
x)^(3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x^3(1-ax)^{3/2}}\,dx$$

$$= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x^3(c-acx)}\,dx}{(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-\frac{9ac}{2}-\frac{7}{2}a^2cx}{x^2\sqrt{1+ax}\,(c-acx)}\,dx}{2c(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{9a\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{\frac{23a^2c^2}{4}+\frac{9}{4}a^3c^2x}{x\sqrt{1+ax}\,(c-acx)}\,dx}{2c^2(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{9a\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx(1-ax)^{3/2}} + \frac{\left(4a^3(c-acx)^{3/2}\right)\int \frac{1}{\sqrt{1+ax}\,(c-acx)}}{(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{9a\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx(1-ax)^{3/2}} + \frac{\left(8a^2(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{2c-cx^2}\right.}{(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{2cx^2(1-ax)^{3/2}} - \frac{9a\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx(1-ax)^{3/2}} - \frac{23a^2(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{1+a\right.}{4c(1-ax)^{3/2}}$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.53

$$-\frac{\sqrt{c-acx}\left(23a^2x^2\tanh^{-1}\left(\sqrt{ax+1}\right)-16\sqrt{2}\,a^2x^2\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)+\sqrt{ax+1}\,(9ax+2)\right)}{4x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] -1/4*(Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(2 + 9*a*x) + 23*a^2*x^2*ArcTanh[Sqrt[
1 + a*x]] - 16*Sqrt[2]*a^2*x^2*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]]))/(x^2*Sqrt[1
 - a*x])
```

fricas [A] time = 0.49, size = 391, normalized size = 2.26

$$\left[\frac{16\sqrt{2}\left(a^3x^3-a^2x^2\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+23\left(a^3x^3-a^2x^2\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-}}{}\right.}{8\left(ax^3-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] [1/8*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*
sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*x
 + 1)) + 23*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a
^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2
+ 1)*sqrt(-a*c*x + c)*(9*a*x + 2))/(a*x^3 - x^2), 1/4*(16*sqrt(2)*(a^3*x^3
- a^2*x^2)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt
(-c)/(a^2*c*x^2 - c)) - 23*(a^3*x^3 - a^2*x^2)*sqrt(-c)*arctan(sqrt(-a^2*x^
2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c)*(9*a*x + 2))/(a*x^3 - x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 131, normalized size = 0.76

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(16\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c-23c\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)x^2a^2-9xa\sqrt{c\,(ax+1)}}{4\sqrt{c}\,\,(ax-1)\sqrt{c\,(ax+1)}\,\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^3,x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(16*2^(1/2)*arctanh(1/2*(c*(a*x+
1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-23*c*arctanh((c*(a*x+1))^(1/2)/c^(1/2)
)*x^2*a^2-9*x*a*(c*(a*x+1))^(1/2)*c^(1/2)-2*(c*(a*x+1))^(1/2)*c^(1/2))/c^(1
/2)/(a*x-1)/(c*(a*x+1))^(1/2)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^3}{x^3\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}\left(ax+1\right)^{3}}{x^{3}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2
)), x)
```

3.409 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=216

$$-\frac{45a^3(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{8c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,a^3(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}-\frac{19a^2\sqrt{ax+1}\,(c-acx)^{3/2}}{8cx(1-ax)^{3/2}}-\frac{\sqrt{ax+1}\,(c-}{3cx^3(1-}$$

```
[Out] -45/8*a^3*(-a*c*x+c)^(3/2)*arctanh((a*x+1)^(1/2))/c/(-a*x+1)^(3/2)+4*a^3*(-
a*c*x+c)^(3/2)*arctanh(1/2*(a*x+1)^(1/2)*2^(1/2))*2^(1/2)/c/(-a*x+1)^(3/2)-
1/3*(-a*c*x+c)^(3/2)*(a*x+1)^(1/2)/c/x^3/(-a*x+1)^(3/2)-13/12*a*(-a*c*x+c)^
(3/2)*(a*x+1)^(1/2)/c/x^2/(-a*x+1)^(3/2)-19/8*a^2*(-a*c*x+c)^(3/2)*(a*x+1)^
(1/2)/c/x/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 98, 151, 156, 63, 208}

$$-\frac{19a^2\sqrt{ax+1}\,(c-acx)^{3/2}}{8cx(1-ax)^{3/2}}-\frac{45a^3(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{8c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,a^3(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}-\frac{13a\sqrt{ax+1}}{12cx^2(1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] -(Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(3*c*x^3*(1 - a*x)^(3/2)) - (13*a*Sqrt[1
 + a*x]*(c - a*c*x)^(3/2))/(12*c*x^2*(1 - a*x)^(3/2)) - (19*a^2*Sqrt[1 + a*
x]*(c - a*c*x)^(3/2))/(8*c*x*(1 - a*x)^(3/2)) - (45*a^3*(c - a*c*x)^(3/2)*A
rcTanh[Sqrt[1 + a*x]])/(8*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a^3*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x^4(1-ax)^{3/2}}\,dx$$
$$= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x^4(c-acx)}\,dx}{(1-ax)^{3/2}}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-\frac{13ac}{2}-\frac{11}{2}a^2cx}{x^3\sqrt{1+ax}\,(c-acx)}\,dx}{3c(1-ax)^{3/2}}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}\,(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{\frac{57a^2c^2}{4}+\frac{39}{4}a^3c^2x}{x^2\sqrt{1+ax}\,(c-acx)}\,dx}{6c^2(1-ax)^{3/2}}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}\,(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} - \frac{19a^2\sqrt{1+ax}\,(c-acx)^{3/2}}{8cx(1-ax)^{3/2}} - \frac{(c-\cdots}{\cdots}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}\,(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} - \frac{19a^2\sqrt{1+ax}\,(c-acx)^{3/2}}{8cx(1-ax)^{3/2}} + \frac{(4a^4\cdots}{\cdots}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}\,(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} - \frac{19a^2\sqrt{1+ax}\,(c-acx)^{3/2}}{8cx(1-ax)^{3/2}} + \frac{(8a^3\cdots}{\cdots}$$
$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{3cx^3(1-ax)^{3/2}} - \frac{13a\sqrt{1+ax}\,(c-acx)^{3/2}}{12cx^2(1-ax)^{3/2}} - \frac{19a^2\sqrt{1+ax}\,(c-acx)^{3/2}}{8cx(1-ax)^{3/2}} - \frac{45a^3\cdots}{\cdots}$$

Mathematica [A] time = 0.07, size = 100, normalized size = 0.46

$$-\frac{\sqrt{c-acx}\left(135a^3x^3\tanh^{-1}\left(\sqrt{ax+1}\right)-96\sqrt{2}\,a^3x^3\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)+\sqrt{ax+1}\left(57a^2x^2+26ax+8\right)\right)}{24x^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] -1/24*(Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(8 + 26*a*x + 57*a^2*x^2) + 135*a^3*x
^3*ArcTanh[Sqrt[1 + a*x]] - 96*Sqrt[2]*a^3*x^3*ArcTanh[Sqrt[1 + a*x]/Sqrt[2
]]))/(x^3*Sqrt[1 - a*x])
```

fricas [A] time = 0.55, size = 407, normalized size = 1.88

$$\left[\frac{96\sqrt{2}\left(a^4x^4-a^3x^3\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3\,c}{a^2x^2-2\,ax+1}\right)+135\left(a^4x^4-a^3x^3\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{}}{}\right.}{48\left(ax^4-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="f
ricas")
```

```
[Out] [1/48*(96*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*a*
x + 1)) + 135*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(
-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*(57*a^2*x^2
+ 26*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x^4 - x^3), 1/24*(96*
sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*arctan(sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 135*(a^4*x^4 - a^3*x^3)*sqrt(-c)*a
rctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (57*a
^2*x^2 + 26*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x^4 - x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 151, normalized size = 0.70

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(96\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^3a^3c-135c\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)x^3a^3-57x^2a^2\sqrt{c\,(ax}\right.}{24\sqrt{c}\,\,(ax-1)\sqrt{c\,(ax+1)}\,\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^4,x)
```

```
[Out] -1/24*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(96*2^(1/2)*arctanh(1/2*(c*(a*x
+1))^(1/2)*2^(1/2)/c^(1/2))*x^3*a^3*c-135*c*arctanh((c*(a*x+1))^(1/2)/c^(1/
```

```
2))*x^3*a^3-57*x^2*a^2*(c*(a*x+1))^(1/2)*c^(1/2)-26*x*a*(c*(a*x+1))^(1/2)*c
^(1/2)-8*(c*(a*x+1))^(1/2)*c^(1/2))/c^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^3}{x^4\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x^4*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x^4*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,\left(ax+1\right)^3}{x^4\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(a*x + 1)**3/(x**4*(-(a*x - 1)*(a*x + 1))**(3/2
)), x)
```

3.410 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=259

$$-\frac{363a^4(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{64c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,a^4(c-acx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{c(1-ax)^{3/2}}-\frac{149a^3\sqrt{ax+1}\,(c-acx)^{3/2}}{64cx(1-ax)^{3/2}}-\frac{107a^2\sqrt{}}{96c}$$

```
[Out] -363/64*a^4*(-a*c*x+c)^(3/2)*arctanh((a*x+1)^(1/2))/c/(-a*x+1)^(3/2)+4*a^4*
(-a*c*x+c)^(3/2)*arctanh(1/2*(a*x+1)^(1/2)*2^(1/2))*2^(1/2)/c/(-a*x+1)^(3/2
)-1/4*(-a*c*x+c)^(3/2)*(a*x+1)^(1/2)/c/x^4/(-a*x+1)^(3/2)-17/24*a*(-a*c*x+c
)^(3/2)*(a*x+1)^(1/2)/c/x^3/(-a*x+1)^(3/2)-107/96*a^2*(-a*c*x+c)^(3/2)*(a*x
+1)^(1/2)/c/x^2/(-a*x+1)^(3/2)-149/64*a^3*(-a*c*x+c)^(3/2)*(a*x+1)^(1/2)/c/
x/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 98, 151, 156, 63, 208}

$$-\frac{107a^2\sqrt{ax+1}\,(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}}-\frac{149a^3\sqrt{ax+1}\,(c-acx)^{3/2}}{64cx(1-ax)^{3/2}}-\frac{363a^4(c-acx)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{64c(1-ax)^{3/2}}+\frac{4\sqrt{2}\,a^4(c-acx)^{3}}{c(1-}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] -(Sqrt[1 + a*x]*(c - a*c*x)^(3/2))/(4*c*x^4*(1 - a*x)^(3/2)) - (17*a*Sqrt[1
 + a*x]*(c - a*c*x)^(3/2))/(24*c*x^3*(1 - a*x)^(3/2)) - (107*a^2*Sqrt[1 + a
*x]*(c - a*c*x)^(3/2))/(96*c*x^2*(1 - a*x)^(3/2)) - (149*a^3*Sqrt[1 + a*x]*
(c - a*c*x)^(3/2))/(64*c*x*(1 - a*x)^(3/2)) - (363*a^4*(c - a*c*x)^(3/2)*Ar
cTanh[Sqrt[1 + a*x]])/(64*c*(1 - a*x)^(3/2)) + (4*Sqrt[2]*a^4*(c - a*c*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]/Sqrt[2]])/(c*(1 - a*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{(1+ax)^{3/2}\sqrt{c-acx}}{x^5(1-ax)^{3/2}}\,dx$$

$$= \frac{(c-acx)^{3/2}\int \frac{(1+ax)^{3/2}}{x^5(c-acx)}\,dx}{(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{(c-acx)^{3/2}\int \frac{-\frac{17ac}{2}-\frac{15}{2}a^2cx}{x^4\sqrt{1+ax}\,(c-acx)}\,dx}{4c(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}\,(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} + \frac{(c-acx)^{3/2}\int \frac{\frac{107a^2c^2}{4}+\frac{85}{4}a^3c^2x}{x^3\sqrt{1+ax}\,(c-acx)}\,dx}{12c^2(1-ax)^{3/2}}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}\,(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}\,(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{(c}{}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}\,(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}\,(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{14}{}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}\,(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}\,(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{14}{}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}\,(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}\,(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{14}{}$$

$$= -\frac{\sqrt{1+ax}\,(c-acx)^{3/2}}{4cx^4(1-ax)^{3/2}} - \frac{17a\sqrt{1+ax}\,(c-acx)^{3/2}}{24cx^3(1-ax)^{3/2}} - \frac{107a^2\sqrt{1+ax}\,(c-acx)^{3/2}}{96cx^2(1-ax)^{3/2}} - \frac{14}{}$$

Mathematica [A] time = 0.08, size = 108, normalized size = 0.42

$$-\frac{\sqrt{c-acx}\left(1089a^4x^4\tanh^{-1}\left(\sqrt{ax+1}\right) - 768\sqrt{2}\,a^4x^4\tanh^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right) + \sqrt{ax+1}\left(447a^3x^3 + 214a^2x^2 + 136ax\right.\right.}{192x^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] -1/192*(Sqrt[c - a*c*x]*(Sqrt[1 + a*x]*(48 + 136*a*x + 214*a^2*x^2 + 447*a^
3*x^3) + 1089*a^4*x^4*ArcTanh[Sqrt[1 + a*x]] - 768*Sqrt[2]*a^4*x^4*ArcTanh[
Sqrt[1 + a*x]/Sqrt[2]]))/(x^4*Sqrt[1 - a*x])
```

fricas [A] time = 0.54, size = 423, normalized size = 1.63

$$\left[\frac{768 \sqrt{2}\left(a^5x^5 - a^4x^4\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-3c}{a^2x^2-2ax+1}\right) + 1089\left(a^5x^5 - a^4x^4\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2}{}\right.}{384\left(ax^5 - x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="f
ricas")
```

```
[Out] [1/384*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(a^2*c*x^2 + 2*a*c*x -
 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a^2*x^2 - 2*
a*x + 1)) + 1089*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sq
rt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*(447*a^3*
x^3 + 214*a^2*x^2 + 136*a*x + 48)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x
^5 - x^4), 1/192*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*arctan(sqrt(2)*s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 1089*(a^5*x^
5 - a^4*x^4)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(
a^2*c*x^2 - c)) + (447*a^3*x^3 + 214*a^2*x^2 + 136*a*x + 48)*sqrt(-a^2*x^2
+ 1)*sqrt(-a*c*x + c))/(a*x^5 - x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 171, normalized size = 0.66

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\,(ax-1)}\,\left(768\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^4a^4c - 1089c\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)x^4a^4 - 447x^3a^3\sqrt{c}\,(\right.}{192\sqrt{c}\,\left(ax-1\right)\sqrt{c\,(ax+1)}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^5,x)
```

```
[Out] -1/192*(-a^2*x^2+1)^(1/2)*(-c*(a*x-1))^(1/2)*(768*2^(1/2)*arctanh(1/2*(c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^4*a^4*c-1089*c*arctanh((c*(a*x+1))^(1/2)/c^
(1/2))*x^4*a^4-447*x^3*a^3*(c*(a*x+1))^(1/2)*c^(1/2)-214*x^2*a^2*(c*(a*x+1)
)^(1/2)*c^(1/2)-136*x*a*(c*(a*x+1))^(1/2)*c^(1/2)-48*(c*(a*x+1))^(1/2)*c^(1
/2))/c^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)/x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^3}{x^5\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x^5*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a*c*x)^(1/2)*(a*x + 1)^3)/(x^5*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

3.411 $\int e^{-\tanh^{-1}(ax)} x^m \sqrt{c-acx}\, dx$

Optimal. Leaf size=114

$$\frac{2(4m+5)(ax+1)x^m\sqrt{c-acx}\,(-ax)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};ax+1\right)}{a(2m+3)\sqrt{1-a^2x^2}}-\frac{2c\sqrt{1-a^2x^2}\,x^{m+1}}{(2m+3)\sqrt{c-acx}}$$

```
[Out] 2*(5+4*m)*x^m*(a*x+1)*hypergeom([1/2, -m],[3/2],a*x+1)*(-a*c*x+c)^(1/2)/a/(
3+2*m)/((-a*x)^m)/(-a^2*x^2+1)^(1/2)-2*c*x^(1+m)*(-a^2*x^2+1)^(1/2)/(3+2*m)
/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6128, 881, 892, 67, 65}

$$\frac{2(4m+5)(ax+1)x^m\sqrt{c-acx}\,(-ax)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};ax+1\right)}{a(2m+3)\sqrt{1-a^2x^2}}-\frac{2c\sqrt{1-a^2x^2}\,x^{m+1}}{(2m+3)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*x^(1 + m)*Sqrt[1 - a^2*x^2])/((3 + 2*m)*Sqrt[c - a*c*x]) + (2*(5 + 4*
m)*x^m*(1 + a*x)*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + a*x])/
(a*(3 + 2*m)*(-(a*x))^m*Sqrt[1 - a^2*x^2])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
```

```
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)} x^m \sqrt{c-acx}\, dx &= \frac{\int \frac{x^m (c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\, dx}{c} \\
&= -\frac{2cx^{1+m}\sqrt{1-a^2x^2}}{(3+2m)\sqrt{c-acx}} + \frac{(5+4m)\int \frac{x^m\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\, dx}{3+2m} \\
&= -\frac{2cx^{1+m}\sqrt{1-a^2x^2}}{(3+2m)\sqrt{c-acx}} + \frac{\left((5+4m)\sqrt{\frac{1}{c}+\frac{ax}{c}}\sqrt{c-acx}\right)\int \frac{x^m}{\sqrt{\frac{1}{c}+\frac{ax}{c}}}\, dx}{(3+2m)\sqrt{1-a^2x^2}} \\
&= -\frac{2cx^{1+m}\sqrt{1-a^2x^2}}{(3+2m)\sqrt{c-acx}} + \frac{\left((5+4m)x^m(-ax)^{-m}\sqrt{\frac{1}{c}+\frac{ax}{c}}\sqrt{c-acx}\right)\int \frac{(-ax)^m}{\sqrt{\frac{1}{c}+\frac{ax}{c}}}\, dx}{(3+2m)\sqrt{1-a^2x^2}} \\
&= -\frac{2cx^{1+m}\sqrt{1-a^2x^2}}{(3+2m)\sqrt{c-acx}} + \frac{2(5+4m)x^m(-ax)^{-m}(1+ax)\sqrt{c-acx}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\right.}{a(3+2m)\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.68

$$-\frac{c\sqrt{1-ax}\,x^{m+1}\left(2(m+1)\sqrt{ax+1}-(4m+5)\,{}_2F_1\left(\frac{1}{2},m+1;m+2;-ax\right)\right)}{(m+1)(2m+3)\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] -((c*x^(1 + m)*Sqrt[1 - a*x]*(2*(1 + m)*Sqrt[1 + a*x] - (5 + 4*m)*Hypergeom
etric2F1[1/2, 1 + m, 2 + m, -(a*x)]))/((1 + m)*(3 + 2*m)*Sqrt[c - a*c*x]))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*x^m/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*x^m/(a*x + 1), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-acx+c}\,\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*x^m/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x^m*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-c\,(ax-1)}\,\sqrt{-\,(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.412 $\int e^{-\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=135

$$\frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}-\frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}}+\frac{104\sqrt{1-a^2x^2}\sqrt{c-acx}}{105a^3}+\frac{104c\sqrt{1-a^2x^2}}{105a^3\sqrt{c-acx}}$$

```
[Out] 104/105*c*(-a^2*x^2+1)^(1/2)/a^3/(-a*c*x+c)^(1/2)+26/35*c*x^2*(-a^2*x^2+1)^
(1/2)/a/(-a*c*x+c)^(1/2)-2/7*c*x^3*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2)+104/
105*(-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] time = 0.21, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6128, 881, 871, 795, 649}

$$-\frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}}+\frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}}+\frac{104\sqrt{1-a^2x^2}\sqrt{c-acx}}{105a^3}+\frac{104c\sqrt{1-a^2x^2}}{105a^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] (104*c*Sqrt[1 - a^2*x^2])/(105*a^3*Sqrt[c - a*c*x]) + (26*c*x^2*Sqrt[1 - a^
2*x^2])/(35*a*Sqrt[c - a*c*x]) - (2*c*x^3*Sqrt[1 - a^2*x^2])/(7*Sqrt[c - a*
c*x]) + (104*Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/(105*a^3)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
```

```
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= \frac{\int \frac{x^2(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}} + \frac{13}{7}\int \frac{x^2\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}} - \frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}} - \frac{52\int \frac{x\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx}{35a}\\
&= \frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}} - \frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}} + \frac{104\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{105a^3} + \frac{52\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx}{105a^2}\\
&= \frac{104c\sqrt{1-a^2x^2}}{105a^3\sqrt{c-acx}} + \frac{26cx^2\sqrt{1-a^2x^2}}{35a\sqrt{c-acx}} - \frac{2cx^3\sqrt{1-a^2x^2}}{7\sqrt{c-acx}} + \frac{104\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{105a^3}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.41

$$-\frac{2c\sqrt{1-a^2x^2}\left(15a^3x^3-39a^2x^2+52ax-104\right)}{105a^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*Sqrt[1 - a^2*x^2]*(-104 + 52*a*x - 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3
*Sqrt[c - a*c*x])
```

fricas [A] time = 0.45, size = 58, normalized size = 0.43

$$\frac{2\left(15\,a^3x^3-39\,a^2x^2+52\,ax-104\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/105*(15*a^3*x^3 - 39*a^2*x^2 + 52*a*x - 104)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c)/(a^4*x - a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 56, normalized size = 0.41

$$\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\left(15x^3a^3-39a^2x^2+52ax-104\right)}{105\left(ax-1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/105*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)*(15*a^3*x^3-39*a^2*x^2+52*a*x-104
)/(a*x-1)/a^3
```

maxima [A] time = 0.33, size = 62, normalized size = 0.46

$$-\frac{2\left(15\,a^3\sqrt{c}\,x^3-39\,a^2\sqrt{c}\,x^2+52\,a\sqrt{c}\,x-104\,\sqrt{c}\right)\sqrt{ax+1}\,(ax-1)}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] -2/105*(15*a^3*sqrt(c)*x^3 - 39*a^2*sqrt(c)*x^2 + 52*a*sqrt(c)*x - 104*sqrt
(c))*sqrt(a*x + 1)*(a*x - 1)/(a^4*x - a^3)
```

mupad [B] time = 0.95, size = 74, normalized size = 0.55

$$\frac{2\,\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}\,\left(15\,a^2\,x^2-24\,a\,x+28\right)}{105\,a^3}-\frac{152\,\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{105\,a^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a*x + 1),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2)*(15*a^2*x^2 - 24*a*x + 28))/(105*a
^3) - (152*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(105*a^3*(a*x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sqrt{-c\,(ax-1)}\,\sqrt{-(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.413 $\int e^{-\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=101

$$-\frac{2\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}{5a^2c}-\frac{2\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{5a^2}-\frac{8c\sqrt{1-a^2x^2}}{5a^2\sqrt{c-acx}}$$

```
[Out] -2/5*(-a*c*x+c)^(3/2)*(-a^2*x^2+1)^(1/2)/a^2/c-8/5*c*(-a^2*x^2+1)^(1/2)/a^2
/(-a*c*x+c)^(1/2)-2/5*(-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6128, 795, 657, 649}

$$-\frac{2\sqrt{1-a^2x^2}\,(c-acx)^{3/2}}{5a^2c}-\frac{2\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{5a^2}-\frac{8c\sqrt{1-a^2x^2}}{5a^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
```

```
[Out] (-8*c*Sqrt[1 - a^2*x^2])/(5*a^2*Sqrt[c - a*c*x]) - (2*Sqrt[c - a*c*x]*Sqrt[
1 - a^2*x^2])/(5*a^2) - (2*(c - a*c*x)^(3/2)*Sqrt[1 - a^2*x^2])/(5*a^2*c)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
```

```
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx &= \frac{\int \frac{x(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{c} \\
&= -\frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a^2c} - \frac{3\int \frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{5ac} \\
&= -\frac{2\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{5a^2} - \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a^2c} - \frac{4\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx}{5a} \\
&= -\frac{8c\sqrt{1-a^2x^2}}{5a^2\sqrt{c-acx}} - \frac{2\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{5a^2} - \frac{2(c-acx)^{3/2}\sqrt{1-a^2x^2}}{5a^2c}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.46

$$-\frac{2c\sqrt{1-a^2x^2}\,\left(a^2x^2-3ax+6\right)}{5a^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^ArcTanh[a*x],x]
[Out] (-2*c*Sqrt[1 - a^2*x^2]*(6 - 3*a*x + a^2*x^2))/(5*a^2*Sqrt[c - a*c*x])
```

fricas [A] time = 0.41, size = 49, normalized size = 0.49

$$\frac{2\left(a^2x^2-3\,ax+6\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{5\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/5*(a^2*x^2 - 3*a*x + 6)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x - a^2)
```

giac [A] time = 0.95, size = 65, normalized size = 0.64

$$\frac{8\sqrt{2}|c|}{5a^2\sqrt{c}} - \frac{4\sqrt{acx+c}|c|}{a^2c} - \frac{2\left((acx+c)^{\frac{5}{2}}|c| - 5(acx+c)^{\frac{3}{2}}c|c|\right)}{5a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] 8/5*sqrt(2)*abs(c)/(a^2*sqrt(c)) - 4*sqrt(a*c*x + c)*abs(c)/(a^2*c) - 2/5*(
(a*c*x + c)^(5/2)*abs(c) - 5*(a*c*x + c)^(3/2)*c*abs(c))/(a^2*c^3)
```

maple [A] time = 0.03, size = 47, normalized size = 0.47

$$\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\left(a^2x^2-3ax+6\right)}{5(ax-1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/5*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)*(a^2*x^2-3*a*x+6)/(a*x-1)/a^2
```

maxima [A] time = 0.37, size = 50, normalized size = 0.50

$$-\frac{2\left(a^2\sqrt{c}\,x^2 - 3\,a\sqrt{c}\,x + 6\sqrt{c}\right)\sqrt{ax+1}\,(ax-1)}{5\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/5*(a^2*sqrt(c)*x^2 - 3*a*sqrt(c)*x + 6*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/
(a^3*x - a^2)
```

mupad [B] time = 0.90, size = 75, normalized size = 0.74

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{12\,\sqrt{1-a^2\,x^2}}{5\,a^3} - \frac{6\,x\,\sqrt{1-a^2\,x^2}}{5\,a^2} + \frac{2\,x^2\,\sqrt{1-a^2\,x^2}}{5\,a}\right)}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a*x + 1),x)
```

```
[Out] ((c - a*c*x)^(1/2)*((12*(1 - a^2*x^2)^(1/2))/(5*a^3) - (6*x*(1 - a^2*x^2)^(
1/2))/(5*a^2) + (2*x^2*(1 - a^2*x^2)^(1/2))/(5*a)))/(x - 1/a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\,(ax-1)}\,\sqrt{-\,(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.414 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=66

$$\frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{3a}$$

```
[Out] 8/3*c*(-a^2*x^2+1)^(1/2)/a/(-a*c*x+c)^(1/2)+2/3*(-a*c*x+c)^(1/2)*(-a^2*x^2+
1)^(1/2)/a
```

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}+\frac{2\sqrt{1-a^2x^2}\,\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcTanh[a*x],x]
```

```
[Out] (8*c*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x]) + (2*Sqrt[c - a*c*x]*Sqrt[1 -
 a^2*x^2])/(3*a)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\int \frac{(c-acx)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{2\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{3a} + \frac{4}{3}\int \frac{\sqrt{c-acx}}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{8c\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}} + \frac{2\sqrt{c-acx}\,\sqrt{1-a^2x^2}}{3a}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.58

$$-\frac{2c(ax-5)\sqrt{1-a^2x^2}}{3a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcTanh[a*x],x]
[Out] (-2*c*(-5 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a*c*x])
```

fricas [A] time = 0.46, size = 39, normalized size = 0.59

$$\frac{2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,(ax-5)}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
[Out] 2/3*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(a*x - 5)/(a^2*x - a)
```

giac [A] time = 0.38, size = 50, normalized size = 0.76

$$-\frac{8\sqrt{2}\,|c|}{3\,a\sqrt{c}} - \frac{2\,(acx+c)^{\frac{3}{2}}|c|}{3\,ac^2} + \frac{4\sqrt{acx+c}\,|c|}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] -8/3*sqrt(2)*abs(c)/(a*sqrt(c)) - 2/3*(a*c*x + c)^(3/2)*abs(c)/(a*c^2) + 4*
sqrt(a*c*x + c)*abs(c)/(a*c)
```

maple [A] time = 0.03, size = 39, normalized size = 0.59

$$\frac{2\sqrt{-a^{2}x^{2}+1}\,\sqrt{-acx+c}\,\left(ax-5\right)}{3\left(ax-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 2/3*(-a^2*x^2+1)^(1/2)*(-a*c*x+c)^(1/2)*(a*x-5)/(a*x-1)/a
```

maxima [A] time = 0.38, size = 37, normalized size = 0.56

$$-\frac{2\left(a\sqrt{c}\,x-5\sqrt{c}\right)\sqrt{ax+1}\,(ax-1)}{3\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(a*sqrt(c)*x - 5*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/(a^2*x - a)
```

mupad [B] time = 0.00, size = 56, normalized size = 0.85

$$-\frac{\sqrt{c-a\,c\,x}\,\left(\frac{10\,\sqrt{1-a^2\,x^2}}{3\,a^2}-\frac{2\,x\,\sqrt{1-a^2\,x^2}}{3\,a}\right)}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a*x + 1),x)
```

```
[Out] -((c - a*c*x)^(1/2)*((10*(1 - a^2*x^2)^(1/2))/(3*a^2) - (2*x*(1 - a^2*x^2)^
(1/2))/(3*a)))/(x - 1/a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(ax-1\right)}\,\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.415 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=68

$$-\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -2*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)-2*c*(-a^2*x
^2+1)^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6128, 881, 875, 208}

$$-\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] (-2*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[
1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] && !IntegerQ[p] && EqQ[m + p - 1, 0] && !LtQ[n, -1] && Integer
```

```
Q[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= \frac{\int \frac{(c-acx)^{3/2}}{x\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}} + \int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}} + \left(2a^2c^2\right)\mathrm{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\
&= -\frac{2c\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.65

$$-\frac{2c\sqrt{1-ax}\left(\sqrt{ax+1}+\tanh^{-1}\left(\sqrt{ax+1}\right)\right)}{\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] (-2*c*Sqrt[1 - a*x]*(Sqrt[1 + a*x] + ArcTanh[Sqrt[1 + a*x]]))/Sqrt[c - a*c*
x]
```

fricas [A] time = 0.46, size = 184, normalized size = 2.71

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{ax-1}, -\frac{2\left((ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x}}{}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*
c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c
))/(a*x - 1), -2*((a*x - 1)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x
+ c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x
- 1)]
```

giac [A] time = 0.15, size = 82, normalized size = 1.21

$$2\left(\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{acx+c}}{c}\right)|c|-\frac{2\left(\sqrt{c}\,|c|\arctan\left(\frac{\sqrt{2}\,\sqrt{c}}{\sqrt{-c}}\right)-\sqrt{2}\,\sqrt{-c}\,|c|\right)}{\sqrt{-c}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*(arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(a*c*x + c)/c)*abs(c) -
2*(sqrt(c)*abs(c)*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) - sqrt(2)*sqrt(-c)*abs(c
))/(sqrt(-c)*sqrt(c))
```

maple [A] time = 0.04, size = 69, normalized size = 1.01

$$\frac{2\sqrt{-c\,(ax-1)}\,\sqrt{-a^2x^2+1}\,\left(\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)+\sqrt{c\,(ax+1)}\right)}{(ax-1)\,\sqrt{c\,(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] 2*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(c^(1/2)*arctanh((c*(a*x+1))^(1/2)/
c^(1/2))+(c*(a*x+1))^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/((a*x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{x\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(x*(a*x + 1)),x)
```

```
[Out] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(x*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{x\left(ax+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(x*(a*x + 1)), x)
```

3.416 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=72

$$3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)-\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}$$

```
[Out] 3*a*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)-c*(-a^2*x^
2+1)^(1/2)/x/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6128, 879, 875, 208}

$$3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)-\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -((c*Sqrt[1 - a^2*x^2])/(x*Sqrt[c - a*c*x])) + 3*a*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
```

```
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= \frac{\int \frac{(c-acx)^{3/2}}{x^2\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}} - \frac{1}{2}(3a)\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}} - \left(3a^3c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\
&= -\frac{c\sqrt{1-a^2x^2}}{x\sqrt{c-acx}} + 3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.72

$$\frac{\sqrt{1-ax}\left(3ac\tanh^{-1}\left(\sqrt{ax+1}\right)-\frac{c\sqrt{ax+1}}{x}\right)}{\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] (Sqrt[1 - a*x]*(-((c*Sqrt[1 + a*x])/x) + 3*a*c*ArcTanh[Sqrt[1 + a*x]]))/Sqr
t[c - a*c*x]
```

fricas [A] time = 0.49, size = 207, normalized size = 2.88

$$\left[\frac{3\left(a^2x^2-ax\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{2\left(ax^2-x\right)}, \frac{3\left(a^2x^2-ax\right)\sqrt{-c}\,\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [1/2*(3*(a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqr
t(-a*c*x + c))/(a*x^2 - x), (3*(a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*x^
2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c))/(a*x^2 - x)]
```

giac [A] time = 1.02, size = 90, normalized size = 1.25

$$-a\left(\frac{3\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{acx+c}}{acx}\right)|c|+\frac{3\,a\sqrt{c}\,|c|\arctan\left(\frac{\sqrt{2}\,\sqrt{c}}{\sqrt{-c}}\right)+\sqrt{2}\,a\sqrt{-c}\,|c|}{\sqrt{-c}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] -a*(3*arctan(sqrt(a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(a*c*x + c)/(a*c*x))*
abs(c) + (3*a*sqrt(c)*abs(c)*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) + sqrt(2)*a*s
qrt(-c)*abs(c))/(sqrt(-c)*sqrt(c))
```

maple [A] time = 0.05, size = 79, normalized size = 1.10

$$\frac{\sqrt{-c\,(ax-1)}\,\sqrt{-a^2x^2+1}\,\left(-3\,\mathrm{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)xac+\sqrt{c\,(ax+1)}\,\sqrt{c}\right)}{(ax-1)\,\sqrt{c\,(ax+1)}\,x\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] (-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(-3*arctanh((c*(a*x+1))^(1/2)/c^(1/2)
)*x*a*c+(c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(c*(a*x+1))^(1/2)/x/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/((a*x + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{x^2\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(x^2*(a*x + 1)),x)
```

```
[Out] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(x^2*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{x^{2}\left(ax+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(x**2*(a*x + 1)), x)
```

3.417 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=112

$$\frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}}-\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}}-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -7/4*a^2*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)-1/2*c
*(-a^2*x^2+1)^(1/2)/x^2/(-a*c*x+c)^(1/2)+7/4*a*c*(-a^2*x^2+1)^(1/2)/x/(-a*c
*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6128, 879, 873, 875, 208}

$$\frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}}-\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}}-\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(2*x^2*Sqrt[c - a*c*x]) + (7*a*c*Sqrt[1 - a^2*x^2])/
(4*x*Sqrt[c - a*c*x]) - (7*a^2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/
Sqrt[c - a*c*x]])/4
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
```

```
Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= \frac{\int \frac{(c-acx)^{3/2}}{x^3\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}} - \frac{1}{4}(7a)\int \frac{\sqrt{c-acx}}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}} + \frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}} + \frac{1}{8}\left(7a^2\right)\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}} + \frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}} + \frac{1}{4}\left(7a^4c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a}}{\sqrt{c-a}}\right.\\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2\sqrt{c-acx}} + \frac{7ac\sqrt{1-a^2x^2}}{4x\sqrt{c-acx}} - \frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.57

$$
-\frac{c\sqrt{1-ax}\left(7a^2x^2\tanh^{-1}\left(\sqrt{ax+1}\right)+(2-7ax)\sqrt{ax+1}\right)}{4x^2\sqrt{c-acx}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] -1/4*(c*Sqrt[1 - a*x]*((2 - 7*a*x)*Sqrt[1 + a*x] + 7*a^2*x^2*ArcTanh[Sqrt[1
 + a*x]]))/(x^2*Sqrt[c - a*c*x])
```

fricas [A] time = 0.49, size = 232, normalized size = 2.07

$$\left[\frac{7\left(a^3x^3-a^2x^2\right)\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\,(7ax-2)}{8\left(ax^3-x^2\right)},-\frac{7\left(a^3x^3-a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/8*(7*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x
^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*sqrt(-a^2*x^2 + 1)
*sqrt(-a*c*x + c)*(7*a*x - 2))/(a*x^3 - x^2), -1/4*(7*(a^3*x^3 - a^2*x^2)*s
qrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)
) + sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*(7*a*x - 2))/(a*x^3 - x^2)]
```

giac [A] time = 0.21, size = 117, normalized size = 1.04

$$\frac{1}{4}a^2c\left(\frac{7\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}+\frac{7\,(acx+c)^{\frac{3}{2}}-9\sqrt{acx+c}\,c}{a^2c^3x^2}\right)|c|-\frac{7a^2c|c|\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+5\sqrt{2}\,a^2\sqrt{-c}\sqrt{c}\,|c|}{4\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/4*a^2*c*(7*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c) + (7*(a*c*x + c)
^(3/2) - 9*sqrt(a*c*x + c)*c)/(a^2*c^3*x^2))*abs(c) - 1/4*(7*a^2*c*abs(c)*a
rctan(sqrt(2)*sqrt(c)/sqrt(-c)) + 5*sqrt(2)*a^2*sqrt(-c)*sqrt(c)*abs(c))/(s
qrt(-c)*c)
```

maple [A] time = 0.05, size = 101, normalized size = 0.90

$$\frac{\sqrt{-c\,(ax-1)}\,\sqrt{-a^2x^2+1}\,\left(7c\operatorname{arctanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)x^2a^2-7xa\sqrt{c\,(ax+1)}\,\sqrt{c}+2\sqrt{c\,(ax+1)}\,\sqrt{c}\right)}{4\sqrt{c}\,(ax-1)\sqrt{c\,(ax+1)}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] 1/4*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(7*c*arctanh((c*(a*x+1))^(1/2)/c^
(1/2))*x^2*a^2-7*x*a*(c*(a*x+1))^(1/2)*c^(1/2)+2*(c*(a*x+1))^(1/2)*c^(1/2))
/c^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/((a*x + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{x^3\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(x^3*(a*x + 1)),x)
```

```
[Out] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(x^3*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}\,\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{x^3\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(x**3*(a*x + 1)), x)
```

3.418 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=148

$$-\frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}}+\frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}}-\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] 11/8*a^3*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)-1/3*c
*(-a^2*x^2+1)^(1/2)/x^3/(-a*c*x+c)^(1/2)+11/12*a*c*(-a^2*x^2+1)^(1/2)/x^2/(
-a*c*x+c)^(1/2)-11/8*a^2*c*(-a^2*x^2+1)^(1/2)/x/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6128, 879, 873, 875, 208}

$$-\frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}}+\frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}}-\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(3*x^3*Sqrt[c - a*c*x]) + (11*a*c*Sqrt[1 - a^2*x^2])
/(12*x^2*Sqrt[c - a*c*x]) - (11*a^2*c*Sqrt[1 - a^2*x^2])/(8*x*Sqrt[c - a*c*
x]) + (11*a^3*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]])
/8
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
```

```
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\int \frac{(c-acx)^{3/2}}{x^4\sqrt{1-a^2x^2}}\,dx}{c}$$

$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}} - \frac{1}{6}(11a)\int \frac{\sqrt{c-acx}}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}} + \frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}} + \frac{1}{8}\left(11a^2\right)\int \frac{\sqrt{c-acx}}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}} + \frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}} - \frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}} - \frac{1}{16}\left(11a^3\right)\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}} + \frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}} - \frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}} - \frac{1}{8}\left(11a^5c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+ \cdots}\right.$$

$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3\sqrt{c-acx}} + \frac{11ac\sqrt{1-a^2x^2}}{12x^2\sqrt{c-acx}} - \frac{11a^2c\sqrt{1-a^2x^2}}{8x\sqrt{c-acx}} + \frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2 \cdots}}{\sqrt{c-acx}}\right.$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.38

$$\frac{c\sqrt{1-a^2x^2}\left(11a^3x^3\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};ax+1\right)-1\right)}{3x^3\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] (c*Sqrt[1 - a^2*x^2]*(-1 + 11*a^3*x^3*Hypergeometric2F1[1/2, 3, 3/2, 1 + a*
x]))/(3*x^3*Sqrt[c - a*c*x])
```

fricas [A] time = 0.60, size = 248, normalized size = 1.68

$$\left[\frac{33\left(a^4x^4-a^3x^3\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)+2\left(33a^2x^2-22ax+8\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{48\left(ax^4-x^3\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/48*(33*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2
*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*(33*a^2*x^2 - 22
*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a*x^4 - x^3), 1/24*(33*(a^4
*x^4 - a^3*x^3)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c
)/(a^2*c*x^2 - c)) + (33*a^2*x^2 - 22*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c))/(a*x^4 - x^3)]
```

giac [A] time = 0.26, size = 132, normalized size = 0.89

$$-\frac{1}{24}a^3c^2\left(\frac{33\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^2}+\frac{33\,(acx+c)^{\frac{5}{2}}-88\,(acx+c)^{\frac{3}{2}}c+63\sqrt{acx+c}\,c^2}{a^3c^5x^3}\right)|c|+\frac{33\,a^3c|c|\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)+}{24\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="gia
c")
```

```
[Out] -1/24*a^3*c^2*(33*arctan(sqrt(a*c*x + c)/sqrt(-c))/(sqrt(-c)*c^2) + (33*(a*
c*x + c)^(5/2) - 88*(a*c*x + c)^(3/2)*c + 63*sqrt(a*c*x + c)*c^2)/(a^3*c^5*
```

```
x^3))*abs(c) + 1/24*(33*a^3*c*abs(c)*arctan(sqrt(2)*sqrt(c)/sqrt(-c)) + 19*
sqrt(2)*a^3*sqrt(-c)*sqrt(c)*abs(c))/(sqrt(-c)*c)
```

maple [A] time = 0.05, size = 121, normalized size = 0.82

$$-\frac{\sqrt{-c\left(ax-1\right)}\,\sqrt{-a^2x^2+1}\,\left(33c\,\mathrm{arctanh}\left(\frac{\sqrt{c\left(ax+1\right)}}{\sqrt{c}}\right)x^3a^3-33x^2a^2\sqrt{c\left(ax+1\right)}\,\sqrt{c}+22xa\sqrt{c\left(ax+1\right)}\,\sqrt{c}-8\sqrt{c\left(\right.}\right.}{24\sqrt{c}\,\left(ax-1\right)\sqrt{c\left(ax+1\right)}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] -1/24*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(33*c*arctanh((c*(a*x+1))^(1/2)
/c^(1/2))*x^3*a^3-33*x^2*a^2*(c*(a*x+1))^(1/2)*c^(1/2)+22*x*a*(c*(a*x+1))^(
1/2)*c^(1/2)-8*(c*(a*x+1))^(1/2)*c^(1/2))/c^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)
/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/((a*x + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{x^4\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(x^4*(a*x + 1)),x)
```

```
[Out] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(x^4*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(ax-1\right)}\,\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{x^4\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/(x**4*(a*x + 1)), x)
```

3.419 $\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=139

$$-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{2(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{5/2}}{5a^4c^2}-\frac{2(c-acx)^{3/2}}{3a^4c}-\frac{4\sqrt{c-acx}}{a^4}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a^4/c-2/5*(-a*c*x+c)^(5/2)/a^4/c^2+2/7*(-a*c*x+c)^(7/
2)/a^4/c^3-2/9*(-a*c*x+c)^(9/2)/a^4/c^4+4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1
/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4-4*(-a*c*x+c)^(1/2)/a^4
```

Rubi [A] time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 88, 50, 63, 206}

$$-\frac{2(c-acx)^{9/2}}{9a^4c^4}+\frac{2(c-acx)^{7/2}}{7a^4c^3}-\frac{2(c-acx)^{5/2}}{5a^4c^2}-\frac{2(c-acx)^{3/2}}{3a^4c}-\frac{4\sqrt{c-acx}}{a^4}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^4 - (2*(c - a*c*x)^(3/2))/(3*a^4*c) - (2*(c - a*c*x)
^(5/2))/(5*a^4*c^2) + (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) - (2*(c - a*c*x)^(9
/2))/(9*a^4*c^4) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt
[c])])/a^4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \int \frac{x^3(1-ax)\sqrt{c-acx}}{1+ax}\,dx$$
$$= \frac{\int \frac{x^3(c-acx)^{3/2}}{1+ax}\,dx}{c}$$
$$= \frac{\int \left(\frac{(c-acx)^{3/2}}{a^3} - \frac{(c-acx)^{3/2}}{a^3(1+ax)} - \frac{(c-acx)^{5/2}}{a^3c} + \frac{(c-acx)^{7/2}}{a^3c^2}\right)dx}{c}$$
$$= -\frac{2(c-acx)^{5/2}}{5a^4c^2} + \frac{2(c-acx)^{7/2}}{7a^4c^3} - \frac{2(c-acx)^{9/2}}{9a^4c^4} - \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^3c}$$
$$= -\frac{2(c-acx)^{3/2}}{3a^4c} - \frac{2(c-acx)^{5/2}}{5a^4c^2} + \frac{2(c-acx)^{7/2}}{7a^4c^3} - \frac{2(c-acx)^{9/2}}{9a^4c^4} - \frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a^3}$$
$$= -\frac{4\sqrt{c-acx}}{a^4} - \frac{2(c-acx)^{3/2}}{3a^4c} - \frac{2(c-acx)^{5/2}}{5a^4c^2} + \frac{2(c-acx)^{7/2}}{7a^4c^3} - \frac{2(c-acx)^{9/2}}{9a^4c^4} - \frac{(4}{}$$
$$= -\frac{4\sqrt{c-acx}}{a^4} - \frac{2(c-acx)^{3/2}}{3a^4c} - \frac{2(c-acx)^{5/2}}{5a^4c^2} + \frac{2(c-acx)^{7/2}}{7a^4c^3} - \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{8}{}$$
$$= -\frac{4\sqrt{c-acx}}{a^4} - \frac{2(c-acx)^{3/2}}{3a^4c} - \frac{2(c-acx)^{5/2}}{5a^4c^2} + \frac{2(c-acx)^{7/2}}{7a^4c^3} - \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{4}{}$$

Mathematica [A] time = 0.13, size = 87, normalized size = 0.63

$$\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4} - \frac{2\left(35a^4x^4 - 95a^3x^3 + 138a^2x^2 - 236ax + 788\right)\sqrt{c-acx}}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(788 - 236*a*x + 138*a^2*x^2 - 95*a^3*x^3 + 35*a^4*x^4)
)/(315*a^4) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
)/a^4
```

fricas [A] time = 0.79, size = 169, normalized size = 1.22

$$\left[\frac{2\left(315\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right) - \left(35a^4x^4 - 95a^3x^3 + 138a^2x^2 - 236ax + 788\right)\sqrt{-acx+c}\right)}{315a^4}, -\frac{2\left(63\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [2/315*(315*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) - (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*x + 788
)*sqrt(-a*c*x + c))/a^4, -2/315*(630*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sq
rt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a
*x + 788)*sqrt(-a*c*x + c))/a^4]
```

giac [A] time = 0.19, size = 159, normalized size = 1.14

$$-\frac{4\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^4\sqrt{-c}}-\frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}\,a^{32}c^{32}+45\,(acx-c)^3\sqrt{-acx+c}\,a^{32}c^{33}+63\,(acx-c)^2\sqrt{}\right.}{315\,a^{36}c^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^4*sqrt(-c)) -
 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^32*c^32 + 45*(a*c*x - c)^3*sqrt
(-a*c*x + c)*a^32*c^33 + 63*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^32*c^34 + 105*
(-a*c*x + c)^(3/2)*a^32*c^35 + 630*sqrt(-a*c*x + c)*a^32*c^36)/(a^36*c^36)
```

maple [A] time = 0.04, size = 101, normalized size = 0.73

$$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}-\frac{c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{5}{2}}c^2}{5}+\frac{c^3(-acx+c)^{\frac{3}{2}}}{3}+2\sqrt{-acx+c}\;c^4-2c^{\frac{9}{2}}\sqrt{2}\;\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\;\sqrt{2}}{2\sqrt{c}}\right)\right)}{c^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -2/c^4/a^4*(1/9*(-a*c*x+c)^(9/2)-1/7*c*(-a*c*x+c)^(7/2)+1/5*(-a*c*x+c)^(5/2
)*c^2+1/3*c^3*(-a*c*x+c)^(3/2)+2*(-a*c*x+c)^(1/2)*c^4-2*c^(9/2)*2^(1/2)*arc
tanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.46, size = 123, normalized size = 0.88

$$-\frac{2\left(315\sqrt{2}\,c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+35\,(-acx+c)^{\frac{9}{2}}-45\,(-acx+c)^{\frac{7}{2}}c+63\,(-acx+c)^{\frac{5}{2}}c^2+105\,(-acx+c)^{\frac{3}{2}}c^3\right.}{315\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] -2/315*(315*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt
(2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) - 45*(-a*c*x + c)^
(7/2)*c + 63*(-a*c*x + c)^(5/2)*c^2 + 105*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt
(-a*c*x + c)*c^4)/(a^4*c^4)
```

mupad [B] time = 0.85, size = 114, normalized size = 0.82

$$\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^4\,c^3}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^4\,c}-\frac{2\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^4\,c^2}-\frac{4\,\sqrt{c-a\,c\,x}}{a^4}-\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a^4\,c^4}-\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,4{}\mathrm{i}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(a*x + 1)^2,x)
```

```
[Out] (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) - (2*(c - a*c*x)^(3/2))/(3*a^4*c) - (2*(c
 - a*c*x)^(5/2))/(5*a^4*c^2) - (4*(c - a*c*x)^(1/2))/a^4 - (2*(c - a*c*x)^(
9/2))/(9*a^4*c^4) - (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2
*c^(1/2)))*4i)/a^4
```

sympy [A] time = 18.58, size = 126, normalized size = 0.91

$$\frac{2\left(-\frac{2\sqrt{2}\,c^{5}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2c^{4}\sqrt{-acx+c}-\frac{c^{3}(-acx+c)^{\frac{3}{2}}}{3}-\frac{c^{2}(-acx+c)^{\frac{5}{2}}}{5}+\frac{c(-acx+c)^{\frac{7}{2}}}{7}-\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^{4}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] 2*(-2*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) - 2
*c**4*sqrt(-a*c*x + c) - c**3*(-a*c*x + c)**(3/2)/3 - c**2*(-a*c*x + c)**(5
/2)/5 + c*(-a*c*x + c)**(7/2)/7 - (-a*c*x + c)**(9/2)/9)/(a**4*c**4)
```

3.420 $\int e^{-2\tanh^{-1}(ax)} x^2 \sqrt{c-acx}\, dx$

Optimal. Leaf size=97

$$\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a^3/c+2/7*(-a*c*x+c)^(7/2)/a^3/c^3-4*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3+4*(-a*c*x+c)^(1/2)/a^3
```

Rubi [A] time = 0.16, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 21, 88, 50, 63, 206}

$$\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^3 + (2*(c - a*c*x)^(3/2))/(3*a^3*c) + (2*(c - a*c*x)^
(7/2))/(7*a^3*c^3) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/a^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\int \frac{x^2(1-ax)\sqrt{c-acx}}{1+ax}\,dx$$
$$=\frac{\int \frac{x^2(c-acx)^{3/2}}{1+ax}\,dx}{c}$$
$$=\frac{\int \left(\frac{(c-acx)^{3/2}}{a^2(1+ax)}-\frac{(c-acx)^{5/2}}{a^2c}\right)dx}{c}$$
$$=\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^2c}$$
$$=\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a^2}$$
$$=\frac{4\sqrt{c-acx}}{a^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a^2}$$
$$=\frac{4\sqrt{c-acx}}{a^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a^3}$$
$$=\frac{4\sqrt{c-acx}}{a^3}+\frac{2(c-acx)^{3/2}}{3a^3c}+\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 0.80

$$\frac{2\left(-3a^3x^3+9a^2x^2-16ax+52\right)\sqrt{c-acx}-84\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{21a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(52 - 16*a*x + 9*a^2*x^2 - 3*a^3*x^3) - 84*Sqrt[2]*Sqrt[
c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(21*a^3)
```

fricas [A] time = 0.55, size = 154, normalized size = 1.59

$$\left[\frac{2\left(21\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\left(3a^3x^3-9a^2x^2+16ax-52\right)\sqrt{-acx+c}\right)}{21a^3}, \frac{2\left(42\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [2/21*(21*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) - (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/
a^3, 2/21*(42*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)
/c) - (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/a^3]
```

giac [A] time = 0.17, size = 105, normalized size = 1.08

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^3\sqrt{-c}}-\frac{2\left(3\,(acx-c)^3\sqrt{-acx+c}\,a^{18}c^{18}-7\,(-acx+c)^{\frac{3}{2}}a^{18}c^{20}-42\sqrt{-acx+c}\,a^{18}c^{21}\right)}{21\,a^{21}c^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^3*sqrt(-c)) -
2/21*(3*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^18*c^18 - 7*(-a*c*x + c)^(3/2)*a^1
8*c^20 - 42*sqrt(-a*c*x + c)*a^18*c^21)/(a^21*c^21)
```

maple [A] time = 0.04, size = 75, normalized size = 0.77

$$\frac{\frac{2(-acx+c)^{\frac{7}{2}}}{7}+\frac{2(-acx+c)^{\frac{3}{2}}c^2}{3}+4\sqrt{-acx+c}\;c^3-4c^{\frac{7}{2}}\sqrt{2}\;\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\;\sqrt{2}}{2\sqrt{c}}\right)}{c^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c^3/a^3*(1/7*(-a*c*x+c)^(7/2)+1/3*(-a*c*x+c)^(3/2)*c^2+2*(-a*c*x+c)^(1/2)
*c^3-2*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.42, size = 97, normalized size = 1.00

$$\frac{2\left(21\sqrt{2}c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3\,(-acx+c)^{\frac{7}{2}}+7\,(-acx+c)^{\frac{3}{2}}c^2+42\sqrt{-acx+c}\,c^3\right)}{21\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] 2/21*(21*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)
*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(7/2) + 7*(-a*c*x + c)^(3/2)
*c^2 + 42*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

mupad [B] time = 0.10, size = 80, normalized size = 0.82

$$\frac{4\sqrt{c-a\,c\,x}}{a^3}+\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^3\,c}+\frac{2\,(c-a\,c\,x)^{7/2}}{7\,a^3\,c^3}+\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(a*x + 1)^2,x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^3 + (2*(c - a*c*x)^(3/2))/(3*a^3*c) + (2*(c - a*c*x
)^(7/2))/(7*a^3*c^3) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)
/(2*c^(1/2)))*4i)/a^3
```

sympy [A] time = 14.87, size = 97, normalized size = 1.00

$$-\frac{2\left(-\frac{2\sqrt{2}c^4\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2c^3\sqrt{-acx+c}-\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -2*(-2*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) -
2*c**3*sqrt(-a*c*x + c) - c**2*(-a*c*x + c)**(3/2)/3 - (-a*c*x + c)**(7/2)/
7)/(a**3*c**3)
```

3.421 $\int e^{-2\tanh^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal. Leaf size=97

$$-\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{3a^2c}-\frac{4\sqrt{c-acx}}{a^2}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a^2/c-2/5*(-a*c*x+c)^(5/2)/a^2/c^2+4*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2-4*(-a*c*x+c)^(1/2)/a^2
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6130, 21, 80, 50, 63, 206}

$$-\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{3a^2c}-\frac{4\sqrt{c-acx}}{a^2}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^2 - (2*(c - a*c*x)^(3/2))/(3*a^2*c) - (2*(c - a*c*x)
^(5/2))/(5*a^2*c^2) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/a^2
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx &= \int \frac{x(1-ax)\sqrt{c-acx}}{1+ax}\,dx \\
&= \frac{\int \frac{x(c-acx)^{3/2}}{1+ax}\,dx}{c} \\
&= -\frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{ac} \\
&= -\frac{2(c-acx)^{3/2}}{3a^2c} - \frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a} \\
&= -\frac{4\sqrt{c-acx}}{a^2} - \frac{2(c-acx)^{3/2}}{3a^2c} - \frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a} \\
&= -\frac{4\sqrt{c-acx}}{a^2} - \frac{2(c-acx)^{3/2}}{3a^2c} - \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a^2} \\
&= -\frac{4\sqrt{c-acx}}{a^2} - \frac{2(c-acx)^{3/2}}{3a^2c} - \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 70, normalized size = 0.72

$$
\frac{60\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) - 2\left(3a^2x^2-11ax+38\right)\sqrt{c-acx}}{15a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x]*(38 - 11*a*x + 3*a^2*x^2) + 60*Sqrt[2]*Sqrt[c]*ArcTanh[
Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a^2)
```

fricas [A] time = 0.46, size = 137, normalized size = 1.41

$$
\left[\frac{2\left(15\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right) - \left(3a^2x^2-11ax+38\right)\sqrt{-acx+c}\right)}{15a^2}, -\frac{2\left(30\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx}}{2c}\right.\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) - (3*a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2, -2/15*(
30*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (3*a^
2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2]
```

giac [A] time = 0.27, size = 105, normalized size = 1.08

$$-\frac{4\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{a^2\sqrt{-c}}-\frac{2\left(3\,(acx-c)^2\sqrt{-acx+c}\,a^8c^8+5\,(-acx+c)^{\frac{3}{2}}a^8c^9+30\,\sqrt{-acx+c}\,a^8c^{10}\right)}{15\,a^{10}c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^2*sqrt(-c)) -
 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^8 + 5*(-a*c*x + c)^(3/2)*a^8*
c^9 + 30*sqrt(-a*c*x + c)*a^8*c^10)/(a^10*c^10)
```

maple [A] time = 0.04, size = 73, normalized size = 0.75

$$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}+\frac{c(-acx+c)^{\frac{3}{2}}}{3}+2\sqrt{-acx+c}\ c^2-2c^{\frac{5}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\ \sqrt{2}}{2\sqrt{c}}\right)\right)}{c^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -2/c^2/a^2*(1/5*(-a*c*x+c)^(5/2)+1/3*c*(-a*c*x+c)^(3/2)+2*(-a*c*x+c)^(1/2)*
c^2-2*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.40, size = 95, normalized size = 0.98

$$-\frac{2\left(15\,\sqrt{2}\,c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+3\,(-acx+c)^{\frac{5}{2}}+5\,(-acx+c)^{\frac{3}{2}}c+30\,\sqrt{-acx+c}\,c^2\right)}{15\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -2/15*(15*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 5*(-a*c*x + c)^(3/2
)*c + 30*sqrt(-a*c*x + c)*c^2)/(a^2*c^2)
```

mupad [B] time = 0.86, size = 80, normalized size = 0.82

$$-\frac{4\,\sqrt{c-a\,c\,x}}{a^2}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^2\,c}-\frac{2\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^2\,c^2}-\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,4{}\mathrm{i}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(a*x + 1)^2,x)
```

```
[Out] - (4*(c - a*c*x)^(1/2))/a^2 - (2*(c - a*c*x)^(3/2))/(3*a^2*c) - (2*(c - a*c
*x)^(5/2))/(5*a^2*c^2) - (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1
i)/(2*c^(1/2)))*4i)/a^2
```

sympy [A] time = 11.87, size = 94, normalized size = 0.97

$$\frac{2\left(-\frac{2\sqrt{2}c^3\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2c^2\sqrt{-acx+c}-\frac{c(-acx+c)^{\frac{3}{2}}}{3}-\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] 2*(-2*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) - 2
*c**2*sqrt(-a*c*x + c) - c*(-a*c*x + c)**(3/2)/3 - (-a*c*x + c)**(5/2)/5)/(
a**2*c**2)
```

3.422 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{c-acx}}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a/c-4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^
(1/2)*c^(1/2)/a+4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6130, 21, 50, 63, 206}

$$\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{c-acx}}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a + (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*Sqrt[2]*Sqrt[c]*
ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx &= \int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= \frac{2(c-acx)^{3/2}}{3ac}+2\int \frac{\sqrt{c-acx}}{1+ax}\,dx\\
&= \frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}+(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= \frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}-\frac{8\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= \frac{4\sqrt{c-acx}}{a}+\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.80

$$
-\frac{2(ax-7)\sqrt{c-acx}+12\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/3*(2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.52, size = 120, normalized size = 1.58

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\sqrt{-acx+c}\,(ax-7)\right)}{3a}, \frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}\right.}{3a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) - sqrt(-a*c*x + c)*(a*x - 7))/a, 2/3*(6*sqrt(2)*sqrt(-c)*arc
tan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))/
a]
```

giac [A] time = 0.17, size = 77, normalized size = 1.01

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}\,a^2c^3\right)}{3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) + 2/
3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

maple [A] time = 0.03, size = 59, normalized size = 0.78

$$\frac{\frac{2(-acx+c)^{\frac{3}{2}}}{3}+4c\sqrt{-acx+c}-4c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh(
1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.40, size = 79, normalized size = 1.04

$$\frac{2\left(3\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*s
qrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)/(a
*c)
```

mupad [B] time = 0.00, size = 61, normalized size = 0.80

$$\frac{4\sqrt{c-a\,c\,x}}{a}+\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c}-\frac{4\sqrt{2}\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(a*x + 1)^2,x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a + (2*(c - a*c*x)^(3/2))/(3*a*c) - (4*2^(1/2)*c^(1/2
)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a
```

sympy [A] time = 8.08, size = 75, normalized size = 0.99

$$-\frac{2\left(-\frac{2\sqrt{2}c^2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2c\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -2*(-2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) -
2*c*sqrt(-a*c*x + c) - (-a*c*x + c)**(3/2)/3)/(a*c)
```

3.423 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=74

$$-2\sqrt{c-acx}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*arctanh(1/2*(-a*c*x+c)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-2*(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 21, 84, 156, 63, 208, 206}

$$-2\sqrt{c-acx}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2]
*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
```

```
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \int \frac{(1-ax)\sqrt{c-acx}}{x(1+ax)}\,dx$$
$$= \frac{\int \frac{(c-acx)^{3/2}}{x(1+ax)}\,dx}{c}$$
$$= -2\sqrt{c-acx} + \frac{\int \frac{ac^2-3a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{ac}$$
$$= -2\sqrt{c-acx} + c\int \frac{1}{x\sqrt{c-acx}}\,dx - (4ac)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$
$$= -2\sqrt{c-acx} + 8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right) - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-a\right.}{a}$$
$$= -2\sqrt{c-acx} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.00

$$-2\sqrt{c-acx} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -2*Sqrt[c - a*c*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2]
*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.48, size = 157, normalized size = 2.12

$$\left[2\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right) + \sqrt{c}\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right) - 2\sqrt{-acx+c}, -4\sqrt{2}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(
a*x + 1)) + sqrt(c)*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) - 2*s
qrt(-a*c*x + c), -4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sq
```

```
rt(-c)/c) + 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x
+ c)]
```

giac [A] time = 0.17, size = 67, normalized size = 0.91

$$-\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*c*a
rctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*sqrt(-a*c*x + c)
```

maple [A] time = 0.04, size = 58, normalized size = 0.78

$$-2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}+4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}-2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] -2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*arctanh(1/2*(-a*c*x+c)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-2*(-a*c*x+c)^(1/2)
```

maxima [A] time = 0.41, size = 97, normalized size = 1.31

$$-2\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+\sqrt{c}\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)-2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(
c) + sqrt(-a*c*x + c))) + sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a
*c*x + c) + sqrt(c))) - 2*sqrt(-a*c*x + c)
```

mupad [B] time = 0.83, size = 57, normalized size = 0.77

$$4\sqrt{2}\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)-2\,\sqrt{c-a\,c\,x}-2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(x*(a*x + 1)^2),x)
```

```
[Out] 4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))) - 2*(c - a
*c*x)^(1/2) - 2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2))
```

sympy [A] time = 10.11, size = 80, normalized size = 1.08

$$\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{4\sqrt{2}\,c\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - 2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x,x)
```

```
[Out] 2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 4*sqrt(2)*c*atan(sqrt(2)*sqr
t(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) - 2*sqrt(-a*c*x + c)
```

3.424 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{c-acx}}{x}+5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 5*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a*arctanh(1/2*(-a*c*x+c)^(1
/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 21, 98, 156, 63, 208, 206}

$$-\frac{\sqrt{c-acx}}{x}+5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a*c*x]/x) + 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqr
t[2]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{(1-ax)\sqrt{c-acx}}{x^2(1+ax)}\,dx$$
$$= \frac{\int \frac{(c-acx)^{3/2}}{x^2(1+ax)}\,dx}{c}$$
$$= -\frac{\sqrt{c-acx}}{x} - \frac{\int \frac{\frac{5ac^2}{2}-\frac{3}{2}a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{c}$$
$$= -\frac{\sqrt{c-acx}}{x} - \frac{1}{2}(5ac)\int \frac{1}{x\sqrt{c-acx}}\,dx + \left(4a^2c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$
$$= -\frac{\sqrt{c-acx}}{x} + 5\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) - (8a)\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x,$$
$$= -\frac{\sqrt{c-acx}}{x} + 5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\,a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.05, size = 79, normalized size = 1.00

$$-\frac{\sqrt{c-acx}}{x} + 5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\,a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a*c*x]/x) + 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqr
t[2]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.63, size = 175, normalized size = 2.22

$$\left[\frac{4\sqrt{2}\,a\sqrt{c}\,x\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right) + 5a\sqrt{c}\,x\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right) - 2\sqrt{-acx+c}}{2x}, \frac{4\sqrt{2}\,a\sqrt{-c}\,x\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(4*sqrt(2)*a*sqrt(c)*x*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + 5*a*sqrt(c)*x*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c)
```

```
- 2*c)/x) - 2*sqrt(-a*c*x + c))/x, (4*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt(
2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 5*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sq
rt(-c)/c) - sqrt(-a*c*x + c))/x]
```

giac [A] time = 0.18, size = 72, normalized size = 0.91

$$\frac{4\sqrt{2}\,ac\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{5\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 5*a*
c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/x
```

maple [A] time = 0.04, size = 71, normalized size = 0.90

$$2ac\left(-\frac{2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\ \sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{-acx+c}}{2xac}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] 2*a*c*(-2*2^(1/2)/c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))-1/2
*(-a*c*x+c)^(1/2)/x/a/c+5/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.44, size = 111, normalized size = 1.41

$$\frac{1}{2}\,ac\left(\frac{4\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{\sqrt{c}}-\frac{5\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\sqrt{-acx+c}}{acx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*a*c*(4*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(
c) + sqrt(-a*c*x + c)))/sqrt(c) - 5*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(
-a*c*x + c) + sqrt(c)))/sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

mupad [B] time = 0.86, size = 62, normalized size = 0.78

$$5\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)-\frac{\sqrt{c-a\,c\,x}}{x}-4\,\sqrt{2}\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(x^2*(a*x + 1)^2),x)
```

```
[Out] 5*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) - (c - a*c*x)^(1/2)/x - 4*2^(1
/2)*a*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

sympy [B] time = 14.77, size = 162, normalized size = 2.05

$$\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{6ac\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}\,ac\,\mathrm{atan}\left(\cdots\right.}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
```

```
[Out] a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 - a*c**2
*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 - 6*a*c*atan(sq
rt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*a*c*atan(sqrt(2)*sqrt(-a*c*x
+ c)/(2*sqrt(-c)))/sqrt(-c) - sqrt(-a*c*x + c)/x
```

3.425 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=106

$$-\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{\sqrt{c-acx}}{2x^2}+\frac{9a\sqrt{c-acx}}{4x}$$

```
[Out] -23/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a^2*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-1/2*(-a*c*x+c)^(1/2)/x^2+9/4*a
*(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6130, 21, 98, 151, 156, 63, 208, 206}

$$-\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{\sqrt{c-acx}}{2x^2}+\frac{9a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -Sqrt[c - a*c*x]/(2*x^2) + (9*a*Sqrt[c - a*c*x])/(4*x) - (23*a^2*Sqrt[c]*Ar
cTanh[Sqrt[c - a*c*x]/Sqrt[c]])/4 + 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c -
a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{(1-ax)\sqrt{c-acx}}{x^3(1+ax)}\,dx$$
$$= \frac{\int \frac{(c-acx)^{3/2}}{x^3(1+ax)}\,dx}{c}$$
$$= -\frac{\sqrt{c-acx}}{2x^2} - \frac{\int \frac{\frac{9ac^2}{2}-\frac{7}{2}a^2c^2x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{2c}$$
$$= -\frac{\sqrt{c-acx}}{2x^2} + \frac{9a\sqrt{c-acx}}{4x} + \frac{\int \frac{\frac{23a^2c^3}{4}-\frac{9}{4}a^3c^3x}{x(1+ax)\sqrt{c-acx}}\,dx}{2c^2}$$
$$= -\frac{\sqrt{c-acx}}{2x^2} + \frac{9a\sqrt{c-acx}}{4x} + \frac{1}{8}\left(23a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx - \left(4a^3c\right)\int \frac{1}{(1+ax)\sqrt{c-}}$$
$$= -\frac{\sqrt{c-acx}}{2x^2} + \frac{9a\sqrt{c-acx}}{4x} - \frac{1}{4}(23a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) + \left(8a^2\right)\text{Sul}$$
$$= -\frac{\sqrt{c-acx}}{2x^2} + \frac{9a\sqrt{c-acx}}{4x} - \frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.$$

Mathematica [A] time = 0.08, size = 93, normalized size = 0.88

$$-\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) + \frac{(9ax-2)\sqrt{c-acx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] ((-2 + 9*a*x)*Sqrt[c - a*c*x])/(4*x^2) - (23*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/Sqrt[c]])/4 + 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*
Sqrt[c])]
```

fricas [A] time = 0.70, size = 204, normalized size = 1.92

$$\left[\frac{16\sqrt{2}\,a^2\sqrt{c}\,x^2\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right) + 23\,a^2\sqrt{c}\,x^2\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right) + 2\sqrt{-acx+c}\,(9\,ax-2)}{8\,x^2}, -\frac{16}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fricas"
)
```

```
[Out] [1/8*(16*sqrt(2)*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sq
rt(c) - 3*c)/(a*x + 1)) + 23*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(-a*c*x + c
)*sqrt(c) - 2*c)/x) + 2*sqrt(-a*c*x + c)*(9*a*x - 2))/x^2, -1/4*(16*sqrt(2)
*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 23*a^2*
sqrt(-c)*x^2*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(9*a*x
- 2))/x^2]
```

giac [A] time = 0.20, size = 106, normalized size = 1.00

$$-\frac{4\sqrt{2}\,a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{23\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}}-\frac{9\,(-acx+c)^{\frac{3}{2}}a^2c-7\sqrt{-acx+c}\,a^2c^2}{4\,a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^2*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2
3/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/4*(9*(-a*c*x + c)^
(3/2)*a^2*c - 7*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

maple [A] time = 0.04, size = 95, normalized size = 0.90

$$-2a^2c^2\left(-\frac{2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\ \sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\frac{-\frac{9(-acx+c)^{\frac{3}{2}}}{8}+\frac{7c\sqrt{-acx+c}}{8}}{x^2a^2c^2}-\frac{23\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)
```

```
[Out] -2*a^2*c^2*(-2/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
)-1/c*((-9/8*(-a*c*x+c)^(3/2)+7/8*c*(-a*c*x+c)^(1/2))/x^2/a^2/c^2-23/8/c^(1
/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

maxima [A] time = 0.40, size = 152, normalized size = 1.43

$$-\frac{1}{8}\,a^2c^2\left(\frac{2\left(9\,(-acx+c)^{\frac{3}{2}}-7\sqrt{-acx+c}\,c\right)}{(acx-c)^2c+2\,(acx-c)c^2+c^3}+\frac{16\sqrt{2}\,\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}-\frac{23\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="maxima"
)
```

```
[Out] -1/8*a^2*c^2*(2*(9*(-a*c*x + c)^(3/2) - 7*sqrt(-a*c*x + c)*c)/((a*c*x - c)^
2*c + 2*(a*c*x - c)*c^2 + c^3) + 16*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a
*c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*c*x + c)))/c^(3/2) - 23*log((sqrt(-a*
c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(3/2))
```

mupad [B] time = 0.11, size = 88, normalized size = 0.83

$$\frac{7\sqrt{c-a\,c\,x}}{4\,x^2}+\frac{a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,23\mathrm{i}}{4}-\frac{9\,{\left(c-a\,c\,x\right)}^{3/2}}{4\,c\,x^2}-\sqrt{2}\,a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(x^3*(a*x + 1)^2),x)
```

```
[Out] (7*(c - a*c*x)^(1/2))/(4*x^2) + (a^2*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/c^
(1/2))*23i)/4 - (9*(c - a*c*x)^(3/2))/(4*c*x^2) - 2^(1/2)*a^2*c^(1/2)*atan(
(2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

sympy [B] time = 25.66, size = 352, normalized size = 3.32

$$-\frac{10a^2c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}+\frac{6a^2c^3\left(-acx+c\right)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}+\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}-\frac{3a^2c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)*
*2) + 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*
x + c)**2) + 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*
x + c))/8 - 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x
+ c))/8 - 3*a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x +
 c))/2 + 3*a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c
))/2 + 8*a**2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 4*sqrt(2)*a**2*c
*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 3*a*sqrt(-a*c*x + c
)/x
```

3.426 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=127

$$\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{19a^2\sqrt{c-acx}}{8x}-\frac{\sqrt{c-acx}}{3x^3}+\frac{13a\sqrt{c-acx}}{12x^2}$$

```
[Out] 45/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a^3*arctanh(1/2*(-a*c*
x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-1/3*(-a*c*x+c)^(1/2)/x^3+13/12*
a*(-a*c*x+c)^(1/2)/x^2-19/8*a^2*(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6130, 21, 98, 151, 156, 63, 208, 206}

$$-\frac{19a^2\sqrt{c-acx}}{8x}+\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{13a\sqrt{c-acx}}{12x^2}-\frac{\sqrt{c-acx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] -Sqrt[c - a*c*x]/(3*x^3) + (13*a*Sqrt[c - a*c*x])/(12*x^2) - (19*a^2*Sqrt[c
 - a*c*x])/(8*x) + (45*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 - 4*
Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{(1-ax)\sqrt{c-acx}}{x^4(1+ax)}\,dx$$

$$= \frac{\int \frac{(c-acx)^{3/2}}{x^4(1+ax)}\,dx}{c}$$

$$= -\frac{\sqrt{c-acx}}{3x^3} - \frac{\int \frac{\frac{13ac^2}{2}-\frac{11}{2}a^2c^2x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{3c}$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} + \frac{\int \frac{\frac{57a^2c^3}{4}-\frac{39}{4}a^3c^3x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{6c^2}$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} - \frac{19a^2\sqrt{c-acx}}{8x} - \frac{\int \frac{\frac{135a^3c^4}{8}-\frac{57}{8}a^4c^4x}{x(1+ax)\sqrt{c-acx}}\,dx}{6c^3}$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} - \frac{19a^2\sqrt{c-acx}}{8x} - \frac{1}{16}\left(45a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx + (4$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} - \frac{19a^2\sqrt{c-acx}}{8x} + \frac{1}{8}\left(45a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x,\right.$$

$$= -\frac{\sqrt{c-acx}}{3x^3} + \frac{13a\sqrt{c-acx}}{12x^2} - \frac{19a^2\sqrt{c-acx}}{8x} + \frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.80

$$\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) + \frac{\left(-57a^2x^2+26ax-8\right)\sqrt{c-acx}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-8 + 26*a*x - 57*a^2*x^2))/(24*x^3) + (45*a^3*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.50, size = 220, normalized size = 1.73

$$\left[\frac{96\sqrt{2}a^3\sqrt{c}x^3\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right) + 135a^3\sqrt{c}x^3\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right) - 2\left(57a^2x^2-26ax+8\right)\sqrt{-}}{48x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/48*(96*sqrt(2)*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c) - 3*c)/(a*x + 1)) + 135*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(-a*c*x +
 c)*sqrt(c) - 2*c)/x) - 2*(57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3,
1/24*(96*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(
-c)/c) - 135*a^3*sqrt(-c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (57*a^2
*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3]
```

giac [A] time = 0.15, size = 133, normalized size = 1.05

$$\frac{4\sqrt{2}\,a^3c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{45\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}}-\frac{57\,(acx-c)^2\sqrt{-acx+c}\,a^3c-88\,(-acx+c)^{\frac{3}{2}}a^3c^2+39\sqrt{}}{24\,a^3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 45
/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/24*(57*(a*c*x - c)^
2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 39*sqrt(-a*c*x +
 c)*a^3*c^3)/(a^3*c^3*x^3)
```

maple [A] time = 0.04, size = 110, normalized size = 0.87

$$2a^3c^3\left(-\frac{2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\ \sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{-\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11c(-acx+c)^{\frac{3}{2}}}{6}-\frac{13\sqrt{-acx+c}\ c^2}{16}}{x^3a^3c^3}-\frac{45\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16\sqrt{c}}}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
```

```
[Out] 2*a^3*c^3*(-2/c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
-1/c^2*(-(-19/16*(-a*c*x+c)^(5/2)+11/6*c*(-a*c*x+c)^(3/2)-13/16*(-a*c*x+c)^
(1/2)*c^2)/x^3/a^3/c^3-45/16/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

maxima [A] time = 0.41, size = 183, normalized size = 1.44

$$-\frac{1}{48}\,a^3c^3\left(\frac{2\left(57\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+39\sqrt{-acx+c}\,c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{96\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}+\frac{135\log\left(\frac{\sqrt{}}{\sqrt{}}\right)}{c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="maxima"
)
```

```
[Out] -1/48*a^3*c^3*(2*(57*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 39*sqrt
(-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)*
c^4 + c^5) - 96*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*
sqrt(c) + sqrt(-a*c*x + c)))/c^(5/2) + 135*log((sqrt(-a*c*x + c) - sqrt(c))
/(sqrt(-a*c*x + c) + sqrt(c)))/c^(5/2))
```

mupad [B] time = 0.13, size = 105, normalized size = 0.83

$$\frac{11\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,c\,x^3}-\frac{a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,45{}\mathrm{i}}{8}-\frac{13\,\sqrt{c-a\,c\,x}}{8\,x^3}-\frac{19\,{\left(c-a\,c\,x\right)}^{5/2}}{8\,c^2\,x^3}+\sqrt{2}\,a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(x^4*(a*x + 1)^2),x)
```

```
[Out] (11*(c - a*c*x)^(3/2))/(3*c*x^3) - (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)
/c^(1/2))*45i)/8 - (13*(c - a*c*x)^(1/2))/(8*x^3) - (19*(c - a*c*x)^(5/2))/
(8*c^2*x^3) + 2^(1/2)*a^3*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^
(1/2)))*4i
```

sympy [B] time = 29.74, size = 614, normalized size = 4.83

$$\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}-\frac{80a^3c^5\left(-acx+c\right)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)
```

```
[Out] 66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x +
 c)**2 + 48*c**3*(-a*c*x + c)**3) - 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144*
a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) +
30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*
x + c)**2 + 48*c**3*(-a*c*x + c)**3) + 30*a**3*c**4*sqrt(-a*c*x + c)/(16*a*
c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 5*a**3*c**4*sqrt(c**(-7))*log(-
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 - 5*a**3*c**4*sqrt(c**(-7))*log(c
**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 - 18*a**3*c**3*(-a*c*x + c)**(3/2)
/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 9*a**3*c**3*sqrt(c**(-5)
)*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 + 9*a**3*c**3*sqrt(c**(-5))
*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 + 2*a**3*c**2*sqrt(c**(-3))*l
```

```
og(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) - 2*a**3*c**2*sqrt(c**(-3))*log(
c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) - 8*a**3*c*atan(sqrt(-a*c*x + c)/sqr
t(-c))/sqrt(-c) + 4*sqrt(2)*a**3*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c
)))/sqrt(-c) - 4*a**2*sqrt(-a*c*x + c)/x
```

3.427 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=148

$$-\frac{363}{64}a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{149a^3\sqrt{c-acx}}{64x}-\frac{107a^2\sqrt{c-acx}}{96x^2}-\frac{\sqrt{c-acx}}{4x^4}+$$

```
[Out] -363/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a^4*arctanh(1/2*(-a
*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-1/4*(-a*c*x+c)^(1/2)/x^4+17/
24*a*(-a*c*x+c)^(1/2)/x^3-107/96*a^2*(-a*c*x+c)^(1/2)/x^2+149/64*a^3*(-a*c*
x+c)^(1/2)/x
```

Rubi [A] time = 0.21, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6130, 21, 98, 151, 156, 63, 208, 206}

$$-\frac{107a^2\sqrt{c-acx}}{96x^2}+\frac{149a^3\sqrt{c-acx}}{64x}-\frac{363}{64}a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{17a\sqrt{c-ac}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] -Sqrt[c - a*c*x]/(4*x^4) + (17*a*Sqrt[c - a*c*x])/(24*x^3) - (107*a^2*Sqrt[
c - a*c*x])/(96*x^2) + (149*a^3*Sqrt[c - a*c*x])/(64*x) - (363*a^4*Sqrt[c]*
ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 + 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c
 - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{(1-ax)\sqrt{c-acx}}{x^5(1+ax)}\,dx$$

$$= \frac{\int \frac{(c-acx)^{3/2}}{x^5(1+ax)}\,dx}{c}$$

$$= -\frac{\sqrt{c-acx}}{4x^4} - \frac{\int \frac{\frac{17ac^2}{2}-\frac{15}{2}a^2c^2x}{x^4(1+ax)\sqrt{c-acx}}\,dx}{4c}$$

$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} + \frac{\int \frac{\frac{107a^2c^3}{4}-\frac{85}{4}a^3c^3x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{12c^2}$$

$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{\int \frac{\frac{447a^3c^4}{8}-\frac{321}{8}a^4c^4x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{24c^3}$$

$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{149a^3\sqrt{c-acx}}{64x} + \frac{\int \frac{\frac{1089a^4c^5}{16}-\frac{447}{16}a}{x(1+ax)\sqrt{c-a}}}{24c^4}$$

$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{149a^3\sqrt{c-acx}}{64x} + \frac{1}{128}\left(363a^4c\right)$$

$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{149a^3\sqrt{c-acx}}{64x} - \frac{1}{64}\left(363a^3\right)\text{Su}$$

$$= -\frac{\sqrt{c-acx}}{4x^4} + \frac{17a\sqrt{c-acx}}{24x^3} - \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{149a^3\sqrt{c-acx}}{64x} - \frac{363}{64}a^4\sqrt{c}\,\tan$$

Mathematica [A] time = 0.11, size = 109, normalized size = 0.74

$$-\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{\left(447a^3x^3-214a^2x^2+136ax-48\right)\sqrt{c-acx}}{192x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-48 + 136*a*x - 214*a^2*x^2 + 447*a^3*x^3))/(192*x^4) - (
363*a^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 + 4*Sqrt[2]*a^4*Sqrt[c
]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.56, size = 236, normalized size = 1.59

$$\left[\frac{768\,\sqrt{2}\,a^4\sqrt{c}\,x^4\log\left(\frac{acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{ax+1}\right)+1089\,a^4\sqrt{c}\,x^4\log\left(\frac{acx+2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)+2\left(447\,a^3x^3-214\,a^2x^2+\right.}{384\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="fricas"
)
```

```
[Out] [1/384*(768*sqrt(2)*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)
*sqrt(c) - 3*c)/(a*x + 1)) + 1089*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(-a*c*
x + c)*sqrt(c) - 2*c)/x) + 2*(447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqr
t(-a*c*x + c))/x^4, -1/192*(768*sqrt(2)*a^4*sqrt(-c)*x^4*arctan(1/2*sqrt(2)
*sqrt(-a*c*x + c)*sqrt(-c)/c) - 1089*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x +
c)*sqrt(-c)/c) - (447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqrt(-a*c*x + c
))/x^4]
```

giac [A] time = 0.18, size = 160, normalized size = 1.08

$$-\frac{4\,\sqrt{2}\,a^4c\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{363\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\,\sqrt{-c}}+\frac{447\,(acx-c)^3\sqrt{-acx+c}\,a^4c+1127\,(acx-c)^2\sqrt{-acx}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 3
63/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/192*(447*(a*c*x
- c)^3*sqrt(-a*c*x + c)*a^4*c + 1127*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^2
 - 1049*(-a*c*x + c)^(3/2)*a^4*c^3 + 321*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*c^4
*x^4)
```

maple [A] time = 0.04, size = 123, normalized size = 0.83

$$-2a^4c^4\left(-\frac{2\sqrt{2}\;\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\;\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{\frac{-\frac{149(-acx+c)^{\frac{7}{2}}}{128}+\frac{1127c(-acx+c)^{\frac{5}{2}}}{384}-\frac{1049(-acx+c)^{\frac{3}{2}}c^2}{384}+\frac{107\sqrt{-acx+c}\;c^3}{128}}{x^4a^4c^4}-\frac{363\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{128\sqrt{c}}}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x)
```

```
[Out] -2*a^4*c^4*(-2/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
)-1/c^3*((-149/128*(-a*c*x+c)^(7/2)+1127/384*c*(-a*c*x+c)^(5/2)-1049/384*(-
a*c*x+c)^(3/2)*c^2+107/128*(-a*c*x+c)^(1/2)*c^3)/x^4/a^4/c^4-363/128/c^(1/2
)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

maxima [A] time = 0.60, size = 212, normalized size = 1.43

$$-\frac{1}{384}\,a^4c^4\left(\frac{2\left(447\,(-acx+c)^{\frac{7}{2}}-1127\,(-acx+c)^{\frac{5}{2}}c+1049\,(-acx+c)^{\frac{3}{2}}c^2-321\,\sqrt{-acx+c}\,c^3\right)}{(acx-c)^4c^3+4\,(acx-c)^3c^4+6\,(acx-c)^2c^5+4\,(acx-c)c^6+c^7}+\frac{768\,\sqrt{2}\,\log\left(-\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="maxima"
)
```

```
[Out] -1/384*a^4*c^4*(2*(447*(-a*c*x + c)^(7/2) - 1127*(-a*c*x + c)^(5/2)*c + 104
9*(-a*c*x + c)^(3/2)*c^2 - 321*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4
*(a*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 768*s
qrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a
*c*x + c)))/c^(7/2) - 1089*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x +
c) + sqrt(c)))/c^(7/2))
```

mupad [B] time = 0.89, size = 122, normalized size = 0.82

$$\frac{107\,\sqrt{c-a\,c\,x}}{64\,x^4}+\frac{a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,363\mathrm{i}}{64}-\frac{1049\,(c-a\,c\,x)^{3/2}}{192\,c\,x^4}+\frac{1127\,(c-a\,c\,x)^{5/2}}{192\,c^2\,x^4}-\frac{149\,(c-a\,c\,x)^{7/2}}{64\,c^3\,x^4}-\sqrt{2}\,a^4\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2*x^2 - 1)*(c - a*c*x)^(1/2))/(x^5*(a*x + 1)^2),x)
```

```
[Out] (107*(c - a*c*x)^(1/2))/(64*x^4) + (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)
/c^(1/2))*363i)/64 - (1049*(c - a*c*x)^(3/2))/(192*c*x^4) + (1127*(c - a*c*
x)^(5/2))/(192*c^2*x^4) - (149*(c - a*c*x)^(7/2))/(64*c^3*x^4) - 2^(1/2)*a^
4*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

sympy [B] time = 49.27, size = 991, normalized size = 6.70

$$-\frac{558a^4c^8\sqrt{-acx+c}}{1536ac^8x-1152c^8+2304c^6\,(-acx+c)^2-1536c^5\,(-acx+c)^3+384c^4\,(-acx+c)^4}+\frac{}{1536ac^8x-1152c^8+230}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**5,x)
```

```
[Out] -558*a**4*c**8*sqrt(-a*c*x + c)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*
c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 1022*
a**4*c**7*(-a*c*x + c)**(3/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*
x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 770*a**
4*c**6*(-a*c*x + c)**(5/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x +
 c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 198*a**4*c
**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 +
48*c**3*(-a*c*x + c)**3) + 210*a**4*c**5*(-a*c*x + c)**(7/2)/(1536*a*c**8*x
 - 1152*c**8 + 2304*c**6*(-a*c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*
c**4*(-a*c*x + c)**4) + 240*a**4*c**5*(-a*c*x + c)**(3/2)/(-144*a*c**6*x +
96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) + 35*a**4*c**
5*sqrt(c**(-9))*log(-c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 - 35*a**4*c
**5*sqrt(c**(-9))*log(c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 - 90*a**4*
c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**
2 + 48*c**3*(-a*c*x + c)**3) - 40*a**4*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x -
 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 15*a**4*c**4*sqrt(c**(-7))*log(-c**4*sq
rt(c**(-7)) + sqrt(-a*c*x + c))/16 + 15*a**4*c**4*sqrt(c**(-7))*log(c**4*sq
rt(c**(-7)) + sqrt(-a*c*x + c))/16 + 24*a**4*c**3*(-a*c*x + c)**(3/2)/(16*a
*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 3*a**4*c**3*sqrt(c**(-5))*log(
-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/2 - 3*a**4*c**3*sqrt(c**(-5))*log(c
**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/2 - 2*a**4*c**2*sqrt(c**(-3))*log(-c*
*2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) + 2*a**4*c**2*sqrt(c**(-3))*log(c**2*s
qrt(c**(-3)) + sqrt(-a*c*x + c)) + 8*a**4*c*atan(sqrt(-a*c*x + c)/sqrt(-c))
/sqrt(-c) - 4*sqrt(2)*a**4*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sq
rt(-c) + 4*a**3*sqrt(-a*c*x + c)/x
```

3.428 $\int e^{-3\tanh^{-1}(ax)} x^3 \sqrt{c-acx}\,dx$

Optimal. Leaf size=235

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{9/2}}{9a^4(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}(ax+1)^{7/2}}{a^4(c-acx)^{3/2}}+\frac{38c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^4(c-acx)^{3/2}}-\frac{50c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^4(c-acx)^{3/2}}+\frac{32c^2}{}$$

```
[Out] -50/3*c^2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)/a^4/(-a*c*x+c)^(3/2)+38/5*c^2*(-a*x+
1)^(3/2)*(a*x+1)^(5/2)/a^4/(-a*c*x+c)^(3/2)-2*c^2*(-a*x+1)^(3/2)*(a*x+1)^(7
/2)/a^4/(-a*c*x+c)^(3/2)+2/9*c^2*(-a*x+1)^(3/2)*(a*x+1)^(9/2)/a^4/(-a*c*x+c
)^(3/2)+8*c^2*(-a*x+1)^(3/2)/a^4/(-a*c*x+c)^(3/2)/(a*x+1)^(1/2)+32*c^2*(-a*
x+1)^(3/2)*(a*x+1)^(1/2)/a^4/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6130, 23, 88}

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{9/2}}{9a^4(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}(ax+1)^{7/2}}{a^4(c-acx)^{3/2}}+\frac{38c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^4(c-acx)^{3/2}}-\frac{50c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^4(c-acx)^{3/2}}+\frac{32c^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*c^2*(1 - a*x)^(3/2))/(a^4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (32*c^2*(1
- a*x)^(3/2)*Sqrt[1 + a*x])/(a^4*(c - a*c*x)^(3/2)) - (50*c^2*(1 - a*x)^(3/
2)*(1 + a*x)^(3/2))/(3*a^4*(c - a*c*x)^(3/2)) + (38*c^2*(1 - a*x)^(3/2)*(1
+ a*x)^(5/2))/(5*a^4*(c - a*c*x)^(3/2)) - (2*c^2*(1 - a*x)^(3/2)*(1 + a*x)^
(7/2))/(a^4*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x)^(3/2)*(1 + a*x)^(9/2))/(9
*a^4*(c - a*c*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \int \frac{x^3(1-ax)^{3/2}\sqrt{c-acx}}{(1+ax)^{3/2}}\,dx$$
$$= \frac{(1-ax)^{3/2}\int \frac{x^3(c-acx)^2}{(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$= \frac{(1-ax)^{3/2}\int \left(-\frac{4c^2}{a^3(1+ax)^{3/2}} + \frac{16c^2}{a^3\sqrt{1+ax}} - \frac{25c^2\sqrt{1+ax}}{a^3} + \frac{19c^2(1+ax)^{3/2}}{a^3} - \frac{7c^2(1+ax)^{5/2}}{a^3} + \frac{c^2(1}{}\right.}{(c-acx)^{3/2}}$$
$$= \frac{8c^2(1-ax)^{3/2}}{a^4\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{32c^2(1-ax)^{3/2}\sqrt{1+ax}}{a^4(c-acx)^{3/2}} - \frac{50c^2(1-ax)^{3/2}(1+ax)^{3/2}}{3a^4(c-acx)^{3/2}} +$$

Mathematica [A] time = 0.05, size = 76, normalized size = 0.32

$$\frac{2c\sqrt{1-ax}\left(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656\right)}{45a^4\sqrt{ax+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(656 + 328*a*x - 82*a^2*x^2 + 41*a^3*x^3 - 20*a^4*x^4 +
5*a^5*x^5))/(45*a^4*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.48, size = 76, normalized size = 0.32

$$-\frac{2\left(5\,a^5x^5-20\,a^4x^4+41\,a^3x^3-82\,a^2x^2+328\,ax+656\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{45\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/45*(5*a^5*x^5 - 20*a^4*x^4 + 41*a^3*x^3 - 82*a^2*x^2 + 328*a*x + 656)*sq
rt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^6*x^2 - a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 79, normalized size = 0.34

$$\frac{2\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{-acx+c}\,\left(5x^{5}a^{5}-20x^{4}a^{4}+41x^{3}a^{3}-82a^{2}x^{2}+328ax+656\right)}{45\left(ax+1\right)^{2}\left(ax-1\right)^{2}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/45*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(5*a^5*x^5-20*a^4*x^4+41*a^3*x^3-8
2*a^2*x^2+328*a*x+656)/(a*x+1)^2/(a*x-1)^2/a^4
```

maxima [A] time = 0.34, size = 86, normalized size = 0.37

$$\frac{2\left(5\,a^{5}\sqrt{c}\,x^{5}-20\,a^{4}\sqrt{c}\,x^{4}+41\,a^{3}\sqrt{c}\,x^{3}-82\,a^{2}\sqrt{c}\,x^{2}+328\,a\sqrt{c}\,x+656\,\sqrt{c}\right)\sqrt{ax+1}\,(ax-1)}{45\left(a^{6}x^{2}-a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] 2/45*(5*a^5*sqrt(c)*x^5 - 20*a^4*sqrt(c)*x^4 + 41*a^3*sqrt(c)*x^3 - 82*a^2*
sqrt(c)*x^2 + 328*a*sqrt(c)*x + 656*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/(a^6*x
^2 - a^4)
```

mupad [B] time = 1.07, size = 115, normalized size = 0.49

$$\frac{4\sqrt{1-a^{2}\,x^{2}}\,\sqrt{c-a\,c\,x}}{a^{4}\,\left(a\,x+1\right)}-\frac{928\sqrt{1-a^{2}\,x^{2}}\,\sqrt{c-a\,c\,x}}{45\,a^{4}\,\left(a\,x-1\right)}-\frac{2\sqrt{1-a^{2}\,x^{2}}\,\sqrt{c-a\,c\,x}\,\left(5\,a^{3}\,x^{3}-20\,a^{2}\,x^{2}+46\,a\,x-102\right)}{45\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(a*x + 1)^3,x)
```

```
[Out] (4*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a^4*(a*x + 1)) - (928*(1 - a^2*x
^2)^(1/2)*(c - a*c*x)^(1/2))/(45*a^4*(a*x - 1)) - (2*(1 - a^2*x^2)^(1/2)*(c
 - a*c*x)^(1/2)*(46*a*x - 20*a^2*x^2 + 5*a^3*x^3 - 102))/(45*a^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt{-c\left(ax-1\right)}\,\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3
, x)
```

3.429 $\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=197

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{7/2}}{7a^3(c-acx)^{3/2}}-\frac{12c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^3(c-acx)^{3/2}}+\frac{26c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^3(c-acx)^{3/2}}-\frac{24c^2(1-ax)^{3/2}\sqrt{ax+1}}{a^3(c-acx)^{3/2}}-\frac{8}{a^3\sqrt{a}}$$

```
[Out] 26/3*c^2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)/a^3/(-a*c*x+c)^(3/2)-12/5*c^2*(-a*x+1
)^(3/2)*(a*x+1)^(5/2)/a^3/(-a*c*x+c)^(3/2)+2/7*c^2*(-a*x+1)^(3/2)*(a*x+1)^(
7/2)/a^3/(-a*c*x+c)^(3/2)-8*c^2*(-a*x+1)^(3/2)/a^3/(-a*c*x+c)^(3/2)/(a*x+1)
^(1/2)-24*c^2*(-a*x+1)^(3/2)*(a*x+1)^(1/2)/a^3/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6130, 23, 88}

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{7/2}}{7a^3(c-acx)^{3/2}}-\frac{12c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^3(c-acx)^{3/2}}+\frac{26c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^3(c-acx)^{3/2}}-\frac{24c^2(1-ax)^{3/2}\sqrt{ax+1}}{a^3(c-acx)^{3/2}}-\frac{8}{a^3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-8*c^2*(1 - a*x)^(3/2))/(a^3*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (24*c^2*(1
 - a*x)^(3/2)*Sqrt[1 + a*x])/(a^3*(c - a*c*x)^(3/2)) + (26*c^2*(1 - a*x)^(3
/2)*(1 + a*x)^(3/2))/(3*a^3*(c - a*c*x)^(3/2)) - (12*c^2*(1 - a*x)^(3/2)*(1
 + a*x)^(5/2))/(5*a^3*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x)^(3/2)*(1 + a*x)
^(7/2))/(7*a^3*(c - a*c*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= \int \frac{x^2(1-ax)^{3/2}\sqrt{c-acx}}{(1+ax)^{3/2}}\,dx\\ &= \frac{(1-ax)^{3/2}\int \frac{x^2(c-acx)^2}{(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}\\ &= \frac{(1-ax)^{3/2}\int \left(\frac{4c^2}{a^2(1+ax)^{3/2}}-\frac{12c^2}{a^2\sqrt{1+ax}}+\frac{13c^2\sqrt{1+ax}}{a^2}-\frac{6c^2(1+ax)^{3/2}}{a^2}+\frac{c^2(1+ax)^{5/2}}{a^2}\right)dx}{(c-acx)^{3/2}}\\ &= -\frac{8c^2(1-ax)^{3/2}}{a^3\sqrt{1+ax}\,(c-acx)^{3/2}}-\frac{24c^2(1-ax)^{3/2}\sqrt{1+ax}}{a^3(c-acx)^{3/2}}+\frac{26c^2(1-ax)^{3/2}(1+ax)^{3/2}}{3a^3(c-acx)^{3/2}}-\end{aligned}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.35

$$\frac{2c\sqrt{1-ax}\left(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336\right)}{105a^3\sqrt{ax+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(-1336 - 668*a*x + 167*a^2*x^2 - 66*a^3*x^3 + 15*a^4*x^4
))/(105*a^3*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.50, size = 68, normalized size = 0.35

$$-\frac{2\left(15\,a^4x^4-66\,a^3x^3+167\,a^2x^2-668\,ax-1336\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{105\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/105*(15*a^4*x^4 - 66*a^3*x^3 + 167*a^2*x^2 - 668*a*x - 1336)*sqrt(-a^2*x
^2 + 1)*sqrt(-a*c*x + c)/(a^5*x^2 - a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 71, normalized size = 0.36

$$\frac{2\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{-acx+c}\,\left(15x^{4}a^{4}-66x^{3}a^{3}+167a^{2}x^{2}-668ax-1336\right)}{105\left(ax+1\right)^{2}\left(ax-1\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/105*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(15*a^4*x^4-66*a^3*x^3+167*a^2*x^
2-668*a*x-1336)/(a*x+1)^2/(a*x-1)^2/a^3
```

maxima [A] time = 0.37, size = 75, normalized size = 0.38

$$\frac{2\left(15\,a^{4}\sqrt{c}\,x^{4}-66\,a^{3}\sqrt{c}\,x^{3}+167\,a^{2}\sqrt{c}\,x^{2}-668\,a\sqrt{c}\,x-1336\,\sqrt{c}\right)\sqrt{ax+1}\,(ax-1)}{105\left(a^{5}x^{2}-a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] 2/105*(15*a^4*sqrt(c)*x^4 - 66*a^3*sqrt(c)*x^3 + 167*a^2*sqrt(c)*x^2 - 668*
a*sqrt(c)*x - 1336*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)/(a^5*x^2 - a^3)
```

mupad [B] time = 1.04, size = 107, normalized size = 0.54

$$\frac{1888\sqrt{1-a^{2}\,x^{2}}\,\sqrt{c-a\,c\,x}}{105\,a^{3}\,\left(a\,x-1\right)}-\frac{2\sqrt{1-a^{2}\,x^{2}}\,\sqrt{c-a\,c\,x}\,\left(15\,a^{2}\,x^{2}-66\,a\,x+182\right)}{105\,a^{3}}-\frac{4\sqrt{1-a^{2}\,x^{2}}\,\sqrt{c-a\,c\,x}}{a^{3}\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(a*x + 1)^3,x)
```

```
[Out] (1888*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(105*a^3*(a*x - 1)) - (2*(1 -
a^2*x^2)^(1/2)*(c - a*c*x)^(1/2)*(15*a^2*x^2 - 66*a*x + 182))/(105*a^3) - (
4*(1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/(a^3*(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\,(ax-1)}\,\left(-\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3
, x)
```

3.430 $\int e^{-3\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=157

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^2(c-acx)^{3/2}}-\frac{10c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^2(c-acx)^{3/2}}+\frac{16c^2(1-ax)^{3/2}\sqrt{ax+1}}{a^2(c-acx)^{3/2}}+\frac{8c^2(1-ax)^{3/2}}{a^2\sqrt{ax+1}\,(c-acx)^{3/2}}$$

```
[Out] -10/3*c^2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)/a^2/(-a*c*x+c)^(3/2)+2/5*c^2*(-a*x+1
)^(3/2)*(a*x+1)^(5/2)/a^2/(-a*c*x+c)^(3/2)+8*c^2*(-a*x+1)^(3/2)/a^2/(-a*c*x
+c)^(3/2)/(a*x+1)^(1/2)+16*c^2*(-a*x+1)^(3/2)*(a*x+1)^(1/2)/a^2/(-a*c*x+c)^
(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6130, 23, 77}

$$\frac{2c^2(1-ax)^{3/2}(ax+1)^{5/2}}{5a^2(c-acx)^{3/2}}-\frac{10c^2(1-ax)^{3/2}(ax+1)^{3/2}}{3a^2(c-acx)^{3/2}}+\frac{16c^2(1-ax)^{3/2}\sqrt{ax+1}}{a^2(c-acx)^{3/2}}+\frac{8c^2(1-ax)^{3/2}}{a^2\sqrt{ax+1}\,(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*c^2*(1 - a*x)^(3/2))/(a^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (16*c^2*(1
- a*x)^(3/2)*Sqrt[1 + a*x])/(a^2*(c - a*c*x)^(3/2)) - (10*c^2*(1 - a*x)^(3/
2)*(1 + a*x)^(3/2))/(3*a^2*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x)^(3/2)*(1 +
 a*x)^(5/2))/(5*a^2*(c - a*c*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx = \int \frac{x(1-ax)^{3/2}\sqrt{c-acx}}{(1+ax)^{3/2}}\,dx$$
$$= \frac{(1-ax)^{3/2}\int \frac{x(c-acx)^2}{(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$= \frac{(1-ax)^{3/2}\int \left(-\frac{4c^2}{a(1+ax)^{3/2}}+\frac{8c^2}{a\sqrt{1+ax}}-\frac{5c^2\sqrt{1+ax}}{a}+\frac{c^2(1+ax)^{3/2}}{a}\right)dx}{(c-acx)^{3/2}}$$
$$= \frac{8c^2(1-ax)^{3/2}}{a^2\sqrt{1+ax}\,(c-acx)^{3/2}}+\frac{16c^2(1-ax)^{3/2}\sqrt{1+ax}}{a^2(c-acx)^{3/2}}-\frac{10c^2(1-ax)^{3/2}(1+ax)^{3/2}}{3a^2(c-acx)^{3/2}}+\frac{2}{}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.38

$$\frac{2c\sqrt{1-ax}\left(3a^3x^3-16a^2x^2+79ax+158\right)}{15a^2\sqrt{ax+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(158 + 79*a*x - 16*a^2*x^2 + 3*a^3*x^3))/(15*a^2*Sqrt[1
+ a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.45, size = 60, normalized size = 0.38

$$-\frac{2\left(3\,a^3x^3-16\,a^2x^2+79\,ax+158\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{15\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fri
cas")
```

```
[Out] -2/15*(3*a^3*x^3 - 16*a^2*x^2 + 79*a*x + 158)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c)/(a^4*x^2 - a^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 63, normalized size = 0.40

$$\frac{2\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \sqrt{-acx+c}\, \left(3x^{3} a^{3}-16a^{2} x^{2}+79ax+158\right)}{15\left(ax+1\right)^{2} \left(ax-1\right)^{2} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/15*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(3*a^3*x^3-16*a^2*x^2+79*a*x+158)/
(a*x+1)^2/(a*x-1)^2/a^2
```

maxima [A] time = 0.40, size = 64, normalized size = 0.41

$$\frac{2\left(3\,a^{3} \sqrt{c}\,x^{3}-16\,a^{2} \sqrt{c}\,x^{2}+79\,a\sqrt{c}\,x+158\,\sqrt{c}\right)\sqrt{ax+1}\,(ax-1)}{15\left(a^{4} x^{2}-a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="max
ima")
```

```
[Out] 2/15*(3*a^3*sqrt(c)*x^3 - 16*a^2*sqrt(c)*x^2 + 79*a*sqrt(c)*x + 158*sqrt(c)
)*sqrt(a*x + 1)*(a*x - 1)/(a^4*x^2 - a^2)
```

mupad [B] time = 1.00, size = 97, normalized size = 0.62

$$\frac{\sqrt{c-a\,c\,x}\,\left(\frac{316\,\sqrt{1-a^2\,x^2}}{15\,a^4}+\frac{158\,x\,\sqrt{1-a^2\,x^2}}{15\,a^3}+\frac{2\,x^3\,\sqrt{1-a^2\,x^2}}{5\,a}-\frac{32\,x^2\,\sqrt{1-a^2\,x^2}}{15\,a^2}\right)}{\frac{1}{a^2}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(a*x + 1)^3,x)
```

```
[Out] ((c - a*c*x)^(1/2)*((316*(1 - a^2*x^2)^(1/2))/(15*a^4) + (158*x*(1 - a^2*x^
2)^(1/2))/(15*a^3) + (2*x^3*(1 - a^2*x^2)^(1/2))/(5*a) - (32*x^2*(1 - a^2*x
^2)^(1/2))/(15*a^2)))/(1/a^2 - x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\,(ax-1)}\,\left(-\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x
)
```

3.431 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=103

$$\frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}+\frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}}-\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] 16/3*(-a*c*x+c)^(3/2)/a/c/(-a^2*x^2+1)^(1/2)+2/3*(-a*c*x+c)^(5/2)/a/c^2/(-a
^2*x^2+1)^(1/2)-64/3*(-a*c*x+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6127, 657, 649}

$$\frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}+\frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}}-\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-64*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - a^2*x^2]) + (16*(c - a*c*x)^(3/2))/(3*a
*c*Sqrt[1 - a^2*x^2]) + (2*(c - a*c*x)^(5/2))/(3*a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6127

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> D
ist[c^n, Int[(c + d*x)^(p - n)*(1 - a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a*c + d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\int \frac{(c-acx)^{7/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}} + \frac{8\int \frac{(c-acx)^{5/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^2}$$

$$= \frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}} + \frac{32\int \frac{(c-acx)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{64\sqrt{c-acx}}{3a\sqrt{1-a^2x^2}} + \frac{16(c-acx)^{3/2}}{3ac\sqrt{1-a^2x^2}} + \frac{2(c-acx)^{5/2}}{3ac^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.50

$$\frac{2c\sqrt{1-ax}\left(a^2x^2-10ax-23\right)}{3a\sqrt{ax+1}\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(-23 - 10*a*x + a^2*x^2))/(3*a*Sqrt[1 + a*x]*Sqrt[c - a*
c*x])
```

fricas [A] time = 0.47, size = 49, normalized size = 0.48

$$-\frac{2\left(a^2x^2-10\,ax-23\right)\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{3\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/(a^3*x^2 -
 a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 54, normalized size = 0.52

$$\frac{2\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{-acx+c}\,\left(a^{2}x^{2}-10ax-23\right)}{3\left(ax+1\right)^{2}\left(ax-1\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2/3*(-a^2*x^2+1)^(3/2)*(-a*c*x+c)^(1/2)*(a^2*x^2-10*a*x-23)/(a*x+1)^2/(a*x-
1)^2/a
```

maxima [A] time = 0.42, size = 50, normalized size = 0.49

$$\frac{2\left(a^{2}\sqrt{c}\,x^{2}-10\,a\sqrt{c}\,x-23\,\sqrt{c}\right)\sqrt{ax+1}\,(ax-1)}{3\left(a^{3}x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*(a^2*sqrt(c)*x^2 - 10*a*sqrt(c)*x - 23*sqrt(c))*sqrt(a*x + 1)*(a*x - 1)
/(a^3*x^2 - a)
```

mupad [B] time = 0.00, size = 40, normalized size = 0.39

$$-\frac{2\sqrt{c-a\,c\,x}\,\left(-a^{2}\,x^{2}+10\,a\,x+23\right)}{3\,a\,\sqrt{1-a^{2}\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(a*x + 1)^3,x)
```

```
[Out] -(2*(c - a*c*x)^(1/2)*(10*a*x - a^2*x^2 + 23))/(3*a*(1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,\left(-\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}{(ax+1)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)
```

3.432 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=107

$$\frac{2c^2(1-ax)^{3/2}\sqrt{ax+1}}{(c-acx)^{3/2}}+\frac{8c^2(1-ax)^{3/2}}{\sqrt{ax+1}\,(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{(c-acx)^{3/2}}$$

```
[Out] -2*c^2*(-a*x+1)^(3/2)*arctanh((a*x+1)^(1/2))/(-a*c*x+c)^(3/2)+8*c^2*(-a*x+1
)^(3/2)/(-a*c*x+c)^(3/2)/(a*x+1)^(1/2)+2*c^2*(-a*x+1)^(3/2)*(a*x+1)^(1/2)/(
-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6130, 23, 87, 63, 208}

$$\frac{2c^2(1-ax)^{3/2}\sqrt{ax+1}}{(c-acx)^{3/2}}+\frac{8c^2(1-ax)^{3/2}}{\sqrt{ax+1}\,(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (8*c^2*(1 - a*x)^(3/2))/(Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (2*c^2*(1 - a*x
)^(3/2)*Sqrt[1 + a*x])/(c - a*c*x)^(3/2) - (2*c^2*(1 - a*x)^(3/2)*ArcTanh[S
qrt[1 + a*x]])/(c - a*c*x)^(3/2)
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x(1+ax)^{3/2}}\,dx\\
&= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}\\
&= \frac{(1-ax)^{3/2}\int \left(-\frac{4ac^2}{(1+ax)^{3/2}}+\frac{ac^2}{\sqrt{1+ax}}+\frac{c^2}{x\sqrt{1+ax}}\right)dx}{(c-acx)^{3/2}}\\
&= \frac{8c^2(1-ax)^{3/2}}{\sqrt{1+ax}\,(c-acx)^{3/2}}+\frac{2c^2(1-ax)^{3/2}\sqrt{1+ax}}{(c-acx)^{3/2}}+\frac{\left(c^2(1-ax)^{3/2}\right)\int \frac{1}{x\sqrt{1+ax}}\,dx}{(c-acx)^{3/2}}\\
&= \frac{8c^2(1-ax)^{3/2}}{\sqrt{1+ax}\,(c-acx)^{3/2}}+\frac{2c^2(1-ax)^{3/2}\sqrt{1+ax}}{(c-acx)^{3/2}}+\frac{\left(2c^2(1-ax)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a}+\frac{x^2}{a}}\,d
}{a(c-acx)^{3/2}}\\
&= \frac{8c^2(1-ax)^{3/2}}{\sqrt{1+ax}\,(c-acx)^{3/2}}+\frac{2c^2(1-ax)^{3/2}\sqrt{1+ax}}{(c-acx)^{3/2}}-\frac{2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{1+ax}\right)}{(c-acx)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 51, normalized size = 0.48

$$\frac{2c\sqrt{1-ax}\left({}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};ax+1\right)+ax+4\right)}{\sqrt{ax+1}\,\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (2*c*Sqrt[1 - a*x]*(4 + a*x + Hypergeometric2F1[-1/2, 1, 1/2, 1 + a*x]))/(S
qrt[1 + a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.47, size = 209, normalized size = 1.95

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right)-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\,(ax+5)}{a^2x^2-1},-\frac{2\left(\left(a^2x^2-1\right)\sqrt{-c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="fri
cas")
```

```
[Out] [((a^2*x^2 - 1)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt
(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x
 + c)*(a*x + 5))/(a^2*x^2 - 1), -2*((a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2
*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 + 1)*s
qrt(-a*c*x + c)*(a*x + 5))/(a^2*x^2 - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 78, normalized size = 0.73

$$\frac{2\sqrt{-c\,(ax-1)}\,\sqrt{-a^2x^2+1}\,\left(\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,(ax+1)}}{\sqrt{c}}\right)\sqrt{c\,(ax+1)}-acx-5c\right)}{(ax-1)\,c\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] 2*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(c^(1/2)*arctanh((c*(a*x+1))^(1/2)/
c^(1/2))*(c*(a*x+1))^(1/2)-a*c*x-5*c)/(a*x-1)/c/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{(ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}\,\sqrt{c-a\,c\,x}}{x\,{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x*(a*x + 1)^3),x)
```

```
[Out] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}\,\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x\,(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x*(a*x + 1)**3),
 x)
```

3.433 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=112

$$-\frac{9ac^2(1-ax)^{3/2}}{\sqrt{ax+1}\,(c-acx)^{3/2}}-\frac{c^2(1-ax)^{3/2}}{x\sqrt{ax+1}\,(c-acx)^{3/2}}+\frac{7ac^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{(c-acx)^{3/2}}$$

```
[Out] 7*a*c^2*(-a*x+1)^(3/2)*arctanh((a*x+1)^(1/2))/(-a*c*x+c)^(3/2)-9*a*c^2*(-a*
x+1)^(3/2)/(-a*c*x+c)^(3/2)/(a*x+1)^(1/2)-c^2*(-a*x+1)^(3/2)/x/(-a*c*x+c)^(
3/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6130, 23, 89, 78, 63, 208}

$$-\frac{9ac^2(1-ax)^{3/2}}{\sqrt{ax+1}\,(c-acx)^{3/2}}-\frac{c^2(1-ax)^{3/2}}{x\sqrt{ax+1}\,(c-acx)^{3/2}}+\frac{7ac^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (-9*a*c^2*(1 - a*x)^(3/2))/(Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (c^2*(1 - a*
x)^(3/2))/(x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (7*a*c^2*(1 - a*x)^(3/2)*Ar
cTanh[Sqrt[1 + a*x]])/(c - a*c*x)^(3/2)
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x^2(1+ax)^{3/2}}\,dx$$
$$= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x^2(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$= -\frac{c^2(1-ax)^{3/2}}{x\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{-\frac{7ac^2}{2}+a^2c^2x}{x(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$= -\frac{9ac^2(1-ax)^{3/2}}{\sqrt{1+ax}\,(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{x\sqrt{1+ax}\,(c-acx)^{3/2}} - \frac{\left(7ac^2(1-ax)^{3/2}\right)\int \frac{1}{x\sqrt{1+ax}}\,dx}{2(c-acx)^{3/2}}$$
$$= -\frac{9ac^2(1-ax)^{3/2}}{\sqrt{1+ax}\,(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{x\sqrt{1+ax}\,(c-acx)^{3/2}} - \frac{\left(7c^2(1-ax)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a}+\frac{x^2}{a}}\right.}{(c-acx)^{3/2}}$$
$$= -\frac{9ac^2(1-ax)^{3/2}}{\sqrt{1+ax}\,(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{x\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{7ac^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{1+ax}\right.}{(c-acx)^{3/2}}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.57

$$\frac{c\sqrt{1-ax}\left(-9ax+7ax\sqrt{ax+1}\,\tanh^{-1}\left(\sqrt{ax+1}\right)-1\right)}{x\sqrt{ax+1}\,\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (c*Sqrt[1 - a*x]*(-1 - 9*a*x + 7*a*x*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 + a*x]]))
/(x*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.74, size = 223, normalized size = 1.99

$$\left[\frac{7\left(a^3x^3-ax\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,(9\,ax+1)}{2\left(a^2x^3-x\right)}, \frac{7\left(a^3x^3-ax\right)\sqrt{\vphantom{c}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] [1/2*(7*(a^3*x^3 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqr
t(-a*c*x + c)*(9*a*x + 1))/(a^2*x^3 - x), (7*(a^3*x^3 - a*x)*sqrt(-c)*arcta
n(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2
*x^2 + 1)*sqrt(-a*c*x + c)*(9*a*x + 1))/(a^2*x^3 - x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 82, normalized size = 0.73

$$\frac{\left(-7\,\mathrm{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)xa\sqrt{c\,(ax+1)}\,+9xa\sqrt{c}\,+\sqrt{c}\right)\sqrt{-c\,(ax-1)}\,\sqrt{-a^2x^2+1}}{(ax-1)\sqrt{c}\,\,(ax+1)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] (-7*arctanh((c*(a*x+1))^(1/2)/c^(1/2))*x*a*(c*(a*x+1))^(1/2)+9*x*a*c^(1/2)+
c^(1/2))*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/c^(1/2)/(a*x+1)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{(ax+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}\,\sqrt{c-a\,c\,x}}{x^2\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x^2*(a*x + 1)^3),x)
```

```
[Out] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x^2*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(ax-1)}\,(-(ax-1)(ax+1))^{\frac{3}{2}}}{x^2(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**2*(a*x + 1)**
3), x)
```

3.434 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=163

$$\frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{ax+1}\,(c-acx)^{3/2}}-\frac{47a^2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{4(c-acx)^{3/2}}-\frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{ax+1}\,(c-acx)^{3/2}}+\frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{ax+1}\,(c-acx)^{3/2}}$$

```
[Out] -47/4*a^2*c^2*(-a*x+1)^(3/2)*arctanh((a*x+1)^(1/2))/(-a*c*x+c)^(3/2)+47/4*a
^2*c^2*(-a*x+1)^(3/2)/(-a*c*x+c)^(3/2)/(a*x+1)^(1/2)-1/2*c^2*(-a*x+1)^(3/2)
/x^2/(-a*c*x+c)^(3/2)/(a*x+1)^(1/2)+13/4*a*c^2*(-a*x+1)^(3/2)/x/(-a*c*x+c)^
(3/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 89, 78, 51, 63, 208}

$$\frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{ax+1}\,(c-acx)^{3/2}}-\frac{47a^2c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{4(c-acx)^{3/2}}-\frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{ax+1}\,(c-acx)^{3/2}}+\frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{ax+1}\,(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (47*a^2*c^2*(1 - a*x)^(3/2))/(4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (c^2*(1
- a*x)^(3/2))/(2*x^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (13*a*c^2*(1 - a*x)
^(3/2))/(4*x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (47*a^2*c^2*(1 - a*x)^(3/2)
*ArcTanh[Sqrt[1 + a*x]])/(4*(c - a*c*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x^3(1+ax)^{3/2}}\,dx$$
$$= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x^3(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$= -\frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{-\frac{13ac^2}{2}+2a^2c^2x}{x^2(1+ax)^{3/2}}\,dx}{2(c-acx)^{3/2}}$$
$$= -\frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{\left(47a^2c^2(1-ax)^{3/2}\right)\int \frac{1}{x(1+ax)^{3}}}{8(c-acx)^{3/2}}$$
$$= \frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{1+ax}\,(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{(47a^2}{}$$
$$= \frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{1+ax}\,(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{(47ac}{}$$
$$= \frac{47a^2c^2(1-ax)^{3/2}}{4\sqrt{1+ax}\,(c-acx)^{3/2}} - \frac{c^2(1-ax)^{3/2}}{2x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{13ac^2(1-ax)^{3/2}}{4x\sqrt{1+ax}\,(c-acx)^{3/2}} - \frac{47a^2c}{}$$

Mathematica [C] time = 0.03, size = 65, normalized size = 0.40

$$\frac{c\sqrt{1-ax}\left(47a^2x^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};ax+1\right)+13ax-2\right)}{4x^2\sqrt{ax+1}\,\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (c*Sqrt[1 - a*x]*(-2 + 13*a*x + 47*a^2*x^2*Hypergeometric2F1[-1/2, 1, 1/2,
1 + a*x]))/(4*x^2*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.47, size = 252, normalized size = 1.55

$$\left[\frac{47\left(a^4x^4-a^2x^2\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-2c}{ax^2-x}\right)-2\left(47a^2x^2+13ax-2\right)\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}}{8\left(a^2x^4-x^2\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] [1/8*(47*(a^4*x^4 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*
x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*(47*a^2*x^2 + 13*
a*x - 2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^2*x^4 - x^2), -1/4*(47*(a^
4*x^4 - a^2*x^2)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-
c)/(a^2*c*x^2 - c)) + (47*a^2*x^2 + 13*a*x - 2)*sqrt(-a^2*x^2 + 1)*sqrt(-a*
c*x + c))/(a^2*x^4 - x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 100, normalized size = 0.61

$$\frac{\sqrt{-c\,(ax-1)}\,\sqrt{-a^2x^2+1}\,\left(47\,\mathrm{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)x^2a^2\sqrt{c\,(ax+1)}-47x^2a^2\sqrt{c}-13xa\sqrt{c}+2\sqrt{c}\right)}{4\sqrt{c}\,(ax-1)\,(ax+1)\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] 1/4*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(47*arctanh((c*(a*x+1))^(1/2)/c^(
1/2))*x^2*a^2*(c*(a*x+1))^(1/2)-47*x^2*a^2*c^(1/2)-13*x*a*c^(1/2)+2*c^(1/2)
)/c^(1/2)/(a*x-1)/(a*x+1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{(ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}\,\sqrt{c-a\,c\,x}}{x^3\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x^3*(a*x + 1)^3),x)
```

```
[Out] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x^3*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}\,\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x^3\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**3*(a*x + 1)**
3), x)
```

3.435 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=206

$$-\frac{119a^3c^2(1-ax)^{3/2}}{8\sqrt{ax+1}\,(c-acx)^{3/2}}+\frac{119a^3c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{8(c-acx)^{3/2}}-\frac{119a^2c^2(1-ax)^{3/2}}{24x\sqrt{ax+1}\,(c-acx)^{3/2}}-\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{ax+1}\,(c-acx)^{3/2}}$$

```
[Out] 119/8*a^3*c^2*(-a*x+1)^(3/2)*arctanh((a*x+1)^(1/2))/(-a*c*x+c)^(3/2)-119/8*
a^3*c^2*(-a*x+1)^(3/2)/(-a*c*x+c)^(3/2)/(a*x+1)^(1/2)-1/3*c^2*(-a*x+1)^(3/2
)/x^3/(-a*c*x+c)^(3/2)/(a*x+1)^(1/2)+19/12*a*c^2*(-a*x+1)^(3/2)/x^2/(-a*c*x
+c)^(3/2)/(a*x+1)^(1/2)-119/24*a^2*c^2*(-a*x+1)^(3/2)/x/(-a*c*x+c)^(3/2)/(a
*x+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 209, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 89, 78, 51, 63, 208}

$$-\frac{119a^2c^2(1-ax)^{3/2}\sqrt{ax+1}}{8x(c-acx)^{3/2}}+\frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{ax+1}\,(c-acx)^{3/2}}+\frac{119a^3c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{8(c-acx)^{3/2}}+\frac{19ac^2(1-ax}{12x^2\sqrt{ax+1}\,(c-}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] -(c^2*(1 - a*x)^(3/2))/(3*x^3*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (19*a*c^2*
(1 - a*x)^(3/2))/(12*x^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (119*a^2*c^2*(1
 - a*x)^(3/2))/(12*x*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (119*a^2*c^2*(1 - a
*x)^(3/2)*Sqrt[1 + a*x])/(8*x*(c - a*c*x)^(3/2)) + (119*a^3*c^2*(1 - a*x)^(
3/2)*ArcTanh[Sqrt[1 + a*x]])/(8*(c - a*c*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x^4(1+ax)^{3/2}}\,dx$$

$$= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x^4(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{-\frac{19ac^2}{2}+3a^2c^2x}{x^3(1+ax)^{3/2}}\,dx}{3(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{\left(119a^2c^2(1-ax)^{3/2}\right)\int \frac{}{x^2}}{24(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{1+ax}\,(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{1+ax}\,(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{1+ax}\,(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{3x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{19ac^2(1-ax)^{3/2}}{12x^2\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{119a^2c^2(1-ax)^{3/2}}{12x\sqrt{1+ax}\,(c-acx)^{3/2}}$$

Mathematica [C] time = 0.03, size = 65, normalized size = 0.32

$$-\frac{c\sqrt{1-ax}\left(119a^3x^3\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};ax+1\right)-19ax+4\right)}{12x^3\sqrt{ax+1}\,\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] -1/12*(c*Sqrt[1 - a*x]*(4 - 19*a*x + 119*a^3*x^3*Hypergeometric2F1[-1/2, 2,
 1/2, 1 + a*x]))/(x^3*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.50, size = 268, normalized size = 1.30

$$\left[\frac{357\left(a^5x^5-a^3x^3\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-2c}{ax^2-x}\right)+2\left(357\,a^3x^3+119\,a^2x^2-38\,ax+8\right)\sqrt{-a^2x^2+1}}{48\left(a^2x^5-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="f
ricas")
```

```
[Out] [1/48*(357*(a^5*x^5 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^
2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*(357*a^3*x^3 +
119*a^2*x^2 - 38*a*x + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^2*x^5 - x
^3), 1/24*(357*(a^5*x^5 - a^3*x^3)*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (357*a^3*x^3 + 119*a^2*x^2 - 38*a*x
 + 8)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c))/(a^2*x^5 - x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 111, normalized size = 0.54

$$-\frac{\sqrt{-c\,(ax-1)}\,\sqrt{-a^2x^2+1}\,\left(357\,\mathrm{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)x^3a^3\sqrt{c\,(ax+1)}-357x^3a^3\sqrt{c}-119x^2a^2\sqrt{c}+38xa\sqrt{c}-8}{24\sqrt{c}\,(ax-1)\,(ax+1)\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] -1/24*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(357*arctanh((c*(a*x+1))^(1/2)/
c^(1/2))*x^3*a^3*(c*(a*x+1))^(1/2)-357*x^3*a^3*c^(1/2)-119*x^2*a^2*c^(1/2)+
38*x*a*c^(1/2)-8*c^(1/2))/c^(1/2)/(a*x-1)/(a*x+1)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{(ax+1)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}\,\sqrt{c-a\,c\,x}}{x^4\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x^4*(a*x + 1)^3),x)
```

```
[Out] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x^4*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)}\,\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x^4\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**4*(a*x + 1)**
3), x)
```

3.436 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=249

$$\frac{1115a^4c^2(1-ax)^{3/2}}{64\sqrt{ax+1}\,(c-acx)^{3/2}}-\frac{1115a^4c^2(1-ax)^{3/2}\tanh^{-1}\left(\sqrt{ax+1}\right)}{64(c-acx)^{3/2}}+\frac{1115a^3c^2(1-ax)^{3/2}}{192x\sqrt{ax+1}\,(c-acx)^{3/2}}-\frac{223a^2c^2(1-ax)^{3/2}}{96x^2\sqrt{ax+1}\,(c-acx)^{3/2}}$$

```
[Out] -1115/64*a^4*c^2*(-a*x+1)^(3/2)*arctanh((a*x+1)^(1/2))/(-a*c*x+c)^(3/2)+111
5/64*a^4*c^2*(-a*x+1)^(3/2)/(-a*c*x+c)^(3/2)/(a*x+1)^(1/2)-1/4*c^2*(-a*x+1)
^(3/2)/x^4/(-a*c*x+c)^(3/2)/(a*x+1)^(1/2)+25/24*a*c^2*(-a*x+1)^(3/2)/x^3/(-
a*c*x+c)^(3/2)/(a*x+1)^(1/2)-223/96*a^2*c^2*(-a*x+1)^(3/2)/x^2/(-a*c*x+c)^(
3/2)/(a*x+1)^(1/2)+1115/192*a^3*c^2*(-a*x+1)^(3/2)/x/(-a*c*x+c)^(3/2)/(a*x+
1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 252, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6130, 23, 89, 78, 51, 63, 208}

$$-\frac{1115a^2c^2(1-ax)^{3/2}\sqrt{ax+1}}{96x^2(c-acx)^{3/2}}+\frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{ax+1}\,(c-acx)^{3/2}}+\frac{1115a^3c^2(1-ax)^{3/2}\sqrt{ax+1}}{64x(c-acx)^{3/2}}-\frac{1115a^4c^2(1-ax)^{3/2}\tanh^{-1}}{64(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] -(c^2*(1 - a*x)^(3/2))/(4*x^4*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (25*a*c^2*
(1 - a*x)^(3/2))/(24*x^3*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) + (223*a^2*c^2*(1
 - a*x)^(3/2))/(24*x^2*Sqrt[1 + a*x]*(c - a*c*x)^(3/2)) - (1115*a^2*c^2*(1
- a*x)^(3/2)*Sqrt[1 + a*x])/(96*x^2*(c - a*c*x)^(3/2)) + (1115*a^3*c^2*(1 -
 a*x)^(3/2)*Sqrt[1 + a*x])/(64*x*(c - a*c*x)^(3/2)) - (1115*a^4*c^2*(1 - a*
x)^(3/2)*ArcTanh[Sqrt[1 + a*x]])/(64*(c - a*c*x)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \int \frac{(1-ax)^{3/2}\sqrt{c-acx}}{x^5(1+ax)^{3/2}}\,dx$$

$$= \frac{(1-ax)^{3/2}\int \frac{(c-acx)^2}{x^5(1+ax)^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{-\frac{25ac^2}{2}+4a^2c^2x}{x^4(1+ax)^{3/2}}\,dx}{4(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{\left(223a^2c^2(1-ax)^{3/2}\right)\int \frac{}{x^3(1}}{48(c-acx)^{3/2}}$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}\,(c-acx)^{3/2}} -$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}\,(c-acx)^{3/2}} -$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}\,(c-acx)^{3/2}} -$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}\,(c-acx)^{3/2}} -$$

$$= -\frac{c^2(1-ax)^{3/2}}{4x^4\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{25ac^2(1-ax)^{3/2}}{24x^3\sqrt{1+ax}\,(c-acx)^{3/2}} + \frac{223a^2c^2(1-ax)^{3/2}}{24x^2\sqrt{1+ax}\,(c-acx)^{3/2}} -$$

Mathematica [C] time = 0.03, size = 65, normalized size = 0.26

$$\frac{c\sqrt{1-ax}\left(223a^4x^4\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};ax+1\right)+25ax-6\right)}{24x^4\sqrt{ax+1}\,\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (c*Sqrt[1 - a*x]*(-6 + 25*a*x + 223*a^4*x^4*Hypergeometric2F1[-1/2, 3, 1/2,
 1 + a*x]))/(24*x^4*Sqrt[1 + a*x]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.47, size = 284, normalized size = 1.14

$$\left[\frac{3345\left(a^{6}x^{6}-a^{4}x^{4}\right)\sqrt{c}\,\log\left(-\frac{a^{2}cx^{2}+acx+2\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{ax^{2}-x}\right)-2\left(3345\,a^{4}x^{4}+1115\,a^{3}x^{3}-446\,a^{2}x^{2}+200\,ax\right.}{384\left(a^{2}x^{6}-x^{4}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="f
ricas")
```

```
[Out] [1/384*(3345*(a^6*x^6 - a^4*x^4)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-
a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) - 2*(3345*a^4*x^4
 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c))/(a^2*x^6 - x^4), -1/192*(3345*(a^6*x^6 - a^4*x^4)*sqrt(-c)*arctan(s
qrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (3345*a^4*x^
4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a^2*x^2 + 1)*sqrt(-a*c
*x + c))/(a^2*x^6 - x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 122, normalized size = 0.49

$$\frac{\sqrt{-c\left(ax-1\right)}\,\sqrt{-a^{2}x^{2}+1}\,\left(3345\,\mathrm{arctanh}\left(\frac{\sqrt{c\left(ax+1\right)}}{\sqrt{c}}\right)x^{4}a^{4}\sqrt{c\left(ax+1\right)}-3345x^{4}a^{4}\sqrt{c}-1115x^{3}a^{3}\sqrt{c}+446x^{2}a^{2}\sqrt{\right.}{192\sqrt{c}\,\left(ax-1\right)\left(ax+1\right)x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] 1/192*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(3345*arctanh((c*(a*x+1))^(1/2)
/c^(1/2))*x^4*a^4*(c*(a*x+1))^(1/2)-3345*x^4*a^4*c^(1/2)-1115*x^3*a^3*c^(1/
2)+446*x^2*a^2*c^(1/2)-200*x*a*c^(1/2)+48*c^(1/2))/c^(1/2)/(a*x-1)/(a*x+1)/
x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{-acx+c}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(-a*c*x + c)/((a*x + 1)^3*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}\,\sqrt{c-a\,c\,x}}{x^5\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x^5*(a*x + 1)^3),x)
```

```
[Out] int(((1 - a^2*x^2)^(3/2)*(c - a*c*x)^(1/2))/(x^5*(a*x + 1)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.437 $\int e^{-2p\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=61

$$-\frac{2^{-p}(1-ax)^p(c-acx)^{p+1}\,{}_2F_1\left(p,2p+1;2(p+1);\frac{1}{2}(1-ax)\right)}{ac(2p+1)}$$

```
[Out] -(-a*x+1)^p*(-a*c*x+c)^(1+p)*hypergeom([p, 1+2*p],[2+2*p],-1/2*a*x+1/2)/(2^
p)/a/c/(1+2*p)
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6130, 23, 69}

$$-\frac{2^{-p}(1-ax)^p(c-acx)^{p+1}\,{}_2F_1\left(p,2p+1;2(p+1);\frac{1}{2}(1-ax)\right)}{ac(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] -(((1 - a*x)^p*(c - a*c*x)^(1 + p)*Hypergeometric2F1[p, 1 + 2*p, 2*(1 + p),
 (1 - a*x)/2])/(2^p*a*c*(1 + 2*p)))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2p\tanh^{-1}(ax)}(c-acx)^p\,dx&=\int(1-ax)^p(1+ax)^{-p}(c-acx)^p\,dx\\&=\left((1-ax)^p(c-acx)^{-p}\right)\int(1+ax)^{-p}(c-acx)^{2p}\,dx\\&=-\frac{2^{-p}(1-ax)^p(c-acx)^{1+p}\,{}_2F_1\left(p,1+2p;2(1+p);\frac{1}{2}(1-ax)\right)}{ac(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.92

$$-\frac{2^{-p}(1-ax)^{p+1}(c-acx)^p\,{}_2F_1\left(p,2p+1;2p+2;\frac{1}{2}-\frac{ax}{2}\right)}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] -((((1 - a*x)^(1 + p)*(c - a*c*x)^p*Hypergeometric2F1[p, 1 + 2*p, 2 + 2*p, 1
/2 - (a*x)/2])/(2^p*(a + 2*a*p)))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(-acx+c)^p}{\left(\frac{ax+1}{ax-1}\right)^p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-acx+c)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int (-acx + c)^p \, e^{-2p \operatorname{arctanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x)
[Out] int((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-acx + c)^p}{\left(\frac{ax+1}{ax-1}\right)^p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{-2\,p\,\operatorname{atanh}(a\,x)} \, (c - a\,c\,x)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*p*atanh(a*x))*(c - a*c*x)^p,x)
[Out] int(exp(-2*p*atanh(a*x))*(c - a*c*x)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-c\,(ax - 1))^p \, e^{-2p \operatorname{atanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p/exp(2*p*atanh(a*x)),x)
[Out] Integral((-c*(a*x - 1))**p*exp(-2*p*atanh(a*x)), x)
```

3.438 $\int e^{2p\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=37

$$\frac{(1-ax)^{-p}(ax+1)^{p+1}(c-acx)^p}{a(p+1)}$$

```
[Out] (a*x+1)^(1+p)*(-a*c*x+c)^p/a/(1+p)/((-a*x+1)^p)
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6130, 23, 32}

$$\frac{(1-ax)^{-p}(ax+1)^{p+1}(c-acx)^p}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((1 + a*x)^(1 + p)*(c - a*c*x)^p)/(a*(1 + p)*(1 - a*x)^p)
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{2p\tanh^{-1}(ax)}(c-acx)^p\,dx = \int (1-ax)^{-p}(1+ax)^p(c-acx)^p\,dx$$
$$= \left((1-ax)^{-p}(c-acx)^p\right)\int (1+ax)^p\,dx$$
$$= \frac{(1-ax)^{-p}(1+ax)^{1+p}(c-acx)^p}{a(1+p)}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.86

$$\frac{(ax+1)(c-acx)^p e^{2p\tanh^{-1}(ax)}}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcTanh[a*x])*(c - a*c*x)^p,x]
[Out] (E^(2*p*ArcTanh[a*x])*(1 + a*x)*(c - a*c*x)^p)/(a*(1 + p))
```

fricas [A] time = 0.56, size = 37, normalized size = 1.00

$$\frac{(ax+1)(-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^p}{ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
[Out] (a*x + 1)*(-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^p/(a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^p, x)
```

maple [A] time = 0.03, size = 32, normalized size = 0.86

$$\frac{(ax+1)\,e^{2p\,\mathrm{arctanh}(ax)}\,(-acx+c)^p}{a\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arctanh(a*x))*(-a*c*x+c)^p,x)
```

```
[Out] (a*x+1)/a/(1+p)*exp(2*p*arctanh(a*x))*(-a*c*x+c)^p
```

maxima [A] time = 0.40, size = 30, normalized size = 0.81

$$\frac{\left(a\,(-c)^{p}\,x+(-c)^{p}\right)(ax+1)^{p}}{a\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] (a*(-c)^p*x + (-c)^p)*(a*x + 1)^p/(a*(p + 1))
```

mupad [B] time = 0.89, size = 37, normalized size = 1.00

$$\frac{(c-a\,c\,x)^{p}\,(a\,x+1)^{p+1}}{a\,(1-a\,x)^{p}\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*atanh(a*x))*(c - a*c*x)^p,x)
```

```
[Out] ((c - a*c*x)^p*(a*x + 1)^(p + 1))/(a*(1 - a*x)^p*(p + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x}{c} & \text{for}\: a = 0 \wedge p = -1 \\ c^{p} x & \text{for}\: a = 0 \\ - \frac{\int \frac{1}{a x e^{2 \operatorname{atanh}{\left(a x \right)}} - e^{2 \operatorname{atanh}{\left(a x \right)}}}\, dx}{c} & \text{for}\: p = -1 \\ \frac{a x \left(- a c x + c\right)^{p} e^{2 p \operatorname{atanh}{\left(a x \right)}}}{a p + a} + \frac{\left(- a c x + c\right)^{p} e^{2 p \operatorname{atanh}{\left(a x \right)}}}{a p + a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*atanh(a*x))*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((x/c, Eq(a, 0) & Eq(p, -1)), (c**p*x, Eq(a, 0)), (-Integral(1/(a*
x*exp(2*atanh(a*x)) - exp(2*atanh(a*x))), x)/c, Eq(p, -1)), (a*x*(-a*c*x +
c)**p*exp(2*p*atanh(a*x))/(a*p + a) + (-a*c*x + c)**p*exp(2*p*atanh(a*x))/(
a*p + a), True))
```

3.439 $\int e^{n\tanh^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=82

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}(c-acx)^{p+1}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{ac(-n+2p+2)}$$

```
[Out] -2^(1+1/2*n)*(-a*c*x+c)^(1+p)*hypergeom([-1/2*n, 1-1/2*n+p],[2-1/2*n+p],-1/
2*a*x+1/2)/a/c/(2-n+2*p)/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6130, 23, 69}

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}(c-acx)^{p+1}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{ac(-n+2p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] -((2^(1 + n/2)*(c - a*c*x)^(1 + p)*Hypergeometric2F1[-n/2, 1 - n/2 + p, 2 -
 n/2 + p, (1 - a*x)/2])/(a*c*(2 - n + 2*p)*(1 - a*x)^(n/2)))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}(c-acx)^p\,dx &= \int (1-ax)^{-n/2}(1+ax)^{n/2}(c-acx)^p\,dx\\ &= \left((1-ax)^{-n/2}(c-acx)^{n/2}\right)\int (1+ax)^{n/2}(c-acx)^{-\frac{n}{2}+p}\,dx\\ &= -\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}(c-acx)^{1+p}\,{}_2F_1\left(-\frac{n}{2},1-\frac{n}{2}+p;2-\frac{n}{2}+p;\frac{1}{2}(1-ax)\right)}{ac(2-n+2p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.94

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}(c-acx)^p\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}-\frac{ax}{2}\right)}{a(n-2(p+1))}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (2^(1 + n/2)*(1 - a*x)^(1 - n/2)*(c - a*c*x)^p*Hypergeometric2F1[-1/2*n, 1
- n/2 + p, 2 - n/2 + p, 1/2 - (a*x)/2])/(a*(n - 2*(1 + p)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-acx+c)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} (-acx + c)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x)
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx + c)^{p} \left(\frac{ax + 1}{ax - 1}\right)^{\frac{1}{2}n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{atanh}(a x)} (c - a c x)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a*c*x)^p,x)
[Out] int(exp(n*atanh(a*x))*(c - a*c*x)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-c(ax - 1))^{p} e^{n \operatorname{atanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p*exp(n*atanh(a*x)), x)
```

3.440 $\int e^{n\tanh^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=68

$$-\frac{c^3 2^{\frac{n}{2}+1}(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(4-\frac{n}{2},-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(8-n)}$$

```
[Out] -2^(1+1/2*n)*c^3*(-a*x+1)^(4-1/2*n)*hypergeom([-1/2*n, 4-1/2*n],[5-1/2*n],-
1/2*a*x+1/2)/a/(8-n)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 69}

$$-\frac{c^3 2^{\frac{n}{2}+1}(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(4-\frac{n}{2},-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(8-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -((2^(1 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[4 - n/2, -n/2, 5 -
 n/2, (1 - a*x)/2])/(a*(8 - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}(c-acx)^3\,dx = c^3\int (1-ax)^{3-\frac{n}{2}}(1+ax)^{n/2}\,dx = -\frac{2^{1+\frac{n}{2}}c^3(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(4-\frac{n}{2},-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(8-n)}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 0.96

$$\frac{c^3 2^{\frac{n}{2}+1}(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(4-\frac{n}{2},-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(n-8)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2^(1 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[4 - n/2, -1/2*n, 5 -
 n/2, (1 - a*x)/2])/(a*(-8 + n))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^3c^3x^3-3\,a^2c^3x^2+3\,ac^3x-c^3\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*((a*x + 1)/(a*x -
 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -(acx-c)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} (-acx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (acx - c)^3 \left(\frac{ax + 1}{ax - 1}\right)^{\frac{1}{2}n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{atanh}(a x)} (c - a c x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a*c*x)^3,x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - a*c*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3 \left(\int 3axe^{n \operatorname{atanh}(ax)} \, dx + \int \left(-3a^2x^2e^{n \operatorname{atanh}(ax)}\right) dx + \int a^3x^3e^{n \operatorname{atanh}(ax)} \, dx + \int \left(-e^{n \operatorname{atanh}(ax)}\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x*exp(n*atanh(a*x)), x) + Integral(-3*a**2*x**2*exp(n*a
tanh(a*x)), x) + Integral(a**3*x**3*exp(n*atanh(a*x)), x) + Integral(-exp(n
*atanh(a*x)), x))
```

3.441 $\int e^{n\tanh^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=68

$$-\frac{c^2 2^{\frac{n}{2}+1}(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(3-\frac{n}{2},-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

```
[Out] -2^(1+1/2*n)*c^2*(-a*x+1)^(3-1/2*n)*hypergeom([-1/2*n, 3-1/2*n],[4-1/2*n],-
1/2*a*x+1/2)/a/(6-n)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 69}

$$-\frac{c^2 2^{\frac{n}{2}+1}(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(3-\frac{n}{2},-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] -((2^(1 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[3 - n/2, -n/2, 4 -
 n/2, (1 - a*x)/2])/(a*(6 - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}(c-acx)^2\,dx = c^2\int (1-ax)^{2-\frac{n}{2}}(1+ax)^{n/2}\,dx$$
$$= -\frac{2^{1+\frac{n}{2}}c^2(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(3-\frac{n}{2},-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 0.96

$$\frac{c^2 2^{\frac{n}{2}+1}(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(3-\frac{n}{2},-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(n-6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (2^(1 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[3 - n/2, -1/2*n, 4 -
 n/2, (1 - a*x)/2])/(a*(-6 + n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^2x^2 - 2\,ac^2x + c^2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (acx-c)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}\,(-acx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (acx-c)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{atanh}(a\,x)}\,{\left(c-a\,c\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a*c*x)^2,x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - a*c*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{2} \left(\int \left(-2axe^{n \operatorname{atanh}{\left(ax \right)}}\right)\, dx + \int a^{2} x^{2} e^{n \operatorname{atanh}{\left(ax \right)}}\, dx + \int e^{n \operatorname{atanh}{\left(ax \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x*exp(n*atanh(a*x)), x) + Integral(a**2*x**2*exp(n*atan
h(a*x)), x) + Integral(exp(n*atanh(a*x)), x))
```

3.442 $\int e^{n\tanh^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=66

$$-\frac{c2^{\frac{n}{2}+1}(1-ax)^{2-\frac{n}{2}}\,_2F_1\left(2-\frac{n}{2},-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

```
[Out] -2^(1+1/2*n)*c*(-a*x+1)^(2-1/2*n)*hypergeom([-1/2*n, 2-1/2*n],[3-1/2*n],-1/
2*a*x+1/2)/a/(4-n)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6129, 69}

$$-\frac{c2^{\frac{n}{2}+1}(1-ax)^{2-\frac{n}{2}}\,_2F_1\left(2-\frac{n}{2},-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] -((2^(1 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[2 - n/2, -n/2, 3 - n
/2, (1 - a*x)/2])/(a*(4 - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}(c-acx)\,dx = c\int (1-ax)^{1-\frac{n}{2}}(1+ax)^{n/2}\,dx$$
$$= -\frac{2^{1+\frac{n}{2}}c(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(2-\frac{n}{2},-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.95

$$\frac{c2^{\frac{n}{2}+1}(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(2-\frac{n}{2},-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(n-4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a*c*x),x]
```

```
[Out] (2^(1 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[2 - n/2, -1/2*n, 3 - n
/2, (1 - a*x)/2])/(a*(-4 + n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}\,(-acx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a*c*x+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a*c*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (acx - c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -integrate((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{n\,\mathrm{atanh}(a\,x)}\,(c - a\,c\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a*c*x),x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int axe^{n\,\mathrm{atanh}\,(ax)}\,dx + \int \left(-e^{n\,\mathrm{atanh}\,(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x*exp(n*atanh(a*x)), x) + Integral(-exp(n*atanh(a*x)), x))
```

3.443 $\int \frac{e^{n\tanh^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=59

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{acn}$$

```
[Out] 2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],-1/2*a*x+1/2)/a/c/n/((-a*x
+1)^(1/2*n))
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a*c*n*(1
 - a*x)^(n/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{c-acx}\,dx = \frac{\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c}$$
$$= \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{acn}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.00

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x),x]
```

```
[Out] (2^(1 + n/2)*Hypergeometric2F1[-1/2*n, -1/2*n, 1 - n/2, (1 - a*x)/2])/(a*c*
n*(1 - a*x)^(n/2))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)}}{-acx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c),x)
[Out] int(exp(n*arctanh(a*x))/(-a*c*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx - c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c),x, algorithm="maxima")
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{n \operatorname{atanh}(a\,x)}}{c - a\,c\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a*c*x),x)
[Out] int(exp(n*atanh(a*x))/(c - a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{e^{n \operatorname{atanh}(ax)}}{ax-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c),x)
[Out] -Integral(exp(n*atanh(a*x))/(a*x - 1), x)/c
```

3.444 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=39

$$\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

```
[Out] (-a*x+1)^(-1-1/2*n)*(a*x+1)^(1+1/2*n)/a/c^2/(2+n)
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6129, 37}

$$\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] ((1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^2*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^2}\\ &= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}+1}}{ac^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^2,x]
```

```
[Out] ((1 - a*x)^(-1 - n/2)*(1 + a*x)^(1 + n/2))/(a*c^2*(2 + n))
```

fricas [A] time = 0.60, size = 58, normalized size = 1.49

$$\frac{(ax+1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n+2\,ac^2-\left(a^2c^2n+2\,a^2c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*x + 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n + 2*a*c^2 - (a^2*c^2*n + 2
*a^2*c^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

maple [A] time = 0.03, size = 33, normalized size = 0.85

$$-\frac{\mathrm{e}^{n\,\mathrm{arctanh}(ax)}\,(ax+1)}{(ax-1)\,c^2\,(2+n)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^2,x)
```

```
[Out] -exp(n*arctanh(a*x))*(a*x+1)/(a*x-1)/c^2/(2+n)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

mupad [B] time = 1.10, size = 37, normalized size = 0.95

$$\frac{(a\,x+1)^{\frac{n}{2}+1}}{a\,c^2\,(1-a\,x)^{\frac{n}{2}+1}\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a*c*x)^2,x)
```

```
[Out] (a*x + 1)^(n/2 + 1)/(a*c^2*(1 - a*x)^(n/2 + 1)*(n + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \text{NaN} & \text{for}\: a = \frac{1}{x} \wedge c = 0 \wedge n = -2 \\ \tilde{\infty} x e^{\infty n} & \text{for}\: a = \frac{1}{x} \\ \tilde{\infty} \int e^{n \operatorname{atanh}{(ax)}}\, dx & \text{for}\: c = 0 \\ - \frac{ax \operatorname{atanh}{(ax)}}{a^2c^2xe^{2 \operatorname{atanh}{(ax)}} - ac^2e^{2 \operatorname{atanh}{(ax)}}} - \frac{\operatorname{atanh}{(ax)}}{a^2c^2xe^{2 \operatorname{atanh}{(ax)}} - ac^2e^{2 \operatorname{atanh}{(ax)}}} & \text{for}\: n = -2 \\ - \frac{axe^{n \operatorname{atanh}{(ax)}}}{a^2c^2nx + 2a^2c^2x - ac^2n - 2ac^2} - \frac{e^{n \operatorname{atanh}{(ax)}}}{a^2c^2nx + 2a^2c^2x - ac^2n - 2ac^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**2,x)
```

```
[Out] Piecewise((nan, Eq(c, 0) & Eq(n, -2) & Eq(a, 1/x)), (zoo*x*exp(oo*n), Eq(a,
 1/x)), (zoo*Integral(exp(n*atanh(a*x)), x), Eq(c, 0)), (-a*x*atanh(a*x)/(a
**2*c**2*x*exp(2*atanh(a*x)) - a*c**2*exp(2*atanh(a*x))) - atanh(a*x)/(a**2
*c**2*x*exp(2*atanh(a*x)) - a*c**2*exp(2*atanh(a*x))), Eq(n, -2)), (-a*x*ex
p(n*atanh(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a*c**2*n - 2*a*c**2) - exp
(n*atanh(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a*c**2*n - 2*a*c**2), True)
)
```

3.445 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=84

$$\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^3\left(n^2+6n+8\right)}+\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-2}}{ac^3(n+4)}$$

```
[Out] (-a*x+1)^(-1-1/2*n)*(a*x+1)^(1+1/2*n)/a/c^3/(n^2+6*n+8)+(-a*x+1)^(-2-1/2*n)
*(a*x+1)^(1+1/2*n)/a/c^3/(4+n)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 45, 37}

$$\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^3\left(n^2+6n+8\right)}+\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-2}}{ac^3(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] ((1 - a*x)^(-2 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^3*(4 + n)) + ((1 - a*x)^(
-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^3*(8 + 6*n + n^2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^3}\,dx &= \frac{\int (1-ax)^{-3-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^3}\\ &= \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^3(4+n)}+\frac{\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^3(4+n)}\\ &= \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^3(4+n)}+\frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.61

$$\frac{(1-ax)^{-\frac{n}{2}-2}(-ax+n+3)(ax+1)^{\frac{n}{2}+1}}{ac^3(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^3,x]
```

```
[Out] ((1 - a*x)^(-2 - n/2)*(3 + n - a*x)*(1 + a*x)^(1 + n/2))/(a*c^3*(2 + n)*(4
+ n))
```

fricas [A] time = 0.44, size = 128, normalized size = 1.52

$$-\frac{\left(a^2x^2-(an+2\,a)x-n-3\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^2+6\,ac^3n+8\,ac^3+\left(a^3c^3n^2+6\,a^3c^3n+8\,a^3c^3\right)x^2-2\left(a^2c^3n^2+6\,a^2c^3n+8\,a^2c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 - (a*n + 2*a)*x - n - 3)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^2
 + 6*a*c^3*n + 8*a*c^3 + (a^3*c^3*n^2 + 6*a^3*c^3*n + 8*a^3*c^3)*x^2 - 2*(a
^2*c^3*n^2 + 6*a^2*c^3*n + 8*a^2*c^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

maple [A] time = 0.03, size = 46, normalized size = 0.55

$$-\frac{e^{n\,\mathrm{arctanh}(ax)}\,(ax-n-3)\,(ax+1)}{(ax-1)^2\,c^3\left(n^2+6n+8\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^3,x)
```

```
[Out] -exp(n*arctanh(a*x))*(a*x-n-3)*(a*x+1)/(a*x-1)^2/c^3/(n^2+6*n+8)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

mupad [B] time = 1.27, size = 100, normalized size = 1.19

$$\frac{e^{\frac{n\,\ln(a\,x+1)}{2}-\frac{n\,\ln(1-a\,x)}{2}}\left(\frac{n+3}{a^3\,c^3\left(n^2+6\,n+8\right)}-\frac{x^2}{a\,c^3\left(n^2+6\,n+8\right)}+\frac{x\,(n+2)}{a^2\,c^3\left(n^2+6\,n+8\right)}\right)}{\frac{1}{a^2}-\frac{2\,x}{a}+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a*c*x)^3,x)
```

```
[Out] (exp((n*log(a*x + 1))/2 - (n*log(1 - a*x))/2)*((n + 3)/(a^3*c^3*(6*n + n^2
+ 8)) - x^2/(a*c^3*(6*n + n^2 + 8)) + (x*(n + 2))/(a^2*c^3*(6*n + n^2 + 8))
))/(1/a^2 - (2*x)/a + x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**3,x)
```

```
[Out] Piecewise((zoo*Integral(exp(n*atanh(a*x)), x), Eq(c, 0)), (a**2*x**2*atanh(
a*x)/(2*a**3*c**3*x**2*exp(4*atanh(a*x)) - 4*a**2*c**3*x*exp(4*atanh(a*x))
+ 2*a*c**3*exp(4*atanh(a*x))) + 2*a*x*atanh(a*x)/(2*a**3*c**3*x**2*exp(4*at
anh(a*x)) - 4*a**2*c**3*x*exp(4*atanh(a*x)) + 2*a*c**3*exp(4*atanh(a*x))) -
 a*x/(2*a**3*c**3*x**2*exp(4*atanh(a*x)) - 4*a**2*c**3*x*exp(4*atanh(a*x))
+ 2*a*c**3*exp(4*atanh(a*x))) + atanh(a*x)/(2*a**3*c**3*x**2*exp(4*atanh(a*
x)) - 4*a**2*c**3*x*exp(4*atanh(a*x)) + 2*a*c**3*exp(4*atanh(a*x))) - 1/(2*
a**3*c**3*x**2*exp(4*atanh(a*x)) - 4*a**2*c**3*x*exp(4*atanh(a*x)) + 2*a*c*
*3*exp(4*atanh(a*x))), Eq(n, -4)), (-a**2*x**2*atanh(a*x)/(2*a**3*c**3*x**2
*exp(2*atanh(a*x)) - 4*a**2*c**3*x*exp(2*atanh(a*x)) + 2*a*c**3*exp(2*atanh
(a*x))) + a*x/(2*a**3*c**3*x**2*exp(2*atanh(a*x)) - 4*a**2*c**3*x*exp(2*ata
nh(a*x)) + 2*a*c**3*exp(2*atanh(a*x))) + atanh(a*x)/(2*a**3*c**3*x**2*exp(2
*atanh(a*x)) - 4*a**2*c**3*x*exp(2*atanh(a*x)) + 2*a*c**3*exp(2*atanh(a*x))
) + 1/(2*a**3*c**3*x**2*exp(2*atanh(a*x)) - 4*a**2*c**3*x*exp(2*atanh(a*x))
 + 2*a*c**3*exp(2*atanh(a*x))), Eq(n, -2)), (-a**2*x**2*exp(n*atanh(a*x))/(
a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n
**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*
c**3) + a*n*x*exp(n*atanh(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 +
 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x
+ a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + 2*a*x*exp(n*atanh(a*x))/(a**3*c**3
*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 1
2*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + n
*exp(n*atanh(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*
x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2
 + 6*a*c**3*n + 8*a*c**3) + 3*exp(n*atanh(a*x))/(a**3*c**3*n**2*x**2 + 6*a*
*3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x -
 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3), True))
```

3.446 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=135

$$\frac{2(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)}+\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-3}}{ac^4(n+6)}+\frac{2(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)}$$

```
[Out] 2*(-a*x+1)^(-2-1/2*n)*(a*x+1)^(1+1/2*n)/a/c^4/(n^2+10*n+24)+2*(-a*x+1)^(-1-
1/2*n)*(a*x+1)^(1+1/2*n)/a/c^4/(n^3+12*n^2+44*n+48)+(-a*x+1)^(-3-1/2*n)*(a*
x+1)^(1+1/2*n)/a/c^4/(6+n)
```

Rubi [A] time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6129, 45, 37}

$$\frac{2(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)}+\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-3}}{ac^4(n+6)}+\frac{2(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] ((1 - a*x)^(-3 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^4*(6 + n)) + (2*(1 - a*x)
^(-2 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^4*(4 + n)*(6 + n)) + (2*(1 - a*x)^(
-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^4*(6 + n)*(8 + 6*n + n^2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{\int (1-ax)^{-4-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^4}\\
&= \frac{(1-ax)^{-3-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(6+n)}+\frac{2\int (1-ax)^{-3-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^4(6+n)}\\
&= \frac{(1-ax)^{-3-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(6+n)}+\frac{2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)}+\frac{2\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^4(4+n)(6+n)}\\
&= \frac{(1-ax)^{-3-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(6+n)}+\frac{2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)}+\frac{2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^4(2+n)(4+n)(6+n)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.55

$$\frac{(1-ax)^{-\frac{n}{2}-3}(ax+1)^{\frac{n}{2}+1}\left(2a^2x^2-2anx-8ax+n^2+8n+14\right)}{ac^4(n+2)(n+4)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a*c*x)^4,x]
```

```
[Out] ((1 - a*x)^(-3 - n/2)*(1 + a*x)^(1 + n/2)*(14 + 8*n + n^2 - 8*a*x - 2*a*n*x
 + 2*a^2*x^2))/(a*c^4*(2 + n)*(4 + n)*(6 + n))
```

fricas [A] time = 0.73, size = 228, normalized size = 1.69

$$\frac{\left(2\,a^3x^3-2\left(a^2n+3\,a^2\right)x^2+n^2+\left(an^2+6\,an+6\,a\right)x+8\,n-}{ac^4n^3+12\,ac^4n^2+44\,ac^4n+48\,ac^4-\left(a^4c^4n^3+12\,a^4c^4n^2+44\,a^4c^4n+48\,a^4c^4\right)x^3+3\left(a^3c^4n^3+12\,a^3c^4n^2+44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] (2*a^3*x^3 - 2*(a^2*n + 3*a^2)*x^2 + n^2 + (a*n^2 + 6*a*n + 6*a)*x + 8*n +
14)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^3 + 12*a*c^4*n^2 + 44*a*c^4*n +
48*a*c^4 - (a^4*c^4*n^3 + 12*a^4*c^4*n^2 + 44*a^4*c^4*n + 48*a^4*c^4)*x^3 +
 3*(a^3*c^4*n^3 + 12*a^3*c^4*n^2 + 44*a^3*c^4*n + 48*a^3*c^4)*x^2 - 3*(a^2*
c^4*n^3 + 12*a^2*c^4*n^2 + 44*a^2*c^4*n + 48*a^2*c^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

maple [A] time = 0.03, size = 68, normalized size = 0.50

$$-\frac{(ax+1)\left(2a^2x^2-2anx-8ax+n^2+8n+14\right)e^{n\operatorname{arctanh}(ax)}}{(ax-1)^3c^4a\left(n^2+8n+12\right)(4+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a*c*x+c)^4,x)
```

```
[Out] -(a*x+1)*(2*a^2*x^2-2*a*n*x-8*a*x+n^2+8*n+14)*exp(n*arctanh(a*x))/(a*x-1)^3
/c^4/a/(n^2+8*n+12)/(4+n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{(acx-c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

mupad [B] time = 1.34, size = 167, normalized size = 1.24

$$-\frac{(a\,x+1)^{n/2}\left(\frac{2\,x^3}{a\,c^4\left(n^3+12\,n^2+44\,n+48\right)}+\frac{n^2+8\,n+14}{a^4\,c^4\left(n^3+12\,n^2+44\,n+48\right)}-\frac{x^2\,(2\,n+6)}{a^2\,c^4\left(n^3+12\,n^2+44\,n+48\right)}+\frac{x\left(n^2+6\,n+6\right)}{a^3\,c^4\left(n^3+12\,n^2+44\,n+48\right)}\right)}{(1-a\,x)^{n/2}\left(\frac{3\,x}{a^2}-\frac{1}{a^3}+x^3-\frac{3\,x^2}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a*c*x)^4,x)
```

```
[Out] -((a*x + 1)^(n/2)*((2*x^3)/(a*c^4*(44*n + 12*n^2 + n^3 + 48)) + (8*n + n^2
+ 14)/(a^4*c^4*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(2*n + 6))/(a^2*c^4*(44*n
 + 12*n^2 + n^3 + 48)) + (x*(6*n + n^2 + 6))/(a^3*c^4*(44*n + 12*n^2 + n^3
+ 48))))/((1 - a*x)^(n/2)*((3*x)/a^2 - 1/a^3 + x^3 - (3*x^2)/a))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a*c*x+c)**4,x)
```

```
[Out] Timed out
```

3.447 $\int e^{\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{x(1-ax)^{-p} F_1\left(1-p; \frac{1}{2}-p, -\frac{1}{2}; 2-p; ax, -ax\right) \left(c - \frac{c}{ax}\right)^p}{1-p}$$

```
[Out] (c-c/a/x)^p*x*AppellF1(1-p,1/2-p,-1/2,2-p,a*x,-a*x)/(1-p)/((-a*x+1)^p)
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6134, 6129, 133}

$$\frac{x(1-ax)^{-p} F_1\left(1-p; \frac{1}{2}-p, -\frac{1}{2}; 2-p; ax, -ax\right) \left(c - \frac{c}{ax}\right)^p}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, 1/2 - p, -1/2, 2 - p, a*x, -(a*x)])/((1
- p)*(1 - a*x)^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int e^{\tanh^{-1}(ax)}x^{-p}(1-ax)^p\,dx\\ &= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int x^{-p}(1-ax)^{-\frac{1}{2}+p}\sqrt{1+ax}\,dx\\ &= \frac{\left(c-\frac{c}{ax}\right)^p x(1-ax)^{-p}F_1\left(1-p;\frac{1}{2}-p,-\frac{1}{2};2-p;ax,-ax\right)}{1-p}\end{aligned}$$

Mathematica [F] time = 1.40, size = 0, normalized size = 0.00

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^p,x]
[Out] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^p, x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(\frac{acx-c}{ax}\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{p}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x)
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{p}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{p}\,(a\,x+1)}{\sqrt{1-a^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
[Out] int(((c - c/(a*x))^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.448 $\int e^{\tanh^{-1}(ax)} \left(c-\frac{c}{ax}\right)^4 \, dx$

Optimal. Leaf size=125

$$-\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}+\frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{3c^4\sin^{-1}(ax)}{a}$$

```
[Out] -1/3*c^4*(-a^2*x^2+1)^(3/2)/a^4/x^3+3/2*c^4*(-a^2*x^2+1)^(3/2)/a^3/x^2-3*c^
4*arcsin(a*x)/a-1/2*c^4*arctanh((-a^2*x^2+1)^(1/2))/a-1/2*c^4*(-a*x+6)*(-a^
2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.26, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6131, 6128, 1807, 813, 844, 216, 266, 63, 208}

$$\frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{3c^4\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] -(c^4*(6 - a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^4*(1 - a^2*x^2)^(3/2))/(3
*a^4*x^3) + (3*c^4*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (3*c^4*ArcSin[a*x])/a
 - (c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
```

```
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int \frac{e^{\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}$$

$$= \frac{c^4\int \frac{(1-ax)^3\sqrt{1-a^2x^2}}{x^4}\,dx}{a^4}$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{c^4\int \frac{\sqrt{1-a^2x^2}\left(9a-9a^2x+3a^3x^2\right)}{x^3}\,dx}{3a^4}$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} + \frac{c^4\int \frac{\left(18a^2+3a^3x\right)\sqrt{1-a^2x^2}}{x^2}\,dx}{6a^4}$$

$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{c^4\int \frac{-6a^3+36a^4x}{x\sqrt{1-a^2x^2}}\,dx}{12a^4}$$

$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \left(3c^4\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,d$$

$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^4\sin^{-1}(ax)}{a} + \frac{c^4\,\mathrm{Su}}{}$$

$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^4\sin^{-1}(ax)}{a} - \frac{c^4\,\mathrm{Su}}{}$$

$$= -\frac{c^4(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^4\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^4\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^4\sin^{-1}(ax)}{a} - \frac{c^4\,\mathrm{ta}}{}$$

Mathematica [A] time = 0.26, size = 86, normalized size = 0.69

$$\frac{c^4\left(-3\log\left(\sqrt{1-a^2x^2}+1\right) - \frac{\sqrt{1-a^2x^2}\left(6a^3x^3+16a^2x^2-9ax+2\right)}{a^3x^3} + 3\log(ax) - 18\sin^{-1}(ax)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*(-((Sqrt[1 - a^2*x^2]*(2 - 9*a*x + 16*a^2*x^2 + 6*a^3*x^3))/(a^3*x^3))
 - 18*ArcSin[a*x] + 3*Log[a*x] - 3*Log[1 + Sqrt[1 - a^2*x^2]]))/(6*a)
```

fricas [A] time = 0.52, size = 132, normalized size = 1.06

$$\frac{36\,a^3c^4x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,a^3c^4x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^4x^3-\left(6\,a^3c^4x^3+16\,a^2c^4x^2-9\,ac^4x+2\,c^4\right)\sqrt{}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(36*a^3*c^4*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a^3*c^4*x^3*
log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^4*x^3 - (6*a^3*c^4*x^3 + 16*a^2*c
^4*x^2 - 9*a*c^4*x + 2*c^4)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

giac [B] time = 0.43, size = 263, normalized size = 2.10

$$\frac{\left(c^4-\frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^4}{a^2x}+\frac{33\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^4}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|}-\frac{3\,c^4\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{|a|}-\frac{c^4\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\sqrt{-a^2x^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 1/24*(c^4 - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) + 33*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) - 3*c^4*arcsin(a*x)*sgn(a)/abs(a) - 1/2*c^4*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^4
/a - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/x - 9*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2*c^4/(a^2*x^2) + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^4/(a^4*x
^3))/(a^2*abs(a))
```

maple [A] time = 0.05, size = 142, normalized size = 1.14

$$-\frac{c^4\sqrt{-a^2x^2+1}}{a}-\frac{3c^4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{c^4\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{8c^4\sqrt{-a^2x^2+1}}{3a^2x}-\frac{c^4\sqrt{-a^2x^2+1}}{3a^4x^3}+\frac{3c^4\sqrt{-a^2x^2}}{2x^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^4,x)
```

```
[Out] -c^4*(-a^2*x^2+1)^(1/2)/a-3*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))-1/2*c^4/a*arctanh(1/(-a^2*x^2+1)^(1/2))-8/3*c^4*(-a^2*x^2+1)^(1/2
)/a^2/x-1/3*c^4*(-a^2*x^2+1)^(1/2)/a^4/x^3+3/2*c^4*(-a^2*x^2+1)^(1/2)/x^2/a
^3
```

maxima [A] time = 0.51, size = 190, normalized size = 1.52

$$-\frac{3\,c^4\arcsin\left(ax\right)}{a}-\frac{2\,c^4\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}-\frac{\sqrt{-a^2x^2+1}\,c^4}{a}+\frac{3\left(a^2\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^4}{2\,a^3}-\frac{2\,\sqrt{-a^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -3*c^4*arcsin(a*x)/a - 2*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a
- sqrt(-a^2*x^2 + 1)*c^4/a + 3/2*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/a
bs(x)) + sqrt(-a^2*x^2 + 1)/x^2)*c^4/a^3 - 2*sqrt(-a^2*x^2 + 1)*c^4/(a^2*x)
 - 1/3*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*c^4/a^4
```

mupad [B] time = 0.05, size = 137, normalized size = 1.10

$$\frac{3\,c^4\,\sqrt{1-a^2\,x^2}}{2\,a^3\,x^2}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{a}-\frac{8\,c^4\,\sqrt{1-a^2\,x^2}}{3\,a^2\,x}-\frac{3\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{3\,a^4\,x^3}+\frac{c^4\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^4*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/(2*a) - (3*c^4*asinh(x*(-a^2)^(1/2)))
/(-a^2)^(1/2) - (c^4*(1 - a^2*x^2)^(1/2))/a - (8*c^4*(1 - a^2*x^2)^(1/2))/(
3*a^2*x) + (3*c^4*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2) - (c^4*(1 - a^2*x^2)^(1/
2))/(3*a^4*x^3)
```

sympy [A] time = 11.93, size = 357, normalized size = 2.86

$$ac^4\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-3c^4\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\ \mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+\frac{2c^4\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**4,x)
```

```
[Out] a*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 - 3*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/
a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*c**4*Piecewise((-acosh(1/(a*x)),
 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + 2*c**4*Piecewise((-I*s
qrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))
/a**2 - 3*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2
))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1
- 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + c**
4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3
*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a*
*2*x**2 + 1)/(3*x**3), True))/a**4
```

3.449 $\int e^{\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^3 dx$

Optimal. Leaf size=97

$$-\frac{c^3(ax+4)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{2c^3\sin^{-1}(ax)}{a}$$

```
[Out] 1/2*c^3*(-a^2*x^2+1)^(3/2)/a^3/x^2-2*c^3*arcsin(a*x)/a+1/2*c^3*arctanh((-a^
2*x^2+1)^(1/2))/a-1/2*c^3*(a*x+4)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.19, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6131, 6128, 1807, 813, 844, 216, 266, 63, 208}

$$\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^3(ax+4)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{2c^3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] -(c^3*(4 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) + (c^3*(1 - a^2*x^2)^(3/2))/(2
*a^3*x^2) - (2*c^3*ArcSin[a*x])/a + (c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
```

```
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3\int\frac{e^{\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}$$
$$=-\frac{c^3\int\frac{(1-ax)^2\sqrt{1-a^2x^2}}{x^3}\,dx}{a^3}$$
$$=\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}+\frac{c^3\int\frac{\left(4a-a^2x\right)\sqrt{1-a^2x^2}}{x^2}\,dx}{2a^3}$$
$$=-\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^3\int\frac{2a^2+8a^3x}{x\sqrt{1-a^2x^2}}\,dx}{4a^3}$$
$$=-\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\left(2c^3\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{c^3\int\frac{1}{x\sqrt{1-a^2x^2}}\,d}{2a}$$
$$=-\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{2c^3\sin^{-1}(ax)}{a}-\frac{c^3\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}\,dx,\right.}{4a}$$
$$=-\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{2c^3\sin^{-1}(ax)}{a}+\frac{c^3\,\text{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\right.}{2a^3}$$
$$=-\frac{c^3(4+ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{2c^3\sin^{-1}(ax)}{a}+\frac{c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}$$

Mathematica [A] time = 0.20, size = 75, normalized size = 0.77

$$\frac{c^3\left(\frac{\sqrt{1-a^2x^2}\left(-2a^2x^2-4ax+1\right)}{a^2x^2}+\log\left(\sqrt{1-a^2x^2}+1\right)-\log(ax)-4\sin^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*(((1 - 4*a*x - 2*a^2*x^2)*Sqrt[1 - a^2*x^2])/(a^2*x^2) - 4*ArcSin[a*x]
 - Log[a*x] + Log[1 + Sqrt[1 - a^2*x^2]]))/(2*a)
```

fricas [A] time = 0.60, size = 121, normalized size = 1.25

$$\frac{8\,a^2c^3x^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-a^2c^3x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-2\,a^2c^3x^2-\left(2\,a^2c^3x^2+4\,ac^3x-c^3\right)\sqrt{-a^2x^2+1}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(8*a^2*c^3*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a^2*c^3*x^2*log
((sqrt(-a^2*x^2 + 1) - 1)/x) - 2*a^2*c^3*x^2 - (2*a^2*c^3*x^2 + 4*a*c^3*x -
 c^3)*sqrt(-a^2*x^2 + 1))/(a^3*x^2)
```

giac [B] time = 0.38, size = 207, normalized size = 2.13

$$-\frac{\left(c^3-\frac{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^3}{a^2x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}-\frac{2\,c^3\arcsin\left(ax\right)\mathrm{sgn}(a)}{|a|}+\frac{c^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\sqrt{-a^2x^2+1}\,c^3}{a}-\frac{8\left(\sqrt{-a^2x^2+1}\,|\right.}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -1/8*(c^3 - 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x))*a^4*x^2/((sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*abs(a)) - 2*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/2*
c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) -
sqrt(-a^2*x^2 + 1)*c^3/a - 1/8*(8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3*abs(a
)/(a^2*x) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*abs(a)/(a^4*x^2))/a^2
```

maple [A] time = 0.04, size = 119, normalized size = 1.23

$$-\frac{c^3\sqrt{-a^2x^2+1}}{a}-\frac{2c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{2c^3\sqrt{-a^2x^2+1}}{a^2x}+\frac{c^3\sqrt{-a^2x^2+1}}{2x^2a^3}+\frac{c^3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^3,x)
```

```
[Out] -c^3*(-a^2*x^2+1)^(1/2)/a-2*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))-2*c^3*(-a^2*x^2+1)^(1/2)/a^2/x+1/2*c^3*(-a^2*x^2+1)^(1/2)/x^2/a^3
+1/2*c^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.57, size = 111, normalized size = 1.14

$$-\frac{2\,c^3\arcsin(ax)}{a}-\frac{\sqrt{-a^2x^2+1}\,c^3}{a}+\frac{\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^3}{2\,a^3}-\frac{2\sqrt{-a^2x^2+1}\,c^3}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -2*c^3*arcsin(a*x)/a - sqrt(-a^2*x^2 + 1)*c^3/a + 1/2*(a^2*log(2*sqrt(-a^2*
x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)/x^2)*c^3/a^3 - 2*sqrt(-a^2
*x^2 + 1)*c^3/(a^2*x)
```

mupad [B] time = 0.80, size = 114, normalized size = 1.18

$$\frac{c^3\sqrt{1-a^2x^2}}{2\,a^3x^2}-\frac{c^3\sqrt{1-a^2x^2}}{a}-\frac{2\,c^3\sqrt{1-a^2x^2}}{a^2x}-\frac{2\,c^3\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{c^3\,\mathrm{atan}\left(\sqrt{1-a^2x^2}\,1\mathrm{i}\right)1\mathrm{i}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^3*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2) - (c^3*atan((1 - a^2*x^2)^(1/2)*1i)*1
i)/(2*a) - (c^3*(1 - a^2*x^2)^(1/2))/a - (2*c^3*(1 - a^2*x^2)^(1/2))/(a^2*x
) - (2*c^3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [A] time = 7.38, size = 228, normalized size = 2.35

$$ac^3\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-2c^3\left(\begin{cases}\sqrt{\frac{1}{a^2}}\,\mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\,\mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+\frac{2c^3\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)}{a^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**3,x)
```

```
[Out] a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 - 2*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/
```

```
a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*c**3*Piecewise((-I*sqrt(a**2*x**
2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2 - c**3
*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Ab
s(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2
))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3
```

3.450 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=65

$$-\frac{c^2(ax+1)\sqrt{1-a^2x^2}}{a^2x}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c^2\sin^{-1}(ax)}{a}$$

```
[Out] -c^2*arcsin(a*x)/a+c^2*arctanh((-a^2*x^2+1)^(1/2))/a-c^2*(a*x+1)*(-a^2*x^2+
1)^(1/2)/a^2/x
```

Rubi [A] time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6131, 6128, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(ax+1)\sqrt{1-a^2x^2}}{a^2x}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*(1 + a*x)*Sqrt[1 - a^2*x^2])/(a^2*x)) - (c^2*ArcSin[a*x])/a + (c^2*A
rcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\int\frac{e^{\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}$$
$$=\frac{c^2\int\frac{(1-ax)\sqrt{1-a^2x^2}}{x^2}\,dx}{a^2}$$
$$=-\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\int\frac{2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}$$
$$=-\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-c^2\int\frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{c^2\int\frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}$$
$$=-\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\sin^{-1}(ax)}{a}-\frac{c^2\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}$$
$$=-\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\sin^{-1}(ax)}{a}+\frac{c^2\,\mathrm{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}$$
$$=-\frac{c^2(1+ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\sin^{-1}(ax)}{a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.11, size = 78, normalized size = 1.20

$$-\frac{c^2\left(ax\sqrt{1-a^2x^2}+\sqrt{1-a^2x^2}-ax\log\left(\sqrt{1-a^2x^2}+1\right)+ax\log(ax)+ax\sin^{-1}(ax)\right)}{a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*(Sqrt[1 - a^2*x^2] + a*x*Sqrt[1 - a^2*x^2] + a*x*ArcSin[a*x] + a*x*L
og[a*x] - a*x*Log[1 + Sqrt[1 - a^2*x^2]]))/(a^2*x))
```

fricas [A] time = 0.46, size = 94, normalized size = 1.45

$$\frac{2\,ac^2x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-ac^2x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-ac^2x-\left(ac^2x+c^2\right)\sqrt{-a^2x^2+1}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (2*a*c^2*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a*c^2*x*log((sqrt(-a^2*
x^2 + 1) - 1)/x) - a*c^2*x - (a*c^2*x + c^2)*sqrt(-a^2*x^2 + 1))/(a^2*x)
```

giac [B] time = 0.44, size = 139, normalized size = 2.14

$$\frac{a^2c^2x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}+\frac{c^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}\,c^2}{a}-\frac{\left(\sqrt{-a^2x^2+1}\,|a|+\right.}{2\,a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*c^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - c^2*arcsin(a*x)*sg
n(a)/abs(a) + c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(
x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^2/a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)*c^2/(a^2*x*abs(a))
```

maple [A] time = 0.04, size = 95, normalized size = 1.46

$$-\frac{c^2\sqrt{-a^2x^2+1}}{a}-\frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{c^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}-\frac{c^2\sqrt{-a^2x^2+1}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^2,x)
```

```
[Out] -c^2*(-a^2*x^2+1)^(1/2)/a-c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))+c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))-c^2*(-a^2*x^2+1)^(1/2)/a^2/x
```

maxima [A] time = 0.54, size = 89, normalized size = 1.37

$$-\frac{c^2\arcsin\left(ax\right)}{a}+\frac{c^2\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}-\frac{\sqrt{-a^2x^2+1}\,c^2}{a}-\frac{\sqrt{-a^2x^2+1}\,c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -c^2*arcsin(a*x)/a + c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a - sq
rt(-a^2*x^2 + 1)*c^2/a - sqrt(-a^2*x^2 + 1)*c^2/(a^2*x)
```

mupad [B] time = 0.04, size = 91, normalized size = 1.40

$$-\frac{c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{a}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{a^2\,x}-\frac{c^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\;1\mathrm{i}\right)\,1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] - (c^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (c^2*atan((1 - a^2*x^2)^(1/2)*
1i)*1i)/a - (c^2*(1 - a^2*x^2)^(1/2))/a - (c^2*(1 - a^2*x^2)^(1/2))/(a^2*x)
```

sympy [A] time = 7.26, size = 151, normalized size = 2.32

$$ac^2\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\,\mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\,\mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)-\frac{c^2\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**2,x)
```

```
[Out] a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 - c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a*
*2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - c**2*Piecewise((-acosh(1/(a*x)), 1/A
bs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + c**2*Piecewise((-I*sqrt(a*
*2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2
```

3.451 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=41

$$\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c\sqrt{1-a^2x^2}}{a}$$

[Out] c*arctanh((-a^2*x^2+1)^(1/2))/a-c*(-a^2*x^2+1)^(1/2)/a

Rubi [A] time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6131, 6128, 266, 50, 63, 208}

$$\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c\sqrt{1-a^2x^2}}{a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[a*x]*(c - c/(a*x)),x]

[Out] -((c*Sqrt[1 - a^2*x^2])/a) + (c*ArcTanh[Sqrt[1 - a^2*x^2]])/a

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\frac{c\int \frac{e^{\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int \frac{\sqrt{1-a^2x^2}}{x}\,dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x}\,dx,x,x^2\right)}{2a}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}-\frac{c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.02

$$-\frac{c\left(\sqrt{1-a^2x^2}-\log\left(\sqrt{1-a^2x^2}+1\right)+\log(x)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x)),x]
[Out] -((c*(Sqrt[1 - a^2*x^2] + Log[x] - Log[1 + Sqrt[1 - a^2*x^2]]))/a)
```

fricas [A] time = 0.43, size = 41, normalized size = 1.00

$$-\frac{c\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\,c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x),x, algorithm="fricas")
[Out] -(c*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*c)/a
```

giac [A] time = 0.22, size = 58, normalized size = 1.41

$$\frac{c\log\left(\sqrt{-a^2x^2+1}+1\right)-c\log\left(-\sqrt{-a^2x^2+1}+1\right)-2\sqrt{-a^2x^2+1}\,c}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x),x, algorithm="giac")
[Out] 1/2*(c*log(sqrt(-a^2*x^2 + 1) + 1) - c*log(-sqrt(-a^2*x^2 + 1) + 1) - 2*sqr
t(-a^2*x^2 + 1)*c)/a
```

maple [A] time = 0.03, size = 34, normalized size = 0.83

$$\frac{c\left(-\sqrt{-a^2x^2+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x),x)
[Out] c/a*(-(-a^2*x^2+1)^(1/2)+arctanh(1/(-a^2*x^2+1)^(1/2)))
```

maxima [A] time = 0.45, size = 50, normalized size = 1.22

$$\frac{c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}-\frac{\sqrt{-a^2x^2+1}\,c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a - sqrt(-a^2*x^2 + 1)*c/a
```

mupad [B] time = 0.82, size = 37, normalized size = 0.90

$$\frac{c\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{a}-\frac{c\,\sqrt{1-a^2\,x^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c*atanh((1 - a^2*x^2)^(1/2)))/a - (c*(1 - a^2*x^2)^(1/2))/a
```

sympy [A] time = 20.41, size = 61, normalized size = 1.49

$$\begin{cases} \frac{-c\sqrt{-a^2x^2+1}+\frac{c\left(-\log\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)+\log\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{2}}{a} & \text{for } a\neq 0 \\ cx+\tilde{\infty}c\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x),x)
```

```
[Out] Piecewise(((-c*sqrt(-a**2*x**2 + 1) + c*(-log(-1 + 1/sqrt(-a**2*x**2 + 1))
+ log(1 + 1/sqrt(-a**2*x**2 + 1)))/2)/a, Ne(a, 0)), (c*x + zoo*c*log(x), Tr
ue))
```

3.452 $\int \frac{e^{\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=65

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}-\frac{2\sqrt{1-a^2x^2}}{ac}+\frac{2\sin^{-1}(ax)}{ac}$$

```
[Out] -(-a^2*x^2+1)^(3/2)/a/c/(-a*x+1)^2+2*arcsin(a*x)/a/c-2*(-a^2*x^2+1)^(1/2)/a
/c
```

Rubi [A] time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6131, 6128, 793, 665, 216}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}-\frac{2\sqrt{1-a^2x^2}}{ac}+\frac{2\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x)),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*c) - (1 - a^2*x^2)^(3/2)/(a*c*(1 - a*x)^2) + (2*A
rcSin[a*x])/(a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
```

```
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\frac{a\int \frac{e^{\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\
&= -\frac{a\int \frac{x\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx}{c}\\
&= -\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx}{c}\\
&= -\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}+\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{\left(1-a^2x^2\right)^{3/2}}{ac(1-ax)^2}+\frac{2\sin^{-1}(ax)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.80

$$
\frac{\frac{(ax-3)\sqrt{ax+1}}{\sqrt{1-ax}}-4\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{ac}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x)),x]
```

```
[Out] (((-3 + a*x)*Sqrt[1 + a*x])/Sqrt[1 - a*x] - 4*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]
)/(a*c)
```

fricas [A] time = 0.52, size = 68, normalized size = 1.05

$$-\frac{3\,ax+4\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,(ax-3)-3}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(3*a*x + 4*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x - 3) - 3)/(a^2*c*x - a*c)
```

giac [A] time = 0.19, size = 73, normalized size = 1.12

$$\frac{2\,\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}-\frac{\sqrt{-a^2x^2+1}}{ac}-\frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2*arcsin(a*x)*sgn(a)/(c*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c) - 4/(c*((sqrt(-a
^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 96, normalized size = 1.48

$$-\frac{\sqrt{-a^2x^2+1}}{ac}+\frac{2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c\sqrt{a^2}}+\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a^2c\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c+2/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(
1/2))+2/a^2/c/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))), x)
```

mupad [B] time = 0.07, size = 91, normalized size = 1.40

$$\frac{2\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{c\sqrt{-a^2}}-\frac{\sqrt{1-a^2x^2}}{a\,c}-\frac{2\sqrt{1-a^2x^2}}{c\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (2*asinh(x*(-a^2)^(1/2)))/(c*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a*c) - (2
*(1 - a^2*x^2)^(1/2))/(c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int\frac{x}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{ax^2}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(x/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integ
ral(a*x**2/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.453 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=104

$$\frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}-\frac{6\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{3\sin^{-1}(ax)}{ac^2}$$

```
[Out] 1/3*(-a^2*x^2+1)^(3/2)/a/c^2/(-a*x+1)^3+(-a^2*x^2+1)^(3/2)/a/c^2/(-a*x+1)^2
+3*arcsin(a*x)/a/c^2-6*(-a^2*x^2+1)^(1/2)/a/c^2/(-a*x+1)
```

Rubi [A] time = 0.19, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6131, 6128, 1639, 793, 663, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3}-\frac{6\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{3\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (-6*Sqrt[1 - a^2*x^2])/(a*c^2*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(3*a*c^2*(1
- a*x)^3) + (1 - a^2*x^2)^(3/2)/(a*c^2*(1 - a*x)^2) + (3*ArcSin[a*x])/(a*c^
2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{e^{\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$

$$= \frac{a^2\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^3}\,dx}{c^2}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2} - \frac{\int \frac{\left(2a^2-3a^3x\right)\sqrt{1-a^2x^2}}{(1-ax)^3}\,dx}{a^2c^2}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2} - \frac{3\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^2}\,dx}{c^2}$$

$$= -\frac{6\sqrt{1-a^2x^2}}{ac^2(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= -\frac{6\sqrt{1-a^2x^2}}{ac^2(1-ax)} + \frac{\left(1-a^2x^2\right)^{3/2}}{3ac^2(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{ac^2(1-ax)^2} + \frac{3\sin^{-1}(ax)}{ac^2}$$

Mathematica [A] time = 0.14, size = 53, normalized size = 0.51

$$\frac{\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2+19ax-14\right)}{(ax-1)^2}+9\sin^{-1}(ax)}{3ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (((-14 + 19*a*x - 3*a^2*x^2)*Sqrt[1 - a^2*x^2])/(-1 + a*x)^2 + 9*ArcSin[a*x
])/(3*a*c^2)
```

fricas [A] time = 0.49, size = 107, normalized size = 1.03

$$-\frac{14\,a^2x^2-28\,ax+18\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^2x^2-19\,ax+14\right)\sqrt{-a^2x^2+1}+14}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(14*a^2*x^2 - 28*a*x + 18*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2
+ 1) - 1)/(a*x)) + (3*a^2*x^2 - 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^3*
c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 140, normalized size = 1.35

$$-\frac{\sqrt{-a^{2}x^{2}+1}}{a\,c^{2}}+\frac{3\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{c^{2}\sqrt{a^{2}}}+\frac{2\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{3a^{3}c^{2}\left(x-\frac{1}{a}\right)^{2}}+\frac{13\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{3a^{2}c^{2}\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^2+3/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+2/3/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+13/3/a^2
/c^2/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^{2}x^{2}+1}\left(c-\frac{c}{ax}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^2), x)
```

mupad [B] time = 0.82, size = 140, normalized size = 1.35

$$\frac{2\,a\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^2\,x^2-2\,a^3\,c^2\,x+a^2\,c^2\right)}+\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^2\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a\,c^2}-\frac{13\,\sqrt{1-a^2\,x^2}}{3\,\sqrt{-a^2}\,\left(c^2\,x\,\sqrt{-a^2}-\frac{c^2\,\sqrt{-a^2}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (2*a*(1 - a^2*x^2)^(1/2))/(3*(a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) + (3*as
inh(x*(-a^2)^(1/2)))/(c^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a*c^2) - (13
*(1 - a^2*x^2)^(1/2))/(3*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^(1/
2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int \frac{x^2}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{a^2x^2\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*(Integral(x**2/(a**2*x**2*sqrt(-a**2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2
 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**3/(a**2*x**2*sqrt(-a**2*x
**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.454 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=136

$$\frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2}+\frac{14\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4}-\frac{8\sqrt{1-a^2x^2}}{ac^3(1-ax)}+\frac{4\sin^{-1}(ax)}{ac^3}$$

```
[Out] -1/5*(-a^2*x^2+1)^(3/2)/a/c^3/(-a*x+1)^4+14/15*(-a^2*x^2+1)^(3/2)/a/c^3/(-a
*x+1)^3+(-a^2*x^2+1)^(3/2)/a/c^3/(-a*x+1)^2+4*arcsin(a*x)/a/c^3-8*(-a^2*x^2
+1)^(1/2)/a/c^3/(-a*x+1)
```

Rubi [A] time = 0.28, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6131, 6128, 1639, 1637, 659, 651, 663, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2}+\frac{14\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4}-\frac{8\sqrt{1-a^2x^2}}{ac^3(1-ax)}+\frac{4\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] (-8*Sqrt[1 - a^2*x^2])/(a*c^3*(1 - a*x)) - (1 - a^2*x^2)^(3/2)/(5*a*c^3*(1
- a*x)^4) + (14*(1 - a^2*x^2)^(3/2))/(15*a*c^3*(1 - a*x)^3) + (1 - a^2*x^2)
^(3/2)/(a*c^3*(1 - a*x)^2) + (4*ArcSin[a*x])/(a*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
```

```
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{a^3\int \frac{e^{\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$

$$= -\frac{a^3\int \frac{x^3\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx}{c^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2} - \frac{\int \frac{\sqrt{1-a^2x^2}\left(2a^2-5a^3x+4a^4x^2\right)}{(1-ax)^4}\,dx}{a^2c^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2} - \frac{\int \left(\frac{a^2\sqrt{1-a^2x^2}}{(-1+ax)^4} + \frac{3a^2\sqrt{1-a^2x^2}}{(-1+ax)^3} + \frac{4a^2\sqrt{1-a^2x^2}}{(-1+ax)^2}\right)dx}{a^2c^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2} - \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{c^3} - \frac{3\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{c^3} - \frac{4\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{c^3}$$

$$= -\frac{8\sqrt{1-a^2x^2}}{ac^3(1-ax)} - \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4} + \frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2} + \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5c^3} + \frac{4\int \frac{1}{\sqrt{1-a^2x^2}}\,d\cdots}{c^3}$$

$$= -\frac{8\sqrt{1-a^2x^2}}{ac^3(1-ax)} - \frac{\left(1-a^2x^2\right)^{3/2}}{5ac^3(1-ax)^4} + \frac{14\left(1-a^2x^2\right)^{3/2}}{15ac^3(1-ax)^3} + \frac{\left(1-a^2x^2\right)^{3/2}}{ac^3(1-ax)^2} + \frac{4\sin^{-1}(ax)}{ac^3}$$

Mathematica [A] time = 0.16, size = 61, normalized size = 0.45

$$\frac{\frac{\sqrt{1-a^2x^2}\left(-15a^3x^3+149a^2x^2-222ax+94\right)}{(ax-1)^3} + 60\sin^{-1}(ax)}{15ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(94 - 222*a*x + 149*a^2*x^2 - 15*a^3*x^3))/(-1 + a*x)^3
 + 60*ArcSin[a*x])/(15*a*c^3)
```

fricas [A] time = 0.47, size = 143, normalized size = 1.05

$$-\frac{94\,a^3x^3 - 282\,a^2x^2 + 282\,ax + 120\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(15\,a^3x^3 - 149\,a^2x^2 + 222\,\cdots\right.}{15\left(a^4c^3x^3 - 3\,a^3c^3x^2 + 3\,a^2c^3x - ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")

[Out] -1/15*(94*a^3*x^3 - 282*a^2*x^2 + 282*a*x + 120*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^3*x^3 - 149*a^2*x^2 + 222*a*x - 94)*sqrt(-a^2*x^2 + 1) - 94)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)

giac [A] time = 0.20, size = 181, normalized size = 1.33

$$\frac{4 \arcsin(ax)\operatorname{sgn}(a)}{c^3|a|}-\frac{\sqrt{-a^2x^2+1}}{ac^3}+\frac{2\left(\frac{335\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{505\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{285\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{60\left(\sqrt{-a^2x^2+\cdots}\right.}{a^8x^4}\right.}{15\,c^3\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="giac")

[Out] 4*arcsin(a*x)*sgn(a)/(c^3*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c^3) + 2/15*(335*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 505*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 285*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 60*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 79)/(c^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))

maple [A] time = 0.05, size = 184, normalized size = 1.35

$$-\frac{\sqrt{-a^2x^2+1}}{a\,c^3}+\frac{4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^3\sqrt{a^2}}+\frac{31\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{15a^3c^3\left(x-\frac{1}{a}\right)^2}+\frac{104\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{15a^2c^3\left(x-\frac{1}{a}\right)}+\frac{2\sqrt{-a\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x)

[Out] -(-a^2*x^2+1)^(1/2)/a/c^3+4/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+31/15/a^3/c^3/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+104/15/a^2/c^3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2/5/a^4/c^3/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^3), x)
```

mupad [B] time = 0.82, size = 225, normalized size = 1.65

$$\frac{31\,a\,\sqrt{1-a^2\,x^2}}{15\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^3\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a\,c^3}-\frac{104\,\sqrt{1-a^2\,x^2}}{15\,\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}\right)}-\frac{}{5\,\sqrt{-a^2}\,\left(3\,c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (31*a*(1 - a^2*x^2)^(1/2))/(15*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (4*
asinh(x*(-a^2)^(1/2)))/(c^3*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a*c^3) - (
104*(1 - a^2*x^2)^(1/2))/(15*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)
^(1/2))/a)) - (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2)
 - (c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)^(1/
2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\left(\int\frac{x^3}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{ax^4}{a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*(Integral(x**3/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**
2*x**2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Inte
gral(a*x**4/(a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 +
 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.455 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=168

$$\frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2}+\frac{184\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}-\frac{26\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}-\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)}+\frac{5\sin^{-1}(ax)}{ac^4}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a/c^4/(-a*x+1)^5-26/35*(-a^2*x^2+1)^(3/2)/a/c^4/(-a*
x+1)^4+184/105*(-a^2*x^2+1)^(3/2)/a/c^4/(-a*x+1)^3+(-a^2*x^2+1)^(3/2)/a/c^4
/(-a*x+1)^2+5*arcsin(a*x)/a/c^4-10*(-a^2*x^2+1)^(1/2)/a/c^4/(-a*x+1)
```

Rubi [A] time = 0.35, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6131, 6128, 1639, 1637, 659, 651, 663, 216}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2}+\frac{184\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}-\frac{26\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}-\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)}+\frac{5\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] (-10*Sqrt[1 - a^2*x^2])/(a*c^4*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(7*a*c^4*(1
 - a*x)^5) - (26*(1 - a^2*x^2)^(3/2))/(35*a*c^4*(1 - a*x)^4) + (184*(1 - a^
2*x^2)^(3/2))/(105*a*c^4*(1 - a*x)^3) + (1 - a^2*x^2)^(3/2)/(a*c^4*(1 - a*x
)^2) + (5*ArcSin[a*x])/(a*c^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
```

```
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= \frac{a^4\int \frac{e^{\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}\\
&= \frac{a^4\int \frac{x^4\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx}{c^4}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2}-\frac{\int \frac{\sqrt{1-a^2x^2}\left(2a^2-7a^3x+9a^4x^2-5a^5x^3\right)}{(1-ax)^5}\,dx}{a^2c^4}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2}-\frac{\int \left(\frac{a^2\sqrt{1-a^2x^2}}{(-1+ax)^5}+\frac{4a^2\sqrt{1-a^2x^2}}{(-1+ax)^4}+\frac{6a^2\sqrt{1-a^2x^2}}{(-1+ax)^3}+\frac{5a^2\sqrt{1-a^2x^2}}{(-1+ax)^2}\right)dx}{a^2c^4}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^5}\,dx}{c^4}-\frac{4\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{c^4}-\frac{5\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{c^4}-\frac{6\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{c^4}\\
&= -\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}-\frac{4\left(1-a^2x^2\right)^{3/2}}{5ac^4(1-ax)^4}+\frac{2\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2}+\cdots\\
&= -\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}-\frac{26\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{26\left(1-a^2x^2\right)^{3/2}}{15ac^4(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2}+\cdots\\
&= -\frac{10\sqrt{1-a^2x^2}}{ac^4(1-ax)}+\frac{\left(1-a^2x^2\right)^{3/2}}{7ac^4(1-ax)^5}-\frac{26\left(1-a^2x^2\right)^{3/2}}{35ac^4(1-ax)^4}+\frac{184\left(1-a^2x^2\right)^{3/2}}{105ac^4(1-ax)^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{ac^4(1-ax)^2}+\cdots
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 69, normalized size = 0.41

$$
\frac{\frac{\sqrt{1-a^2x^2}\left(-105a^4x^4+1444a^3x^3-3256a^2x^2+2771ax-824\right)}{(ax-1)^4}+525\sin^{-1}(ax)}{105ac^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-824 + 2771*a*x - 3256*a^2*x^2 + 1444*a^3*x^3 - 105*a^
4*x^4))/(-1 + a*x)^4 + 525*ArcSin[a*x])/(105*a*c^4)
```

fricas [A] time = 0.48, size = 177, normalized size = 1.05

$$-\frac{824\,a^{4}x^{4}-3296\,a^{3}x^{3}+4944\,a^{2}x^{2}-3296\,ax+1050\left(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1\right)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)+}{105\left(a^{5}c^{4}x^{4}-4\,a^{4}c^{4}x^{3}+6\,a^{3}c^{4}x^{2}-4\,a^{2}c^{4}x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(824*a^4*x^4 - 3296*a^3*x^3 + 4944*a^2*x^2 - 3296*a*x + 1050*(a^4*x^
4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)) + (105*a^4*x^4 - 1444*a^3*x^3 + 3256*a^2*x^2 - 2771*a*x + 824)*sqrt(-a^2
*x^2 + 1) + 824)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 228, normalized size = 1.36

$$-\frac{\sqrt{-a^{2}x^{2}+1}}{a\,c^{4}}+\frac{5\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{c^{4}\sqrt{a^{2}}}+\frac{446\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{105a^{3}c^{4}\left(x-\frac{1}{a}\right)^{2}}+\frac{1024\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{105a^{2}c^{4}\left(x-\frac{1}{a}\right)}+\frac{57\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^4+5/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+446/105/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1024
/105/a^2/c^4/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+57/35/a^4/c^4/(x-1/
a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+2/7/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^4), x)
```

mupad [B] time = 0.83, size = 389, normalized size = 2.32

$$\frac{16\,a\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^4\,x^2-2\,a^3\,c^4\,x+a^2\,c^4\right)}+\frac{5\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^4\,\sqrt{-a^2}}+\frac{4\,a^3\,\sqrt{1-a^2\,x^2}}{35\,\left(a^6\,c^4\,x^2-2\,a^5\,c^4\,x+a^4\,c^4\right)}-\frac{6\,a^4\,\sqrt{1-a^2\,x^2}}{5\,\left(a^7\,c^4\,x^2-2\,a^6\,c^4\,x+a^5\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^4*(1 - a^2*x^2)^(1/2)),x)
[Out] (16*a*(1 - a^2*x^2)^(1/2))/(3*(a^2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) + (5*a
sinh(x*(-a^2)^(1/2)))/(c^4*(-a^2)^(1/2)) + (4*a^3*(1 - a^2*x^2)^(1/2))/(35*
(a^4*c^4 - 2*a^5*c^4*x + a^6*c^4*x^2)) - (6*a^4*(1 - a^2*x^2)^(1/2))/(5*(a^
5*c^4 - 2*a^6*c^4*x + a^7*c^4*x^2)) - (1 - a^2*x^2)^(1/2)/(a*c^4) + (2*a*(1
 - a^2*x^2)^(1/2))/(7*(a^2*c^4 - 4*a^3*c^4*x + 6*a^4*c^4*x^2 - 4*a^5*c^4*x^
3 + a^6*c^4*x^4)) - (1024*(1 - a^2*x^2)^(1/2))/(105*(-a^2)^(1/2)*(c^4*x*(-a
^2)^(1/2) - (c^4*(-a^2)^(1/2))/a)) - (57*(1 - a^2*x^2)^(1/2))/(35*(-a^2)^(1
/2)*(3*c^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1/2)
 - 3*a*c^4*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\left(\int \frac{x^4}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}+6a^2x^2\sqrt{-a\ldots}}\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**4,x)
[Out] a**4*(Integral(x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*x**3*sqrt(-a**
2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**5/(a**4*x**4*sqrt(-a**2*x**2 +
 1) - 4*a**3*x**3*sqrt(-a**2*x**2 + 1) + 6*a**2*x**2*sqrt(-a**2*x**2 + 1) -
 4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.456 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=59

$$-\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}-x\left(c-\frac{c}{ax}\right)^p$$

```
[Out] -(c-c/a/x)^p*x-(2-p)*(c-c/a/x)^p*hypergeom([1, p],[1+p],1-1/a/x)/a/p
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6133, 25, 514, 375, 78, 65}

$$-\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}-x\left(c-\frac{c}{ax}\right)^p$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -((c - c/(a*x))^p*x) - ((2 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 +
 p, 1 - 1/(a*x)])/(a*p)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = \int \frac{\left(c-\frac{c}{ax}\right)^p(1+ax)}{1-ax}\,dx$$
$$= -\frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{-1+p}(1+ax)}{x}\,dx}{a}$$
$$= -\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{-1+p}\,dx}{a}$$
$$= \frac{c\,\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{-1+p}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}$$
$$= -\left(c-\frac{c}{ax}\right)^p x + \frac{(c(2-p))\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{-1+p}}{x}\,dx,x,\frac{1}{x}\right)}{a}$$
$$= -\left(c-\frac{c}{ax}\right)^p x - \frac{(2-p)\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(1,p;1+p;1-\frac{1}{ax}\right)}{ap}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.78

$$\frac{\left(c-\frac{c}{ax}\right)^p\left((p-2)\,{}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)-apx\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*(-(a*p*x) + (-2 + p)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/
(a*x)]))/(a*p)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax+1)\left(\frac{acx-c}{ax}\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral(-(a*x + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate(-(a*x + 1)^2*(c - c/(a*x))^p/(a^2*x^2 - 1), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^p}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x)
[Out] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^p,x, algorithm="maxima")
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^p/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\left(c-\frac{c}{a\,x}\right)^p\,(a\,x+1)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^p*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] int(-((c - c/(a*x))^p*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 7.93, size = 274, normalized size = 4.64

$$-a\left(\begin{cases}\frac{0^px}{a}+\frac{0^p\log(ax-1)}{a^2}-\frac{a^{-p}c^ppx^2x^{-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{for } |ax|>1\\ \frac{0^px}{a}+\frac{0^p\log(-ax+1)}{a^2}-\frac{a^{-p}c^ppx^2x^{-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{otherwise}\end{cases}\right)-\begin{cases}\frac{0^p\log(ax-1)}{a}-\frac{a^{-p}c^ppxx^{-p}e^{i\pi p}\Gamma(p)\Gamma(\cdots}{\Gamma(2\cdots}\\ \frac{0^p\log(-ax+1)}{a}-\frac{a^{-p}c^ppxx^{-p}e^{i\pi p}\Gamma(p)\Gamma\cdots}{\Gamma(\cdots}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**p,x)
```

```
[Out] -a*Piecewise((0**p*x/a + 0**p*log(a*x - 1)/a**2 - a**(-p)*c**p*p*x**2*x**(-
p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), a*x)/(
gamma(3 - p)*gamma(p + 1)), Abs(a*x) > 1), (0**p*x/a + 0**p*log(-a*x + 1)/a
**2 - a**(-p)*c**p*p*x**2*x**(-p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((
1 - p, 2 - p), (3 - p,), a*x)/(gamma(3 - p)*gamma(p + 1)), True)) - Piecewi
se((0**p*log(a*x - 1)/a - a**(-p)*c**p*p*x*x**(-p)*exp(I*pi*p)*gamma(p)*gam
ma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(gamma(2 - p)*gamma(p + 1)),
 Abs(a*x) > 1), (0**p*log(-a*x + 1)/a - a**(-p)*c**p*p*x*x**(-p)*exp(I*pi*p
)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(gamma(2 - p)*
gamma(p + 1)), True))
```

3.457 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal. Leaf size=62

$$\frac{c^5}{4a^5x^4}-\frac{c^5}{a^4x^3}+\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+\frac{3c^5\log(x)}{a}+c^5(-x)$$

```
[Out] 1/4*c^5/a^5/x^4-c^5/a^4/x^3+c^5/a^3/x^2+2*c^5/a^2/x-c^5*x+3*c^5*ln(x)/a
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 75}

$$\frac{c^5}{a^3x^2}-\frac{c^5}{a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}+\frac{3c^5\log(x)}{a}+c^5(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(a^4*x^3) + c^5/(a^3*x^2) + (2*c^5)/(a^2*x) - c^5*x +
 (3*c^5*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx = -\frac{c^5\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^5}{x^5}\,dx}{a^5}$$
$$= -\frac{c^5\int\frac{(1-ax)^4(1+ax)}{x^5}\,dx}{a^5}$$
$$= -\frac{c^5\int\left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5}$$
$$= \frac{c^5}{4a^5x^4}-\frac{c^5}{a^4x^3}+\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}-c^5x+\frac{3c^5\log(x)}{a}$$

Mathematica [A] time = 0.23, size = 64, normalized size = 1.03

$$\frac{c^5}{4a^5x^4}-\frac{c^5}{a^4x^3}+\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+\frac{3c^5\log(ax)}{a}+c^5(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(a^4*x^3) + c^5/(a^3*x^2) + (2*c^5)/(a^2*x) - c^5*x +
 (3*c^5*Log[a*x])/a
```

fricas [A] time = 0.55, size = 67, normalized size = 1.08

$$-\frac{4\,a^5c^5x^5-12\,a^4c^5x^4\log(x)-8\,a^3c^5x^3-4\,a^2c^5x^2+4\,ac^5x-c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) - 8*a^3*c^5*x^3 - 4*a^2*c^5*x^2
 + 4*a*c^5*x - c^5)/(a^5*x^4)
```

giac [A] time = 0.16, size = 59, normalized size = 0.95

$$-c^5x+\frac{3\,c^5\log\left(|x|\right)}{a}+\frac{8\,a^3c^5x^3+4\,a^2c^5x^2-4\,ac^5x+c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] -c^5*x + 3*c^5*log(abs(x))/a + 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5
*x + c^5)/(a^5*x^4)
```

maple [A] time = 0.03, size = 61, normalized size = 0.98

$$\frac{c^5}{4a^5x^4} - \frac{c^5}{a^4x^3} + \frac{c^5}{x^2a^3} + \frac{2c^5}{a^2x} - c^5x + \frac{3c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^5,x)
[Out] 1/4*c^5/a^5/x^4-c^5/a^4/x^3+c^5/x^2/a^3+2*c^5/a^2/x-c^5*x+3*c^5*ln(x)/a
```

maxima [A] time = 0.47, size = 58, normalized size = 0.94

$$-c^5x + \frac{3\,c^5\log(x)}{a} + \frac{8\,a^3c^5x^3 + 4\,a^2c^5x^2 - 4\,ac^5x + c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^5,x, algorithm="maxima")
[Out] -c^5*x + 3*c^5*log(x)/a + 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*x +
c^5)/(a^5*x^4)
```

mupad [B] time = 0.85, size = 49, normalized size = 0.79

$$\frac{c^5\left(a^2\,x^2 - a\,x + 2\,a^3\,x^3 - a^5\,x^5 + 3\,a^4\,x^4\,\ln(x) + \frac{1}{4}\right)}{a^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^5*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] (c^5*(a^2*x^2 - a*x + 2*a^3*x^3 - a^5*x^5 + 3*a^4*x^4*log(x) + 1/4))/(a^5*x
^4)
```

sympy [A] time = 0.28, size = 63, normalized size = 1.02

$$\frac{-a^5c^5x + 3a^4c^5\log{(x)} - \frac{-8a^3c^5x^3 - 4a^2c^5x^2 + 4ac^5x - c^5}{4x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**5,x)
[Out] (-a**5*c**5*x + 3*a**4*c**5*log(x) - (-8*a**3*c**5*x**3 - 4*a**2*c**5*x**2
+ 4*a*c**5*x - c**5)/(4*x**4))/a**5
```

3.458 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=40

$$-\frac{c^4}{3a^4x^3}+\frac{c^4}{a^3x^2}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

```
[Out] -1/3*c^4/a^4/x^3+c^4/a^3/x^2-c^4*x+2*c^4*ln(x)/a
```

Rubi [A] time = 0.10, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 75}

$$\frac{c^4}{a^3x^2}-\frac{c^4}{3a^4x^3}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -c^4/(3*a^4*x^3) + c^4/(a^3*x^2) - c^4*x + (2*c^4*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$
$$= \frac{c^4\int\frac{(1-ax)^3(1+ax)}{x^4}dx}{a^4}$$
$$= \frac{c^4\int\left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}$$
$$= -\frac{c^4}{3a^4x^3}+\frac{c^4}{a^3x^2}-c^4x+\frac{2c^4\log(x)}{a}$$

Mathematica [A] time = 0.16, size = 42, normalized size = 1.05

$$-\frac{c^4}{3a^4x^3}+\frac{c^4}{a^3x^2}+\frac{2c^4\log(ax)}{a}+c^4(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^4,x]
[Out] -1/3*c^4/(a^4*x^3) + c^4/(a^3*x^2) - c^4*x + (2*c^4*Log[a*x])/a
```

fricas [A] time = 0.57, size = 43, normalized size = 1.08

$$-\frac{3\,a^4c^4x^4-6\,a^3c^4x^3\log(x)-3\,ac^4x+c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^4,x, algorithm="fricas")
[Out] -1/3*(3*a^4*c^4*x^4 - 6*a^3*c^4*x^3*log(x) - 3*a*c^4*x + c^4)/(a^4*x^3)
```

giac [A] time = 0.20, size = 39, normalized size = 0.98

$$-c^4x+\frac{2\,c^4\log(|x|)}{a}+\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^4,x, algorithm="giac")
[Out] -c^4*x + 2*c^4*log(abs(x))/a + 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

maple [A] time = 0.03, size = 39, normalized size = 0.98

$$-\frac{c^4}{3a^4x^3}+\frac{c^4}{x^2a^3}-c^4x+\frac{2c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^4,x)
[Out] -1/3*c^4/a^4/x^3+c^4/x^2/a^3-c^4*x+2*c^4*ln(x)/a
```

maxima [A] time = 0.30, size = 38, normalized size = 0.95

$$-c^4x+\frac{2\,c^4\log(x)}{a}+\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^4,x, algorithm="maxima")
[Out] -c^4*x + 2*c^4*log(x)/a + 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

mupad [B] time = 0.81, size = 35, normalized size = 0.88

$$\frac{c^4\,\left(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln(x)-1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^4*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] (c^4*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

sympy [A] time = 0.19, size = 39, normalized size = 0.98

$$\frac{-a^4c^4x+2a^3c^4\log{(x)}-\frac{-3ac^4x+c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**4,x)
[Out] (-a**4*c**4*x + 2*a**3*c**4*log(x) - (-3*a*c**4*x + c**4)/(3*x**3))/a**4
```

3.459 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=40

$$\frac{c^3}{2a^3x^2}-\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3(-x)$$

```
[Out] 1/2*c^3/a^3/x^2-c^3/a^2/x-c^3*x+c^3*ln(x)/a
```

Rubi [A] time = 0.10, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 75}

$$\frac{c^3}{2a^3x^2}-\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) - c^3/(a^2*x) - c^3*x + (c^3*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx = -\frac{c^3\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}$$
$$= -\frac{c^3\int\frac{(1-ax)^2(1+ax)}{x^3}\,dx}{a^3}$$
$$= -\frac{c^3\int\left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3}$$
$$= \frac{c^3}{2a^3x^2}-\frac{c^3}{a^2x}-c^3x+\frac{c^3\log(x)}{a}$$

Mathematica [A] time = 0.13, size = 42, normalized size = 1.05

$$\frac{c^3}{2a^3x^2}-\frac{c^3}{a^2x}+\frac{c^3\log(ax)}{a}+c^3(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^3,x]
[Out] c^3/(2*a^3*x^2) - c^3/(a^2*x) - c^3*x + (c^3*Log[a*x])/a
```

fricas [A] time = 0.41, size = 45, normalized size = 1.12

$$-\frac{2\,a^3c^3x^3-2\,a^2c^3x^2\log(x)+2\,ac^3x-c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^3,x, algorithm="fricas")
[Out] -1/2*(2*a^3*c^3*x^3 - 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x - c^3)/(a^3*x^2)
```

giac [A] time = 0.20, size = 38, normalized size = 0.95

$$-c^3x+\frac{c^3\log(|x|)}{a}-\frac{2\,ac^3x-c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^3,x, algorithm="giac")
[Out] -c^3*x + c^3*log(abs(x))/a - 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

maple [A] time = 0.03, size = 39, normalized size = 0.98

$$\frac{c^3}{2x^2a^3} - \frac{c^3}{a^2x} - c^3x + \frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^3,x)
[Out] 1/2*c^3/x^2/a^3-c^3/a^2/x-c^3*x+c^3*ln(x)/a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 0.05, size = 35, normalized size = 0.88

$$-\frac{c^3\,\left(2\,a\,x+2\,a^3\,x^3-2\,a^2\,x^2\,\ln(x)-1\right)}{2\,a^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] -(c^3*(2*a*x + 2*a^3*x^3 - 2*a^2*x^2*log(x) - 1))/(2*a^3*x^2)
```

sympy [A] time = 0.17, size = 37, normalized size = 0.92

$$\frac{-a^3c^3x + a^2c^3\log{(x)} - \frac{2ac^3x-c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**3,x)
[Out] (-a**3*c**3*x + a**2*c**3*log(x) - (2*a*c**3*x - c**3)/(2*x**2))/a**3
```

3.460 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=18

$$c^2(-x)-\frac{c^2}{a^2x}$$

[Out] -c^2/a^2/x-c^2*x

Rubi [A] time = 0.09, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6129, 73, 14}

$$c^2(-x)-\frac{c^2}{a^2x}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^2,x]

[Out] -(c^2/(a^2*x)) - c^2*x

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$
$$= \frac{c^2\int\frac{(1-ax)(1+ax)}{x^2}dx}{a^2}$$
$$= \frac{c^2\int\frac{1-a^2x^2}{x^2}dx}{a^2}$$
$$= \frac{c^2\int\left(-a^2+\frac{1}{x^2}\right)dx}{a^2}$$
$$= -\frac{c^2}{a^2x}-c^2x$$

Mathematica [A] time = 0.10, size = 18, normalized size = 1.00

$$c^2(-x)-\frac{c^2}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^2,x]
[Out] -(c^2/(a^2*x)) - c^2*x
```

fricas [A] time = 0.56, size = 22, normalized size = 1.22

$$-\frac{a^2c^2x^2+c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^2,x, algorithm="fricas")
[Out] -(a^2*c^2*x^2 + c^2)/(a^2*x)
```

giac [A] time = 0.20, size = 18, normalized size = 1.00

$$-c^2x-\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^2,x, algorithm="giac")
[Out] -c^2*x - c^2/(a^2*x)
```

maple [A] time = 0.03, size = 20, normalized size = 1.11

$$\frac{c^2\left(-a^2x-\frac{1}{x}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^2,x)
[Out] c^2/a^2*(-a^2*x-1/x)
```

maxima [A] time = 0.35, size = 18, normalized size = 1.00

$$-c^2x-\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^2,x, algorithm="maxima")
[Out] -c^2*x - c^2/(a^2*x)
```

mupad [B] time = 0.04, size = 20, normalized size = 1.11

$$-\frac{c^2\,\left(a^2\,x^2+1\right)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] -(c^2*(a^2*x^2 + 1))/(a^2*x)
```

sympy [A] time = 0.10, size = 17, normalized size = 0.94

$$\frac{-a^2c^2x-\frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**2,x)
[Out] (-a**2*c**2*x - c**2/x)/a**2
```

3.461 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=13

$$-\frac{c\log(x)}{a}-cx$$

[Out] -c*x-c*ln(x)/a

Rubi [A] time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6131, 6129, 43}

$$-\frac{c\log(x)}{a}-cx$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x)),x]

[Out] -(c*x) - (c*Log[x])/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{c\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$= -\frac{c\int\frac{1+ax}{x}\,dx}{a}$$
$$= -\frac{c\int\left(a+\frac{1}{x}\right)dx}{a}$$
$$= -cx-\frac{c\log(x)}{a}$$

Mathematica [A] time = 0.04, size = 13, normalized size = 1.00

$$-\frac{c\log(x)}{a}-cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x)),x]
[Out] -(c*x) - (c*Log[x])/a
```

fricas [A] time = 0.51, size = 14, normalized size = 1.08

$$-\frac{acx+c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x),x, algorithm="fricas")
[Out] -(a*c*x + c*log(x))/a
```

giac [A] time = 0.18, size = 14, normalized size = 1.08

$$-cx-\frac{c\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x),x, algorithm="giac")
[Out] -c*x - c*log(abs(x))/a
```

maple [A] time = 0.03, size = 14, normalized size = 1.08

$$-cx - \frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x),x)
[Out] -c*x-c*ln(x)/a
```

maxima [A] time = 0.41, size = 13, normalized size = 1.00

$$-cx - \frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x),x, algorithm="maxima")
[Out] -c*x - c*log(x)/a
```

mupad [B] time = 0.03, size = 12, normalized size = 0.92

$$-\frac{c\,\left(\ln(x)+a\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] -(c*(log(x) + a*x))/a
```

sympy [A] time = 0.09, size = 12, normalized size = 0.92

$$\frac{-acx - c\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x),x)
[Out] (-a*c*x - c*log(x))/a
```

3.462 $\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=38

$$-\frac{2}{ac(1-ax)}-\frac{3\log(1-ax)}{ac}-\frac{x}{c}$$

```
[Out] -x/c-2/a/c/(-a*x+1)-3*ln(-a*x+1)/a/c
```

Rubi [A] time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$-\frac{2}{ac(1-ax)}-\frac{3\log(1-ax)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] -(x/c) - 2/(a*c*(1 - a*x)) - (3*Log[1 - a*x])/(a*c)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{a\int \frac{e^{2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= -\frac{a\int \frac{x(1+ax)}{(1-ax)^2}\,dx}{c}$$
$$= -\frac{a\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{c}$$
$$= -\frac{x}{c}-\frac{2}{ac(1-ax)}-\frac{3\log(1-ax)}{ac}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.82

$$-\frac{ax+\frac{2}{1-ax}+3\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x)),x]
[Out] -((a*x + 2/(1 - a*x) + 3*Log[1 - a*x])/(a*c))
```

fricas [A] time = 0.46, size = 41, normalized size = 1.08

$$-\frac{a^2x^2-ax+3\,(ax-1)\log\,(ax-1)-2}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x),x, algorithm="fricas")
[Out] -(a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

giac [A] time = 0.36, size = 37, normalized size = 0.97

$$-\frac{x}{c}-\frac{3\,\log\,(|ax-1|)}{ac}+\frac{2}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x),x, algorithm="giac")
[Out] -x/c - 3*log(abs(a*x - 1))/(a*c) + 2/((a*x - 1)*a*c)
```

maple [A] time = 0.03, size = 37, normalized size = 0.97

$$-\frac{x}{c}+\frac{2}{ca\left(ax-1\right)}-\frac{3\ln\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x),x)
[Out] -x/c+2/c/a/(a*x-1)-3/a/c*ln(a*x-1)
```

maxima [A] time = 0.35, size = 36, normalized size = 0.95

$$-\frac{x}{c}+\frac{2}{a^2cx-ac}-\frac{3\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x),x, algorithm="maxima")
[Out] -x/c + 2/(a^2*c*x - a*c) - 3*log(a*x - 1)/(a*c)
```

mupad [B] time = 0.06, size = 35, normalized size = 0.92

$$-\frac{x}{c}-\frac{2}{a\left(c-a\,c\,x\right)}-\frac{3\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a*x))*(a^2*x^2 - 1)),x)
[Out] - x/c - 2/(a*(c - a*c*x)) - (3*log(a*x - 1))/(a*c)
```

sympy [A] time = 0.17, size = 26, normalized size = 0.68

$$\frac{2}{a^2cx-ac}-\frac{x}{c}-\frac{3\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x),x)
[Out] 2/(a**2*c*x - a*c) - x/c - 3*log(a*x - 1)/(a*c)
```

3.463 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=53

$$-\frac{5}{ac^2(1-ax)}+\frac{1}{ac^2(1-ax)^2}-\frac{4\log(1-ax)}{ac^2}-\frac{x}{c^2}$$

```
[Out] -x/c^2+1/a/c^2/(-a*x+1)^2-5/a/c^2/(-a*x+1)-4*ln(-a*x+1)/a/c^2
```

Rubi [A] time = 0.11, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$-\frac{5}{ac^2(1-ax)}+\frac{1}{ac^2(1-ax)^2}-\frac{4\log(1-ax)}{ac^2}-\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] -(x/c^2) + 1/(a*c^2*(1 - a*x)^2) - 5/(a*c^2*(1 - a*x)) - (4*Log[1 - a*x])/(
a*c^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
```

```
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= \frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c^2}$$
$$= \frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c^2}$$
$$= -\frac{x}{c^2}+\frac{1}{ac^2(1-ax)^2}-\frac{5}{ac^2(1-ax)}-\frac{4\log(1-ax)}{ac^2}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.96

$$\frac{5}{ac^2(ax-1)}+\frac{1}{ac^2(ax-1)^2}-\frac{4\log(1-ax)}{ac^2}-\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^2,x]
[Out] -(x/c^2) + 1/(a*c^2*(-1 + a*x)^2) + 5/(a*c^2*(-1 + a*x)) - (4*Log[1 - a*x])
/(a*c^2)
```

fricas [A] time = 0.42, size = 71, normalized size = 1.34

$$-\frac{a^3x^3-2\,a^2x^2-4\,ax+4\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+4}{a^3c^2x^2-2\,a^2c^2x+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="fricas")
[Out] -(a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(
a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.16, size = 42, normalized size = 0.79

$$-\frac{x}{c^2}-\frac{4\,\log\left(|ax-1|\right)}{ac^2}+\frac{5\,ax-4}{(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -x/c^2 - 4*log(abs(a*x - 1))/(a*c^2) + (5*a*x - 4)/((a*x - 1)^2*a*c^2)
```

maple [A] time = 0.03, size = 51, normalized size = 0.96

$$-\frac{x}{c^2}+\frac{5}{a\,c^2\,(ax-1)}-\frac{4\ln\,(ax-1)}{a\,c^2}+\frac{1}{a\,c^2\,(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^2,x)
```

```
[Out] -x/c^2+5/a/c^2/(a*x-1)-4/a/c^2*ln(a*x-1)+1/a/c^2/(a*x-1)^2
```

maxima [A] time = 0.31, size = 55, normalized size = 1.04

$$\frac{5\,ax-4}{a^3c^2x^2-2\,a^2c^2x+ac^2}-\frac{x}{c^2}-\frac{4\,\log\,(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] (5*a*x - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - x/c^2 - 4*log(a*x - 1)/(a
*c^2)
```

mupad [B] time = 0.07, size = 54, normalized size = 1.02

$$\frac{5\,x-\frac{4}{a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}-\frac{x}{c^2}-\frac{4\,\ln\,(a\,x-1)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a*x))^2*(a^2*x^2 - 1)),x)
```

```
[Out] (5*x - 4/a)/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) - x/c^2 - (4*log(a*x - 1))/(a*c
^2)
```

sympy [A] time = 0.25, size = 51, normalized size = 0.96

$$-\frac{-5ax+4}{a^3c^2x^2-2a^2c^2x+ac^2}-\frac{x}{c^2}-\frac{4\log\,(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**2,x)
```

```
[Out] -(-5*a*x + 4)/(a**3*c**2*x**2 - 2*a**2*c**2*x + a*c**2) - x/c**2 - 4*log(a*
x - 1)/(a*c**2)
```

3.464 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=74

$$-\frac{9}{ac^3(1-ax)}+\frac{7}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}-\frac{5\log(1-ax)}{ac^3}-\frac{x}{c^3}$$

```
[Out] -x/c^3-2/3/a/c^3/(-a*x+1)^3+7/2/a/c^3/(-a*x+1)^2-9/a/c^3/(-a*x+1)-5*ln(-a*x
+1)/a/c^3
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$-\frac{9}{ac^3(1-ax)}+\frac{7}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}-\frac{5\log(1-ax)}{ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] -(x/c^3) - 2/(3*a*c^3*(1 - a*x)^3) + 7/(2*a*c^3*(1 - a*x)^2) - 9/(a*c^3*(1
- a*x)) - (5*Log[1 - a*x])/(a*c^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
```

```
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= -\frac{a^3\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}\\ &= -\frac{a^3\int \frac{x^3(1+ax)}{(1-ax)^4}\,dx}{c^3}\\ &= -\frac{a^3\int \left(\frac{1}{a^3}+\frac{2}{a^3(-1+ax)^4}+\frac{7}{a^3(-1+ax)^3}+\frac{9}{a^3(-1+ax)^2}+\frac{5}{a^3(-1+ax)}\right)dx}{c^3}\\ &= -\frac{x}{c^3}-\frac{2}{3ac^3(1-ax)^3}+\frac{7}{2ac^3(1-ax)^2}-\frac{9}{ac^3(1-ax)}-\frac{5\log(1-ax)}{ac^3}\end{aligned}$$

Mathematica [A] time = 0.13, size = 63, normalized size = 0.85

$$\frac{-6a^4x^4+18a^3x^3+36a^2x^2-81ax-30(ax-1)^3\log(1-ax)+37}{6ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (37 - 81*a*x + 36*a^2*x^2 + 18*a^3*x^3 - 6*a^4*x^4 - 30*(-1 + a*x)^3*Log[1
 - a*x])/(6*a*c^3*(-1 + a*x)^3)
```

fricas [A] time = 0.49, size = 100, normalized size = 1.35

$$-\frac{6\,a^4x^4-18\,a^3x^3-36\,a^2x^2+81\,ax+30\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-37}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^4*x^4 - 18*a^3*x^3 - 36*a^2*x^2 + 81*a*x + 30*(a^3*x^3 - 3*a^2*x^
2 + 3*a*x - 1)*log(a*x - 1) - 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3)
```

giac [A] time = 0.18, size = 51, normalized size = 0.69

$$-\frac{x}{c^3} - \frac{5\log\left(|ax-1|\right)}{ac^3} + \frac{54\,a^2x^2 - 87\,ax + 37}{6\,(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -x/c^3 - 5*log(abs(a*x - 1))/(a*c^3) + 1/6*(54*a^2*x^2 - 87*a*x + 37)/((a*x
 - 1)^3*a*c^3)
```

maple [A] time = 0.04, size = 67, normalized size = 0.91

$$-\frac{x}{c^3} + \frac{9}{a\,c^3\,(ax-1)} - \frac{5\ln(ax-1)}{c^3a} + \frac{7}{2a\,c^3\,(ax-1)^2} + \frac{2}{3a\,c^3\,(ax-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^3,x)
```

```
[Out] -x/c^3+9/a/c^3/(a*x-1)-5/c^3/a*ln(a*x-1)+7/2/a/c^3/(a*x-1)^2+2/3/a/c^3/(a*x
-1)^3
```

maxima [A] time = 0.36, size = 76, normalized size = 1.03

$$\frac{54\,a^2x^2 - 87\,ax + 37}{6\left(a^4c^3x^3 - 3\,a^3c^3x^2 + 3\,a^2c^3x - ac^3\right)} - \frac{x}{c^3} - \frac{5\log\left(ax-1\right)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(54*a^2*x^2 - 87*a*x + 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x -
 a*c^3) - x/c^3 - 5*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.84, size = 73, normalized size = 0.99

$$-\frac{9\,a\,x^2 - \frac{29\,x}{2} + \frac{37}{6\,a}}{-a^3\,c^3\,x^3 + 3\,a^2\,c^3\,x^2 - 3\,a\,c^3\,x + c^3} - \frac{x}{c^3} - \frac{5\,\ln\left(a\,x-1\right)}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a*x))^3*(a^2*x^2 - 1)),x)
```

```
[Out] - (9*a*x^2 - (29*x)/2 + 37/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*
c^3*x) - x/c^3 - (5*log(a*x - 1))/(a*c^3)
```

sympy [A] time = 0.35, size = 75, normalized size = 1.01

$$-\frac{-54a^2x^2+87ax-37}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}-\frac{x}{c^3}-\frac{5\log(ax-1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**3,x)
```

```
[Out] -(-54*a**2*x**2 + 87*a*x - 37)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a
**2*c**3*x - 6*a*c**3) - x/c**3 - 5*log(a*x - 1)/(a*c**3)
```

3.465 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=88

$$-\frac{14}{ac^4(1-ax)}+\frac{8}{ac^4(1-ax)^2}-\frac{3}{ac^4(1-ax)^3}+\frac{1}{2ac^4(1-ax)^4}-\frac{6\log(1-ax)}{ac^4}-\frac{x}{c^4}$$

```
[Out] -x/c^4+1/2/a/c^4/(-a*x+1)^4-3/a/c^4/(-a*x+1)^3+8/a/c^4/(-a*x+1)^2-14/a/c^4/
(-a*x+1)-6*ln(-a*x+1)/a/c^4
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$-\frac{14}{ac^4(1-ax)}+\frac{8}{ac^4(1-ax)^2}-\frac{3}{ac^4(1-ax)^3}+\frac{1}{2ac^4(1-ax)^4}-\frac{6\log(1-ax)}{ac^4}-\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] -(x/c^4) + 1/(2*a*c^4*(1 - a*x)^4) - 3/(a*c^4*(1 - a*x)^3) + 8/(a*c^4*(1 -
a*x)^2) - 14/(a*c^4*(1 - a*x)) - (6*Log[1 - a*x])/(a*c^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
```

```
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$
$$= \frac{a^4\int \frac{x^4(1+ax)}{(1-ax)^5}\,dx}{c^4}$$
$$= \frac{a^4\int \left(-\frac{1}{a^4}-\frac{2}{a^4(-1+ax)^5}-\frac{9}{a^4(-1+ax)^4}-\frac{16}{a^4(-1+ax)^3}-\frac{14}{a^4(-1+ax)^2}-\frac{6}{a^4(-1+ax)}\right)dx}{c^4}$$
$$= -\frac{x}{c^4}+\frac{1}{2ac^4(1-ax)^4}-\frac{3}{ac^4(1-ax)^3}+\frac{8}{ac^4(1-ax)^2}-\frac{14}{ac^4(1-ax)}-\frac{6\log(1-ax)}{ac^4}$$

Mathematica [A] time = 0.15, size = 71, normalized size = 0.81

$$\frac{-2a^5x^5+8a^4x^4+16a^3x^3-60a^2x^2+56ax-12(ax-1)^4\log(1-ax)-17}{2ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-17 + 56*a*x - 60*a^2*x^2 + 16*a^3*x^3 + 8*a^4*x^4 - 2*a^5*x^5 - 12*(-1 +
a*x)^4*Log[1 - a*x])/(2*a*c^4*(-1 + a*x)^4)
```

fricas [A] time = 0.42, size = 126, normalized size = 1.43

$$-\frac{2\,a^5x^5-8\,a^4x^4-16\,a^3x^3+60\,a^2x^2-56\,ax+12\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax-1\right)+17}{2\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 60*a^2*x^2 - 56*a*x + 12*(a^4*x^
4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 17)/(a^5*c^4*x^4 - 4*
a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.17, size = 59, normalized size = 0.67

$$-\frac{x}{c^4}-\frac{6\,\log\left(|ax-1|\right)}{ac^4}+\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\,(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -x/c^4 - 6*log(abs(a*x - 1))/(a*c^4) + 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*
x - 17)/((a*x - 1)^4*a*c^4)
```

maple [A] time = 0.04, size = 82, normalized size = 0.93

$$-\frac{x}{c^4}+\frac{14}{a\,c^4\,(ax-1)}-\frac{6\ln\left(ax-1\right)}{a\,c^4}+\frac{8}{a\,c^4\,(ax-1)^2}+\frac{1}{2a\,c^4\,(ax-1)^4}+\frac{3}{a\,c^4\,(ax-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^4,x)
```

```
[Out] -x/c^4+14/a/c^4/(a*x-1)-6/a/c^4*ln(a*x-1)+8/a/c^4/(a*x-1)^2+1/2/a/c^4/(a*x-
1)^4+3/a/c^4/(a*x-1)^3
```

maxima [A] time = 0.36, size = 94, normalized size = 1.07

$$\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}-\frac{x}{c^4}-\frac{6\,\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x - 17)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 +
6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4) - x/c^4 - 6*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.85, size = 90, normalized size = 1.02

$$\frac{29\,x-34\,a\,x^2-\frac{17}{2\,a}+14\,a^2\,x^3}{a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+6\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}-\frac{x}{c^4}-\frac{6\,\ln\left(a\,x-1\right)}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a*x))^4*(a^2*x^2 - 1)),x)
```

```
[Out] (29*x - 34*a*x^2 - 17/(2*a) + 14*a^2*x^3)/(c^4 + 6*a^2*c^4*x^2 - 4*a^3*c^4*
x^3 + a^4*c^4*x^4 - 4*a*c^4*x) - x/c^4 - (6*log(a*x - 1))/(a*c^4)
```

sympy [A] time = 0.45, size = 95, normalized size = 1.08

$$-\frac{-28a^3x^3 + 68a^2x^2 - 58ax + 17}{2a^5c^4x^4 - 8a^4c^4x^3 + 12a^3c^4x^2 - 8a^2c^4x + 2ac^4} - \frac{x}{c^4} - \frac{6\log(ax-1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**4,x)
```

```
[Out] -(-28*a**3*x**3 + 68*a**2*x**2 - 58*a*x + 17)/(2*a**5*c**4*x**4 - 8*a**4*c*
*4*x**3 + 12*a**3*c**4*x**2 - 8*a**2*c**4*x + 2*a*c**4) - x/c**4 - 6*log(a*
x - 1)/(a*c**4)
```

3.466 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=103

$$\frac{c^4(3ax+2)\sqrt{1-a^2x^2}}{2a^2x}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^4\sin^{-1}(ax)}{a}$$

```
[Out] -1/6*c^4*(-3*a*x+2)*(-a^2*x^2+1)^(3/2)/a^4/x^3+c^4*arcsin(a*x)/a-3/2*c^4*ar
ctanh((-a^2*x^2+1)^(1/2))/a+1/2*c^4*(3*a*x+2)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^4(3ax+2)\sqrt{1-a^2x^2}}{2a^2x}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^4\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*(2 + 3*a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^4*(2 - 3*a*x)*(1 - a^2*x
^2)^(3/2))/(6*a^4*x^3) + (c^4*ArcSin[a*x])/a - (3*c^4*ArcTanh[Sqrt[1 - a^2*
x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx &= \frac{c^4\int \frac{e^{3\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}\\
&= \frac{c^4\int \frac{(1-ax)\left(1-a^2x^2\right)^{3/2}}{x^4}\,dx}{a^4}\\
&= -\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}-\frac{c^4\int \frac{\left(4a^2-6a^3x\right)\sqrt{1-a^2x^2}}{x^2}\,dx}{4a^4}\\
&= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^4\int \frac{12a^3+8a^4x}{x\sqrt{1-a^2x^2}}\,dx}{8a^4}\\
&= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+c^4\int \frac{1}{\sqrt{1-a^2x^2}}\,dx+\frac{\left(3c^4\right)\int
\end{aligned}$$

$$\begin{aligned}
&= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^4\sin^{-1}(ax)}{a}+\frac{\left(3c^4\right)\text{Subst}\left(\int\right.}{}\\
&= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^4\sin^{-1}(ax)}{a}-\frac{\left(3c^4\right)\text{Subst}\left(\int\right.}{}\\
&= \frac{c^4(2+3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^4(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^4\sin^{-1}(ax)}{a}-\frac{3c^4\tanh^{-1}\left(\sqrt{1\right.}}{2a}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 85, normalized size = 0.83

$$\frac{c^4\left(-9\log\left(\sqrt{1-a^2x^2}+1\right)+\frac{\sqrt{1-a^2x^2}\left(6a^3x^3+8a^2x^2+3ax-2\right)}{a^3x^3}+9\log(ax)+6\sin^{-1}(ax)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^4,x]
```

[Out] (c^4*((Sqrt[1 - a^2*x^2]*(-2 + 3*a*x + 8*a^2*x^2 + 6*a^3*x^3))/(a^3*x^3) + 6*ArcSin[a*x] + 9*Log[a*x] - 9*Log[1 + Sqrt[1 - a^2*x^2]]))/(6*a)

fricas [A] time = 0.65, size = 132, normalized size = 1.28

$$-\frac{12\,a^{3}c^{4}x^{3}\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-9\,a^{3}c^{4}x^{3}\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)-6\,a^{3}c^{4}x^{3}-\left(6\,a^{3}c^{4}x^{3}+8\,a^{2}c^{4}x^{2}+3\,ac^{4}x-2\,c^{4}\right)\sqrt{\cdots}}{6\,a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^4,x, algorithm="fricas")

[Out] -1/6*(12*a^3*c^4*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 9*a^3*c^4*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^4*x^3 - (6*a^3*c^4*x^3 + 8*a^2*c^4*x^2 + 3*a*c^4*x - 2*c^4)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)

giac [B] time = 0.22, size = 262, normalized size = 2.54

$$\frac{\left(c^{4}-\frac{3\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)c^{4}}{a^{2}x}-\frac{15\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{2}c^{4}}{a^{4}x^{2}}\right)a^{6}x^{3}}{24\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{3}|a|}+\frac{c^{4}\arcsin\left(ax\right)\mathrm{sgn}(a)}{|a|}-\frac{3\,c^{4}\log\left(\frac{\left|-2\,\sqrt{-a^{2}x^{2}+1}\,|a|-2\,a\right|}{2\,a^{2}|x|}\right)}{2\,|a|}+\frac{\sqrt{-a^{2}x^{2}\cdots}}{a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^4,x, algorithm="giac")

[Out] 1/24*(c^4 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*abs(a)) + c^4*arcsin(a*x)*sgn(a)/abs(a) - 3/2*c^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^4/a + 1/24*(15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/x + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^2*x^2) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^4/(a^4*x^3))/(a^2*abs(a))

maple [A] time = 0.05, size = 180, normalized size = 1.75

$$-\frac{c^{4}a\,x^{2}}{\sqrt{-a^{2}x^{2}+1}}+\frac{c^{4}}{2a\sqrt{-a^{2}x^{2}+1}}-\frac{4c^{4}x}{3\sqrt{-a^{2}x^{2}+1}}+\frac{c^{4}\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{\sqrt{a^{2}}}-\frac{3c^{4}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)}{2a}+\frac{5c^{4}}{3a^{2}x\sqrt{-a^{2}x^{2}+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^4,x)

```
[Out] -c^4*a*x^2/(-a^2*x^2+1)^(1/2)+1/2*c^4/a/(-a^2*x^2+1)^(1/2)-4/3*c^4*x/(-a^2*
x^2+1)^(1/2)+c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-3/2*c
^4/a*arctanh(1/(-a^2*x^2+1)^(1/2))+5/3*c^4/a^2/x/(-a^2*x^2+1)^(1/2)-1/3*c^4
/a^4/x^3/(-a^2*x^2+1)^(1/2)+1/2*c^4/a^3/x^2/(-a^2*x^2+1)^(1/2)
```

maxima [B] time = 0.50, size = 349, normalized size = 3.39

$$-a^3c^4\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)-a^2c^4\left(\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin(ax)}{a^3}\right)+\frac{3\,c^4x}{\sqrt{-a^2x^2+1}}+\frac{3\,c^4\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -a^3*c^4*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) - a^2*
c^4*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3) + 3*c^4*x/sqrt(-a^2*x^2
+ 1) + 3*c^4*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/ab
s(x)))/a - 3*(2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*c^4/a^
2 - 3*c^4/(sqrt(-a^2*x^2 + 1)*a) + 1/2*(3*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(
x) + 2/abs(x)) - 3*a^2/sqrt(-a^2*x^2 + 1) + 1/(sqrt(-a^2*x^2 + 1)*x^2))*c^4
/a^3 + 1/3*(8*a^4*x/sqrt(-a^2*x^2 + 1) - 4*a^2/(sqrt(-a^2*x^2 + 1)*x) - 1/(
sqrt(-a^2*x^2 + 1)*x^3))*c^4/a^4
```

mupad [B] time = 0.05, size = 135, normalized size = 1.31

$$\frac{c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{c^4\,\sqrt{1-a^2\,x^2}}{a}+\frac{4\,c^4\,\sqrt{1-a^2\,x^2}}{3\,a^2\,x}+\frac{c^4\,\sqrt{1-a^2\,x^2}}{2\,a^3\,x^2}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{3\,a^4\,x^3}+\frac{c^4\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c^4*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) + (c^4*atan((1 - a^2*x^2)^(1/2)*1i
)*3i)/(2*a) + (c^4*(1 - a^2*x^2)^(1/2))/a + (4*c^4*(1 - a^2*x^2)^(1/2))/(3*
a^2*x) + (c^4*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2) - (c^4*(1 - a^2*x^2)^(1/2))/
(3*a^4*x^3)
```

sympy [A] time = 18.87, size = 354, normalized size = 3.44

$$-ac^4\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+c^4\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)+\frac{2c^4\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**4,x)
```

```
[Out] -a*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) + c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a
**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 2*c**4*Piecewise((-acosh(1/(a*x)),
1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - 2*c**4*Piecewise((-I*sq
rt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/
a**2 - c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/
(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1
/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + c**4*P
iecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x*
*3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*
x**2 + 1)/(3*x**3), True))/a**4
```

3.467 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=77

$$\frac{3c^3\sqrt{1-a^2x^2}}{2a}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}$$

```
[Out] 1/2*c^3*(-a^2*x^2+1)^(3/2)/a^3/x^2-3/2*c^3*arctanh((-a^2*x^2+1)^(1/2))/a+3/
2*c^3*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6131, 6128, 266, 47, 50, 63, 208}

$$\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}+\frac{3c^3\sqrt{1-a^2x^2}}{2a}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] (3*c^3*Sqrt[1 - a^2*x^2])/(2*a) + (c^3*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (
3*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3\int\frac{e^{3\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}$$
$$=-\frac{c^3\int\frac{\left(1-a^2x^2\right)^{3/2}}{x^3}\,dx}{a^3}$$
$$=-\frac{c^3\,\text{Subst}\left(\int\frac{\left(1-a^2x\right)^{3/2}}{x^2}\,dx,x,x^2\right)}{2a^3}$$
$$=\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}+\frac{\left(3c^3\right)\text{Subst}\left(\int\frac{\sqrt{1-a^2x}}{x}\,dx,x,x^2\right)}{4a}$$
$$=\frac{3c^3\sqrt{1-a^2x^2}}{2a}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}+\frac{\left(3c^3\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{4a}$$
$$=\frac{3c^3\sqrt{1-a^2x^2}}{2a}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{\left(3c^3\right)\text{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{2a^3}$$
$$=\frac{3c^3\sqrt{1-a^2x^2}}{2a}+\frac{c^3\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}$$

Mathematica [A] time = 0.14, size = 77, normalized size = 1.00

$$\frac{\sqrt{1-a^2x^2}\left(\frac{c^3}{2a^2x^2}+c^3\right)}{a}-\frac{3c^3\log\left(\sqrt{1-a^2x^2}+1\right)}{2a}+\frac{3c^3\log(ax)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] ((c^3 + c^3/(2*a^2*x^2))*Sqrt[1 - a^2*x^2])/a + (3*c^3*Log[a*x])/(2*a) - (3
*c^3*Log[1 + Sqrt[1 - a^2*x^2]])/(2*a)
```

fricas [A] time = 0.49, size = 78, normalized size = 1.01

$$\frac{3\,a^2c^3x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+2\,a^2c^3x^2+\left(2\,a^2c^3x^2+c^3\right)\sqrt{-a^2x^2+1}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*c^3*x^2*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 2*a^2*c^3*x^2 + (2*a^2
*c^3*x^2 + c^3)*sqrt(-a^2*x^2 + 1))/(a^3*x^2)
```

giac [A] time = 0.36, size = 97, normalized size = 1.26

$$-\frac{3\,a^4c^3\log\left(\sqrt{-a^2x^2+1}+1\right)-3\,a^4c^3\log\left(-\sqrt{-a^2x^2+1}+1\right)-4\,\sqrt{-a^2x^2+1}\,a^4c^3-\frac{2\,\sqrt{-a^2x^2+1}\,a^2c^3}{x^2}}{4\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*a^4*c^3*log(sqrt(-a^2*x^2 + 1) + 1) - 3*a^4*c^3*log(-sqrt(-a^2*x^2
+ 1) + 1) - 4*sqrt(-a^2*x^2 + 1)*a^4*c^3 - 2*sqrt(-a^2*x^2 + 1)*a^2*c^3/x^2
)/a^5
```

maple [A] time = 0.04, size = 118, normalized size = 1.53

$$\frac{c^3\left(a^6\left(-\frac{x^2}{a^2\sqrt{-a^2x^2+1}}+\frac{2}{a^4\sqrt{-a^2x^2+1}}\right)-\frac{3a^2}{\sqrt{-a^2x^2+1}}+\frac{3a^2\left(\frac{1}{\sqrt{-a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{2}+\frac{1}{2x^2\sqrt{-a^2x^2+1}}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^3,x)
```

```
[Out] c^3/a^3*(a^6*(-1/a^2*x^2/(-a^2*x^2+1)^(1/2)+2/a^4/(-a^2*x^2+1)^(1/2))-3*a^2
/(-a^2*x^2+1)^(1/2)+3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1
/2)))+1/2/x^2/(-a^2*x^2+1)^(1/2))
```

maxima [B] time = 0.41, size = 188, normalized size = 2.44

$$-a^3c^3\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)+\frac{3\,c^3\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)\right)}{a}-\frac{3\,c^3}{\sqrt{-a^2x^2+1}\,a}+\frac{\left(3\,a^2\log\left(\frac{2\sqrt{}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -a^3*c^3*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*c^
3*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)))/a -
3*c^3/(sqrt(-a^2*x^2 + 1)*a) + 1/2*(3*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) +
 2/abs(x)) - 3*a^2/sqrt(-a^2*x^2 + 1) + 1/(sqrt(-a^2*x^2 + 1)*x^2))*c^3/a^3
```

mupad [B] time = 0.06, size = 64, normalized size = 0.83

$$\frac{c^3\sqrt{1-a^2x^2}}{a}-\frac{3c^3\,\mathrm{atanh}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c^3*(1 - a^2*x^2)^(1/2))/a - (3*c^3*atanh((1 - a^2*x^2)^(1/2)))/(2*a) + (c
^3*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2)
```

sympy [A] time = 35.55, size = 104, normalized size = 1.35

$$\frac{2c^3\sqrt{-a^2x^2+1}+\frac{3c^3\log\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{3c^3\log\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{c^3}{2\left(1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}+\frac{c^3}{2\left(-1+\frac{1}{\sqrt{-a^2x^2+1}}\right)}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**3,x)
```

```
[Out] (2*c**3*sqrt(-a**2*x**2 + 1) + 3*c**3*log(-1 + 1/sqrt(-a**2*x**2 + 1))/2 -
3*c**3*log(1 + 1/sqrt(-a**2*x**2 + 1))/2 + c**3/(2*(1 + 1/sqrt(-a**2*x**2 +
 1))) + c**3/(2*(-1 + 1/sqrt(-a**2*x**2 + 1))))/(2*a)
```

3.468 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=67

$$-\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c^2\sin^{-1}(ax)}{a}$$

```
[Out] -c^2*arcsin(a*x)/a-c^2*arctanh((-a^2*x^2+1)^(1/2))/a-c^2*(-a*x+1)*(-a^2*x^2
+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.16, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 850, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*(1 - a*x)*Sqrt[1 - a^2*x^2])/(a^2*x)) - (c^2*ArcSin[a*x])/a - (c^2*A
rcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\int \frac{e^{3\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}$$
$$= \frac{c^2\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^2(1-ax)}\,dx}{a^2}$$
$$= \frac{c^2\int \frac{(1+ax)\sqrt{1-a^2x^2}}{x^2}\,dx}{a^2}$$
$$= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x} - \frac{c^2\int \frac{-2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}$$
$$= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x} - c^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx + \frac{c^2\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}$$
$$= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x} - \frac{c^2\sin^{-1}(ax)}{a} + \frac{c^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x} - \frac{c^2\sin^{-1}(ax)}{a} - \frac{c^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^3}$$
$$= -\frac{c^2(1-ax)\sqrt{1-a^2x^2}}{a^2x} - \frac{c^2\sin^{-1}(ax)}{a} - \frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.11, size = 82, normalized size = 1.22

$$\frac{\sqrt{1-a^2x^2}\left(c^2-\frac{c^2}{ax}\right)}{a} - \frac{c^2\log\left(\sqrt{1-a^2x^2}+1\right)}{a} + \frac{c^2\log(ax)}{a} - \frac{c^2\sin^{-1}(ax)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] ((c^2 - c^2/(a*x))*Sqrt[1 - a^2*x^2])/a - (c^2*ArcSin[a*x])/a + (c^2*Log[a*
x])/a - (c^2*Log[1 + Sqrt[1 - a^2*x^2]])/a
```

fricas [A] time = 0.58, size = 93, normalized size = 1.39

$$\frac{2\,ac^2x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + ac^2x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + ac^2x + \left(ac^2x-c^2\right)\sqrt{-a^2x^2+1}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (2*a*c^2*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*c^2*x*log((sqrt(-a^2*
x^2 + 1) - 1)/x) + a*c^2*x + (a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1))/(a^2*x)
```

giac [B] time = 0.25, size = 139, normalized size = 2.07

$$\frac{a^2c^2x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}-\frac{c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}\,c^2}{a}-\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right.}{2\,a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*c^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - c^2*arcsin(a*x)*sg
n(a)/abs(a) - c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(
x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^2/a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)*c^2/(a^2*x*abs(a))
```

maple [B] time = 0.05, size = 133, normalized size = 1.99

$$-\frac{c^2a\,x^2}{\sqrt{-a^2x^2+1}}+\frac{c^2}{a\sqrt{-a^2x^2+1}}+\frac{c^2x}{\sqrt{-a^2x^2+1}}-\frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{c^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}-\frac{c^2}{a^2x\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^2,x)
```

```
[Out] -c^2*a*x^2/(-a^2*x^2+1)^(1/2)+c^2/a/(-a^2*x^2+1)^(1/2)+c^2*x/(-a^2*x^2+1)^(
1/2)-c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-c^2/a*arctanh
(1/(-a^2*x^2+1)^(1/2))-c^2/a^2/x/(-a^2*x^2+1)^(1/2)
```

maxima [B] time = 0.43, size = 209, normalized size = 3.12

$$-a^3c^2\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)+a^2c^2\left(\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin\left(ax\right)}{a^3}\right)-\frac{2\,c^2x}{\sqrt{-a^2x^2+1}}+\frac{c^2\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a^3*c^2*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + a^2*
c^2*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3) - 2*c^2*x/sqrt(-a^2*x^2
+ 1) + c^2*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(
x)))/a + (2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*c^2/a^2 -
2*c^2/(sqrt(-a^2*x^2 + 1)*a)
```

mupad [B] time = 0.81, size = 90, normalized size = 1.34

$$\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\frac{c^2\,\mathrm{atan}\left(\sqrt{1-a^2x^2}\,1\mathrm{i}\right)1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c^2*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/a - (c^2*asinh(x*(-a^2)^(1/2)))/(-a^2
)^(1/2) + (c^2*(1 - a^2*x^2)^(1/2))/a - (c^2*(1 - a^2*x^2)^(1/2))/(a^2*x)
```

sympy [A] time = 13.95, size = 151, normalized size = 2.25

$$-ac^2\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\,\mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\,\mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+\frac{c^2\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**2,x)
```

```
[Out] -a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) - c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a
**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + c**2*Piecewise((-acosh(1/(a*x)), 1/
Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + c**2*Piecewise((-I*sqrt(a
**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2
```

3.469 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=50

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{2c\sin^{-1}(ax)}{a}$$

```
[Out] -2*c*arcsin(a*x)/a+c*arctanh((-a^2*x^2+1)^(1/2))/a+c*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.16, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6131, 6128, 852, 1809, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{2c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*Sqrt[1 - a^2*x^2])/a - (2*c*ArcSin[a*x])/a + (c*ArcTanh[Sqrt[1 - a^2*x^2
]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx=-\frac{c\int\frac{e^{3\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$=-\frac{c\int\frac{\left(1-a^2x^2\right)^{3/2}}{x(1-ax)^2}\,dx}{a}$$
$$=-\frac{c\int\frac{(1+ax)^2}{x\sqrt{1-a^2x^2}}\,dx}{a}$$
$$=\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\int\frac{-a^2-2a^3x}{x\sqrt{1-a^2x^2}}\,dx}{a^3}$$
$$=\frac{c\sqrt{1-a^2x^2}}{a}-(2c)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{c\int\frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}$$
$$=\frac{c\sqrt{1-a^2x^2}}{a}-\frac{2c\sin^{-1}(ax)}{a}-\frac{c\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}$$
$$=\frac{c\sqrt{1-a^2x^2}}{a}-\frac{2c\sin^{-1}(ax)}{a}+\frac{c\,\text{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}$$
$$=\frac{c\sqrt{1-a^2x^2}}{a}-\frac{2c\sin^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 0.94

$$\frac{c\left(\sqrt{1-a^2x^2}+\log\left(\sqrt{1-a^2x^2}+1\right)-2\sin^{-1}(ax)-\log(x)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*(Sqrt[1 - a^2*x^2] - 2*ArcSin[a*x] - Log[x] + Log[1 + Sqrt[1 - a^2*x^2]]
))/a
```

fricas [A] time = 0.43, size = 66, normalized size = 1.32

$$\frac{4\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-c\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\,c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (4*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - c*log((sqrt(-a^2*x^2 + 1) - 1
)/x) + sqrt(-a^2*x^2 + 1)*c)/a
```

giac [A] time = 2.10, size = 68, normalized size = 1.36

$$-\frac{2\,c\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}+\frac{c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}\,c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] -2*c*arcsin(a*x)*sgn(a)/abs(a) + c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a)
 - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c/a
```

maple [A] time = 0.04, size = 84, normalized size = 1.68

$$-\frac{ca\,x^2}{\sqrt{-a^2x^2+1}}+\frac{c}{a\sqrt{-a^2x^2+1}}-\frac{2c\arctan\left(\frac{\sqrt{a^2}\;x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{c\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x),x)
```

```
[Out] -c*a*x^2/(-a^2*x^2+1)^(1/2)+c/a/(-a^2*x^2+1)^(1/2)-2*c/(a^2)^(1/2)*arctan((
a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+c/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [B] time = 0.47, size = 140, normalized size = 2.80

$$-a^3c\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)+2\,a^2c\left(\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin\left(ax\right)}{a^3}\right)-\frac{2\,cx}{\sqrt{-a^2x^2+1}}-\frac{c\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(\cdots\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -a^3*c*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 2*a^2*
c*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3) - 2*c*x/sqrt(-a^2*x^2 + 1)
 - c*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)))/a
```

mupad [B] time = 0.05, size = 56, normalized size = 1.12

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\,\mathrm{atanh}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{2\,c\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c*(1 - a^2*x^2)^(1/2))/a + (c*atanh((1 - a^2*x^2)^(1/2)))/a - (2*c*asinh(x
*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [A] time = 10.25, size = 104, normalized size = 2.08

$$-ac\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-2c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\ \mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)-\frac{c\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x),x)
```

```
[Out] -a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) -
 2*c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)
*asinh(x*sqrt(-a**2)), a**2 < 0)) - c*Piecewise((-acosh(1/(a*x)), 1/Abs(a**
2*x**2) > 1), (I*asin(1/(a*x)), True))/a
```

3.470 $\int \frac{e^{3\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=99

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4}+\frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2}+\frac{4\sqrt{1-a^2x^2}}{ac}-\frac{4\sin^{-1}(ax)}{ac}$$

```
[Out] 8/3*(-a^2*x^2+1)^(3/2)/a/c/(-a*x+1)^2-1/3*(-a^2*x^2+1)^(5/2)/a/c/(-a*x+1)^4
-4*arcsin(a*x)/a/c+4*(-a^2*x^2+1)^(1/2)/a/c
```

Rubi [A] time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 793, 663, 665, 216}

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4}+\frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2}+\frac{4\sqrt{1-a^2x^2}}{ac}-\frac{4\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(a*c) + (8*(1 - a^2*x^2)^(3/2))/(3*a*c*(1 - a*x)^2) -
 (1 - a^2*x^2)^(5/2)/(3*a*c*(1 - a*x)^4) - (4*ArcSin[a*x])/(a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
```

```
|| EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{a\int \frac{e^{3\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$

$$= -\frac{a\int \frac{x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^4}\,dx}{c}$$

$$= -\frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4} + \frac{4\int \frac{\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}\,dx}{3c}$$

$$= \frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2} - \frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4} - \frac{4\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx}{c}$$

$$= \frac{4\sqrt{1-a^2x^2}}{ac} + \frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2} - \frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4} - \frac{4\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}$$

$$= \frac{4\sqrt{1-a^2x^2}}{ac} + \frac{8\left(1-a^2x^2\right)^{3/2}}{3ac(1-ax)^2} - \frac{\left(1-a^2x^2\right)^{5/2}}{3ac(1-ax)^4} - \frac{4\sin^{-1}(ax)}{ac}$$

Mathematica [C] time = 0.04, size = 62, normalized size = 0.63

$$-\frac{16\sqrt{2}\,(ax-1)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-ax)\right)+(ax+1)^{5/2}}{3ac(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] -1/3*((1 + a*x)^(5/2) + 16*Sqrt[2]*(-1 + a*x)*Hypergeometric2F1[-3/2, -1/2,
 1/2, (1 - a*x)/2])/(a*c*(1 - a*x)^(3/2))
```

fricas [A] time = 0.47, size = 101, normalized size = 1.02

$$\frac{19\,a^2x^2-38\,ax+24\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^2x^2-26\,ax+19\right)\sqrt{-a^2x^2+1}+19}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] 1/3*(19*a^2*x^2 - 38*a*x + 24*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)) + (3*a^2*x^2 - 26*a*x + 19)*sqrt(-a^2*x^2 + 1) + 19)/(a^3*c
*x^2 - 2*a^2*c*x + a*c)
```

giac [A] time = 0.20, size = 126, normalized size = 1.27

$$-\frac{4 \arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}+\frac{\sqrt{-a^2x^2+1}}{ac}-\frac{8\left(\frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}-4\right)}{3\,c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] -4*arcsin(a*x)*sgn(a)/(c*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c) - 8/3*(9*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^
4*x^2) - 4)/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a))
```

maple [A] time = 0.04, size = 168, normalized size = 1.70

$$-\frac{a\,x^2}{c\sqrt{-a^2x^2+1}}+\frac{9}{ca\sqrt{-a^2x^2+1}}+\frac{12x}{c\sqrt{-a^2x^2+1}}-\frac{4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c\sqrt{a^2}}+\frac{8}{3c\,a^2\left(x-\frac{1}{a}\right)\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x),x)
```

```
[Out] -a/c*x^2/(-a^2*x^2+1)^(1/2)+9/c/a/(-a^2*x^2+1)^(1/2)+12/c*x/(-a^2*x^2+1)^(1
/2)-4/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+8/3/c/a^2/(x-1
/a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-16/3/c/(-a^2*(x-1/a)^2-2*a*(x-1/a))^
(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))), x)
```

mupad [B] time = 0.06, size = 129, normalized size = 1.30

$$\frac{\sqrt{1-a^2\,x^2}}{a\,c}-\frac{20\,\sqrt{1-a^2\,x^2}}{3\,\left(\frac{c\,\sqrt{-a^2}}{a}-c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c\,\sqrt{-a^2}}-\frac{4\,a\,\sqrt{1-a^2\,x^2}}{3\,\left(c\,a^4\,x^2-2\,c\,a^3\,x+c\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a*x))*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(a*c) - (20*(1 - a^2*x^2)^(1/2))/(3*((c*(-a^2)^(1/2))/a
 - c*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (4*asinh(x*(-a^2)^(1/2)))/(c*(-a^2)^(1
/2)) - (4*a*(1 - a^2*x^2)^(1/2))/(3*(a^2*c + a^4*c*x^2 - 2*a^3*c*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int\frac{x}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{3ax^2}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(x/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2
+ 1) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*a*
x**2/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*
x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x**3/(
-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sqrt
(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**4/(-a**3*x*
*3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sqrt(-a**2*x
**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.471 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=128

$$\frac{(ax+1)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}}-\frac{2(ax+1)^4}{3ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{10(ax+1)^2}{3ac^2\sqrt{1-a^2x^2}}+\frac{5\sqrt{1-a^2x^2}}{ac^2}-\frac{5\sin^{-1}(ax)}{ac^2}$$

```
[Out] 1/5*(a*x+1)^5/a/c^2/(-a^2*x^2+1)^(5/2)-2/3*(a*x+1)^4/a/c^2/(-a^2*x^2+1)^(3/
2)-5*arcsin(a*x)/a/c^2+10/3*(a*x+1)^2/a/c^2/(-a^2*x^2+1)^(1/2)+5*(-a^2*x^2+
1)^(1/2)/a/c^2
```

Rubi [A] time = 0.26, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6131, 6128, 852, 1635, 21, 669, 641, 216}

$$\frac{(ax+1)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}}-\frac{2(ax+1)^4}{3ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{10(ax+1)^2}{3ac^2\sqrt{1-a^2x^2}}+\frac{5\sqrt{1-a^2x^2}}{ac^2}-\frac{5\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (1 + a*x)^5/(5*a*c^2*(1 - a^2*x^2)^(5/2)) - (2*(1 + a*x)^4)/(3*a*c^2*(1 - a
^2*x^2)^(3/2)) + (10*(1 + a*x)^2)/(3*a*c^2*Sqrt[1 - a^2*x^2]) + (5*Sqrt[1 -
 a^2*x^2])/(a*c^2) - (5*ArcSin[a*x])/(a*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{e^{3\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$

$$= \frac{a^2\int \frac{x^2\left(1-a^2x^2\right)^{3/2}}{(1-ax)^5}\,dx}{c^2}$$

$$= \frac{a^2\int \frac{x^2(1+ax)^5}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{a^2\int \frac{\left(\frac{5}{a^2}+\frac{5x}{a}\right)(1+ax)^4}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^2}$$

$$= \frac{(1+ax)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{(1+ax)^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{2(1+ax)^4}{3ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{5\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^2}$$

$$= \frac{(1+ax)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{2(1+ax)^4}{3ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{10(1+ax)^2}{3ac^2\sqrt{1-a^2x^2}} - \frac{5\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{2(1+ax)^4}{3ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{10(1+ax)^2}{3ac^2\sqrt{1-a^2x^2}} + \frac{5\sqrt{1-a^2x^2}}{ac^2} - \frac{5\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^5}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{2(1+ax)^4}{3ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{10(1+ax)^2}{3ac^2\sqrt{1-a^2x^2}} + \frac{5\sqrt{1-a^2x^2}}{ac^2} - \frac{5\sin^{-1}(ax)}{ac^2}$$

Mathematica [A] time = 0.13, size = 61, normalized size = 0.48

$$\frac{\frac{\sqrt{1-a^2x^2}\left(15a^3x^3-188a^2x^2+279ax-118\right)}{(ax-1)^3} - 75\sin^{-1}(ax)}{15ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-118 + 279*a*x - 188*a^2*x^2 + 15*a^3*x^3))/(-1 + a*x)
^3 - 75*ArcSin[a*x])/(15*a*c^2)
```

fricas [A] time = 0.49, size = 143, normalized size = 1.12

$$\frac{118\,a^3x^3 - 354\,a^2x^2 + 354\,ax + 150\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(15\,a^3x^3 - 188\,a^2x^2 + 279\right.}{15\left(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(118*a^3*x^3 - 354*a^2*x^2 + 354*a*x + 150*(a^3*x^3 - 3*a^2*x^2 + 3*a*
x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^3*x^3 - 188*a^2*x^2 +
 279*a*x - 118)*sqrt(-a^2*x^2 + 1) - 118)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*
a^2*c^2*x - a*c^2)
```

giac [A] time = 0.21, size = 180, normalized size = 1.41

$$-\frac{5\,\arcsin\left(ax\right)\mathrm{sgn}(a)}{c^2|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^2}-\frac{2\left(\frac{440\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{670\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{360\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{75\left(\sqrt{-a^2x^2}\right.}{a^8}\right.}{15\,c^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -5*arcsin(a*x)*sgn(a)/(c^2*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^2) - 2/15*(440
*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 670*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^2/(a^4*x^2) + 360*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 75*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 103)/(c^2*((sqrt(-a^2*x^2 + 1)*ab
s(a) + a)/(a^2*x) - 1)^5*abs(a))
```

maple [A] time = 0.05, size = 212, normalized size = 1.66

$$-\frac{a\,x^2}{c^2\sqrt{-a^2x^2+1}}+\frac{14}{a\,c^2\sqrt{-a^2x^2+1}}+\frac{25x}{c^2\sqrt{-a^2x^2+1}}-\frac{5\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^2\sqrt{a^2}}+\frac{8}{5a^3c^2\left(x-\frac{1}{a}\right)^2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x)
```

```
[Out] -a/c^2*x^2/(-a^2*x^2+1)^(1/2)+14/a/c^2/(-a^2*x^2+1)^(1/2)+25*x/c^2/(-a^2*x^
2+1)^(1/2)-5/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+8/5/a
^3/c^2/(x-1/a)^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+116/15/a^2/c^2/(x-1/a)/
(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-232/15/c^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^
(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^2), x)
```

mupad [B] time = 0.82, size = 270, normalized size = 2.11

$$\frac{8\,a^2\,\sqrt{1-a^2\,x^2}}{15\,\left(a^5\,c^2\,x^2-2\,a^4\,c^2\,x+a^3\,c^2\right)}-\frac{5\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^2\,\sqrt{-a^2}}-\frac{4\,a\,\sqrt{1-a^2\,x^2}}{a^4\,c^2\,x^2-2\,a^3\,c^2\,x+a^2\,c^2}+\frac{\sqrt{1-a^2\,x^2}}{a\,c^2}+\frac{143\,\sqrt{1-a^2}}{15\,\sqrt{-a^2}\,\left(c^2\,x\,\sqrt{-a^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a*x))^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (8*a^2*(1 - a^2*x^2)^(1/2))/(15*(a^3*c^2 - 2*a^4*c^2*x + a^5*c^2*x^2)) - (5
*asinh(x*(-a^2)^(1/2)))/(c^2*(-a^2)^(1/2)) - (4*a*(1 - a^2*x^2)^(1/2))/(a^2
*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2) + (1 - a^2*x^2)^(1/2)/(a*c^2) + (143*(1 -
 a^2*x^2)^(1/2))/(15*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^(1/2))/
a)) + (4*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(3*c^2*x*(-a^2)^(1/2) - (c^2*
(-a^2)^(1/2))/a + a^2*c^2*x^3*(-a^2)^(1/2) - 3*a*c^2*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int\frac{x^2}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int\frac{3ax^3}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*(Integral(x**2/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a*
*2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Int
egral(3*a*x**3/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x*
*2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral
(3*a**2*x**4/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2
 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a
**3*x**5/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2 + 1
) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.472 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{(ax+1)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}}+\frac{4(ax+1)^5}{7ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{(ax+1)^4}{ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{4(ax+1)^2}{ac^3\sqrt{1-a^2x^2}}+\frac{6\sqrt{1-a^2x^2}}{ac^3}-\frac{6\sin^{-1}(ax)}{ac^3}$$

```
[Out] -1/7*(a*x+1)^6/a/c^3/(-a^2*x^2+1)^(7/2)+4/7*(a*x+1)^5/a/c^3/(-a^2*x^2+1)^(5
/2)-(a*x+1)^4/a/c^3/(-a^2*x^2+1)^(3/2)-6*arcsin(a*x)/a/c^3+4*(a*x+1)^2/a/c^
3/(-a^2*x^2+1)^(1/2)+6*(-a^2*x^2+1)^(1/2)/a/c^3
```

Rubi [A] time = 0.33, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6131, 6128, 852, 1635, 789, 669, 641, 216}

$$-\frac{(ax+1)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}}+\frac{4(ax+1)^5}{7ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{(ax+1)^4}{ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{4(ax+1)^2}{ac^3\sqrt{1-a^2x^2}}+\frac{6\sqrt{1-a^2x^2}}{ac^3}-\frac{6\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] -(1 + a*x)^6/(7*a*c^3*(1 - a^2*x^2)^(7/2)) + (4*(1 + a*x)^5)/(7*a*c^3*(1 -
a^2*x^2)^(5/2)) - (1 + a*x)^4/(a*c^3*(1 - a^2*x^2)^(3/2)) + (4*(1 + a*x)^2)
/(a*c^3*Sqrt[1 - a^2*x^2]) + (6*Sqrt[1 - a^2*x^2])/(a*c^3) - (6*ArcSin[a*x]
)/(a*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
```

```
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{a^3\int \frac{e^{3\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$

$$= -\frac{a^3\int \frac{x^3\left(1-a^2x^2\right)^{3/2}}{(1-ax)^6}\,dx}{c^3}$$

$$= -\frac{a^3\int \frac{x^3(1+ax)^6}{\left(1-a^2x^2\right)^{9/2}}\,dx}{c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{a^3\int \frac{(1+ax)^5\left(\frac{6}{a^3}+\frac{7x}{a^2}+\frac{7x^2}{a}\right)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{4(1+ax)^5}{7ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{a^3\int \frac{\left(\frac{70}{a^3}+\frac{35x}{a^2}\right)(1+ax)^4}{\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{4(1+ax)^5}{7ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{(1+ax)^4}{ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{2\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{4(1+ax)^5}{7ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{(1+ax)^4}{ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{4(1+ax)^2}{ac^3\sqrt{1-a^2x^2}} - \frac{6\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,d}{c^3}$$

$$= -\frac{(1+ax)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{4(1+ax)^5}{7ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{(1+ax)^4}{ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{4(1+ax)^2}{ac^3\sqrt{1-a^2x^2}} + \frac{6\sqrt{1-a^2x^2}}{ac^3}$$

$$= -\frac{(1+ax)^6}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{4(1+ax)^5}{7ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{(1+ax)^4}{ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{4(1+ax)^2}{ac^3\sqrt{1-a^2x^2}} + \frac{6\sqrt{1-a^2x^2}}{ac^3}$$

Mathematica [A] time = 0.16, size = 69, normalized size = 0.45

$$\frac{\frac{\sqrt{1-a^2x^2}\left(7a^4x^4-116a^3x^3+261a^2x^2-222ax+66\right)}{(ax-1)^4}-42\sin^{-1}(ax)}{7ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(66 - 222*a*x + 261*a^2*x^2 - 116*a^3*x^3 + 7*a^4*x^4))
/(-1 + a*x)^4 - 42*ArcSin[a*x])/(7*a*c^3)
```

fricas [A] time = 0.51, size = 177, normalized size = 1.14

$$\frac{66\,a^4x^4-264\,a^3x^3+396\,a^2x^2-264\,ax+84\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(7\,a^4x^4\right.}{7\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/7*(66*a^4*x^4 - 264*a^3*x^3 + 396*a^2*x^2 - 264*a*x + 84*(a^4*x^4 - 4*a^3
*x^3 + 6*a^2*x^2 - 4*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (7*a
^4*x^4 - 116*a^3*x^3 + 261*a^2*x^2 - 222*a*x + 66)*sqrt(-a^2*x^2 + 1) + 66)
/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 256, normalized size = 1.65

$$-\frac{a\,x^2}{c^3\sqrt{-a^2x^2+1}}+\frac{20}{a\,c^3\sqrt{-a^2x^2+1}}+\frac{44x}{c^3\sqrt{-a^2x^2+1}}-\frac{6\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^3\sqrt{a^2}}+\frac{44}{7a^3c^3\left(x-\frac{1}{a}\right)^2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x)
```

```
[Out] -a/c^3*x^2/(-a^2*x^2+1)^(1/2)+20/a/c^3/(-a^2*x^2+1)^(1/2)+44*x/c^3/(-a^2*x^
2+1)^(1/2)-6/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+44/7/
a^3/c^3/(x-1/a)^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+110/7/a^2/c^3/(x-1/a)/
```

(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-220/7/c^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(
1/2)*x+8/7/a^4/c^3/(x-1/a)^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{(-a^2x^2+1)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")

[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^3), x)

mupad [B] time = 0.07, size = 340, normalized size = 2.19

$$\frac{\sqrt{1-a^2\,x^2}}{a\,c^3}-\frac{6\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^3\,\sqrt{-a^2}}-\frac{8\,a^3\,\sqrt{1-a^2\,x^2}}{35\,\left(a^6\,c^3\,x^2-2\,a^5\,c^3\,x+a^4\,c^3\right)}-\frac{31\,a\,\sqrt{1-a^2\,x^2}}{5\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{}{7\,\left(a^6\,c^3\,x^4-4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + 1)^3/((c - c/(a*x))^3*(1 - a^2*x^2)^(3/2)),x)

[Out] (1 - a^2*x^2)^(1/2)/(a*c^3) - (6*asinh(x*(-a^2)^(1/2)))/(c^3*(-a^2)^(1/2))
- (8*a^3*(1 - a^2*x^2)^(1/2))/(35*(a^4*c^3 - 2*a^5*c^3*x + a^6*c^3*x^2)) -
(31*a*(1 - a^2*x^2)^(1/2))/(5*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) - (4*a
*(1 - a^2*x^2)^(1/2))/(7*(a^2*c^3 - 4*a^3*c^3*x + 6*a^4*c^3*x^2 - 4*a^5*c^3
*x^3 + a^6*c^3*x^4)) + (88*(1 - a^2*x^2)^(1/2))/(7*(-a^2)^(1/2)*(c^3*x*(-a^
2)^(1/2) - (c^3*(-a^2)^(1/2))/a)) + (20*(1 - a^2*x^2)^(1/2))/(7*(-a^2)^(1/2
)*(3*c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) -
 3*a*c^3*x^2*(-a^2)^(1/2)))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{x^3}{-a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**3,x)

[Out] a**3*(Integral(x**3/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a*
*2*x**2 + 1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x*

```
*2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral
(3*a*x**4/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 +
1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) +
3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x*
*5/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*
a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*x*s
qrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**6/(-a**5
*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**3*x**3
*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*x*sqrt(-a**2
*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.473 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=190

$$\frac{(ax+1)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}}-\frac{34(ax+1)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}}+\frac{344(ax+1)^5}{315ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{4(ax+1)^4}{3ac^4\left(1-a^2x^2\right)^{3/2}}+\frac{14(ax+1)^2}{3ac^4\sqrt{1-a^2x^2}}+\frac{7\sqrt{1-a^2x^2}}{ac^4}-$$

```
[Out] 1/9*(a*x+1)^7/a/c^4/(-a^2*x^2+1)^(9/2)-34/63*(a*x+1)^6/a/c^4/(-a^2*x^2+1)^(
7/2)+344/315*(a*x+1)^5/a/c^4/(-a^2*x^2+1)^(5/2)-4/3*(a*x+1)^4/a/c^4/(-a^2*x
^2+1)^(3/2)-7*arcsin(a*x)/a/c^4+14/3*(a*x+1)^2/a/c^4/(-a^2*x^2+1)^(1/2)+7*(
-a^2*x^2+1)^(1/2)/a/c^4
```

Rubi [A] time = 0.44, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6131, 6128, 852, 1635, 789, 669, 641, 216}

$$\frac{(ax+1)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}}-\frac{34(ax+1)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}}+\frac{344(ax+1)^5}{315ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{4(ax+1)^4}{3ac^4\left(1-a^2x^2\right)^{3/2}}+\frac{14(ax+1)^2}{3ac^4\sqrt{1-a^2x^2}}+\frac{7\sqrt{1-a^2x^2}}{ac^4}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (1 + a*x)^7/(9*a*c^4*(1 - a^2*x^2)^(9/2)) - (34*(1 + a*x)^6)/(63*a*c^4*(1 -
 a^2*x^2)^(7/2)) + (344*(1 + a*x)^5)/(315*a*c^4*(1 - a^2*x^2)^(5/2)) - (4*(
1 + a*x)^4)/(3*a*c^4*(1 - a^2*x^2)^(3/2)) + (14*(1 + a*x)^2)/(3*a*c^4*Sqrt[
1 - a^2*x^2]) + (7*Sqrt[1 - a^2*x^2])/(a*c^4) - (7*ArcSin[a*x])/(a*c^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
```

```
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{e^{3\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4\left(1-a^2x^2\right)^{3/2}}{(1-ax)^7}\,dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4(1+ax)^7}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^4}$$

$$= \frac{(1+ax)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{a^4\int \frac{(1+ax)^6\left(\frac{7}{a^4}+\frac{9x}{a^3}+\frac{9x^2}{a^2}+\frac{9x^3}{a}\right)}{\left(1-a^2x^2\right)^{9/2}}\,dx}{9c^4}$$

$$= \frac{(1+ax)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{34(1+ax)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{a^4\int \frac{(1+ax)^5\left(\frac{155}{a^4}+\frac{126x}{a^3}+\frac{63x^2}{a^2}\right)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{63c^4}$$

$$= \frac{(1+ax)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{34(1+ax)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{344(1+ax)^5}{315ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{a^4\int \frac{\left(\frac{945}{a^4}+\frac{315x}{a^3}\right)(1+ax)^4}{\left(1-a^2x^2\right)^{5/2}}\,dx}{315c^4}$$

$$= \frac{(1+ax)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{34(1+ax)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{344(1+ax)^5}{315ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{4(1+ax)^4}{3ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{7\int \frac{(}{(1-}}{}$$

$$= \frac{(1+ax)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{34(1+ax)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{344(1+ax)^5}{315ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{4(1+ax)^4}{3ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{14(1}{3ac^4\sqrt{}}$$

$$= \frac{(1+ax)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{34(1+ax)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{344(1+ax)^5}{315ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{4(1+ax)^4}{3ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{14(1}{3ac^4\sqrt{}}$$

$$= \frac{(1+ax)^7}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{34(1+ax)^6}{63ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{344(1+ax)^5}{315ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{4(1+ax)^4}{3ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{14(1}{3ac^4\sqrt{}}$$

Mathematica [A] time = 0.20, size = 77, normalized size = 0.41

$$\frac{\frac{\sqrt{1-a^2x^2}\left(315a^5x^5-6539a^4x^4+19780a^3x^3-25347a^2x^2+15115ax-3464\right)}{(ax-1)^5}-2205\sin^{-1}(ax)}{315ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-3464 + 15115*a*x - 25347*a^2*x^2 + 19780*a^3*x^3 - 65
39*a^4*x^4 + 315*a^5*x^5))/(-1 + a*x)^5 - 2205*ArcSin[a*x])/(315*a*c^4)
```

fricas [A] time = 0.79, size = 213, normalized size = 1.12

$$\frac{3464\,a^5x^5-17320\,a^4x^4+34640\,a^3x^3-34640\,a^2x^2+17320\,ax+4410\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax\right.}{315\left(a^6c^4x^5-5\,a^5c^4x^4+10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(3464*a^5*x^5 - 17320*a^4*x^4 + 34640*a^3*x^3 - 34640*a^2*x^2 + 17320
*a*x + 4410*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*arc
tan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (315*a^5*x^5 - 6539*a^4*x^4 + 19780*a
^3*x^3 - 25347*a^2*x^2 + 15115*a*x - 3464)*sqrt(-a^2*x^2 + 1) - 3464)/(a^6*
c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a
*c^4)
```

giac [A] time = 0.67, size = 288, normalized size = 1.52

$$-\frac{7\arcsin(ax)\,\mathrm{sgn}(a)}{c^4|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^4}-\frac{2\left(\frac{26136\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{93834\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{188706\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -7*arcsin(a*x)*sgn(a)/(c^4*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^4) - 2/315*(26
136*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 93834*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^2/(a^4*x^2) + 188706*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) -
 229194*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 167580*(sqrt(-a^2*x^2
```

```
 + 1)*abs(a) + a)^5/(a^10*x^5) - 75810*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^6/(a
^12*x^6) + 19530*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7/(a^14*x^7) - 2205*(sqrt(
-a^2*x^2 + 1)*abs(a) + a)^8/(a^16*x^8) - 3149)/(c^4*((sqrt(-a^2*x^2 + 1)*ab
s(a) + a)/(a^2*x) - 1)^9*abs(a))
```

maple [A] time = 0.06, size = 300, normalized size = 1.58

$$-\frac{a\,x^2}{c^4\sqrt{-a^2x^2+1}}+\frac{27}{a\,c^4\sqrt{-a^2x^2+1}}+\frac{70x}{c^4\sqrt{-a^2x^2+1}}-\frac{7\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^4\sqrt{a^2}}+\frac{5002}{315a^3c^4\left(x-\frac{1}{a}\right)^2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x)
```

```
[Out] -a/c^4*x^2/(-a^2*x^2+1)^(1/2)+27/a/c^4/(-a^2*x^2+1)^(1/2)+70*x/c^4/(-a^2*x^
2+1)^(1/2)-7/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+5002/
315/a^3/c^4/(x-1/a)^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+8543/315/a^2/c^4/(
x-1/a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-17086/315/c^4/(-a^2*(x-1/a)^2-2*a
*(x-1/a))^(1/2)*x+356/63/a^4/c^4/(x-1/a)^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/
2)+8/9/a^5/c^4/(x-1/a)^4/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^4), x)
```

mupad [B] time = 0.87, size = 579, normalized size = 3.05

$$\frac{4\sqrt{1-a^2\,x^2}}{9\sqrt{-a^2}\,\left(5\,c^4\,x\,\sqrt{-a^2}-\frac{c^4\,\sqrt{-a^2}}{a}+10\,a^2\,c^4\,x^3\,\sqrt{-a^2}-5\,a^3\,c^4\,x^4\,\sqrt{-a^2}+a^4\,c^4\,x^5\,\sqrt{-a^2}-10\,a\,c^4\,x^2\,\sqrt{-a^2}\right)}-\frac{}{3\,\left(a^4\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a*x))^4*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (4*(1 - a^2*x^2)^(1/2))/(9*(-a^2)^(1/2)*(5*c^4*x*(-a^2)^(1/2) - (c^4*(-a^2)
^(1/2))/a + 10*a^2*c^4*x^3*(-a^2)^(1/2) - 5*a^3*c^4*x^4*(-a^2)^(1/2) + a^4*
c^4*x^5*(-a^2)^(1/2) - 10*a*c^4*x^2*(-a^2)^(1/2))) - (44*a*(1 - a^2*x^2)^(1
/2))/(3*(a^2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) - (7*asinh(x*(-a^2)^(1/2)))/
(c^4*(-a^2)^(1/2)) - (8*a^3*(1 - a^2*x^2)^(1/2))/(7*(a^4*c^4 - 2*a^5*c^4*x
+ a^6*c^4*x^2)) + (1754*a^4*(1 - a^2*x^2)^(1/2))/(315*(a^5*c^4 - 2*a^6*c^4*
x + a^7*c^4*x^2)) + (1 - a^2*x^2)^(1/2)/(a*c^4) - (20*a*(1 - a^2*x^2)^(1/2)
)/(7*(a^2*c^4 - 4*a^3*c^4*x + 6*a^4*c^4*x^2 - 4*a^5*c^4*x^3 + a^6*c^4*x^4))
 + (4964*(1 - a^2*x^2)^(1/2))/(315*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2) - (c^4*
(-a^2)^(1/2))/a)) + (697*(1 - a^2*x^2)^(1/2))/(105*(-a^2)^(1/2)*(3*c^4*x*(-
a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1/2) - 3*a*c^4*x^2*
(-a^2)^(1/2))) + (16*a^6*(1 - a^2*x^2)^(1/2))/(63*(a^7*c^4 - 4*a^8*c^4*x +
6*a^9*c^4*x^2 - 4*a^10*c^4*x^3 + a^11*c^4*x^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \frac{x^4}{-a^6x^6\sqrt{-a^2x^2+1}+4a^5x^5\sqrt{-a^2x^2+1}-5a^4x^4\sqrt{-a^2x^2+1}+5a^2x^2\sqrt{-a^2x^2+1}-4ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^6x^6\sqrt{-a^2x^2+1}+4a^5x^5\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*(Integral(x**4/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a*
*2*x**2 + 1) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x*
*2 + 1) - 4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral
(3*a*x**5/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 +
1) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) -
4*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x*
*6/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*
a**4*x**4*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*s
qrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**7/(-a**6
*x**6*sqrt(-a**2*x**2 + 1) + 4*a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**4*x**4
*sqrt(-a**2*x**2 + 1) + 5*a**2*x**2*sqrt(-a**2*x**2 + 1) - 4*a*x*sqrt(-a**2
*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.474 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=93

$$\frac{(4-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}-\frac{c(5-p)\left(c-\frac{c}{ax}\right)^{p-1}}{a(1-p)}+cx\left(c-\frac{c}{ax}\right)^{p-1}$$

```
[Out] -c*(5-p)*(c-c/a/x)^(-1+p)/a/(1-p)+c*(c-c/a/x)^(-1+p)*x+(4-p)*(c-c/a/x)^p*hy
pergeom([1, p],[1+p],1-1/a/x)/a/p
```

Rubi [A] time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6133, 25, 514, 375, 89, 79, 65}

$$\frac{(4-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}-\frac{c(5-p)\left(c-\frac{c}{ax}\right)^{p-1}}{a(1-p)}+cx\left(c-\frac{c}{ax}\right)^{p-1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -((c*(5 - p)*(c - c/(a*x))^(-1 + p))/(a*(1 - p))) + c*(c - c/(a*x))^(-1 + p
)*x + ((4 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a*x)])
/(a*p)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\int\frac{\left(c-\frac{c}{ax}\right)^p(1+ax)^2}{(1-ax)^2}\,dx$$
$$=\frac{c^2\int\frac{\left(c-\frac{c}{ax}\right)^{-2+p}(1+ax)^2}{x^2}\,dx}{a^2}$$
$$=\frac{c^2\int\left(a+\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{-2+p}\,dx}{a^2}$$
$$=-\frac{c^2\,\text{Subst}\left(\int\frac{(a+x)^2\left(c-\frac{cx}{a}\right)^{-2+p}}{x^2}\,dx,x,\frac{1}{x}\right)}{a^2}$$
$$=c\left(c-\frac{c}{ax}\right)^{-1+p}x-\frac{c\,\text{Subst}\left(\int\frac{(ac(4-p)+cx)\left(c-\frac{cx}{a}\right)^{-2+p}}{x}\,dx,x,\frac{1}{x}\right)}{a^2}$$
$$=-\frac{c(5-p)\left(c-\frac{c}{ax}\right)^{-1+p}}{a(1-p)}+c\left(c-\frac{c}{ax}\right)^{-1+p}x-\frac{(c(4-p))\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{-1+p}}{x}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=-\frac{c(5-p)\left(c-\frac{c}{ax}\right)^{-1+p}}{a(1-p)}+c\left(c-\frac{c}{ax}\right)^{-1+p}x+\frac{(4-p)\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(1,p;1+p;1-\frac{1}{ax}\right)}{ap}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 0.87

$$\frac{\left(c-\frac{c}{ax}\right)^p\left(apx(p(ax-1)-ax+5)-\left(p^2-5p+4\right)(ax-1)\,{}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)\right)}{a(p-1)p(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*(a*p*x*(5 - a*x + p*(-1 + a*x)) - (4 - 5*p + p^2)*(-1 + a*
x)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a*x)]))/(a*(-1 + p)*p*(-1 + a*x))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)\left(\frac{acx-c}{ax}\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*((a*c*x - c)/(a*x))^p/(a^2*x^2 - 2*a*x + 1),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4\left(c-\frac{c}{ax}\right)^p}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^4*(c - c/(a*x))^p/(a^2*x^2 - 1)^2, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4\left(c-\frac{c}{ax}\right)^p}{\left(-a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x)
```

```
[Out] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4\left(c-\frac{c}{ax}\right)^p}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^4*(c - c/(a*x))^p/(a^2*x^2 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^p\,(a\,x+1)^4}{\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^p*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] int(((c - c/(a*x))^p*(a*x + 1)^4)/(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p}(ax+1)^{2}}{(ax-1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

3.475 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal. Leaf size=64

$$\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

```
[Out] 1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/a^3/x^2+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$-\frac{c^5}{a^3x^2}-\frac{c^5}{3a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = -\frac{c^5\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^5}{x^5}dx}{a^5}$$
$$= -\frac{c^5\int \frac{(1-ax)^3(1+ax)^2}{x^5}dx}{a^5}$$
$$= -\frac{c^5\int \left(-a^5+\frac{1}{x^5}-\frac{a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{a^5}$$
$$= \frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}$$

Mathematica [A] time = 0.28, size = 66, normalized size = 1.03

$$\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}-\frac{c^5\log(ax)}{a}+c^5x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[a*x])/a
```

fricas [A] time = 0.44, size = 67, normalized size = 1.05

$$\frac{12\,a^5c^5x^5-12\,a^4c^5x^4\log(x)+24\,a^3c^5x^3-12\,a^2c^5x^2-4\,ac^5x+3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) + 24*a^3*c^5*x^3 - 12*a^2*c^5*
x^2 - 4*a*c^5*x + 3*c^5)/(a^5*x^4)
```

giac [A] time = 0.20, size = 60, normalized size = 0.94

$$c^5x-\frac{c^5\log(|x|)}{a}+\frac{24\,a^3c^5x^3-12\,a^2c^5x^2-4\,ac^5x+3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*x - c^5*log(abs(x))/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5
*x + 3*c^5)/(a^5*x^4)
```

maple [A] time = 0.03, size = 61, normalized size = 0.95

$$\frac{c^5}{4a^5x^4} - \frac{c^5}{3a^4x^3} - \frac{c^5}{x^2a^3} + \frac{2c^5}{a^2x} + c^5x - \frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^5,x)
```

```
[Out] 1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/x^2/a^3+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

maxima [A] time = 0.35, size = 59, normalized size = 0.92

$$c^5x - \frac{c^5\log(x)}{a} + \frac{24\,a^3c^5x^3 - 12\,a^2c^5x^2 - 4\,ac^5x + 3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - c^5*log(x)/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5*x +
3*c^5)/(a^5*x^4)
```

mupad [B] time = 0.07, size = 51, normalized size = 0.80

$$-\frac{c^5\,\left(4\,a\,x+12\,a^2\,x^2-24\,a^3\,x^3-12\,a^5\,x^5+12\,a^4\,x^4\,\ln(x)-3\right)}{12\,a^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^5*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] -(c^5*(4*a*x + 12*a^2*x^2 - 24*a^3*x^3 - 12*a^5*x^5 + 12*a^4*x^4*log(x) - 3
))/(12*a^5*x^4)
```

sympy [A] time = 0.28, size = 63, normalized size = 0.98

$$\frac{a^5c^5x - a^4c^5\log{(x)} + \frac{24a^3c^5x^3 - 12a^2c^5x^2 - 4ac^5x + 3c^5}{12x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**5,x)
```

```
[Out] (a**5*c**5*x - a**4*c**5*log(x) + (24*a**3*c**5*x**3 - 12*a**2*c**5*x**2 -
4*a*c**5*x + 3*c**5)/(12*x**4))/a**5
```

3.476 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=30

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

[Out] -1/3*c^4/a^4/x^3+2*c^4/a^2/x+c^4*x

Rubi [A] time = 0.10, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6129, 73, 270}

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^4,x]

[Out] -c^4/(3*a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^4}{x^4} dx}{a^4}$$
$$= \frac{c^4\int \frac{(1-ax)^2(1+ax)^2}{x^4} dx}{a^4}$$
$$= \frac{c^4\int \frac{\left(1-a^2x^2\right)^2}{x^4} dx}{a^4}$$
$$= \frac{c^4\int \left(a^4+\frac{1}{x^4}-\frac{2a^2}{x^2}\right) dx}{a^4}$$
$$= -\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Mathematica [A] time = 0.19, size = 30, normalized size = 1.00

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^4,x]
[Out] -1/3*c^4/(a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

fricas [A] time = 0.49, size = 36, normalized size = 1.20

$$\frac{3\,a^4c^4x^4+6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^4,x, algorithm="fricas")
[Out] 1/3*(3*a^4*c^4*x^4 + 6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

giac [A] time = 0.23, size = 31, normalized size = 1.03

$$c^4x+\frac{6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

maple [A] time = 0.03, size = 27, normalized size = 0.90

$$\frac{c^4 \left(x\,a^4+\frac{2a^2}{x}-\frac{1}{3x^3}\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^4,x)
```

```
[Out] c^4/a^4*(x*a^4+2*a^2/x-1/3/x^3)
```

maxima [A] time = 0.37, size = 31, normalized size = 1.03

$$c^4x+\frac{6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

mupad [B] time = 0.81, size = 27, normalized size = 0.90

$$\frac{c^4\,\left(a^4\,x^4+2\,a^2\,x^2-\frac{1}{3}\right)}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (c^4*(2*a^2*x^2 + a^4*x^4 - 1/3))/(a^4*x^3)
```

sympy [A] time = 0.16, size = 31, normalized size = 1.03

$$\frac{a^4c^4x+\frac{6a^2c^4x^2-c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**4,x)
```

```
[Out] (a**4*c**4*x + (6*a**2*c**4*x**2 - c**4)/(3*x**3))/a**4
```

3.477 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=38

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

```
[Out] 1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x+c^3*ln(x)/a
```

Rubi [A] time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 75}

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{c^3\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$
$$= -\frac{c^3\int \frac{(1-ax)(1+ax)^2}{x^3}dx}{a^3}$$
$$= -\frac{c^3\int \left(-a^3+\frac{1}{x^3}+\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3}$$
$$= \frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}$$

Mathematica [A] time = 0.14, size = 40, normalized size = 1.05

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(ax)}{a}+c^3x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^3,x]
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[a*x])/a
```

fricas [A] time = 0.55, size = 43, normalized size = 1.13

$$\frac{2\,a^3c^3x^3+2\,a^2c^3x^2\log(x)+2\,ac^3x+c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^3,x, algorithm="fricas")
[Out] 1/2*(2*a^3*c^3*x^3 + 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x + c^3)/(a^3*x^2)
```

giac [A] time = 0.18, size = 35, normalized size = 0.92

$$c^3x+\frac{c^3\log(|x|)}{a}+\frac{2\,ac^3x+c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^3,x, algorithm="giac")
[Out] c^3*x + c^3*log(abs(x))/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

maple [A] time = 0.03, size = 37, normalized size = 0.97

$$\frac{c^3}{2x^2a^3} + \frac{c^3}{a^2x} + c^3x + \frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^3,x)
[Out] 1/2*c^3/x^2/a^3+c^3/a^2/x+c^3*x+c^3*ln(x)/a
```

maxima [A] time = 0.38, size = 34, normalized size = 0.89

$$c^3x + \frac{c^3\log(x)}{a} + \frac{2\,ac^3x + c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^3,x, algorithm="maxima")
[Out] c^3*x + c^3*log(x)/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

mupad [B] time = 0.81, size = 31, normalized size = 0.82

$$\frac{c^3\,\left(a\,x + a^3\,x^3 + a^2\,x^2\,\ln(x) + \frac{1}{2}\right)}{a^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] (c^3*(a*x + a^3*x^3 + a^2*x^2*log(x) + 1/2))/(a^3*x^2)
```

sympy [A] time = 0.18, size = 37, normalized size = 0.97

$$\frac{a^3c^3x + a^2c^3\log{(x)} + \frac{2ac^3x+c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**3,x)
[Out] (a**3*c**3*x + a**2*c**3*log(x) + (2*a*c**3*x + c**3)/(2*x**2))/a**3
```

3.478 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=27

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

[Out] -c^2/a^2/x+c^2*x+2*c^2*ln(x)/a

Rubi [A] time = 0.10, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 43}

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x))^2,x]

[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[x])/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$
$$= \frac{c^2\int \frac{(1+ax)^2}{x^2}dx}{a^2}$$
$$= \frac{c^2\int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx}{a^2}$$
$$= -\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}$$

Mathematica [A] time = 0.10, size = 29, normalized size = 1.07

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(ax)}{a}+c^2x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x))^2,x]
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[a*x])/a
```

fricas [A] time = 0.41, size = 32, normalized size = 1.19

$$\frac{a^2c^2x^2+2\,ac^2x\log(x)-c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^2,x, algorithm="fricas")
[Out] (a^2*c^2*x^2 + 2*a*c^2*x*log(x) - c^2)/(a^2*x)
```

giac [A] time = 0.20, size = 28, normalized size = 1.04

$$c^2x+\frac{2\,c^2\log\left(|x|\right)}{a}-\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^2,x, algorithm="giac")
[Out] c^2*x + 2*c^2*log(abs(x))/a - c^2/(a^2*x)
```

maple [A] time = 0.03, size = 28, normalized size = 1.04

$$-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^2,x)
[Out] -c^2/a^2/x+c^2*x+2*c^2*ln(x)/a
```

maxima [A] time = 0.35, size = 27, normalized size = 1.00

$$c^2x+\frac{2\,c^2\log(x)}{a}-\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x)^2,x, algorithm="maxima")
[Out] c^2*x + 2*c^2*log(x)/a - c^2/(a^2*x)
```

mupad [B] time = 0.81, size = 25, normalized size = 0.93

$$\frac{c^2\,\left(a^2\,x^2+2\,a\,x\,\ln(x)-1\right)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] (c^2*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

sympy [A] time = 0.13, size = 26, normalized size = 0.96

$$\frac{a^2c^2x+2ac^2\log(x)-\frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x)**2,x)
[Out] (a**2*c**2*x + 2*a*c**2*log(x) - c**2/x)/a**2
```

3.479 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=25

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

```
[Out] c*x-c*ln(x)/a+4*c*ln(-a*x+1)/a
```

Rubi [A] time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6131, 6129, 72}

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{c\int\frac{e^{4\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$= -\frac{c\int\frac{(1+ax)^2}{x(1-ax)}\,dx}{a}$$
$$= -\frac{c\int\left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)dx}{a}$$
$$= cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 1.00

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a*x)),x]
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

fricas [A] time = 0.52, size = 23, normalized size = 0.92

$$\frac{acx+4\,c\log\left(ax-1\right)-c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x),x, algorithm="fricas")
[Out] (a*c*x + 4*c*log(a*x - 1) - c*log(x))/a
```

giac [A] time = 0.19, size = 26, normalized size = 1.04

$$cx+\frac{4\,c\log\left(|ax-1|\right)}{a}-\frac{c\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x),x, algorithm="giac")
[Out] c*x + 4*c*log(abs(a*x - 1))/a - c*log(abs(x))/a
```

maple [A] time = 0.03, size = 25, normalized size = 1.00

$$cx - \frac{c\ln(x)}{a} + \frac{4c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x),x)
[Out] c*x-c*ln(x)/a+4*c/a*ln(a*x-1)
```

maxima [A] time = 0.33, size = 24, normalized size = 0.96

$$cx + \frac{4\,c\log(ax-1)}{a} - \frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a/x),x, algorithm="maxima")
[Out] c*x + 4*c*log(a*x - 1)/a - c*log(x)/a
```

mupad [B] time = 0.08, size = 24, normalized size = 0.96

$$c\,x - \frac{c\,\ln(x)}{a} + \frac{4\,c\,\ln(a\,x-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] c*x - (c*log(x))/a + (4*c*log(a*x - 1))/a
```

sympy [A] time = 0.23, size = 17, normalized size = 0.68

$$cx + \frac{c\left(-\log(x) + 4\log\left(x - \frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a/x),x)
[Out] c*x + c*(-log(x) + 4*log(x - 1/a))/a
```

3.480 $\int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=53

$$\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

```
[Out] x/c-2/a/c/(-a*x+1)^2+8/a/c/(-a*x+1)+5*ln(-a*x+1)/a/c
```

Rubi [A] time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 77}

$$\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] x/c - 2/(a*c*(1 - a*x)^2) + 8/(a*c*(1 - a*x)) + (5*Log[1 - a*x])/(a*c)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\frac{a\int \frac{e^{4\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\
&= -\frac{a\int \frac{x(1+ax)^2}{(1-ax)^3}\,dx}{c}\\
&= -\frac{a\int \left(-\frac{1}{a}-\frac{4}{a(-1+ax)^3}-\frac{8}{a(-1+ax)^2}-\frac{5}{a(-1+ax)}\right)dx}{c}\\
&= \frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.96

$$-\frac{a\left(-\frac{8}{a^2(1-ax)}+\frac{2}{a^2(1-ax)^2}-\frac{5\log(1-ax)}{a^2}-\frac{x}{a}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] -((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/a
^2))/c)
```

fricas [A] time = 0.50, size = 64, normalized size = 1.21

$$\frac{a^3x^3-2\,a^2x^2-7\,ax+5\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+6}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 7*a*x + 5*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 6)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

giac [A] time = 0.18, size = 42, normalized size = 0.79

$$\frac{x}{c}+\frac{5\,\log\left(|ax-1|\right)}{ac}-\frac{2\left(4\,ax-3\right)}{\left(ax-1\right)^2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x),x, algorithm="giac")
```

```
[Out] x/c + 5*log(abs(a*x - 1))/(a*c) - 2*(4*a*x - 3)/((a*x - 1)^2*a*c)
```

maple [A] time = 0.04, size = 51, normalized size = 0.96

$$\frac{x}{c} - \frac{8}{ca\,(ax-1)} + \frac{5\ln(ax-1)}{ac} - \frac{2}{ac\,(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x),x)
```

```
[Out] x/c-8/c/a/(a*x-1)+5/a/c*ln(a*x-1)-2/a/c/(a*x-1)^2
```

maxima [A] time = 0.34, size = 49, normalized size = 0.92

$$-\frac{2\,(4\,ax-3)}{a^3cx^2-2\,a^2cx+ac} + \frac{x}{c} + \frac{5\,\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*(4*a*x - 3)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

mupad [B] time = 0.07, size = 48, normalized size = 0.91

$$\frac{x}{c} - \frac{8\,x-\frac{6}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c} + \frac{5\,\ln(a\,x-1)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - c/(a*x))*(a^2*x^2 - 1)^2),x)
```

```
[Out] x/c - (8*x - 6/a)/(c + a^2*c*x^2 - 2*a*c*x) + (5*log(a*x - 1))/(a*c)
```

sympy [A] time = 0.25, size = 41, normalized size = 0.77

$$\frac{-8ax+6}{a^3cx^2-2a^2cx+ac} + \frac{x}{c} + \frac{5\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a/x),x)
```

```
[Out] (-8*a*x + 6)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

3.481 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2+4/3/a/c^2/(-a*x+1)^3-6/a/c^2/(-a*x+1)^2+13/a/c^2/(-a*x+1)+6*ln(-a*x+1
)/a/c^2
```

Rubi [A] time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 + 4/(3*a*c^2*(1 - a*x)^3) - 6/(a*c^2*(1 - a*x)^2) + 13/(a*c^2*(1 - a*
x)) + (6*Log[1 - a*x])/(a*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= \frac{a^2\int \frac{e^{4\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}\\
&= \frac{a^2\int \frac{x^2(1+ax)^2}{(1-ax)^4}\,dx}{c^2}\\
&= \frac{a^2\int \left(\frac{1}{a^2}+\frac{4}{a^2(-1+ax)^4}+\frac{12}{a^2(-1+ax)^3}+\frac{13}{a^2(-1+ax)^2}+\frac{6}{a^2(-1+ax)}\right)dx}{c^2}\\
&= \frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 63, normalized size = 0.89

$$\frac{3a^4x^4-9a^3x^3-30a^2x^2+57ax+18(ax-1)^3\log(1-ax)-25}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-25 + 57*a*x - 30*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 18*(-1 + a*x)^3*Log[1
- a*x])/(3*a*c^2*(-1 + a*x)^3)
```

fricas [A] time = 0.45, size = 100, normalized size = 1.41

$$\frac{3\,a^4x^4-9\,a^3x^3-30\,a^2x^2+57\,ax+18\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-25}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 30*a^2*x^2 + 57*a*x + 18*(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)*log(a*x - 1) - 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2)
```

giac [A] time = 0.13, size = 50, normalized size = 0.70

$$\frac{x}{c^2}+\frac{6\,\log\left(|ax-1|\right)}{ac^2}-\frac{39\,a^2x^2-60\,ax+25}{3\left(ax-1\right)^3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] x/c^2 + 6*log(abs(a*x - 1))/(a*c^2) - 1/3*(39*a^2*x^2 - 60*a*x + 25)/((a*x
- 1)^3*a*c^2)
```

maple [A] time = 0.03, size = 66, normalized size = 0.93

$$\frac{x}{c^2} - \frac{13}{a\,c^2\,(ax-1)} + \frac{6\ln(ax-1)}{a\,c^2} - \frac{6}{a\,c^2\,(ax-1)^2} - \frac{4}{3a\,c^2\,(ax-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^2,x)
```

```
[Out] x/c^2-13/a/c^2/(a*x-1)+6/a/c^2*ln(a*x-1)-6/a/c^2/(a*x-1)^2-4/3/a/c^2/(a*x-1
)^3
```

maxima [A] time = 0.31, size = 75, normalized size = 1.06

$$-\frac{39\,a^2x^2 - 60\,ax + 25}{3\left(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2\right)} + \frac{x}{c^2} + \frac{6\,\log(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(39*a^2*x^2 - 60*a*x + 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 6*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.84, size = 71, normalized size = 1.00

$$\frac{13\,a\,x^2 - 20\,x + \frac{25}{3\,a}}{-a^3\,c^2\,x^3 + 3\,a^2\,c^2\,x^2 - 3\,a\,c^2\,x + c^2} + \frac{x}{c^2} + \frac{6\,\ln(a\,x-1)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - c/(a*x))^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] (13*a*x^2 - 20*x + 25/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x
) + x/c^2 + (6*log(a*x - 1))/(a*c^2)
```

sympy [A] time = 0.34, size = 73, normalized size = 1.03

$$\frac{-39a^2x^2 + 60ax - 25}{3a^4c^2x^3 - 9a^3c^2x^2 + 9a^2c^2x - 3ac^2} + \frac{x}{c^2} + \frac{6\log(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a/x)**2,x)
```

```
[Out] (-39*a**2*x**2 + 60*a*x - 25)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2
*c**2*x - 3*a*c**2) + x/c**2 + 6*log(a*x - 1)/(a*c**2)
```

3.482 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=89

$$\frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3-1/a/c^3/(-a*x+1)^4+16/3/a/c^3/(-a*x+1)^3-25/2/a/c^3/(-a*x+1)^2+19/a/c
^3/(-a*x+1)+7*ln(-a*x+1)/a/c^3
```

Rubi [A] time = 0.15, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 - 1/(a*c^3*(1 - a*x)^4) + 16/(3*a*c^3*(1 - a*x)^3) - 25/(2*a*c^3*(1 -
 a*x)^2) + 19/(a*c^3*(1 - a*x)) + (7*Log[1 - a*x])/(a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{a^3\int \frac{e^{4\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$

$$= -\frac{a^3\int \frac{x^3(1+ax)^2}{(1-ax)^5}\,dx}{c^3}$$

$$= -\frac{a^3\int \left(-\frac{1}{a^3}-\frac{4}{a^3(-1+ax)^5}-\frac{16}{a^3(-1+ax)^4}-\frac{25}{a^3(-1+ax)^3}-\frac{19}{a^3(-1+ax)^2}-\frac{7}{a^3(-1+ax)}\right)dx}{c^3}$$

$$= \frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}$$

Mathematica [A] time = 0.14, size = 71, normalized size = 0.80

$$\frac{6a^5x^5-24a^4x^4-78a^3x^3+243a^2x^2-218ax+42(ax-1)^4\log(1-ax)+65}{6ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (65 - 218*a*x + 243*a^2*x^2 - 78*a^3*x^3 - 24*a^4*x^4 + 6*a^5*x^5 + 42*(-1
+ a*x)^4*Log[1 - a*x])/(6*a*c^3*(-1 + a*x)^4)
```

fricas [A] time = 0.51, size = 126, normalized size = 1.42

$$\frac{6\,a^5x^5-24\,a^4x^4-78\,a^3x^3+243\,a^2x^2-218\,ax+42\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax-1\right)+65}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^5*x^5 - 24*a^4*x^4 - 78*a^3*x^3 + 243*a^2*x^2 - 218*a*x + 42*(a^4*
x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 65)/(a^5*c^3*x^4 -
4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.33, size = 58, normalized size = 0.65

$$\frac{x}{c^3}+\frac{7\,\log\left(|ax-1|\right)}{ac^3}-\frac{114\,a^3x^3-267\,a^2x^2+224\,ax-65}{6\left(ax-1\right)^4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 + 7*log(abs(a*x - 1))/(a*c^3) - 1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*
a*x - 65)/((a*x - 1)^4*a*c^3)
```

maple [A] time = 0.03, size = 81, normalized size = 0.91

$$\frac{x}{c^{3}}-\frac{19}{a\,c^{3}\left(a x -1\right)}+\frac{7 \ln \left(a x -1\right)}{c^{3} a}-\frac{25}{2 a\,c^{3}\left(a x -1\right)^{2}}-\frac{1}{a\,c^{3}\left(a x -1\right)^{4}}-\frac{16}{3 a\,c^{3}\left(a x -1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^3,x)
```

```
[Out] x/c^3-19/a/c^3/(a*x-1)+7/c^3/a*ln(a*x-1)-25/2/a/c^3/(a*x-1)^2-1/a/c^3/(a*x-
1)^4-16/3/a/c^3/(a*x-1)^3
```

maxima [A] time = 0.34, size = 93, normalized size = 1.04

$$-\frac{114\,a^{3}x^{3}-267\,a^{2}x^{2}+224\,ax-65}{6\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3}\right)}+\frac{x}{c^{3}}+\frac{7\,\log\left(ax-1\right)}{ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*a*x - 65)/(a^5*c^3*x^4 - 4*a^4*c^3*x^
3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 + 7*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.08, size = 90, normalized size = 1.01

$$\frac{x}{c^{3}}-\frac{\frac{112\,x}{3}-\frac{89\,a\,x^{2}}{2}-\frac{65}{6\,a}+19\,a^{2}\,x^{3}}{a^{4}\,c^{3}\,x^{4}-4\,a^{3}\,c^{3}\,x^{3}+6\,a^{2}\,c^{3}\,x^{2}-4\,a\,c^{3}\,x+c^{3}}+\frac{7\,\ln\left(a\,x-1\right)}{a\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - c/(a*x))^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] x/c^3 - ((112*x)/3 - (89*a*x^2)/2 - 65/(6*a) + 19*a^2*x^3)/(c^3 + 6*a^2*c^3
*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (7*log(a*x - 1))/(a*c^3)
```

sympy [A] time = 0.44, size = 94, normalized size = 1.06

$$\frac{-114a^{3}x^{3}+267a^{2}x^{2}-224ax+65}{6a^{5}c^{3}x^{4}-24a^{4}c^{3}x^{3}+36a^{3}c^{3}x^{2}-24a^{2}c^{3}x+6ac^{3}}+\frac{x}{c^{3}}+\frac{7\log\left(ax-1\right)}{ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a/x)**3,x)
```

```
[Out] (-114*a**3*x**3 + 267*a**2*x**2 - 224*a*x + 65)/(6*a**5*c**3*x**4 - 24*a**4
*c**3*x**3 + 36*a**3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3) + x/c**3 + 7*lo
g(a*x - 1)/(a*c**3)
```

3.483 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=105

$$\frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

```
[Out] x/c^4+4/5/a/c^4/(-a*x+1)^5-5/a/c^4/(-a*x+1)^4+41/3/a/c^4/(-a*x+1)^3-22/a/c^
4/(-a*x+1)^2+26/a/c^4/(-a*x+1)+8*ln(-a*x+1)/a/c^4
```

Rubi [A] time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 + 4/(5*a*c^4*(1 - a*x)^5) - 5/(a*c^4*(1 - a*x)^4) + 41/(3*a*c^4*(1 -
a*x)^3) - 22/(a*c^4*(1 - a*x)^2) + 26/(a*c^4*(1 - a*x)) + (8*Log[1 - a*x])/
(a*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
```

```
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4(1+ax)^2}{(1-ax)^6}\,dx}{c^4}$$

$$= \frac{a^4\int \left(\frac{1}{a^4}+\frac{4}{a^4(-1+ax)^6}+\frac{20}{a^4(-1+ax)^5}+\frac{41}{a^4(-1+ax)^4}+\frac{44}{a^4(-1+ax)^3}+\frac{26}{a^4(-1+ax)^2}+\frac{8}{a^4(-1+ax)}\right)dx}{c^4}$$

$$= \frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3}-\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8\log(1-}{ac^4}$$

Mathematica [A] time = 0.17, size = 79, normalized size = 0.75

$$\frac{15a^6x^6-75a^5x^5-240a^4x^4+1080a^3x^3-1480a^2x^2+890ax+120(ax-1)^5\log(1-ax)-202}{15ac^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-202 + 890*a*x - 1480*a^2*x^2 + 1080*a^3*x^3 - 240*a^4*x^4 - 75*a^5*x^5 +
15*a^6*x^6 + 120*(-1 + a*x)^5*Log[1 - a*x])/(15*a*c^4*(-1 + a*x)^5)
```

fricas [A] time = 0.43, size = 154, normalized size = 1.47

$$\frac{15\,a^6x^6-75\,a^5x^5-240\,a^4x^4+1080\,a^3x^3-1480\,a^2x^2+890\,ax+120\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-\right.}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*x^6 - 75*a^5*x^5 - 240*a^4*x^4 + 1080*a^3*x^3 - 1480*a^2*x^2 +
 890*a*x + 120*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*
log(a*x - 1) - 202)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.21, size = 66, normalized size = 0.63

$$\frac{x}{c^4} + \frac{8\,\log\left(|ax-1|\right)}{ac^4} - \frac{390\,a^4x^4 - 1230\,a^3x^3 + 1555\,a^2x^2 - 905\,ax + 202}{15\,(ax-1)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 + 8*log(abs(a*x - 1))/(a*c^4) - 1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 15
55*a^2*x^2 - 905*a*x + 202)/((a*x - 1)^5*a*c^4)
```

maple [A] time = 0.04, size = 96, normalized size = 0.91

$$\frac{x}{c^4} - \frac{26}{a\,c^4\,(ax-1)} + \frac{8\ln(ax-1)}{a\,c^4} - \frac{22}{a\,c^4\,(ax-1)^2} - \frac{4}{5a\,c^4\,(ax-1)^5} - \frac{5}{a\,c^4\,(ax-1)^4} - \frac{41}{3a\,c^4\,(ax-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^4,x)
```

```
[Out] x/c^4-26/a/c^4/(a*x-1)+8/a/c^4*ln(a*x-1)-22/a/c^4/(a*x-1)^2-4/5/a/c^4/(a*x-
1)^5-5/a/c^4/(a*x-1)^4-41/3/a/c^4/(a*x-1)^3
```

maxima [A] time = 0.33, size = 113, normalized size = 1.08

$$-\frac{390\,a^4x^4 - 1230\,a^3x^3 + 1555\,a^2x^2 - 905\,ax + 202}{15\left(a^6c^4x^5 - 5\,a^5c^4x^4 + 10\,a^4c^4x^3 - 10\,a^3c^4x^2 + 5\,a^2c^4x - ac^4\right)} + \frac{x}{c^4} + \frac{8\,\log(ax-1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 1555*a^2*x^2 - 905*a*x + 202)/(a^6*c^4*
x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4
) + x/c^4 + 8*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.10, size = 109, normalized size = 1.04

$$\frac{x}{c^4} + \frac{\frac{311\,a\,x^2}{3} - \frac{181\,x}{3} + \frac{202}{15\,a} - 82\,a^2\,x^3 + 26\,a^3\,x^4}{-a^5\,c^4\,x^5 + 5\,a^4\,c^4\,x^4 - 10\,a^3\,c^4\,x^3 + 10\,a^2\,c^4\,x^2 - 5\,a\,c^4\,x + c^4} + \frac{8\,\ln(a\,x-1)}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - c/(a*x))^4*(a^2*x^2 - 1)^2),x)
```

```
[Out] x/c^4 + ((311*a*x^2)/3 - (181*x)/3 + 202/(15*a) - 82*a^2*x^3 + 26*a^3*x^4)/
(c^4 + 10*a^2*c^4*x^2 - 10*a^3*c^4*x^3 + 5*a^4*c^4*x^4 - a^5*c^4*x^5 - 5*a*
c^4*x) + (8*log(a*x - 1))/(a*c^4)
```

sympy [A] time = 0.55, size = 114, normalized size = 1.09

$$\frac{-390a^4x^4+1230a^3x^3-1555a^2x^2+905ax-202}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4}+\frac{x}{c^4}+\frac{8\log{(ax-1)}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a/x)**4,x)
```

```
[Out] (-390*a**4*x**4 + 1230*a**3*x**3 - 1555*a**2*x**2 + 905*a*x - 202)/(15*a**6
*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**4*c**4*x**3 - 150*a**3*c**4*x**2 +
75*a**2*c**4*x - 15*a*c**4) + x/c**4 + 8*log(a*x - 1)/(a*c**4)
```

3.484 $\int e^{-\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{x(1-ax)^{-p} F_1\left(1-p; -p-\frac{1}{2}, \frac{1}{2}; 2-p; ax, -ax\right) \left(c - \frac{c}{ax}\right)^p}{1-p}$$

```
[Out] (c-c/a/x)^p*x*AppellF1(1-p,-1/2-p,1/2,2-p,a*x,-a*x)/(1-p)/((-a*x+1)^p)
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6134, 6129, 133}

$$\frac{x(1-ax)^{-p} F_1\left(1-p; -p-\frac{1}{2}, \frac{1}{2}; 2-p; ax, -ax\right) \left(c - \frac{c}{ax}\right)^p}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^ArcTanh[a*x],x]
```

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, -1/2 - p, 1/2, 2 - p, a*x, -(a*x)])/((1
- p)*(1 - a*x)^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx&=\left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int e^{-\tanh^{-1}(ax)}x^{-p}(1-ax)^p\,dx\\&=\left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int\frac{x^{-p}(1-ax)^{\frac{1}{2}+p}}{\sqrt{1+ax}}\,dx\\&=\frac{\left(c-\frac{c}{ax}\right)^p x(1-ax)^{-p}F_1\left(1-p;-\frac{1}{2}-p,\frac{1}{2};2-p;ax,-ax\right)}{1-p}\end{aligned}$$

Mathematica [F] time = 1.12, size = 0, normalized size = 0.00

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^ArcTanh[a*x],x]
[Out] Integrate[(c - c/(a*x))^p/E^ArcTanh[a*x], x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(\frac{acx-c}{ax}\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^{p} \sqrt{-a^{2}x^{2}+1}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}\left(c-\frac{c}{ax}\right)^{p}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{p} \sqrt{1-a^{2}\,x^{2}}}{a\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p} \sqrt{-(ax-1)(ax+1)}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.485 $\int e^{-\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^4 dx$

Optimal. Leaf size=140

$$\frac{c^4\sqrt{1-a^2x^2}}{a}-\frac{32c^4\sqrt{1-a^2x^2}}{3a^2x}+\frac{25c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3}+\frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2}+\frac{5c^4\sin^{-1}(ax)}{a}$$

```
[Out] 5*c^4*arcsin(a*x)/a+25/2*c^4*arctanh((-a^2*x^2+1)^(1/2))/a+c^4*(-a^2*x^2+1)
^(1/2)/a-1/3*c^4*(-a^2*x^2+1)^(1/2)/a^4/x^3+5/2*c^4*(-a^2*x^2+1)^(1/2)/a^3/
x^2-32/3*c^4*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.42, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{c^4\sqrt{1-a^2x^2}}{a}-\frac{32c^4\sqrt{1-a^2x^2}}{3a^2x}+\frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3}+\frac{25c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{5c^4\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*Sqrt[1 - a^2*x^2])/a - (c^4*Sqrt[1 - a^2*x^2])/(3*a^4*x^3) + (5*c^4*Sq
rt[1 - a^2*x^2])/(2*a^3*x^2) - (32*c^4*Sqrt[1 - a^2*x^2])/(3*a^2*x) + (5*c^
4*ArcSin[a*x])/a + (25*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
```

```
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int \frac{e^{-\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}$$

$$= \frac{c^4\int \frac{(1-ax)^5}{x^4\sqrt{1-a^2x^2}}\,dx}{a^4}$$

$$= -\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} - \frac{c^4\int \frac{15a-32a^2x+30a^3x^2-15a^4x^3+3a^5x^4}{x^3\sqrt{1-a^2x^2}}\,dx}{3a^4}$$

$$= -\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} + \frac{c^4\int \frac{64a^2-75a^3x+30a^4x^2-6a^5x^3}{x^2\sqrt{1-a^2x^2}}\,dx}{6a^4}$$

$$= -\frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} - \frac{c^4\int \frac{75a^3-30a^4x+6a^5x^2}{x\sqrt{1-a^2x^2}}\,dx}{6a^4}$$

$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \frac{c^4\int \frac{-75a^5+30a^6x}{x\sqrt{1-a^2x^2}}}{6a^6}$$

$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \left(5c^4\right)\int \frac{1}{\sqrt{1-a^2}}$$

$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \frac{5c^4\sin^{-1}(ax)}{a} -$$

$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \frac{5c^4\sin^{-1}(ax)}{a} +$$

$$= \frac{c^4\sqrt{1-a^2x^2}}{a} - \frac{c^4\sqrt{1-a^2x^2}}{3a^4x^3} + \frac{5c^4\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{32c^4\sqrt{1-a^2x^2}}{3a^2x} + \frac{5c^4\sin^{-1}(ax)}{a} +$$

Mathematica [A] time = 0.25, size = 85, normalized size = 0.61

$$\frac{c^4\left(75\log\left(\sqrt{1-a^2x^2}+1\right)+\frac{\sqrt{1-a^2x^2}\left(6a^3x^3-64a^2x^2+15ax-2\right)}{a^3x^3}-75\log(ax)+30\sin^{-1}(ax)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*((Sqrt[1 - a^2*x^2]*(-2 + 15*a*x - 64*a^2*x^2 + 6*a^3*x^3))/(a^3*x^3)
+ 30*ArcSin[a*x] - 75*Log[a*x] + 75*Log[1 + Sqrt[1 - a^2*x^2]]))/(6*a)
```

fricas [A] time = 0.43, size = 132, normalized size = 0.94

$$-\frac{60\,a^3c^4x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+75\,a^3c^4x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^4x^3-\left(6\,a^3c^4x^3-64\,a^2c^4x^2+15\,ac^4x-2\,c\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(60*a^3*c^4*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 75*a^3*c^4*x^
3*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^4*x^3 - (6*a^3*c^4*x^3 - 64*a^2
*c^4*x^2 + 15*a*c^4*x - 2*c^4)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

giac [B] time = 0.20, size = 262, normalized size = 1.87

$$\frac{\left(c^4-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^4}{a^2x}+\frac{129\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^4}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|}+\frac{5\,c^4\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}+\frac{25\,c^4\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\frac{\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(c^4 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) + 129*(sqrt(-a^2
*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^3*abs(a)) + 5*c^4*arcsin(a*x)*sgn(a)/abs(a) + 25/2*c^4*log(1/2*abs(-2*
sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*
c^4/a - 1/24*(129*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/x - 15*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^2*c^4/(a^2*x^2) + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^4/(
a^4*x^3))/(a^2*abs(a))
```

maple [A] time = 0.05, size = 232, normalized size = 1.66

$$-\frac{25c^4\sqrt{-a^2x^2+1}}{2a}+\frac{25c^4\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{11c^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^2x}-11c^4x\sqrt{-a^2x^2+1}-\frac{11c^4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -25/2*c^4*(-a^2*x^2+1)^(1/2)/a+25/2*c^4/a*arctanh(1/(-a^2*x^2+1)^(1/2))-11*
c^4/a^2/x*(-a^2*x^2+1)^(3/2)-11*c^4*x*(-a^2*x^2+1)^(1/2)-11*c^4/(a^2)^(1/2)
*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/3*c^4*(-a^2*x^2+1)^(3/2)/a^4/x^
3+5/2*c^4*(-a^2*x^2+1)^(3/2)/x^2/a^3+16*c^4/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^
(1/2)+16*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$4\,ac^4\left(\frac{\arcsin(ax)}{a^2}+\frac{\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a^2}\right)+c^4\left(\frac{\arcsin(ax)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}\right)+\int\frac{\left(6\,a^2c^4x^2-4\,ac^4x+c^4\right)\sqrt{ax+1}}{a^5x^5+a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 4*a*c^4*(arcsin(a*x)/a^2 + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a^2)
 + c^4*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) + integrate((6*a^2*c^4*x^2 -
4*a*c^4*x + c^4)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^5*x^5 + a^4*x^4), x)
```

mupad [B] time = 0.85, size = 136, normalized size = 0.97

$$\frac{5\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{c^4\,\sqrt{1-a^2\,x^2}}{a}-\frac{32\,c^4\,\sqrt{1-a^2\,x^2}}{3\,a^2\,x}+\frac{5\,c^4\,\sqrt{1-a^2\,x^2}}{2\,a^3\,x^2}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{3\,a^4\,x^3}-\frac{c^4\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (5*c^4*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (c^4*atan((1 - a^2*x^2)^(1/2)*
1i)*25i)/(2*a) + (c^4*(1 - a^2*x^2)^(1/2))/a - (32*c^4*(1 - a^2*x^2)^(1/2))
/(3*a^2*x) + (5*c^4*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2) - (c^4*(1 - a^2*x^2)^(
1/2))/(3*a^4*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{\sqrt{-a^2x^2+1}}{ax^5+x^4}\,dx+\int\left(-\frac{4ax\sqrt{-a^2x^2+1}}{ax^5+x^4}\right)dx+\int\frac{6a^2x^2\sqrt{-a^2x^2+1}}{ax^5+x^4}\,dx+\int\left(-\frac{4a^3x^3\sqrt{-a^2x^2+1}}{ax^5+x^4}\right)dx+\int\frac{a^4x^4\sqrt{-a^2x^2+1}}{ax^5+x^4}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**4*(Integral(sqrt(-a**2*x**2 + 1)/(a*x**5 + x**4), x) + Integral(-4*a*x*s
qrt(-a**2*x**2 + 1)/(a*x**5 + x**4), x) + Integral(6*a**2*x**2*sqrt(-a**2*x
**2 + 1)/(a*x**5 + x**4), x) + Integral(-4*a**3*x**3*sqrt(-a**2*x**2 + 1)/(
a*x**5 + x**4), x) + Integral(a**4*x**4*sqrt(-a**2*x**2 + 1)/(a*x**5 + x**4
), x))/a**4
```

3.486 $\int e^{-\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^3 dx$

Optimal. Leaf size=111

$$\frac{c^3\sqrt{1-a^2x^2}}{a} - \frac{4c^3\sqrt{1-a^2x^2}}{a^2x} + \frac{13c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} + \frac{4c^3\sin^{-1}(ax)}{a}$$

```
[Out] 4*c^3*arcsin(a*x)/a+13/2*c^3*arctanh((-a^2*x^2+1)^(1/2))/a+c^3*(-a^2*x^2+1)
^(1/2)/a+1/2*c^3*(-a^2*x^2+1)^(1/2)/a^3/x^2-4*c^3*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.31, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{c^3\sqrt{1-a^2x^2}}{a} - \frac{4c^3\sqrt{1-a^2x^2}}{a^2x} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} + \frac{13c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a} + \frac{4c^3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*Sqrt[1 - a^2*x^2])/a + (c^3*Sqrt[1 - a^2*x^2])/(2*a^3*x^2) - (4*c^3*Sq
rt[1 - a^2*x^2])/(a^2*x) + (4*c^3*ArcSin[a*x])/a + (13*c^3*ArcTanh[Sqrt[1 -
 a^2*x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{c^3\int \frac{e^{-\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}$$
$$= -\frac{c^3\int \frac{(1-ax)^4}{x^3\sqrt{1-a^2x^2}}\,dx}{a^3}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}+\frac{c^3\int \frac{8a-13a^2x+8a^3x^2-2a^4x^3}{x^2\sqrt{1-a^2x^2}}\,dx}{2a^3}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}-\frac{c^3\int \frac{13a^2-8a^3x+2a^4x^2}{x\sqrt{1-a^2x^2}}\,dx}{2a^3}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{c^3\int \frac{-13a^4+8a^5x}{x\sqrt{1-a^2x^2}}\,dx}{2a^5}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\left(4c^3\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{\left(13c^3\right)\int}{}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{4c^3\sin^{-1}(ax)}{a}-\frac{\left(13c^3\right)\text{Subst}\left(\int\right.}{4a}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{4c^3\sin^{-1}(ax)}{a}+\frac{\left(13c^3\right)\text{Subst}\left(\int\right.}{}$$
$$= \frac{c^3\sqrt{1-a^2x^2}}{a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{4c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{4c^3\sin^{-1}(ax)}{a}+\frac{13c^3\tanh^{-1}\left(\sqrt{1-}\right.}{2a}$$

Mathematica [A] time = 0.19, size = 77, normalized size = 0.69

$$\frac{c^3\left(\frac{\sqrt{1-a^2x^2}\left(2a^2x^2-8ax+1\right)}{a^2x^2}+13\log\left(\sqrt{1-a^2x^2}+1\right)-13\log(ax)+8\sin^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*((Sqrt[1 - a^2*x^2]*(1 - 8*a*x + 2*a^2*x^2))/(a^2*x^2) + 8*ArcSin[a*x]
 - 13*Log[a*x] + 13*Log[1 + Sqrt[1 - a^2*x^2]]))/(2*a)
```

fricas [A] time = 0.43, size = 119, normalized size = 1.07

$$-\frac{16\,a^2c^3x^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+13\,a^2c^3x^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-2\,a^2c^3x^2-\left(2\,a^2c^3x^2-8\,ac^3x+c^3\right)\sqrt{-a^2x^2+1}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(16*a^2*c^3*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 13*a^2*c^3*x^
2*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 2*a^2*c^3*x^2 - (2*a^2*c^3*x^2 - 8*a*c^
3*x + c^3)*sqrt(-a^2*x^2 + 1))/(a^3*x^2)
```

giac [B] time = 0.21, size = 206, normalized size = 1.86

$$-\frac{\left(c^3-\frac{16\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^3}{a^2x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}+\frac{4\,c^3\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{|a|}+\frac{13\,c^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\frac{\sqrt{-a^2x^2+1}\,c^3}{a}-\frac{16\left(\sqrt{-a^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(c^3 - 16*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x))*a^4*x^2/((sqrt(
-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)) + 4*c^3*arcsin(a*x)*sgn(a)/abs(a) + 13/
2*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a)
+ sqrt(-a^2*x^2 + 1)*c^3/a - 1/8*(16*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3*ab
s(a)/(a^2*x) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3*abs(a)/(a^4*x^2))/a^2
```

maple [B] time = 0.04, size = 209, normalized size = 1.88

$$-\frac{13c^3\sqrt{-a^2x^2+1}}{2a}+\frac{13c^3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{4c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^2x}-4c^3x\sqrt{-a^2x^2+1}-\frac{4c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{c^3\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -13/2*c^3*(-a^2*x^2+1)^(1/2)/a+13/2*c^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))-4*c
^3/a^2/x*(-a^2*x^2+1)^(3/2)-4*c^3*x*(-a^2*x^2+1)^(1/2)-4*c^3/(a^2)^(1/2)*ar
ctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/2*c^3*(-a^2*x^2+1)^(3/2)/x^2/a^3+8
```

```
*c^3/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+8*c^3/(a^2)^(1/2)*arctan((a^2)^(1
/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3\,ac^3\left(\frac{\arcsin(ax)}{a^2}+\frac{\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a^2}\right)+c^3\left(\frac{\arcsin(ax)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}\right)+\int\frac{\left(3\,ac^3x-c^3\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{a^4x^4+a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 3*a*c^3*(arcsin(a*x)/a^2 + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a^2)
 + c^3*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) + integrate((3*a*c^3*x - c^3)
*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^4*x^4 + a^3*x^3), x)
```

mupad [B] time = 0.04, size = 113, normalized size = 1.02

$$\frac{4\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{c^3\,\sqrt{1-a^2\,x^2}}{a}-\frac{4\,c^3\,\sqrt{1-a^2\,x^2}}{a^2\,x}+\frac{c^3\,\sqrt{1-a^2\,x^2}}{2\,a^3\,x^2}-\frac{c^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,13\mathrm{i}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (4*c^3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (c^3*atan((1 - a^2*x^2)^(1/2)*
1i)*13i)/(2*a) + (c^3*(1 - a^2*x^2)^(1/2))/a - (4*c^3*(1 - a^2*x^2)^(1/2))/
(a^2*x) + (c^3*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\left(-\frac{\sqrt{-a^2x^2+1}}{ax^4+x^3}\right)dx+\int\frac{3ax\sqrt{-a^2x^2+1}}{ax^4+x^3}\,dx+\int\left(-\frac{3a^2x^2\sqrt{-a^2x^2+1}}{ax^4+x^3}\right)dx+\int\frac{a^3x^3\sqrt{-a^2x^2+1}}{ax^4+x^3}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**3*(Integral(-sqrt(-a**2*x**2 + 1)/(a*x**4 + x**3), x) + Integral(3*a*x*s
qrt(-a**2*x**2 + 1)/(a*x**4 + x**3), x) + Integral(-3*a**2*x**2*sqrt(-a**2*
x**2 + 1)/(a*x**4 + x**3), x) + Integral(a**3*x**3*sqrt(-a**2*x**2 + 1)/(a*
x**4 + x**3), x))/a**3
```

3.487 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{3c^2\sin^{-1}(ax)}{a}$$

[Out] 3*c^2*arcsin(a*x)/a+3*c^2*arctanh((-a^2*x^2+1)^(1/2))/a+c^2*(-a^2*x^2+1)^(1/2)/a-c^2*(-a^2*x^2+1)^(1/2)/a^2/x

Rubi [A] time = 0.24, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6131, 6128, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{3c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[(c - c/(a*x))^2/E^ArcTanh[a*x],x]

[Out] (c^2*Sqrt[1 - a^2*x^2])/a - (c^2*Sqrt[1 - a^2*x^2])/(a^2*x) + (3*c^2*ArcSin[a*x])/a + (3*c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/a

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\frac{c^2\int\frac{e^{-\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}$$
$$=\frac{c^2\int\frac{(1-ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$=-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}-\frac{c^2\int\frac{3a-3a^2x+a^3x^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$=\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\frac{c^2\int\frac{-3a^3+3a^4x}{x\sqrt{1-a^2x^2}}\,dx}{a^4}$$
$$=\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\left(3c^2\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{\left(3c^2\right)\int\frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}$$
$$=\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\frac{3c^2\sin^{-1}(ax)}{a}-\frac{\left(3c^2\right)\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}$$
$$=\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\frac{3c^2\sin^{-1}(ax)}{a}+\frac{\left(3c^2\right)\mathrm{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-\right.}{a^3}$$
$$=\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}+\frac{3c^2\sin^{-1}(ax)}{a}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.13, size = 83, normalized size = 1.01

$$\frac{\sqrt{1-a^2x^2}\left(c^2-\frac{c^2}{ax}\right)}{a}+\frac{3c^2\log\left(\sqrt{1-a^2x^2}+1\right)}{a}-\frac{3c^2\log(ax)}{a}+\frac{3c^2\sin^{-1}(ax)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^ArcTanh[a*x],x]
```

```
[Out] ((c^2 - c^2/(a*x))*Sqrt[1 - a^2*x^2])/a + (3*c^2*ArcSin[a*x])/a - (3*c^2*Lo
g[a*x])/a + (3*c^2*Log[1 + Sqrt[1 - a^2*x^2]])/a
```

fricas [A] time = 0.48, size = 97, normalized size = 1.18

$$-\frac{6\,ac^2x\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,ac^2x\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-ac^2x-\left(ac^2x-c^2\right)\sqrt{-a^2x^2+1}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(6*a*c^2*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a*c^2*x*log((sqrt(-a
^2*x^2 + 1) - 1)/x) - a*c^2*x - (a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1))/(a^2*x)
```

giac [A] time = 0.43, size = 139, normalized size = 1.70

$$\frac{a^2c^2x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}+\frac{3\,c^2\arcsin(ax)\operatorname{sgn}(a)}{|a|}+\frac{3\,c^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}\,c^2}{a}-\frac{\left(\sqrt{-a^2x^2+1}\,|a|\right.}{2\,a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*c^2*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + 3*c^2*arcsin(a*x)*
sgn(a)/abs(a) + 3*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*
abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^2/a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)*c^2/(a^2*x*abs(a))
```

maple [B] time = 0.04, size = 186, normalized size = 2.27

$$-\frac{3c^2\sqrt{-a^2x^2+1}}{a}+\frac{3c^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}-\frac{c^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^2x}-c^2x\sqrt{-a^2x^2+1}-\frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{4c^2\sqrt{-a^2\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -3*c^2*(-a^2*x^2+1)^(1/2)/a+3*c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))-c^2/a^2/x
*(-a^2*x^2+1)^(3/2)-c^2*x*(-a^2*x^2+1)^(1/2)-c^2/(a^2)^(1/2)*arctan((a^2)^(
1/2)*x/(-a^2*x^2+1)^(1/2))+4*c^2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+4*c^2
/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,ac^2\left(\frac{\arcsin(ax)}{a^2}+\frac{\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a^2}\right)+c^2\left(\frac{\arcsin(ax)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}\right)+c^2\int\frac{\sqrt{ax+1}\,\sqrt{-ax+1}}{a^3x^3+a^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*a*c^2*(arcsin(a*x)/a^2 + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a^2)
 + c^2*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) + c^2*integrate(sqrt(a*x + 1)
*sqrt(-a*x + 1)/(a^3*x^3 + a^2*x^2), x)
```

mupad [B] time = 0.82, size = 90, normalized size = 1.10

$$\frac{3\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{c^2\,\sqrt{1-a^2\,x^2}}{a}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{a^2\,x}-\frac{c^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (3*c^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (c^2*atan((1 - a^2*x^2)^(1/2)*
1i)*3i)/a + (c^2*(1 - a^2*x^2)^(1/2))/a - (c^2*(1 - a^2*x^2)^(1/2))/(a^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int \frac{\sqrt{-a^2x^2+1}}{ax^3+x^2}\,dx+\int\left(-\frac{2ax\sqrt{-a^2x^2+1}}{ax^3+x^2}\right)dx+\int\frac{a^2x^2\sqrt{-a^2x^2+1}}{ax^3+x^2}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**2*(Integral(sqrt(-a**2*x**2 + 1)/(a*x**3 + x**2), x) + Integral(-2*a*x*s
qrt(-a**2*x**2 + 1)/(a*x**3 + x**2), x) + Integral(a**2*x**2*sqrt(-a**2*x**
2 + 1)/(a*x**3 + x**2), x))/a**2
```

3.488 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=50

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{2c\sin^{-1}(ax)}{a}$$

```
[Out] 2*c*arcsin(a*x)/a+c*arctanh((-a^2*x^2+1)^(1/2))/a+c*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.15, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6131, 6128, 1809, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{2c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^ArcTanh[a*x],x]
```

```
[Out] (c*Sqrt[1 - a^2*x^2])/a + (2*c*ArcSin[a*x])/a + (c*ArcTanh[Sqrt[1 - a^2*x^2
]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{c\int \frac{e^{-\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$= -\frac{c\int \frac{(1-ax)^2}{x\sqrt{1-a^2x^2}}\,dx}{a}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\int \frac{-a^2+2a^3x}{x\sqrt{1-a^2x^2}}\,dx}{a^3}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+(2c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{c\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{2c\sin^{-1}(ax)}{a}-\frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{2c\sin^{-1}(ax)}{a}+\frac{c\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{2c\sin^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 0.94

$$\frac{c\left(\sqrt{1-a^2x^2}+\log\left(\sqrt{1-a^2x^2}+1\right)+2\sin^{-1}(ax)-\log(x)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^ArcTanh[a*x],x]
```

```
[Out] (c*(Sqrt[1 - a^2*x^2] + 2*ArcSin[a*x] - Log[x] + Log[1 + Sqrt[1 - a^2*x^2]]
))/a
```

fricas [A] time = 0.52, size = 67, normalized size = 1.34

$$-\frac{4\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+c\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}\,c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(4*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c*log((sqrt(-a^2*x^2 + 1) -
1)/x) - sqrt(-a^2*x^2 + 1)*c)/a
```

giac [A] time = 0.21, size = 68, normalized size = 1.36

$$\frac{2\,c\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}+\frac{c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}\,c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*arcsin(a*x)*sgn(a)/abs(a) + c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a)
- 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c/a
```

maple [B] time = 0.04, size = 106, normalized size = 2.12

$$-\frac{c\sqrt{-a^2x^2+1}}{a}+\frac{c\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}+\frac{2c\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}{a}+\frac{2c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -c*(-a^2*x^2+1)^(1/2)/a+c/a*arctanh(1/(-a^2*x^2+1)^(1/2))+2*c/a*(-a^2*(x+1/
a)^2+2*a*(x+1/a))^(1/2)+2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^
2+2*a*(x+1/a))^(1/2))
```

maxima [A] time = 0.46, size = 70, normalized size = 1.40

$$ac\left(\frac{\arcsin\left(ax\right)}{a^2}+\frac{\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a^2}\right)+c\left(\frac{\arcsin\left(ax\right)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*c*(arcsin(a*x)/a^2 + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a^2) + c
*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a)
```

mupad [B] time = 0.06, size = 56, normalized size = 1.12

$$\frac{c\,\sqrt{1-a^2\,x^2}}{a}+\frac{c\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{a}+\frac{2\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (c*(1 - a^2*x^2)^(1/2))/a + (c*atanh((1 - a^2*x^2)^(1/2)))/a + (2*c*asinh(x
*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\left(-\frac{\sqrt{-a^2x^2+1}}{ax^2+x}\right)dx+\int\frac{ax\sqrt{-a^2x^2+1}}{ax^2+x}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c*(Integral(-sqrt(-a**2*x**2 + 1)/(a*x**2 + x), x) + Integral(a*x*sqrt(-a**
2*x**2 + 1)/(a*x**2 + x), x))/a
```

3.489 $\int \frac{e^{-\tanh^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=21

$$\frac{\sqrt{1-a^2x^2}}{ac}$$

[Out] (-a^2*x^2+1)^(1/2)/a/c

Rubi [A] time = 0.07, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6128, 261}

$$\frac{\sqrt{1-a^2x^2}}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))),x]

[Out] Sqrt[1 - a^2*x^2]/(a*c)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{a\int \frac{e^{-\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= -\frac{a\int \frac{x}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{\sqrt{1-a^2x^2}}{ac}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{\sqrt{1-a^2x^2}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))),x]
[Out] Sqrt[1 - a^2*x^2]/(a*c)
```

fricas [A] time = 0.55, size = 19, normalized size = 0.90

$$\frac{\sqrt{-a^2x^2+1}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="fricas")
[Out] sqrt(-a^2*x^2 + 1)/(a*c)
```

giac [A] time = 0.16, size = 19, normalized size = 0.90

$$\frac{\sqrt{-a^2x^2+1}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="giac")
[Out] sqrt(-a^2*x^2 + 1)/(a*c)
```

maple [A] time = 0.03, size = 20, normalized size = 0.95

$$\frac{\sqrt{-a^2x^2+1}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)/a/c
```

maxima [A] time = 0.33, size = 22, normalized size = 1.05

$$\frac{\sqrt{ax+1}\sqrt{-ax+1}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] sqrt(a*x + 1)*sqrt(-a*x + 1)/(a*c)
```

mupad [B] time = 0.04, size = 19, normalized size = 0.90

$$\frac{\sqrt{1-a^2\,x^2}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))*(a*x + 1)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(a*c)
```

sympy [A] time = 9.02, size = 53, normalized size = 2.52

$$a\left(\begin{cases} \tilde{\infty}\left(\begin{cases} \frac{x^2}{2} & \text{for } a^2=0 \\ -\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3a^2} & \text{otherwise} \end{cases}\right) & \text{for } c=0 \\ -\frac{x^2}{2c} & \text{for } a^2=0 \\ \frac{\sqrt{-a^2x^2+1}}{a^2c} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x),x)
```

```
[Out] a*Piecewise((zoo*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)
/(3*a**2), True)), Eq(c, 0)), (-x**2/(2*c), Eq(a**2, 0)), (sqrt(-a**2*x**2
+ 1)/(a**2*c), True))
```

3.490 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\sin^{-1}(ax)}{ac^2}$$

```
[Out] -arcsin(a*x)/a/c^2+(-a^2*x^2+1)^(1/2)/a/c^2+(-a^2*x^2+1)^(1/2)/a/c^2/(-a*x+
1)
```

Rubi [A] time = 0.18, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 1639, 12, 793, 216}

$$\frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)}+\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*c^2) + Sqrt[1 - a^2*x^2]/(a*c^2*(1 - a*x)) - ArcSin[a*
x]/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{e^{-\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= \frac{a^2\int \frac{x^2}{(1-ax)\sqrt{1-a^2x^2}}\,dx}{c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{ac^2} + \frac{\int \frac{a^3x}{(1-ax)\sqrt{1-a^2x^2}}\,dx}{a^2c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{ac^2} + \frac{a\int \frac{x}{(1-ax)\sqrt{1-a^2x^2}}\,dx}{c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{ac^2} + \frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$
$$= \frac{\sqrt{1-a^2x^2}}{ac^2} + \frac{\sqrt{1-a^2x^2}}{ac^2(1-ax)} - \frac{\sin^{-1}(ax)}{ac^2}$$

Mathematica [A] time = 0.10, size = 47, normalized size = 0.75

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{c^2}-\frac{1}{c^2(ax-1)}\right)}{a} - \frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^2),x]
[Out] (Sqrt[1 - a^2*x^2]*(c^(-2) - 1/(c^2*(-1 + a*x))))/a - ArcSin[a*x]/(a*c^2)
```

fricas [A] time = 0.47, size = 71, normalized size = 1.13

$$\frac{2\,ax + 2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(ax-2) - 2}{a^2c^2x - ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
[Out] (2*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(a*x - 2) - 2)/(a^2*c^2*x - a*c^2)
```

giac [A] time = 2.11, size = 72, normalized size = 1.14

$$-\frac{\arcsin(ax)\operatorname{sgn}(a)}{c^2|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^2}+\frac{2}{c^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c^2*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^2) + 2/(c^2*((sq
rt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [B] time = 0.05, size = 198, normalized size = 3.14

$$\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{2a^3c^2\left(x-\frac{1}{a}\right)^2}+\frac{5\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{4a\,c^2}-\frac{5\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{4c^2\sqrt{a^2}}+\frac{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2 \cdots}}{4a\,c^2 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x)
```

```
[Out] 1/2/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+5/4/a/c^2*(-a^2*(x
-1/a)^2-2*a*(x-1/a))^(1/2)-5/4/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(
x-1/a)^2-2*a*(x-1/a))^(1/2))+1/4/a/c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1
/4/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^2), x)
```

mupad [B] time = 0.86, size = 89, normalized size = 1.41

$$\frac{\sqrt{1-a^2\,x^2}}{a\,c^2}-\frac{\operatorname{asinh}\left(x\,\sqrt{-a^2}\right)}{c^2\,\sqrt{-a^2}}+\frac{\sqrt{1-a^2\,x^2}}{c^2\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^2*(a*x + 1)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(a*c^2) - asinh(x*(-a^2)^(1/2))/(c^2*(-a^2)^(1/2)) + (1
 - a^2*x^2)^(1/2)/(c^2*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2 \sqrt{-a^2x^2+1}}{a^3x^3-a^2x^2-ax+1}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 - a**2*x**2 - a*x + 1),
x)/c**2
```

3.491 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=94

$$-\frac{(ax+1)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{8(ax+1)}{3ac^3\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{2\sin^{-1}(ax)}{ac^3}$$

```
[Out] -1/3*(a*x+1)^2/a/c^3/(-a^2*x^2+1)^(3/2)-2*arcsin(a*x)/a/c^3+8/3*(a*x+1)/a/c
^3/(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1)^(1/2)/a/c^3
```

Rubi [A] time = 0.26, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 852, 1635, 641, 216}

$$-\frac{(ax+1)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{8(ax+1)}{3ac^3\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{2\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] -(1 + a*x)^2/(3*a*c^3*(1 - a^2*x^2)^(3/2)) + (8*(1 + a*x))/(3*a*c^3*Sqrt[1
- a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c^3) - (2*ArcSin[a*x])/(a*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{a^3\int \frac{e^{-\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$
$$= -\frac{a^3\int \frac{x^3}{(1-ax)^2\sqrt{1-a^2x^2}}\,dx}{c^3}$$
$$= -\frac{a^3\int \frac{x^3(1+ax)^2}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^3}$$
$$= -\frac{(1+ax)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^3\int \frac{(1+ax)\left(\frac{2}{a^3}+\frac{3x}{a^2}+\frac{3x^2}{a}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^3}$$
$$= -\frac{(1+ax)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{8(1+ax)}{3ac^3\sqrt{1-a^2x^2}} - \frac{a^3\int \frac{\frac{6}{a^3}+\frac{3x}{a^2}}{\sqrt{1-a^2x^2}}\,dx}{3c^3}$$
$$= -\frac{(1+ax)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{8(1+ax)}{3ac^3\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^3} - \frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^3}$$
$$= -\frac{(1+ax)^2}{3ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{8(1+ax)}{3ac^3\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^3} - \frac{2\sin^{-1}(ax)}{ac^3}$$

Mathematica [A] time = 0.14, size = 53, normalized size = 0.56

$$\frac{\frac{\sqrt{1-a^2x^2}\left(3a^2x^2-14ax+10\right)}{(ax-1)^2} - 6\sin^{-1}(ax)}{3ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(10 - 14*a*x + 3*a^2*x^2))/(-1 + a*x)^2 - 6*ArcSin[a*x]
)/(3*a*c^3)
```

fricas [A] time = 0.64, size = 107, normalized size = 1.14

$$\frac{10\,a^2x^2 - 20\,ax + 12\left(a^2x^2 - 2\,ax + 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(3\,a^2x^2 - 14\,ax + 10\right)\sqrt{-a^2x^2+1} + 10}{3\left(a^3c^3x^2 - 2\,a^2c^3x + ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(10*a^2*x^2 - 20*a*x + 12*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)) + (3*a^2*x^2 - 14*a*x + 10)*sqrt(-a^2*x^2 + 1) + 10)/(a^3*c
^3*x^2 - 2*a^2*c^3*x + a*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 242, normalized size = 2.57

$$\frac{5\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{4a^{3}c^{3}\left(x-\frac{1}{a}\right)^{2}}+\frac{17\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{8a\,c^{3}}-\frac{17\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}\right)}{8c^{3}\sqrt{a^{2}}}+\frac{\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}\right.}{6a^{4}c^{3}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x)
```

```
[Out] 5/4/a^3/c^3/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+17/8/a/c^3*(-a^2*(
x-1/a)^2-2*a*(x-1/a))^(1/2)-17/8/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/6/a^4/c^3/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x
-1/a))^(3/2)+1/8/a/c^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/8/c^3/(a^2)^(1/
2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^3), x)
```

mupad [B] time = 0.06, size = 139, normalized size = 1.48

$$\frac{\sqrt{1-a^2\,x^2}}{a\,c^3}-\frac{2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^3\,\sqrt{-a^2}}-\frac{a\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{8\,\sqrt{1-a^2\,x^2}}{3\,\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-a^2}-\frac{c^3\,\sqrt{-a^2}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^3*(a*x + 1)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(a*c^3) - (2*asinh(x*(-a^2)^(1/2)))/(c^3*(-a^2)^(1/2))
- (a*(1 - a^2*x^2)^(1/2))/(3*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (8*(1
 - a^2*x^2)^(1/2))/(3*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))
/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\int\frac{x^3\sqrt{-a^2x^2+1}}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3*sqrt(-a**2*x**2 + 1)/(a**4*x**4 - 2*a**3*x**3 + 2*a*x -
1), x)/c**3
```

3.492 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=125

$$\frac{(ax+1)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{6(ax+1)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}}+\frac{24(ax+1)}{5ac^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^4}-\frac{3\sin^{-1}(ax)}{ac^4}$$

```
[Out] 1/5*(a*x+1)^3/a/c^4/(-a^2*x^2+1)^(5/2)-6/5*(a*x+1)^2/a/c^4/(-a^2*x^2+1)^(3/
2)-3*arcsin(a*x)/a/c^4+24/5*(a*x+1)/a/c^4/(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1)^(
1/2)/a/c^4
```

Rubi [A] time = 0.35, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 852, 1635, 641, 216}

$$\frac{(ax+1)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{6(ax+1)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}}+\frac{24(ax+1)}{5ac^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^4}-\frac{3\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] (1 + a*x)^3/(5*a*c^4*(1 - a^2*x^2)^(5/2)) - (6*(1 + a*x)^2)/(5*a*c^4*(1 - a
^2*x^2)^(3/2)) + (24*(1 + a*x))/(5*a*c^4*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*
x^2]/(a*c^4) - (3*ArcSin[a*x])/(a*c^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{e^{-\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4}{(1-ax)^3\sqrt{1-a^2x^2}}\,dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4(1+ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^4}$$

$$= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{a^4\int \frac{(1+ax)^2\left(\frac{3}{a^4}+\frac{5x}{a^3}+\frac{5x^2}{a^2}+\frac{5x^3}{a}\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^4}$$

$$= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{a^4\int \frac{(1+ax)\left(\frac{27}{a^4}+\frac{30x}{a^3}+\frac{15x^2}{a^2}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^4}$$

$$= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^4\sqrt{1-a^2x^2}} - \frac{a^4\int \frac{\frac{45}{a^4}+\frac{15x}{a^3}}{\sqrt{1-a^2x^2}}\,dx}{15c^4}$$

$$= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^4} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^4}$$

$$= \frac{(1+ax)^3}{5ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^4\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^4} - \frac{3\sin^{-1}(ax)}{ac^4}$$

Mathematica [A] time = 0.17, size = 61, normalized size = 0.49

$$\frac{\frac{\sqrt{1-a^2x^2}\left(5a^3x^3-39a^2x^2+57ax-24\right)}{(ax-1)^3} - 15\sin^{-1}(ax)}{5ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-24 + 57*a*x - 39*a^2*x^2 + 5*a^3*x^3))/(-1 + a*x)^3 -
 15*ArcSin[a*x])/(5*a*c^4)
```

fricas [A] time = 0.53, size = 143, normalized size = 1.14

$$\frac{24\,a^3x^3 - 72\,a^2x^2 + 72\,ax + 30\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(5\,a^3x^3 - 39\,a^2x^2 + 57\,ax - 24\right.}{5\left(a^4c^4x^3 - 3\,a^3c^4x^2 + 3\,a^2c^4x - ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(24*a^3*x^3 - 72*a^2*x^2 + 72*a*x + 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1
)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (5*a^3*x^3 - 39*a^2*x^2 + 57*a*x
 - 24)*sqrt(-a^2*x^2 + 1) - 24)/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x
- a*c^4)
```

giac [A] time = 0.19, size = 180, normalized size = 1.44

$$-\frac{3\,\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{c^4|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^4}-\frac{2\left(\frac{80\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{120\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{70\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+\right.}{a^8x^4}\right.}{5\,c^4\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/(c^4*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^4) - 2/5*(80*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 120*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^2/(a^4*x^2) + 70*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 15*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 19)/(c^4*((sqrt(-a^2*x^2 + 1)*abs(a)
 + a)/(a^2*x) - 1)^5*abs(a))
```

maple [B] time = 0.05, size = 286, normalized size = 2.29

$$\frac{17\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{8a^3c^4\left(x-\frac{1}{a}\right)^2}+\frac{49\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{16a\,c^4}-\frac{49\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{16c^4\sqrt{a^2}}+\frac{11\left(-a^2\left(x\right.\right.}{20a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x)
```

```
[Out] 17/8/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+49/16/a/c^4*(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2)-49/16/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-
a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+11/20/a^4/c^4/(x-1/a)^3*(-a^2*(x-1/a)^2-2
*a*(x-1/a))^(3/2)+1/10/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)
+1/16/a/c^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/16/c^4/(a^2)^(1/2)*arctan(
(a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^4), x)
```

mupad [B] time = 0.84, size = 272, normalized size = 2.18

$$\frac{2\,a^4\,\sqrt{1-a^2\,x^2}}{15\,\left(a^7\,c^4\,x^2-2\,a^6\,c^4\,x+a^5\,c^4\right)}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^4\,\sqrt{-a^2}}-\frac{4\,a\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^4\,x^2-2\,a^3\,c^4\,x+a^2\,c^4\right)}+\frac{\sqrt{1-a^2\,x^2}}{a\,c^4}+\frac{24\,\sqrt{1-}}{5\,\sqrt{-a^2}\,\left(c^4\,x\,\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^4*(a*x + 1)),x)
```

```
[Out] (2*a^4*(1 - a^2*x^2)^(1/2))/(15*(a^5*c^4 - 2*a^6*c^4*x + a^7*c^4*x^2)) - (3
*asinh(x*(-a^2)^(1/2)))/(c^4*(-a^2)^(1/2)) - (4*a*(1 - a^2*x^2)^(1/2))/(3*(
a^2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) + (1 - a^2*x^2)^(1/2)/(a*c^4) + (24*(
1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2)
)/a)) + (1 - a^2*x^2)^(1/2)/(5*(-a^2)^(1/2)*(3*c^4*x*(-a^2)^(1/2) - (c^4*(-
a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1/2) - 3*a*c^4*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4\sqrt{-a^2x^2+1}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4*sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x
**3 + 2*a**2*x**2 - 3*a*x + 1), x)/c**4
```

3.493 $\int e^{-2\tanh^{-1}(ax)} \left(c-\frac{c}{ax}\right)^p \, dx$

Optimal. Leaf size=114

$$-\frac{\left(c-\frac{c}{ax}\right)^{p+2} {}_2F_1\left(1,p+2;p+3;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}+\frac{\left(c-\frac{c}{ax}\right)^{p+2} {}_2F_1\left(1,p+2;p+3;1-\frac{1}{ax}\right)}{ac^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{c^2}$$

```
[Out] -(c-c/a/x)^(2+p)*x/c^2-1/2*(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1/2*(a-
1/x)/a)/a/c^2/(2+p)+(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1-1/a/x)/a/c^2
```

Rubi [A] time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6133, 25, 514, 375, 103, 156, 65, 68}

$$-\frac{\left(c-\frac{c}{ax}\right)^{p+2} {}_2F_1\left(1,p+2;p+3;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}+\frac{\left(c-\frac{c}{ax}\right)^{p+2} {}_2F_1\left(1,p+2;p+3;1-\frac{1}{ax}\right)}{ac^2}-\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(((c - c/(a*x))^(2 + p)*x)/c^2) - ((c - c/(a*x))^(2 + p)*Hypergeometric2F1
[1, 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*a*c^2*(2 + p)) + ((c - c/(a*x))^(
2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/(a*c^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \int \frac{\left(c-\frac{c}{ax}\right)^p(1-ax)}{1+ax}dx$$
$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}x}{1+ax}dx}{c}$$
$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}}{a+\frac{1}{x}}dx}{c}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x^2(a+x)}dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2} - \frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}\left(c(2+p)+\frac{c(1+p)x}{a}\right)}{x(a+x)}dx, x, \frac{1}{x}\right)}{c^2}$$
$$= -\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2} + \frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{a+x}dx, x, \frac{1}{x}\right)}{ac} - \frac{(2+p)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x}dx, x, \frac{1}{x}\right)}{ac}$$
$$= -\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2} - \frac{\left(c-\frac{c}{ax}\right)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)} + \frac{\left(c-\frac{c}{ax}\right)^{2+p}\,{}_2F_1\left(1,2+\right.}{ac^2}$$

Mathematica [A] time = 0.05, size = 87, normalized size = 0.76

$$-\frac{(ax-1)^2\left(c-\frac{c}{ax}\right)^p\left({}_2F_1\left(1,p+2;p+3;\frac{a-\frac{1}{x}}{2a}\right)+2(p+2)\left(ax-{}_2F_1\left(1,p+2;p+3;1-\frac{1}{ax}\right)\right)\right)}{2a^3(p+2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/2*((c - c/(a*x))^p*(-1 + a*x)^2*(Hypergeometric2F1[1, 2 + p, 3 + p, (a -
 x^(-1))/(2*a)] + 2*(2 + p)*(a*x - Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1
/(a*x)])))/(a^3*(2 + p)*x^2)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax-1)\left(\frac{acx-c}{ax}\right)^p}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-(a*x - 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] integrate(-(a^2*x^2 - 1)*(c - c/(a*x))^p/(a*x + 1)^2, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^p\left(-a^2x^2+1\right)}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x)
[Out] int((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^p/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-\frac{c}{a\,x}\right)^p\,\left(a^2\,x^2-1\right)}{(a\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^p*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a*x))^p*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\left(c-\frac{c}{ax}\right)^{p}}{ax+1}\right)dx-\int\frac{ax\left(c-\frac{c}{ax}\right)^{p}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-(c - c/(a*x))**p/(a*x + 1), x) - Integral(a*x*(c - c/(a*x))**p/(
a*x + 1), x)
```

3.494 $\int e^{-2\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^4 \, dx$

Optimal. Leaf size=66

$$-\frac{c^4}{3a^4x^3} + \frac{3c^4}{a^3x^2} - \frac{16c^4}{a^2x} - \frac{26c^4\log(x)}{a} + \frac{32c^4\log(ax+1)}{a} + c^4(-x)$$

```
[Out] -1/3*c^4/a^4/x^3+3*c^4/a^3/x^2-16*c^4/a^2/x-c^4*x-26*c^4*ln(x)/a+32*c^4*ln(
a*x+1)/a
```

Rubi [A] time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{3c^4}{a^3x^2} - \frac{c^4}{3a^4x^3} - \frac{16c^4}{a^2x} - \frac{26c^4\log(x)}{a} + \frac{32c^4\log(ax+1)}{a} + c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -c^4/(3*a^4*x^3) + (3*c^4)/(a^3*x^2) - (16*c^4)/(a^2*x) - c^4*x - (26*c^4*L
og[x])/a + (32*c^4*Log[1 + a*x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = \frac{c^4\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$
$$= \frac{c^4\int\frac{(1-ax)^5}{x^4(1+ax)}dx}{a^4}$$
$$= \frac{c^4\int\left(-a^4+\frac{1}{x^4}-\frac{6a}{x^3}+\frac{16a^2}{x^2}-\frac{26a^3}{x}+\frac{32a^4}{1+ax}\right)dx}{a^4}$$
$$= -\frac{c^4}{3a^4x^3}+\frac{3c^4}{a^3x^2}-\frac{16c^4}{a^2x}-c^4x-\frac{26c^4\log(x)}{a}+\frac{32c^4\log(1+ax)}{a}$$

Mathematica [A] time = 0.18, size = 68, normalized size = 1.03

$$-\frac{c^4}{3a^4x^3}+\frac{3c^4}{a^3x^2}-\frac{16c^4}{a^2x}-\frac{26c^4\log(ax)}{a}+\frac{32c^4\log(ax+1)}{a}+c^4(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/3*c^4/(a^4*x^3) + (3*c^4)/(a^3*x^2) - (16*c^4)/(a^2*x) - c^4*x - (26*c^4
*Log[a*x])/a + (32*c^4*Log[1 + a*x])/a
```

fricas [A] time = 0.45, size = 71, normalized size = 1.08

$$-\frac{3\,a^4c^4x^4-96\,a^3c^4x^3\log(ax+1)+78\,a^3c^4x^3\log(x)+48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^4*c^4*x^4 - 96*a^3*c^4*x^3*log(a*x + 1) + 78*a^3*c^4*x^3*log(x) +
 48*a^2*c^4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

giac [A] time = 0.17, size = 112, normalized size = 1.70

$$-\frac{6\,c^4\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}-\frac{26\,c^4\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}+\frac{\left(3\,c^4+\frac{49\,c^4}{ax+1}-\frac{117\,c^4}{(ax+1)^2}+\frac{66\,c^4}{(ax+1)^3}\right)(ax+1)}{3\,a\left(\frac{1}{ax+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -6*c^4*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a - 26*c^4*log(abs(-1/(a*x +
1) + 1))/a + 1/3*(3*c^4 + 49*c^4/(a*x + 1) - 117*c^4/(a*x + 1)^2 + 66*c^4/(
a*x + 1)^3)*(a*x + 1)/(a*(1/(a*x + 1) - 1)^3)
```

maple [A] time = 0.04, size = 65, normalized size = 0.98

$$-\frac{c^4}{3a^4x^3}+\frac{3c^4}{x^2a^3}-\frac{16c^4}{a^2x}-c^4x-\frac{26c^4\ln(x)}{a}+\frac{32c^4\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/3*c^4/a^4/x^3+3*c^4/x^2/a^3-16*c^4/a^2/x-c^4*x-26*c^4*ln(x)/a+32*c^4*ln(
a*x+1)/a
```

maxima [A] time = 0.31, size = 61, normalized size = 0.92

$$-c^4x+\frac{32\,c^4\log(ax+1)}{a}-\frac{26\,c^4\log(x)}{a}-\frac{48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^4*x + 32*c^4*log(a*x + 1)/a - 26*c^4*log(x)/a - 1/3*(48*a^2*c^4*x^2 - 9*
a*c^4*x + c^4)/(a^4*x^3)
```

mupad [B] time = 0.11, size = 63, normalized size = 0.95

$$\frac{32\,c^4\,\ln(a\,x+1)}{a}-\frac{16\,a^2\,c^4\,x^2-3\,a\,c^4\,x+\frac{c^4}{3}}{a^4\,x^3}-\frac{26\,c^4\,\ln(x)}{a}-c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^4*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] (32*c^4*log(a*x + 1))/a - (c^4/3 + 16*a^2*c^4*x^2 - 3*a*c^4*x)/(a^4*x^3) -
(26*c^4*log(x))/a - c^4*x
```

sympy [A] time = 0.47, size = 58, normalized size = 0.88

$$-c^4x-\frac{2c^4\left(13\log(x)-16\log\left(x+\frac{1}{a}\right)\right)}{a}-\frac{48a^2c^4x^2-9ac^4x+c^4}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**4*x - 2*c**4*(13*log(x) - 16*log(x + 1/a))/a - (48*a**2*c**4*x**2 - 9*a
*c**4*x + c**4)/(3*a**4*x**3)
```

3.495 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=55

$$\frac{c^3}{2a^3x^2}-\frac{5c^3}{a^2x}-\frac{11c^3\log(x)}{a}+\frac{16c^3\log(ax+1)}{a}+c^3(-x)$$

```
[Out] 1/2*c^3/a^3/x^2-5*c^3/a^2/x-c^3*x-11*c^3*ln(x)/a+16*c^3*ln(a*x+1)/a
```

Rubi [A] time = 0.12, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$\frac{c^3}{2a^3x^2}-\frac{5c^3}{a^2x}-\frac{11c^3\log(x)}{a}+\frac{16c^3\log(ax+1)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c^3/(2*a^3*x^2) - (5*c^3)/(a^2*x) - c^3*x - (11*c^3*Log[x])/a + (16*c^3*Log
[1 + a*x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\frac{c^3\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}$$
$$=-\frac{c^3\int\frac{(1-ax)^4}{x^3(1+ax)}\,dx}{a^3}$$
$$=-\frac{c^3\int\left(a^3+\frac{1}{x^3}-\frac{5a}{x^2}+\frac{11a^2}{x}-\frac{16a^3}{1+ax}\right)dx}{a^3}$$
$$=\frac{c^3}{2a^3x^2}-\frac{5c^3}{a^2x}-c^3x-\frac{11c^3\log(x)}{a}+\frac{16c^3\log(1+ax)}{a}$$

Mathematica [A] time = 0.13, size = 57, normalized size = 1.04

$$\frac{c^3}{2a^3x^2}-\frac{5c^3}{a^2x}-\frac{11c^3\log(ax)}{a}+\frac{16c^3\log(ax+1)}{a}+c^3(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c^3/(2*a^3*x^2) - (5*c^3)/(a^2*x) - c^3*x - (11*c^3*Log[a*x])/a + (16*c^3*L
og[1 + a*x])/a
```

fricas [A] time = 0.42, size = 62, normalized size = 1.13

$$-\frac{2\,a^3c^3x^3-32\,a^2c^3x^2\log(ax+1)+22\,a^2c^3x^2\log(x)+10\,ac^3x-c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*c^3*x^3 - 32*a^2*c^3*x^2*log(a*x + 1) + 22*a^2*c^3*x^2*log(x) +
 10*a*c^3*x - c^3)/(a^3*x^2)
```

giac [A] time = 3.79, size = 100, normalized size = 1.82

$$-\frac{5\,c^3\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}-\frac{11\,c^3\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}-\frac{\left(2\,c^3+\frac{7\,c^3}{ax+1}-\frac{10\,c^3}{(ax+1)^2}\right)(ax+1)}{2\,a\left(\frac{1}{ax+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -5*c^3*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a - 11*c^3*log(abs(-1/(a*x +
1) + 1))/a - 1/2*(2*c^3 + 7*c^3/(a*x + 1) - 10*c^3/(a*x + 1)^2)*(a*x + 1)/(
a*(1/(a*x + 1) - 1)^2)
```

maple [A] time = 0.04, size = 54, normalized size = 0.98

$$\frac{c^3}{2x^2a^3} - \frac{5c^3}{a^2x} - c^3x - \frac{11c^3\ln(x)}{a} + \frac{16c^3\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/2*c^3/x^2/a^3-5*c^3/a^2/x-c^3*x-11*c^3*ln(x)/a+16*c^3*ln(a*x+1)/a
```

maxima [A] time = 0.31, size = 52, normalized size = 0.95

$$-c^3x + \frac{16\,c^3\log(ax+1)}{a} - \frac{11\,c^3\log(x)}{a} - \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^3*x + 16*c^3*log(a*x + 1)/a - 11*c^3*log(x)/a - 1/2*(10*a*c^3*x - c^3)/(
a^3*x^2)
```

mupad [B] time = 0.88, size = 51, normalized size = 0.93

$$\frac{\frac{c^3}{2} - 5\,a\,c^3\,x}{a^3\,x^2} - c^3\,x - \frac{11\,c^3\,\ln(x)}{a} + \frac{16\,c^3\,\ln(a\,x+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^3*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] (c^3/2 - 5*a*c^3*x)/(a^3*x^2) - c^3*x - (11*c^3*log(x))/a + (16*c^3*log(a*x
 + 1))/a
```

sympy [A] time = 0.36, size = 44, normalized size = 0.80

$$-c^3x - \frac{c^3\left(11\log(x) - 16\log\left(x + \frac{1}{a}\right)\right)}{a} - \frac{10ac^3x - c^3}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**3*x - c**3*(11*log(x) - 16*log(x + 1/a))/a - (10*a*c**3*x - c**3)/(2*a*
*3*x**2)
```

3.496 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=42

$$-\frac{c^2}{a^2x}-\frac{4c^2\log(x)}{a}+\frac{8c^2\log(ax+1)}{a}+c^2(-x)$$

```
[Out] -c^2/a^2/x-c^2*x-4*c^2*ln(x)/a+8*c^2*ln(a*x+1)/a
```

Rubi [A] time = 0.11, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$-\frac{c^2}{a^2x}-\frac{4c^2\log(x)}{a}+\frac{8c^2\log(ax+1)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^2/(a^2*x)) - c^2*x - (4*c^2*Log[x])/a + (8*c^2*Log[1 + a*x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$
$$= \frac{c^2\int \frac{(1-ax)^3}{x^2(1+ax)}dx}{a^2}$$
$$= \frac{c^2\int \left(-a^2+\frac{1}{x^2}-\frac{4a}{x}+\frac{8a^2}{1+ax}\right)dx}{a^2}$$
$$= -\frac{c^2}{a^2x}-c^2x-\frac{4c^2\log(x)}{a}+\frac{8c^2\log(1+ax)}{a}$$

Mathematica [A]   time = 0.10, size = 44, normalized size = 1.05

$$-\frac{c^2}{a^2x}-\frac{4c^2\log(ax)}{a}+\frac{8c^2\log(ax+1)}{a}+c^2(-x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^(2*ArcTanh[a*x]),x]
[Out] -(c^2/(a^2*x)) - c^2*x - (4*c^2*Log[a*x])/a + (8*c^2*Log[1 + a*x])/a
```

fricas [A]   time = 0.43, size = 44, normalized size = 1.05

$$-\frac{a^2c^2x^2-8\,ac^2x\log(ax+1)+4\,ac^2x\log(x)+c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -(a^2*c^2*x^2 - 8*a*c^2*x*log(a*x + 1) + 4*a*c^2*x*log(x) + c^2)/(a^2*x)
```

giac [A]   time = 1.98, size = 72, normalized size = 1.71

$$-\frac{4\,c^2\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}-\frac{4\,c^2\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}+\frac{(ax+1)c^2}{a\left(\frac{1}{ax+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

[Out] -4*c^2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a - 4*c^2*log(abs(-1/(a*x + 1
) + 1))/a + (a*x + 1)*c^2/(a*(1/(a*x + 1) - 1))

maple [A] time = 0.03, size = 43, normalized size = 1.02

$$-\frac{c^2}{a^2 x}-c^2 x-\frac{4c^2 \ln(x)}{a}+\frac{8c^2 \ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a/x)^2/(a*x+1)^2*(-a^2*x^2+1),x)

[Out] -c^2/a^2/x-c^2*x-4*c^2*ln(x)/a+8*c^2*ln(a*x+1)/a

maxima [A] time = 0.31, size = 42, normalized size = 1.00

$$-c^2 x+\frac{8\,c^2 \log(ax+1)}{a}-\frac{4\,c^2 \log(x)}{a}-\frac{c^2}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")

[Out] -c^2*x + 8*c^2*log(a*x + 1)/a - 4*c^2*log(x)/a - c^2/(a^2*x)

mupad [B] time = 0.87, size = 42, normalized size = 1.00

$$\frac{8\,c^2\,\ln(a\,x+1)}{a}-\frac{c^2}{a^2\,x}-\frac{4\,c^2\,\ln(x)}{a}-c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((c - c/(a*x))^2*(a^2*x^2 - 1))/(a*x + 1)^2,x)

[Out] (8*c^2*log(a*x + 1))/a - c^2/(a^2*x) - (4*c^2*log(x))/a - c^2*x

sympy [A] time = 0.29, size = 32, normalized size = 0.76

$$-c^2 x-\frac{4c^2\left(\log(x)-2\log\left(x+\frac{1}{a}\right)\right)}{a}-\frac{c^2}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)**2/(a*x+1)**2*(-a**2*x**2+1),x)

[Out] -c**2*x - 4*c**2*(log(x) - 2*log(x + 1/a))/a - c**2/(a**2*x)

3.497 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=25

$$-\frac{c\log(x)}{a}+\frac{4c\log(ax+1)}{a}-cx$$

```
[Out] -c*x-c*ln(x)/a+4*c*ln(a*x+1)/a
```

Rubi [A] time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6131, 6129, 72}

$$-\frac{c\log(x)}{a}+\frac{4c\log(ax+1)}{a}-cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c*x) - (c*Log[x])/a + (4*c*Log[1 + a*x])/a
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{c\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$= -\frac{c\int \frac{(1-ax)^2}{x(1+ax)}\,dx}{a}$$
$$= -\frac{c\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a}$$
$$= -cx-\frac{c\log(x)}{a}+\frac{4c\log(1+ax)}{a}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 1.00

$$-\frac{c\log(x)}{a}+\frac{4c\log(ax+1)}{a}-cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))/E^(2*ArcTanh[a*x]),x]
[Out] -(c*x) - (c*Log[x])/a + (4*c*Log[1 + a*x])/a
```

fricas [A] time = 0.51, size = 23, normalized size = 0.92

$$-\frac{acx-4\,c\log\left(ax+1\right)+c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -(a*c*x - 4*c*log(a*x + 1) + c*log(x))/a
```

giac [B] time = 0.16, size = 56, normalized size = 2.24

$$-\frac{(ax+1)c}{a}-\frac{3\,c\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}-\frac{c\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] -(a*x + 1)*c/a - 3*c*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a - c*log(abs(-
1/(a*x + 1) + 1))/a
```

maple [A] time = 0.03, size = 26, normalized size = 1.04

$$-cx-\frac{c\ln(x)}{a}+\frac{4c\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -c*x-c*ln(x)/a+4*c*ln(a*x+1)/a
```

maxima [A] time = 0.31, size = 25, normalized size = 1.00

$$-cx+\frac{4c\log(ax+1)}{a}-\frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c*x + 4*c*log(a*x + 1)/a - c*log(x)/a
```

mupad [B] time = 0.07, size = 25, normalized size = 1.00

$$\frac{4c\ln(ax+1)}{a}-\frac{c\ln(x)}{a}-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] (4*c*log(a*x + 1))/a - (c*log(x))/a - c*x
```

sympy [A] time = 0.22, size = 19, normalized size = 0.76

$$-cx-\frac{c\left(\log(x)-4\log\left(x+\frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c*x - c*(log(x) - 4*log(x + 1/a))/a
```

3.498 $\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=20

$$\frac{\log(ax+1)}{ac}-\frac{x}{c}$$

[Out] -x/c+ln(a*x+1)/a/c

Rubi [A] time = 0.09, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 43}

$$\frac{\log(ax+1)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))),x]

[Out] -(x/c) + Log[1 + a*x]/(a*c)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{a\int \frac{e^{-2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= -\frac{a\int \frac{x}{1+ax}\,dx}{c}$$
$$= -\frac{a\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{c}$$
$$= -\frac{x}{c}+\frac{\log(1+ax)}{ac}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.90

$$\frac{\log(ax+1)-ax}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))),x]
[Out] (-(a*x) + Log[1 + a*x])/(a*c)
```

fricas [A] time = 0.55, size = 20, normalized size = 1.00

$$-\frac{ax-\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x),x, algorithm="fricas")
[Out] -(a*x - log(a*x + 1))/(a*c)
```

giac [B] time = 1.77, size = 41, normalized size = 2.05

$$-\frac{ax+1}{ac}-\frac{\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x),x, algorithm="giac")
[Out] -(a*x + 1)/(a*c) - log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c)
```

maple [A] time = 0.03, size = 21, normalized size = 1.05

$$-\frac{x}{c}+\frac{\ln \left(a x +1\right)}{a c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x),x)
[Out] -x/c+ln(a*x+1)/a/c
```

maxima [A] time = 0.35, size = 20, normalized size = 1.00

$$-\frac{x}{c}+\frac{\log \left(a x+1\right)}{a c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x),x, algorithm="maxima")
[Out] -x/c + log(a*x + 1)/(a*c)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.90

$$\frac{\ln \left(a\,x+1\right)-a\,x}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a*x))*(a*x + 1)^2),x)
[Out] (log(a*x + 1) - a*x)/(a*c)
```

sympy [A] time = 0.11, size = 19, normalized size = 0.95

$$-a \left(\frac{x}{a c} - \frac{\log{\left(a x + 1 \right)}}{a^{2} c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x),x)
[Out] -a*(x/(a*c) - log(a*x + 1)/(a**2*c))
```

3.499 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=18

$$\frac{\tanh^{-1}(ax)}{ac^2} - \frac{x}{c^2}$$

```
[Out] -x/c^2+arctanh(a*x)/a/c^2
```

Rubi [A] time = 0.11, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6129, 72, 207}

$$\frac{\tanh^{-1}(ax)}{ac^2} - \frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] -(x/c^2) + ArcTanh[a*x]/(a*c^2)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
```

```
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= \frac{a^2\int \frac{x^2}{(1-ax)(1+ax)}\,dx}{c^2}$$
$$= \frac{a^2\int \left(-\frac{1}{a^2}-\frac{1}{a^2\left(-1+a^2x^2\right)}\right)dx}{c^2}$$
$$= -\frac{x}{c^2}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{c^2}$$
$$= -\frac{x}{c^2}+\frac{\tanh^{-1}(ax)}{ac^2}$$

Mathematica [B] time = 0.08, size = 40, normalized size = 2.22

$$-\frac{\log(1-ax)}{2ac^2}+\frac{\log(ax+1)}{2ac^2}-\frac{x}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^2),x]
[Out] -(x/c^2) - Log[1 - a*x]/(2*a*c^2) + Log[1 + a*x]/(2*a*c^2)
```

fricas [A] time = 0.52, size = 27, normalized size = 1.50

$$-\frac{2\,ax-\log\left(ax+1\right)+\log\left(ax-1\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="fricas")
[Out] -1/2*(2*a*x - log(a*x + 1) + log(a*x - 1))/(a*c^2)
```

giac [A] time = 0.16, size = 35, normalized size = 1.94

$$-\frac{ax+1}{ac^2}-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -(a*x + 1)/(a*c^2) - 1/2*log(abs(-2/(a*x + 1) + 1))/(a*c^2)
```

maple [A] time = 0.03, size = 36, normalized size = 2.00

$$-\frac{x}{c^2}-\frac{\ln \left(a x-1\right)}{2 a\, c^2}+\frac{\ln \left(a x+1\right)}{2 a\, c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^2,x)
```

```
[Out] -x/c^2-1/2/a/c^2*ln(a*x-1)+1/2*ln(a*x+1)/a/c^2
```

maxima [A] time = 0.38, size = 35, normalized size = 1.94

$$-\frac{x}{c^2}+\frac{\log \left(a x+1\right)}{2\, a c^2}-\frac{\log \left(a x-1\right)}{2\, a c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -x/c^2 + 1/2*log(a*x + 1)/(a*c^2) - 1/2*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.86, size = 16, normalized size = 0.89

$$\frac{\mathrm{atanh}\left(a\,x\right)-a\,x}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a*x))^2*(a*x + 1)^2),x)
```

```
[Out] (atanh(a*x) - a*x)/(a*c^2)
```

sympy [B] time = 0.15, size = 36, normalized size = 2.00

$$-a^{2}\left(\frac{x}{a^{2}c^{2}}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{a^{3}c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**2,x)
```

```
[Out] -a**2*(x/(a**2*c**2) + (log(x - 1/a)/2 - log(x + 1/a)/2)/(a**3*c**2))
```

3.500 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=58

$$-\frac{1}{2ac^3(1-ax)}-\frac{5\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

```
[Out] -x/c^3-1/2/a/c^3/(-a*x+1)-5/4*ln(-a*x+1)/a/c^3+1/4*ln(a*x+1)/a/c^3
```

Rubi [A] time = 0.12, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$-\frac{1}{2ac^3(1-ax)}-\frac{5\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -(x/c^3) - 1/(2*a*c^3*(1 - a*x)) - (5*Log[1 - a*x])/(4*a*c^3) + Log[1 + a*x
]/(4*a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{a^3\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$

$$= -\frac{a^3\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{c^3}$$

$$= -\frac{a^3\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c^3}$$

$$= -\frac{x}{c^3}-\frac{1}{2ac^3(1-ax)}-\frac{5\log(1-ax)}{4ac^3}+\frac{\log(1+ax)}{4ac^3}$$

Mathematica [A] time = 0.11, size = 57, normalized size = 0.98

$$\frac{1}{2ac^3(ax-1)}-\frac{5\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -(x/c^3) + 1/(2*a*c^3*(-1 + a*x)) - (5*Log[1 - a*x])/(4*a*c^3) + Log[1 + a*
x]/(4*a*c^3)
```

fricas [A] time = 0.40, size = 59, normalized size = 1.02

$$-\frac{4\,a^2x^2-4\,ax-(ax-1)\log(ax+1)+5\,(ax-1)\log(ax-1)-2}{4\left(a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
 - 2)/(a^2*c^3*x - a*c^3)
```

giac [A] time = 0.20, size = 85, normalized size = 1.47

$$-\frac{(ax+1)\left(\frac{9}{ax+1}-4\right)}{4\,ac^3\left(\frac{2}{ax+1}-1\right)}+\frac{\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac^3}-\frac{5\,\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a*x + 1)*(9/(a*x + 1) - 4)/(a*c^3*(2/(a*x + 1) - 1)) + log(abs(a*x +
1)/((a*x + 1)^2*abs(a)))/(a*c^3) - 5/4*log(abs(-2/(a*x + 1) + 1))/(a*c^3)
```

maple [A] time = 0.03, size = 51, normalized size = 0.88

$$-\frac{x}{c^{3}}+\frac{1}{2 a\,c^{3} \left(a x -1\right)}-\frac{5 \ln \left(a x -1\right)}{4 c^{3} a}+\frac{\ln \left(a x +1\right)}{4 a\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^3,x)
```

```
[Out] -x/c^3+1/2/a/c^3/(a*x-1)-5/4/c^3/a*ln(a*x-1)+1/4*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.34, size = 54, normalized size = 0.93

$$\frac{1}{2 \left(a^{2} c^{3} x-a c^{3}\right)}-\frac{x}{c^{3}}+\frac{\log \left(a x+1\right)}{4 \, a c^{3}}-\frac{5 \, \log \left(a x-1\right)}{4 \, a c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/2/(a^2*c^3*x - a*c^3) - x/c^3 + 1/4*log(a*x + 1)/(a*c^3) - 5/4*log(a*x -
1)/(a*c^3)
```

mupad [B] time = 0.10, size = 53, normalized size = 0.91

$$\frac{\ln \left(a\,x+1\right)}{4\,a\,c^3}-\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)}-\frac{5\,\ln \left(a\,x-1\right)}{4\,a\,c^3}-\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a*x))^3*(a*x + 1)^2),x)
```

```
[Out] log(a*x + 1)/(4*a*c^3) - 1/(2*a*(c^3 - a*c^3*x)) - (5*log(a*x - 1))/(4*a*c^
3) - x/c^3
```

sympy [A] time = 0.31, size = 58, normalized size = 1.00

$$- a^{3} \left(- \frac{1}{2 a^{5} c^{3} x - 2 a^{4} c^{3}} + \frac{x}{a^{3} c^{3}} + \frac{\frac{5 \log {\left(x - \frac{1}{a} \right)}}{4} - \frac{\log {\left(x + \frac{1}{a} \right)}}{4}}{a^{4} c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**3,x)
```

```
[Out] -a**3*(-1/(2*a**5*c**3*x - 2*a**4*c**3) + x/(a**3*c**3) + (5*log(x - 1/a)/4
 - log(x + 1/a)/4)/(a**4*c**3))
```

3.501 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} \, dx$

Optimal. Leaf size=76

$$-\frac{7}{4ac^4(1-ax)}+\frac{1}{4ac^4(1-ax)^2}-\frac{17\log(1-ax)}{8ac^4}+\frac{\log(ax+1)}{8ac^4}-\frac{x}{c^4}$$

```
[Out] -x/c^4+1/4/a/c^4/(-a*x+1)^2-7/4/a/c^4/(-a*x+1)-17/8*ln(-a*x+1)/a/c^4+1/8*ln
(a*x+1)/a/c^4
```

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 88}

$$-\frac{7}{4ac^4(1-ax)}+\frac{1}{4ac^4(1-ax)^2}-\frac{17\log(1-ax)}{8ac^4}+\frac{\log(ax+1)}{8ac^4}-\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] -(x/c^4) + 1/(4*a*c^4*(1 - a*x)^2) - 7/(4*a*c^4*(1 - a*x)) - (17*Log[1 - a*
x])/(8*a*c^4) + Log[1 + a*x]/(8*a*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^4}$$

$$= \frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^4}$$

$$= -\frac{x}{c^4}+\frac{1}{4ac^4(1-ax)^2}-\frac{7}{4ac^4(1-ax)}-\frac{17\log(1-ax)}{8ac^4}+\frac{\log(1+ax)}{8ac^4}$$

Mathematica [A] time = 0.15, size = 69, normalized size = 0.91

$$\frac{-8a^3x^3+16a^2x^2+6ax-17(ax-1)^2\log(1-ax)+(ax-1)^2\log(ax+1)-12}{8ac^4(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (-12 + 6*a*x + 16*a^2*x^2 - 8*a^3*x^3 - 17*(-1 + a*x)^2*Log[1 - a*x] + (-1
+ a*x)^2*Log[1 + a*x])/(8*a*c^4*(-1 + a*x)^2)
```

fricas [A] time = 0.41, size = 93, normalized size = 1.22

$$-\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+17\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+12}{8\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a
*c^4)
```

giac [A] time = 0.18, size = 95, normalized size = 1.25

$$\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac^4}-\frac{17\,\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^4}+\frac{(ax+1)\left(\frac{77}{ax+1}-\frac{88}{(ax+1)^2}-16\right)}{16\,ac^4\left(\frac{2}{ax+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c^4) - 17/8*log(abs(-2/(a*x + 1
) + 1))/(a*c^4) + 1/16*(a*x + 1)*(77/(a*x + 1) - 88/(a*x + 1)^2 - 16)/(a*c^
4*(2/(a*x + 1) - 1)^2)
```

maple [A] time = 0.04, size = 66, normalized size = 0.87

$$-\frac{x}{c^{4}}+\frac{1}{4 a\,c^{4} \left(a x -1\right)^{2}}+\frac{7}{4 a\,c^{4} \left(a x -1\right)}-\frac{17 \ln \left(a x -1\right)}{8 a\,c^{4}}+\frac{\ln \left(a x +1\right)}{8 a\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^4,x)
```

```
[Out] -x/c^4+1/4/a/c^4/(a*x-1)^2+7/4/a/c^4/(a*x-1)-17/8/a/c^4*ln(a*x-1)+1/8*ln(a*
x+1)/a/c^4
```

maxima [A] time = 0.32, size = 70, normalized size = 0.92

$$\frac{7 \, a x - 6}{4 \, {\left(a^{3} c^{4} x^{2} - 2 \, a^{2} c^{4} x + a c^{4}\right)}} - \frac{x}{c^{4}} + \frac{\log\left(a x + 1\right)}{8 \, a c^{4}} - \frac{17 \, \log\left(a x - 1\right)}{8 \, a c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/4*(7*a*x - 6)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) - x/c^4 + 1/8*log(a*x +
 1)/(a*c^4) - 17/8*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.10, size = 68, normalized size = 0.89

$$\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}-\frac{x}{c^4}-\frac{17\,\ln\left(a\,x-1\right)}{8\,a\,c^4}+\frac{\ln\left(a\,x+1\right)}{8\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a*x))^4*(a*x + 1)^2),x)
```

```
[Out] ((7*x)/4 - 3/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) - x/c^4 - (17*log(a*x -
 1))/(8*a*c^4) + log(a*x + 1)/(8*a*c^4)
```

sympy [A] time = 0.39, size = 75, normalized size = 0.99

$$- a^{4} \left(\frac{- 7 a x + 6}{4 a^{7} c^{4} x^{2} - 8 a^{6} c^{4} x + 4 a^{5} c^{4}} + \frac{x}{a^{4} c^{4}} + \frac{\frac{17 \log{\left(x - \frac{1}{a} \right)}}{8} - \frac{\log{\left(x + \frac{1}{a} \right)}}{8}}{a^{5} c^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**4,x)
```

```
[Out] -a**4*((-7*a*x + 6)/(4*a**7*c**4*x**2 - 8*a**6*c**4*x + 4*a**5*c**4) + x/(a
**4*c**4) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**4))
```

3.502 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=140

$$-\frac{c^3\sqrt{1-a^2x^2}}{a}-\frac{6c^3\sqrt{1-a^2x^2}}{a^2x}-\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{33c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{6c^3\sin^{-1}(ax)}{a}$$

```
[Out] -6*c^3*arcsin(a*x)/a+33/2*c^3*arctanh((-a^2*x^2+1)^(1/2))/a-32*c^3*(-a*x+1)
/a/(-a^2*x^2+1)^(1/2)-c^3*(-a^2*x^2+1)^(1/2)/a+1/2*c^3*(-a^2*x^2+1)^(1/2)/a
^3/x^2-6*c^3*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.37, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6131, 6128, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$-\frac{c^3\sqrt{1-a^2x^2}}{a}-\frac{6c^3\sqrt{1-a^2x^2}}{a^2x}+\frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2}-\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{33c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{6c^3\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-32*c^3*(1 - a*x))/(a*Sqrt[1 - a^2*x^2]) - (c^3*Sqrt[1 - a^2*x^2])/a + (c^
3*Sqrt[1 - a^2*x^2])/(2*a^3*x^2) - (6*c^3*Sqrt[1 - a^2*x^2])/(a^2*x) - (6*c
^3*ArcSin[a*x])/a + (33*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
```

```
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{c^3\int\frac{e^{-3\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$

$$= -\frac{c^3\int\frac{(1-ax)^6}{x^3(1-a^2x^2)^{3/2}}dx}{a^3}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} + \frac{c^3\int\frac{-1+6ax-16a^2x^2-6a^3x^3+a^4x^4}{x^3\sqrt{1-a^2x^2}}dx}{a^3}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{c^3\int\frac{-12a+33a^2x+12a^3x^2-2a^4x^3}{x^2\sqrt{1-a^2x^2}}dx}{2a^3}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} + \frac{c^3\int\frac{-33a^2-12a^3x+2a^4x^2}{x\sqrt{1-a^2x^2}}dx}{2a^3}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \frac{c^3\int\frac{33a^4+12a^5x}{x\sqrt{1-a^2x^2}}}{2a^5}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \left(6c^3\right)\int\frac{1}{\sqrt{1-}}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \frac{6c^3\sin^{-1}(ax)}{a}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \frac{6c^3\sin^{-1}(ax)}{a}$$

$$= -\frac{32c^3(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^3\sqrt{1-a^2x^2}}{a} + \frac{c^3\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{6c^3\sqrt{1-a^2x^2}}{a^2x} - \frac{6c^3\sin^{-1}(ax)}{a}$$

Mathematica [A] time = 0.30, size = 93, normalized size = 0.66

$$\frac{c^3\left(33\log\left(\sqrt{1-a^2x^2}+1\right)-\frac{\sqrt{1-a^2x^2}\left(2a^3x^3+78a^2x^2+11ax-1\right)}{a^2x^2(ax+1)}-33\log(ax)-12\sin^{-1}(ax)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(-((Sqrt[1 - a^2*x^2]*(-1 + 11*a*x + 78*a^2*x^2 + 2*a^3*x^3))/(a^2*x^2
*(1 + a*x))) - 12*ArcSin[a*x] - 33*Log[a*x] + 33*Log[1 + Sqrt[1 - a^2*x^2]]
))/(2*a)
```

fricas [A] time = 0.47, size = 177, normalized size = 1.26

$$-\frac{66\,a^3c^3x^3+66\,a^2c^3x^2-24\left(a^3c^3x^3+a^2c^3x^2\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+33\left(a^3c^3x^3+a^2c^3x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+(2}{2\left(a^4x^3+a^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(66*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 24*(a^3*c^3*x^3 + a^2*c^3*x^2)*arct
an((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 33*(a^3*c^3*x^3 + a^2*c^3*x^2)*log((sq
rt(-a^2*x^2 + 1) - 1)/x) + (2*a^3*c^3*x^3 + 78*a^2*c^3*x^2 + 11*a*c^3*x - c
^3)*sqrt(-a^2*x^2 + 1))/(a^4*x^3 + a^3*x^2)
```

giac [B] time = 0.20, size = 265, normalized size = 1.89

$$-\frac{6\,c^3\arcsin\left(ax\right)\mathrm{sgn}(a)}{|a|}-\frac{\left(c^3-\frac{23\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^3}{a^2x}-\frac{536\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^3}{a^4x^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}+\frac{33\,c^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -6*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/8*(c^3 - 23*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)*c^3/(a^2*x) - 536*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2))*a^
4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(
a^2*x) + 1)*abs(a)) + 33/2*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2
*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^3/a - 1/8*(24*(sqrt(-a^2*x^
2 + 1)*abs(a) + a)*c^3*abs(a)/(a^2*x) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c
^3*abs(a)/(a^4*x^2))/a^2
```

maple [B] time = 0.06, size = 352, normalized size = 2.51

$$-\frac{11c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{2a}-\frac{33c^3\sqrt{-a^2x^2+1}}{2a}+\frac{33c^3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{6c^3\left(-a^2x^2+1\right)^{\frac{5}{2}}}{a^2x}-6c^3x\left(-a^2x^2+1\right)^{\frac{3}{2}}-9c^3x\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -11/2*c^3*(-a^2*x^2+1)^(3/2)/a-33/2*c^3*(-a^2*x^2+1)^(1/2)/a+33/2*c^3/a*arc
tanh(1/(-a^2*x^2+1)^(1/2))-6*c^3/a^2/x*(-a^2*x^2+1)^(5/2)-6*c^3*x*(-a^2*x^2
+1)^(3/2)-9*c^3*x*(-a^2*x^2+1)^(1/2)-9*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x
/(-a^2*x^2+1)^(1/2))+1/2*c^3/a^3/x^2*(-a^2*x^2+1)^(5/2)-8*c^3/a^4/(x+1/a)^3
*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-4*c^3/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a
*(x+1/a))^(5/2)+2*c^3/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+3*c^3*(-a^2*(x+1
/a)^2+2*a*(x+1/a))^(1/2)*x+3*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+
1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^3}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^3/(a*x + 1)^3, x)
```

mupad [B] time = 0.06, size = 161, normalized size = 1.15

$$\frac{32\,c^3\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{c^3\,\sqrt{1-a^2\,x^2}}{a}-\frac{6\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{6\,c^3\,\sqrt{1-a^2\,x^2}}{a^2\,x}+\frac{c^3\,\sqrt{1-a^2\,x^2}}{2\,a^3\,x^2}-\frac{c^3\,\mathrm{atan}\left(\sqrt{1-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (32*c^3*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2
)) - (c^3*atan((1 - a^2*x^2)^(1/2)*1i)*33i)/(2*a) - (c^3*(1 - a^2*x^2)^(1/2
))/a - (6*c^3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (6*c^3*(1 - a^2*x^2)^(1
/2))/(a^2*x) + (c^3*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\left(-\frac{\sqrt{-a^2x^2+1}}{a^3x^6+3a^2x^5+3ax^4+x^3}\right)dx+\int\frac{3ax\sqrt{-a^2x^2+1}}{a^3x^6+3a^2x^5+3ax^4+x^3}\,dx+\int\left(-\frac{2a^2x^2\sqrt{-a^2x^2+1}}{a^3x^6+3a^2x^5+3ax^4+x^3}\right)dx+\int\left(-\frac{2a^3x^3\sqrt{-a^2x^2+1}}{a^3x^6+3a^2x^5+3ax^4+x^3}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c**3*(Integral(-sqrt(-a**2*x**2 + 1)/(a**3*x**6 + 3*a**2*x**5 + 3*a*x**4 +
x**3), x) + Integral(3*a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**6 + 3*a**2*x**5 +
3*a*x**4 + x**3), x) + Integral(-2*a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**
6 + 3*a**2*x**5 + 3*a*x**4 + x**3), x) + Integral(-2*a**3*x**3*sqrt(-a**2*x
**2 + 1)/(a**3*x**6 + 3*a**2*x**5 + 3*a*x**4 + x**3), x) + Integral(3*a**4*
x**4*sqrt(-a**2*x**2 + 1)/(a**3*x**6 + 3*a**2*x**5 + 3*a*x**4 + x**3), x) +
 Integral(-a**5*x**5*sqrt(-a**2*x**2 + 1)/(a**3*x**6 + 3*a**2*x**5 + 3*a*x*
*4 + x**3), x))/a**3
```

3.503 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=111

$$-\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}-\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{5c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{5c^2\sin^{-1}(ax)}{a}$$

```
[Out] -5*c^2*arcsin(a*x)/a+5*c^2*arctanh((-a^2*x^2+1)^(1/2))/a-16*c^2*(-a*x+1)/a/
(-a^2*x^2+1)^(1/2)-c^2*(-a^2*x^2+1)^(1/2)/a-c^2*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.30, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6131, 6128, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$-\frac{c^2\sqrt{1-a^2x^2}}{a}-\frac{c^2\sqrt{1-a^2x^2}}{a^2x}-\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{5c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{5c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-16*c^2*(1 - a*x))/(a*Sqrt[1 - a^2*x^2]) - (c^2*Sqrt[1 - a^2*x^2])/a - (c^
2*Sqrt[1 - a^2*x^2])/(a^2*x) - (5*c^2*ArcSin[a*x])/a + (5*c^2*ArcTanh[Sqrt[
1 - a^2*x^2]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
```

```
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \frac{c^2\int \frac{e^{-3\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}$$

$$= \frac{c^2\int \frac{(1-ax)^5}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\int \frac{-1+5ax+5a^2x^2-a^3x^3}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} + \frac{c^2\int \frac{-5a-5a^2x+a^3x^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \frac{c^2\int \frac{5a^3+5a^4x}{x\sqrt{1-a^2x^2}}\,dx}{a^4}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \left(5c^2\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx - \frac{\left(5c^2\right)\int}{}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \frac{5c^2\sin^{-1}(ax)}{a} - \frac{\left(5c^2\right)\text{Subst}\left(\int\right.}{2}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \frac{5c^2\sin^{-1}(ax)}{a} + \frac{\left(5c^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{16c^2(1-ax)}{a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} - \frac{c^2\sqrt{1-a^2x^2}}{a^2x} - \frac{5c^2\sin^{-1}(ax)}{a} + \frac{5c^2\tanh^{-1}\left(\sqrt{1\right.}{a}$$

Mathematica [A] time = 0.23, size = 81, normalized size = 0.73

$$\frac{c^2\left(-\frac{\sqrt{1-a^2x^2}\left(a^2x^2+18ax+1\right)}{ax(ax+1)}+5\log\left(\sqrt{1-a^2x^2}+1\right)-5\log(ax)-5\sin^{-1}(ax)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^2*(-((Sqrt[1 - a^2*x^2]*(1 + 18*a*x + a^2*x^2))/(a*x*(1 + a*x))) - 5*Arc
Sin[a*x] - 5*Log[a*x] + 5*Log[1 + Sqrt[1 - a^2*x^2]]))/a
```

fricas [A] time = 0.60, size = 149, normalized size = 1.34

$$-\frac{17\,a^2c^2x^2+17\,ac^2x-10\left(a^2c^2x^2+ac^2x\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+5\left(a^2c^2x^2+ac^2x\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(a^2c^2x^2+\right.}{a^3x^2+a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(17*a^2*c^2*x^2 + 17*a*c^2*x - 10*(a^2*c^2*x^2 + a*c^2*x)*arctan((sqrt(-a^
2*x^2 + 1) - 1)/(a*x)) + 5*(a^2*c^2*x^2 + a*c^2*x)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) + (a^2*c^2*x^2 + 18*a*c^2*x + c^2)*sqrt(-a^2*x^2 + 1))/(a^3*x^2 + a
^2*x)
```

giac [A] time = 0.20, size = 197, normalized size = 1.77

$$-\frac{5\,c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}+\frac{5\,c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}\,c^2}{a}+\frac{\left(c^2+\frac{65\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^2}{a^2x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -5*c^2*arcsin(a*x)*sgn(a)/abs(a) + 5*c^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*
abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^2/a + 1/2*(c^2 +
65*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x))*a^2*x/((sqrt(-a^2*x^2 + 1)*
abs(a) + a)*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a)) - 1/2*(sq
rt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x*abs(a))
```

maple [B] time = 0.05, size = 329, normalized size = 2.96

$$-\frac{5c^{2}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{3a}-\frac{5c^{2}\sqrt{-a^{2}x^{2}+1}}{a}+\frac{5c^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)}{a}-\frac{c^{2}\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}}{a^{2}x}-c^{2}x\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}-\frac{3c^{2}x\sqrt{-a^{2}x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -5/3*c^2*(-a^2*x^2+1)^(3/2)/a-5*c^2*(-a^2*x^2+1)^(1/2)/a+5*c^2/a*arctanh(1/
(-a^2*x^2+1)^(1/2))-c^2/a^2/x*(-a^2*x^2+1)^(5/2)-c^2*x*(-a^2*x^2+1)^(3/2)-3
/2*c^2*x*(-a^2*x^2+1)^(1/2)-3/2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*
x^2+1)^(1/2))-4*c^2/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-4*c^2/
a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-7/3*c^2/a*(-a^2*(x+1/a)^2+
2*a*(x+1/a))^(3/2)-7/2*c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-7/2*c^2/(a^
2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{2}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^2/(a*x + 1)^3, x)
```

mupad [B] time = 0.83, size = 138, normalized size = 1.24

$$\frac{16\,c^{2}\,\sqrt{1-a^{2}\,x^{2}}}{\left(x\,\sqrt{-a^{2}}+\frac{\sqrt{-a^{2}}}{a}\right)\,\sqrt{-a^{2}}}-\frac{c^{2}\,\sqrt{1-a^{2}\,x^{2}}}{a}-\frac{5\,c^{2}\,\operatorname{asinh}\left(x\,\sqrt{-a^{2}}\right)}{\sqrt{-a^{2}}}-\frac{c^{2}\,\sqrt{1-a^{2}\,x^{2}}}{a^{2}\,x}-\frac{c^{2}\,\operatorname{atan}\left(\sqrt{1-a^{2}\,x^{2}}\,1\mathrm{i}\right)\,5\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (16*c^2*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2
)) - (c^2*atan((1 - a^2*x^2)^(1/2)*1i)*5i)/a - (c^2*(1 - a^2*x^2)^(1/2))/a
- (5*c^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (c^2*(1 - a^2*x^2)^(1/2))/(a
^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\frac{\sqrt{-a^2x^2+1}}{a^3x^5+3a^2x^4+3ax^3+x^2}\,dx+\int\left(-\frac{2ax\sqrt{-a^2x^2+1}}{a^3x^5+3a^2x^4+3ax^3+x^2}\right)dx+\int\frac{2a^3x^3\sqrt{-a^2x^2+1}}{a^3x^5+3a^2x^4+3ax^3+x^2}\,dx+\int\left(-\frac{a^4x^4\sqrt{-a^2x^2+1}}{a^3x^5+3a^2x^4+3ax^3+x^2}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c**2*(Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**5 + 3*a**2*x**4 + 3*a*x**3 + x
**2), x) + Integral(-2*a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**5 + 3*a**2*x**4 +
3*a*x**3 + x**2), x) + Integral(2*a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x**5
 + 3*a**2*x**4 + 3*a*x**3 + x**2), x) + Integral(-a**4*x**4*sqrt(-a**2*x**2
 + 1)/(a**3*x**5 + 3*a**2*x**4 + 3*a*x**3 + x**2), x))/a**2
```

3.504 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=77

$$-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{4c\sin^{-1}(ax)}{a}$$

```
[Out] -4*c*arcsin(a*x)/a+c*arctanh((-a^2*x^2+1)^(1/2))/a-8*c*(-a*x+1)/a/(-a^2*x^2
+1)^(1/2)-c*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.21, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6131, 6128, 1805, 1809, 844, 216, 266, 63, 208}

$$-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{4c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-8*c*(1 - a*x))/(a*Sqrt[1 - a^2*x^2]) - (c*Sqrt[1 - a^2*x^2])/a - (4*c*Arc
Sin[a*x])/a + (c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\frac{c\int\frac{e^{-3\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int\frac{(1-ax)^4}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{a}\\
&= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}+\frac{c\int\frac{-1-4ax+a^2x^2}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{c\int\frac{a^2+4a^3x}{x\sqrt{1-a^2x^2}}\,dx}{a^3}\\
&= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-(4c)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{c\int\frac{1}{x\sqrt{1-a^2x^2}}\,dx}{a}\\
&= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{4c\sin^{-1}(ax)}{a}-\frac{c\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{4c\sin^{-1}(ax)}{a}+\frac{c\,\text{Subst}\left(\int\frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^3}\\
&= -\frac{8c(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{c\sqrt{1-a^2x^2}}{a}-\frac{4c\sin^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 61, normalized size = 0.79

$$
\frac{c\left(-\frac{\sqrt{1-a^2x^2}\,(ax+9)}{ax+1}+\log\left(\sqrt{1-a^2x^2}+1\right)-4\sin^{-1}(ax)-\log(x)\right)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c*(-(((9 + a*x)*Sqrt[1 - a^2*x^2])/(1 + a*x)) - 4*ArcSin[a*x] - Log[x] + L
og[1 + Sqrt[1 - a^2*x^2]]))/a
```

fricas [A] time = 0.43, size = 97, normalized size = 1.26

$$-\frac{9\,acx-8\,(acx+c)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(acx+c)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\,(acx+9\,c)+9\,c}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(9*a*c*x - 8*(a*c*x + c)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*c*x +
 c)*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(a*c*x + 9*c) + 9*
c)/(a^2*x + a)
```

giac [A] time = 4.04, size = 104, normalized size = 1.35

$$-\frac{4\,c\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}+\frac{c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}\,c}{a}+\frac{16\,c}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -4*c*arcsin(a*x)*sgn(a)/abs(a) + c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a)
 - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c/a + 16*c/(((sqrt(-a^2*x
^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [B] time = 0.05, size = 223, normalized size = 2.90

$$-\frac{c\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3a}-\frac{c\sqrt{-a^2x^2+1}}{a}+\frac{c\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}-\frac{2c\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^4\left(x+\frac{1}{a}\right)^3}-\frac{3c\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\right.}{a^3\left(x+\frac{1}{a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/3*c*(-a^2*x^2+1)^(3/2)/a-c*(-a^2*x^2+1)^(1/2)/a+c/a*arctanh(1/(-a^2*x^2+
1)^(1/2))-2*c/a^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-3*c/a^3/(x+1
/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-8/3*c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a
))^(3/2)-4*c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-4*c/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))/(a*x + 1)^3, x)
```

mupad [B] time = 0.84, size = 102, normalized size = 1.32

$$\frac{c\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{a}-\frac{c\,\sqrt{1-a^2\,x^2}}{a}-\frac{4\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{8\,c\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (c*atanh((1 - a^2*x^2)^(1/2)))/a - (c*(1 - a^2*x^2)^(1/2))/a - (4*c*asinh(x
*(-a^2)^(1/2)))/(-a^2)^(1/2) + (8*c*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) +
 (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\left(-\frac{\sqrt{-a^2x^2+1}}{a^3x^4+3a^2x^3+3ax^2+x}\right)dx+\int\frac{ax\sqrt{-a^2x^2+1}}{a^3x^4+3a^2x^3+3ax^2+x}\,dx+\int\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^3x^4+3a^2x^3+3ax^2+x}\,dx+\int\left(-\frac{a^3x^3\sqrt{-a^2x^2+1}}{a^3x^4+3a^2x^3+3ax^2+x}\right)dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c*(Integral(-sqrt(-a**2*x**2 + 1)/(a**3*x**4 + 3*a**2*x**3 + 3*a*x**2 + x),
 x) + Integral(a*x*sqrt(-a**2*x**2 + 1)/(a**3*x**4 + 3*a**2*x**3 + 3*a*x**2
 + x), x) + Integral(a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**4 + 3*a**2*x**
3 + 3*a*x**2 + x), x) + Integral(-a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**3*x**4
 + 3*a**2*x**3 + 3*a*x**2 + x), x))/a
```

3.505 $\int \frac{e^{-3\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=65

$$-\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{2\sin^{-1}(ax)}{ac}$$

```
[Out] -2*arcsin(a*x)/a/c-(-a*x+1)^2/a/c/(-a^2*x^2+1)^(1/2)-2*(-a^2*x^2+1)^(1/2)/a
/c
```

Rubi [A] time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6131, 6128, 789, 641, 216}

$$-\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{2\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))),x]
```

```
[Out] -((1 - a*x)^2/(a*c*Sqrt[1 - a^2*x^2])) - (2*Sqrt[1 - a^2*x^2])/(a*c) - (2*A
rcSin[a*x])/(a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\frac{a\int \frac{e^{-3\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\
&= -\frac{a\int \frac{x(1-ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= -\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{ac}-\frac{2\sin^{-1}(ax)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.03

$$\frac{a^2x^2+4\sqrt{1-a^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)+2ax-3}{ac\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))),x]
```

[Out] (-3 + 2*a*x + a^2*x^2 + 4*Sqrt[1 - a^2*x^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/(a*c*Sqrt[1 - a^2*x^2])

fricas [A] time = 0.47, size = 67, normalized size = 1.03

$$-\frac{3\,ax-4\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,(ax+3)+3}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="fricas")

[Out] -(3*a*x - 4*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*(a*x + 3) + 3)/(a^2*c*x + a*c)

giac [A] time = 1.82, size = 73, normalized size = 1.12

$$-\frac{2\,\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|}-\frac{\sqrt{-a^2x^2+1}}{ac}+\frac{4}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="giac")

[Out] -2*arcsin(a*x)*sgn(a)/(c*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c) + 4/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))

maple [B] time = 0.05, size = 292, normalized size = 4.49

$$\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{24ac}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}\,x}{16c}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{16c\sqrt{a^2}}-\frac{\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\right.}{2a^4c\left(x+\frac{1}{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x),x)

[Out] 1/24/a/c*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-1/16/c*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x-1/16/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-1/2/a^4/c/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-5/4/c/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-31/24/c/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-31/16/c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-31/16/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))), x)
```

mupad [B] time = 0.06, size = 90, normalized size = 1.38

$$\frac{2\sqrt{1-a^2\,x^2}}{c\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a\,c}-\frac{2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))*(a*x + 1)^3),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(c*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (1 - a^2*x^2)^(1/2)/(a*c) - (2*asinh(x*(-a^2)^(1/2)))/(c*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int \frac{x\sqrt{-a^2x^2+1}}{a^4x^4+2a^3x^3-2ax-1}\,dx+\int \left(-\frac{a^2x^3\sqrt{-a^2x^2+1}}{a^4x^4+2a^3x^3-2ax-1}\right)dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(x*sqrt(-a**2*x**2 + 1)/(a**4*x**4 + 2*a**3*x**3 - 2*a*x - 1), x
) + Integral(-a**2*x**3*sqrt(-a**2*x**2 + 1)/(a**4*x**4 + 2*a**3*x**3 - 2*a
*x - 1), x))/c
```

3.506 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\sin^{-1}(ax)}{ac^2}$$

```
[Out] -arcsin(a*x)/a/c^2+(a*x-1)/a/c^2/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a/c^
2
```

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 797, 641, 216, 637}

$$-\frac{1-ax}{ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] -((1 - a*x)/(a*c^2*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/(a*c^2) - ArcSin
[a*x]/(a*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
```

```
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= \frac{a^2\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^2}$$
$$= \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^2} - \frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$
$$= -\frac{1-ax}{ac^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{ac^2} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$
$$= -\frac{1-ax}{ac^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{ac^2} - \frac{\sin^{-1}(ax)}{ac^2}$$

Mathematica [A] time = 0.15, size = 46, normalized size = 0.73

$$-\frac{\sqrt{1-a^2x^2}\,(ax+2)+(ax+1)\sin^{-1}(ax)}{ac^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] -(((2 + a*x)*Sqrt[1 - a^2*x^2] + (1 + a*x)*ArcSin[a*x])/(a*c^2*(1 + a*x)))
```

fricas [A] time = 0.54, size = 71, normalized size = 1.13

$$-\frac{2\,ax - 2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(ax+2) + 2}{a^2c^2x + ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^2*c^2*x + a*c^2)
```

giac [A] time = 0.18, size = 73, normalized size = 1.16

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c^2|a|} - \frac{\sqrt{-a^2x^2+1}}{ac^2} + \frac{2}{c^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(c^2*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c^2) + 2/(c^2*((sq
rt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [B] time = 0.05, size = 336, normalized size = 5.33

$$-\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{5}{2}}}{8a^3c^2\left(x-\frac{1}{a}\right)^2} - \frac{5\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{48a\,c^2} + \frac{5\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}\;x}{32c^2} + \frac{5\arctan\left(\cdots\right)}{32c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x)
```

```
[Out] -1/8/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-5/48/a/c^2*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(3/2)+5/32/c^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x+
5/32/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2
))-1/4/a^4/c^2/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-3/4/a^3/c^2/(x+
1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-37/48/a/c^2*(-a^2*(x+1/a)^2+2*a*(
```

```
x+1/a))^(3/2)-37/32/c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-37/32/c^2/(a^2
)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^2), x)
```

mupad [B] time = 0.06, size = 89, normalized size = 1.41

$$\frac{\sqrt{1-a^2\,x^2}}{c^2\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a\,c^2}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^2\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^2*(a*x + 1)^3),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(c^2*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2)) -
(1 - a^2*x^2)^(1/2)/(a*c^2) - asinh(x*(-a^2)^(1/2))/(c^2*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int \frac{x^2\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx+\int \left(-\frac{a^2x^4\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*(Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 + a**4*x**4 - 2*a**3*x*
*3 - 2*a**2*x**2 + a*x + 1), x) + Integral(-a**2*x**4*sqrt(-a**2*x**2 + 1)/
(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x))/c**2
```

3.507 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{1}{ac^3\sqrt{1-a^2x^2}}$$

```
[Out] -1/a/c^3/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a/c^3
```

Rubi [A] time = 0.12, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6128, 266, 43}

$$-\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{1}{ac^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -(1/(a*c^3*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/(a*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= -\frac{a^3\int \frac{e^{-3\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}\\
&= -\frac{a^3\int \frac{x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}\\
&= -\frac{a^3\,\text{Subst}\left(\int \frac{x}{\left(1-a^2x\right)^{3/2}}\,dx,x,x^2\right)}{2c^3}\\
&= -\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{a^2\left(1-a^2x\right)^{3/2}}-\frac{1}{a^2\sqrt{1-a^2x}}\right)dx,x,x^2\right)}{2c^3}\\
&= -\frac{1}{ac^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 30, normalized size = 0.67

$$\frac{a^2x^2-2}{ac^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^3),x]
[Out] (-2 + a^2*x^2)/(a*c^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 1.37, size = 53, normalized size = 1.18

$$-\frac{2\,a^2x^2+\left(a^2x^2-2\right)\sqrt{-a^2x^2+1}-2}{a^3c^3x^2-ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -(2*a^2*x^2 + (a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1) - 2)/(a^3*c^3*x^2 - a*c^3)
```

giac [A] time = 0.26, size = 38, normalized size = 0.84

$$-\frac{\frac{\sqrt{-a^2x^2+1}}{c^3}+\frac{1}{\sqrt{-a^2x^2+1}\,c^3}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
[Out] -(sqrt(-a^2*x^2 + 1)/c^3 + 1/(sqrt(-a^2*x^2 + 1)*c^3))/a
```

maple [A] time = 0.03, size = 43, normalized size = 0.96

$$\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(a^2x^2-2\right)}{a\left(ax-1\right)^2c^3\left(ax+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x)
[Out] 1/a*(-a^2*x^2+1)^(3/2)*(a^2*x^2-2)/(a*x-1)^2/c^3/(a*x+1)^2
```

maxima [A] time = 0.33, size = 45, normalized size = 1.00

$$-\frac{\left(a^2x^2-2\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{a^3c^3x^2-ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
[Out] -(a^2*x^2 - 2)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^3*c^3*x^2 - a*c^3)
```

mupad [B] time = 0.86, size = 116, normalized size = 2.58

$$\frac{\sqrt{1-a^2\,x^2}}{2\,c^3\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a\,c^3}-\frac{\sqrt{1-a^2\,x^2}}{2\,c^3\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^3*(a*x + 1)^3),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(2*c^3*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (1 - a^2*x^2)^(1/2)/(a*c^3) - (1 - a^2*x^2)^(1/2)/(2*c^3*(x*(-a^2)^(1/2)
- (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [A] time = 23.32, size = 34, normalized size = 0.76

$$-\frac{2\left(\frac{\sqrt{-a^2x^2+1}}{2c^3}+\frac{1}{2c^3\sqrt{-a^2x^2+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**3,x)
```

```
[Out] -2*(sqrt(-a**2*x**2 + 1)/(2*c**3) + 1/(2*c**3*sqrt(-a**2*x**2 + 1)))/a
```

3.508 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=96

$$-\frac{x(4ax+3)}{3c^4\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^4}+\frac{a^2x^3(ax+1)}{3c^4\left(1-a^2x^2\right)^{3/2}}+\frac{\sin^{-1}(ax)}{ac^4}$$

```
[Out] 1/3*a^2*x^3*(a*x+1)/c^4/(-a^2*x^2+1)^(3/2)+arcsin(a*x)/a/c^4-1/3*x*(4*a*x+3
)/c^4/(-a^2*x^2+1)^(1/2)-8/3*(-a^2*x^2+1)^(1/2)/a/c^4
```

Rubi [A] time = 0.16, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6131, 6128, 850, 819, 641, 216}

$$\frac{a^2x^3(ax+1)}{3c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(4ax+3)}{3c^4\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^4}+\frac{\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (a^2*x^3*(1 + a*x))/(3*c^4*(1 - a^2*x^2)^(3/2)) - (x*(3 + 4*a*x))/(3*c^4*Sq
rt[1 - a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*a*c^4) + ArcSin[a*x]/(a*c^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
```

```
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \frac{a^4\int \frac{e^{-3\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4}{(1-ax)\left(1-a^2x^2\right)^{3/2}}\,dx}{c^4}$$

$$= \frac{a^4\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^4}$$

$$= \frac{a^2x^3(1+ax)}{3c^4\left(1-a^2x^2\right)^{3/2}} - \frac{a^2\int \frac{x^2(3+4ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^4}$$

$$= \frac{a^2x^3(1+ax)}{3c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{3+8ax}{\sqrt{1-a^2x^2}}\,dx}{3c^4}$$

$$= \frac{a^2x^3(1+ax)}{3c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3c^4\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3ac^4} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^4}$$

$$= \frac{a^2x^3(1+ax)}{3c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3c^4\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3ac^4} + \frac{\sin^{-1}(ax)}{ac^4}$$

Mathematica [A] time = 0.19, size = 68, normalized size = 0.71

$$\frac{\frac{\sqrt{1-a^2x^2}\left(-3a^3x^3+7a^2x^2+5ax-8\right)}{(ax-1)^2(ax+1)}+3\sin^{-1}(ax)}{3ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(-8 + 5*a*x + 7*a^2*x^2 - 3*a^3*x^3))/((-1 + a*x)^2*(1
+ a*x)) + 3*ArcSin[a*x])/(3*a*c^4)
```

fricas [A] time = 0.85, size = 142, normalized size = 1.48

$$-\frac{8\,a^3x^3-8\,a^2x^2-8\,ax+6\left(a^3x^3-a^2x^2-ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax+8\right)\sqrt{-a^2x^2+1}}{3\left(a^4c^4x^3-a^3c^4x^2-a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(8*a^3*x^3 - 8*a^2*x^2 - 8*a*x + 6*(a^3*x^3 - a^2*x^2 - a*x + 1)*arcta
n((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^3*x^3 - 7*a^2*x^2 - 5*a*x + 8)*sqr
t(-a^2*x^2 + 1) + 8)/(a^4*c^4*x^3 - a^3*c^4*x^2 - a^2*c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^4), x)
```

maple [B] time = 0.06, size = 424, normalized size = 4.42

$$-\frac{23\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{96a\,c^4}-\frac{29\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{32a\,c^4}+\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{5}{2}}}{24a^5c^4\left(x-\frac{1}{a}\right)^4}+\frac{17\left(-a^2\left(x\right.\right.}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x)
```

```
[Out] -23/96/a/c^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-29/32/a/c^4*(-a^2*(x-1/a)^2
-2*a*(x-1/a))^(3/2)+1/24/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/
2)+17/48/a^4/c^4/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-1/16/a^4/c^4/
(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-1/4/a^3/c^4/(x+1/a)^2*(-a^2*(x
+1/a)^2+2*a*(x+1/a))^(5/2)-23/64/c^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-2
3/64/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2
))-43/48/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+87/64/c^4*(-a
^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x+87/64/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*
x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^4), x)
```

mupad [B] time = 0.07, size = 188, normalized size = 1.96

$$\frac{a\sqrt{1-a^2x^2}}{6\left(a^4c^4x^2-2a^3c^4x+a^2c^4\right)}+\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{c^4\sqrt{-a^2}}-\frac{\sqrt{1-a^2x^2}}{a\,c^4}+\frac{\sqrt{1-a^2x^2}}{4\sqrt{-a^2}\left(c^4x\sqrt{-a^2}+\frac{c^4\sqrt{-a^2}}{a}\right)}-\frac{19\sqrt{1-}}{12\sqrt{-a^2}\left(c^4x\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^4*(a*x + 1)^3),x)
```

```
[Out] (a*(1 - a^2*x^2)^(1/2))/(6*(a^2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) + asinh(x
*(-a^2)^(1/2))/(c^4*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a*c^4) + (1 - a^2*
x^2)^(1/2)/(4*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2) + (c^4*(-a^2)^(1/2))/a)) - (
19*(1 - a^2*x^2)^(1/2))/(12*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^
(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\left(\int\frac{x^4\sqrt{-a^2x^2+1}}{a^7x^7-a^6x^6-3a^5x^5+3a^4x^4+3a^3x^3-3a^2x^2-ax+1}\,dx+\int\left(-\frac{a^2x^6\sqrt{-a^2x^2+1}}{a^7x^7-a^6x^6-3a^5x^5+3a^4x^4+3a^3x^3-3a^2x^2-ax+1}\right)dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*(Integral(x**4*sqrt(-a**2*x**2 + 1)/(a**7*x**7 - a**6*x**6 - 3*a**5*x*
*5 + 3*a**4*x**4 + 3*a**3*x**3 - 3*a**2*x**2 - a*x + 1), x) + Integral(-a**
2*x**6*sqrt(-a**2*x**2 + 1)/(a**7*x**7 - a**6*x**6 - 3*a**5*x**5 + 3*a**4*x
**4 + 3*a**3*x**3 - 3*a**2*x**2 - a*x + 1), x))/c**4
```

3.509 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$

Optimal. Leaf size=125

$$-\frac{(ax+1)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}}+\frac{22(ax+1)}{15ac^5\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{ac^5}-\frac{2(23ax+30)}{15ac^5\sqrt{1-a^2x^2}}+\frac{2\sin^{-1}(ax)}{ac^5}$$

```
[Out] -1/5*(a*x+1)^2/a/c^5/(-a^2*x^2+1)^(5/2)+22/15*(a*x+1)/a/c^5/(-a^2*x^2+1)^(3
/2)+2*arcsin(a*x)/a/c^5-2/15*(23*a*x+30)/a/c^5/(-a^2*x^2+1)^(1/2)-(-a^2*x^2
+1)^(1/2)/a/c^5
```

Rubi [A] time = 0.32, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6131, 6128, 852, 1635, 1814, 641, 216}

$$-\frac{(ax+1)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}}+\frac{22(ax+1)}{15ac^5\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{ac^5}-\frac{2(23ax+30)}{15ac^5\sqrt{1-a^2x^2}}+\frac{2\sin^{-1}(ax)}{ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] -(1 + a*x)^2/(5*a*c^5*(1 - a^2*x^2)^(5/2)) + (22*(1 + a*x))/(15*a*c^5*(1 -
a^2*x^2)^(3/2)) - (2*(30 + 23*a*x))/(15*a*c^5*Sqrt[1 - a^2*x^2]) - Sqrt[1 -
 a^2*x^2]/(a*c^5) + (2*ArcSin[a*x])/(a*c^5)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = -\frac{a^5\int \frac{e^{-3\tanh^{-1}(ax)}x^5}{(1-ax)^5}\,dx}{c^5}$$

$$= -\frac{a^5\int \frac{x^5}{(1-ax)^2\left(1-a^2x^2\right)^{3/2}}\,dx}{c^5}$$

$$= -\frac{a^5\int \frac{x^5(1+ax)^2}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^5}$$

$$= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}} + \frac{a^5\int \frac{(1+ax)\left(\frac{2}{a^5}+\frac{5x}{a^4}+\frac{5x^2}{a^3}+\frac{5x^3}{a^2}+\frac{5x^4}{a}\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^5}$$

$$= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}} + \frac{22(1+ax)}{15ac^5\left(1-a^2x^2\right)^{3/2}} - \frac{a^5\int \frac{\frac{16}{a^5}+\frac{45x}{a^4}+\frac{30x^2}{a^3}+\frac{15x^3}{a^2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^5}$$

$$= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}} + \frac{22(1+ax)}{15ac^5\left(1-a^2x^2\right)^{3/2}} - \frac{2(30+23ax)}{15ac^5\sqrt{1-a^2x^2}} + \frac{a^5\int \frac{\frac{30}{a^5}+\frac{15x}{a^4}}{\sqrt{1-a^2x^2}}\,dx}{15c^5}$$

$$= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}} + \frac{22(1+ax)}{15ac^5\left(1-a^2x^2\right)^{3/2}} - \frac{2(30+23ax)}{15ac^5\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{ac^5} + \frac{2\int \frac{1}{\sqrt{1-a^2x^2}}\,d}{c^5}$$

$$= -\frac{(1+ax)^2}{5ac^5\left(1-a^2x^2\right)^{5/2}} + \frac{22(1+ax)}{15ac^5\left(1-a^2x^2\right)^{3/2}} - \frac{2(30+23ax)}{15ac^5\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{ac^5} + \frac{2\sin^{-1}(ax)}{ac^5}$$

Mathematica [A] time = 0.24, size = 76, normalized size = 0.61

$$\frac{\frac{\sqrt{1-a^2x^2}\left(-15a^4x^4+76a^3x^3-32a^2x^2-82ax+56\right)}{(ax-1)^3(ax+1)}+30\sin^{-1}(ax)}{15ac^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] ((Sqrt[1 - a^2*x^2]*(56 - 82*a*x - 32*a^2*x^2 + 76*a^3*x^3 - 15*a^4*x^4))/(
(-1 + a*x)^3*(1 + a*x)) + 30*ArcSin[a*x])/(15*a*c^5)
```

fricas [A] time = 0.58, size = 151, normalized size = 1.21

$$-\frac{56\,a^{4}x^{4}-112\,a^{3}x^{3}+112\,ax+60\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1\right)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)+\left(15\,a^{4}x^{4}-76\,a^{3}x^{3}+32\,a^{2}x^{2}\right.}{15\left(a^{5}c^{5}x^{4}-2\,a^{4}c^{5}x^{3}+2\,a^{2}c^{5}x-ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(56*a^4*x^4 - 112*a^3*x^3 + 112*a*x + 60*(a^4*x^4 - 2*a^3*x^3 + 2*a*x
 - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (15*a^4*x^4 - 76*a^3*x^3 + 3
2*a^2*x^2 + 82*a*x - 56)*sqrt(-a^2*x^2 + 1) - 56)/(a^5*c^5*x^4 - 2*a^4*c^5*
x^3 + 2*a^2*c^5*x - a*c^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.06, size = 468, normalized size = 3.74

$$-\frac{49\left(-a^{2}\left(x+\frac{1}{a}\right)^{2}+2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{384a\,c^{5}}+\frac{31\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)\right)^{\frac{5}{2}}}{48a^{4}c^{5}\left(x-\frac{1}{a}\right)^{3}}-\frac{\left(-a^{2}\left(x+\frac{1}{a}\right)^{2}+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{32a^{4}c^{5}\left(x+\frac{1}{a}\right)^{3}}-\frac{9\left(-a^{2}\left(x+\right.\right.}{64a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^5,x)
```

```
[Out] -49/384/a/c^5*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+31/48/a^4/c^5/(x-1/a)^3*(-
a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-1/32/a^4/c^5/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a
*(x+1/a))^(5/2)-9/64/a^3/c^5/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-4
9/256/c^5*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-49/256/c^5/(a^2)^(1/2)*arcta
n((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-139/96/a^3/c^5/(x-1/a)^
2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+561/256/c^5*(-a^2*(x-1/a)^2-2*a*(x-1/a
))^(1/2)*x+561/256/c^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a
```

```
*(x-1/a))^(1/2))+1/40/a^6/c^5/(x-1/a)^5*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+
7/48/a^5/c^5/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-187/128/a/c^5*(-a
^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^5), x)
```

mupad [B] time = 0.91, size = 275, normalized size = 2.20

$$\frac{41\,a\,\sqrt{1-a^2\,x^2}}{60\,\left(a^4\,c^5\,x^2-2\,a^3\,c^5\,x+a^2\,c^5\right)}+\frac{2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^5\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a\,c^5}+\frac{\sqrt{1-a^2\,x^2}}{8\,\sqrt{-a^2}\,\left(c^5\,x\,\sqrt{-a^2}+\frac{c^5\,\sqrt{-a^2}}{a}\right)}-\frac{383}{120\,\sqrt{-a^2}\,\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^5*(a*x + 1)^3),x)
```

```
[Out] (41*a*(1 - a^2*x^2)^(1/2))/(60*(a^2*c^5 - 2*a^3*c^5*x + a^4*c^5*x^2)) + (2*
asinh(x*(-a^2)^(1/2)))/(c^5*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a*c^5) + (
1 - a^2*x^2)^(1/2)/(8*(-a^2)^(1/2)*(c^5*x*(-a^2)^(1/2) + (c^5*(-a^2)^(1/2))
/a)) - (383*(1 - a^2*x^2)^(1/2))/(120*(-a^2)^(1/2)*(c^5*x*(-a^2)^(1/2) - (c
^5*(-a^2)^(1/2))/a)) - (1 - a^2*x^2)^(1/2)/(10*(-a^2)^(1/2)*(3*c^5*x*(-a^2)
^(1/2) - (c^5*(-a^2)^(1/2))/a + a^2*c^5*x^3*(-a^2)^(1/2) - 3*a*c^5*x^2*(-a^
2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^5\left(\int \frac{x^5\sqrt{-a^2x^2+1}}{a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1}\,dx+\int\left(-\frac{a^2x^7\sqrt{-a^2x^2+1}}{a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1}\right)dx\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**5,x)
```

```
[Out] a**5*(Integral(x**5*sqrt(-a**2*x**2 + 1)/(a**8*x**8 - 2*a**7*x**7 - 2*a**6*
x**6 + 6*a**5*x**5 - 6*a**3*x**3 + 2*a**2*x**2 + 2*a*x - 1), x) + Integral(
-a**2*x**7*sqrt(-a**2*x**2 + 1)/(a**8*x**8 - 2*a**7*x**7 - 2*a**6*x**6 + 6*
a**5*x**5 - 6*a**3*x**3 + 2*a**2*x**2 + 2*a*x - 1), x))/c**5
```

3.510 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=225

$$-\frac{7a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}-\frac{a^3x^4(54-227ax)\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{105(1-ax)^{9/2}}-\frac{10a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{21(1-ax)^{5/2}}+2ax^2\sqrt{}$$

```
[Out] -7*a^(7/2)*(c-c/a/x)^(9/2)*x^(9/2)*arcsinh(a^(1/2)*x^(1/2))/(-a*x+1)^(9/2)-
1/105*a^3*(c-c/a/x)^(9/2)*x^4*(-227*a*x+54)*(a*x+1)^(1/2)/(-a*x+1)^(9/2)-10
/21*a^2*(c-c/a/x)^(9/2)*x^3*(a*x+1)^(1/2)/(-a*x+1)^(5/2)+2/5*a*(c-c/a/x)^(9
/2)*x^2*(a*x+1)^(1/2)/(-a*x+1)^(3/2)-2/7*(c-c/a/x)^(9/2)*x*(a*x+1)^(1/2)/(-
a*x+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6134, 6129, 97, 150, 143, 54, 215}

$$-\frac{10a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{21(1-ax)^{5/2}}-\frac{a^3x^4(54-227ax)\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{105(1-ax)^{9/2}}-\frac{7a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+2ax^2\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] -(a^3*(c - c/(a*x))^(9/2)*x^4*(54 - 227*a*x)*Sqrt[1 + a*x])/(105*(1 - a*x)^
(9/2)) - (10*a^2*(c - c/(a*x))^(9/2)*x^3*Sqrt[1 + a*x])/(21*(1 - a*x)^(5/2)
) + (2*a*(c - c/(a*x))^(9/2)*x^2*Sqrt[1 + a*x])/(5*(1 - a*x)^(3/2)) - (2*(c
 - c/(a*x))^(9/2)*x*Sqrt[1 + a*x])/(7*Sqrt[1 - a*x]) - (7*a^(7/2)*(c - c/(a
*x))^(9/2)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{e^{\tanh^{-1}(ax)}(1-ax)^{9/2}}{x^{9/2}}dx}{(1-ax)^{9/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{(1-ax)^4\sqrt{1+ax}}{x^{9/2}}dx}{(1-ax)^{9/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} + \frac{\left(2\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{(1-ax)^3\left(-\frac{7a}{2}-\frac{9a^2x}{2}\right)}{x^{7/2}\sqrt{1+ax}}dx}{7(1-ax)^{9/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{9/2}x^2\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} + \frac{\left(4\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{(1-ax)^2\left(25\right.}{x^{5/2}\sqrt{}}}{35(1-ax)^{9/2}}$$

$$= -\frac{10a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)^{9/2}x^2\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}}$$

$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4(54-227ax)\sqrt{1+ax}}{105(1-ax)^{9/2}} - \frac{10a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)}{5(1}$$

$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4(54-227ax)\sqrt{1+ax}}{105(1-ax)^{9/2}} - \frac{10a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)}{5(1}$$

$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4(54-227ax)\sqrt{1+ax}}{105(1-ax)^{9/2}} - \frac{10a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{2a\left(c-\frac{c}{ax}\right)}{5(1}$$

Mathematica [C] time = 0.08, size = 101, normalized size = 0.45

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(245a^2x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-ax\right)+\sqrt{ax+1}\left(105a^4x^4-688a^3x^3-601a^2x^2+162ax-30\right)\right)}{105a^4x^3\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(-30 + 162*a*x - 601*a^2*x^2 - 688*a^
3*x^3 + 105*a^4*x^4) + 245*a^2*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(a*
x)]))/(105*a^4*x^3*Sqrt[1 - a*x])
```

fricas [A] time = 0.55, size = 386, normalized size = 1.72

$$\left[\frac{735\left(a^{4}c^{4}x^{4}-a^{3}c^{4}x^{3}\right)\sqrt{-c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\left(2\,a^{2}x^{2}+ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(105\,a^{4}c^{4}x^{4}+292\,a^{3}c^{4}x^{3}-35\right.}{420\left(a^{5}x^{4}-a^{4}x^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*
x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))
 - c)/(a*x - 1)) - 4*(105*a^4*c^4*x^4 + 292*a^3*c^4*x^3 - 356*a^2*c^4*x^2 +
 162*a*c^4*x - 30*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4
 - a^4*x^3), 1/210*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*arctan(2*sqrt(-
a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c))
 - 2*(105*a^4*c^4*x^4 + 292*a^3*c^4*x^3 - 356*a^2*c^4*x^2 + 162*a*c^4*x - 3
0*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(9/2)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.10, size = 172, normalized size = 0.76

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^{4}\sqrt{-a^{2}x^{2}+1}\,\left(210a^{\frac{9}{2}}\sqrt{-(ax+1)\,x}\,x^{4}+735\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)x^{4}a^{4}+584a^{\frac{7}{2}}x^{3}\sqrt{-(ax+1)\,x}\,-\right.}{210x^{3}a^{\frac{9}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(9/2),x)
```

```
[Out] -1/210*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(-a^2*x^2+1)^(1/2)*(210*a^(9/2
)*(-(a*x+1)*x)^(1/2)*x^4+735*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2
))*x^4*a^4+584*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)-712*a^(5/2)*x^2*(-(a*x+1)*x)^
```

```
(1/2)+324*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-60*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-
1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^(9/2)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{9/2}\,(a\,x+1)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(9/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
[Out] int(((c - c/(a*x))^(9/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(9/2),x)
[Out] Timed out
```

3.511 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=181

$$\frac{5a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}-\frac{a^2x^3\sqrt{ax+1}\,(31ax+18)\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{7/2}}+\frac{2ax^2\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{7/2}}{3(1-ax)^{3/2}}-\frac{2x\sqrt{ax+}}{5\sqrt{}}$$

```
[Out] 5*a^(5/2)*(c-c/a/x)^(7/2)*x^(7/2)*arcsinh(a^(1/2)*x^(1/2))/(-a*x+1)^(7/2)+2
/3*a*(c-c/a/x)^(7/2)*x^2*(a*x+1)^(1/2)/(-a*x+1)^(3/2)-1/15*a^2*(c-c/a/x)^(7
/2)*x^3*(31*a*x+18)*(a*x+1)^(1/2)/(-a*x+1)^(7/2)-2/5*(c-c/a/x)^(7/2)*x*(a*x
+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6134, 6129, 97, 150, 143, 54, 215}

$$-\frac{a^2x^3\sqrt{ax+1}\,(31ax+18)\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{7/2}}+\frac{5a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{2ax^2\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{7/2}}{3(1-ax)^{3/2}}-\frac{2x\sqrt{ax}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^(7/2),x]
```

```
[Out] (2*a*(c - c/(a*x))^(7/2)*x^2*Sqrt[1 + a*x])/(3*(1 - a*x)^(3/2)) - (2*(c - c
/(a*x))^(7/2)*x*Sqrt[1 + a*x])/(5*Sqrt[1 - a*x]) - (a^2*(c - c/(a*x))^(7/2)
*x^3*Sqrt[1 + a*x]*(18 + 31*a*x))/(15*(1 - a*x)^(7/2)) + (5*a^(5/2)*(c - c/
(a*x))^(7/2)*x^(7/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{e^{\tanh^{-1}(ax)}(1-ax)^{7/2}}{x^{7/2}}\,dx}{(1-ax)^{7/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)^3\sqrt{1+ax}}{x^{7/2}}\,dx}{(1-ax)^{7/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} + \frac{\left(2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)^2\left(-\frac{5a}{2}-\frac{7a^2x}{2}\right)}{x^{5/2}\sqrt{1+ax}}\,dx}{5(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{3(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} + \frac{\left(4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)\left(\ldots\right.}{x^{3/2}\ldots}}{15(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{3(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}\,(18\ldots}{15(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{3(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}\,(18\ldots}{15(1-ax)^{7/2}}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{3(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}\,(18\ldots}{15(1-ax)^{7/2}}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 0.52

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}\left(15a^3x^3+56a^2x^2-28ax+6\right)-75a^{5/2}x^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{15a^3x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^(7/2),x]

[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(6 - 28*a*x + 56*a^2*x^2 + 15*a^3*x^3
) - 75*a^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]]))/(15*a^3*x^2*Sqrt[1 - a*x]
)
```

fricas [A] time = 0.70, size = 364, normalized size = 2.01

$$\left[\frac{75\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(15\,a^3c^3x^3+56\,a^2c^3x^2-28\,ac^3x\right.}{60\left(a^4x^3-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) - 4*(15*a^3*c^3*x^3 + 56*a^2*c^3*x^2 - 28*a*c^3*x + 6*c^3)*s
qrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(75*(a
^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(15*a^3*c^3*x^3 + 56*
a^2*c^3*x^2 - 28*a*c^3*x + 6*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)
))/(a^4*x^3 - a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 154, normalized size = 0.85

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\sqrt{-a^2x^2+1}\,\left(30a^{\frac{7}{2}}x^3\sqrt{-(ax+1)\,x}+75\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)x^3a^3+112a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x}-56a\right.}{30x^2a^{\frac{7}{2}}\,(ax-1)\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(7/2),x)
```

```
[Out] -1/30*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(-a^2*x^2+1)^(1/2)*(30*a^(7/2)*
x^3*(-(a*x+1)*x)^(1/2)+75*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*
```

```
x^3*a^3+112*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)-56*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+
12*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^(7/2)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x+1)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(7/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
[Out] int(((c - c/(a*x))^(7/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(7/2),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**(7/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x
)
```

3.512 $\int e^{\tanh^{-1}(ax)} \left(c - \frac{c}{ax}\right)^{5/2} dx$

Optimal. Leaf size=171

$$-\frac{3a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}-\frac{3a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}+\frac{4ax^2(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{2x(ax+1)^{3/2}\left(c-\right.}{3(1-ax)^{5/2}}$$

```
[Out] -2/3*(c-c/a/x)^(5/2)*x*(a*x+1)^(3/2)/(-a*x+1)^(5/2)+4*a*(c-c/a/x)^(5/2)*x^2
*(a*x+1)^(3/2)/(-a*x+1)^(5/2)-3*a^(3/2)*(c-c/a/x)^(5/2)*x^(5/2)*arcsinh(a^(
1/2)*x^(1/2))/(-a*x+1)^(5/2)-3*a^2*(c-c/a/x)^(5/2)*x^3*(a*x+1)^(1/2)/(-a*x+
1)^(5/2)
```

Rubi [A] time = 0.16, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6134, 6129, 89, 78, 50, 54, 215}

$$-\frac{3a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{3a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{4ax^2(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{2x(ax+1)^{3/2}\left(c-\right.}{3(1-ax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (-3*a^2*(c - c/(a*x))^(5/2)*x^3*Sqrt[1 + a*x])/(1 - a*x)^(5/2) - (2*(c - c/
(a*x))^(5/2)*x*(1 + a*x)^(3/2))/(3*(1 - a*x)^(5/2)) + (4*a*(c - c/(a*x))^(5
/2)*x^2*(1 + a*x)^(3/2))/(1 - a*x)^(5/2) - (3*a^(3/2)*(c - c/(a*x))^(5/2)*x
^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{e^{\tanh^{-1}(ax)}(1-ax)^{5/2}}{x^{5/2}}dx}{(1-ax)^{5/2}}$$
$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1-ax)^2\sqrt{1+ax}}{x^{5/2}}dx}{(1-ax)^{5/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{\left(2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{\sqrt{1+ax}\left(-3a+\frac{3a^2x}{2}\right)}{x^{3/2}}dx}{3(1-ax)^{5/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{(1-ax)^{5/2}}-\frac{\left(3a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int}{(1-ax)^{5/2}}$$
$$= -\frac{3a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3}}{(1-ax)^{5/2}}$$
$$= -\frac{3a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3}}{(1-ax)^{5/2}}$$
$$= -\frac{3a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x(1+ax)^{3/2}}{3(1-ax)^{5/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3}}{(1-ax)^{5/2}}$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.51

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}\left(3a^2x^2+10ax-2\right)-9a^{3/2}x^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{3a^2x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(-2 + 10*a*x + 3*a^2*x^2) - 9*a^(3/2)
*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]]))/(3*a^2*x*Sqrt[1 - a*x])
```

fricas [A] time = 1.68, size = 330, normalized size = 1.93

$$\left[\frac{9\left(a^2c^2x^2-ac^2x\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(3a^2c^2x^2+10ac^2x-2c^2\right)\sqrt{-a^2x^2}}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(3*a^2*c^2*x^2 + 10*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1)*sqrt(
(a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x), 1/6*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(
c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c
*x^2 - a*c*x - c)) - 2*(3*a^2*c^2*x^2 + 10*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 +
 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(5/2)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.06, size = 136, normalized size = 0.80

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^2\sqrt{-a^2x^2+1}\,\left(6a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x}+9\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)x^2a^2+20a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}-4\sqrt{a}\,\sqrt{}\right.}{6x\,a^{\frac{5}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(5/2),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(-a^2*x^2+1)^(1/2)*(6*a^(5/2)*x^2*
(-(a*x+1)*x)^(1/2)+9*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2*a
^2+20*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-4*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-
(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^(5/2)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(5/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
[Out] int(((c - c/(a*x))^(5/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(5/2),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x
)
```

3.513 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2} dx$

Optimal. Leaf size=128

$$-\frac{2x\left(1-a^2x^2\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^3}+\frac{\sqrt{a}\,x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{(1-ax)^{3/2}}+\frac{ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}$$

```
[Out] -2*(c-c/a/x)^(3/2)*x*(-a^2*x^2+1)^(3/2)/(-a*x+1)^3+(c-c/a/x)^(3/2)*x^(3/2)*
arcsinh(a^(1/2)*x^(1/2))*a^(1/2)/(-a*x+1)^(3/2)+a*(c-c/a/x)^(3/2)*x^2*(a*x+
1)^(1/2)/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6134, 6128, 879, 848, 50, 54, 215}

$$-\frac{2x\left(1-a^2x^2\right)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^3}+\frac{ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}+\frac{\sqrt{a}\,x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (a*(c - c/(a*x))^(3/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (2*(c - c/(a*x)
)^(3/2)*x*(1 - a^2*x^2)^(3/2))/(1 - a*x)^3 + (Sqrt[a]*(c - c/(a*x))^(3/2)*x
^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{e^{\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}dx}{(1-ax)^{3/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{\sqrt{1-ax}\sqrt{1-a^2x^2}}{x^{3/2}}dx}{(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\left(a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}dx}{(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\left(a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{\sqrt{1+ax}}{\sqrt{x}}dx}{(1-ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\left(a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{}{\sqrt{x}}}{2(1-ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\left(a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\text{Subst}}{(1-a}$$

$$= \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\left(1-a^2x^2\right)^{3/2}}{(1-ax)^3}+\frac{\sqrt{a}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\sinh^{-}}{(1-ax)^{3/2}}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.55

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}(ax+2)-\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{a\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(2 + a*x) - Sqrt[a]*Sqrt[x]*ArcSinh[Sqr
t[a]*Sqrt[x]]))/(a*Sqrt[1 - a*x])
```

fricas [A] time = 0.60, size = 266, normalized size = 2.08

$$\left[\frac{(acx-c)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\sqrt{-a^2x^2+1}(acx+2c)\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)}, \frac{(acx-c)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*c*x - c)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x
)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) - 4*s
qrt(-a^2*x^2 + 1)*(a*c*x + 2*c)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(
(a*c*x - c)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*(a*c*x + 2*c)*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.05, size = 108, normalized size = 0.84

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\sqrt{-a^2x^2+1}\,\left(2a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}+\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)xa+4\sqrt{a}\,\sqrt{-(ax+1)\,x}\right)}{2a^{\frac{3}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-a^2*x^2+1)^(1/2)*(2*a^(3/2)*x*(-(a*x
+1)*x)^(1/2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+4*a^(1/2)
*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^(3/2)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(3/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
[Out] int(((c - c/(a*x))^(3/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(3/2),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x
)
```

3.514 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{c\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(1/2)/(-a*x+1)^(1/2)-c*(
-a^2*x^2+1)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6134, 6128, 848, 50, 54, 215}

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x] + (Sqrt[c - c/(a*x)]*Sqrt
[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{1-ax}}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.82

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{a}\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]
*Sqrt[x]]))/(Sqrt[a]*Sqrt[1 - a*x])
```

fricas [A] time = 0.59, size = 249, normalized size = 2.96

$$\left[-\frac{4\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx-c}{ax}} - (ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4\left(a^2x-a\right)}, -\frac{2\sqrt{-a^2x^2+1}\,ax\sqrt{\vphantom{x}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - (a*x - 1)*sqrt(-c
)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt
(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^
2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + (a*x - 1)*sqrt(c)*arctan(2*sqrt(-a
^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)))
/(a^2*x - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.05, size = 89, normalized size = 1.06

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(-2\sqrt{a}\,\sqrt{-(ax+1)\,x}+\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\right)}{2\,(ax-1)\,\sqrt{-(ax+1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(-2*a^(1/2)*(-(a*x+1)*x)^(1/
2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^(
1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.515 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=157

$$-\frac{3\sqrt{1-ax}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}}+\frac{2\sqrt{2}\,\sqrt{1-ax}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{1-ax}\,\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -3*arcsinh(a^(1/2)*x^(1/2))*(-a*x+1)^(1/2)/a^(3/2)/(c-c/a/x)^(1/2)/x^(1/2)+
2*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))*2^(1/2)*(-a*x+1)^(1/2)/a^(
3/2)/(c-c/a/x)^(1/2)/x^(1/2)-(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6134, 6129, 101, 157, 54, 215, 93, 206}

$$-\frac{3\sqrt{1-ax}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}}+\frac{2\sqrt{2}\,\sqrt{1-ax}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{1-ax}\,\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/Sqrt[c - c/(a*x)],x]
```

```
[Out] -((Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - c/(a*x)])) - (3*Sqrt[1 - a*x]*A
rcSinh[Sqrt[a]*Sqrt[x]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]) + (2*Sqrt[2]*S
qrt[1 - a*x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)*Sqr
t[c - c/(a*x)]*Sqrt[x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-ax}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}\sqrt{1+ax}}{1-ax}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{1-ax}\int \frac{\frac{1}{2}+\frac{3ax}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-ax}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a\sqrt{c-\frac{c}{ax}}\sqrt{x}} + \frac{\left(2\sqrt{1-ax}\right)\int \frac{1}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-ax}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}} + \frac{\left(4\sqrt{1-ax}\right)\text{Subst}\left(\int \frac{1}{1-2ax^2}\right.}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{3\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}} + \frac{2\sqrt{2}\sqrt{1-ax}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.67

$$-\frac{\sqrt{1-ax}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+3\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/Sqrt[c - c/(a*x)],x]
```

```
[Out] -((Sqrt[1 - a*x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + 3*ArcSinh[Sqrt[a]*Sqrt[x]
] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(a^(3/2)*S
qrt[c - c/(a*x)]*Sqrt[x]))
```

fricas [A] time = 0.57, size = 447, normalized size = 2.85

$$\left[-\frac{4\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx-c}{ax}}-2\sqrt{2}\,(acx-c)\sqrt{-\frac{1}{c}}\log\left(-\frac{17\,a^3x^3-3\,a^2x^2-4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-\frac{1}{c}}\,\sqrt{\frac{acx-c}{ax}}-13\,ax-1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+3}{4\left(a^2cx-ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - 2*sqrt(2)*(a*c*x
- c)*sqrt(-1/c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 4*sqrt(2)*(3*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x)) - 13*a*x - 1)/(a^3*x^
3 - 3*a^2*x^2 + 3*a*x - 1)) + 3*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*
c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x
)) - c)/(a*x - 1)))/(a^2*c*x - a*c), -1/2*(2*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a
*c*x - c)/(a*x)) + 3*(a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c
)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*sqrt(2)*(a*c*x -
 c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x))/((3*a^2
*x^2 - 2*a*x - 1)*sqrt(c)))/sqrt(c))/(a^2*c*x - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))), x)
```

maple [A] time = 0.05, size = 168, normalized size = 1.07

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\sqrt{-a^2x^2+1}\,\left(-2\sqrt{-(ax+1)\,x}\,a^{\frac{3}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}+3\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)a\sqrt{2}\,\sqrt{-\frac{1}{a}}-4\ln\left(\frac{2\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(a}}{ax-}\right.\right.}{4a^{\frac{3}{2}}c\,(ax-1)\,\sqrt{-(ax+1)\,x}\,\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c*(-a^2*x^2+1)^(1/2)*(-2*(-(a*x+1)*x)^(
1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*
x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2)-4*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^
(1/2)*a-3*a*x-1)/(a*x-1))*a^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)*2^(1/2)/(-1/a
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2)),x)
[Out] int((a*x + 1)/((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(1/2),x)
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.516 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=198

$$\frac{5(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{2}\,a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2\sqrt{ax+1}\,(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\sqrt{ax+1}\,\sqrt{1-ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] 5*(-a*x+1)^(3/2)*arcsinh(a^(1/2)*x^(1/2))/a^(5/2)/(c-c/a/x)^(3/2)/x^(3/2)-7
/2*(-a*x+1)^(3/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))/a^(5/2)/(c
-c/a/x)^(3/2)/x^(3/2)*2^(1/2)+2*(-a*x+1)^(3/2)*(a*x+1)^(1/2)/a^2/(c-c/a/x)^
(3/2)/x+(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a/(c-c/a/x)^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6134, 6129, 97, 154, 157, 54, 215, 93, 206}

$$\frac{5(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{2}\,a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2\sqrt{ax+1}\,(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\sqrt{ax+1}\,\sqrt{1-ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*(c - c/(a*x))^(3/2)) + (2*(1 - a*x)^(3/2)*
Sqrt[1 + a*x])/(a^2*(c - c/(a*x))^(3/2)*x) + (5*(1 - a*x)^(3/2)*ArcSinh[Sqr
t[a]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2)) - (7*(1 - a*x)^(3/2)*A
rcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(Sqrt[2]*a^(5/2)*(c - c/(a
*x))^(3/2)*x^(3/2))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{(1-ax)^{3/2}\int \frac{e^{\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}\sqrt{1+ax}}{(1-ax)^2}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(1-ax)^{3/2}\int \frac{\sqrt{x}\left(\frac{3}{2}+2ax\right)}{(1-ax)\sqrt{1+ax}}\,dx}{a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1-ax)^{3/2}\int \frac{-a-\frac{5a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a^3\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{\left(5(1-ax)^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} - \frac{\left(7(1-ax)^{3/2}\right)\int \frac{}{\sqrt{\ }}}{2a^2\left(c-\frac{c}{ax}\right.}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{\left(5(1-ax)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} - \frac{(7(1}{}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{5(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} - \frac{7(1-ax)^{3/2}\tanh^{-}}{\sqrt{2}\,a^{5/2}\left(c-\frac{c}{a}\right.}$$

Mathematica [A] time = 0.16, size = 126, normalized size = 0.64

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{ax+1}(ax-2)+10(ax-1)\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-7\sqrt{2}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{2a^{3/2}c\sqrt{x}\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*(-2 + a*x)*Sqrt[1 + a*x] + 10*(-1 + a*x)*ArcSinh[Sqrt[a]
*Sqrt[x]] - 7*Sqrt[2]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 +
 a*x]])/(2*a^(3/2)*c*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 - a*x])
```

fricas [A] time = 0.57, size = 526, normalized size = 2.66

$$\left[-\frac{7\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+10\left(a^2x^2-2\,ax+\right.}{8\left(a^3c^2x^2-2\,a^2c^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*
sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 10*(a^2*x
^2 - 2*a*x + 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(a^
2*x^2 - 2*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2
*a^2*c^2*x + a*c^2), 1/4*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arctan(2*
sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2
 - 2*a*c*x - c)) - 10*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2
+ 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 4*(a^
2*x^2 - 2*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2
*a^2*c^2*x + a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(3/2)), x)
```

maple [A] time = 0.06, size = 276, normalized size = 1.39

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(2a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\;x-5a^2\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\sqrt{2}\,\sqrt{-\frac{1}{a}}\;x-4\sqrt{-(ax+1)\,x}\cdots\right.}{4a^{\frac{3}{2}}c^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^2*(-a^2*x^2+1)^(1/2)*(2*a^(5/2)*2^(1
/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*x-5*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-
(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x-4*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/
2)*(-1/a)^(1/2)+7*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3
*a*x-1)/(a*x-1))*x+5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(
1/2)*(-1/a)^(1/2)-7*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1
)/(a*x-1))*a^(1/2))*2^(1/2)/(a*x-1)^2/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a*x))^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(3/2)*sqrt(-(a*x - 1)*(a*x + 1))),
 x)
```

3.517 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{7(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{79(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{8\sqrt{2}\,a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\sqrt{ax+1}\,(1-ax)^{5/2}}{8a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11\sqrt{ax+1}\,(1-ax)^{3/2}}{8a^2x\left(c-\frac{c}{ax}\right)^{5/2}}+$$

```
[Out] -7*(-a*x+1)^(5/2)*arcsinh(a^(1/2)*x^(1/2))/a^(7/2)/(c-c/a/x)^(5/2)/x^(5/2)+
79/16*(-a*x+1)^(5/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))/a^(7/2)
/(c-c/a/x)^(5/2)/x^(5/2)*2^(1/2)-11/8*(-a*x+1)^(3/2)*(a*x+1)^(1/2)/a^2/(c-c
/a/x)^(5/2)/x-23/8*(-a*x+1)^(5/2)*(a*x+1)^(1/2)/a^3/(c-c/a/x)^(5/2)/x^2+1/2
*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a/(c-c/a/x)^(5/2)
```

Rubi [A] time = 0.20, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6134, 6129, 97, 149, 154, 157, 54, 215, 93, 206}

$$-\frac{23\sqrt{ax+1}\,(1-ax)^{5/2}}{8a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{7(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{79(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{8\sqrt{2}\,a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11\sqrt{ax+1}\,(1-ax)^{3/2}}{8a^2x\left(c-\frac{c}{ax}\right)^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - a*x]*Sqrt[1 + a*x])/(2*a*(c - c/(a*x))^(5/2)) - (11*(1 - a*x)^(3/
2)*Sqrt[1 + a*x])/(8*a^2*(c - c/(a*x))^(5/2)*x) - (23*(1 - a*x)^(5/2)*Sqrt[
1 + a*x])/(8*a^3*(c - c/(a*x))^(5/2)*x^2) - (7*(1 - a*x)^(5/2)*ArcSinh[Sqrt
[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) + (79*(1 - a*x)^(5/2)*A
rcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(8*Sqrt[2]*a^(7/2)*(c - c/
(a*x))^(5/2)*x^(5/2))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{(1-ax)^{5/2}\int \frac{e^{\tanh^{-1}(ax)}x^{5/2}}{(1-ax)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}\int \frac{x^{5/2}\sqrt{1+ax}}{(1-ax)^3}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{(1-ax)^{5/2}\int \frac{x^{3/2}\left(\frac{5}{2}+3ax\right)}{(1-ax)^2\sqrt{1+ax}}\,dx}{2a\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{(1-ax)^{5/2}\int \frac{\sqrt{x}\left(-\frac{33a}{4}-\frac{23a^2x}{2}\right)}{(1-ax)\sqrt{1+ax}}\,dx}{4a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{23(1-ax)^{5/2}\sqrt{1+ax}}{8a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\int \frac{\frac{23a^2}{4}+14a}{\sqrt{x}(1-ax)\sqrt{}}}{4a^5\left(c-\frac{c}{ax}\right)^{5/2}x^5}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{23(1-ax)^{5/2}\sqrt{1+ax}}{8a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} - \frac{\left(7(1-ax)^{5/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+}}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/}}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{23(1-ax)^{5/2}\sqrt{1+ax}}{8a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} - \frac{\left(7(1-ax)^{5/2}\right)\text{Subst}\left(\int\right.}{a^3\left(c-\frac{c}{ax}\right.}$$

$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{23(1-ax)^{5/2}\sqrt{1+ax}}{8a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} - \frac{7(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{\right.}}{a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/}}$$

Mathematica [A] time = 0.18, size = 139, normalized size = 0.56

$$\frac{-2\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(8a^2x^2-35ax+23\right)-112(ax-1)^2\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+79\sqrt{2}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16a^{3/2}c^2\sqrt{x}(1-ax)^{3/2}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(23 - 35*a*x + 8*a^2*x^2) - 112*(-1 + a*x
)^2*ArcSinh[Sqrt[a]*Sqrt[x]] + 79*Sqrt[2]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqr
t[a]*Sqrt[x])/Sqrt[1 + a*x]])/(16*a^(3/2)*c^2*Sqrt[c - c/(a*x)]*Sqrt[x]*(1
- a*x)^(3/2))
```

fricas [A] time = 0.64, size = 600, normalized size = 2.41

$$\left[-\frac{79\,\sqrt{2}\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\sqrt{-c}\,\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx+4\,\sqrt{2}\left(3\,a^{2}x^{2}+ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+112\left(a^{3}x^{3}\right.}{64\left(a^{4}c^{3}x^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(79*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(17*a^3*
c*x^3 - 3*a^2*c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2
+ 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1
)) + 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a
*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x)) - c)/(a*x - 1)) + 8*(8*a^3*x^3 - 35*a^2*x^2 + 23*a*x)*sqrt(-a^2*x^2 + 1
)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c
^3), 1/32*(79*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan(2*sq
rt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan(2*sqr
t(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
c)) - 4*(8*a^3*x^3 - 35*a^2*x^2 + 23*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x -
c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.07, size = 390, normalized size = 1.57

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(16a^{\frac{7}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x^2-70a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x+79a^{\frac{5}{2}}\ln\left(\frac{2\sqrt{2}\,\sqrt{-\frac{1}{a}}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/32*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(16*a^(7/2)*2^(1/2)*(-1/a)
^(1/2)*(-(a*x+1)*x)^(1/2)*x^2-70*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^
(1/2)*x+79*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)
/(a*x-1))*x^2-56*a^3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/
2)*(-1/a)^(1/2)*x^2+46*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)+112*
a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x
-158*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-
1))*x-56*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^
(1/2)+79*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*
a^(1/2))*2^(1/2)/a^(3/2)/c^3/(a*x-1)^3/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a*x))^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(5/2)*sqrt(-(a*x - 1)*(a*x + 1))),
 x)
```

3.518 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=145

$$\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-x\left(c-\frac{c}{ax}\right)^{9/2}$$

```
[Out] -5/3*c^3*(c-c/a/x)^(3/2)/a-c^2*(c-c/a/x)^(5/2)/a-5/7*c*(c-c/a/x)^(7/2)/a-(c
-c/a/x)^(9/2)*x+5*c^(9/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a-5*c^4*(c-c/a/x
)^(1/2)/a
```

Rubi [A] time = 0.19, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 50, 63, 208}

$$-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-x\left(c-\frac{c}{ax}\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (-5*c^4*Sqrt[c - c/(a*x)])/a - (5*c^3*(c - c/(a*x))^(3/2))/(3*a) - (c^2*(c
- c/(a*x))^(5/2))/a - (5*c*(c - c/(a*x))^(7/2))/(7*a) - (c - c/(a*x))^(9/2)
*x + (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}{1-ax}\,dx$$

$$=-\frac{c\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{x}\,dx}{a}$$

$$=-\frac{c\int\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}dx}{a}$$

$$=\frac{c\,\text{Subst}\left(\int\frac{(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{(5c)\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{\left(5c^2\right)\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{\left(5c^3\right)\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}\,dx,x,\frac{1}{x}\right.}{2a}$$

$$=-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{\left(5c^4\right)\text{Subst}\left(\right.}{}$$

$$=-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-$$

$$=-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x+$$

$$=-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x+$$

Mathematica [A] time = 0.15, size = 91, normalized size = 0.63

$$\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c^4\left(21a^4x^4+92a^3x^3+4a^2x^2-18ax+6\right)\sqrt{c-\frac{c}{ax}}}{21a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] -1/21*(c^4*Sqrt[c - c/(a*x)]*(6 - 18*a*x + 4*a^2*x^2 + 92*a^3*x^3 + 21*a^4*
x^4))/(a^4*x^3) + (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

fricas [A] time = 0.46, size = 234, normalized size = 1.61

$$\left[\frac{105\,a^3c^{\frac{9}{2}}x^3\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(21\,a^4c^4x^4+92\,a^3c^4x^3+4\,a^2c^4x^2-18\,ac^4x+6\,c^4\right)\sqrt{\frac{acx-c}{ax}}}{42\,a^4x^3},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/42*(105*a^3*c^(9/2)*x^3*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + c) - 2*(21*a^4*c^4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x
 + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3), -1/21*(105*a^3*sqrt(-c)*c^4*x
^3*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (21*a^4*c^4*x^4 + 92*a^3*c^
4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x
^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-97,36.6646323889,7]W
```

```
arning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[
0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,
[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-89,7.79369851155,-49]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-64,-30,70]Warning, c
hoosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [22,42,56]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4
,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-9,-13,46]Warning, choosing root of [1,0,%%
%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%
%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] a
t parameters values [24,49,-6]Warning, choosing root of [1,0,%%%{-2,[2,1,2]
%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3
]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters
 values [-49,-33,-70]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{
2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%
{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [
8,63,-64]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%
}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%
%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [2,62,-37]War
ning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,
1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1
,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-80,-23,65]Warning, choo
sing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0
,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+
%%%{1,[0,2,0]%%%}] at parameters values [-85,28,-44]Warning, choosing root
of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,
2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2
,0]%%%}] at parameters values [-22,93,91]Warning, choosing root of [1,0,%%%
{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%
%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at
 parameters values [31,-21,88]Warning, choosing root of [1,0,%%%{-2,[2,1,2]
%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3
]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters
 values [76,-66,66]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,
[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-
1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [5,
-23,79]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}
+%%%{2,[1,1,1]%%%}+%%%{2,[1,0,1]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}
+%%%{1,[4,0,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{4,[3,1,3]%%%}+%%%{-2,[3,0,3]%%%}+
%%%{1,[2,2,2]%%%}+%%%{-2,[2,1,2]%%%}+%%%{1,[2,0,2]%%%}] at parameters value
s [-88,9,6]Warning, choosing root of [1,0,%%%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%
```

```
%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]
%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters
values [-69,-8,31]Warning, choosing root of [1,0,%%%{-2,[2,1,0]%%%}+%%%{2,[
1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,[3,2,1]%%%}+%%%{1,
[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at para
meters values [89,2,97]Warning, choosing root of [1,0,%%%{-2,[2,1,0]%%%}+%%
%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,[3,2,1]%%%}+%
%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at
 parameters values [-92,80,-24]Warning, choosing root of [1,0,%%%{-2,[2,1,0
]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,[3,2,
1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]
%%%}] at parameters values [-17,41,64]Warning, choosing root of [1,0,%%%{2,
[1,1,1]%%%}+%%%{-2,[0,2,1]%%%}+%%%{-2,[0,0,1]%%%},0,%%%{1,[2,2,2]%%%}+%%%{-
2,[1,3,2]%%%}+%%%{2,[1,1,2]%%%}+%%%{1,[0,4,2]%%%}+%%%{-2,[0,2,2]%%%}+%%%{1,
[0,0,2]%%%}] at parameters values [18,-51,-42]Warning, choosing root of [1,
0,%%%{2,[1,1,1]%%%}+%%%{-2,[0,2,1]%%%}+%%%{-2,[0,0,1]%%%},0,%%%{1,[2,2,2]%%
%}+%%%{-2,[1,3,2]%%%}+%%%{2,[1,1,2]%%%}+%%%{1,[0,4,2]%%%}+%%%{-2,[0,2,2]%%%
}+%%%{1,[0,0,2]%%%}] at parameters values [-65,22,-94]Sign error (%%%{sqrt(
c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+%%%{-2*sqrt(c)*a^2,1%%%}+%%%{-a*
sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*c)*abs(a)/4,5/2%%%}+%%%{undef,7/
2%%%})Evaluation time: 1.79Limit: Max order reached or unable to make serie
s expansion Error: Bad Argument Value
```

maple [A] time = 0.05, size = 163, normalized size = 1.12

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^4 \left(-210a^{\frac{9}{2}}\sqrt{a\,x^2-x}\;x^5+105\ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^5a^4+168a^{\frac{7}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^3-16a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^2-\right.}{42x^4\sqrt{(ax-1)\,x}\;a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(9/2),x)
```

```
[Out] 1/42*(c*(a*x-1)/a/x)^(1/2)/x^4*c^4*(-210*a^(9/2)*(a*x^2-x)^(1/2)*x^5+105*ln
(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^5*a^4+168*a^(7/2)*(a*x^
2-x)^(3/2)*x^3-16*a^(5/2)*(a*x^2-x)^(3/2)*x^2-24*a^(3/2)*(a*x^2-x)^(3/2)*x+
12*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1)*x)^(1/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(9/2),x, algorithm="maxima")
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^(9/2)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(c-\frac{c}{a\,x}\right)^{9/2}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(9/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] int(-((c - c/(a*x))^(9/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 31.19, size = 2421, normalized size = 16.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(9/2),x)
[Out] -c**4*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqr
t(a)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (
I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a
), True)) - 4*c**5*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - 4*c**4*s
qrt(c - c/(a*x))/a - 2*c**4*Piecewise((4*I*a**(11/2)*sqrt(c)*x**(7/2)/(-15*
I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(5/2)) - 4*I*a**(9/2)*sqrt(c)*x**(5/
2)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(5/2)) - 4*I*a**5*sqrt(c)*x*
*3*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(5/2)) + 2*I*a
**4*sqrt(c)*x**2*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**
(5/2)) + 8*I*a**3*sqrt(c)*x*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a
**(5/2)*x**(5/2)) - 6*I*a**2*sqrt(c)*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2)
 + 15*I*a**(5/2)*x**(5/2)), Abs(a*x) > 1), (-4*I*a**(11/2)*sqrt(c)*x**(7/2)
/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(5/2)) + 4*I*a**(9/2)*sqrt(c)*x
**(5/2)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(5/2)) - 4*a**5*sqrt(c)*
x**3*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(5/2)) + 2*a
**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**
(5/2)) + 8*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**
(5/2)*x**(5/2)) - 6*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 1
5*I*a**(5/2)*x**(5/2)), True))/a**3 + c**4*Piecewise((16*I*a**(19/2)*sqrt(c
)*x**(13/2)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I
*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)) - 48*I*a**(17/2)*sqrt(c)*x**(
11/2)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9
/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)) + 48*I*a**(15/2)*sqrt(c)*x**(9/2)/(
```

```
-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**
(9/2) + 105*I*a**(7/2)*x**(7/2)) - 16*I*a**(13/2)*sqrt(c)*x**(7/2)/(-105*I*
a**(13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) +
 105*I*a**(7/2)*x**(7/2)) - 16*I*a**9*sqrt(c)*x**6*sqrt(a*x - 1)/(-105*I*a*
*(13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 1
05*I*a**(7/2)*x**(7/2)) + 40*I*a**8*sqrt(c)*x**5*sqrt(a*x - 1)/(-105*I*a**(
13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105
*I*a**(7/2)*x**(7/2)) - 30*I*a**7*sqrt(c)*x**4*sqrt(a*x - 1)/(-105*I*a**(13
/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I
*a**(7/2)*x**(7/2)) + 40*I*a**6*sqrt(c)*x**3*sqrt(a*x - 1)/(-105*I*a**(13/2
)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a
**(7/2)*x**(7/2)) - 100*I*a**5*sqrt(c)*x**2*sqrt(a*x - 1)/(-105*I*a**(13/2)
*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a*
*(7/2)*x**(7/2)) + 96*I*a**4*sqrt(c)*x*sqrt(a*x - 1)/(-105*I*a**(13/2)*x**(
13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2
)*x**(7/2)) - 30*I*a**3*sqrt(c)*sqrt(a*x - 1)/(-105*I*a**(13/2)*x**(13/2) +
 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7
/2)), Abs(a*x) > 1), (16*I*a**(19/2)*sqrt(c)*x**(13/2)/(-105*I*a**(13/2)*x*
*(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7
/2)*x**(7/2)) - 48*I*a**(17/2)*sqrt(c)*x**(11/2)/(-105*I*a**(13/2)*x**(13/2
) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x*
*(7/2)) + 48*I*a**(15/2)*sqrt(c)*x**(9/2)/(-105*I*a**(13/2)*x**(13/2) + 315
*I*a**(11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2))
 - 16*I*a**(13/2)*sqrt(c)*x**(7/2)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(
11/2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)) + 16*a
**9*sqrt(c)*x**6*sqrt(-a*x + 1)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/
2)*x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)) - 40*a**8
*sqrt(c)*x**5*sqrt(-a*x + 1)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/2)*
x**(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)) + 30*a**7*sq
rt(c)*x**4*sqrt(-a*x + 1)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/2)*x**
(11/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)) - 40*a**6*sqrt(
c)*x**3*sqrt(-a*x + 1)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11
/2) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)) + 100*a**5*sqrt(c)
*x**2*sqrt(-a*x + 1)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2
) - 315*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)) - 96*a**4*sqrt(c)*x*
sqrt(-a*x + 1)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 31
5*I*a**(9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)) + 30*a**3*sqrt(c)*sqrt(-a*
x + 1)/(-105*I*a**(13/2)*x**(13/2) + 315*I*a**(11/2)*x**(11/2) - 315*I*a**(
9/2)*x**(9/2) + 105*I*a**(7/2)*x**(7/2)), True))/a**4
```

3.519 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=120

$$\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[Out] -c^2*(c-c/a/x)^(3/2)/a-3/5*c*(c-c/a/x)^(5/2)/a-(c-c/a/x)^(7/2)*x+3*c^(7/2)*
arctanh((c-c/a/x)^(1/2)/c^(1/2))/a-3*c^3*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.16, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 50, 63, 208}

$$-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (-3*c^3*Sqrt[c - c/(a*x)])/a - (c^2*(c - c/(a*x))^(3/2))/a - (3*c*(c - c/(a
*x))^(5/2))/(5*a) - (c - c/(a*x))^(7/2)*x + (3*c^(7/2)*ArcTanh[Sqrt[c - c/(
a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{1-ax}dx$$
$$=-\frac{c\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{x}dx}{a}$$
$$=-\frac{c\int\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}dx}{a}$$
$$=\frac{c\,\text{Subst}\left(\int\frac{(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}{x^2}dx,x,\frac{1}{x}\right)}{a}$$
$$=-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{(3c)\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{\left(3c^2\right)\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{\left(3c^3\right)\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{\left(3c^4\right)\text{Subst}\left(\int\right.}{2}$$
$$=-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x+\left(3c^3\right)\text{Subst}\left(\int\right.$$
$$=-\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{3c^{7/2}\tanh^{-1}\left(\sqrt{\right.}}{a}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.69

$$\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c^3\left(5a^3x^3+8a^2x^2+4ax-2\right)\sqrt{c-\frac{c}{ax}}}{5a^3x^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(7/2),x]

[Out] -1/5*(c^3*Sqrt[c - c/(a*x)]*(-2 + 4*a*x + 8*a^2*x^2 + 5*a^3*x^3))/(a^3*x^2) + (3*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a

fricas [A] time = 0.41, size = 212, normalized size = 1.77

$$\left[\frac{15\,a^{2}c^{\frac{7}{2}}x^{2}\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(5\,a^{3}c^{3}x^{3}+8\,a^{2}c^{3}x^{2}+4\,ac^{3}x-2\,c^{3}\right)\sqrt{\frac{acx-c}{ax}}}{10\,a^{3}x^{2}},-\frac{15\,a^{2}\sqrt{-c}\,c^{3}x^{2}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(7/2),x, algorithm="fricas")

[Out] [1/10*(15*a^2*c^(7/2)*x^2*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2), -1/5*(15*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(7/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-97,36.6646323889,7]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-89,7.79369851155,-49] Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-64,-30,70]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [22,42,56]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4

,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-9,-13,46]Warning, choosing root of [1,0,%%
%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%
%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] a
t parameters values [24,49,-6]Warning, choosing root of [1,0,%%%{-2,[2,1,2]
%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3
]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters
 values [-49,-33,-70]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{
2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%
{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [
8,63,-64]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%
}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%
%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [2,62,-37]War
ning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,
1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1
,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-80,-23,65]Warning, choo
sing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0
,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+
%%%{1,[0,2,0]%%%}] at parameters values [-85,28,-44]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,
2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2
,0]%%%}] at parameters values [-22,93,91]Warning, choosing root of [1,0,%%%
{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%
%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at
 parameters values [31,-21,88]Warning, choosing root of [1,0,%%%{-2,[2,1,2]
%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{2,[1,0,1]%%%},0,%%%{1,[4,2,4]
%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{4,[3,1,3]%
%%}+%%%{-2,[3,0,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,1,2]%%%}+%%%{1,[2,0,2]%%
%}] at parameters values [76,-66,66]Warning, choosing root of [1,0,%%%{-2,[
2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,
[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0
,2,0]%%%}] at parameters values [5,-23,79]Warning, choosing root of [1,0,%%
%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%
%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%
%{1,[0,2,0]%%%}] at parameters values [-88,9,6]Warning, choosing root of [1
,0,%%%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%
%%}+%%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [-69,-8,31]Warning, choosing roo
t of [1,0,%%%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[
4,2,0]%%%}+%%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1
,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [89,2,97]Warning, choosin
g root of [1,0,%%%{2,[1,1,1]%%%}+%%%{-2,[0,2,1]%%%}+%%%{-2,[0,0,1]%%%},0,%%
%{1,[2,2,2]%%%}+%%%{-2,[1,3,2]%%%}+%%%{2,[1,1,2]%%%}+%%%{1,[0,4,2]%%%}+%%%{
-2,[0,2,2]%%%}+%%%{1,[0,0,2]%%%}] at parameters values [-92,80,-24]Warning,
 choosing root of [1,0,%%%{2,[1,1,1]%%%}+%%%{-2,[0,2,1]%%%}+%%%{-2,[0,0,1]%

```
%%},0,%%%{1,[2,2,2]%%%}+%%%{-2,[1,3,2]%%%}+%%%{2,[1,1,2]%%%}+%%%{1,[0,4,2]%
%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,0,2]%%%}] at parameters values [-17,41,64]S
ign error (%%%{sqrt(c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+%%%{-2*sqrt(
c)*a^2,1%%%}+%%%{-a*sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*c)*abs(a)/4,
5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or unable to make series
 expansion Error: Bad Argument Value
```

maple [A] time = 0.04, size = 144, normalized size = 1.20

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^3 \left(-30\sqrt{a\,x^2-x}\; a^{\frac{7}{2}} x^4 + 20 a^{\frac{5}{2}} \left(a\,x^2-x\right)^{\frac{3}{2}} x^2 + 15\ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right) x^4 a^3 + 4a^{\frac{3}{2}} \left(a\,x^2-x\right)^{\frac{3}{2}} x - 4\left(a\right.\right.}{10x^3\sqrt{(ax-1)\,x}\; a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(7/2),x)
```

```
[Out] 1/10*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3*(-30*(a*x^2-x)^(1/2)*a^(7/2)*x^4+20*a^(5
/2)*(a*x^2-x)^(3/2)*x^2+15*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/
2))*x^4*a^3+4*a^(3/2)*(a*x^2-x)^(3/2)*x-4*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1)
*x)^(1/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^(7/2)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x+1)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(7/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - c/(a*x))^(7/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 25.99, size = 777, normalized size = 6.48

$$-c^3\left(\begin{cases}\frac{\sqrt{a}\sqrt{c}\,x^{\frac{3}{2}}}{\sqrt{ax-1}} - \frac{\sqrt{c}\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} - \frac{\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{ax-1}} & \text{for } |ax| > 1 \\ \frac{i\sqrt{c}\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} + \frac{i\sqrt{c}\sqrt{x}\sqrt{-ax+1}}{\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{2c^4\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2c^3\sqrt{c-\frac{c}{ax}}}{a} + \frac{c^3\left(\begin{cases}0 & \text{f} \\ \frac{2a\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{3c} & \text{c}\end{cases}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(7/2),x)
```

```
[Out] -c**3*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqr
t(a)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (
I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a
), True)) - 2*c**4*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) - 2*c**3*s
qrt(c - c/(a*x))/a + c**3*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2
)/(3*c), True))/a**2 - c**3*Piecewise((4*I*a**(11/2)*sqrt(c)*x**(7/2)/(-15*
I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(5/2)) - 4*I*a**(9/2)*sqrt(c)*x**(5/
2)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(5/2)) - 4*I*a**5*sqrt(c)*x*
*3*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(5/2)) + 2*I*a
**4*sqrt(c)*x**2*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**
(5/2)) + 8*I*a**3*sqrt(c)*x*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a
**(5/2)*x**(5/2)) - 6*I*a**2*sqrt(c)*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2)
 + 15*I*a**(5/2)*x**(5/2)), Abs(a*x) > 1), (-4*I*a**(11/2)*sqrt(c)*x**(7/2)
/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(5/2)) + 4*I*a**(9/2)*sqrt(c)*x
**(5/2)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(5/2)) - 4*a**5*sqrt(c)*
x**3*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(5/2)) + 2*a
**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**
(5/2)) + 8*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**
(5/2)*x**(5/2)) - 6*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 1
5*I*a**(5/2)*x**(5/2)), True))/a**3
```

3.520 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=96

$$\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[Out] -1/3*c*(c-c/a/x)^(3/2)/a-(c-c/a/x)^(5/2)*x+c^(5/2)*arctanh((c-c/a/x)^(1/2)/
c^(1/2))/a-c^2*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 50, 63, 208}

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] -((c^2*Sqrt[c - c/(a*x)])/a) - (c*(c - c/(a*x))^(3/2))/(3*a) - (c - c/(a*x)
)^(5/2)*x + (c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{1-ax}dx$$

$$= -\frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{x}dx}{a}$$

$$= -\frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}dx}{a}$$

$$= \frac{c\,\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}{x^2}dx, x, \frac{1}{x}\right)}{a}$$

$$= -\left(c-\frac{c}{ax}\right)^{5/2}x - \frac{c\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} - \left(c-\frac{c}{ax}\right)^{5/2}x - \frac{c^2\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c^2\sqrt{c-\frac{c}{ax}}}{a} - \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} - \left(c-\frac{c}{ax}\right)^{5/2}x - \frac{c^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c^2\sqrt{c-\frac{c}{ax}}}{a} - \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} - \left(c-\frac{c}{ax}\right)^{5/2}x + c^2\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}dx, x, \sqrt{c-\frac{c}{ax}}\right.$$

$$= -\frac{c^2\sqrt{c-\frac{c}{ax}}}{a} - \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} - \left(c-\frac{c}{ax}\right)^{5/2}x + \frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.78

$$\frac{c^2\left(-3a^2x^2+2ax-2\right)\sqrt{c-\frac{c}{ax}}+3ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 2*a*x - 3*a^2*x^2) + 3*a*c^(5/2)*x*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/(3*a^2*x)
```

fricas [A] time = 0.54, size = 182, normalized size = 1.90

$$\left[\frac{3\,ac^{\frac{5}{2}}x\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(3\,a^2c^2x^2-2\,ac^2x+2\,c^2\right)\sqrt{\frac{acx-c}{ax}}}{6\,a^2x},-\frac{3\,a\sqrt{-c}\,c^2x\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*c^(5/2)*x*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c) - 2*(3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)
, -1/3*(3*a*sqrt(-c)*c^2*x*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (3*
a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [86,-97,-82]Warning, c
hoosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [7,-27,26]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4
,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-89,63,-49]Warning, choosing root of [1,0,%
%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+
%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]
at parameters values [-86,-64,-30]Warning, choosing root of [1,0,%%%{-2,[2,
1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3
,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parame
ters values [70,22,42]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%
```

```
{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%
%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values
[56,-9,-13]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%
%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2
]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [46,24,49]W
arning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[
0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,
[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [18,-49,-33]Warning, ch
oosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%}
,0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%
}+%%%{1,[0,2,0]%%%}] at parameters values [-70,8,63]Warning, choosing root
of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{2,[1,0,
1]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{-2,[3,2
,3]%%%}+%%%{4,[3,1,3]%%%}+%%%{-2,[3,0,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,1,
2]%%%}+%%%{1,[2,0,2]%%%}] at parameters values [-64,2,62]Sign error (%%%{sq
rt(c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+%%%{-2*sqrt(c)*a^2,1%%%}+%%%{
-a*sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*c)*abs(a)/4,5/2%%%}+%%%{undef
,7/2%%%})Limit: Max order reached or unable to make series expansion Error:
 Bad Argument Value
```

maple [A] time = 0.04, size = 108, normalized size = 1.12

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^2 \left(-6\sqrt{a\,x^2-x}\, a^{\frac{5}{2}} x^3+3\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) x^3 a^2+4\left(a\,x^2-x\right)^{\frac{3}{2}}\sqrt{a}\right)}{6x^2\sqrt{(ax-1)\,x}\, a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(5/2),x)
```

```
[Out] 1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2*(-6*(a*x^2-x)^(1/2)*a^(5/2)*x^3+3*ln(1/2*
(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^2+4*(a*x^2-x)^(3/2)*a^(1
/2))/((a*x-1)*x)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^(5/2)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(c-\frac{c}{a\,x}\right)^{5/2}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - c/(a*x))^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 6.73, size = 143, normalized size = 1.49

$$-c^{2}\left(\begin{cases}\frac{\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{ax-1}}-\frac{\sqrt{c}\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}-\frac{\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{ax-1}} & \text{for}\: \left|{ax}\right| > 1 \\\frac{i\sqrt{c}\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}\sqrt{-ax+1}}{\sqrt{a}} & \text{otherwise} \end{cases}\right)+\frac{c^{2}\left(\begin{cases}0 & \text{for}\: c = 0 \\\frac{2a\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(5/2),x)
```

```
[Out] -c**2*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqr
t(a)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (
I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a
), True)) + c**2*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)/(3*c),
True))/a**2
```

3.521 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=71

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c\sqrt{c-\frac{c}{ax}}}{a}-x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[Out] -(c-c/a/x)^(3/2)*x-c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+c*(c-c/a/x)^(
1/2)/a
```

Rubi [A] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 50, 63, 208}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c\sqrt{c-\frac{c}{ax}}}{a}-x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)])/a - (c - c/(a*x))^(3/2)*x - (c^(3/2)*ArcTanh[Sqrt[c -
 c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{1-ax}dx$$
$$=-\frac{c\int\frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x}dx}{a}$$
$$=-\frac{c\int\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}\,dx}{a}$$
$$=\frac{c\,\text{Subst}\left(\int\frac{(a+x)\sqrt{c-\frac{cx}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{a}$$
$$=-\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c^2\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}$$
$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x-c\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)$$
$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.80

$$\frac{c(2-ax)\sqrt{c-\frac{c}{ax}}-c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(2 - a*x) - c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
)/a
```

fricas [A] time = 0.44, size = 136, normalized size = 1.92

$$\left[\frac{c^{\frac{3}{2}}\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},\ \frac{\sqrt{-c}\,c\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c^(3/2)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2
*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, (sqrt(-c)*c*arctan(sqrt(-c)*sqrt
((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [86,-97,-82]Warning, c
hoosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [7,-27,26]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4
,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-89,63,-49]Warning, choosing root of [1,0,%
%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+
%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]
at parameters values [-86,-64,-30]Warning, choosing root of [1,0,%%%{-2,[2,
1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3
,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parame
ters values [70,22,42]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%
{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%
%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values
[56,-9,-13]Sign error (%%%{sqrt(c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+
%%%{-2*sqrt(c)*a^2,1%%%}+%%%{-a*sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*
```

```
c)*abs(a)/4,5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

maple [A] time = 0.04, size = 103, normalized size = 1.45

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\left(-2\sqrt{a\,x^2-x}\; a^{\frac{3}{2}} x^2+4\left(a\,x^2-x\right)^{\frac{3}{2}}\sqrt{a}\;+\ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right) x^2 a\right)}{2x\sqrt{(ax-1)\,x}\; a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)/x*c*(-2*(a*x^2-x)^(1/2)*a^(3/2)*x^2+4*(a*x^2-x)^
(3/2)*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a)/((
a*x-1)*x)^(1/2)/a^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a*x))^(3/2)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - c/(a*x))^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 30.51, size = 163, normalized size = 2.30

$$-c\left(\begin{cases}\frac{\sqrt{a}\,\sqrt{c}\,x^{\frac{3}{2}}}{\sqrt{ax-1}}-\frac{\sqrt{c}\,\operatorname{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{a}-\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{a}\,\sqrt{ax-1}} & \text{for } |ax|>1\\ \frac{i\sqrt{c}\,\operatorname{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\,\sqrt{x}\,\sqrt{-ax+1}}{\sqrt{a}} & \text{otherwise}\end{cases}\right)+\frac{2c^2\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2c\sqrt{c-\frac{c}{ax}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(3/2),x)
```

```
[Out] -c*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqrt(a
)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (I*s
qrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a),
True)) + 2*c**2*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) + 2*c*sqrt(c
- c/(a*x))/a
```

3.522 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=51

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] -3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a-x*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6133, 25, 514, 375, 78, 63, 208}

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{1-ax}\,dx$$
$$=-\frac{c\int\frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x}\,dx}{a}$$
$$=-\frac{c\int\frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=\frac{c\,\text{Subst}\left(\int\frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=-\sqrt{c-\frac{c}{ax}}\,x+\frac{(3c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$
$$=-\sqrt{c-\frac{c}{ax}}\,x-3\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)$$
$$=-\sqrt{c-\frac{c}{ax}}\,x-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [**A**] time = 0.03, size = 51, normalized size = 1.00

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

fricas [A] time = 0.45, size = 125, normalized size = 2.45

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\,-\frac{ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a, -(a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(
-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

giac [B] time = 4.94, size = 97, normalized size = 1.90

$$-\frac{3\,\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{2\,a}+\frac{3\,\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\,|a|+ac\right|\right)}{2\,a\mathrm{sgn}(x)}-\frac{\sqrt{a^2cx^2-acx}\,|a|}{a^2\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a + 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) - sqrt(a
^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

maple [B] time = 0.04, size = 120, normalized size = 2.35

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^2-x}\,\sqrt{a}-4\sqrt{(ax-1)\,x}\,\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-2\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{(ax-1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(1/2)-4*((a*x-1)*x)^(1/2)*
a^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-2*ln(1/2*(2*((a
*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c - \frac{c}{ax}}}{ax - 1}\, dx - \int \frac{ax\sqrt{c - \frac{c}{ax}}}{ax - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x*sqrt(c - c/(a*x))/
(a*x - 1), x)
```

3.523 $\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=71

$$-\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] -5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)+5/a/(c-c/a/x)^(1/2)-x/(c-c/a/
x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 51, 63, 208}

$$-\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] 5/(a*Sqrt[c - c/(a*x)]) - x/Sqrt[c - c/(a*x)] - (5*ArcTanh[Sqrt[c - c/(a*x)
]/Sqrt[c]])/(a*Sqrt[c])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,(1-ax)}\,dx$$
$$= -\frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}x}\,dx}{a}$$
$$= -\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{a}$$
$$= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{(5c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{5}{a\sqrt{c-\frac{c}{ax}}} - \frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{5\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{5}{a\sqrt{c-\frac{c}{ax}}} - \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}$$
$$= \frac{5}{a\sqrt{c-\frac{c}{ax}}} - \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Mathematica [C] time = 0.03, size = 44, normalized size = 0.62

$$\frac{5\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{1}{ax}\right)-ax}{a\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (-(a*x) + 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*Sqrt[c - c/(a*
x)])
```

fricas [A] time = 0.44, size = 175, normalized size = 2.46

$$\left[\frac{5\,(ax-1)\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(a^2x^2-5\,ax\right)\sqrt{\frac{acx-c}{ax}}}{2\left(a^2cx-ac\right)},\ \frac{5\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(\cdots}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(5*(a*x - 1)*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) - 2*(a^2*x^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c), (5
*(a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a^2*x^2 -
 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

giac [B] time = 2.17, size = 123, normalized size = 1.73

$$\frac{a\left(\frac{5\,c\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}+\frac{4\,c^2-\frac{5\,(acx-c)c}{ax}}{\left(c\sqrt{\frac{acx-c}{ax}}-\frac{(acx-c)\sqrt{\frac{acx-c}{ax}}}{ax}\right)a^2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] a*(5*c*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)) + (4*c^2 - 5
*(a*c*x - c)*c/(a*x))/((c*sqrt((a*c*x - c)/(a*x)) - (a*c*x - c)*sqrt((a*c*x
 - c)/(a*x))/(a*x))*a^2))/c
```

maple [B] time = 0.04, size = 194, normalized size = 2.73

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(10a^{\frac{5}{2}}\sqrt{(ax-1)\,x}\;x^2+5\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^2a^2-8a^{\frac{3}{2}}\left((ax-1)\,x\right)^{\frac{3}{2}}-20a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\;x-10\ln\cdots\right.}{2\sqrt{(ax-1)\,x}\;c\,(ax-1)^2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(1/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(10*a^(5/2)*((a*x-1)*x)^(1/2)*x^2+5*ln(1/2*(2*
((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2-8*a^(3/2)*((a*x-1)*x)^(
3/2)-20*a^(3/2)*((a*x-1)*x)^(1/2)*x-10*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*x*a+10*((a*x-1)*x)^(1/2)*a^(1/2)+5*ln(1/2*(2*((a*x-1)*x)^
(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c/(a*x-1)^2/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{(a^2x^2-1)\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*sqrt(c - c/(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a\,x+1)^2}{\sqrt{c-\frac{c}{a\,x}}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a*x))^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(a*x + 1)^2/((c - c/(a*x))^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax\sqrt{c-\frac{c}{ax}}-\sqrt{c-\frac{c}{ax}}}\,dx-\int \frac{1}{ax\sqrt{c-\frac{c}{ax}}-\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(c - c/(a*x)) - sqrt(c - c/(a*x))), x) - Integral(1/
(a*x*sqrt(c - c/(a*x)) - sqrt(c - c/(a*x))), x)
```

3.524 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7}{ac\sqrt{c-\frac{c}{ax}}}+\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] 7/3/a/(c-c/a/x)^(3/2)-x/(c-c/a/x)^(3/2)-7*arctanh((c-c/a/x)^(1/2)/c^(1/2))/
a/c^(3/2)+7/a/c/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 51, 63, 208}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7}{ac\sqrt{c-\frac{c}{ax}}}+\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] 7/(3*a*(c - c/(a*x))^(3/2)) + 7/(a*c*Sqrt[c - c/(a*x)]) - x/(c - c/(a*x))^(
3/2) - (7*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(3/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}\,dx$$

$$= -\frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}x}\,dx}{a}$$

$$= -\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{a}$$

$$= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{(7c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{ac\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{ac\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2}$$

$$= \frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{ac\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 0.57

$$\frac{x\left(7\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{1}{ax}\right)-3ax\right)}{3c(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (x*(-3*a*x + 7*Hypergeometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c*Sqrt[c
- c/(a*x)]*(-1 + a*x))
```

fricas [A] time = 0.66, size = 238, normalized size = 2.48

$$\left[\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(3\,a^3x^3-28\,a^2x^2+21\,ax\right)\sqrt{\frac{acx-c}{ax}}}{6\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)},\ \frac{21\left(a^2x^2-2\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(21*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x)) + c) - 2*(3*a^3*x^3 - 28*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c
)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), 1/3*(21*(a^2*x^2 - 2*a*x + 1
)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^3*x^3 - 28*a^2
*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
]
```

giac [A] time = 0.20, size = 136, normalized size = 1.42

$$\frac{a\left(\frac{2\left(2\,c+\frac{9\,(acx-c)}{ax}\right)x}{(acx-c)a\sqrt{\frac{acx-c}{ax}}}+\frac{21\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}-\frac{3\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*a*(2*(2*c + 9*(a*c*x - c)/(a*x))*x/((a*c*x - c)*a*sqrt((a*c*x - c)/(a*x
))) + 21*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)) - 3*sqrt((
a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))))/c
```

maple [B] time = 0.05, size = 260, normalized size = 2.71

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(42a^{\frac{7}{2}}\sqrt{(ax-1)\,x}\;x^3+21\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^3a^3-36a^{\frac{5}{2}}\left((ax-1)\,x\right)^{\frac{3}{2}}x-126a^{\frac{5}{2}}\sqrt{(ax-1)\,x}\;x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)*x*(42*a^(7/2)*((a*x-1)*x)^(1/2)*x^3+21*ln(1/2*(2
*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3-36*a^(5/2)*((a*x-1)*x)
^(3/2)*x-126*a^(5/2)*((a*x-1)*x)^(1/2)*x^2-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a
^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2+28*a^(3/2)*((a*x-1)*x)^(3/2)+126*a^(3/2)*(
(a*x-1)*x)^(1/2)*x+63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*x*a-42*((a*x-1)*x)^(1/2)*a^(1/2)-21*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*
a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^2/(a*x-1)^3/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{(a^2x^2-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a*x))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a\,x+1)^2}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a^2\,x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a*x))^(3/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(a*x + 1)^2/((c - c/(a*x))^(3/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{acx\sqrt{c-\frac{c}{ax}}-2c\sqrt{c-\frac{c}{ax}}+\frac{c\sqrt{c-\frac{c}{ax}}}{ax}}\,dx-\int \frac{1}{acx\sqrt{c-\frac{c}{ax}}-2c\sqrt{c-\frac{c}{ax}}+\frac{c\sqrt{c-\frac{c}{ax}}}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] -Integral(a*x/(a*c*x*sqrt(c - c/(a*x)) - 2*c*sqrt(c - c/(a*x)) + c*sqrt(c -
 c/(a*x))/(a*x)), x) - Integral(1/(a*c*x*sqrt(c - c/(a*x)) - 2*c*sqrt(c - c
/(a*x)) + c*sqrt(c - c/(a*x))/(a*x)), x)
```

3.525 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 9/5/a/(c-c/a/x)^(5/2)+3/a/c/(c-c/a/x)^(3/2)-x/(c-c/a/x)^(5/2)-9*arctanh((c-
c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)+9/a/c^2/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 51, 63, 208}

$$\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] 9/(5*a*(c - c/(a*x))^(5/2)) + 3/(a*c*(c - c/(a*x))^(3/2)) + 9/(a*c^2*Sqrt[c
 - c/(a*x)]) - x/(c - c/(a*x))^(5/2) - (9*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]
])/(a*c^(5/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx &= \int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}\,dx \\
&= -\frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}x}\,dx}{a} \\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{a} \\
&= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{(9c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac} \\
&= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2} \\
&= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3} \\
&= \frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.50

$$\frac{9\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{1}{ax}\right)}{5a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] -(x/(c - c/(a*x))^(5/2)) + (9*Hypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]
)/(5*a*(c - c/(a*x))^(5/2))
```

fricas [A] time = 0.44, size = 294, normalized size = 2.47

$$\left[\frac{45\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(5\,a^4x^4-69\,a^3x^3+105\,a^2x^2-45\,ax\right)\sqrt{\frac{ac}{}}}{10\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/10*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x + 2*a*sqrt
(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^
2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c
^3*x - a*c^3), 1/5*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(sq
rt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^2 -
 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3)]
```

giac [A] time = 0.19, size = 165, normalized size = 1.39

$$\frac{a\left(\frac{2\left(2\,c^2+\frac{5\,(acx-c)c}{ax}+\frac{20\,(acx-c)^2}{a^2x^2}\right)x^2}{(acx-c)^2c\sqrt{\frac{acx-c}{ax}}}+\frac{45\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c}-\frac{5\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c}\right)}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/5*a*(2*(2*c^2 + 5*(a*c*x - c)*c/(a*x) + 20*(a*c*x - c)^2/(a^2*x^2))*x^2/(
(a*c*x - c)^2*c*sqrt((a*c*x - c)/(a*x))) + 45*arctan(sqrt((a*c*x - c)/(a*x)
```

```
)/sqrt(-c))/(a^2*sqrt(-c)*c) - 5*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x -
 c)/(a*x))*c))/c
```

maple [B] time = 0.05, size = 328, normalized size = 2.76

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(90a^{\frac{9}{2}}\sqrt{(ax-1)\,x}\; x^4+45\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^4a^4-80a^{\frac{7}{2}}\left((ax-1)\,x\right)^{\frac{3}{2}}x^2-360a^{\frac{7}{2}}\sqrt{(ax-1)\,x}\;x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/10*(c*(a*x-1)/a/x)^(1/2)*x*(90*a^(9/2)*((a*x-1)*x)^(1/2)*x^4+45*ln(1/2*(
2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^4*a^4-80*a^(7/2)*((a*x-1)*x
)^(3/2)*x^2-360*a^(7/2)*((a*x-1)*x)^(1/2)*x^3-180*ln(1/2*(2*((a*x-1)*x)^(1/
2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3+132*a^(5/2)*((a*x-1)*x)^(3/2)*x+540*a^
(5/2)*((a*x-1)*x)^(1/2)*x^2+270*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))*x^2*a^2-60*a^(3/2)*((a*x-1)*x)^(3/2)-360*a^(3/2)*((a*x-1)*x)^(1/
2)*x-180*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x*a+90*((a*x
-1)*x)^(1/2)*a^(1/2)+45*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2
)))/((a*x-1)*x)^(1/2)/c^3/(a*x-1)^4/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a*x))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a\,x+1)^2}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a*x))^(5/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(a*x + 1)^2/((c - c/(a*x))^(5/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^2x\sqrt{c-\frac{c}{ax}}-3c^2\sqrt{c-\frac{c}{ax}}+\frac{3c^2\sqrt{c-\frac{c}{ax}}}{ax}-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a^2x^2}}\,dx-\int \frac{1}{ac^2x\sqrt{c-\frac{c}{ax}}-3c^2\sqrt{c-\frac{c}{ax}}+\frac{3c^2\sqrt{c-\frac{c}{ax}}}{ax}-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**(5/2),x)
```

```
[Out] -Integral(a*x/(a*c**2*x*sqrt(c - c/(a*x)) - 3*c**2*sqrt(c - c/(a*x)) + 3*c*
*2*sqrt(c - c/(a*x))/(a*x) - c**2*sqrt(c - c/(a*x))/(a**2*x**2)), x) - Inte
gral(1/(a*c**2*x*sqrt(c - c/(a*x)) - 3*c**2*sqrt(c - c/(a*x)) + 3*c**2*sqrt
(c - c/(a*x))/(a*x) - c**2*sqrt(c - c/(a*x))/(a**2*x**2)), x)
```

3.526 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}+\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[Out] 11/7/a/(c-c/a/x)^(7/2)+11/5/a/c/(c-c/a/x)^(5/2)+11/3/a/c^2/(c-c/a/x)^(3/2)-
x/(c-c/a/x)^(7/2)-11*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(7/2)+11/a/c^3/(c
-c/a/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 78, 51, 63, 208}

$$\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}+\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] 11/(7*a*(c - c/(a*x))^(7/2)) + 11/(5*a*c*(c - c/(a*x))^(5/2)) + 11/(3*a*c^2
*(c - c/(a*x))^(3/2)) + 11/(a*c^3*Sqrt[c - c/(a*x)]) - x/(c - c/(a*x))^(7/2
) - (11*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}\,dx$$

$$= -\frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{9/2}x}\,dx}{a}$$

$$= -\frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx}{a}$$

$$= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{(11c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x,\right.}{2ac^2}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\tanh^{-1}\left(\right.}{ac^{7/2}}$$

Mathematica [C] time = 0.04, size = 46, normalized size = 0.32

$$\frac{\frac{11\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};1-\frac{1}{ax}\right)}{a}-7x}{7\left(c-\frac{c}{ax}\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] (-7*x + (11*Hypergeometric2F1[-7/2, 1, -5/2, 1 - 1/(a*x)])/a)/(7*(c - c/(a*
x))^(7/2))
```

fricas [A] time = 0.44, size = 346, normalized size = 2.37

$$\left[\frac{1155\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(105\,a^5x^5-1936\,a^4x^4+446\right.}{210\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/210*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log(-2*a
*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(105*a^5*x^5 - 1936*a
^4*x^4 + 4466*a^3*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(
a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), 1/105*(
1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(sqrt(-c)
*sqrt((a*c*x - c)/(a*x))/c) - (105*a^5*x^5 - 1936*a^4*x^4 + 4466*a^3*x^3 -
3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^5*c^4*x^4 - 4*a^4*c^4*
x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

giac [A] time = 0.23, size = 187, normalized size = 1.28

$$\frac{a\left(\frac{2\left(30\,c^3+\frac{63\,(acx-c)c^2}{ax}+\frac{140\,(acx-c)^2c}{a^2x^2}+\frac{525\,(acx-c)^3}{a^3x^3}\right)ax^3}{(acx-c)^3c^2\sqrt{\frac{acx-c}{ax}}}+\frac{1155\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^2}-\frac{105\,\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^2}\right)}{105\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/105*a*(2*(30*c^3 + 63*(a*c*x - c)*c^2/(a*x) + 140*(a*c*x - c)^2*c/(a^2*x^
2) + 525*(a*c*x - c)^3/(a^3*x^3))*a*x^3/((a*c*x - c)^3*c^2*sqrt((a*c*x - c)
/(a*x))) + 1155*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^2)
 - 105*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^2))/c
```

maple [B] time = 0.05, size = 396, normalized size = 2.71

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(2310a^{\frac{11}{2}}\sqrt{(ax-1)\,x}\; x^5 + 1155\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right) x^5a^5 - 2100a^{\frac{9}{2}}\left((ax-1)\,x\right)^{\frac{3}{2}} x^3 - 11550a^{\frac{9}{2}}\sqrt{(a\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(7/2),x)
```

```
[Out] -1/210*(c*(a*x-1)/a/x)^(1/2)*x*(2310*a^(11/2)*((a*x-1)*x)^(1/2)*x^5+1155*ln
(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^5*a^5-2100*a^(9/2)*((
a*x-1)*x)^(3/2)*x^3-11550*a^(9/2)*((a*x-1)*x)^(1/2)*x^4-5775*ln(1/2*(2*((a*
x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^4*a^4+5368*a^(7/2)*((a*x-1)*x)^(3
/2)*x^2+23100*a^(7/2)*((a*x-1)*x)^(1/2)*x^3+11550*ln(1/2*(2*((a*x-1)*x)^(1/
2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3-4928*a^(5/2)*((a*x-1)*x)^(3/2)*x-23100
*a^(5/2)*((a*x-1)*x)^(1/2)*x^2-11550*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*
a*x-1)/a^(1/2))*x^2*a^2+1540*a^(3/2)*((a*x-1)*x)^(3/2)+11550*a^(3/2)*((a*x-
1)*x)^(1/2)*x+5775*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x*
a-2310*((a*x-1)*x)^(1/2)*a^(1/2)-1155*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^4/(a*x-1)^5/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a*x))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a\,x+1)^2}{\left(c-\frac{c}{a\,x}\right)^{7/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a*x))^(7/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(a*x + 1)^2/((c - c/(a*x))^(7/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^3x\sqrt{c-\frac{c}{ax}}-4c^3\sqrt{c-\frac{c}{ax}}+\frac{6c^3\sqrt{c-\frac{c}{ax}}}{ax}-\frac{4c^3\sqrt{c-\frac{c}{ax}}}{a^2x^2}+\frac{c^3\sqrt{c-\frac{c}{ax}}}{a^3x^3}}\,dx-\int \frac{1}{ac^3x\sqrt{c-\frac{c}{ax}}-4c^3\sqrt{c-\frac{c}{ax}}+\frac{6c^3\sqrt{c-\frac{c}{a}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a/x)**(7/2),x)
```

```
[Out] -Integral(a*x/(a*c**3*x*sqrt(c - c/(a*x)) - 4*c**3*sqrt(c - c/(a*x)) + 6*c*
*3*sqrt(c - c/(a*x))/(a*x) - 4*c**3*sqrt(c - c/(a*x))/(a**2*x**2) + c**3*sq
rt(c - c/(a*x))/(a**3*x**3)), x) - Integral(1/(a*c**3*x*sqrt(c - c/(a*x)) -
 4*c**3*sqrt(c - c/(a*x)) + 6*c**3*sqrt(c - c/(a*x))/(a*x) - 4*c**3*sqrt(c
- c/(a*x))/(a**2*x**2) + c**3*sqrt(c - c/(a*x))/(a**3*x**3)), x)
```

3.527 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=223

$$\frac{3a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}-\frac{3a^3x^4\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{(1-ax)^{9/2}}+\frac{3a^2x^3(6-17ax)(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35(1-ax)^{9/2}}+\frac{6ax^2(ax}{3}$$

```
[Out] 3/35*a^2*(c-c/a/x)^(9/2)*x^3*(-17*a*x+6)*(a*x+1)^(3/2)/(-a*x+1)^(9/2)+6/35*
a*(c-c/a/x)^(9/2)*x^2*(a*x+1)^(3/2)/(-a*x+1)^(5/2)-2/7*(c-c/a/x)^(9/2)*x*(a
*x+1)^(3/2)/(-a*x+1)^(3/2)+3*a^(7/2)*(c-c/a/x)^(9/2)*x^(9/2)*arcsinh(a^(1/2
)*x^(1/2))/(-a*x+1)^(9/2)-3*a^3*(c-c/a/x)^(9/2)*x^4*(a*x+1)^(1/2)/(-a*x+1)^
(9/2)
```

Rubi [A] time = 0.18, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 97, 150, 143, 47, 54, 215}

$$\frac{3a^2x^3(6-17ax)(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35(1-ax)^{9/2}}-\frac{3a^3x^4\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{9/2}}{(1-ax)^{9/2}}+\frac{3a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{6ax^2(ax}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (-3*a^3*(c - c/(a*x))^(9/2)*x^4*Sqrt[1 + a*x])/(1 - a*x)^(9/2) + (3*a^2*(c
- c/(a*x))^(9/2)*x^3*(6 - 17*a*x)*(1 + a*x)^(3/2))/(35*(1 - a*x)^(9/2)) + (
6*a*(c - c/(a*x))^(9/2)*x^2*(1 + a*x)^(3/2))/(35*(1 - a*x)^(5/2)) - (2*(c -
 c/(a*x))^(9/2)*x*(1 + a*x)^(3/2))/(7*(1 - a*x)^(3/2)) + (3*a^(7/2)*(c - c/
(a*x))^(9/2)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
```

```
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{e^{3\tanh^{-1}(ax)}(1-ax)^{9/2}}{x^{9/2}}\,dx}{(1-ax)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^3(1+ax)^{3/2}}{x^{9/2}}\,dx}{(1-ax)^{9/2}}$$

$$=-\frac{2\left(c-\frac{c}{ax}\right)^{9/2}x(1+ax)^{3/2}}{7(1-ax)^{3/2}}+\frac{\left(2\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^2\sqrt{1+ax}\left(-\frac{3a}{2}-\frac{9a^2x}{2}\right)}{x^{7/2}}\,dx}{7(1-ax)^{9/2}}$$

$$=\frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2(1+ax)^{3/2}}{35(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{9/2}x(1+ax)^{3/2}}{7(1-ax)^{3/2}}+\frac{\left(4\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-a}{}}{35(1-}$$

$$=\frac{3a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3(6-17ax)(1+ax)^{3/2}}{35(1-ax)^{9/2}}+\frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^2(1+ax)^{3/2}}{35(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{9}}{7(1-}$$

$$=-\frac{3a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4\sqrt{1+ax}}{(1-ax)^{9/2}}+\frac{3a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3(6-17ax)(1+ax)^{3/2}}{35(1-ax)^{9/2}}+\frac{6a\left(c-\frac{c}{ax}\right.}{35(}$$

$$=-\frac{3a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4\sqrt{1+ax}}{(1-ax)^{9/2}}+\frac{3a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3(6-17ax)(1+ax)^{3/2}}{35(1-ax)^{9/2}}+\frac{6a\left(c-\frac{c}{ax}\right.}{35(}$$

$$=-\frac{3a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4\sqrt{1+ax}}{(1-ax)^{9/2}}+\frac{3a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3(6-17ax)(1+ax)^{3/2}}{35(1-ax)^{9/2}}+\frac{6a\left(c-\frac{c}{ax}\right.}{35(}$$

Mathematica [C] time = 0.07, size = 85, normalized size = 0.38

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(35a^2x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-ax\right)+\left(35a^2x^2-46ax+10\right)(ax+1)^{5/2}\right)}{35a^4x^3\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(9/2),x]
```

[Out] -1/35*(c^4*Sqrt[c - c/(a*x)]*((1 + a*x)^(5/2)*(10 - 46*a*x + 35*a^2*x^2) + 35*a^2*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(a*x)]))/(a^4*x^3*Sqrt[1 - a*x])

fricas [A] time = 0.60, size = 386, normalized size = 1.73

$$\left[\frac{105\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(35\,a^4c^4x^4+164\,a^3c^4x^3-12\right.}{140\left(a^5x^4-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")

[Out] [1/140*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(35*a^4*c^4*x^4 + 164*a^3*c^4*x^3 - 12*a^2*c^4*x^2 - 26*a*c^4*x + 10*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), -1/70*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(35*a^4*c^4*x^4 + 164*a^3*c^4*x^3 - 12*a^2*c^4*x^2 - 26*a*c^4*x + 10*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(9/2),x, algorithm="giac")

[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(9/2)/(-a^2*x^2 + 1)^(3/2), x)

maple [A] time = 0.06, size = 172, normalized size = 0.77

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;c^4\sqrt{-a^2x^2+1}\;\left(70a^{\frac{9}{2}}\sqrt{-(ax+1)\,x}\;x^4+105\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right)x^4a^4+328a^{\frac{7}{2}}x^3\sqrt{-(ax+1)\,x}\;-2\right.}{70x^3a^{\frac{9}{2}}\,(ax-1)\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(9/2),x)
```

```
[Out] 1/70*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(-a^2*x^2+1)^(1/2)*(70*a^(9/2)*(
-(a*x+1)*x)^(1/2)*x^4+105*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*
x^4*a^4+328*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)-24*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2
)-52*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+20*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-
(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(9/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(9/2)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{9/2}\,(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(9/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a*x))^(9/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{9}{2}}(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(9/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3
/2), x)
```

3.528 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=217

$$-\frac{a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}-\frac{a^3x^4\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}{(1-ax)^{7/2}}+\frac{2a^2x^3(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3(1-ax)^{7/2}}+\frac{4ax^2(ax+1)^{5/2}\left(c\right.}{3(1-ax)^{7/}}$$

```
[Out] 2/3*a^2*(c-c/a/x)^(7/2)*x^3*(a*x+1)^(3/2)/(-a*x+1)^(7/2)-2/5*(c-c/a/x)^(7/2
)*x*(a*x+1)^(5/2)/(-a*x+1)^(7/2)+4/3*a*(c-c/a/x)^(7/2)*x^2*(a*x+1)^(5/2)/(-
a*x+1)^(7/2)-a^(5/2)*(c-c/a/x)^(7/2)*x^(7/2)*arcsinh(a^(1/2)*x^(1/2))/(-a*x
+1)^(7/2)-a^3*(c-c/a/x)^(7/2)*x^4*(a*x+1)^(1/2)/(-a*x+1)^(7/2)
```

Rubi [A] time = 0.17, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 89, 78, 47, 50, 54, 215}

$$-\frac{a^3x^4\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}{(1-ax)^{7/2}}+\frac{2a^2x^3(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3(1-ax)^{7/2}}-\frac{a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{4ax^2(ax+1)^{5/2}\left(c\right.}{3(1-ax)^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] -((a^3*(c - c/(a*x))^(7/2)*x^4*Sqrt[1 + a*x])/(1 - a*x)^(7/2)) + (2*a^2*(c
- c/(a*x))^(7/2)*x^3*(1 + a*x)^(3/2))/(3*(1 - a*x)^(7/2)) - (2*(c - c/(a*x)
)^(7/2)*x*(1 + a*x)^(5/2))/(5*(1 - a*x)^(7/2)) + (4*a*(c - c/(a*x))^(7/2)*x
^2*(1 + a*x)^(5/2))/(3*(1 - a*x)^(7/2)) - (a^(5/2)*(c - c/(a*x))^(7/2)*x^(7
/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
```

```
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{e^{3\tanh^{-1}(ax)}(1-ax)^{7/2}}{x^{7/2}}dx}{(1-ax)^{7/2}}$$
$$=\frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)^2(1+ax)^{3/2}}{x^{7/2}}dx}{(1-ax)^{7/2}}$$
$$=-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax)^{5/2}}{5(1-ax)^{7/2}}+\frac{\left(2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1+ax)^{3/2}\left(-5a+\frac{5a^2x}{2}\right)}{x^{5/2}}dx}{5(1-ax)^{7/2}}$$
$$=-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax)^{5/2}}{5(1-ax)^{7/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{7/2}x^2(1+ax)^{5/2}}{3(1-ax)^{7/2}}-\frac{\left(a^2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int}{3(1-ax)^{7/2}}$$
$$=\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3(1+ax)^{3/2}}{3(1-ax)^{7/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax)^{5/2}}{5(1-ax)^{7/2}}+\frac{4a\left(c-\frac{c}{ax}\right)^{7/2}x^2(1+a}{3(1-ax)^{7/2}}$$
$$=-\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4\sqrt{1+ax}}{(1-ax)^{7/2}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3(1+ax)^{3/2}}{3(1-ax)^{7/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax}{5(1-ax)^{7/2}}$$
$$=-\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4\sqrt{1+ax}}{(1-ax)^{7/2}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3(1+ax)^{3/2}}{3(1-ax)^{7/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax}{5(1-ax)^{7/2}}$$
$$=-\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4\sqrt{1+ax}}{(1-ax)^{7/2}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3(1+ax)^{3/2}}{3(1-ax)^{7/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x(1+ax}{5(1-ax)^{7/2}}$$

Mathematica [C] time = 0.06, size = 77, normalized size = 0.35

$$-\frac{2c^3\sqrt{c-\frac{c}{ax}}\left(5a^2x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-ax\right)+(10ax-3)(ax+1)^{5/2}\right)}{15a^3x^2\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (-2*c^3*Sqrt[c - c/(a*x)]*((1 + a*x)^(5/2)*(-3 + 10*a*x) + 5*a^2*x^2*Hyperg
eometric2F1[-3/2, -1/2, 1/2, -(a*x)]))/(15*a^3*x^2*Sqrt[1 - a*x])
```

fricas [A] time = 0.47, size = 364, normalized size = 1.68

$$\left[\frac{15\left(a^{3}c^{3}x^{3}-a^{2}c^{3}x^{2}\right)\sqrt{-c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\left(2\,a^{2}x^{2}+ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15\,a^{3}c^{3}x^{3}+44\,a^{2}c^{3}x^{2}+8\,ac^{3}x\right.}{60\left(a^{4}x^{3}-a^{3}x^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(7/2),x, algorithm="fricas
")
```

```
[Out] [1/60*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
+ 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(15*a^3*c^3*x^3 + 44*a^2*c^3*x^2 + 8*a*c^3*x - 6*c^3)*sq
rt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2), -1/30*(15*(a
^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(15*a^3*c^3*x^3 + 44*
a^2*c^3*x^2 + 8*a*c^3*x - 6*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))
)/(a^4*x^3 - a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 154, normalized size = 0.71

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;c^{3}\sqrt{-a^{2}x^{2}+1}\;\left(30a^{\frac{7}{2}}x^{3}\sqrt{-(ax+1)\,x}\;+15\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right)x^{3}a^{3}+88a^{\frac{5}{2}}x^{2}\sqrt{-(ax+1)\,x}\;+16a^{\frac{3}{2}}x\right.}{30x^{2}a^{\frac{7}{2}}\,(ax-1)\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(7/2),x)
```

```
[Out] 1/30*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(-a^2*x^2+1)^(1/2)*(30*a^(7/2)*x
^3*(-(a*x+1)*x)^(1/2)+15*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x
^3*a^3+88*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)+16*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-12
*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(7/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(7/2)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(7/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a*x))^(7/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(7/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(7/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3
/2), x)
```

3.529 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=176

$$-\frac{a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}-\frac{a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{2x\left(1-a^2x^2\right)^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^5}+\frac{2ax^2(ax+1)^{3/2}\left(c-\right.}{3(1-ax)^{5/2}}$$

```
[Out] 2/3*a*(c-c/a/x)^(5/2)*x^2*(a*x+1)^(3/2)/(-a*x+1)^(5/2)-2/3*(c-c/a/x)^(5/2)*
x*(-a^2*x^2+1)^(5/2)/(-a*x+1)^5-a^(3/2)*(c-c/a/x)^(5/2)*x^(5/2)*arcsinh(a^(
1/2)*x^(1/2))/(-a*x+1)^(5/2)-a^2*(c-c/a/x)^(5/2)*x^3*(a*x+1)^(1/2)/(-a*x+1)
^(5/2)
```

Rubi [A] time = 0.21, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6128, 879, 848, 47, 50, 54, 215}

$$-\frac{a^2x^3\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{(1-ax)^{5/2}}-\frac{2x\left(1-a^2x^2\right)^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^5}-\frac{a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{2ax^2(ax+1)^{3/2}\left(c-\right.}{3(1-ax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] -((a^2*(c - c/(a*x))^(5/2)*x^3*Sqrt[1 + a*x])/(1 - a*x)^(5/2)) + (2*a*(c -
c/(a*x))^(5/2)*x^2*(1 + a*x)^(3/2))/(3*(1 - a*x)^(5/2)) - (2*(c - c/(a*x))^
(5/2)*x*(1 - a^2*x^2)^(5/2))/(3*(1 - a*x)^5) - (a^(3/2)*(c - c/(a*x))^(5/2)
*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{e^{3\tanh^{-1}(ax)}(1-ax)^{5/2}}{x^{5/2}}\,dx}{(1-ax)^{5/2}}$$

$$=\frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{\left(1-a^2x^2\right)^{3/2}}{x^{5/2}\sqrt{1-ax}}\,dx}{(1-ax)^{5/2}}$$

$$=-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x^2\right)^{5/2}}{3(1-ax)^5}-\frac{\left(a\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{\left(1-a^2x^2\right)^{3/2}}{x^{3/2}(1-ax)^{3/2}}\,dx}{3(1-ax)^{5/2}}$$

$$=-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x^2\right)^{5/2}}{3(1-ax)^5}-\frac{\left(a\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1+ax)^{3/2}}{x^{3/2}}\,dx}{3(1-ax)^{5/2}}$$

$$=\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{3(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x^2\right)^{5/2}}{3(1-ax)^5}-\frac{\left(a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int}{(1-ax)^{5/2}}$$

$$=-\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}}+\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{3(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x\right.}{3(1-ax)^5}$$

$$=-\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}}+\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{3(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x\right.}{3(1-ax)^5}$$

$$=-\frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3\sqrt{1+ax}}{(1-ax)^{5/2}}+\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}x^2(1+ax)^{3/2}}{3(1-ax)^{5/2}}-\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\left(1-a^2x\right.}{3(1-ax)^5}$$

Mathematica [C] time = 0.04, size = 66, normalized size = 0.38

$$-\frac{2c^2\sqrt{c-\frac{c}{ax}}\left((ax+1)^{5/2}-ax\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-ax\right)\right)}{3a^2x\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (-2*c^2*Sqrt[c - c/(a*x)]*((1 + a*x)^(5/2) - a*x*Hypergeometric2F1[-3/2, -1
/2, 1/2, -(a*x)]))/(3*a^2*x*Sqrt[1 - a*x])
```

fricas [A] time = 0.53, size = 330, normalized size = 1.88

$$\left[\frac{3\left(a^{2}c^{2}x^{2}-ac^{2}x\right)\sqrt{-c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\left(2\,a^{2}x^{2}+ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3\,a^{2}c^{2}x^{2}+2\,ac^{2}x+2\,c^{2}\right)\sqrt{-a^{2}x^{2}}}{12\left(a^{3}x^{2}-a^{2}x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(5/2),x, algorithm="fricas
")
```

```
[Out] [1/12*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(3*a^2*c^2*x^2 + 2*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1)*sqrt((
a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x), 1/6*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c
)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*
x^2 - a*c*x - c)) + 2*(3*a^2*c^2*x^2 + 2*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1
)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^{3}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(5/2)/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.05, size = 136, normalized size = 0.77

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^{2}\sqrt{-a^{2}x^{2}+1}\,\left(-6a^{\frac{5}{2}}x^{2}\sqrt{-(ax+1)\,x}+3\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)x^{2}a^{2}-4a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}-4\sqrt{a}\,\sqrt{}\right.}{6x\,a^{\frac{5}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(5/2),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(-a^2*x^2+1)^(1/2)*(-6*a^(5/2)*x^2
*(-(a*x+1)*x)^(1/2)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2*
```

```
a^2-4*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-4*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-
(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(5/2)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}\,(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(5/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a*x))^(5/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3
/2), x)
```

3.530 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$\frac{3\sqrt{a}\,x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}+\frac{3ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}-\frac{2x(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}$$

```
[Out] -2*(c-c/a/x)^(3/2)*x*(a*x+1)^(3/2)/(-a*x+1)^(3/2)+3*(c-c/a/x)^(3/2)*x^(3/2)
*arcsinh(a^(1/2)*x^(1/2))*a^(1/2)/(-a*x+1)^(3/2)+3*a*(c-c/a/x)^(3/2)*x^2*(a
*x+1)^(1/2)/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6128, 848, 47, 50, 54, 215}

$$\frac{3ax^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}+\frac{3\sqrt{a}\,x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{2x(ax+1)^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (3*a*(c - c/(a*x))^(3/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (2*(c - c/(a*
x))^(3/2)*x*(1 + a*x)^(3/2))/(1 - a*x)^(3/2) + (3*Sqrt[a]*(c - c/(a*x))^(3/
2)*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{3\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}dx}{(1-ax)^{3/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{(1-a^2x^2)^{3/2}}{x^{3/2}(1-ax)^{3/2}}dx}{(1-ax)^{3/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{(1+ax)^{3/2}}{x^{3/2}}dx}{(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x(1+ax)^{3/2}}{(1-ax)^{3/2}} + \frac{\left(3a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{\sqrt{1+ax}}{\sqrt{x}}dx}{(1-ax)^{3/2}}$$

$$= \frac{3a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{3/2}x(1+ax)^{3/2}}{(1-ax)^{3/2}} + \frac{\left(3a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{}{\sqrt{x}}}{2(1-ax)^{3/2}}$$

$$= \frac{3a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{3/2}x(1+ax)^{3/2}}{(1-ax)^{3/2}} + \frac{\left(3a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\text{Subs}}{(1-a}$$

$$= \frac{3a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{3/2}x(1+ax)^{3/2}}{(1-ax)^{3/2}} + \frac{3\sqrt{a}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\sinh}{(1-ax)^{3/2}}$$

Mathematica [C] time = 0.03, size = 42, normalized size = 0.33

$$-\frac{2x\left(c-\frac{c}{ax}\right)^{3/2} {}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-ax\right)}{(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*Hypergeometric2F1[-3/2, -1/2, 1/2, -(a*x)])/(1 - a*x)^(3/2)
```

fricas [A] time = 0.54, size = 268, normalized size = 2.11

$$\left[\frac{3(acx-c)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\sqrt{-a^2x^2+1}(acx-2c)\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)}, \frac{3(acx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(3*(a*c*x - c)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a
*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4
*sqrt(-a^2*x^2 + 1)*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2
*(3*(a*c*x - c)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*sqrt(-a^2*x^2 + 1)*(a*c*x - 2*c
)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.05, size = 109, normalized size = 0.86

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\sqrt{-a^2x^2+1}\, \left(-2a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\, +3\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)xa+4\sqrt{a}\,\sqrt{-(ax+1)\,x}\right)}{2a^{\frac{3}{2}}\,(ax-1)\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-a^2*x^2+1)^(1/2)*(-2*a^(3/2)*x*(-(a*
x+1)*x)^(1/2)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+4*a^(1
/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a*x))^(3/2)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(3/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a*x))^(3/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- c \left(-1 + \frac{1}{a x}\right)\right)^{\frac{3}{2}} \left(a x + 1\right)^{3}}{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3
/2), x)
```

3.531 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=155

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{5\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] -5*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(1/2)/(-a*x+1)^(1/2)+
4*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)*x^
(1/2)/a^(1/2)/(-a*x+1)^(1/2)-x*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 102, 157, 54, 215, 93, 206}

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{5\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x]) - (5*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqr
t[c - c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(S
qrt[a]*Sqrt[1 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{\sqrt{x}(1-ax)}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{3a}{2}-\frac{5a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}} + \frac{\left(4\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}(1-ax)\sqrt{\quad}}}{\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{1-ax}} + \frac{\left(8\sqrt{c-\frac{c}{ax}}\sqrt{x}\right.}{}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{5\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}} + \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\sqrt{x}\tanh^{-1}\left(\right.}{\sqrt{a}\sqrt{1-ax}}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.68

$$-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+5\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + 5*ArcSinh[Sqr
t[a]*Sqrt[x]] - 4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])
)/(Sqrt[a]*Sqrt[1 - a*x]))
```

fricas [A] time = 0.53, size = 442, normalized size = 2.85

$$\left[\frac{4\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx-c}{ax}}+4\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx+4\sqrt{2}\left(3a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+5(a}{4\left(a^2x-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*(a*x - 1
)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x
^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c
*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)
) - c)/(a*x - 1)))/(a^2*x - a), 1/2*(2*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x -
 c)/(a*x)) - 4*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + 5*(a*x
 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x
))/(2*a^2*c*x^2 - a*c*x - c)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 165, normalized size = 1.06

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^{2}x^{2}+1}\, \left(2\sqrt{-(ax+1)\,x}\; a^{\frac{3}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}} - 5\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right) a\sqrt{2}\,\sqrt{-\frac{1}{a}} + 8\ln\left(\frac{2\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(a \cdots}{ax \cdots}\right.\right.}{4\,(ax-1)\sqrt{-(ax+1)\,x}\; a^{\frac{3}{2}}\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*(-(a*x+1)*x)^(1/2)*a^(3/2
)*2^(1/2)*(-1/a)^(1/2)-5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a
*2^(1/2)*(-1/a)^(1/2)+8*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a
*x-1)/(a*x-1))*a^(1/2))*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/a^(3/2)/(-1/a)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

3.532 $\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=195

$$\frac{7\sqrt{1-ax}\,\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-ax}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}+\frac{(ax+1)^{3/2}}{a\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}+\frac{2\sqrt{1-ax}\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] (a*x+1)^(3/2)/a/(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)+7*arcsinh(a^(1/2)*x^(1/2))*(
-a*x+1)^(1/2)/a^(3/2)/(c-c/a/x)^(1/2)/x^(1/2)-5*arctanh(2^(1/2)*a^(1/2)*x^(
1/2)/(a*x+1)^(1/2))*2^(1/2)*(-a*x+1)^(1/2)/a^(3/2)/(c-c/a/x)^(1/2)/x^(1/2)+
2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6134, 6129, 97, 154, 157, 54, 215, 93, 206}

$$\frac{7\sqrt{1-ax}\,\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-ax}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}+\frac{(ax+1)^{3/2}}{a\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}+\frac{2\sqrt{1-ax}\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (2*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - c/(a*x)]) + (1 + a*x)^(3/2)/(a*
Sqrt[c - c/(a*x)]*Sqrt[1 - a*x]) + (7*Sqrt[1 - a*x]*ArcSinh[Sqrt[a]*Sqrt[x]
])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]) - (5*Sqrt[2]*Sqrt[1 - a*x]*ArcTanh[(
Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]
)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-ax}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}(1+ax)^{3/2}}{(1-ax)^2}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} - \frac{\sqrt{1-ax}\int \frac{\sqrt{1+ax}\left(\frac{1}{2}+2ax\right)}{\sqrt{x}(1-ax)}\,dx}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{2\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} + \frac{\sqrt{1-ax}\int \frac{-\frac{3a}{2}-\frac{7a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{2\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} + \frac{\left(7\sqrt{1-ax}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a\sqrt{c-\frac{c}{ax}}\sqrt{x}} - \frac{\left(5\sqrt{1-ax}\right)\int \frac{}{\sqrt{x}(1}}{a\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{2\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} + \frac{\left(7\sqrt{1-ax}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}} - \frac{(10\sqrt{}}{}$$

$$= \frac{2\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} + \frac{(1+ax)^{3/2}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1-ax}} + \frac{7\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}} - \frac{5\sqrt{2}\sqrt{1-ax}\tanh}{a^{3/2}\sqrt{c-}}$$

Mathematica [A] time = 0.11, size = 120, normalized size = 0.62

$$\frac{\sqrt{a}\sqrt{x}\sqrt{ax+1}\,(3-ax)+(7-7ax)\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+5\sqrt{2}\,(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{x}\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*(3 - a*x)*Sqrt[1 + a*x] + (7 - 7*a*x)*ArcSinh[Sqrt[a]*Sqrt
[x]] + 5*Sqrt[2]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]
])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 - a*x])
```

fricas [A] time = 0.83, size = 513, normalized size = 2.63

$$\left[\frac{5\sqrt{2}\left(a^2cx^2-2\,acx+c\right)\sqrt{-\frac{1}{c}}\,\log\left(-\frac{17\,a^3x^3-3\,a^2x^2+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-\frac{1}{c}}\,\sqrt{\frac{acx-c}{ax}}-13\,ax-1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-7\left(a^2x^2-2\,ax+1\right)\cdot}{4\left(a^3cx^2-2\,a^2cx+ac\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(5*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*log(-(17*a^3*x^3 - 3*a
^2*x^2 + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-1/c)*sqrt((a*
c*x - c)/(a*x)) - 13*a*x - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - 7*(a^2*x
^2 - 2*a*x + 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(a^
2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c*x^2 - 2*a
^2*c*x + a*c), 1/2*(7*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2
+ 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a^
2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)) - 5*sqrt(2)*(a^2*
c*x^2 - 2*a*c*x + c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x -
c)/(a*x))/((3*a^2*x^2 - 2*a*x - 1)*sqrt(c)))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*
x + a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))), x)
```

maple [A] time = 0.06, size = 276, normalized size = 1.42

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\left(2a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x-6\sqrt{-(ax+1)\,x}\,a^{\frac{3}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}-7a^2\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right.\right.}{4a^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*a^(5/2)*2^(1/2)*(-1/a)^(1
/2)*(-(a*x+1)*x)^(1/2)*x-6*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)-
7*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)
*x+10*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x
-1))*x+7*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^
(1/2)-10*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*
a^(1/2))*2^(1/2)/a^(3/2)/c/(a*x-1)^2/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,x+1)^3}{\sqrt{c-\frac{c}{a\,x}}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2)),x)
```

[Out] int((a*x + 1)^3/((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(1/2),x)

[Out] Integral((a*x + 1)**3/(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2
)), x)

3.533 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=249

$$-\frac{9(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{51(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4\sqrt{2}\,a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{21\sqrt{ax+1}\,(1-ax)^{3/2}}{8a^2x\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9(ax+1)^{3/2}\sqrt{1-ax}}{8a^2x\left(c-\frac{c}{ax}\right)^{3/2}}+$$

```
[Out] -9*(-a*x+1)^(3/2)*arcsinh(a^(1/2)*x^(1/2))/a^(5/2)/(c-c/a/x)^(3/2)/x^(3/2)+
51/8*(-a*x+1)^(3/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))/a^(5/2)/
(c-c/a/x)^(3/2)/x^(3/2)*2^(1/2)+1/2*(a*x+1)^(3/2)/a/(c-c/a/x)^(3/2)/(-a*x+1
)^(1/2)-9/8*(a*x+1)^(3/2)*(-a*x+1)^(1/2)/a^2/(c-c/a/x)^(3/2)/x-21/8*(-a*x+1
)^(3/2)*(a*x+1)^(1/2)/a^2/(c-c/a/x)^(3/2)/x
```

Rubi [A] time = 0.21, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6134, 6129, 97, 149, 154, 157, 54, 215, 93, 206}

$$-\frac{9(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{51(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4\sqrt{2}\,a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{21\sqrt{ax+1}\,(1-ax)^{3/2}}{8a^2x\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9(ax+1)^{3/2}\sqrt{1-ax}}{8a^2x\left(c-\frac{c}{ax}\right)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (-21*(1 - a*x)^(3/2)*Sqrt[1 + a*x])/(8*a^2*(c - c/(a*x))^(3/2)*x) + (1 + a*
x)^(3/2)/(2*a*(c - c/(a*x))^(3/2)*Sqrt[1 - a*x]) - (9*Sqrt[1 - a*x]*(1 + a*
x)^(3/2))/(8*a^2*(c - c/(a*x))^(3/2)*x) - (9*(1 - a*x)^(3/2)*ArcSinh[Sqrt[a
]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2)) + (51*(1 - a*x)^(3/2)*Arc
Tanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(4*Sqrt[2]*a^(5/2)*(c - c/(a
*x))^(3/2)*x^(3/2))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{(1-ax)^{3/2}\int \frac{e^{3\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}(1+ax)^{3/2}}{(1-ax)^{3}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{(1-ax)^{3/2}\int \frac{\sqrt{x}\,\sqrt{1+ax}\left(\frac{3}{2}+3ax\right)}{(1-ax)^{2}}\,dx}{2a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}\,(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\int \frac{\sqrt{1+ax}\left(-\frac{9a}{4}-\frac{21a^2x}{2}\right)}{\sqrt{x}\,(1-ax)}\,dx}{4a^3\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{21(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}\,(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1-ax)^{3/2}\int \frac{\frac{15a^{\cdots}}{2}}{\sqrt{x}\,(1-\cdots}}{4a^4\left(c-\frac{c}{ax}\right)^{\cdots}}$$

$$= -\frac{21(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}\,(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{\left(9(1-ax)^{3/2}\right)\int \cdots}{2a^2\left(c-\frac{c}{ax}\right)^{3/\cdots}}$$

$$= -\frac{21(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}\,(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{\left(9(1-ax)^{3/2}\right)\text{Sub}\cdots}{a^2\left(c-\cdots\right.}$$

$$= -\frac{21(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}\,(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{9(1-ax)^{3/2}\sinh^{-\cdots}}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/\cdots}}$$

Mathematica [A] time = 0.14, size = 139, normalized size = 0.56

$$-\frac{-2\sqrt{a}\,\sqrt{x}\,\sqrt{ax+1}\,\left(4a^2x^2-23ax+15\right)-72(ax-1)^2\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)+51\sqrt{2}\,(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)}{8a^{3/2}c\sqrt{x}\,(1-ax)^{3/2}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -1/8*(-2*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(15 - 23*a*x + 4*a^2*x^2) - 72*(-1 +
 a*x)^2*ArcSinh[Sqrt[a]*Sqrt[x]] + 51*Sqrt[2]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]
*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)*c*Sqrt[c - c/(a*x)]*Sqrt[x]*(1 -
 a*x)^(3/2))
```

fricas [A] time = 0.60, size = 600, normalized size = 2.41

$$\left[-\frac{51\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+72\left(a^3x^3 \right.}{32\left(a^4c^2x^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas
")
```

```
[Out] [-1/32*(51*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(17*a^3*
c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2
+ 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1
)) + 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*
c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x
)) - c)/(a*x - 1)) - 8*(4*a^3*x^3 - 23*a^2*x^2 + 15*a*x)*sqrt(-a^2*x^2 + 1)
*sqrt((a*c*x - c)/(a*x)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^
2), -1/16*(51*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan(2*sq
rt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan(2*sqrt
(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c
)) - 4*(4*a^3*x^3 - 23*a^2*x^2 + 15*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c
)/(a*x)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(3/2)), x)
```

maple [B] time = 0.07, size = 390, normalized size = 1.57

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(8a^{\frac{7}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x^2-36a^3\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\sqrt{2}\,\sqrt{-\frac{1}{a}}\,x^2-46a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/16*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^2*(-a^2*x^2+1)^(1/2)*(8*a^(7/2)*2^(1
/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*x^2-36*a^3*arctan(1/2/a^(1/2)*(2*a*x+1)
/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x^2-46*a^(5/2)*2^(1/2)*(-1/a)^(1/
2)*(-(a*x+1)*x)^(1/2)*x+51*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^
(1/2)*a-3*a*x-1)/(a*x-1))*x^2+72*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)
*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x+30*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1
/a)^(1/2)-102*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x
-1)/(a*x-1))*x-36*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2
)*(-1/a)^(1/2)+51*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/
(a*x-1))*a^(1/2))*2^(1/2)/(a*x-1)^3/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a\,x+1)^3}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a*x))^(3/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((c - c/(a*x))^(3/2)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-c*(-1 + 1/(a*x)))**(3/2)*(-(a*x - 1)*(a*x + 1))**(
3/2)), x)
```

3.534 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=293

$$\frac{11(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{249(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16\sqrt{2}\,a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{103\sqrt{ax+1}\,(1-ax)^{5/2}}{32a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{43(ax+1)^{3/2}(1-ax)^{3/}}{32a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 43/32*(-a*x+1)^(3/2)*(a*x+1)^(3/2)/a^3/(c-c/a/x)^(5/2)/x^2+11*(-a*x+1)^(5/2
)*arcsinh(a^(1/2)*x^(1/2))/a^(7/2)/(c-c/a/x)^(5/2)/x^(5/2)-249/32*(-a*x+1)^
(5/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))/a^(7/2)/(c-c/a/x)^(5/2
)/x^(5/2)*2^(1/2)+1/3*(a*x+1)^(3/2)/a/(c-c/a/x)^(5/2)/(-a*x+1)^(1/2)-13/24*
(a*x+1)^(3/2)*(-a*x+1)^(1/2)/a^2/(c-c/a/x)^(5/2)/x+103/32*(-a*x+1)^(5/2)*(a
*x+1)^(1/2)/a^3/(c-c/a/x)^(5/2)/x^2
```

Rubi [A] time = 0.23, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6134, 6129, 97, 149, 154, 157, 54, 215, 93, 206}

$$\frac{103\sqrt{ax+1}\,(1-ax)^{5/2}}{32a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{43(ax+1)^{3/2}(1-ax)^{3/2}}{32a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{249(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16\sqrt{2}\,a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (103*(1 - a*x)^(5/2)*Sqrt[1 + a*x])/(32*a^3*(c - c/(a*x))^(5/2)*x^2) + (1 +
 a*x)^(3/2)/(3*a*(c - c/(a*x))^(5/2)*Sqrt[1 - a*x]) - (13*Sqrt[1 - a*x]*(1
+ a*x)^(3/2))/(24*a^2*(c - c/(a*x))^(5/2)*x) + (43*(1 - a*x)^(3/2)*(1 + a*x
)^(3/2))/(32*a^3*(c - c/(a*x))^(5/2)*x^2) + (11*(1 - a*x)^(5/2)*ArcSinh[Sqr
t[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) - (249*(1 - a*x)^(5/2)
*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(16*Sqrt[2]*a^(7/2)*(c -
 c/(a*x))^(5/2)*x^(5/2))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{(1-ax)^{5/2}\int \frac{e^{3\tanh^{-1}(ax)}x^{5/2}}{(1-ax)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}\int \frac{x^{5/2}(1+ax)^{3/2}}{(1-ax)^{4}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{(1-ax)^{5/2}\int \frac{x^{3/2}\sqrt{1+ax}\left(\frac{5}{2}+4ax\right)}{(1-ax)^{3}}\,dx}{3a\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}\,(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{(1-ax)^{5/2}\int \frac{\sqrt{x}\sqrt{1+ax}\left(-\frac{39a}{4}-\frac{45a^2x}{2}\right)}{(1-ax)^{2}}\,dx}{12a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}\,(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1+ax)^{3/2}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} - \frac{(1-ax)^{5/2}\int}{24a^5\left(c\right.}$$

$$= \frac{103(1-ax)^{5/2}\sqrt{1+ax}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}\,(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1}{32a^3\left(c-\frac{c}{ax}\right)}$$

$$= \frac{103(1-ax)^{5/2}\sqrt{1+ax}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}\,(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1}{32a^3\left(c-\frac{c}{ax}\right)}$$

$$= \frac{103(1-ax)^{5/2}\sqrt{1+ax}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}\,(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1}{32a^3\left(c-\frac{c}{ax}\right)}$$

$$= \frac{103(1-ax)^{5/2}\sqrt{1+ax}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}\,(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1}{32a^3\left(c-\frac{c}{ax}\right)}$$

Mathematica [A] time = 0.17, size = 147, normalized size = 0.50

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(-48a^3x^3+415a^2x^2-554ax+219\right)-1056(ax-1)^3\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+747\sqrt{2}\,(ax-1)^3\tanh}{96a^{3/2}c^2\sqrt{x}\,(1-ax)^{5/2}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(219 - 554*a*x + 415*a^2*x^2 - 48*a^3*x^3)
 - 1056*(-1 + a*x)^3*ArcSinh[Sqrt[a]*Sqrt[x]] + 747*Sqrt[2]*(-1 + a*x)^3*Ar
cTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(96*a^(3/2)*c^2*Sqrt[c - c/
(a*x)]*Sqrt[x]*(1 - a*x)^(5/2))
```

fricas [A] time = 0.53, size = 668, normalized size = 2.28

$$\left[-\frac{747\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas
")
```

```
[Out] [-1/384*(747*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)
*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x
^2 + 3*a*x - 1)) + 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(
-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sq
rt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) - 8*(48*a^4*x^4 - 415*a^3*x^
3 + 554*a^2*x^2 - 219*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5
*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), -1/192*(74
7*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(2*sq
rt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)
*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x
^2 - a*c*x - c)) - 4*(48*a^4*x^4 - 415*a^3*x^3 + 554*a^2*x^2 - 219*a*x)*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a
^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(5/2)), x)
```

maple [B] time = 0.07, size = 504, normalized size = 1.72

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(96a^{\frac{9}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x^3-830a^{\frac{7}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x^2+747a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{2}\,\sqrt{-}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/192*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(96*a^(9/2)*2^(1/2)*(-1/a)
^(1/2)*(-(a*x+1)*x)^(1/2)*x^3-830*a^(7/2)*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)
^(1/2)*x^2+747*a^(7/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*
x-1)/(a*x-1))*x^3-528*a^4*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*
2^(1/2)*(-1/a)^(1/2)*x^3+1108*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/
2)*x+1584*a^3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/
a)^(1/2)*x^2-2241*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3
*a*x-1)/(a*x-1))*x^2-438*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)-15
84*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2
)*x+2241*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(
a*x-1))*x+528*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-
1/a)^(1/2)-747*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*
x-1))*a^(1/2))*2^(1/2)/a^(3/2)/c^3/(a*x-1)^4/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a\,x+1)^3}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a*x))^(5/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((c - c/(a*x))^(5/2)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-c*(-1 + 1/(a*x)))**(5/2)*(-(a*x - 1)*(a*x + 1))**(
3/2)), x)
```

3.535 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=225

$$\frac{11a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{a^3x^4\sqrt{ax+1}\,(521ax+2718)\left(c-\frac{c}{ax}\right)^{9/2}}{105(1-ax)^{9/2}}-\frac{94a^2x^3\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{9/2}}{21(1-ax)^{5/2}}+\frac{6a}{}$$

```
[Out] 11*a^(7/2)*(c-c/a/x)^(9/2)*x^(9/2)*arcsinh(a^(1/2)*x^(1/2))/(-a*x+1)^(9/2)-
94/21*a^2*(c-c/a/x)^(9/2)*x^3*(a*x+1)^(1/2)/(-a*x+1)^(5/2)+6/5*a*(c-c/a/x)^
(9/2)*x^2*(a*x+1)^(1/2)/(-a*x+1)^(3/2)+1/105*a^3*(c-c/a/x)^(9/2)*x^4*(521*a
*x+2718)*(a*x+1)^(1/2)/(-a*x+1)^(9/2)-2/7*(c-c/a/x)^(9/2)*x*(a*x+1)^(1/2)/(
-a*x+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$\frac{a^3x^4\sqrt{ax+1}\,(521ax+2718)\left(c-\frac{c}{ax}\right)^{9/2}}{105(1-ax)^{9/2}}-\frac{94a^2x^3\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{9/2}}{21(1-ax)^{5/2}}+\frac{11a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{6a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-94*a^2*(c - c/(a*x))^(9/2)*x^3*Sqrt[1 + a*x])/(21*(1 - a*x)^(5/2)) + (6*a
*(c - c/(a*x))^(9/2)*x^2*Sqrt[1 + a*x])/(5*(1 - a*x)^(3/2)) - (2*(c - c/(a*
x))^(9/2)*x*Sqrt[1 + a*x])/(7*Sqrt[1 - a*x]) + (a^3*(c - c/(a*x))^(9/2)*x^4
*Sqrt[1 + a*x]*(2718 + 521*a*x))/(105*(1 - a*x)^(9/2)) + (11*a^(7/2)*(c - c
/(a*x))^(9/2)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{e^{-\tanh^{-1}(ax)}(1-ax)^{9/2}}{x^{9/2}}dx}{(1-ax)^{9/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{(1-ax)^{5}}{x^{9/2}\sqrt{1+ax}}dx}{(1-ax)^{9/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{(1-ax)^{3}\left(\frac{21a}{2}-\frac{5a^2x}{2}\right)}{x^{7/2}\sqrt{1+ax}}dx}{7(1-ax)^{9/2}}$$

$$= \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^{2}\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+ax}}{7\sqrt{1-ax}} - \frac{\left(4\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int \frac{(1-ax)^{2}\cdots}{x\cdots}}{35(1-ax)^{9\cdots}}$$

$$= -\frac{94a^2\left(c-\frac{c}{ax}\right)^{9/2}x^{3}\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^{2}\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+a\cdots}}{7\sqrt{1-ax}}\cdots$$

$$= -\frac{94a^2\left(c-\frac{c}{ax}\right)^{9/2}x^{3}\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^{2}\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+a\cdots}}{7\sqrt{1-ax}}\cdots$$

$$= -\frac{94a^2\left(c-\frac{c}{ax}\right)^{9/2}x^{3}\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^{2}\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+a\cdots}}{7\sqrt{1-ax}}\cdots$$

$$= -\frac{94a^2\left(c-\frac{c}{ax}\right)^{9/2}x^{3}\sqrt{1+ax}}{21(1-ax)^{5/2}} + \frac{6a\left(c-\frac{c}{ax}\right)^{9/2}x^{2}\sqrt{1+ax}}{5(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1+a\cdots}}{7\sqrt{1-ax}}\cdots$$

Mathematica [A] time = 2.83, size = 108, normalized size = 0.48

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{-ax(ax+1)}\left(105a^4x^4-4156a^3x^3+1028a^2x^2-246ax+30\right)-1155a^4x^4\sin^{-1}\left(\sqrt{-ax}\right)\right)}{105a^3x^2(-ax)^{3/2}\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[-(a*x*(1 + a*x))]*(30 - 246*a*x + 1028*a^2*x^2
 - 4156*a^3*x^3 + 105*a^4*x^4) - 1155*a^4*x^4*ArcSin[Sqrt[-(a*x)]]))/(105*a
^3*x^2*(-(a*x))^(3/2)*Sqrt[1 - a*x])
```

fricas [A] time = 0.46, size = 386, normalized size = 1.72

$$\left[\frac{1155\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(105\,a^4c^4x^4-4156\,a^3c^4x^3+1\right.}{420\left(a^5x^4-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/420*(1155*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c
*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)
) - c)/(a*x - 1)) + 4*(105*a^4*c^4*x^4 - 4156*a^3*c^4*x^3 + 1028*a^2*c^4*x^
2 - 246*a*c^4*x + 30*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*
x^4 - a^4*x^3), -1/210*(1155*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*arctan(2*s
qrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x
- c)) - 2*(105*a^4*c^4*x^4 - 4156*a^3*c^4*x^3 + 1028*a^2*c^4*x^2 - 246*a*c^
4*x + 30*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^
3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 172, normalized size = 0.76

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;c^4\sqrt{-a^2x^2+1}\;\left(210a^{\frac{9}{2}}\sqrt{-(ax+1)\,x}\;x^4+1155\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right)x^4a^4-8312a^{\frac{7}{2}}x^3\sqrt{-(ax+1)\,x}\;+\right.}{210x^3a^{\frac{9}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/210*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(-a^2*x^2+1)^(1/2)*(210*a^(9/2)
*(-(a*x+1)*x)^(1/2)*x^4+1155*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2
))*x^4*a^4-8312*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)+2056*a^(5/2)*x^2*(-(a*x+1)*x
)^(1/2)-492*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+60*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*
x-1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(9/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{9/2}\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(9/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(9/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(9/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
[Out] Timed out
```

3.536 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=179

$$-\frac{9a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}-\frac{a^2x^3\sqrt{ax+1}\,(7ax+66)\left(c-\frac{c}{ax}\right)^{7/2}}{5(1-ax)^{7/2}}+\frac{2ax^2\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{7/2}}{(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}}{5\sqrt{1}}$$

```
[Out] -9*a^(5/2)*(c-c/a/x)^(7/2)*x^(7/2)*arcsinh(a^(1/2)*x^(1/2))/(-a*x+1)^(7/2)+
2*a*(c-c/a/x)^(7/2)*x^2*(a*x+1)^(1/2)/(-a*x+1)^(3/2)-1/5*a^2*(c-c/a/x)^(7/2
)*x^3*(7*a*x+66)*(a*x+1)^(1/2)/(-a*x+1)^(7/2)-2/5*(c-c/a/x)^(7/2)*x*(a*x+1)
^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$-\frac{a^2x^3\sqrt{ax+1}\,(7ax+66)\left(c-\frac{c}{ax}\right)^{7/2}}{5(1-ax)^{7/2}}-\frac{9a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{2ax^2\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{7/2}}{(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}}{5\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] (2*a*(c - c/(a*x))^(7/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (2*(c - c/(a*
x))^(7/2)*x*Sqrt[1 + a*x])/(5*Sqrt[1 - a*x]) - (a^2*(c - c/(a*x))^(7/2)*x^3
*Sqrt[1 + a*x]*(66 + 7*a*x))/(5*(1 - a*x)^(7/2)) - (9*a^(5/2)*(c - c/(a*x))
^(7/2)*x^(7/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{-\tanh^{-1}(ax)}(1-ax)^{7/2}}{x^{7/2}}dx}{(1-ax)^{7/2}}$$
$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{(1-ax)^4}{x^{7/2}\sqrt{1+ax}}dx}{(1-ax)^{7/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{(1-ax)^2\left(\frac{15a}{2}-\frac{3a^2x}{2}\right)}{x^{5/2}\sqrt{1+ax}}dx}{5(1-ax)^{7/2}}$$
$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{\left(4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{(1-ax)\left(-\right.}{x^{3/2}}}{15(1-ax)^{7/2}}$$
$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}\,(66}{5(1-ax)^{7/2}}$$
$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}\,(66}{5(1-ax)^{7/2}}$$
$$= \frac{2a\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}}{5\sqrt{1-ax}} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3\sqrt{1+ax}\,(66}{5(1-ax)^{7/2}}$$

Mathematica [A] time = 0.10, size = 95, normalized size = 0.53

$$-\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}\left(5a^3x^3-92a^2x^2+16ax-2\right)-45a^{5/2}x^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{5a^3x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] -1/5*(c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(-2 + 16*a*x - 92*a^2*x^2 + 5*a^
3*x^3) - 45*a^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]]))/(a^3*x^2*Sqrt[1 - a*
x])
```

fricas [A] time = 0.70, size = 364, normalized size = 2.03

$$\left[\frac{45\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(5\,a^3c^3x^3-92\,a^2c^3x^2+16\,ac^3\right.}{20\left(a^4x^3-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
+ 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(5*a^3*c^3*x^3 - 92*a^2*c^3*x^2 + 16*a*c^3*x - 2*c^3)*sq
rt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2), -1/10*(45*(a
^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(5*a^3*c^3*x^3 - 92*a
^2*c^3*x^2 + 16*a*c^3*x - 2*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))
)/(a^4*x^3 - a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 154, normalized size = 0.86

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;c^3\sqrt{-a^2x^2+1}\;\left(10a^{\frac{7}{2}}x^3\sqrt{-(ax+1)\,x}\;+45\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right)x^3a^3-184a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x}\;+32a\right.}{10x^2a^{\frac{7}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/10*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(-a^2*x^2+1)^(1/2)*(10*a^(7/2)*x
^3*(-(a*x+1)*x)^(1/2)+45*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x
```

```
^3*a^3-184*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)+32*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-4
*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(7/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(7/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(7/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**(7/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x
)
```

3.537 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{7a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{ax^2(18-ax)\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}}-\frac{2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-ax}}$$

```
[Out] 7*a^(3/2)*(c-c/a/x)^(5/2)*x^(5/2)*arcsinh(a^(1/2)*x^(1/2))/(-a*x+1)^(5/2)+1
/3*a*(c-c/a/x)^(5/2)*x^2*(-a*x+18)*(a*x+1)^(1/2)/(-a*x+1)^(5/2)-2/3*(c-c/a/
x)^(5/2)*x*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6134, 6129, 98, 143, 54, 215}

$$\frac{7a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{ax^2(18-ax)\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}}-\frac{2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*(c - c/(a*x))^(5/2)*x*Sqrt[1 + a*x])/(3*Sqrt[1 - a*x]) + (a*(c - c/(a*x
))^(5/2)*x^2*(18 - a*x)*Sqrt[1 + a*x])/(3*(1 - a*x)^(5/2)) + (7*a^(3/2)*(c
 - c/(a*x))^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{e^{-\tanh^{-1}(ax)}(1-ax)^{5/2}}{x^{5/2}}dx}{(1-ax)^{5/2}}$$
$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1-ax)^3}{x^{5/2}\sqrt{1+ax}}dx}{(1-ax)^{5/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}}{3\sqrt{1-ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int\frac{(1-ax)\left(\frac{9a}{2}-\frac{a^2x}{2}\right)}{x^{3/2}\sqrt{1+ax}}dx}{3(1-ax)^{5/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{a\left(c-\frac{c}{ax}\right)^{5/2}x^2(18-ax)\sqrt{1+ax}}{3(1-ax)^{5/2}} + \frac{\left(7a^2\left(c-\frac{c}{ax}\right)^{5/2}x\right.}{2(1-}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{a\left(c-\frac{c}{ax}\right)^{5/2}x^2(18-ax)\sqrt{1+ax}}{3(1-ax)^{5/2}} + \frac{\left(7a^2\left(c-\frac{c}{ax}\right)^{5/2}x\right.}{}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{a\left(c-\frac{c}{ax}\right)^{5/2}x^2(18-ax)\sqrt{1+ax}}{3(1-ax)^{5/2}} + \frac{7a^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{(1-}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.64

$$-\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}\left(3a^2x^2-22ax+2\right)-21a^{3/2}x^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{3a^2x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] -1/3*(c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x]*(2 - 22*a*x + 3*a^2*x^2) - 21*a^
(3/2)*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]]))/(a^2*x*Sqrt[1 - a*x])
```

fricas [A] time = 0.67, size = 330, normalized size = 2.41

$$\left[\frac{21\left(a^2c^2x^2-ac^2x\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^2c^2x^2-22ac^2x+2c^2\right)\sqrt{-a^2}}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*
(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) + 4*(3*a^2*c^2*x^2 - 22*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1)*sqrt
((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x), -1/6*(21*(a^2*c^2*x^2 - a*c^2*x)*sq
rt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^
2*c*x^2 - a*c*x - c)) - 2*(3*a^2*c^2*x^2 - 22*a*c^2*x + 2*c^2)*sqrt(-a^2*x^
2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 136, normalized size = 0.99

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^2\sqrt{-a^2x^2+1}\, \left(6a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x}\, + 21 \arctan\left(\frac{2ax+1}{2\sqrt{a}\, \sqrt{-(ax+1)x}}\right) x^2a^2 - 44a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\, + 4\sqrt{a}\, \sqrt{-}}{6x\, a^{\frac{5}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/6*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(-a^2*x^2+1)^(1/2)*(6*a^(5/2)*x^2*(
-(a*x+1)*x)^(1/2)+21*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2*a
^2-44*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+4*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-
(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(5/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(5/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^(5/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(5/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x
)
```

3.538 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{5\sqrt{a}\,x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{(1-ax)^{3/2}}+\frac{ax^2\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}$$

```
[Out] -5*(c-c/a/x)^(3/2)*x^(3/2)*arcsinh(a^(1/2)*x^(1/2))*a^(1/2)/(-a*x+1)^(3/2)-
2*(c-c/a/x)^(3/2)*x*(a*x+1)^(1/2)/(-a*x+1)^(3/2)+a*(c-c/a/x)^(3/2)*x^2*(a*x
+1)^(1/2)/(-a*x+1)^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6134, 6129, 89, 80, 54, 215}

$$\frac{ax^2\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}-\frac{5\sqrt{a}\,x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{2x\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*Sqrt[1 + a*x])/(1 - a*x)^(3/2) + (a*(c - c/(a*x))
^(3/2)*x^2*Sqrt[1 + a*x])/(1 - a*x)^(3/2) - (5*Sqrt[a]*(c - c/(a*x))^(3/2)*
x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{-\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}dx}{(1-ax)^{3/2}}$$
$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{(1-ax)^2}{x^{3/2}\sqrt{1+ax}}dx}{(1-ax)^{3/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1+ax}}{(1-ax)^{3/2}} + \frac{\left(2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{-a+\frac{a^2x}{2}}{\sqrt{x}\sqrt{1+ax}}dx}{(1-ax)^{3/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1+ax}}{(1-ax)^{3/2}} + \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{\left(5a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1}}}{2(1-ax)^{3/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1+ax}}{(1-ax)^{3/2}} + \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{\left(5a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\text{Subst}\left(\right.}{(1-ax)^{3}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1+ax}}{(1-ax)^{3/2}} + \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2\sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{5\sqrt{a}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\sinh^{-1}}{(1-ax)^{3/2}}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.56

$$-\frac{c\sqrt{c-\frac{c}{ax}}\left((ax-2)\sqrt{ax+1}-5\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^ArcTanh[a*x],x]
[Out] -((c*Sqrt[c - c/(a*x)]*((-2 + a*x)*Sqrt[1 + a*x] - 5*Sqrt[a]*Sqrt[x]*ArcSin
h[Sqrt[a]*Sqrt[x]]))/(a*Sqrt[1 - a*x]))
```

fricas [A] time = 0.59, size = 268, normalized size = 2.13

$$\left[\frac{5(acx-c)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\sqrt{-a^2x^2+1}(acx-2c)\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)}, -\frac{5(acx-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a*c*x - c)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a
*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4
*sqrt(-a^2*x^2 + 1)*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a), -1/
2*(5*(a*c*x - c)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*
x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*(a*c*x - 2*
c)*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 109, normalized size = 0.87

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\sqrt{-a^2x^2+1}\, \left(2a^{\frac{3}{2}}x\sqrt{-\left(ax+1\right)x}\, +5\arctan\left(\frac{2ax+1}{2\sqrt{a}\, \sqrt{-(ax+1)x}}\right)xa-4\sqrt{a}\, \sqrt{-\left(ax+1\right)x}\right)}{2a^{\frac{3}{2}}\left(ax-1\right)\sqrt{-\left(ax+1\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-a^2*x^2+1)^(1/2)*(2*a^(3/2)*x*(-(a*x+
1)*x)^(1/2)+5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a-4*a^(1/2
)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a*x))^(3/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(3/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(3/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x
)
```

3.539 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}$$

```
[Out] 3*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(1/2)/(-a*x+1)^(1/2)-x
*(c-c/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

Rubi [A] time = 0.16, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6134, 6128, 881, 848, 54, 215}

$$\frac{3\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 - a^2*x^2])/(1 - a*x)) + (3*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax} + \frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax} + \frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax} + \frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax} + \frac{3\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.74

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\frac{3\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}-\sqrt{x}\sqrt{ax+1}\right)}{\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(-(Sqrt[x]*Sqrt[1 + a*x]) + (3*ArcSinh[Sqrt[a]*S
qrt[x]])/Sqrt[a]))/Sqrt[1 - a*x]
```

fricas [A] time = 0.45, size = 250, normalized size = 2.78

$$\left[\frac{4\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx-c}{ax}}+3(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4\left(a^2x-a\right)}, \frac{2\sqrt{-a^2x^2+1}\,ax\sqrt{\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + 3*(a*x - 1)*sqrt(-
c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*x - a), 1/2*(2*sqrt(-a^
2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - 3*(a*x - 1)*sqrt(c)*arctan(2*sqrt(
-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)
))/(a^2*x - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

maple [A] time = 0.05, size = 91, normalized size = 1.01

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\sqrt{-a^2x^2+1}\,\left(2\sqrt{a}\,\sqrt{-(ax+1)x}+3\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\right)}{2(ax-1)\sqrt{-(ax+1)x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*a^(1/2)*(-(a*x+1)*x)^(1/2
)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^
(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.540 $\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{1-ax}\,\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{1-ax}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -arcsinh(a^(1/2)*x^(1/2))*(-a*x+1)^(1/2)/a^(3/2)/(c-c/a/x)^(1/2)/x^(1/2)+(-
a*x+1)^(1/2)*(a*x+1)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6134, 6128, 848, 50, 54, 215}

$$\frac{\sqrt{1-ax}\,\sqrt{ax+1}}{a\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{1-ax}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - c/(a*x)]) - (Sqrt[1 - a*x]*ArcSin
h[Sqrt[a]*Sqrt[x]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-ax}\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$
$$= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}\sqrt{1-ax}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$
$$= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$
$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-ax}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$
$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-ax}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$
$$= \frac{\sqrt{1-ax}\sqrt{1+ax}}{a\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.81

$$\frac{\sqrt{1-ax}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}-\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{a^{3/2}\sqrt{x}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]),x]
[Out] (Sqrt[1 - a*x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] - ArcSinh[Sqrt[a]*Sqrt[x]]))/
(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x])
```

fricas [A] time = 0.50, size = 254, normalized size = 2.89

$$\left[\frac{4\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx-c}{ax}} - (ax-1)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4\left(a^2cx-ac\right)}, \frac{2\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx}{a}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - (a*x - 1)*sqrt(-c)
*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(
-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*c*x - a*c), 1/2*(2*sqrt(-
a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - (a*x - 1)*sqrt(c)*arctan(2*sqrt(
-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)
))/(a^2*c*x - a*c)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 92, normalized size = 1.05

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^{2}x^{2}+1}\, \left(2\sqrt{a}\, \sqrt{-\left(ax+1\right)x}\, +\arctan\left(\frac{2ax+1}{2\sqrt{a}\, \sqrt{-(ax+1)x}}\right)\right)}{2\sqrt{a}\, c\left(ax-1\right)\sqrt{-\left(ax+1\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c*(-a^2*x^2+1)^(1/2)*(2*a^(1/2)*(-(a*x+
1)*x)^(1/2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*
x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}}{\left(ax+1\right)\sqrt{c-\frac{c}{ax}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*sqrt(c - c/(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}}{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^(1/2)*(a*x + 1)),x)
[Out] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^(1/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(1/2),x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)), x)
```

3.541 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\sqrt{2}(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{ax+1}(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -(-a*x+1)^(3/2)*arcsinh(a^(1/2)*x^(1/2))/a^(5/2)/(c-c/a/x)^(3/2)/x^(3/2)+(-
a*x+1)^(3/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))/a^(5/2)/(c-c/a/
x)^(3/2)/x^(3/2)*2^(1/2)-(-a*x+1)^(3/2)*(a*x+1)^(1/2)/a^2/(c-c/a/x)^(3/2)/x
```

Rubi [A] time = 0.18, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6134, 6129, 102, 21, 105, 54, 215, 93, 206}

$$-\frac{(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\sqrt{2}(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{ax+1}(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] -(((1 - a*x)^(3/2)*Sqrt[1 + a*x])/(a^2*(c - c/(a*x))^(3/2)*x)) - ((1 - a*x)
^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2)) + (S
qrt[2]*(1 - a*x)^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a
^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{(1-ax)^{3/2}\int \frac{e^{-\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}}{(1-ax)\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\int \frac{-\frac{1}{2}-\frac{ax}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1-ax)^{3/2}\int \frac{\sqrt{1+ax}}{\sqrt{x}(1-ax)}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} + \frac{(1-ax)^{3/2}\int \frac{1}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} + \frac{\left(2(1-ax)^{3/2}\right)\mathrm{Subst}\left(\int \frac{}{1-\cdots}\right.}{a^2\left(c-\frac{c}{ax}\right)^{3/2}\cdots}$$

$$= -\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} + \frac{\sqrt{2}(1-ax)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right)}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.66

$$\frac{\sqrt{1-ax}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{a^{3/2}c\sqrt{x}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[1 - a*x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]] -
Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(a^(3/2)*c*Sqrt[
c - c/(a*x)]*Sqrt[x])
```

fricas [A] time = 0.56, size = 453, normalized size = 2.85

$$\left[\frac{4\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx-c}{ax}}+\sqrt{2}\,(acx-c)\sqrt{-\frac{1}{c}}\log\left(-\frac{17\,a^3x^3-3\,a^2x^2+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-\frac{1}{c}}\,\sqrt{\frac{acx-c}{ax}}-13\,ax-1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-(ax-}{4\left(a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + sqrt(2)*(a*c*x - c
)*sqrt(-1/c)*log(-(17*a^3*x^3 - 3*a^2*x^2 + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqr
t(-a^2*x^2 + 1)*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x)) - 13*a*x - 1)/(a^3*x^3 -
 3*a^2*x^2 + 3*a*x - 1)) - (a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x +
 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)))/(a^2*c^2*x - a*c^2), 1/2*(2*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c
*x - c)/(a*x)) + (a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*
sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - sqrt(2)*(a*c*x - c)*ar
ctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x))/((3*a^2*x^2 -
 2*a*x - 1)*sqrt(c)))/sqrt(c))/(a^2*c^2*x - a*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 168, normalized size = 1.06

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(2\sqrt{-(ax+1)\,x}\,a^{\frac{3}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}-\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)a\sqrt{2}\,\sqrt{-\frac{1}{a}}+2\ln\left(\frac{2\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+}}{ax-1}\right.\right.}{4a^{\frac{3}{2}}c^2\,(ax-1)\,\sqrt{-(ax+1)\,x}\,\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*(-(a*x+1)*x)^(1/2)*a^(3/2
)*2^(1/2)*(-1/a)^(1/2)-arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2
^(1/2)*(-1/a)^(1/2)+2*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x
-1)/(a*x-1))*a^(1/2))*2^(1/2)/a^(3/2)/c^2/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^(3/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^(3/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)),
 x)
```

3.542 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}} dx$

Optimal. Leaf size=208

$$\frac{3(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{2\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3\sqrt{ax+1}(1-ax)^{5/2}}{2a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\sqrt{ax+1}(1-ax)^{3/2}}{2a^2x\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 3*(-a*x+1)^(5/2)*arcsinh(a^(1/2)*x^(1/2))/a^(7/2)/(c-c/a/x)^(5/2)/x^(5/2)-9
/4*(-a*x+1)^(5/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))/a^(7/2)/(c
-c/a/x)^(5/2)/x^(5/2)*2^(1/2)+1/2*(-a*x+1)^(3/2)*(a*x+1)^(1/2)/a^2/(c-c/a/x
)^(5/2)/x+3/2*(-a*x+1)^(5/2)*(a*x+1)^(1/2)/a^3/(c-c/a/x)^(5/2)/x^2
```

Rubi [A] time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6134, 6129, 98, 154, 157, 54, 215, 93, 206}

$$\frac{3\sqrt{ax+1}(1-ax)^{5/2}}{2a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{2\sqrt{2}a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\sqrt{ax+1}(1-ax)^{3/2}}{2a^2x\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] ((1 - a*x)^(3/2)*Sqrt[1 + a*x])/(2*a^2*(c - c/(a*x))^(5/2)*x) + (3*(1 - a*x
)^(5/2)*Sqrt[1 + a*x])/(2*a^3*(c - c/(a*x))^(5/2)*x^2) + (3*(1 - a*x)^(5/2)
*ArcSinh[Sqrt[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) - (9*(1 -
a*x)^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(2*Sqrt[2]*a^(
7/2)*(c - c/(a*x))^(5/2)*x^(5/2))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{(1-ax)^{5/2}\int \frac{e^{-\tanh^{-1}(ax)}x^{5/2}}{(1-ax)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}\int \frac{x^{5/2}}{(1-ax)^2\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{(1-ax)^{5/2}\int \frac{\sqrt{x}\left(\frac{3}{2}+3ax\right)}{(1-ax)\sqrt{1+ax}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{3(1-ax)^{5/2}\sqrt{1+ax}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\int \frac{-\frac{3a}{2}-3a^2x}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{2a^4\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{3(1-ax)^{5/2}\sqrt{1+ax}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{\left(3(1-ax)^{5/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} - \frac{\left(9(1-ax)^{5/2}\right)\ldots}{4a^3\left(c-\ldots\right.}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{3(1-ax)^{5/2}\sqrt{1+ax}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{\left(3(1-ax)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} - \ldots$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{3(1-ax)^{5/2}\sqrt{1+ax}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{3(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} - \frac{9(1-ax)^{5/2}\text{ta}\ldots}{2\sqrt{2}\,a^{7/2}\left(c\ldots\right.}$$

Mathematica [A] time = 0.18, size = 127, normalized size = 0.61

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{ax+1}(3-2ax)-12(ax-1)\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+9\sqrt{2}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4a^{3/2}c^2\sqrt{x}\sqrt{1-ax}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*(3 - 2*a*x)*Sqrt[1 + a*x] - 12*(-1 + a*x)*ArcSinh[Sqrt[a
]*Sqrt[x]] + 9*Sqrt[2]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1
+ a*x]])/(4*a^(3/2)*c^2*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 - a*x])
```

fricas [A] time = 0.57, size = 528, normalized size = 2.54

$$\left[-\frac{9\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+12\left(a^2x^2-2\,ax+1\right.}{16\left(a^3c^3x^2-2\,a^2c^3x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas
")
```

```
[Out] [-1/16*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2
*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)
*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 12*(a^2*
x^2 - 2*a*x + 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)
*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) - 8*(2
*a^2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2
- 2*a^2*c^3*x + a*c^3), -1/8*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arcta
n(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c
*x^2 - 2*a*c*x - c)) - 12*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arctan(2*sqrt(-a^2*
x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 4
*(2*a^2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x
^2 - 2*a^2*c^3*x + a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

maple [A] time = 0.06, size = 276, normalized size = 1.33

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\left(4a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)x}\,x-6\sqrt{-(ax+1)x}\,a^{\frac{3}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}-6a^2\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right.\right.}{8a^{\frac{3}{2}}c^3(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(4*a^(5/2)*2^(1/2)*(-1/a)^(1
/2)*(-(a*x+1)*x)^(1/2)*x-6*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)-
6*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)
*x+9*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-
1))*x+6*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(
1/2)-9*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^
(1/2))*2^(1/2)/a^(3/2)/c^3/(a*x-1)^2/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^(5/2)*(a*x + 1)),x)
```

[Out] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^(5/2)*(a*x + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(5/2),x)

[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)

3.543 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{5(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{115(1-ax)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16\sqrt{2}\,a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{35\sqrt{ax+1}\,(1-ax)^{7/2}}{16a^4x^3\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{15\sqrt{ax+1}\,(1-ax)^{5/2}}{16a^3x^2\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[Out] -5*(-a*x+1)^(7/2)*arcsinh(a^(1/2)*x^(1/2))/a^(9/2)/(c-c/a/x)^(7/2)/x^(7/2)+
115/32*(-a*x+1)^(7/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))/a^(9/2
)/(c-c/a/x)^(7/2)/x^(7/2)*2^(1/2)+1/4*(-a*x+1)^(3/2)*(a*x+1)^(1/2)/a^2/(c-c
/a/x)^(7/2)/x-15/16*(-a*x+1)^(5/2)*(a*x+1)^(1/2)/a^3/(c-c/a/x)^(7/2)/x^2-35
/16*(-a*x+1)^(7/2)*(a*x+1)^(1/2)/a^4/(c-c/a/x)^(7/2)/x^3
```

Rubi [A] time = 0.20, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6134, 6129, 98, 149, 154, 157, 54, 215, 93, 206}

$$-\frac{35\sqrt{ax+1}\,(1-ax)^{7/2}}{16a^4x^3\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{15\sqrt{ax+1}\,(1-ax)^{5/2}}{16a^3x^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{5(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{115(1-ax)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{16\sqrt{2}\,a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] ((1 - a*x)^(3/2)*Sqrt[1 + a*x])/(4*a^2*(c - c/(a*x))^(7/2)*x) - (15*(1 - a*
x)^(5/2)*Sqrt[1 + a*x])/(16*a^3*(c - c/(a*x))^(7/2)*x^2) - (35*(1 - a*x)^(7
/2)*Sqrt[1 + a*x])/(16*a^4*(c - c/(a*x))^(7/2)*x^3) - (5*(1 - a*x)^(7/2)*Ar
cSinh[Sqrt[a]*Sqrt[x]])/(a^(9/2)*(c - c/(a*x))^(7/2)*x^(7/2)) + (115*(1 - a
*x)^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(16*Sqrt[2]*a^(
9/2)*(c - c/(a*x))^(7/2)*x^(7/2))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{(1-ax)^{7/2}\int \frac{e^{-\tanh^{-1}(ax)}x^{7/2}}{(1-ax)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{7/2}\int \frac{x^{7/2}}{(1-ax)^3\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{(1-ax)^{7/2}\int \frac{x^{3/2}\left(\frac{5}{2}+5ax\right)}{(1-ax)^2\sqrt{1+ax}}\,dx}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{(1-ax)^{7/2}\int \frac{\sqrt{x}\left(-\frac{45a}{4}-\frac{35a^2x}{2}\right)}{(1-ax)\sqrt{1+ax}}\,dx}{8a^4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{35(1-ax)^{7/2}\sqrt{1+ax}}{16a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\int \frac{\frac{35a^2}{4}+20}{\sqrt{x}(1-ax)}}{8a^6\left(c-\frac{c}{ax}\right)^{7/2}x}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{35(1-ax)^{7/2}\sqrt{1+ax}}{16a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} - \frac{\left(5(1-ax)^{7/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1}}}{2a^4\left(c-\frac{c}{ax}\right)^{7/2}x^7}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{35(1-ax)^{7/2}\sqrt{1+ax}}{16a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} - \frac{\left(5(1-ax)^{7/2}\right)\text{Subst}\left(\right.}{a^4\left(c-\frac{c}{ax}\right.}$$

$$= \frac{(1-ax)^{3/2}\sqrt{1+ax}}{4a^2\left(c-\frac{c}{ax}\right)^{7/2}x} - \frac{15(1-ax)^{5/2}\sqrt{1+ax}}{16a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2} - \frac{35(1-ax)^{7/2}\sqrt{1+ax}}{16a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} - \frac{5(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{\right.}}{a^{9/2}\left(c-\frac{c}{ax}\right)^{7/2}x^7}$$

Mathematica [A] time = 0.23, size = 139, normalized size = 0.55

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(16a^2x^2-55ax+35\right)+160(ax-1)^2\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-115\sqrt{2}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{32a^{3/2}c^3\sqrt{x}(1-ax)^{3/2}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(35 - 55*a*x + 16*a^2*x^2) + 160*(-1 + a*x
)^2*ArcSinh[Sqrt[a]*Sqrt[x]] - 115*Sqrt[2]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sq
rt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(32*a^(3/2)*c^3*Sqrt[c - c/(a*x)]*Sqrt[x]*(1
 - a*x)^(3/2))
```

fricas [A] time = 0.60, size = 600, normalized size = 2.38

$$\left[-\frac{115\sqrt{2}\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{-c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+160\left(a^{3}\right.}{128\left(a^4c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(7/2),x, algorithm="fricas
")
```

```
[Out] [-1/128*(115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(17*a^
3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^
2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7
*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(
a*x)) - c)/(a*x - 1)) - 8*(16*a^3*x^3 - 55*a^2*x^2 + 35*a*x)*sqrt(-a^2*x^2
+ 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x -
a*c^4), -1/64*(115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan
(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*
x^2 - 2*a*c*x - c)) - 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan(
2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c
*x - c)) - 4*(16*a^3*x^3 - 55*a^2*x^2 + 35*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*
c*x - c)/(a*x)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 390, normalized size = 1.55

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(32a^{\frac{7}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x^2-110a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x-80a^3\arctan\left(\frac{2ax}{2\sqrt{a}\,\sqrt{-}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(7/2),x)
```

```
[Out] 1/64*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(32*a^(7/2)*2^(1/2)*(-1/a)^
(1/2)*(-(a*x+1)*x)^(1/2)*x^2-110*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^
(1/2)*x-80*a^3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1
/a)^(1/2)*x^2+115*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3
*a*x-1)/(a*x-1))*x^2+70*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)+160
*a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*
x-230*a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x
-1))*x-80*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)
^(1/2)+115*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1)
)*a^(1/2))*2^(1/2)/a^(3/2)/c^4/(a*x-1)^3/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a/x)^(7/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a*x))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-\frac{c}{a\,x}\right)}^{7/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^(7/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - c/(a*x))^(7/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a/x)**(7/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x + 1)),
 x)
```

3.544 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$

Optimal. Leaf size=189

$$\frac{13c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{64\sqrt{2}\,c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/}}{7a}$$

```
[Out] 19/3*c^3*(c-c/a/x)^(3/2)/a+3/5*c^2*(c-c/a/x)^(5/2)/a-5/7*c*(c-c/a/x)^(7/2)/
a-(c-c/a/x)^(9/2)*x+13*c^(9/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a-64*c^(9/2
)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a+51*c^4*(c-c/a/x)^(
1/2)/a
```

Rubi [A] time = 0.26, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\frac{13c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{64\sqrt{2}\,c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(9/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (51*c^4*Sqrt[c - c/(a*x)])/a + (19*c^3*(c - c/(a*x))^(3/2))/(3*a) + (3*c^2*
(c - c/(a*x))^(5/2))/(5*a) - (5*c*(c - c/(a*x))^(7/2))/(7*a) - (c - c/(a*x)
)^(9/2)*x + (13*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a - (64*Sqrt[2]
*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
```

```
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}(1-ax)}{1+ax}dx$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{11/2}x}{1+ax}dx}{c}$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{11/2}}{a+\frac{1}{x}}dx}{c}$$

$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{11/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}\left(\frac{13c^2}{2}+\frac{5c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{2\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}\left(\frac{91c^3}{4}-\frac{21c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{7c}$$

$$=\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{4\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{455c^4}{8}-\frac{665c^4x}{8a}\right.}{x(a+x)}\right.}{35c}$$

$$=\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{8\,\text{Subst}\left(\int\right.}{}$$

$$=\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}$$

$$=\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}$$

$$=\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}$$

$$=\frac{51c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{19c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}-\left(c-\frac{c}{ax}\right)^{9/2}$$

Mathematica [A] time = 0.35, size = 133, normalized size = 0.70

$$\frac{c^4\left(-105a^4x^4+6428a^3x^3-1196a^2x^2+258ax-30\right)\sqrt{c-\frac{c}{ax}}}{105a^4x^3}+\frac{13c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{64\sqrt{2}\,c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(-30 + 258*a*x - 1196*a^2*x^2 + 6428*a^3*x^3 - 105*a
^4*x^4))/(105*a^4*x^3) + (13*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
- (64*Sqrt[2]*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.59, size = 342, normalized size = 1.81

$$\left[\frac{6720\sqrt{2}\,a^3c^{\frac{9}{2}}x^3\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+1365\,a^3c^{\frac{9}{2}}x^3\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(105\,a^4c^4x^4-\cdots\right.}{210\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/210*(6720*sqrt(2)*a^3*c^(9/2)*x^3*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 1365*a^3*c^(9/2)*x^3*log(-2*a*c*x
- 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(105*a^4*c^4*x^4 - 6428*a^
3*c^4*x^3 + 1196*a^2*c^4*x^2 - 258*a*c^4*x + 30*c^4)*sqrt((a*c*x - c)/(a*x)
))/(a^4*x^3), 1/105*(6720*sqrt(2)*a^3*sqrt(-c)*c^4*x^3*arctan(1/2*sqrt(2)*s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 1365*a^3*sqrt(-c)*c^4*x^3*arctan(sqrt(
-c)*sqrt((a*c*x - c)/(a*x))/c) - (105*a^4*c^4*x^4 - 6428*a^3*c^4*x^3 + 1196
*a^2*c^4*x^2 - 258*a*c^4*x + 30*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
```

```
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.05, size = 305, normalized size = 1.61

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{4}\left(-17430\sqrt{a\,x^{2}-x}\,\sqrt{\frac{1}{a}}\,a^{\frac{9}{2}}x^{5}+6720\sqrt{\frac{1}{a}}\,a^{\frac{9}{2}}\sqrt{(ax-1)\,x}\,x^{5}+10920\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,a^{\frac{7}{2}}x^{3}+8715\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/210*(c*(a*x-1)/a/x)^(1/2)/x^4*c^4/a^(9/2)*(-17430*(a*x^2-x)^(1/2)*(1/a)^
(1/2)*a^(9/2)*x^5+6720*(1/a)^(1/2)*a^(9/2)*((a*x-1)*x)^(1/2)*x^5+10920*(a*x
^2-x)^(3/2)*(1/a)^(1/2)*a^(7/2)*x^3+8715*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^5*a^4-10080*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(
1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^5*a^4-6720*ln((2*2^(1/2)*(1/a)^(1/2)*(
(a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*2^(1/2)*a^(7/2)*x^5-1936*a^(5/2)*(a*x^
2-x)^(3/2)*x^2*(1/a)^(1/2)+456*a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)-60*(a*
x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^{2}x^{2}-1\right)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{(ax+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^(9/2)/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-\frac{c}{a\,x}\right)^{9/2}\,\left(a^{2}\,x^{2}-1\right)}{\left(a\,x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(9/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a*x))^(9/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{5c^4\sqrt{c-\frac{c}{ax}}}{ax+1}\right)dx-\int\frac{10c^4\sqrt{c-\frac{c}{ax}}}{a^2x^2+ax}\,dx-\int\left(-\frac{10c^4\sqrt{c-\frac{c}{ax}}}{a^3x^3+a^2x^2}\right)dx-\int\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a^4x^4+a^3x^3}\,dx-\int\left(-\frac{c^4\sqrt{c-\frac{c}{ax}}}{a^5x^5+a^4x^4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(9/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-5*c**4*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(10*c**4*sqrt(c
 - c/(a*x))/(a**2*x**2 + a*x), x) - Integral(-10*c**4*sqrt(c - c/(a*x))/(a*
*3*x**3 + a**2*x**2), x) - Integral(5*c**4*sqrt(c - c/(a*x))/(a**4*x**4 + a
**3*x**3), x) - Integral(-c**4*sqrt(c - c/(a*x))/(a**5*x**5 + a**4*x**4), x
) - Integral(a*c**4*x*sqrt(c - c/(a*x))/(a*x + 1), x)
```

3.545 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=164

$$\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[Out] 5/3*c^2*(c-c/a/x)^(3/2)/a-3/5*c*(c-c/a/x)^(5/2)/a-(c-c/a/x)^(7/2)*x+11*c^(7
/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a-32*c^(7/2)*arctanh(1/2*(c-c/a/x)^(1/
2)*2^(1/2)/c^(1/2))*2^(1/2)/a+21*c^3*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (21*c^3*Sqrt[c - c/(a*x)])/a + (5*c^2*(c - c/(a*x))^(3/2))/(3*a) - (3*c*(c
 - c/(a*x))^(5/2))/(5*a) - (c - c/(a*x))^(7/2)*x + (11*c^(7/2)*ArcTanh[Sqrt[
c - c/(a*x)]/Sqrt[c]])/a - (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
```

```
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}{1+ax}dx$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}x}{1+ax}dx}{c}$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}}{a+\frac{1}{x}}dx}{c}$$

$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{9/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}\left(\frac{11c^2}{2}+\frac{3c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{2\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{55c^3}{4}-\frac{25c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{5c}$$

$$=\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{4\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{165c^4}{8}-\frac{315c^4x}{8a}\right)}{x(a+x)}d\right.}{15c}$$

$$=\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{8\,\text{Subst}\left(\int\frac{\frac{165c}{16}}{x(a+}\right.}{1}$$

$$=\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{\left(11c^4\right)\text{Subst}\left(\int\right.}{}$$

$$=\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x+\left(11c^3\right)\text{Subst}\left(\int\right.$$

$$=\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}-\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{11c^{7/2}\tanh^{-1}\left(\right.}{a}$$

Mathematica [A] time = 0.18, size = 125, normalized size = 0.76

$$\frac{c^3\left(-15a^3x^3+376a^2x^2-52ax+6\right)\sqrt{c-\frac{c}{ax}}}{15a^3x^2}+\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(6 - 52*a*x + 376*a^2*x^2 - 15*a^3*x^3))/(15*a^3*x^2
) + (11*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a - (32*Sqrt[2]*c^(7/2)
*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.50, size = 320, normalized size = 1.95

$$\left[\frac{480\sqrt{2}\,a^2c^{\frac{7}{2}}x^2\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+165\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(15\,a^3c^3x^3-37\right.}{30\,a^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/30*(480*sqrt(2)*a^2*c^(7/2)*x^2*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 165*a^2*c^(7/2)*x^2*log(-2*a*c*x - 2
*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(15*a^3*c^3*x^3 - 376*a^2*c^3
*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2), 1/15*(480*sq
rt(2)*a^2*sqrt(-c)*c^3*x^2*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) - 165*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c
) - (15*a^3*c^3*x^3 - 376*a^2*c^3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c
)/(a*x)))/(a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
```

```
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [B] time = 0.05, size = 281, normalized size = 1.71

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^3 \left(-1110\sqrt{a\,x^2-x}\; a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\; x^4+480 a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\;\sqrt{(ax-1)\,x}\; x^4+660 a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}} x^2\sqrt{\frac{1}{a}}\;+555\ln\left(\frac{2\sqrt{a\,x^2-}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/30*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3/a^(7/2)*(-1110*(a*x^2-x)^(1/2)*a^(7/2)*
(1/a)^(1/2)*x^4+480*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^4+660*a^(5/2)*(
a*x^2-x)^(3/2)*x^2*(1/a)^(1/2)+555*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))*(1/a)^(1/2)*x^4*a^3-480*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/
2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^4-720*ln(1/2*(2*((a*x-1)*x)^(1/2
)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^4*a^3-92*a^(3/2)*(a*x^2-x)^(3/2)*
x*(1/a)^(1/2)+12*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/(1/
a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^(7/2)/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\left(a^2\,x^2-1\right)}{(a\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(7/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a*x))^(7/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{4c^3\sqrt{c-\frac{c}{ax}}}{ax+1}\right)dx-\int\frac{6c^3\sqrt{c-\frac{c}{ax}}}{a^2x^2+ax}\,dx-\int\left(-\frac{4c^3\sqrt{c-\frac{c}{ax}}}{a^3x^3+a^2x^2}\right)dx-\int\frac{c^3\sqrt{c-\frac{c}{ax}}}{a^4x^4+a^3x^3}\,dx-\int\frac{ac^3x\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-4*c**3*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(6*c**3*sqrt(c
- c/(a*x))/(a**2*x**2 + a*x), x) - Integral(-4*c**3*sqrt(c - c/(a*x))/(a**3
*x**3 + a**2*x**2), x) - Integral(c**3*sqrt(c - c/(a*x))/(a**4*x**4 + a**3*
x**3), x) - Integral(a*c**3*x*sqrt(c - c/(a*x))/(a*x + 1), x)
```

3.546 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=139

$$\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[Out] -1/3*c*(c-c/a/x)^(3/2)/a-(c-c/a/x)^(5/2)*x+9*c^(5/2)*arctanh((c-c/a/x)^(1/2
)/c^(1/2))/a-16*c^(5/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2
)/a+7*c^2*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (7*c^2*Sqrt[c - c/(a*x)])/a - (c*(c - c/(a*x))^(3/2))/(3*a) - (c - c/(a*x))
^(5/2)*x + (9*c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a - (16*Sqrt[2]*c
^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}{1+ax}dx$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}x}{1+ax}dx}{c}$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}dx}{c}$$

$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{9c^2}{2}+\frac{c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{2\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{27c^3}{4}-\frac{21c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{3c}$$

$$=\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{4\,\text{Subst}\left(\int\frac{\frac{27c^4}{8}-\frac{69c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{3c}$$

$$=\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{\left(9c^3\right)\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}+$$

$$=\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x+\left(9c^2\right)\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c\right.$$

$$=\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{16\sqrt{2}\,c^{5/2}}{}$$

Mathematica [A] time = 0.11, size = 116, normalized size = 0.83

$$\frac{c^2\left(-3a^2x^2+26ax-2\right)\sqrt{c-\frac{c}{ax}}+27ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-48\sqrt{2}\,ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 26*a*x - 3*a^2*x^2) + 27*a*c^(5/2)*x*ArcTanh[S
qrt[c - c/(a*x)]/Sqrt[c]] - 48*Sqrt[2]*a*c^(5/2)*x*ArcTanh[Sqrt[c - c/(a*x)
]/(Sqrt[2]*Sqrt[c])])/(3*a^2*x)
```

fricas [A] time = 0.48, size = 282, normalized size = 2.03

$$\left[\frac{48\sqrt{2}\,ac^{\frac{5}{2}}x\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+27\,ac^{\frac{5}{2}}x\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\left(3\,a^2c^2x^2-26\,ac^2x+2\right.}{6\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/6*(48*sqrt(2)*a*c^(5/2)*x*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) - 3*a*c*x + c)/(a*x + 1)) + 27*a*c^(5/2)*x*log(-2*a*c*x - 2*a*sqrt(c)*
x*sqrt((a*c*x - c)/(a*x)) + c) - 2*(3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*sqr
t((a*c*x - c)/(a*x)))/(a^2*x), 1/3*(48*sqrt(2)*a*sqrt(-c)*c^2*x*arctan(1/2*
sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 27*a*sqrt(-c)*c^2*x*arctan(sq
rt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*sq
rt((a*c*x - c)/(a*x)))/(a^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [B] time = 0.05, size = 257, normalized size = 1.85

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^2 \left(-90\sqrt{a\,x^2-x}\; a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\; x^3+48a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\;\sqrt{(ax-1)\,x}\; x^3+48a^{\frac{3}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}} x\sqrt{\frac{1}{a}}\;+45\ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}+}{2\sqrt{a}}\right.\right.}{6x^2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2/a^(5/2)*(-90*(a*x^2-x)^(1/2)*a^(5/2)*(1/
a)^(1/2)*x^3+48*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3+48*a^(3/2)*(a*x^2
-x)^(3/2)*x*(1/a)^(1/2)+45*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/
2))*(1/a)^(1/2)*x^3*a^2-48*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-
1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3-72*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^3*a^2-4*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2)
)/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{(ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^(5/2)/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\left(a^2\,x^2-1\right)}{(a\,x+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(5/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a*x))^(5/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{3c^2\sqrt{c-\frac{c}{ax}}}{ax+1}\right) dx-\int \frac{3c^2\sqrt{c-\frac{c}{ax}}}{a^2x^2+ax}\, dx-\int \left(-\frac{c^2\sqrt{c-\frac{c}{ax}}}{a^3x^3+a^2x^2}\right) dx-\int \frac{ac^2x\sqrt{c-\frac{c}{ax}}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-3*c**2*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(3*c**2*sqrt(c
- c/(a*x))/(a**2*x**2 + a*x), x) - Integral(-c**2*sqrt(c - c/(a*x))/(a**3*x
**3 + a**2*x**2), x) - Integral(a*c**2*x*sqrt(c - c/(a*x))/(a*x + 1), x)
```

3.547 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=113

$$\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{c\sqrt{c-\frac{c}{ax}}}{a}-x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[Out] -(c-c/a/x)^(3/2)*x+7*c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a-8*c^(3/2)*a
rctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a+c*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.21, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{c\sqrt{c-\frac{c}{ax}}}{a}-x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)])/a - (c - c/(a*x))^(3/2)*x + (7*c^(3/2)*ArcTanh[Sqrt[c
 - c/(a*x)]/Sqrt[c]])/a - (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqr
t[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}{1+ax}dx$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}x}{1+ax}dx}{c}$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}dx}{c}$$

$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{7c^2}{2}-\frac{c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{2\,\text{Subst}\left(\int\frac{\frac{7c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{\left(7c^2\right)\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}+\frac{\left(8c^2\right)\text{Subst}\left(\int\cdots\right.}{}$$

$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x+(7c)\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)-(16c)\,\text{Sub}\cdots$$

$$=\frac{c\sqrt{c-\frac{c}{ax}}}{a}-\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.08, size = 96, normalized size = 0.85

$$\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)+c(2-ax)\sqrt{c-\frac{c}{ax}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(2 - a*x) + 7*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c
]] - 8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.48, size = 231, normalized size = 2.04

$$\left[\frac{8\sqrt{2}\,c^{\frac{3}{2}}\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+7\,c^{\frac{3}{2}}\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)-2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},\frac{8\sqrt{2}\sqrt{-c}\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/2*(8*sqrt(2)*c^(3/2)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
- 3*a*c*x + c)/(a*x + 1)) + 7*c^(3/2)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*
c*x - c)/(a*x)) + c) - 2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, (8*sqrt(
2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 7*sq
rt(-c)*c*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt((a
*c*x - c)/(a*x)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [B] time = 0.04, size = 229, normalized size = 2.03

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\left(-10\sqrt{a\,x^2-x}\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\, x^2+8a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}\, x^2+4\left(a\,x^2-x\right)^{\frac{3}{2}}\sqrt{a}\,\sqrt{\frac{1}{a}}+5\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right.\right.}{2x\,a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)/x*c/a^(3/2)*(-10*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(
1/2)*x^2+8*a^(3/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^2+4*(a*x^2-x)^(3/2)*a^(1
/2)*(1/a)^(1/2)+5*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)
^(1/2)*x^2*a-8*a^(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*
a-3*a*x+1)/(a*x+1))*x^2-12*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(
1/2))*(1/a)^(1/2)*x^2*a)/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a*x))^(3/2)/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\left(a^2\,x^2-1\right)}{(a\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(3/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a*x))^(3/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2c\sqrt{c-\frac{c}{ax}}}{ax+1}\right)dx-\int\frac{c\sqrt{c-\frac{c}{ax}}}{a^2x^2+ax}\,dx-\int\frac{acx\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-2*c*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(c*sqrt(c - c/(a*x
))/(a**2*x**2 + a*x), x) - Integral(a*c*x*sqrt(c - c/(a*x))/(a*x + 1), x)
```

3.548 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=93

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] 5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a-4*arctanh(1/2*(c-c/a/x)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a-x*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 98, 156, 63, 208}

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{1+ax}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\sqrt{c-\frac{c}{ax}}\,x - \frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\sqrt{c-\frac{c}{ax}}\,x - \frac{(5c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} + \frac{(4c)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\sqrt{c-\frac{c}{ax}}\,x + 5\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) - 8\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x,\right.$$

$$= -\sqrt{c-\frac{c}{ax}}\,x + \frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} - \frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 93, normalized size = 1.00

$$x\left(-\sqrt{c-\frac{c}{ax}}\right) + \frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} - \frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.45, size = 217, normalized size = 2.33

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{2}\,\sqrt{c}\,\log\left(\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)-5\,\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},-\frac{ax\sqrt{\frac{acx-c}{ax}}-4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 4*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) - 5*sqrt(c)*log(
-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, -(a*x*sqrt((a*c*x
 - c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
 c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.05, size = 190, normalized size = 2.04

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-4\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2a}{2\sqrt{a}}\right.\right.}{2\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*((a*x-
1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1
)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*
a*x-1)/a^(1/2))*a*(1/a)^(1/2)+6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^{2} x^{2}-1\right) \sqrt{c-\frac{c}{a x}}}{(a x+1)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
[Out] -int(((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{c-\frac{c}{a x}}}{a x+1}\right) dx - \int \frac{a x \sqrt{c-\frac{c}{a x}}}{a x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x*sqrt(c - c/(a*x))
/(a*x + 1), x)
```

3.549 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=96

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] 3*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)-2*arctanh(1/2*(c-c/a/x)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)-x*(c-c/a/x)^(1/2)/c
```

Rubi [A] time = 0.16, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 99, 156, 63, 208}

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt
[c]) - (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \int \frac{1-ax}{\sqrt{c-\frac{c}{ax}}\,(1+ax)}\,dx$$
$$= -\frac{a\int \frac{\sqrt{c-\frac{c}{ax}}\,x}{1+ax}\,dx}{c}$$
$$= -\frac{a\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}\,dx}{c}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c} + \frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}+\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} + \frac{2\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c} + \frac{3\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c} - \frac{4\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}} - \frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.05, size = 96, normalized size = 1.00

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}} - \frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt
[c]) - (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

fricas [A] time = 0.44, size = 232, normalized size = 2.42

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}} - 2\,\sqrt{2}\,\sqrt{c}\,\log\left(\frac{\frac{2\,\sqrt{2}\,ax\,\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}-3\,ax+1}{ax+1}\right) - 3\,\sqrt{c}\,\log\left(-2\,acx - 2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}} + c\right)}{2\,ac}, \frac{2\,\sqrt{2}\,c\sqrt{-\frac{1}{c}}\,\text{arctan}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 2*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*x
*sqrt((a*c*x - c)/(a*x))/sqrt(c) - 3*a*x + 1)/(a*x + 1)) - 3*sqrt(c)*log(-2
*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c), (2*sqrt(2)*c*sq
rt(-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1)) -
 a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/
(a*x))/c))/(a*c)]
```

giac [A] time = 0.22, size = 130, normalized size = 1.35

$$ac\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{acx-c}{ax}}}{2\,\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c} - \frac{3\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c} - \frac{\sqrt{\frac{acx-c}{ax}}}{a^2\left(c - \frac{acx-c}{ax}\right)c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] a*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sq
rt(-c)*c) - 3*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c) - s
qrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c))
```

maple [A] time = 0.04, size = 136, normalized size = 1.42

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}} + 3\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}} + 2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}\right)}{2\sqrt{(ax-1)\,x}\,c\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+3*ln(
1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+2*2^(1/2)*
ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/((
a*x-1)*x)^(1/2)/c/a^(3/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2-1}{(ax+1)^2\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*sqrt(c - c/(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a^2\,x^2-1}{\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax\sqrt{c-\frac{c}{ax}}+\sqrt{c-\frac{c}{ax}}}\,dx-\int \left(-\frac{1}{ax\sqrt{c-\frac{c}{ax}}+\sqrt{c-\frac{c}{ax}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(c - c/(a*x)) + sqrt(c - c/(a*x))), x) - Integral(-1
/(a*x*sqrt(c - c/(a*x)) + sqrt(c - c/(a*x))), x)
```

3.550 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}-\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}$$

```
[Out] arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(3/2)-arctanh(1/2*(c-c/a/x)^(1/2)*2^(1
/2)/c^(1/2))*2^(1/2)/a/c^(3/2)-x*(c-c/a/x)^(1/2)/c^2
```

Rubi [A] time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 103, 21, 83, 63, 208}

$$-\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c^2) + ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2
)) - (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
```

```
tQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx &= \int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx \\
&= -\frac{a\int \frac{x}{\sqrt{c-\frac{c}{ax}}\,(1+ax)}\,dx}{c} \\
&= -\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}\,dx}{c} \\
&= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c} \\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\mathrm{Subst}\left(\int \frac{\frac{c}{2}-\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2} \\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{2c^2} \\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac} + \frac{\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{ac} \\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2} \\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} - \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 95, normalized size = 1.00

$$
\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} - \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}} - \frac{x\sqrt{c-\frac{c}{ax}}}{c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x)/c^2) + ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2
)) - (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

fricas [A] time = 0.43, size = 231, normalized size = 2.43

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-\sqrt{2}\,\sqrt{c}\,\log\left(\frac{\frac{2\,\sqrt{2}\,ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}-3\,ax+1}{ax+1}\right)-\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac^{2}},\ \frac{\sqrt{2}\,c\sqrt{-\frac{1}{c}}\,\arctan\left(\frac{\sqrt{2}\,a\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*x*s
qrt((a*c*x - c)/(a*x))/sqrt(c) - 3*a*x + 1)/(a*x + 1)) - sqrt(c)*log(-2*a*c
*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c^2), (sqrt(2)*c*sqrt(-
1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1)) - a*x
*sqrt((a*c*x - c)/(a*x)) - sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))
/c))/(a*c^2)]
```

giac [A] time = 0.18, size = 129, normalized size = 1.36

$$ac\left(\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{acx-c}{ax}}}{2\,\sqrt{-c}}\right)}{a^{2}\sqrt{-c}\,c^{2}}-\frac{\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^{2}\sqrt{-c}\,c^{2}}-\frac{\sqrt{\frac{acx-c}{ax}}}{a^{2}\left(c-\frac{acx-c}{ax}\right)c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] a*c*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt
(-c)*c^2) - arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^2) - s
qrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^2))
```

maple [A] time = 0.04, size = 134, normalized size = 1.41

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(-2\sqrt{(ax-1)\,x}\; a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a\sqrt{\frac{1}{a}}+\sqrt{2}\;\ln\left(\frac{2\sqrt{2}\;\sqrt{\frac{1}{a}}\;\sqrt{(ax-1)x}\;a-3ax+1}{ax+1}\right)\sqrt{a}\right)}{2a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\; c^{2}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/
2)+ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+2^(1
/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2)
)/((a*x-1)*x)^(1/2)/c^2/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^{2}x^{2}-1}{(ax+1)^{2}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a*x))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a^{2}\,x^{2}-1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{acx\sqrt{c-\frac{c}{ax}}-\frac{c\sqrt{c-\frac{c}{ax}}}{ax}}\,dx-\int\left(-\frac{1}{acx\sqrt{c-\frac{c}{ax}}-\frac{c\sqrt{c-\frac{c}{ax}}}{ax}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] -Integral(a*x/(a*c*x*sqrt(c - c/(a*x)) - c*sqrt(c - c/(a*x))/(a*x)), x) - I
ntegral(-1/(a*c*x*sqrt(c - c/(a*x)) - c*sqrt(c - c/(a*x))/(a*x)), x)
```

3.551 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}-\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)-1/2*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)+2/a/c^2/(c-c/a/x)^(1/2)-x/c^2/(c-c/a/x)
^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$-\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] 2/(a*c^2*Sqrt[c - c/(a*x)]) - x/(c^2*Sqrt[c - c/(a*x)]) - ArcTanh[Sqrt[c -
c/(a*x)]/Sqrt[c]]/(a*c^(5/2)) - ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])
]/(Sqrt[2]*a*c^(5/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx &= \int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx \\
&= -\frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx}{c} \\
&= -\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{c} \\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c} \\
&= -\frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{-\frac{c}{2}-\frac{3cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2} \\
&= \frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{\frac{c^2}{2}+\frac{c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^4} \\
&= \frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2} + \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2} \\
&= \frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3} - \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3} \\
&= \frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 67, normalized size = 0.56

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a-\frac{1}{x}}{2a}\right)+{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{1}{ax}\right)-ax}{ac^2\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (-(a*x) + Hypergeometric2F1[-1/2, 1, 1/2, (a - x^(-1))/(2*a)] + Hypergeomet
ric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*c^2*Sqrt[c - c/(a*x)])
```

fricas [A] time = 0.44, size = 284, normalized size = 2.39

$$\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+2(ax-1)\sqrt{c}\log\left(-2acx+2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-4\left(a^2x^2-2ax\right)}{4\left(a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)
/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 2*(a*x - 1)*sqrt(c)*log(-2*a*c*x + 2*a*
sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 4*(a^2*x^2 - 2*a*x)*sqrt((a*c*x -
c)/(a*x)))/(a^2*c^3*x - a*c^3), 1/2*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(1/2*
sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*(a*x - 1)*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(a^2*x^2 - 2*a*x)*sqrt((a*c*x - c)/(
a*x)))/(a^2*c^3*x - a*c^3)]
```

giac [A] time = 0.20, size = 166, normalized size = 1.39

$$\frac{1}{2}ac\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^3}+\frac{2\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^3}+\frac{2\left(c-\frac{2(acx-c)}{ax}\right)}{\left(c\sqrt{\frac{acx-c}{ax}}-\frac{(acx-c)\sqrt{\frac{acx-c}{ax}}}{ax}\right)a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

[Out] 1/2*a*c*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*
sqrt(-c)*c^3) + 2*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^
3) + 2*(c - 2*(a*c*x - c)/(a*x))/((c*sqrt((a*c*x - c)/(a*x)) - (a*c*x - c)*
sqrt((a*c*x - c)/(a*x))/(a*x))*a^2*c^3))

maple [B] time = 0.05, size = 370, normalized size = 3.11

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(8a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}\;x^2+2\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\;x^2a^3-a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(5/2),x)

[Out] -1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(8*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/
2)*x^2+2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*
x^2*a^3-a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x
+1)/(a*x+1))*x^2-4*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)-16*a^(5/2)*(1/a)^(
1/2)*((a*x-1)*x)^(1/2)*x-4*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(
1/2))*(1/a)^(1/2)*x*a^2+2*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1
)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x+8*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+2*
ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/2)
*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/(
(a*x-1)*x)^(1/2)/c^3/(a*x-1)^2/(1/a)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")

[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a*x))^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a^2\,x^2-1}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^2x\sqrt{c-\frac{c}{ax}}-c^2\sqrt{c-\frac{c}{ax}}-\frac{c^2\sqrt{c-\frac{c}{ax}}}{ax}+\frac{c^2\sqrt{c-\frac{c}{ax}}}{a^2x^2}}\,dx-\int\left(-\frac{1}{ac^2x\sqrt{c-\frac{c}{ax}}-c^2\sqrt{c-\frac{c}{ax}}-\frac{c^2\sqrt{c-\frac{c}{ax}}}{ax}+\frac{c^2\sqrt{c-\frac{c}{ax}}}{a^2x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(5/2),x)
```

```
[Out] -Integral(a*x/(a*c**2*x*sqrt(c - c/(a*x)) - c**2*sqrt(c - c/(a*x)) - c**2*s
qrt(c - c/(a*x))/(a*x) + c**2*sqrt(c - c/(a*x))/(a**2*x**2)), x) - Integral
(-1/(a*c**2*x*sqrt(c - c/(a*x)) - c**2*sqrt(c - c/(a*x)) - c**2*sqrt(c - c/
(a*x))/(a*x) + c**2*sqrt(c - c/(a*x))/(a**2*x**2)), x)
```

3.552 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}} dx$

Optimal. Leaf size=148

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}+\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] 4/3/a/c^2/(c-c/a/x)^(3/2)-x/c^2/(c-c/a/x)^(3/2)-3*arctanh((c-c/a/x)^(1/2)/c
^(1/2))/a/c^(7/2)-1/4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a/c^(7/2
)*2^(1/2)+7/2/a/c^3/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] 4/(3*a*c^2*(c - c/(a*x))^(3/2)) + 7/(2*a*c^3*Sqrt[c - c/(a*x)]) - x/(c^2*(c
 - c/(a*x))^(3/2)) - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2)) - A
rcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx$$

$$= -\frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}-\frac{5cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{9c^2}{2}+\frac{6c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^4}$$

$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{9c^3}{2}-\frac{21c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{3c^6}$$

$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4ac^3} + \frac{3\,\text{Subst}}{}$$

$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{2c^4} - \frac{3\,\text{Su}}{}$$

$$= \frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} - \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}$$

Mathematica [C] time = 0.06, size = 77, normalized size = 0.52

$$\frac{x\left({}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a-\frac{1}{x}}{2a}\right)+3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{1}{ax}\right)-3ax\right)}{3c^3(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (x*(-3*a*x + Hypergeometric2F1[-3/2, 1, -1/2, (a - x^(-1))/(2*a)] + 3*Hyper
geometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c^3*Sqrt[c - c/(a*x)]*(-1 + a
*x))
```

fricas [A] time = 0.44, size = 356, normalized size = 2.41

$$\left[\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+36\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+\right.}{24\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 36*(a^2*x^2 - 2*a*x + 1)
*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 4*(6*a
^3*x^3 - 29*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2
*c^4*x + a*c^4), 1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(1/2*
sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 36*(a^2*x^2 - 2*a*x + 1)*sqrt
(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(6*a^3*x^3 - 29*a^2*x^2
 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

giac [A] time = 0.19, size = 187, normalized size = 1.26

$$\frac{1}{12}\,ac\left(\frac{2\left(2\,c+\frac{15\,(acx-c)}{ax}\right)x}{(acx-c)ac^4\sqrt{\frac{acx-c}{ax}}}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^4}+\frac{36\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^4}-\frac{12\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/12*a*c*(2*(2*c + 15*(a*c*x - c)/(a*x))*x/((a*c*x - c)*a*c^4*sqrt((a*c*x -
 c)/(a*x))) + 3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c)
)/(a^2*sqrt(-c)*c^4) + 36*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqr
t(-c)*c^4) - 12*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^4))
```

maple [B] time = 0.05, size = 497, normalized size = 3.36

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(84a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}\;x^{3}+36\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\;x^{3}a^{4}-3a^{\frac{7}{2}}\sqrt{2}\;\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\;a-3ax+}{ax+1}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(7/2),x)
```

```
[Out] -1/24*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(84*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(
1/2)*x^3+36*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/
2)*x^3*a^4-3*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-
3*a*x+1)/(a*x+1))*x^3-60*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)*x-252*a^(7/2
)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^2-108*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)
+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^2*a^3+9*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a
)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^2+52*a^(5/2)*(1/a)^(1/2)*((
a*x-1)*x)^(3/2)+252*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x+108*ln(1/2*(2*(
(a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x*a^2-9*a^(3/2)*2^(1
/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x-84*((
a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-36*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+3*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1
)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^4/(a*x-1)^3/(1/
a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^{2}x^{2}-1}{(ax+1)^{2}\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a*x))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a^2 x^2 - 1}{\left(c - \frac{c}{a x}\right)^{7/2} (a x + 1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^3x\sqrt{c-\frac{c}{ax}} - 2c^3\sqrt{c-\frac{c}{ax}} + \frac{2c^3\sqrt{c-\frac{c}{ax}}}{a^2x^2} - \frac{c^3\sqrt{c-\frac{c}{ax}}}{a^3x^3}}\,dx - \int \left(-\frac{1}{ac^3x\sqrt{c-\frac{c}{ax}} - 2c^3\sqrt{c-\frac{c}{ax}} + \frac{2c^3\sqrt{c-\frac{c}{ax}}}{a^2x^2} - \frac{c^3\sqrt{c-\frac{c}{ax}}}{a^3x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(7/2),x)
```

```
[Out] -Integral(a*x/(a*c**3*x*sqrt(c - c/(a*x)) - 2*c**3*sqrt(c - c/(a*x)) + 2*c*
*3*sqrt(c - c/(a*x))/(a**2*x**2) - c**3*sqrt(c - c/(a*x))/(a**3*x**3)), x)
- Integral(-1/(a*c**3*x*sqrt(c - c/(a*x)) - 2*c**3*sqrt(c - c/(a*x)) + 2*c*
*3*sqrt(c - c/(a*x))/(a**2*x**2) - c**3*sqrt(c - c/(a*x))/(a**3*x**3)), x)
```

3.553 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}+\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}+\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 6/5/a/c^2/(c-c/a/x)^(5/2)+11/6/a/c^3/(c-c/a/x)^(3/2)-x/c^2/(c-c/a/x)^(5/2)-
5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(9/2)-1/8*arctanh(1/2*(c-c/a/x)^(1/2
)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)+21/4/a/c^4/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}+\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] 6/(5*a*c^2*(c - c/(a*x))^(5/2)) + 11/(6*a*c^3*(c - c/(a*x))^(3/2)) + 21/(4*
a*c^4*Sqrt[c - c/(a*x)]) - x/(c^2*(c - c/(a*x))^(5/2)) - (5*ArcTanh[Sqrt[c
- c/(a*x)]/Sqrt[c]])/(a*c^(9/2)) - ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[
c])]/(4*Sqrt[2]*a*c^(9/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx=\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}\,dx$$

$$=-\frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx}{c}$$

$$=-\frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{c}$$

$$=\frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\text{Subst}\left(\int \frac{-\frac{5c}{2}-\frac{7cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$=\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\text{Subst}\left(\int \frac{\frac{25c^2}{2}+\frac{15c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{5c^4}$$

$$=\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\text{Subst}\left(\int \frac{-\frac{75c^3}{2}-\frac{165c^3x}{4a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{15c^6}$$

$$=\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\text{Subst}\left(\int \frac{\frac{75c^4}{2}+\frac{315c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,d\right.}{15c^8}$$

$$=\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx\right.}{8ac^4}$$

$$=\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\right.}{4c^5}$$

$$=\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}-\text{ta}$$

Mathematica [C] time = 0.06, size = 82, normalized size = 0.47

$$-\frac{ax^2\left(-{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{a-\frac{1}{x}}{2a}\right)-5\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{1}{ax}\right)+5ax\right)}{5c^4(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] -1/5*(a*x^2*(5*a*x - Hypergeometric2F1[-5/2, 1, -3/2, (a - x^(-1))/(2*a)] -
 5*Hypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]))/(c^4*Sqrt[c - c/(a*x)]*(
-1 + a*x)^2)
```

fricas [A] time = 0.42, size = 428, normalized size = 2.47

$$\left[\frac{15\sqrt{2}\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{c}\,\log\left(\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+600\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{c}\,\log\left(-2\,ac\right.}{240\left(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((2*sqrt(2)
*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 600*(a^3*x
^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*
x - c)/(a*x)) + c) - 4*(60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*s
qrt((a*c*x - c)/(a*x)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
, 1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/2*s
qrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 600*(a^3*x^3 - 3*a^2*x^2 + 3*a
*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(60*a^4*x^4
 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^5*x
^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

giac [A] time = 0.19, size = 207, normalized size = 1.20

$$\frac{1}{120}\,ac\left(\frac{2\left(12\,c^2+\frac{50\,(acx-c)c}{ax}+\frac{255\,(acx-c)^2}{a^2x^2}\right)x^2}{(acx-c)^2c^5\sqrt{\frac{acx-c}{ax}}}+\frac{15\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{acx-c}{ax}}}{2\,\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^5}+\frac{600\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^5}-\frac{120\,\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/120*a*c*(2*(12*c^2 + 50*(a*c*x - c)*c/(a*x) + 255*(a*c*x - c)^2/(a^2*x^2)
)*x^2/((a*c*x - c)^2*c^5*sqrt((a*c*x - c)/(a*x))) + 15*sqrt(2)*arctan(1/2*s
qrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^5) + 600*arctan(sq
rt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^5) - 120*sqrt((a*c*x - c)/(
a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^5))
```

maple [B] time = 0.05, size = 626, normalized size = 3.62

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(1260a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}\; x^4 + 600\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\; x^4a^5 - 15a^{\frac{9}{2}}\sqrt{2}\;\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\;a-}{ax+1}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(9/2),x)
```

```
[Out] -1/240*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(1260*a^(11/2)*(1/a)^(1/2)*((a*x-1)*
x)^(1/2)*x^4+600*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a
)^(1/2)*x^4*a^5-15*a^(9/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1
/2)*a-3*a*x+1)/(a*x+1))*x^4-1020*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)*x^2-
5040*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3-2400*ln(1/2*(2*((a*x-1)*x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^3*a^4+60*a^(7/2)*2^(1/2)*ln((2
*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3+1792*a^(7/2)
*(1/a)^(1/2)*((a*x-1)*x)^(3/2)*x+7560*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)
*x^2+3600*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)
*x^2*a^3-90*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3
*a*x+1)/(a*x+1))*x^2-820*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)-5040*a^(5/2)
*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x-2400*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*(1/a)^(1/2)*x*a^2+60*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(
1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x+1260*((a*x-1)*x)^(1/2)*a^(3/2)
*(1/a)^(1/2)+600*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1
/a)^(1/2)-15*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)
/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^5/(a*x-1)^4/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a*x))^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a^2\,x^2-1}{{\left(c-\frac{c}{a\,x}\right)}^{9/2}\,{\left(a\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^4x\sqrt{c-\frac{c}{ax}}-3c^4\sqrt{c-\frac{c}{ax}}+\frac{2c^4\sqrt{c-\frac{c}{ax}}}{ax}+\frac{2c^4\sqrt{c-\frac{c}{ax}}}{a^2x^2}-\frac{3c^4\sqrt{c-\frac{c}{ax}}}{a^3x^3}+\frac{c^4\sqrt{c-\frac{c}{ax}}}{a^4x^4}}\,dx-\int\left(-\frac{}{ac^4x\sqrt{c-\frac{c}{ax}}-3c^4\sqrt{c-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a/x)**(9/2),x)
```

```
[Out] -Integral(a*x/(a*c**4*x*sqrt(c - c/(a*x)) - 3*c**4*sqrt(c - c/(a*x)) + 2*c*
*4*sqrt(c - c/(a*x))/(a*x) + 2*c**4*sqrt(c - c/(a*x))/(a**2*x**2) - 3*c**4*
sqrt(c - c/(a*x))/(a**3*x**3) + c**4*sqrt(c - c/(a*x))/(a**4*x**4)), x) - I
ntegral(-1/(a*c**4*x*sqrt(c - c/(a*x)) - 3*c**4*sqrt(c - c/(a*x)) + 2*c**4*
sqrt(c - c/(a*x))/(a*x) + 2*c**4*sqrt(c - c/(a*x))/(a**2*x**2) - 3*c**4*sqr
t(c - c/(a*x))/(a**3*x**3) + c**4*sqrt(c - c/(a*x))/(a**4*x**4)), x)
```

3.554 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=267

$$-\frac{15a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}+\frac{5a^4x^5(587-109ax)\left(c-\frac{c}{ax}\right)^{9/2}}{7(1-ax)^{9/2}\sqrt{ax+1}}+\frac{70a^3x^4\left(c-\frac{c}{ax}\right)^{9/2}}{(1-ax)^{5/2}\sqrt{ax+1}}-\frac{50a^2x^3\left(c-\frac{c}{ax}\right)^{9/2}}{7(1-ax)^{3/2}\sqrt{ax+}}$$

```
[Out] -15*a^(7/2)*(c-c/a/x)^(9/2)*x^(9/2)*arcsinh(a^(1/2)*x^(1/2))/(-a*x+1)^(9/2)
+5/7*a^4*(c-c/a/x)^(9/2)*x^5*(-109*a*x+587)/(-a*x+1)^(9/2)/(a*x+1)^(1/2)+70
*a^3*(c-c/a/x)^(9/2)*x^4/(-a*x+1)^(5/2)/(a*x+1)^(1/2)-50/7*a^2*(c-c/a/x)^(9
/2)*x^3/(-a*x+1)^(3/2)/(a*x+1)^(1/2)+10/7*a*(c-c/a/x)^(9/2)*x^2/(-a*x+1)^(1
/2)/(a*x+1)^(1/2)-2/7*(c-c/a/x)^(9/2)*x*(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$\frac{5a^4x^5(587-109ax)\left(c-\frac{c}{ax}\right)^{9/2}}{7(1-ax)^{9/2}\sqrt{ax+1}}+\frac{70a^3x^4\left(c-\frac{c}{ax}\right)^{9/2}}{(1-ax)^{5/2}\sqrt{ax+1}}-\frac{50a^2x^3\left(c-\frac{c}{ax}\right)^{9/2}}{7(1-ax)^{3/2}\sqrt{ax+1}}-\frac{15a^{7/2}x^{9/2}\left(c-\frac{c}{ax}\right)^{9/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (5*a^4*(c - c/(a*x))^(9/2)*x^5*(587 - 109*a*x))/(7*(1 - a*x)^(9/2)*Sqrt[1 +
 a*x]) + (70*a^3*(c - c/(a*x))^(9/2)*x^4)/((1 - a*x)^(5/2)*Sqrt[1 + a*x]) -
 (50*a^2*(c - c/(a*x))^(9/2)*x^3)/(7*(1 - a*x)^(3/2)*Sqrt[1 + a*x]) + (10*a
*(c - c/(a*x))^(9/2)*x^2)/(7*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*(c - c/(a*x)
)^(9/2)*x*Sqrt[1 - a*x])/(7*Sqrt[1 + a*x]) - (15*a^(7/2)*(c - c/(a*x))^(9/2
)*x^(9/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(9/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{e^{-3\tanh^{-1}(ax)}(1-ax)^{9/2}}{x^{9/2}}\,dx}{(1-ax)^{9/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^6}{x^{9/2}(1+ax)^{3/2}}\,dx}{(1-ax)^{9/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1-ax}}{7\sqrt{1+ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^4\left(\frac{25a}{2}-\frac{5a^2x}{2}\right)}{x^{7/2}(1+ax)^{3/2}}\,dx}{7(1-ax)^{9/2}}$$

$$= \frac{10a\left(c-\frac{c}{ax}\right)^{9/2}x^2}{7\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1-ax}}{7\sqrt{1+ax}} - \frac{\left(4\left(c-\frac{c}{ax}\right)^{9/2}x^{9/2}\right)\int\frac{(1-ax)^3\left(-\frac{375a^2}{4}\right.}{x^{5/2}(1+ax)}}{35(1-ax)^{9/2}}$$

$$= -\frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} + \frac{10a\left(c-\frac{c}{ax}\right)^{9/2}x^2}{7\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}x\sqrt{1-ax}}{7\sqrt{1+ax}} - \frac{\left(8\left(c-\right.\right.}{}$$

$$= \frac{70a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4}{(1-ax)^{5/2}\sqrt{1+ax}} - \frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} + \frac{10a\left(c-\frac{c}{ax}\right)^{9/2}x^2}{7\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{9/2}}{7\sqrt{1+}}$$

$$= \frac{5a^4\left(c-\frac{c}{ax}\right)^{9/2}x^5(587-109ax)}{7(1-ax)^{9/2}\sqrt{1+ax}} + \frac{70a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4}{(1-ax)^{5/2}\sqrt{1+ax}} - \frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} +$$

$$= \frac{5a^4\left(c-\frac{c}{ax}\right)^{9/2}x^5(587-109ax)}{7(1-ax)^{9/2}\sqrt{1+ax}} + \frac{70a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4}{(1-ax)^{5/2}\sqrt{1+ax}} - \frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} +$$

$$= \frac{5a^4\left(c-\frac{c}{ax}\right)^{9/2}x^5(587-109ax)}{7(1-ax)^{9/2}\sqrt{1+ax}} + \frac{70a^3\left(c-\frac{c}{ax}\right)^{9/2}x^4}{(1-ax)^{5/2}\sqrt{1+ax}} - \frac{50a^2\left(c-\frac{c}{ax}\right)^{9/2}x^3}{7(1-ax)^{3/2}\sqrt{1+ax}} +$$

Mathematica [C] time = 10.48, size = 234, normalized size = 0.88

$$-\frac{c^4\sqrt{c-\frac{c}{ax}}\left(-\frac{7168(-ax(ax+1))^{5/2}(ax-1)^4\,{}_5F_4\left(-\frac{3}{2},2,2,2,\frac{5}{2};1,1,1,\frac{7}{2};-ax\right)}{(ax+1)^{3/2}}+105\left(101a^5x^5+209a^4x^4-54a^3x^3-54a^2x^2+81ax\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/896*(c^4*Sqrt[c - c/(a*x)]*(Sqrt[-(a*x*(1 + a*x))]*(3091 - 12955*a*x + 3
4100*a^2*x^2 - 59750*a^3*x^3 - 375805*a^4*x^4 + 84329*a^5*x^5 + 165830*a^6*
x^6 - 214760*a^7*x^7 + 70000*a^8*x^8) + 105*(-27 + 81*a*x - 54*a^2*x^2 - 54
*a^3*x^3 + 209*a^4*x^4 + 101*a^5*x^5)*ArcSin[Sqrt[-(a*x)]] - (7168*(-1 + a*
x)^4*(-(a*x*(1 + a*x)))^(5/2)*HypergeometricPFQ[{-3/2, 2, 2, 2, 5/2}, {1, 1
, 1, 7/2}, -(a*x)])/(1 + a*x)^(3/2)))/(a^4*x^3*Sqrt[-(a*x*(1 + a*x))]*Sqrt[
1 - a^2*x^2])
```

fricas [A] time = 0.47, size = 408, normalized size = 1.53

$$\left[\frac{105\left(a^5c^4x^5-a^3c^4x^3\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(7\,a^5c^4x^5+1755\,a^4c^4x^4+720\right.}{28\left(a^6x^5-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/28*(105*(a^5*c^4*x^5 - a^3*c^4*x^3)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) - 4*(7*a^5*c^4*x^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 1
10*a^2*c^4*x^2 + 20*a*c^4*x - 2*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a
*x)))/(a^6*x^5 - a^4*x^3), 1/14*(105*(a^5*c^4*x^5 - a^3*c^4*x^3)*sqrt(c)*ar
ctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) - 2*(7*a^5*c^4*x^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 110
*a^2*c^4*x^2 + 20*a*c^4*x - 2*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x
)))/(a^6*x^5 - a^4*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.07, size = 227, normalized size = 0.85

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^4 \left(14\sqrt{-(ax+1)\,x}\; a^{\frac{11}{2}} x^5 + 105 \arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right) x^5 a^5 + 3510 a^{\frac{9}{2}} \sqrt{-(ax+1)\,x}\; x^4 + 105 \arctan\left(\right.\right.}{14x^3 a^{\frac{9}{2}} (ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/14*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(14*(-(a*x+1)*x)^(1/2)*a^(11/2)
*x^5+105*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^5*a^5+3510*a^(9
/2)*(-(a*x+1)*x)^(1/2)*x^4+105*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1
/2))*x^4*a^4+1440*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)-220*a^(5/2)*x^2*(-(a*x+1)*
x)^(1/2)+40*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-4*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^
2*x^2+1)^(1/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}} \left(c - \frac{c}{ax}\right)^{\frac{9}{2}}}{(ax+1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(9/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c - \frac{c}{a\,x}\right)^{9/2} \left(1 - a^2\,x^2\right)^{3/2}}{(a\,x+1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(9/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a*x))^(9/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(9/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

3.555 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=225

$$\frac{13a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}-\frac{a^3x^4(2525-427ax)\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{7/2}\sqrt{ax+1}}-\frac{398a^2x^3\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{3/2}\sqrt{ax+1}}+\frac{38ax^2\left(c-\frac{c}{ax}\right)^{7/2}}{15\sqrt{1-ax}\sqrt{ax}}$$

```
[Out] 13*a^(5/2)*(c-c/a/x)^(7/2)*x^(7/2)*arcsinh(a^(1/2)*x^(1/2))/(-a*x+1)^(7/2)-
1/15*a^3*(c-c/a/x)^(7/2)*x^4*(-427*a*x+2525)/(-a*x+1)^(7/2)/(a*x+1)^(1/2)-3
98/15*a^2*(c-c/a/x)^(7/2)*x^3/(-a*x+1)^(3/2)/(a*x+1)^(1/2)+38/15*a*(c-c/a/x
)^(7/2)*x^2/(-a*x+1)^(1/2)/(a*x+1)^(1/2)-2/5*(c-c/a/x)^(7/2)*x*(-a*x+1)^(1/
2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$-\frac{a^3x^4(2525-427ax)\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{7/2}\sqrt{ax+1}}-\frac{398a^2x^3\left(c-\frac{c}{ax}\right)^{7/2}}{15(1-ax)^{3/2}\sqrt{ax+1}}+\frac{13a^{5/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{7/2}}+\frac{38ax^2\left(c-\frac{c}{ax}\right)}{15\sqrt{1-ax}\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(a^3*(c - c/(a*x))^(7/2)*x^4*(2525 - 427*a*x))/(15*(1 - a*x)^(7/2)*Sqrt[1
+ a*x]) - (398*a^2*(c - c/(a*x))^(7/2)*x^3)/(15*(1 - a*x)^(3/2)*Sqrt[1 + a*
x]) + (38*a*(c - c/(a*x))^(7/2)*x^2)/(15*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*
(c - c/(a*x))^(7/2)*x*Sqrt[1 - a*x])/(5*Sqrt[1 + a*x]) + (13*a^(5/2)*(c - c
/(a*x))^(7/2)*x^(7/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(7/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{e^{-3\tanh^{-1}(ax)}(1-ax)^{7/2}}{x^{7/2}}dx}{(1-ax)^{7/2}}$$
$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)^5}{x^{7/2}(1+ax)^{3/2}}dx}{(1-ax)^{7/2}}$$
$$= -\frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1-ax}}{5\sqrt{1+ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)^3\left(\frac{19a}{2}-\frac{3a^2x}{2}\right)}{x^{5/2}(1+ax)^{3/2}}dx}{5(1-ax)^{7/2}}$$
$$= \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1-ax}}{5\sqrt{1+ax}} - \frac{\left(4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}\right)\int\frac{(1-ax)^2\left(-\frac{19}{}\right.}{x^{3/2}(1+}}{15(1-ax)^{7/2}}$$
$$= -\frac{398a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3}{15(1-ax)^{3/2}\sqrt{1+ax}} + \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1-ax}}{5\sqrt{1+ax}} - \frac{\left(8\left(c\right.\right.}{}$$
$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4(2525-427ax)}{15(1-ax)^{7/2}\sqrt{1+ax}} - \frac{398a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3}{15(1-ax)^{3/2}\sqrt{1+ax}} + \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+a}}$$
$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4(2525-427ax)}{15(1-ax)^{7/2}\sqrt{1+ax}} - \frac{398a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3}{15(1-ax)^{3/2}\sqrt{1+ax}} + \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+a}}$$
$$= -\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}x^4(2525-427ax)}{15(1-ax)^{7/2}\sqrt{1+ax}} - \frac{398a^2\left(c-\frac{c}{ax}\right)^{7/2}x^3}{15(1-ax)^{3/2}\sqrt{1+ax}} + \frac{38a\left(c-\frac{c}{ax}\right)^{7/2}x^2}{15\sqrt{1-ax}\sqrt{1+a}}$$

Mathematica [C] time = 3.76, size = 200, normalized size = 0.89

$$-\frac{c^3x\sqrt{c-\frac{c}{ax}}\left(1040a^4x^4(ax-1)^3\sqrt{ax+1}\sqrt{-ax(ax+1)}\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};-ax\right)+585\left(19a^4x^4-86a^3x^3+70ax-35\right.\right.}{720(-ax)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/720*(c^3*Sqrt[c - c/(a*x)]*x*(-3*Sqrt[-(a*x*(1 + a*x))]*(-6921 + 19192*a
*x - 21508*a^2*x^2 - 28706*a^3*x^3 + 6325*a^4*x^4 - 2470*a^5*x^5 + 520*a^6*
x^6) + 585*(-35 + 70*a*x - 86*a^3*x^3 + 19*a^4*x^4)*ArcSin[Sqrt[-(a*x)]] +
```

```
1040*a^4*x^4*(-1 + a*x)^3*Sqrt[1 + a*x]*Sqrt[-(a*x*(1 + a*x))]*Hypergeometr
ic2F1[3/2, 9/2, 11/2, -(a*x)]))/((-(a*x))^(7/2)*Sqrt[1 + a*x]*Sqrt[1 - a^2*
x^2])
```

fricas [A] time = 0.55, size = 386, normalized size = 1.72

$$\left[\frac{195\left(a^4c^3x^4-a^2c^3x^2\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(15\,a^4c^3x^4+1591\,a^3c^3x^3+548\right.}{60\left(a^5x^4-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/60*(195*(a^4*c^3*x^4 - a^2*c^3*x^2)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))
- c)/(a*x - 1)) - 4*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3 + 548*a^2*c^3*x^2 -
62*a*c^3*x + 6*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 -
a^3*x^2), 1/30*(195*(a^4*c^3*x^4 - a^2*c^3*x^2)*sqrt(c)*arctan(2*sqrt(-a^2*
x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2
*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3 + 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)
*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 209, normalized size = 0.93

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(30a^{\frac{9}{2}}\sqrt{-(ax+1)\,x}\,x^4+195\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)x^4a^4+3182a^{\frac{7}{2}}x^3\sqrt{-(ax+1)\,x}+195\arctan\left(\right.\right.}{30x^2a^{\frac{7}{2}}\,(ax+1)\sqrt{-(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/30*(c*(a*x-1)/a/x)^(1/2)/x^2*c^3/a^(7/2)*(30*a^(9/2)*(-(a*x+1)*x)^(1/2)*
x^4+195*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^4*a^4+3182*a^(7/
2)*x^3*(-(a*x+1)*x)^(1/2)+195*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/
2))*x^3*a^3+1096*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)-124*a^(3/2)*x*(-(a*x+1)*x)^
(1/2)+12*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^2*x^2+1)^(1/2)/(a*x+1)/(-(a*x+1)*x
)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(7/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{7/2}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(7/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a*x))^(7/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

3.556 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$

Optimal. Leaf size=181

$$-\frac{11a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{a^2x^3(191-25ax)\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}\sqrt{ax+1}}+\frac{26ax^2\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-ax}\sqrt{ax+1}}-\frac{2x\sqrt{1-ax}\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{ax+1}}$$

```
[Out] -11*a^(3/2)*(c-c/a/x)^(5/2)*x^(5/2)*arcsinh(a^(1/2)*x^(1/2))/(-a*x+1)^(5/2)
+1/3*a^2*(c-c/a/x)^(5/2)*x^3*(-25*a*x+191)/(-a*x+1)^(5/2)/(a*x+1)^(1/2)+26/
3*a*(c-c/a/x)^(5/2)*x^2/(-a*x+1)^(1/2)/(a*x+1)^(1/2)-2/3*(c-c/a/x)^(5/2)*x*
(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6129, 98, 150, 143, 54, 215}

$$\frac{a^2x^3(191-25ax)\left(c-\frac{c}{ax}\right)^{5/2}}{3(1-ax)^{5/2}\sqrt{ax+1}}-\frac{11a^{3/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{5/2}}+\frac{26ax^2\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{1-ax}\sqrt{ax+1}}-\frac{2x\sqrt{1-ax}\left(c-\frac{c}{ax}\right)^{5/2}}{3\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (a^2*(c - c/(a*x))^(5/2)*x^3*(191 - 25*a*x))/(3*(1 - a*x)^(5/2)*Sqrt[1 + a*
x]) + (26*a*(c - c/(a*x))^(5/2)*x^2)/(3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*(
c - c/(a*x))^(5/2)*x*Sqrt[1 - a*x])/(3*Sqrt[1 + a*x]) - (11*a^(3/2)*(c - c/
(a*x))^(5/2)*x^(5/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(5/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{-3\tanh^{-1}(ax)}(1-ax)^{5/2}}{x^{5/2}}\,dx}{(1-ax)^{5/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{(1-ax)^4}{x^{5/2}(1+ax)^{3/2}}\,dx}{(1-ax)^{5/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{(1-ax)^2\left(\frac{13a}{2}-\frac{a^2x}{2}\right)}{x^{3/2}(1+ax)^{3/2}}\,dx}{3(1-ax)^{5/2}}$$

$$= \frac{26a\left(c-\frac{c}{ax}\right)^{5/2}x^2}{3\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}} - \frac{\left(4\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{(1-ax)\left(-\frac{79a^2}{4}-\cdots\right)}{\sqrt{x}\,(1+ax)^{3/\cdots}}}{3(1-ax)^{5/2}}$$

$$= \frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3(191-25ax)}{3(1-ax)^{5/2}\sqrt{1+ax}} + \frac{26a\left(c-\frac{c}{ax}\right)^{5/2}x^2}{3\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}} - \cdots$$

$$= \frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3(191-25ax)}{3(1-ax)^{5/2}\sqrt{1+ax}} + \frac{26a\left(c-\frac{c}{ax}\right)^{5/2}x^2}{3\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}} - \cdots$$

$$= \frac{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^3(191-25ax)}{3(1-ax)^{5/2}\sqrt{1+ax}} + \frac{26a\left(c-\frac{c}{ax}\right)^{5/2}x^2}{3\sqrt{1-ax}\sqrt{1+ax}} - \frac{2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1-ax}}{3\sqrt{1+ax}} - \cdots$$

Mathematica [A] time = 0.08, size = 97, normalized size = 0.54

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-33a^{3/2}x^{3/2}\sqrt{ax+1}\,\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+3a^3x^3+133a^2x^2+32ax-2\right)}{3a^2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 32*a*x + 133*a^2*x^2 + 3*a^3*x^3 - 33*a^(3/2)*
x^(3/2)*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]]))/(3*a^2*x*Sqrt[1 - a^2*x^2]
)
```

fricas [A] time = 0.55, size = 352, normalized size = 1.94

$$\left[\frac{33\left(a^3c^2x^3-ac^2x\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(3\,a^3c^2x^3+133\,a^2c^2x^2+32\,ac^2x\right.}{12\left(a^4x^3-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/12*(33*(a^3*c^2*x^3 - a*c^2*x)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*
(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) - 4*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2*x - 2*c^2)*sqrt(
-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3 - a^2*x), 1/6*(33*(a^3*c^2*
x^3 - a*c^2*x)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2
 + 32*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3
 - a^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.07, size = 191, normalized size = 1.06

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(6a^{\frac{7}{2}}x^3\sqrt{-(ax+1)\,x}+33\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)x^3a^3+266a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x}+33\arctan\left(\frac{}{2\sqrt{a}}\right.\right.}{6x\,a^{\frac{5}{2}}\,(ax+1)\,\sqrt{-(ax+1)\,x}\,\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(5/2)*(6*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)+33*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^3*a^3+266*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)+33*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2*a^2+64*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-4*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^2*x^2+1)^(1/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} x^{2} + 1\right)^{\frac{3}{2}} \left(c - \frac{c}{a x}\right)^{\frac{5}{2}}}{\left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(5/2)/(a*x + 1)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - c/(a*x))^(5/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)

[Out] int(((c - c/(a*x))^(5/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- c \left(-1 + \frac{1}{a x}\right)\right)^{\frac{5}{2}} \left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}{\left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)**(5/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)

[Out] Integral((-c*(-1 + 1/(a*x)))**(5/2)*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)

3.557 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=133

$$\frac{9\sqrt{a}\,x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{ax^2(23-ax)\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}\sqrt{ax+1}}-\frac{2x\sqrt{1-ax}\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{ax+1}}$$

```
[Out] 9*(c-c/a/x)^(3/2)*x^(3/2)*arcsinh(a^(1/2)*x^(1/2))*a^(1/2)/(-a*x+1)^(3/2)-a
*(c-c/a/x)^(3/2)*x^2*(-a*x+23)/(-a*x+1)^(3/2)/(a*x+1)^(1/2)-2*(c-c/a/x)^(3/
2)*x*(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6134, 6129, 98, 143, 54, 215}

$$-\frac{ax^2(23-ax)\left(c-\frac{c}{ax}\right)^{3/2}}{(1-ax)^{3/2}\sqrt{ax+1}}+\frac{9\sqrt{a}\,x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{(1-ax)^{3/2}}-\frac{2x\sqrt{1-ax}\left(c-\frac{c}{ax}\right)^{3/2}}{\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*Sqrt[1 - a*x])/Sqrt[1 + a*x] - (a*(c - c/(a*x))^(
3/2)*x^2*(23 - a*x))/((1 - a*x)^(3/2)*Sqrt[1 + a*x]) + (9*Sqrt[a]*(c - c/(a
*x))^(3/2)*x^(3/2)*ArcSinh[Sqrt[a]*Sqrt[x]])/(1 - a*x)^(3/2)
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{-3\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}dx}{(1-ax)^{3/2}}$$

$$= \frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{(1-ax)^3}{x^{3/2}(1+ax)^{3/2}}dx}{(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\left(2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{(1-ax)\left(\frac{7a}{2}+\frac{a^2x}{2}\right)}{\sqrt{x}(1+ax)^{3/2}}dx}{(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2(23-ax)}{(1-ax)^{3/2}\sqrt{1+ax}} + \frac{\left(9a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{}{\sqrt{x}}}{2(1-ax)^{3/2}}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2(23-ax)}{(1-ax)^{3/2}\sqrt{1+ax}} + \frac{\left(9a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\text{Subs}}{(1-a}$$

$$= -\frac{2\left(c-\frac{c}{ax}\right)^{3/2}x\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{a\left(c-\frac{c}{ax}\right)^{3/2}x^2(23-ax)}{(1-ax)^{3/2}\sqrt{1+ax}} + \frac{9\sqrt{a}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\sinh}{(1-ax)^{3/2}}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.60

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(a^2x^2+19ax-9\sqrt{a}\sqrt{x}\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+2\right)}{a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(3*ArcTanh[a*x]),x]
[Out] (c*Sqrt[c - c/(a*x)]*(2 + 19*a*x + a^2*x^2 - 9*Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x
]*ArcSinh[Sqrt[a]*Sqrt[x]]))/(a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.56, size = 298, normalized size = 2.24

$$\left[\frac{9\left(a^2cx^2-c\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(a^2cx^2+19acx+2c\right)\sqrt{-a^2x^2+1}\sqrt{\frac{ac}{a}}}{4\left(a^3x^2-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(9*(a^2*c*x^2 - c)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2
 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 - 4*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)
))/(a^3*x^2 - a), 1/2*(9*(a^2*c*x^2 - c)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*c
*x^2 + 19*a*c*x + 2*c)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2
 - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 164, normalized size = 1.23

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c \left(2a^{\frac{5}{2}}x^2\sqrt{-(ax+1)x} + 9\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)x^2a^2 + 38a^{\frac{3}{2}}x\sqrt{-(ax+1)x} + 9\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)}}\cdots\right.\right.}{2a^{\frac{3}{2}}(ax+1)\sqrt{-(ax+1)x}\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(2*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)+9*ar
ctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2*a^2+38*a^(3/2)*x*(-(a*x+
1)*x)^(1/2)+9*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+4*a^(1/2
)*(-(a*x+1)*x)^(1/2))*(-a^2*x^2+1)^(1/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))^(3/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\left(1-a^2\,x^2\right)^{3/2}}{\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(3/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a*x))^(3/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)
**3, x)
```

3.558 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=123

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{8x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] -7*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(1/2)/(-a*x+1)^(1/2)+
8*x*(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)+x*(c-c/a/x)^(1/2)*(a*x+1)^
(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6134, 6129, 89, 80, 54, 215}

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{8x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*Sqrt[c - c/(a*x)]*x)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a*x)]*
x*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (7*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[
a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{\sqrt{x}(1+ax)^{3/2}}dx}{\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\frac{3a^2}{2}-\frac{a^3x}{2}}{\sqrt{x}\sqrt{1+ax}}dx}{a^2\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}dx}{2\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}dx, x, \sqrt{\right.}{\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{7\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}(ax+9)-7\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{a}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(9 + a*x) - 7*Sqrt[1 + a*x]*Arc
Sinh[Sqrt[a]*Sqrt[x]]))/(Sqrt[a]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.51, size = 282, normalized size = 2.29

$$\left[\frac{7\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(a^2x^2+9ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{4\left(a^3x^2-a\right)}, \frac{7\left(a^2x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(7*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 +
 a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) -
 4*(a^2*x^2 + 9*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 -
 a), 1/2*(7*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*x^2 + 9*a*x)*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 140, normalized size = 1.14

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\; x \left(2a^{\frac{3}{2}} x\sqrt{-(ax+1)\,x} + 7\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right) xa + 18\sqrt{a}\;\sqrt{-(ax+1)\,x} + 7\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right.\right.}{2\sqrt{a}\;(ax+1)\sqrt{-(ax+1)\,x}\;(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+7*arctan(1/2/a
^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+18*a^(1/2)*(-(a*x+1)*x)^(1/2)+7*ar
ctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/2)/a^(1/2)/
(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3
, x)
```

3.559 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=127

$$\frac{5\sqrt{1-ax}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}}-\frac{x(1-ax)}{\sqrt{1-a^2x^2}\,\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{1-ax}}{a\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 5*arcsinh(a^(1/2)*x^(1/2))*(-a*x+1)^(1/2)/a^(3/2)/(c-c/a/x)^(1/2)/x^(1/2)-5
*(-a*x+1)^(1/2)/a/(c-c/a/x)^(1/2)/(a*x+1)^(1/2)-x*(-a*x+1)/(c-c/a/x)^(1/2)/
(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6128, 881, 848, 47, 54, 215}

$$-\frac{x(1-ax)}{\sqrt{1-a^2x^2}\,\sqrt{c-\frac{c}{ax}}}+\frac{5\sqrt{1-ax}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{1-ax}}{a\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (-5*Sqrt[1 - a*x])/(a*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]) - (x*(1 - a*x))/(Sqr
t[c - c/(a*x)]*Sqrt[1 - a^2*x^2]) + (5*Sqrt[1 - a*x]*ArcSinh[Sqrt[a]*Sqrt[x
]])/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-ax}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{\sqrt{1-ax}\int \frac{\sqrt{x}(1-ax)^{5/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{\left(5\sqrt{1-ax}\right)\int \frac{\sqrt{x}(1-ax)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{2\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{\left(5\sqrt{1-ax}\right)\int \frac{\sqrt{x}}{(1+ax)^{3/2}}\,dx}{2\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{5\sqrt{1-ax}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}} - \frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{\left(5\sqrt{1-ax}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{5\sqrt{1-ax}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}} - \frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{\left(5\sqrt{1-ax}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= -\frac{5\sqrt{1-ax}}{a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}} - \frac{x(1-ax)}{\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}} + \frac{5\sqrt{1-ax}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.68

$$\frac{\sqrt{1-ax}\left(5\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-\sqrt{a}\sqrt{x}(ax+5)\right)}{a^{3/2}\sqrt{x}\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - a*x]*(-(Sqrt[a]*Sqrt[x]*(5 + a*x)) + 5*Sqrt[1 + a*x]*ArcSinh[Sqrt
[a]*Sqrt[x]]))/(a^(3/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 + a*x])
```

fricas [A] time = 0.53, size = 286, normalized size = 2.25

$$\left[-\frac{5\left(a^{2}x^{2}-1\right)\sqrt{-c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\left(2\,a^{2}x^{2}+ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^{2}x^{2}+5\,ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^{3}cx^{2}-ac\right)},\ \frac{5\left(a^{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/4*(5*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2
+ a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
+ 4*(a^2*x^2 + 5*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c*x^
2 - a*c), 1/2*(5*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c
)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*x^2 + 5*a*x
)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c*x^2 - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{(ax+1)^{3}\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*sqrt(c - c/(a*x))), x)
```

maple [A] time = 0.06, size = 143, normalized size = 1.13

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}+5\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)xa+10\sqrt{a}\,\sqrt{-(ax+1)\,x}+5\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\right.}{2\sqrt{a}\,c\,(ax+1)\sqrt{-(ax+1)\,x}\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c*(2*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+5*ar
ctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+10*a^(1/2)*(-(a*x+1)*x)^
```

```
(1/2)+5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/2
)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*sqrt(c - c/(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^(1/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^(1/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**
3), x)
```

3.560 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{3(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\sqrt{ax+1}\,(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2(1-ax)^{3/2}}{a\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -3*(-a*x+1)^(3/2)*arcsinh(a^(1/2)*x^(1/2))/a^(5/2)/(c-c/a/x)^(3/2)/x^(3/2)-
2*(-a*x+1)^(3/2)/a/(c-c/a/x)^(3/2)/(a*x+1)^(1/2)+3*(-a*x+1)^(3/2)*(a*x+1)^(
1/2)/a^2/(c-c/a/x)^(3/2)/x
```

Rubi [A] time = 0.19, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6134, 6128, 848, 47, 50, 54, 215}

$$-\frac{3(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}x^{3/2}\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\sqrt{ax+1}\,(1-ax)^{3/2}}{a^2x\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2(1-ax)^{3/2}}{a\sqrt{ax+1}\,\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (-2*(1 - a*x)^(3/2))/(a*(c - c/(a*x))^(3/2)*Sqrt[1 + a*x]) + (3*(1 - a*x)^(
3/2)*Sqrt[1 + a*x])/(a^2*(c - c/(a*x))^(3/2)*x) - (3*(1 - a*x)^(3/2)*ArcSin
h[Sqrt[a]*Sqrt[x]])/(a^(5/2)*(c - c/(a*x))^(3/2)*x^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{(1-ax)^{3/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}(1-ax)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= \frac{(1-ax)^{3/2}\int \frac{x^{3/2}}{(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{2(1-ax)^{3/2}}{a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1+ax}} + \frac{\left(3(1-ax)^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{a\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{2(1-ax)^{3/2}}{a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1+ax}} + \frac{3(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{\left(3(1-ax)^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{2(1-ax)^{3/2}}{a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1+ax}} + \frac{3(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{\left(3(1-ax)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{2(1-ax)^{3/2}}{a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1+ax}} + \frac{3(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{3(1-ax)^{3/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Mathematica [C] time = 0.04, size = 44, normalized size = 0.34

$$\frac{2x(1-ax)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-ax\right)}{5\left(c-\frac{c}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (2*x*(1 - a*x)^(3/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -(a*x)])/(5*(c - c/(a
*x))^(3/2))
```

fricas [A] time = 0.45, size = 294, normalized size = 2.24

$$\left[-\frac{3\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+3\,ax\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^3c^2x^2-ac^2\right)},\ \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fric
as")
```

```
[Out] [-1/4*(3*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2
+ a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
+ 4*(a^2*x^2 + 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*
x^2 - a*c^2), 1/2*(3*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sq
rt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*x^2 + 3
*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - a*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 143, normalized size = 1.09

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(2a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\;+3\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right)xa+6\sqrt{a}\;\sqrt{-(ax+1)\,x}\;+3\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right)\ldots\right.}{2\sqrt{a}\;c^2\,(ax+1)\,\sqrt{-(ax+1)\,x}\;\;(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c^2*(2*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+3*
arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+6*a^(1/2)*(-(a*x+1)*x)
^(1/2)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/
2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^(3/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^(3/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1
)**3), x)
```

3.561 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{2}\,a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{2\sqrt{ax+1}\,(1-ax)^{5/2}}{a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}}{a^2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] (-a*x+1)^(5/2)*arcsinh(a^(1/2)*x^(1/2))/a^(7/2)/(c-c/a/x)^(5/2)/x^(5/2)+1/2
*(-a*x+1)^(5/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))/a^(7/2)/(c-c
/a/x)^(5/2)/x^(5/2)*2^(1/2)+(-a*x+1)^(5/2)/a^2/(c-c/a/x)^(5/2)/x/(a*x+1)^(1
/2)-2*(-a*x+1)^(5/2)*(a*x+1)^(1/2)/a^3/(c-c/a/x)^(5/2)/x^2
```

Rubi [A] time = 0.21, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6134, 6129, 98, 154, 157, 54, 215, 93, 206}

$$-\frac{2\sqrt{ax+1}\,(1-ax)^{5/2}}{a^3x^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{2}\,a^{7/2}x^{5/2}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{(1-ax)^{5/2}}{a^2x\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (1 - a*x)^(5/2)/(a^2*(c - c/(a*x))^(5/2)*x*Sqrt[1 + a*x]) - (2*(1 - a*x)^(5
/2)*Sqrt[1 + a*x])/(a^3*(c - c/(a*x))^(5/2)*x^2) + ((1 - a*x)^(5/2)*ArcSinh
[Sqrt[a]*Sqrt[x]])/(a^(7/2)*(c - c/(a*x))^(5/2)*x^(5/2)) + ((1 - a*x)^(5/2)
*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(Sqrt[2]*a^(7/2)*(c - c/
(a*x))^(5/2)*x^(5/2))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{(1-ax)^{5/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^{5/2}}{(1-ax)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}\int \frac{x^{5/2}}{(1-ax)(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{(1-ax)^{5/2}\int \frac{\sqrt{x}\left(\frac{3}{2}-2ax\right)}{(1-ax)\sqrt{1+ax}}\,dx}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\int \frac{a-\frac{a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a^4\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} + \frac{(1-ax)^{5/2}\int}{2a^3\left(c\right.}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\,\text{Subst}\left(\int \frac{1}{1-2ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{1+ax}}\right)}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

$$= \frac{(1-ax)^{5/2}}{a^2\left(c-\frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} + \frac{(1-ax)^{5/2}}{\sqrt{2}\,a^{7/2}}$$

Mathematica [C] time = 0.16, size = 162, normalized size = 0.81

$$\frac{\sqrt{1-ax}\left(5\left(2\sqrt{a}\sqrt{x}-4\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+\sqrt{2ax+2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)-4a^{5/2}x^{5/2}\sqrt{ax+1}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\right.\right.}{10a^{3/2}c^2\sqrt{x}\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (Sqrt[1 - a*x]*(5*(2*Sqrt[a]*Sqrt[x] - 4*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt
[x]] + Sqrt[2 + 2*a*x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]) -
4*a^(5/2)*x^(5/2)*Sqrt[1 + a*x]*Hypergeometric2F1[3/2, 5/2, 7/2, -(a*x)]))/
(10*a^(3/2)*c^2*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 + a*x])
```

fricas [A] time = 0.68, size = 492, normalized size = 2.47

$$\left[-\frac{\sqrt{2}\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8\,a^3c\cdots}{\cdots}\right.}{8\left(a^3c^3x^2-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fric
as")
```

```
[Out] [-1/8*(sqrt(2)*(a^2*x^2 - 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13
*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*
x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(a^2*x^2 - 1)*sqr
t(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*
sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(a^2*x^2 + 2*a*x)*sqrt
(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - a*c^3), 1/4*(sqrt(2)
*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt
((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + 2*(a^2*x^2 - 1)*sqrt(c)*
arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^
2 - a*c*x - c)) - 4*(a^2*x^2 + 2*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(
a*x)))/(a^3*c^3*x^2 - a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(5/2)), x)
```

maple [A] time = 0.06, size = 279, normalized size = 1.40

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{2}\left(2a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\; x+a^{2}\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\sqrt{2}\,\sqrt{-\frac{1}{a}}\; x+4\sqrt{-(ax+1)\,x}\; a^{\frac{3}{2}}\sqrt{2}\right.}{4a^{\frac{3}{2}}c^{3}(ax+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^3*2^(1/2)*(2*a^(5/2)*2^(1/2)*(-1/a)^
(1/2)*(-(a*x+1)*x)^(1/2)*x+a^2*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1
/2))*2^(1/2)*(-1/a)^(1/2)*x+4*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/
2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2)+
a^(3/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*x
+ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1/2))
*(-a^2*x^2+1)^(1/2)/(a*x+1)/(-1/a)^(1/2)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{(ax+1)^{3}\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^(5/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^(5/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1
)**3), x)
```

3.562 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=251

$$\frac{(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11(1-ax)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4\sqrt{2}\,a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{7\sqrt{ax+1}\,(1-ax)^{7/2}}{4a^4x^3\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{(1-ax)^{7/2}}{4a^3x^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}+$$

```
[Out] (-a*x+1)^(7/2)*arcsinh(a^(1/2)*x^(1/2))/a^(9/2)/(c-c/a/x)^(7/2)/x^(7/2)-11/
8*(-a*x+1)^(7/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))/a^(9/2)/(c-
c/a/x)^(7/2)/x^(7/2)*2^(1/2)+1/2*(-a*x+1)^(5/2)/a^2/(c-c/a/x)^(7/2)/x/(a*x+
1)^(1/2)-1/4*(-a*x+1)^(7/2)/a^3/(c-c/a/x)^(7/2)/x^2/(a*x+1)^(1/2)+7/4*(-a*x
+1)^(7/2)*(a*x+1)^(1/2)/a^4/(c-c/a/x)^(7/2)/x^3
```

Rubi [A] time = 0.22, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6134, 6129, 98, 150, 154, 157, 54, 215, 93, 206}

$$\frac{7\sqrt{ax+1}\,(1-ax)^{7/2}}{4a^4x^3\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{(1-ax)^{7/2}}{4a^3x^2\sqrt{ax+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{(1-ax)^{7/2}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11(1-ax)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{4\sqrt{2}\,a^{9/2}x^{7/2}\left(c-\frac{c}{ax}\right)^{7/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (1 - a*x)^(5/2)/(2*a^2*(c - c/(a*x))^(7/2)*x*Sqrt[1 + a*x]) - (1 - a*x)^(7/
2)/(4*a^3*(c - c/(a*x))^(7/2)*x^2*Sqrt[1 + a*x]) + (7*(1 - a*x)^(7/2)*Sqrt[
1 + a*x])/(4*a^4*(c - c/(a*x))^(7/2)*x^3) + ((1 - a*x)^(7/2)*ArcSinh[Sqrt[a
]*Sqrt[x]])/(a^(9/2)*(c - c/(a*x))^(7/2)*x^(7/2)) - (11*(1 - a*x)^(7/2)*Arc
Tanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(4*Sqrt[2]*a^(9/2)*(c - c/(a
*x))^(7/2)*x^(7/2))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{(1-ax)^{7/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^{7/2}}{(1-ax)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{7/2}\int \frac{x^{7/2}}{(1-ax)^2(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}\int \frac{x^{3/2}\left(\frac{5}{2}+3ax\right)}{(1-ax)(1+ax)^{3/2}}\,dx}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} - \frac{(1-ax)^{7/2}\int \frac{\sqrt{x}\left(-\frac{3a}{4}+\frac{7a^2x}{2}\right)}{(1-ax)\sqrt{1+ax}}\,dx}{2a^4\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} + \frac{7(1-ax)^{7/2}\sqrt{1+ax}}{4a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\int}{2a^6\left(c\right.}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} + \frac{7(1-ax)^{7/2}\sqrt{1+ax}}{4a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\int}{2a^4\left(c-\right.}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} + \frac{7(1-ax)^{7/2}\sqrt{1+ax}}{4a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\,\mathrm{S}}{a}$$

$$= \frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} + \frac{7(1-ax)^{7/2}\sqrt{1+ax}}{4a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\,\mathrm{s}}{a^{9/2}\left(c-\right.}$$

Mathematica [C] time = 0.65, size = 234, normalized size = 0.93

$$\frac{-40a^{7/2}x^{7/2}(ax-1)\sqrt{ax+1}\;{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-ax\right) - 56a^{5/2}x^{5/2}(ax-1)\sqrt{ax+1}\;{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-ax\right) + 35\left(\sqrt{a}\sqrt{x}\left(2a^3x^3\right.\right.}{560a^{3/2}c^3\sqrt{x}\sqrt{1-a^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (35*(Sqrt[a]*Sqrt[x]*(-25 + 2*a*x + 13*a^2*x^2 + 2*a^3*x^3) + 19*(-1 + a*x)
*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]] - 22*(-1 + a*x)*Sqrt[2 + 2*a*x]*Arc
Tanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]) - 56*a^(5/2)*x^(5/2)*(-1 + a
*x)*Sqrt[1 + a*x]*Hypergeometric2F1[3/2, 5/2, 7/2, -(a*x)] - 40*a^(7/2)*x^(
7/2)*(-1 + a*x)*Sqrt[1 + a*x]*Hypergeometric2F1[3/2, 7/2, 9/2, -(a*x)])/(56
0*a^(3/2)*c^3*Sqrt[c - c/(a*x)]*Sqrt[x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.59, size = 596, normalized size = 2.37

$$\left[-\frac{11\sqrt{2}\left(a^3x^3-a^2x^2-ax+1\right)\sqrt{-c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+8\left(a^3x^3-a^2 \cdots\right.}{32\left(a^4c^4x^3-a^3 \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(7/2),x, algorithm="fric
as")
```

```
[Out] [-1/32*(11*sqrt(2)*(a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(-c)*log(-(17*a^3*c*x^
3 - 3*a^2*c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)
*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) +
 8*(a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 8*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x
- c)/(a*x)))/(a^4*c^4*x^3 - a^3*c^4*x^2 - a^2*c^4*x + a*c^4), 1/16*(11*sqrt
(2)*(a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 +
1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 8*(a^
3*x^3 - a^2*x^2 - a*x + 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*
sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 4*(4*a^3*x^3 + a^2*x^2
 - 7*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^4*x^3 - a^3*c^
4*x^2 - a^2*c^4*x + a*c^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(7/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 315, normalized size = 1.25

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{2}\,\left(8a^{\frac{7}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x^2+2a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x-4a^3\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(7/2),x)
```

```
[Out] -1/16*(c*(a*x-1)/a/x)^(1/2)*x*2^(1/2)*(8*a^(7/2)*2^(1/2)*(-1/a)^(1/2)*(-(a*
x+1)*x)^(1/2)*x^2+2*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*x-4*a^3
*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x^2+
11*a^(5/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1)
)*x^2-14*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)+4*arctan(1/2/a^(1/
2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2)-11*ln((2*2^(1/2)*(-
1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1/2))*(-a^2*x^2+1)^(1/
2)/a^(3/2)/c^4/(a*x+1)/(-1/a)^(1/2)/(-(a*x+1)*x)^(1/2)/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a/x)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a*x))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{7/2}\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^(7/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - c/(a*x))^(7/2)*(a*x + 1)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a/x)**(7/2),x)
[Out] Timed out
```

3.563 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)x^3}\,dx$

Optimal. Leaf size=105

$$-\frac{4a^2(4ax+3)}{3c\sqrt{1-a^2x^2}}-\frac{8a^2(ax+1)}{3c\left(1-a^2x^2\right)^{3/2}}+\frac{a\sqrt{1-a^2x^2}}{cx}+\frac{4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] -8/3*a^2*(a*x+1)/c/(-a^2*x^2+1)^(3/2)+4*a^2*arctanh((-a^2*x^2+1)^(1/2))/c-4
/3*a^2*(4*a*x+3)/c/(-a^2*x^2+1)^(1/2)+a*(-a^2*x^2+1)^(1/2)/c/x
```

Rubi [A] time = 0.32, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6131, 6128, 852, 1805, 807, 266, 63, 208}

$$-\frac{4a^2(4ax+3)}{3c\sqrt{1-a^2x^2}}-\frac{8a^2(ax+1)}{3c\left(1-a^2x^2\right)^{3/2}}+\frac{a\sqrt{1-a^2x^2}}{cx}+\frac{4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/((c - c/(a*x))*x^3),x]
```

```
[Out] (-8*a^2*(1 + a*x))/(3*c*(1 - a^2*x^2)^(3/2)) - (4*a^2*(3 + 4*a*x))/(3*c*Sqr
t[1 - a^2*x^2]) + (a*Sqrt[1 - a^2*x^2])/(c*x) + (4*a^2*ArcTanh[Sqrt[1 - a^2
*x^2]])/c
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)x^3}\,dx = -\frac{a\int \frac{e^{3\tanh^{-1}(ax)}}{x^2(1-ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^2(1-ax)^4}\,dx}{c}$$

$$= -\frac{a\int \frac{(1+ax)^4}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} + \frac{a\int \frac{-3-12ax-13a^2x^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} - \frac{a\int \frac{3+12ax}{x^2\sqrt{1-a^2x^2}}\,dx}{3c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} + \frac{a\sqrt{1-a^2x^2}}{cx} - \frac{\left(4a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} + \frac{a\sqrt{1-a^2x^2}}{cx} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} + \frac{a\sqrt{1-a^2x^2}}{cx} + \frac{4\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{c}$$

$$= -\frac{8a^2(1+ax)}{3c\left(1-a^2x^2\right)^{3/2}} - \frac{4a^2(3+4ax)}{3c\sqrt{1-a^2x^2}} + \frac{a\sqrt{1-a^2x^2}}{cx} + \frac{4a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Mathematica [**A**] time = 0.04, size = 92, normalized size = 0.88

$$\frac{a\left(-19a^3x^3+7a^2x^2+12ax(ax-1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+23ax-3\right)}{3cx(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/((c - c/(a*x))*x^3),x]
```

```
[Out] (a*(-3 + 23*a*x + 7*a^2*x^2 - 19*a^3*x^3 + 12*a*x*(-1 + a*x)*Sqrt[1 - a^2*x
^2]*ArcTanh[Sqrt[1 - a^2*x^2]]))/(3*c*x*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.43, size = 120, normalized size = 1.14

$$-\frac{20\,a^4x^3-40\,a^3x^2+20\,a^2x+12\left(a^4x^3-2\,a^3x^2+a^2x\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(19\,a^3x^2-26\,a^2x+3\,a\right)\sqrt{-a^2x^2+1}}{3\left(a^2cx^3-2\,acx^2+cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/3*(20*a^4*x^3 - 40*a^3*x^2 + 20*a^2*x + 12*(a^4*x^3 - 2*a^3*x^2 + a^2*x)
*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (19*a^3*x^2 - 26*a^2*x + 3*a)*sqrt(-a^2*
x^2 + 1))/(a^2*c*x^3 - 2*a*c*x^2 + c*x)
```

giac [B] time = 0.22, size = 217, normalized size = 2.07

$$\frac{4\,a^3\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c|a|}+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{2\,cx|a|}+\frac{\left(3\,a^3-\frac{89\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x}+\frac{153\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{ax^2}-\frac{99\left(\sqrt{-a^2x^2\right.}}{a^3x}\right.}{6\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)/x^3,x, algorithm="giac")
```

```
[Out] 4*a^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(
a)) + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/(c*x*abs(a)) + 1/6*(3*a^3 - 89*
(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x + 153*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2
/(a*x^2) - 99*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^3*x^3))*a^2*x/((sqrt(-a^
2*x^2 + 1)*abs(a) + a)*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*ab
s(a))
```

maple [A] time = 0.04, size = 165, normalized size = 1.57

$$\frac{a\left(-\frac{a^2x}{\sqrt{-a^2x^2+1}}-4a\left(\frac{1}{\sqrt{-a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)+\frac{1}{x\sqrt{-a^2x^2+1}}+8a\left(\frac{1}{3a\left(x-\frac{1}{a}\right)\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}+\frac{-2a^2\left(x-\right.}{3a\sqrt{-a^2\left(x-\frac{1}{a}\right.}}\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)/x^3,x)
```

```
[Out] a/c*(-a^2*x/(-a^2*x^2+1)^(1/2)-4*a*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^
2+1)^(1/2)))+1/x/(-a^2*x^2+1)^(1/2)+8*a*(1/3/a/(x-1/a)/(-a^2*(x-1/a)^2-2*a*
(x-1/a))^(1/2)+1/3/a*(-2*a^2*(x-1/a)-2*a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{ax}\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a/x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a*x))*x^3), x)
```

mupad [B] time = 0.87, size = 138, normalized size = 1.31

$$\frac{a\,\sqrt{1-a^2\,x^2}}{c\,x}-\frac{4\,a^4\,\sqrt{1-a^2\,x^2}}{3\,\left(c\,a^4\,x^2-2\,c\,a^3\,x+c\,a^2\right)}+\frac{16\,a^3\,\sqrt{1-a^2\,x^2}}{3\,\left(\frac{c\,\sqrt{-a^2}}{a}-c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1{}\mathrm{i}\right)\,4{}\mathrm{i}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/(x^3*(c - c/(a*x))*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (a*(1 - a^2*x^2)^(1/2))/(c*x) - (4*a^4*(1 - a^2*x^2)^(1/2))/(3*(a^2*c + a^4
*c*x^2 - 2*a^3*c*x)) - (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*4i)/c + (16*a^3*(1
 - a^2*x^2)^(1/2))/(3*((c*(-a^2)^(1/2))/a - c*x*(-a^2)^(1/2))*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int \frac{3ax}{-a^3x^5\sqrt{-a^2x^2+1}+a^2x^4\sqrt{-a^2x^2+1}+ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^2}{-a^3x^5\sqrt{-a^2x^2+1}+a^2x^4\sqrt{-a^2x^2+1}+ax^3\sqrt{-a^2x^2+1}-x^2\sqrt{-a^2x^2+1}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a/x)/x**3,x)
```

```
[Out] a*(Integral(3*a*x/(-a**3*x**5*sqrt(-a**2*x**2 + 1) + a**2*x**4*sqrt(-a**2*x
**2 + 1) + a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x) + I
ntegral(3*a**2*x**2/(-a**3*x**5*sqrt(-a**2*x**2 + 1) + a**2*x**4*sqrt(-a**2
*x**2 + 1) + a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x) +
```

```
Integral(a**3*x**3/(-a**3*x**5*sqrt(-a**2*x**2 + 1) + a**2*x**4*sqrt(-a**2
*x**2 + 1) + a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x) +
 Integral(1/(-a**3*x**5*sqrt(-a**2*x**2 + 1) + a**2*x**4*sqrt(-a**2*x**2 +
1) + a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x))/c
```

3.564 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx$

Optimal. Leaf size=57

$$\frac{2x^{m+1}\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(-\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};-ax\right)}{(2m+1)\sqrt{1-ax}}$$

```
[Out] 2*x^(1+m)*hypergeom([-1/2, 1/2+m],[3/2+m],-a*x)*(c-c/a/x)^(1/2)/(1+2*m)/(-a
*x+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6134, 6128, 848, 64}

$$\frac{2x^{m+1}\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(-\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};-ax\right)}{(2m+1)\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, 1/2 + m, 3/2 + m, -(
a*x)])/((1 + 2*m)*Sqrt[1 - a*x])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
```

```
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx=\frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int e^{\tanh^{-1}(ax)}x^{-\frac{1}{2}+m}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int\frac{x^{-\frac{1}{2}+m}\sqrt{1-a^2x^2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int x^{-\frac{1}{2}+m}\sqrt{1+ax}\,dx}{\sqrt{1-ax}}$$
$$=\frac{2\sqrt{c-\frac{c}{ax}}\,x^{1+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;-ax\right)}{(1+2m)\sqrt{1-ax}}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.98

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(-\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};-ax\right)}{\left(m+\frac{1}{2}\right)\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, 1/2 + m, 3/2 + m, -(a*
x)])/((1/2 + m)*Sqrt[1 - a*x])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^m\sqrt{\frac{acx-c}{ax}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^m*sqrt((a*c*x - c)/(a*x))/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}\,x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^m/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^m\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}\,x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^m/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^m*(c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)),
 x)
```

3.565 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=179

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}+\frac{x^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-ax}}$$

```
[Out] 1/8*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(5/2)/(-a*x+1)^(1/2)
-1/8*x*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/a^2/(-a*x+1)^(1/2)+1/12*x^2*(c-c/a/x)^
(1/2)*(a*x+1)^(1/2)/a/(-a*x+1)^(1/2)+1/3*x^3*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/
(-a*x+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6134, 6128, 848, 50, 54, 215}

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}+\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{x^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) + (Sqrt[c - c/(a
*x)]*x^2*Sqrt[1 + a*x])/(12*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^3*Sqrt[
1 + a*x])/(3*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sq
rt[x]])/(8*a^(5/2)*Sqrt[1 - a*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int e^{\tanh^{-1}(ax)}x^{3/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{1-a^2x^2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int x^{3/2}\sqrt{1+ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{x^{3/2}}{\sqrt{1+ax}}\,dx}{6\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1+ax}}{3\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{8a\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\cdots}{16a^2\sqrt{\cdots}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\cdots}{\cdots}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\,\mathrm{si}\cdots}{8a^{5/2}\sqrt{\cdots}}$$

Mathematica [**A**] time = 0.06, size = 88, normalized size = 0.49

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\,\sqrt{x}\,\sqrt{ax+1}\left(8a^2x^2+2ax-3\right)+3\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)\right)}{24a^{5/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(-3 + 2*a*x + 8*a
^2*x^2) + 3*ArcSinh[Sqrt[a]*Sqrt[x]]))/(24*a^(5/2)*Sqrt[1 - a*x])
```

fricas [A] time = 0.64, size = 292, normalized size = 1.63

$$\left[\frac{3\,(ax-1)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(8\,a^3x^3+2\,a^2x^2-3\,ax\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{acx-}{ax}}}{96\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/96*(3*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*
x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) - 4*
(8*a^3*x^3 + 2*a^2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))
/(a^4*x - a^3), -1/48*(3*(a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sq
rt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^3*x^3 +
 2*a^2*x^2 - 3*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^
3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.04, size = 125, normalized size = 0.70

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\sqrt{-a^2x^2+1}\;\left(16a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x}\;+4a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\;-6\sqrt{a}\;\sqrt{-(ax+1)\,x}\;-3\arctan\left(\frac{2ax+}{2\sqrt{a}\;\sqrt{-(a}}\right.\right.}{48a^{\frac{5}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/48*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(16*a^(5/2)*x^2*(-(a*x+1)*
x)^(1/2)+4*a^(3/2)*x*(-(a*x+1)*x)^(1/2)-6*a^(1/2)*(-(a*x+1)*x)^(1/2)-3*arct
```

```
an(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/a^(5/2)/(a*x-1)/(-(a*x+1)*x)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)),
 x)
```

3.566 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=135

$$-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-ax}}+\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

```
[Out] -1/4*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(3/2)/(-a*x+1)^(1/2
)+1/4*x*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/a/(-a*x+1)^(1/2)+1/2*x^2*(c-c/a/x)^(1
/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6134, 6128, 848, 50, 54, 215}

$$-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-ax}}+\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)
]*x^2*Sqrt[1 + a*x])/(2*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh
[Sqrt[a]*Sqrt[x]])/(4*a^(3/2)*Sqrt[1 - a*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{\tanh^{-1}(ax)}\sqrt{x}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \sqrt{x}\sqrt{1+ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{2\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{2\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{8a\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{2\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \cdot\right)}{4a\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.59

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}\,(2ax+1)-\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{4a^{3/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(1 + 2*a*x) - Arc
Sinh[Sqrt[a]*Sqrt[x]]))/(4*a^(3/2)*Sqrt[1 - a*x])
```

fricas [A] time = 0.51, size = 272, normalized size = 2.01

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{16\left(a^3x-a^2\right)}, \frac{(ax-1)\cdot}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/16*((a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)
*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) - 4*(2
*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2),
1/8*((a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2
+ 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.05, size = 105, normalized size = 0.78

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\sqrt{-a^2x^2+1}\,\left(4a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}+2\sqrt{a}\,\sqrt{-(ax+1)\,x}+\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\right)}{8a^{\frac{3}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/8*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)/a^(3/2)*(4*a^(3/2)*x*(-(a*x
+1)*x)^(1/2)+2*a^(1/2)*(-(a*x+1)*x)^(1/2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a
*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
[Out] int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(c-c/a/x)**(1/2),x)
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.567 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=85

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(1/2)/(-a*x+1)^(1/2)+x*(
c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6134, 6128, 848, 50, 54, 215}

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x] + (Sqrt[c - c/(a*x)]*Sqrt
[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.81

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]
*Sqrt[x]]))/(Sqrt[a]*Sqrt[1 - a*x])
```

fricas [A] time = 0.48, size = 249, normalized size = 2.93

$$\left[-\frac{4\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx-c}{ax}} - (ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4\left(a^2x-a\right)}, -\frac{2\sqrt{-a^2x^2+1}\,ax\sqrt{\vphantom{x}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")

[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - (a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + (a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)))/(a^2*x - a)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")

[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/sqrt(-a^2*x^2 + 1), x)

maple [A] time = 0.04, size = 91, normalized size = 1.07

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(2\sqrt{a}\,\sqrt{-(ax+1)\,x}-\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\right)}{2\,(ax-1)\,\sqrt{-(ax+1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x)

[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*a^(1/2)*(-(a*x+1)*x)^(1/2)-arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^(1/2)/a^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")

[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/sqrt(-a^2*x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.568 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=86

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{1-ax}}-\frac{2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}$$

```
[Out] 2*arcsinh(a^(1/2)*x^(1/2))*a^(1/2)*(c-c/a/x)^(1/2)*x^(1/2)/(-a*x+1)^(1/2)-2
*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6134, 6128, 848, 47, 54, 215}

$$\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{1-ax}}-\frac{2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] + (2*Sqrt[a]*Sqrt[c - c/
(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{3/2}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.71

$$-\frac{2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}-\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x,x]
[Out] (-2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x] - Sqrt[a]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt
[x]]))/Sqrt[1 - a*x]
```

fricas [A] time = 0.93, size = 236, normalized size = 2.74

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, -\frac{(ax-1)\sqrt{c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [1/2*((a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x - 1), -((a*x - 1)*sqrt(c)*arc
tan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.04, size = 90, normalized size = 1.05

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{-a^2x^2+1}\,\left(\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)xa+2\sqrt{a}\,\sqrt{-(ax+1)\,x}\right)}{(ax-1)\sqrt{-(ax+1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] (c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)*(arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a
*x+1)*x)^(1/2))*x*a+2*a^(1/2)*(-(a*x+1)*x)^(1/2))/(a*x-1)/(-(a*x+1)*x)^(1/2
)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))),
x)
```

3.569 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

```
[Out] -2/3*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/x/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6134, 6128, 848, 37}

$$-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{5/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{5/2}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{5/2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\,(1+ax)^{3/2}}{3x\sqrt{1-ax}}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.00

$$-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x])
```

fricas [A] time = 0.50, size = 47, normalized size = 1.15

$$\frac{2\sqrt{-a^2x^2+1}\,(ax+1)\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")

[Out] 2/3*sqrt(-a^2*x^2 + 1)*(a*x + 1)*sqrt((a*c*x - c)/(a*x))/(a*x^2 - x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^2), x)

maple [A] time = 0.03, size = 40, normalized size = 0.98

$$-\frac{2\,(ax+1)^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^2,x)

[Out] -2/3*(a*x+1)^2/x/(-a^2*x^2+1)^(1/2)*(c*(a*x-1)/a/x)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^2), x)

mupad [B] time = 1.12, size = 44, normalized size = 1.07

$$-\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{2\,a^2\,x^2}{3}+\frac{4\,a\,x}{3}+\frac{2}{3}\right)}{x\,\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] -((c - c/(a*x))^(1/2)*((4*a*x)/3 + (2*a^2*x^2)/3 + 2/3))/(x*(1 - a^2*x^2)^(
1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))
), x)
```

3.570 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{4a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}$$

```
[Out] -2/5*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/x^2/(-a*x+1)^(1/2)+4/15*a*(a*x+1)^(3/2)*
(c-c/a/x)^(1/2)/x/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6134, 6128, 848, 45, 37}

$$\frac{4a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(5*x^2*Sqrt[1 - a*x]) + (4*a*Sqrt[c
- c/(a*x)]*(1 + a*x)^(3/2))/(15*x*Sqrt[1 - a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{7/2}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\,(1+ax)^{3/2}}{5x^2\sqrt{1-ax}}-\frac{\left(2a\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{5/2}}\,dx}{5\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\,(1+ax)^{3/2}}{5x^2\sqrt{1-ax}}+\frac{4a\sqrt{c-\frac{c}{ax}}\,(1+ax)^{3/2}}{15x\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.56

$$\frac{2(ax+1)^{3/2}(2ax-3)\sqrt{c-\frac{c}{ax}}}{15x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2)*(-3 + 2*a*x))/(15*x^2*Sqrt[1 - a*x])
```

fricas [A] time = 0.76, size = 58, normalized size = 0.69

$$-\frac{2\left(2a^2x^2-ax-3\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] -2/15*(2*a^2*x^2 - a*x - 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))/(a*x
^3 - x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.03, size = 46, normalized size = 0.55

$$\frac{2\left(ax+1\right)^2\left(2ax-3\right)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^3,x)
```

[Out] 2/15*(a*x+1)^2*(2*a*x-3)*(c*(a*x-1)/a/x)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^3), x)

mupad [B] time = 1.05, size = 52, normalized size = 0.62

$$-\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(-\frac{4\,a^3\,x^3}{15}-\frac{2\,a^2\,x^2}{15}+\frac{8\,a\,x}{15}+\frac{2}{5}\right)}{x^2\,\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^3*(1 - a^2*x^2)^(1/2)),x)

[Out] -((c - c/(a*x))^(1/2)*((8*a*x)/15 - (2*a^2*x^2)/15 - (4*a^3*x^3)/15 + 2/5))/(x^2*(1 - a^2*x^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x**3,x)

[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)

3.571 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=128

$$-\frac{16a^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}}+\frac{8a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}}$$

```
[Out] -2/7*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/x^3/(-a*x+1)^(1/2)+8/35*a*(a*x+1)^(3/2)*
(c-c/a/x)^(1/2)/x^2/(-a*x+1)^(1/2)-16/105*a^2*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)
/x/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6134, 6128, 848, 45, 37}

$$-\frac{16a^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{8a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(7*x^3*Sqrt[1 - a*x]) + (8*a*Sqrt[c
- c/(a*x)]*(1 + a*x)^(3/2))/(35*x^2*Sqrt[1 - a*x]) - (16*a^2*Sqrt[c - c/(a*
x)]*(1 + a*x)^(3/2))/(105*x*Sqrt[1 - a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{9/2}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{7x^3\sqrt{1-ax}} - \frac{\left(4a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{7/2}}\,dx}{7\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{7x^3\sqrt{1-ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{35x^2\sqrt{1-ax}} + \frac{\left(8a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{5/2}}\,dx}{35\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{7x^3\sqrt{1-ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{35x^2\sqrt{1-ax}} - \frac{16a^2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{105x\sqrt{1-ax}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.43

$$-\frac{2(ax+1)^{3/2}\left(8a^2x^2-12ax+15\right)\sqrt{c-\frac{c}{ax}}}{105x^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2)*(15 - 12*a*x + 8*a^2*x^2))/(105*x^3*S
qrt[1 - a*x])
```

fricas [A] time = 1.12, size = 66, normalized size = 0.52

$$\frac{2\left(8\,a^3x^3-4\,a^2x^2+3\,ax+15\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] 2/105*(8*a^3*x^3 - 4*a^2*x^2 + 3*a*x + 15)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x -
 c)/(a*x))/(a*x^4 - x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^4), x)
```

maple [A] time = 0.03, size = 54, normalized size = 0.42

$$-\frac{2\,(ax+1)^2\left(8a^2x^2-12ax+15\right)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] -2/105*(a*x+1)^2*(8*a^2*x^2-12*a*x+15)*(c*(a*x-1)/a/x)^(1/2)/x^3/(-a^2*x^2+
1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^4), x)
```

mupad [B] time = 1.14, size = 60, normalized size = 0.47

$$-\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{16\,a^4\,x^4}{105}+\frac{8\,a^3\,x^3}{105}-\frac{2\,a^2\,x^2}{105}+\frac{12\,a\,x}{35}+\frac{2}{7}\right)}{x^3\,\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^4*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] -((c - c/(a*x))^(1/2)*((12*a*x)/35 - (2*a^2*x^2)/105 + (8*a^3*x^3)/105 + (1
6*a^4*x^4)/105 + 2/7))/(x^3*(1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^4\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**4*sqrt(-(a*x - 1)*(a*x + 1))
), x)
```

3.572 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=172

$$\frac{32a^3(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{315x\sqrt{1-ax}}-\frac{16a^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{105x^2\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}}+\frac{4a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{21x^3\sqrt{1-ax}}$$

```
[Out] -2/9*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/x^4/(-a*x+1)^(1/2)+4/21*a*(a*x+1)^(3/2)*
(c-c/a/x)^(1/2)/x^3/(-a*x+1)^(1/2)-16/105*a^2*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)
/x^2/(-a*x+1)^(1/2)+32/315*a^3*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/x/(-a*x+1)^(1/
2)
```

Rubi [A] time = 0.23, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6134, 6128, 848, 45, 37}

$$-\frac{16a^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{105x^2\sqrt{1-ax}}+\frac{32a^3(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{315x\sqrt{1-ax}}+\frac{4a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{21x^3\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2))/(9*x^4*Sqrt[1 - a*x]) + (4*a*Sqrt[c
 - c/(a*x)]*(1 + a*x)^(3/2))/(21*x^3*Sqrt[1 - a*x]) - (16*a^2*Sqrt[c - c/(a*
x)]*(1 + a*x)^(3/2))/(105*x^2*Sqrt[1 - a*x]) + (32*a^3*Sqrt[c - c/(a*x)]*(1
 + a*x)^(3/2))/(315*x*Sqrt[1 - a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{11/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-a^2x^2}}{x^{11/2}\sqrt{1-ax}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{11/2}}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{9x^4\sqrt{1-ax}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{9/2}}\,dx}{3\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{9x^4\sqrt{1-ax}} + \frac{4a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{21x^3\sqrt{1-ax}} + \frac{\left(8a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{7/2}}\,dx}{21\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{9x^4\sqrt{1-ax}} + \frac{4a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{21x^3\sqrt{1-ax}} - \frac{16a^2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{105x^2\sqrt{1-ax}} - \frac{\left(16a^3\sqrt{\ldots}\right.}{\ldots}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{9x^4\sqrt{1-ax}} + \frac{4a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{21x^3\sqrt{1-ax}} - \frac{16a^2\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{105x^2\sqrt{1-ax}} + \frac{32a^3\sqrt{c\ldots}}{31\ldots}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 0.37

$$\frac{2(ax+1)^{3/2}\left(16a^3x^3-24a^2x^2+30ax-35\right)\sqrt{c-\frac{c}{ax}}}{315x^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(1 + a*x)^(3/2)*(-35 + 30*a*x - 24*a^2*x^2 + 16*a^3*x^
3))/(315*x^4*Sqrt[1 - a*x])
```

fricas [A] time = 0.40, size = 74, normalized size = 0.43

$$-\frac{2\left(16\,a^4x^4-8\,a^3x^3+6\,a^2x^2-5\,ax-35\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{315\left(ax^5-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fric
as")
```

```
[Out] -2/315*(16*a^4*x^4 - 8*a^3*x^3 + 6*a^2*x^2 - 5*a*x - 35)*sqrt(-a^2*x^2 + 1)
*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^5), x)
```

maple [A] time = 0.03, size = 62, normalized size = 0.36

$$\frac{2\,(ax+1)^2\left(16x^3a^3-24a^2x^2+30ax-35\right)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] 2/315*(a*x+1)^2*(16*a^3*x^3-24*a^2*x^2+30*a*x-35)*(c*(a*x-1)/a/x)^(1/2)/x^4
/(-a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{\sqrt{-a^2x^2+1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(sqrt(-a^2*x^2 + 1)*x^5), x)
```

mupad [B] time = 1.13, size = 68, normalized size = 0.40

$$-\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(-\frac{32\,a^5\,x^5}{315}-\frac{16\,a^4\,x^4}{315}+\frac{4\,a^3\,x^3}{315}-\frac{2\,a^2\,x^2}{315}+\frac{16\,a\,x}{63}+\frac{2}{9}\right)}{x^4\,\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^5*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] -((c - c/(a*x))^(1/2)*((16*a*x)/63 - (2*a^2*x^2)/315 + (4*a^3*x^3)/315 - (1
6*a^4*x^4)/315 - (32*a^5*x^5)/315 + 2/9))/(x^4*(1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^5\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**5*sqrt(-(a*x - 1)*(a*x + 1))
), x)
```

3.573 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=130

$$-\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}-\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}-\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

```
[Out] -75/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4-75/64*x*(c-c/a/x)^(1/2)
/a^3-25/32*x^2*(c-c/a/x)^(1/2)/a^2-5/8*x^3*(c-c/a/x)^(1/2)/a-1/4*x^4*(c-c/a
/x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 78, 51, 63, 208}

$$-\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}-\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}-\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (-75*Sqrt[c - c/(a*x)]*x)/(64*a^3) - (25*Sqrt[c - c/(a*x)]*x^2)/(32*a^2) -
(5*Sqrt[c - c/(a*x)]*x^3)/(8*a) - (Sqrt[c - c/(a*x)]*x^4)/4 - (75*Sqrt[c]*A
rcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}\,x^3(1+ax)}{1-ax}\,dx$$

$$=-\frac{c\int\frac{x^2(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=-\frac{c\int\frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=\frac{c\,\text{Subst}\left(\int\frac{a+x}{x^5\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=-\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{(15c)\,\text{Subst}\left(\int\frac{1}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a}$$

$$=-\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{(25c)\,\text{Subst}\left(\int\frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^2}$$

$$=-\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}-\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{(75c)\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{64a^3}$$

$$=-\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}-\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}-\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{(75c)\,\text{Subst}\left(\int \ldots\right.}{128a\ldots}$$

$$=-\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}-\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}-\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{75\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\ldots\right.}{64\ldots}$$

$$=-\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}-\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}-\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}-\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{75\sqrt{c}\,\tanh^{-1}\left(\ldots\right.}{64a^4}$$

Mathematica [C] time = 0.03, size = 50, normalized size = 0.38

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(a^4x^4+15\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\frac{1}{ax}\right)\right)}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] -1/4*(Sqrt[c - c/(a*x)]*(a^4*x^4 + 15*Hypergeometric2F1[1/2, 4, 3/2, 1 - 1/
(a*x)]))/a^4
```

fricas [A] time = 0.45, size = 179, normalized size = 1.38

$$\left[-\frac{2\left(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2+75\,ax\right)\sqrt{\frac{acx-c}{ax}}-75\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{128\,a^4},-\frac{\left(16\,a^4x^4+40\,a^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x - c)
/(a*x)) - 75*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
 c))/a^4, -1/64*((16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*
x - c)/(a*x)) - 75*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4
]
```

giac [A] time = 0.21, size = 142, normalized size = 1.09

$$-\frac{1}{64}\sqrt{a^2cx^2-acx}\left(2\left(4\,x\left(\frac{2\,x|a|}{a^2\mathrm{sgn}(x)}+\frac{5\,|a|}{a^3\mathrm{sgn}(x)}\right)+\frac{25\,|a|}{a^4\mathrm{sgn}(x)}\right)x+\frac{75\,|a|}{a^5\mathrm{sgn}(x)}\right)-\frac{75\,\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{128\,a^4}+\frac{75\,\sqrt{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*sqrt(a^2*c*x^2 - a*c*x)*(2*(4*x*(2*x*abs(a)/(a^2*sgn(x)) + 5*abs(a)/(
a^3*sgn(x))) + 25*abs(a)/(a^4*sgn(x)))*x + 75*abs(a)/(a^5*sgn(x))) - 75/128
*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^4 + 75/128*sqrt(c)*log(abs(-2*(sqrt(a^
2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^4*sgn(x))
```

maple [A] time = 0.04, size = 172, normalized size = 1.32

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32x\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{7}{2}}+112\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{5}{2}}+212\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x-106\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+256a^{\frac{3}{2}}\sqrt{(ax-1)}\right.}{128\sqrt{(ax-1)\,x}\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/128*(c*(a*x-1)/a/x)^(1/2)*x*(32*x*(a*x^2-x)^(3/2)*a^(7/2)+112*(a*x^2-x)^
(3/2)*a^(5/2)+212*(a*x^2-x)^(1/2)*a^(5/2)*x-106*(a*x^2-x)^(1/2)*a^(3/2)+256
*a^(3/2)*((a*x-1)*x)^(1/2)+128*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))-53*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x
-1)*x)^(1/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2 \sqrt{c-\frac{c}{ax}}\, x^3}{a^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))*x^3/(a^2*x^2 - 1), x)
```

mupad [B] time = 1.35, size = 111, normalized size = 0.85

$$\frac{365\,x^4\left(c-\frac{c}{a\,x}\right)^{3/2}}{64\,c}-\frac{181\,x^4\,\sqrt{c-\frac{c}{a\,x}}}{64}-\frac{275\,x^4\left(c-\frac{c}{a\,x}\right)^{5/2}}{64\,c^2}+\frac{75\,x^4\left(c-\frac{c}{a\,x}\right)^{7/2}}{64\,c^3}+\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{a\,x}}\,1\mathrm{i}}{\sqrt{c}}\right)\,75\mathrm{i}}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - c/(a*x))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (365*x^4*(c - c/(a*x))^(3/2))/(64*c) - (181*x^4*(c - c/(a*x))^(1/2))/64 - (
275*x^4*(c - c/(a*x))^(5/2))/(64*c^2) + (75*x^4*(c - c/(a*x))^(7/2))/(64*c^
3) + (c^(1/2)*atan(((c - c/(a*x))^(1/2)*1i)/c^(1/2))*75i)/(64*a^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3\sqrt{c-\frac{c}{ax}}}{ax-1}\, dx - \int \frac{ax^4\sqrt{c-\frac{c}{ax}}}{ax-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(x**3*sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x**4*sqrt(c - c
/(a*x))/(a*x - 1), x)
```

3.574 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=105

$$-\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}-\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}-\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[Out] -11/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3-11/8*x*(c-c/a/x)^(1/2)/a
^2-11/12*x^2*(c-c/a/x)^(1/2)/a-1/3*x^3*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 78, 51, 63, 208}

$$-\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}-\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}-\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (-11*Sqrt[c - c/(a*x)]*x)/(8*a^2) - (11*Sqrt[c - c/(a*x)]*x^2)/(12*a) - (Sq
rt[c - c/(a*x)]*x^3)/3 - (11*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8
*a^3)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)}{1-ax}\,dx$$
$$=-\frac{c\int\frac{x(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=-\frac{c\int\frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=\frac{c\,\mathrm{Subst}\left(\int\frac{a+x}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{(11c)\,\mathrm{Subst}\left(\int\frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{6a}$$
$$=-\frac{11\sqrt{c-\frac{c}{ax}}\,x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{(11c)\,\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$
$$=-\frac{11\sqrt{c-\frac{c}{ax}}\,x}{8a^2}-\frac{11\sqrt{c-\frac{c}{ax}}\,x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{(11c)\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^3}$$
$$=-\frac{11\sqrt{c-\frac{c}{ax}}\,x}{8a^2}-\frac{11\sqrt{c-\frac{c}{ax}}\,x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3-\frac{11\,\mathrm{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{8a^2}$$
$$=-\frac{11\sqrt{c-\frac{c}{ax}}\,x}{8a^2}-\frac{11\sqrt{c-\frac{c}{ax}}\,x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3-\frac{11\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}$$

Mathematica [C] time = 0.03, size = 50, normalized size = 0.48

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(a^3x^3+11\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{1}{ax}\right)\right)}{3a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -1/3*(Sqrt[c - c/(a*x)]*(a^3*x^3 + 11*Hypergeometric2F1[1/2, 3, 3/2, 1 - 1/
(a*x)]))/a^3
```

fricas [A] time = 0.46, size = 163, normalized size = 1.55

$$\left[-\frac{2\left(8\,a^3x^3+22\,a^2x^2+33\,ax\right)\sqrt{\frac{acx-c}{ax}}-33\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{48\,a^3},-\frac{\left(8\,a^3x^3+22\,a^2x^2+33\,ax\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sq
rt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^3, -1/24
*((8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sqrt(-c)*a
rctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

giac [A] time = 0.25, size = 127, normalized size = 1.21

$$-\frac{1}{24}\,\sqrt{a^2cx^2-acx}\left(2\,x\left(\frac{4\,x|a|}{a^2\mathrm{sgn}(x)}+\frac{11\,|a|}{a^3\mathrm{sgn}(x)}\right)+\frac{33\,|a|}{a^4\mathrm{sgn}(x)}\right)-\frac{11\,\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{16\,a^3}+\frac{11\,\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\right.\right.\right.}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(a^2*c*x^2 - a*c*x)*(2*x*(4*x*abs(a)/(a^2*sgn(x)) + 11*abs(a)/(a^
3*sgn(x))) + 33*abs(a)/(a^4*sgn(x))) - 11/16*sqrt(c)*log(abs(a)*abs(c))*sgn
(x)/a^3 + 11/16*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x)
)*sqrt(c)*abs(a) + a*c))/(a^3*sgn(x))
```

maple [A] time = 0.04, size = 155, normalized size = 1.48

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(-16\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{5}{2}}-60\sqrt{a\,x^2-x}\;a^{\frac{5}{2}}x+30\sqrt{a\,x^2-x}\;a^{\frac{3}{2}}-96a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\;-48a\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2a}{2\sqrt{a}}\right.\right.}{48\sqrt{(ax-1)\,x}\;a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a/x)^(1/2),x)
```

[Out] 1/48*(c*(a*x-1)/a/x)^(1/2)*x*(-16*(a*x^2-x)^(3/2)*a^(5/2)-60*(a*x^2-x)^(1/2)*a^(5/2)*x+30*(a*x^2-x)^(1/2)*a^(3/2)-96*a^(3/2)*((a*x-1)*x)^(1/2)-48*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))+15*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x-1)*x)^(1/2)/a^(7/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}\,x^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxima")

[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))*x^2/(a^2*x^2 - 1), x)

mupad [B] time = 1.33, size = 90, normalized size = 0.86

$$\frac{11\,x^3\,{\left(c-\frac{c}{a\,x}\right)}^{3/2}}{3\,c}-\frac{21\,x^3\,\sqrt{c-\frac{c}{a\,x}}}{8}-\frac{11\,x^3\,{\left(c-\frac{c}{a\,x}\right)}^{5/2}}{8\,c^2}+\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{a\,x}}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,11{}\mathrm{i}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2*(c - c/(a*x))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)

[Out] (11*x^3*(c - c/(a*x))^(3/2))/(3*c) - (21*x^3*(c - c/(a*x))^(1/2))/8 - (11*x^3*(c - c/(a*x))^(5/2))/(8*c^2) + (c^(1/2)*atan(((c - c/(a*x))^(1/2)*1i)/c^(1/2))*11i)/(8*a^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx-\int \frac{ax^3\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(c-c/a/x)**(1/2),x)

[Out] -Integral(x**2*sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x**3*sqrt(c - c/(a*x))/(a*x - 1), x)

3.575 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=80

$$-\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[Out] -7/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2-7/4*x*(c-c/a/x)^(1/2)/a-1
/2*x^2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6133, 25, 434, 446, 78, 51, 63, 208}

$$-\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (-7*Sqrt[c - c/(a*x)]*x)/(4*a) - (Sqrt[c - c/(a*x)]*x^2)/2 - (7*Sqrt[c]*Arc
Tanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=\int \frac{\sqrt{c-\frac{c}{ax}}\,x(1+ax)}{1-ax}\,dx$$
$$=-\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=-\frac{c\int \frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=-\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{(7c)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{4a}$$
$$=-\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{(7c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$
$$=-\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{7\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{4a}$$
$$=-\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{7\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}$$

Mathematica [A] time = 0.07, size = 77, normalized size = 0.96

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(ax\sqrt{1-\frac{1}{ax}}(2ax+7)+7\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\right)\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] -1/4*(Sqrt[c - c/(a*x)]*(a*Sqrt[1 - 1/(a*x)]*x*(7 + 2*a*x) + 7*ArcTanh[Sqrt
[1 - 1/(a*x)]]))/(a^2*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.43, size = 147, normalized size = 1.84

$$\left[-\frac{2\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}-7\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{8\,a^2},-\frac{\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}-7\,\sqrt{-c}\,\arctan\right.}{4\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(2*a^2*x^2 + 7*a*x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(c)*log(-2*a*c
*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2, -1/4*((2*a^2*x^2 + 7*
a*x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/
(a*x))/c))/a^2]
```

giac [A] time = 0.22, size = 112, normalized size = 1.40

$$-\frac{1}{4}\sqrt{a^2cx^2-acx}\left(\frac{2\,x|a|}{a^2\mathrm{sgn}(x)}+\frac{7\,|a|}{a^3\mathrm{sgn}(x)}\right)-\frac{7\,\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{8\,a^2}+\frac{7\,\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\,\right|\right.}{8\,a^2\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(a^2*c*x^2 - a*c*x)*(2*x*abs(a)/(a^2*sgn(x)) + 7*abs(a)/(a^3*sgn(x
))) - 7/8*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^2 + 7/8*sqrt(c)*log(abs(-2*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^2*sgn(x))
```

maple [B] time = 0.04, size = 139, normalized size = 1.74

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(4\sqrt{a\,x^2-x}\;a^{\frac{5}{2}}x-2\sqrt{a\,x^2-x}\;a^{\frac{3}{2}}+16a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\;+8a\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-\ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}}{2\sqrt{a}}\right.\right.}{8\sqrt{(ax-1)\,x}\;a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*(a*x^2-x)^(1/2)*a^(5/2)*x-2*(a*x^2-x)^(1/2)
*a^(3/2)+16*a^(3/2)*((a*x-1)*x)^(1/2)+8*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/
2)+2*a*x-1)/a^(1/2))-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)
/((a*x-1)*x)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}\,x}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))*x/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - c/(a*x))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x*(c - c/(a*x))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx - \int \frac{ax^2\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(x*sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x**2*sqrt(c - c/(a
*x))/(a*x - 1), x)
```

3.576 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=51

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] -3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a-x*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6133, 25, 514, 375, 78, 63, 208}

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{1-ax}\,dx$$

$$= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x}\,dx}{a}$$

$$= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\sqrt{c-\frac{c}{ax}}\,x + \frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\sqrt{c-\frac{c}{ax}}\,x - 3\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= -\sqrt{c-\frac{c}{ax}}\,x - \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 1.00

$$x\left(-\sqrt{c-\frac{c}{ax}}\right) - \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
[Out] -(Sqrt[c - c/(a*x)]*x) - (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

fricas [A] time = 0.53, size = 125, normalized size = 2.45

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},-\frac{ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a, -(a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(
-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

giac [B] time = 0.23, size = 97, normalized size = 1.90

$$-\frac{3\,\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{2\,a}+\frac{3\,\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\,|a|+ac\right|\right)}{2\,a\mathrm{sgn}(x)}-\frac{\sqrt{a^2cx^2-acx}\,|a|}{a^2\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a + 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) - sqrt(a
^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

maple [B] time = 0.04, size = 120, normalized size = 2.35

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^2-x}\,\sqrt{a}-4\sqrt{(ax-1)\,x}\,\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-2\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{(ax-1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(1/2)-4*((a*x-1)*x)^(1/2)*
a^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-2*ln(1/2*(2*((a
*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c - \frac{c}{ax}}}{ax - 1}\, dx - \int \frac{ax\sqrt{c - \frac{c}{ax}}}{ax - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2),x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x - 1), x) - Integral(a*x*sqrt(c - c/(a*x))/
(a*x - 1), x)
```

3.577 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=47

$$-2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

```
[Out] -2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)-2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6133, 25, 514, 446, 80, 63, 208}

$$-2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x(1-ax)}\,dx$$
$$= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^2}\,dx}{a}$$
$$= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x}\,dx}{a}$$
$$= \frac{c\,\text{Subst}\left(\int \frac{a+x}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -2\sqrt{c-\frac{c}{ax}} + c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= -2\sqrt{c-\frac{c}{ax}} - (2a)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$
$$= -2\sqrt{c-\frac{c}{ax}} - 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.00

$$-2\sqrt{c-\frac{c}{ax}} - 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

fricas [A] time = 0.51, size = 111, normalized size = 2.36

$$\left[\sqrt{c}\log\left(-2\,acx + 2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}} + c\right) - 2\sqrt{\frac{acx-c}{ax}}, 2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right) - 2\sqrt{\frac{acx-c}{ax}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) - 2*sqrt
((a*c*x - c)/(a*x)), 2*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c)
- 2*sqrt((a*c*x - c)/(a*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [86,-97,-82]Warning, c
hoosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [7,-27,26]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4
,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-89,63,-49]Warning, choosing root of [1,0,%
%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+
%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]
at parameters values [-86,-64,-30]Warning, choosing root of [1,0,%%%{-2,[2,
1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3
,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parame
ters values [70,22,42]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%
{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%
%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values
[56,-9,-13]Sign error (%%%{sqrt(c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+
%%%{-2*sqrt(c)*a^2,1%%%}+%%%{-a*sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*
c)*abs(a)/4,5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

maple [B] time = 0.04, size = 98, normalized size = 2.09

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\left(-2a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\;x^{2}+2\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}\;-\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^{2}a\right)}{x\sqrt{(ax-1)\,x}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] (c*(a*x-1)/a/x)^(1/2)/x*(-2*a^(3/2)*((a*x-1)*x)^(1/2)*x^2+2*(a*x^2-x)^(3/2)
*a^(1/2)-ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a)/((a*x
-1)*x)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{(a^2x^2-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^2}{x\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)), x)
```

sympy [A] time = 9.65, size = 39, normalized size = 0.83

$$\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] 2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) - 2*sqrt(c - c/(a*x))
```

3.578 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=42

$$\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}$$

[Out] 2/3*a*(c-c/a/x)^(3/2)/c-4*a*(c-c/a/x)^(1/2)

Rubi [A] time = 0.22, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6133, 25, 514, 444, 43}

$$\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^2,x]

[Out] -4*a*Sqrt[c - c/(a*x)] + (2*a*(c - c/(a*x))^(3/2))/(3*c)

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^2(1-ax)}\,dx \\
&= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^3}\,dx}{a} \\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^2}\,dx}{a} \\
&= \frac{c\,\mathrm{Subst}\left(\int \frac{a+x}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{c\,\mathrm{Subst}\left(\int \left(\frac{2a}{\sqrt{c-\frac{cx}{a}}}-\frac{a\sqrt{c-\frac{cx}{a}}}{c}\right)dx, x, \frac{1}{x}\right)}{a} \\
&= -4a\sqrt{c-\frac{c}{ax}}+\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.67

$$
-\frac{2(5ax+1)\sqrt{c-\frac{c}{ax}}}{3x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + 5*a*x))/(3*x)
```

fricas [A] time = 0.41, size = 28, normalized size = 0.67

$$-\frac{2\,(5\,ax+1)\sqrt{\frac{acx-c}{ax}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -2/3*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%%{6,[2,1,5]%%%}+%%%{-6
,[1,1,4]%%%}+%%%{-6,[0,1,3]%%%},[4]%%%}+%%%{%%{[%%%{-9,[2,0,4]%%%}+%%%{9,[1
,0,3]%%%}+%%%{9,[0,0,2]%%%},0,%%%{9,[4,1,6]%%%}+%%%{-18,[3,1,5]%%%}+%%%{9,[
2,1,4]%%%}+%%%{-9,[0,1,2]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%
%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+
%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[3]%%%}+%%%{%%%{18,[4,1,5]%%%}+%%%{
-36,[3,1,4]%%%}+%%%{18,[1,1,2]%%%},[2]%%%}+%%%{%%{[%%%{-3,[4,0,4]%%%}+%%%{6
,[3,0,3]%%%}+%%%{-3,[1,0,1]%%%},0,%%%{3,[6,1,6]%%%}+%%%{-9,[5,1,5]%%%}+%%%{
9,[4,1,4]%%%}+%%%{-3,[3,1,3]%%%}+%%%{-3,[2,1,2]%%%}+%%%{3,[1,1,1]%%%}]:[1,0
,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%
}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}
]%%},[1]%%%} / %%%{%%%{256,[8,8,8]%%%}+%%%{-1024,[7,8,7]%%%}+%%%{1536,[6,8,
6]%%%}+%%%{-1024,[5,8,5]%%%}+%%%{256,[4,8,4]%%%},[4]%%%}+%%%{%%{[%%%{-384,[
8,7,7]%%%}+%%%{1536,[7,7,6]%%%}+%%%{-2304,[6,7,5]%%%}+%%%{1536,[5,7,4]%%%}+
%%%{-384,[4,7,3]%%%},0,%%%{384,[10,8,9]%%%}+%%%{-1920,[9,8,8]%%%}+%%%{4224,
[8,8,7]%%%}+%%%{-5376,[7,8,6]%%%}+%%%{4224,[6,8,5]%%%}+%%%{-1920,[5,8,4]%%%
}+%%%{384,[4,8,3]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,
1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1
,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[3]%%%}+%%%{%%%{768,[10,8,8]%%%}+%%%{-3840,
[9,8,7]%%%}+%%%{7680,[8,8,6]%%%}+%%%{-7680,[7,8,5]%%%}+%%%{3840,[6,8,4]%%%}
```

```
+%%%{-768,[5,8,3]%%%},[2]%%%}+%%%{%%{[%%%{-128,[10,7,7]%%%}+%%%{640,[9,7,6]
%%%}+%%%{-1280,[8,7,5]%%%}+%%%{1280,[7,7,4]%%%}+%%%{-640,[6,7,3]%%%}+%%%{12
8,[5,7,2]%%%},0,%%%{128,[12,8,9]%%%}+%%%{-768,[11,8,8]%%%}+%%%{2048,[10,8,7
]%%%}+%%%{-3200,[9,8,6]%%%}+%%%{3200,[8,8,5]%%%}+%%%{-2048,[7,8,4]%%%}+%%%{
768,[6,8,3]%%%}+%%%{-128,[5,8,2]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]
%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,
2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[1]%%%} Error: Bad Argument
Value
```

maple [A] time = 0.03, size = 27, normalized size = 0.64

$$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(5ax+1)}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^2,x)
[Out] -2/3*(c*(a*x-1)/a/x)^(1/2)*(5*a*x+1)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x^2), x)
```

mupad [B] time = 0.90, size = 24, normalized size = 0.57

$$-\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,\left(5\,a\,x+1\right)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)),x)
[Out] -(2*(c - c/(a*x))^(1/2)*(5*a*x + 1))/(3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c-\frac{c}{ax}}}{ax^3-x^2}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^3-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x**3 - x**2), x) - Integral(a*x*sqrt(c - c/(
a*x))/(a*x**3 - x**2), x)
```

3.579 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=69

$$-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}-4a^2\sqrt{c-\frac{c}{ax}}$$

```
[Out] 2*a^2*(c-c/a/x)^(3/2)/c-2/5*a^2*(c-c/a/x)^(5/2)/c^2-4*a^2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6133, 25, 514, 446, 77}

$$-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}-4a^2\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] -4*a^2*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/c - (2*a^2*(c - c/(a
*x))^(5/2))/(5*c^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^3(1-ax)}\,dx\\
&= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^4}\,dx}{a}\\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^3}\,dx}{a}\\
&= \frac{c\,\text{Subst}\left(\int \frac{x(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \frac{c\,\text{Subst}\left(\int \left(\frac{2a^2}{\sqrt{c-\frac{cx}{a}}}-\frac{3a^2\sqrt{c-\frac{cx}{a}}}{c}+\frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{c^2}\right)dx,x,\frac{1}{x}\right)}{a}\\
&= -4a^2\sqrt{c-\frac{c}{ax}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.52

$$-\frac{2\left(6a^2x^2+3ax+1\right)\sqrt{c-\frac{c}{ax}}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + 3*a*x + 6*a^2*x^2))/(5*x^2)
```

fricas [A] time = 0.44, size = 36, normalized size = 0.52

$$-\frac{2\left(6\,a^2x^2+3\,ax+1\right)\sqrt{\frac{acx-c}{ax}}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fricas")
[Out] -2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%%{%%{[-5,0]:[1,0,%%%{-
1,[1]%%%}]%%},[0,5]%%%},[6]%%%}+%%%{%%{[%%%{25,[0,4]%%%},0]:[1,0,%%%{%%%{-1
,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[5]%%%}+%%%{%%%{%%{[-50,
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,5]%%%}+%%%{%%{[50,0]:[1,0,%%%{-1,[1]%%%}]%%},
[1,4]%%%},[4]%%%}+%%%{%%{[%%%{50,[2,4]%%%}+%%%{-50,[1,3]%%%},0]:[1,0,%%%{%%
%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[3]%%%}+%%%{%%%{%%{[
-25,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,5]%%%}+%%%{%%{[50,0]:[1,0,%%%{-1,[1]%%%}]
%%},[3,4]%%%}+%%%{%%{[-25,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3]%%%},[2]%%%}+%%%{
%%{[%%%{5,[4,4]%%%}+%%%{-10,[3,3]%%%}+%%%{5,[2,2]%%%},0]:[1,0,%%%{%%%{-1,[1
]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[1]%%%} / %%%{%%%{%%{poly1[
%%%{5,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,12]%%%}+%%%{%%{poly1[%%%{-30,[
6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,11]%%%}+%%%{%%{poly1[%%%{75,[6]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[10,10]%%%}+%%%{%%{poly1[%%%{-100,[6]%%%},0]:[1,0
```

```
,%%%{-1,[1]%%%}]%%},[9,9]%%%}+%%%{%%{poly1[%%%{75,[6]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[8,8]%%%}+%%%{%%{poly1[%%%{-30,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[7,7]%%%}+%%%{%%{poly1[%%%{5,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,6]%%%},
[6]%%%}+%%%{%%{[%%%{%%%{-25,[6]%%%},[12,11]%%%}+%%%{%%%{150,[6]%%%},[11,10]
%%%}+%%%{%%%{-375,[6]%%%},[10,9]%%%}+%%%{%%%{500,[6]%%%},[9,8]%%%}+%%%{%%%{
-375,[6]%%%},[8,7]%%%}+%%%{%%%{150,[6]%%%},[7,6]%%%}+%%%{%%%{-25,[6]%%%},[6
,5]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%
},[5]%%%}+%%%{%%%{%%{poly1[%%%{50,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,12
]%%%}+%%%{%%{poly1[%%%{-350,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[13,11]%%%}+
%%%{%%{poly1[%%%{1050,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,10]%%%}+%%%{%%
{poly1[%%%{-1750,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,9]%%%}+%%%{%%{poly1
[%%%{1750,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,8]%%%}+%%%{%%{poly1[%%%{-1
050,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,7]%%%}+%%%{%%{poly1[%%%{350,[6]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,6]%%%}+%%%{%%{poly1[%%%{-50,[6]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[7,5]%%%},[4]%%%}+%%%{%%{[%%%{%%%{-50,[6]%%%},[14,11]%
%%}+%%%{%%%{350,[6]%%%},[13,10]%%%}+%%%{%%%{-1050,[6]%%%},[12,9]%%%}+%%%{%%
%{1750,[6]%%%},[11,8]%%%}+%%%{%%%{-1750,[6]%%%},[10,7]%%%}+%%%{%%%{1050,[6]
%%%},[9,6]%%%}+%%%{%%%{-350,[6]%%%},[8,5]%%%}+%%%{%%%{50,[6]%%%},[7,4]%%%},
0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[3]%%%
}+%%%{%%%{%%{poly1[%%%{25,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[16,12]%%%}+%%
%{%%{poly1[%%%{-200,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,11]%%%}+%%%{%%{p
oly1[%%%{700,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,10]%%%}+%%%{%%{poly1[%%
%{-1400,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[13,9]%%%}+%%%{%%{poly1[%%%{1750
,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,8]%%%}+%%%{%%{poly1[%%%{-1400,[6]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,7]%%%}+%%%{%%{poly1[%%%{700,[6]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[10,6]%%%}+%%%{%%{poly1[%%%{-200,[6]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[9,5]%%%}+%%%{%%{poly1[%%%{25,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[8,4]%%%},[2]%%%}+%%%{%%{[%%%{%%%{-5,[6]%%%},[16,11]%%%}+%%%{%%%{40,[6
]%%%},[15,10]%%%}+%%%{%%%{-140,[6]%%%},[14,9]%%%}+%%%{%%%{280,[6]%%%},[13,8
]%%%}+%%%{%%%{-350,[6]%%%},[12,7]%%%}+%%%{%%%{280,[6]%%%},[11,6]%%%}+%%%{%%
%{-140,[6]%%%},[10,5]%%%}+%%%{%%%{40,[6]%%%},[9,4]%%%}+%%%{%%%{-5,[6]%%%},[
8,3]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%
%},[1]%%%} Error: Bad Argument Value
```

maple [A] time = 0.03, size = 35, normalized size = 0.51

$$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,\left(6a^2x^2+3ax+1\right)}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] -2/5*(c*(a*x-1)/a/x)^(1/2)*(6*a^2*x^2+3*a*x+1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{(a^2x^2-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxima")
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x^3), x)
```

mupad [B] time = 0.91, size = 32, normalized size = 0.46

$$-\frac{2\sqrt{c-\frac{c}{a\,x}}\,\left(6\,a^2\,x^2+3\,a\,x+1\right)}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)),x)
[Out] -(2*(c - c/(a*x))^(1/2)*(3*a*x + 6*a^2*x^2 + 1))/(5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c-\frac{c}{ax}}}{ax^4-x^3}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^4-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x**3,x)
[Out] -Integral(sqrt(c - c/(a*x))/(a*x**4 - x**3), x) - Integral(a*x*sqrt(c - c/(
a*x))/(a*x**4 - x**3), x)
```

3.580 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=96

$$\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}$$

```
[Out] 10/3*a^3*(c-c/a/x)^(3/2)/c-8/5*a^3*(c-c/a/x)^(5/2)/c^2+2/7*a^3*(c-c/a/x)^(7
/2)/c^3-4*a^3*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6133, 25, 514, 446, 77}

$$\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] -4*a^3*Sqrt[c - c/(a*x)] + (10*a^3*(c - c/(a*x))^(3/2))/(3*c) - (8*a^3*(c -
 c/(a*x))^(5/2))/(5*c^2) + (2*a^3*(c - c/(a*x))^(7/2))/(7*c^3)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^4(1-ax)}\,dx$$

$$= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^5}\,dx}{a}$$

$$= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^4}\,dx}{a}$$

$$= \frac{c\,\text{Subst}\left(\int \frac{x^2(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c\,\text{Subst}\left(\int \left(\frac{2a^3}{\sqrt{c-\frac{cx}{a}}} - \frac{5a^3\sqrt{c-\frac{cx}{a}}}{c} + \frac{4a^3\left(c-\frac{cx}{a}\right)^{3/2}}{c^2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{5/2}}{c^3}\right)dx, x, \frac{1}{x}\right)}{a}$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} + \frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.46

$$-\frac{2\left(104a^3x^3+52a^2x^2+39ax+15\right)\sqrt{c-\frac{c}{ax}}}{105x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
[Out] (-2*Sqrt[c - c/(a*x)]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*x^3)
```

fricas [A] time = 0.50, size = 44, normalized size = 0.46

$$-\frac{2\left(104\,a^3x^3+52\,a^2x^2+39\,ax+15\right)\sqrt{\frac{acx-c}{ax}}}{105\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fricas")
[Out] -2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt((a*c*x - c)/(a*x))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Evaluation time: 0.45Unable to divide, perhaps due to rounding error%%%{%%%
{210,[2,1,9]%%%}+%%%{-210,[1,1,8]%%%}+%%%{-210,[0,1,7]%%%},[8]%%%}+%%%{%%{[
%%%{-735,[2,0,8]%%%}+%%%{735,[1,0,7]%%%}+%%%{735,[0,0,6]%%%},0,%%%{735,[4,1
,10]%%%}+%%%{-1470,[3,1,9]%%%}+%%%{735,[2,1,8]%%%}+%%%{-735,[0,1,6]%%%}]:[1
,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%
%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%
%}]%%},[7]%%%}+%%%{%%%{4410,[4,1,9]%%%}+%%%{-8820,[3,1,8]%%%}+%%%{4410,[1,1
,6]%%%},[6]%%%}+%%%{%%{[%%%{-3675,[4,0,8]%%%}+%%%{7350,[3,0,7]%%%}+%%%{-367
5,[1,0,5]%%%},0,%%%{3675,[6,1,10]%%%}+%%%{-11025,[5,1,9]%%%}+%%%{11025,[4,1
,8]%%%}+%%%{-3675,[3,1,7]%%%}+%%%{-3675,[2,1,6]%%%}+%%%{3675,[1,1,5]%%%}]:[
1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]
%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%
%%}]%%},[5]%%%}+%%%{%%%{7350,[6,1,9]%%%}+%%%{-22050,[5,1,8]%%%}+%%%{14700,[
```

4,1,7]%%%}+%%%{7350,[3,1,6]%%%}+%%%{-7350,[2,1,5]%%%},[4]%%%}+%%%{%%{[%%%{-
2205,[6,0,8]%%%}+%%%{6615,[5,0,7]%%%}+%%%{-4410,[4,0,6]%%%}+%%%{-2205,[3,0,
5]%%%}+%%%{2205,[2,0,4]%%%},0,%%%{2205,[8,1,10]%%%}+%%%{-8820,[7,1,9]%%%}+%
%%{13230,[6,1,8]%%%}+%%%{-8820,[5,1,7]%%%}+%%%{4410,[3,1,5]%%%}+%%%{-2205,[
2,1,4]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,
%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%
%%{1,[0,2,0]%%%}]%%},[3]%%%}+%%%{%%%{1470,[8,1,9]%%%}+%%%{-5880,[7,1,8]%%%}
+%%%{7350,[6,1,7]%%%}+%%%{-1470,[5,1,6]%%%}+%%%{-2940,[4,1,5]%%%}+%%%{1470,
[3,1,4]%%%},[2]%%%}+%%%{%%{[%%%{-105,[8,0,8]%%%}+%%%{420,[7,0,7]%%%}+%%%{-5
25,[6,0,6]%%%}+%%%{105,[5,0,5]%%%}+%%%{210,[4,0,4]%%%}+%%%{-105,[3,0,3]%%%}
,0,%%%{105,[10,1,10]%%%}+%%%{-525,[9,1,9]%%%}+%%%{1050,[8,1,8]%%%}+%%%{-105
0,[7,1,7]%%%}+%%%{420,[6,1,6]%%%}+%%%{210,[5,1,5]%%%}+%%%{-315,[4,1,4]%%%}+
%%%{105,[3,1,3]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,
0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2
,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[1]%%%} / %%%{%%%{65536,[16,16,16]%%%}+%%%{-5
24288,[15,16,15]%%%}+%%%{1835008,[14,16,14]%%%}+%%%{-3670016,[13,16,13]%%%}
+%%%{4587520,[12,16,12]%%%}+%%%{-3670016,[11,16,11]%%%}+%%%{1835008,[10,16,
10]%%%}+%%%{-524288,[9,16,9]%%%}+%%%{65536,[8,16,8]%%%},[8]%%%}+%%%{%%{[%%%
{-229376,[16,15,15]%%%}+%%%{1835008,[15,15,14]%%%}+%%%{-6422528,[14,15,13]%
%%}+%%%{12845056,[13,15,12]%%%}+%%%{-16056320,[12,15,11]%%%}+%%%{12845056,[
11,15,10]%%%}+%%%{-6422528,[10,15,9]%%%}+%%%{1835008,[9,15,8]%%%}+%%%{-2293
76,[8,15,7]%%%},0,%%%{229376,[18,16,17]%%%}+%%%{-2064384,[17,16,16]%%%}+%%%
{8486912,[16,16,15]%%%}+%%%{-21102592,[15,16,14]%%%}+%%%{35323904,[14,16,13
]%%%}+%%%{-41746432,[13,16,12]%%%}+%%%{35323904,[12,16,11]%%%}+%%%{-2110259
2,[11,16,10]%%%}+%%%{8486912,[10,16,9]%%%}+%%%{-2064384,[9,16,8]%%%}+%%%{22
9376,[8,16,7]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]
%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1
]%%%}+%%%{1,[0,2,0]%%%}]%%},[7]%%%}+%%%{%%%{1376256,[18,16,16]%%%}+%%%{-123
86304,[17,16,15]%%%}+%%%{49545216,[16,16,14]%%%}+%%%{-115605504,[15,16,13]%
%%}+%%%{173408256,[14,16,12]%%%}+%%%{-173408256,[13,16,11]%%%}+%%%{11560550
4,[12,16,10]%%%}+%%%{-49545216,[11,16,9]%%%}+%%%{12386304,[10,16,8]%%%}+%%%
{-1376256,[9,16,7]%%%},[6]%%%}+%%%{%%{[%%%{-1146880,[18,15,15]%%%}+%%%{1032
1920,[17,15,14]%%%}+%%%{-41287680,[16,15,13]%%%}+%%%{96337920,[15,15,12]%%%
}+%%%{-144506880,[14,15,11]%%%}+%%%{144506880,[13,15,10]%%%}+%%%{-96337920,
[12,15,9]%%%}+%%%{41287680,[11,15,8]%%%}+%%%{-10321920,[10,15,7]%%%}+%%%{11
46880,[9,15,6]%%%},0,%%%{1146880,[20,16,17]%%%}+%%%{-11468800,[19,16,16]%%%
}+%%%{52756480,[18,16,15]%%%}+%%%{-147947520,[17,16,14]%%%}+%%%{282132480,[
16,16,13]%%%}+%%%{-385351680,[15,16,12]%%%}+%%%{385351680,[14,16,11]%%%}+%%
%{-282132480,[13,16,10]%%%}+%%%{147947520,[12,16,9]%%%}+%%%{-52756480,[11,1
6,8]%%%}+%%%{11468800,[10,16,7]%%%}+%%%{-1146880,[9,16,6]%%%}]:[1,0,%%%{-2,
[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2
,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[5]
%%%}+%%%{%%%{2293760,[20,16,16]%%%}+%%%{-22937600,[19,16,15]%%%}+%%%{103219
200,[18,16,14]%%%}+%%%{-275251200,[17,16,13]%%%}+%%%{481689600,[16,16,12]%%
%}+%%%{-578027520,[15,16,11]%%%}+%%%{481689600,[14,16,10]%%%}+%%%{-27525120

0,[13,16,9]%%%}+%%%{103219200,[12,16,8]%%%}+%%%{-22937600,[11,16,7]%%%}+%%%
{2293760,[10,16,6]%%%},[4]%%%}+%%%{%%{[%%%{-688128,[20,15,15]%%%}+%%%{68812
80,[19,15,14]%%%}+%%%{-30965760,[18,15,13]%%%}+%%%{82575360,[17,15,12]%%%}+
%%%{-144506880,[16,15,11]%%%}+%%%{173408256,[15,15,10]%%%}+%%%{-144506880,[
14,15,9]%%%}+%%%{82575360,[13,15,8]%%%}+%%%{-30965760,[12,15,7]%%%}+%%%{688
1280,[11,15,6]%%%}+%%%{-688128,[10,15,5]%%%},0,%%%{688128,[22,16,17]%%%}+%%
%{-7569408,[21,16,16]%%%}+%%%{38535168,[20,16,15]%%%}+%%%{-120422400,[19,16
,14]%%%}+%%%{258048000,[18,16,13]%%%}+%%%{-400490496,[17,16,12]%%%}+%%%{462
422016,[16,16,11]%%%}+%%%{-400490496,[15,16,10]%%%}+%%%{258048000,[14,16,9]
%%%}+%%%{-120422400,[13,16,8]%%%}+%%%{38535168,[12,16,7]%%%}+%%%{-7569408,[
11,16,6]%%%}+%%%{688128,[10,16,5]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1
]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2
,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[3]%%%}+%%%{%%%{458752,[22,
16,16]%%%}+%%%{-5046272,[21,16,15]%%%}+%%%{25231360,[20,16,14]%%%}+%%%{-756
94080,[19,16,13]%%%}+%%%{151388160,[18,16,12]%%%}+%%%{-211943424,[17,16,11]
%%%}+%%%{211943424,[16,16,10]%%%}+%%%{-151388160,[15,16,9]%%%}+%%%{75694080
,[14,16,8]%%%}+%%%{-25231360,[13,16,7]%%%}+%%%{5046272,[12,16,6]%%%}+%%%{-4
58752,[11,16,5]%%%},[2]%%%}+%%%{%%{[%%%{-32768,[22,15,15]%%%}+%%%{360448,[2
1,15,14]%%%}+%%%{-1802240,[20,15,13]%%%}+%%%{5406720,[19,15,12]%%%}+%%%{-10
813440,[18,15,11]%%%}+%%%{15138816,[17,15,10]%%%}+%%%{-15138816,[16,15,9]%%
%}+%%%{10813440,[15,15,8]%%%}+%%%{-5406720,[14,15,7]%%%}+%%%{1802240,[13,15
,6]%%%}+%%%{-360448,[12,15,5]%%%}+%%%{32768,[11,15,4]%%%},0,%%%{32768,[24,1
6,17]%%%}+%%%{-393216,[23,16,16]%%%}+%%%{2195456,[22,16,15]%%%}+%%%{-756940
8,[21,16,14]%%%}+%%%{18022400,[20,16,13]%%%}+%%%{-31358976,[19,16,12]%%%}+%
%%{41091072,[18,16,11]%%%}+%%%{-41091072,[17,16,10]%%%}+%%%{31358976,[16,16
,9]%%%}+%%%{-18022400,[15,16,8]%%%}+%%%{7569408,[14,16,7]%%%}+%%%{-2195456,
[13,16,6]%%%}+%%%{393216,[12,16,5]%%%}+%%%{-32768,[11,16,4]%%%}]:[1,0,%%%{-
2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{
-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[
1]%%%} Error: Bad Argument Value

maple [A] time = 0.03, size = 43, normalized size = 0.45

$$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,\left(104x^3a^3+52a^2x^2+39ax+15\right)}{105x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] -2/105*(c*(a*x-1)/a/x)^(1/2)*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{\left(a^2x^2-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x^4), x)
```

mupad [B] time = 0.91, size = 77, normalized size = 0.80

$$-\frac{208\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{105}-\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{7\,x^3}-\frac{26\,a\,\sqrt{c-\frac{c}{a\,x}}}{35\,x^2}-\frac{104\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] - (208*a^3*(c - c/(a*x))^(1/2))/105 - (2*(c - c/(a*x))^(1/2))/(7*x^3) - (26
*a*(c - c/(a*x))^(1/2))/(35*x^2) - (104*a^2*(c - c/(a*x))^(1/2))/(105*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c-\frac{c}{ax}}}{ax^5-x^4}\,dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^5-x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x**5 - x**4), x) - Integral(a*x*sqrt(c - c/(
a*x))/(a*x**5 - x**4), x)
```

3.581 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=121

$$-\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}+\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^4\sqrt{c-\frac{c}{ax}}$$

```
[Out] 14/3*a^4*(c-c/a/x)^(3/2)/c-18/5*a^4*(c-c/a/x)^(5/2)/c^2+10/7*a^4*(c-c/a/x)^
(7/2)/c^3-2/9*a^4*(c-c/a/x)^(9/2)/c^4-4*a^4*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6133, 25, 514, 446, 77}

$$-\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}+\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^4\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] -4*a^4*Sqrt[c - c/(a*x)] + (14*a^4*(c - c/(a*x))^(3/2))/(3*c) - (18*a^4*(c
- c/(a*x))^(5/2))/(5*c^2) + (10*a^4*(c - c/(a*x))^(7/2))/(7*c^3) - (2*a^4*(
c - c/(a*x))^(9/2))/(9*c^4)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx &= \int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^5(1-ax)}\,dx\\
&= -\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^6}\,dx}{a}\\
&= -\frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^5}\,dx}{a}\\
&= \frac{c\,\text{Subst}\left(\int \frac{x^3(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \frac{c\,\text{Subst}\left(\int \left(\frac{2a^4}{\sqrt{c-\frac{cx}{a}}}-\frac{7a^4\sqrt{c-\frac{cx}{a}}}{c}+\frac{9a^4\left(c-\frac{cx}{a}\right)^{3/2}}{c^2}-\frac{5a^4\left(c-\frac{cx}{a}\right)^{5/2}}{c^3}+\frac{a^4\left(c-\frac{cx}{a}\right)^{7/2}}{c^4}\right)dx,x,\frac{1}{x}\right)}{a}\\
&= -4a^4\sqrt{c-\frac{c}{ax}}+\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right.}{9c^4}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 52, normalized size = 0.43

$$-\frac{2\left(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35\right)\sqrt{c-\frac{c}{ax}}}{315x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*a^4*x^
4))/(315*x^4)
```

fricas [A] time = 0.62, size = 52, normalized size = 0.43

$$-\frac{2\left(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35\right)\sqrt{\frac{acx-c}{ax}}}{315\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt((a*c*x
- c)/(a*x))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%%{%%{[-315,0]:[1,0,%%%
{-1,[1]%%%}]%%},[0,9]%%%},[10]%%%}+%%%{%%{[%%%{2835,[0,8]%%%},0]:[1,0,%%%{%
%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[9]%%%}+%%%{%%%{%%{
[-11340,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,9]%%%}+%%%{%%{[11340,0]:[1,0,%%%{-1,[
1]%%%}]%%},[1,8]%%%},[8]%%%}+%%%{%%{[%%%{26460,[2,8]%%%}+%%%{-26460,[1,7]%%
%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[7]
%%%}+%%%{%%%{%%{[-39690,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,9]%%%}+%%%{%%{[79380,
0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8]%%%}+%%%{%%{[-39690,0]:[1,0,%%%{-1,[1]%%%}]
%%},[2,7]%%%},[6]%%%}+%%%{%%{[%%%{39690,[4,8]%%%}+%%%{-79380,[3,7]%%%}+%%%{
39690,[2,6]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1
]%%%}]%%},[5]%%%}+%%%{%%%{%%{[-26460,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,9]%%%}+%
```

%%{%%{[79380,0]:[1,0,%%%{-1,[1]%%%}]%%},[5,8]%%%}+%%%{%%{[-79380,0]:[1,0,%%
%{-1,[1]%%%}]%%},[4,7]%%%}+%%%{%%{[26460,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,6]%%
%},[4]%%%}+%%%{%%{[%%%{11340,[6,8]%%%}+%%%{-34020,[5,7]%%%}+%%%{34020,[4,6]
%%%}+%%%{-11340,[3,5]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1
]%%%},[1,1]%%%}]%%},[3]%%%}+%%%{%%%{%%{[-2835,0]:[1,0,%%%{-1,[1]%%%}]%%},[8
,9]%%%}+%%%{%%{[11340,0]:[1,0,%%%{-1,[1]%%%}]%%},[7,8]%%%}+%%%{%%{[-17010,0
]:[1,0,%%%{-1,[1]%%%}]%%},[6,7]%%%}+%%%{%%{[11340,0]:[1,0,%%%{-1,[1]%%%}]%%
},[5,6]%%%}+%%%{%%{[-2835,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,5]%%%},[2]%%%}+%%%{
%%{[%%%{315,[8,8]%%%}+%%%{-1260,[7,7]%%%}+%%%{1890,[6,6]%%%}+%%%{-1260,[5,5
]%%%}+%%%{315,[4,4]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%
%%},[1,1]%%%}]%%},[1]%%%} / %%%{%%%{%%{poly1[%%%{315,[10]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[20,20]%%%}+%%%{%%{poly1[%%%{-3150,[10]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[19,19]%%%}+%%%{%%{poly1[%%%{14175,[10]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[18,18]%%%}+%%%{%%{poly1[%%%{-37800,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[17,17]%%%}+%%%{%%{poly1[%%%{66150,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
16,16]%%%}+%%%{%%{poly1[%%%{-79380,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,
15]%%%}+%%%{%%{poly1[%%%{66150,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,14]%
%%}+%%%{%%{poly1[%%%{-37800,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[13,13]%%%}
+%%%{%%{poly1[%%%{14175,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,12]%%%}+%%%
{%%{poly1[%%%{-3150,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,11]%%%}+%%%{%%{
poly1[%%%{315,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,10]%%%},[10]%%%}+%%%{
%%{[%%%{%%%{-2835,[10]%%%},[20,19]%%%}+%%%{%%%{28350,[10]%%%},[19,18]%%%}+%
%%{%%%{-127575,[10]%%%},[18,17]%%%}+%%%{%%%{340200,[10]%%%},[17,16]%%%}+%%%
{%%%{-595350,[10]%%%},[16,15]%%%}+%%%{%%%{714420,[10]%%%},[15,14]%%%}+%%%{%
%%{-595350,[10]%%%},[14,13]%%%}+%%%{%%%{340200,[10]%%%},[13,12]%%%}+%%%{%%%
{-127575,[10]%%%},[12,11]%%%}+%%%{%%%{28350,[10]%%%},[11,10]%%%}+%%%{%%%{-2
835,[10]%%%},[10,9]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%
%%},[1,1]%%%}]%%},[9]%%%}+%%%{%%%{%%{poly1[%%%{11340,[10]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[22,20]%%%}+%%%{%%{poly1[%%%{-124740,[10]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[21,19]%%%}+%%%{%%{poly1[%%%{623700,[10]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[20,18]%%%}+%%%{%%{poly1[%%%{-1871100,[10]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[19,17]%%%}+%%%{%%{poly1[%%%{3742200,[10]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[18,16]%%%}+%%%{%%{poly1[%%%{-5239080,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[17,15]%%%}+%%%{%%{poly1[%%%{5239080,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[16,14]%%%}+%%%{%%{poly1[%%%{-3742200,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[15,13]%%%}+%%%{%%{poly1[%%%{1871100,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[14,12]%%%}+%%%{%%{poly1[%%%{-623700,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1
3,11]%%%}+%%%{%%{poly1[%%%{124740,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,1
0]%%%}+%%%{%%{poly1[%%%{-11340,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,9]%%
%},[8]%%%}+%%%{%%{[%%%{%%%{-26460,[10]%%%},[22,19]%%%}+%%%{%%%{291060,[10]%
%%},[21,18]%%%}+%%%{%%%{-1455300,[10]%%%},[20,17]%%%}+%%%{%%%{4365900,[10]%
%%},[19,16]%%%}+%%%{%%%{-8731800,[10]%%%},[18,15]%%%}+%%%{%%%{12224520,[10]
%%%},[17,14]%%%}+%%%{%%%{-12224520,[10]%%%},[16,13]%%%}+%%%{%%%{8731800,[10
]%%%},[15,12]%%%}+%%%{%%%{-4365900,[10]%%%},[14,11]%%%}+%%%{%%%{1455300,[10
]%%%},[13,10]%%%}+%%%{%%%{-291060,[10]%%%},[12,9]%%%}+%%%{%%%{26460,[10]%%%

},[11,8]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%
%}]%%},[7]%%%}+%%%{%%%{%%{poly1[%%%{39690,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[24,20]%%%}+%%%{%%{poly1[%%%{-476280,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[23,19]%%%}+%%%{%%{poly1[%%%{2619540,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
22,18]%%%}+%%%{%%{poly1[%%%{-8731800,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2
1,17]%%%}+%%%{%%{poly1[%%%{19646550,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[20
,16]%%%}+%%%{%%{poly1[%%%{-31434480,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[19
,15]%%%}+%%%{%%{poly1[%%%{36673560,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[18,
14]%%%}+%%%{%%{poly1[%%%{-31434480,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[17,
13]%%%}+%%%{%%{poly1[%%%{19646550,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[16,1
2]%%%}+%%%{%%{poly1[%%%{-8731800,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,11
]%%%}+%%%{%%{poly1[%%%{2619540,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,10]%
%%}+%%%{%%{poly1[%%%{-476280,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[13,9]%%%}
+%%%{%%{poly1[%%%{39690,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,8]%%%},[6]%
%%}+%%%{%%{[%%%{%%%{-39690,[10]%%%},[24,19]%%%}+%%%{%%%{476280,[10]%%%},[23
,18]%%%}+%%%{%%%{-2619540,[10]%%%},[22,17]%%%}+%%%{%%%{8731800,[10]%%%},[21
,16]%%%}+%%%{%%%{-19646550,[10]%%%},[20,15]%%%}+%%%{%%%{31434480,[10]%%%},[
19,14]%%%}+%%%{%%%{-36673560,[10]%%%},[18,13]%%%}+%%%{%%%{31434480,[10]%%%}
,[17,12]%%%}+%%%{%%%{-19646550,[10]%%%},[16,11]%%%}+%%%{%%%{8731800,[10]%%%
},[15,10]%%%}+%%%{%%%{-2619540,[10]%%%},[14,9]%%%}+%%%{%%%{476280,[10]%%%},
[13,8]%%%}+%%%{%%%{-39690,[10]%%%},[12,7]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2
,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[5]%%%}+%%%{%%%{%%{poly1[%%%{26460,
[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[26,20]%%%}+%%%{%%{poly1[%%%{-343980,[1
0]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[25,19]%%%}+%%%{%%{poly1[%%%{2063880,[10]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[24,18]%%%}+%%%{%%{poly1[%%%{-7567560,[10]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[23,17]%%%}+%%%{%%{poly1[%%%{18918900,[10]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[22,16]%%%}+%%%{%%{poly1[%%%{-34054020,[10]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[21,15]%%%}+%%%{%%{poly1[%%%{45405360,[10]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[20,14]%%%}+%%%{%%{poly1[%%%{-45405360,[10]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[19,13]%%%}+%%%{%%{poly1[%%%{34054020,[10]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[18,12]%%%}+%%%{%%{poly1[%%%{-18918900,[10]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[17,11]%%%}+%%%{%%{poly1[%%%{7567560,[10]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[16,10]%%%}+%%%{%%{poly1[%%%{-2063880,[10]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[15,9]%%%}+%%%{%%{poly1[%%%{343980,[10]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[14,8]%%%}+%%%{%%{poly1[%%%{-26460,[10]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[13,7]%%%},[4]%%%}+%%%{%%{[%%%{%%%{-11340,[10]%%%},[26,19]
%%%}+%%%{%%%{147420,[10]%%%},[25,18]%%%}+%%%{%%%{-884520,[10]%%%},[24,17]%%
%}+%%%{%%%{3243240,[10]%%%},[23,16]%%%}+%%%{%%%{-8108100,[10]%%%},[22,15]%%
%}+%%%{%%%{14594580,[10]%%%},[21,14]%%%}+%%%{%%%{-19459440,[10]%%%},[20,13]
%%%}+%%%{%%%{19459440,[10]%%%},[19,12]%%%}+%%%{%%%{-14594580,[10]%%%},[18,1
1]%%%}+%%%{%%%{8108100,[10]%%%},[17,10]%%%}+%%%{%%%{-3243240,[10]%%%},[16,9
]%%%}+%%%{%%%{884520,[10]%%%},[15,8]%%%}+%%%{%%%{-147420,[10]%%%},[14,7]%%%
}+%%%{%%%{11340,[10]%%%},[13,6]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%
%{%%%{1,[1]%%%},[1,1]%%%}]%%},[3]%%%}+%%%{%%%{%%{poly1[%%%{2835,[10]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[28,20]%%%}+%%%{%%{poly1[%%%{-39690,[10]%%%},0]:[1

,0,%%%{-1,[1]%%%}]%%},[27,19]%%%}+%%%{%%{poly1[%%%{257985,[10]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[26,18]%%%}+%%%{%%{poly1[%%%{-1031940,[10]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[25,17]%%%}+%%%{%%{poly1[%%%{2837835,[10]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[24,16]%%%}+%%%{%%{poly1[%%%{-5675670,[10]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[23,15]%%%}+%%%{%%{poly1[%%%{8513505,[10]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[22,14]%%%}+%%%{%%{poly1[%%%{-9729720,[10]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[21,13]%%%}+%%%{%%{poly1[%%%{8513505,[10]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[20,12]%%%}+%%%{%%{poly1[%%%{-5675670,[10]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[19,11]%%%}+%%%{%%{poly1[%%%{2837835,[10]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[18,10]%%%}+%%%{%%{poly1[%%%{-1031940,[10]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[17,9]%%%}+%%%{%%{poly1[%%%{257985,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[16,8]%%%}+%%%{%%{poly1[%%%{-39690,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1
5,7]%%%}+%%%{%%{poly1[%%%{2835,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,6]%%
%},[2]%%%}+%%%{%%{[%%%{%%%{-315,[10]%%%},[28,19]%%%}+%%%{%%%{4410,[10]%%%},
[27,18]%%%}+%%%{%%%{-28665,[10]%%%},[26,17]%%%}+%%%{%%%{114660,[10]%%%},[25
,16]%%%}+%%%{%%%{-315315,[10]%%%},[24,15]%%%}+%%%{%%%{630630,[10]%%%},[23,1
4]%%%}+%%%{%%%{-945945,[10]%%%},[22,13]%%%}+%%%{%%%{1081080,[10]%%%},[21,12
]%%%}+%%%{%%%{-945945,[10]%%%},[20,11]%%%}+%%%{%%%{630630,[10]%%%},[19,10]%
%%}+%%%{%%%{-315315,[10]%%%},[18,9]%%%}+%%%{%%%{114660,[10]%%%},[17,8]%%%}+
%%%{%%%{-28665,[10]%%%},[16,7]%%%}+%%%{%%%{4410,[10]%%%},[15,6]%%%}+%%%{%%%
{-315,[10]%%%},[14,5]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1
]%%%},[1,1]%%%}]%%},[1]%%%} Error: Bad Argument Value

maple [A] time = 0.03, size = 51, normalized size = 0.42

$$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,\left(272x^4a^4+136x^3a^3+102a^2x^2+85ax+35\right)}{315x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] -2/315*(c*(a*x-1)/a/x)^(1/2)*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35
)/x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{(ax+1)^2\sqrt{c-\frac{c}{ax}}}{\left(a^2x^2-1\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a*x))/((a^2*x^2 - 1)*x^5), x)
```

mupad [B] time = 0.90, size = 98, normalized size = 0.81

$$-\frac{544\,a^4\,\sqrt{c-\frac{c}{a\,x}}}{315}-\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{9\,x^4}-\frac{34\,a\,\sqrt{c-\frac{c}{a\,x}}}{63\,x^3}-\frac{68\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x^2}-\frac{272\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{315\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)),x)
```

```
[Out] - (544*a^4*(c - c/(a*x))^(1/2))/315 - (2*(c - c/(a*x))^(1/2))/(9*x^4) - (34
*a*(c - c/(a*x))^(1/2))/(63*x^3) - (68*a^2*(c - c/(a*x))^(1/2))/(105*x^2) -
 (272*a^3*(c - c/(a*x))^(1/2))/(315*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{c-\frac{c}{ax}}}{ax^6-x^5}\,dx-\int\frac{ax\sqrt{c-\frac{c}{ax}}}{ax^6-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] -Integral(sqrt(c - c/(a*x))/(a*x**6 - x**5), x) - Integral(a*x*sqrt(c - c/(
a*x))/(a*x**6 - x**5), x)
```

3.582 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=292

$$-\frac{363\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{7/2}\sqrt{1-ax}}-\frac{21x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{32a^3\sqrt{1-ax}}-\frac{107x\sqrt{ax+1}\ \cdots}{64a^3\sqrt{1-\cdots}}$$

```
[Out] -21/32*x*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/a^3/(-a*x+1)^(1/2)-11/24*x^2*(a*x+1)
^(3/2)*(c-c/a/x)^(1/2)/a^2/(-a*x+1)^(1/2)-1/4*x^3*(a*x+1)^(3/2)*(c-c/a/x)^(
1/2)/a/(-a*x+1)^(1/2)-363/64*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/
2)/a^(7/2)/(-a*x+1)^(1/2)+4*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))*
2^(1/2)*(c-c/a/x)^(1/2)*x^(1/2)/a^(7/2)/(-a*x+1)^(1/2)-107/64*x*(c-c/a/x)^(
1/2)*(a*x+1)^(1/2)/a^3/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 101, 154, 157, 54, 215, 93, 206}

$$-\frac{11x^2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{24a^2\sqrt{1-ax}}-\frac{21x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{32a^3\sqrt{1-ax}}-\frac{107x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-ax}}-\frac{363\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{1-ax}}+\frac{4\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (-107*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(64*a^3*Sqrt[1 - a*x]) - (21*Sqrt[
c - c/(a*x)]*x*(1 + a*x)^(3/2))/(32*a^3*Sqrt[1 - a*x]) - (11*Sqrt[c - c/(a*
x)]*x^2*(1 + a*x)^(3/2))/(24*a^2*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*x^3*(1
 + a*x)^(3/2))/(4*a*Sqrt[1 - a*x]) - (363*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh
[Sqrt[a]*Sqrt[x]])/(64*a^(7/2)*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqrt[c - c/(a*x)
]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(7/2)*Sqrt[1
 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{3\tanh^{-1}(ax)}x^{5/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{5/2}(1+ax)^{3/2}}{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x^3(1+ax)^{3/2}}{4a\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{1+ax}\left(\frac{5}{2}+\frac{11ax}{2}\right)}{1-ax}\,dx}{4a\sqrt{1-ax}}$$

$$= -\frac{11\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^3(1+ax)^{3/2}}{4a\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{1+ax}\left(-\right.}{1-a}}{12a^3\sqrt{1-ax}}$$

$$= -\frac{21\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^3(1+ax)^{3/2}}{4a\sqrt{1-ax}} + \frac{\left(\right.}{}$$

$$= -\frac{107\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} - \frac{21\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{}}{}$$

$$= -\frac{107\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} - \frac{21\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{}}{}$$

$$= -\frac{107\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} - \frac{21\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{}}{}$$

$$= -\frac{107\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} - \frac{21\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \frac{\sqrt{}}{}$$

<u>**Mathematica**</u> **[A]** time = 0.10, size = 130, normalized size = 0.45

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\,x\sqrt{ax+1}\left(48a^3x^3+136a^2x^2+214ax+447\right)+1089\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-768\sqrt{2}\sqrt{x}\tanh^{-1}\left(\right.\right.}{192a^{7/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] -1/192*(Sqrt[c - c/(a*x)]*(Sqrt[a]*x*Sqrt[1 + a*x]*(447 + 214*a*x + 136*a^2
*x^2 + 48*a^3*x^3) + 1089*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]] - 768*Sqrt[2]*Sq
rt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(a^(7/2)*Sqrt[1 -
a*x])
```

fricas [A] time = 0.55, size = 500, normalized size = 1.71

$$\left[\frac{768\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+1089\,(ax-1)\sqrt{-c}\,\log\left(-\frac{8\,a}{}\right.}{768\left(a^5x-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [1/768*(768*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 1
3*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c
*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1089*(a*x - 1)*sqr
t(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*
sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(48*a^4*x^4 + 136*a^3*
x^3 + 214*a^2*x^2 + 447*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a
^5*x - a^4), -1/384*(768*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a
^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)
) - 1089*(a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*
c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(48*a^4*x^4 + 136*a^3*x^3 +
214*a^2*x^2 + 447*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x -
 a^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}\,x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.06, size = 247, normalized size = 0.85

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\, \left(96a^{\frac{9}{2}}\sqrt{2}\, \sqrt{-\frac{1}{a}}\, \sqrt{-(ax+1)\,x}\, x^3+272a^{\frac{7}{2}}\sqrt{2}\, \sqrt{-\frac{1}{a}}\, \sqrt{-(ax+1)\,x}\, x^2+428a^{\frac{5}{2}}\sqrt{2}\, \sqrt{-\frac{1}{a}}\right.}{768a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/768*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(96*a^(9/2)*2^(1/2)*(-1/a)
^(1/2)*(-(a*x+1)*x)^(1/2)*x^3+272*a^(7/2)*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)
^(1/2)*x^2+428*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*x+894*(-(a*x
+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)-1089*arctan(1/2/a^(1/2)*(2*a*x+1)
/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2)+1536*ln((2*2^(1/2)*(-1/a)^(1/2)
*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1/2))*2^(1/2)/a^(9/2)/(a*x-1)/(-
(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 \sqrt{c-\frac{c}{ax}}\, x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\, \sqrt{c-\frac{c}{a\,x}}\, (a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**
(3/2), x)
```

3.583 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=248

$$-\frac{45\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{5/2}\sqrt{1-ax}}-\frac{3x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-ax}}-\frac{13x\sqrt{ax+1}\sqrt{c}}{8a^2\sqrt{1-ax}}$$

```
[Out] -3/4*x*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/a^2/(-a*x+1)^(1/2)-1/3*x^2*(a*x+1)^(3/
2)*(c-c/a/x)^(1/2)/a/(-a*x+1)^(1/2)-45/8*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)
^(1/2)*x^(1/2)/a^(5/2)/(-a*x+1)^(1/2)+4*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*
x+1)^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)*x^(1/2)/a^(5/2)/(-a*x+1)^(1/2)-13/8*x*(
c-c/a/x)^(1/2)*(a*x+1)^(1/2)/a^2/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 101, 154, 157, 54, 215, 93, 206}

$$-\frac{3x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-ax}}-\frac{13x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}-\frac{45\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (-13*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) - (3*Sqrt[c -
 c/(a*x)]*x*(1 + a*x)^(3/2))/(4*a^2*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*x^2
*(1 + a*x)^(3/2))/(3*a*Sqrt[1 - a*x]) - (45*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSi
nh[Sqrt[a]*Sqrt[x]])/(8*a^(5/2)*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqrt[c - c/(a*x
)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(5/2)*Sqrt[
1 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{3\tanh^{-1}(ax)}x^{3/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}(1+ax)^{3/2}}{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{1+ax}\left(\frac{3}{2}+\frac{9ax}{2}\right)}{1-ax}\,dx}{3a\sqrt{1-ax}}$$

$$= -\frac{3\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}\left(-\frac{9a}{4}-\frac{39}{\cdots}\right.}{\sqrt{x}(1-ax)}}{6a^3\sqrt{1-ax}}$$

$$= -\frac{13\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{3\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}} + \frac{\left(\sqrt{c-\cdots}\right.}{\cdots}$$

$$= -\frac{13\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{3\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}} - \frac{\left(45\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{13\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{3\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}} - \frac{\left(45\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{13\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{3\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{4a^2\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)^{3/2}}{3a\sqrt{1-ax}} - \frac{45\sqrt{c\cdots}}{\cdots}$$

Mathematica [A] time = 0.09, size = 122, normalized size = 0.49

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\,x\sqrt{ax+1}\left(8a^2x^2+26ax+57\right)+135\sqrt{x}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)-96\sqrt{2}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{24a^{5/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -1/24*(Sqrt[c - c/(a*x)]*(Sqrt[a]*x*Sqrt[1 + a*x]*(57 + 26*a*x + 8*a^2*x^2)
 + 135*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]] - 96*Sqrt[2]*Sqrt[x]*ArcTanh[(Sqrt[
2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(a^(5/2)*Sqrt[1 - a*x])
```

fricas [A] time = 0.65, size = 484, normalized size = 1.95

$$\left[\frac{96\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+135\,(ax-1)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3 \cdots}{\cdots}\right)\cdots}{96\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [1/96*(96*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*
a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x
 - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 135*(a*x - 1)*sqrt(-
c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(8*a^3*x^3 + 26*a^2*x^2 +
 57*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), -1/48*(
96*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*
x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 135*(a*x - 1)*sqrt
(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*
c*x^2 - a*c*x - c)) - 2*(8*a^3*x^3 + 26*a^2*x^2 + 57*a*x)*sqrt(-a^2*x^2 + 1
)*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 219, normalized size = 0.88

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(16a^{\frac{7}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)x}\,x^2+52a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)x}\,x+114\sqrt{-(ax+1)x}\,a\right.}{96a^{\frac{7}{2}}(ax-1)\sqrt{-(ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/96*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(16*a^(7/2)*2^(1/2)*(-1/a)^
(1/2)*(-(a*x+1)*x)^(1/2)*x^2+52*a^(5/2)*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(
1/2)*x+114*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2)-135*arctan(1/2/a
^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2)+192*ln((2*2^(1/
2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1/2))*2^(1/2)/a^(
7/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}\,x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**
(3/2), x)
```

3.584 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=204

$$-\frac{23\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{1-ax}}-\frac{x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{2a\sqrt{1-ax}}-\frac{7x\sqrt{ax+1}\sqrt{c-}}{4a\sqrt{1-ax}}$$

```
[Out] -1/2*x*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/a/(-a*x+1)^(1/2)-23/4*arcsinh(a^(1/2)*
x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(3/2)/(-a*x+1)^(1/2)+4*arctanh(2^(1/2)*a
^(1/2)*x^(1/2)/(a*x+1)^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)*x^(1/2)/a^(3/2)/(-a*x
+1)^(1/2)-7/4*x*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/a/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6134, 6129, 101, 154, 157, 54, 215, 93, 206}

$$-\frac{23\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{a^{3/2}\sqrt{1-ax}}-\frac{x(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{2a\sqrt{1-ax}}-\frac{7x\sqrt{ax+1}\sqrt{c-}}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (-7*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) - (Sqrt[c - c/(a
*x)]*x*(1 + a*x)^(3/2))/(2*a*Sqrt[1 - a*x]) - (23*Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcSinh[Sqrt[a]*Sqrt[x]])/(4*a^(3/2)*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqrt[c -
c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(a^(3/2)
*Sqrt[1 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
```

```
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{3\tanh^{-1}(ax)}\sqrt{x}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}(1+ax)^{3/2}}{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{2a\sqrt{1-ax}} + \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}\left(\frac{1}{2}+\frac{7ax}{2}\right)}{\sqrt{x}(1-ax)}\,dx}{2a\sqrt{1-ax}}$$

$$= -\frac{7\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{2a\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{9a}{4}-\frac{23a^2x}{4}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{2a^2\sqrt{1-ax}}$$

$$= -\frac{7\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{2a\sqrt{1-ax}} - \frac{\left(23\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{8a\sqrt{1-ax}} +$$

$$= -\frac{7\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{2a\sqrt{1-ax}} - \frac{\left(23\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\right.}{4a\sqrt{1-ax}}$$

$$= -\frac{7\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x(1+ax)^{3/2}}{2a\sqrt{1-ax}} - \frac{23\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}} +$$

Mathematica [A] time = 0.07, size = 114, normalized size = 0.56

$$-\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\,x\sqrt{ax+1}\,(2ax+9)+23\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-16\sqrt{2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{4a^{3/2}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] -1/4*(Sqrt[c - c/(a*x)]*(Sqrt[a]*x*Sqrt[1 + a*x]*(9 + 2*a*x) + 23*Sqrt[x]*A
rcSinh[Sqrt[a]*Sqrt[x]] - 16*Sqrt[2]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[
x])/Sqrt[1 + a*x]]))/(a^(3/2)*Sqrt[1 - a*x])
```

fricas [A] time = 0.61, size = 468, normalized size = 2.29

$$\left[\frac{16\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx+4\sqrt{2}\left(3a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+23(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-}{}\right.}{16\left(a^3x-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/16*(16*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*
a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x
 - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 23*(a*x - 1)*sqrt(-c
)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt
(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(2*a^2*x^2 + 9*a*x)*sqrt(-
a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(16*sqrt(2)*(a*x
- 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 23*(a*x - 1)*sqrt(c)*arctan(2*sqrt
(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c
)) - 2*(2*a^2*x^2 + 9*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3
*x - a^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.05, size = 191, normalized size = 0.94

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(4a^{\frac{5}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)\,x}\,x+18\sqrt{-(ax+1)\,x}\,a^{\frac{3}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}-23\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)}}\right.\right.}{16a^{\frac{5}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}\,\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/16*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(4*a^(5/2)*2^(1/2)*(-1/a)^(
1/2)*(-(a*x+1)*x)^(1/2)*x+18*(-(a*x+1)*x)^(1/2)*a^(3/2)*2^(1/2)*(-1/a)^(1/2
)-23*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a*2^(1/2)*(-1/a)^(1/2
)+32*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*x-1)/(a*x-1))*a^(1
/2))*2^(1/2)/a^(5/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}\,x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))*x/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x*(c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/
2), x)
```

3.585 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=155

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{5\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] -5*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(1/2)/(-a*x+1)^(1/2)+
4*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)*x^
(1/2)/a^(1/2)/(-a*x+1)^(1/2)-x*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6134, 6129, 102, 157, 54, 215, 93, 206}

$$-\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{5\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/Sqrt[1 - a*x]) - (5*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x]) + (4*Sqrt[2]*Sqr
t[c - c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/(S
qrt[a]*Sqrt[1 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{\sqrt{x}(1-ax)}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{3a}{2}-\frac{5a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}} + \frac{\left(4\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}(1-ax)}}{\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{1-ax}} + \frac{\left(8\sqrt{c-\frac{c}{ax}}\right.}{}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{5\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}} + \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\sqrt{x}\tanh^{-1}\left(\right.}{\sqrt{a}\sqrt{1-ax}}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.68

$$-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}+5\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
[Out] -((Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x] + 5*ArcSinh[Sqr
t[a]*Sqrt[x]] - 4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])
)/(Sqrt[a]*Sqrt[1 - a*x]))
```

fricas [A] time = 0.56, size = 442, normalized size = 2.85

$$\left[\frac{4\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx-c}{ax}}+4\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx+4\sqrt{2}\left(3a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+5}{4\left(a^2x-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*(a*x - 1
)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x
^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c
*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)
) - c)/(a*x - 1)))/(a^2*x - a), 1/2*(2*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x -
 c)/(a*x)) - 4*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + 5*(a*x
 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x
))/(2*a^2*c*x^2 - a*c*x - c)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 165, normalized size = 1.06

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\,\left(2\sqrt{-(ax+1)\,x}\; a^{\frac{3}{2}}\sqrt{2}\,\sqrt{-\frac{1}{a}}\, -5\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right) a\sqrt{2}\,\sqrt{-\frac{1}{a}}\, +8\ln\left(\frac{2\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax\cdots}{ax-1\cdots}\right.\right.}{4\,(ax-1)\sqrt{-(ax+1)\,x}\; a^{\frac{3}{2}}\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*(-(a*x+1)*x)^(1/2)*a^(3/2
)*2^(1/2)*(-1/a)^(1/2)-5*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*a
*2^(1/2)*(-1/a)^(1/2)+8*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a
*x-1)/(a*x-1))*a^(1/2))*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/a^(3/2)/(-1/a)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

3.586 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=154

$$-\frac{2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}$$

```
[Out] -2*arcsinh(a^(1/2)*x^(1/2))*a^(1/2)*(c-c/a/x)^(1/2)*x^(1/2)/(-a*x+1)^(1/2)+
4*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))*2^(1/2)*a^(1/2)*(c-c/a/x)^
(1/2)*x^(1/2)/(-a*x+1)^(1/2)-2*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6134, 6129, 98, 157, 54, 215, 93, 206}

$$-\frac{2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}+\frac{4\sqrt{2}\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (2*Sqrt[a]*Sqrt[c - c/
(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x] + (4*Sqrt[2]*Sqrt[a]
*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]
)/Sqrt[1 - a*x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{3/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{3a}{2}-\frac{a^2x}{2}}{\sqrt{x}(1-ax)\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}} + \frac{\left(4a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}(1-ax)\sqrt{}}}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{1-ax}} + \frac{\left(8a\sqrt{c-\frac{c}{ax}}\right.}{}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}} + \frac{4\sqrt{2}\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\tan}{\sqrt{1-ax}}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 0.67

$$-\frac{2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}+\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)-2\sqrt{2}\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x] + Sqrt[a]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt
[x]] - 2*Sqrt[2]*Sqrt[a]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 +
 a*x]]))/Sqrt[1 - a*x]
```

fricas [A] time = 0.91, size = 429, normalized size = 2.79

$$\left[\frac{2\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx+4\sqrt{2}\left(3a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx}{}\right.}{2(ax-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fric
as")
```

```
[Out] [1/2*(2*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*
c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x -
 c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + (a*x - 1)*sqrt(-c)*log
(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*
sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x
- c)/(a*x)))/(a*x - 1), -(2*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(2*sqrt(2)*sqrt
(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x -
 c)) - (a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*
x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x
 - c)/(a*x)))/(a*x - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 166, normalized size = 1.08

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{-a^{2}x^{2}+1}\,\left(-\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\sqrt{2}\,\sqrt{-\frac{1}{a}}\,xa+2\sqrt{-(ax+1)\,x}\,\sqrt{a}\,\sqrt{2}\,\sqrt{-\frac{1}{a}}+4\sqrt{a}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-}}{}\right)\right)}{2\,(ax-1)\,\sqrt{-(ax+1)\,x}\,\sqrt{a}\,\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)*(-arctan(1/2/a^(1/2)*(2*a*x+1)
/(-(a*x+1)*x)^(1/2))*2^(1/2)*(-1/a)^(1/2)*x*a+2*(-(a*x+1)*x)^(1/2)*a^(1/2)*
2^(1/2)*(-1/a)^(1/2)+4*a^(1/2)*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2
)*a-3*a*x-1)/(a*x-1))*x)*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/a^(1/2)/(-1/a)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^3}{x\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x\left(-(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3
/2)), x)
```

3.587 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=147

$$\frac{4\sqrt{2}\,a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{4a\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

```
[Out] -2/3*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/x/(-a*x+1)^(1/2)+4*a^(3/2)*arctanh(2^(1/
2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)*x^(1/2)/(-a*x+1)^
(1/2)-4*a*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6134, 6129, 94, 93, 206}

$$\frac{4\sqrt{2}\,a^{3/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{4a\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{2(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-4*a*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (2*Sqrt[c - c/(a*x)]
*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(3/2)*Sqrt[c - c/(a*x)
]*Sqrt[x]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{5/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{5/2}(1-ax)}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\,(1+ax)^{3/2}}{3x\sqrt{1-ax}}+\frac{\left(2a\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{3/2}(1-ax)}\,dx}{\sqrt{1-ax}}\\
&= -\frac{4a\sqrt{c-\frac{c}{ax}}\,\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}\,(1+ax)^{3/2}}{3x\sqrt{1-ax}}+\frac{\left(4a^2\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{1}{\sqrt{x}\,(1-ax)\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{4a\sqrt{c-\frac{c}{ax}}\,\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}\,(1+ax)^{3/2}}{3x\sqrt{1-ax}}+\frac{\left(8a^2\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\mathrm{Subst}\left(\int \frac{1}{1-2ax^2}\,dx\right.}{\sqrt{1-ax}}\\
&= -\frac{4a\sqrt{c-\frac{c}{ax}}\,\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}\,(1+ax)^{3/2}}{3x\sqrt{1-ax}}+\frac{4\sqrt{2}\,a^{3/2}\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{}}{\sqrt{1+ax}}\right.}{\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 93, normalized size = 0.63

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(6\sqrt{2}a^{3/2}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)-\sqrt{ax+1}\,(7ax+1)\right)}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-(Sqrt[1 + a*x]*(1 + 7*a*x)) + 6*Sqrt[2]*a^(3/2)*x^(3
/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(3*x*Sqrt[1 - a*x])
```

fricas [A] time = 0.55, size = 309, normalized size = 2.10

$$\left[\frac{3\sqrt{2}\left(a^2x^2-ax\right)\sqrt{-c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,\sqrt{-a^2x^2+1}\,(7\,ax+1)\sqrt{\vphantom{x}}}{3\left(ax^2-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] [1/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 -
 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a
*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*sqrt(-a^2*x^2
+ 1)*(7*a*x + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x), -2/3*(3*sqrt(2)*(a^2
*x^2 - a*x)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - sqrt(-a^2*x^2 + 1)*(7*a*x +
 1)*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
```

```
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A]   time = 0.06, size = 150, normalized size = 1.02

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{-a^2x^2+1}\,\left(7a\sqrt{2}\,\sqrt{-\frac{1}{a}}\,x\sqrt{-(ax+1)\,x}+6a\ln\left(\frac{2\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)x}\,a-3ax-1}{ax-1}\right)x^2+\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1}\right.}{3x\,(ax-1)\,\sqrt{-(ax+1)\,x}\,\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] 1/3*(c*(a*x-1)/a/x)^(1/2)/x*(-a^2*x^2+1)^(1/2)*(7*a*2^(1/2)*(-1/a)^(1/2)*x*
(-(a*x+1)*x)^(1/2)+6*a*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)*a-3*a*
x-1)/(a*x-1))*x^2+2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2))*2^(1/2)/(a*x-1)/
(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^3}{x^2\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^2\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x + 1))*
*(3/2)), x)
```

3.588 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=191

$$\frac{4\sqrt{2}\,a^{5/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{4a^2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}-\frac{2(ax+1)^{5/2}\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}-\frac{2a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

```
[Out] -2/3*a*(a*x+1)^(3/2)*(c-c/a/x)^(1/2)/x/(-a*x+1)^(1/2)-2/5*(a*x+1)^(5/2)*(c-
c/a/x)^(1/2)/x^2/(-a*x+1)^(1/2)+4*a^(5/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(
a*x+1)^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)*x^(1/2)/(-a*x+1)^(1/2)-4*a^2*(c-c/a/x
)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 96, 94, 93, 206}

$$-\frac{4a^2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{4\sqrt{2}\,a^{5/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{2(ax+1)^{5/2}\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}-\frac{2a(ax+1)^{3/2}\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (-4*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (2*a*Sqrt[c - c/(a
*x)]*(1 + a*x)^(3/2))/(3*x*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a*x)]*(1 + a*x)^
(5/2))/(5*x^2*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(5/2)*Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{7/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{5/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{\left(2a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1+ax}}{x^{3/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{4a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{\left(4a^3\sqrt{c-\right.}{}$$
$$= -\frac{4a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{\left(8a^3\sqrt{c-\right.}{}$$
$$= -\frac{4a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2a\sqrt{c-\frac{c}{ax}}(1+ax)^{3/2}}{3x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}(1+ax)^{5/2}}{5x^2\sqrt{1-ax}} + \frac{4\sqrt{2}a^{5/2}\sqrt{}}{}$$

Mathematica [A] time = 0.06, size = 101, normalized size = 0.53

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(30\sqrt{2}a^{5/2}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)-\sqrt{ax+1}\left(38a^2x^2+11ax+3\right)\right)}{15x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-(Sqrt[1 + a*x]*(3 + 11*a*x + 38*a^2*x^2)) + 30*Sqrt[
2]*a^(5/2)*x^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]]))/(15*x
^2*Sqrt[1 - a*x])
```

fricas [A] time = 0.53, size = 337, normalized size = 1.76

$$\left[\frac{15\sqrt{2}\left(a^3x^3-a^2x^2\right)\sqrt{-c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx+4\sqrt{2}\left(3a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+2\left(38a^2x^2+11ax+3\right)}{15\left(ax^3-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fr
icas")
```

```
[Out] [1/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*s
qrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(38*a^2*
x^2 + 11*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x^3 - x^2)
, -2/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*
x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) -
 (38*a^2*x^2 + 11*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x
^3 - x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 181, normalized size = 0.95

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{-a^2x^2+1}\,\left(38a^2\sqrt{2}\,\sqrt{-\frac{1}{a}}\,x^2\sqrt{-(ax+1)x}+30a^2\ln\left(\frac{2\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)x}\,a-3ax-1}{ax-1}\right)x^3+11a\sqrt{2}\,\sqrt{-\frac{1}{a}}\,x}{15x^2(ax-1)\sqrt{-(ax+1)x}\,\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] 1/15*(c*(a*x-1)/a/x)^(1/2)/x^2*(-a^2*x^2+1)^(1/2)*(38*a^2*2^(1/2)*(-1/a)^(1
/2)*x^2*(-(a*x+1)*x)^(1/2)+30*a^2*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(
1/2)*a-3*a*x-1)/(a*x-1))*x^3+11*a*2^(1/2)*(-1/a)^(1/2)*x*(-(a*x+1)*x)^(1/2)
+3*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2))*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1
/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^3}{x^3\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^3\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x + 1))*
*(3/2)), x)
```

3.589 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=237

$$\frac{4\sqrt{2}\,a^{7/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{104a^3\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{21\sqrt{1-ax}}-\frac{32a^2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{21x\sqrt{1-ax}}-\frac{2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}}-$$

```
[Out] 4*a^(7/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))*2^(1/2)*(c-c/a/x)^
(1/2)*x^(1/2)/(-a*x+1)^(1/2)-104/21*a^3*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x
+1)^(1/2)-2/7*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/x^3/(-a*x+1)^(1/2)-6/7*a*(c-c/a
/x)^(1/2)*(a*x+1)^(1/2)/x^2/(-a*x+1)^(1/2)-32/21*a^2*(c-c/a/x)^(1/2)*(a*x+1
)^(1/2)/x/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6129, 98, 152, 12, 93, 206}

$$-\frac{104a^3\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{21\sqrt{1-ax}}-\frac{32a^2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{21x\sqrt{1-ax}}+\frac{4\sqrt{2}\,a^{7/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{6a\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{7x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-104*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(21*Sqrt[1 - a*x]) - (2*Sqrt[c -
 c/(a*x)]*Sqrt[1 + a*x])/(7*x^3*Sqrt[1 - a*x]) - (6*a*Sqrt[c - c/(a*x)]*Sqr
t[1 + a*x])/(7*x^2*Sqrt[1 - a*x]) - (32*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]
)/(21*x*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(7/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcTa
nh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{9/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{15a}{2}-\frac{13a^2x}{2}}{x^{7/2}(1-ax)\sqrt{1+ax}}\,dx}{7\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} + \frac{\left(4\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{20a^2+15a^3x}{x^{5/2}(1-ax)\sqrt{1+ax}}\,dx}{35\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21x\sqrt{1-ax}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right.}{1}$$

$$= -\frac{104a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-}}{21x\sqrt{1}}$$

$$= -\frac{104a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-}}{21x\sqrt{1}}$$

$$= -\frac{104a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-}}{21x\sqrt{1}}$$

$$= -\frac{104a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{21\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^3\sqrt{1-ax}} - \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{7x^2\sqrt{1-ax}} - \frac{32a^2\sqrt{c-}}{21x\sqrt{1}}$$

Mathematica [A] time = 0.07, size = 109, normalized size = 0.46

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(42\sqrt{2}\,a^{7/2}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)-\sqrt{ax+1}\left(52a^3x^3+16a^2x^2+9ax+3\right)\right)}{21x^3\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
[Out] (2*Sqrt[c - c/(a*x)]*(-(Sqrt[1 + a*x]*(3 + 9*a*x + 16*a^2*x^2 + 52*a^3*x^3)
```

```
) + 42*Sqrt[2]*a^(7/2)*x^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a
*x]]))/(21*x^3*Sqrt[1 - a*x])
```

fricas [A] time = 0.58, size = 353, normalized size = 1.49

$$\left[\frac{21\sqrt{2}\left(a^4x^4-a^3x^3\right)\sqrt{-c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx+4\,\sqrt{2}\left(3\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\left(52\,a^3x^3+16\,a^2x^2+\right.}{21\left(ax^4-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fr
icas")
```

```
[Out] [1/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*s
qrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(52*a^3*
x^3 + 16*a^2*x^2 + 9*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(
a*x^4 - x^3), -2/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(2*sqrt(2
)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a
*c*x - c)) - (52*a^3*x^3 + 16*a^2*x^2 + 9*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt(
(a*c*x - c)/(a*x)))/(a*x^4 - x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 209, normalized size = 0.88

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{-a^2x^2+1}\,\left(52x^3\sqrt{-(ax+1)\,x}\;a^3\sqrt{2}\,\sqrt{-\frac{1}{a}}+42a^3\ln\left(\frac{2\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)x}\;a-3ax-1}{ax-1}\right)x^4+16a^2\sqrt{2}\,\sqrt{-\frac{1}{a}}\;x\right.}{21x^3\,(ax-1)\,\sqrt{-(ax+1)\,x}\,\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] 1/21*(c*(a*x-1)/a/x)^(1/2)/x^3*(-a^2*x^2+1)^(1/2)*(52*x^3*(-(a*x+1)*x)^(1/2
)*a^3*2^(1/2)*(-1/a)^(1/2)+42*a^3*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(
1/2)*a-3*a*x-1)/(a*x-1))*x^4+16*a^2*2^(1/2)*(-1/a)^(1/2)*x^2*(-(a*x+1)*x)^(
1/2)+9*a*2^(1/2)*(-1/a)^(1/2)*x*(-(a*x+1)*x)^(1/2)+3*2^(1/2)*(-1/a)^(1/2)*(
-(a*x+1)*x)^(1/2))*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^3}{x^4\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x^4*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x^4*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^4\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x**4*(-(a*x - 1)*(a*x + 1))*
*(3/2)), x)
```

3.590 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=281

$$\frac{4\sqrt{2}\,a^{9/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}-\frac{1576a^4\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{315\sqrt{1-ax}}-\frac{472a^3\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{315x\sqrt{1-ax}}-\frac{92a^2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{105x^2\sqrt{1-ax}}$$

```
[Out] 4*a^(9/2)*arctanh(2^(1/2)*a^(1/2)*x^(1/2)/(a*x+1)^(1/2))*2^(1/2)*(c-c/a/x)^
(1/2)*x^(1/2)/(-a*x+1)^(1/2)-1576/315*a^4*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a
*x+1)^(1/2)-2/9*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/x^4/(-a*x+1)^(1/2)-38/63*a*(c
-c/a/x)^(1/2)*(a*x+1)^(1/2)/x^3/(-a*x+1)^(1/2)-92/105*a^2*(c-c/a/x)^(1/2)*(
a*x+1)^(1/2)/x^2/(-a*x+1)^(1/2)-472/315*a^3*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/x
/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6129, 98, 152, 12, 93, 206}

$$-\frac{92a^2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{105x^2\sqrt{1-ax}}-\frac{1576a^4\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{315\sqrt{1-ax}}-\frac{472a^3\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{315x\sqrt{1-ax}}+\frac{4\sqrt{2}\,a^{9/2}\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (-1576*a^4*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(315*Sqrt[1 - a*x]) - (2*Sqrt[c
 - c/(a*x)]*Sqrt[1 + a*x])/(9*x^4*Sqrt[1 - a*x]) - (38*a*Sqrt[c - c/(a*x)]*
Sqrt[1 + a*x])/(63*x^3*Sqrt[1 - a*x]) - (92*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 +
a*x])/(105*x^2*Sqrt[1 - a*x]) - (472*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(
315*x*Sqrt[1 - a*x]) + (4*Sqrt[2]*a^(9/2)*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcTanh
[(Sqrt[2]*Sqrt[a]*Sqrt[x])/Sqrt[1 + a*x]])/Sqrt[1 - a*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{11/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1+ax)^{3/2}}{x^{11/2}(1-ax)}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-\frac{19a}{2}-\frac{17a^2x}{2}}{x^{9/2}(1-ax)\sqrt{1+ax}}\,dx}{9\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} + \frac{\left(4\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\frac{69a^2}{2}+\frac{57a^3x}{2}}{x^{7/2}(1-ax)\sqrt{1+ax}}\,dx}{63\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105x^2\sqrt{1-ax}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right.}{3}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105x^2\sqrt{1-ax}} - \frac{472a^3\sqrt{c-}}{315x\sqrt{1}}$$

$$= -\frac{1576a^4\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{315\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-}}{105x^2}$$

$$= -\frac{1576a^4\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{315\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-}}{105x^2}$$

$$= -\frac{1576a^4\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{315\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-}}{105x^2}$$

$$= -\frac{1576a^4\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{315\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{9x^4\sqrt{1-ax}} - \frac{38a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{63x^3\sqrt{1-ax}} - \frac{92a^2\sqrt{c-}}{105x^2}$$

Mathematica [A] time = 0.08, size = 117, normalized size = 0.42

$$\frac{2\sqrt{c-\frac{c}{ax}}\left(630\sqrt{2}\,a^{9/2}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{ax+1}}\right)-\sqrt{ax+1}\left(788a^4x^4+236a^3x^3+138a^2x^2+95ax+35\right)\right)}{315x^4\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-(Sqrt[1 + a*x]*(35 + 95*a*x + 138*a^2*x^2 + 236*a^3*
x^3 + 788*a^4*x^4)) + 630*Sqrt[2]*a^(9/2)*x^(9/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*
Sqrt[x])/Sqrt[1 + a*x]]))/(315*x^4*Sqrt[1 - a*x])
```

fricas [A] time = 0.48, size = 369, normalized size = 1.31

$$\left[\frac{315\,\sqrt{2}\left(a^{5}x^{5}-a^{4}x^{4}\right)\sqrt{-c}\,\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx+4\,\sqrt{2}\left(3\,a^{2}x^{2}+ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+2\left(788\,a^{4}x^{4}+236\,a^{3}\right.}{315\left(ax^{5}-x^{4}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fr
icas")
```

```
[Out] [1/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(17*a^3*c*x^3 - 3*a^2
*c*x^2 - 13*a*c*x + 4*sqrt(2)*(3*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)
*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(788*a
^4*x^4 + 236*a^3*x^3 + 138*a^2*x^2 + 95*a*x + 35)*sqrt(-a^2*x^2 + 1)*sqrt((
a*c*x - c)/(a*x)))/(a*x^5 - x^4), -2/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*s
qrt(c)*arctan(2*sqrt(2)*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - (788*a^4*x^4 + 236*a^3*x^3 + 138*a^2*x^2
 + 95*a*x + 35)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x^5 - x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 237, normalized size = 0.84

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{-a^2x^2+1}\,\left(788x^4\sqrt{-(ax+1)x}\,a^4\sqrt{2}\,\sqrt{-\frac{1}{a}}+630a^4\ln\left(\frac{2\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\sqrt{-(ax+1)x}\,a-3ax-1}{ax-1}\right)x^5+236x^3\sqrt{-(ax}{315x^4(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] 1/315*(c*(a*x-1)/a/x)^(1/2)/x^4*(-a^2*x^2+1)^(1/2)*(788*x^4*(-(a*x+1)*x)^(1
/2)*a^4*2^(1/2)*(-1/a)^(1/2)+630*a^4*ln((2*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x
)^(1/2)*a-3*a*x-1)/(a*x-1))*x^5+236*x^3*(-(a*x+1)*x)^(1/2)*a^3*2^(1/2)*(-1/
a)^(1/2)+138*a^2*2^(1/2)*(-1/a)^(1/2)*x^2*(-(a*x+1)*x)^(1/2)+95*a*2^(1/2)*(
-1/a)^(1/2)*x*(-(a*x+1)*x)^(1/2)+35*2^(1/2)*(-1/a)^(1/2)*(-(a*x+1)*x)^(1/2)
)*2^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)/(-1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{ax}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a*x))/((-a^2*x^2 + 1)^(3/2)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)^3}{x^5\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x^5*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1)^3)/(x^5*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)^3}{x^5\,(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)**3/(x**5*(-(a*x - 1)*(a*x + 1))*
*(3/2)), x)
```

3.591 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx$

Optimal. Leaf size=116

$$\frac{(4m+3)x^{m+1}\sqrt{c-\frac{c}{ax}}\ {}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};-ax\right)}{(m+1)(2m+1)\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)(1-ax)}$$

```
[Out] (3+4*m)*x^(1+m)*hypergeom([1/2, 1/2+m],[3/2+m],-a*x)*(c-c/a/x)^(1/2)/(2*m^2
+3*m+1)/(-a*x+1)^(1/2)-x^(1+m)*(c-c/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/(1+m)/(-a
*x+1)
```

Rubi [A] time = 0.29, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6134, 6128, 881, 848, 64}

$$\frac{(4m+3)x^{m+1}\sqrt{c-\frac{c}{ax}}\ {}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};-ax\right)}{(m+1)(2m+1)\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\,x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^m)/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x^(1 + m)*Sqrt[1 - a^2*x^2])/((1 + m)*(1 - a*x))) + ((
3 + 4*m)*Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[1/2, 1/2 + m, 3/2 +
m, -(a*x)])/((1 + m)*(1 + 2*m)*Sqrt[1 - a*x])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
```

```
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-\tanh^{-1}(ax)}x^{-\frac{1}{2}+m}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}(1-ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x^{1+m}\sqrt{1-a^2x^2}}{(1+m)(1-ax)}+\frac{\left((3+4m)\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}\sqrt{1-ax}}{\sqrt{1-a^2x^2}}\,dx}{2(1+m)\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x^{1+m}\sqrt{1-a^2x^2}}{(1+m)(1-ax)}+\frac{\left((3+4m)\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}}{\sqrt{1+ax}}\,dx}{2(1+m)\sqrt{1-ax}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,x^{1+m}\sqrt{1-a^2x^2}}{(1+m)(1-ax)}+\frac{(3+4m)\sqrt{c-\frac{c}{ax}}\,x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;-ax\right)}{(1+m)(1+2m)\sqrt{1-ax}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.75

$$-\frac{2x^{m+1}\sqrt{c-\frac{c}{ax}}\left(a(4m+3)x\,{}_2F_1\left(\frac{1}{2},m+\frac{3}{2};m+\frac{5}{2};-ax\right)-(2m+3)\sqrt{ax+1}\right)}{(4m^2+8m+3)\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^m)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*x^(1 + m)*(-((3 + 2*m)*Sqrt[1 + a*x]) + a*(3 + 4*m)*x
*Hypergeometric2F1[1/2, 3/2 + m, 5/2 + m, -(a*x)]))/((3 + 8*m + 4*m^2)*Sqrt
[1 - a*x])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m\sqrt{\frac{acx-c}{ax}}}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*sqrt((a*c*x - c)/(a*x))/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}\,x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x^m/(a*x + 1), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{c-\frac{c}{ax}}\,\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}\,x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x^m/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\sqrt{c-\frac{c}{a\,x}}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x^m*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1),
 x)
```

3.592 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=182

$$\frac{11\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{3(1-ax)}-\frac{11x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}+\frac{11x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-ax}}$$

```
[Out] 11/8*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(5/2)/(-a*x+1)^(1/2
)-11/8*x*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/a^2/(-a*x+1)^(1/2)+11/12*x^2*(c-c/a/
x)^(1/2)*(a*x+1)^(1/2)/a/(-a*x+1)^(1/2)-1/3*x^3*(c-c/a/x)^(1/2)*(-a^2*x^2+1
)^(1/2)/(-a*x+1)
```

Rubi [A] time = 0.31, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6128, 881, 848, 50, 54, 215}

$$-\frac{x^3\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{3(1-ax)}-\frac{11x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}+\frac{11\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{11x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (-11*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) + (11*Sqrt[c
 - c/(a*x)]*x^2*Sqrt[1 + a*x])/(12*a*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*x)]*x^3
*Sqrt[1 - a^2*x^2])/(3*(1 - a*x)) + (11*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[S
qrt[a]*Sqrt[x]])/(8*a^(5/2)*Sqrt[1 - a*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-\tanh^{-1}(ax)}x^{3/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}(1-ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1-a^2x^2}}{3(1-ax)} + \frac{\left(11\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{1-ax}}{\sqrt{1-a^2x^2}}\,dx}{6\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1-a^2x^2}}{3(1-ax)} + \frac{\left(11\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}}{\sqrt{1+ax}}\,dx}{6\sqrt{1-ax}}$$

$$= \frac{11\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1-a^2x^2}}{3(1-ax)} - \frac{\left(11\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}\,dx}{8a\sqrt{1-ax}}$$

$$= -\frac{11\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{11\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1-a^2x^2}}{3(1-ax)} + \frac{\left(11\sqrt{c\right.}}{}$$

$$= -\frac{11\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{11\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1-a^2x^2}}{3(1-ax)} + \frac{\left(11\sqrt{c\right.}}{}$$

$$= -\frac{11\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{11\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1-a^2x^2}}{3(1-ax)} + \frac{11\sqrt{c-}}{}$$

Mathematica [A] time = 0.09, size = 88, normalized size = 0.48

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}\left(-8a^2x^2+22ax-33\right)+33\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{24a^{5/2}\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(-33 + 22*a*x - 8
*a^2*x^2) + 33*ArcSinh[Sqrt[a]*Sqrt[x]]))/(24*a^(5/2)*Sqrt[1 - a*x])
```

fricas [A] time = 0.49, size = 292, normalized size = 1.60

$$\left[\frac{33\,(ax-1)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(8\,a^3x^3-22\,a^2x^2+33\,ax\right)\sqrt{-a^2x^2+1}\,\sqrt{}}{96\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/96*(33*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a
*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4
*(8*a^3*x^3 - 22*a^2*x^2 + 33*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x
)))/(a^4*x - a^3), -1/48*(33*(a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*
a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(8*a^3*x
^3 - 22*a^2*x^2 + 33*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^4*
x - a^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[2,0,2]%%%}+%%%{2,[
1,1,1]%%%}+%%%{-2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{2,[4,1,4]%%%}+%%%{1,[
4,0,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-2,[3,1,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[
2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{1,[0,0,0]%%%}] at param
eters values [86,-97,-82]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+
%%%{2,[2,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{-2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}
+%%%{2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[3,2,3]%%%}+%%%{2,[3,1,3]%%%}+%%
%{1,[2,2,2]%%%}+%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%
%{1,[0,0,0]%%%}] at parameters values [7,-27,26]sym2poly/r2sym(const gen &
e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 125, normalized size = 0.69

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\sqrt{-a^2x^2+1}\, \left(16a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x}\, -44a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\, +66\sqrt{a}\, \sqrt{-(ax+1)\,x}\, +33\arctan\left(\frac{2ax}{2\sqrt{a}\, \sqrt{-(}}\right.\right.}{48a^{\frac{5}{2}}\,(ax-1)\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/48*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)/a^(5/2)*(16*a^(5/2)*x^2*(-(
a*x+1)*x)^(1/2)-44*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+66*a^(1/2)*(-(a*x+1)*x)^(1/
2)+33*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}\,x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x^2*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1),
 x)
```

3.593 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=138

$$-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{2(1-ax)}+\frac{7x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

```
[Out] -7/4*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(3/2)/(-a*x+1)^(1/2
)+7/4*x*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/a/(-a*x+1)^(1/2)-1/2*x^2*(c-c/a/x)^(1
/2)*(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

Rubi [A] time = 0.23, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6134, 6128, 881, 848, 50, 54, 215}

$$-\frac{x^2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{2(1-ax)}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{7x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^ArcTanh[a*x],x]
```

```
[Out] (7*Sqrt[c - c/(a*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) - (Sqrt[c - c/(a*
x)]*x^2*Sqrt[1 - a^2*x^2])/(2*(1 - a*x)) - (7*Sqrt[c - c/(a*x)]*Sqrt[x]*Arc
Sinh[Sqrt[a]*Sqrt[x]])/(4*a^(3/2)*Sqrt[1 - a*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-\tanh^{-1}(ax)}\sqrt{x}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}(1-ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1-a^2x^2}}{2(1-ax)} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}\sqrt{1-ax}}{\sqrt{1-a^2x^2}}dx}{4\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1-a^2x^2}}{2(1-ax)} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}dx}{4\sqrt{1-ax}}$$
$$= \frac{7\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1-a^2x^2}}{2(1-ax)} - \frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}dx}{8a\sqrt{1-ax}}$$
$$= \frac{7\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1-a^2x^2}}{2(1-ax)} - \frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}dx\right.}{4a\sqrt{1-ax}}$$
$$= \frac{7\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} - \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1-a^2x^2}}{2(1-ax)} - \frac{7\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.58

$$-\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\sqrt{ax+1}(2ax-7)+7\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{4a^{3/2}\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^ArcTanh[a*x],x]
```

```
[Out] -1/4*(Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*(-7 + 2*a*x)
 + 7*ArcSinh[Sqrt[a]*Sqrt[x]]))/(a^(3/2)*Sqrt[1 - a*x])
```

fricas [A] time = 0.54, size = 276, normalized size = 2.00

$$\left[\frac{7(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(2a^2x^2-7ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{16\left(a^3x-a^2\right)}, \frac{7(ax-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/16*(7*(a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*
x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*
(2*a^2*x^2 - 7*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^
2), 1/8*(7*(a*x - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((
a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(2*a^2*x^2 - 7*a*x)*sqrt(-
a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}\,x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

maple [A] time = 0.05, size = 107, normalized size = 0.78

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\sqrt{-a^2x^2+1}\,\left(4a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\,-14\sqrt{a}\,\sqrt{-(ax+1)\,x}\,-7\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\right)}{8a^{\frac{3}{2}}\,(ax-1)\,\sqrt{-(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(4*a^(3/2)*x*(-(a*x+1)*x)^(1
/2)-14*a^(1/2)*(-(a*x+1)*x)^(1/2)-7*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*
x)^(1/2)))/a^(3/2)/(a*x-1)/(-(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}\,x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
[Out] int((x*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.594 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}$$

```
[Out] 3*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(1/2)/(-a*x+1)^(1/2)-x
*(c-c/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

Rubi [A] time = 0.18, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6134, 6128, 881, 848, 54, 215}

$$\frac{3\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a*x)]*x*Sqrt[1 - a^2*x^2])/(1 - a*x)) + (3*Sqrt[c - c/(a*x)]
*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax} + \frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax} + \frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax} + \frac{\left(3\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{1-ax}}$$
$$= -\frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1-a^2x^2}}{1-ax} + \frac{3\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.74

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\frac{3\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} - \sqrt{x}\sqrt{ax+1}\right)}{\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(-(Sqrt[x]*Sqrt[1 + a*x]) + (3*ArcSinh[Sqrt[a]*S
qrt[x]])/Sqrt[a]))/Sqrt[1 - a*x]
```

fricas [A] time = 0.55, size = 250, normalized size = 2.78

$$\left[\frac{4\sqrt{-a^2x^2+1}\,ax\sqrt{\frac{acx-c}{ax}} + 3\,(ax-1)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)}{4\left(a^2x-a\right)}, \frac{2\sqrt{-a^2x^2+1}\,ax\sqrt{\vphantom{x}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) + 3*(a*x - 1)*sqrt(-
c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)))/(a^2*x - a), 1/2*(2*sqrt(-a^
2*x^2 + 1)*a*x*sqrt((a*c*x - c)/(a*x)) - 3*(a*x - 1)*sqrt(c)*arctan(2*sqrt(
-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)
))/(a^2*x - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

maple [A] time = 0.05, size = 91, normalized size = 1.01

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\sqrt{-a^2x^2+1}\;\left(2\sqrt{a}\;\sqrt{-(ax+1)\,x}\;+3\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right)\right)}{2\,(ax-1)\sqrt{-(ax+1)\,x}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*a^(1/2)*(-(a*x+1)*x)^(1/2
)+3*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/(a*x-1)/(-(a*x+1)*x)^
(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\;\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(a*x + 1), x)
```

3.595 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=89

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}-\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}$$

```
[Out] -2*arcsinh(a^(1/2)*x^(1/2))*a^(1/2)*(c-c/a/x)^(1/2)*x^(1/2)/(-a*x+1)^(1/2)-
2*(c-c/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

Rubi [A] time = 0.27, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6128, 879, 848, 54, 215}

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{1-ax}-\frac{2\sqrt{a}\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 - a^2*x^2])/(1 - a*x) - (2*Sqrt[a]*Sqrt[c - c/
(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{x^{3/2}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{1-ax} - \frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{1-ax} - \frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{1-ax} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{1-ax} - \frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.67

$$-\frac{2\sqrt{c-\frac{c}{ax}}\left(\sqrt{ax+1}+\sqrt{a}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(Sqrt[1 + a*x] + Sqrt[a]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt
[x]]))/Sqrt[1 - a*x]
```

fricas [A] time = 0.63, size = 235, normalized size = 2.64

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, \frac{(ax-1)\sqrt{c}\arctan\left(\frac{2\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [1/2*((a*x - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x - 1), ((a*x - 1)*sqrt(c)*arct
an(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
a*c*x - c)) + 2*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

maple [A] time = 0.05, size = 91, normalized size = 1.02

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\left(-\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)xa+2\sqrt{a}\,\sqrt{-\left(ax+1\right)x}\right)\sqrt{-a^2x^2+1}}{\left(ax-1\right)\sqrt{-\left(ax+1\right)x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (c*(a*x-1)/a/x)^(1/2)*(-arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*
a+2*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^2*x^2+1)^(1/2)/(a*x-1)/(-(a*x+1)*x)^(1/
2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{1-a^2\,x^2}}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,\sqrt{-(ax-1)\,(ax+1)}}{x\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(x*(a*x + 1)),
x)
```

3.596 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=84

$$\frac{10a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{3x(1-ax)}$$

```
[Out] 10/3*a*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)-2/3*(c-c/a/x)^(1/2)*(-a
^2*x^2+1)^(1/2)/x/(-a*x+1)
```

Rubi [A] time = 0.26, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6134, 6128, 879, 848, 37}

$$\frac{10a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{3x(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] (10*a*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(3*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a
*x)]*Sqrt[1 - a^2*x^2])/(3*x*(1 - a*x))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
```

```
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{5/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{x^{5/2}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{3x(1-ax)}-\frac{\left(5a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{x^{3/2}\sqrt{1-a^2x^2}}\,dx}{3\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{3x(1-ax)}-\frac{\left(5a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{1+ax}}\,dx}{3\sqrt{1-ax}}\\
&= \frac{10a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{3\sqrt{1-ax}}-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{3x(1-ax)}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.56

$$\frac{2\sqrt{ax+1}(5ax-1)\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]*(-1 + 5*a*x))/(3*x*Sqrt[1 - a*x])
```

fricas [A] time = 0.46, size = 48, normalized size = 0.57

$$-\frac{2\sqrt{-a^2x^2+1}(5\,ax-1)\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] -2/3*sqrt(-a^2*x^2 + 1)*(5*a*x - 1)*sqrt((a*c*x - c)/(a*x))/(a*x^2 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)
```

maple [A] time = 0.03, size = 46, normalized size = 0.55

$$-\frac{2\,(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{-a^2x^2+1}}{3\,(ax-1)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -2/3*(5*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)
```

mupad [B] time = 1.01, size = 60, normalized size = 0.71

$$\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{10\,x\,\sqrt{1-a^2\,x^2}}{3}-\frac{2\,\sqrt{1-a^2\,x^2}}{3\,a}\right)}{\frac{x}{a}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(x^2*(a*x + 1)),x)
```

```
[Out] ((c - c/(a*x))^(1/2)*((10*x*(1 - a^2*x^2)^(1/2))/3 - (2*(1 - a^2*x^2)^(1/2)
)/(3*a)))/(x/a - x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,\sqrt{-(ax-1)(ax+1)}}{x^2\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(x**2*(a*x + 1)
), x)
```

3.597 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=128

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{5x^2(1-ax)}-\frac{12a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-ax}}+\frac{6a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{5x\sqrt{1-ax}}$$

```
[Out] -12/5*a^2*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)+6/5*a*(c-c/a/x)^(1/2
)*(a*x+1)^(1/2)/x/(-a*x+1)^(1/2)-2/5*(c-c/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2
/(-a*x+1)
```

Rubi [A] time = 0.27, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6128, 879, 848, 45, 37}

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{5x^2(1-ax)}-\frac{12a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-ax}}+\frac{6a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{5x\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] (-12*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(5*Sqrt[1 - a*x]) + (6*a*Sqrt[c -
 c/(a*x)]*Sqrt[1 + a*x])/(5*x*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a*x)]*Sqrt[1
- a^2*x^2])/(5*x^2*(1 - a*x))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{x^{7/2}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{5x^2(1-ax)} - \frac{\left(9a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{x^{5/2}\sqrt{1-a^2x^2}}\,dx}{5\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{5x^2(1-ax)} - \frac{\left(9a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}}$$
$$= \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{5x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{5x^2(1-ax)} + \frac{\left(6a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}}$$
$$= -\frac{12a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{5\sqrt{1-ax}} + \frac{6a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{5x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{5x^2(1-ax)}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.43

$$-\frac{2\sqrt{ax+1}\left(6a^2x^2-3ax+1\right)\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^3),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]*(1 - 3*a*x + 6*a^2*x^2))/(5*x^2*Sqrt[1
- a*x])
```

fricas [A] time = 0.43, size = 58, normalized size = 0.45

$$\frac{2\left(6\,a^2x^2-3\,ax+1\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{acx-c}{ax}}}{5\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] 2/5*(6*a^2*x^2 - 3*a*x + 1)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))/(a*x
^3 - x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

maple [A] time = 0.03, size = 54, normalized size = 0.42

$$\frac{2\left(6a^2x^2-3ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{-a^2x^2+1}}{5x^2\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] 2/5*(6*a^2*x^2-3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2/(a*x-1
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{ax}}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

mupad [B] time = 1.05, size = 79, normalized size = 0.62

$$\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{2\,\sqrt{1-a^2\,x^2}}{5\,a}-\frac{6\,x\,\sqrt{1-a^2\,x^2}}{5}+\frac{12\,a\,x^2\,\sqrt{1-a^2\,x^2}}{5}\right)}{x^3-\frac{x^2}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(x^3*(a*x + 1)),x)
```

```
[Out] ((c - c/(a*x))^(1/2)*((2*(1 - a^2*x^2)^(1/2))/(5*a) - (6*x*(1 - a^2*x^2)^(1
/2))/5 + (12*a*x^2*(1 - a^2*x^2)^(1/2))/5))/(x^3 - x^2/a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{x^{3}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(x**3*(a*x + 1)
), x)
```

3.598 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=172

$$\frac{208a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105\sqrt{1-ax}}-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{7x^3(1-ax)}-\frac{104a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{26a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}}$$

```
[Out] 208/105*a^3*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)+26/35*a*(c-c/a/x)^
(1/2)*(a*x+1)^(1/2)/x^2/(-a*x+1)^(1/2)-104/105*a^2*(c-c/a/x)^(1/2)*(a*x+1)^
(1/2)/x/(-a*x+1)^(1/2)-2/7*(c-c/a/x)^(1/2)*(-a^2*x^2+1)^(1/2)/x^3/(-a*x+1)
```

Rubi [A] time = 0.26, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6128, 879, 848, 45, 37}

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{c-\frac{c}{ax}}}{7x^3(1-ax)}+\frac{208a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105\sqrt{1-ax}}-\frac{104a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{26a\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] (208*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(105*Sqrt[1 - a*x]) + (26*a*Sqrt[
c - c/(a*x)]*Sqrt[1 + a*x])/(35*x^2*Sqrt[1 - a*x]) - (104*a^2*Sqrt[c - c/(a
*x)]*Sqrt[1 + a*x])/(105*x*Sqrt[1 - a*x]) - (2*Sqrt[c - c/(a*x)]*Sqrt[1 - a
^2*x^2])/(7*x^3*(1 - a*x))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6128

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.)*((e_.) + (f_.)*
(x_))^(m_.), x_Symbol] :> Dist[c^n, Int[(e + f*x)^m*(c + d*x)^(p - n)*(1 -
a^2*x^2)^(n/2), x], x] /; FreeQ[{a, c, d, e, f, m, p}, x] && EqQ[a*c + d, 0
] && IntegerQ[(n - 1)/2] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p - n/2 - 1,
 0]) && IntegerQ[2*p]
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^{3/2}}{x^{9/2}\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{7x^3(1-ax)} - \frac{\left(13a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{1-ax}}{x^{7/2}\sqrt{1-a^2x^2}}\,dx}{7\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{7x^3(1-ax)} - \frac{\left(13a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{7/2}\sqrt{1+ax}}\,dx}{7\sqrt{1-ax}}$$
$$= \frac{26a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{35x^2\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{7x^3(1-ax)} + \frac{\left(52a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{1+ax}}\,dx}{35\sqrt{1-ax}}$$
$$= \frac{26a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{35x^2\sqrt{1-ax}} - \frac{104a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105x\sqrt{1-ax}} - \frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{7x^3(1-ax)} - \frac{\left(104a^3\sqrt{c\right.}}{}$$
$$= \frac{208a^3\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105\sqrt{1-ax}} + \frac{26a\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{35x^2\sqrt{1-ax}} - \frac{104a^2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{105x\sqrt{1-ax}} - \frac{2\sqrt{c-}}{7x^3}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.37

$$\frac{2\sqrt{ax+1}\left(104a^3x^3-52a^2x^2+39ax-15\right)\sqrt{c-\frac{c}{ax}}}{105x^3\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcTanh[a*x]*x^4),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]*(-15 + 39*a*x - 52*a^2*x^2 + 104*a^3*x^3
))/(105*x^3*Sqrt[1 - a*x])
```

fricas [A] time = 0.54, size = 66, normalized size = 0.38

$$-\frac{2\left(104\,a^3x^3-52\,a^2x^2+39\,ax-15\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] -2/105*(104*a^3*x^3 - 52*a^2*x^2 + 39*a*x - 15)*sqrt(-a^2*x^2 + 1)*sqrt((a*
c*x - c)/(a*x))/(a*x^4 - x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

maple [A] time = 0.03, size = 62, normalized size = 0.36

$$-\frac{2\left(104x^3a^3-52a^2x^2+39ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{-a^2x^2+1}}{105x^3\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] -2/105*(104*a^3*x^3-52*a^2*x^2+39*a*x-15)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1
)^(1/2)/x^3/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{ax}}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

mupad [B] time = 1.09, size = 86, normalized size = 0.50

$$-\frac{152\,\sqrt{1-a^2\,x^2}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3\,(a\,x-1)}-\frac{26\,\sqrt{1-a^2\,x^2}\,\left(8\,a^2\,x^2+4\,a\,x+7\right)\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(1/2))/(x^4*(a*x + 1)),x)
```

```
[Out] - (152*(1 - a^2*x^2)^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
 - (26*(1 - a^2*x^2)^(1/2)*(4*a*x + 8*a^2*x^2 + 7)*((c*(a*x - 1))/(a*x))^(1
/2))/(105*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\sqrt{-(ax-1)(ax+1)}}{x^4(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*sqrt(-(a*x - 1)*(a*x + 1))/(x**4*(a*x + 1)
), x)
```

3.599 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=172

$$-\frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}+\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}-\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{17x^3\sqrt{c-\frac{c}{ax}}}{24a}$$

```
[Out] -363/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4+4*arctanh(1/2*(c-c/a/x
)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4+149/64*x*(c-c/a/x)^(1/2)/a^3-1
07/96*x^2*(c-c/a/x)^(1/2)/a^2+17/24*x^3*(c-c/a/x)^(1/2)/a-1/4*x^4*(c-c/a/x)
^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$-\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}+\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}-\frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}-\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{17x^3\sqrt{c-\frac{c}{ax}}}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (149*Sqrt[c - c/(a*x)]*x)/(64*a^3) - (107*Sqrt[c - c/(a*x)]*x^2)/(96*a^2) +
 (17*Sqrt[c - c/(a*x)]*x^3)/(24*a) - (Sqrt[c - c/(a*x)]*x^4)/4 - (363*Sqrt[
c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4) + (4*Sqrt[2]*Sqrt[c]*ArcTan
h[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^4
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,x^3(1-ax)}{1+ax}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^4}{1+ax}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^5(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 - \frac{\text{Subst}\left(\int \frac{\frac{17c^2}{2}-\frac{15c^2x}{2a}}{x^4(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$= \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} - \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 + \frac{\text{Subst}\left(\int \frac{\frac{107c^3}{4}-\frac{85c^3x}{4a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{12ac^2}$$

$$= -\frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} + \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} - \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 - \frac{\text{Subst}\left(\int \frac{\frac{447c^4}{8}-\frac{321c^4x}{8a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{24a^2c^3}$$

$$= \frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3} - \frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} + \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} - \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 + \frac{\text{Subst}\left(\int \frac{\frac{1089c}{16}\ldots}{x(a+\ldots}\right.}{24\ldots}$$

$$= \frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3} - \frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} + \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} - \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 + \frac{(363c)\,\text{Subst}\left(\ldots\right.}{1\ldots}$$

$$= \frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3} - \frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} + \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} - \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 - \frac{363\,\text{Subst}\left(\int\ldots\right.}{\ldots}$$

$$= \frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3} - \frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} + \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} - \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 - \frac{363\sqrt{c}\,\tanh^{-1}\ldots}{64a^4\ldots}$$

Mathematica [A] time = 0.15, size = 116, normalized size = 0.67

$$\frac{ax\left(-48a^3x^3+136a^2x^2-214ax+447\right)\sqrt{c-\frac{c}{ax}}-1089\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+768\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{192a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(447 - 214*a*x + 136*a^2*x^2 - 48*a^3*x^3) - 1089*Sq
rt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] + 768*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(192*a^4)
```

fricas [A] time = 0.63, size = 274, normalized size = 1.59

$$\left[\frac{768\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)-2\left(48a^4x^4-136a^3x^3+214a^2x^2-447ax\right)\sqrt{\frac{acx-c}{ax}}+1089\sqrt{c}\log\left(\right.}{384a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/384*(768*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + 3*a*c*x - c)/(a*x + 1)) - 2*(48*a^4*x^4 - 136*a^3*x^3 + 214*a^2*x^2
- 447*a*x)*sqrt((a*c*x - c)/(a*x)) + 1089*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c
)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^4, -1/192*(768*sqrt(2)*sqrt(-c)*arctan(
1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (48*a^4*x^4 - 136*a^3*x^3
 + 214*a^2*x^2 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) - 1089*sqrt(-c)*arctan(sq
rt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [A] time = 0.04, size = 259, normalized size = 1.51

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(96x \left(a\,x^2-x\right)^{\frac{3}{2}} a^{\frac{9}{2}} \sqrt{\frac{1}{a}} - 176 \left(a\,x^2-x\right)^{\frac{3}{2}} a^{\frac{7}{2}} \sqrt{\frac{1}{a}} + 252\sqrt{a\,x^2-x}\, a^{\frac{7}{2}} \sqrt{\frac{1}{a}}\, x - 126\sqrt{a\,x^2-x}\, a^{\frac{5}{2}} \sqrt{\frac{1}{a}} - 7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/384*(c*(a*x-1)/a/x)^(1/2)*x*(96*x*(a*x^2-x)^(3/2)*a^(9/2)*(1/a)^(1/2)-17
6*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1/2)+252*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/
2)*x-126*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)-768*a^(5/2)*(1/a)^(1/2)*((a*x-
1)*x)^(1/2)+768*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)
*a-3*a*x+1)/(a*x+1))+1152*a^2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/
a^(1/2))*(1/a)^(1/2)-63*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(11/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}\,x^3}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))*x^3/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\sqrt{c-\frac{c}{a\,x}}\,\left(a^2\,x^2-1\right)}{(a\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x^3*(c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x^3\sqrt{c-\frac{c}{ax}}}{ax+1}\right)dx-\int \frac{ax^4\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**3*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x**4*sqrt(c -
c/(a*x))/(a*x + 1), x)
```

3.600 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=147

$$\frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}-\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}-\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[Out] 45/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3-4*arctanh(1/2*(c-c/a/x)^(
1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3-19/8*x*(c-c/a/x)^(1/2)/a^2+13/12*
x^2*(c-c/a/x)^(1/2)/a-1/3*x^3*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$-\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}-\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-19*Sqrt[c - c/(a*x)]*x)/(8*a^2) + (13*Sqrt[c - c/(a*x)]*x^2)/(12*a) - (Sq
rt[c - c/(a*x)]*x^3)/3 + (45*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8
*a^3) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^
3
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}\,x^2(1-ax)}{1+ax}\,dx$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{1+ax}\,dx}{c}$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}\,dx}{c}$$

$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3-\frac{\text{Subst}\left(\int\frac{\frac{13c^2}{2}-\frac{11c^2x}{2a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{3c}$$

$$=\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{\text{Subst}\left(\int\frac{\frac{57c^3}{4}-\frac{39c^3x}{4a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{6ac^2}$$

$$=-\frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2}+\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3-\frac{\text{Subst}\left(\int\frac{\frac{135c^4}{8}-\frac{57c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{6a^2c^3}$$

$$=-\frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2}+\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3-\frac{(45c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right.}{16a^3}$$

$$=-\frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2}+\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{45\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\right.}{8a^2}$$

$$=-\frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2}+\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}-\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{45\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}-\frac{4\sqrt{2}}{}$$

Mathematica [A] time = 0.10, size = 108, normalized size = 0.73

$$\frac{ax\left(-8a^2x^2+26ax-57\right)\sqrt{c-\frac{c}{ax}}+135\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-96\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(-57 + 26*a*x - 8*a^2*x^2) + 135*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]] - 96*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqr
t[2]*Sqrt[c])])/(24*a^3)
```

fricas [A] time = 0.58, size = 256, normalized size = 1.74

$$\left[\frac{96\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)-2\left(8a^3x^3-26a^2x^2+57ax\right)\sqrt{\frac{acx-c}{ax}}+135\sqrt{c}\log\left(-2acx-2a\sqrt{c}x\sqrt{\frac{ac}{}}\right.}{48a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)
) - 3*a*c*x + c)/(a*x + 1)) - 2*(8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*c
*x - c)/(a*x)) + 135*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/
(a*x)) + c))/a^3, 1/24*(96*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqr
t((a*c*x - c)/(a*x))/c) - (8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*c*x - c
)/(a*x)) - 135*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [A] time = 0.04, size = 237, normalized size = 1.61

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(16\left(a\,x^2-x\right)^{\frac{3}{2}} a^{\frac{7}{2}}\sqrt{\frac{1}{a}}-36\sqrt{a\,x^2-x}\; a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\; x+18\sqrt{a\,x^2-x}\; a^{\frac{5}{2}}\sqrt{\frac{1}{a}}+96a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\;\sqrt{(ax-1)\,x}-96a^{\frac{3}{2}}\right.}{48\sqrt{(ax-1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1/2)-36*(a
*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x+18*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)+
96*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)-96*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(
1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))-144*a^2*ln(1/2*(2*((a*x-1)
*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)+9*ln(1/2*(2*(a*x^2-x)^(1/2)
*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(9/2)/(1/a)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}\,x^2}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\;\left(a^2\,x^2-1\right)}{\left(a\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x^2*(c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x^2\sqrt{c-\frac{c}{ax}}}{ax+1}\right)dx-\int\frac{ax^3\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**2*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x**3*sqrt(c -
c/(a*x))/(a*x + 1), x)
```

3.601 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=122

$$-\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}-\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[Out] -23/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2+4*arctanh(1/2*(c-c/a/x)^
(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2+9/4*x*(c-c/a/x)^(1/2)/a-1/2*x^2*
(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$-\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}-\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (9*Sqrt[c - c/(a*x)]*x)/(4*a) - (Sqrt[c - c/(a*x)]*x^2)/2 - (23*Sqrt[c]*Arc
Tanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[
c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^2
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=\int\frac{\sqrt{c-\frac{c}{ax}}\,x(1-ax)}{1+ax}\,dx$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{1+ax}\,dx}{c}$$

$$=-\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}\,dx}{c}$$

$$=\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{\text{Subst}\left(\int\frac{\frac{9c^2}{2}-\frac{7c^2x}{2a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2c}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{\text{Subst}\left(\int\frac{\frac{23c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2ac^2}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{(23c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2}-\frac{(4c)\,\text{Subst}\left(\int\frac{}{(a+}\right.}{a}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{23\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{4a}+\frac{8\,\text{Subst}\left(\int\frac{}{2a-}\right.}{}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a}-\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{23\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{4\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Mathematica [**A**] time = 0.08, size = 100, normalized size = 0.82

$$\frac{ax(9-2ax)\sqrt{c-\frac{c}{ax}}-23\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+16\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(9 - 2*a*x) - 23*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/S
qrt[c]] + 16*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/
(4*a^2)
```

fricas [A] time = 0.69, size = 242, normalized size = 1.98

$$\left[\frac{16\,\sqrt{2}\,\sqrt{c}\,\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-2\left(2\,a^{2}x^{2}-9\,ax\right)\sqrt{\frac{acx-c}{ax}}+23\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{8\,a^{2}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)
) + 3*a*c*x - c)/(a*x + 1)) - 2*(2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x))
 + 23*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^
2, -1/4*(16*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(
a*x))/c) + (2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)) - 23*sqrt(-c)*arctan
(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.04, size = 216, normalized size = 1.77

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{a\,x^{2}-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x-2\sqrt{a\,x^{2}-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}-16a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}+16a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-}{ax+1}\right.\right.}{8\sqrt{(ax-1)\,x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x-2*(a*
x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)-16*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)+1
6*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a
*x+1))+24*a^2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(
1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/(
(a*x-1)*x)^(1/2)/a^(7/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}\,x}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))*x/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x*(c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x\sqrt{c-\frac{c}{ax}}}{ax+1}\right)dx-\int \frac{ax^2\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x*sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x**2*sqrt(c - c/(
a*x))/(a*x + 1), x)
```

3.602 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=93

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] 5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a-4*arctanh(1/2*(c-c/a/x)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a-x*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6133, 25, 514, 375, 98, 156, 63, 208}

$$x\left(-\sqrt{c-\frac{c}{ax}}\right)+\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{1+ax}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\sqrt{c-\frac{c}{ax}}\,x - \frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\sqrt{c-\frac{c}{ax}}\,x - \frac{(5c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} + \frac{(4c)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\sqrt{c-\frac{c}{ax}}\,x + 5\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) - 8\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x,\right.$$

$$= -\sqrt{c-\frac{c}{ax}}\,x + \frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} - \frac{4\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 93, normalized size = 1.00

$$x\left(-\sqrt{c-\frac{c}{ax}}\right) + \frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} - \frac{4\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a*x)]*x) + (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a -
 (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.47, size = 217, normalized size = 2.33

$$\left[-\frac{2\,ax\sqrt{\frac{acx-c}{ax}} - 4\,\sqrt{2}\,\sqrt{c}\,\log\left(\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}} - 3\,acx+c}{ax+1}\right) - 5\,\sqrt{c}\,\log\left(-2\,acx - 2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}} + c\right)}{2\,a}, -\frac{ax\sqrt{\frac{acx-c}{ax}} - 4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) - 4*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) - 5*sqrt(c)*log(
-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, -(a*x*sqrt((a*c*x
 - c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
 c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.04, size = 190, normalized size = 2.04

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(2\sqrt{a\,x^2-x}\;a^{\frac{3}{2}}\sqrt{\frac{1}{a}} - 4\sqrt{(ax-1)\,x}\;a^{\frac{3}{2}}\sqrt{\frac{1}{a}} + 4\sqrt{2}\;\ln\left(\frac{2\sqrt{2}\;\sqrt{\frac{1}{a}}\;\sqrt{(ax-1)x}\;a-3ax+1}{ax+1}\right)\sqrt{a} - \ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}+2a}{2\sqrt{a}}\right.\right.}{2\sqrt{(ax-1)\,x}\;a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*((a*x-
1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1
)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*
a*x-1)/a^(1/2))*a*(1/a)^(1/2)+6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a^2\,x^2-1\right)}{(a\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{c-\frac{c}{ax}}}{ax+1}\right)dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x + 1), x) - Integral(a*x*sqrt(c - c/(a*x))
/(a*x + 1), x)
```

3.603 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=86

$$-2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)+4*arctanh(1/2*(c-c/a/x)^(1/2)*2
^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 434, 446, 84, 156, 63, 208}

$$-2\sqrt{c-\frac{c}{ax}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] + 4*Sqr
t[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x(1+ax)}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{1+ax}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}\,dx}{c}$$

$$= \frac{a\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -2\sqrt{c-\frac{c}{ax}} + \frac{a\,\mathrm{Subst}\left(\int \frac{c^2-\frac{3c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -2\sqrt{c-\frac{c}{ax}} + c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right) - (4c)\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,\right.$$

$$= -2\sqrt{c-\frac{c}{ax}} - (2a)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) + (8a)\,\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx\right.$$

$$= -2\sqrt{c-\frac{c}{ax}} - 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) + 4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Mathematica [A] time = 0.04, size = 86, normalized size = 1.00

$$-2\sqrt{c-\frac{c}{ax}} - 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) + 4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -2*Sqrt[c - c/(a*x)] - 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] + 4*Sqr
t[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.57, size = 206, normalized size = 2.40

$$\left[2\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)+\sqrt{c}\log\left(-2acx+2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)-2\sqrt{\frac{acx-c}{ax}},-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*
a*c*x - c)/(a*x + 1)) + sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) + c) - 2*sqrt((a*c*x - c)/(a*x)), -4*sqrt(2)*sqrt(-c)*arctan(1/2*
sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*sqrt(-c)*arctan(sqrt(-c)*sq
rt((a*c*x - c)/(a*x))/c) - 2*sqrt((a*c*x - c)/(a*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [B] time = 0.05, size = 227, normalized size = 2.64

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-4\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,x^2+2\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}x^2\sqrt{\frac{1}{a}}+2\left(a\,x^2-x\right)^{\frac{3}{2}}\sqrt{a}\,\sqrt{\frac{1}{a}}+2\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\right.}{x\sqrt{(ax-1)\,x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] (c*(a*x-1)/a/x)^(1/2)/x*(-4*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)*x^2+2*((a*x
-1)*x)^(1/2)*a^(3/2)*x^2*(1/a)^(1/2)+2*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2)+
2*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^2*a-2*a
^(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+
```

```
1))*x^2-3*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)
*x^2*a)/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a^2\,x^2-1\right)}{x\,\left(a\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(x*(a*x + 1)^2),x)
```

```
[Out] -int(((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(x*(a*x + 1)^2), x)
```

sympy [A] time = 12.33, size = 80, normalized size = 0.93

$$\frac{2c\operatorname{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{4\sqrt{2}\,c\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{c-\frac{c}{ax}}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x,x)
```

```
[Out] 2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) - 4*sqrt(2)*c*atan(sqrt(2)*sq
rt(c - c/(a*x))/(2*sqrt(-c)))/sqrt(-c) - 2*sqrt(c - c/(a*x))
```

3.604 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=82

$$\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

```
[Out] 2/3*a*(c-c/a/x)^(3/2)/c-4*a*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^
(1/2)*c^(1/2)+4*a*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6133, 25, 514, 444, 50, 63, 208}

$$\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] 4*a*Sqrt[c - c/(a*x)] + (2*a*(c - c/(a*x))^(3/2))/(3*c) - 4*Sqrt[2]*a*Sqrt[
c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^2(1+ax)}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x(1+ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + (2a)\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$

$$= 4a\sqrt{c-\frac{c}{ax}} + \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + (4ac)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 4a\sqrt{c-\frac{c}{ax}} + \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \left(8a^2\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= 4a\sqrt{c-\frac{c}{ax}} + \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - 4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.84

$$\frac{2(7ax-1)\sqrt{c-\frac{c}{ax}}}{3x} - 4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-1 + 7*a*x))/(3*x) - 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.63, size = 157, normalized size = 1.91

$$\left[\frac{2\left(3\sqrt{2}\,a\sqrt{c}\,x\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x},\frac{2\left(6\sqrt{2}\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+(7\,ax-\right.}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a*sqrt(c)*x*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) - 3*a*c*x + c)/(a*x + 1)) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x, 2/3
*(6*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)
)/c) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m operator + Error: Bad Argument Value
```

maple [B] time = 0.04, size = 254, normalized size = 3.10

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-18\sqrt{a\,x^2-x}\;a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\;x^3+6a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\;\sqrt{(ax-1)\,x}\;x^3+12a^{\frac{3}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x\sqrt{\frac{1}{a}}\;+9\ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right.}{3x^2\sqrt{(ax-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/3*(c*(a*x-1)/a/x)^(1/2)/x^2*(-18*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x^3
+6*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3+12*a^(3/2)*(a*x^2-x)^(3/2)*x*(
1/a)^(1/2)+9*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2
)*x^3*a^2-6*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3
*a*x+1)/(a*x+1))*x^3-9*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
```

```
)*(1/a)^(1/2)*x^3*a^2-2*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1
/2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x^2), x)
```

mupad [B] time = 1.42, size = 67, normalized size = 0.82

$$4\,a\,\sqrt{c-\frac{c}{a\,x}}+\frac{2\,a\,\left(c-\frac{c}{a\,x}\right)^{3/2}}{3\,c}-4\,\sqrt{2}\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-\frac{c}{a\,x}}}{2\,\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(x^2*(a*x + 1)^2),x)
[Out] 4*a*(c - c/(a*x))^(1/2) + (2*a*(c - c/(a*x))^(3/2))/(3*c) - 4*2^(1/2)*a*c^(
1/2)*atanh((2^(1/2)*(c - c/(a*x))^(1/2))/(2*c^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{c-\frac{c}{ax}}}{ax^3+x^2}\right)dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x**3 + x**2), x) - Integral(a*x*sqrt(c - c/
(a*x))/(a*x**3 + x**2), x)
```

3.605 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=113

$$-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^2\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

```
[Out] -2/3*a^2*(c-c/a/x)^(3/2)/c-2/5*a^2*(c-c/a/x)^(5/2)/c^2+4*a^2*arctanh(1/2*(c
-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-4*a^2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 80, 50, 63, 208}

$$-\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^2\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -4*a^2*Sqrt[c - c/(a*x)] - (2*a^2*(c - c/(a*x))^(3/2))/(3*c) - (2*a^2*(c -
c/(a*x))^(5/2))/(5*c^2) + 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^3(1+ax)}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^2(1+ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= -\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{a^2\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= -\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \left(2a^2\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx,x,\frac{1}{x}\right)$$

$$= -4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \left(4a^2c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\right.$$

$$= -4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \left(8a^3\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\right.$$

$$= -4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + 4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.08, size = 79, normalized size = 0.70

$$4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right) - \frac{2\left(38a^2x^2-11ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(3 - 11*a*x + 38*a^2*x^2))/(15*x^2) + 4*Sqrt[2]*a^2*S
qrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.57, size = 185, normalized size = 1.64

$$\left[\frac{2\left(15\sqrt{2}\,a^2\sqrt{c}\,x^2\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-\left(38\,a^2x^2-11\,ax+3\right)\sqrt{\frac{acx-c}{ax}}\right)}{15\,x^2}, -\frac{2\left(30\sqrt{2}\,a^2\sqrt{-c}\,x^2\arctan\left(\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*a^2*sqrt(c)*x^2*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*
x - c)/(a*x)))/x^2, -2/15*(30*sqrt(2)*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x
- c)/(a*x)))/x^2]
```

giac [B] time = 1.18, size = 278, normalized size = 2.46

$$-\frac{4\sqrt{2}\,a^3c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}\,|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}\,|a|\mathrm{sgn}(x)}-\frac{2\left(60\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^4a^5c-45\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-\cdots}\right.\right.}{15\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/15*(60*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^5*c - 45*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^3*a^4*c^(3/2)*abs(a) + 35*(sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - a*c*x))^2*a^5*c^2 - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^4*
c^(5/2)*abs(a) + 3*a^5*c^3)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*a^
2*abs(a)*sgn(x))
```

maple [B] time = 0.05, size = 278, normalized size = 2.46

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-90\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^4+30a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}\,x^4+60a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^2\sqrt{\frac{1}{a}}+45\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax\cdots}{2\sqrt{a}}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)
```

```
[Out] 1/15*(c*(a*x-1)/a/x)^(1/2)/x^3*(-90*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^4
+30*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^4+60*a^(5/2)*(a*x^2-x)^(3/2)*x^
2*(1/a)^(1/2)+45*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^
(1/2)*x^4*a^3-30*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2
)*a-3*a*x+1)/(a*x+1))*x^4-45*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a
^(1/2))*(1/a)^(1/2)*x^4*a^3-16*a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)+6*(a*x
^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x^3), x)
```

mupad [B] time = 1.74, size = 96, normalized size = 0.85

$$-4\,a^2\,\sqrt{c-\frac{c}{a\,x}}-\frac{2\,a^2\,{\left(c-\frac{c}{a\,x}\right)}^{3/2}}{3\,c}-\frac{2\,a^2\,{\left(c-\frac{c}{a\,x}\right)}^{5/2}}{5\,c^2}-\sqrt{2}\,a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-\frac{c}{a\,x}}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(x^3*(a*x + 1)^2),x)
```

```
[Out] - 4*a^2*(c - c/(a*x))^(1/2) - (2*a^2*(c - c/(a*x))^(3/2))/(3*c) - (2*a^2*(c
 - c/(a*x))^(5/2))/(5*c^2) - 2^(1/2)*a^2*c^(1/2)*atan((2^(1/2)*(c - c/(a*x)
)^(1/2)*1i)/(2*c^(1/2)))*4i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{c-\frac{c}{ax}}}{ax^4+x^3}\right)dx-\int \frac{ax\sqrt{c-\frac{c}{ax}}}{ax^4+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x**4 + x**3), x) - Integral(a*x*sqrt(c - c/
(a*x))/(a*x**4 + x**3), x)
```

3.606 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=113

$$\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2/3*a^3*(c-c/a/x)^(3/2)/c+2/7*a^3*(c-c/a/x)^(7/2)/c^3-4*a^3*arctanh(1/2*(c-
c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+4*a^3*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 88, 50, 63, 208}

$$\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] 4*a^3*Sqrt[c - c/(a*x)] + (2*a^3*(c - c/(a*x))^(3/2))/(3*c) + (2*a^3*(c - c
/(a*x))^(7/2))/(7*c^3) - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^4(1+ax)}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^3(1+ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^4}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \left(\frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}-\frac{a\left(c-\frac{cx}{a}\right)^{5/2}}{c}\right)dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{a^3\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \left(2a^3\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx,x,\frac{1}{x}\right)$$

$$= 4a^3\sqrt{c-\frac{c}{ax}} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \left(4a^3c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\right.$$

$$= 4a^3\sqrt{c-\frac{c}{ax}} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \left(8a^4\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\right.$$

$$= 4a^3\sqrt{c-\frac{c}{ax}} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - 4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.13, size = 87, normalized size = 0.77

$$\frac{2\left(52a^3x^3-16a^2x^2+9ax-3\right)\sqrt{c-\frac{c}{ax}}}{21x^3} - 4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-3 + 9*a*x - 16*a^2*x^2 + 52*a^3*x^3))/(21*x^3) - 4*S
qrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.64, size = 197, normalized size = 1.74

$$\left[\frac{2\left(21\sqrt{2}a^3\sqrt{c}x^3\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+\left(52a^3x^3-16a^2x^2+9ax-3\right)\sqrt{\frac{acx-c}{ax}}\right)}{21x^3}, \frac{2\left(42\sqrt{2}a^3\sqrt{-c}x^3\operatorname{arc}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*a^3*sqrt(c)*x^3*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (52*a^3*x^3 - 16*a^2*x^2 + 9*a*x - 3)
*sqrt((a*c*x - c)/(a*x)))/x^3, 2/21*(42*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1/2
*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (52*a^3*x^3 - 16*a^2*x^2 + 9
*a*x - 3)*sqrt((a*c*x - c)/(a*x)))/x^3]
```

giac [B] time = 1.23, size = 356, normalized size = 3.15

$$\frac{4\sqrt{2}a^4c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}|a|\mathrm{sgn}(x)}+\frac{2\left(84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)^6a^7c-84\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/21*(84*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a^7*c - 84*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - a*c*x))^5*a^6*c^(3/2)*abs(a) + 112*(sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - a*c*x))^4*a^7*c^2 - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a
^6*c^(5/2)*abs(a) + 63*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^7*c^3
- 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^6*c^(7/2)*abs(a) + 3*a^7*c
^4)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^3*abs(a)*sgn(x))
```

maple [B] time = 0.05, size = 302, normalized size = 2.67

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-126\sqrt{a\,x^2-x}\,\sqrt{\frac{1}{a}}\,a^{\frac{9}{2}}x^5+42\sqrt{\frac{1}{a}}\,a^{\frac{9}{2}}\sqrt{(ax-1)\,x}\,x^5+84\left(a\,x^2-x\right)^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,a^{\frac{7}{2}}x^3+63\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2}{2\sqrt{a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
```

```
[Out] -1/21*(c*(a*x-1)/a/x)^(1/2)/x^4*(-126*(a*x^2-x)^(1/2)*(1/a)^(1/2)*a^(9/2)*x
^5+42*(1/a)^(1/2)*a^(9/2)*((a*x-1)*x)^(1/2)*x^5+84*(a*x^2-x)^(3/2)*(1/a)^(1
/2)*a^(7/2)*x^3+63*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a
)^(1/2)*x^5*a^4-42*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(
a*x+1))*2^(1/2)*a^(7/2)*x^5-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*(1/a)^(1/2)*x^5*a^4-20*a^(5/2)*(a*x^2-x)^(3/2)*x^2*(1/a)^(1/2)+12
*a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)-6*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2
))/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x^4), x)
```

mupad [B] time = 2.11, size = 96, normalized size = 0.85

$$4\,a^3\,\sqrt{c-\frac{c}{a\,x}}+\frac{2\,a^3\,{\left(c-\frac{c}{a\,x}\right)}^{3/2}}{3\,c}+\frac{2\,a^3\,{\left(c-\frac{c}{a\,x}\right)}^{7/2}}{7\,c^3}+\sqrt{2}\,a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-\frac{c}{a\,x}}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,4{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(x^4*(a*x + 1)^2),x)
```

```
[Out] 4*a^3*(c - c/(a*x))^(1/2) + (2*a^3*(c - c/(a*x))^(3/2))/(3*c) + (2*a^3*(c -
 c/(a*x))^(7/2))/(7*c^3) + 2^(1/2)*a^3*c^(1/2)*atan((2^(1/2)*(c - c/(a*x))^
(1/2)*1i)/(2*c^(1/2)))*4i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{c-\frac{c}{ax}}}{ax^5+x^4}\right)dx-\int\frac{ax\sqrt{c-\frac{c}{ax}}}{ax^5+x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)
```

```
[Out] -Integral(-sqrt(c - c/(a*x))/(a*x**5 + x**4), x) - Integral(a*x*sqrt(c - c/
(a*x))/(a*x**5 + x**4), x)
```

3.607 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=163

$$-\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^4\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

```
[Out] -2/3*a^4*(c-c/a/x)^(3/2)/c-2/5*a^4*(c-c/a/x)^(5/2)/c^2+2/7*a^4*(c-c/a/x)^(7
/2)/c^3-2/9*a^4*(c-c/a/x)^(9/2)/c^4+4*a^4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/
2)/c^(1/2))*2^(1/2)*c^(1/2)-4*a^4*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6133, 25, 514, 446, 88, 50, 63, 208}

$$-\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^4\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] -4*a^4*Sqrt[c - c/(a*x)] - (2*a^4*(c - c/(a*x))^(3/2))/(3*c) - (2*a^4*(c -
c/(a*x))^(5/2))/(5*c^2) + (2*a^4*(c - c/(a*x))^(7/2))/(7*c^3) - (2*a^4*(c -
 c/(a*x))^(9/2))/(9*c^4) + 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/
(Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^5(1+ax)}\,dx$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^4(1+ax)}\,dx}{c}$$

$$= -\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^5}\,dx}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{a\,\text{Subst}\left(\int \left(a^2\left(c-\frac{cx}{a}\right)^{3/2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x} - \frac{a^2\left(c-\frac{cx}{a}\right)^{5/2}}{c} + \frac{a^2\left(c-\frac{cx}{a}\right)^{7/2}}{c^2}\right)dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - \frac{a^4\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} - \left(2a^4\right)\text{Subst}$$

$$= -4a^4\sqrt{c-\frac{c}{ax}} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}$$

$$= -4a^4\sqrt{c-\frac{c}{ax}} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}$$

$$= -4a^4\sqrt{c-\frac{c}{ax}} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}$$

Mathematica [A] time = 0.14, size = 95, normalized size = 0.58

$$4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right) - \frac{2\left(788a^4x^4 - 236a^3x^3 + 138a^2x^2 - 95ax + 35\right)\sqrt{c-\frac{c}{ax}}}{315x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(35 - 95*a*x + 138*a^2*x^2 - 236*a^3*x^3 + 788*a^4*x^
4))/(315*x^4) + 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sq
rt[c])]
```

fricas [A] time = 0.51, size = 217, normalized size = 1.33

$$\left[\frac{2\left(315\sqrt{2}\,a^4\sqrt{c}\,x^4\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-\left(788\,a^4x^4-236\,a^3x^3+138\,a^2x^2-95\,ax+35\right)\sqrt{\frac{acx-c}{ax}}\right)}{315\,x^4},-\frac{2\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="fricas")
```

```
[Out] [2/315*(315*sqrt(2)*a^4*sqrt(c)*x^4*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (788*a^4*x^4 - 236*a^3*x^3 + 138*a
^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/x^4, -2/315*(630*sqrt(2)*a^4
*sqrt(-c)*x^4*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (788
*a^4*x^4 - 236*a^3*x^3 + 138*a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x))
)/x^4]
```

giac [B] time = 2.35, size = 434, normalized size = 2.66

$$-\frac{4\sqrt{2}\,a^5c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}\,|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}\,|a|\mathrm{sgn}(x)}-\frac{2\left(1260\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^8a^9c-1260\left(\sqrt{a^2c}\,x-\sqrt{a^2c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^5*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/315*(126
0*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^8*a^9*c - 1260*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - a*c*x))^7*a^8*c^(3/2)*abs(a) + 2100*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^6*a^9*c^2 - 3150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c
*x))^5*a^8*c^(5/2)*abs(a) + 3528*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^
4*a^9*c^3 - 2625*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^8*c^(7/2)*ab
s(a) + 1215*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^9*c^4 - 315*(sqrt
```

(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^8*c^(9/2)*abs(a) + 35*a^9*c^5)/((sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*a^4*abs(a)*sgn(x))

maple [B] time = 0.05, size = 326, normalized size = 2.00

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(630\sqrt{(ax-1)\,x}\;a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\;x^{6}-1890\sqrt{a\,x^{2}-x}\;a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\;x^{6}+1260\left(a\,x^{2}-x\right)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\;x^{4}+945\ln\left(\frac{2\sqrt{a\,x^{2}-x}\;\cdots}{2\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x)

[Out] 1/315*(c*(a*x-1)/a/x)^(1/2)/x^5*(630*((a*x-1)*x)^(1/2)*a^(11/2)*(1/a)^(1/2)
x^6-1890(a*x^2-x)^(1/2)*a^(11/2)*(1/a)^(1/2)*x^6+1260*(a*x^2-x)^(3/2)*a^(
9/2)*(1/a)^(1/2)*x^4+945*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*(1/a)^(1/2)*x^6*a^5-630*a^(9/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1
)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^6-945*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^6*a^5-316*(a*x^2-x)^(3/2)*(1/a)^(1/2)*a^(7/
2)*x^3+156*a^(5/2)*(a*x^2-x)^(3/2)*x^2*(1/a)^(1/2)-120*a^(3/2)*(a*x^2-x)^(3
/2)*x*(1/a)^(1/2)+70*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)
/a^(1/2)/(1/a)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^{2}x^{2}-1\right)\sqrt{c-\frac{c}{ax}}}{(ax+1)^{2}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="maxima")

[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a*x))/((a*x + 1)^2*x^5), x)

mupad [B] time = 2.62, size = 138, normalized size = 0.85

$$\frac{2\,a^4\left(c-\frac{c}{a\,x}\right)^{7/2}}{7\,c^3}-\frac{2\,a^4\left(c-\frac{c}{a\,x}\right)^{3/2}}{3\,c}-\frac{2\,a^4\left(c-\frac{c}{a\,x}\right)^{5/2}}{5\,c^2}-4\,a^4\sqrt{c-\frac{c}{a\,x}}-\frac{2\,a^4\left(c-\frac{c}{a\,x}\right)^{9/2}}{9\,c^4}-\sqrt{2}\,a^4\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-\frac{c}{a\,x}}}{2\,\sqrt{c}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((c - c/(a*x))^(1/2)*(a^2*x^2 - 1))/(x^5*(a*x + 1)^2),x)

[Out] (2*a^4*(c - c/(a*x))^(7/2))/(7*c^3) - (2*a^4*(c - c/(a*x))^(3/2))/(3*c) - (2*a^4*(c - c/(a*x))^(5/2))/(5*c^2) - 4*a^4*(c - c/(a*x))^(1/2) - (2*a^4*(c - c/(a*x))^(9/2))/(9*c^4) - 2^(1/2)*a^4*c^(1/2)*atan((2^(1/2)*(c - c/(a*x))^(1/2)*1i)/(2*c^(1/2)))*4i

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{c-\frac{c}{ax}}}{ax^6+x^5}\right)dx-\int\frac{ax\sqrt{c-\frac{c}{ax}}}{ax^6+x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**5,x)

[Out] -Integral(-sqrt(c - c/(a*x))/(a*x**6 + x**5), x) - Integral(a*x*sqrt(c - c/(a*x))/(a*x**6 + x**5), x)

3.608 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=262

$$\frac{1115\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{1-ax}}-\frac{1115x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-ax}}+\frac{1115x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-ax}}+\frac{x^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4\sqrt{1-ax}}+\frac{8x}{\sqrt{1-}}$$

```
[Out] 1115/64*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(7/2)/(-a*x+1)^(
1/2)+8*x^4*(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)-1115/64*x*(c-c/a/x)
^(1/2)*(a*x+1)^(1/2)/a^3/(-a*x+1)^(1/2)+1115/96*x^2*(c-c/a/x)^(1/2)*(a*x+1)
^(1/2)/a^2/(-a*x+1)^(1/2)-223/24*x^3*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/a/(-a*x+
1)^(1/2)+1/4*x^4*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6129, 89, 80, 50, 54, 215}

$$\frac{1115x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-ax}}-\frac{1115x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-ax}}+\frac{1115\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{1-ax}}+\frac{x^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4\sqrt{1-ax}}+\frac{8x}{\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*Sqrt[c - c/(a*x)]*x^4)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (1115*Sqrt[c - c/
(a*x)]*x*Sqrt[1 + a*x])/(64*a^3*Sqrt[1 - a*x]) + (1115*Sqrt[c - c/(a*x)]*x^
2*Sqrt[1 + a*x])/(96*a^2*Sqrt[1 - a*x]) - (223*Sqrt[c - c/(a*x)]*x^3*Sqrt[1
 + a*x])/(24*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^4*Sqrt[1 + a*x])/(4*Sq
rt[1 - a*x]) + (1115*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]])/(6
4*a^(7/2)*Sqrt[1 - a*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int e^{-3\tanh^{-1}(ax)}x^{5/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{x^{5/2}(1-ax)^2}{(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^4}{\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{x^{5/2}\left(\frac{27a^2}{2}-\frac{a^3x}{2}\right)}{\sqrt{1+ax}}\,dx}{a^2\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^4}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^4\sqrt{1+ax}}{4\sqrt{1-ax}} - \frac{\left(223\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{x^{5/2}}{\sqrt{1+ax}}\,dx}{8\sqrt{1-ax}}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^4}{\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{223\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1+ax}}{24a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^4\sqrt{1+ax}}{4\sqrt{1-ax}} + \frac{\left(1115\sqrt{c-\right.}{48}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^4}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{1115\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{96a^2\sqrt{1-ax}} - \frac{223\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1+ax}}{24a\sqrt{1-ax}} + \frac{\sqrt{c-}}{4}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^4}{\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{1115\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} + \frac{1115\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{96a^2\sqrt{1-ax}} - \frac{223\sqrt{c}}{2}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^4}{\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{1115\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} + \frac{1115\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{96a^2\sqrt{1-ax}} - \frac{223\sqrt{c}}{2}$$
$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^4}{\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{1115\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} + \frac{1115\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{96a^2\sqrt{1-ax}} - \frac{223\sqrt{c}}{2}$$

Mathematica [A] time = 0.06, size = 108, normalized size = 0.41

$$\frac{\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\left(\sqrt{a}\,\sqrt{x}\,\left(48a^4x^4-200a^3x^3+446a^2x^2-1115ax-3345\right)+3345\sqrt{ax+1}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\,\right)\right)}{192a^{7/2}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(-3345 - 1115*a*x + 446*a^2*x^2
 - 200*a^3*x^3 + 48*a^4*x^4) + 3345*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]])
)/(192*a^(7/2)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.69, size = 336, normalized size = 1.28

$$\left[\frac{3345\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(48\,a^5x^5-200\,a^4x^4+446\,a^3x^3-11\right.}{768\left(a^6x^2-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/768*(3345*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*
x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) - 4*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2 - 3345*a*x)*
sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^6*x^2 - a^4), -1/384*(3345*(
a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^
3*x^3 - 1115*a^2*x^2 - 3345*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))
)/(a^6*x^2 - a^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 194, normalized size = 0.74

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(96a^{\frac{9}{2}}\sqrt{-(ax+1)\,x}\;x^4-400a^{\frac{7}{2}}x^3\sqrt{-(ax+1)\,x}\;+892a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x}\;-2230a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\right.}{384a^{\frac{7}{2}}\,(ax+1)\,\sqrt{-(ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/384*(c*(a*x-1)/a/x)^(1/2)*x*(96*a^(9/2)*(-(a*x+1)*x)^(1/2)*x^4-400*a^(7/
2)*x^3*(-(a*x+1)*x)^(1/2)+892*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)-2230*a^(3/2)*x
*(-(a*x+1)*x)^(1/2)-3345*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x
*a-6690*a^(1/2)*(-(a*x+1)*x)^(1/2)-3345*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x
+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/2)/a^(7/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}\,x^3}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))*x^3/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,\left(1-a^2\,x^2\right)^{3/2}}{\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int((x^3*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x +
1)**3, x)
```

3.609 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=218

$$-\frac{119\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{119x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}+\frac{x^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}+\frac{8x^3\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{119x^2\sqrt{ax}}{12a\sqrt{}}$$

```
[Out] -119/8*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(5/2)/(-a*x+1)^(1
/2)+8*x^3*(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)+119/8*x*(c-c/a/x)^(1
/2)*(a*x+1)^(1/2)/a^2/(-a*x+1)^(1/2)-119/12*x^2*(c-c/a/x)^(1/2)*(a*x+1)^(1/
2)/a/(-a*x+1)^(1/2)+1/3*x^3*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6134, 6129, 89, 80, 50, 54, 215}

$$\frac{119x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-ax}}-\frac{119\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{8a^{5/2}\sqrt{1-ax}}+\frac{x^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}}+\frac{8x^3\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{119x^2\sqrt{ax+}}{12a\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*Sqrt[c - c/(a*x)]*x^3)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (119*Sqrt[c - c/(
a*x)]*x*Sqrt[1 + a*x])/(8*a^2*Sqrt[1 - a*x]) - (119*Sqrt[c - c/(a*x)]*x^2*S
qrt[1 + a*x])/(12*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^3*Sqrt[1 + a*x])/
(3*Sqrt[1 - a*x]) - (119*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sqrt[x]]
)/(8*a^(5/2)*Sqrt[1 - a*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-3\tanh^{-1}(ax)}x^{3/2}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}(1-ax)^2}{(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^3}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}\left(\frac{19a^2}{2}-\frac{a^3x}{2}\right)}{\sqrt{1+ax}}\,dx}{a^2\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^3}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1+ax}}{3\sqrt{1-ax}} - \frac{\left(119\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{x^{3/2}}{\sqrt{1+ax}}\,dx}{6\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^3}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{119\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\left(119\sqrt{c-\right.}{8}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^3}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{119\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{119\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{a}}}{3\sqrt{}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^3}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{119\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{119\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{a}}}{3\sqrt{}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^3}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{119\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} - \frac{119\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{a}}}{3\sqrt{}}$$

<u>**Mathematica**</u> **[A]** time = 0.06, size = 100, normalized size = 0.46

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\left(8a^3x^3-38a^2x^2+119ax+357\right)-357\sqrt{ax+1}\,\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{24a^{5/2}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(357 + 119*a*x - 38*a^2*x^2 + 8
*a^3*x^3) - 357*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]]))/(24*a^(5/2)*Sqrt[1
 - a^2*x^2])
```

fricas [A] time = 0.66, size = 320, normalized size = 1.47

$$\left[\frac{357\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^2x^2+ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(8\,a^4x^4-38\,a^3x^3+119\,a^2x^2+357\,ax\right)}{96\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/96*(357*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^
2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)
) - 4*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt(-a^2*x^2 + 1)*s
qrt((a*c*x - c)/(a*x)))/(a^5*x^2 - a^3), 1/48*(357*(a^2*x^2 - 1)*sqrt(c)*ar
ctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) - 2*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt(-a^
2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^5*x^2 - a^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 176, normalized size = 0.81

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}x^3\sqrt{-(ax+1)\,x}\;-76a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x}\;+238a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\;+357\arctan\left(\frac{2ax+1}{2\sqrt{a}\;\sqrt{-(ax+1)x}}\right)\right.}{48a^{\frac{5}{2}}\left(ax+1\right)\sqrt{-(ax+1)\,x}\;\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

[Out] -1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^(7/2)*x^3*(-(a*x+1)*x)^(1/2)-76*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)+238*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+357*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+714*a^(1/2)*(-(a*x+1)*x)^(1/2)+357*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))/a^(5/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}\,x^2}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))*x^2/(a*x + 1)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,\left(1-a^2\,x^2\right)^{3/2}}{(a\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)

[Out] int((x^2*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)

[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3, x)

3.610 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=174

$$\frac{47\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-ax}}+\frac{8x^2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{47x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

```
[Out] 47/4*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(3/2)/(-a*x+1)^(1/2
)+8*x^2*(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)-47/4*x*(c-c/a/x)^(1/2)
*(a*x+1)^(1/2)/a/(-a*x+1)^(1/2)+1/2*x^2*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x
+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6134, 6129, 89, 80, 50, 54, 215}

$$\frac{47\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{1-ax}}+\frac{x^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-ax}}+\frac{8x^2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{47x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*Sqrt[c - c/(a*x)]*x^2)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (47*Sqrt[c - c/(a
*x)]*x*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a*x]) + (Sqrt[c - c/(a*x)]*x^2*Sqrt[1 +
 a*x])/(2*Sqrt[1 - a*x]) + (47*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[a]*Sq
rt[x]])/(4*a^(3/2)*Sqrt[1 - a*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int e^{-3\tanh^{-1}(ax)}\sqrt{x}\sqrt{1-ax}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}(1-ax)^2}{(1+ax)^{3/2}}dx}{\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}\left(\frac{11a^2}{2}-\frac{a^3x}{2}\right)}{\sqrt{1+ax}}dx}{a^2\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{2\sqrt{1-ax}} - \frac{\left(47\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{\sqrt{x}}{\sqrt{1+ax}}dx}{4\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{47\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{2\sqrt{1-ax}} + \frac{\left(47\sqrt{c-\frac{c}{ax}}\sqrt{\right.}{8a\sqrt{}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{47\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{2\sqrt{1-ax}} + \frac{\left(47\sqrt{c-\frac{c}{ax}}\sqrt{\right.}{}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{47\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{4a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x^2\sqrt{1+ax}}{2\sqrt{1-ax}} + \frac{47\sqrt{c-\frac{c}{ax}}\sqrt{x}}{4a^{3/2}}$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.53

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}\left(2a^2x^2-13ax-47\right)+47\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{4a^{3/2}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(-47 - 13*a*x + 2*a^2*x^2) + 47
*Sqrt[1 + a*x]*ArcSinh[Sqrt[a]*Sqrt[x]]))/(4*a^(3/2)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.63, size = 304, normalized size = 1.75

$$\left[\frac{47\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(2a^3x^3-13a^2x^2-47ax\right)\sqrt{-a^2x^2+1}}{16\left(a^4x^2-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] [1/16*(47*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2
 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 - 4*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^2 - a^2), -1/8*(47*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*
x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2
*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x
)))/(a^4*x^2 - a^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 158, normalized size = 0.91

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(4a^{\frac{5}{2}}x^2\sqrt{-(ax+1)\,x} - 26a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x} - 47\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)xa - 94\sqrt{a}\,\sqrt{-(ax+1)\,x}\right.}{8a^{\frac{3}{2}}\,(ax+1)\sqrt{-(ax+1)\,x}\,\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(5/2)*x^2*(-(a*x+1)*x)^(1/2)-26*a^(3/2)*x
*(-(a*x+1)*x)^(1/2)-47*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a
-94*a^(1/2)*(-(a*x+1)*x)^(1/2)-47*arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)
^(1/2)))*(-a^2*x^2+1)^(1/2)/a^(3/2)/(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}\,x}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))*x/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{c-\frac{c}{a\,x}}\,\left(1-a^2\,x^2\right)^{3/2}}{{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int((x*(c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)*
*3, x)
```

3.611 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=123

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{8x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

```
[Out] -7*arcsinh(a^(1/2)*x^(1/2))*(c-c/a/x)^(1/2)*x^(1/2)/a^(1/2)/(-a*x+1)^(1/2)+
8*x*(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)+x*(c-c/a/x)^(1/2)*(a*x+1)^
(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6134, 6129, 89, 80, 54, 215}

$$\frac{x\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}}+\frac{8x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{7\sqrt{x}\sqrt{c-\frac{c}{ax}}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (8*Sqrt[c - c/(a*x)]*x)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a*x)]*
x*Sqrt[1 + a*x])/Sqrt[1 - a*x] - (7*Sqrt[c - c/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[
a]*Sqrt[x]])/(Sqrt[a]*Sqrt[1 - a*x])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{(1-ax)^2}{\sqrt{x}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{\frac{3a^2}{2}-\frac{a^3x}{2}}{\sqrt{x}\sqrt{1+ax}}\,dx}{a^2\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{\left(7\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+ax^2}}\,dx,x,\right.}{\sqrt{1-ax}}$$

$$= \frac{8\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{ax}}\,x\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{7\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{1-ax}}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{c-\frac{c}{ax}}\left(\sqrt{a}\sqrt{x}(ax+9)-7\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{a}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*Sqrt[x]*(Sqrt[a]*Sqrt[x]*(9 + a*x) - 7*Sqrt[1 + a*x]*Arc
Sinh[Sqrt[a]*Sqrt[x]]))/(Sqrt[a]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.70, size = 282, normalized size = 2.29

$$\left[\frac{7\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(a^2x^2+9ax\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{4\left(a^3x^2-a\right)}, \frac{7\left(a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(7*(a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^2*x^2 +
 a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) -
 4*(a^2*x^2 + 9*a*x)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 -
 a), 1/2*(7*(a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*s
qrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a^2*x^2 + 9*a*x)*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 140, normalized size = 1.14

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\, + 7\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)xa + 18\sqrt{a}\,\sqrt{-(ax+1)\,x}\, + 7\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)\right.}{2\sqrt{a}\,(ax+1)\sqrt{-(ax+1)\,x}\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+7*arctan(1/2/a
^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x*a+18*a^(1/2)*(-(a*x+1)*x)^(1/2)+7*ar
ctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2)))*(-a^2*x^2+1)^(1/2)/a^(1/2)/
(a*x+1)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(a*x + 1)**3
, x)
```

3.612 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=124

$$-\frac{10ax\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{1-ax}}$$

```
[Out] 2*arcsinh(a^(1/2)*x^(1/2))*a^(1/2)*(c-c/a/x)^(1/2)*x^(1/2)/(-a*x+1)^(1/2)-2
*(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)-10*a*x*(c-c/a/x)^(1/2)/(-a*x+
1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 89, 78, 54, 215}

$$-\frac{10ax\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{2\sqrt{a}\,\sqrt{x}\,\sqrt{c-\frac{c}{ax}}\,\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)])/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (10*a*Sqrt[c - c/(a*
x)]*x)/(Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (2*Sqrt[a]*Sqrt[c - c/(a*x)]*Sqrt[x]
*ArcSinh[Sqrt[a]*Sqrt[x]])/Sqrt[1 - a*x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{3/2}}\,dx}{\sqrt{1-ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{3/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-2a+\frac{a^2x}{2}}{\sqrt{x}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{10a\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{10a\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \ldots\right)}{\sqrt{1-ax}}$$
$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{10a\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{1-ax}}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.56

$$-\frac{2\sqrt{c-\frac{c}{ax}}\left(5ax-\sqrt{a}\sqrt{x}\sqrt{ax+1}\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)+1\right)}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x),x]
[Out] (-2*Sqrt[c - c/(a*x)]*(1 + 5*a*x - Sqrt[a]*Sqrt[x]*Sqrt[1 + a*x]*ArcSinh[Sq
rt[a]*Sqrt[x]]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.58, size = 264, normalized size = 2.13

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^2x^2+ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\sqrt{-a^2x^2+1}\,(5ax+1)\sqrt{\frac{acx-c}{ax}}}{2\left(a^2x^2-1\right)}, -\frac{\left(a^2x^2-1\right)\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^2*x^2 + a
*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4
*sqrt(-a^2*x^2 + 1)*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x)))/(a^2*x^2 - 1), -((
a^2*x^2 - 1)*sqrt(c)*arctan(2*sqrt(-a^2*x^2 + 1)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*sqrt(-a^2*x^2 + 1)*(5*a*x + 1)*sqr
t((a*c*x - c)/(a*x)))/(a^2*x^2 - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 142, normalized size = 1.15

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\left(\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)x^2a^2+10a^{\frac{3}{2}}x\sqrt{-(ax+1)\,x}\,+\arctan\left(\frac{2ax+1}{2\sqrt{a}\,\sqrt{-(ax+1)x}}\right)xa+2\sqrt{a}\,\sqrt{-(ax+1)\,x}}{(ax+1)\,\sqrt{a}\,\sqrt{-(ax+1)\,x}\,\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] (c*(a*x-1)/a/x)^(1/2)*(arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)*x)^(1/2))*x^2
*a^2+10*a^(3/2)*x*(-(a*x+1)*x)^(1/2)+arctan(1/2/a^(1/2)*(2*a*x+1)/(-(a*x+1)
*x)^(1/2))*x*a+2*a^(1/2)*(-(a*x+1)*x)^(1/2))*(-a^2*x^2+1)^(1/2)/(a*x+1)/a^(
1/2)/(-(a*x+1)*x)^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(1-a^2\,x^2\right)^{3/2}}{x\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(x*(a*x + 1)^3),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(x*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x*(a*x + 1)
**3), x)
```

3.613 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=123

$$\frac{46a^2x\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{20a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] 20/3*a*(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)-2/3*(c-c/a/x)^(1/2)/x/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)+46/3*a^2*x*(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)/(a*x+
1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6134, 6129, 89, 78, 37}

$$\frac{46a^2x\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{20a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (20*a*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*Sqrt[c - c/(a
*x)])/(3*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (46*a^2*Sqrt[c - c/(a*x)]*x)/(3*S
qrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{5/2}}\,dx}{\sqrt{1-ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{5/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}\\
&= -\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-5a+\frac{3a^2x}{2}}{x^{3/2}(1+ax)^{3/2}}\,dx}{3\sqrt{1-ax}}\\
&= \frac{20a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\sqrt{1+ax}}+\frac{\left(23a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}(1+ax)^{3/2}}\,dx}{3\sqrt{1-ax}}\\
&= \frac{20a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-ax}\sqrt{1+ax}}-\frac{2\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-ax}\sqrt{1+ax}}+\frac{46a^2\sqrt{c-\frac{c}{ax}}\,x}{3\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.41

$$\frac{2\left(23a^2x^2+10ax-1\right)\sqrt{c-\frac{c}{ax}}}{3x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-1 + 10*a*x + 23*a^2*x^2))/(3*x*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 58, normalized size = 0.47

$$-\frac{2\left(23\,a^2x^2+10\,ax-1\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{acx-c}{ax}}}{3\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] -2/3*(23*a^2*x^2 + 10*a*x - 1)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))/(
a^2*x^3 - x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.03, size = 61, normalized size = 0.50

$$\frac{2\left(23a^2x^2+10ax-1\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3\left(ax+1\right)^2x\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] 2/3*(23*a^2*x^2+10*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^
2/x/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{(ax+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^2), x)
```

mupad [B] time = 1.12, size = 80, normalized size = 0.65

$$\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{46\,x^2\,\sqrt{1-a^2\,x^2}}{3}-\frac{2\,\sqrt{1-a^2\,x^2}}{3\,a^2}+\frac{20\,x\,\sqrt{1-a^2\,x^2}}{3\,a}\right)}{\frac{x}{a^2}-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^2*(a*x + 1)^3),x)
```

```
[Out] ((c - c/(a*x))^(1/2)*((46*x^2*(1 - a^2*x^2)^(1/2))/3 - (2*(1 - a^2*x^2)^(1/
2))/(3*a^2) + (20*x*(1 - a^2*x^2)^(1/2))/(3*a)))/(x/a^2 - x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}{x^2(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**2*(a*x +
 1)**3), x)
```

3.614 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=166

$$-\frac{316a^2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}}+\frac{158a^2\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] 158/15*a^2*(c-c/a/x)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)-2/5*(c-c/a/x)^(1/2)
/x^2/(-a*x+1)^(1/2)/(a*x+1)^(1/2)+32/15*a*(c-c/a/x)^(1/2)/x/(-a*x+1)^(1/2)/
(a*x+1)^(1/2)-316/15*a^2*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 89, 78, 45, 37}

$$-\frac{316a^2\sqrt{ax+1}\,\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}}+\frac{158a^2\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (158*a^2*Sqrt[c - c/(a*x)])/(15*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (2*Sqrt[c -
c/(a*x)])/(5*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (32*a*Sqrt[c - c/(a*x)])/(1
5*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (316*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x]
)/(15*Sqrt[1 - a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{7/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{7/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(2\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{-8a+\frac{5a^2x}{2}}{x^{5/2}(1+ax)^{3/2}}\,dx}{5\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(79a^2\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{1}{x^{3/2}(1+ax)^{3/2}}}{15\sqrt{1-ax}}$$

$$= \frac{158a^2\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(158a^2\sqrt{c-\right.}{}$$

$$= \frac{158a^2\sqrt{c-\frac{c}{ax}}}{15\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{2\sqrt{c-\frac{c}{ax}}}{5x^2\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{32a\sqrt{c-\frac{c}{ax}}}{15x\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{316a^2\sqrt{c-}}{15\sqrt{1}}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.35

$$-\frac{2\left(158a^3x^3+79a^2x^2-16ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(3 - 16*a*x + 79*a^2*x^2 + 158*a^3*x^3))/(15*x^2*Sqrt
[1 - a^2*x^2])
```

fricas [A] time = 0.55, size = 68, normalized size = 0.41

$$\frac{2\left(158\,a^3x^3+79\,a^2x^2-16\,ax+3\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{acx-c}{ax}}}{15\left(a^2x^4-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="fr
icas")
```

```
[Out] 2/15*(158*a^3*x^3 + 79*a^2*x^2 - 16*a*x + 3)*sqrt(-a^2*x^2 + 1)*sqrt((a*c*x
 - c)/(a*x))/(a^2*x^4 - x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.03, size = 69, normalized size = 0.42

$$-\frac{2\left(158x^3a^3+79a^2x^2-16ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(-a^2x^2+1\right)^{\frac{3}{2}}}{15\left(ax+1\right)^2x^2\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] -2/15*(158*a^3*x^3+79*a^2*x^2-16*a*x+3)*(c*(a*x-1)/a/x)^(1/2)*(-a^2*x^2+1)^
(3/2)/(a*x+1)^2/x^2/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{\left(ax+1\right)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^3), x)
```

mupad [B] time = 1.20, size = 99, normalized size = 0.60

$$\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{2\,\sqrt{1-a^2\,x^2}}{5\,a^2}+\frac{158\,x^2\,\sqrt{1-a^2\,x^2}}{15}-\frac{32\,x\,\sqrt{1-a^2\,x^2}}{15\,a}+\frac{316\,a\,x^3\,\sqrt{1-a^2\,x^2}}{15}\right)}{x^4-\frac{x^2}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^3*(a*x + 1)^3),x)
```

```
[Out] ((c - c/(a*x))^(1/2)*((2*(1 - a^2*x^2)^(1/2))/(5*a^2) + (158*x^2*(1 - a^2*x
^2)^(1/2))/15 - (32*x*(1 - a^2*x^2)^(1/2))/(15*a) + (316*a*x^3*(1 - a^2*x^2
)^(1/2))/15))/(x^4 - x^2/a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(-(ax-1)(ax+1))^{\frac{3}{2}}}{x^3(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**3*(a*x +
 1)**3), x)
```

3.615 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=213

$$\frac{2672a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105\sqrt{1-ax}}-\frac{1336a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{ax+1}}+\frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -2/7*(c-c/a/x)^(1/2)/x^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)+44/35*a*(c-c/a/x)^(1/
2)/x^2/(-a*x+1)^(1/2)/(a*x+1)^(1/2)+334/35*a^2*(c-c/a/x)^(1/2)/x/(-a*x+1)^(
1/2)/(a*x+1)^(1/2)+2672/105*a^3*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2
)-1336/105*a^2*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/x/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 89, 78, 45, 37}

$$\frac{2672a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105\sqrt{1-ax}}-\frac{1336a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{105x\sqrt{1-ax}}+\frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{ax+1}}+\frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)])/(7*x^3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (44*a*Sqrt[c -
 c/(a*x)])/(35*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (334*a^2*Sqrt[c - c/(a*x)
])/(35*x*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (2672*a^3*Sqrt[c - c/(a*x)]*Sqrt[1
+ a*x])/(105*Sqrt[1 - a*x]) - (1336*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(1
05*x*Sqrt[1 - a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
```

```
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{9/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{9/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{-11a+\frac{7a^2x}{2}}{x^{7/2}(1+ax)^{3/2}}\,dx}{7\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(167a^2\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int \frac{1}{x^{5/2}(1+ax)^{3/2}}}{35\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{1+ax}} + \frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(668a^2\right.}{}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{1+ax}} + \frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{1+ax}} - \frac{1336a^2}{105}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{7x^3\sqrt{1-ax}\sqrt{1+ax}} + \frac{44a\sqrt{c-\frac{c}{ax}}}{35x^2\sqrt{1-ax}\sqrt{1+ax}} + \frac{334a^2\sqrt{c-\frac{c}{ax}}}{35x\sqrt{1-ax}\sqrt{1+ax}} + \frac{2672a^3}{10}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.31

$$\frac{2\left(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15\right)\sqrt{c-\frac{c}{ax}}}{105x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(-15 + 66*a*x - 167*a^2*x^2 + 668*a^3*x^3 + 1336*a^4*x
^4))/(105*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.65, size = 76, normalized size = 0.36

$$-\frac{2\left(1336\,a^4x^4+668\,a^3x^3-167\,a^2x^2+66\,ax-15\right)\sqrt{-a^2x^2+1}\sqrt{\frac{acx-c}{ax}}}{105\left(a^2x^5-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="fr
icas")
```

```
[Out] -2/105*(1336*a^4*x^4 + 668*a^3*x^3 - 167*a^2*x^2 + 66*a*x - 15)*sqrt(-a^2*x
^2 + 1)*sqrt((a*c*x - c)/(a*x))/(a^2*x^5 - x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.03, size = 77, normalized size = 0.36

$$\frac{2\left(1336x^{4}a^{4}+668x^{3}a^{3}-167a^{2}x^{2}+66ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{105\left(ax+1\right)^{2}x^{3}\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] 2/105*(1336*a^4*x^4+668*a^3*x^3-167*a^2*x^2+66*a*x-15)*(c*(a*x-1)/a/x)^(1/2
)*(-a^2*x^2+1)^(3/2)/(a*x+1)^2/x^3/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{ax}}}{\left(ax+1\right)^{3}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^4), x)
```

mupad [B] time = 1.21, size = 120, normalized size = 0.56

$$-\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{44\,x\,\sqrt{1-a^2\,x^2}}{35\,a}-\frac{334\,x^2\,\sqrt{1-a^2\,x^2}}{105}-\frac{2\,\sqrt{1-a^2\,x^2}}{7\,a^2}+\frac{1336\,a\,x^3\,\sqrt{1-a^2\,x^2}}{105}+\frac{2672\,a^2\,x^4\,\sqrt{1-a^2\,x^2}}{105}\right)}{x^5-\frac{x^3}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^4*(a*x + 1)^3),x)
```

```
[Out] -((c - c/(a*x))^(1/2)*((44*x*(1 - a^2*x^2)^(1/2))/(35*a) - (334*x^2*(1 - a^
2*x^2)^(1/2))/105 - (2*(1 - a^2*x^2)^(1/2))/(7*a^2) + (1336*a*x^3*(1 - a^2*
x^2)^(1/2))/105 + (2672*a^2*x^4*(1 - a^2*x^2)^(1/2))/105))/(x^5 - x^3/a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x^4\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**4*(a*x +
 1)**3), x)
```

3.616 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=257

$$-\frac{1312a^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{45\sqrt{1-ax}}+\frac{656a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{45x\sqrt{1-ax}}-\frac{164a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{15x^2\sqrt{1-ax}}+\frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\sqrt{ax+1}}-\frac{2\sqrt{c}}{9x^4\sqrt{1-a}}$$

```
[Out] -2/9*(c-c/a/x)^(1/2)/x^4/(-a*x+1)^(1/2)/(a*x+1)^(1/2)+8/9*a*(c-c/a/x)^(1/2)
/x^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)+82/9*a^2*(c-c/a/x)^(1/2)/x^2/(-a*x+1)^(1/
2)/(a*x+1)^(1/2)-1312/45*a^4*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/(-a*x+1)^(1/2)-1
64/15*a^2*(c-c/a/x)^(1/2)*(a*x+1)^(1/2)/x^2/(-a*x+1)^(1/2)+656/45*a^3*(c-c/
a/x)^(1/2)*(a*x+1)^(1/2)/x/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6134, 6129, 89, 78, 45, 37}

$$-\frac{164a^2\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{15x^2\sqrt{1-ax}}+\frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\sqrt{ax+1}}-\frac{1312a^4\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{45\sqrt{1-ax}}+\frac{656a^3\sqrt{ax+1}\sqrt{c-\frac{c}{ax}}}{45x\sqrt{1-ax}}+\frac{8a\sqrt{c}}{9x^3\sqrt{1-a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)])/(9*x^4*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (8*a*Sqrt[c -
c/(a*x)])/(9*x^3*Sqrt[1 - a*x]*Sqrt[1 + a*x]) + (82*a^2*Sqrt[c - c/(a*x)])/
(9*x^2*Sqrt[1 - a*x]*Sqrt[1 + a*x]) - (1312*a^4*Sqrt[c - c/(a*x)]*Sqrt[1 +
a*x])/(45*Sqrt[1 - a*x]) - (164*a^2*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(15*x^
2*Sqrt[1 - a*x]) + (656*a^3*Sqrt[c - c/(a*x)]*Sqrt[1 + a*x])/(45*x*Sqrt[1 -
 a*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
```

```
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-ax}}{x^{11/2}}\,dx}{\sqrt{1-ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{(1-ax)^2}{x^{11/2}(1+ax)^{3/2}}\,dx}{\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(2\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{-14a+\frac{9a^2x}{2}}{x^{9/2}(1+ax)^{3/2}}\,dx}{9\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(41a^2\sqrt{c-\frac{c}{ax}}\,\sqrt{x}\right)\int \frac{1}{x^{7/2}(1+ax)^{3/2}} \cdots}{9\sqrt{1-ax}}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\,\sqrt{1+ax}} + \cdots$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\,\sqrt{1+ax}} - \cdots$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\,\sqrt{1+ax}} - \cdots$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}}{9x^4\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{8a\sqrt{c-\frac{c}{ax}}}{9x^3\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{82a^2\sqrt{c-\frac{c}{ax}}}{9x^2\sqrt{1-ax}\,\sqrt{1+ax}} - \cdots$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.29

$$-\frac{2\left(656a^5x^5+328a^4x^4-82a^3x^3+41a^2x^2-20ax+5\right)\sqrt{c-\frac{c}{ax}}}{45x^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (-2*Sqrt[c - c/(a*x)]*(5 - 20*a*x + 41*a^2*x^2 - 82*a^3*x^3 + 328*a^4*x^4 +
 656*a^5*x^5))/(45*x^4*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.54, size = 84, normalized size = 0.33

$$\frac{2\left(656\,a^5x^5+328\,a^4x^4-82\,a^3x^3+41\,a^2x^2-20\,ax+5\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{acx-c}{ax}}}{45\left(a^2x^6-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="fr
icas")
```

```
[Out] 2/45*(656*a^5*x^5 + 328*a^4*x^4 - 82*a^3*x^3 + 41*a^2*x^2 - 20*a*x + 5)*sqr
t(-a^2*x^2 + 1)*sqrt((a*c*x - c)/(a*x))/(a^2*x^6 - x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.03, size = 85, normalized size = 0.33

$$-\frac{2 \left(656 x^{5} a^{5}+328 x^{4} a^{4}-82 x^{3} a^{3}+41 a^{2} x^{2}-20 a x +5\right) \sqrt{\frac{c \left(a x -1\right)}{a x}}\, \left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{45 \left(a x +1\right)^{2} x^{4} \left(a x -1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] -2/45*(656*a^5*x^5+328*a^4*x^4-82*a^3*x^3+41*a^2*x^2-20*a*x+5)*(c*(a*x-1)/a
/x)^(1/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^2/x^4/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \sqrt{c-\frac{c}{a x}}}{\left(a x+1\right)^{3} x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a*x))/((a*x + 1)^3*x^5), x)
```

mupad [B] time = 1.21, size = 139, normalized size = 0.54

$$\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(\frac{2\,\sqrt{1-a^2\,x^2}}{9\,a^2}+\frac{82\,x^2\,\sqrt{1-a^2\,x^2}}{45}-\frac{8\,x\,\sqrt{1-a^2\,x^2}}{9\,a}-\frac{164\,a\,x^3\,\sqrt{1-a^2\,x^2}}{45}+\frac{656\,a^2\,x^4\,\sqrt{1-a^2\,x^2}}{45}+\frac{1312\,a^3\,x^5\,\sqrt{1-a^2\,x^2}}{45}\right)}{x^6-\frac{x^4}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^5*(a*x + 1)^3),x)
```

```
[Out] ((c - c/(a*x))^(1/2)*((2*(1 - a^2*x^2)^(1/2))/(9*a^2) + (82*x^2*(1 - a^2*x^
2)^(1/2))/45 - (8*x*(1 - a^2*x^2)^(1/2))/(9*a) - (164*a*x^3*(1 - a^2*x^2)^(
1/2))/45 + (656*a^2*x^4*(1 - a^2*x^2)^(1/2))/45 + (1312*a^3*x^5*(1 - a^2*x^
2)^(1/2))/45))/(x^6 - x^4/a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{x^5\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(-(a*x - 1)*(a*x + 1))**(3/2)/(x**5*(a*x +
 1)**3), x)
```

3.617 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=64

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^pF_1\left(1-p;\frac{1}{2}(n-2p),-\frac{n}{2};2-p;ax,-ax\right)}{1-p}$$

```
[Out] (c-c/a/x)^p*x*AppellF1(1-p,1/2*n-p,-1/2*n,2-p,a*x,-a*x)/(1-p)/((-a*x+1)^p)
```

Rubi [A] time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6134, 6129, 133}

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^pF_1\left(1-p;\frac{1}{2}(n-2p),-\frac{n}{2};2-p;ax,-ax\right)}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, (n - 2*p)/2, -n/2, 2 - p, a*x, -(a*x)])/
((1 - p)*(1 - a*x)^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int e^{n\tanh^{-1}(ax)}x^{-p}(1-ax)^p\,dx\\
&= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int x^{-p}(1-ax)^{-\frac{n}{2}+p}(1+ax)^{n/2}\,dx\\
&= \frac{\left(c-\frac{c}{ax}\right)^p x(1-ax)^{-p}F_1\left(1-p;\frac{1}{2}(n-2p),-\frac{n}{2};2-p;ax,-ax\right)}{1-p}
\end{aligned}
$$

Mathematica [F] time = 0.64, size = 0, normalized size = 0.00

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x))^p,x]
[Out] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x))^p, x]
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\left(\frac{acx-c}{ax}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a*c*x - c)/(a*x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} \left(c - \frac{c}{ax}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x)^p,x)
[Out] int(exp(n*arctanh(a*x))*(c-c/a/x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{n \operatorname{atanh}(a\,x)} \left(c - \frac{c}{a\,x}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a*x))^p,x)
[Out] int(exp(n*atanh(a*x))*(c - c/(a*x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1 + \frac{1}{ax}\right)\right)^p e^{n \operatorname{atanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a/x)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(n*atanh(a*x)), x)
```

3.618 $\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=54

$$\frac{x(1-ax)^{-p}F_1\left(1-p;-2p,p;2-p;ax,-ax\right)\left(c-\frac{c}{ax}\right)^p}{1-p}$$

```
[Out] (c-c/a/x)^p*x*AppellF1(1-p,-2*p,p,2-p,a*x,-a*x)/(1-p)/((-a*x+1)^p)
```

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6134, 6129, 133}

$$\frac{x(1-ax)^{-p}F_1\left(1-p;-2p,p;2-p;ax,-ax\right)\left(c-\frac{c}{ax}\right)^p}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a*x))^p*x*AppellF1[1 - p, -2*p, p, 2 - p, a*x, -(a*x)])/((1 - p)*(
1 - a*x)^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int e^{-2p\tanh^{-1}(ax)}x^{-p}(1-ax)^p\,dx$$
$$= \left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int x^{-p}(1-ax)^{2p}(1+ax)^{-p}\,dx$$
$$= \frac{\left(c-\frac{c}{ax}\right)^p x(1-ax)^{-p}F_1(1-p;-2p,p;2-p;ax,-ax)}{1-p}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*p*ArcTanh[a*x]),x]
[Out] Integrate[(c - c/(a*x))^p/E^(2*p*ArcTanh[a*x]), x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{acx-c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x, algorithm="fricas")
[Out] integral(((a*c*x - c)/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x, algorithm="giac")
```

`[Out] integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)`

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(c-\frac{c}{ax}\right)^{p} \mathrm{e}^{-2p\,\mathrm{arctanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x)`

`[Out] int((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^{p}}{\left(\frac{ax+1}{ax-1}\right)^{p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((c-c/a/x)^p/exp(2*p*arctanh(a*x)),x, algorithm="maxima")`

`[Out] integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{-2\,p\,\mathrm{atanh}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(exp(-2*p*atanh(a*x))*(c - c/(a*x))^p,x)`

`[Out] int(exp(-2*p*atanh(a*x))*(c - c/(a*x))^p, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^{p} e^{-2p\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((c-c/a/x)**p/exp(2*p*atanh(a*x)),x)`

`[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(-2*p*atanh(a*x)), x)`

3.619 $\int e^{2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=50

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^p\,{}_2F_1(1-p,-p;2-p;-ax)}{1-p}$$

```
[Out] (c-c/a/x)^p*x*hypergeom([-p, 1-p],[2-p],-a*x)/(1-p)/((-a*x+1)^p)
```

Rubi [A] time = 0.10, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6134, 6129, 64}

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^p\,{}_2F_1(1-p,-p;2-p;-ax)}{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*x*Hypergeometric2F1[1 - p, -p, 2 - p, -(a*x)])/((1 - p)*(1
 - a*x)^p)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int e^{2p\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=\left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int e^{2p\tanh^{-1}(ax)}x^{-p}(1-ax)^p\,dx$$
$$=\left(\left(c-\frac{c}{ax}\right)^p x^p(1-ax)^{-p}\right)\int x^{-p}(1+ax)^p\,dx$$
$$=\frac{\left(c-\frac{c}{ax}\right)^p x(1-ax)^{-p}\,{}_2F_1(1-p,-p;2-p;-ax)}{1-p}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$\frac{x(1-ax)^{-p}\left(c-\frac{c}{ax}\right)^p\,{}_2F_1(1-p,-p;2-p;-ax)}{1-p}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcTanh[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*x*Hypergeometric2F1[1 - p, -p, 2 - p, -(a*x)])/((1 - p)*(1
 - a*x)^p)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax+1}{ax-1}\right)^p\left(\frac{acx-c}{ax}\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^p*((a*c*x - c)/(a*x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int e^{2p\,\mathrm{arctanh}(ax)} \left(c - \frac{c}{ax}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x)
[Out] int(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^{p} \left(\frac{ax+1}{ax-1}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{2\,p\,\mathrm{atanh}(a\,x)} \left(c - \frac{c}{a\,x}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*atanh(a*x))*(c - c/(a*x))^p,x)
[Out] int(exp(2*p*atanh(a*x))*(c - c/(a*x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1 + \frac{1}{ax}\right)\right)^{p} e^{2p\,\mathrm{atanh}\,(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*atanh(a*x))*(c-c/a/x)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(2*p*atanh(a*x)), x)
```

3.620 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=130

$$\frac{c^2 2^{n/2}(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},2-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}+\frac{4c^2(ax+1)^{n/2}(1-ax)^{-n/2}\,{}_2F_1\left(2,\frac{n}{2};\frac{n+2}{2};\frac{ax+1}{1-ax}\right)}{an}$$

```
[Out] 4*c^2*(a*x+1)^(1/2*n)*hypergeom([2, 1/2*n],[1+1/2*n],(a*x+1)/(-a*x+1))/a/n/
((-a*x+1)^(1/2*n))+2^(1/2*n)*c^2*(-a*x+1)^(2-1/2*n)*hypergeom([2-1/2*n, 1-1
/2*n],[3-1/2*n],-1/2*a*x+1/2)/a/(4-n)
```

Rubi [C] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 0.55, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6131, 6129, 136}

$$\frac{c^2 2^{3-\frac{n}{2}}(ax+1)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-4}{2},2;\frac{n+4}{2};\frac{1}{2}(ax+1),ax+1\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] (2^(3 - n/2)*c^2*(1 + a*x)^((2 + n)/2)*AppellF1[(2 + n)/2, (-4 + n)/2, 2, (
4 + n)/2, (1 + a*x)/2, 1 + a*x])/(a*(2 + n))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx=\frac{c^2\int\frac{e^{n\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}$$
$$=\frac{c^2\int\frac{(1-ax)^{2-\frac{n}{2}}(1+ax)^{n/2}}{x^2}\,dx}{a^2}$$
$$=\frac{2^{3-\frac{n}{2}}c^2(1+ax)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{1}{2}(-4+n),2;\frac{4+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{a(2+n)}$$

Mathematica [B] time = 0.50, size = 262, normalized size = 2.02

$$-\frac{c^2e^{n\tanh^{-1}(ax)}\left(an^2x\,{}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\tanh^{-1}(ax)}\right)-2anxe^{2\tanh^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\tanh^{-1}(ax)}\right)+a(n-2)n\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*E^(n*ArcTanh[a*x])*(2*n + n^2 - 2*a*E^(2*ArcTanh[a*x])*n*x*Hypergeom
etric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])] + a*E^(2*ArcTanh[a*x])*(
-2 + n)*n*x*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcTanh[a*x])] + 4*
a*x*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcTanh[a*x])] + 2*a*n*x*Hyper
geometric2F1[1, n/2, 1 + n/2, -E^(2*ArcTanh[a*x])] - 4*a*x*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcTanh[a*x])] + a*n^2*x*Hypergeometric2F1[1, n/2,
1 + n/2, E^(2*ArcTanh[a*x])] - 4*a*E^(2*ArcTanh[a*x])*n*x*Hypergeometric2F1
[2, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])]))/(a^2*n*(2 + n)*x))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^2x^2-2\,ac^2x+c^2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2
*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} \left(c - \frac{c}{ax}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a/x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{atanh}(a\,x)} \left(c - \frac{c}{a\,x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a*x))^2,x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - c/(a*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int a^2e^{n\operatorname{atanh}(ax)}\,dx+\int\frac{e^{n\operatorname{atanh}(ax)}}{x^2}\,dx+\int\left(-\frac{2ae^{n\operatorname{atanh}(ax)}}{x}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a/x)**2,x)
```

```
[Out] c**2*(Integral(a**2*exp(n*atanh(a*x)), x) + Integral(exp(n*atanh(a*x))/x**2
, x) + Integral(-2*a*exp(n*atanh(a*x))/x, x))/a**2
```

3.621 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=187

$$-\frac{2c(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1,\frac{n-2}{2};\frac{n}{2};\frac{ax+1}{1-ax}\right)}{a(2-n)}+\frac{c2^{n/2}(1-n)(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(\frac{2-n}{2},2-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)(4-n)}+\frac{c(ax}{}$$

```
[Out] c*(-a*x+1)^(2-1/2*n)*(a*x+1)^(-1+1/2*n)/a/(2-n)-2*c*(-a*x+1)^(1-1/2*n)*(a*x
+1)^(-1+1/2*n)*hypergeom([1, -1+1/2*n],[1/2*n],(a*x+1)/(-a*x+1))/a/(2-n)+2^
(1/2*n)*c*(1-n)*(-a*x+1)^(2-1/2*n)*hypergeom([2-1/2*n, 1-1/2*n],[3-1/2*n],-
1/2*a*x+1/2)/a/(n^2-6*n+8)
```

Rubi [A] time = 0.13, antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6131, 6129, 105, 69, 131}

$$-\frac{c2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)}-\frac{2c(ax+1)^{n/2}(1-ax)^{-n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{an}+\frac{c2^{\frac{n}{2}+1}(1}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] (-2*c*(1 + a*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (1 - a*x)/(1 + a*
x)])/(a*n*(1 - a*x)^(n/2)) - (2^(1 + n/2)*c*(1 - a*x)^(1 - n/2)*Hypergeomet
ric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a*(2 - n)) + (2^(1 + n/2)*c*H
ypergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a*n*(1 - a*x)^(n/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\frac{c\int\frac{e^{n\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int\frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{n/2}}{x}\,dx}{a}\\
&= c\int(1-ax)^{-n/2}(1+ax)^{n/2}\,dx-\frac{c\int\frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{x}\,dx}{a}\\
&= -\frac{2^{1+\frac{n}{2}}c(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)}+c\int(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx\\
&= -\frac{2c(1-ax)^{-n/2}(1+ax)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{an}-\frac{2^{1+\frac{n}{2}}c(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2}\right.}{a(2-n)}
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 180, normalized size = 0.96

$$\frac{ce^{n\tanh^{-1}(ax)}\left(e^{2\tanh^{-1}(ax)}{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\tanh^{-1}(ax)}\right)+e^{2\tanh^{-1}(ax)}{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\tanh^{-1}(ax)}\right)-\frac{(n+2)\left(\right.}{}\right.}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*E^(n*ArcTanh[a*x])*(E^(2*ArcTanh[a*x])*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, -E^(2*ArcTanh[a*x])] + E^(2*ArcTanh[a*x])*Hypergeometric2F1[1, 1 + n/
2, 2 + n/2, E^(2*ArcTanh[a*x])] - ((2 + n)*(Hypergeometric2F1[1, n/2, 1 + n
/2, -E^(2*ArcTanh[a*x])] - Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcTanh[
a*x])]))/n + 4*E^(2*ArcTanh[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E
^(2*ArcTanh[a*x])]))/(a*(2 + n))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}\left(c-\frac{c}{ax}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a/x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{atanh}(a\,x)}\,\left(c - \frac{c}{a\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a*x)),x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - c/(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int ae^{n\,\mathrm{atanh}\,(ax)}\,dx + \int \left(-\frac{e^{n\,\mathrm{atanh}\,(ax)}}{x}\right) dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a/x),x)
```

```
[Out] c*(Integral(a*exp(n*atanh(a*x)), x) + Integral(-exp(n*atanh(a*x))/x, x))/a
```

3.622 $\int \frac{e^{n \tanh^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{n}{2}+1}(n+1)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac(2-n)n}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-n/2}}{acn}$$

```
[Out] -(a*x+1)^(1+1/2*n)/a/c/n/((-a*x+1)^(1/2*n))-2^(1+1/2*n)*(1+n)*(-a*x+1)^(1-1
/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],-1/2*a*x+1/2)/a/c/(2-n)/n
```

Rubi [A] time = 0.13, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6131, 6129, 79, 69}

$$-\frac{2^{\frac{n}{2}+1}(n+1)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac(2-n)n}-\frac{(ax+1)^{\frac{n+2}{2}}(1-ax)^{-n/2}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] -((1 + a*x)^((2 + n)/2)/(a*c*n*(1 - a*x)^(n/2))) - (2^(1 + n/2)*(1 + n)*(1
- a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1 - a*x)/2])/(a
*c*(2 - n)*n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\frac{a\int \frac{e^{n\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\
&= -\frac{a\int x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c}\\
&= -\frac{(1-ax)^{-n/2}(1+ax)^{\frac{2+n}{2}}}{acn}+\frac{(1+n)\int(1-ax)^{-n/2}(1+ax)^{n/2}\,dx}{cn}\\
&= -\frac{(1-ax)^{-n/2}(1+ax)^{\frac{2+n}{2}}}{acn}-\frac{2^{1+\frac{n}{2}}(1+n)(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac(2-n)n}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 95, normalized size = 0.86

$$\frac{(1-ax)^{-n/2}\left(-2^{\frac{n}{2}+1}(n+1)(ax-1)\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)-\left((n-2)(ax+1)^{\frac{n}{2}+1}\right)\right)}{ac(n-2)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a*x)),x]
```

```
[Out] (-((-2 + n)*(1 + a*x)^(1 + n/2)) - 2^(1 + n/2)*(1 + n)*(-1 + a*x)*Hypergeom
etric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - a*x)/2])/(a*c*(-2 + n)*n*(1 - a*x)^
(n/2))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x),x, algorithm="fricas")
[Out] integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a/x),x)
[Out] int(exp(n*arctanh(a*x))/(c-c/a/x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{c-\frac{c}{a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - c/(a*x)),x)
[Out] int(exp(n*atanh(a*x))/(c - c/(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int \frac{xe^{n\,\mathrm{atanh}\,(ax)}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a/x),x)
[Out] a*Integral(x*exp(n*atanh(a*x))/(a*x - 1), x)/c
```

3.623 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=139

$$-\frac{2^{\frac{n}{2}+1}(n+2)(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac^2n}+\frac{(n+3)(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^2(n+2)}-\frac{x(ax+1)^{\frac{n+2}{2}}(1-ax)^{-}}{c^2}$$

```
[Out] (3+n)*(-a*x+1)^(-1-1/2*n)*(a*x+1)^(1+1/2*n)/a/c^2/(2+n)-x*(-a*x+1)^(-1-1/2*
n)*(a*x+1)^(1+1/2*n)/c^2-2^(1+1/2*n)*(2+n)*hypergeom([-1/2*n, -1/2*n],[1-1/
2*n],-1/2*a*x+1/2)/a/c^2/n/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6131, 6129, 90, 79, 69}

$$-\frac{2^{\frac{n}{2}+1}(n+2)(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac^2n}+\frac{(n+3)(ax+1)^{\frac{n+2}{2}}(1-ax)^{-\frac{n}{2}-1}}{ac^2(n+2)}-\frac{x(ax+1)^{\frac{n+2}{2}}(1-ax)^{-}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] ((3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a*c^2*(2 + n)) - (x*(
1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/c^2 - (2^(1 + n/2)*(2 + n)*Hyper
geometric2F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a*c^2*n*(1 - a*x)^(n/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \frac{a^2\int \frac{e^{n\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$

$$= \frac{a^2\int x^2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c^2}$$

$$= -\frac{x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{c^2} - \frac{\int (1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}(-1-a(2+n)x)\,dx}{c^2}$$

$$= \frac{(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^2(2+n)} - \frac{x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{c^2} - \frac{(2+n)\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/}}{c^2}$$

$$= \frac{(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{ac^2(2+n)} - \frac{x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{c^2} - \frac{2^{1+\frac{n}{2}}(2+n)(1-ax)^{-n/2}\,{}_2F_1\left(-\right.}{ac^2n}$$

Mathematica [A] time = 0.47, size = 194, normalized size = 1.40

$$\frac{e^{n\tanh^{-1}(ax)}\left(-2n(ax-1)e^{2\tanh^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\tanh^{-1}(ax)}\right)-4ne^{2\tanh^{-1}(ax)}\,{}_2F_1\left(2,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\tan\ldots}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (E^(n*ArcTanh[a*x])*(4 + n - 4*a*x - 3*a*n*x - 2*E^(2*ArcTanh[a*x])*n*(-1 +
 a*x)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])] + 2*(2 +
n)*(-1 + a*x)*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcTanh[a*x])] - 4*E
^(2*ArcTanh[a*x])*n*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*
x])] + 4*a*E^(2*ArcTanh[a*x])*n*x*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E
^(2*ArcTanh[a*x])]))/(a*c^2*n*(2 + n)*(-1 + a*x))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2x^2-2\,ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*c^2*x + c
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{e^{n\,\mathrm{arctanh}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a/x)^2,x)
[Out] int(exp(n*arctanh(a*x))/(c-c/a/x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{\left(c-\frac{c}{a\,x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - c/(a*x))^2,x)
[Out] int(exp(n*atanh(a*x))/(c - c/(a*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2 e^{n\,\mathrm{atanh}\,(ax)}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a/x)**2,x)
[Out] a**2*Integral(x**2*exp(n*atanh(a*x))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

3.624 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$

Optimal. Leaf size=54

$$-\frac{2x\left(c-\frac{c}{ax}\right)^{3/2}F_1\left(-\frac{1}{2};\frac{n-3}{2},-\frac{n}{2};\frac{1}{2};ax,-ax\right)}{(1-ax)^{3/2}}$$

```
[Out] -2*(c-c/a/x)^(3/2)*x*AppellF1(-1/2,-3/2+1/2*n,-1/2*n,1/2,a*x,-a*x)/(-a*x+1)
^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6134, 6129, 133}

$$-\frac{2x\left(c-\frac{c}{ax}\right)^{3/2}F_1\left(-\frac{1}{2};\frac{n-3}{2},-\frac{n}{2};\frac{1}{2};ax,-ax\right)}{(1-ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*(c - c/(a*x))^(3/2)*x*AppellF1[-1/2, (-3 + n)/2, -n/2, 1/2, a*x, -(a*x)
])/(1 - a*x)^(3/2)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
```

```
0] && !IntegerQ[p]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{e^{n\tanh^{-1}(ax)}(1-ax)^{3/2}}{x^{3/2}}\,dx}{(1-ax)^{3/2}}$$
$$=\frac{\left(\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}\right)\int\frac{(1-ax)^{\frac{3}{2}-\frac{n}{2}}(1+ax)^{n/2}}{x^{3/2}}\,dx}{(1-ax)^{3/2}}$$
$$=-\frac{2\left(c-\frac{c}{ax}\right)^{3/2}xF_1\left(-\frac{1}{2};\frac{1}{2}(-3+n),-\frac{n}{2};\frac{1}{2};ax,-ax\right)}{(1-ax)^{3/2}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a*x))^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(acx-c)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x, algorithm="fricas")
[Out] integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/
(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} \left(c - \frac{c}{ax}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x)
[Out] int(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^{\frac{3}{2}} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{n \operatorname{atanh}(a\,x)} \left(c - \frac{c}{a\,x}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a*x))^(3/2),x)
[Out] int(exp(n*atanh(a*x))*(c - c/(a*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a/x)**(3/2),x)
[Out] Timed out
```

3.625 $\int e^{n \tanh^{-1}(ax)} \sqrt{c - \frac{c}{ax}} \, dx$

Optimal. Leaf size=54

$$\frac{2x\sqrt{c - \frac{c}{ax}} F_1\left(\frac{1}{2}; \frac{n-1}{2}, -\frac{n}{2}; \frac{3}{2}; ax, -ax\right)}{\sqrt{1 - ax}}$$

```
[Out] 2*x*AppellF1(1/2,-1/2+1/2*n,-1/2*n,3/2,a*x,-a*x)*(c-c/a/x)^(1/2)/(-a*x+1)^(
1/2)
```

Rubi [A] time = 0.15, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6134, 6129, 133}

$$\frac{2x\sqrt{c - \frac{c}{ax}} F_1\left(\frac{1}{2}; \frac{n-1}{2}, -\frac{n}{2}; \frac{3}{2}; ax, -ax\right)}{\sqrt{1 - ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*x*AppellF1[1/2, (-1 + n)/2, -n/2, 3/2, a*x, -(a*x)])/S
qrt[1 - a*x]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
```

```
0] && !IntegerQ[p]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{e^{n\tanh^{-1}(ax)}\sqrt{1-ax}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{ax}}\sqrt{x}\right)\int\frac{(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{n/2}}{\sqrt{x}}\,dx}{\sqrt{1-ax}}$$
$$=\frac{2\sqrt{c-\frac{c}{ax}}\,xF_1\left(\frac{1}{2};\frac{1}{2}(-1+n),-\frac{n}{2};\frac{3}{2};ax,-ax\right)}{\sqrt{1-ax}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*Sqrt[c - c/(a*x)],x]
[Out] $Aborted
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\sqrt{\frac{acx-c}{ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x)
[Out] int(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c-\frac{c}{ax}}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{n\,\mathrm{atanh}(a\,x)}\,\sqrt{c-\frac{c}{a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a*x))^(1/2),x)
[Out] int(exp(n*atanh(a*x))*(c - c/(a*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(-1+\frac{1}{ax}\right)}e^{n\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a/x)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*exp(n*atanh(a*x)), x)
```

3.626 $\int \frac{e^{n \tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}} \, dx$

Optimal. Leaf size=56

$$\frac{2x\sqrt{1-ax}\,F_1\left(\frac{3}{2};\frac{n+1}{2},-\frac{n}{2};\frac{5}{2};ax,-ax\right)}{3\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 2/3*x*AppellF1(3/2,1/2+1/2*n,-1/2*n,5/2,a*x,-a*x)*(-a*x+1)^(1/2)/(c-c/a/x)^
(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6134, 6129, 133}

$$\frac{2x\sqrt{1-ax}\,F_1\left(\frac{3}{2};\frac{n+1}{2},-\frac{n}{2};\frac{5}{2};ax,-ax\right)}{3\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (2*x*Sqrt[1 - a*x]*AppellF1[3/2, (1 + n)/2, -n/2, 5/2, a*x, -(a*x)])/(3*Sqr
t[c - c/(a*x)])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-ax}\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{x}}{\sqrt{1-ax}}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{\sqrt{1-ax}\int \sqrt{x}\,(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{n/2}\,dx}{\sqrt{c-\frac{c}{ax}}\sqrt{x}}$$

$$= \frac{2x\sqrt{1-ax}\,F_1\left(\frac{3}{2};\frac{1+n}{2},-\frac{n}{2};\frac{5}{2};ax,-ax\right)}{3\sqrt{c-\frac{c}{ax}}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{acx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a*c*x -
 c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x)
[Out] int(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - c/(a*x))^(1/2),x)
```

```
[Out] int(exp(n*atanh(a*x))/(c - c/(a*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/sqrt(-c*(-1 + 1/(a*x))), x)
```

3.627 $\int \frac{e^{n \tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}} dx$

Optimal. Leaf size=56

$$\frac{2x(1-ax)^{3/2}F_1\left(\frac{5}{2};\frac{n+3}{2},-\frac{n}{2};\frac{7}{2};ax,-ax\right)}{5\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] 2/5*x*(-a*x+1)^(3/2)*AppellF1(5/2,3/2+1/2*n,-1/2*n,7/2,a*x,-a*x)/(c-c/a/x)^
(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6134, 6129, 133}

$$\frac{2x(1-ax)^{3/2}F_1\left(\frac{5}{2};\frac{n+3}{2},-\frac{n}{2};\frac{7}{2};ax,-ax\right)}{5\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (2*x*(1 - a*x)^(3/2)*AppellF1[5/2, (3 + n)/2, -n/2, 7/2, a*x, -(a*x)])/(5*(
c - c/(a*x))^(3/2))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6134

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(x^p*(c + d/x)^p)/(1 + (c*x)/d)^p, Int[(u*(1 + (c*x)/d)^p*E^(n*Arc
```

```
Tanh[a*x]))/x^p, x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2,
0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{(1-ax)^{3/2}\int \frac{e^{n\tanh^{-1}(ax)}x^{3/2}}{(1-ax)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= \frac{(1-ax)^{3/2}\int x^{3/2}(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{n/2}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$
$$= \frac{2x(1-ax)^{3/2}F_1\left(\frac{5}{2};\frac{3+n}{2},-\frac{n}{2};\frac{7}{2};ax,-ax\right)}{5\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{a^2c^2x^2-2\,ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a^2
*c^2*x^2 - 2*a*c^2*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c - \frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{\left(c - \frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x)
[Out] int(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c - \frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{\left(c - \frac{c}{a\,x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - c/(a*x))^(3/2),x)
```

```
[Out] int(exp(n*atanh(a*x))/(c - c/(a*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a/x)**(3/2),x)
[Out] Integral(exp(n*atanh(a*x))/(-c*(-1 + 1/(a*x)))**(3/2), x)
```

3.628 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=169

$$\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}+\frac{35c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{16a}-\frac{c^4(7ax+6)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}+\frac{c^4(35ax+24)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4}{}$$

```
[Out] -1/48*c^4*(35*a*x+16)*(-a^2*x^2+1)^(3/2)/a^4/x^3+1/120*c^4*(35*a*x+24)*(-a^
2*x^2+1)^(5/2)/a^6/x^5-1/42*c^4*(7*a*x+6)*(-a^2*x^2+1)^(7/2)/a^8/x^7+c^4*ar
csin(a*x)/a+35/16*c^4*arctanh((-a^2*x^2+1)^(1/2))/a+1/16*c^4*(-35*a*x+16)*(
-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6157, 6148, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^4(7ax+6)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}+\frac{c^4(35ax+24)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4(35ax+16)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*(16 - 35*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) - (c^4*(16 + 35*a*x)*(1 -
a^2*x^2)^(3/2))/(48*a^4*x^3) + (c^4*(24 + 35*a*x)*(1 - a^2*x^2)^(5/2))/(120
*a^6*x^5) - (c^4*(6 + 7*a*x)*(1 - a^2*x^2)^(7/2))/(42*a^8*x^7) + (c^4*ArcSi
n[a*x])/a + (35*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(16*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
```

```
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx=\frac{c^4\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$=\frac{c^4\int\frac{(1+ax)\left(1-a^2x^2\right)^{7/2}}{x^8}dx}{a^8}$$

$$=-\frac{c^4(6+7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}-\frac{c^4\int\frac{\left(12a^2+14a^3x\right)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{12a^8}$$

$$=\frac{c^4(24+35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4(6+7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7}+\frac{c^4\int\frac{\left(96a^4+140a^5x\right)\left(1-a^2x^2\right.}{x^4}}{96a^8}$$

$$=-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5}-\frac{c^4(6+7ax)\left(1-a^2\right.}{42a^8x^7}$$

$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x\right.}{120a^6x^5}$$

$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x\right.}{120a^6x^5}$$

$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x\right.}{120a^6x^5}$$

$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x\right.}{120a^6x^5}$$

$$=\frac{c^4(16-35ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{c^4(16+35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3}+\frac{c^4(24+35ax)\left(1-a^2x\right.}{120a^6x^5}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.41

$$\frac{c^4\left(-\frac{9\,{}_2F_1\left(-\frac{7}{2},-\frac{7}{2};-\frac{5}{2};a^2x^2\right)}{x^7}-7a^7\left(1-a^2x^2\right)^{9/2}\,{}_2F_1\left(4,\frac{9}{2};\frac{11}{2};1-a^2x^2\right)\right)}{63a^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*((-9*Hypergeometric2F1[-7/2, -7/2, -5/2, a^2*x^2])/x^7 - 7*a^7*(1 - a^
2*x^2)^(9/2)*Hypergeometric2F1[4, 9/2, 11/2, 1 - a^2*x^2]))/(63*a^8)
```

fricas [A] time = 0.64, size = 175, normalized size = 1.04

$$-\frac{3360\,a^7c^4x^7\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3675\,a^7c^4x^7\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+1680\,a^7c^4x^7+\left(1680\,a^7c^4x^7-2816\,a^6c^4x^6+\cdots\right.}{1680\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/1680*(3360*a^7*c^4*x^7*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3675*a^7
*c^4*x^7*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 1680*a^7*c^4*x^7 + (1680*a^7*c^4
*x^7 - 2816*a^6*c^4*x^6 + 3045*a^5*c^4*x^5 + 1952*a^4*c^4*x^4 - 1330*a^3*c^
4*x^3 - 1056*a^2*c^4*x^2 + 280*a*c^4*x + 240*c^4)*sqrt(-a^2*x^2 + 1))/(a^8*
x^7)
```

giac [B] time = 0.21, size = 505, normalized size = 2.99

$$\frac{\left(15\,c^4+\frac{35\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^4}{a^2x}-\frac{189\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^4}{a^4x^2}-\frac{525\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^4}{a^6x^3}+\frac{1295\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^4}{a^8x^4}+\frac{4935\left(\sqrt{-a^2x^2+1}\,|a|\cdots\right.}{a^{10}x^5}\cdots}{13440\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/13440*(15*c^4 + 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) - 189*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2) - 525*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)^3*c^4/(a^6*x^3) + 1295*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^8*x^
4) + 4935*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^10*x^5) - 9765*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^6*c^4/(a^12*x^6))*a^14*x^7/((sqrt(-a^2*x^2 + 1)*abs(
```

```
a) + a)^7*abs(a)) + c^4*arcsin(a*x)*sgn(a)/abs(a) + 35/16*c^4*log(1/2*abs(-
2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1
)*c^4/a + 1/13440*(9765*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x - 4935*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 1295*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^3*c^4/x^3 + 525*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4) + 1
89*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^4*x^5) - 35*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^6*c^4/(a^6*x^6) - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7*c^4/(a^
8*x^7))/(a^6*abs(a))
```

maple [A] time = 0.06, size = 233, normalized size = 1.38

$$-\frac{c^4\sqrt{-a^2x^2+1}}{a}+\frac{c^4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{35c^4\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{16a}+\frac{176c^4\sqrt{-a^2x^2+1}}{105a^2x}-\frac{122c^4\sqrt{-a^2x^2+1}}{105a^4x^3}-\frac{29c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^4,x)
```

```
[Out] -c^4*(-a^2*x^2+1)^(1/2)/a+c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))+35/16*c^4/a*arctanh(1/(-a^2*x^2+1)^(1/2))+176/105*c^4*(-a^2*x^2+1)^
(1/2)/a^2/x-122/105*c^4*(-a^2*x^2+1)^(1/2)/a^4/x^3-29/16*c^4*(-a^2*x^2+1)^(
1/2)/x^2/a^3-1/7*c^4/a^8/x^7*(-a^2*x^2+1)^(1/2)+22/35*c^4/a^6/x^5*(-a^2*x^2
+1)^(1/2)-1/6*c^4/a^7/x^6*(-a^2*x^2+1)^(1/2)+19/24*c^4/a^5/x^4*(-a^2*x^2+1)
^(1/2)
```

maxima [B] time = 0.42, size = 515, normalized size = 3.05

$$\frac{c^4\arcsin(ax)}{a}+\frac{4c^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}-\frac{\sqrt{-a^2x^2+1}\,c^4}{a}-\frac{3\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^4}{a^3}+\frac{4\sqrt{-a^2x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*arcsin(a*x)/a + 4*c^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a - s
qrt(-a^2*x^2 + 1)*c^4/a - 3*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)
) + sqrt(-a^2*x^2 + 1)/x^2)*c^4/a^3 + 4*sqrt(-a^2*x^2 + 1)*c^4/(a^2*x) - 2*
(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*c^4/a^4 + 1/2*(3*a^4*
log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 3*sqrt(-a^2*x^2 + 1)*a^2/x^2
+ 2*sqrt(-a^2*x^2 + 1)/x^4)*c^4/a^5 + 4/15*(8*sqrt(-a^2*x^2 + 1)*a^4/x + 4*
sqrt(-a^2*x^2 + 1)*a^2/x^3 + 3*sqrt(-a^2*x^2 + 1)/x^5)*c^4/a^6 - 1/48*(15*a
^6*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 15*sqrt(-a^2*x^2 + 1)*a^4/
x^2 + 10*sqrt(-a^2*x^2 + 1)*a^2/x^4 + 8*sqrt(-a^2*x^2 + 1)/x^6)*c^4/a^7 - 1
/35*(16*sqrt(-a^2*x^2 + 1)*a^6/x + 8*sqrt(-a^2*x^2 + 1)*a^4/x^3 + 6*sqrt(-a
^2*x^2 + 1)*a^2/x^5 + 5*sqrt(-a^2*x^2 + 1)/x^7)*c^4/a^8
```

mupad [B] time = 0.88, size = 228, normalized size = 1.35

$$\frac{c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{a}+\frac{176\,c^4\,\sqrt{1-a^2\,x^2}}{105\,a^2\,x}-\frac{29\,c^4\,\sqrt{1-a^2\,x^2}}{16\,a^3\,x^2}-\frac{122\,c^4\,\sqrt{1-a^2\,x^2}}{105\,a^4\,x^3}+\frac{19\,c^4\,\sqrt{1-a^2\,x^2}}{24\,a^5\,x^4}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^4*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (c^4*atan((1 - a^2*x^2)^(1/2)*1i
)*35i)/(16*a) - (c^4*(1 - a^2*x^2)^(1/2))/a + (176*c^4*(1 - a^2*x^2)^(1/2))
/(105*a^2*x) - (29*c^4*(1 - a^2*x^2)^(1/2))/(16*a^3*x^2) - (122*c^4*(1 - a^
2*x^2)^(1/2))/(105*a^4*x^3) + (19*c^4*(1 - a^2*x^2)^(1/2))/(24*a^5*x^4) + (
22*c^4*(1 - a^2*x^2)^(1/2))/(35*a^6*x^5) - (c^4*(1 - a^2*x^2)^(1/2))/(6*a^7
*x^6) - (c^4*(1 - a^2*x^2)^(1/2))/(7*a^8*x^7)
```

sympy [A] time = 26.21, size = 1119, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] a*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 + c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a*
*2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 4*c**4*Piecewise((-acosh(1/(a*x)), 1
/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - 4*c**4*Piecewise((-I*sqr
t(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a
**2 + 6*c**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))
/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 -
1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + 6*c**
4*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3
*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a*
*2*x**2 + 1)/(3*x**3), True))/a**4 - 4*c**4*Piecewise((-3*a**4*acosh(1/(a*x
))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*
x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*
I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x*
*3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))/
a**5 - 4*c**4*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(
-1 + 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(
a**2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 -
1/(a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))/a**
6 + c**4*Piecewise((-5*a**6*acosh(1/(a*x))/16 + 5*a**5/(16*x*sqrt(-1 + 1/(a
**2*x**2))) - 5*a**3/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) - a/(24*x**5*sqrt(-
```

```
1 + 1/(a**2*x**2))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**
2) > 1), (5*I*a**6*asin(1/(a*x))/16 - 5*I*a**5/(16*x*sqrt(1 - 1/(a**2*x**2)
)) + 5*I*a**3/(48*x**3*sqrt(1 - 1/(a**2*x**2))) + I*a/(24*x**5*sqrt(1 - 1/(
a**2*x**2))) + I/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**7 + c**4*Pie
cewise((-16*a**7*sqrt(-1 + 1/(a**2*x**2))/35 - 8*a**5*sqrt(-1 + 1/(a**2*x**
2))/(35*x**2) - 6*a**3*sqrt(-1 + 1/(a**2*x**2))/(35*x**4) - a*sqrt(-1 + 1/(
a**2*x**2))/(7*x**6), 1/Abs(a**2*x**2) > 1), (-16*I*a**7*sqrt(1 - 1/(a**2*x
**2))/35 - 8*I*a**5*sqrt(1 - 1/(a**2*x**2))/(35*x**2) - 6*I*a**3*sqrt(1 - 1
/(a**2*x**2))/(35*x**4) - I*a*sqrt(1 - 1/(a**2*x**2))/(7*x**6), True))/a**8
```

3.629 $\int e^{\tanh^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^3 dx$

Optimal. Leaf size=136

$$\frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{15c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3(5ax+4)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}-\frac{c^3(15ax+8)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3\sin^{-1}}{a}$$

```
[Out] -1/24*c^3*(15*a*x+8)*(-a^2*x^2+1)^(3/2)/a^4/x^3+1/20*c^3*(5*a*x+4)*(-a^2*x^
2+1)^(5/2)/a^6/x^5+c^3*arcsin(a*x)/a+15/8*c^3*arctanh((-a^2*x^2+1)^(1/2))/a
+1/8*c^3*(-15*a*x+8)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.19, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6157, 6148, 811, 813, 844, 216, 266, 63, 208}

$$\frac{c^3(5ax+4)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}-\frac{c^3(15ax+8)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x}+\frac{15c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3\sin^{-1}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*(8 - 15*a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) - (c^3*(8 + 15*a*x)*(1 - a^2
*x^2)^(3/2))/(24*a^4*x^3) + (c^3*(4 + 5*a*x)*(1 - a^2*x^2)^(5/2))/(20*a^6*x
^5) + (c^3*ArcSin[a*x])/a + (15*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(8*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$

$$= -\frac{c^3\int\frac{(1+ax)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{a^6}$$

$$= \frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}+\frac{c^3\int\frac{\left(8a^2+10a^3x\right)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{8a^6}$$

$$= -\frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}-\frac{c^3\int\frac{\left(32a^4+60a^5x\right)\sqrt{1-a^2x^2}}{x^2}}{32a^6}$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x}-\frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}+$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x}-\frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}+$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x}-\frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}+$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x}-\frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}+$$

$$= \frac{c^3(8-15ax)\sqrt{1-a^2x^2}}{8a^2x}-\frac{c^3(8+15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(4+5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}+$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.51

$$\frac{c^3\left(\frac{7\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};a^2x^2\right)}{x^5}+5a^5\left(1-a^2x^2\right)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-a^2x^2\right)\right)}{35a^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*((7*Hypergeometric2F1[-5/2, -5/2, -3/2, a^2*x^2])/x^5 + 5*a^5*(1 - a^2
*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 - a^2*x^2]))/(35*a^6)
```

fricas [A] time = 0.62, size = 153, normalized size = 1.12

$$-\frac{240\,a^5c^3x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+225\,a^5c^3x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+120\,a^5c^3x^5+\left(120\,a^5c^3x^5-184\,a^4c^3x^4+135\right.}{120\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/120*(240*a^5*c^3*x^5*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 225*a^5*c^
3*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 120*a^5*c^3*x^5 + (120*a^5*c^3*x^5
- 184*a^4*c^3*x^4 + 135*a^3*c^3*x^3 + 88*a^2*c^3*x^2 - 30*a*c^3*x - 24*c^3)
*sqrt(-a^2*x^2 + 1))/(a^6*x^5)
```

giac [B] time = 0.27, size = 385, normalized size = 2.83

$$-\frac{\left(6\,c^3+\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^3}{a^2x}-\frac{70\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^3}{a^4x^2}-\frac{240\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^3}{a^6x^3}+\frac{660\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^3}{a^8x^4}\right)a^{10}x^5}{960\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5|a|}+\frac{c^3\arcsin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/960*(6*c^3 + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x) - 70*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2) - 240*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^3/(a^6*x^3) + 660*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3/(a^8*x^4))*
a^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) + c^3*arcsin(a*x)*sgn(a
)/abs(a) + 15/8*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*ab
s(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^3/a + 1/960*(660*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)*a^2*c^3/x - 240*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2 - 70*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/(a^2*x^3) + 15*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^4*c^3/(a^4*x^4) + 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/(a^6*x^5
))/(a^4*abs(a))
```

maple [A] time = 0.05, size = 187, normalized size = 1.38

$$-\frac{c^3\sqrt{-a^2x^2+1}}{a}+\frac{c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{15c^3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{8a}+\frac{23c^3\sqrt{-a^2x^2+1}}{15a^2x}-\frac{11c^3\sqrt{-a^2x^2+1}}{15a^4x^3}-\frac{9c^3\sqrt{-}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^3,x)
```

```
[Out] -c^3*(-a^2*x^2+1)^(1/2)/a+c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))+15/8*c^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))+23/15*c^3*(-a^2*x^2+1)^(1/
2)/a^2/x-11/15*c^3/a^4/x^3*(-a^2*x^2+1)^(1/2)-9/8*c^3*(-a^2*x^2+1)^(1/2)/x^
2/a^3+1/4*c^3/a^5/x^4*(-a^2*x^2+1)^(1/2)+1/5*c^3/a^6/x^5*(-a^2*x^2+1)^(1/2)
```

maxima [B] time = 0.42, size = 332, normalized size = 2.44

$$\frac{c^3 \arcsin(ax)}{a} + \frac{3\,c^3 \log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)}{a} - \frac{\sqrt{-a^2x^2+1}\,c^3}{a} - \frac{3\left(a^2 \log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^3}{2\,a^3} + \frac{3\,\sqrt{-a^2x^2}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*arcsin(a*x)/a + 3*c^3*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a - s
qrt(-a^2*x^2 + 1)*c^3/a - 3/2*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(
x)) + sqrt(-a^2*x^2 + 1)/x^2)*c^3/a^3 + 3*sqrt(-a^2*x^2 + 1)*c^3/(a^2*x) -
(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*c^3/a^4 + 1/8*(3*a^4*
log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 3*sqrt(-a^2*x^2 + 1)*a^2/x^2
+ 2*sqrt(-a^2*x^2 + 1)/x^4)*c^3/a^5 + 1/15*(8*sqrt(-a^2*x^2 + 1)*a^4/x + 4*
sqrt(-a^2*x^2 + 1)*a^2/x^3 + 3*sqrt(-a^2*x^2 + 1)/x^5)*c^3/a^6
```

mupad [B] time = 0.05, size = 182, normalized size = 1.34

$$\frac{c^3 \operatorname{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}} - \frac{c^3\,\sqrt{1-a^2\,x^2}}{a} + \frac{23\,c^3\,\sqrt{1-a^2\,x^2}}{15\,a^2\,x} - \frac{9\,c^3\,\sqrt{1-a^2\,x^2}}{8\,a^3\,x^2} - \frac{11\,c^3\,\sqrt{1-a^2\,x^2}}{15\,a^4\,x^3} + \frac{c^3\,\sqrt{1-a^2\,x^2}}{4\,a^5\,x^4} + \frac{c^3\,\sqrt{1-}}{5\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (c^3*atan((1 - a^2*x^2)^(1/2)*1i
)*15i)/(8*a) - (c^3*(1 - a^2*x^2)^(1/2))/a + (23*c^3*(1 - a^2*x^2)^(1/2))/(
15*a^2*x) - (9*c^3*(1 - a^2*x^2)^(1/2))/(8*a^3*x^2) - (11*c^3*(1 - a^2*x^2)
^(1/2))/(15*a^4*x^3) + (c^3*(1 - a^2*x^2)^(1/2))/(4*a^5*x^4) + (c^3*(1 - a^
2*x^2)^(1/2))/(5*a^6*x^5)
```

sympy [A] time = 16.35, size = 687, normalized size = 5.05

$$ac^3\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+c^3\left(\begin{cases}\sqrt{\frac{1}{a^2}}\,\text{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\,\text{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)-\frac{3c^3\left(\begin{cases}-\text{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\,\text{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 + c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a*
*2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 3*c**3*Piecewise((-acosh(1/(a*x)), 1
/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - 3*c**3*Piecewise((-I*sqr
t(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a
**2 + 3*c**3*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))
/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 -
1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + 3*c**
3*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3
*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a*
*2*x**2 + 1)/(3*x**3), True))/a**4 - c**3*Piecewise((-3*a**4*acosh(1/(a*x))
/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x*
*2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*
a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3
*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a*
*5 - c**3*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 +
 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2
*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a
**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))/a**6
```

3.630 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=103

$$\frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{c^2(3ax+2)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2\sin^{-1}(ax)}{a}$$

```
[Out] -1/6*c^2*(3*a*x+2)*(-a^2*x^2+1)^(3/2)/a^4/x^3+c^2*arcsin(a*x)/a+3/2*c^2*arc
tanh((-a^2*x^2+1)^(1/2))/a+1/2*c^2*(-3*a*x+2)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.16, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6157, 6148, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(3ax+2)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*(2 - 3*a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(2 + 3*a*x)*(1 - a^2*x
^2)^(3/2))/(6*a^4*x^3) + (c^2*ArcSin[a*x])/a + (3*c^2*ArcTanh[Sqrt[1 - a^2*
x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
```

```
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}$$

$$=\frac{c^2\int\frac{(1+ax)\left(1-a^2x^2\right)^{3/2}}{x^4}\,dx}{a^4}$$

$$=-\frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}-\frac{c^2\int\frac{\left(4a^2+6a^3x\right)\sqrt{1-a^2x^2}}{x^2}\,dx}{4a^4}$$

$$=\frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2\int\frac{-12a^3+8a^4x}{x\sqrt{1-a^2x^2}}\,dx}{8a^4}$$

$$=\frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+c^2\int\frac{1}{\sqrt{1-a^2x^2}}\,dx-\frac{\left(3c^2\right)\int}{}$$

$$=\frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2\sin^{-1}(ax)}{a}-\frac{\left(3c^2\right)\text{Subst}\left(\int\right.}{}$$

$$=\frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2\sin^{-1}(ax)}{a}+\frac{\left(3c^2\right)\text{Subst}\left(\int\right.}{}$$

$$=\frac{c^2(2-3ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2(2+3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2\sin^{-1}(ax)}{a}+\frac{3c^2\tanh^{-1}\left(\sqrt{\right.}}{2a}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.68

$$\frac{c^2\left(-\frac{5\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};a^2x^2\right)}{x^3}-3a^3\left(1-a^2x^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-a^2x^2\right)\right)}{15a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*((-5*Hypergeometric2F1[-3/2, -3/2, -1/2, a^2*x^2])/x^3 - 3*a^3*(1 - a^
2*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 - a^2*x^2]))/(15*a^4)
```

fricas [A] time = 0.40, size = 131, normalized size = 1.27

$$-\frac{12\,a^3c^2x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+9\,a^3c^2x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+6\,a^3c^2x^3+\left(6\,a^3c^2x^3-8\,a^2c^2x^2+3\,ac^2x+2\,c^2\right)\sqrt{\ldots}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a^3*c^2*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 9*a^3*c^2*x^3
*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 6*a^3*c^2*x^3 + (6*a^3*c^2*x^3 - 8*a^2*c
^2*x^2 + 3*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

giac [B] time = 0.57, size = 263, normalized size = 2.55

$$\frac{\left(c^2+\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^2}{a^2x}-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^2}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|}+\frac{c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}+\frac{3\,c^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\sqrt{-a^2x^2\ldots}}{a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(c^2 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x) - 15*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^2/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) + c^2*arcsin(a*x)*sgn(a)/abs(a) + 3/2*c^2*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^2/a
 + 1/24*(15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/x - 3*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2*c^2/(a^2*x^2) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a^4*x^3
))/(a^2*abs(a))
```

maple [A] time = 0.04, size = 141, normalized size = 1.37

$$-\frac{c^2\sqrt{-a^2x^2+1}}{a}+\frac{c^2\arctan\left(\frac{\sqrt{a^2}\;x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{3c^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2a}+\frac{4c^2\sqrt{-a^2x^2+1}}{3a^2x}-\frac{c^2\sqrt{-a^2x^2+1}}{3a^4x^3}-\frac{c^2\sqrt{-a^2x^2\ldots}}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^2,x)
```

```
[Out] -c^2*(-a^2*x^2+1)^(1/2)/a+c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))+3/2*c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))+4/3*c^2*(-a^2*x^2+1)^(1/2)/
a^2/x-1/3*c^2/a^4/x^3*(-a^2*x^2+1)^(1/2)-1/2*c^2/a^3/x^2*(-a^2*x^2+1)^(1/2)
```

maxima [B] time = 0.41, size = 189, normalized size = 1.83

$$\frac{c^2 \arcsin(ax)}{a} + \frac{2c^2 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)}{a} - \frac{\sqrt{-a^2x^2+1}\,c^2}{a} - \frac{\left(a^2 \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-a^2x^2+1}}{x^2}\right)c^2}{2a^3} + \frac{2\sqrt{-a^2x^2+}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*arcsin(a*x)/a + 2*c^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a - s
qrt(-a^2*x^2 + 1)*c^2/a - 1/2*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(
x)) + sqrt(-a^2*x^2 + 1)/x^2)*c^2/a^3 + 2*sqrt(-a^2*x^2 + 1)*c^2/(a^2*x) -
1/3*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*c^2/a^4
```

mupad [B] time = 0.04, size = 136, normalized size = 1.32

$$\frac{c^2 \operatorname{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}} - \frac{c^2\sqrt{1-a^2x^2}}{a} + \frac{4c^2\sqrt{1-a^2x^2}}{3a^2x} - \frac{c^2\sqrt{1-a^2x^2}}{2a^3x^2} - \frac{c^2\sqrt{1-a^2x^2}}{3a^4x^3} - \frac{c^2 \operatorname{atan}\left(\sqrt{1-a^2x^2}\,1i\right)3i}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (c^2*atan((1 - a^2*x^2)^(1/2)*1i
)*3i)/(2*a) - (c^2*(1 - a^2*x^2)^(1/2))/a + (4*c^2*(1 - a^2*x^2)^(1/2))/(3*
a^2*x) - (c^2*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2) - (c^2*(1 - a^2*x^2)^(1/2))/
(3*a^4*x^3)
```

sympy [A] time = 9.86, size = 354, normalized size = 3.44

$$ac^2\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2 = 0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right) + c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2 > 0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2 < 0\end{cases}\right) - \frac{2c^2\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|} > 1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 + c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a*
*2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 2*c**2*Piecewise((-acosh(1/(a*x)), 1
/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - 2*c**2*Piecewise((-I*sqr
t(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a
**2 + c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(
2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/
(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + c**2*Pi
ecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**
3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x
**2 + 1)/(3*x**3), True))/a**4
```

3.631 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=58

$$\frac{c\sqrt{1-a^2x^2}\,(1-ax)}{a^2x}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{c\sin^{-1}(ax)}{a}$$

```
[Out] c*arcsin(a*x)/a+c*arctanh((-a^2*x^2+1)^(1/2))/a+c*(-a*x+1)*(-a^2*x^2+1)^(1/
2)/a^2/x
```

Rubi [A] time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6157, 6148, 813, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}\,(1-ax)}{a^2x}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(a^2*x) + (c*ArcSin[a*x])/a + (c*ArcTanh[Sq
rt[1 - a^2*x^2]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\frac{c\int \frac{e^{\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$

$$= -\frac{c\int \frac{(1+ax)\sqrt{1-a^2x^2}}{x^2}dx}{a^2}$$

$$= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\int \frac{-2a+2a^2x}{x\sqrt{1-a^2x^2}}dx}{2a^2}$$

$$= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+c\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{c\int \frac{1}{x\sqrt{1-a^2x^2}}dx}{a}$$

$$= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\sin^{-1}(ax)}{a}-\frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}dx,x,x^2\right)}{2a}$$

$$= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\sin^{-1}(ax)}{a}+\frac{c\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}dx,x,\sqrt{1-a^2x^2}\right)}{a^3}$$

$$= \frac{c(1-ax)\sqrt{1-a^2x^2}}{a^2x}+\frac{c\sin^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{c\left(\sqrt{1-a^2x^2}\,(1-ax)+ax\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ax\sin^{-1}(ax)\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*((1 - a*x)*Sqrt[1 - a^2*x^2] + a*x*ArcSin[a*x] + a*x*ArcTanh[Sqrt[1 - a^
2*x^2]]))/(a^2*x)
```

fricas [A] time = 0.42, size = 84, normalized size = 1.45

$$-\frac{2\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+acx+\sqrt{-a^2x^2+1}\,(acx-c)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2),x, algorithm="fricas")

[Out] -(2*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*c*x*log((sqrt(-a^2*x^2 + 1) - 1)/x) + a*c*x + sqrt(-a^2*x^2 + 1)*(a*c*x - c))/(a^2*x)

giac [B] time = 0.26, size = 128, normalized size = 2.21

$$-\frac{a^2cx}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}+\frac{c\arcsin\left(ax\right)\mathrm{sgn}(a)}{|a|}+\frac{c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}\,c}{a}+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right.}{2\,a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2),x, algorithm="giac")

[Out] -1/2*a^2*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + c*arcsin(a*x)*sgn(a)/abs(a) + c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c/a + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c/(a^2*x*abs(a))

maple [A] time = 0.04, size = 85, normalized size = 1.47

$$-\frac{c\sqrt{-a^2x^2+1}}{a}+\frac{c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{c\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}+\frac{c\sqrt{-a^2x^2+1}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2),x)

[Out] -c*(-a^2*x^2+1)^(1/2)/a+c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+c/a*arctanh(1/(-a^2*x^2+1)^(1/2))+c*(-a^2*x^2+1)^(1/2)/a^2/x

maxima [A] time = 0.40, size = 79, normalized size = 1.36

$$\frac{c\arcsin\left(ax\right)}{a}+\frac{c\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)}{a}-\frac{\sqrt{-a^2x^2+1}\,c}{a}+\frac{\sqrt{-a^2x^2+1}\,c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2),x, algorithm="maxima")

[Out] c*arcsin(a*x)/a + c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))/a - sqrt(-a^2*x^2 + 1)*c/a + sqrt(-a^2*x^2 + 1)*c/(a^2*x)

mupad [B] time = 0.83, size = 76, normalized size = 1.31

$$\frac{c\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{a}-\frac{c\,\sqrt{1-a^2\,x^2}}{a}+\frac{c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{c\,\sqrt{1-a^2\,x^2}}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (c*atanh((1 - a^2*x^2)^(1/2)))/a - (c*(1 - a^2*x^2)^(1/2))/a + (c*asinh(x*(
-a^2)^(1/2)))/(-a^2)^(1/2) + (c*(1 - a^2*x^2)^(1/2))/(a^2*x)
```

sympy [A] time = 5.60, size = 144, normalized size = 2.48

$$ac\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\ \mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)-\frac{c\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}-\frac{c\left(\{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2),x)
```

```
[Out] a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) +
c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*as
inh(x*sqrt(-a**2)), a**2 < 0)) - c*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x
**2) > 1), (I*asin(1/(a*x)), True))/a - c*Piecewise((-I*sqrt(a**2*x**2 - 1)
/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2
```

3.632 $\int \frac{e^{\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{ax+1}{ac\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] arcsin(a*x)/a/c+(-a*x-1)/a/c/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a/c
```

Rubi [A] time = 0.13, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6157, 6148, 797, 641, 216, 637}

$$-\frac{ax+1}{ac\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] -((1 + a*x)/(a*c*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/(a*c) + ArcSin[a*x
]/(a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
```

```
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int \frac{x^2(1+ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= -\frac{\int \frac{1+ax}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}+\frac{\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{1+ax}{ac\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= -\frac{1+ax}{ac\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.87

$$\frac{a^2x^2+\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-ax-2}{ac\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2)),x]
```

[Out] (-2 - a*x + a^2*x^2 + Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(a*c*Sqrt[1 - a^2*x^2]
)

fricas [A] time = 0.77, size = 68, normalized size = 1.11

$$-\frac{2\,ax + 2\,(ax - 1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(ax - 2) - 2}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")

[Out] -(2*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x - 2) - 2)/(a^2*c*x - a*c)

giac [A] time = 0.20, size = 72, normalized size = 1.18

$$\frac{\arcsin(ax)\operatorname{sgn}(a)}{c|a|} - \frac{\sqrt{-a^2x^2+1}}{ac} - \frac{2}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")

[Out] arcsin(a*x)*sgn(a)/(c*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c) - 2/(c*((sqrt(-a^2
*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))

maple [A] time = 0.04, size = 94, normalized size = 1.54

$$-\frac{\sqrt{-a^2x^2+1}}{ac} + \frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c\sqrt{a^2}} + \frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2 - 2a\left(x-\frac{1}{a}\right)}}{a^2c\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x)

[Out] -(-a^2*x^2+1)^(1/2)/a/c+1/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(
1/2))+1/a^2/c/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))), x)
```

mupad [B] time = 0.84, size = 90, normalized size = 1.48

$$\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{c\sqrt{-a^2}}-\frac{\sqrt{1-a^2x^2}}{a\,c}-\frac{\sqrt{1-a^2x^2}}{c\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] asinh(x*(-a^2)^(1/2))/(c*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a*c) - (1 - a
^2*x^2)^(1/2)/(c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2}{ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2/(a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x)/c
```

3.633 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx$

Optimal. Leaf size=96

$$-\frac{x(4ax+3)}{3c^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{a^2x^3(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{\sin^{-1}(ax)}{ac^2}$$

```
[Out] 1/3*a^2*x^3*(a*x+1)/c^2/(-a^2*x^2+1)^(3/2)+arcsin(a*x)/a/c^2-1/3*x*(4*a*x+3
)/c^2/(-a^2*x^2+1)^(1/2)-8/3*(-a^2*x^2+1)^(1/2)/a/c^2
```

Rubi [A] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6157, 6148, 819, 641, 216}

$$\frac{a^2x^3(ax+1)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(4ax+3)}{3c^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (a^2*x^3*(1 + a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) - (x*(3 + 4*a*x))/(3*c^2*Sq
rt[1 - a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*a*c^2) + ArcSin[a*x]/(a*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
```

```
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= \frac{a^4\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}\\
&= \frac{a^4\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{a^2x^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2\int \frac{x^2(3+4ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^2}\\
&= \frac{a^2x^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3+4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{\int \frac{3+8ax}{\sqrt{1-a^2x^2}}\,dx}{3c^2}\\
&= \frac{a^2x^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3+4ax)}{3c^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= \frac{a^2x^3(1+ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3+4ax)}{3c^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\sin^{-1}(ax)}{ac^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.81

$$\frac{3a^3x^3-7a^2x^2+3(ax-1)\sqrt{1-a^2x^2}\sin^{-1}(ax)-5ax+8}{3ac^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[
a*x])/(3*a*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.42, size = 142, normalized size = 1.48

$$-\frac{8a^3x^3-8a^2x^2-8ax+6\left(a^3x^3-a^2x^2-ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3a^3x^3-7a^2x^2-5ax+8\right)\sqrt{-a^2x^2+}}{3\left(a^4c^2x^3-a^3c^2x^2-a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(8*a^3*x^3 - 8*a^2*x^2 - 8*a*x + 6*(a^3*x^3 - a^2*x^2 - a*x + 1)*arcta
n((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^3*x^3 - 7*a^2*x^2 - 5*a*x + 8)*sqr
t(-a^2*x^2 + 1) + 8)/(a^4*c^2*x^3 - a^3*c^2*x^2 - a^2*c^2*x + a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^2), x)
```

maple [B] time = 0.05, size = 177, normalized size = 1.84

$$-\frac{\sqrt{-a^2x^2+1}}{a\,c^2}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^2\sqrt{a^2}}+\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{6a^3c^2\left(x-\frac{1}{a}\right)^2}+\frac{19\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{12a^2c^2\left(x-\frac{1}{a}\right)}-\frac{\sqrt{-a^2\left(x+\right.}}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^2+1/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+1/6/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+19/12/a^
2/c^2/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/4/a^2/c^2/(x+1/a)*(-a^2*
(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^2), x)
```

mupad [B] time = 0.07, size = 188, normalized size = 1.96

$$\frac{a\sqrt{1-a^2x^2}}{6\left(a^4c^2x^2-2a^3c^2x+a^2c^2\right)}+\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{c^2\sqrt{-a^2}}-\frac{\sqrt{1-a^2x^2}}{a\,c^2}+\frac{\sqrt{1-a^2x^2}}{4\sqrt{-a^2}\left(c^2x\sqrt{-a^2}+\frac{c^2\sqrt{-a^2}}{a}\right)}-\frac{19\sqrt{1-}}{12\sqrt{-a^2}\left(c^2x\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a*(1 - a^2*x^2)^(1/2))/(6*(a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) + asinh(x
*(-a^2)^(1/2))/(c^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a*c^2) + (1 - a^2*
x^2)^(1/2)/(4*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) + (c^2*(-a^2)^(1/2))/a)) - (
19*(1 - a^2*x^2)^(1/2))/(12*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^
(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int \frac{x^4}{a^3x^3\sqrt{-a^2x^2+1}-a^2x^2\sqrt{-a^2x^2+1}-ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4/(a**3*x**3*sqrt(-a**2*x**2 + 1) - a**2*x**2*sqrt(-a**2*x
**2 + 1) - a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**2
```

3.634 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=129

$$-\frac{x(8ax+5)}{5c^3\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{a^2x^3(6ax+5)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{a^4x^5(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{\sin^{-1}(ax)}{ac^3}$$

```
[Out] -1/5*a^4*x^5*(a*x+1)/c^3/(-a^2*x^2+1)^(5/2)+1/15*a^2*x^3*(6*a*x+5)/c^3/(-a^
2*x^2+1)^(3/2)+arcsin(a*x)/a/c^3-1/5*x*(8*a*x+5)/c^3/(-a^2*x^2+1)^(1/2)-16/
5*(-a^2*x^2+1)^(1/2)/a/c^3
```

Rubi [A] time = 0.20, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6157, 6148, 819, 641, 216}

$$-\frac{a^4x^5(ax+1)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(6ax+5)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(8ax+5)}{5c^3\sqrt{1-a^2x^2}}-\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -(a^4*x^5*(1 + a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(5 + 6*a*x))/(1
5*c^3*(1 - a^2*x^2)^(3/2)) - (x*(5 + 8*a*x))/(5*c^3*Sqrt[1 - a^2*x^2]) - (1
6*Sqrt[1 - a^2*x^2])/(5*a*c^3) + ArcSin[a*x]/(a*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
```

```
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}$$

$$= -\frac{a^6\int \frac{x^6(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^4\int \frac{x^4(5+6ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^3}$$

$$= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(5+6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} - \frac{a^2\int \frac{x^2(15+24ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^3}$$

$$= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(5+6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} - \frac{x(5+8ax)}{5c^3\sqrt{1-a^2x^2}} + \frac{\int \frac{15+48ax}{\sqrt{1-a^2x^2}}\,dx}{15c^3}$$

$$= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(5+6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} - \frac{x(5+8ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{16\sqrt{1-a^2x^2}}{5ac^3} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^3}$$

$$= -\frac{a^4x^5(1+ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(5+6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} - \frac{x(5+8ax)}{5c^3\sqrt{1-a^2x^2}} - \frac{16\sqrt{1-a^2x^2}}{5ac^3} + \frac{\sin^{-1}(ax)}{ac^3}$$

Mathematica [A] time = 0.07, size = 108, normalized size = 0.84

$$\frac{15a^5x^5-38a^4x^4-52a^3x^3+87a^2x^2+15(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+33ax-48}{15ac^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-48 + 33*a*x + 87*a^2*x^2 - 52*a^3*x^3 - 38*a^4*x^4 + 15*a^5*x^5 + 15*(-1
+ a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(15*a*c^3*(-1 + a*x)^2*(1
 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.53, size = 211, normalized size = 1.64

$$-\frac{48\,a^5x^5-48\,a^4x^4-96\,a^3x^3+96\,a^2x^2+48\,ax+30\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right.}{15\left(a^6c^3x^5-a^5c^3x^4-2\,a^4c^3x^3+2\,a^3c^3x^2+a^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(48*a^5*x^5 - 48*a^4*x^4 - 96*a^3*x^3 + 96*a^2*x^2 + 48*a*x + 30*(a^5
*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*arctan((sqrt(-a^2*x^2 + 1
) - 1)/(a*x)) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x
 - 48)*sqrt(-a^2*x^2 + 1) - 48)/(a^6*c^3*x^5 - a^5*c^3*x^4 - 2*a^4*c^3*x^3
+ 2*a^3*c^3*x^2 + a^2*c^3*x - a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^3), x)
```

maple [B] time = 0.05, size = 259, normalized size = 2.01

$$-\frac{\sqrt{-a^2x^2+1}}{a\,c^3}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^3\sqrt{a^2}}+\frac{23\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{60a^3c^3\left(x-\frac{1}{a}\right)^2}+\frac{493\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{240a^2c^3\left(x-\frac{1}{a}\right)}+\frac{\sqrt{-a^2\left(x\right.}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^3+1/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+23/60/a^3/c^3/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+493/24
0/a^2/c^3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/20/a^4/c^3/(x-1/a)^3
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/24/a^3/c^3/(x+1/a)^2*(-a^2*(x+1/a)^2+
2*a*(x+1/a))^(1/2)-25/48/a^2/c^3/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^3), x)
```

mupad [B] time = 0.88, size = 367, normalized size = 2.84

$$\frac{5\,a\,\sqrt{1-a^2\,x^2}}{12\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{a\,\sqrt{1-a^2\,x^2}}{24\,\left(a^4\,c^3\,x^2+2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^3\,\sqrt{-a^2}}-\frac{a^6\,\sqrt{1-a^2\,x^2}}{30\,\left(a^9\,c^3\,x^2-2\,a^8\,c^3\,x+a^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (5*a*(1 - a^2*x^2)^(1/2))/(12*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (a*(
1 - a^2*x^2)^(1/2))/(24*(a^2*c^3 + 2*a^3*c^3*x + a^4*c^3*x^2)) + asinh(x*(-
a^2)^(1/2))/(c^3*(-a^2)^(1/2)) - (a^6*(1 - a^2*x^2)^(1/2))/(30*(a^7*c^3 - 2
*a^8*c^3*x + a^9*c^3*x^2)) - (1 - a^2*x^2)^(1/2)/(a*c^3) + (25*(1 - a^2*x^2
)^(1/2))/(48*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) + (c^3*(-a^2)^(1/2))/a)) - (4
93*(1 - a^2*x^2)^(1/2))/(240*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)
^(1/2))/a)) - (1 - a^2*x^2)^(1/2)/(20*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2) -
(c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)^(1/2))
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6\int\frac{x^6}{a^5x^5\sqrt{-a^2x^2+1}-a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}+2a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6/(a**5*x**5*sqrt(-a**2*x**2 + 1) - a**4*x**4*sqrt(-a**2*x
**2 + 1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) + 2*a**2*x**2*sqrt(-a**2*x**2 +
 1) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x)/c**3
```

3.635 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=162

$$-\frac{x(64ax+35)}{35c^4\sqrt{1-a^2x^2}}-\frac{128\sqrt{1-a^2x^2}}{35ac^4}+\frac{a^2x^3(48ax+35)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{a^6x^7(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{a^4x^5(8ax+7)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{\sin^{-1}(ax)}{ac^4}$$

```
[Out] 1/7*a^6*x^7*(a*x+1)/c^4/(-a^2*x^2+1)^(7/2)-1/35*a^4*x^5*(8*a*x+7)/c^4/(-a^2
*x^2+1)^(5/2)+1/105*a^2*x^3*(48*a*x+35)/c^4/(-a^2*x^2+1)^(3/2)+arcsin(a*x)/
a/c^4-1/35*x*(64*a*x+35)/c^4/(-a^2*x^2+1)^(1/2)-128/35*(-a^2*x^2+1)^(1/2)/a
/c^4
```

Rubi [A] time = 0.22, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6157, 6148, 819, 641, 216}

$$\frac{a^6x^7(ax+1)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{a^4x^5(8ax+7)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(48ax+35)}{105c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(64ax+35)}{35c^4\sqrt{1-a^2x^2}}-\frac{128\sqrt{1-a^2x^2}}{35ac^4}+\frac{\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (a^6*x^7*(1 + a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) - (a^4*x^5*(7 + 8*a*x))/(35
*c^4*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(35 + 48*a*x))/(105*c^4*(1 - a^2*x^2)^
(3/2)) - (x*(35 + 64*a*x))/(35*c^4*Sqrt[1 - a^2*x^2]) - (128*Sqrt[1 - a^2*x
^2])/(35*a*c^4) + ArcSin[a*x]/(a*c^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8(1+ax)}{\left(1-a^2x^2\right)^{9/2}}\,dx}{c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^6\int \frac{x^6(7+8ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^4\int \frac{x^4(35+48ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35+48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{a^2\int \frac{x^2(105+192ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{105c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35+48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(35+64ax)}{35c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{105+384ax}{\sqrt{1-a^2x^2}}\,d}{105c^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35+48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(35+64ax)}{35c^4\sqrt{1-a^2x^2}} - \frac{128\sqrt{1-a^2x}}{35ac^4}$$

$$= \frac{a^6x^7(1+ax)}{7c^4\left(1-a^2x^2\right)^{7/2}} - \frac{a^4x^5(7+8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(35+48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}} - \frac{x(35+64ax)}{35c^4\sqrt{1-a^2x^2}} - \frac{128\sqrt{1-a^2x}}{35ac^4}$$

Mathematica [A] time = 0.10, size = 126, normalized size = 0.78

$$\frac{105a^7x^7-281a^6x^6-559a^5x^5+965a^4x^4+715a^3x^3-1065a^2x^2+105(ax-1)^3(ax+1)^2\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-27}{105ac^4(ax-1)^3(ax+1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (384 - 279*a*x - 1065*a^2*x^2 + 715*a^3*x^3 + 965*a^4*x^4 - 559*a^5*x^5 - 2
81*a^6*x^6 + 105*a^7*x^7 + 105*(-1 + a*x)^3*(1 + a*x)^2*Sqrt[1 - a^2*x^2]*A
rcSin[a*x])/(105*a*c^4*(-1 + a*x)^3*(1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.63, size = 282, normalized size = 1.74

$$-\frac{384\,a^7x^7 - 384\,a^6x^6 - 1152\,a^5x^5 + 1152\,a^4x^4 + 1152\,a^3x^3 - 1152\,a^2x^2 - 384\,ax + 210\left(a^7x^7 - a^6x^6 - 3\,a^5x^5 + 3\right.}{105\left(a^8c^4x^7 - a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(384*a^7*x^7 - 384*a^6*x^6 - 1152*a^5*x^5 + 1152*a^4*x^4 + 1152*a^3*
x^3 - 1152*a^2*x^2 - 384*a*x + 210*(a^7*x^7 - a^6*x^6 - 3*a^5*x^5 + 3*a^4*x
^4 + 3*a^3*x^3 - 3*a^2*x^2 - a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)
) + (105*a^7*x^7 - 281*a^6*x^6 - 559*a^5*x^5 + 965*a^4*x^4 + 715*a^3*x^3 -
1065*a^2*x^2 - 279*a*x + 384)*sqrt(-a^2*x^2 + 1) + 384)/(a^8*c^4*x^7 - a^7*
c^4*x^6 - 3*a^6*c^4*x^5 + 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 - 3*a^3*c^4*x^2 - a
^2*c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^4), x)
```

maple [B] time = 0.06, size = 341, normalized size = 2.10

$$-\frac{\sqrt{-a^2x^2+1}}{a\,c^4}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^4\sqrt{a^2}}+\frac{211\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{336a^3c^4\left(x-\frac{1}{a}\right)^2}+\frac{1657\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{672a^2c^4\left(x-\frac{1}{a}\right)}+\frac{\sqrt{-a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a/c^4+1/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+
1)^(1/2))+211/336/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1657
/672/a^2/c^4/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/56/a^5/c^4/(x-1/a
)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+17/112/a^4/c^4/(x-1/a)^3*(-a^2*(x-1/
a)^2-2*a*(x-1/a))^(1/2)+7/60/a^3/c^4/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))
```

```
^(1/2)-379/480/a^2/c^4/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-1/80/a^4/
c^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^4), x)
```

mupad [B] time = 0.97, size = 613, normalized size = 3.78

$$\frac{35\,a\,\sqrt{1-a^2\,x^2}}{48\,\left(a^4\,c^4\,x^2-2\,a^3\,c^4\,x+a^2\,c^4\right)}+\frac{a\,\sqrt{1-a^2\,x^2}}{8\,\left(a^4\,c^4\,x^2+2\,a^3\,c^4\,x+a^2\,c^4\right)}+\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^4\,\sqrt{-a^2}}+\frac{a^3\,\sqrt{1-a^2\,x^2}}{140\,\left(a^6\,c^4\,x^2-2\,a^5\,c^4\,x+a^4\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^4*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (35*a*(1 - a^2*x^2)^(1/2))/(48*(a^2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) + (a*
(1 - a^2*x^2)^(1/2))/(8*(a^2*c^4 + 2*a^3*c^4*x + a^4*c^4*x^2)) + asinh(x*(-
a^2)^(1/2))/(c^4*(-a^2)^(1/2)) + (a^3*(1 - a^2*x^2)^(1/2))/(140*(a^4*c^4 -
2*a^5*c^4*x + a^6*c^4*x^2)) - (13*a^8*(1 - a^2*x^2)^(1/2))/(120*(a^9*c^4 -
2*a^10*c^4*x + a^11*c^4*x^2)) - (a^8*(1 - a^2*x^2)^(1/2))/(120*(a^9*c^4 + 2
*a^10*c^4*x + a^11*c^4*x^2)) - (1 - a^2*x^2)^(1/2)/(a*c^4) + (a*(1 - a^2*x^
2)^(1/2))/(56*(a^2*c^4 - 4*a^3*c^4*x + 6*a^4*c^4*x^2 - 4*a^5*c^4*x^3 + a^6*
c^4*x^4)) + (379*(1 - a^2*x^2)^(1/2))/(480*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2)
 + (c^4*(-a^2)^(1/2))/a)) - (1657*(1 - a^2*x^2)^(1/2))/(672*(-a^2)^(1/2)*(c
^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a)) + (1 - a^2*x^2)^(1/2)/(80*(-a^2)
^(1/2)*(3*c^4*x*(-a^2)^(1/2) + (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1
/2) + 3*a*c^4*x^2*(-a^2)^(1/2))) - (17*(1 - a^2*x^2)^(1/2))/(112*(-a^2)^(1/
2)*(3*c^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1/2)
- 3*a*c^4*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^8\int \frac{x^8}{a^7x^7\sqrt{-a^2x^2+1}-a^6x^6\sqrt{-a^2x^2+1}-3a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}+3a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}-ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8/(a**7*x**7*sqrt(-a**2*x**2 + 1) - a**6*x**6*sqrt(-a**2*x
**2 + 1) - 3*a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 +
 1) + 3*a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) -
 a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**4
```

3.636 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal. Leaf size=128

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{4a^9x^8}-\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}+\frac{2c^5}{5a^6x^5}+\frac{3c^5}{a^5x^4}+\frac{2c^5}{3a^4x^3}-\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}-\frac{2c^5\log(x)}{a}+c^5(-x)$$

```
[Out] 1/9*c^5/a^10/x^9+1/4*c^5/a^9/x^8-3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6+2/5*c^5/a^
6/x^5+3*c^5/a^5/x^4+2/3*c^5/a^4/x^3-4*c^5/a^3/x^2-3*c^5/a^2/x-c^5*x-2*c^5*l
n(x)/a
```

Rubi [A] time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{4c^5}{a^3x^2}+\frac{2c^5}{3a^4x^3}+\frac{3c^5}{a^5x^4}+\frac{2c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}-\frac{3c^5}{7a^8x^7}+\frac{c^5}{4a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}-\frac{2c^5\log(x)}{a}+c^5(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(4*a^9*x^8) - (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) + (2*c^5)/(5*a^6*x^5) + (3*c^5)/(a^5*x^4) + (2*c^5)/(3*a^4*x^3) - (4*c^
5)/(a^3*x^2) - (3*c^5)/(a^2*x) - c^5*x - (2*c^5*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = -\frac{c^5\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}\,dx}{a^{10}}$$
$$= -\frac{c^5\int \frac{(1-ax)^4(1+ax)^6}{x^{10}}\,dx}{a^{10}}$$
$$= -\frac{c^5\int\left(a^{10}+\frac{1}{x^{10}}+\frac{2a}{x^9}-\frac{3a^2}{x^8}-\frac{8a^3}{x^7}+\frac{2a^4}{x^6}+\frac{12a^5}{x^5}+\frac{2a^6}{x^4}-\frac{8a^7}{x^3}-\frac{3a^8}{x^2}+\frac{2a^9}{x}\right)dx}{a^{10}}$$
$$= \frac{c^5}{9a^{10}x^9}+\frac{c^5}{4a^9x^8}-\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}+\frac{2c^5}{5a^6x^5}+\frac{3c^5}{a^5x^4}+\frac{2c^5}{3a^4x^3}-\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}-c^5x$$

Mathematica [A] time = 0.04, size = 128, normalized size = 1.00

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{4a^9x^8}-\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}+\frac{2c^5}{5a^6x^5}+\frac{3c^5}{a^5x^4}+\frac{2c^5}{3a^4x^3}-\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}-\frac{2c^5\log(x)}{a}+c^5(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(4*a^9*x^8) - (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) + (2*c^5)/(5*a^6*x^5) + (3*c^5)/(a^5*x^4) + (2*c^5)/(3*a^4*x^3) - (4*c^
5)/(a^3*x^2) - (3*c^5)/(a^2*x) - c^5*x - (2*c^5*Log[x])/a
```

fricas [A] time = 0.41, size = 122, normalized size = 0.95

$$-\frac{1260\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log(x)+3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1\ldots}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] -1/1260*(1260*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) + 3780*a^8*c^5*x^8 +
5040*a^7*c^5*x^7 - 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1
680*a^3*c^5*x^3 + 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

giac [A] time = 0.19, size = 116, normalized size = 0.91

$$-c^5x-\frac{2\,c^5\log(|x|)}{a}-\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2\ldots}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] -c^5*x - 2*c^5*log(abs(x))/a - 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7
- 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 +
 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

maple [A] time = 0.04, size = 117, normalized size = 0.91

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{4a^9x^8}-\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}+\frac{2c^5}{5a^6x^5}+\frac{3c^5}{a^5x^4}+\frac{2c^5}{3a^4x^3}-\frac{4c^5}{x^2a^3}-\frac{3c^5}{a^2x}-c^5x-\frac{2c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^5,x)
```

```
[Out] 1/9*c^5/a^10/x^9+1/4*c^5/a^9/x^8-3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6+2/5*c^5/a^
6/x^5+3*c^5/a^5/x^4+2/3*c^5/a^4/x^3-4*c^5/x^2/a^3-3*c^5/a^2/x-c^5*x-2*c^5*l
n(x)/a
```

maxima [A] time = 0.31, size = 115, normalized size = 0.90

$$-c^5x-\frac{2\,c^5\log(x)}{a}-\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] -c^5*x - 2*c^5*log(x)/a - 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 - 840
*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 + 540*
a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

mupad [B] time = 0.89, size = 90, normalized size = 0.70

$$-\frac{c^5\,\left(\frac{3\,a^2\,x^2}{7}-\frac{a\,x}{4}+\frac{4\,a^3\,x^3}{3}-\frac{2\,a^4\,x^4}{5}-3\,a^5\,x^5-\frac{2\,a^6\,x^6}{3}+4\,a^7\,x^7+3\,a^8\,x^8+a^{10}\,x^{10}+2\,a^9\,x^9\,\ln(x)-\frac{1}{9}\right)}{a^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^5*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -(c^5*((3*a^2*x^2)/7 - (a*x)/4 + (4*a^3*x^3)/3 - (2*a^4*x^4)/5 - 3*a^5*x^5
- (2*a^6*x^6)/3 + 4*a^7*x^7 + 3*a^8*x^8 + a^10*x^10 + 2*a^9*x^9*log(x) - 1/
9))/(a^10*x^9)
```

sympy [A] time = 0.82, size = 126, normalized size = 0.98

$$\frac{-a^{10}c^5x - 2a^9c^5\log(x) - \frac{3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-315ac^5x-140c^5}{1260x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**5,x)
```

```
[Out] (-a**10*c**5*x - 2*a**9*c**5*log(x) - (3780*a**8*c**5*x**8 + 5040*a**7*c**5
*x**7 - 840*a**6*c**5*x**6 - 3780*a**5*c**5*x**5 - 504*a**4*c**5*x**4 + 168
0*a**3*c**5*x**3 + 540*a**2*c**5*x**2 - 315*a*c**5*x - 140*c**5)/(1260*x**9
))/a**10
```

3.637 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=91

$$-\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}-\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4(-x)$$

```
[Out] -1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6+2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/a^3/
x^2-2*c^4/a^2/x-c^4*x-2*c^4*ln(x)/a
```

Rubi [A] time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{3c^4}{a^3x^2}+\frac{3c^4}{2a^5x^4}+\frac{2c^4}{5a^6x^5}-\frac{c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x
^4) - (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x - (2*c^4*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$
$$= \frac{c^4\int \frac{(1-ax)^3(1+ax)^5}{x^8}dx}{a^8}$$
$$= \frac{c^4\int\left(-a^8+\frac{1}{x^8}+\frac{2a}{x^7}-\frac{2a^2}{x^6}-\frac{6a^3}{x^5}+\frac{6a^5}{x^3}+\frac{2a^6}{x^2}-\frac{2a^7}{x}\right)dx}{a^8}$$
$$= -\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}-\frac{2c^4}{a^2x}-c^4x-\frac{2c^4\log(x)}{a}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.00

$$-\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}-\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -1/7*c^4/(a^8*x^7) - c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5
*x^4) - (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x - (2*c^4*Log[x])/a
```

fricas [A] time = 0.61, size = 89, normalized size = 0.98

$$-\frac{210\,a^8c^4x^8+420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/210*(210*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 + 630*a^
5*c^4*x^5 - 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^
7)
```

giac [A] time = 0.20, size = 83, normalized size = 0.91

$$-c^4x-\frac{2\,c^4\log\left(|x|\right)}{a}-\frac{420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -c^4*x - 2*c^4*log(abs(x))/a - 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 3
15*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

maple [A] time = 0.03, size = 84, normalized size = 0.92

$$-\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{x^2a^3}-\frac{2c^4}{a^2x}-c^4x-\frac{2c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6+2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/x^2/
a^3-2*c^4/a^2/x-c^4*x-2*c^4*ln(x)/a
```

maxima [A] time = 0.31, size = 82, normalized size = 0.90

$$-c^4x-\frac{2\,c^4\log(x)}{a}-\frac{420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -c^4*x - 2*c^4*log(x)/a - 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 315*a^
3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

mupad [B] time = 0.07, size = 66, normalized size = 0.73

$$-\frac{c^4\,\left(\frac{a\,x}{3}-\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5+2\,a^6\,x^6+a^8\,x^8+2\,a^7\,x^7\,\ln(x)+\frac{1}{7}\right)}{a^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^4*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -(c^4*((a*x)/3 - (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 + 2*a^6*x^6 + a^
8*x^8 + 2*a^7*x^7*log(x) + 1/7))/(a^8*x^7)
```

sympy [A] time = 0.49, size = 90, normalized size = 0.99

$$\frac{-a^8c^4x-2a^7c^4\log{(x)}-\frac{420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70ac^4x+30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**4,x)
```

```
[Out] (-a**8*c**4*x - 2*a**7*c**4*log(x) - (420*a**6*c**4*x**6 + 630*a**5*c**4*x*
*5 - 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 + 70*a*c**4*x + 30*c**4)/(210*x
**7))/a**8
```

3.638 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=78

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}-\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}-\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3(-x)$$

```
[Out] 1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4-1/3*c^3/a^4/x^3-2*c^3/a^3/x^2-c^3/a^2/x-c^3
*x-2*c^3*ln(x)/a
```

Rubi [A] time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{2c^3}{a^3x^2}-\frac{c^3}{3a^4x^3}+\frac{c^3}{2a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x - (2*c^3*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \frac{(1-ax)^2(1+ax)^4}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \left(a^6+\frac{1}{x^6}+\frac{2a}{x^5}-\frac{a^2}{x^4}-\frac{4a^3}{x^3}-\frac{a^4}{x^2}+\frac{2a^5}{x}\right)dx}{a^6}$$
$$= \frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}-\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}-\frac{c^3}{a^2x}-c^3x-\frac{2c^3\log(x)}{a}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 1.00

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}-\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}-\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x - (2*c^3*Log[x])/a
```

fricas [A] time = 0.73, size = 78, normalized size = 1.00

$$-\frac{30\,a^6c^3x^6+60\,a^5c^3x^5\log(x)+30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(30*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 + 60*a^3*c^3
*x^3 + 10*a^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

giac [A] time = 0.19, size = 72, normalized size = 0.92

$$-c^3x-\frac{2\,c^3\log\left(|x|\right)}{a}-\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -c^3*x - 2*c^3*log(abs(x))/a - 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a
^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

maple [A] time = 0.03, size = 73, normalized size = 0.94

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}-\frac{c^3}{3a^4x^3}-\frac{2c^3}{x^2a^3}-\frac{c^3}{a^2x}-c^3x-\frac{2c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4-1/3*c^3/a^4/x^3-2*c^3/x^2/a^3-c^3/a^2/x-c^3
*x-2*c^3*ln(x)/a
```

maxima [A] time = 0.31, size = 71, normalized size = 0.91

$$-c^3x-\frac{2\,c^3\log(x)}{a}-\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -c^3*x - 2*c^3*log(x)/a - 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^2*c^
3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

mupad [B] time = 0.88, size = 57, normalized size = 0.73

$$-\frac{c^3\,\left(\frac{a^2\,x^2}{3}-\frac{a\,x}{2}+2\,a^3\,x^3+a^4\,x^4+a^6\,x^6+2\,a^5\,x^5\,\ln(x)-\frac{1}{5}\right)}{a^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -(c^3*((a^2*x^2)/3 - (a*x)/2 + 2*a^3*x^3 + a^4*x^4 + a^6*x^6 + 2*a^5*x^5*lo
g(x) - 1/5))/(a^6*x^5)
```

sympy [A] time = 0.35, size = 78, normalized size = 1.00

$$\frac{-a^6c^3x-2a^5c^3\log{(x)}-\frac{30a^4c^3x^4+60a^3c^3x^3+10a^2c^3x^2-15ac^3x-6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**3,x)
```

```
[Out] (-a**6*c**3*x - 2*a**5*c**3*log(x) - (30*a**4*c**3*x**4 + 60*a**3*c**3*x**3
 + 10*a**2*c**3*x**2 - 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

3.639 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=41

$$-\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}-\frac{2c^2\log(x)}{a}+c^2(-x)$$

[Out] -1/3*c^2/a^4/x^3-c^2/a^3/x^2-c^2*x-2*c^2*ln(x)/a

Rubi [A] time = 0.11, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 75}

$$-\frac{c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{2c^2\log(x)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]

[Out] -c^2/(3*a^4*x^3) - c^2/(a^3*x^2) - c^2*x - (2*c^2*Log[x])/a

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\int \frac{e^{2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$
$$= \frac{c^2\int \frac{(1-ax)(1+ax)^3}{x^4}dx}{a^4}$$
$$= \frac{c^2\int \left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{2a^3}{x}\right)dx}{a^4}$$
$$= -\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}-c^2x-\frac{2c^2\log(x)}{a}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.00

$$-\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}-\frac{2c^2\log(x)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
[Out] -1/3*c^2/(a^4*x^3) - c^2/(a^3*x^2) - c^2*x - (2*c^2*Log[x])/a
```

fricas [A] time = 0.68, size = 43, normalized size = 1.05

$$-\frac{3\,a^4c^2x^4+6\,a^3c^2x^3\log(x)+3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
[Out] -1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/(a^4*x^3)
```

giac [A] time = 0.19, size = 37, normalized size = 0.90

$$-c^2x-\frac{2\,c^2\log\left(|x|\right)}{a}-\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^2,x, algorithm="giac")
[Out] -c^2*x - 2*c^2*log(abs(x))/a - 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

maple [A] time = 0.03, size = 40, normalized size = 0.98

$$-\frac{c^2}{3a^4x^3}-\frac{c^2}{x^2a^3}-c^2x-\frac{2c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/3*c^2/a^4/x^3-c^2/x^2/a^3-c^2*x-2*c^2*ln(x)/a
```

maxima [A] time = 0.31, size = 36, normalized size = 0.88

$$-c^2x-\frac{2\,c^2\log(x)}{a}-\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -c^2*x - 2*c^2*log(x)/a - 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.85

$$-\frac{c^2\,\left(3\,a\,x+3\,a^4\,x^4+6\,a^3\,x^3\,\ln(x)+1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -(c^2*(3*a*x + 3*a^4*x^4 + 6*a^3*x^3*log(x) + 1))/(3*a^4*x^3)
```

sympy [A] time = 0.20, size = 41, normalized size = 1.00

$$\frac{-a^4c^2x-2a^3c^2\log{(x)}-\frac{3ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**2,x)
```

```
[Out] (-a**4*c**2*x - 2*a**3*c**2*log(x) - (3*a*c**2*x + c**2)/(3*x**3))/a**4
```

3.640 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=21

$$\frac{c}{a^2x}-\frac{2c\log(x)}{a}+c(-x)$$

[Out] c/a^2/x-c*x-2*c*ln(x)/a

Rubi [A] time = 0.07, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6157, 6150, 43}

$$\frac{c}{a^2x}-\frac{2c\log(x)}{a}+c(-x)$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]

[Out] c/(a^2*x) - c*x - (2*c*Log[x])/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx=-\frac{c\int\frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$
$$=-\frac{c\int\frac{(1+ax)^2}{x^2}dx}{a^2}$$
$$=-\frac{c\int\left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx}{a^2}$$
$$=\frac{c}{a^2x}-cx-\frac{2c\log(x)}{a}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{c}{a^2x}-\frac{2c\log(x)}{a}+c(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
[Out] c/(a^2*x) - c*x - (2*c*Log[x])/a
```

fricas [A] time = 0.47, size = 27, normalized size = 1.29

$$-\frac{a^2cx^2+2\,acx\log(x)-c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2),x, algorithm="fricas")
[Out] -(a^2*c*x^2 + 2*a*c*x*log(x) - c)/(a^2*x)
```

giac [A] time = 0.19, size = 22, normalized size = 1.05

$$-cx-\frac{2\,c\log(|x|)}{a}+\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2),x, algorithm="giac")
[Out] -c*x - 2*c*log(abs(x))/a + c/(a^2*x)
```

maple [A] time = 0.03, size = 22, normalized size = 1.05

$$\frac{c}{a^{2}x}-cx-\frac{2c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2),x)
```

```
[Out] c/a^2/x-c*x-2*c*ln(x)/a
```

maxima [A] time = 0.31, size = 21, normalized size = 1.00

$$-cx-\frac{2\,c\log(x)}{a}+\frac{c}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -c*x - 2*c*log(x)/a + c/(a^2*x)
```

mupad [B] time = 0.84, size = 24, normalized size = 1.14

$$-\frac{c\,\left(a^{2}\,x^{2}+2\,a\,x\,\ln(x)-1\right)}{a^{2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] -(c*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

sympy [A] time = 0.12, size = 20, normalized size = 0.95

$$\frac{-a^{2}cx-2ac\log{(x)}+\frac{c}{x}}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2),x)
```

```
[Out] (-a**2*c*x - 2*a*c*log(x) + c/x)/a**2
```

3.641 $\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{ac(1-ax)}-\frac{2\log(1-ax)}{ac}-\frac{x}{c}$$

```
[Out] -x/c-1/a/c/(-a*x+1)-2*ln(-a*x+1)/a/c
```

Rubi [A] time = 0.12, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 43}

$$-\frac{1}{ac(1-ax)}-\frac{2\log(1-ax)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(x/c) - 1/(a*c*(1 - a*x)) - (2*Log[1 - a*x])/(a*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{a^2\int \frac{e^{2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}$$
$$= -\frac{a^2\int \frac{x^2}{(1-ax)^2}\,dx}{c}$$
$$= -\frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(-1+ax)^2}+\frac{2}{a^2(-1+ax)}\right)dx}{c}$$
$$= -\frac{x}{c}-\frac{1}{ac(1-ax)}-\frac{2\log(1-ax)}{ac}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$-\frac{1}{ac(1-ax)}-\frac{2\log(1-ax)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
[Out] -(x/c) - 1/(a*c*(1 - a*x)) - (2*Log[1 - a*x])/(a*c)
```

fricas [A] time = 0.48, size = 41, normalized size = 1.08

$$-\frac{a^2x^2-ax+2\,(ax-1)\log\left(ax-1\right)-1}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] -(a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^2*c*x - a*c)
```

giac [A] time = 0.18, size = 36, normalized size = 0.95

$$-\frac{x}{c}-\frac{2\,\log\left(|ax-1|\right)}{ac}+\frac{1}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="giac")
[Out] -x/c - 2*log(abs(a*x - 1))/(a*c) + 1/((a*x - 1)*a*c)
```

maple [A] time = 0.03, size = 36, normalized size = 0.95

$$-\frac{x}{c}+\frac{1}{ca\left(ax-1\right)}-\frac{2\ln\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2),x)
[Out] -x/c+1/c/a/(a*x-1)-2/a/c*ln(a*x-1)
```

maxima [A] time = 0.30, size = 34, normalized size = 0.89

$$-\frac{x}{c}+\frac{1}{a^{2}cx-ac}-\frac{2\,\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] -x/c + 1/(a^2*c*x - a*c) - 2*log(a*x - 1)/(a*c)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.92

$$-\frac{x}{c}-\frac{1}{a\,\left(c-a\,c\,x\right)}-\frac{2\,\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a^2*x^2))*(a^2*x^2 - 1)),x)
[Out] - x/c - 1/(a*(c - a*c*x)) - (2*log(a*x - 1))/(a*c)
```

sympy [A] time = 0.17, size = 37, normalized size = 0.97

$$-a^{2}\left(-\frac{1}{a^{4}cx-a^{3}c}+\frac{x}{a^{2}c}+\frac{2\log\left(ax-1\right)}{a^{3}c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2),x)
[Out] -a**2*(-1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c))
```

3.642 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx$

Optimal. Leaf size=76

$$-\frac{7}{4ac^2(1-ax)}+\frac{1}{4ac^2(1-ax)^2}-\frac{17\log(1-ax)}{8ac^2}+\frac{\log(ax+1)}{8ac^2}-\frac{x}{c^2}$$

```
[Out] -x/c^2+1/4/a/c^2/(-a*x+1)^2-7/4/a/c^2/(-a*x+1)-17/8*ln(-a*x+1)/a/c^2+1/8*ln
(a*x+1)/a/c^2
```

Rubi [A] time = 0.15, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{7}{4ac^2(1-ax)}+\frac{1}{4ac^2(1-ax)^2}-\frac{17\log(1-ax)}{8ac^2}+\frac{\log(ax+1)}{8ac^2}-\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(x/c^2) + 1/(4*a*c^2*(1 - a*x)^2) - 7/(4*a*c^2*(1 - a*x)) - (17*Log[1 - a*
x])/(8*a*c^2) + Log[1 + a*x]/(8*a*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}$$
$$= \frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^2}$$
$$= -\frac{x}{c^2}+\frac{1}{4ac^2(1-ax)^2}-\frac{7}{4ac^2(1-ax)}-\frac{17\log(1-ax)}{8ac^2}+\frac{\log(1+ax)}{8ac^2}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.91

$$\frac{-8a^3x^3+16a^2x^2+6ax-17(ax-1)^2\log(1-ax)+(ax-1)^2\log(ax+1)-12}{8ac^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-12 + 6*a*x + 16*a^2*x^2 - 8*a^3*x^3 - 17*(-1 + a*x)^2*Log[1 - a*x] + (-1
+ a*x)^2*Log[1 + a*x])/(8*a*c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.49, size = 93, normalized size = 1.22

$$-\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+17\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+12}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a
*c^2)
```

giac [A] time = 0.18, size = 58, normalized size = 0.76

$$-\frac{x}{c^2}+\frac{\log\left(|ax+1|\right)}{8\,ac^2}-\frac{17\,\log\left(|ax-1|\right)}{8\,ac^2}+\frac{7\,ax-6}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -x/c^2 + 1/8*log(abs(a*x + 1))/(a*c^2) - 17/8*log(abs(a*x - 1))/(a*c^2) + 1
/4*(7*a*x - 6)/((a*x - 1)^2*a*c^2)
```

maple [A] time = 0.04, size = 66, normalized size = 0.87

$$-\frac{x}{c^2}+\frac{1}{4a\,c^2\,(ax-1)^2}+\frac{7}{4a\,c^2\,(ax-1)}-\frac{17\ln(ax-1)}{8a\,c^2}+\frac{\ln(ax+1)}{8a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -x/c^2+1/4/a/c^2/(a*x-1)^2+7/4/a/c^2/(a*x-1)-17/8/a/c^2*ln(a*x-1)+1/8*ln(a*
x+1)/a/c^2
```

maxima [A] time = 0.31, size = 70, normalized size = 0.92

$$\frac{7\,ax-6}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}-\frac{x}{c^2}+\frac{\log(ax+1)}{8\,ac^2}-\frac{17\,\log(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(7*a*x - 6)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - x/c^2 + 1/8*log(a*x +
 1)/(a*c^2) - 17/8*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.10, size = 68, normalized size = 0.89

$$\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}-\frac{x}{c^2}-\frac{17\,\ln(a\,x-1)}{8\,a\,c^2}+\frac{\ln(a\,x+1)}{8\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^2*(a^2*x^2 - 1)),x)
```

```
[Out] ((7*x)/4 - 3/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) - x/c^2 - (17*log(a*x -
 1))/(8*a*c^2) + log(a*x + 1)/(8*a*c^2)
```

sympy [A] time = 0.41, size = 75, normalized size = 0.99

$$-a^4\left(\frac{-7ax+6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] -a**4*((-7*a*x + 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) + x/(a
**4*c**2) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2))
```

3.643 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=111

$$-\frac{39}{16ac^3(1-ax)}+\frac{1}{16ac^3(ax+1)}+\frac{5}{8ac^3(1-ax)^2}-\frac{1}{12ac^3(1-ax)^3}-\frac{9\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

```
[Out] -x/c^3-1/12/a/c^3/(-a*x+1)^3+5/8/a/c^3/(-a*x+1)^2-39/16/a/c^3/(-a*x+1)+1/16
/a/c^3/(a*x+1)-9/4*ln(-a*x+1)/a/c^3+1/4*ln(a*x+1)/a/c^3
```

Rubi [A] time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{39}{16ac^3(1-ax)}+\frac{1}{16ac^3(ax+1)}+\frac{5}{8ac^3(1-ax)^2}-\frac{1}{12ac^3(1-ax)^3}-\frac{9\log(1-ax)}{4ac^3}+\frac{\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -(x/c^3) - 1/(12*a*c^3*(1 - a*x)^3) + 5/(8*a*c^3*(1 - a*x)^2) - 39/(16*a*c^
3*(1 - a*x)) + 1/(16*a*c^3*(1 + a*x)) - (9*Log[1 - a*x])/(4*a*c^3) + Log[1
+ a*x]/(4*a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
```

```
/; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{2\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}$$

$$= -\frac{a^6\int \frac{x^6}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$

$$= -\frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{4a^6(-1+ax)^4}+\frac{5}{4a^6(-1+ax)^3}+\frac{39}{16a^6(-1+ax)^2}+\frac{9}{4a^6(-1+ax)}+\frac{1}{16a^6(1+ax)^2}-\frac{1}{4a^6(1+ax)}\right)dx}{c^3}$$

$$= -\frac{x}{c^3}-\frac{1}{12ac^3(1-ax)^3}+\frac{5}{8ac^3(1-ax)^2}-\frac{39}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}-\frac{9\log(1-ax)}{4ac^3}+ -$$

Mathematica [A] time = 0.06, size = 103, normalized size = 0.93

$$\frac{-12a^5x^5+24a^4x^4+30a^3x^3-48a^2x^2-14ax-27(ax-1)^3(ax+1)\log(1-ax)+3(ax-1)^3(ax+1)\log(ax+1)}{12ac^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (22 - 14*a*x - 48*a^2*x^2 + 30*a^3*x^3 + 24*a^4*x^4 - 12*a^5*x^5 - 27*(-1 +
 a*x)^3*(1 + a*x)*Log[1 - a*x] + 3*(-1 + a*x)^3*(1 + a*x)*Log[1 + a*x])/(12
*a*c^3*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.57, size = 137, normalized size = 1.23

$$-\frac{12\,a^5x^5-24\,a^4x^4-30\,a^3x^3+48\,a^2x^2+14\,ax-3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log(ax+1)+27\left(a^4x^4-2\,a^3x^3+\right.}{12\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(12*a^5*x^5 - 24*a^4*x^4 - 30*a^3*x^3 + 48*a^2*x^2 + 14*a*x - 3*(a^4*
x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x + 1) + 27*(a^4*x^4 - 2*a^3*x^3 + 2*a*x
 - 1)*log(a*x - 1) - 22)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3
)
```

giac [A] time = 0.17, size = 81, normalized size = 0.73

$$-\frac{x}{c^3}+\frac{\log(|ax+1|)}{4\,ac^3}-\frac{9\,\log(|ax-1|)}{4\,ac^3}+\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\,(ax+1)(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -x/c^3 + 1/4*log(abs(a*x + 1))/(a*c^3) - 9/4*log(abs(a*x - 1))/(a*c^3) + 1/
6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

maple [A] time = 0.04, size = 96, normalized size = 0.86

$$-\frac{x}{c^3}+\frac{1}{12a\,c^3\,(ax-1)^3}+\frac{5}{8a\,c^3\,(ax-1)^2}+\frac{39}{16a\,c^3\,(ax-1)}-\frac{9\ln(ax-1)}{4a\,c^3}+\frac{1}{16a\,c^3\,(ax+1)}+\frac{\ln(ax+1)}{4a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -x/c^3+1/12/a/c^3/(a*x-1)^3+5/8/a/c^3/(a*x-1)^2+39/16/a/c^3/(a*x-1)-9/4/a/c
^3*ln(a*x-1)+1/16/a/c^3/(a*x+1)+1/4*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.31, size = 98, normalized size = 0.88

$$\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}-\frac{x}{c^3}+\frac{\log(ax+1)}{4\,ac^3}-\frac{9\,\log(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 +
2*a^2*c^3*x - a*c^3) - x/c^3 + 1/4*log(a*x + 1)/(a*c^3) - 9/4*log(a*x - 1)/
(a*c^3)
```

mupad [B] time = 0.12, size = 94, normalized size = 0.85

$$\frac{\frac{13\,x}{6}+2\,a\,x^2-\frac{11}{6\,a}-\frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}-\frac{x}{c^3}-\frac{9\,\ln(a\,x-1)}{4\,a\,c^3}+\frac{\ln(a\,x+1)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^3*(a^2*x^2 - 1)),x)
```

```
[Out] ((13*x)/6 + 2*a*x^2 - 11/(6*a) - (5*a^2*x^3)/2)/(c^3 + 2*a^3*c^3*x^3 - a^4*
c^3*x^4 - 2*a*c^3*x) - x/c^3 - (9*log(a*x - 1))/(4*a*c^3) + log(a*x + 1)/(4
*a*c^3)
```

sympy [A] time = 0.62, size = 104, normalized size = 0.94

$$-a^6\left(\frac{-15a^3x^3+12a^2x^2+13ax-11}{6a^{11}c^3x^4-12a^{10}c^3x^3+12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{9\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] -a**6*((-15*a**3*x**3 + 12*a**2*x**2 + 13*a*x - 11)/(6*a**11*c**3*x**4 - 12
*a**10*c**3*x**3 + 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (9*log(x
 - 1/a)/4 - log(x + 1/a)/4)/(a**7*c**3))
```

3.644 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=146

$$-\frac{99}{32ac^4(1-ax)}+\frac{11}{64ac^4(ax+1)}+\frac{35}{32ac^4(1-ax)^2}-\frac{1}{64ac^4(ax+1)^2}-\frac{13}{48ac^4(1-ax)^3}+\frac{1}{32ac^4(1-ax)^4}-\frac{303\log(1-a}{128ac^4}$$

```
[Out] -x/c^4+1/32/a/c^4/(-a*x+1)^4-13/48/a/c^4/(-a*x+1)^3+35/32/a/c^4/(-a*x+1)^2-
99/32/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)^2+11/64/a/c^4/(a*x+1)-303/128*ln(-a
*x+1)/a/c^4+47/128*ln(a*x+1)/a/c^4
```

Rubi [A] time = 0.20, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{99}{32ac^4(1-ax)}+\frac{11}{64ac^4(ax+1)}+\frac{35}{32ac^4(1-ax)^2}-\frac{1}{64ac^4(ax+1)^2}-\frac{13}{48ac^4(1-ax)^3}+\frac{1}{32ac^4(1-ax)^4}-\frac{303\log(1-a}{128ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] -(x/c^4) + 1/(32*a*c^4*(1 - a*x)^4) - 13/(48*a*c^4*(1 - a*x)^3) + 35/(32*a*
c^4*(1 - a*x)^2) - 99/(32*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 + a*x)^2) + 11/
(64*a*c^4*(1 + a*x)) - (303*Log[1 - a*x])/(128*a*c^4) + (47*Log[1 + a*x])/(
128*a*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{2\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$

$$= \frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{8a^8(-1+ax)^5}-\frac{13}{16a^8(-1+ax)^4}-\frac{35}{16a^8(-1+ax)^3}-\frac{99}{32a^8(-1+ax)^2}-\frac{303}{128a^8(-1+ax)}+\frac{1}{32a^8(1+ax)^3}\right.}{c^4}$$

$$= -\frac{x}{c^4}+\frac{1}{32ac^4(1-ax)^4}-\frac{13}{48ac^4(1-ax)^3}+\frac{35}{32ac^4(1-ax)^2}-\frac{99}{32ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)^2}$$

Mathematica [A] time = 0.09, size = 123, normalized size = 0.84

$$\frac{-384a^7x^7+768a^6x^6+1638a^5x^5-2508a^4x^4-1732a^3x^3+2516a^2x^2+550ax-909(ax-1)^4(ax+1)^2\log(1-a}{384ac^4(ax-1)^4(ax+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-800 + 550*a*x + 2516*a^2*x^2 - 1732*a^3*x^3 - 2508*a^4*x^4 + 1638*a^5*x^5
 + 768*a^6*x^6 - 384*a^7*x^7 - 909*(-1 + a*x)^4*(1 + a*x)^2*Log[1 - a*x] +
141*(-1 + a*x)^4*(1 + a*x)^2*Log[1 + a*x])/(384*a*c^4*(-1 + a*x)^4*(1 + a*x
)^2)
```

fricas [A] time = 0.57, size = 233, normalized size = 1.60

$$-\frac{384\,a^7x^7-768\,a^6x^6-1638\,a^5x^5+2508\,a^4x^4+1732\,a^3x^3-2516\,a^2x^2-550\,ax-141\left(a^6x^6-2\,a^5x^5-a^4x^4+\right.}{384\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/384*(384*a^7*x^7 - 768*a^6*x^6 - 1638*a^5*x^5 + 2508*a^4*x^4 + 1732*a^3*
x^3 - 2516*a^2*x^2 - 550*a*x - 141*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x
^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 909*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4
 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 800)/(a^7*c^4*x^6 - 2*a^
6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4
)
```

giac [A] time = 0.40, size = 97, normalized size = 0.66

$$-\frac{x}{c^4}+\frac{47\,\log\left(|ax+1|\right)}{128\,ac^4}-\frac{303\,\log\left(|ax-1|\right)}{128\,ac^4}+\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\,(ax+1)^2(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -x/c^4 + 47/128*log(abs(a*x + 1))/(a*c^4) - 303/128*log(abs(a*x - 1))/(a*c^
4) + 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*
x - 400)/((a*x + 1)^2*(a*x - 1)^4*a*c^4)
```

maple [A] time = 0.04, size = 126, normalized size = 0.86

$$-\frac{x}{c^4}+\frac{1}{32a\,c^4\,(ax-1)^4}+\frac{13}{48a\,c^4\,(ax-1)^3}+\frac{35}{32a\,c^4\,(ax-1)^2}+\frac{99}{32a\,c^4\,(ax-1)}-\frac{303\ln(ax-1)}{128a\,c^4}-\frac{1}{64a\,c^4\,(ax+1)^2}+\frac{}{64a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -x/c^4+1/32/a/c^4/(a*x-1)^4+13/48/a/c^4/(a*x-1)^3+35/32/a/c^4/(a*x-1)^2+99/
32/a/c^4/(a*x-1)-303/128/a/c^4*ln(a*x-1)-1/64/a/c^4/(a*x+1)^2+11/64/a/c^4/(
a*x+1)+47/128*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.31, size = 146, normalized size = 1.00

$$\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}-\frac{x}{c^4}+\frac{47\,\log\left(ax+1\right)}{128\,ac^4}-\frac{303\,\log\left(ax-1\right)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x - 4
00)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^
2 - 2*a^2*c^4*x + a*c^4) - x/c^4 + 47/128*log(a*x + 1)/(a*c^4) - 303/128*lo
g(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.15, size = 144, normalized size = 0.99

$$\frac{47\ln(ax+1)}{128\,a\,c^4}-\frac{\frac{467\,x}{192}+\frac{437\,a\,x^2}{96}-\frac{25}{12\,a}-\frac{529\,a^2\,x^3}{96}-\frac{81\,a^3\,x^4}{32}+\frac{209\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5+a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+a^2\,c^4\,x^2+2\,a\,c^4\,x-c^4}-\frac{303\ln(ax-1)}{128\,a\,c^4}-\frac{x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^4*(a^2*x^2 - 1)),x)
```

```
[Out] (47*log(a*x + 1))/(128*a*c^4) - ((467*x)/192 + (437*a*x^2)/96 - 25/(12*a) -
 (529*a^2*x^3)/96 - (81*a^3*x^4)/32 + (209*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4
- 4*a^3*c^4*x^3 + a^4*c^4*x^4 + 2*a^5*c^4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) -
(303*log(a*x - 1))/(128*a*c^4) - x/c^4
```

sympy [A] time = 0.89, size = 158, normalized size = 1.08

$$-a^8\left(\frac{-627a^5x^5+486a^4x^4+1058a^3x^3-874a^2x^2-467ax+400}{192a^{15}c^4x^6-384a^{14}c^4x^5-192a^{13}c^4x^4+768a^{12}c^4x^3-192a^{11}c^4x^2-384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{303\log}{12}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] -a**8*((-627*a**5*x**5 + 486*a**4*x**4 + 1058*a**3*x**3 - 874*a**2*x**2 - 4
67*a*x + 400)/(192*a**15*c**4*x**6 - 384*a**14*c**4*x**5 - 192*a**13*c**4*x
**4 + 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 - 384*a**10*c**4*x + 192*a*
*9*c**4) + x/(a**8*c**4) + (303*log(x - 1/a)/128 - 47*log(x + 1/a)/128)/(a*
*9*c**4))
```

3.645 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=191

$$-\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x}-\frac{15c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{16a}-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7}-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6}-\frac{c^4(5ax+24)\left(1-a^2x^2\right)^{5}}{40a^6x^5}$$

```
[Out] 1/16*c^4*(5*a*x+16)*(-a^2*x^2+1)^(3/2)/a^4/x^3-1/40*c^4*(5*a*x+24)*(-a^2*x^
2+1)^(5/2)/a^6/x^5-1/7*c^4*(-a^2*x^2+1)^(7/2)/a^8/x^7-1/2*c^4*(-a^2*x^2+1)^
(7/2)/a^7/x^6-3*c^4*arcsin(a*x)/a-15/16*c^4*arctanh((-a^2*x^2+1)^(1/2))/a-3
/16*c^4*(-5*a*x+16)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.37, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6157, 6148, 1807, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6}-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7}-\frac{c^4(5ax+24)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5}+\frac{c^4(5ax+16)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3}-\frac{3c^4(16-5ax)\sqrt{1-a^2}}{16a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-3*c^4*(16 - 5*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) + (c^4*(16 + 5*a*x)*(1 -
 a^2*x^2)^(3/2))/(16*a^4*x^3) - (c^4*(24 + 5*a*x)*(1 - a^2*x^2)^(5/2))/(40*
a^6*x^5) - (c^4*(1 - a^2*x^2)^(7/2))/(7*a^8*x^7) - (c^4*(1 - a^2*x^2)^(7/2)
)/(2*a^7*x^6) - (3*c^4*ArcSin[a*x])/a - (15*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])
/(16*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$= \frac{c^4\int \frac{(1+ax)^3\left(1-a^2x^2\right)^{5/2}}{x^8}dx}{a^8}$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} - \frac{c^4\int \frac{\left(1-a^2x^2\right)^{5/2}\left(-21a-21a^2x-7a^3x^2\right)}{x^7}dx}{7a^8}$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} - \frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6} + \frac{c^4\int \frac{\left(126a^2+21a^3x\right)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{42a^8}$$

$$= -\frac{c^4(24+5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} - \frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6} - \frac{c^4\int \frac{(1008a^4+}{}}{}$$

$$= \frac{c^4(16+5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} - \frac{c^4(}{}$$

$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)\left(1-a^2x\right.}{40a^6x^5}$$

$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)\left(1-a^2x\right.}{40a^6x^5}$$

$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)\left(1-a^2x\right.}{40a^6x^5}$$

$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)\left(1-a^2x\right.}{40a^6x^5}$$

$$= -\frac{3c^4(16-5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16+5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24+5ax)\left(1-a^2x\right.}{40a^6x^5}$$

Mathematica [C] time = 0.21, size = 191, normalized size = 1.00

$$\frac{c^4\left(-336a^2x^2\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};a^2x^2\right)-\frac{5\left(16a^8x^8-231a^7x^7-64a^6x^6+413a^5x^5+96a^4x^4-238a^3x^3-64a^2x^2+16a^7x^7\left(a^2x^2-1\right)^4\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\right.\right.}{\sqrt{1-a^2x^2}}\right.}{560a^8x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*(-336*a^2*x^2*Hypergeometric2F1[-5/2, -5/2, -3/2, a^2*x^2] - (5*(16 +
56*a*x - 64*a^2*x^2 - 238*a^3*x^3 + 96*a^4*x^4 + 413*a^5*x^5 - 64*a^6*x^6 -
 231*a^7*x^7 + 16*a^8*x^8 - 105*a^7*x^7*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 -
a^2*x^2]] + 16*a^7*x^7*(-1 + a^2*x^2)^4*Hypergeometric2F1[3, 7/2, 9/2, 1 -
a^2*x^2]))/Sqrt[1 - a^2*x^2]))/(560*a^8*x^7)
```

fricas [A] time = 1.31, size = 175, normalized size = 0.92

$$\frac{3360\,a^7c^4x^7\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+525\,a^7c^4x^7\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+560\,a^7c^4x^7+\left(560\,a^7c^4x^7-2496\,a^6c^4x^6-525\right.}{560\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas
")
```

```
[Out] 1/560*(3360*a^7*c^4*x^7*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 525*a^7*c^
4*x^7*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 560*a^7*c^4*x^7 + (560*a^7*c^4*x^7
- 2496*a^6*c^4*x^6 - 525*a^5*c^4*x^5 + 992*a^4*c^4*x^4 + 770*a^3*c^4*x^3 -
96*a^2*c^4*x^2 - 280*a*c^4*x - 80*c^4)*sqrt(-a^2*x^2 + 1))/(a^8*x^7)
```

giac [B] time = 0.24, size = 505, normalized size = 2.64

$$\frac{\left(5\,c^4+\frac{35\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^4}{a^2x}+\frac{49\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^4}{a^4x^2}-\frac{245\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^4}{a^6x^3}-\frac{875\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^4}{a^8x^4}+\frac{455\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5c}{a^{10}x^5}\right.}{4480\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/4480*(5*c^4 + 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) + 49*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2) - 245*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^4/(a^6*x^3) - 875*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^8*x^4) +
 455*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^10*x^5) + 9065*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^6*c^4/(a^12*x^6))*a^14*x^7/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^7*abs(a)) - 3*c^4*arcsin(a*x)*sgn(a)/abs(a) - 15/16*c^4*log(1/2*abs(-2*s
qrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c
^4/a - 1/4480*(9065*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x + 455*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 875*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^3*c^4/x^3 - 245*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4) + 49*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^4*x^5) + 35*(sqrt(-a^2*x^2 + 1)*abs(a)
```

```
 + a)^6*c^4/(a^6*x^6) + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7*c^4/(a^8*x^7))/
(a^6*abs(a))
```

maple [A] time = 0.09, size = 273, normalized size = 1.43

$$-\frac{218c^4}{35a^2x\sqrt{-a^2x^2+1}}+\frac{15c^4}{8a^5x^4\sqrt{-a^2x^2+1}}-\frac{c^4}{7a^8x^7\sqrt{-a^2x^2+1}}-\frac{c^4}{35a^6x^5\sqrt{-a^2x^2+1}}+\frac{68c^4}{35a^4x^3\sqrt{-a^2x^2+1}}-\frac{}{16a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^4,x)
```

```
[Out] -218/35*c^4/a^2/x/(-a^2*x^2+1)^(1/2)+15/8*c^4/a^5/x^4/(-a^2*x^2+1)^(1/2)-1/
7*c^4/a^8/x^7/(-a^2*x^2+1)^(1/2)-1/35*c^4/a^6/x^5/(-a^2*x^2+1)^(1/2)+68/35*
c^4/a^4/x^3/(-a^2*x^2+1)^(1/2)-37/16*c^4/a^3/x^2/(-a^2*x^2+1)^(1/2)-1/2*c^4
/a^7/x^6/(-a^2*x^2+1)^(1/2)+156/35*c^4*x/(-a^2*x^2+1)^(1/2)-c^4*a*x^2/(-a^2
*x^2+1)^(1/2)-3*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+31
/16*c^4/a/(-a^2*x^2+1)^(1/2)-15/16*c^4/a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [B] time = 0.42, size = 745, normalized size = 3.90

$$-a^3c^4\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)+3\,a^2c^4\left(\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin\left(ax\right)}{a^3}\right)-\frac{11\,c^4x}{\sqrt{-a^2x^2+1}}-\frac{6\,c^4\left(\frac{1}{\sqrt{-a^2x^2+1}}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima
")
```

```
[Out] -a^3*c^4*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*a^
2*c^4*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3) - 11*c^4*x/sqrt(-a^2*x
^2 + 1) - 6*c^4*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2
/abs(x)))/a + 14*(2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*c^
4/a^2 - c^4/(sqrt(-a^2*x^2 + 1)*a) - 7*(3*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(
x) + 2/abs(x)) - 3*a^2/sqrt(-a^2*x^2 + 1) + 1/(sqrt(-a^2*x^2 + 1)*x^2))*c^4
/a^3 - 2*(8*a^4*x/sqrt(-a^2*x^2 + 1) - 4*a^2/(sqrt(-a^2*x^2 + 1)*x) - 1/(sq
rt(-a^2*x^2 + 1)*x^3))*c^4/a^4 + 11/8*(15*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(
x) + 2/abs(x)) - 15*a^4/sqrt(-a^2*x^2 + 1) + 5*a^2/(sqrt(-a^2*x^2 + 1)*x^2)
 + 2/(sqrt(-a^2*x^2 + 1)*x^4))*c^4/a^5 - 1/5*(16*a^6*x/sqrt(-a^2*x^2 + 1) -
 8*a^4/(sqrt(-a^2*x^2 + 1)*x) - 2*a^2/(sqrt(-a^2*x^2 + 1)*x^3) - 1/(sqrt(-a
^2*x^2 + 1)*x^5))*c^4/a^6 - 1/16*(105*a^6*log(2*sqrt(-a^2*x^2 + 1)/abs(x) +
 2/abs(x)) - 105*a^6/sqrt(-a^2*x^2 + 1) + 35*a^4/(sqrt(-a^2*x^2 + 1)*x^2) +
 14*a^2/(sqrt(-a^2*x^2 + 1)*x^4) + 8/(sqrt(-a^2*x^2 + 1)*x^6))*c^4/a^7 + 1/
35*(128*a^8*x/sqrt(-a^2*x^2 + 1) - 64*a^6/(sqrt(-a^2*x^2 + 1)*x) - 16*a^4/(
```

```
sqrt(-a^2*x^2 + 1)*x^3) - 8*a^2/(sqrt(-a^2*x^2 + 1)*x^5) - 5/(sqrt(-a^2*x^2
 + 1)*x^7))*c^4/a^8
```

mupad [B] time = 0.89, size = 228, normalized size = 1.19

$$\frac{c^4\sqrt{1-a^2x^2}}{a}-\frac{3c^4\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{156c^4\sqrt{1-a^2x^2}}{35a^2x}-\frac{15c^4\sqrt{1-a^2x^2}}{16a^3x^2}+\frac{62c^4\sqrt{1-a^2x^2}}{35a^4x^3}+\frac{11c^4\sqrt{1-a^2x^2}}{8a^5x^4}-\frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c^4*atan((1 - a^2*x^2)^(1/2)*1i)*15i)/(16*a) - (3*c^4*asinh(x*(-a^2)^(1/2)
))/(-a^2)^(1/2) + (c^4*(1 - a^2*x^2)^(1/2))/a - (156*c^4*(1 - a^2*x^2)^(1/2
))/(35*a^2*x) - (15*c^4*(1 - a^2*x^2)^(1/2))/(16*a^3*x^2) + (62*c^4*(1 - a^
2*x^2)^(1/2))/(35*a^4*x^3) + (11*c^4*(1 - a^2*x^2)^(1/2))/(8*a^5*x^4) - (6*
c^4*(1 - a^2*x^2)^(1/2))/(35*a^6*x^5) - (c^4*(1 - a^2*x^2)^(1/2))/(2*a^7*x^
6) - (c^4*(1 - a^2*x^2)^(1/2))/(7*a^8*x^7)
```

sympy [A] time = 30.60, size = 935, normalized size = 4.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] -a*c**4*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) - 3*c**4*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1
/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 8*c**4*Piecewise((-I*sqrt(a**2*x*
*2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2 + 6*c
**4*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1
/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x
**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 - 6*c**4*Piecewi
se((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), A
bs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 +
 1)/(3*x**3), True))/a**4 - 8*c**4*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*
a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) -
 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*as
in(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1
 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**5 + 3*
c**4*Piecewise((-5*a**6*acosh(1/(a*x))/16 + 5*a**5/(16*x*sqrt(-1 + 1/(a**2*
x**2))) - 5*a**3/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) - a/(24*x**5*sqrt(-1 +
1/(a**2*x**2))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) >
 1), (5*I*a**6*asin(1/(a*x))/16 - 5*I*a**5/(16*x*sqrt(1 - 1/(a**2*x**2))) +
```

```
 5*I*a**3/(48*x**3*sqrt(1 - 1/(a**2*x**2))) + I*a/(24*x**5*sqrt(1 - 1/(a**2
*x**2))) + I/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**7 + c**4*Piecewi
se((-16*a**7*sqrt(-1 + 1/(a**2*x**2))/35 - 8*a**5*sqrt(-1 + 1/(a**2*x**2))/
(35*x**2) - 6*a**3*sqrt(-1 + 1/(a**2*x**2))/(35*x**4) - a*sqrt(-1 + 1/(a**2
*x**2))/(7*x**6), 1/Abs(a**2*x**2) > 1), (-16*I*a**7*sqrt(1 - 1/(a**2*x**2)
)/35 - 8*I*a**5*sqrt(1 - 1/(a**2*x**2))/(35*x**2) - 6*I*a**3*sqrt(1 - 1/(a*
*2*x**2))/(35*x**4) - I*a*sqrt(1 - 1/(a**2*x**2))/(7*x**6), True))/a**8
```

3.646 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=157

$$-\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3(ax+8)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}-$$

```
[Out] 1/8*c^3*(a*x+8)*(-a^2*x^2+1)^(3/2)/a^4/x^3+1/5*c^3*(-a^2*x^2+1)^(5/2)/a^6/x
^5+3/4*c^3*(-a^2*x^2+1)^(5/2)/a^5/x^4-3*c^3*arcsin(a*x)/a-3/8*c^3*arctanh((
-a^2*x^2+1)^(1/2))/a-3/8*c^3*(-a*x+8)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.32, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6157, 6148, 1807, 811, 813, 844, 216, 266, 63, 208}

$$\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{c^3(ax+8)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}-\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}-3c$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-3*c^3*(8 - a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) + (c^3*(8 + a*x)*(1 - a^2*x^
2)^(3/2))/(8*a^4*x^3) + (c^3*(1 - a^2*x^2)^(5/2))/(5*a^6*x^5) + (3*c^3*(1 -
 a^2*x^2)^(5/2))/(4*a^5*x^4) - (3*c^3*ArcSin[a*x])/a - (3*c^3*ArcTanh[Sqrt[
1 - a^2*x^2]])/(8*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\frac{c^3\int\frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}\,dx}{a^6}$$

$$= -\frac{c^3\int\frac{(1+ax)^3\left(1-a^2x^2\right)^{3/2}}{x^6}\,dx}{a^6}$$

$$= \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} + \frac{c^3\int\frac{\left(1-a^2x^2\right)^{3/2}\left(-15a-15a^2x-5a^3x^2\right)}{x^5}\,dx}{5a^6}$$

$$= \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} + \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4} - \frac{c^3\int\frac{\left(60a^2+5a^3x\right)\left(1-a^2x^2\right)^{3/2}}{x^4}\,dx}{20a^6}$$

$$= \frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} + \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4} + \frac{c^3\int\frac{\left(240a^4+30a^5\right.}{x^{2}}}{80a}$$

$$= -\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} + \frac{3c^3\left(1-\right.}{4a}$$

$$= -\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} + \frac{3c^3\left(1-\right.}{4a}$$

$$= -\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} + \frac{3c^3\left(1-\right.}{4a}$$

$$= -\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} + \frac{3c^3\left(1-\right.}{4a}$$

$$= -\frac{3c^3(8-ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8+ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} + \frac{3c^3\left(1-\right.}{4a}$$

Mathematica [C] time = 0.10, size = 186, normalized size = 1.18

$$\frac{c^3\left(-8a^6x^6+75a^5x^5+24a^4x^4-105a^3x^3+40a^2x^2\sqrt{1-a^2x^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};a^2x^2\right)-24a^2x^2-8a^5x^5\left(a^2x^2-1\right.\right.}{40a^6x^5\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*(8 + 30*a*x - 24*a^2*x^2 - 105*a^3*x^3 + 24*a^4*x^4 + 75*a^5*x^5 - 8*a
^6*x^6 + 45*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]] + 40*a^2*x
^2*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, a^2*x^2] - 8*a^5*x
^5*(-1 + a^2*x^2)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 - a^2*x^2]))/(40*a^6*x
^5*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.66, size = 153, normalized size = 0.97

$$\frac{240\,a^5c^3x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+15\,a^5c^3x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+40\,a^5c^3x^5+\left(40\,a^5c^3x^5-152\,a^4c^3x^4-55\,a^3c^3x^3\cdots\right.}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas
")
```

```
[Out] 1/40*(240*a^5*c^3*x^5*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 15*a^5*c^3*x
^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + 40*a^5*c^3*x^5 + (40*a^5*c^3*x^5 - 152
*a^4*c^3*x^4 - 55*a^3*c^3*x^3 + 24*a^2*c^3*x^2 + 30*a*c^3*x + 8*c^3)*sqrt(-
a^2*x^2 + 1))/(a^6*x^5)
```

giac [B] time = 0.51, size = 385, normalized size = 2.45

$$-\frac{\left(2\,c^3+\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^3}{a^2x}+\frac{30\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^3}{a^4x^2}-\frac{80\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^3}{a^6x^3}-\frac{580\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^3}{a^8x^4}\right)a^{10}x^5}{320\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5|a|}-\frac{3\,c^3\arcsin\left(\cdots\right.}{|a|\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/320*(2*c^3 + 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x) + 30*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2) - 80*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*c^3/(a^6*x^3) - 580*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3/(a^8*x^4))*a
^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 3*c^3*arcsin(a*x)*sgn(
a)/abs(a) - 3/8*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*ab
s(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^3/a - 1/320*(580*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)*a^2*c^3/x + 80*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2 - 30*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/(a^2*x^3) - 15*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^4*c^3/(a^4*x^4) - 2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/(a^6*x^5)
)/(a^4*abs(a))
```

maple [A] time = 0.06, size = 227, normalized size = 1.45

$$-\frac{c^3 a x^2}{\sqrt{-a^2x^2+1}}+\frac{19c^3}{8a\sqrt{-a^2x^2+1}}+\frac{19c^3x}{5\sqrt{-a^2x^2+1}}-\frac{3c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{3c^3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{8a}-\frac{22c^3}{5a^2x\sqrt{-a^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^3,x)
```

```
[Out] -c^3*a*x^2/(-a^2*x^2+1)^(1/2)+19/8*c^3/a/(-a^2*x^2+1)^(1/2)+19/5*c^3*x/(-a^
2*x^2+1)^(1/2)-3*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-3
/8*c^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))-22/5*c^3/a^2/x/(-a^2*x^2+1)^(1/2)-17
/8*c^3/a^3/x^2/(-a^2*x^2+1)^(1/2)+3/4*c^3/a^5/x^4/(-a^2*x^2+1)^(1/2)+1/5*c^
3/a^6/x^5/(-a^2*x^2+1)^(1/2)+2/5*c^3/a^4/x^3/(-a^2*x^2+1)^(1/2)
```

maxima [B] time = 0.42, size = 443, normalized size = 2.82

$$-a^3c^3\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)+3\,a^2c^3\left(\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin(ax)}{a^3}\right)-\frac{8\,c^3x}{\sqrt{-a^2x^2+1}}-\frac{6\,c^3\left(\frac{1}{\sqrt{-a^2x^2+1}}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima
")
```

```
[Out] -a^3*c^3*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*a^
2*c^3*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3) - 8*c^3*x/sqrt(-a^2*x^
2 + 1) - 6*c^3*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/
abs(x)))/a + 6*(2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*c^3/
a^2 - 4*(3*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 3*a^2/sqrt(-a^
2*x^2 + 1) + 1/(sqrt(-a^2*x^2 + 1)*x^2))*c^3/a^3 + 3/8*(15*a^4*log(2*sqrt(-
a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 15*a^4/sqrt(-a^2*x^2 + 1) + 5*a^2/(sqrt(-
a^2*x^2 + 1)*x^2) + 2/(sqrt(-a^2*x^2 + 1)*x^4))*c^3/a^5 - 1/5*(16*a^6*x/sqr
t(-a^2*x^2 + 1) - 8*a^4/(sqrt(-a^2*x^2 + 1)*x) - 2*a^2/(sqrt(-a^2*x^2 + 1)*
x^3) - 1/(sqrt(-a^2*x^2 + 1)*x^5))*c^3/a^6
```

mupad [B] time = 0.07, size = 182, normalized size = 1.16

$$\frac{c^3\sqrt{1-a^2x^2}}{a}-\frac{3\,c^3\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{19\,c^3\sqrt{1-a^2x^2}}{5\,a^2x}-\frac{11\,c^3\sqrt{1-a^2x^2}}{8\,a^3x^2}+\frac{3\,c^3\sqrt{1-a^2x^2}}{5\,a^4x^3}+\frac{3\,c^3\sqrt{1-a^2x^2}}{4\,a^5x^4}+\frac{c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c^3*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/(8*a) - (3*c^3*asinh(x*(-a^2)^(1/2)))
/(-a^2)^(1/2) + (c^3*(1 - a^2*x^2)^(1/2))/a - (19*c^3*(1 - a^2*x^2)^(1/2))/
(5*a^2*x) - (11*c^3*(1 - a^2*x^2)^(1/2))/(8*a^3*x^2) + (3*c^3*(1 - a^2*x^2)
^(1/2))/(5*a^4*x^3) + (3*c^3*(1 - a^2*x^2)^(1/2))/(4*a^5*x^4) + (c^3*(1 - a
^2*x^2)^(1/2))/(5*a^6*x^5)
```

sympy [A] time = 19.49, size = 687, normalized size = 4.38

$$-ac^3\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-3c^3\left(\begin{cases}\sqrt{\frac{1}{a^2}}\operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)-\frac{c^3\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] -a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) - 3*c**3*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1
/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - c**3*Piecewise((-acosh(1/(a*x)),
1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + 5*c**3*Piecewise((-I*sq
rt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/
a**2 + 5*c**3*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2)
)/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 -
 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 - c**3
*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*
x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**
2*x**2 + 1)/(3*x**3), True))/a**4 - 3*c**3*Piecewise((-3*a**4*acosh(1/(a*x)
)/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x
**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I
*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**
3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a
**5 - c**3*Piecewise((-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1
+ 1/(a**2*x**2))/(15*x**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**
2*x**2) > 1), (-8*I*a**5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(
a**2*x**2))/(15*x**2) - I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))/a**6
```

3.647 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$

Optimal. Leaf size=124

$$-\frac{c^2(ax+6)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{3c^2\sin^{-1}(ax)}{a}$$

```
[Out] -1/3*c^2*(-a^2*x^2+1)^(3/2)/a^4/x^3-3/2*c^2*(-a^2*x^2+1)^(3/2)/a^3/x^2-3*c^
2*arcsin(a*x)/a+1/2*c^2*arctanh((-a^2*x^2+1)^(1/2))/a-1/2*c^2*(a*x+6)*(-a^2
*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.27, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6148, 1807, 813, 844, 216, 266, 63, 208}

$$-\frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{c^2(ax+6)\sqrt{1-a^2x^2}}{2a^2x}+\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{3c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(c^2*(6 + a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(1 - a^2*x^2)^(3/2))/(3
*a^4*x^3) - (3*c^2*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (3*c^2*ArcSin[a*x])/a
 + (c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
```

```
/; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}$$
$$= \frac{c^2\int \frac{(1+ax)^3\sqrt{1-a^2x^2}}{x^4}\,dx}{a^4}$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{c^2\int \frac{\sqrt{1-a^2x^2}\left(-9a-9a^2x-3a^3x^2\right)}{x^3}\,dx}{3a^4}$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} + \frac{c^2\int \frac{\left(18a^2-3a^3x\right)\sqrt{1-a^2x^2}}{x^2}\,dx}{6a^4}$$
$$= -\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{c^2\int \frac{6a^3+36a^4x}{x\sqrt{1-a^2x^2}}\,dx}{12a^4}$$
$$= -\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \left(3c^2\right)\int \frac{1}{\sqrt{1-a^2}}$$
$$= -\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^2\sin^{-1}(ax)}{a} -$$
$$= -\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^2\sin^{-1}(ax)}{a} +$$
$$= -\frac{c^2(6+ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^2\sin^{-1}(ax)}{a} +$$

Mathematica [A] time = 0.07, size = 128, normalized size = 1.03

$$-\frac{c^2\left(6a^5x^5-16a^4x^4-15a^3x^3+14a^2x^2+18a^3x^3\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-3a^3x^3\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\right.}{6a^4x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -1/6*(c^2*(2 + 9*a*x + 14*a^2*x^2 - 15*a^3*x^3 - 16*a^4*x^4 + 6*a^5*x^5 + 1
8*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x] - 3*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTa
nh[Sqrt[1 - a^2*x^2]]))/(a^4*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.58, size = 131, normalized size = 1.06

$$\frac{36\,a^3c^2x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-3\,a^3c^2x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+6\,a^3c^2x^3+\left(6\,a^3c^2x^3-16\,a^2c^2x^2-9\,ac^2x-2\,c^2\right)\sqrt{-}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(36*a^3*c^2*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 3*a^3*c^2*x^3*
log((sqrt(-a^2*x^2 + 1) - 1)/x) + 6*a^3*c^2*x^3 + (6*a^3*c^2*x^3 - 16*a^2*c
^2*x^2 - 9*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

giac [B] time = 0.31, size = 262, normalized size = 2.11

$$\frac{\left(c^2+\frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^2}{a^2x}+\frac{33\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^2}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|}-\frac{3\,c^2\arcsin\left(ax\right)\mathrm{sgn}(a)}{|a|}+\frac{c^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\frac{\sqrt{-a^2x^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(c^2 + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x) + 33*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^2/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) - 3*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/2*c^2*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^2
/a - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/x + 9*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2*c^2/(a^2*x^2) + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a^4*x
^3))/(a^2*abs(a))
```

maple [A] time = 0.05, size = 181, normalized size = 1.46

$$-\frac{c^2a\,x^2}{\sqrt{-a^2x^2+1}}+\frac{5c^2}{2a\sqrt{-a^2x^2+1}}+\frac{8c^2x}{3\sqrt{-a^2x^2+1}}-\frac{3c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{c^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{7c^2}{3a^2x\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^2,x)
```

```
[Out] -c^2*a*x^2/(-a^2*x^2+1)^(1/2)+5/2*c^2/a/(-a^2*x^2+1)^(1/2)+8/3*c^2*x/(-a^2*
x^2+1)^(1/2)-3*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/2
*c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))-7/3*c^2/a^2/x/(-a^2*x^2+1)^(1/2)-1/3*c
^2/a^4/x^3/(-a^2*x^2+1)^(1/2)-3/2*c^2/a^3/x^2/(-a^2*x^2+1)^(1/2)
```

maxima [B] time = 0.42, size = 347, normalized size = 2.80

$$-a^3c^2\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)+3\,a^2c^2\left(\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin\left(ax\right)}{a^3}\right)-\frac{5\,c^2x}{\sqrt{-a^2x^2+1}}-\frac{5\,c^2\left(\frac{1}{\sqrt{-a^2x^2+1}}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima
")
```

```
[Out] -a^3*c^2*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*a^
2*c^2*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3) - 5*c^2*x/sqrt(-a^2*x^
2 + 1) - 5*c^2*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/
abs(x)))/a + (2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*c^2/a^
2 + c^2/(sqrt(-a^2*x^2 + 1)*a) - 3/2*(3*a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x)
 + 2/abs(x)) - 3*a^2/sqrt(-a^2*x^2 + 1) + 1/(sqrt(-a^2*x^2 + 1)*x^2))*c^2/a
^3 + 1/3*(8*a^4*x/sqrt(-a^2*x^2 + 1) - 4*a^2/(sqrt(-a^2*x^2 + 1)*x) - 1/(sq
rt(-a^2*x^2 + 1)*x^3))*c^2/a^4
```

mupad [B] time = 0.82, size = 136, normalized size = 1.10

$$\frac{c^2\,\sqrt{1-a^2\,x^2}}{a}-\frac{3\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{8\,c^2\,\sqrt{1-a^2\,x^2}}{3\,a^2\,x}-\frac{3\,c^2\,\sqrt{1-a^2\,x^2}}{2\,a^3\,x^2}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{3\,a^4\,x^3}-\frac{c^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1{}\mathrm{i}\right)\,1}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c^2*(1 - a^2*x^2)^(1/2))/a - (c^2*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/(2*a) -
 (3*c^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (8*c^2*(1 - a^2*x^2)^(1/2))/(
3*a^2*x) - (3*c^2*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2) - (c^2*(1 - a^2*x^2)^(1/
2))/(3*a^4*x^3)
```

sympy [A] time = 16.83, size = 357, normalized size = 2.88

$$-ac^2\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-3c^2\left(\begin{cases}\sqrt{\frac{1}{a^2}}\,\text{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2>0\\ \sqrt{-\frac{1}{a^2}}\,\text{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2<0\end{cases}\right)-\frac{2c^2\left(\begin{cases}-\text{acosh}\left(\frac{1}{ax}\right) & \text{for } \frac{1}{|a^2x^2|}>1\\ i\,\text{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] -a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)
) - 3*c**2*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1
/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) - 2*c**2*Piecewise((-acosh(1/(a*x))
, 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True))/a + 2*c**2*Piecewise((-I*
sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)
)/a**2 + 3*c**2*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**
2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1
 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))/a**3 + c*
*2*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(
3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a
**2*x**2 + 1)/(3*x**3), True))/a**4
```

3.648 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=73

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}+\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{3c\sin^{-1}(ax)}{a}$$

```
[Out] -3*c*arcsin(a*x)/a+3*c*arctanh((-a^2*x^2+1)^(1/2))/a+c*(-a^2*x^2+1)^(1/2)/a
+c*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.21, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6157, 6148, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}+\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{3c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*Sqrt[1 - a^2*x^2])/a + (c*Sqrt[1 - a^2*x^2])/(a^2*x) - (3*c*ArcSin[a*x])
/a + (3*c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\frac{c\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}dx}{a^2}$$
$$= -\frac{c\int \frac{(1+ax)^3}{x^2\sqrt{1-a^2x^2}}dx}{a^2}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a^2x}+\frac{c\int \frac{-3a-3a^2x-a^3x^2}{x\sqrt{1-a^2x^2}}dx}{a^2}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{c\int \frac{3a^3+3a^4x}{x\sqrt{1-a^2x^2}}dx}{a^4}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-(3c)\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{(3c)\int \frac{1}{x\sqrt{1-a^2x^2}}dx}{a}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}-\frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}dx,x,x^2\right)}{2a}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}dx,x,\sqrt{1-a^2x^2}\right)}{a^3}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.77

$$\frac{c\left(\sqrt{1-a^2x^2}\,(ax+1)+3ax\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-3ax\sin^{-1}(ax)\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*((1 + a*x)*Sqrt[1 - a^2*x^2] - 3*a*x*ArcSin[a*x] + 3*a*x*ArcTanh[Sqrt[1
 - a^2*x^2]]))/(a^2*x)
```

fricas [A] time = 0.60, size = 82, normalized size = 1.12

$$\frac{6\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-3\,acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+acx+\sqrt{-a^2x^2+1}\,(acx+c)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 3*a*c*x*log((sqrt(-a^2*x^
2 + 1) - 1)/x) + a*c*x + sqrt(-a^2*x^2 + 1)*(a*c*x + c))/(a^2*x)
```

giac [A] time = 0.25, size = 129, normalized size = 1.77

$$-\frac{a^2cx}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{3\,c\arcsin\left(ax\right)\mathrm{sgn}(a)}{|a|}+\frac{3\,c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}\,c}{a}+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+\right.}{2\,a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -1/2*a^2*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 3*c*arcsin(a*x)*sgn
(a)/abs(a) + 3*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x
)))/abs(a) + sqrt(-a^2*x^2 + 1)*c/a + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c
/(a^2*x*abs(a))
```

maple [A] time = 0.05, size = 121, normalized size = 1.66

$$-\frac{ca\,x^2}{\sqrt{-a^2x^2+1}}+\frac{c}{a\sqrt{-a^2x^2+1}}-\frac{cx}{\sqrt{-a^2x^2+1}}-\frac{3c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{3c\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}+\frac{c}{a^2x\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2),x)
```

```
[Out] -c*a*x^2/(-a^2*x^2+1)^(1/2)+c/a/(-a^2*x^2+1)^(1/2)-c*x/(-a^2*x^2+1)^(1/2)-3
*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3*c/a*arctanh(1/(-a
^2*x^2+1)^(1/2))+c/a^2/x/(-a^2*x^2+1)^(1/2)
```

maxima [B] time = 0.42, size = 200, normalized size = 2.74

$$-a^3c\left(\frac{x^2}{\sqrt{-a^2x^2+1}\,a^2}-\frac{2}{\sqrt{-a^2x^2+1}\,a^4}\right)+3\,a^2c\left(\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin\left(ax\right)}{a^3}\right)-\frac{2\,cx}{\sqrt{-a^2x^2+1}}-\frac{3\,c\left(\frac{1}{\sqrt{-a^2x^2+1}}-\log\left(\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a^3*c*(x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqrt(-a^2*x^2 + 1)*a^4)) + 3*a^2*
c*(x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3) - 2*c*x/sqrt(-a^2*x^2 + 1)
 - 3*c*(1/sqrt(-a^2*x^2 + 1) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)))
/a - (2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*c/a^2 + 2*c/(s
qrt(-a^2*x^2 + 1)*a)
```

mupad [B] time = 0.04, size = 81, normalized size = 1.11

$$\frac{c\sqrt{1-a^2x^2}}{a}-\frac{3\,c\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{c\,\mathrm{atan}\left(\sqrt{1-a^2x^2}\,1\mathrm{i}\right)3\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c*(1 - a^2*x^2)^(1/2))/a - (3*c*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - (c*a
tan((1 - a^2*x^2)^(1/2)*1i)*3i)/a + (c*(1 - a^2*x^2)^(1/2))/(a^2*x)
```

sympy [A] time = 12.33, size = 150, normalized size = 2.05

$$-ac\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)-3c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\,\mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\,\mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)-\frac{3c\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2),x)
```

```
[Out] -a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) -
 3*c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)
*asinh(x*sqrt(-a**2)), a**2 < 0)) - 3*c*Piecewise((-acosh(1/(a*x)), 1/Abs(a
**2*x**2) > 1), (I*asin(1/(a*x)), True))/a - c*Piecewise((-I*sqrt(a**2*x**2
 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))/a**2
```

3.649 $\int \frac{e^{3\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=95

$$-\frac{(ax+1)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{2(ax+1)^2}{ac\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}$$

```
[Out] -1/3*(a*x+1)^3/a/c/(-a^2*x^2+1)^(3/2)-3*arcsin(a*x)/a/c+2*(a*x+1)^2/a/c/(-a
^2*x^2+1)^(1/2)+3*(-a^2*x^2+1)^(1/2)/a/c
```

Rubi [A] time = 0.23, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6157, 6148, 1635, 21, 669, 641, 216}

$$-\frac{(ax+1)^3}{3ac\left(1-a^2x^2\right)^{3/2}}+\frac{2(ax+1)^2}{ac\sqrt{1-a^2x^2}}+\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(1 + a*x)^3/(3*a*c*(1 - a^2*x^2)^(3/2)) + (2*(1 + a*x)^2)/(a*c*Sqrt[1 - a^
2*x^2]) + (3*Sqrt[1 - a^2*x^2])/(a*c) - (3*ArcSin[a*x])/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{a^2\int \frac{e^{3\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}$$

$$= -\frac{a^2\int \frac{x^2(1+ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c}$$

$$= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} + \frac{a^2\int \frac{\left(\frac{3}{a^2}+\frac{3x}{a}\right)(1+ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}$$

$$= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} + \frac{2(1+ax)^2}{ac\sqrt{1-a^2x^2}} - \frac{3\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{c}$$

$$= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} + \frac{2(1+ax)^2}{ac\sqrt{1-a^2x^2}} + \frac{3\sqrt{1-a^2x^2}}{ac} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}$$

$$= -\frac{(1+ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} + \frac{2(1+ax)^2}{ac\sqrt{1-a^2x^2}} + \frac{3\sqrt{1-a^2x^2}}{ac} - \frac{3\sin^{-1}(ax)}{ac}$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.82

$$\frac{-3a^3x^3+16a^2x^2-9(ax-1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+5ax-14}{3ac(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] (-14 + 5*a*x + 16*a^2*x^2 - 3*a^3*x^3 - 9*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcS
in[a*x])/(3*a*c*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.56, size = 101, normalized size = 1.06

$$\frac{14\,a^2x^2-28\,ax+18\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^2x^2-19\,ax+14\right)\sqrt{-a^2x^2+1}\,+14}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] 1/3*(14*a^2*x^2 - 28*a*x + 18*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)) + (3*a^2*x^2 - 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^3*c
*x^2 - 2*a^2*c*x + a*c)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 168, normalized size = 1.77

$$-\frac{a\,x^{2}}{c\sqrt{-a^{2}x^{2}+1}}+\frac{6}{ca\sqrt{-a^{2}x^{2}+1}}+\frac{7x}{c\sqrt{-a^{2}x^{2}+1}}-\frac{3\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{c\sqrt{a^{2}}}+\frac{4}{3c\,a^{2}\left(x-\frac{1}{a}\right)\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x)
```

```
[Out] -a/c*x^2/(-a^2*x^2+1)^(1/2)+6/c/a/(-a^2*x^2+1)^(1/2)+7/c*x/(-a^2*x^2+1)^(1/
2)-3/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+4/3/c/a^2/(x-1/
a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-8/3/c/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^{2}x^{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))), x)
```

mupad [B] time = 0.07, size = 129, normalized size = 1.36

$$\frac{\sqrt{1-a^2\,x^2}}{a\,c}-\frac{13\,\sqrt{1-a^2\,x^2}}{3\,\left(\frac{c\,\sqrt{-a^2}}{a}-c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c\,\sqrt{-a^2}}-\frac{2\,a\,\sqrt{1-a^2\,x^2}}{3\,\left(c\,a^4\,x^2-2\,c\,a^3\,x+c\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a^2*x^2))*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(a*c) - (13*(1 - a^2*x^2)^(1/2))/(3*((c*(-a^2)^(1/2))/a
 - c*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (3*asinh(x*(-a^2)^(1/2)))/(c*(-a^2)^(1
/2)) - (2*a*(1 - a^2*x^2)^(1/2))/(3*(a^2*c + a^4*c*x^2 - 2*a^3*c*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int\frac{x^2}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx+\int\frac{2ax^3}{-a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(Integral(x**2/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2
*x**2 + 1) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integra
l(2*a*x**3/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1
) + a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x) + Integral(a**2*x*
*4/(-a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*
sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x))/c
```

3.650 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=125

$$\frac{(ax+1)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}-\frac{6(ax+1)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{24(ax+1)}{5ac^2\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\sin^{-1}(ax)}{ac^2}$$

```
[Out] 1/5*(a*x+1)^3/a/c^2/(-a^2*x^2+1)^(5/2)-6/5*(a*x+1)^2/a/c^2/(-a^2*x^2+1)^(3/
2)-3*arcsin(a*x)/a/c^2+24/5*(a*x+1)/a/c^2/(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1)^(
1/2)/a/c^2
```

Rubi [A] time = 0.33, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6148, 1635, 641, 216}

$$\frac{(ax+1)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}-\frac{6(ax+1)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}+\frac{24(ax+1)}{5ac^2\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (1 + a*x)^3/(5*a*c^2*(1 - a^2*x^2)^(5/2)) - (6*(1 + a*x)^2)/(5*a*c^2*(1 - a
^2*x^2)^(3/2)) + (24*(1 + a*x))/(5*a*c^2*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*
x^2]/(a*c^2) - (3*ArcSin[a*x])/(a*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
```

```
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{3\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}$$

$$= \frac{a^4\int \frac{x^4(1+ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{a^4\int \frac{(1+ax)^2\left(\frac{3}{a^4}+\frac{5x}{a^3}+\frac{5x^2}{a^2}+\frac{5x^3}{a}\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{a^4\int \frac{(1+ax)\left(\frac{27}{a^4}+\frac{30x}{a^3}+\frac{15x^2}{a^2}\right)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^2\sqrt{1-a^2x^2}} - \frac{a^4\int \frac{\frac{45}{a^4}+\frac{15x}{a^3}}{\sqrt{1-a^2x^2}}\,dx}{15c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^2\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^2} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{(1+ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}} - \frac{6(1+ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{24(1+ax)}{5ac^2\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{ac^2} - \frac{3\sin^{-1}(ax)}{ac^2}$$

Mathematica [A] time = 0.10, size = 88, normalized size = 0.70

$$\frac{-5a^4x^4+34a^3x^3-18a^2x^2-15(ax-1)^2\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-33ax+24}{5ac^2(ax-1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (24 - 33*a*x - 18*a^2*x^2 + 34*a^3*x^3 - 5*a^4*x^4 - 15*(-1 + a*x)^2*Sqrt[1
 - a^2*x^2]*ArcSin[a*x])/(5*a*c^2*(-1 + a*x)^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 143, normalized size = 1.14

$$\frac{24\,a^3x^3-72\,a^2x^2+72\,ax+30\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(5\,a^3x^3-39\,a^2x^2+57\,ax-24\right)\cdot}{5\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas
")
```

```
[Out] 1/5*(24*a^3*x^3 - 72*a^2*x^2 + 72*a*x + 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1
)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (5*a^3*x^3 - 39*a^2*x^2 + 57*a*x
 - 24)*sqrt(-a^2*x^2 + 1) - 24)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2)
```

giac [A] time = 0.24, size = 180, normalized size = 1.44

$$-\frac{3\,\arcsin\left(ax\right)\mathrm{sgn}(a)}{c^2|a|}+\frac{\sqrt{-a^2x^2+1}}{ac^2}-\frac{2\left(\frac{80\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{120\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{70\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}\,|\right.}{a^8x^4}\right.}{5\,c^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/(c^2*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c^2) - 2/5*(80*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 120*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^2/(a^4*x^2) + 70*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 15*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 19)/(c^2*((sqrt(-a^2*x^2 + 1)*abs(a)
 + a)/(a^2*x) - 1)^5*abs(a))
```

maple [A] time = 0.05, size = 212, normalized size = 1.70

$$-\frac{a\,x^2}{c^2\sqrt{-a^2x^2+1}}+\frac{7}{a\,c^2\sqrt{-a^2x^2+1}}+\frac{10x}{c^2\sqrt{-a^2x^2+1}}-\frac{3\arctan\left(\frac{\sqrt{a^2}\;x}{\sqrt{-a^2x^2+1}}\right)}{c^2\sqrt{a^2}}+\frac{2}{5a^3c^2\left(x-\frac{1}{a}\right)^2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -a/c^2*x^2/(-a^2*x^2+1)^(1/2)+7/a/c^2/(-a^2*x^2+1)^(1/2)+10*x/c^2/(-a^2*x^2
+1)^(1/2)-3/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+2/5/a^
3/c^2/(x-1/a)^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+13/5/a^2/c^2/(x-1/a)/(-a
^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-26/5/c^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^2), x)
```

mupad [B] time = 0.84, size = 272, normalized size = 2.18

$$\frac{2\,a^4\,\sqrt{1-a^2\,x^2}}{15\,\left(a^7\,c^2\,x^2-2\,a^6\,c^2\,x+a^5\,c^2\right)}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^2\,\sqrt{-a^2}}-\frac{4\,a\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^2\,x^2-2\,a^3\,c^2\,x+a^2\,c^2\right)}+\frac{\sqrt{1-a^2\,x^2}}{a\,c^2}+\frac{24\,\sqrt{1}}{5\,\sqrt{-a^2}\,\left(c^2\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a^2*x^2))^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (2*a^4*(1 - a^2*x^2)^(1/2))/(15*(a^5*c^2 - 2*a^6*c^2*x + a^7*c^2*x^2)) - (3
*asinh(x*(-a^2)^(1/2)))/(c^2*(-a^2)^(1/2)) - (4*a*(1 - a^2*x^2)^(1/2))/(3*(
a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) + (1 - a^2*x^2)^(1/2)/(a*c^2) + (24*(
1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^(1/2)
)/a)) + (1 - a^2*x^2)^(1/2)/(5*(-a^2)^(1/2)*(3*c^2*x*(-a^2)^(1/2) - (c^2*(-
a^2)^(1/2))/a + a^2*c^2*x^3*(-a^2)^(1/2) - 3*a*c^2*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\left(\int \frac{x^4}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{-a^4x^4\sqrt{-a^2x^2+1}+2a^3x^3\sqrt{-a^2x^2+1}-2ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+}}\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(Integral(x**4/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a*
*2*x**2 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Int
egral(a*x**5/(-a**4*x**4*sqrt(-a**2*x**2 + 1) + 2*a**3*x**3*sqrt(-a**2*x**2
 + 1) - 2*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.651 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{(ax+1)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}}+\frac{38(ax+1)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{137(ax+1)}{35ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}+\frac{181ax+245}{35ac^3\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{ac^3}$$

```
[Out] -1/7*(a*x+1)^3/a/c^3/(-a^2*x^2+1)^(7/2)+38/35*(a*x+1)^2/a/c^3/(-a^2*x^2+1)^
(5/2)-137/35*(a*x+1)/a/c^3/(-a^2*x^2+1)^(3/2)-3*arcsin(a*x)/a/c^3+1/35*(181
*a*x+245)/a/c^3/(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1)^(1/2)/a/c^3
```

Rubi [A] time = 0.44, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6148, 1635, 1814, 641, 216}

$$-\frac{(ax+1)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}}+\frac{38(ax+1)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{137(ax+1)}{35ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{\sqrt{1-a^2x^2}}{ac^3}+\frac{181ax+245}{35ac^3\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -(1 + a*x)^3/(7*a*c^3*(1 - a^2*x^2)^(7/2)) + (38*(1 + a*x)^2)/(35*a*c^3*(1
 - a^2*x^2)^(5/2)) - (137*(1 + a*x))/(35*a*c^3*(1 - a^2*x^2)^(3/2)) + (245 +
 181*a*x)/(35*a*c^3*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c^3) - (3*Arc
Sin[a*x])/(a*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
```

```
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{3\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}$$

$$= -\frac{a^6\int \frac{x^6(1+ax)^3}{(1-a^2x^2)^{9/2}}\,dx}{c^3}$$

$$= -\frac{(1+ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{a^6\int \frac{(1+ax)^2\left(\frac{3}{a^6}+\frac{7x}{a^5}+\frac{7x^2}{a^4}+\frac{7x^3}{a^3}+\frac{7x^4}{a^2}+\frac{7x^5}{a}\right)}{(1-a^2x^2)^{7/2}}\,dx}{7c^3}$$

$$= -\frac{(1+ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{38(1+ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{a^6\int \frac{(1+ax)\left(\frac{61}{a^6}+\frac{140x}{a^5}+\frac{105x^2}{a^4}+\frac{70x^3}{a^3}+\frac{35x^4}{a^2}\right)}{(1-a^2x^2)^{5/2}}\,dx}{35c^3}$$

$$= -\frac{(1+ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{38(1+ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{137(1+ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{a^6\int \frac{\frac{228}{a^6}+\frac{630x}{a^5}+\frac{315x^2}{a^4}+\frac{105x^3}{a^3}}{(1-a^2x^2)^{3/2}}\,d}{105c^3}$$

$$= -\frac{(1+ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{38(1+ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{137(1+ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{245+181ax}{35ac^3\sqrt{1-a^2x^2}} - \frac{a^6\int}{1}$$

$$= -\frac{(1+ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{38(1+ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{137(1+ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{245+181ax}{35ac^3\sqrt{1-a^2x^2}} + \frac{\sqrt{1-}}{ac}$$

$$= -\frac{(1+ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}} + \frac{38(1+ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{137(1+ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}} + \frac{245+181ax}{35ac^3\sqrt{1-a^2x^2}} + \frac{\sqrt{1-}}{ac}$$

Mathematica [A] time = 0.12, size = 96, normalized size = 0.62

$$\frac{-35a^5x^5+286a^4x^4-368a^3x^3-125a^2x^2-105(ax-1)^3\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+423ax-176}{35ac^3(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-176 + 423*a*x - 125*a^2*x^2 - 368*a^3*x^3 + 286*a^4*x^4 - 35*a^5*x^5 - 10
5*(-1 + a*x)^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(35*a*c^3*(-1 + a*x)^3*Sqrt[1
 - a^2*x^2])
```

fricas [A] time = 0.53, size = 212, normalized size = 1.37

$$\frac{176\,a^5x^5 - 528\,a^4x^4 + 352\,a^3x^3 + 352\,a^2x^2 - 528\,ax + 210\left(a^5x^5 - 3\,a^4x^4 + 2\,a^3x^3 + 2\,a^2x^2 - 3\,ax + 1\right)\arctan\left(\frac{\sqrt{}}{}\right.}{35\left(a^6c^3x^5 - 3\,a^5c^3x^4 + 2\,a^4c^3x^3 + 2\,a^3c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(176*a^5*x^5 - 528*a^4*x^4 + 352*a^3*x^3 + 352*a^2*x^2 - 528*a*x + 210
*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*arctan((sqrt(-a^
2*x^2 + 1) - 1)/(a*x)) + (35*a^5*x^5 - 286*a^4*x^4 + 368*a^3*x^3 + 125*a^2*
x^2 - 423*a*x + 176)*sqrt(-a^2*x^2 + 1) + 176)/(a^6*c^3*x^5 - 3*a^5*c^3*x^4
 + 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 - 3*a^2*c^3*x + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^3), x)
```

maple [A] time = 0.05, size = 256, normalized size = 1.65

$$-\frac{a\,x^2}{c^3\sqrt{-a^2x^2+1}}+\frac{8}{a\,c^3\sqrt{-a^2x^2+1}}+\frac{13x}{c^3\sqrt{-a^2x^2+1}}-\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^3\sqrt{a^2}}+\frac{38}{35a^3c^3\left(x-\frac{1}{a}\right)^2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -a/c^3*x^2/(-a^2*x^2+1)^(1/2)+8/a/c^3/(-a^2*x^2+1)^(1/2)+13*x/c^3/(-a^2*x^2
+1)^(1/2)-3/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+38/35/
```

```
a^3/c^3/(x-1/a)^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+137/35/a^2/c^3/(x-1/a)
/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-274/35/c^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))
^(1/2)*x+1/7/a^4/c^3/(x-1/a)^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{(-a^2x^2+1)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^3), x)
```

mupad [B] time = 2.61, size = 1548, normalized size = 9.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a^2*x^2))^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (71*(1 - a^2*x^2)^(1/2))/(280*(-a^2)^(1/2)*(c^3*1i + 3*c^3*x*(-a^2)^(1/2) +
 a^2*c^3*x^2*3i + a^2*c^3*x^3*(-a^2)^(1/2))) - (3*a^7*(1 - a^2*x^2)^(1/2))/
(112*(a^8*c^3 + c^3*x*(-a^2)^(9/2)*4i + 6*a^10*c^3*x^2 + a^12*c^3*x^4 + a^2
*c^3*x^3*(-a^2)^(9/2)*4i)) - (a^9*(1 - a^2*x^2)^(1/2))/(112*(a^10*c^3 - c^3
*x*(-a^2)^(11/2)*4i + 6*a^12*c^3*x^2 + a^14*c^3*x^4 - a^2*c^3*x^3*(-a^2)^(1
1/2)*4i)) - (a^9*(1 - a^2*x^2)^(1/2))/(112*(a^10*c^3 + c^3*x*(-a^2)^(11/2)*
4i + 6*a^12*c^3*x^2 + a^14*c^3*x^4 + a^2*c^3*x^3*(-a^2)^(11/2)*4i)) - (537*
a^7*(1 - a^2*x^2)^(1/2))/(1120*(a^8*c^3 - c^3*x*(-a^2)^(9/2)*2i + a^10*c^3*
x^2)) - (537*a^7*(1 - a^2*x^2)^(1/2))/(1120*(a^8*c^3 + c^3*x*(-a^2)^(9/2)*2
i + a^10*c^3*x^2)) - (417*a^9*(1 - a^2*x^2)^(1/2))/(1120*(a^10*c^3 - c^3*x*
(-a^2)^(11/2)*2i + a^12*c^3*x^2)) - (417*a^9*(1 - a^2*x^2)^(1/2))/(1120*(a^
10*c^3 + c^3*x*(-a^2)^(11/2)*2i + a^12*c^3*x^2)) - (3*asinh(x*(-a^2)^(1/2))
)/(c^3*(-a^2)^(1/2)) - (3*a^7*(1 - a^2*x^2)^(1/2))/(112*(a^8*c^3 - c^3*x*(-
a^2)^(9/2)*4i + 6*a^10*c^3*x^2 + a^12*c^3*x^4 - a^2*c^3*x^3*(-a^2)^(9/2)*4i
)) - (71*(1 - a^2*x^2)^(1/2))/(280*(-a^2)^(1/2)*(c^3*1i - 3*c^3*x*(-a^2)^(1
/2) + a^2*c^3*x^2*3i - a^2*c^3*x^3*(-a^2)^(1/2))) + (a^7*(1 - a^2*x^2)^(1/2
)*6i)/(35*(a^8*c^3*1i + 3*c^3*x*(-a^2)^(9/2) + a^10*c^3*x^2*3i + a^2*c^3*x^
3*(-a^2)^(9/2))) + (a^7*(1 - a^2*x^2)^(1/2)*6i)/(35*(a^8*c^3*1i - 3*c^3*x*(
-a^2)^(9/2) + a^10*c^3*x^2*3i - a^2*c^3*x^3*(-a^2)^(9/2))) + (a^9*(1 - a^2*
x^2)^(1/2)*23i)/(280*(a^10*c^3*1i + 3*c^3*x*(-a^2)^(11/2) + a^12*c^3*x^2*3i
 + a^2*c^3*x^3*(-a^2)^(11/2))) + (a^9*(1 - a^2*x^2)^(1/2)*23i)/(280*(a^10*c
```

```
^3*1i - 3*c^3*x*(-a^2)^(11/2) + a^12*c^3*x^2*3i - a^2*c^3*x^3*(-a^2)^(11/2)
)) + (181*(1 - a^2*x^2)^(1/2))/(70*(c^3*1i + c^3*x*(-a^2)^(1/2))*(-a^2)^(1/
2)) - (181*(1 - a^2*x^2)^(1/2))/(70*(c^3*1i - c^3*x*(-a^2)^(1/2))*(-a^2)^(1
/2)) + (a^7*(1 - a^2*x^2)^(1/2)*1143i)/(1120*(a^8*c^3*1i + c^3*x*(-a^2)^(9/
2))) + (a^7*(1 - a^2*x^2)^(1/2)*1143i)/(1120*(a^8*c^3*1i - c^3*x*(-a^2)^(9/
2))) + (a^9*(1 - a^2*x^2)^(1/2)*1823i)/(1120*(a^10*c^3*1i + c^3*x*(-a^2)^(1
1/2))) + (a^9*(1 - a^2*x^2)^(1/2)*1823i)/(1120*(a^10*c^3*1i - c^3*x*(-a^2)^
(11/2))) + (1 - a^2*x^2)^(1/2)/(a*c^3) + ((1 - a^2*x^2)^(1/2)*1i)/(28*(-a^2
)^(1/2)*(c^3 - c^3*x*(-a^2)^(1/2)*4i + 6*a^2*c^3*x^2 + a^4*c^3*x^4 - a^2*c^
3*x^3*(-a^2)^(1/2)*4i)) - ((1 - a^2*x^2)^(1/2)*1i)/(28*(-a^2)^(1/2)*(c^3 +
c^3*x*(-a^2)^(1/2)*4i + 6*a^2*c^3*x^2 + a^4*c^3*x^4 + a^2*c^3*x^3*(-a^2)^(1
/2)*4i)) + ((1 - a^2*x^2)^(1/2)*477i)/(560*(-a^2)^(1/2)*(c^3 - c^3*x*(-a^2)
^(1/2)*2i + a^2*c^3*x^2)) - ((1 - a^2*x^2)^(1/2)*477i)/(560*(-a^2)^(1/2)*(c
^3 + c^3*x*(-a^2)^(1/2)*2i + a^2*c^3*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6 \int \frac{x^6}{-a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}-\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6/(-a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**
2*x**2 + 1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + 3*a*x*sqrt(-a**2*x**2 + 1) - sqrt(-a**2*x**2 + 1)), x)/c**3
```

3.652 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=185

$$\frac{(ax+1)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}}-\frac{22(ax+1)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}}+\frac{478(ax+1)}{105ac^4\left(1-a^2x^2\right)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{ac^4}+\frac{4(431ax+630)}{315ac^4\sqrt{1-a^2x^2}}-\frac{2(829ax+1155}{315ac^4\left(1-a^2x^2\right.}$$

```
[Out] 1/9*(a*x+1)^3/a/c^4/(-a^2*x^2+1)^(9/2)-22/21*(a*x+1)^2/a/c^4/(-a^2*x^2+1)^(
7/2)+478/105*(a*x+1)/a/c^4/(-a^2*x^2+1)^(5/2)-2/315*(829*a*x+1155)/a/c^4/(-
a^2*x^2+1)^(3/2)-3*arcsin(a*x)/a/c^4+4/315*(431*a*x+630)/a/c^4/(-a^2*x^2+1)
^(1/2)+(-a^2*x^2+1)^(1/2)/a/c^4
```

Rubi [A] time = 0.56, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6148, 1635, 1814, 641, 216}

$$\frac{(ax+1)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}}-\frac{22(ax+1)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}}+\frac{478(ax+1)}{105ac^4\left(1-a^2x^2\right)^{5/2}}+\frac{\sqrt{1-a^2x^2}}{ac^4}+\frac{4(431ax+630)}{315ac^4\sqrt{1-a^2x^2}}-\frac{2(829ax+1155}{315ac^4\left(1-a^2x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (1 + a*x)^3/(9*a*c^4*(1 - a^2*x^2)^(9/2)) - (22*(1 + a*x)^2)/(21*a*c^4*(1 -
 a^2*x^2)^(7/2)) + (478*(1 + a*x))/(105*a*c^4*(1 - a^2*x^2)^(5/2)) - (2*(11
55 + 829*a*x))/(315*a*c^4*(1 - a^2*x^2)^(3/2)) + (4*(630 + 431*a*x))/(315*a
*c^4*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c^4) - (3*ArcSin[a*x])/(a*c^
4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{3\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8(1+ax)^3}{(1-a^2x^2)^{11/2}}\,dx}{c^4}$$

$$= \frac{(1+ax)^3}{9ac^4(1-a^2x^2)^{9/2}} - \frac{a^8\int \frac{(1+ax)^2\left(\frac{3}{a^8}+\frac{9x}{a^7}+\frac{9x^2}{a^6}+\frac{9x^3}{a^5}+\frac{9x^4}{a^4}+\frac{9x^5}{a^3}+\frac{9x^6}{a^2}+\frac{9x^7}{a}\right)}{(1-a^2x^2)^{9/2}}\,dx}{9c^4}$$

$$= \frac{(1+ax)^3}{9ac^4(1-a^2x^2)^{9/2}} - \frac{22(1+ax)^2}{21ac^4(1-a^2x^2)^{7/2}} + \frac{a^8\int \frac{(1+ax)\left(\frac{111}{a^8}+\frac{378x}{a^7}+\frac{315x^2}{a^6}+\frac{252x^3}{a^5}+\frac{189x^4}{a^4}+\frac{126x^5}{a^3}+\frac{63x^6}{a^2}\right)}{(1-a^2x^2)^{7/2}}\,dx}{63c^4}$$

$$= \frac{(1+ax)^3}{9ac^4(1-a^2x^2)^{9/2}} - \frac{22(1+ax)^2}{21ac^4(1-a^2x^2)^{7/2}} + \frac{478(1+ax)}{105ac^4(1-a^2x^2)^{5/2}} - \frac{a^8\int \frac{\frac{879}{a^8}+\frac{4725x}{a^7}+\frac{3150x^2}{a^6}+\frac{1890x^3}{a^5}}{(1-a^2x^2)^{5/2}}}{315c^4}$$

$$= \frac{(1+ax)^3}{9ac^4(1-a^2x^2)^{9/2}} - \frac{22(1+ax)^2}{21ac^4(1-a^2x^2)^{7/2}} + \frac{478(1+ax)}{105ac^4(1-a^2x^2)^{5/2}} - \frac{2(1155+829ax)}{315ac^4(1-a^2x^2)^{3/2}} + \frac{a^8}{}$$

$$= \frac{(1+ax)^3}{9ac^4(1-a^2x^2)^{9/2}} - \frac{22(1+ax)^2}{21ac^4(1-a^2x^2)^{7/2}} + \frac{478(1+ax)}{105ac^4(1-a^2x^2)^{5/2}} - \frac{2(1155+829ax)}{315ac^4(1-a^2x^2)^{3/2}} + \frac{4}{31}$$

$$= \frac{(1+ax)^3}{9ac^4(1-a^2x^2)^{9/2}} - \frac{22(1+ax)^2}{21ac^4(1-a^2x^2)^{7/2}} + \frac{478(1+ax)}{105ac^4(1-a^2x^2)^{5/2}} - \frac{2(1155+829ax)}{315ac^4(1-a^2x^2)^{3/2}} + \frac{4}{31}$$

$$= \frac{(1+ax)^3}{9ac^4(1-a^2x^2)^{9/2}} - \frac{22(1+ax)^2}{21ac^4(1-a^2x^2)^{7/2}} + \frac{478(1+ax)}{105ac^4(1-a^2x^2)^{5/2}} - \frac{2(1155+829ax)}{315ac^4(1-a^2x^2)^{3/2}} + \frac{4}{31}$$

Mathematica [A] time = 0.15, size = 124, normalized size = 0.67

$$\frac{-315a^7x^7+2669a^6x^6-2967a^5x^5-4029a^4x^4+7399a^3x^3-339a^2x^2-945(ax-1)^4(ax+1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax}{315ac^4(ax-1)^4(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] (1664 - 4047*a*x - 339*a^2*x^2 + 7399*a^3*x^3 - 4029*a^4*x^4 - 2967*a^5*x^5
 + 2669*a^6*x^6 - 315*a^7*x^7 - 945*(-1 + a*x)^4*(1 + a*x)*Sqrt[1 - a^2*x^2
]*ArcSin[a*x])/(315*a*c^4*(-1 + a*x)^4*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.72, size = 281, normalized size = 1.52

$$\frac{1664\,a^7x^7 - 4992\,a^6x^6 + 1664\,a^5x^5 + 8320\,a^4x^4 - 8320\,a^3x^3 - 1664\,a^2x^2 + 4992\,ax + 1890\left(a^7x^7 - 3\,a^6x^6 + a^5x^5\right.}{315\left(a^8c^4x^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas
")
```

```
[Out] 1/315*(1664*a^7*x^7 - 4992*a^6*x^6 + 1664*a^5*x^5 + 8320*a^4*x^4 - 8320*a^3
*x^3 - 1664*a^2*x^2 + 4992*a*x + 1890*(a^7*x^7 - 3*a^6*x^6 + a^5*x^5 + 5*a^
4*x^4 - 5*a^3*x^3 - a^2*x^2 + 3*a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a
*x)) + (315*a^7*x^7 - 2669*a^6*x^6 + 2967*a^5*x^5 + 4029*a^4*x^4 - 7399*a^3
*x^3 + 339*a^2*x^2 + 4047*a*x - 1664)*sqrt(-a^2*x^2 + 1) - 1664)/(a^8*c^4*x
^7 - 3*a^7*c^4*x^6 + a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 - a^3*c^4*
x^2 + 3*a^2*c^4*x - a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^4), x)
```

maple [B] time = 0.06, size = 367, normalized size = 1.98

$$-\frac{a\,x^2}{c^4\sqrt{-a^2x^2+1}}+\frac{9}{a\,c^4\sqrt{-a^2x^2+1}}+\frac{16x}{c^4\sqrt{-a^2x^2+1}}-\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^4\sqrt{a^2}}+\frac{5111}{2520a^3c^4\left(x-\frac{1}{a}\right)^2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -a/c^4*x^2/(-a^2*x^2+1)^(1/2)+9/a/c^4/(-a^2*x^2+1)^(1/2)+16*x/c^4/(-a^2*x^2
+1)^(1/2)-3/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+5111/2
520/a^3/c^4/(x-1/a)^2/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+26633/5040/a^2/c^4
/(x-1/a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-26633/2520/c^4/(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)*x+1/18/a^5/c^4/(x-1/a)^4/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2)+125/252/a^4/c^4/(x-1/a)^3/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/48/a^2/c
^4/(x+1/a)/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+1/24/c^4/(-a^2*(x+1/a)^2+2*a*
(x+1/a))^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^4), x)
```

mupad [B] time = 4.43, size = 2165, normalized size = 11.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a^2*x^2))^4*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(72*(-a^2)^(1/2)*(c^4*1i + 5*c^4*x*(-a^2)^(1/2) + a^2*c
^4*x^2*10i + a^4*c^4*x^4*5i + 10*a^2*c^4*x^3*(-a^2)^(1/2) + a^4*c^4*x^5*(-a
^2)^(1/2))) - (109*a^9*(1 - a^2*x^2)^(1/2))/(1344*(a^10*c^4 + c^4*x*(-a^2)^
(11/2)*4i + 6*a^12*c^4*x^2 + a^14*c^4*x^4 + a^2*c^4*x^3*(-a^2)^(11/2)*4i))
- (145*a^11*(1 - a^2*x^2)^(1/2))/(4032*(a^12*c^4 - c^4*x*(-a^2)^(13/2)*4i +
 6*a^14*c^4*x^2 + a^16*c^4*x^4 - a^2*c^4*x^3*(-a^2)^(13/2)*4i)) - (145*a^11
*(1 - a^2*x^2)^(1/2))/(4032*(a^12*c^4 + c^4*x*(-a^2)^(13/2)*4i + 6*a^14*c^4
*x^2 + a^16*c^4*x^4 + a^2*c^4*x^3*(-a^2)^(13/2)*4i)) - (14711*a^9*(1 - a^2*
x^2)^(1/2))/(26880*(a^10*c^4 - c^4*x*(-a^2)^(11/2)*2i + a^12*c^4*x^2)) - (1
4711*a^9*(1 - a^2*x^2)^(1/2))/(26880*(a^10*c^4 + c^4*x*(-a^2)^(11/2)*2i + a
^12*c^4*x^2)) - (8947*a^11*(1 - a^2*x^2)^(1/2))/(16128*(a^12*c^4 - c^4*x*(-
a^2)^(13/2)*2i + a^14*c^4*x^2)) - (8947*a^11*(1 - a^2*x^2)^(1/2))/(16128*(a
^12*c^4 + c^4*x*(-a^2)^(13/2)*2i + a^14*c^4*x^2)) - (3*asinh(x*(-a^2)^(1/2)
))/(c^4*(-a^2)^(1/2)) - (109*a^9*(1 - a^2*x^2)^(1/2))/(1344*(a^10*c^4 - c^4
*x*(-a^2)^(11/2)*4i + 6*a^12*c^4*x^2 + a^14*c^4*x^4 - a^2*c^4*x^3*(-a^2)^(1
```

```
1/2)*4i)) - (1 - a^2*x^2)^(1/2)/(72*(-a^2)^(1/2)*(c^4*1i - 5*c^4*x*(-a^2)^(
1/2) + a^2*c^4*x^2*10i + a^4*c^4*x^4*5i - 10*a^2*c^4*x^3*(-a^2)^(1/2) - a^4
*c^4*x^5*(-a^2)^(1/2))) + (a^9*(1 - a^2*x^2)^(1/2)*1i)/(96*(a^10*c^4*1i + 5
*c^4*x*(-a^2)^(11/2) + a^12*c^4*x^2*10i + a^14*c^4*x^4*5i + 10*a^2*c^4*x^3*
(-a^2)^(11/2) + a^4*c^4*x^5*(-a^2)^(11/2))) + (a^9*(1 - a^2*x^2)^(1/2)*1i)/
(96*(a^10*c^4*1i - 5*c^4*x*(-a^2)^(11/2) + a^12*c^4*x^2*10i + a^14*c^4*x^4*
5i - 10*a^2*c^4*x^3*(-a^2)^(11/2) - a^4*c^4*x^5*(-a^2)^(11/2))) + (a^11*(1
- a^2*x^2)^(1/2)*1i)/(288*(a^12*c^4*1i + 5*c^4*x*(-a^2)^(13/2) + a^14*c^4*x
^2*10i + a^16*c^4*x^4*5i + 10*a^2*c^4*x^3*(-a^2)^(13/2) + a^4*c^4*x^5*(-a^2
)^(13/2))) + (a^11*(1 - a^2*x^2)^(1/2)*1i)/(288*(a^12*c^4*1i - 5*c^4*x*(-a^
2)^(13/2) + a^14*c^4*x^2*10i + a^16*c^4*x^4*5i - 10*a^2*c^4*x^3*(-a^2)^(13/
2) - a^4*c^4*x^5*(-a^2)^(13/2))) + (1507*(1 - a^2*x^2)^(1/2))/(3360*(-a^2)^
(1/2)*(c^4*1i + 3*c^4*x*(-a^2)^(1/2) + a^2*c^4*x^2*3i + a^2*c^4*x^3*(-a^2)^
(1/2))) - (1507*(1 - a^2*x^2)^(1/2))/(3360*(-a^2)^(1/2)*(c^4*1i - 3*c^4*x*(
-a^2)^(1/2) + a^2*c^4*x^2*3i - a^2*c^4*x^3*(-a^2)^(1/2))) + (a^9*(1 - a^2*x
^2)^(1/2)*1231i)/(4480*(a^10*c^4*1i + 3*c^4*x*(-a^2)^(11/2) + a^12*c^4*x^2*
3i + a^2*c^4*x^3*(-a^2)^(11/2))) + (a^9*(1 - a^2*x^2)^(1/2)*1231i)/(4480*(a
^10*c^4*1i - 3*c^4*x*(-a^2)^(11/2) + a^12*c^4*x^2*3i - a^2*c^4*x^3*(-a^2)^(
11/2))) + (a^11*(1 - a^2*x^2)^(1/2)*467i)/(2688*(a^12*c^4*1i + 3*c^4*x*(-a^
2)^(13/2) + a^14*c^4*x^2*3i + a^2*c^4*x^3*(-a^2)^(13/2))) + (a^11*(1 - a^2*
x^2)^(1/2)*467i)/(2688*(a^12*c^4*1i - 3*c^4*x*(-a^2)^(13/2) + a^14*c^4*x^2*
3i - a^2*c^4*x^3*(-a^2)^(13/2))) + (862*(1 - a^2*x^2)^(1/2))/(315*(c^4*1i +
 c^4*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (862*(1 - a^2*x^2)^(1/2))/(315*(c^4*1i
 - c^4*x*(-a^2)^(1/2))*(-a^2)^(1/2)) + (a^9*(1 - a^2*x^2)^(1/2)*25609i)/(26
880*(a^10*c^4*1i + c^4*x*(-a^2)^(11/2))) + (a^9*(1 - a^2*x^2)^(1/2)*25609i)
/(26880*(a^10*c^4*1i - c^4*x*(-a^2)^(11/2))) + (a^11*(1 - a^2*x^2)^(1/2)*31
373i)/(16128*(a^12*c^4*1i + c^4*x*(-a^2)^(13/2))) + (a^11*(1 - a^2*x^2)^(1/
2)*31373i)/(16128*(a^12*c^4*1i - c^4*x*(-a^2)^(13/2))) + (1 - a^2*x^2)^(1/2
)/(a*c^4) + ((1 - a^2*x^2)^(1/2)*59i)/(504*(-a^2)^(1/2)*(c^4 - c^4*x*(-a^2)
^(1/2)*4i + 6*a^2*c^4*x^2 + a^4*c^4*x^4 - a^2*c^4*x^3*(-a^2)^(1/2)*4i)) - (
(1 - a^2*x^2)^(1/2)*59i)/(504*(-a^2)^(1/2)*(c^4 + c^4*x*(-a^2)^(1/2)*4i + 6
*a^2*c^4*x^2 + a^4*c^4*x^4 + a^2*c^4*x^3*(-a^2)^(1/2)*4i)) + ((1 - a^2*x^2)
^(1/2)*22007i)/(20160*(-a^2)^(1/2)*(c^4 - c^4*x*(-a^2)^(1/2)*2i + a^2*c^4*x
^2)) - ((1 - a^2*x^2)^(1/2)*22007i)/(20160*(-a^2)^(1/2)*(c^4 + c^4*x*(-a^2)
^(1/2)*2i + a^2*c^4*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

3.653 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal. Leaf size=116

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

```
[Out] 1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/a
^6/x^5+14/3*c^5/a^4/x^3+4*c^5/a^3/x^2-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

Rubi [A] time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{4c^5}{a^3x^2}+\frac{14c^5}{3a^4x^3}-\frac{14c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}+\frac{c^5}{2a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\frac{c^5\int\frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^5}{x^{10}}\,dx}{a^{10}}$$
$$=-\frac{c^5\int\frac{(1-ax)^3(1+ax)^7}{x^{10}}\,dx}{a^{10}}$$
$$=-\frac{c^5\int\left(-a^{10}+\frac{1}{x^{10}}+\frac{4a}{x^9}+\frac{3a^2}{x^8}-\frac{8a^3}{x^7}-\frac{14a^4}{x^6}+\frac{14a^6}{x^4}+\frac{8a^7}{x^3}-\frac{3a^8}{x^2}-\frac{4a^9}{x}\right)dx}{a^{10}}$$
$$=\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\,\text{lo}}{a}$$

Mathematica [A] time = 0.04, size = 116, normalized size = 1.00

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

fricas [A] time = 0.56, size = 111, normalized size = 0.96

$$\frac{630\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log(x)-1890\,a^8c^5x^8+2520\,a^7c^5x^7+2940\,a^6c^5x^6-1764\,a^4c^5x^4-840\,a^3c^5x^3+270\,}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/630*(630*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) - 1890*a^8*c^5*x^8 + 252
0*a^7*c^5*x^7 + 2940*a^6*c^5*x^6 - 1764*a^4*c^5*x^4 - 840*a^3*c^5*x^3 + 270
*a^2*c^5*x^2 + 315*a*c^5*x + 70*c^5)/(a^10*x^9)
```

giac [A] time = 0.20, size = 104, normalized size = 0.90

$$c^5x+\frac{4\,c^5\log\left(|x|\right)}{a}-\frac{1890\,a^8c^5x^8-2520\,a^7c^5x^7-2940\,a^6c^5x^6+1764\,a^4c^5x^4+840\,a^3c^5x^3-270\,a^2c^5x^2-315\,ac^5x}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] c^5*x + 4*c^5*log(abs(x))/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 -
2940*a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 3
15*a*c^5*x - 70*c^5)/(a^10*x^9)
```

maple [A] time = 0.04, size = 105, normalized size = 0.91

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{x^2a^3}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^5,x)
```

```
[Out] 1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/a
^6/x^5+14/3*c^5/a^4/x^3+4*c^5/x^2/a^3-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

maxima [A] time = 0.31, size = 103, normalized size = 0.89

$$c^5x+\frac{4\,c^5\log(x)}{a}-\frac{1890\,a^8c^5x^8-2520\,a^7c^5x^7-2940\,a^6c^5x^6+1764\,a^4c^5x^4+840\,a^3c^5x^3-270\,a^2c^5x^2-315\,ac^5x}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 4*c^5*log(x)/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 - 2940*
a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 315*a*
c^5*x - 70*c^5)/(a^10*x^9)
```

mupad [B] time = 0.09, size = 81, normalized size = 0.70

$$\frac{c^5\left(\frac{a\,x}{2}+\frac{3\,a^2\,x^2}{7}-\frac{4\,a^3\,x^3}{3}-\frac{14\,a^4\,x^4}{5}+\frac{14\,a^6\,x^6}{3}+4\,a^7\,x^7-3\,a^8\,x^8+a^{10}\,x^{10}+4\,a^9\,x^9\,\ln(x)+\frac{1}{9}\right)}{a^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^5*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (c^5*((a*x)/2 + (3*a^2*x^2)/7 - (4*a^3*x^3)/3 - (14*a^4*x^4)/5 + (14*a^6*x^
6)/3 + 4*a^7*x^7 - 3*a^8*x^8 + a^10*x^10 + 4*a^9*x^9*log(x) + 1/9))/(a^10*x
^9)
```

sympy [A] time = 0.66, size = 112, normalized size = 0.97

$$\frac{a^{10}c^5x + 4a^9c^5\log(x) + \frac{-1890a^8c^5x^8+2520a^7c^5x^7+2940a^6c^5x^6-1764a^4c^5x^4-840a^3c^5x^3+270a^2c^5x^2+315ac^5x+70c^5}{630x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 4*a**9*c**5*log(x) + (-1890*a**8*c**5*x**8 + 2520*a**7*c**5
*x**7 + 2940*a**6*c**5*x**6 - 1764*a**4*c**5*x**4 - 840*a**3*c**5*x**3 + 27
0*a**2*c**5*x**2 + 315*a*c**5*x + 70*c**5)/(630*x**9))/a**10
```

3.654 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=100

$$-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

```
[Out] -1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/x
^3+2*c^4/a^3/x^2-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

Rubi [A] time = 0.14, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{2c^4}{a^3x^2}+\frac{10c^4}{3a^4x^3}+\frac{c^4}{a^5x^4}-\frac{4c^4}{5a^6x^5}-\frac{2c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -c^4/(7*a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x^4
) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x + (4
*c^4*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int\frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}\,dx}{a^8}$$
$$=\frac{c^4\int\frac{(1-ax)^2(1+ax)^6}{x^8}\,dx}{a^8}$$
$$=\frac{c^4\int\left(a^8+\frac{1}{x^8}+\frac{4a}{x^7}+\frac{4a^2}{x^6}-\frac{4a^3}{x^5}-\frac{10a^4}{x^4}-\frac{4a^5}{x^3}+\frac{4a^6}{x^2}+\frac{4a^7}{x}\right)dx}{a^8}$$
$$=-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

Mathematica [A] time = 0.03, size = 100, normalized size = 1.00

$$-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -1/7*c^4/(a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x
^4) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x +
(4*c^4*Log[x])/a
```

fricas [A] time = 0.48, size = 100, normalized size = 1.00

$$\frac{105\,a^8c^4x^8+420\,a^7c^4x^7\log(x)-420\,a^6c^4x^6+210\,a^5c^4x^5+350\,a^4c^4x^4+105\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x-15\,c}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) - 420*a^6*c^4*x^6 + 210*a^5
*c^4*x^5 + 350*a^4*c^4*x^4 + 105*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x
- 15*c^4)/(a^8*x^7)
```

giac [A] time = 0.18, size = 93, normalized size = 0.93

$$c^4x+\frac{4\,c^4\log(|x|)}{a}-\frac{420\,a^6c^4x^6-210\,a^5c^4x^5-350\,a^4c^4x^4-105\,a^3c^4x^3+84\,a^2c^4x^2+70\,ac^4x+15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] c^4*x + 4*c^4*log(abs(x))/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 35
0*a^4*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^
8*x^7)
```

maple [A] time = 0.03, size = 93, normalized size = 0.93

$$-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{x^2a^3}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/x
^3+2*c^4/x^2/a^3-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

maxima [A] time = 0.30, size = 92, normalized size = 0.92

$$c^4x+\frac{4\,c^4\log(x)}{a}-\frac{420\,a^6c^4x^6-210\,a^5c^4x^5-350\,a^4c^4x^4-105\,a^3c^4x^3+84\,a^2c^4x^2+70\,ac^4x+15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 4*c^4*log(x)/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 350*a^4
*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^8*x^7
)
```

mupad [B] time = 0.86, size = 72, normalized size = 0.72

$$\frac{c^4\,\left(a^3\,x^3-\frac{4\,a^2\,x^2}{5}-\frac{2\,a\,x}{3}+\frac{10\,a^4\,x^4}{3}+2\,a^5\,x^5-4\,a^6\,x^6+a^8\,x^8+4\,a^7\,x^7\,\ln(x)-\frac{1}{7}\right)}{a^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (c^4*(a^3*x^3 - (4*a^2*x^2)/5 - (2*a*x)/3 + (10*a^4*x^4)/3 + 2*a^5*x^5 - 4*
a^6*x^6 + a^8*x^8 + 4*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

sympy [A] time = 0.51, size = 100, normalized size = 1.00

$$\frac{a^8c^4x+4a^7c^4\log(x)+\frac{-420a^6c^4x^6+210a^5c^4x^5+350a^4c^4x^4+105a^3c^4x^3-84a^2c^4x^2-70ac^4x-15c^4}{105x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**4,x)
```

```
[Out] (a**8*c**4*x + 4*a**7*c**4*log(x) + (-420*a**6*c**4*x**6 + 210*a**5*c**4*x*
*5 + 350*a**4*c**4*x**4 + 105*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**
4*x - 15*c**4)/(105*x**7))/a**8
```

3.655 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=63

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

```
[Out] 1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/a
```

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 75}

$$\frac{5c^3}{3a^4x^3}+\frac{c^3}{a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \frac{(1-ax)(1+ax)^5}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \left(-a^6+\frac{1}{x^6}+\frac{4a}{x^5}+\frac{5a^2}{x^4}-\frac{5a^4}{x^2}-\frac{4a^5}{x}\right)dx}{a^6}$$
$$= \frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 1.00

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

fricas [A] time = 0.64, size = 67, normalized size = 1.06

$$\frac{15\,a^6c^3x^6+60\,a^5c^3x^5\log(x)-75\,a^4c^3x^4+25\,a^2c^3x^2+15\,ac^3x+3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) - 75*a^4*c^3*x^4 + 25*a^2*c^3*
x^2 + 15*a*c^3*x + 3*c^3)/(a^6*x^5)
```

giac [A] time = 0.14, size = 60, normalized size = 0.95

$$c^3x+\frac{4\,c^3\log\left(|x|\right)}{a}-\frac{75\,a^4c^3x^4-25\,a^2c^3x^2-15\,ac^3x-3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] c^3*x + 4*c^3*log(abs(x))/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*
c^3*x - 3*c^3)/(a^6*x^5)
```

maple [A] time = 0.04, size = 60, normalized size = 0.95

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/a
```

maxima [A] time = 0.31, size = 59, normalized size = 0.94

$$c^3x+\frac{4\,c^3\log(x)}{a}-\frac{75\,a^4c^3x^4-25\,a^2c^3x^2-15\,ac^3x-3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 4*c^3*log(x)/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*c^3*x
 - 3*c^3)/(a^6*x^5)
```

mupad [B] time = 0.06, size = 48, normalized size = 0.76

$$\frac{c^3\,\left(a\,x+\frac{5\,a^2\,x^2}{3}-5\,a^4\,x^4+a^6\,x^6+4\,a^5\,x^5\,\ln(x)+\frac{1}{5}\right)}{a^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] (c^3*(a*x + (5*a^2*x^2)/3 - 5*a^4*x^4 + a^6*x^6 + 4*a^5*x^5*log(x) + 1/5))/
(a^6*x^5)
```

sympy [A] time = 0.32, size = 65, normalized size = 1.03

$$\frac{a^6c^3x+4a^5c^3\log{(x)}+\frac{-75a^4c^3x^4+25a^2c^3x^2+15ac^3x+3c^3}{15x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 4*a**5*c**3*log(x) + (-75*a**4*c**3*x**4 + 25*a**2*c**3*x**2
 + 15*a*c**3*x + 3*c**3)/(15*x**5))/a**6
```

3.656 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

```
[Out] -1/3*c^2/a^4/x^3-2*c^2/a^3/x^2-6*c^2/a^2/x+c^2*x+4*c^2*ln(x)/a
```

Rubi [A] time = 0.12, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 43}

$$-\frac{2c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -c^2/(3*a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*Log
[x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4} dx}{a^4}$$
$$= \frac{c^2\int \frac{(1+ax)^4}{x^4} dx}{a^4}$$
$$= \frac{c^2\int \left(a^4+\frac{1}{x^4}+\frac{4a}{x^3}+\frac{6a^2}{x^2}+\frac{4a^3}{x}\right) dx}{a^4}$$
$$= -\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -1/3*c^2/(a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*L
og[x])/a
```

fricas [A] time = 0.68, size = 56, normalized size = 1.10

$$\frac{3\,a^4c^2x^4+12\,a^3c^2x^3\log(x)-18\,a^2c^2x^2-6\,ac^2x-c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^2*x^4 + 12*a^3*c^2*x^3*log(x) - 18*a^2*c^2*x^2 - 6*a*c^2*x - c
^2)/(a^4*x^3)
```

giac [A] time = 0.20, size = 47, normalized size = 0.92

$$c^2x+\frac{4\,c^2\log\left(|x|\right)}{a}-\frac{18\,a^2c^2x^2+6\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] c^2*x + 4*c^2*log(abs(x))/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x
^3)
```

maple [A] time = 0.03, size = 50, normalized size = 0.98

$$-\frac{c^2}{3a^4x^3} - \frac{2c^2}{x^2a^3} - \frac{6c^2}{a^2x} + c^2x + \frac{4c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/3*c^2/a^4/x^3-2*c^2/x^2/a^3-6*c^2/a^2/x+c^2*x+4*c^2*ln(x)/a
```

maxima [A] time = 0.30, size = 46, normalized size = 0.90

$$c^2x + \frac{4\,c^2\log(x)}{a} - \frac{18\,a^2c^2x^2 + 6\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 4*c^2*log(x)/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x^3)
```

mupad [B] time = 0.85, size = 43, normalized size = 0.84

$$-\frac{c^2\,\left(6\,a\,x+18\,a^2\,x^2-3\,a^4\,x^4-12\,a^3\,x^3\,\ln(x)+1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] -(c^2*(6*a*x + 18*a^2*x^2 - 3*a^4*x^4 - 12*a^3*x^3*log(x) + 1))/(3*a^4*x^3)
```

sympy [A] time = 0.22, size = 53, normalized size = 1.04

$$\frac{a^4c^2x + 4a^3c^2\log{(x)} + \frac{-18a^2c^2x^2-6ac^2x-c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**2,x)
```

```
[Out] (a**4*c**2*x + 4*a**3*c**2*log(x) + (-18*a**2*c**2*x**2 - 6*a*c**2*x - c**2
)/(3*x**3))/a**4
```

3.657 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=33

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

```
[Out] c/a^2/x+c*x-4*c*ln(x)/a+8*c*ln(-a*x+1)/a
```

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6157, 6150, 88}

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\frac{c\int\frac{e^{4\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$
$$= -\frac{c\int\frac{(1+ax)^3}{x^2(1-ax)}dx}{a^2}$$
$$= -\frac{c\int\left(-a^2+\frac{1}{x^2}+\frac{4a}{x}-\frac{8a^2}{-1+ax}\right)dx}{a^2}$$
$$= \frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

fricas [A] time = 0.60, size = 35, normalized size = 1.06

$$\frac{a^2cx^2+8\,acx\log\left(ax-1\right)-4\,acx\log(x)+c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*c*x^2 + 8*a*c*x*log(a*x - 1) - 4*a*c*x*log(x) + c)/(a^2*x)
```

giac [A] time = 0.15, size = 34, normalized size = 1.03

$$cx+\frac{8\,c\log\left(|ax-1|\right)}{a}-\frac{4\,c\log\left(|x|\right)}{a}+\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2),x, algorithm="giac")
[Out] c*x + 8*c*log(abs(a*x - 1))/a - 4*c*log(abs(x))/a + c/(a^2*x)
```

maple [A] time = 0.03, size = 33, normalized size = 1.00

$$cx+\frac{c}{a^{2}x}-\frac{4c\ln(x)}{a}+\frac{8c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2),x)
[Out] c*x+c/a^2/x-4*c*ln(x)/a+8*c/a*ln(a*x-1)
```

maxima [A] time = 0.30, size = 32, normalized size = 0.97

$$cx+\frac{8\,c\log(ax-1)}{a}-\frac{4\,c\log(x)}{a}+\frac{c}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2),x, algorithm="maxima")
[Out] c*x + 8*c*log(a*x - 1)/a - 4*c*log(x)/a + c/(a^2*x)
```

mupad [B] time = 0.07, size = 32, normalized size = 0.97

$$c\,x+\frac{c}{a^{2}\,x}-\frac{4\,c\,\ln(x)}{a}+\frac{8\,c\,\ln(a\,x-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] c*x + c/(a^2*x) - (4*c*log(x))/a + (8*c*log(a*x - 1))/a
```

sympy [A] time = 0.29, size = 26, normalized size = 0.79

$$cx+\frac{4c\left(-\log(x)+2\log\left(x-\frac{1}{a}\right)\right)}{a}+\frac{c}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2),x)
[Out] c*x + 4*c*(-log(x) + 2*log(x - 1/a))/a + c/(a**2*x)
```

3.658 $\int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

```
[Out] x/c-1/a/c/(-a*x+1)^2+5/a/c/(-a*x+1)+4*ln(-a*x+1)/a/c
```

Rubi [A] time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 77}

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{4\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c}\\
&= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c}\\
&= \frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.00

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

fricas [A] time = 1.00, size = 64, normalized size = 1.21

$$\frac{a^3x^3-2\,a^2x^2-4\,ax+4\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+4}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

giac [A] time = 0.17, size = 42, normalized size = 0.79

$$\frac{x}{c}+\frac{4\,\log\left(|ax-1|\right)}{ac}-\frac{5\,ax-4}{(ax-1)^2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] x/c + 4*log(abs(a*x - 1))/(a*c) - (5*a*x - 4)/((a*x - 1)^2*a*c)
```

maple [A] time = 0.03, size = 51, normalized size = 0.96

$$\frac{x}{c}-\frac{5}{ca\left(ax-1\right)}+\frac{4\ln\left(ax-1\right)}{ac}-\frac{1}{ac\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2),x)
```

```
[Out] x/c-5/c/a/(a*x-1)+4/a/c*ln(a*x-1)-1/a/c/(a*x-1)^2
```

maxima [A] time = 0.31, size = 49, normalized size = 0.92

$$-\frac{5\,ax-4}{a^{3}cx^{2}-2\,a^{2}cx+ac}+\frac{x}{c}+\frac{4\,\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -(5*a*x - 4)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

mupad [B] time = 0.07, size = 48, normalized size = 0.91

$$\frac{x}{c}-\frac{5\,x-\frac{4}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - c/(a^2*x^2))*(a^2*x^2 - 1)^2),x)
```

```
[Out] x/c - (5*x - 4/a)/(c + a^2*c*x^2 - 2*a*c*x) + (4*log(a*x - 1))/(a*c)
```

sympy [A] time = 0.24, size = 41, normalized size = 0.77

$$\frac{-5ax+4}{a^{3}cx^{2}-2a^{2}cx+ac}+\frac{x}{c}+\frac{4\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a**2/x**2),x)
```

```
[Out] (-5*a*x + 4)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

3.659 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2+1/3/a/c^2/(-a*x+1)^3-2/a/c^2/(-a*x+1)^2+6/a/c^2/(-a*x+1)+4*ln(-a*x+1)
/a/c^2
```

Rubi [A] time = 0.14, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 43}

$$\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 + 1/(3*a*c^2*(1 - a*x)^3) - 2/(a*c^2*(1 - a*x)^2) + 6/(a*c^2*(1 - a*x
)) + (4*Log[1 - a*x])/(a*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}$$

$$= \frac{a^4\int \frac{x^4}{(1-ax)^4}\,dx}{c^2}$$

$$= \frac{a^4\int \left(\frac{1}{a^4}+\frac{1}{a^4(-1+ax)^4}+\frac{4}{a^4(-1+ax)^3}+\frac{6}{a^4(-1+ax)^2}+\frac{4}{a^4(-1+ax)}\right)dx}{c^2}$$

$$= \frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.89

$$\frac{3a^4x^4-9a^3x^3-9a^2x^2+27ax+12(ax-1)^3\log(1-ax)-13}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-13 + 27*a*x - 9*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 12*(-1 + a*x)^3*Log[1 -
 a*x])/(3*a*c^2*(-1 + a*x)^3)
```

fricas [A] time = 0.55, size = 100, normalized size = 1.41

$$\frac{3\,a^4x^4-9\,a^3x^3-9\,a^2x^2+27\,ax+12\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-13}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 9*a^2*x^2 + 27*a*x + 12*(a^3*x^3 - 3*a^2*x^2 +
 3*a*x - 1)*log(a*x - 1) - 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x -
 a*c^2)
```

giac [A] time = 0.17, size = 50, normalized size = 0.70

$$\frac{x}{c^2}+\frac{4\,\log\left(|ax-1|\right)}{ac^2}-\frac{18\,a^2x^2-30\,ax+13}{3\,(ax-1)^3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 + 4*log(abs(a*x - 1))/(a*c^2) - 1/3*(18*a^2*x^2 - 30*a*x + 13)/((a*x
- 1)^3*a*c^2)
```

maple [A] time = 0.03, size = 66, normalized size = 0.93

$$\frac{x}{c^2}-\frac{6}{a\,c^2\,(ax-1)}-\frac{1}{3a\,c^2\,(ax-1)^3}+\frac{4\ln(ax-1)}{a\,c^2}-\frac{2}{a\,c^2\,(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^2,x)
```

```
[Out] x/c^2-6/a/c^2/(a*x-1)-1/3/a/c^2/(a*x-1)^3+4/a/c^2*ln(a*x-1)-2/a/c^2/(a*x-1)
^2
```

maxima [A] time = 0.31, size = 75, normalized size = 1.06

$$-\frac{18\,a^2x^2-30\,ax+13}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}+\frac{x}{c^2}+\frac{4\,\log(ax-1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*x^2 - 30*a*x + 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 4*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.07, size = 71, normalized size = 1.00

$$\frac{6\,a\,x^2-10\,x+\frac{13}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{4\,\ln(a\,x-1)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - c/(a^2*x^2))^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] (6*a*x^2 - 10*x + 13/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x)
 + x/c^2 + (4*log(a*x - 1))/(a*c^2)
```

sympy [A] time = 0.33, size = 83, normalized size = 1.17

$$a^4\left(\frac{-18a^2x^2+30ax-13}{3a^8c^2x^3-9a^7c^2x^2+9a^6c^2x-3a^5c^2}+\frac{x}{a^4c^2}+\frac{4\log(ax-1)}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-18*a**2*x**2 + 30*a*x - 13)/(3*a**8*c**2*x**3 - 9*a**7*c**2*x**2 +
9*a**6*c**2*x - 3*a**5*c**2) + x/(a**4*c**2) + 4*log(a*x - 1)/(a**5*c**2))
```

3.660 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3} dx$

Optimal. Leaf size=111

$$\frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3-1/8/a/c^3/(-a*x+1)^4+11/12/a/c^3/(-a*x+1)^3-49/16/a/c^3/(-a*x+1)^2+11
1/16/a/c^3/(-a*x+1)+129/32*ln(-a*x+1)/a/c^3-1/32*ln(a*x+1)/a/c^3
```

Rubi [A] time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 - 1/(8*a*c^3*(1 - a*x)^4) + 11/(12*a*c^3*(1 - a*x)^3) - 49/(16*a*c^3*
(1 - a*x)^2) + 111/(16*a*c^3*(1 - a*x)) + (129*Log[1 - a*x])/(32*a*c^3) - L
og[1 + a*x]/(32*a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
```

```
/; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{4\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}$$
$$= -\frac{a^6\int \frac{x^6}{(1-ax)^5(1+ax)}\,dx}{c^3}$$
$$= -\frac{a^6\int \left(-\frac{1}{a^6}-\frac{1}{2a^6(-1+ax)^5}-\frac{11}{4a^6(-1+ax)^4}-\frac{49}{8a^6(-1+ax)^3}-\frac{111}{16a^6(-1+ax)^2}-\frac{129}{32a^6(-1+ax)}+\frac{1}{32a^6(1+ax)}\right)dx}{c^3}$$
$$= \frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2}+\frac{111}{16ac^3(1-ax)}+\frac{129\log(1-ax)}{32ac^3}-$$

Mathematica [A] time = 0.06, size = 89, normalized size = 0.80

$$\frac{2\left(48a^5x^5-192a^4x^4-45a^3x^3+660a^2x^2-701ax+224\right)+387(ax-1)^4\log(1-ax)-3(ax-1)^4\log(ax+1)}{96ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (2*(224 - 701*a*x + 660*a^2*x^2 - 45*a^3*x^3 - 192*a^4*x^4 + 48*a^5*x^5) +
387*(-1 + a*x)^4*Log[1 - a*x] - 3*(-1 + a*x)^4*Log[1 + a*x])/(96*a*c^3*(-1
+ a*x)^4)
```

fricas [A] time = 0.50, size = 163, normalized size = 1.47

$$\frac{96\,a^5x^5-384\,a^4x^4-90\,a^3x^3+1320\,a^2x^2-1402\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log(ax+1)+387\left(a^4\right.}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(96*a^5*x^5 - 384*a^4*x^4 - 90*a^3*x^3 + 1320*a^2*x^2 - 1402*a*x - 3*(
a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x + 1) + 387*(a^4*x^4 -
4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 448)/(a^5*c^3*x^4 - 4*a^4
*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.16, size = 73, normalized size = 0.66

$$\frac{x}{c^3} - \frac{\log\left(|ax+1|\right)}{32\,ac^3} + \frac{129\,\log\left(|ax-1|\right)}{32\,ac^3} - \frac{333\,a^3x^3 - 852\,a^2x^2 + 749\,ax - 224}{48\,(ax-1)^4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/32*log(abs(a*x + 1))/(a*c^3) + 129/32*log(abs(a*x - 1))/(a*c^3) -
 1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/((a*x - 1)^4*a*c^3)
```

maple [A] time = 0.03, size = 95, normalized size = 0.86

$$\frac{x}{c^3} - \frac{1}{8a\,c^3\,(ax-1)^4} - \frac{11}{12a\,c^3\,(ax-1)^3} - \frac{49}{16a\,c^3\,(ax-1)^2} - \frac{111}{16a\,c^3\,(ax-1)} + \frac{129\ln(ax-1)}{32a\,c^3} - \frac{\ln(ax+1)}{32a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^3,x)
```

```
[Out] x/c^3-1/8/a/c^3/(a*x-1)^4-11/12/a/c^3/(a*x-1)^3-49/16/a/c^3/(a*x-1)^2-111/1
6/a/c^3/(a*x-1)+129/32/a/c^3*ln(a*x-1)-1/32*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.31, size = 107, normalized size = 0.96

$$-\frac{333\,a^3x^3 - 852\,a^2x^2 + 749\,ax - 224}{48\left(a^5c^3x^4 - 4\,a^4c^3x^3 + 6\,a^3c^3x^2 - 4\,a^2c^3x + ac^3\right)} + \frac{x}{c^3} - \frac{\log(ax+1)}{32\,ac^3} + \frac{129\,\log(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 - 1/32*log(a*x + 1)/(a*c
^3) + 129/32*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.91, size = 104, normalized size = 0.94

$$\frac{x}{c^3} - \frac{\frac{749\,x}{48} - \frac{71\,a\,x^2}{4} - \frac{14}{3\,a} + \frac{111\,a^2\,x^3}{16}}{a^4\,c^3\,x^4 - 4\,a^3\,c^3\,x^3 + 6\,a^2\,c^3\,x^2 - 4\,a\,c^3\,x + c^3} + \frac{129\,\ln(a\,x-1)}{32\,a\,c^3} - \frac{\ln(a\,x+1)}{32\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - c/(a^2*x^2))^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] x/c^3 - ((749*x)/48 - (71*a*x^2)/4 - 14/(3*a) + (111*a^2*x^3)/16)/(c^3 + 6*
a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (129*log(a*x - 1))
/(32*a*c^3) - log(a*x + 1)/(32*a*c^3)
```

sympy [A] time = 0.63, size = 114, normalized size = 1.03

$$a^6\left(\frac{-333a^3x^3+852a^2x^2-749ax+224}{48a^{11}c^3x^4-192a^{10}c^3x^3+288a^9c^3x^2-192a^8c^3x+48a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{129\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*((-333*a**3*x**3 + 852*a**2*x**2 - 749*a*x + 224)/(48*a**11*c**3*x**4
- 192*a**10*c**3*x**3 + 288*a**9*c**3*x**2 - 192*a**8*c**3*x + 48*a**7*c**3
) + x/(a**6*c**3) + (129*log(x - 1/a)/32 - log(x + 1/a)/32)/(a**7*c**3))
```

3.661 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=146

$$\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}-\frac{67}{16ac^4(1-ax)^2}+\frac{83}{48ac^4(1-ax)^3}-\frac{7}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{261\log(1-a}{64ac^4}$$

```
[Out] x/c^4+1/20/a/c^4/(-a*x+1)^5-7/16/a/c^4/(-a*x+1)^4+83/48/a/c^4/(-a*x+1)^3-67
/16/a/c^4/(-a*x+1)^2+501/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+261/64*ln(-a*
x+1)/a/c^4-5/64*ln(a*x+1)/a/c^4
```

Rubi [A] time = 0.19, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}-\frac{67}{16ac^4(1-ax)^2}+\frac{83}{48ac^4(1-ax)^3}-\frac{7}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{261\log(1-a}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] x/c^4 + 1/(20*a*c^4*(1 - a*x)^5) - 7/(16*a*c^4*(1 - a*x)^4) + 83/(48*a*c^4*
(1 - a*x)^3) - 67/(16*a*c^4*(1 - a*x)^2) + 501/(64*a*c^4*(1 - a*x)) - 1/(64
*a*c^4*(1 + a*x)) + (261*Log[1 - a*x])/(64*a*c^4) - (5*Log[1 + a*x])/(64*a*
c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{4\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$

$$= \frac{a^8\int \left(\frac{1}{a^8}+\frac{1}{4a^8(-1+ax)^6}+\frac{7}{4a^8(-1+ax)^5}+\frac{83}{16a^8(-1+ax)^4}+\frac{67}{8a^8(-1+ax)^3}+\frac{501}{64a^8(-1+ax)^2}+\frac{261}{64a^8(-1+ax)}+\frac{}{64a}\right.}{c^4}$$

$$= \frac{x}{c^4}+\frac{1}{20ac^4(1-ax)^5}-\frac{7}{16ac^4(1-ax)^4}+\frac{83}{48ac^4(1-ax)^3}-\frac{67}{16ac^4(1-ax)^2}+\frac{501}{64ac^4(1-ax)}-$$

Mathematica [A] time = 0.10, size = 98, normalized size = 0.67

$$\frac{\frac{2\left(480a^7x^7-1920a^6x^6-1365a^5x^5+9300a^4x^4-6800a^3x^3-4900a^2x^2+7541ax-2384\right)}{(ax-1)^5(ax+1)}+3915\log(1-ax)-75\log(ax+1)}{960ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((2*(-2384 + 7541*a*x - 4900*a^2*x^2 - 6800*a^3*x^3 + 9300*a^4*x^4 - 1365*a
^5*x^5 - 1920*a^6*x^6 + 480*a^7*x^7))/((-1 + a*x)^5*(1 + a*x)) + 3915*Log[1
 - a*x] - 75*Log[1 + a*x])/(960*a*c^4)
```

fricas [A] time = 0.66, size = 207, normalized size = 1.42

$$\frac{960\,a^7x^7-3840\,a^6x^6-2730\,a^5x^5+18600\,a^4x^4-13600\,a^3x^3-9800\,a^2x^2+15082\,ax-75\left(a^6x^6-4\,a^5x^5+5\,a^4x\right.}{960\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/960*(960*a^7*x^7 - 3840*a^6*x^6 - 2730*a^5*x^5 + 18600*a^4*x^4 - 13600*a^
3*x^3 - 9800*a^2*x^2 + 15082*a*x - 75*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*
a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 3915*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 -
 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) - 4768)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 +
 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.18, size = 96, normalized size = 0.66

$$\frac{x}{c^4}-\frac{5\,\log\left(|ax+1|\right)}{64\,ac^4}+\frac{261\,\log\left(|ax-1|\right)}{64\,ac^4}-\frac{3765\,a^5x^5-9300\,a^4x^4+4400\,a^3x^3+6820\,a^2x^2-8021\,ax+2384}{480\,(ax+1)(ax-1)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 5/64*log(abs(a*x + 1))/(a*c^4) + 261/64*log(abs(a*x - 1))/(a*c^4) -
 1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a*
x + 2384)/((a*x + 1)*(a*x - 1)^5*a*c^4)
```

maple [A] time = 0.05, size = 125, normalized size = 0.86

$$\frac{x}{c^4}-\frac{1}{20a\,c^4\,(ax-1)^5}-\frac{7}{16a\,c^4\,(ax-1)^4}-\frac{83}{48a\,c^4\,(ax-1)^3}-\frac{67}{16a\,c^4\,(ax-1)^2}-\frac{501}{64a\,c^4\,(ax-1)}+\frac{261\ln(ax-1)}{64a\,c^4}-\frac{}{64a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^4,x)
```

```
[Out] x/c^4-1/20/a/c^4/(a*x-1)^5-7/16/a/c^4/(a*x-1)^4-83/48/a/c^4/(a*x-1)^3-67/16
/a/c^4/(a*x-1)^2-501/64/a/c^4/(a*x-1)+261/64/a/c^4*ln(a*x-1)-1/64/a/c^4/(a*
x+1)-5/64*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.32, size = 135, normalized size = 0.92

$$-\frac{3765\,a^5x^5-9300\,a^4x^4+4400\,a^3x^3+6820\,a^2x^2-8021\,ax+2384}{480\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4\right)}+\frac{x}{c^4}-\frac{5\,\log\left(ax+1\right)}{64\,ac^4}+\frac{261\,\log\left(ax-1\right)}{64\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a*
x + 2384)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*
a^2*c^4*x - a*c^4) + x/c^4 - 5/64*log(a*x + 1)/(a*c^4) + 261/64*log(a*x - 1
)/(a*c^4)
```

mupad [B] time = 0.94, size = 131, normalized size = 0.90

$$\frac{\frac{341\,a\,x^2}{24}-\frac{8021\,x}{480}+\frac{149}{30\,a}+\frac{55\,a^2\,x^3}{6}-\frac{155\,a^3\,x^4}{8}+\frac{251\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}+\frac{x}{c^4}+\frac{261\,\ln\left(a\,x-1\right)}{64\,a\,c^4}-\frac{5\,\ln\left(a\,x+1\right)}{64\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - c/(a^2*x^2))^4*(a^2*x^2 - 1)^2),x)
```

```
[Out] ((341*a*x^2)/24 - (8021*x)/480 + 149/(30*a) + (55*a^2*x^3)/6 - (155*a^3*x^4
)/8 + (251*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 + 4*a^5*c^4*x^
5 - a^6*c^4*x^6 - 4*a*c^4*x) + x/c^4 + (261*log(a*x - 1))/(64*a*c^4) - (5*l
og(a*x + 1))/(64*a*c^4)
```

sympy [A] time = 0.90, size = 144, normalized size = 0.99

$$a^8\left(\frac{-3765a^5x^5+9300a^4x^4-4400a^3x^3-6820a^2x^2+8021ax-2384}{480a^{15}c^4x^6-1920a^{14}c^4x^5+2400a^{13}c^4x^4-2400a^{11}c^4x^2+1920a^{10}c^4x-480a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{261\log\left(x-\frac{1}{a}\right)}{64}-\frac{5\log}{}}{a^9c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-3765*a**5*x**5 + 9300*a**4*x**4 - 4400*a**3*x**3 - 6820*a**2*x**2 +
 8021*a*x - 2384)/(480*a**15*c**4*x**6 - 1920*a**14*c**4*x**5 + 2400*a**13*
c**4*x**4 - 2400*a**11*c**4*x**2 + 1920*a**10*c**4*x - 480*a**9*c**4) + x/(
a**8*c**4) + (261*log(x - 1/a)/64 - 5*log(x + 1/a)/64)/(a**9*c**4))
```

3.662 $\int e^{-\tanh^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^4 dx$

Optimal. Leaf size=169

$$\frac{c^4(35ax+16)\sqrt{1-a^2x^2}}{16a^2x} - \frac{35c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{16a} - \frac{c^4(6-7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7} + \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c^4(}{}$$

```
[Out] -1/48*c^4*(-35*a*x+16)*(-a^2*x^2+1)^(3/2)/a^4/x^3+1/120*c^4*(-35*a*x+24)*(-
a^2*x^2+1)^(5/2)/a^6/x^5-1/42*c^4*(-7*a*x+6)*(-a^2*x^2+1)^(7/2)/a^8/x^7+c^4
*arcsin(a*x)/a-35/16*c^4*arctanh((-a^2*x^2+1)^(1/2))/a+1/16*c^4*(35*a*x+16)
*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.23, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6149, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^4(6-7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7} + \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(35ax+16)\sqrt{1-a^2x^2}}{16a^2x} -$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*(16 + 35*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) - (c^4*(16 - 35*a*x)*(1 -
a^2*x^2)^(3/2))/(48*a^4*x^3) + (c^4*(24 - 35*a*x)*(1 - a^2*x^2)^(5/2))/(120
*a^6*x^5) - (c^4*(6 - 7*a*x)*(1 - a^2*x^2)^(7/2))/(42*a^8*x^7) + (c^4*ArcSi
n[a*x])/a - (35*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])/(16*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
```

```
0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$= \frac{c^4\int \frac{(1-ax)\left(1-a^2x^2\right)^{7/2}}{x^8}dx}{a^8}$$

$$= -\frac{c^4(6-7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7} - \frac{c^4\int \frac{\left(12a^2-14a^3x\right)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{12a^8}$$

$$= \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c^4(6-7ax)\left(1-a^2x^2\right)^{7/2}}{42a^8x^7} + \frac{c^4\int \frac{\left(96a^4-140a^5x\right)\left(1-a^2x\right.}{x^4}}{96a^8}$$

$$= -\frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2x^2\right)^{5/2}}{120a^6x^5} - \frac{c^4(6-7ax)\left(1-a\right.}{42a^8x^7}$$

$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2\right.}{120a^6x^5}$$

$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2\right.}{120a^6x^5}$$

$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2\right.}{120a^6x^5}$$

$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2\right.}{120a^6x^5}$$

$$= \frac{c^4(16+35ax)\sqrt{1-a^2x^2}}{16a^2x} - \frac{c^4(16-35ax)\left(1-a^2x^2\right)^{3/2}}{48a^4x^3} + \frac{c^4(24-35ax)\left(1-a^2\right.}{120a^6x^5}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.41

$$\frac{c^4\left(7a^7\left(1-a^2x^2\right)^{9/2}\,{}_2F_1\left(4,\frac{9}{2};\frac{11}{2};1-a^2x^2\right)-\frac{9\,{}_2F_1\left(-\frac{7}{2},-\frac{7}{2};-\frac{5}{2};a^2x^2\right)}{x^7}\right)}{63a^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*((-9*Hypergeometric2F1[-7/2, -7/2, -5/2, a^2*x^2])/x^7 + 7*a^7*(1 - a^
2*x^2)^(9/2)*Hypergeometric2F1[4, 9/2, 11/2, 1 - a^2*x^2]))/(63*a^8)
```

fricas [A] time = 0.84, size = 176, normalized size = 1.04

$$-\frac{3360\,a^7c^4x^7\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-3675\,a^7c^4x^7\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-1680\,a^7c^4x^7-\left(1680\,a^7c^4x^7+2816\,a^6c^4x^6+\cdots\right.}{1680\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(3360*a^7*c^4*x^7*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 3675*a^7
*c^4*x^7*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 1680*a^7*c^4*x^7 - (1680*a^7*c^4
*x^7 + 2816*a^6*c^4*x^6 + 3045*a^5*c^4*x^5 - 1952*a^4*c^4*x^4 - 1330*a^3*c^
4*x^3 + 1056*a^2*c^4*x^2 + 280*a*c^4*x - 240*c^4)*sqrt(-a^2*x^2 + 1))/(a^8*
x^7)
```

giac [B] time = 0.17, size = 504, normalized size = 2.98

$$\frac{\left(15\,c^4-\frac{35\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^4}{a^2x}-\frac{189\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^4}{a^4x^2}+\frac{525\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^4}{a^6x^3}+\frac{1295\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^4}{a^8x^4}-\frac{4935\left(\sqrt{-a^2x^2+1}\,|a|\cdots\right.}{a^{10}x^5}\cdots}{13440\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13440*(15*c^4 - 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) - 189*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2) + 525*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)^3*c^4/(a^6*x^3) + 1295*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^8*x^
4) - 4935*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^10*x^5) - 9765*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^6*c^4/(a^12*x^6))*a^14*x^7/((sqrt(-a^2*x^2 + 1)*abs(
```

```
a) + a)^7*abs(a)) + c^4*arcsin(a*x)*sgn(a)/abs(a) - 35/16*c^4*log(1/2*abs(-
2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1
)*c^4/a + 1/13440*(9765*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x + 4935*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 1295*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^3*c^4/x^3 - 525*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4) + 1
89*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^4*x^5) + 35*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^6*c^4/(a^6*x^6) - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7*c^4/(a^
8*x^7))/(a^6*abs(a))
```

maple [A] time = 0.08, size = 249, normalized size = 1.47

$$\frac{35c^4\sqrt{-a^2x^2+1}}{16a}-\frac{35c^4\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{16a}+\frac{c^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^2x}+c^4x\sqrt{-a^2x^2+1}+\frac{c^4\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{c^4\left(-a^2\right.}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 35/16*c^4*(-a^2*x^2+1)^(1/2)/a-35/16*c^4/a*arctanh(1/(-a^2*x^2+1)^(1/2))+c^
4/a^2/x*(-a^2*x^2+1)^(3/2)+c^4*x*(-a^2*x^2+1)^(1/2)+c^4/(a^2)^(1/2)*arctan(
(a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/6*c^4/a^7/x^6*(-a^2*x^2+1)^(3/2)-5/8*c^
4/a^5/x^4*(-a^2*x^2+1)^(3/2)+19/16*c^4*(-a^2*x^2+1)^(3/2)/x^2/a^3-71/105*c^
4*(-a^2*x^2+1)^(3/2)/a^4/x^3-1/7*c^4/a^8/x^7*(-a^2*x^2+1)^(3/2)+17/35*c^4/a
^6/x^5*(-a^2*x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\frac{\arcsin\left(ax\right)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}\right)-\int\frac{\left(4\,a^6c^4x^6-6\,a^4c^4x^4+4\,a^2c^4x^2-c^4\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{a^9x^9+a^8x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] c^4*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) - integrate((4*a^6*c^4*x^6 - 6*a
^4*c^4*x^4 + 4*a^2*c^4*x^2 - c^4)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^9*x^9 + a
^8*x^8), x)
```

mupad [B] time = 0.90, size = 227, normalized size = 1.34

$$\frac{c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{c^4\,\sqrt{1-a^2\,x^2}}{a}+\frac{176\,c^4\,\sqrt{1-a^2\,x^2}}{105\,a^2\,x}+\frac{29\,c^4\,\sqrt{1-a^2\,x^2}}{16\,a^3\,x^2}-\frac{122\,c^4\,\sqrt{1-a^2\,x^2}}{105\,a^4\,x^3}-\frac{19\,c^4\,\sqrt{1-a^2\,x^2}}{24\,a^5\,x^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (c^4*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) + (c^4*atan((1 - a^2*x^2)^(1/2)*1i
)*35i)/(16*a) + (c^4*(1 - a^2*x^2)^(1/2))/a + (176*c^4*(1 - a^2*x^2)^(1/2))
/(105*a^2*x) + (29*c^4*(1 - a^2*x^2)^(1/2))/(16*a^3*x^2) - (122*c^4*(1 - a^
2*x^2)^(1/2))/(105*a^4*x^3) - (19*c^4*(1 - a^2*x^2)^(1/2))/(24*a^5*x^4) + (
22*c^4*(1 - a^2*x^2)^(1/2))/(35*a^6*x^5) + (c^4*(1 - a^2*x^2)^(1/2))/(6*a^7
*x^6) - (c^4*(1 - a^2*x^2)^(1/2))/(7*a^8*x^7)
```

sympy [C] time = 16.18, size = 1110, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**4*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asi
n(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 -
 log(sqrt(-a**2*x**2 + 1) + 1), True))/a - c**4*Piecewise((-I*a**2*x/sqrt(a
**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2)
> 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1
)), True))/a**2 - 3*c**4*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(-1
+ 1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
 > 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), True))
/a**3 + 3*c**4*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/(
a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2)
)/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4 + 3*c**4*Piecewise(
(a**4*acosh(1/(a*x))/8 - a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*a/(8*x**3*
sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a*
*2*x**2) > 1), (-I*a**4*asin(1/(a*x))/8 + I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2
))) - 3*I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2
*x**2))), True))/a**5 - 3*c**4*Piecewise((2*I*a**4*sqrt(a**2*x**2 - 1)/(15*
x) + I*a**2*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(a**2*x**2 - 1)/(5*x**5),
 Abs(a**2*x**2) > 1), (2*a**4*sqrt(-a**2*x**2 + 1)/(15*x) + a**2*sqrt(-a**2
*x**2 + 1)/(15*x**3) - sqrt(-a**2*x**2 + 1)/(5*x**5), True))/a**6 - c**4*Pi
ecewise((a**6*acosh(1/(a*x))/16 - a**5/(16*x*sqrt(-1 + 1/(a**2*x**2))) + a*
*3/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*a/(24*x**5*sqrt(-1 + 1/(a**2*x**2
))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**
6*asin(1/(a*x))/16 + I*a**5/(16*x*sqrt(1 - 1/(a**2*x**2))) - I*a**3/(48*x**
3*sqrt(1 - 1/(a**2*x**2))) - 5*I*a/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + I/(6
*a*x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**7 + c**4*Piecewise((8*a**7*sqrt
(-1 + 1/(a**2*x**2))/105 + 4*a**5*sqrt(-1 + 1/(a**2*x**2))/(105*x**2) + a**
3*sqrt(-1 + 1/(a**2*x**2))/(35*x**4) - a*sqrt(-1 + 1/(a**2*x**2))/(7*x**6),
 1/Abs(a**2*x**2) > 1), (8*I*a**7*sqrt(1 - 1/(a**2*x**2))/105 + 4*I*a**5*sq
rt(1 - 1/(a**2*x**2))/(105*x**2) + I*a**3*sqrt(1 - 1/(a**2*x**2))/(35*x**4)
 - I*a*sqrt(1 - 1/(a**2*x**2))/(7*x**6), True))/a**8
```

3.663 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=136

$$\frac{c^3(15ax+8)\sqrt{1-a^2x^2}}{8a^2x}-\frac{15c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}-\frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3\sin}{}$$

```
[Out] -1/24*c^3*(-15*a*x+8)*(-a^2*x^2+1)^(3/2)/a^4/x^3+1/20*c^3*(-5*a*x+4)*(-a^2*
x^2+1)^(5/2)/a^6/x^5+c^3*arcsin(a*x)/a-15/8*c^3*arctanh((-a^2*x^2+1)^(1/2))
/a+1/8*c^3*(15*a*x+8)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.22, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6149, 811, 813, 844, 216, 266, 63, 208}

$$\frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}-\frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3}+\frac{c^3(15ax+8)\sqrt{1-a^2x^2}}{8a^2x}-\frac{15c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*(8 + 15*a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) - (c^3*(8 - 15*a*x)*(1 - a^2
*x^2)^(3/2))/(24*a^4*x^3) + (c^3*(4 - 5*a*x)*(1 - a^2*x^2)^(5/2))/(20*a^6*x
^5) + (c^3*ArcSin[a*x])/a - (15*c^3*ArcTanh[Sqrt[1 - a^2*x^2]])/(8*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$

$$= -\frac{c^3\int \frac{(1-ax)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{a^6}$$

$$= \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} + \frac{c^3\int \frac{\left(8a^2-10a^3x\right)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{8a^6}$$

$$= -\frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5} - \frac{c^3\int \frac{\left(32a^4-60a^5x\right)\sqrt{1-a^2x}}{x^2}}{32a^6}$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}$$

$$= \frac{c^3(8+15ax)\sqrt{1-a^2x^2}}{8a^2x} - \frac{c^3(8-15ax)\left(1-a^2x^2\right)^{3/2}}{24a^4x^3} + \frac{c^3(4-5ax)\left(1-a^2x^2\right)^{5/2}}{20a^6x^5}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.51

$$\frac{c^3\left(\frac{7\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};a^2x^2\right)}{x^5}-5a^5\left(1-a^2x^2\right)^{7/2}{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-a^2x^2\right)\right)}{35a^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*((7*Hypergeometric2F1[-5/2, -5/2, -3/2, a^2*x^2])/x^5 - 5*a^5*(1 - a^2
*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 - a^2*x^2]))/(35*a^6)
```

fricas [A] time = 1.15, size = 154, normalized size = 1.13

$$-\frac{240\,a^5c^3x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-225\,a^5c^3x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-120\,a^5c^3x^5-\left(120\,a^5c^3x^5+184\,a^4c^3x^4+135\,a^3}{120\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(240*a^5*c^3*x^5*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 225*a^5*c^
3*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 120*a^5*c^3*x^5 - (120*a^5*c^3*x^5
+ 184*a^4*c^3*x^4 + 135*a^3*c^3*x^3 - 88*a^2*c^3*x^2 - 30*a*c^3*x + 24*c^3)
*sqrt(-a^2*x^2 + 1))/(a^6*x^5)
```

giac [B] time = 0.22, size = 384, normalized size = 2.82

$$-\frac{\left(6\,c^3-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^3}{a^2x}-\frac{70\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^3}{a^4x^2}+\frac{240\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^3}{a^6x^3}+\frac{660\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^3}{a^8x^4}\right)a^{10}x^5}{960\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5|a|}+\frac{c^3\arcsin\left(a\right.}{|a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/960*(6*c^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x) - 70*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2) + 240*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^3/(a^6*x^3) + 660*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3/(a^8*x^4))*
a^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) + c^3*arcsin(a*x)*sgn(a
)/abs(a) - 15/8*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*ab
s(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^3/a + 1/960*(660*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)*a^2*c^3/x + 240*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2 - 70*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/(a^2*x^3) - 15*(sqrt(-a^2*x^2 + 1)*abs
(a) + a)^4*c^3/(a^4*x^4) + 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/(a^6*x^5
))/(a^4*abs(a))
```

maple [A] time = 0.05, size = 203, normalized size = 1.49

$$\frac{15c^3\sqrt{-a^2x^2+1}}{8a}-\frac{15c^3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{8a}+\frac{c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^2x}+c^3x\sqrt{-a^2x^2+1}+\frac{c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{8c^3\left(-a^2x\right.}{15a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 15/8*c^3*(-a^2*x^2+1)^(1/2)/a-15/8*c^3/a*arctanh(1/(-a^2*x^2+1)^(1/2))+c^3/
a^2/x*(-a^2*x^2+1)^(3/2)+c^3*x*(-a^2*x^2+1)^(1/2)+c^3/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-8/15*c^3/a^4/x^3*(-a^2*x^2+1)^(3/2)+7/8*c^3
*(-a^2*x^2+1)^(3/2)/x^2/a^3-1/4*c^3/a^5/x^4*(-a^2*x^2+1)^(3/2)+1/5*c^3/a^6/
x^5*(-a^2*x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^3\left(\frac{\arcsin(ax)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}\right)-\int\frac{\left(3\,a^4c^3x^4-3\,a^2c^3x^2+c^3\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{a^7x^7+a^6x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] c^3*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) - integrate((3*a^4*c^3*x^4 - 3*a
^2*c^3*x^2 + c^3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^7*x^7 + a^6*x^6), x)
```

mupad [B] time = 0.87, size = 181, normalized size = 1.33

$$\frac{c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{c^3\,\sqrt{1-a^2\,x^2}}{a}+\frac{23\,c^3\,\sqrt{1-a^2\,x^2}}{15\,a^2\,x}+\frac{9\,c^3\,\sqrt{1-a^2\,x^2}}{8\,a^3\,x^2}-\frac{11\,c^3\,\sqrt{1-a^2\,x^2}}{15\,a^4\,x^3}-\frac{c^3\,\sqrt{1-a^2\,x^2}}{4\,a^5\,x^4}+\frac{c^3\,\sqrt{1}}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (c^3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) + (c^3*atan((1 - a^2*x^2)^(1/2)*1i
)*15i)/(8*a) + (c^3*(1 - a^2*x^2)^(1/2))/a + (23*c^3*(1 - a^2*x^2)^(1/2))/(
15*a^2*x) + (9*c^3*(1 - a^2*x^2)^(1/2))/(8*a^3*x^2) - (11*c^3*(1 - a^2*x^2)
^(1/2))/(15*a^4*x^3) - (c^3*(1 - a^2*x^2)^(1/2))/(4*a^5*x^4) + (c^3*(1 - a^
2*x^2)^(1/2))/(5*a^6*x^5)
```

sympy [C] time = 10.02, size = 692, normalized size = 5.09

$$\frac{c^3\left(\begin{cases}i\sqrt{a^2x^2-1}-\log(ax)+\frac{\log\left(a^2x^2\right)}{2}+i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for } \left|a^2x^2\right|>1\\ \sqrt{-a^2x^2+1}+\frac{\log\left(a^2x^2\right)}{2}-\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise}\end{cases}\right)}{a}-\frac{c^3\left(\begin{cases}-\frac{ia^2x}{\sqrt{a^2x^2-1}}+ia\,\mathrm{acosh}(ax)+\frac{i}{x\sqrt{a^2x^2}} \\ \frac{a^2x}{\sqrt{-a^2x^2+1}}-a\,\mathrm{asin}(ax)-\frac{1}{x\sqrt{-a^2x^2+1}}\end{cases}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**3*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asi
n(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 -
 log(sqrt(-a**2*x**2 + 1) + 1), True))/a - c**3*Piecewise((-I*a**2*x/sqrt(a
**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2)
> 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1
)), True))/a**2 - 2*c**3*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(-1
+ 1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
 > 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), True))
/a**3 + 2*c**3*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/(
a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2)
)/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4 + c**3*Piecewise((a
**4*acosh(1/(a*x))/8 - a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*a/(8*x**3*sq
rt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2
*x**2) > 1), (-I*a**4*asin(1/(a*x))/8 + I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))
) - 3*I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x
**2))), True))/a**5 - c**3*Piecewise((2*I*a**4*sqrt(a**2*x**2 - 1)/(15*x) +
 I*a**2*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(a**2*x**2 - 1)/(5*x**5), Abs
(a**2*x**2) > 1), (2*a**4*sqrt(-a**2*x**2 + 1)/(15*x) + a**2*sqrt(-a**2*x**
2 + 1)/(15*x**3) - sqrt(-a**2*x**2 + 1)/(5*x**5), True))/a**6
```

3.664 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=103

$$\frac{c^2(3ax+2)\sqrt{1-a^2x^2}}{2a^2x}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2\sin^{-1}(ax)}{a}$$

```
[Out] -1/6*c^2*(-3*a*x+2)*(-a^2*x^2+1)^(3/2)/a^4/x^3+c^2*arcsin(a*x)/a-3/2*c^2*ar
ctanh((-a^2*x^2+1)^(1/2))/a+1/2*c^2*(3*a*x+2)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6149, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3}+\frac{c^2(3ax+2)\sqrt{1-a^2x^2}}{2a^2x}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}+\frac{c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^ArcTanh[a*x],x]
```

```
[Out] (c^2*(2 + 3*a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(2 - 3*a*x)*(1 - a^2*x
^2)^(3/2))/(6*a^4*x^3) + (c^2*ArcSin[a*x])/a - (3*c^2*ArcTanh[Sqrt[1 - a^2*
x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
```

```
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$

$$= \frac{c^2\int \frac{(1-ax)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{a^4}$$

$$= -\frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} - \frac{c^2\int \frac{\left(4a^2-6a^3x\right)\sqrt{1-a^2x^2}}{x^2}dx}{4a^4}$$

$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\int \frac{12a^3+8a^4x}{x\sqrt{1-a^2x^2}}dx}{8a^4}$$

$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + c^2\int \frac{1}{\sqrt{1-a^2x^2}}dx + \frac{(3c^2}{}$$

$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\sin^{-1}(ax)}{a} + \frac{\left(3c^2\right)\text{Subs}}{}$$

$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\sin^{-1}(ax)}{a} - \frac{\left(3c^2\right)\text{Subs}}{}$$

$$= \frac{c^2(2+3ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2(2-3ax)\left(1-a^2x^2\right)^{3/2}}{6a^4x^3} + \frac{c^2\sin^{-1}(ax)}{a} - \frac{3c^2\tanh^{-1}}{2}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.68

$$\frac{c^2\left(3a^3\left(1-a^2x^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-a^2x^2\right)-\frac{5\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};a^2x^2\right)}{x^3}\right)}{15a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^ArcTanh[a*x],x]
```

```
[Out] (c^2*((-5*Hypergeometric2F1[-3/2, -3/2, -1/2, a^2*x^2])/x^3 + 3*a^3*(1 - a^
2*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 - a^2*x^2]))/(15*a^4)
```

fricas [A] time = 0.55, size = 132, normalized size = 1.28

$$-\frac{12\,a^3c^2x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-9\,a^3c^2x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^2x^3-\left(6\,a^3c^2x^3+8\,a^2c^2x^2+3\,ac^2x-2\,c^2\right)\sqrt{-}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a^3*c^2*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 9*a^3*c^2*x^3
*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^2*x^3 - (6*a^3*c^2*x^3 + 8*a^2*c
^2*x^2 + 3*a*c^2*x - 2*c^2)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

giac [B] time = 0.24, size = 262, normalized size = 2.54

$$\frac{\left(c^2-\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^2}{a^2x}-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^2}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|}+\frac{c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}-\frac{3\,c^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}+\frac{\sqrt{-a^2x^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(c^2 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x) - 15*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^2/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) + c^2*arcsin(a*x)*sgn(a)/abs(a) - 3/2*c^2*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + sqrt(-a^2*x^2 + 1)*c^2/a
 + 1/24*(15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/x + 3*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2*c^2/(a^2*x^2) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a^4*x^3
))/(a^2*abs(a))
```

maple [A] time = 0.04, size = 157, normalized size = 1.52

$$\frac{3c^2\sqrt{-a^2x^2+1}}{2a}-\frac{3c^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2a}+\frac{c^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^2x}+c^2x\sqrt{-a^2x^2+1}+\frac{c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{c^2\left(-a^2x^2+\right.}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 3/2*c^2*(-a^2*x^2+1)^(1/2)/a-3/2*c^2/a*arctanh(1/(-a^2*x^2+1)^(1/2))+c^2/a^
2/x*(-a^2*x^2+1)^(3/2)+c^2*x*(-a^2*x^2+1)^(1/2)+c^2/(a^2)^(1/2)*arctan((a^2
```

```
)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/3*c^2*(-a^2*x^2+1)^(3/2)/a^4/x^3+1/2*c^2*(-
a^2*x^2+1)^(3/2)/x^2/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\frac{\arcsin\left(ax\right)}{a}+\frac{\sqrt{-a^{2}x^{2}+1}}{a}\right)-\int\frac{\left(2\,a^{2}c^{2}x^{2}-c^{2}\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{a^{5}x^{5}+a^{4}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] c^2*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) - integrate((2*a^2*c^2*x^2 - c^2
)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^5*x^5 + a^4*x^4), x)
```

mupad [B] time = 0.05, size = 135, normalized size = 1.31

$$\frac{c^{2}\,\mathrm{asinh}\left(x\,\sqrt{-a^{2}}\right)}{\sqrt{-a^{2}}}+\frac{c^{2}\,\sqrt{1-a^{2}\,x^{2}}}{a}+\frac{4\,c^{2}\,\sqrt{1-a^{2}\,x^{2}}}{3\,a^{2}\,x}+\frac{c^{2}\,\sqrt{1-a^{2}\,x^{2}}}{2\,a^{3}\,x^{2}}-\frac{c^{2}\,\sqrt{1-a^{2}\,x^{2}}}{3\,a^{4}\,x^{3}}+\frac{c^{2}\,\mathrm{atan}\left(\sqrt{1-a^{2}\,x^{2}}\,1\mathrm{i}\right)\,3\mathrm{i}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (c^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) + (c^2*atan((1 - a^2*x^2)^(1/2)*1i
)*3i)/(2*a) + (c^2*(1 - a^2*x^2)^(1/2))/a + (4*c^2*(1 - a^2*x^2)^(1/2))/(3*
a^2*x) + (c^2*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2) - (c^2*(1 - a^2*x^2)^(1/2))/
(3*a^4*x^3)
```

sympy [C] time = 6.34, size = 381, normalized size = 3.70

$$\frac{c^{2}\left(\begin{cases}i\sqrt{a^{2}x^{2}-1}-\log\left(ax\right)+\frac{\log\left(a^{2}x^{2}\right)}{2}+i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for}\ \left|a^{2}x^{2}\right|>1\\ \sqrt{-a^{2}x^{2}+1}+\frac{\log\left(a^{2}x^{2}\right)}{2}-\log\left(\sqrt{-a^{2}x^{2}+1}+1\right) & \text{otherwise}\end{cases}\right)}{a}-\frac{c^{2}\left(\begin{cases}-\frac{ia^{2}x}{\sqrt{a^{2}x^{2}-1}}+ia\,\mathrm{acosh}\left(ax\right)+\frac{i}{x\sqrt{a^{2}x^{2}\ldots}} \\ \frac{a^{2}x}{\sqrt{-a^{2}x^{2}+1}}-a\,\mathrm{asin}\left(ax\right)-\frac{1}{x\sqrt{-a^{2}x^{2}+\ldots}}\end{cases}\right.}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c**2*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asi
n(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 -
```

```
 log(sqrt(-a**2*x**2 + 1) + 1), True))/a - c**2*Piecewise((-I*a**2*x/sqrt(a
**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2)
> 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1
)), True))/a**2 - c**2*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(-1 +
1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) >
 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
**3 + c**2*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/(a**2
*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2))/3
- I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4
```

3.665 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=58

$$\frac{c\sqrt{1-a^2x^2}\,(ax+1)}{a^2x}-\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{c\sin^{-1}(ax)}{a}$$

```
[Out] c*arcsin(a*x)/a-c*arctanh((-a^2*x^2+1)^(1/2))/a+c*(a*x+1)*(-a^2*x^2+1)^(1/2
)/a^2/x
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6157, 6149, 813, 844, 216, 266, 63, 208}

$$\frac{c\sqrt{1-a^2x^2}\,(ax+1)}{a^2x}-\frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}+\frac{c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^ArcTanh[a*x],x]
```

```
[Out] (c*(1 + a*x)*Sqrt[1 - a^2*x^2])/(a^2*x) + (c*ArcSin[a*x])/a - (c*ArcTanh[Sq
rt[1 - a^2*x^2]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\frac{c\int \frac{e^{-\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$
$$= -\frac{c\int \frac{(1-ax)\sqrt{1-a^2x^2}}{x^2}dx}{a^2}$$
$$= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x} + \frac{c\int \frac{2a+2a^2x}{x\sqrt{1-a^2x^2}}dx}{2a^2}$$
$$= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x} + c\int \frac{1}{\sqrt{1-a^2x^2}}dx + \frac{c\int \frac{1}{x\sqrt{1-a^2x^2}}dx}{a}$$
$$= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x} + \frac{c\sin^{-1}(ax)}{a} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}dx,x,x^2\right)}{2a}$$
$$= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x} + \frac{c\sin^{-1}(ax)}{a} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}dx,x,\sqrt{1-a^2x^2}\right)}{a^3}$$
$$= \frac{c(1+ax)\sqrt{1-a^2x^2}}{a^2x} + \frac{c\sin^{-1}(ax)}{a} - \frac{c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{c\left(\sqrt{1-a^2x^2}\,(ax+1)-ax\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ax\sin^{-1}(ax)\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))/E^ArcTanh[a*x],x]
[Out] (c*((1 + a*x)*Sqrt[1 - a^2*x^2] + a*x*ArcSin[a*x] - a*x*ArcTanh[Sqrt[1 - a^
2*x^2]]))/(a^2*x)
```

fricas [A] time = 0.46, size = 85, normalized size = 1.47

$$-\frac{2\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-acx-\sqrt{-a^2x^2+1}\,(acx+c)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - a*c*x*log((sqrt(-a^2*x^2
 + 1) - 1)/x) - a*c*x - sqrt(-a^2*x^2 + 1)*(a*c*x + c))/(a^2*x)
```

giac [B] time = 0.43, size = 128, normalized size = 2.21

$$-\frac{a^2cx}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}+\frac{c\arcsin(ax)\operatorname{sgn}(a)}{|a|}-\frac{c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{|a|}+\frac{\sqrt{-a^2x^2+1}\,c}{a}+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{2a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a^2*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + c*arcsin(a*x)*sgn(a
)/abs(a) - c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/
abs(a) + sqrt(-a^2*x^2 + 1)*c/a + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c/(a^
2*x*abs(a))
```

maple [A] time = 0.04, size = 100, normalized size = 1.72

$$\frac{c\sqrt{-a^2x^2+1}}{a}-\frac{c\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}+\frac{c\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a^2x}+cx\sqrt{-a^2x^2+1}+\frac{c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] c*(-a^2*x^2+1)^(1/2)/a-c/a*arctanh(1/(-a^2*x^2+1)^(1/2))+c/a^2/x*(-a^2*x^2+
1)^(3/2)+c*x*(-a^2*x^2+1)^(1/2)+c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\frac{\arcsin(ax)}{a}+\frac{\sqrt{-a^2x^2+1}}{a}\right)-c\int\frac{\sqrt{ax+1}\,\sqrt{-ax+1}}{a^3x^3+a^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] c*(arcsin(a*x)/a + sqrt(-a^2*x^2 + 1)/a) - c*integrate(sqrt(a*x + 1)*sqrt(-
a*x + 1)/(a^3*x^3 + a^2*x^2), x)
```

mupad [B] time = 0.04, size = 76, normalized size = 1.31

$$\frac{c\sqrt{1-a^2\,x^2}}{a}-\frac{c\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{a}+\frac{c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{c\sqrt{1-a^2\,x^2}}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (c*(1 - a^2*x^2)^(1/2))/a - (c*atanh((1 - a^2*x^2)^(1/2)))/a + (c*asinh(x*(
-a^2)^(1/2)))/(-a^2)^(1/2) + (c*(1 - a^2*x^2)^(1/2))/(a^2*x)
```

sympy [C] time = 4.91, size = 177, normalized size = 3.05

$$\frac{c\left(\begin{cases} i\sqrt{a^2x^2-1}-\log(ax)+\frac{\log(a^2x^2)}{2}+i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for } \left|a^2x^2\right|>1 \\ \sqrt{-a^2x^2+1}+\frac{\log(a^2x^2)}{2}-\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right)}{a}-\frac{c\left(\begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}}+ia\,\mathrm{acosh}(ax)+\frac{i}{x\sqrt{a^2x^2-1}} \\ \frac{a^2x}{\sqrt{-a^2x^2+1}}-a\,\mathrm{asin}(ax)-\frac{1}{x\sqrt{-a^2x^2+1}} \end{cases}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] c*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1
/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - lo
g(sqrt(-a**2*x**2 + 1) + 1), True))/a - c*Piecewise((-I*a**2*x/sqrt(a**2*x*
*2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1),
(a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1)), Tr
ue))/a**2
```

3.666 $\int \frac{e^{-\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}} dx$

Optimal. Leaf size=60

$$\frac{1-ax}{ac\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}$$

```
[Out] arcsin(a*x)/a/c+(-a*x+1)/a/c/(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1)^(1/2)/a/c
```

Rubi [A] time = 0.14, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6149, 797, 641, 216, 637}

$$\frac{1-ax}{ac\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] (1 - a*x)/(a*c*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a*c) + ArcSin[a*x]/(
a*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
```

```
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{-\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= -\frac{a^2\int \frac{x^2(1-ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= -\frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}+\frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{1-ax}{ac\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{1-ax}{ac\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{ac}+\frac{\sin^{-1}(ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.90

$$\frac{-a^2x^2+\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-ax+2}{ac\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))),x]
```

[Out] (2 - a*x - a^2*x^2 + Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(a*c*Sqrt[1 - a^2*x^2])

fricas [A] time = 0.42, size = 66, normalized size = 1.10

$$\frac{2\,ax - 2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(ax+2) + 2}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")

[Out] (2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*(a*x + 2) + 2)/(a^2*c*x + a*c)

giac [A] time = 0.20, size = 71, normalized size = 1.18

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{c|a|} + \frac{\sqrt{-a^2x^2+1}}{ac} - \frac{2}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")

[Out] arcsin(a*x)*sgn(a)/(c*abs(a)) + sqrt(-a^2*x^2 + 1)/(a*c) - 2/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))

maple [B] time = 0.04, size = 192, normalized size = 3.20

$$\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2 - 2a\left(x-\frac{1}{a}\right)}}{4ac} - \frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{4c\sqrt{a^2}} + \frac{\left(-a^2\left(x+\frac{1}{a}\right)^2 + 2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{2a^3c\left(x+\frac{1}{a}\right)^2} + \frac{5\sqrt{-a^2\left(x+\frac{1}{a}\right)^2 + 2}}{4ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x)

[Out] 1/4/a/c*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/4/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/2/a^3/c/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+5/4/a/c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+5/4/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))), x)
```

mupad [B] time = 0.06, size = 88, normalized size = 1.47

$$\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{c\sqrt{-a^2}}+\frac{\sqrt{1-a^2x^2}}{a\,c}-\frac{\sqrt{1-a^2x^2}}{c\left(x\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))*(a*x + 1)),x)
[Out] asinh(x*(-a^2)^(1/2))/(c*(-a^2)^(1/2)) + (1 - a^2*x^2)^(1/2)/(a*c) - (1 - a
^2*x^2)^(1/2)/(c*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2\sqrt{-a^2x^2+1}}{a^3x^3+a^2x^2-ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2),x)
[Out] a**2*Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + a**2*x**2 - a*x - 1),
x)/c
```

3.667 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{\sin^{-1}(ax)}{ac^2}$$

```
[Out] 1/3*a^2*x^3*(-a*x+1)/c^2/(-a^2*x^2+1)^(3/2)+arcsin(a*x)/a/c^2-1/3*x*(-4*a*x
+3)/c^2/(-a^2*x^2+1)^(1/2)+8/3*(-a^2*x^2+1)^(1/2)/a/c^2
```

Rubi [A] time = 0.16, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6149, 819, 641, 216}

$$\frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (a^2*x^3*(1 - a*x))/(3*c^2*(1 - a^2*x^2)^(3/2)) - (x*(3 - 4*a*x))/(3*c^2*Sq
rt[1 - a^2*x^2]) + (8*Sqrt[1 - a^2*x^2])/(3*a*c^2) + ArcSin[a*x]/(a*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
```

```
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= \frac{a^4\int \frac{e^{-\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}\\
&= \frac{a^4\int \frac{x^4(1-ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2\int \frac{x^2(3-4ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^2}\\
&= \frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{\int \frac{3-8ax}{\sqrt{1-a^2x^2}}\,dx}{3c^2}\\
&= \frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= \frac{a^2x^3(1-ax)}{3c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(3-4ax)}{3c^2\sqrt{1-a^2x^2}}+\frac{8\sqrt{1-a^2x^2}}{3ac^2}+\frac{\sin^{-1}(ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.80

$$\frac{-3a^3x^3-7a^2x^2+3(ax+1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+5ax+8}{3ac^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (8 + 5*a*x - 7*a^2*x^2 - 3*a^3*x^3 + 3*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a
*x])/(3*a*c^2*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.57, size = 141, normalized size = 1.45

$$\frac{8\,a^3x^3+8\,a^2x^2-8\,ax-6\left(a^3x^3+a^2x^2-ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^3x^3+7\,a^2x^2-5\,ax-8\right)\sqrt{-a^2x^2+1}}{3\left(a^4c^2x^3+a^3c^2x^2-a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas
")
```

```
[Out] 1/3*(8*a^3*x^3 + 8*a^2*x^2 - 8*a*x - 6*(a^3*x^3 + a^2*x^2 - a*x - 1)*arctan
((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^3*x^3 + 7*a^2*x^2 - 5*a*x - 8)*sqrt
(-a^2*x^2 + 1) - 8)/(a^4*c^2*x^3 + a^3*c^2*x^2 - a^2*c^2*x - a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^2), x)
```

maple [B] time = 0.05, size = 274, normalized size = 2.82

$$\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{8a^3c^2\left(x-\frac{1}{a}\right)^2}+\frac{7\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{16a\,c^2}-\frac{7\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{16c^2\sqrt{a^2}}+\frac{3\left(-a^2\left(x+\frac{1}{a}\right)^2+\right.}{4a^3c^2\left(x-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] 1/8/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+7/16/a/c^2*(-a^2*(
x-1/a)^2-2*a*(x-1/a))^(1/2)-7/16/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2))+3/4/a^3/c^2/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x
+1/a))^(3/2)+23/16/a/c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+23/16/c^2/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-1/12/a^4/c^
2/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^2), x)
```

mupad [B] time = 1.16, size = 188, normalized size = 1.94

$$\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^2\,\sqrt{-a^2}}-\frac{a\,\sqrt{1-a^2\,x^2}}{6\,\left(a^4\,c^2\,x^2+2\,a^3\,c^2\,x+a^2\,c^2\right)}+\frac{\sqrt{1-a^2\,x^2}}{a\,c^2}-\frac{19\,\sqrt{1-a^2\,x^2}}{12\,\sqrt{-a^2}\,\left(c^2\,x\,\sqrt{-a^2}+\frac{c^2\,\sqrt{-a^2}}{a}\right)}+\frac{\sqrt{1-}}{4\,\sqrt{-a^2}\,\left(c^2\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^2*(a*x + 1)),x)
```

```
[Out] asinh(x*(-a^2)^(1/2))/(c^2*(-a^2)^(1/2)) - (a*(1 - a^2*x^2)^(1/2))/(6*(a^2*
c^2 + 2*a^3*c^2*x + a^4*c^2*x^2)) + (1 - a^2*x^2)^(1/2)/(a*c^2) - (19*(1 -
a^2*x^2)^(1/2))/(12*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) + (c^2*(-a^2)^(1/2))/a
)) + (1 - a^2*x^2)^(1/2)/(4*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^
(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int \frac{x^4\sqrt{-a^2x^2+1}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4*sqrt(-a**2*x**2 + 1)/(a**5*x**5 + a**4*x**4 - 2*a**3*x**
3 - 2*a**2*x**2 + a*x + 1), x)/c**2
```

3.668 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3} dx$

Optimal. Leaf size=130

$$-\frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}}+\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{\sin^{-1}(ax)}{ac^3}$$

```
[Out] -1/5*a^4*x^5*(-a*x+1)/c^3/(-a^2*x^2+1)^(5/2)+1/15*a^2*x^3*(-6*a*x+5)/c^3/(-
a^2*x^2+1)^(3/2)+arcsin(a*x)/a/c^3-1/5*x*(-8*a*x+5)/c^3/(-a^2*x^2+1)^(1/2)+
16/5*(-a^2*x^2+1)^(1/2)/a/c^3
```

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6149, 819, 641, 216}

$$-\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}}+\frac{16\sqrt{1-a^2x^2}}{5ac^3}+\frac{\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -(a^4*x^5*(1 - a*x))/(5*c^3*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(5 - 6*a*x))/(1
5*c^3*(1 - a^2*x^2)^(3/2)) - (x*(5 - 8*a*x))/(5*c^3*Sqrt[1 - a^2*x^2]) + (1
6*Sqrt[1 - a^2*x^2])/(5*a*c^3) + ArcSin[a*x]/(a*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
```

```
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{-\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}$$

$$= -\frac{a^6\int \frac{x^6(1-ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^4\int \frac{x^4(5-6ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^3}$$

$$= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} - \frac{a^2\int \frac{x^2(15-24ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^3}$$

$$= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} - \frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}} + \frac{\int \frac{15-48ax}{\sqrt{1-a^2x^2}}\,dx}{15c^3}$$

$$= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} - \frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}} + \frac{16\sqrt{1-a^2x^2}}{5ac^3} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^3}$$

$$= -\frac{a^4x^5(1-ax)}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{a^2x^3(5-6ax)}{15c^3\left(1-a^2x^2\right)^{3/2}} - \frac{x(5-8ax)}{5c^3\sqrt{1-a^2x^2}} + \frac{16\sqrt{1-a^2x^2}}{5ac^3} + \frac{\sin^{-1}(ax)}{ac^3}$$

Mathematica [A] time = 0.08, size = 108, normalized size = 0.83

$$\frac{-15a^5x^5-38a^4x^4+52a^3x^3+87a^2x^2+15(ax-1)(ax+1)^2\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-33ax-48}{15ac^3(ax-1)(ax+1)^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (-48 - 33*a*x + 87*a^2*x^2 + 52*a^3*x^3 - 38*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)*(1 + a*x)^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(15*a*c^3*(-1 + a*x)*(1 +
 a*x)^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.50, size = 208, normalized size = 1.60

$$\frac{48\,a^5x^5+48\,a^4x^4-96\,a^3x^3-96\,a^2x^2+48\,ax-30\left(a^5x^5+a^4x^4-2\,a^3x^3-2\,a^2x^2+ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{15\left(a^6c^3x^5+a^5c^3x^4-2\,a^4c^3x^3-2\,a^3c^3x^2+a^2c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas
")
```

```
[Out] 1/15*(48*a^5*x^5 + 48*a^4*x^4 - 96*a^3*x^3 - 96*a^2*x^2 + 48*a*x - 30*(a^5*
x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*arctan((sqrt(-a^2*x^2 + 1)
 - 1)/(a*x)) + (15*a^5*x^5 + 38*a^4*x^4 - 52*a^3*x^3 - 87*a^2*x^2 + 33*a*x
+ 48)*sqrt(-a^2*x^2 + 1) + 48)/(a^6*c^3*x^5 + a^5*c^3*x^4 - 2*a^4*c^3*x^3 -
 2*a^3*c^3*x^2 + a^2*c^3*x + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^3), x)
```

maple [B] time = 0.06, size = 356, normalized size = 2.74

$$\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{4a^3c^3\left(x-\frac{1}{a}\right)^2}+\frac{19\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{32a\,c^3}-\frac{19\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{32c^3\sqrt{a^2}}+\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-\right.}{48a^4c^3\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/4/a^3/c^3/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+19/32/a/c^3*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(1/2)-19/32/c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/48/a^4/c^3/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a
*(x-1/a))^(3/2)+15/16/a^3/c^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+
51/32/a/c^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+51/32/c^3/(a^2)^(1/2)*arctan
```

```
((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-43/240/a^4/c^3/(x+1/a)^3
*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+1/40/a^5/c^3/(x+1/a)^4*(-a^2*(x+1/a)^2+
2*a*(x+1/a))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^3), x)
```

mupad [B] time = 1.28, size = 365, normalized size = 2.81

$$\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{c^3\sqrt{-a^2}}-\frac{5\,a\,\sqrt{1-a^2\,x^2}}{12\,\left(a^4\,c^3\,x^2+2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{a\,\sqrt{1-a^2\,x^2}}{24\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{a^6\,\sqrt{1-a^2\,x^2}}{30\,\left(a^9\,c^3\,x^2+2\,a^8\,c^3\,x+a^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^3*(a*x + 1)),x)
```

```
[Out] asinh(x*(-a^2)^(1/2))/(c^3*(-a^2)^(1/2)) - (5*a*(1 - a^2*x^2)^(1/2))/(12*(a
^2*c^3 + 2*a^3*c^3*x + a^4*c^3*x^2)) - (a*(1 - a^2*x^2)^(1/2))/(24*(a^2*c^3
 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (a^6*(1 - a^2*x^2)^(1/2))/(30*(a^7*c^3 + 2
*a^8*c^3*x + a^9*c^3*x^2)) + (1 - a^2*x^2)^(1/2)/(a*c^3) - (493*(1 - a^2*x^
2)^(1/2))/(240*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) + (c^3*(-a^2)^(1/2))/a)) +
(25*(1 - a^2*x^2)^(1/2))/(48*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)
^(1/2))/a)) - (1 - a^2*x^2)^(1/2)/(20*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2) +
(c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) + 3*a*c^3*x^2*(-a^2)^(1/2))
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6\int \frac{x^6\sqrt{-a^2x^2+1}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6*sqrt(-a**2*x**2 + 1)/(a**7*x**7 + a**6*x**6 - 3*a**5*x**
5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x)/c**3
```

3.669 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx$

Optimal. Leaf size=163

$$-\frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}}+\frac{128\sqrt{1-a^2x^2}}{35ac^4}+\frac{a^2x^3(35-48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}}+\frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{a^4x^5(7-8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{\sin^{-1}(ax)}{ac^4}$$

```
[Out] 1/7*a^6*x^7*(-a*x+1)/c^4/(-a^2*x^2+1)^(7/2)-1/35*a^4*x^5*(-8*a*x+7)/c^4/(-a
^2*x^2+1)^(5/2)+1/105*a^2*x^3*(-48*a*x+35)/c^4/(-a^2*x^2+1)^(3/2)+arcsin(a*
x)/a/c^4-1/35*x*(-64*a*x+35)/c^4/(-a^2*x^2+1)^(1/2)+128/35*(-a^2*x^2+1)^(1/
2)/a/c^4
```

Rubi [A] time = 0.24, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6149, 819, 641, 216}

$$\frac{a^6x^7(1-ax)}{7c^4\left(1-a^2x^2\right)^{7/2}}-\frac{a^4x^5(7-8ax)}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3(35-48ax)}{105c^4\left(1-a^2x^2\right)^{3/2}}-\frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}}+\frac{128\sqrt{1-a^2x^2}}{35ac^4}+\frac{\sin^{-1}(ax)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (a^6*x^7*(1 - a*x))/(7*c^4*(1 - a^2*x^2)^(7/2)) - (a^4*x^5*(7 - 8*a*x))/(35
*c^4*(1 - a^2*x^2)^(5/2)) + (a^2*x^3*(35 - 48*a*x))/(105*c^4*(1 - a^2*x^2)^
(3/2)) - (x*(35 - 64*a*x))/(35*c^4*Sqrt[1 - a^2*x^2]) + (128*Sqrt[1 - a^2*x
^2])/(35*a*c^4) + ArcSin[a*x]/(a*c^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{-\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8(1-ax)}{(1-a^2x^2)^{9/2}}\,dx}{c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4(1-a^2x^2)^{7/2}} - \frac{a^6\int \frac{x^6(7-8ax)}{(1-a^2x^2)^{7/2}}\,dx}{7c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4(1-a^2x^2)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4(1-a^2x^2)^{5/2}} + \frac{a^4\int \frac{x^4(35-48ax)}{(1-a^2x^2)^{5/2}}\,dx}{35c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4(1-a^2x^2)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4(1-a^2x^2)^{5/2}} + \frac{a^2x^3(35-48ax)}{105c^4(1-a^2x^2)^{3/2}} - \frac{a^2\int \frac{x^2(105-192ax)}{(1-a^2x^2)^{3/2}}\,dx}{105c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4(1-a^2x^2)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4(1-a^2x^2)^{5/2}} + \frac{a^2x^3(35-48ax)}{105c^4(1-a^2x^2)^{3/2}} - \frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}} + \frac{\int \frac{105-384ax}{\sqrt{1-a^2x^2}}\,d}{105c^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4(1-a^2x^2)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4(1-a^2x^2)^{5/2}} + \frac{a^2x^3(35-48ax)}{105c^4(1-a^2x^2)^{3/2}} - \frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}} + \frac{128\sqrt{1-a^2x}}{35ac^4}$$

$$= \frac{a^6x^7(1-ax)}{7c^4(1-a^2x^2)^{7/2}} - \frac{a^4x^5(7-8ax)}{35c^4(1-a^2x^2)^{5/2}} + \frac{a^2x^3(35-48ax)}{105c^4(1-a^2x^2)^{3/2}} - \frac{x(35-64ax)}{35c^4\sqrt{1-a^2x^2}} + \frac{128\sqrt{1-a^2x}}{35ac^4}$$

Mathematica [A] time = 0.10, size = 126, normalized size = 0.77

$$\frac{-105a^7x^7-281a^6x^6+559a^5x^5+965a^4x^4-715a^3x^3-1065a^2x^2+105(ax-1)^2(ax+1)^3\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+2}{105ac^4(ax-1)^2(ax+1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (384 + 279*a*x - 1065*a^2*x^2 - 715*a^3*x^3 + 965*a^4*x^4 + 559*a^5*x^5 - 2
81*a^6*x^6 - 105*a^7*x^7 + 105*(-1 + a*x)^2*(1 + a*x)^3*Sqrt[1 - a^2*x^2]*A
rcSin[a*x])/(105*a*c^4*(-1 + a*x)^2*(1 + a*x)^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 1.09, size = 281, normalized size = 1.72

$$\frac{384\,a^{7}x^{7}+384\,a^{6}x^{6}-1152\,a^{5}x^{5}-1152\,a^{4}x^{4}+1152\,a^{3}x^{3}+1152\,a^{2}x^{2}-384\,ax-210\left(a^{7}x^{7}+a^{6}x^{6}-3\,a^{5}x^{5}-3\,\cdots\right.}{105\left(a^{8}c^{4}x^{7}+a^{7}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas
")
```

```
[Out] 1/105*(384*a^7*x^7 + 384*a^6*x^6 - 1152*a^5*x^5 - 1152*a^4*x^4 + 1152*a^3*x
^3 + 1152*a^2*x^2 - 384*a*x - 210*(a^7*x^7 + a^6*x^6 - 3*a^5*x^5 - 3*a^4*x^
4 + 3*a^3*x^3 + 3*a^2*x^2 - a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x))
 + (105*a^7*x^7 + 281*a^6*x^6 - 559*a^5*x^5 - 965*a^4*x^4 + 715*a^3*x^3 + 1
065*a^2*x^2 - 279*a*x - 384)*sqrt(-a^2*x^2 + 1) - 384)/(a^8*c^4*x^7 + a^7*c
^4*x^6 - 3*a^6*c^4*x^5 - 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 + 3*a^3*c^4*x^2 - a^
2*c^4*x - a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^4), x)
```

maple [B] time = 0.06, size = 438, normalized size = 2.69

$$\frac{47\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{128a^{3}c^{4}\left(x-\frac{1}{a}\right)^{2}}+\frac{187\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{256a\,c^{4}}-\frac{187\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}\right)}{256c^{4}\sqrt{a^{2}}}+\frac{\left(-a^{2}\left(x-\cdots\right.\right.}{16\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] 47/128/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+187/256/a/c^4*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-187/256/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2
)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/160/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/
```

```
a)^2-2*a*(x-1/a))^(3/2)+53/960/a^4/c^4/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a
))^(3/2)+35/32/a^3/c^4/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+443/256
/a/c^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)+443/256/c^4/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+1/14/a^5/c^4/(x+1/a)^4*(-a^
2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-187/672/a^4/c^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*
a*(x+1/a))^(3/2)-1/112/a^6/c^4/(x+1/a)^5*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^4), x)
```

mupad [B] time = 1.54, size = 614, normalized size = 3.77

$$\frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{c^4\sqrt{-a^2}}-\frac{35\,a\,\sqrt{1-a^2\,x^2}}{48\,\left(a^4\,c^4\,x^2+2\,a^3\,c^4\,x+a^2\,c^4\right)}-\frac{a\,\sqrt{1-a^2\,x^2}}{8\,\left(a^4\,c^4\,x^2-2\,a^3\,c^4\,x+a^2\,c^4\right)}-\frac{a^3\,\sqrt{1-a^2\,x^2}}{140\,\left(a^6\,c^4\,x^2+2\,a^5\,c^4\,x+a^4\,c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^4*(a*x + 1)),x)
```

```
[Out] asinh(x*(-a^2)^(1/2))/(c^4*(-a^2)^(1/2)) - (35*a*(1 - a^2*x^2)^(1/2))/(48*(
a^2*c^4 + 2*a^3*c^4*x + a^4*c^4*x^2)) - (a*(1 - a^2*x^2)^(1/2))/(8*(a^2*c^4
 - 2*a^3*c^4*x + a^4*c^4*x^2)) - (a^3*(1 - a^2*x^2)^(1/2))/(140*(a^4*c^4 +
2*a^5*c^4*x + a^6*c^4*x^2)) + (a^8*(1 - a^2*x^2)^(1/2))/(120*(a^9*c^4 - 2*a
^10*c^4*x + a^11*c^4*x^2)) + (13*a^8*(1 - a^2*x^2)^(1/2))/(120*(a^9*c^4 + 2
*a^10*c^4*x + a^11*c^4*x^2)) + (1 - a^2*x^2)^(1/2)/(a*c^4) - (a*(1 - a^2*x^
2)^(1/2))/(56*(a^2*c^4 + 4*a^3*c^4*x + 6*a^4*c^4*x^2 + 4*a^5*c^4*x^3 + a^6*
c^4*x^4)) - (1657*(1 - a^2*x^2)^(1/2))/(672*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2
) + (c^4*(-a^2)^(1/2))/a)) + (379*(1 - a^2*x^2)^(1/2))/(480*(-a^2)^(1/2)*(c
^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a)) - (17*(1 - a^2*x^2)^(1/2))/(112*
(-a^2)^(1/2)*(3*c^4*x*(-a^2)^(1/2) + (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a
^2)^(1/2) + 3*a*c^4*x^2*(-a^2)^(1/2))) + (1 - a^2*x^2)^(1/2)/(80*(-a^2)^(1/
2)*(3*c^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1/2)
- 3*a*c^4*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^8 \int \frac{x^8\sqrt{-a^2x^2+1}}{a^9x^9+a^8x^8-4a^7x^7-4a^6x^6+6a^5x^5+6a^4x^4-4a^3x^3-4a^2x^2+ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8*sqrt(-a**2*x**2 + 1)/(a**9*x**9 + a**8*x**8 - 4*a**7*x**
7 - 4*a**6*x**6 + 6*a**5*x**5 + 6*a**4*x**4 - 4*a**3*x**3 - 4*a**2*x**2 + a
*x + 1), x)/c**4
```

3.670 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=91

$$-\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}-\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

```
[Out] -1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6+2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/a^3/
x^2-2*c^4/a^2/x-c^4*x+2*c^4*ln(x)/a
```

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{3c^4}{a^3x^2}-\frac{3c^4}{2a^5x^4}+\frac{2c^4}{5a^6x^5}+\frac{c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x
^4) + (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x + (2*c^4*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int \frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$
$$= \frac{c^4\int \frac{(1-ax)^5(1+ax)^3}{x^8}dx}{a^8}$$
$$= \frac{c^4\int \left(-a^8+\frac{1}{x^8}-\frac{2a}{x^7}-\frac{2a^2}{x^6}+\frac{6a^3}{x^5}-\frac{6a^5}{x^3}+\frac{2a^6}{x^2}+\frac{2a^7}{x}\right)dx}{a^8}$$
$$= -\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}-\frac{2c^4}{a^2x}-c^4x+\frac{2c^4\log(x)}{a}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.00

$$-\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}-\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/7*c^4/(a^8*x^7) + c^4/(3*a^7*x^6) + (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5
*x^4) + (3*c^4)/(a^3*x^2) - (2*c^4)/(a^2*x) - c^4*x + (2*c^4*Log[x])/a
```

fricas [A] time = 0.47, size = 89, normalized size = 0.98

$$-\frac{210\,a^8c^4x^8-420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/210*(210*a^8*c^4*x^8 - 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 - 630*a^
5*c^4*x^5 + 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^
7)
```

giac [A] time = 0.18, size = 160, normalized size = 1.76

$$-\frac{2\,c^4\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}+\frac{2\,c^4\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}+\frac{\left(210\,c^4-\frac{719\,c^4}{ax+1}-\frac{427\,c^4}{(ax+1)^2}+\frac{5271\,c^4}{(ax+1)^3}-\frac{9485\,c^4}{(ax+1)^4}+\frac{7490\,c^4}{(ax+1)^5}-\frac{2730\,c^4}{(ax+1)^6}+42\cdots\right.}{210\,a\left(\frac{1}{ax+1}-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*c^4*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + 2*c^4*log(abs(-1/(a*x + 1
) + 1))/a + 1/210*(210*c^4 - 719*c^4/(a*x + 1) - 427*c^4/(a*x + 1)^2 + 5271
*c^4/(a*x + 1)^3 - 9485*c^4/(a*x + 1)^4 + 7490*c^4/(a*x + 1)^5 - 2730*c^4/(
a*x + 1)^6 + 420*c^4/(a*x + 1)^7)*(a*x + 1)/(a*(1/(a*x + 1) - 1)^7)
```

maple [A] time = 0.04, size = 84, normalized size = 0.92

$$-\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}+\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{x^2a^3}-\frac{2c^4}{a^2x}-c^4x+\frac{2c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6+2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/x^2/
a^3-2*c^4/a^2/x-c^4*x+2*c^4*ln(x)/a
```

maxima [A] time = 0.31, size = 82, normalized size = 0.90

$$-c^4x+\frac{2\,c^4\log(x)}{a}-\frac{420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^4*x + 2*c^4*log(x)/a - 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 315*a^
3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

mupad [B] time = 0.08, size = 66, normalized size = 0.73

$$\frac{c^4\,\left(\frac{a\,x}{3}+\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5-2\,a^6\,x^6-a^8\,x^8+2\,a^7\,x^7\,\ln(x)-\frac{1}{7}\right)}{a^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^4*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] (c^4*((a*x)/3 + (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 - 2*a^6*x^6 - a^8
*x^8 + 2*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

sympy [A] time = 0.46, size = 88, normalized size = 0.97

$$\frac{-a^8c^4x+2a^7c^4\log{(x)}-\frac{420a^6c^4x^6-630a^5c^4x^5+315a^3c^4x^3-84a^2c^4x^2-70ac^4x+30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] (-a**8*c**4*x + 2*a**7*c**4*log(x) - (420*a**6*c**4*x**6 - 630*a**5*c**4*x*
*5 + 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**4*x + 30*c**4)/(210*x
**7))/a**8
```

3.671 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=78

$$\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}-\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}-\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3(-x)$$

```
[Out] 1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4-1/3*c^3/a^4/x^3+2*c^3/a^3/x^2-c^3/a^2/x-c^3
*x+2*c^3*ln(x)/a
```

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{2c^3}{a^3x^2}-\frac{c^3}{3a^4x^3}-\frac{c^3}{2a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c^3/(5*a^6*x^5) - c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x + (2*c^3*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx=-\frac{c^3\int\frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}\,dx}{a^6}$$
$$=-\frac{c^3\int\frac{(1-ax)^4(1+ax)^2}{x^6}\,dx}{a^6}$$
$$=-\frac{c^3\int\left(a^6+\frac{1}{x^6}-\frac{2a}{x^5}-\frac{a^2}{x^4}+\frac{4a^3}{x^3}-\frac{a^4}{x^2}-\frac{2a^5}{x}\right)dx}{a^6}$$
$$=\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}-\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}-\frac{c^3}{a^2x}-c^3x+\frac{2c^3\log(x)}{a}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 1.00

$$\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}-\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}-\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c^3/(5*a^6*x^5) - c^3/(2*a^5*x^4) - c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2) - c
^3/(a^2*x) - c^3*x + (2*c^3*Log[x])/a
```

fricas [A] time = 0.93, size = 78, normalized size = 1.00

$$-\frac{30\,a^6c^3x^6-60\,a^5c^3x^5\log(x)+30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/30*(30*a^6*c^3*x^6 - 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 - 60*a^3*c^3
*x^3 + 10*a^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

giac [A] time = 0.29, size = 136, normalized size = 1.74

$$-\frac{2\,c^3\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}+\frac{2\,c^3\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}+\frac{\left(30\,c^3-\frac{71\,c^3}{ax+1}-\frac{65\,c^3}{(ax+1)^2}+\frac{310\,c^3}{(ax+1)^3}-\frac{270\,c^3}{(ax+1)^4}+\frac{60\,c^3}{(ax+1)^5}\right)(ax+1)}{30\,a\left(\frac{1}{ax+1}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*c^3*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + 2*c^3*log(abs(-1/(a*x + 1
) + 1))/a + 1/30*(30*c^3 - 71*c^3/(a*x + 1) - 65*c^3/(a*x + 1)^2 + 310*c^3/
(a*x + 1)^3 - 270*c^3/(a*x + 1)^4 + 60*c^3/(a*x + 1)^5)*(a*x + 1)/(a*(1/(a*
x + 1) - 1)^5)
```

maple [A] time = 0.04, size = 73, normalized size = 0.94

$$\frac{c^3}{5a^6x^5} - \frac{c^3}{2a^5x^4} - \frac{c^3}{3a^4x^3} + \frac{2c^3}{x^2a^3} - \frac{c^3}{a^2x} - c^3x + \frac{2c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4-1/3*c^3/a^4/x^3+2*c^3/x^2/a^3-c^3/a^2/x-c^3
*x+2*c^3*ln(x)/a
```

maxima [A] time = 0.49, size = 71, normalized size = 0.91

$$-c^3x + \frac{2\,c^3\log(x)}{a} - \frac{30\,a^4c^3x^4 - 60\,a^3c^3x^3 + 10\,a^2c^3x^2 + 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^3*x + 2*c^3*log(x)/a - 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^2*c^
3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

mupad [B] time = 0.85, size = 57, normalized size = 0.73

$$-\frac{c^3\,\left(\frac{a\,x}{2}+\frac{a^2\,x^2}{3}-2\,a^3\,x^3+a^4\,x^4+a^6\,x^6-2\,a^5\,x^5\,\ln(x)-\frac{1}{5}\right)}{a^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^3*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -(c^3*((a*x)/2 + (a^2*x^2)/3 - 2*a^3*x^3 + a^4*x^4 + a^6*x^6 - 2*a^5*x^5*lo
g(x) - 1/5))/(a^6*x^5)
```

sympy [A] time = 0.33, size = 76, normalized size = 0.97

$$\frac{-a^6c^3x + 2a^5c^3\log{(x)} - \frac{30a^4c^3x^4 - 60a^3c^3x^3 + 10a^2c^3x^2 + 15ac^3x - 6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] (-a**6*c**3*x + 2*a**5*c**3*log(x) - (30*a**4*c**3*x**4 - 60*a**3*c**3*x**3
 + 10*a**2*c**3*x**2 + 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

3.672 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$

Optimal. Leaf size=40

$$-\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+\frac{2c^2\log(x)}{a}+c^2(-x)$$

```
[Out] -1/3*c^2/a^4/x^3+c^2/a^3/x^2-c^2*x+2*c^2*ln(x)/a
```

Rubi [A] time = 0.12, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 75}

$$\frac{c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}+\frac{2c^2\log(x)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -c^2/(3*a^4*x^3) + c^2/(a^3*x^2) - c^2*x + (2*c^2*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\int\frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}$$
$$=\frac{c^2\int\frac{(1-ax)^3(1+ax)}{x^4}\,dx}{a^4}$$
$$=\frac{c^2\int\left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}$$
$$=-\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}-c^2x+\frac{2c^2\log(x)}{a}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$-\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+\frac{2c^2\log(x)}{a}+c^2(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(2*ArcTanh[a*x]),x]
[Out] -1/3*c^2/(a^4*x^3) + c^2/(a^3*x^2) - c^2*x + (2*c^2*Log[x])/a
```

fricas [A] time = 0.52, size = 43, normalized size = 1.08

$$-\frac{3\,a^4c^2x^4-6\,a^3c^2x^3\log(x)-3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -1/3*(3*a^4*c^2*x^4 - 6*a^3*c^2*x^3*log(x) - 3*a*c^2*x + c^2)/(a^4*x^3)
```

giac [B] time = 0.22, size = 112, normalized size = 2.80

$$-\frac{2\,c^2\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}+\frac{2\,c^2\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}+\frac{\left(3\,c^2-\frac{5\,c^2}{ax+1}-\frac{3\,c^2}{(ax+1)^2}+\frac{6\,c^2}{(ax+1)^3}\right)(ax+1)}{3\,a\left(\frac{1}{ax+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*c^2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + 2*c^2*log(abs(-1/(a*x + 1
) + 1))/a + 1/3*(3*c^2 - 5*c^2/(a*x + 1) - 3*c^2/(a*x + 1)^2 + 6*c^2/(a*x +
 1)^3)*(a*x + 1)/(a*(1/(a*x + 1) - 1)^3)
```

maple [A] time = 0.03, size = 39, normalized size = 0.98

$$-\frac{c^2}{3a^4x^3}+\frac{c^2}{x^2a^3}-c^2x+\frac{2c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/3*c^2/a^4/x^3+c^2/x^2/a^3-c^2*x+2*c^2*ln(x)/a
```

maxima [A] time = 0.31, size = 38, normalized size = 0.95

$$-c^2x+\frac{2\,c^2\log(x)}{a}+\frac{3\,ac^2x-c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c^2*x + 2*c^2*log(x)/a + 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

mupad [B] time = 0.89, size = 35, normalized size = 0.88

$$\frac{c^2\,\left(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln(x)-1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^2*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] (c^2*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

sympy [A] time = 0.19, size = 39, normalized size = 0.98

$$\frac{-a^4c^2x+2a^3c^2\log{(x)}-\frac{-3ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] (-a**4*c**2*x + 2*a**3*c**2*log(x) - (-3*a*c**2*x + c**2)/(3*x**3))/a**4
```

3.673 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=21

$$\frac{c}{a^2x}+\frac{2c\log(x)}{a}+c(-x)$$

```
[Out] c/a^2/x-c*x+2*c*ln(x)/a
```

Rubi [A] time = 0.07, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6157, 6150, 43}

$$\frac{c}{a^2x}+\frac{2c\log(x)}{a}+c(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^(2*ArcTanh[a*x]),x]
```

```
[Out] c/(a^2*x) - c*x + (2*c*Log[x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\frac{c\int \frac{e^{-2\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$
$$= -\frac{c\int \frac{(1-ax)^2}{x^2}dx}{a^2}$$
$$= -\frac{c\int \left(a^2+\frac{1}{x^2}-\frac{2a}{x}\right)dx}{a^2}$$
$$= \frac{c}{a^2x} - cx + \frac{2c\log(x)}{a}$$

Mathematica [**A**] time = 0.01, size = 21, normalized size = 1.00

$$\frac{c}{a^2x}+\frac{2c\log(x)}{a}+c(-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(2*ArcTanh[a*x]),x]
[Out] c/(a^2*x) - c*x + (2*c*Log[x])/a
```

fricas [**A**] time = 0.42, size = 27, normalized size = 1.29

$$-\frac{a^2cx^2-2\,acx\log(x)-c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -(a^2*c*x^2 - 2*a*c*x*log(x) - c)/(a^2*x)
```

giac [**B**] time = 0.17, size = 88, normalized size = 4.19

$$-\frac{2\,c\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{a}+\frac{2\,c\log\left(\left|-\frac{1}{ax+1}+1\right|\right)}{a}-\frac{c-\frac{2\,c}{ax+1}}{a^2\left(\frac{1}{(ax+1)a}-\frac{1}{(ax+1)^2a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -2*c*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/a + 2*c*log(abs(-1/(a*x + 1) + 1))/a - (c - 2*c/(a*x + 1))/(a^2*(1/((a*x + 1)*a) - 1/((a*x + 1)^2*a)))
```

maple [A] time = 0.03, size = 22, normalized size = 1.05

$$\frac{c}{a^2x} - cx + \frac{2c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] c/a^2/x-c*x+2*c*ln(x)/a
```

maxima [A] time = 0.30, size = 21, normalized size = 1.00

$$-cx + \frac{2\,c\log(x)}{a} + \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -c*x + 2*c*log(x)/a + c/(a^2*x)
```

mupad [B] time = 0.05, size = 24, normalized size = 1.14

$$\frac{c\,\left(2\,a\,x\,\ln(x) - a^2\,x^2 + 1\right)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] (c*(2*a*x*log(x) - a^2*x^2 + 1))/(a^2*x)
```

sympy [A] time = 0.11, size = 20, normalized size = 0.95

$$\frac{-a^2cx + 2ac\log{(x)} + \frac{c}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] (-a**2*c*x + 2*a*c*log(x) + c/x)/a**2
```

3.674 $\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{ac(ax+1)}+\frac{2\log(ax+1)}{ac}-\frac{x}{c}$$

```
[Out] -x/c+1/a/c/(a*x+1)+2*ln(a*x+1)/a/c
```

Rubi [A] time = 0.13, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 43}

$$\frac{1}{ac(ax+1)}+\frac{2\log(ax+1)}{ac}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] -(x/c) + 1/(a*c*(1 + a*x)) + (2*Log[1 + a*x])/(a*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}$$
$$= -\frac{a^2\int \frac{x^2}{(1+ax)^2}\,dx}{c}$$
$$= -\frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(1+ax)^2}-\frac{2}{a^2(1+ax)}\right)dx}{c}$$
$$= -\frac{x}{c}+\frac{1}{ac(1+ax)}+\frac{2\log(1+ax)}{ac}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.80

$$\frac{\frac{1}{a^2x+a}+\frac{2\log(ax+1)}{a}-x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))),x]
[Out] (-x + (a + a^2*x)^(-1) + (2*Log[1 + a*x])/a)/c
```

fricas [A] time = 0.70, size = 39, normalized size = 1.11

$$-\frac{a^2x^2+ax-2\,(ax+1)\log\left(ax+1\right)-1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] -(a^2*x^2 + a*x - 2*(a*x + 1)*log(a*x + 1) - 1)/(a^2*c*x + a*c)
```

giac [A] time = 0.18, size = 55, normalized size = 1.57

$$-\frac{ax+1}{ac}-\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac}+\frac{1}{(ax+1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -(a*x + 1)/(a*c) - 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c) + 1/((a*x
 + 1)*a*c)
```

maple [A] time = 0.03, size = 36, normalized size = 1.03

$$-\frac{x}{c}+\frac{1}{ac\left(ax+1\right)}+\frac{2\ln\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2),x)
```

```
[Out] -x/c+1/a/c/(a*x+1)+2*ln(a*x+1)/a/c
```

maxima [A] time = 0.30, size = 33, normalized size = 0.94

$$-\frac{x}{c}+\frac{1}{a^2cx+ac}+\frac{2\,\log\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -x/c + 1/(a^2*c*x + a*c) + 2*log(a*x + 1)/(a*c)
```

mupad [B] time = 0.05, size = 33, normalized size = 0.94

$$\frac{1}{a\,\left(c+a\,c\,x\right)}-\frac{x}{c}+\frac{2\,\ln\left(a\,x+1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a^2*x^2))*(a*x + 1)^2),x)
```

```
[Out] 1/(a*(c + a*c*x)) - x/c + (2*log(a*x + 1))/(a*c)
```

sympy [A] time = 0.15, size = 37, normalized size = 1.06

$$-a^2\left(-\frac{1}{a^4cx+a^3c}+\frac{x}{a^2c}-\frac{2\log\left(ax+1\right)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2),x)
```

```
[Out] -a**2*(-1/(a**4*c*x + a**3*c) + x/(a**2*c) - 2*log(a*x + 1)/(a**3*c))
```

3.675 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{7}{4ac^2(ax+1)}-\frac{1}{4ac^2(ax+1)^2}-\frac{\log(1-ax)}{8ac^2}+\frac{17\log(ax+1)}{8ac^2}-\frac{x}{c^2}$$

```
[Out] -x/c^2-1/4/a/c^2/(a*x+1)^2+7/4/a/c^2/(a*x+1)-1/8*ln(-a*x+1)/a/c^2+17/8*ln(a
*x+1)/a/c^2
```

Rubi [A] time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$\frac{7}{4ac^2(ax+1)}-\frac{1}{4ac^2(ax+1)^2}-\frac{\log(1-ax)}{8ac^2}+\frac{17\log(ax+1)}{8ac^2}-\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] -(x/c^2) - 1/(4*a*c^2*(1 + a*x)^2) + 7/(4*a*c^2*(1 + a*x)) - Log[1 - a*x]/(
8*a*c^2) + (17*Log[1 + a*x])/(8*a*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}$$
$$= \frac{a^4\int \frac{x^4}{(1-ax)(1+ax)^3}\,dx}{c^2}$$
$$= \frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{8a^4(-1+ax)}+\frac{1}{2a^4(1+ax)^3}-\frac{7}{4a^4(1+ax)^2}+\frac{17}{8a^4(1+ax)}\right)dx}{c^2}$$
$$= -\frac{x}{c^2}-\frac{1}{4ac^2(1+ax)^2}+\frac{7}{4ac^2(1+ax)}-\frac{\log(1-ax)}{8ac^2}+\frac{17\log(1+ax)}{8ac^2}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.92

$$\frac{-8a^3x^3-16a^2x^2+6ax-(ax+1)^2\log(1-ax)+17(ax+1)^2\log(ax+1)+12}{8a(acx+c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (12 + 6*a*x - 16*a^2*x^2 - 8*a^3*x^3 - (1 + a*x)^2*Log[1 - a*x] + 17*(1 + a
*x)^2*Log[1 + a*x])/(8*a*(c + a*c*x)^2)
```

fricas [A] time = 0.59, size = 92, normalized size = 1.24

$$-\frac{8\,a^3x^3+16\,a^2x^2-6\,ax-17\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2+2\,ax+1\right)\log\left(ax-1\right)-12}{8\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^3*x^3 + 16*a^2*x^2 - 6*a*x - 17*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1
) + (a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 12)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a
*c^2)
```

giac [A] time = 0.16, size = 101, normalized size = 1.36

$$-\frac{ax+1}{ac^2}-\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac^2}-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^2}+\frac{\frac{7\,a^5c^2}{ax+1}-\frac{a^5c^2}{(ax+1)^2}}{4\,a^6c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -(a*x + 1)/(a*c^2) - 2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c^2) - 1/8
*log(abs(-2/(a*x + 1) + 1))/(a*c^2) + 1/4*(7*a^5*c^2/(a*x + 1) - a^5*c^2/(a
*x + 1)^2)/(a^6*c^4)
```

maple [A] time = 0.04, size = 66, normalized size = 0.89

$$-\frac{x}{c^2}-\frac{\ln(ax-1)}{8a\,c^2}-\frac{1}{4a\,c^2\,(ax+1)^2}+\frac{7}{4a\,c^2\,(ax+1)}+\frac{17\ln(ax+1)}{8a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -x/c^2-1/8/a/c^2*ln(a*x-1)-1/4/a/c^2/(a*x+1)^2+7/4/a/c^2/(a*x+1)+17/8*ln(a*
x+1)/a/c^2
```

maxima [A] time = 0.31, size = 70, normalized size = 0.95

$$\frac{7\,ax+6}{4\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}-\frac{x}{c^2}+\frac{17\,\log(ax+1)}{8\,ac^2}-\frac{\log(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(7*a*x + 6)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) - x/c^2 + 17/8*log(a*x
+ 1)/(a*c^2) - 1/8*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.89, size = 68, normalized size = 0.92

$$\frac{\frac{7\,x}{4}+\frac{3}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2}-\frac{x}{c^2}-\frac{\ln(a\,x-1)}{8\,a\,c^2}+\frac{17\,\ln(a\,x+1)}{8\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a^2*x^2))^2*(a*x + 1)^2),x)
```

```
[Out] ((7*x)/4 + 3/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) - x/c^2 - log(a*x - 1)/
(8*a*c^2) + (17*log(a*x + 1))/(8*a*c^2)
```

sympy [A] time = 0.39, size = 76, normalized size = 1.03

$$-a^4\left(\frac{-7ax-6}{4a^7c^2x^2+8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{17\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] -a**4*((-7*a*x - 6)/(4*a**7*c**2*x**2 + 8*a**6*c**2*x + 4*a**5*c**2) + x/(a
**4*c**2) + (log(x - 1/a)/8 - 17*log(x + 1/a)/8)/(a**5*c**2))
```

3.676 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3} dx$

Optimal. Leaf size=109

$$-\frac{1}{16ac^3(1-ax)}+\frac{39}{16ac^3(ax+1)}-\frac{5}{8ac^3(ax+1)^2}+\frac{1}{12ac^3(ax+1)^3}-\frac{\log(1-ax)}{4ac^3}+\frac{9\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

```
[Out] -x/c^3-1/16/a/c^3/(-a*x+1)+1/12/a/c^3/(a*x+1)^3-5/8/a/c^3/(a*x+1)^2+39/16/a
/c^3/(a*x+1)-1/4*ln(-a*x+1)/a/c^3+9/4*ln(a*x+1)/a/c^3
```

Rubi [A] time = 0.17, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{1}{16ac^3(1-ax)}+\frac{39}{16ac^3(ax+1)}-\frac{5}{8ac^3(ax+1)^2}+\frac{1}{12ac^3(ax+1)^3}-\frac{\log(1-ax)}{4ac^3}+\frac{9\log(ax+1)}{4ac^3}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -(x/c^3) - 1/(16*a*c^3*(1 - a*x)) + 1/(12*a*c^3*(1 + a*x)^3) - 5/(8*a*c^3*(
1 + a*x)^2) + 39/(16*a*c^3*(1 + a*x)) - Log[1 - a*x]/(4*a*c^3) + (9*Log[1 +
 a*x])/(4*a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
```

```
/; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{-2\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^3}\,dx}{c^3}$$

$$= -\frac{a^6\int \frac{x^6}{(1-ax)^2(1+ax)^4}\,dx}{c^3}$$

$$= -\frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{16a^6(-1+ax)^2}+\frac{1}{4a^6(-1+ax)}+\frac{1}{4a^6(1+ax)^4}-\frac{5}{4a^6(1+ax)^3}+\frac{39}{16a^6(1+ax)^2}-\frac{9}{4a^6(1+ax)}\right)dx}{c^3}$$

$$= -\frac{x}{c^3}-\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}-\frac{5}{8ac^3(1+ax)^2}+\frac{39}{16ac^3(1+ax)}-\frac{\log(1-ax)}{4ac^3}+\frac{91}{}$$

Mathematica [A] time = 0.06, size = 104, normalized size = 0.95

$$\frac{-2\left(6a^5x^5+12a^4x^4-15a^3x^3-24a^2x^2+7ax+11\right)-3(ax-1)(ax+1)^3\log(1-ax)+27(ax-1)(ax+1)^3\log(ax}{12a(ax-1)(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (-2*(11 + 7*a*x - 24*a^2*x^2 - 15*a^3*x^3 + 12*a^4*x^4 + 6*a^5*x^5) - 3*(-1
 + a*x)*(1 + a*x)^3*Log[1 - a*x] + 27*(-1 + a*x)*(1 + a*x)^3*Log[1 + a*x])/
(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

fricas [A] time = 0.64, size = 137, normalized size = 1.26

$$-\frac{12\,a^5x^5+24\,a^4x^4-30\,a^3x^3-48\,a^2x^2+14\,ax-27\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log(ax+1)+3\left(a^4x^4+2\,a^3x^3-2\right.}{12\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(12*a^5*x^5 + 24*a^4*x^4 - 30*a^3*x^3 - 48*a^2*x^2 + 14*a*x - 27*(a^4
*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x
 - 1)*log(a*x - 1) + 22)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3
)
```

giac [A] time = 0.17, size = 140, normalized size = 1.28

$$-\frac{(ax+1)\left(\frac{65}{ax+1}-32\right)}{32\,ac^3\left(\frac{2}{ax+1}-1\right)}-\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac^3}-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{4\,ac^3}+\frac{\frac{117\,a^{11}c^6}{ax+1}-\frac{30\,a^{11}c^6}{(ax+1)^2}+\frac{4\,a^{11}c^6}{(ax+1)^3}}{48\,a^{12}c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/32*(a*x + 1)*(65/(a*x + 1) - 32)/(a*c^3*(2/(a*x + 1) - 1)) - 2*log(abs(a
*x + 1)/((a*x + 1)^2*abs(a)))/(a*c^3) - 1/4*log(abs(-2/(a*x + 1) + 1))/(a*c
^3) + 1/48*(117*a^11*c^6/(a*x + 1) - 30*a^11*c^6/(a*x + 1)^2 + 4*a^11*c^6/(
a*x + 1)^3)/(a^12*c^9)
```

maple [A] time = 0.04, size = 96, normalized size = 0.88

$$-\frac{x}{c^3}+\frac{1}{16a\,c^3\,(ax-1)}-\frac{\ln(ax-1)}{4a\,c^3}+\frac{1}{12a\,c^3\,(ax+1)^3}-\frac{5}{8a\,c^3\,(ax+1)^2}+\frac{39}{16a\,c^3\,(ax+1)}+\frac{9\ln(ax+1)}{4a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -x/c^3+1/16/a/c^3/(a*x-1)-1/4/a/c^3*ln(a*x-1)+1/12/a/c^3/(a*x+1)^3-5/8/a/c^
3/(a*x+1)^2+39/16/a/c^3/(a*x+1)+9/4*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.31, size = 98, normalized size = 0.90

$$\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}-\frac{x}{c^3}+\frac{9\,\log(ax+1)}{4\,ac^3}-\frac{\log(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 -
2*a^2*c^3*x - a*c^3) - x/c^3 + 9/4*log(a*x + 1)/(a*c^3) - 1/4*log(a*x - 1)/
(a*c^3)
```

mupad [B] time = 0.11, size = 94, normalized size = 0.86

$$\frac{\frac{13\,x}{6}-2\,a\,x^2+\frac{11}{6\,a}-\frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3}-\frac{x}{c^3}-\frac{\ln(a\,x-1)}{4\,a\,c^3}+\frac{9\,\ln(a\,x+1)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a^2*x^2))^3*(a*x + 1)^2),x)
```

```
[Out] ((13*x)/6 - 2*a*x^2 + 11/(6*a) - (5*a^2*x^3)/2)/(c^3 - 2*a^3*c^3*x^3 - a^4*
c^3*x^4 + 2*a*c^3*x) - x/c^3 - log(a*x - 1)/(4*a*c^3) + (9*log(a*x + 1))/(4
*a*c^3)
```

sympy [A] time = 0.63, size = 104, normalized size = 0.95

$$-a^{6}\left(\frac{-15a^{3}x^{3}-12a^{2}x^{2}+13ax+11}{6a^{11}c^{3}x^{4}+12a^{10}c^{3}x^{3}-12a^{8}c^{3}x-6a^{7}c^{3}}+\frac{x}{a^{6}c^{3}}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{4}-\frac{9\log\left(x+\frac{1}{a}\right)}{4}}{a^{7}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] -a**6*((-15*a**3*x**3 - 12*a**2*x**2 + 13*a*x + 11)/(6*a**11*c**3*x**4 + 12
*a**10*c**3*x**3 - 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (log(x -
 1/a)/4 - 9*log(x + 1/a)/4)/(a**7*c**3))
```

3.677 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=144

$$-\frac{11}{64ac^4(1-ax)}+\frac{99}{32ac^4(ax+1)}+\frac{1}{64ac^4(1-ax)^2}-\frac{35}{32ac^4(ax+1)^2}+\frac{13}{48ac^4(ax+1)^3}-\frac{1}{32ac^4(ax+1)^4}-\frac{47\log(1-}{128ac^4}$$

```
[Out] -x/c^4+1/64/a/c^4/(-a*x+1)^2-11/64/a/c^4/(-a*x+1)-1/32/a/c^4/(a*x+1)^4+13/4
8/a/c^4/(a*x+1)^3-35/32/a/c^4/(a*x+1)^2+99/32/a/c^4/(a*x+1)-47/128*ln(-a*x+
1)/a/c^4+303/128*ln(a*x+1)/a/c^4
```

Rubi [A] time = 0.20, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 88}

$$-\frac{11}{64ac^4(1-ax)}+\frac{99}{32ac^4(ax+1)}+\frac{1}{64ac^4(1-ax)^2}-\frac{35}{32ac^4(ax+1)^2}+\frac{13}{48ac^4(ax+1)^3}-\frac{1}{32ac^4(ax+1)^4}-\frac{47\log(1-}{128ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] -(x/c^4) + 1/(64*a*c^4*(1 - a*x)^2) - 11/(64*a*c^4*(1 - a*x)) - 1/(32*a*c^4
*(1 + a*x)^4) + 13/(48*a*c^4*(1 + a*x)^3) - 35/(32*a*c^4*(1 + a*x)^2) + 99/
(32*a*c^4*(1 + a*x)) - (47*Log[1 - a*x])/(128*a*c^4) + (303*Log[1 + a*x])/(
128*a*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{-2\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8}{(1-ax)^3(1+ax)^5}\,dx}{c^4}$$

$$= \frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{32a^8(-1+ax)^3}-\frac{11}{64a^8(-1+ax)^2}-\frac{47}{128a^8(-1+ax)}+\frac{1}{8a^8(1+ax)^5}-\frac{13}{16a^8(1+ax)^4}+\frac{35}{16a^8(1+ax)^3}-\frac{3}{}\right.}{c^4}$$

$$= -\frac{x}{c^4}+\frac{1}{64ac^4(1-ax)^2}-\frac{11}{64ac^4(1-ax)}-\frac{1}{32ac^4(1+ax)^4}+\frac{13}{48ac^4(1+ax)^3}-\frac{35}{32ac^4(1+ax)^2}$$

Mathematica [A] time = 0.09, size = 121, normalized size = 0.84

$$\frac{-384a^7x^7-768a^6x^6+1638a^5x^5+2508a^4x^4-1732a^3x^3-2516a^2x^2+550ax-141(ax-1)^2(ax+1)^4\log(1-ax}{384a(ax-1)^2(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (800 + 550*a*x - 2516*a^2*x^2 - 1732*a^3*x^3 + 2508*a^4*x^4 + 1638*a^5*x^5
- 768*a^6*x^6 - 384*a^7*x^7 - 141*(-1 + a*x)^2*(1 + a*x)^4*Log[1 - a*x] + 9
09*(-1 + a*x)^2*(1 + a*x)^4*Log[1 + a*x])/(384*a*(-1 + a*x)^2*(c + a*c*x)^4
)
```

fricas [A] time = 0.49, size = 233, normalized size = 1.62

$$-\frac{384\,a^7x^7+768\,a^6x^6-1638\,a^5x^5-2508\,a^4x^4+1732\,a^3x^3+2516\,a^2x^2-550\,ax-909\left(a^6x^6+2\,a^5x^5-a^4x^4-4\right.}{384\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/384*(384*a^7*x^7 + 768*a^6*x^6 - 1638*a^5*x^5 - 2508*a^4*x^4 + 1732*a^3*
x^3 + 2516*a^2*x^2 - 550*a*x - 909*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x
^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + 141*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 800)/(a^7*c^4*x^6 + 2*a^
6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4
)
```

giac [A] time = 0.17, size = 164, normalized size = 1.14

$$-\frac{2\,\log\left(\frac{|ax+1|}{(ax+1)^2|a|}\right)}{ac^4}-\frac{47\,\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{128\,ac^4}+\frac{(ax+1)\left(\frac{1045}{ax+1}-\frac{1064}{(ax+1)^2}-256\right)}{256\,ac^4\left(\frac{2}{ax+1}-1\right)^2}+\frac{\frac{297\,a^{19}c^{12}}{ax+1}-\frac{105\,a^{19}c^{12}}{(ax+1)^2}+\frac{26\,a^{19}c^{12}}{(ax+1)^3}-\frac{3\,a^{19}c}{(ax+1}}{96\,a^{20}c^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -2*log(abs(a*x + 1)/((a*x + 1)^2*abs(a)))/(a*c^4) - 47/128*log(abs(-2/(a*x
+ 1) + 1))/(a*c^4) + 1/256*(a*x + 1)*(1045/(a*x + 1) - 1064/(a*x + 1)^2 - 2
56)/(a*c^4*(2/(a*x + 1) - 1)^2) + 1/96*(297*a^19*c^12/(a*x + 1) - 105*a^19*
c^12/(a*x + 1)^2 + 26*a^19*c^12/(a*x + 1)^3 - 3*a^19*c^12/(a*x + 1)^4)/(a^2
0*c^16)
```

maple [A] time = 0.04, size = 126, normalized size = 0.88

$$-\frac{x}{c^4}+\frac{1}{64a\,c^4\,(ax-1)^2}+\frac{11}{64a\,c^4\,(ax-1)}-\frac{47\ln(ax-1)}{128a\,c^4}-\frac{1}{32a\,c^4\,(ax+1)^4}+\frac{13}{48a\,c^4\,(ax+1)^3}-\frac{35}{32a\,c^4\,(ax+1)^2}+\frac{}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -x/c^4+1/64/a/c^4/(a*x-1)^2+11/64/a/c^4/(a*x-1)-47/128/a/c^4*ln(a*x-1)-1/32
/a/c^4/(a*x+1)^4+13/48/a/c^4/(a*x+1)^3-35/32/a/c^4/(a*x+1)^2+99/32/a/c^4/(a
*x+1)+303/128*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.31, size = 146, normalized size = 1.01

$$\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}-\frac{x}{c^4}+\frac{303\,\log(ax+1)}{128\,ac^4}-\frac{47\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x + 4
00)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^
2 + 2*a^2*c^4*x + a*c^4) - x/c^4 + 303/128*log(a*x + 1)/(a*c^4) - 47/128*lo
g(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.95, size = 142, normalized size = 0.99

$$\frac{\frac{467\,x}{192}-\frac{437\,a\,x^2}{96}+\frac{25}{12\,a}-\frac{529\,a^2\,x^3}{96}+\frac{81\,a^3\,x^4}{32}+\frac{209\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4}-\frac{x}{c^4}-\frac{47\,\ln\left(a\,x-1\right)}{128\,a\,c^4}+\frac{303\,\ln\left(a\,x+1\right)}{128\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a^2*x^2))^4*(a*x + 1)^2),x)
```

```
[Out] ((467*x)/192 - (437*a*x^2)/96 + 25/(12*a) - (529*a^2*x^3)/96 + (81*a^3*x^4)
/32 + (209*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^4*x^4 +
2*a^5*c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) - x/c^4 - (47*log(a*x - 1))/(128*a
*c^4) + (303*log(a*x + 1))/(128*a*c^4)
```

sympy [A] time = 0.88, size = 158, normalized size = 1.10

$$-a^8\left(\frac{-627a^5x^5-486a^4x^4+1058a^3x^3+874a^2x^2-467ax-400}{192a^{15}c^4x^6+384a^{14}c^4x^5-192a^{13}c^4x^4-768a^{12}c^4x^3-192a^{11}c^4x^2+384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{47\log\left(x\right.}{128}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] -a**8*((-627*a**5*x**5 - 486*a**4*x**4 + 1058*a**3*x**3 + 874*a**2*x**2 - 4
67*a*x - 400)/(192*a**15*c**4*x**6 + 384*a**14*c**4*x**5 - 192*a**13*c**4*x
**4 - 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 + 384*a**10*c**4*x + 192*a*
*9*c**4) + x/(a**8*c**4) + (47*log(x - 1/a)/128 - 303*log(x + 1/a)/128)/(a*
*9*c**4))
```

3.678 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=191

$$-\frac{3c^4(5ax+16)\sqrt{1-a^2x^2}}{16a^2x}+\frac{15c^4\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{16a}-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7}+\frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6}-\frac{c^4(24-5ax)\left(1-a^2x^2\right.}{40a^6x^5}$$

```
[Out] 1/16*c^4*(-5*a*x+16)*(-a^2*x^2+1)^(3/2)/a^4/x^3-1/40*c^4*(-5*a*x+24)*(-a^2*
x^2+1)^(5/2)/a^6/x^5-1/7*c^4*(-a^2*x^2+1)^(7/2)/a^8/x^7+1/2*c^4*(-a^2*x^2+1
)^(7/2)/a^7/x^6-3*c^4*arcsin(a*x)/a+15/16*c^4*arctanh((-a^2*x^2+1)^(1/2))/a
-3/16*c^4*(5*a*x+16)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.36, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6157, 6149, 1807, 811, 813, 844, 216, 266, 63, 208}

$$\frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6}-\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7}-\frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5}+\frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3}-\frac{3c^4(5ax+16)\sqrt{1-a^2}}{16a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*c^4*(16 + 5*a*x)*Sqrt[1 - a^2*x^2])/(16*a^2*x) + (c^4*(16 - 5*a*x)*(1 -
 a^2*x^2)^(3/2))/(16*a^4*x^3) - (c^4*(24 - 5*a*x)*(1 - a^2*x^2)^(5/2))/(40*
a^6*x^5) - (c^4*(1 - a^2*x^2)^(7/2))/(7*a^8*x^7) + (c^4*(1 - a^2*x^2)^(7/2)
)/(2*a^7*x^6) - (3*c^4*ArcSin[a*x])/a + (15*c^4*ArcTanh[Sqrt[1 - a^2*x^2]])
/(16*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = \frac{c^4\int \frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$

$$= \frac{c^4\int \frac{(1-ax)^3\left(1-a^2x^2\right)^{5/2}}{x^8}dx}{a^8}$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} - \frac{c^4\int \frac{\left(1-a^2x^2\right)^{5/2}\left(21a-21a^2x+7a^3x^2\right)}{x^7}dx}{7a^8}$$

$$= -\frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} + \frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6} + \frac{c^4\int \frac{\left(126a^2-21a^3x\right)\left(1-a^2x^2\right)^{5/2}}{x^6}dx}{42a^8}$$

$$= -\frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} + \frac{c^4\left(1-a^2x^2\right)^{7/2}}{2a^7x^6} - \frac{c^4\int \frac{(1008a^4-2}{}}{3}$$

$$= \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right)^{5/2}}{40a^6x^5} - \frac{c^4\left(1-a^2x^2\right)^{7/2}}{7a^8x^7} + \frac{c^4(1}{}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right.}{40a^6x^5}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right.}{40a^6x^5}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right.}{40a^6x^5}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right.}{40a^6x^5}$$

$$= -\frac{3c^4(16+5ax)\sqrt{1-a^2x^2}}{16a^2x} + \frac{c^4(16-5ax)\left(1-a^2x^2\right)^{3/2}}{16a^4x^3} - \frac{c^4(24-5ax)\left(1-a^2x^2\right.}{40a^6x^5}$$

Mathematica [C] time = 0.18, size = 191, normalized size = 1.00

$$\frac{c^4\left(\frac{5\left(-16a^8x^8-231a^7x^7+64a^6x^6+413a^5x^5-96a^4x^4-238a^3x^3+64a^2x^2+16a^7x^7\left(a^2x^2-1\right)^4{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-a^2x^2\right)-105a^7x^7\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+5\right.}{\sqrt{1-a^2x^2}}\right.}{560a^8x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^4*(-336*a^2*x^2*Hypergeometric2F1[-5/2, -5/2, -3/2, a^2*x^2] + (5*(-16 +
 56*a*x + 64*a^2*x^2 - 238*a^3*x^3 - 96*a^4*x^4 + 413*a^5*x^5 + 64*a^6*x^6
- 231*a^7*x^7 - 16*a^8*x^8 - 105*a^7*x^7*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 -
 a^2*x^2]] + 16*a^7*x^7*(-1 + a^2*x^2)^4*Hypergeometric2F1[3, 7/2, 9/2, 1 -
 a^2*x^2]))/Sqrt[1 - a^2*x^2]))/(560*a^8*x^7)
```

fricas [A] time = 0.52, size = 176, normalized size = 0.92

$$\frac{3360\,a^7c^4x^7\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-525\,a^7c^4x^7\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-560\,a^7c^4x^7-\left(560\,a^7c^4x^7+2496\,a^6c^4x^6-525\right.}{560\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/560*(3360*a^7*c^4*x^7*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 525*a^7*c^
4*x^7*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 560*a^7*c^4*x^7 - (560*a^7*c^4*x^7
+ 2496*a^6*c^4*x^6 - 525*a^5*c^4*x^5 - 992*a^4*c^4*x^4 + 770*a^3*c^4*x^3 +
96*a^2*c^4*x^2 - 280*a*c^4*x + 80*c^4)*sqrt(-a^2*x^2 + 1))/(a^8*x^7)
```

giac [B] time = 0.30, size = 506, normalized size = 2.65

$$\frac{\left(5\,c^4-\frac{35\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^4}{a^2x}+\frac{49\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^4}{a^4x^2}+\frac{245\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^4}{a^6x^3}-\frac{875\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^4}{a^8x^4}-\frac{455\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^{10}x^5}\right.}{4480\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^7|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4480*(5*c^4 - 35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^4/(a^2*x) + 49*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^4/(a^4*x^2) + 245*(sqrt(-a^2*x^2 + 1)*abs(a) +
 a)^3*c^4/(a^6*x^3) - 875*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^8*x^4) -
 455*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^10*x^5) + 9065*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^6*c^4/(a^12*x^6))*a^14*x^7/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^7*abs(a)) - 3*c^4*arcsin(a*x)*sgn(a)/abs(a) + 15/16*c^4*log(1/2*abs(-2*s
qrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c
^4/a - 1/4480*(9065*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c^4/x - 455*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^4/x^2 - 875*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^3*c^4/x^3 + 245*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^4/(a^2*x^4) + 49*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^5*c^4/(a^4*x^5) - 35*(sqrt(-a^2*x^2 + 1)*abs(a)
```

```
 + a)^6*c^4/(a^6*x^6) + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^7*c^4/(a^8*x^7))/
(a^6*abs(a))
```

maple [A] time = 0.10, size = 289, normalized size = 1.51

$$\frac{c^4\left(-a^2x^2+1\right)^{\frac{5}{2}}}{2a^7x^6}-\frac{3c^4\left(-a^2x^2+1\right)^{\frac{5}{2}}}{8a^5x^4}-\frac{5c^4\left(-a^2x^2+1\right)^{\frac{5}{2}}}{16a^3x^2}-\frac{5c^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}{16a}-\frac{15c^4\sqrt{-a^2x^2+1}}{16a}+\frac{15c^4\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/2*c^4/a^7/x^6*(-a^2*x^2+1)^(5/2)-3/8*c^4/a^5/x^4*(-a^2*x^2+1)^(5/2)-5/16*
c^4/a^3/x^2*(-a^2*x^2+1)^(5/2)-5/16*c^4*(-a^2*x^2+1)^(3/2)/a-15/16*c^4*(-a^
2*x^2+1)^(1/2)/a+15/16*c^4/a*arctanh(1/(-a^2*x^2+1)^(1/2))+c^4/a^4/x^3*(-a^
2*x^2+1)^(5/2)-2*c^4/a^2/x*(-a^2*x^2+1)^(5/2)-2*c^4*x*(-a^2*x^2+1)^(3/2)-3*
c^4*x*(-a^2*x^2+1)^(1/2)-3*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1
)^(1/2))-1/7*c^4/a^8/x^7*(-a^2*x^2+1)^(5/2)-16/35*c^4/a^6/x^5*(-a^2*x^2+1)^
(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^4}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^4/(a*x + 1)^3, x)
```

mupad [B] time = 0.10, size = 229, normalized size = 1.20

$$\frac{15\,c^4\,\sqrt{1-a^2\,x^2}}{16\,a^3\,x^2}-\frac{c^4\,\sqrt{1-a^2\,x^2}}{a}-\frac{156\,c^4\,\sqrt{1-a^2\,x^2}}{35\,a^2\,x}-\frac{3\,c^4\,\operatorname{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{62\,c^4\,\sqrt{1-a^2\,x^2}}{35\,a^4\,x^3}-\frac{11\,c^4\,\sqrt{1-a^2\,x^2}}{8\,a^5\,x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (15*c^4*(1 - a^2*x^2)^(1/2))/(16*a^3*x^2) - (c^4*atan((1 - a^2*x^2)^(1/2)*1
i)*15i)/(16*a) - (c^4*(1 - a^2*x^2)^(1/2))/a - (156*c^4*(1 - a^2*x^2)^(1/2)
)/(35*a^2*x) - (3*c^4*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) + (62*c^4*(1 - a^
```

```
2*x^2)^(1/2))/(35*a^4*x^3) - (11*c^4*(1 - a^2*x^2)^(1/2))/(8*a^5*x^4) - (6*
c^4*(1 - a^2*x^2)^(1/2))/(35*a^6*x^5) + (c^4*(1 - a^2*x^2)^(1/2))/(2*a^7*x^
6) - (c^4*(1 - a^2*x^2)^(1/2))/(7*a^8*x^7)
```

sympy [C] time = 18.71, size = 1110, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -c**4*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*as
in(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2
- log(sqrt(-a**2*x**2 + 1) + 1), True))/a + 3*c**4*Piecewise((-I*a**2*x/sqr
t(a**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**
2) > 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2
+ 1)), True))/a**2 - c**4*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(-1
 + 1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2
) > 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), True)
)/a**3 - 5*c**4*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/
(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2
))/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4 + 5*c**4*Piecewise
((a**4*acosh(1/(a*x))/8 - a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*a/(8*x**3
*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a
**2*x**2) > 1), (-I*a**4*asin(1/(a*x))/8 + I*a**3/(8*x*sqrt(1 - 1/(a**2*x**
2))) - 3*I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**
2*x**2))), True))/a**5 + c**4*Piecewise((2*I*a**4*sqrt(a**2*x**2 - 1)/(15*x
) + I*a**2*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(a**2*x**2 - 1)/(5*x**5),
Abs(a**2*x**2) > 1), (2*a**4*sqrt(-a**2*x**2 + 1)/(15*x) + a**2*sqrt(-a**2*
x**2 + 1)/(15*x**3) - sqrt(-a**2*x**2 + 1)/(5*x**5), True))/a**6 - 3*c**4*P
iecewise((a**6*acosh(1/(a*x))/16 - a**5/(16*x*sqrt(-1 + 1/(a**2*x**2))) + a
**3/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*a/(24*x**5*sqrt(-1 + 1/(a**2*x**
2))) - 1/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a*
*6*asin(1/(a*x))/16 + I*a**5/(16*x*sqrt(1 - 1/(a**2*x**2))) - I*a**3/(48*x*
*3*sqrt(1 - 1/(a**2*x**2))) - 5*I*a/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + I/(
6*a*x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**7 + c**4*Piecewise((8*a**7*sqr
t(-1 + 1/(a**2*x**2))/105 + 4*a**5*sqrt(-1 + 1/(a**2*x**2))/(105*x**2) + a*
*3*sqrt(-1 + 1/(a**2*x**2))/(35*x**4) - a*sqrt(-1 + 1/(a**2*x**2))/(7*x**6)
, 1/Abs(a**2*x**2) > 1), (8*I*a**7*sqrt(1 - 1/(a**2*x**2))/105 + 4*I*a**5*s
qrt(1 - 1/(a**2*x**2))/(105*x**2) + I*a**3*sqrt(1 - 1/(a**2*x**2))/(35*x**4
) - I*a*sqrt(1 - 1/(a**2*x**2))/(7*x**6), True))/a**8
```

3.679 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$

Optimal. Leaf size=157

$$-\frac{3c^3(ax+8)\sqrt{1-a^2x^2}}{8a^2x}+\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}-$$

```
[Out] 1/8*c^3*(-a*x+8)*(-a^2*x^2+1)^(3/2)/a^4/x^3+1/5*c^3*(-a^2*x^2+1)^(5/2)/a^6/
x^5-3/4*c^3*(-a^2*x^2+1)^(5/2)/a^5/x^4-3*c^3*arcsin(a*x)/a+3/8*c^3*arctanh(
(-a^2*x^2+1)^(1/2))/a-3/8*c^3*(a*x+8)*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.32, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6157, 6149, 1807, 811, 813, 844, 216, 266, 63, 208}

$$-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4}+\frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5}+\frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3}-\frac{3c^3(ax+8)\sqrt{1-a^2x^2}}{8a^2x}+\frac{3c^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{8a}-$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*c^3*(8 + a*x)*Sqrt[1 - a^2*x^2])/(8*a^2*x) + (c^3*(8 - a*x)*(1 - a^2*x^
2)^(3/2))/(8*a^4*x^3) + (c^3*(1 - a^2*x^2)^(5/2))/(5*a^6*x^5) - (3*c^3*(1 -
 a^2*x^2)^(5/2))/(4*a^5*x^4) - (3*c^3*ArcSin[a*x])/a + (3*c^3*ArcTanh[Sqrt[
1 - a^2*x^2]])/(8*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\frac{c^3\int \frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$

$$= -\frac{c^3\int \frac{(1-ax)^3\left(1-a^2x^2\right)^{3/2}}{x^6}dx}{a^6}$$

$$= \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} + \frac{c^3\int \frac{\left(1-a^2x^2\right)^{3/2}\left(15a-15a^2x+5a^3x^2\right)}{x^5}dx}{5a^6}$$

$$= \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} - \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4} - \frac{c^3\int \frac{\left(60a^2-5a^3x\right)\left(1-a^2x^2\right)^{3/2}}{x^4}dx}{20a^6}$$

$$= \frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} - \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{4a^5x^4} + \frac{c^3\int \frac{(240a^4-30a}{x}}{80a}$$

$$= -\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} - \frac{3c^3\left(1-\right.}{4}$$

$$= -\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} - \frac{3c^3\left(1-\right.}{4}$$

$$= -\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} - \frac{3c^3\left(1-\right.}{4}$$

$$= -\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} - \frac{3c^3\left(1-\right.}{4}$$

$$= -\frac{3c^3(8+ax)\sqrt{1-a^2x^2}}{8a^2x} + \frac{c^3(8-ax)\left(1-a^2x^2\right)^{3/2}}{8a^4x^3} + \frac{c^3\left(1-a^2x^2\right)^{5/2}}{5a^6x^5} - \frac{3c^3\left(1-\right.}{4}$$

Mathematica [C] time = 0.10, size = 186, normalized size = 1.18

$$\frac{c^3\left(-8a^6x^6-75a^5x^5+24a^4x^4+105a^3x^3+40a^2x^2\sqrt{1-a^2x^2}\ {}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};a^2x^2\right)-24a^2x^2+8a^5x^5\left(a^2x^2-1\right.}{40a^6x^5\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(8 - 30*a*x - 24*a^2*x^2 + 105*a^3*x^3 + 24*a^4*x^4 - 75*a^5*x^5 - 8*a
^6*x^6 - 45*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]] + 40*a^2*x
^2*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, a^2*x^2] + 8*a^5*x
^5*(-1 + a^2*x^2)^3*Hypergeometric2F1[2, 5/2, 7/2, 1 - a^2*x^2]))/(40*a^6*x
^5*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 154, normalized size = 0.98

$$\frac{240\,a^5c^3x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-15\,a^5c^3x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-40\,a^5c^3x^5-\left(40\,a^5c^3x^5+152\,a^4c^3x^4-55\,a^3c^3x^3-\right.}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/40*(240*a^5*c^3*x^5*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 15*a^5*c^3*x
^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 40*a^5*c^3*x^5 - (40*a^5*c^3*x^5 + 152
*a^4*c^3*x^4 - 55*a^3*c^3*x^3 - 24*a^2*c^3*x^2 + 30*a*c^3*x - 8*c^3)*sqrt(-
a^2*x^2 + 1))/(a^6*x^5)
```

giac [B] time = 0.21, size = 386, normalized size = 2.46

$$-\frac{\left(2\,c^3-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^3}{a^2x}+\frac{30\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^3}{a^4x^2}+\frac{80\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c^3}{a^6x^3}-\frac{580\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4c^3}{a^8x^4}\right)a^{10}x^5}{320\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^5|a|}-\frac{3\,c^3\arcsin\left(\right.}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/320*(2*c^3 - 15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^3/(a^2*x) + 30*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^2*c^3/(a^4*x^2) + 80*(sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*c^3/(a^6*x^3) - 580*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c^3/(a^8*x^4))*a
^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 3*c^3*arcsin(a*x)*sgn(
a)/abs(a) + 3/8*c^3*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*ab
s(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^3/a - 1/320*(580*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)*a^2*c^3/x - 80*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c^3/x^2 - 30*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^3/(a^2*x^3) + 15*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^4*c^3/(a^4*x^4) - 2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c^3/(a^6*x^5)
)/(a^4*abs(a))
```

maple [A] time = 0.06, size = 243, normalized size = 1.55

$$\frac{c^3\left(-a^2x^2+1\right)^{\frac{5}{2}}}{a^4x^3}-\frac{2c^3\left(-a^2x^2+1\right)^{\frac{5}{2}}}{a^2x}-2c^3x\left(-a^2x^2+1\right)^{\frac{3}{2}}-3c^3x\sqrt{-a^2x^2+1}-\frac{3c^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{c^3\left(-a^2x^2+1\right)^{\frac{5}{2}}}{8a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] c^3/a^4/x^3*(-a^2*x^2+1)^(5/2)-2*c^3/a^2/x*(-a^2*x^2+1)^(5/2)-2*c^3*x*(-a^2
*x^2+1)^(3/2)-3*c^3*x*(-a^2*x^2+1)^(1/2)-3*c^3/(a^2)^(1/2)*arctan((a^2)^(1/
2)*x/(-a^2*x^2+1)^(1/2))-1/8*c^3/a^3/x^2*(-a^2*x^2+1)^(5/2)-1/8*c^3*(-a^2*x
^2+1)^(3/2)/a-3/8*c^3*(-a^2*x^2+1)^(1/2)/a+3/8*c^3/a*arctanh(1/(-a^2*x^2+1)
^(1/2))-3/4*c^3*(-a^2*x^2+1)^(5/2)/a^5/x^4+1/5*c^3*(-a^2*x^2+1)^(5/2)/a^6/x
^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^3}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^3/(a*x + 1)^3, x)
```

mupad [B] time = 0.85, size = 183, normalized size = 1.17

$$\frac{11\,c^3\,\sqrt{1-a^2\,x^2}}{8\,a^3\,x^2}-\frac{c^3\,\sqrt{1-a^2\,x^2}}{a}-\frac{19\,c^3\,\sqrt{1-a^2\,x^2}}{5\,a^2\,x}-\frac{3\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+\frac{3\,c^3\,\sqrt{1-a^2\,x^2}}{5\,a^4\,x^3}-\frac{3\,c^3\,\sqrt{1-a^2\,x^2}}{4\,a^5\,x^4}+\frac{c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (11*c^3*(1 - a^2*x^2)^(1/2))/(8*a^3*x^2) - (c^3*atan((1 - a^2*x^2)^(1/2)*1i
)*3i)/(8*a) - (c^3*(1 - a^2*x^2)^(1/2))/a - (19*c^3*(1 - a^2*x^2)^(1/2))/(5
*a^2*x) - (3*c^3*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) + (3*c^3*(1 - a^2*x^2)
^(1/2))/(5*a^4*x^3) - (3*c^3*(1 - a^2*x^2)^(1/2))/(4*a^5*x^4) + (c^3*(1 - a
^2*x^2)^(1/2))/(5*a^6*x^5)
```

sympy [C] time = 11.65, size = 695, normalized size = 4.43

$$-\frac{c^3\left(\begin{cases} i\sqrt{a^2x^2-1}-\log(ax)+\frac{\log(a^2x^2)}{2}+i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for } |a^2x^2|>1 \\ \sqrt{-a^2x^2+1}+\frac{\log(a^2x^2)}{2}-\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right)}{a}+\frac{3c^3\left(\begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}}+ia\,\mathrm{acosh}(ax)+\frac{i}{x\sqrt{a^2 \ldots}} \\ \frac{a^2x}{\sqrt{-a^2x^2+1}}-a\,\mathrm{asin}(ax)-\frac{1}{x\sqrt{-a^2x^2 \ldots}} \end{cases}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -c**3*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*as
in(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2
- log(sqrt(-a**2*x**2 + 1) + 1), True))/a + 3*c**3*Piecewise((-I*a**2*x/sqr
t(a**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**
2) > 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2
+ 1)), True))/a**2 - 2*c**3*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(
-1 + 1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x*
*2) > 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), Tru
e))/a**3 - 2*c**3*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 +
1/(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x*
*2))/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4 + 3*c**3*Piecewi
se((a**4*acosh(1/(a*x))/8 - a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*a/(8*x*
*3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs
(a**2*x**2) > 1), (-I*a**4*asin(1/(a*x))/8 + I*a**3/(8*x*sqrt(1 - 1/(a**2*x
**2))) - 3*I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a
**2*x**2))), True))/a**5 - c**3*Piecewise((2*I*a**4*sqrt(a**2*x**2 - 1)/(15
*x) + I*a**2*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(a**2*x**2 - 1)/(5*x**5)
, Abs(a**2*x**2) > 1), (2*a**4*sqrt(-a**2*x**2 + 1)/(15*x) + a**2*sqrt(-a**
2*x**2 + 1)/(15*x**3) - sqrt(-a**2*x**2 + 1)/(5*x**5), True))/a**6
```

3.680 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=125

$$-\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}+\frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{3c^2\sin^{-1}(ax)}{a}$$

```
[Out] -1/3*c^2*(-a^2*x^2+1)^(3/2)/a^4/x^3+3/2*c^2*(-a^2*x^2+1)^(3/2)/a^3/x^2-3*c^
2*arcsin(a*x)/a-1/2*c^2*arctanh((-a^2*x^2+1)^(1/2))/a-1/2*c^2*(-a*x+6)*(-a^
2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.28, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6157, 6149, 1807, 813, 844, 216, 266, 63, 208}

$$\frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3}-\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x}-\frac{c^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2a}-\frac{3c^2\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -(c^2*(6 - a*x)*Sqrt[1 - a^2*x^2])/(2*a^2*x) - (c^2*(1 - a^2*x^2)^(3/2))/(3
*a^4*x^3) + (3*c^2*(1 - a^2*x^2)^(3/2))/(2*a^3*x^2) - (3*c^2*ArcSin[a*x])/a
 - (c^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
```

```
/; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = \frac{c^2\int \frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}dx}{a^4}$$
$$= \frac{c^2\int \frac{(1-ax)^3\sqrt{1-a^2x^2}}{x^4}dx}{a^4}$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} - \frac{c^2\int \frac{\sqrt{1-a^2x^2}\left(9a-9a^2x+3a^3x^2\right)}{x^3}dx}{3a^4}$$
$$= -\frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} + \frac{c^2\int \frac{\left(18a^2+3a^3x\right)\sqrt{1-a^2x^2}}{x^2}dx}{6a^4}$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{c^2\int \frac{-6a^3+36a^4x}{x\sqrt{1-a^2x^2}}}{12a^4}$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \left(3c^2\right)\int \frac{1}{\sqrt{1-a}}$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^2\sin^{-1}(ax)}{a} +$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^2\sin^{-1}(ax)}{a} -$$
$$= -\frac{c^2(6-ax)\sqrt{1-a^2x^2}}{2a^2x} - \frac{c^2\left(1-a^2x^2\right)^{3/2}}{3a^4x^3} + \frac{3c^2\left(1-a^2x^2\right)^{3/2}}{2a^3x^2} - \frac{3c^2\sin^{-1}(ax)}{a} -$$

Mathematica [A] time = 0.08, size = 128, normalized size = 1.02

$$-\frac{c^2\left(-6a^5x^5-16a^4x^4+15a^3x^3+14a^2x^2+18a^3x^3\sqrt{1-a^2x^2}\sin^{-1}(ax)+3a^3x^3\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\right.}{6a^4x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/6*(c^2*(2 - 9*a*x + 14*a^2*x^2 + 15*a^3*x^3 - 16*a^4*x^4 - 6*a^5*x^5 + 1
8*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x] + 3*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTa
nh[Sqrt[1 - a^2*x^2]]))/(a^4*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.51, size = 132, normalized size = 1.06

$$\frac{36\,a^3c^2x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,a^3c^2x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-6\,a^3c^2x^3-\left(6\,a^3c^2x^3+16\,a^2c^2x^2-9\,ac^2x+2\,c^2\right)\sqrt{-}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/6*(36*a^3*c^2*x^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a^3*c^2*x^3*
log((sqrt(-a^2*x^2 + 1) - 1)/x) - 6*a^3*c^2*x^3 - (6*a^3*c^2*x^3 + 16*a^2*c
^2*x^2 - 9*a*c^2*x + 2*c^2)*sqrt(-a^2*x^2 + 1))/(a^4*x^3)
```

giac [B] time = 0.18, size = 263, normalized size = 2.10

$$\frac{\left(c^2-\frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c^2}{a^2x}+\frac{33\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c^2}{a^4x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|}-\frac{3\,c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}-\frac{c^2\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\sqrt{-a^2x^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(c^2 - 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/(a^2*x) + 33*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^2*c^2/(a^4*x^2))*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) +
a)^3*abs(a)) - 3*c^2*arcsin(a*x)*sgn(a)/abs(a) - 1/2*c^2*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*c^2
/a - 1/24*(33*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c^2/x - 9*(sqrt(-a^2*x^2 + 1)
*abs(a) + a)^2*c^2/(a^2*x^2) + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c^2/(a^4*x
^3))/(a^2*abs(a))
```

maple [B] time = 0.05, size = 299, normalized size = 2.39

$$\frac{c^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{6a}+\frac{c^2\sqrt{-a^2x^2+1}}{2a}-\frac{c^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2a}-\frac{10c^2\left(-a^2x^2+1\right)^{\frac{5}{2}}}{3a^2x}-\frac{10c^2x\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-5c^2x\sqrt{-a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/6*c^2*(-a^2*x^2+1)^(3/2)/a+1/2*c^2*(-a^2*x^2+1)^(1/2)/a-1/2*c^2/a*arctanh
(1/(-a^2*x^2+1)^(1/2))-10/3*c^2/a^2/x*(-a^2*x^2+1)^(5/2)-10/3*c^2*x*(-a^2*x
^2+1)^(3/2)-5*c^2*x*(-a^2*x^2+1)^(1/2)-5*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)
*x/(-a^2*x^2+1)^(1/2))-1/3*c^2/a^4/x^3*(-a^2*x^2+1)^(5/2)+3/2*c^2/a^3/x^2*(
-a^2*x^2+1)^(5/2)+4/3*c^2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+2*c^2*(-a^2*
(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^2}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^2/(a*x + 1)^3, x)
```

mupad [B] time = 0.86, size = 137, normalized size = 1.10

$$\frac{3\,c^2\,\sqrt{1-a^2\,x^2}}{2\,a^3\,x^2}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{a}-\frac{8\,c^2\,\sqrt{1-a^2\,x^2}}{3\,a^2\,x}-\frac{3\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{c^2\,\sqrt{1-a^2\,x^2}}{3\,a^4\,x^3}+\frac{c^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (c^2*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/(2*a) - (3*c^2*asinh(x*(-a^2)^(1/2)))
/(-a^2)^(1/2) - (c^2*(1 - a^2*x^2)^(1/2))/a - (8*c^2*(1 - a^2*x^2)^(1/2))/(
3*a^2*x) + (3*c^2*(1 - a^2*x^2)^(1/2))/(2*a^3*x^2) - (c^2*(1 - a^2*x^2)^(1/
2))/(3*a^4*x^3)
```

sympy [C] time = 7.63, size = 384, normalized size = 3.07

$$-\frac{c^2\left(\begin{cases} i\sqrt{a^2x^2-1}-\log\left(ax\right)+\frac{\log\left(a^2x^2\right)}{2}+i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{for } \left|a^2x^2\right|>1 \\ \sqrt{-a^2x^2+1}+\frac{\log\left(a^2x^2\right)}{2}-\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right)}{a}+\frac{3c^2\left(\begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}}+ia\,\mathrm{acosh}\left(ax\right)+\frac{}{x\sqrt{}} \\ \frac{a^2x}{\sqrt{-a^2x^2+1}}-a\,\mathrm{asin}\left(ax\right)-\frac{1}{x\sqrt{-a^2}} \end{cases}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -c**2*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*as
in(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2
- log(sqrt(-a**2*x**2 + 1) + 1), True))/a + 3*c**2*Piecewise((-I*a**2*x/sqr
t(a**2*x**2 - 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**
2) > 1), (a**2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2
+ 1)), True))/a**2 - 3*c**2*Piecewise((a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(
-1 + 1/(a**2*x**2))) - 1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x*
*2) > 1), (-I*a**2*asin(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), Tru
e))/a**3 + c**2*Piecewise((a**3*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(-1 + 1/
(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(1 - 1/(a**2*x**2
))/3 - I*a*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))/a**4
```

3.681 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=74

$$-\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{3c\sin^{-1}(ax)}{a}$$

```
[Out] -3*c*arcsin(a*x)/a-3*c*arctanh((-a^2*x^2+1)^(1/2))/a-c*(-a^2*x^2+1)^(1/2)/a
+c*(-a^2*x^2+1)^(1/2)/a^2/x
```

Rubi [A] time = 0.21, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6157, 6149, 1807, 1809, 844, 216, 266, 63, 208}

$$-\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}-\frac{3c\sin^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c*Sqrt[1 - a^2*x^2])/a) + (c*Sqrt[1 - a^2*x^2])/(a^2*x) - (3*c*ArcSin[a*
x])/a - (3*c*ArcTanh[Sqrt[1 - a^2*x^2]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\frac{c\int \frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}dx}{a^2}$$
$$= -\frac{c\int \frac{(1-ax)^3}{x^2\sqrt{1-a^2x^2}}dx}{a^2}$$
$$= \frac{c\sqrt{1-a^2x^2}}{a^2x}+\frac{c\int \frac{3a-3a^2x+a^3x^2}{x\sqrt{1-a^2x^2}}dx}{a^2}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{c\int \frac{-3a^3+3a^4x}{x\sqrt{1-a^2x^2}}dx}{a^4}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-(3c)\int \frac{1}{\sqrt{1-a^2x^2}}dx+\frac{(3c)\int \frac{1}{x\sqrt{1-a^2x^2}}dx}{a}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}dx,x,x^2\right)}{2a}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}-\frac{(3c)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}dx,x,\sqrt{1-a\cdots}\right)}{a^3}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{a}+\frac{c\sqrt{1-a^2x^2}}{a^2x}-\frac{3c\sin^{-1}(ax)}{a}-\frac{3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{a}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.77

$$-\frac{c\left(\sqrt{1-a^2x^2}\,(ax-1)+3ax\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ax\sin^{-1}(ax)\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c*((-1 + a*x)*Sqrt[1 - a^2*x^2] + 3*a*x*ArcSin[a*x] + 3*a*x*ArcTanh[Sqrt
[1 - a^2*x^2]]))/(a^2*x))
```

fricas [A] time = 0.71, size = 86, normalized size = 1.16

$$\frac{6\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+3\,acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-acx-\sqrt{-a^2x^2+1}\,(acx-c)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + 3*a*c*x*log((sqrt(-a^2*x^
2 + 1) - 1)/x) - a*c*x - sqrt(-a^2*x^2 + 1)*(a*c*x - c))/(a^2*x)
```

giac [A] time = 0.23, size = 130, normalized size = 1.76

$$-\frac{a^2cx}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{3\,c\arcsin\left(ax\right)\mathrm{sgn}(a)}{|a|}-\frac{3\,c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{\sqrt{-a^2x^2+1}\,c}{a}+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+\right.}{2\,a^2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*a^2*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 3*c*arcsin(a*x)*sgn
(a)/abs(a) - 3*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x
)))/abs(a) - sqrt(-a^2*x^2 + 1)*c/a + 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*c
/(a^2*x*abs(a))
```

maple [B] time = 0.05, size = 266, normalized size = 3.59

$$\frac{c\left(-a^2x^2+1\right)^{\frac{3}{2}}}{a}+\frac{3c\sqrt{-a^2x^2+1}}{a}-\frac{3c\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{a}+\frac{c\left(-a^2x^2+1\right)^{\frac{5}{2}}}{a^2x}+cx\left(-a^2x^2+1\right)^{\frac{3}{2}}+\frac{3cx\sqrt{-a^2x^2+1}}{2}+\frac{3c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] c*(-a^2*x^2+1)^(3/2)/a+3*c*(-a^2*x^2+1)^(1/2)/a-3*c/a*arctanh(1/(-a^2*x^2+1
)^(1/2))+c/a^2/x*(-a^2*x^2+1)^(5/2)+c*x*(-a^2*x^2+1)^(3/2)+3/2*c*x*(-a^2*x^
2+1)^(1/2)+3/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-2*c/a
^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-3*c/a*(-a^2*(x+1/a)^2+2*a*(
x+1/a))^(3/2)-9/2*c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-9/2*c/(a^2)^(1/2)*
arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

mupad [B] time = 0.04, size = 82, normalized size = 1.11

$$\frac{c\sqrt{1-a^2\,x^2}}{a^2\,x}-\frac{3\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{c\sqrt{1-a^2\,x^2}}{a}+\frac{c\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,3\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (c*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/a - (3*c*asinh(x*(-a^2)^(1/2)))/(-a^2)^
(1/2) - (c*(1 - a^2*x^2)^(1/2))/a + (c*(1 - a^2*x^2)^(1/2))/(a^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\left(-\frac{\sqrt{-a^2x^2+1}}{a^2x^4+2ax^3+x^2}\right)dx+\int\frac{ax\sqrt{-a^2x^2+1}}{a^2x^4+2ax^3+x^2}\,dx+\int\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^2x^4+2ax^3+x^2}\,dx+\int\left(-\frac{a^3x^3\sqrt{-a^2x^2+1}}{a^2x^4+2ax^3+x^2}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c*(Integral(-sqrt(-a**2*x**2 + 1)/(a**2*x**4 + 2*a*x**3 + x**2), x) + Integ
ral(a*x*sqrt(-a**2*x**2 + 1)/(a**2*x**4 + 2*a*x**3 + x**2), x) + Integral(a
**2*x**2*sqrt(-a**2*x**2 + 1)/(a**2*x**4 + 2*a*x**3 + x**2), x) + Integral(
-a**3*x**3*sqrt(-a**2*x**2 + 1)/(a**2*x**4 + 2*a*x**3 + x**2), x))/a**2
```

3.682 $\int \frac{e^{-3\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=97

$$\frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}-\frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}$$

```
[Out] 1/3*(-a*x+1)^3/a/c/(-a^2*x^2+1)^(3/2)-3*arcsin(a*x)/a/c-2*(-a*x+1)^2/a/c/(-
a^2*x^2+1)^(1/2)-3*(-a^2*x^2+1)^(1/2)/a/c
```

Rubi [A] time = 0.23, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6157, 6149, 1635, 21, 669, 641, 216}

$$\frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}}-\frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{ac}-\frac{3\sin^{-1}(ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] (1 - a*x)^3/(3*a*c*(1 - a^2*x^2)^(3/2)) - (2*(1 - a*x)^2)/(a*c*Sqrt[1 - a^2
*x^2]) - (3*Sqrt[1 - a^2*x^2])/(a*c) - (3*ArcSin[a*x])/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{a^2\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}$$
$$= -\frac{a^2\int \frac{x^2(1-ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c}$$
$$= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} + \frac{a^2\int \frac{\left(\frac{3}{a^2}-\frac{3x}{a}\right)(1-ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c}$$
$$= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}$$
$$= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} - \frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}} - \frac{3\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} - \frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{ac} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c}$$
$$= \frac{(1-ax)^3}{3ac\left(1-a^2x^2\right)^{3/2}} - \frac{2(1-ax)^2}{ac\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{ac} - \frac{3\sin^{-1}(ax)}{ac}$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.80

$$\frac{3a^3x^3+16a^2x^2-9(ax+1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-5ax-14}{3ac(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] (-14 - 5*a*x + 16*a^2*x^2 + 3*a^3*x^3 - 9*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSi
n[a*x])/(3*a*c*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.63, size = 101, normalized size = 1.04

$$-\frac{14\,a^2x^2+28\,ax-18\left(a^2x^2+2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(3\,a^2x^2+19\,ax+14\right)\sqrt{-a^2x^2+1}\,+14}{3\left(a^3cx^2+2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas
")
```

```
[Out] -1/3*(14*a^2*x^2 + 28*a*x - 18*(a^2*x^2 + 2*a*x + 1)*arctan((sqrt(-a^2*x^2
+ 1) - 1)/(a*x)) + (3*a^2*x^2 + 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^3*
c*x^2 + 2*a^2*c*x + a*c)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 330, normalized size = 3.40

$$\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{48ac}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}\,x}{32c}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{32c\sqrt{a^2}}-\frac{47\left(-a^2\left(x+\frac{1}{a}\right)^2\right.}{24c\,a^3\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x)
```

```
[Out] 1/48/a/c*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-1/32/c*(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(1/2)*x-1/32/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x
-1/a))^(1/2))-47/24/c/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-95/4
8/a/c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)-95/32/c*(-a^2*(x+1/a)^2+2*a*(x+1/a
))^(1/2)*x-95/32/c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+
1/a))^(1/2))+1/6/a^5/c/(x+1/a)^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-11/12/a
^4/c/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))), x)
```

mupad [B] time = 0.85, size = 129, normalized size = 1.33

$$\frac{2\,a\,\sqrt{1-a^2\,x^2}}{3\,\left(c\,a^4\,x^2+2\,c\,a^3\,x+c\,a^2\right)}+\frac{13\,\sqrt{1-a^2\,x^2}}{3\,\left(\frac{c\,\sqrt{-a^2}}{a}+c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))*(a*x + 1)^3),x)
```

```
[Out] (2*a*(1 - a^2*x^2)^(1/2))/(3*(a^2*c + a^4*c*x^2 + 2*a^3*c*x)) + (13*(1 - a^
2*x^2)^(1/2))/(3*((c*(-a^2)^(1/2))/a + c*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (3
*asinh(x*(-a^2)^(1/2)))/(c*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{2}\left(\int \frac{x^{2}\sqrt{-a^{2}x^{2}+1}}{a^{5}x^{5}+3a^{4}x^{4}+2a^{3}x^{3}-2a^{2}x^{2}-3ax-1}\,dx+\int \left(-\frac{a^{2}x^{4}\sqrt{-a^{2}x^{2}+1}}{a^{5}x^{5}+3a^{4}x^{4}+2a^{3}x^{3}-2a^{2}x^{2}-3ax-1}\right)dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 + 3*a**4*x**4 + 2*a**3*
x**3 - 2*a**2*x**2 - 3*a*x - 1), x) + Integral(-a**2*x**4*sqrt(-a**2*x**2 +
 1)/(a**5*x**5 + 3*a**4*x**4 + 2*a**3*x**3 - 2*a**2*x**2 - 3*a*x - 1), x))/
c
```

3.683 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=129

$$-\frac{(1-ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}+\frac{6(1-ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\sin^{-1}(ax)}{ac^2}$$

```
[Out] -1/5*(-a*x+1)^3/a/c^2/(-a^2*x^2+1)^(5/2)+6/5*(-a*x+1)^2/a/c^2/(-a^2*x^2+1)^
(3/2)-3*arcsin(a*x)/a/c^2-24/5*(-a*x+1)/a/c^2/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+
1)^(1/2)/a/c^2
```

Rubi [A] time = 0.36, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6149, 1635, 641, 216}

$$-\frac{(1-ax)^3}{5ac^2\left(1-a^2x^2\right)^{5/2}}+\frac{6(1-ax)^2}{5ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{ac^2}-\frac{3\sin^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] -(1 - a*x)^3/(5*a*c^2*(1 - a^2*x^2)^(5/2)) + (6*(1 - a*x)^2)/(5*a*c^2*(1 -
a^2*x^2)^(3/2)) - (24*(1 - a*x))/(5*a*c^2*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2
*x^2]/(a*c^2) - (3*ArcSin[a*x])/(a*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
```

```
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{-3\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}$$

$$= \frac{a^4\int \frac{x^4(1-ax)^3}{(1-a^2x^2)^{7/2}}\,dx}{c^2}$$

$$= -\frac{(1-ax)^3}{5ac^2(1-a^2x^2)^{5/2}} - \frac{a^4\int \frac{(1-ax)^2\left(\frac{3}{a^4}-\frac{5x}{a^3}+\frac{5x^2}{a^2}-\frac{5x^3}{a}\right)}{(1-a^2x^2)^{5/2}}\,dx}{5c^2}$$

$$= -\frac{(1-ax)^3}{5ac^2(1-a^2x^2)^{5/2}} + \frac{6(1-ax)^2}{5ac^2(1-a^2x^2)^{3/2}} + \frac{a^4\int \frac{(1-ax)\left(\frac{27}{a^4}-\frac{30x}{a^3}+\frac{15x^2}{a^2}\right)}{(1-a^2x^2)^{3/2}}\,dx}{15c^2}$$

$$= -\frac{(1-ax)^3}{5ac^2(1-a^2x^2)^{5/2}} + \frac{6(1-ax)^2}{5ac^2(1-a^2x^2)^{3/2}} - \frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}} - \frac{a^4\int \frac{\frac{45}{a^4}-\frac{15x}{a^3}}{\sqrt{1-a^2x^2}}\,dx}{15c^2}$$

$$= -\frac{(1-ax)^3}{5ac^2(1-a^2x^2)^{5/2}} + \frac{6(1-ax)^2}{5ac^2(1-a^2x^2)^{3/2}} - \frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{ac^2} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= -\frac{(1-ax)^3}{5ac^2(1-a^2x^2)^{5/2}} + \frac{6(1-ax)^2}{5ac^2(1-a^2x^2)^{3/2}} - \frac{24(1-ax)}{5ac^2\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{ac^2} - \frac{3\sin^{-1}(ax)}{ac^2}$$

Mathematica [A] time = 0.11, size = 86, normalized size = 0.67

$$\frac{5a^4x^4+34a^3x^3+18a^2x^2-15(ax+1)^2\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-33ax-24}{5a\sqrt{1-a^2x^2}\,(acx+c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (-24 - 33*a*x + 18*a^2*x^2 + 34*a^3*x^3 + 5*a^4*x^4 - 15*(1 + a*x)^2*Sqrt[1
 - a^2*x^2]*ArcSin[a*x])/(5*a*(c + a*c*x)^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.88, size = 142, normalized size = 1.10

$$-\frac{24\,a^{3}x^{3}+72\,a^{2}x^{2}+72\,ax-30\left(a^{3}x^{3}+3\,a^{2}x^{2}+3\,ax+1\right)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)+\left(5\,a^{3}x^{3}+39\,a^{2}x^{2}+57\,ax+24\right.}{5\left(a^{4}c^{2}x^{3}+3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x+ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fric
as")
```

```
[Out] -1/5*(24*a^3*x^3 + 72*a^2*x^2 + 72*a*x - 30*(a^3*x^3 + 3*a^2*x^2 + 3*a*x +
1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (5*a^3*x^3 + 39*a^2*x^2 + 57*a*
x + 24)*sqrt(-a^2*x^2 + 1) + 24)/(a^4*c^2*x^3 + 3*a^3*c^2*x^2 + 3*a^2*c^2*x
 + a*c^2)
```

giac [A] time = 0.24, size = 181, normalized size = 1.40

$$-\frac{3\,\arcsin\left(ax\right)\mathrm{sgn}(a)}{c^{2}|a|}-\frac{\sqrt{-a^{2}x^{2}+1}}{ac^{2}}+\frac{2\left(\frac{80\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)}{a^{2}x}+\frac{120\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{2}}{a^{4}x^{2}}+\frac{70\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{3}}{a^{6}x^{3}}+\frac{15\left(\sqrt{-a^{2}x^{2}+1}\right.}{a^{8}x^{4}}\right.}{5\,c^{2}\left(\frac{\sqrt{-a^{2}x^{2}+1}\,|a|+a}{a^{2}x}+1\right)^{5}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac
")
```

```
[Out] -3*arcsin(a*x)*sgn(a)/(c^2*abs(a)) - sqrt(-a^2*x^2 + 1)/(a*c^2) + 2/5*(80*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 120*(sqrt(-a^2*x^2 + 1)*abs(a) + a
)^2/(a^4*x^2) + 70*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) + 15*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 19)/(c^2*((sqrt(-a^2*x^2 + 1)*abs(a)
 + a)/(a^2*x) + 1)^5*abs(a))
```

maple [B] time = 0.06, size = 412, normalized size = 3.19

$$-\frac{\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{64a\,c^{2}}-\frac{2\left(-a^{2}\left(x+\frac{1}{a}\right)^{2}+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^{3}c^{2}\left(x+\frac{1}{a}\right)^{2}}-\frac{387\sqrt{-a^{2}\left(x+\frac{1}{a}\right)^{2}+2a\left(x+\frac{1}{a}\right)}\,x}{128c^{2}}-\frac{387\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/64/a/c^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-2/a^3/c^2/(x+1/a)^2*(-a^2*(x
+1/a)^2+2*a*(x+1/a))^(5/2)-387/128/c^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x
-387/128/c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^
(1/2))-1/32/a^3/c^2/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+3/128/c^2*
(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x+3/128/c^2/(a^2)^(1/2)*arctan((a^2)^(1/
2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-1/20/a^6/c^2/(x+1/a)^5*(-a^2*(x+1/
a)^2+2*a*(x+1/a))^(5/2)+1/4/a^5/c^2/(x+1/a)^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^
(5/2)-15/16/a^4/c^2/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-129/64/a/c
^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^2), x)
```

mupad [B] time = 0.06, size = 271, normalized size = 2.10

$$\frac{4\,a\,\sqrt{1-a^2\,x^2}}{3\,\left(a^4\,c^2\,x^2+2\,a^3\,c^2\,x+a^2\,c^2\right)}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^2\,\sqrt{-a^2}}-\frac{2\,a^4\,\sqrt{1-a^2\,x^2}}{15\,\left(a^7\,c^2\,x^2+2\,a^6\,c^2\,x+a^5\,c^2\right)}-\frac{\sqrt{1-a^2\,x^2}}{a\,c^2}+\frac{24\,\sqrt{1}}{5\,\sqrt{-a^2}\,\left(c^2\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^2*(a*x + 1)^3),x)
```

```
[Out] (4*a*(1 - a^2*x^2)^(1/2))/(3*(a^2*c^2 + 2*a^3*c^2*x + a^4*c^2*x^2)) - (3*as
inh(x*(-a^2)^(1/2)))/(c^2*(-a^2)^(1/2)) - (2*a^4*(1 - a^2*x^2)^(1/2))/(15*(
a^5*c^2 + 2*a^6*c^2*x + a^7*c^2*x^2)) - (1 - a^2*x^2)^(1/2)/(a*c^2) + (24*(
1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) + (c^2*(-a^2)^(1/2)
)/a)) + (1 - a^2*x^2)^(1/2)/(5*(-a^2)^(1/2)*(3*c^2*x*(-a^2)^(1/2) + (c^2*(-
a^2)^(1/2))/a + a^2*c^2*x^3*(-a^2)^(1/2) + 3*a*c^2*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\left(\int \frac{x^4\sqrt{-a^2x^2+1}}{a^7x^7+3a^6x^6+a^5x^5-5a^4x^4-5a^3x^3+a^2x^2+3ax+1}\,dx+\int\left(-\frac{a^2x^6\sqrt{-a^2x^2+1}}{a^7x^7+3a^6x^6+a^5x^5-5a^4x^4-5a^3x^3+a^2x^2+3ax+1}\right)dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(Integral(x**4*sqrt(-a**2*x**2 + 1)/(a**7*x**7 + 3*a**6*x**6 + a**5*x*
*5 - 5*a**4*x**4 - 5*a**3*x**3 + a**2*x**2 + 3*a*x + 1), x) + Integral(-a**
2*x**6*sqrt(-a**2*x**2 + 1)/(a**7*x**7 + 3*a**6*x**6 + a**5*x**5 - 5*a**4*x
**4 - 5*a**3*x**3 + a**2*x**2 + 3*a*x + 1), x))/c**2
```

3.684 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=159

$$\frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}}-\frac{38(1-ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{137(1-ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{ac^3}$$

```
[Out] 1/7*(-a*x+1)^3/a/c^3/(-a^2*x^2+1)^(7/2)-38/35*(-a*x+1)^2/a/c^3/(-a^2*x^2+1)
^(5/2)+137/35*(-a*x+1)/a/c^3/(-a^2*x^2+1)^(3/2)-3*arcsin(a*x)/a/c^3+1/35*(1
81*a*x-245)/a/c^3/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a/c^3
```

Rubi [A] time = 0.45, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6149, 1635, 1814, 641, 216}

$$\frac{(1-ax)^3}{7ac^3\left(1-a^2x^2\right)^{7/2}}-\frac{38(1-ax)^2}{35ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{137(1-ax)}{35ac^3\left(1-a^2x^2\right)^{3/2}}-\frac{\sqrt{1-a^2x^2}}{ac^3}-\frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}}-\frac{3\sin^{-1}(ax)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (1 - a*x)^3/(7*a*c^3*(1 - a^2*x^2)^(7/2)) - (38*(1 - a*x)^2)/(35*a*c^3*(1 -
 a^2*x^2)^(5/2)) + (137*(1 - a*x))/(35*a*c^3*(1 - a^2*x^2)^(3/2)) - (245 -
181*a*x)/(35*a*c^3*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(a*c^3) - (3*ArcS
in[a*x])/(a*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
```

```
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{a^6\int \frac{e^{-3\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}$$

$$= -\frac{a^6\int \frac{x^6(1-ax)^3}{(1-a^2x^2)^{9/2}}\,dx}{c^3}$$

$$= \frac{(1-ax)^3}{7ac^3(1-a^2x^2)^{7/2}} + \frac{a^6\int \frac{(1-ax)^2\left(\frac{3}{a^6}-\frac{7x}{a^5}+\frac{7x^2}{a^4}-\frac{7x^3}{a^3}+\frac{7x^4}{a^2}-\frac{7x^5}{a}\right)}{(1-a^2x^2)^{7/2}}\,dx}{7c^3}$$

$$= \frac{(1-ax)^3}{7ac^3(1-a^2x^2)^{7/2}} - \frac{38(1-ax)^2}{35ac^3(1-a^2x^2)^{5/2}} - \frac{a^6\int \frac{(1-ax)\left(\frac{61}{a^6}-\frac{140x}{a^5}+\frac{105x^2}{a^4}-\frac{70x^3}{a^3}+\frac{35x^4}{a^2}\right)}{(1-a^2x^2)^{5/2}}\,dx}{35c^3}$$

$$= \frac{(1-ax)^3}{7ac^3(1-a^2x^2)^{7/2}} - \frac{38(1-ax)^2}{35ac^3(1-a^2x^2)^{5/2}} + \frac{137(1-ax)}{35ac^3(1-a^2x^2)^{3/2}} + \frac{a^6\int \frac{\frac{228}{a^6}-\frac{630x}{a^5}+\frac{315x^2}{a^4}-\frac{105x^3}{a^3}}{(1-a^2x^2)^{3/2}}\,d}{105c^3}$$

$$= \frac{(1-ax)^3}{7ac^3(1-a^2x^2)^{7/2}} - \frac{38(1-ax)^2}{35ac^3(1-a^2x^2)^{5/2}} + \frac{137(1-ax)}{35ac^3(1-a^2x^2)^{3/2}} - \frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}} - \frac{a^6\int}{1}$$

$$= \frac{(1-ax)^3}{7ac^3(1-a^2x^2)^{7/2}} - \frac{38(1-ax)^2}{35ac^3(1-a^2x^2)^{5/2}} + \frac{137(1-ax)}{35ac^3(1-a^2x^2)^{3/2}} - \frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-}}{ac}$$

$$= \frac{(1-ax)^3}{7ac^3(1-a^2x^2)^{7/2}} - \frac{38(1-ax)^2}{35ac^3(1-a^2x^2)^{5/2}} + \frac{137(1-ax)}{35ac^3(1-a^2x^2)^{3/2}} - \frac{245-181ax}{35ac^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-}}{ac}$$

Mathematica [A] time = 0.13, size = 94, normalized size = 0.59

$$\frac{35a^5x^5+286a^4x^4+368a^3x^3-125a^2x^2-105(ax+1)^3\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-423ax-176}{35a\sqrt{1-a^2x^2}\,(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^3),x]
```

[Out] (-176 - 423*a*x - 125*a^2*x^2 + 368*a^3*x^3 + 286*a^4*x^4 + 35*a^5*x^5 - 10
5*(1 + a*x)^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(35*a*(c + a*c*x)^3*Sqrt[1 - a
^2*x^2])

fricas [A] time = 0.57, size = 213, normalized size = 1.34

$$-\frac{176\,a^5x^5+528\,a^4x^4+352\,a^3x^3-352\,a^2x^2-528\,ax-210\left(a^5x^5+3\,a^4x^4+2\,a^3x^3-2\,a^2x^2-3\,ax-1\right)\arctan\left(\right.}{35\left(a^6c^3x^5+3\,a^5c^3x^4+2\,a^4c^3x^3-2\,a^3c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fric
as")
```

```
[Out] -1/35*(176*a^5*x^5 + 528*a^4*x^4 + 352*a^3*x^3 - 352*a^2*x^2 - 528*a*x - 21
0*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*arctan((sqrt(-a
^2*x^2 + 1) - 1)/(a*x)) + (35*a^5*x^5 + 286*a^4*x^4 + 368*a^3*x^3 - 125*a^2
*x^2 - 423*a*x - 176)*sqrt(-a^2*x^2 + 1) - 176)/(a^6*c^3*x^5 + 3*a^5*c^3*x^
4 + 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - 3*a^2*c^3*x - a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^3), x)
```

maple [B] time = 0.07, size = 494, normalized size = 3.11

$$-\frac{529\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{256a\,c^3}+\frac{5\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{16a^5c^3\left(x+\frac{1}{a}\right)^4}-\frac{31\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{32a^4c^3\left(x+\frac{1}{a}\right)^3}-\frac{263\left(-a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -529/256/a/c^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+5/16/a^5/c^3/(x+1/a)^4*(-
a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-31/32/a^4/c^3/(x+1/a)^3*(-a^2*(x+1/a)^2+2*
a*(x+1/a))^(5/2)-263/128/a^3/c^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/
2)-1587/512/c^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x-1587/512/c^3/(a^2)^(1/
2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))+1/56/a^7/c^3/(x
+1/a)^6*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-61/560/a^6/c^3/(x+1/a)^5*(-a^2*(
x+1/a)^2+2*a*(x+1/a))^(5/2)-5/64/a^3/c^3/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(5/2)+51/512/c^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x+51/512/c^3/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+1/64/a^4/c^
3/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-17/256/a/c^3*(-a^2*(x-1/a)^2
-2*a*(x-1/a))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^3), x)
```

mupad [B] time = 1.44, size = 436, normalized size = 2.74

$$\frac{49\,a\,\sqrt{1-a^2\,x^2}}{24\,\left(a^4\,c^3\,x^2+2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{c^3\,\sqrt{-a^2}}+\frac{a^3\,\sqrt{1-a^2\,x^2}}{35\,\left(a^6\,c^3\,x^2+2\,a^5\,c^3\,x+a^4\,c^3\right)}-\frac{11\,a^6\,\sqrt{1-a^2\,x^2}}{30\,\left(a^9\,c^3\,x^2+2\,a^8\,c^3\,x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^3*(a*x + 1)^3),x)
```

```
[Out] (49*a*(1 - a^2*x^2)^(1/2))/(24*(a^2*c^3 + 2*a^3*c^3*x + a^4*c^3*x^2)) - (3*
asinh(x*(-a^2)^(1/2)))/(c^3*(-a^2)^(1/2)) + (a^3*(1 - a^2*x^2)^(1/2))/(35*(
a^4*c^3 + 2*a^5*c^3*x + a^6*c^3*x^2)) - (11*a^6*(1 - a^2*x^2)^(1/2))/(30*(a
^7*c^3 + 2*a^8*c^3*x + a^9*c^3*x^2)) - (1 - a^2*x^2)^(1/2)/(a*c^3) + (a*(1
- a^2*x^2)^(1/2))/(14*(a^2*c^3 + 4*a^3*c^3*x + 6*a^4*c^3*x^2 + 4*a^5*c^3*x^
3 + a^6*c^3*x^4)) + (2931*(1 - a^2*x^2)^(1/2))/(560*(-a^2)^(1/2)*(c^3*x*(-a
^2)^(1/2) + (c^3*(-a^2)^(1/2))/a)) - (1 - a^2*x^2)^(1/2)/(16*(-a^2)^(1/2)*(
c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a)) + (71*(1 - a^2*x^2)^(1/2))/(140
*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2) + (c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-
a^2)^(1/2) + 3*a*c^3*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6\left(\int \frac{x^6\sqrt{-a^2x^2+1}}{a^9x^9+3a^8x^8-8a^6x^6-6a^5x^5+6a^4x^4+8a^3x^3-3ax-1}\,dx+\int \left(-\frac{a^2x^8\sqrt{-a^2x^2+1}}{a^9x^9+3a^8x^8-8a^6x^6-6a^5x^5+6a^4x^4+8a^3x^3-3ax-1}\right)dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*(Integral(x**6*sqrt(-a**2*x**2 + 1)/(a**9*x**9 + 3*a**8*x**8 - 8*a**6*
x**6 - 6*a**5*x**5 + 6*a**4*x**4 + 8*a**3*x**3 - 3*a*x - 1), x) + Integral(
-a**2*x**8*sqrt(-a**2*x**2 + 1)/(a**9*x**9 + 3*a**8*x**8 - 8*a**6*x**6 - 6*
a**5*x**5 + 6*a**4*x**4 + 8*a**3*x**3 - 3*a*x - 1), x))/c**3
```

3.685 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=189

$$-\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}}+\frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}}-\frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{ac^4}-\frac{4(630-431ax)}{315ac^4\sqrt{1-a^2x^2}}+\frac{2(1155-829a}{315ac^4\left(1-a^2x\right.}$$

```
[Out] -1/9*(-a*x+1)^3/a/c^4/(-a^2*x^2+1)^(9/2)+22/21*(-a*x+1)^2/a/c^4/(-a^2*x^2+1
)^(7/2)-478/105*(-a*x+1)/a/c^4/(-a^2*x^2+1)^(5/2)+2/315*(-829*a*x+1155)/a/c
^4/(-a^2*x^2+1)^(3/2)-3*arcsin(a*x)/a/c^4-4/315*(-431*a*x+630)/a/c^4/(-a^2*
x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a/c^4
```

Rubi [A] time = 0.64, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6157, 6149, 1635, 1814, 641, 216}

$$-\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}}+\frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}}-\frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}}-\frac{\sqrt{1-a^2x^2}}{ac^4}-\frac{4(630-431ax)}{315ac^4\sqrt{1-a^2x^2}}+\frac{2(1155-829a}{315ac^4\left(1-a^2x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] -(1 - a*x)^3/(9*a*c^4*(1 - a^2*x^2)^(9/2)) + (22*(1 - a*x)^2)/(21*a*c^4*(1
- a^2*x^2)^(7/2)) - (478*(1 - a*x))/(105*a*c^4*(1 - a^2*x^2)^(5/2)) + (2*(1
155 - 829*a*x))/(315*a*c^4*(1 - a^2*x^2)^(3/2)) - (4*(630 - 431*a*x))/(315*
a*c^4*Sqrt[1 - a^2*x^2]) - Sqrt[1 - a^2*x^2]/(a*c^4) - (3*ArcSin[a*x])/(a*c
^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \frac{a^8\int \frac{e^{-3\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= \frac{a^8\int \frac{x^8(1-ax)^3}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^4}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} - \frac{a^8\int \frac{(1-ax)^2\left(\frac{3}{a^8}-\frac{9x}{a^7}+\frac{9x^2}{a^6}-\frac{9x^3}{a^5}+\frac{9x^4}{a^4}-\frac{9x^5}{a^3}+\frac{9x^6}{a^2}-\frac{9x^7}{a}\right)}{\left(1-a^2x^2\right)^{9/2}}\,dx}{9c^4}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{a^8\int \frac{(1-ax)\left(\frac{111}{a^8}-\frac{378x}{a^7}+\frac{315x^2}{a^6}-\frac{252x^3}{a^5}+\frac{189x^4}{a^4}-\frac{126x^5}{a^3}+\frac{63x^6}{a^2}\right)}{\left(1-a^2x^2\right)^{7/2}}}{63c^4}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} - \frac{a^8\int \frac{\frac{879}{a^8}-\frac{4725x}{a^7}+\frac{3150x^2}{a^6}-\frac{189}{a}}{\left(1-a^2x^2\right)}}{315c^4}$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} + \frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} +$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} + \frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} -$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} + \frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} -$$

$$= -\frac{(1-ax)^3}{9ac^4\left(1-a^2x^2\right)^{9/2}} + \frac{22(1-ax)^2}{21ac^4\left(1-a^2x^2\right)^{7/2}} - \frac{478(1-ax)}{105ac^4\left(1-a^2x^2\right)^{5/2}} + \frac{2(1155-829ax)}{315ac^4\left(1-a^2x^2\right)^{3/2}} -$$

Mathematica [A] time = 0.16, size = 122, normalized size = 0.65

$$\frac{315a^7x^7+2669a^6x^6+2967a^5x^5-4029a^4x^4-7399a^3x^3-339a^2x^2-945(ax-1)(ax+1)^4\sqrt{1-a^2x^2}\,\sin^{-1}(ax)}{315a(ax-1)\sqrt{1-a^2x^2}\,(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (1664 + 4047*a*x - 339*a^2*x^2 - 7399*a^3*x^3 - 4029*a^4*x^4 + 2967*a^5*x^5
 + 2669*a^6*x^6 + 315*a^7*x^7 - 945*(-1 + a*x)*(1 + a*x)^4*Sqrt[1 - a^2*x^2
]*ArcSin[a*x])/(315*a*(-1 + a*x)*(c + a*c*x)^4*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.49, size = 278, normalized size = 1.47

$$-\frac{1664\,a^7x^7+4992\,a^6x^6+1664\,a^5x^5-8320\,a^4x^4-8320\,a^3x^3+1664\,a^2x^2+4992\,ax-1890\left(a^7x^7+3\,a^6x^6+a^5x\right.}{315\left(a^8c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fric
as")
```

```
[Out] -1/315*(1664*a^7*x^7 + 4992*a^6*x^6 + 1664*a^5*x^5 - 8320*a^4*x^4 - 8320*a^
3*x^3 + 1664*a^2*x^2 + 4992*a*x - 1890*(a^7*x^7 + 3*a^6*x^6 + a^5*x^5 - 5*a
^4*x^4 - 5*a^3*x^3 + a^2*x^2 + 3*a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(
a*x)) + (315*a^7*x^7 + 2669*a^6*x^6 + 2967*a^5*x^5 - 4029*a^4*x^4 - 7399*a^
3*x^3 - 339*a^2*x^2 + 4047*a*x + 1664)*sqrt(-a^2*x^2 + 1) + 1664)/(a^8*c^4*
x^7 + 3*a^7*c^4*x^6 + a^6*c^4*x^5 - 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 + a^3*c^4
*x^2 + 3*a^2*c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^4), x)
```

maple [B] time = 0.08, size = 576, normalized size = 3.05

$$-\frac{\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{144a^8c^4\left(x+\frac{1}{a}\right)^7}+\frac{13\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{252a^7c^4\left(x+\frac{1}{a}\right)^6}-\frac{1629\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}\right)}{512c^4\sqrt{a^2}}-\frac{811\left(-a^2\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x)

[Out] -1/144/a^8/c^4/(x+1/a)^7*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+13/252/a^7/c^4/(x+1/a)^6*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-1629/512/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-811/384/a^3/c^4/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-1629/512/c^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+1/384/a^5/c^4/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+29/768/a^4/c^4/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-25/192/a^3/c^4/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)+93/512/c^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x+93/512/c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-1723/10080/a^6/c^4/(x+1/a)^5*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+35/96/a^5/c^4/(x+1/a)^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-769/768/a^4/c^4/(x+1/a)^3*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)-31/256/a/c^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-543/256/a/c^4*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")

[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^4), x)

mupad [B] time = 1.79, size = 671, normalized size = 3.55

$$\frac{a\sqrt{1-a^2x^2}}{96\left(a^4c^4x^2-2a^3c^4x+a^2c^4\right)}+\frac{67a\sqrt{1-a^2x^2}}{24\left(a^4c^4x^2+2a^3c^4x+a^2c^4\right)}+\frac{}{36\sqrt{-a^2}\left(5c^4x\sqrt{-a^2}+\frac{c^4\sqrt{-a^2}}{a}+10a^2c^4x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^4*(a*x + 1)^3),x)

[Out] (a*(1 - a^2*x^2)^(1/2))/(96*(a^2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) + (67*a*(1 - a^2*x^2)^(1/2))/(24*(a^2*c^4 + 2*a^3*c^4*x + a^4*c^4*x^2)) + (1 - a^2*x^2)^(1/2)/(36*(-a^2)^(1/2)*(5*c^4*x*(-a^2)^(1/2) + (c^4*(-a^2)^(1/2))/a + 10*a^2*c^4*x^3*(-a^2)^(1/2) + 5*a^3*c^4*x^4*(-a^2)^(1/2) + a^4*c^4*x^5*(-a^2)^(1/2) + 10*a*c^4*x^2*(-a^2)^(1/2))) - (3*asinh(x*(-a^2)^(1/2)))/(c^4*(-a

```
^2)^(1/2)) + (a^3*(1 - a^2*x^2)^(1/2))/(10*(a^4*c^4 + 2*a^5*c^4*x + a^6*c^4
*x^2)) - (1759*a^8*(1 - a^2*x^2)^(1/2))/(2520*(a^9*c^4 + 2*a^10*c^4*x + a^1
1*c^4*x^2)) - (1 - a^2*x^2)^(1/2)/(a*c^4) + (a*(1 - a^2*x^2)^(1/2))/(4*(a^2
*c^4 + 4*a^3*c^4*x + 6*a^4*c^4*x^2 + 4*a^5*c^4*x^3 + a^6*c^4*x^4)) + (11359
1*(1 - a^2*x^2)^(1/2))/(20160*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2) + (c^4*(-a^2
)^(1/2))/a)) - (31*(1 - a^2*x^2)^(1/2))/(192*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/
2) - (c^4*(-a^2)^(1/2))/a)) + (1507*(1 - a^2*x^2)^(1/2))/(1680*(-a^2)^(1/2)
*(3*c^4*x*(-a^2)^(1/2) + (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1/2) +
3*a*c^4*x^2*(-a^2)^(1/2))) - (a^10*(1 - a^2*x^2)^(1/2))/(63*(a^11*c^4 + 4*a
^12*c^4*x + 6*a^13*c^4*x^2 + 4*a^14*c^4*x^3 + a^15*c^4*x^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^8\left(\int \frac{x^8\sqrt{-a^2x^2+1}}{a^{11}x^{11}+3a^{10}x^{10}-a^9x^9-11a^8x^8-6a^7x^7+14a^6x^6+14a^5x^5-6a^4x^4-11a^3x^3-a^2x^2+3ax+1}\,dx + \int \left(-\frac{a^2x^{10}\sqrt{-}}{a^{11}x^{11}+3a^{10}x^{10}-a^9x^9-11a^8x^8-6a^7x^7+14a}\right.\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*(Integral(x**8*sqrt(-a**2*x**2 + 1)/(a**11*x**11 + 3*a**10*x**10 - a**
9*x**9 - 11*a**8*x**8 - 6*a**7*x**7 + 14*a**6*x**6 + 14*a**5*x**5 - 6*a**4*
x**4 - 11*a**3*x**3 - a**2*x**2 + 3*a*x + 1), x) + Integral(-a**2*x**10*sqr
t(-a**2*x**2 + 1)/(a**11*x**11 + 3*a**10*x**10 - a**9*x**9 - 11*a**8*x**8 -
 6*a**7*x**7 + 14*a**6*x**6 + 14*a**5*x**5 - 6*a**4*x**4 - 11*a**3*x**3 - a
**2*x**2 + 3*a*x + 1), x))/c**4
```

3.686 $\int e^{\tanh^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^{9/2} dx$

Optimal. Leaf size=374

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{7\left(1-a^2x^2\right)^{9/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{8\left(1-a^2x^2\right)^{9/2}}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{3\left(1-a^2x^2\right)^{9/2}}+\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{a^8x^9\log(x)\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{4a^7x^8\left(c-\right.}{\left(1-a^2\right.}$$

```
[Out] -1/8*(c-c/a^2/x^2)^(9/2)*x/(-a^2*x^2+1)^(9/2)-1/7*a*(c-c/a^2/x^2)^(9/2)*x^2
/(-a^2*x^2+1)^(9/2)+2/3*a^2*(c-c/a^2/x^2)^(9/2)*x^3/(-a^2*x^2+1)^(9/2)+4/5*
a^3*(c-c/a^2/x^2)^(9/2)*x^4/(-a^2*x^2+1)^(9/2)-3/2*a^4*(c-c/a^2/x^2)^(9/2)*
x^5/(-a^2*x^2+1)^(9/2)-2*a^5*(c-c/a^2/x^2)^(9/2)*x^6/(-a^2*x^2+1)^(9/2)+2*a
^6*(c-c/a^2/x^2)^(9/2)*x^7/(-a^2*x^2+1)^(9/2)+4*a^7*(c-c/a^2/x^2)^(9/2)*x^8
/(-a^2*x^2+1)^(9/2)+a^9*(c-c/a^2/x^2)^(9/2)*x^10/(-a^2*x^2+1)^(9/2)+a^8*(c-
c/a^2/x^2)^(9/2)*x^9*ln(x)/(-a^2*x^2+1)^(9/2)
```

Rubi [A] time = 0.20, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{4a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}+\frac{4a^3x^4\left(c-\right.}{5\left(1-a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) - (a*(c - c/(a^2*x^2))
^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) + (2*a^2*(c - c/(a^2*x^2))^(9/2)*x^3)/(
3*(1 - a^2*x^2)^(9/2)) + (4*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)/(5*(1 - a^2*x^
2)^(9/2)) - (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*x^2)^(9/2)) - (
2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) + (2*a^6*(c - c/(a^2
*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) + (4*a^7*(c - c/(a^2*x^2))^(9/2)*x^8)
/(1 - a^2*x^2)^(9/2) + (a^9*(c - c/(a^2*x^2))^(9/2)*x^10)/(1 - a^2*x^2)^(9/
2) + (a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log[x])/(1 - a^2*x^2)^(9/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}}{x^9}\,dx}{\left(1-a^2x^2\right)^{9/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^4(1+ax)^5}{x^9}\,dx}{\left(1-a^2x^2\right)^{9/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\left(a^9+\frac{1}{x^9}+\frac{a}{x^8}-\frac{4a^2}{x^7}-\frac{4a^3}{x^6}+\frac{6a^4}{x^5}+\frac{6a^5}{x^4}-\frac{4a^6}{x^3}-\frac{4a^7}{x^2}+\frac{a^8}{x}\right)dx}{\left(1-a^2x^2\right)^{9/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{8\left(1-a^2x^2\right)^{9/2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{7\left(1-a^2x^2\right)^{9/2}}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3}{3\left(1-a^2x^2\right)^{9/2}}+\frac{4a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5\left(1-a^2x^2\right)^{9/2}}$$

Mathematica [A] time = 0.10, size = 114, normalized size = 0.30

$$\frac{x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(a^9x+a^8\log(x)+\frac{4a^7}{x}+\frac{2a^6}{x^2}-\frac{2a^5}{x^3}-\frac{3a^4}{2x^4}+\frac{4a^3}{5x^5}+\frac{2a^2}{3x^6}-\frac{a}{7x^7}-\frac{1}{8x^8}\right)}{\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*x^9*(-1/8*1/x^8 - a/(7*x^7) + (2*a^2)/(3*x^6) + (4
*a^3)/(5*x^5) - (3*a^4)/(2*x^4) - (2*a^5)/x^3 + (2*a^6)/x^2 + (4*a^7)/x + a
^9*x + a^8*Log[x]))/(1 - a^2*x^2)^(9/2)
```

fricas [A] time = 0.70, size = 606, normalized size = 1.62

$$\left[\frac{420\left(a^{9}c^{4}x^{9}-a^{7}c^{4}x^{7}\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}+\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)-\left(840\,a^{9}c^{4}x^{9}+3360\,a^{7}c^{4}x^{7}+16\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="fric
as")
```

```
[Out] [1/840*(420*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2
 - c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - c)/(a^2*x^4 - x^2)) - (840*a^9*c^4*x^9 + 3360*a^7*c^4*x^7 + 168
0*a^6*c^4*x^6 - 1680*a^5*c^4*x^5 - (840*a^9 + 3360*a^7 + 1680*a^6 - 1680*a^
5 - 1260*a^4 + 672*a^3 + 560*a^2 - 120*a - 105)*c^4*x^8 - 1260*a^4*c^4*x^4
+ 672*a^3*c^4*x^3 + 560*a^2*c^4*x^2 - 120*a*c^4*x - 105*c^4)*sqrt(-a^2*x^2
+ 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7), -1/840*(840*(a^
9*c^4*x^9 - a^7*c^4*x^7)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sq
rt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) +
(840*a^9*c^4*x^9 + 3360*a^7*c^4*x^7 + 1680*a^6*c^4*x^6 - 1680*a^5*c^4*x^5 -
 (840*a^9 + 3360*a^7 + 1680*a^6 - 1680*a^5 - 1260*a^4 + 672*a^3 + 560*a^2 -
 120*a - 105)*c^4*x^8 - 1260*a^4*c^4*x^4 + 672*a^3*c^4*x^3 + 560*a^2*c^4*x^
2 - 120*a*c^4*x - 105*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^10*x^9 - a^8*x^7)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{9}{2}}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(9/2)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.09, size = 118, normalized size = 0.32

$$-\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{9}{2}} x\sqrt{-a^{2}x^{2}+1}\,\left(840a^{9}x^{9}+840a^{8}\ln(x)x^{8}+3360a^{7}x^{7}+1680x^{6}a^{6}-1680x^{5}a^{5}-1260x^{4}a^{4}+672x^{3}a^{3}\right.}{840\left(a^{2}x^{2}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] -1/840*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a^2*x^2-1)^5*(-a^2*x^2+1)^(1/2)*(84
0*a^9*x^9+840*a^8*ln(x)*x^8+3360*a^7*x^7+1680*x^6*a^6-1680*x^5*a^5-1260*x^4
*a^4+672*x^3*a^3+560*a^2*x^2-120*a*x-105)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{9}{2}}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(9/2)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(9/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(9/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(9/2),x)
```

```
[Out] Timed out
```

3.687 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=299

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}-\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{a^6x^7\log(x)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^5x^6\left(c-\right.}{\left(1-a^2x\right.}$$

```
[Out] -1/6*(c-c/a^2/x^2)^(7/2)*x/(-a^2*x^2+1)^(7/2)-1/5*a*(c-c/a^2/x^2)^(7/2)*x^2
/(-a^2*x^2+1)^(7/2)+3/4*a^2*(c-c/a^2/x^2)^(7/2)*x^3/(-a^2*x^2+1)^(7/2)+a^3*
(c-c/a^2/x^2)^(7/2)*x^4/(-a^2*x^2+1)^(7/2)-3/2*a^4*(c-c/a^2/x^2)^(7/2)*x^5/
(-a^2*x^2+1)^(7/2)-3*a^5*(c-c/a^2/x^2)^(7/2)*x^6/(-a^2*x^2+1)^(7/2)-a^7*(c-
c/a^2/x^2)^(7/2)*x^8/(-a^2*x^2+1)^(7/2)-a^6*(c-c/a^2/x^2)^(7/2)*x^7*ln(x)/(
-a^2*x^2+1)^(7/2)
```

Rubi [A] time = 0.19, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$-\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}+\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}-\frac{ax^2\left(c-\frac{c}{a^2x}\right.}{5\left(1-a^2x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) - (a*(c - c/(a^2*x^2))
^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) + (3*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) + (a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(1 - a^2*x^2)^(7
/2) - (3*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) - (3*a^5*
(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) - (a^7*(c - c/(a^2*x^2))^(
7/2)*x^8)/(1 - a^2*x^2)^(7/2) - (a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[x])/(1
 - a^2*x^2)^(7/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx &= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}\\
&= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^3(1+ax)^4}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}\\
&= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\left(-a^7+\frac{1}{x^7}+\frac{a}{x^6}-\frac{3a^2}{x^5}-\frac{3a^3}{x^4}+\frac{3a^4}{x^3}+\frac{3a^5}{x^2}-\frac{a^6}{x}\right)dx}{\left(1-a^2x^2\right)^{7/2}}\\
&= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{6\left(1-a^2x^2\right)^{7/2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{5\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{4\left(1-a^2x^2\right)^{7/2}}+\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{\left(1-a^2x^2\right)^{7/2}}-
\end{aligned}$$

Mathematica [A] time = 0.06, size = 98, normalized size = 0.33

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(60a^7x^7+60a^6x^6\log(x)+180a^5x^5+90a^4x^4-60a^3x^3-45a^2x^2+12ax+10\right)}{60a^6x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(10 + 12*a*x - 45*a^2*x^2 - 60*a^3*x^3 + 90*a^4*
x^4 + 180*a^5*x^5 + 60*a^7*x^7 + 60*a^6*x^6*Log[x]))/(60*a^6*x^5*Sqrt[1 - a
^2*x^2])
```

fricas [A] time = 0.79, size = 542, normalized size = 1.81

$$\left[\frac{30\left(a^{7}c^{3}x^{7}-a^{5}c^{3}x^{5}\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}+\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)-\left(60\,a^{7}c^{3}x^{7}+180\,a^{5}c^{3}x^{5}+90\,a^{4}c\right.}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="fric
as")
```

```
[Out] [1/60*(30*(a^7*c^3*x^7 - a^5*c^3*x^5)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
 c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) - c)/(a^2*x^4 - x^2)) - (60*a^7*c^3*x^7 + 180*a^5*c^3*x^5 + 90*a^4*
c^3*x^4 - (60*a^7 + 180*a^5 + 90*a^4 - 60*a^3 - 45*a^2 + 12*a + 10)*c^3*x^6
 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 12*a*c^3*x + 10*c^3)*sqrt(-a^2*x^2 + 1
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7 - a^6*x^5), -1/60*(60*(a^7*c^3*
x^7 - a^5*c^3*x^5)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt(c)*
sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (60*a^
7*c^3*x^7 + 180*a^5*c^3*x^5 + 90*a^4*c^3*x^4 - (60*a^7 + 180*a^5 + 90*a^4 -
 60*a^3 - 45*a^2 + 12*a + 10)*c^3*x^6 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 1
2*a*c^3*x + 10*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^
8*x^7 - a^6*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.05, size = 102, normalized size = 0.34

$$-\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}} x\sqrt{-a^{2}x^{2}+1}\,\left(60a^{7}x^{7}+60a^{6}\ln(x)x^{6}+180x^{5}a^{5}+90x^{4}a^{4}-60x^{3}a^{3}-45a^{2}x^{2}+12ax+10\right)}{60\left(a^{2}x^{2}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a^2*x^2-1)^4*(-a^2*x^2+1)^(1/2)*(60*
a^7*x^7+60*a^6*ln(x)*x^6+180*x^5*a^5+90*x^4*a^4-60*x^3*a^3-45*a^2*x^2+12*a*
x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,(a\,x+1)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x + 1)/sqrt(-(a*x - 1)
*(a*x + 1)), x)
```

3.688 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=219

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{3\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -1/4*(c-c/a^2/x^2)^(5/2)*x/(-a^2*x^2+1)^(5/2)-1/3*a*(c-c/a^2/x^2)^(5/2)*x^2
/(-a^2*x^2+1)^(5/2)+a^2*(c-c/a^2/x^2)^(5/2)*x^3/(-a^2*x^2+1)^(5/2)+2*a^3*(c
-c/a^2/x^2)^(5/2)*x^4/(-a^2*x^2+1)^(5/2)+a^5*(c-c/a^2/x^2)^(5/2)*x^6/(-a^2*
x^2+1)^(5/2)+a^4*(c-c/a^2/x^2)^(5/2)*x^5*ln(x)/(-a^2*x^2+1)^(5/2)
```

Rubi [A] time = 0.18, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{3\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) - (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(3*(1 - a^2*x^2)^(5/2)) + (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1
- a^2*x^2)^(5/2) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2)
+ (a^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) + (a^4*(c - c/(a^2*
x^2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^2(1+ax)^3}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\left(a^5+\frac{1}{x^5}+\frac{a}{x^4}-\frac{2a^2}{x^3}-\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}{4\left(1-a^2x^2\right)^{5/2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{\left(1-a^2x^2\right)^{5/2}}+$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.37

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(12a^5x^5+12a^4x^4\log(x)+24a^3x^3+12a^2x^2-4ax-3\right)}{12a^4x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(-3 - 4*a*x + 12*a^2*x^2 + 24*a^3*x^3 + 12*a^5*x
^5 + 12*a^4*x^4*Log[x]))/(12*a^4*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 1.71, size = 478, normalized size = 2.18

$$\left[\frac{6\left(a^5c^2x^5 - a^3c^2x^3\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) - \left(12\,a^5c^2x^5 + 24\,a^3c^2x^3 - \left(12\,a^5 + \right.\right.}{12\left(a^6x^5 - a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2
*x^2)) - c)/(a^2*x^4 - x^2)) - (12*a^5*c^2*x^5 + 24*a^3*c^2*x^3 - (12*a^5 +
 24*a^3 + 12*a^2 - 4*a - 3)*c^2*x^4 + 12*a^2*c^2*x^2 - 4*a*c^2*x - 3*c^2)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3), -1/
12*(12*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3
 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^
2 + c)) + (12*a^5*c^2*x^5 + 24*a^3*c^2*x^3 - (12*a^5 + 24*a^3 + 12*a^2 - 4*
a - 3)*c^2*x^4 + 12*a^2*c^2*x^2 - 4*a*c^2*x - 3*c^2)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c - \frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.05, size = 86, normalized size = 0.39

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}} x\sqrt{-a^2x^2+1}\,\left(12x^5a^5 + 12a^4\ln(x)x^4 + 24x^3a^3 + 12a^2x^2 - 4ax - 3\right)}{12\left(a^2x^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/12*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/(a^2*x^2-1)^3*(-a^2*x^2+1)^(1/2)*(12*
x^5*a^5+12*a^4*ln(x)*x^4+24*x^3*a^3+12*a^2*x^2-4*a*x-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x+1)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x + 1)/sqrt(-(a*x - 1)
*(a*x + 1)), x)
```

3.689 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=146

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -1/2*(c-c/a^2/x^2)^(3/2)*x/(-a^2*x^2+1)^(3/2)-a*(c-c/a^2/x^2)^(3/2)*x^2/(-a
^2*x^2+1)^(3/2)-a^3*(c-c/a^2/x^2)^(3/2)*x^4/(-a^2*x^2+1)^(3/2)-a^2*(c-c/a^2
/x^2)^(3/2)*x^3*ln(x)/(-a^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 75}

$$-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) - (a*(c - c/(a^2*x^2))
^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) - (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 - a^
2*x^2)^(3/2) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(3/2)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
```

```
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)(1+ax)^2}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\left(-a^3+\frac{1}{x^3}+\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{2\left(1-a^2x^2\right)^{3/2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{\left(1-a^2x^2\right)^{3/2}}-\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^4}{\left(1-a^2x^2\right)^{3/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\log(}{\left(1-a^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.49

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(2a^3x^3+3a^2x^2+2a^2x^2\log(x)+2ax+1\right)}{2a^2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(3/2),x]
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(1 + 2*a*x + 3*a^2*x^2 + 2*a^3*x^3 + 2*a^2*x^2*Log
[x]))/(2*a^2*x*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 1.53, size = 373, normalized size = 2.55

$$\left[\frac{\left(a^3cx^3-acx\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-\left(2a^3cx^3-\left(2a^3+2a+1\right)cx^2+2acx+c\right.}{2\left(a^4x^3-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^3*c*x^3 - a*c*x)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*
x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)
/(a^2*x^4 - x^2)) - (2*a^3*c*x^3 - (2*a^3 + 2*a + 1)*c*x^2 + 2*a*c*x + c)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x), -1/2*
(2*(a^3*c*x^3 - a*c*x)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt
(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (2
*a^3*c*x^3 - (2*a^3 + 2*a + 1)*c*x^2 + 2*a*c*x + c)*sqrt(-a^2*x^2 + 1)*sqrt
((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(3/2)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.05, size = 70, normalized size = 0.48

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}} x\sqrt{-a^2x^2+1}\,\left(2x^3a^3+2a^2\ln(x)x^2+2ax+1\right)}{2\left(a^2x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a^2*x^2-1)^2*(-a^2*x^2+1)^(1/2)*(2*x^
3*a^3+2*a^2*ln(x)*x^2+2*a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(3/2)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)/sqrt(-(a*x - 1)
*(a*x + 1)), x)
```

3.690 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

[Out] a*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)

Rubi [A] time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 43}

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)],x]

[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1+ax}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(a+\frac{1}{x}\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.54

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,(ax+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(a*x + Log[x]))/Sqrt[1 - a^2*x^2]
```

fricas [B] time = 0.69, size = 320, normalized size = 4.71

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-2\left(a^2x^2-a^2x\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^3x^2-a\right)}, -\frac{\left(a^2x^2 \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
```

```
x^4 - x^2)) - 2*(a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^3*x^2 - a), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)
*(a*x^3 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 +
1)*c*x^2 + c)) + (a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^3*x^2 - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.04, size = 52, normalized size = 0.76

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(ax+\ln(x)\right)\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(a*x+ln(x))*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

maxima [C] time = 0.39, size = 17, normalized size = 0.25

$$-i\sqrt{c}\,x-\frac{i\sqrt{c}\,\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -I*sqrt(c)*x - I*sqrt(c)*log(x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a
*x + 1)), x)
```

3.691 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] -(-a^2*x^2+1)^(1/2)/a/(c-c/a^2/x^2)^(1/2)-ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/a^2
/x/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 43}

$$-\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)])) - (Sqrt[1 - a^2*x^2]*Log[1 -
 a*x])/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
```

```
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{x}{1-ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\ &= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a}-\frac{1}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\ &= -\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.61

$$-\frac{\sqrt{1-a^2x^2}\,(ax+\log(1-ax))}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/Sqrt[c - c/(a^2*x^2)],x]
[Out] -((Sqrt[1 - a^2*x^2]*(a*x + Log[1 - a*x]))/(a^2*Sqrt[c - c/(a^2*x^2)]*x))
```

fricas [B] time = 1.41, size = 366, normalized size = 4.63

$$\left[-\frac{2\sqrt{-a^2x^2+1}\,a^2x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx-\left(a^5x^5-4\,a^4x^4+6\,a^3x^3-4\,a^2x^2\right)\sqrt{-a^2x^2+}}{a^4x^4-2\,a^3x^3+2\,ax-1}\right.}{2\left(a^3cx^2-ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/2*(2*sqrt(-a^2*x^2 + 1)*a^2*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + (a^2*
x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2
+ 4*a*c*x - (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2)*sqrt(-a^2*x^2 + 1
)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*
a*x - 1)))/(a^3*c*x^2 - a*c), -(sqrt(-a^2*x^2 + 1)*a^2*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2)) + (a^2*x^2 - 1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x + 2)*sqrt(-
a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 - 2*a^2*c*x
^2 - a*c*x + 2*c)))/(a^3*c*x^2 - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))), x)
```

maple [A] time = 0.04, size = 50, normalized size = 0.63

$$-\frac{\sqrt{-a^2x^2+1}\,\left(ax+\ln\left(ax-1\right)\right)}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(a*x+ln(a*x-1))/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

maxima [C] time = 0.43, size = 21, normalized size = 0.27

$$-\frac{i\,x}{\sqrt{c}}-\frac{i\,\log\left(ax-1\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -I*x/sqrt(c) - I*log(a*x - 1)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2)),x)
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(1/2),x)
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 +
1/(a*x)))), x)
```

3.692 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=175

$$\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] (-a^2*x^2+1)^(3/2)/a^3/(c-c/a^2/x^2)^(3/2)/x^2+1/2*(-a^2*x^2+1)^(3/2)/a^4/(
c-c/a^2/x^2)^(3/2)/x^3/(-a*x+1)+5/4*(-a^2*x^2+1)^(3/2)*ln(-a*x+1)/a^4/(c-c/
a^2/x^2)^(3/2)/x^3-1/4*(-a^2*x^2+1)^(3/2)*ln(a*x+1)/a^4/(c-c/a^2/x^2)^(3/2)
/x^3
```

Rubi [A] time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2) + (1 - a^2*x^2)^(3/2)
/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)) + (5*(1 - a^2*x^2)^(3/2)*Log
[1 - a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - ((1 - a^2*x^2)^(3/2)*Log[1
 + a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1-ax)}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{\left(1-a^2x^2\right)^{3/}}{4a^4\left(c-\right.}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.52

$$-\frac{\sqrt{1-a^2x^2}\left(a^2x^2-1\right)\left(\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}+\frac{x}{a^3}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(-1 + a^2*x^2)*(x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1
 - a*x])/(4*a^4) - Log[1 + a*x]/(4*a^4)))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,a^4x^4\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^4*x^4*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^5*c
^2*x^5 - a^4*c^2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

maple [A] time = 0.05, size = 93, normalized size = 0.53

$$-\frac{\left(4a^2x^2+5\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)-4ax-5\ln\left(ax-1\right)+\ln\left(ax+1\right)-2\right)\left(ax+1\right)\sqrt{-a^2x^2+1}}{4a^4x^3\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/4*(4*a^2*x^2+5*ln(a*x-1)*x*a-a*x*ln(a*x+1)-4*a*x-5*ln(a*x-1)+ln(a*x+1)-2
)*(a*x+1)*(-a^2*x^2+1)^(1/2)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a
*x)))**(3/2)), x)
```

3.693 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{23\left(1-a^2x^2\right)^{5/2}\log(1-a}{16a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

```
[Out] -(-a^2*x^2+1)^(5/2)/a^5/(c-c/a^2/x^2)^(5/2)/x^4+1/8*(-a^2*x^2+1)^(5/2)/a^6/
(c-c/a^2/x^2)^(5/2)/x^5/(-a*x+1)^2-(-a^2*x^2+1)^(5/2)/a^6/(c-c/a^2/x^2)^(5/
2)/x^5/(-a*x+1)+1/8*(-a^2*x^2+1)^(5/2)/a^6/(c-c/a^2/x^2)^(5/2)/x^5/(a*x+1)-
23/16*(-a^2*x^2+1)^(5/2)*ln(-a*x+1)/a^6/(c-c/a^2/x^2)^(5/2)/x^5+7/16*(-a^2*
x^2+1)^(5/2)*ln(a*x+1)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.20, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{23\left(1-\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -((1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4)) + (1 - a^2*x^2)^(5
/2)/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)^2) - (1 - a^2*x^2)^(5/2)/(
a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)) + (1 - a^2*x^2)^(5/2)/(8*a^6*(c
- c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)) - (23*(1 - a^2*x^2)^(5/2)*Log[1 - a*x])
/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (7*(1 - a^2*x^2)^(5/2)*Log[1 + a*x]
)/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(1-ax)^3(1+ax)^2}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(1-a^2x^2\right)^{5/2}\int \left(-\frac{1}{a^5}-\frac{1}{4a^5(-1+ax)^3}-\frac{1}{a^5(-1+ax)^2}-\frac{23}{16a^5(-1+ax)}-\frac{1}{8a^5(1+ax)^2}+\frac{7}{16a^5(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\ &= -\frac{\left(1-a^2x^2\right)^{5/2}}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)^2}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)}+\frac{(1-}{8a^6\left(c-\frac{}{a^2}\right.}\end{aligned}$$

Mathematica [A] time = 0.14, size = 87, normalized size = 0.33

$$\frac{\left(1-a^2x^2\right)^{5/2}\left(2\left(-8ax+\frac{8}{ax-1}+\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-23\log(1-ax)+7\log(ax+1)\right)}{16a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - a^2*x^2)^(5/2)*(2*(-8*a*x + (-1 + a*x)^(-2) + 8/(-1 + a*x) + (1 + a*x
)^(-1)) - 23*Log[1 - a*x] + 7*Log[1 + a*x]))/(16*a^6*(c - c/(a^2*x^2))^(5/2
)*x^5)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,a^6x^6\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^6*x^6*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^7*c
^3*x^7 - a^6*c^3*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^
2*c^3*x^2 - a*c^3*x + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

maple [A] time = 0.06, size = 167, normalized size = 0.63

$$-\frac{\sqrt{-a^2x^2+1}\;(ax+1)\left(16x^4a^4+23\ln(ax-1)\,x^3a^3-7a^3x^3\ln(ax+1)-16x^3a^3-23\ln(ax-1)\,x^2a^2+7\ln(ax\right.}{16a^6x^5\left(\frac{c(a^2x^2-}{a^2x^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(a*x+1)*(16*x^4*a^4+23*ln(a*x-1)*x^3*a^3-7*a^3*x^3
*ln(a*x+1)-16*x^3*a^3-23*ln(a*x-1)*x^2*a^2+7*ln(a*x+1)*x^2*a^2-34*a^2*x^2-2
3*ln(a*x-1)*x*a+7*a*x*ln(a*x+1)+18*a*x+23*ln(a*x-1)-7*ln(a*x+1)+12)/a^6/x^5
/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\sqrt{1-a^2\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a
*x)))**(5/2)), x)
```

3.694 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=361

$$\frac{3\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x}\right.}$$

```
[Out] (-a^2*x^2+1)^(7/2)/a^7/(c-c/a^2/x^2)^(7/2)/x^6+1/24*(-a^2*x^2+1)^(7/2)/a^8/
(c-c/a^2/x^2)^(7/2)/x^7/(-a*x+1)^3-11/32*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^
2)^(7/2)/x^7/(-a*x+1)^2+3/2*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/
(-a*x+1)+1/32*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(a*x+1)^2-5/16
*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(a*x+1)+51/32*(-a^2*x^2+1)^
(7/2)*ln(-a*x+1)/a^8/(c-c/a^2/x^2)^(7/2)/x^7-19/32*(-a^2*x^2+1)^(7/2)*ln(a*
x+1)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.30, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 88}

$$\frac{3\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6) + (1 - a^2*x^2)^(7/2)
/(24*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^3) - (11*(1 - a^2*x^2)^(7/2)
)/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^2) + (3*(1 - a^2*x^2)^(7/2)
)/(2*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(32*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^2) - (5*(1 - a^2*x^2)^(7/2))/(16*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) + (51*(1 - a^2*x^2)^(7/2)*Log[1 -
 a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) - (19*(1 - a^2*x^2)^(7/2)*Log[1
 + a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx &= \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{e^{\tanh^{-1}(ax)}x^7}{\left(1-a^2x^2\right)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(1-ax)^4(1+ax)^3}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{\left(1-a^2x^2\right)^{7/2}\int \left(\frac{1}{a^7}+\frac{1}{8a^7(-1+ax)^4}+\frac{11}{16a^7(-1+ax)^3}+\frac{3}{2a^7(-1+ax)^2}+\frac{51}{32a^7(-1+ax)}-\frac{1}{16a^7(1+ax)^3}+\frac{5}{16a^7(}\right.}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{\left(1-a^2x^2\right)^{7/2}}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}+\frac{\left(1-a^2x^2\right)^{7/2}}{24a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^3}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^2}+\frac{3}{2a^8\left(c-\right.}\end{aligned}$$

Mathematica [A] time = 0.12, size = 147, normalized size = 0.41

$$\frac{\sqrt{1-a^2x^2}\left(-96a^6x^6+96a^5x^5+366a^4x^4-222a^3x^3-338a^2x^2+122ax-153(ax-1)^3(ax+1)^2\log(1-ax)+57\right.}{96a^2c^3x(ax-1)^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

[Out] (Sqrt[1 - a^2*x^2]*(88 + 122*a*x - 338*a^2*x^2 - 222*a^3*x^3 + 366*a^4*x^4 + 96*a^5*x^5 - 96*a^6*x^6 - 153*(-1 + a*x)^3*(1 + a*x)^2*Log[1 - a*x] + 57*(-1 + a*x)^3*(1 + a*x)^2*Log[1 + a*x]))/(96*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3*(1 + a*x)^2)

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,a^8x^8\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^9c^4x^9-a^8c^4x^8-4\,a^7c^4x^7+4\,a^6c^4x^6+6\,a^5c^4x^5-6\,a^4c^4x^4-4\,a^3c^4x^3+4\,a^2c^4x^2+ac^4x-c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^8*x^8*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^9*c^4*x^9 - a^8*c^4*x^8 - 4*a^7*c^4*x^7 + 4*a^6*c^4*x^6 + 6*a^5*c^4*x^5 - 6*a^4*c^4*x^4 - 4*a^3*c^4*x^3 + 4*a^2*c^4*x^2 + a*c^4*x - c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

maple [A] time = 0.06, size = 239, normalized size = 0.66

$$-\frac{\sqrt{-a^2x^2+1}\;(ax+1)\left(96x^6a^6+153\ln(ax-1)\,x^5a^5-57\ln(ax+1)\,x^5a^5-96x^5a^5-153\ln(ax-1)\,x^4a^4+57\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/96*(-a^2*x^2+1)^(1/2)*(a*x+1)*(96*x^6*a^6+153*ln(a*x-1)*x^5*a^5-57*ln(a*x+1)*x^5*a^5-96*x^5*a^5-153*ln(a*x-1)*x^4*a^4+57*ln(a*x+1)*x^4*a^4-366*x^4*a^4-306*ln(a*x-1)*x^3*a^3+114*a^3*x^3*ln(a*x+1)+222*x^3*a^3+306*ln(a*x-1)*x
```

```
^2*a^2-114*ln(a*x+1)*x^2*a^2+338*a^2*x^2+153*ln(a*x-1)*x*a-57*a*x*ln(a*x+1)
-122*a*x-153*ln(a*x-1)+57*ln(a*x+1)-88)/a^8/x^7/(c*(a^2*x^2-1)/a^2/x^2)^(7/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a
*x)))**(7/2)), x)
```

3.695 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=450

$$-\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{28(1-ax)^2}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{8(1-ax)}+\frac{71a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{336(1-ax)^3}-\frac{501a^8x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{128(1-ax)^4(ax+1)^4}+\frac{2a^8x^9\,(\ldots}{(1-\ldots}$$

```
[Out] 295/1344*a^4*(c-c/a^2/x^2)^(9/2)*x^5/(-a*x+1)^4-501/128*a^8*(c-c/a^2/x^2)^(
9/2)*x^9/(-a*x+1)^4/(a*x+1)^4+373/192*a^7*(c-c/a^2/x^2)^(9/2)*x^8/(-a*x+1)^
4/(a*x+1)^3+501/640*a^6*(c-c/a^2/x^2)^(9/2)*x^7/(-a*x+1)^4/(a*x+1)^2+661/16
80*a^5*(c-c/a^2/x^2)^(9/2)*x^6/(-a*x+1)^4/(a*x+1)-127/420*a^3*(c-c/a^2/x^2)
^(9/2)*x^4*(a*x+1)/(-a*x+1)^4+71/336*a^2*(c-c/a^2/x^2)^(9/2)*x^3*(a*x+1)/(-
a*x+1)^3-1/28*a*(c-c/a^2/x^2)^(9/2)*x^2*(a*x+1)/(-a*x+1)^2-1/8*(c-c/a^2/x^2
)^(9/2)*x*(a*x+1)/(-a*x+1)+2*a^8*(c-c/a^2/x^2)^(9/2)*x^9*arcsin(a*x)/(-a*x+
1)^(9/2)/(a*x+1)^(9/2)+245/128*a^8*(c-c/a^2/x^2)^(9/2)*x^9*arctanh((-a*x+1)
^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(9/2)/(a*x+1)^(9/2)
```

Rubi [A] time = 0.56, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{501a^8x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{128(1-ax)^4(ax+1)^4}+\frac{373a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{192(1-ax)^4(ax+1)^3}+\frac{501a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{640(1-ax)^4(ax+1)^2}+\frac{661a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{1680(1-ax)^4(ax+1)}+\frac{295a^4x^5\,(c-\ldots}{1344(1-a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] (295*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(1344*(1 - a*x)^4) - (501*a^8*(c - c/
(a^2*x^2))^(9/2)*x^9)/(128*(1 - a*x)^4*(1 + a*x)^4) + (373*a^7*(c - c/(a^2*
x^2))^(9/2)*x^8)/(192*(1 - a*x)^4*(1 + a*x)^3) + (501*a^6*(c - c/(a^2*x^2))
^(9/2)*x^7)/(640*(1 - a*x)^4*(1 + a*x)^2) + (661*a^5*(c - c/(a^2*x^2))^(9/2
)*x^6)/(1680*(1 - a*x)^4*(1 + a*x)) - (127*a^3*(c - c/(a^2*x^2))^(9/2)*x^4*
(1 + a*x))/(420*(1 - a*x)^4) + (71*a^2*(c - c/(a^2*x^2))^(9/2)*x^3*(1 + a*x
))/(336*(1 - a*x)^3) - (a*(c - c/(a^2*x^2))^(9/2)*x^2*(1 + a*x))/(28*(1 - a
*x)^2) - ((c - c/(a^2*x^2))^(9/2)*x*(1 + a*x))/(8*(1 - a*x)) + (2*a^8*(c -
c/(a^2*x^2))^(9/2)*x^9*ArcSin[a*x])/((1 - a*x)^(9/2)*(1 + a*x)^(9/2)) + (24
5*a^8*(c - c/(a^2*x^2))^(9/2)*x^9*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(12
8*(1 - a*x)^(9/2)*(1 + a*x)^(9/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

```
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{9/2}(1+ax)^{9/2}}{x^9}dx}{(1-ax)^{9/2}(1+ax)^{9/2}}$$

$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{7/2}(1+ax)^{11/2}}{x^9}dx}{(1-ax)^{9/2}(1+ax)^{9/2}}$$

$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1+ax)}{8(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{5/2}(1+ax)^{9/2}(2a-9a^2x)}{x^8}dx}{8(1-ax)^{9/2}(1+ax)^{9/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2(1+ax)}{28(1-ax)^2} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1+ax)}{8(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{3}}{}}{56(1-ax)^{9/2}}$$

$$= \frac{71a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3(1+ax)}{336(1-ax)^3} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2(1+ax)}{28(1-ax)^2} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1+ax}{8(1-ax)}$$

$$= -\frac{127a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4(1+ax)}{420(1-ax)^4} + \frac{71a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3(1+ax)}{336(1-ax)^3} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{28(1-a}$$

$$= \frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4} - \frac{127a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4(1+ax)}{420(1-ax)^4} + \frac{71a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3(1}{336(1-ax)^3}$$

$$= \frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4} + \frac{661a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^6}{1680(1-ax)^4(1+ax)} - \frac{127a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4(1+ax)}{420(1-ax)^4}$$

$$= \frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4} + \frac{501a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2} + \frac{661a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^6}{1680(1-ax)^4(1+ax)} - 127a^3$$

$$= \frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4} + \frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3} + \frac{501a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2} + \frac{661a}{1680}$$

$$= \frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4} - \frac{501a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4} + \frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3} + \frac{501a}{640(1}$$

$$= \frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4} - \frac{501a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4} + \frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3} + \frac{501a}{640(1}$$

$$= \frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4} - \frac{501a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4} + \frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3} + \frac{501a}{640(1}$$

$$= \frac{295a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{1344(1-ax)^4} - \frac{501a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4} + \frac{373a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{192(1-ax)^4(1+ax)^3} + \frac{501a}{640(1}$$

Mathematica [A] time = 0.20, size = 166, normalized size = 0.37

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(26880a^8x^8\log\left(\sqrt{a^2x^2-1}+ax\right)+25725a^8x^8\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(13440a^8x^8-4505\right.\right.}{13440a^8x^7\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] -1/13440*(c^4*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1680 + 3840*a*x -
4760*a^2*x^2 - 16896*a^3*x^3 + 770*a^4*x^4 + 31232*a^5*x^5 + 14595*a^6*x^6
- 45056*a^7*x^7 + 13440*a^8*x^8) + 25725*a^8*x^8*ArcTan[1/Sqrt[-1 + a^2*x^2
]] + 26880*a^8*x^8*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(a^8*x^7*Sqrt[-1 + a^2*x
^2])
```

fricas [A] time = 0.77, size = 482, normalized size = 1.07

$$\left[\frac{53760\,a^7\sqrt{-c}\,c^4x^7\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+25725\,a^7\sqrt{-c}\,c^4x^7\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(13440\,a^8c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/26880*(53760*a^7*sqrt(-c)*c^4*x^7*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^
2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 25725*a^7*sqrt(-c)*c^4*x^7*log(-(a^2*c
*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(1344
0*a^8*c^4*x^8 - 45056*a^7*c^4*x^7 + 14595*a^6*c^4*x^6 + 31232*a^5*c^4*x^5 +
 770*a^4*c^4*x^4 - 16896*a^3*c^4*x^3 - 4760*a^2*c^4*x^2 + 3840*a*c^4*x + 16
80*c^4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7), -1/13440*(25725*a^7*c^(
9/2)*x^7*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)
) - 13440*a^7*c^(9/2)*x^7*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)) - c) + (13440*a^8*c^4*x^8 - 45056*a^7*c^4*x^7 + 14595*a^
6*c^4*x^6 + 31232*a^5*c^4*x^5 + 770*a^4*c^4*x^4 - 16896*a^3*c^4*x^3 - 4760*
a^2*c^4*x^2 + 3840*a*c^4*x + 1680*c^4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^
8*x^7)]
```

giac [A] time = 137.06, size = 707, normalized size = 1.57

$$\frac{1}{6720}\left(\frac{25725\,c^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{13440\,c^{\frac{9}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{6720\,\sqrt{a^2c\ldots}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac")
```

```
[Out] 1/6720*(25725*c^(9/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x)/a^2 + 13440*c^(9/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))
*sgn(x)/(a*abs(a)) - 6720*sqrt(a^2*c*x^2 - c)*c^4*sgn(x)/a^2 + (14595*(sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 - c))^15*c^5*abs(a)*sgn(x) + 107520*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^14*a*c^(11/2)*sgn(x) + 76055*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^13*c^6*abs(a)*sgn(x) + 430080*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^12*a*c^(13/2)*sgn(x) + 64435*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^11*c^7*abs(a)*sgn(x) + 1111040*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^10*
a*c^(15/2)*sgn(x) + 110495*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*c^8*abs(
a)*sgn(x) + 1576960*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*c^(17/2)*sgn(
x) - 110495*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^9*abs(a)*sgn(x) + 141
2096*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(19/2)*sgn(x) - 64435*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^10*abs(a)*sgn(x) + 831488*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(21/2)*sgn(x) - 76055*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^11*abs(a)*sgn(x) + 252928*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^2*a*c^(23/2)*sgn(x) - 14595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)*c^12*abs(a)*sgn(x) + 45056*a*c^(25/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^8*a^2*abs(a)))*abs(a)
```

maple [B] time = 0.12, size = 965, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] -1/40320*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/a^2*(-5040*a^4*(c*(a^2*x^2-1)/a^2)
^(11/2)*(-c/a^2)^(1/2)+77175*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-c)/a^2/x)*x^8*c^6+23808*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^7*
a^11-17535*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^6*a^10-13056*(-c/a^2
)^(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^5*a^9-6510*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(11/2)*x^4*a^8-6912*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^3*a
^7-10920*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^2*a^6-11520*(-c/a^2)^(
```

```
1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x*a^5+58590*(-c/a^2)^(1/2)*c^(11/2)*ln(x*c^
(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^8*a+22050*(-c/a^2)^(1/2)*c^(11/2)*ln((c^
(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^8*a-23808*(-c/a^2)^(1/2
)*(c*(a^2*x^2-1)/a^2)^(9/2)*x^9*a^11*c+8575*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(9/2)*x^8*a^10*c+8960*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(9/2)*x^8*
a^10*c+26784*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^9*a^9*c^2+10080*(-c
/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^9*a^9*c^2-11025*(-c/a^2)^(1/2)*
(c*(a^2*x^2-1)/a^2)^(7/2)*x^8*a^8*c^2-31248*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(5/2)*x^9*a^7*c^3-11760*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^
9*a^7*c^3+15435*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^8*a^6*c^3+39060*
(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^9*a^5*c^4+14700*(-c/a^2)^(1/2)*(
(a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^9*a^5*c^4-25725*(-c/a^2)^(1/2)*(c*(a^2*x^2-1
)/a^2)^(3/2)*x^8*a^4*c^4-58590*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^9
*a^3*c^5-22050*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^9*a^3*c^5+771
75*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^8*a^2*c^5)/(-c/a^2)^(1/2)/(c*
(a^2*x^2-1)/a^2)^(9/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxima")
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^(9/2)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\,(a\,x+1)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(9/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] int(-((c - c/(a^2*x^2))^(9/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(9/2),x)
[Out] Exception raised: TypeError
```

3.696 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=372

$$-\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{15(1-ax)^2}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(1-ax)}+\frac{13a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{40(1-ax)^3}+\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}-\frac{2a^6x^7\left(c-\right.}{(1-ax}$$

```
[Out] -11/30*a^3*(c-c/a^2/x^2)^(7/2)*x^4/(-a*x+1)^3+57/16*a^6*(c-c/a^2/x^2)^(7/2)
*x^7/(-a*x+1)^3/(a*x+1)^3-41/24*a^5*(c-c/a^2/x^2)^(7/2)*x^6/(-a*x+1)^3/(a*x
+1)^2-57/80*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)+13/40*a^2*(c-c/a
^2/x^2)^(7/2)*x^3*(a*x+1)/(-a*x+1)^3-1/15*a*(c-c/a^2/x^2)^(7/2)*x^2*(a*x+1)
/(-a*x+1)^2-1/6*(c-c/a^2/x^2)^(7/2)*x*(a*x+1)/(-a*x+1)-2*a^6*(c-c/a^2/x^2)^
(7/2)*x^7*arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)-25/16*a^6*(c-c/a^2/x^2)^
(7/2)*x^7*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1)^(7/2
)
```

Rubi [A] time = 0.50, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}-\frac{41a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{24(1-ax)^3(ax+1)^2}-\frac{57a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{80(1-ax)^3(ax+1)}-\frac{11a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{30(1-ax)^3}+\frac{13a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7}}{40(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (-11*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(30*(1 - a*x)^3) + (57*a^6*(c - c/(a^
2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) - (41*a^5*(c - c/(a^2*x^2))
^(7/2)*x^6)/(24*(1 - a*x)^3*(1 + a*x)^2) - (57*a^4*(c - c/(a^2*x^2))^(7/2)*
x^5)/(80*(1 - a*x)^3*(1 + a*x)) + (13*a^2*(c - c/(a^2*x^2))^(7/2)*x^3*(1 +
a*x))/(40*(1 - a*x)^3) - (a*(c - c/(a^2*x^2))^(7/2)*x^2*(1 + a*x))/(15*(1 -
 a*x)^2) - ((c - c/(a^2*x^2))^(7/2)*x*(1 + a*x))/(6*(1 - a*x)) - (2*a^6*(c
- c/(a^2*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) - (
25*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(1
6*(1 - a*x)^(7/2)*(1 + a*x)^(7/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}\,dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{5/2}(1+ax)^{9/2}}{x^7}\,dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}\left(2a-7a^2x\right)}{x^6}\,dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{\sqrt{1-a\cdots}}{\cdots}}{30(1-ax)^{7/\cdots}}$$

$$=\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+a\cdots}{6(1-ax)}$$

$$=-\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}+\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+a\cdots}{15(1-ax)^2}$$

$$=-\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}+\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3}-\cdots$$

$$=-\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}-\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}+\frac{13a^2\left(c\cdots\right.}{\cdots}$$

$$=-\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}+\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}-\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}-\frac{57a^4\left(c\cdots\right.}{80(1-\cdots}$$

$$=-\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}+\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}-\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}-\frac{57a^4\left(c\cdots\right.}{80(1-\cdots}$$

$$=-\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}+\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}-\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}-\frac{57a^4\left(c\cdots\right.}{80(1-\cdots}$$

$$=-\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}+\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}-\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}-\frac{57a^4\left(c\cdots\right.}{80(1-\cdots}$$

Mathematica [A] time = 0.16, size = 150, normalized size = 0.40

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(480a^6x^6\log\left(\sqrt{a^2x^2-1}+ax\right)+375a^6x^6\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(240a^6x^6-736a^5x^5+105\right.\right.}{240a^6x^5\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -1/240*(c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 - 96*a*x + 70*a^
2*x^2 + 352*a^3*x^3 + 105*a^4*x^4 - 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^
6*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]])
)/(a^6*x^5*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.64, size = 438, normalized size = 1.18

$$\left[\frac{960\,a^5\sqrt{-c}\,c^3x^5\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+375\,a^5\sqrt{-c}\,c^3x^5\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(240\,a^6c^3x^6-73\right.}{480\,a^6x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2 +
 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(240*a^6*c^
3*x^6 - 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^
2 - 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), -1/240
*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a
^2*c*x^2 - c)) - 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 - 736*a^5*c^3*x^5 + 1
05*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 - 96*a*c^3*x - 40*c^3)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.07, size = 795, normalized size = 2.14

$$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}} x \left(-2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}} \sqrt{-\frac{c}{a^2}}\, x^7a^9c+2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{9}{2}} \sqrt{-\frac{c}{a^2}}\, x^5a^9+375\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}} \sqrt{-\frac{c}{a^2}}\, x^6a^8c-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(-2016*(c*(a^2*x^2-1)/a^2)^(7/2)
*(-c/a^2)^(1/2)*x^7*a^9*c+2016*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^5
*a^9+375*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*x^6*a^8*c-480*(-c/a^2)^(1
/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a^8*c+105*(c*(a^2*x^2-1)/a^2)^(9/2)*(
-c/a^2)^(1/2)*x^4*a^8+2352*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*x^7*a^7
*c^2-560*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7*c^2+224*(c*(a
^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^3*a^7-525*(c*(a^2*x^2-1)/a^2)^(5/2)*(
-c/a^2)^(1/2)*x^6*a^6*c^2-2940*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^7
*a^5*c^3+700*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^7*a^5*c^3+630*(
c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^2*a^6+875*(c*(a^2*x^2-1)/a^2)^(3/
2)*(-c/a^2)^(1/2)*x^6*a^4*c^3+672*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*
x*a^5+4410*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^7*a^3*c^4-1050*(-c/a^
2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^7*a^3*c^4+280*a^4*(c*(a^2*x^2-1)/a
^2)^(9/2)*(-c/a^2)^(1/2)-2625*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^6*
a^2*c^4-4410*(-c/a^2)^(1/2)*c^(9/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))
*x^6*a+1050*(-c/a^2)^(1/2)*c^(9/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2
)+c*x)/c^(1/2))*x^6*a-2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a
^2-c)/a^2/x)*x^6*c^5)/(c*(a^2*x^2-1)/a^2)^(7/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^(7/2)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(7/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] int(-((c - c/(a^2*x^2))^(7/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(7/2),x)
[Out] Exception raised: TypeError
```

3.697 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2} dx$

Optimal. Leaf size=294

$$-\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(1-ax)}+\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2}-\frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}+\frac{2a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^{5/2}(ax \cdots}$$

```
[Out] 5/8*a^2*(c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)^2-25/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5
/(-a*x+1)^2/(a*x+1)^2+17/12*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+1)-
1/6*a*(c-c/a^2/x^2)^(5/2)*x^2*(a*x+1)/(-a*x+1)^2-1/4*(c-c/a^2/x^2)^(5/2)*x*
(a*x+1)/(-a*x+1)+2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5/2)/(
a*x+1)^(5/2)+9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(a*x+1)
^(1/2))/(-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

Rubi [A] time = 0.45, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}+\frac{17a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{12(1-ax)^2(ax+1)}+\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2}-\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/\cdots}}{4(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (5*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(8*(1 - a*x)^2) - (25*a^4*(c - c/(a^2*x
^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) + (17*a^3*(c - c/(a^2*x^2))^(5/
2)*x^4)/(12*(1 - a*x)^2*(1 + a*x)) - (a*(c - c/(a^2*x^2))^(5/2)*x^2*(1 + a*
x))/(6*(1 - a*x)^2) - ((c - c/(a^2*x^2))^(5/2)*x*(1 + a*x))/(4*(1 - a*x)) +
 (2*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)
^(5/2)) + (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 +
 a*x]])/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{\sqrt{1-ax}\,(1+ax)^{5/2}\left(2a-5a^2x\right)}{x^4}dx}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1+ax)^{5}}{}}{12(1-ax)^{5/2}(1+}$$

$$=\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)}+\frac{\left(\left(c-\right.\right.}{}$$

$$=\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}+\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}-\frac{\left(c-\frac{c}{a^2x}\right.}{4}$$

$$=\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}-\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}+\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5}}{6(1-}$$

$$=\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}-\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}+\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5}}{6(1-}$$

$$=\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}-\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}+\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5}}{6(1-}$$

$$=\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}-\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}+\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5}}{6(1-}$$

Mathematica [A] time = 0.13, size = 134, normalized size = 0.46

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(48a^4x^4\log\left(\sqrt{a^2x^2-1}+ax\right)+27a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(24a^4x^4-64a^3x^3-3a^2x^2+\right.\right.}{24a^4x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

[Out] -1/24*(c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 64*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 48*a^4*x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(a^4*x^3*Sqrt[-1 + a^2*x^2])

fricas [A] time = 0.50, size = 394, normalized size = 1.34

$$\left[\frac{96\,a^3\sqrt{-c}\,c^2x^3\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+27\,a^3\sqrt{-c}\,c^2x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(24\,a^4c^2x^4-64\,a^3\right.}{48\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")

[Out] [1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*c^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3), -1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 24*a^3*c^(5/2)*x^3*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3)]

giac [A] time = 6.54, size = 416, normalized size = 1.41

$$\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2-c}\,c^2\mathrm{sgn}}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac")

[Out] 1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2 + 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) - 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4*abs(a)*sgn(x) - 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) - 160*(sqr

```
t(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) - 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

maple [B] time = 0.05, size = 625, normalized size = 2.13

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}} x \left(-80\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}} x^5a^7c + 80\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}} x^3a^7 + 48\sqrt{-\frac{c}{a^2}}\,\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}} x^4a^6c + 27\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(5/2)*x^5*a^7*c+80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^3*a^7
+48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^6*c+27*(-c/a^2)^(1/2
)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^6*c+60*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/
a^2)^(3/2)*x^5*a^5*c^2-75*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^2*a^6+
100*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^5*a^5*c^2-80*(-c/a^2)^(1/2)*
(c*(a^2*x^2-1)/a^2)^(7/2)*x*a^5-45*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)
*x^4*a^4*c^2-90*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c^3-15
0*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c^3-30*a^4*(c*(a^2*x^2-1
)/a^2)^(7/2)*(-c/a^2)^(1/2)+150*(-c/a^2)^(1/2)*c^(7/2)*ln(x*c^(1/2)+(c*(a^2
*x^2-1)/a^2)^(1/2))*x^4*a+90*(-c/a^2)^(1/2)*c^(7/2)*ln((c^(1/2)*((a*x-1)*(a
*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^4*a+135*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^
2)^(1/2)*x^4*a^2*c^3+135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2
-c)/a^2/x)*x^4*c^4)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(5/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^(5/2)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - c/(a^2*x^2))^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

3.698 $\int e^{2\tanh^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{3/2} dx$

Optimal. Leaf size=214

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}+\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}-\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\text{ta}}{2(1-ax}$$

```
[Out] -a*(c-c/a^2/x^2)^(3/2)*x^2/(-a*x+1)+5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1
)/(a*x+1)-1/2*(c-c/a^2/x^2)^(3/2)*x*(a*x+1)/(-a*x+1)-2*a^2*(c-c/a^2/x^2)^(3
/2)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)-1/2*a^2*(c-c/a^2/x^2)^(3/2
)*x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

Rubi [A] time = 0.39, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}-\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}-\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\text{tan}}{2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 - a*x)) + (5*a^2*(c - c/(a^2*x^2))^(3/
2)*x^3)/(2*(1 - a*x)*(1 + a*x)) - ((c - c/(a^2*x^2))^(3/2)*x*(1 + a*x))/(2*
(1 - a*x)) - (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/
2)*(1 + a*x)^(3/2)) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x
]*Sqrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{\sqrt{1-ax}\,(1+ax)^{5/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1+ax)^{3/2}\left(2a-3a^2x\right)}{x^2\sqrt{1-ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{\sqrt{1+ax}\left(a^2-5a^3x\right)}{x\sqrt{1-ax}}}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}+\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}\right.}{2a(1-}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}+\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}+\frac{\left(a^2\left(c-\frac{c}{a^2x^2}\right.\right.}{2(1}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}+\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}-\frac{\left(a^3\left(c-\frac{c}{a^2x^2}\right.\right.}{}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}+\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}-\frac{2a^2\left(c-\frac{c}{a^2x^2}\right.}{(1-ax)}$$

Mathematica [A] time = 0.10, size = 115, normalized size = 0.54

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(2a^2x^2-4ax-1\right)+4a^2x^2\log\left(\sqrt{a^2x^2-1}+ax\right)+a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2a^2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -1/2*(c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 - 4*a*x + 2*a^2*x^2)
+ a^2*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*
x^2]]))/(a^2*x*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 1.39, size = 316, normalized size = 1.48

$$\left[\frac{8\,a\sqrt{-c}\,cx\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+a\sqrt{-c}\,cx\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(2\,a^2cx^2-4\,acx-c\right)\sqrt{\frac{a^2cx^2-}{a^2x^2}}}{4\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c)) + a*sqrt(-c)*c*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(2*a^2*c*x^2 - 4*a*c*x - c)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), -1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x
*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 2*a*c^(3/2)*x*log(2*a^2
*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*
x^2 - 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

giac [A] time = 0.45, size = 265, normalized size = 1.24

$$\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}+\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}\,c\,\mathrm{sgn}(x)}{a^2}-\frac{(\sqrt{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
+ 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a)
) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^3*c^2*abs(a)*sgn(x) - 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/2
)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) - 4*a*c^
(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a)))
*abs(a)
```

maple [B] time = 0.05, size = 455, normalized size = 2.13

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}} x\left(12\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}} x^3a^5c-12\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}} x\,a^5+4\sqrt{-\frac{c}{a^2}}\,\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}} x^2a^4c-\sqrt{-\frac{c}{a^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/
a^2)^(3/2)*x^3*a^5*c-12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^5+4*(-
c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^2*a^4*c-(-c/a^2)^(1/2)*(c*(a^2
*x^2-1)/a^2)^(3/2)*x^2*a^4*c+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)
*x^3*a^3*c^2-3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)-18*(-c/a^2)^(1/
2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^3*c^2+18*(-c/a^2)^(1/2)*c^(5/2)*ln(x*c^(
1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^2*a-6*(-c/a^2)^(1/2)*c^(5/2)*ln((c^(1/2)*
((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a+3*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(1/2)*x^2*a^2*c^2+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1
/2)*a^2-c)/a^2/x)*x^2*c^3)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(3/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^(3/2)/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] int(-((c - c/(a^2*x^2))^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(3/2),x)
[Out] Exception raised: TypeError
```

3.699 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=118

$$-x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] -x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(
a*x+1)^(1/2)-x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-
a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6159, 6129, 102, 157, 41, 216, 92, 208}

$$-x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-a-2a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}}{\sqrt{1-ax}\,\sqrt{1+ax}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(2a\sqrt{c}\right.}{\sqrt{}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

Mathematica [A] time = 0.08, size = 81, normalized size = 0.69

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2
]] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.56, size = 270, normalized size = 2.29

$$\left[-\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\sqrt{-c}\,\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, -\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\,\text{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(-c)*log(-(a^
2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, -(
a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*
c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sq
rt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 0.05, size = 198, normalized size = 1.68

$$-\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\, x \left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^{2}}}\, a^{2}\sqrt{-\frac{c}{a^{2}}}+2\sqrt{c}\, \ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^{2}}}+cx}{\sqrt{c}}\right) a\sqrt{-\frac{c}{a^{2}}}-\sqrt{-\frac{c}{a^{2}}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\, a^{2}-c\ln\left(\frac{2\cdots}{\cdots}\right.\right.}{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\, a^{2}\sqrt{-\frac{c}{a^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)+2*c^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))
*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^{2}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{a^{2}x^{2}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x*sqrt(c - c/(
a**2*x**2))/(a*x - 1), x)
```

3.700 $\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=110

$$\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-ax}\,\sqrt{ax+1}\,\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] 2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^2)^(1/2)+(a*x+1)^2/a^2/x/(c-c/a^2/x^2)^
(1/2)-2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6159, 6129, 78, 50, 41, 216}

$$\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-ax}\,\sqrt{ax+1}\,\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[c - c/(a^2*x^2)]*x) + (1 + a*x)^2/(a^2*Sq
rt[c - c/(a^2*x^2)]*x) - (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*S
qrt[c - c/(a^2*x^2)]*x)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{e^{2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= \frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{x\sqrt{1+ax}}{(1-ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2\sqrt{1-ax}\,\sqrt{1+ax}\,\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.63

$$\frac{-a^2x^2-2\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax+3}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (3 + 2*a*x - a^2*x^2 - 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/
(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

fricas [A] time = 1.55, size = 216, normalized size = 1.96

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(a^2x^2-3\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac},\ \frac{2\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\cdots}{a^2cx-ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
*c*x - a*c), (2*(a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^2*c*x - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2}{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))), x)
```

maple [A] time = 0.04, size = 178, normalized size = 1.62

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\left(\sqrt{c}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x\,a^2+2\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)xac-2a\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,\sqrt{c}-\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a\sqrt{c}-2\ln\cdots\right)}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,c^{\frac{3}{2}}a\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2)^(1/2)*(c^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2+2*ln(x*
c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a*c-2*a*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*
```

```
c^(1/2)-(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2)-2*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a
^2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{(a^2x^2-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a\,x+1)^2}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1)),x)
[Out] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax\sqrt{c-\frac{c}{a^2x^2}}-\sqrt{c-\frac{c}{a^2x^2}}}\,dx-\int \frac{1}{ax\sqrt{c-\frac{c}{a^2x^2}}-\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(1/2),x)
[Out] -Integral(a*x/(a*x*sqrt(c - c/(a**2*x**2)) - sqrt(c - c/(a**2*x**2))), x) -
 Integral(1/(a*x*sqrt(c - c/(a**2*x**2)) - sqrt(c - c/(a**2*x**2))), x)
```

3.701 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(1-ax)^{3/2}(ax+1)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] 1/3*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x-2/3*(-2*a*x+5)*(-a*x+1)*(a*x+1)^2/a
^4/(c-c/a^2/x^2)^(3/2)/x^3+2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^4/(
c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.37, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6159, 6129, 98, 143, 41, 216}

$$-\frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(1-ax)^{3/2}(ax+1)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (1 + a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) - (2*(5 - 2*a*x)*(1 - a*x)*(1
 + a*x)^2)/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (2*(1 - a*x)^(3/2)*(1 + a*
x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && !(SumSimplerQ[n, 1] && !SumSimplerQ[m, 1])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{(1-ax)^{5/2}\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2+4ax)}{(1-ax)^{3/2}\sqrt{1+ax}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,d}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A] time = 0.08, size = 95, normalized size = 0.77

$$\frac{-3a^3x^3+11a^2x^2-6(ax-1)\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)+4ax-10}{3a^2cx(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (-10 + 4*a*x + 11*a^2*x^2 - 3*a^3*x^3 - 6*(-1 + a*x)*Sqrt[-1 + a^2*x^2]*Log
[a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x))
```

fricas [A] time = 0.98, size = 281, normalized size = 2.28

$$\left[\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(3\,a^3x^3-14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}, \frac{6\left(a^2x^2-2\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")

[Out] [1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), 1/3*(6*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")

[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(3/2)), x)

maple [B] time = 0.05, size = 326, normalized size = 2.65

$$-\frac{\left(3c^{\frac{3}{2}}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,x^3a^3-15x^2a^2c^{\frac{3}{2}}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+4c^{\frac{3}{2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^2a^2+6\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{\frac{(ax-1)(ax\cdots}{a^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x)

[Out] -1/3*(3*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^3-15*x^2*a^2*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2+6*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2*c-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a-6*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a*c+12*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)-2*(c*(a^2*x^2-1)/a^2)^(1/2)*c^(3/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(1/2)/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a\,x+1)^2}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(3/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(3/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{acx\sqrt{c-\frac{c}{a^2x^2}}-c\sqrt{c-\frac{c}{a^2x^2}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{ax}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}}\,dx-\int \frac{1}{acx\sqrt{c-\frac{c}{a^2x^2}}-c\sqrt{c-\frac{c}{a^2x^2}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{ax}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] -Integral(a*x/(a*c*x*sqrt(c - c/(a**2*x**2)) - c*sqrt(c - c/(a**2*x**2)) -
c*sqrt(c - c/(a**2*x**2))/(a*x) + c*sqrt(c - c/(a**2*x**2))/(a**2*x**2)), x
) - Integral(1/(a*c*x*sqrt(c - c/(a**2*x**2)) - c*sqrt(c - c/(a**2*x**2)) -
 c*sqrt(c - c/(a**2*x**2))/(a*x) + c*sqrt(c - c/(a**2*x**2))/(a**2*x**2)),
x)
```

3.702 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-ax)^{5/2}\sin^{-1}(ax)}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(a}{3a}$$

```
[Out] 1/5*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x-2/3*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a^2
/x^2)^(5/2)/x^2+58/15*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(5/2)/x^3+2/15
*(-a*x+1)^3*(a*x+1)^2*(43*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5-2*(-a*x+1)^(5
/2)*(a*x+1)^(5/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.41, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(1-ax)}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-}{a^6x^5\left(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (1 + a*x)^2/(5*a^2*(c - c/(a^2*x^2))^(5/2)*x) - (2*(1 - a*x)*(1 + a*x)^2)/(
3*a^3*(c - c/(a^2*x^2))^(5/2)*x^2) + (58*(1 - a*x)^2*(1 + a*x)^2)/(15*a^4*(
c - c/(a^2*x^2))^(5/2)*x^3) + (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 43*a*x))/(15
*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) - (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcS
in[a*x])/(a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{7/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+6ax)}{(1-ax)^{5/2}(1+ax)^{3/2}}\,dx}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2\left(-30a-28a^2x\right)}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int}{15a^6\left(c-\frac{c}{a^2x^2}\right.} \\
&= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28-}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} \\
&= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28-}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} \\
&= \frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28-}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 105, normalized size = 0.52

$$
\frac{-15a^4x^4+76a^3x^3-32a^2x^2-30(ax-1)^2\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)-82ax+56}{15a^2c^2x(ax-1)^2\sqrt{c-\frac{c}{a^2x^2}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

[Out] (56 - 82*a*x - 32*a^2*x^2 + 76*a^3*x^3 - 15*a^4*x^4 - 30*(-1 + a*x)^2*Sqrt[
-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x^
2)]*x*(-1 + a*x)^2)

fricas [A] time = 0.63, size = 353, normalized size = 1.74

$$\left[\frac{15\left(a^4x^4-2a^3x^3+2ax-1\right)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(15a^5x^5-76a^4x^4+32a^3x^3+82a^2x^2\right.}{15\left(a^5c^3x^4-2a^4c^3x^3+2a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")

[Out] [1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (15*a^5*x^5 - 76*a^4*x^
4 + 32*a^3*x^3 + 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3), 1/15*(30*(a^4*x^4 - 2*a^3*
x^3 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2
*x^2))/(a^2*c*x^2 - c)) - (15*a^5*x^5 - 76*a^4*x^4 + 32*a^3*x^3 + 82*a^2*x^
2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 +
 2*a^2*c^3*x - a*c^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")

[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(5/2)), x)

maple [B] time = 0.05, size = 462, normalized size = 2.28

$$-\frac{\left(15c^{\frac{5}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^5a^5-45x^4c^{\frac{5}{2}}a^4\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}-16c^{\frac{5}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^4a^4-60c^{\frac{5}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^3a^3+16c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/15*(15*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^5*a^5-45*x^4*c^(5/2)*a^4*
((a*x-1)*(a*x+1)*c/a^2)^(3/2)-16*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4-
60*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^3*a^3+16*c^(5/2)*(c*(a^2*x^2-1)/
a^2)^(3/2)*x^3*a^3+30*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x
+1)*c/a^2)^(3/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^4*c+90*c^(5/2)*((a*x-1)*(a*x
+1)*c/a^2)^(3/2)*x^2*a^2+24*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^2-30*ln
(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(c*(a^2
*x^2-1)/a^2)^(3/2)*a^3*c+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x*a-24*c^
(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a-50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2
)-6*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(3/2
)/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(a\,x+1)^2}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(5/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(5/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^2x\sqrt{c-\frac{c}{a^2x^2}} - c^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{ax} + \frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2} + \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3} - \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4}}\,dx - \int \frac{}{ac^2x\sqrt{c-\frac{c}{a^2x^2}} - c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] -Integral(a*x/(a*c**2*x*sqrt(c - c/(a**2*x**2)) - c**2*sqrt(c - c/(a**2*x**
2)) - 2*c**2*sqrt(c - c/(a**2*x**2))/(a*x) + 2*c**2*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + c**2*sqrt(c - c/(a**2*x**2))/(a**3*x**3) - c**2*sqrt(c - c/(
a**2*x**2))/(a**4*x**4)), x) - Integral(1/(a*c**2*x*sqrt(c - c/(a**2*x**2))
 - c**2*sqrt(c - c/(a**2*x**2)) - 2*c**2*sqrt(c - c/(a**2*x**2))/(a*x) + 2*
c**2*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + c**2*sqrt(c - c/(a**2*x**2))/(a*
*3*x**3) - c**2*sqrt(c - c/(a**2*x**2))/(a**4*x**4)), x)
```

3.703 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=283

$$\frac{(ax+1)^2}{7a^2x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^{7/2}(1-ax)^{7/2}\sin^{-1}(ax)}{a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-$$

```
[Out] 1/7*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x-2/5*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a^2
/x^2)^(7/2)/x^2+124/105*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(7/2)/x^3-78
2/105*(-a*x+1)^3*(a*x+1)^2/a^5/(c-c/a^2/x^2)^(7/2)/x^4-142/35*(-a*x+1)^4*(a
*x+1)^2/a^6/(c-c/a^2/x^2)^(7/2)/x^5-2/35*(-a*x+1)^4*(a*x+1)^3*(107*a*x+72)/
a^8/(c-c/a^2/x^2)^(7/2)/x^7+2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x)/a^8/
(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.43, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$-\frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{782(ax+1)^2(1-ax)^3}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{124(ax+1)^2(1-ax)^2}{105a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax}{5a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (1 + a*x)^2/(7*a^2*(c - c/(a^2*x^2))^(7/2)*x) - (2*(1 - a*x)*(1 + a*x)^2)/(
5*a^3*(c - c/(a^2*x^2))^(7/2)*x^2) + (124*(1 - a*x)^2*(1 + a*x)^2)/(105*a^4
*(c - c/(a^2*x^2))^(7/2)*x^3) - (782*(1 - a*x)^3*(1 + a*x)^2)/(105*a^5*(c -
 c/(a^2*x^2))^(7/2)*x^4) - (142*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^
2*x^2))^(7/2)*x^5) - (2*(1 - a*x)^4*(1 + a*x)^3*(72 + 107*a*x))/(35*a^8*(c
- c/(a^2*x^2))^(7/2)*x^7) + (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])
/(a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
```

```
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{9/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+8ax)}{(1-ax)^{7/2}(1+ax)^{5/2}}\,dx}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4\left(-70a-54a^2x\right)}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{35a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)}{105a^6\left(c-\right.}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= \frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

Mathematica [A] time = 0.12, size = 133, normalized size = 0.47

$$\frac{-105a^6x^6+562a^5x^5-74a^4x^4-1226a^3x^3+636a^2x^2-210(ax-1)^3(ax+1)\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)+6}{105a^2c^3x(ax-1)^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (-432 + 654*a*x + 636*a^2*x^2 - 1226*a^3*x^3 - 74*a^4*x^4 + 562*a^5*x^5 - 1
05*a^6*x^6 - 210*(-1 + a*x)^3*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-
1 + a^2*x^2]])/(105*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.86, size = 497, normalized size = 1.76

$$\left[\frac{105\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(105\,a^7x^7-56\right.}{105\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4 - 636*a^3
*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x
+ a*c^4), 1/105*(210*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 -
 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
)/(a^2*c*x^2 - c)) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4
 - 636*a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*
a^2*c^4*x + a*c^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

maple [B] time = 0.06, size = 572, normalized size = 2.02

$$-\frac{\left(105c^{\frac{7}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}}x^7a^7+96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^6a^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}}-96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^5a^5-39\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/105*(105*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7+96*(c*(a^2*x^2-1)
/a^2)^(5/2)*c^(7/2)*x^6*a^6-553*x^6*c^(7/2)*a^6*((a*x-1)*(a*x+1)*c/a^2)^(5/
2)-96*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^5*a^5-392*c^(7/2)*((a*x-1)*(a*x+1
)*c/a^2)^(5/2)*x^5*a^5-240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^4*a^4+1540*c
^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^4+210*(c*(a^2*x^2-1)/a^2)^(5/2)*
ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x*a^6
*c+240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^3*a^3+350*c^(7/2)*((a*x-1)*(a*x+
1)*c/a^2)^(5/2)*x^3*a^3-210*(c*(a^2*x^2-1)/a^2)^(5/2)*ln(x*c^(1/2)+(c*(a^2*
x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*a^5*c+180*(c*(a^2*x^2-1)/a
^2)^(5/2)*c^(7/2)*x^2*a^2-1470*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^2*a^
2-180*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x*a-42*c^(7/2)*((a*x-1)*(a*x+1)*c/a
^2)^(5/2)*x*a-30*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)+462*c^(7/2)*((a*x-1)*(a*
x+1)*c/a^2)^(5/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(5/2)/x^7/(c*(a^2*x^2-1)
/a^2/x^2)^(7/2)/a^8/c^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(a\,x+1)^2}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(7/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(7/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^3x\sqrt{c-\frac{c}{a^2x^2}}-c^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{ax}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^5x^5}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^6x^6}}\, d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] -Integral(a*x/(a*c**3*x*sqrt(c - c/(a**2*x**2)) - c**3*sqrt(c - c/(a**2*x**
2)) - 3*c**3*sqrt(c - c/(a**2*x**2))/(a*x) + 3*c**3*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + 3*c**3*sqrt(c - c/(a**2*x**2))/(a**3*x**3) - 3*c**3*sqrt(c -
 c/(a**2*x**2))/(a**4*x**4) - c**3*sqrt(c - c/(a**2*x**2))/(a**5*x**5) + c*
*3*sqrt(c - c/(a**2*x**2))/(a**6*x**6)), x) - Integral(1/(a*c**3*x*sqrt(c -
 c/(a**2*x**2)) - c**3*sqrt(c - c/(a**2*x**2)) - 3*c**3*sqrt(c - c/(a**2*x*
*2))/(a*x) + 3*c**3*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + 3*c**3*sqrt(c - c
/(a**2*x**2))/(a**3*x**3) - 3*c**3*sqrt(c - c/(a**2*x**2))/(a**4*x**4) - c*
*3*sqrt(c - c/(a**2*x**2))/(a**5*x**5) + c**3*sqrt(c - c/(a**2*x**2))/(a**6
*x**6)), x)
```

3.704 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{9/2}}\,dx$

Optimal. Leaf size=363

$$\frac{(ax+1)^2}{9a^2x\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{2(ax+1)^4(1019ax+704)(1-ax)^5}{315a^{10}x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}-\frac{2(ax+1)^{9/2}(1-ax)^{9/2}\sin^{-1}(ax)}{a^{10}x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{1334(ax+1)^3(1-ax)}{315a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}$$

```
[Out] 1/9*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(9/2)/x-2/7*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a^2
/x^2)^(9/2)/x^2+214/315*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(9/2)/x^3-64
6/315*(-a*x+1)^3*(a*x+1)^2/a^5/(c-c/a^2/x^2)^(9/2)/x^4+302/21*(-a*x+1)^4*(a
*x+1)^2/a^6/(c-c/a^2/x^2)^(9/2)/x^5+2458/315*(-a*x+1)^5*(a*x+1)^2/a^7/(c-c/
a^2/x^2)^(9/2)/x^6+1334/315*(-a*x+1)^5*(a*x+1)^3/a^8/(c-c/a^2/x^2)^(9/2)/x^
7+2/315*(-a*x+1)^5*(a*x+1)^4*(1019*a*x+704)/a^10/(c-c/a^2/x^2)^(9/2)/x^9-2*
(-a*x+1)^(9/2)*(a*x+1)^(9/2)*arcsin(a*x)/a^10/(c-c/a^2/x^2)^(9/2)/x^9
```

Rubi [A] time = 0.48, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$\frac{1334(ax+1)^3(1-ax)^5}{315a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{2458(ax+1)^2(1-ax)^5}{315a^7x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{2(ax+1)^4(1019ax+704)(1-ax)^5}{315a^{10}x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}+\frac{302(ax+1)^2(1-ax)^4}{21a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}-\frac{6}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] (1 + a*x)^2/(9*a^2*(c - c/(a^2*x^2))^(9/2)*x) - (2*(1 - a*x)*(1 + a*x)^2)/(
7*a^3*(c - c/(a^2*x^2))^(9/2)*x^2) + (214*(1 - a*x)^2*(1 + a*x)^2)/(315*a^4
*(c - c/(a^2*x^2))^(9/2)*x^3) - (646*(1 - a*x)^3*(1 + a*x)^2)/(315*a^5*(c -
 c/(a^2*x^2))^(9/2)*x^4) + (302*(1 - a*x)^4*(1 + a*x)^2)/(21*a^6*(c - c/(a^
2*x^2))^(9/2)*x^5) + (2458*(1 - a*x)^5*(1 + a*x)^2)/(315*a^7*(c - c/(a^2*x^
2))^(9/2)*x^6) + (1334*(1 - a*x)^5*(1 + a*x)^3)/(315*a^8*(c - c/(a^2*x^2))^
(9/2)*x^7) + (2*(1 - a*x)^5*(1 + a*x)^4*(704 + 1019*a*x))/(315*a^10*(c - c/
(a^2*x^2))^(9/2)*x^9) - (2*(1 - a*x)^(9/2)*(1 + a*x)^(9/2)*ArcSin[a*x])/(a^
10*(c - c/(a^2*x^2))^(9/2)*x^9)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{9/2}}\,dx = \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^9}{(1-ax)^{9/2}(1+ax)^{9/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int \frac{x^9}{(1-ax)^{11/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int \frac{x^7(8+10ax)}{(1-ax)^{9/2}(1+ax)^{7/2}}\,dx}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} - \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int \frac{x^6\left(-126a-88a^2x\right)}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{63a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{\left((1-ax)^{9/2}(1+ax)^{9/2}\right)\int}{315a^6\left(c-\frac{c}{a^2}\right.}$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} -$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} +$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} +$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} +$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} +$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} +$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} +$$

$$= \frac{(1+ax)^2}{9a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x} - \frac{2(1-ax)(1+ax)^2}{7a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2} + \frac{214(1-ax)^2(1+ax)^2}{315a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3} - \frac{646(1-ax)^3(1+ax)^2}{315a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4} +$$

Mathematica [A] time = 0.15, size = 151, normalized size = 0.42

$$\frac{-315a^8x^8+1756a^7x^7+268a^6x^6-5784a^5x^5+2060a^4x^4+6200a^3x^3-3372a^2x^2-630(ax-1)^4(ax+1)^2\sqrt{a^2x^2}}{315a^2c^4x(ax-1)^4(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] (1408 - 2186*a*x - 3372*a^2*x^2 + 6200*a^3*x^3 + 2060*a^4*x^4 - 5784*a^5*x^
5 + 268*a^6*x^6 + 1756*a^7*x^7 - 315*a^8*x^8 - 630*(-1 + a*x)^4*(1 + a*x)^2
*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(315*a^2*c^4*Sqrt[c - c/
(a^2*x^2)]*x*(-1 + a*x)^4*(1 + a*x)^2)
```

fricas [A] time = 0.99, size = 569, normalized size = 1.57

$$\left[\frac{315\left(a^8x^8-2a^7x^7-2a^6x^6+6a^5x^5-6a^3x^3+2a^2x^2+2ax-1\right)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-(}{315\left(a^9c^5x^8-2a^8c^5x^7-2a^7c^5x^6+6a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a^8*x^8 - 2*a^7*x^7 - 2*a^6*x^6 + 6*a^5*x^5 - 6*a^3*x^3 + 2*a^
2*x^2 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*
x^2 - c)/(a^2*x^2)) - c) - (315*a^9*x^9 - 1756*a^8*x^8 - 268*a^7*x^7 + 5784
*a^6*x^6 - 2060*a^5*x^5 - 6200*a^4*x^4 + 3372*a^3*x^3 + 2186*a^2*x^2 - 1408
*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 - 2*a^7
*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5*x - a*
c^5), 1/315*(630*(a^8*x^8 - 2*a^7*x^7 - 2*a^6*x^6 + 6*a^5*x^5 - 6*a^3*x^3 +
 2*a^2*x^2 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (315*a^9*x^9 - 1756*a^8*x^8 - 268*a^7*x^7
+ 5784*a^6*x^6 - 2060*a^5*x^5 - 6200*a^4*x^4 + 3372*a^3*x^3 + 2186*a^2*x^2
- 1408*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 -
 2*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5*
x - a*c^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2}{(a^2x^2-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(9/2)), x)
```

maple [B] time = 0.11, size = 682, normalized size = 1.88

$$\left(-315c^{\frac{9}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{7}{2}}x^9a^9+256\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}c^{\frac{9}{2}}x^8a^8+1185x^8c^{\frac{9}{2}}a^8\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{7}{2}}-256\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}c^{\frac{9}{2}}x^7a^7+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] 1/315*(-315*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^9*a^9+256*(c*(a^2*x^2-1
)/a^2)^(7/2)*c^(9/2)*x^8*a^8+1185*x^8*c^(9/2)*a^8*((a*x-1)*(a*x+1)*c/a^2)^(
7/2)-256*(c*(a^2*x^2-1)/a^2)^(7/2)*c^(9/2)*x^7*a^7+2280*c^(9/2)*((a*x-1)*(a
*x+1)*c/a^2)^(7/2)*x^7*a^7-896*(c*(a^2*x^2-1)/a^2)^(7/2)*c^(9/2)*x^6*a^6-46
20*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a^6+896*(c*(a^2*x^2-1)/a^2)^(7
/2)*c^(9/2)*x^5*a^5-4620*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^5*a^5-630*
(c*(a^2*x^2-1)/a^2)^(7/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*
(a*x+1)*c/a^2)^(7/2)*x*a^8*c+1120*(c*(a^2*x^2-1)/a^2)^(7/2)*c^(9/2)*x^4*a^4
+7140*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^4*a^4+630*(c*(a^2*x^2-1)/a^2)
^(7/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(7/2
)*a^7*c-1120*(c*(a^2*x^2-1)/a^2)^(7/2)*c^(9/2)*x^3*a^3+3948*c^(9/2)*((a*x-1
)*(a*x+1)*c/a^2)^(7/2)*x^3*a^3-560*(c*(a^2*x^2-1)/a^2)^(7/2)*c^(9/2)*x^2*a^
2-4998*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^2*a^2+560*(c*(a^2*x^2-1)/a^2
)^(7/2)*c^(9/2)*x*a-1338*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x*a+70*(c*(a
^2*x^2-1)/a^2)^(7/2)*c^(9/2)+1338*c^(9/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2))*(a
*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(7/2)/x^9/(c*(a^2*x^2-1)/a^2/x^2)^(9/2)/a^10/
c^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(c-c/a^2/x^2)^(9/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2/((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(a\,x+1)^2}{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(9/2)*(a^2*x^2 - 1)),x)
[Out] int(-(a*x + 1)^2/((c - c/(a^2*x^2))^(9/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^4x\sqrt{c-\frac{c}{a^2x^2}} - c^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{ax} + \frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2} + \frac{6c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3} - \frac{6c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4} - \frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^5x^5} + \frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(c-c/a**2/x**2)**(9/2),x)
[Out] -Integral(a*x/(a*c**4*x*sqrt(c - c/(a**2*x**2)) - c**4*sqrt(c - c/(a**2*x**
2)) - 4*c**4*sqrt(c - c/(a**2*x**2))/(a*x) + 4*c**4*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + 6*c**4*sqrt(c - c/(a**2*x**2))/(a**3*x**3) - 6*c**4*sqrt(c -
 c/(a**2*x**2))/(a**4*x**4) - 4*c**4*sqrt(c - c/(a**2*x**2))/(a**5*x**5) +
4*c**4*sqrt(c - c/(a**2*x**2))/(a**6*x**6) + c**4*sqrt(c - c/(a**2*x**2))/(
a**7*x**7) - c**4*sqrt(c - c/(a**2*x**2))/(a**8*x**8)), x) - Integral(1/(a*
c**4*x*sqrt(c - c/(a**2*x**2)) - c**4*sqrt(c - c/(a**2*x**2)) - 4*c**4*sqrt
(c - c/(a**2*x**2))/(a*x) + 4*c**4*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + 6*
c**4*sqrt(c - c/(a**2*x**2))/(a**3*x**3) - 6*c**4*sqrt(c - c/(a**2*x**2))/(
a**4*x**4) - 4*c**4*sqrt(c - c/(a**2*x**2))/(a**5*x**5) + 4*c**4*sqrt(c - c
/(a**2*x**2))/(a**6*x**6) + c**4*sqrt(c - c/(a**2*x**2))/(a**7*x**7) - c**4
*sqrt(c - c/(a**2*x**2))/(a**8*x**8)), x)
```

3.705 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=300

$$-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{7\left(1-a^2x^2\right)^{9/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{8\left(1-a^2x^2\right)^{9/2}}-\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{3a^8x^9\log(x)\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{2a^5x^6\left(c\right.}{\left(1-a\right.}$$

```
[Out] -1/8*(c-c/a^2/x^2)^(9/2)*x/(-a^2*x^2+1)^(9/2)-3/7*a*(c-c/a^2/x^2)^(9/2)*x^2
/(-a^2*x^2+1)^(9/2)+8/5*a^3*(c-c/a^2/x^2)^(9/2)*x^4/(-a^2*x^2+1)^(9/2)+3/2*
a^4*(c-c/a^2/x^2)^(9/2)*x^5/(-a^2*x^2+1)^(9/2)-2*a^5*(c-c/a^2/x^2)^(9/2)*x^
6/(-a^2*x^2+1)^(9/2)-4*a^6*(c-c/a^2/x^2)^(9/2)*x^7/(-a^2*x^2+1)^(9/2)-a^9*(
c-c/a^2/x^2)^(9/2)*x^10/(-a^2*x^2+1)^(9/2)-3*a^8*(c-c/a^2/x^2)^(9/2)*x^9*ln
(x)/(-a^2*x^2+1)^(9/2)
```

Rubi [A] time = 0.19, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}+\frac{8a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{5\left(1-a^2x^2\right)^{9/2}}-\frac{3ax^2\left(c-\right.}{7\left(1-a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) - (3*a*(c - c/(a^2*x^2
))^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) + (8*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)
/(5*(1 - a^2*x^2)^(9/2)) + (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*
x^2)^(9/2)) - (2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) - (4*
a^6*(c - c/(a^2*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) - (a^9*(c - c/(a^2*x^2
))^(9/2)*x^10)/(1 - a^2*x^2)^(9/2) - (3*a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log
[x])/(1 - a^2*x^2)^(9/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^3(1+ax)^6}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\left(-a^9+\frac{1}{x^9}+\frac{3a}{x^8}-\frac{8a^3}{x^6}-\frac{6a^4}{x^5}+\frac{6a^5}{x^4}+\frac{8a^6}{x^3}-\frac{3a^8}{x}\right)dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{8\left(1-a^2x^2\right)^{9/2}}-\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{7\left(1-a^2x^2\right)^{9/2}}+\frac{8a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}$$

Mathematica [A] time = 0.07, size = 98, normalized size = 0.33

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(280a^9x^9+840a^8x^8\log(x)+1120a^6x^6+560a^5x^5-420a^4x^4-448a^3x^3+120ax+35\right)}{280a^8x^7\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] -1/280*(c^4*Sqrt[c - c/(a^2*x^2)]*(35 + 120*a*x - 448*a^3*x^3 - 420*a^4*x^4
 + 560*a^5*x^5 + 1120*a^6*x^6 + 280*a^9*x^9 + 840*a^8*x^8*Log[x]))/(a^8*x^7
*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.79, size = 542, normalized size = 1.81

$$\left[\frac{420\left(a^{9}c^{4}x^{9}-a^{7}c^{4}x^{7}\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}-\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)+\left(280\,a^{9}c^{4}x^{9}+1120\,a^{6}c^{4}x^{6}+560\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="fr
icas")
```

```
[Out] [1/280*(420*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2
 - c*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - c)/(a^2*x^4 - x^2)) + (280*a^9*c^4*x^9 + 1120*a^6*c^4*x^6 + 560
*a^5*c^4*x^5 - (280*a^9 + 1120*a^6 + 560*a^5 - 420*a^4 - 448*a^3 + 120*a +
35)*c^4*x^8 - 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqr
t(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7), 1/28
0*(840*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3
 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^
2 + c)) + (280*a^9*c^4*x^9 + 1120*a^6*c^4*x^6 + 560*a^5*c^4*x^5 - (280*a^9
+ 1120*a^6 + 560*a^5 - 420*a^4 - 448*a^3 + 120*a + 35)*c^4*x^8 - 420*a^4*c^
4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a^
2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{3}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{9}{2}}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(9/2)/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.05, size = 102, normalized size = 0.34

$$\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{9}{2}} x\sqrt{-a^{2}x^{2}+1}\,\left(280a^{9}x^{9}+840a^{8}\ln(x)x^{8}+1120x^{6}a^{6}+560x^{5}a^{5}-420x^{4}a^{4}-448x^{3}a^{3}+120ax+35\right)}{280\left(a^{2}x^{2}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] 1/280*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a^2*x^2-1)^5*(-a^2*x^2+1)^(1/2)*(280
*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6+560*x^5*a^5-420*x^4*a^4-448*x^3*a^3
+120*a*x+35)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 \left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(9/2)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(9/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(9/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{9}{2}}(ax+1)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(9/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(9/2)*(a*x + 1)**3/(-(a*x - 1)*
(a*x + 1))**(3/2), x)
```

3.706 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=301

$$-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6\left(1-a^2x^2\right)^{7/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}+\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^6x^7\log(x)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}$$

```
[Out] -1/6*(c-c/a^2/x^2)^(7/2)*x/(-a^2*x^2+1)^(7/2)-3/5*a*(c-c/a^2/x^2)^(7/2)*x^2
/(-a^2*x^2+1)^(7/2)-1/4*a^2*(c-c/a^2/x^2)^(7/2)*x^3/(-a^2*x^2+1)^(7/2)+5/3*
a^3*(c-c/a^2/x^2)^(7/2)*x^4/(-a^2*x^2+1)^(7/2)+5/2*a^4*(c-c/a^2/x^2)^(7/2)*
x^5/(-a^2*x^2+1)^(7/2)-a^5*(c-c/a^2/x^2)^(7/2)*x^6/(-a^2*x^2+1)^(7/2)+a^7*(
c-c/a^2/x^2)^(7/2)*x^8/(-a^2*x^2+1)^(7/2)+3*a^6*(c-c/a^2/x^2)^(7/2)*x^7*ln(
x)/(-a^2*x^2+1)^(7/2)
```

Rubi [A] time = 0.20, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{5a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}+\frac{5a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3\left(1-a^2x^2\right)^{7/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) - (3*a*(c - c/(a^2*x^2
))^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) - (a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) + (5*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a^2*x^
2)^(7/2)) + (5*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) - (
a^5*(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) + (a^7*(c - c/(a^2*x^2
))^(7/2)*x^8)/(1 - a^2*x^2)^(7/2) + (3*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[
x])/(1 - a^2*x^2)^(7/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^2(1+ax)^5}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\left(a^7+\frac{1}{x^7}+\frac{3a}{x^6}+\frac{a^2}{x^5}-\frac{5a^3}{x^4}-\frac{5a^4}{x^3}+\frac{a^5}{x^2}+\frac{3a^6}{x}\right)dx}{\left(1-a^2x^2\right)^{7/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{6\left(1-a^2x^2\right)^{7/2}}-\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{5\left(1-a^2x^2\right)^{7/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{4\left(1-a^2x^2\right)^{7/2}}+\frac{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{3\left(1-a^2x^2\right)^{7/2}}$$

Mathematica [A] time = 0.06, size = 98, normalized size = 0.33

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(60a^7x^7+180a^6x^6\log(x)-60a^5x^5+150a^4x^4+100a^3x^3-15a^2x^2-36ax-10\right)}{60a^6x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -1/60*(c^3*Sqrt[c - c/(a^2*x^2)]*(-10 - 36*a*x - 15*a^2*x^2 + 100*a^3*x^3 +
 150*a^4*x^4 - 60*a^5*x^5 + 60*a^7*x^7 + 180*a^6*x^6*Log[x]))/(a^6*x^5*Sqrt
[1 - a^2*x^2])
```

fricas [A] time = 0.77, size = 542, normalized size = 1.80

$$\left[\frac{90\left(a^{7}c^{3}x^{7}-a^{5}c^{3}x^{5}\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}-\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)+\left(60\,a^{7}c^{3}x^{7}-60\,a^{5}c^{3}x^{5}+150\,a^{4}c^{3}\right.}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="fr
icas")
```

```
[Out] [1/60*(90*(a^7*c^3*x^7 - a^5*c^3*x^5)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
 c*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) - c)/(a^2*x^4 - x^2)) + (60*a^7*c^3*x^7 - 60*a^5*c^3*x^5 + 150*a^4*
c^3*x^4 - (60*a^7 - 60*a^5 + 150*a^4 + 100*a^3 - 15*a^2 - 36*a - 10)*c^3*x^
6 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2 - 36*a*c^3*x - 10*c^3)*sqrt(-a^2*x^2 +
 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7 - a^6*x^5), 1/60*(180*(a^7*c^
3*x^7 - a^5*c^3*x^5)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt(c
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (60*
a^7*c^3*x^7 - 60*a^5*c^3*x^5 + 150*a^4*c^3*x^4 - (60*a^7 - 60*a^5 + 150*a^4
 + 100*a^3 - 15*a^2 - 36*a - 10)*c^3*x^6 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2
 - 36*a*c^3*x - 10*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))
/(a^8*x^7 - a^6*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{3}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(7/2)/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.05, size = 102, normalized size = 0.34

$$\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}} x\sqrt{-a^{2}x^{2}+1}\,\left(60a^{7}x^{7}+180a^{6}\ln(x)x^{6}-60x^{5}a^{5}+150x^{4}a^{4}+100x^{3}a^{3}-15a^{2}x^{2}-36ax-10\right)}{60\left(a^{2}x^{2}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a^2*x^2-1)^4*(-a^2*x^2+1)^(1/2)*(60*a
^7*x^7+180*a^6*ln(x)*x^6-60*x^5*a^5+150*x^4*a^4+100*x^3*a^3-15*a^2*x^2-36*a
*x-10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(7/2)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(a\,x+1\right)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x + 1)**3/(-(a*x - 1)*
(a*x + 1))**(3/2), x)
```

3.707 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=220

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{3a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -1/4*(c-c/a^2/x^2)^(5/2)*x/(-a^2*x^2+1)^(5/2)-a*(c-c/a^2/x^2)^(5/2)*x^2/(-a
^2*x^2+1)^(5/2)-a^2*(c-c/a^2/x^2)^(5/2)*x^3/(-a^2*x^2+1)^(5/2)+2*a^3*(c-c/a
^2/x^2)^(5/2)*x^4/(-a^2*x^2+1)^(5/2)-a^5*(c-c/a^2/x^2)^(5/2)*x^6/(-a^2*x^2+
1)^(5/2)-3*a^4*(c-c/a^2/x^2)^(5/2)*x^5*ln(x)/(-a^2*x^2+1)^(5/2)
```

Rubi [A] time = 0.18, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 75}

$$-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}-\frac{3a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) - (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(1 - a^2*x^2)^(5/2) - (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1 - a^
2*x^2)^(5/2) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2) - (a
^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) - (3*a^4*(c - c/(a^2*x^
2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx &= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}}{x^5}\,dx}{\left(1-a^2x^2\right)^{5/2}}\\
&= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)(1+ax)^4}{x^5}\,dx}{\left(1-a^2x^2\right)^{5/2}}\\
&= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\left(-a^5+\frac{1}{x^5}+\frac{3a}{x^4}+\frac{2a^2}{x^3}-\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{\left(1-a^2x^2\right)^{5/2}}\\
&= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}{4\left(1-a^2x^2\right)^{5/2}}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{\left(1-a^2x^2\right)^{5/2}}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{\left(1-a^2x^2\right)^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.41

$$
-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(4a^5x^5+5a^4x^4+12a^4x^4\log(x)-8a^3x^3+4a^2x^2+4ax+1\right)}{4a^4x^3\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -1/4*(c^2*Sqrt[c - c/(a^2*x^2)]*(1 + 4*a*x + 4*a^2*x^2 - 8*a^3*x^3 + 5*a^4*
x^4 + 4*a^5*x^5 + 12*a^4*x^4*Log[x]))/(a^4*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 1.11, size = 474, normalized size = 2.15

$$\left[\frac{6\left(a^{5}c^{2}x^{5}-a^{3}c^{2}x^{3}\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}-\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)+\left(4\,a^{5}c^{2}x^{5}-8\,a^{3}c^{2}x^{3}-\left(4\,a^{5}-8\,a^{3}\right.\right.}{4\left(a^{6}x^{5}-a^{4}x^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="fr
icas")
```

```
[Out] [1/4*(6*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c
*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - c)/(a^2*x^4 - x^2)) + (4*a^5*c^2*x^5 - 8*a^3*c^2*x^3 - (4*a^5 - 8*a
^3 + 4*a^2 + 4*a + 1)*c^2*x^4 + 4*a^2*c^2*x^2 + 4*a*c^2*x + c^2)*sqrt(-a^2*
x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3), 1/4*(12*(a^5
*c^2*x^5 - a^3*c^2*x^3)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (
4*a^5*c^2*x^5 - 8*a^3*c^2*x^3 - (4*a^5 - 8*a^3 + 4*a^2 + 4*a + 1)*c^2*x^4 +
 4*a^2*c^2*x^2 + 4*a*c^2*x + c^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^6*x^5 - a^4*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{3}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{5}{2}}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(5/2)/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.05, size = 86, normalized size = 0.39

$$\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{5}{2}} x\sqrt{-a^{2}x^{2}+1}\,\left(4x^{5}a^{5}+12a^{4}\ln(x)x^{4}-8x^{3}a^{3}+4a^{2}x^{2}+4ax+1\right)}{4\left(a^{2}x^{2}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/4*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/(a^2*x^2-1)^3*(-a^2*x^2+1)^(1/2)*(4*x^5
*a^5+12*a^4*ln(x)*x^4-8*x^3*a^3+4*a^2*x^2+4*a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(5/2)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,(a\,x+1)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x + 1)**3/(-(a*x - 1)*
(a*x + 1))**(3/2), x)
```

3.708 $\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$

Optimal. Leaf size=145

$$-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}+\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -1/2*(c-c/a^2/x^2)^(3/2)*x/(-a^2*x^2+1)^(3/2)-3*a*(c-c/a^2/x^2)^(3/2)*x^2/(
-a^2*x^2+1)^(3/2)+a^3*(c-c/a^2/x^2)^(3/2)*x^4/(-a^2*x^2+1)^(3/2)+3*a^2*(c-c
/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) - (3*a*(c - c/(a^2*x^2
))^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) + (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 -
a^2*x^2)^(3/2) + (3*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(
3/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1+ax)^3}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\left(a^3+\frac{1}{x^3}+\frac{3a}{x^2}+\frac{3a^2}{x}\right)dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{2\left(1-a^2x^2\right)^{3/2}}-\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{\left(1-a^2x^2\right)^{3/2}}+\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^4}{\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{\left(1-a^2x^2\right)^{3}}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.44

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(2a^3x^3+6a^2x^2\log(x)-6ax-1\right)}{2a^2x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -1/2*(c*Sqrt[c - c/(a^2*x^2)]*(-1 - 6*a*x + 2*a^3*x^3 + 6*a^2*x^2*Log[x]))/
(a^2*x*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.60, size = 378, normalized size = 2.61

$$\frac{3\left(a^3cx^3-acx\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)+\left(2a^3cx^3-\left(2a^3-6a-1\right)cx^2-6acx\right.}{2\left(a^4x^3-a^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(3*(a^3*c*x^3 - a*c*x)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - (
a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) -
c)/(a^2*x^4 - x^2)) + (2*a^3*c*x^3 - (2*a^3 - 6*a - 1)*c*x^2 - 6*a*c*x - c)
*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x), 1/2
*(6*(a^3*c*x^3 - a*c*x)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + (
2*a^3*c*x^3 - (2*a^3 - 6*a - 1)*c*x^2 - 6*a*c*x - c)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(3/2)/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.05, size = 70, normalized size = 0.48

$$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}} x\sqrt{-a^2x^2+1}\,\left(2x^3a^3+6a^2\ln(x)x^2-6ax-1\right)}{2\left(a^2x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a^2*x^2-1)^2*(-a^2*x^2+1)^(1/2)*(2*x^3
*a^3+6*a^2*ln(x)*x^2-6*a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^(3/2)/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- c \left(-1 + \frac{1}{a x}\right) \left(1 + \frac{1}{a x}\right)\right)^{\frac{3}{2}} \left(a x + 1\right)^{3}}{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)**3/(-(a*x - 1)*
(a*x + 1))**(3/2), x)
```

3.709 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=108

$$-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -a*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+x*ln(x)*(c-c/a^2/x^2)^(1/2)/(
-a^2*x^2+1)^(1/2)-4*x*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 72}

$$-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqr
t[1 - a^2*x^2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1+ax)^2}{x(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.44

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,(-ax-4\log(1-ax)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/Sqrt[1 - a^2*x
^2]
```

fricas [F] time = 3.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
x^2 - 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.05, size = 61, normalized size = 0.56

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-ax+\ln(x)-4\ln\left(ax-1\right)\right)\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a*x+ln(x)-4*ln(a*x-1))*(-a^2*x^2+1)^(1/2
)/(a^2*x^2-1)
```

maxima [C] time = 0.45, size = 148, normalized size = 1.37

$$-\frac{1}{2}a^3\left(-\frac{2i\sqrt{c}\,x}{a^3}+\frac{i\sqrt{c}\,\log\left(ax+1\right)}{a^4}-\frac{i\sqrt{c}\,\log\left(ax-1\right)}{a^4}\right)-\frac{3}{2}a^2\left(-\frac{i\sqrt{c}\,\log\left(ax+1\right)}{a^3}-\frac{i\sqrt{c}\,\log\left(ax-1\right)}{a^3}\right)-\frac{3}{2}a\left(\frac{i\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*a^3*(-2*I*sqrt(c)*x/a^3 + I*sqrt(c)*log(a*x + 1)/a^4 - I*sqrt(c)*log(a
*x - 1)/a^4) - 3/2*a^2*(-I*sqrt(c)*log(a*x + 1)/a^3 - I*sqrt(c)*log(a*x - 1
)/a^3) - 3/2*a*(I*sqrt(c)*log(a*x + 1)/a^2 - I*sqrt(c)*log(a*x - 1)/a^2) +
1/2*I*sqrt(c)*log(a*x + 1)/a + 1/2*I*sqrt(c)*log(a*x - 1)/a - I*sqrt(c)*log
(x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*
x + 1))**(3/2), x)
```

3.710 $\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-a^2x^2}\,\log(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (-a^2*x^2+1)^(1/2)/a/(c-c/a^2/x^2)^(1/2)+2*(-a^2*x^2+1)^(1/2)/a^2/x/(-a*x+1
)/(c-c/a^2/x^2)^(1/2)+3*ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(
1/2)
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 77}

$$\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-a^2x^2}\,\log(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)]) + (2*Sqrt[1 - a^2*x^2])/(a^2*Sq
rt[c - c/(a^2*x^2)]*x*(1 - a*x)) + (3*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a^2*
Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x(1+ax)}{(1-ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}+\frac{3\sqrt{1-a^2x^2}\,\log(1-ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.48

$$\frac{\sqrt{1-a^2x^2}\left(ax+\frac{2}{1-ax}+3\log(1-ax)\right)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(a*x + 2/(1 - a*x) + 3*Log[1 - a*x]))/(a^2*Sqrt[c - c/(a
^2*x^2)]*x)
```

fricas [A] time = 2.05, size = 440, normalized size = 3.58

$$\left[-\frac{3\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\sqrt{-c}\log\left(\frac{a^{6}cx^{6}-4\,a^{5}cx^{5}+5\,a^{4}cx^{4}-4\,a^{2}cx^{2}+4\,acx+\left(a^{5}x^{5}-4\,a^{4}x^{4}+6\,a^{3}x^{3}-4\,a^{2}x^{2}\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1}\right)}{2\left(a^{4}cx^{3}-a^{3}cx^{2}-a^{2}cx+ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [-1/2*(3*(a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^
5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 + 4*a*c*x + (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3
- 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) -
2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)) - 2*(a^3*x^3 - 3*a^2*x^2)*sqrt(-a^2
*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*c*x^3 - a^3*c*x^2 - a^2*c*x
 + a*c), (3*(a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x +
 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 -
 2*a^2*c*x^2 - a*c*x + 2*c)) + (a^3*x^3 - 3*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*c*x^3 - a^3*c*x^2 - a^2*c*x + a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^{2}x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))), x)
```

maple [A] time = 0.05, size = 77, normalized size = 0.63

$$\frac{\sqrt{-a^{2}x^{2}+1}\,\left(a^{2}x^{2}+3\ln(ax-1)\,xa-ax-3\ln(ax-1)-2\right)}{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\;x\,a^{2}\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-a^2*x^2+1)^(1/2)/a^2*(a^2*x^2+3*ln(a*x-1)*x*a-a*x-3*ln(a*x-1)-2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i}{2\,(ax+1)(ax-1)a\sqrt{c}}-\int\frac{a^4x^4+3\,a^3x^3+3\,a^2x^2}{\left(i\,a^2\sqrt{c}\,x^2-i\,\sqrt{c}\right)(ax+1)(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I/((a*x + 1)*(a*x - 1)*a*sqrt(c)) - integrate((a^4*x^4 + 3*a^3*x^3 + 3*a^2*x^2)/((I*a^2*sqrt(c)*x^2 - I*sqrt(c))*(a*x + 1)*(a*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a\,x+1)^3}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^3}{(-(ax-1)\,(ax+1))^{\frac{3}{2}}\,\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))), x)
```

3.711 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4x^3(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{3\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] -(-a^2*x^2+1)^(3/2)/a^3/(c-c/a^2/x^2)^(3/2)/x^2+1/2*(-a^2*x^2+1)^(3/2)/a^4/
(c-c/a^2/x^2)^(3/2)/x^3/(-a*x+1)^2-3*(-a^2*x^2+1)^(3/2)/a^4/(c-c/a^2/x^2)^(
3/2)/x^3/(-a*x+1)-3*(-a^2*x^2+1)^(3/2)*ln(-a*x+1)/a^4/(c-c/a^2/x^2)^(3/2)/x
^3
```

Rubi [A] time = 0.19, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4x^3(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{3\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2)) + (1 - a^2*x^2)^(3
/2)/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)^2) - (3*(1 - a^2*x^2)^(3/2
))/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 - a*x)) - (3*(1 - a^2*x^2)^(3/2)*Log
[1 - a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{3\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(1-ax)^3}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}\int \left(-\frac{1}{a^3}-\frac{1}{a^3(-1+ax)^3}-\frac{3}{a^3(-1+ax)^2}-\frac{3}{a^3(-1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1-ax)^2}-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1-ax)}-\frac{3\left(1-a^2\right.}{a^4\left(c\right.}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.50

$$\frac{\sqrt{1-a^2x^2}\left(2a^3x^3-4a^2x^2-4ax+6(ax-1)^2\log(1-ax)+5\right)}{2a^2cx(ax-1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(5 - 4*a*x - 4*a^2*x^2 + 2*a^3*x^3 + 6*(-1 + a*x)^2*Log[
1 - a*x]))/(2*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^2)
```

fricas [A] time = 2.47, size = 477, normalized size = 2.74

$$\left[-\frac{3\left(a^4x^4-2a^3x^3+2ax-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx+\left(a^5x^5-4a^4x^4+6a^3x^3-4a^2x^2\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^4x^4-2a^3x^3+2ax-1}\right.}{2\left(a^5c^2x^4-2a^4c^2x^3+2a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [-1/2*(3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*
c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 + 4*a*c*x + (a^5*x^5 - 4*a^4*x^4 + 6*a^3*
x^3 - 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)) - (2*a^4*x^4 - 9*a^3*x^3 + 6*a^
2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^2*x^4 - 2
*a^4*c^2*x^3 + 2*a^2*c^2*x - a*c^2), 1/2*(6*(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^
2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 - 2*a^2*c*x^2 - a*c*x + 2*c)) + (2*a^4*x
^4 - 9*a^3*x^3 + 6*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)))/(a^5*c^2*x^4 - 2*a^4*c^2*x^3 + 2*a^2*c^2*x - a*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 106, normalized size = 0.61

$$\frac{\left(2x^3a^3+6\ln\left(ax-1\right)x^2a^2-4a^2x^2-12\ln\left(ax-1\right)xa-4ax+6\ln\left(ax-1\right)+5\right)\left(ax+1\right)\sqrt{-a^2x^2+1}}{2\left(ax-1\right)a^4x^3\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(2*x^3*a^3+6*ln(a*x-1)*x^2*a^2-4*a^2*x^2-12*ln(a*x-1)*x*a-4*a*x+6*ln(a*
x-1)+5)*(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^
(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,x+1)^3}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a^2*x^2))^(3/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((c - c/(a^2*x^2))^(3/2)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(-1 + 1/(a*x))*(1
+ 1/(a*x)))**(3/2)), x)
```

3.712 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=269

$$\frac{31\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6x^5(1-ax)^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{49\left(1-a^2x^2\right)^{5/2}\log(1-ax}{16a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

```
[Out] (-a^2*x^2+1)^(5/2)/a^5/(c-c/a^2/x^2)^(5/2)/x^4+1/6*(-a^2*x^2+1)^(5/2)/a^6/(
c-c/a^2/x^2)^(5/2)/x^5/(-a*x+1)^3-9/8*(-a^2*x^2+1)^(5/2)/a^6/(c-c/a^2/x^2)^
(5/2)/x^5/(-a*x+1)^2+31/8*(-a^2*x^2+1)^(5/2)/a^6/(c-c/a^2/x^2)^(5/2)/x^5/(-
a*x+1)+49/16*(-a^2*x^2+1)^(5/2)*ln(-a*x+1)/a^6/(c-c/a^2/x^2)^(5/2)/x^5-1/16
*(-a^2*x^2+1)^(5/2)*ln(a*x+1)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.22, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{31\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6x^5(1-ax)^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{49\left(1-\right.}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4) + (1 - a^2*x^2)^(5/2)
/(6*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)^3) - (9*(1 - a^2*x^2)^(5/2))/
(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)^2) + (31*(1 - a^2*x^2)^(5/2))/
(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)) + (49*(1 - a^2*x^2)^(5/2)*Log
[1 - a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) - ((1 - a^2*x^2)^(5/2)*Log[
1 + a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{3\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(1-ax)^4(1+ax)}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \left(\frac{1}{a^5}+\frac{1}{2a^5(-1+ax)^4}+\frac{9}{4a^5(-1+ax)^3}+\frac{31}{8a^5(-1+ax)^2}+\frac{49}{16a^5(-1+ax)}-\frac{1}{16a^5(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{\left(1-a^2x^2\right)^{5/2}}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)^3}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)^2}+\frac{31}{8a^6\left(c-\right.}$$

Mathematica [A] time = 0.08, size = 113, normalized size = 0.42

$$\frac{\sqrt{1-a^2x^2}\left(2\left(24a^4x^4-72a^3x^3-21a^2x^2+135ax-70\right)+147(ax-1)^3\log(1-ax)-3(ax-1)^3\log(ax+1)\right)}{48a^2c^2x(ax-1)^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*(-70 + 135*a*x - 21*a^2*x^2 - 72*a^3*x^3 + 24*a^4*x^4
) + 147*(-1 + a*x)^3*Log[1 - a*x] - 3*(-1 + a*x)^3*Log[1 + a*x]))/(48*a^2*c
^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3)
```

fricas [F] time = 2.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,a^6x^6\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^7c^3x^7-3\,a^6c^3x^6+a^5c^3x^5+5\,a^4c^3x^4-5\,a^3c^3x^3-a^2c^3x^2+3\,ac^3x-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^6*x^6*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^7*c^
3*x^7 - 3*a^6*c^3*x^6 + a^5*c^3*x^5 + 5*a^4*c^3*x^4 - 5*a^3*c^3*x^3 - a^2*c
^3*x^2 + 3*a*c^3*x - c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(5/2)), x)
```

maple [A] time = 0.06, size = 176, normalized size = 0.65

$$\frac{\left(48x^4a^4+147\ln(ax-1)\,x^3a^3-3a^3x^3\ln(ax+1)-144x^3a^3-441\ln(ax-1)\,x^2a^2+9\ln(ax+1)\,x^2a^2-42a^2x^2\right.}{48\,(ax-1)\,a^6x^5\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/48*(48*x^4*a^4+147*ln(a*x-1)*x^3*a^3-3*a^3*x^3*ln(a*x+1)-144*x^3*a^3-441*
ln(a*x-1)*x^2*a^2+9*ln(a*x+1)*x^2*a^2-42*a^2*x^2+441*ln(a*x-1)*x*a-9*a*x*ln
(a*x+1)+270*a*x-147*ln(a*x-1)+3*ln(a*x+1)-140)*(a*x+1)^2*(-a^2*x^2+1)^(1/2)
/(a*x-1)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a\,x+1)^3}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a^2*x^2))^(5/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((c - c/(a^2*x^2))^(5/2)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(-1 + 1/(a*x))*(1
+ 1/(a*x)))**(5/2)), x)
```

3.713 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=363

$$-\frac{75\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{59\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(1-ax)^3\left(c-\frac{c}{a^2x^2}\right.}$$

```
[Out] -(-a^2*x^2+1)^(7/2)/a^7/(c-c/a^2/x^2)^(7/2)/x^6+1/16*(-a^2*x^2+1)^(7/2)/a^8
/(c-c/a^2/x^2)^(7/2)/x^7/(-a*x+1)^4-1/2*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2
)^(7/2)/x^7/(-a*x+1)^3+59/32*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
/(-a*x+1)^2-75/16*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(-a*x+1)+1
/32*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(a*x+1)-201/64*(-a^2*x^2
+1)^(7/2)*ln(-a*x+1)/a^8/(c-c/a^2/x^2)^(7/2)/x^7+9/64*(-a^2*x^2+1)^(7/2)*ln
(a*x+1)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.25, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{75\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{59\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(1-ax)^3\left(c-\frac{c}{a^2x^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -((1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6)) + (1 - a^2*x^2)^(7
/2)/(16*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^4) - (1 - a^2*x^2)^(7/2)/
(2*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^3) + (59*(1 - a^2*x^2)^(7/2))/
(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^2) - (75*(1 - a^2*x^2)^(7/2))
/(16*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(32*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) - (201*(1 - a^2*x^2)^(7/2)*Log[1
- a*x])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) + (9*(1 - a^2*x^2)^(7/2)*Log[1
 + a*x])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{e^{3\tanh^{-1}(ax)}x^7}{\left(1-a^2x^2\right)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(1-ax)^5(1+ax)^2}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \left(-\frac{1}{a^7}-\frac{1}{4a^7(-1+ax)^5}-\frac{3}{2a^7(-1+ax)^4}-\frac{59}{16a^7(-1+ax)^3}-\frac{75}{16a^7(-1+ax)^2}-\frac{201}{64a^7(-1+ax)}-\frac{3\ldots}{}\right.}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{\left(1-a^2x^2\right)^{7/2}}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}+\frac{\left(1-a^2x^2\right)^{7/2}}{16a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^4}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^3}+\frac{}{32a^8\ldots}$$

Mathematica [A] time = 0.12, size = 146, normalized size = 0.40

$$\frac{\sqrt{1-a^2x^2}\left(2\left(32a^6x^6-96a^5x^5-87a^4x^4+309a^3x^3-59a^2x^2-207ax+104\right)+201(ax+1)(ax-1)^4\log(1-ax\ldots\right.}{64a^2c^3x(ax-1)^4(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*(104 - 207*a*x - 59*a^2*x^2 + 309*a^3*x^3 - 87*a^4*x^
4 - 96*a^5*x^5 + 32*a^6*x^6) + 201*(-1 + a*x)^4*(1 + a*x)*Log[1 - a*x] - 9*
(-1 + a*x)^4*(1 + a*x)*Log[1 + a*x]))/(64*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(
-1 + a*x)^4*(1 + a*x))
```

fricas [F] time = 2.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,a^8x^8\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^9c^4x^9-3\,a^8c^4x^8+8\,a^6c^4x^6-6\,a^5c^4x^5-6\,a^4c^4x^4+8\,a^3c^4x^3-3\,ac^4x+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^8*x^8*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^9*c^
4*x^9 - 3*a^8*c^4*x^8 + 8*a^6*c^4*x^6 - 6*a^5*c^4*x^5 - 6*a^4*c^4*x^4 + 8*a
^3*c^4*x^3 - 3*a*c^4*x + c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(7/2)), x)
```

maple [A] time = 0.06, size = 248, normalized size = 0.68

$$\frac{\sqrt{-a^2x^2+1}\;(ax+1)^2\left(64x^6a^6+201\ln(ax-1)\,x^5a^5-9\ln(ax+1)\,x^5a^5-192x^5a^5-603\ln(ax-1)\,x^4a^4+27\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/64*(-a^2*x^2+1)^(1/2)*(a*x+1)^2*(64*x^6*a^6+201*ln(a*x-1)*x^5*a^5-9*ln(a*
x+1)*x^5*a^5-192*x^5*a^5-603*ln(a*x-1)*x^4*a^4+27*ln(a*x+1)*x^4*a^4-174*x^4
```

```
*a^4+402*ln(a*x-1)*x^3*a^3-18*a^3*x^3*ln(a*x+1)+618*x^3*a^3+402*ln(a*x-1)*x
^2*a^2-18*ln(a*x+1)*x^2*a^2-118*a^2*x^2-603*ln(a*x-1)*x*a+27*a*x*ln(a*x+1)-
414*a*x+201*ln(a*x-1)-9*ln(a*x+1)+208)/(a*x-1)/a^8/x^7/(c*(a^2*x^2-1)/a^2/x
^2)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a\,x+1)^3}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - c/(a^2*x^2))^(7/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((c - c/(a^2*x^2))^(7/2)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(-1 + 1/(a*x))*(1
+ 1/(a*x)))**(7/2)), x)
```

3.714 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=375

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{7\left(1-a^2x^2\right)^{9/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{8\left(1-a^2x^2\right)^{9/2}}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{3\left(1-a^2x^2\right)^{9/2}}-\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{a^8x^9\log(x)\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}$$

```
[Out] -1/8*(c-c/a^2/x^2)^(9/2)*x/(-a^2*x^2+1)^(9/2)+1/7*a*(c-c/a^2/x^2)^(9/2)*x^2
/(-a^2*x^2+1)^(9/2)+2/3*a^2*(c-c/a^2/x^2)^(9/2)*x^3/(-a^2*x^2+1)^(9/2)-4/5*
a^3*(c-c/a^2/x^2)^(9/2)*x^4/(-a^2*x^2+1)^(9/2)-3/2*a^4*(c-c/a^2/x^2)^(9/2)*
x^5/(-a^2*x^2+1)^(9/2)+2*a^5*(c-c/a^2/x^2)^(9/2)*x^6/(-a^2*x^2+1)^(9/2)+2*a
^6*(c-c/a^2/x^2)^(9/2)*x^7/(-a^2*x^2+1)^(9/2)-4*a^7*(c-c/a^2/x^2)^(9/2)*x^8
/(-a^2*x^2+1)^(9/2)-a^9*(c-c/a^2/x^2)^(9/2)*x^10/(-a^2*x^2+1)^(9/2)+a^8*(c-
c/a^2/x^2)^(9/2)*x^9*ln(x)/(-a^2*x^2+1)^(9/2)
```

Rubi [A] time = 0.21, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}-\frac{4a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{5\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) + (a*(c - c/(a^2*x^2))
^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) + (2*a^2*(c - c/(a^2*x^2))^(9/2)*x^3)/(
3*(1 - a^2*x^2)^(9/2)) - (4*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)/(5*(1 - a^2*x^
2)^(9/2)) - (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*x^2)^(9/2)) + (
2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) + (2*a^6*(c - c/(a^2
*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) - (4*a^7*(c - c/(a^2*x^2))^(9/2)*x^8)
/(1 - a^2*x^2)^(9/2) - (a^9*(c - c/(a^2*x^2))^(9/2)*x^10)/(1 - a^2*x^2)^(9/
2) + (a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log[x])/(1 - a^2*x^2)^(9/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^5(1+ax)^4}{x^9}dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\left(-a^9+\frac{1}{x^9}-\frac{a}{x^8}-\frac{4a^2}{x^7}+\frac{4a^3}{x^6}+\frac{6a^4}{x^5}-\frac{6a^5}{x^4}-\frac{4a^6}{x^3}+\frac{4a^7}{x^2}+\frac{a^8}{x}\right)dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{8\left(1-a^2x^2\right)^{9/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{7\left(1-a^2x^2\right)^{9/2}}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3}{3\left(1-a^2x^2\right)^{9/2}}-\frac{4a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{5\left(1-a^2x^2\right)^{9/2}}$$

Mathematica [A] time = 0.11, size = 115, normalized size = 0.31

$$\frac{x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(a^9(-x)+a^8\log(x)-\frac{4a^7}{x}+\frac{2a^6}{x^2}+\frac{2a^5}{x^3}-\frac{3a^4}{2x^4}-\frac{4a^3}{5x^5}+\frac{2a^2}{3x^6}+\frac{a}{7x^7}-\frac{1}{8x^8}\right)}{\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*x^9*(-1/8*1/x^8 + a/(7*x^7) + (2*a^2)/(3*x^6) - (4
*a^3)/(5*x^5) - (3*a^4)/(2*x^4) + (2*a^5)/x^3 + (2*a^6)/x^2 - (4*a^7)/x - a
^9*x + a^8*Log[x]))/(1 - a^2*x^2)^(9/2)
```

fricas [A] time = 2.28, size = 606, normalized size = 1.62

$$\left[\frac{420\left(a^9c^4x^9-a^7c^4x^7\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)+\left(840\,a^9c^4x^9+3360\,a^7c^4x^7-168\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/840*(420*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2
 - c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - c)/(a^2*x^4 - x^2)) + (840*a^9*c^4*x^9 + 3360*a^7*c^4*x^7 - 168
0*a^6*c^4*x^6 - 1680*a^5*c^4*x^5 - (840*a^9 + 3360*a^7 - 1680*a^6 - 1680*a^
5 + 1260*a^4 + 672*a^3 - 560*a^2 - 120*a + 105)*c^4*x^8 + 1260*a^4*c^4*x^4
+ 672*a^3*c^4*x^3 - 560*a^2*c^4*x^2 - 120*a*c^4*x + 105*c^4)*sqrt(-a^2*x^2
+ 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7), -1/840*(840*(a^
9*c^4*x^9 - a^7*c^4*x^7)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sq
rt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) -
(840*a^9*c^4*x^9 + 3360*a^7*c^4*x^7 - 1680*a^6*c^4*x^6 - 1680*a^5*c^4*x^5 -
 (840*a^9 + 3360*a^7 - 1680*a^6 - 1680*a^5 + 1260*a^4 + 672*a^3 - 560*a^2 -
 120*a + 105)*c^4*x^8 + 1260*a^4*c^4*x^4 + 672*a^3*c^4*x^3 - 560*a^2*c^4*x^
2 - 120*a*c^4*x + 105*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^10*x^9 - a^8*x^7)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1), x)
```

maple [A] time = 0.05, size = 118, normalized size = 0.31

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{9}{2}}x\sqrt{-a^2x^2+1}\,\left(-840a^9x^9+840a^8\ln(x)x^8-3360a^7x^7+1680x^6a^6+1680x^5a^5-1260x^4a^4-672x^3a\right.}{840\left(a^2x^2-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/840*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a^2*x^2-1)^5*(-a^2*x^2+1)^(1/2)*(-8
40*a^9*x^9+840*a^8*ln(x)*x^8-3360*a^7*x^7+1680*x^6*a^6+1680*x^5*a^5-1260*x^
4*a^4-672*x^3*a^3+560*a^2*x^2+120*a*x-105)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(9/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(9/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(9/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

3.715 $\int e^{-\tanh^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{7/2} dx$

Optimal. Leaf size=299

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}+\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{a^6x^7\log(x)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}$$

```
[Out] -1/6*(c-c/a^2/x^2)^(7/2)*x/(-a^2*x^2+1)^(7/2)+1/5*a*(c-c/a^2/x^2)^(7/2)*x^2
/(-a^2*x^2+1)^(7/2)+3/4*a^2*(c-c/a^2/x^2)^(7/2)*x^3/(-a^2*x^2+1)^(7/2)-a^3*
(c-c/a^2/x^2)^(7/2)*x^4/(-a^2*x^2+1)^(7/2)-3/2*a^4*(c-c/a^2/x^2)^(7/2)*x^5/
(-a^2*x^2+1)^(7/2)+3*a^5*(c-c/a^2/x^2)^(7/2)*x^6/(-a^2*x^2+1)^(7/2)+a^7*(c-
c/a^2/x^2)^(7/2)*x^8/(-a^2*x^2+1)^(7/2)-a^6*(c-c/a^2/x^2)^(7/2)*x^7*ln(x)/(
-a^2*x^2+1)^(7/2)
```

Rubi [A] time = 0.19, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}-\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) + (a*(c - c/(a^2*x^2))
^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) + (3*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) - (a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(1 - a^2*x^2)^(7
/2) - (3*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) + (3*a^5*
(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) + (a^7*(c - c/(a^2*x^2))^(
7/2)*x^8)/(1 - a^2*x^2)^(7/2) - (a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[x])/(1
 - a^2*x^2)^(7/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^4(1+ax)^3}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\left(a^7+\frac{1}{x^7}-\frac{a}{x^6}-\frac{3a^2}{x^5}+\frac{3a^3}{x^4}+\frac{3a^4}{x^3}-\frac{3a^5}{x^2}-\frac{a^6}{x}\right)dx}{\left(1-a^2x^2\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{6\left(1-a^2x^2\right)^{7/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{5\left(1-a^2x^2\right)^{7/2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{4\left(1-a^2x^2\right)^{7/2}}-\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{\left(1-a^2x^2\right)^{7/2}}$$

Mathematica [A] time = 0.06, size = 98, normalized size = 0.33

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(60a^7x^7-60a^6x^6\log(x)+180a^5x^5-90a^4x^4-60a^3x^3+45a^2x^2+12ax-10\right)}{60a^6x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] -1/60*(c^3*Sqrt[c - c/(a^2*x^2)]*(-10 + 12*a*x + 45*a^2*x^2 - 60*a^3*x^3 -
90*a^4*x^4 + 180*a^5*x^5 + 60*a^7*x^7 - 60*a^6*x^6*Log[x]))/(a^6*x^5*Sqrt[1
 - a^2*x^2])
```

fricas [A] time = 2.26, size = 542, normalized size = 1.81

$$\left[\frac{30\left(a^{7}c^{3}x^{7}-a^{5}c^{3}x^{5}\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}+\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)+\left(60\,a^{7}c^{3}x^{7}+180\,a^{5}c^{3}x^{5}-90\,a^{4}c^{3}\cdots\right.}{60\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/60*(30*(a^7*c^3*x^7 - a^5*c^3*x^5)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
 c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) - c)/(a^2*x^4 - x^2)) + (60*a^7*c^3*x^7 + 180*a^5*c^3*x^5 - 90*a^4*
c^3*x^4 - (60*a^7 + 180*a^5 - 90*a^4 - 60*a^3 + 45*a^2 + 12*a - 10)*c^3*x^6
 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 12*a*c^3*x - 10*c^3)*sqrt(-a^2*x^2 + 1
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7 - a^6*x^5), -1/60*(60*(a^7*c^3*
x^7 - a^5*c^3*x^5)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt(c)*
sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (60*a^
7*c^3*x^7 + 180*a^5*c^3*x^5 - 90*a^4*c^3*x^4 - (60*a^7 + 180*a^5 - 90*a^4 -
 60*a^3 + 45*a^2 + 12*a - 10)*c^3*x^6 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 1
2*a*c^3*x - 10*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^
8*x^7 - a^6*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

maple [A] time = 0.05, size = 102, normalized size = 0.34

$$-\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}x\sqrt{-a^{2}x^{2}+1}\,\left(-60a^{7}x^{7}+60a^{6}\ln(x)x^{6}-180x^{5}a^{5}+90x^{4}a^{4}+60x^{3}a^{3}-45a^{2}x^{2}-12ax+10\right)}{60\left(a^{2}x^{2}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a^2*x^2-1)^4*(-a^2*x^2+1)^(1/2)*(-60
*a^7*x^7+60*a^6*ln(x)*x^6-180*x^5*a^5+90*x^4*a^4+60*x^3*a^3-45*a^2*x^2-12*a
*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(7/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2
)/(a*x + 1), x)
```

3.716 $\int e^{-\tanh^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{5/2} dx$

Optimal. Leaf size=220

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{3\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -1/4*(c-c/a^2/x^2)^(5/2)*x/(-a^2*x^2+1)^(5/2)+1/3*a*(c-c/a^2/x^2)^(5/2)*x^2
/(-a^2*x^2+1)^(5/2)+a^2*(c-c/a^2/x^2)^(5/2)*x^3/(-a^2*x^2+1)^(5/2)-2*a^3*(c
-c/a^2/x^2)^(5/2)*x^4/(-a^2*x^2+1)^(5/2)-a^5*(c-c/a^2/x^2)^(5/2)*x^6/(-a^2*
x^2+1)^(5/2)+a^4*(c-c/a^2/x^2)^(5/2)*x^5*ln(x)/(-a^2*x^2+1)^(5/2)
```

Rubi [A] time = 0.18, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{3\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) + (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(3*(1 - a^2*x^2)^(5/2)) + (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1
- a^2*x^2)^(5/2) - (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2)
- (a^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) + (a^4*(c - c/(a^2*
x^2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^3(1+ax)^2}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\left(-a^5+\frac{1}{x^5}-\frac{a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{\left(1-a^2x^2\right)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{3\left(1-a^2x^2\right)^{5/2}}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{\left(1-a^2x^2\right)^{5/2}}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.37

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(12a^5x^5-12a^4x^4\log(x)+24a^3x^3-12a^2x^2-4ax+3\right)}{12a^4x^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] -1/12*(c^2*Sqrt[c - c/(a^2*x^2)]*(3 - 4*a*x - 12*a^2*x^2 + 24*a^3*x^3 + 12*
a^5*x^5 - 12*a^4*x^4*Log[x]))/(a^4*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.75, size = 478, normalized size = 2.17

$$\left[\frac{6\left(a^5c^2x^5 - a^3c^2x^3\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) + \left(12\,a^5c^2x^5 + 24\,a^3c^2x^3 - \left(12\,a^5 + 2\right.\right.}{12\left(a^6x^5 - a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
c*x^4 + (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2
*x^2)) - c)/(a^2*x^4 - x^2)) + (12*a^5*c^2*x^5 + 24*a^3*c^2*x^3 - (12*a^5 +
 24*a^3 - 12*a^2 - 4*a + 3)*c^2*x^4 - 12*a^2*c^2*x^2 - 4*a*c^2*x + 3*c^2)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3), -1/
12*(12*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3
 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^
2 + c)) - (12*a^5*c^2*x^5 + 24*a^3*c^2*x^3 - (12*a^5 + 24*a^3 - 12*a^2 - 4*
a + 3)*c^2*x^4 - 12*a^2*c^2*x^2 - 4*a*c^2*x + 3*c^2)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)
```

maple [A] time = 0.05, size = 86, normalized size = 0.39

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}} x\sqrt{-a^2x^2+1}\,\left(-12x^5a^5+12a^4\ln(x)x^4-24x^3a^3+12a^2x^2+4ax-3\right)}{12\left(a^2x^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/12*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/(a^2*x^2-1)^3*(-a^2*x^2+1)^(1/2)*(-12
*x^5*a^5+12*a^4*ln(x)*x^4-24*x^3*a^3+12*a^2*x^2+4*a*x-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(5/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(5/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2
)/(a*x + 1), x)
```

3.717 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$

Optimal. Leaf size=144

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}+\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -1/2*(c-c/a^2/x^2)^(3/2)*x/(-a^2*x^2+1)^(3/2)+a*(c-c/a^2/x^2)^(3/2)*x^2/(-a
^2*x^2+1)^(3/2)+a^3*(c-c/a^2/x^2)^(3/2)*x^4/(-a^2*x^2+1)^(3/2)-a^2*(c-c/a^2
/x^2)^(3/2)*x^3*ln(x)/(-a^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 75}

$$\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}-\frac{a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) + (a*(c - c/(a^2*x^2))
^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) + (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 - a^
2*x^2)^(3/2) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(3/2)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
```

```
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^2(1+ax)}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{2\left(1-a^2x^2\right)^{3/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{\left(1-a^2x^2\right)^{3/2}}+\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^4}{\left(1-a^2x^2\right)^{3/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3 1}{\left(1-a^2x^2\right)^{3/2}}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.50

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(2a^3x^3-3a^2x^2-2a^2x^2\log(x)+2ax-1\right)}{2a^2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^ArcTanh[a*x],x]
[Out] -1/2*(c*Sqrt[c - c/(a^2*x^2)]*(-1 + 2*a*x - 3*a^2*x^2 + 2*a^3*x^3 - 2*a^2*x
^2*Log[x]))/(a^2*x*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.77, size = 377, normalized size = 2.62

$$\left[\frac{\left(a^3cx^3-acx\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)+\left(2\,a^3cx^3-\left(2\,a^3+2\,a-1\right)cx^2+2\,acx-\right.}{2\left(a^4x^3-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^3*c*x^3 - a*c*x)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*
x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)
/(a^2*x^4 - x^2)) + (2*a^3*c*x^3 - (2*a^3 + 2*a - 1)*c*x^2 + 2*a*c*x - c)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x), -1/2*
(2*(a^3*c*x^3 - a*c*x)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt
(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (2
*a^3*c*x^3 - (2*a^3 + 2*a - 1)*c*x^2 + 2*a*c*x - c)*sqrt(-a^2*x^2 + 1)*sqrt
((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

maple [A] time = 0.05, size = 70, normalized size = 0.49

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}} x\sqrt{-a^2x^2+1}\,\left(-2x^3a^3+2a^2\ln(x)x^2-2ax+1\right)}{2\left(a^2x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a^2*x^2-1)^2*(-a^2*x^2+1)^(1/2)*(-2*x
^3*a^3+2*a^2*ln(x)*x^2-2*a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(3/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2
)/(a*x + 1), x)
```

3.718 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] -a*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+x*ln(x)*(c-c/a^2/x^2)^(1/2)/(
-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcTanh[a*x],x]
```

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1-ax}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(-a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.55

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,(\log(x)-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-(a*x) + Log[x]))/Sqrt[1 - a^2*x^2]
```

fricas [B] time = 1.24, size = 321, normalized size = 4.65

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)+2\left(a^2x^2-a^2x\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^3x^2-a\right)}, -\frac{\left(a^2x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
```

```
x^4 - x^2)) + 2*(a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^3*x^2 - a), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)
*(a*x^3 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 +
1)*c*x^2 + c)) - (a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^3*x^2 - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

maple [A] time = 0.04, size = 53, normalized size = 0.77

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-ax+\ln(x)\right)\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a*x+ln(x))*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(
a*x + 1), x)
```

3.719 $\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (-a^2*x^2+1)^(1/2)/a/(c-c/a^2/x^2)^(1/2)-ln(a*x+1)*(-a^2*x^2+1)^(1/2)/a^2/x
/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - a^2*x^2]*Log[1 + a*
x])/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
```

```
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x}{1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-a^2x^2}\log(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.65

$$\frac{\sqrt{1-a^2x^2}\left(\frac{x}{a}-\frac{\log(ax+1)}{a^2}\right)}{x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
[Out] (Sqrt[1 - a^2*x^2]*(x/a - Log[1 + a*x]/a^2))/(Sqrt[c - c/(a^2*x^2)]*x)
```

fricas [B] time = 0.66, size = 367, normalized size = 4.77

$$\left[\frac{2\sqrt{-a^2x^2+1}\,a^2x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - \left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^6cx^6+4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2-4\,acx-\left(a^5x^5+4\,a^4x^4+6\,a^3x^3+4\,a^2x^2\right)\sqrt{-a^2x^2+}}{a^4x^4+2\,a^3x^3-2\,ax-1}\right.}{2\left(a^3cx^2-ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(2*sqrt(-a^2*x^2 + 1)*a^2*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - (a^2*x
^2 - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 -
 4*a*c*x - (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3 + 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)
*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a
*x - 1)))/(a^3*c*x^2 - a*c), (sqrt(-a^2*x^2 + 1)*a^2*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - (a^2*x^2 - 1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x + 2)*sqrt(-a^
2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2
 - a*c*x - 2*c)))/(a^3*c*x^2 - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

maple [A] time = 0.04, size = 51, normalized size = 0.66

$$-\frac{\sqrt{-a^2x^2+1}\;(-ax+\ln(ax+1))}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\;x\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(-a*x+ln(a*x+1))/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

maxima [C] time = 0.38, size = 21, normalized size = 0.27

$$\frac{i\,x}{\sqrt{c}}-\frac{i\,\log(ax+1)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] I*x/sqrt(c) - I*log(a*x + 1)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2 x^2}}{\sqrt{c-\frac{c}{a^2 x^2}}\,(a x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*
(a*x + 1)), x)
```

3.720 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=175

$$\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] -(-a^2*x^2+1)^(3/2)/a^3/(c-c/a^2/x^2)^(3/2)/x^2+1/2*(-a^2*x^2+1)^(3/2)/a^4/
(c-c/a^2/x^2)^(3/2)/x^3/(a*x+1)-1/4*(-a^2*x^2+1)^(3/2)*ln(-a*x+1)/a^4/(c-c/
a^2/x^2)^(3/2)/x^3+5/4*(-a^2*x^2+1)^(3/2)*ln(a*x+1)/a^4/(c-c/a^2/x^2)^(3/2)
/x^3
```

Rubi [A] time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{5\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{4a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] -((1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2)) + (1 - a^2*x^2)^(3
/2)/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)) - ((1 - a^2*x^2)^(3/2)*Lo
g[1 - a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (5*(1 - a^2*x^2)^(3/2)*Lo
g[1 + a*x])/(4*a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{-\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(1-ax)(1+ax)^2}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \left(-\frac{1}{a^3}-\frac{1}{4a^3(-1+ax)}-\frac{1}{2a^3(1+ax)^2}+\frac{5}{4a^3(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1+ax)}-\frac{\left(1-a^2x^2\right)^{3/2}\log(1-ax)}{4a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{5\left(1-a^2x^2\right.}{4a^4\left(c\right.}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.52

$$-\frac{\sqrt{1-a^2x^2}\left(a^2x^2-1\right)\left(\frac{1}{2a^4(ax+1)}-\frac{\log(1-ax)}{4a^4}+\frac{5\log(ax+1)}{4a^4}-\frac{x}{a^3}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(-1 + a^2*x^2)*(-(x/a^3) + 1/(2*a^4*(1 + a*x)) - Log[1
 - a*x]/(4*a^4) + (5*Log[1 + a*x])/(4*a^4)))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

fricas [F] time = 1.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,a^4x^4\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^5c^2x^5+a^4c^2x^4-2\,a^3c^2x^3-2\,a^2c^2x^2+ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^4*x^4*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^5*c^
2*x^5 + a^4*c^2*x^4 - 2*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + a*c^2*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

maple [A] time = 0.05, size = 92, normalized size = 0.53

$$\frac{\left(4a^2x^2+\ln(ax-1)\,xa-5ax\ln(ax+1)+4ax+\ln(ax-1)-5\ln(ax+1)-2\right)(ax-1)\sqrt{-a^2x^2+1}}{4a^4x^3\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/4*(4*a^2*x^2+ln(a*x-1)*x*a-5*a*x*ln(a*x+1)+4*a*x+ln(a*x-1)-5*ln(a*x+1)-2)
*(a*x-1)*(-a^2*x^2+1)^(1/2)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/
2)*(a*x + 1)), x)
```

3.721 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=265

$$\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{7\left(1-a^2x^2\right)^{5/2}\log(1-ax)}{16a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-$$

```
[Out] (-a^2*x^2+1)^(5/2)/a^5/(c-c/a^2/x^2)^(5/2)/x^4+1/8*(-a^2*x^2+1)^(5/2)/a^6/(
c-c/a^2/x^2)^(5/2)/x^5/(-a*x+1)+1/8*(-a^2*x^2+1)^(5/2)/a^6/(c-c/a^2/x^2)^(5
/2)/x^5/(a*x+1)^2-(-a^2*x^2+1)^(5/2)/a^6/(c-c/a^2/x^2)^(5/2)/x^5/(a*x+1)+7/
16*(-a^2*x^2+1)^(5/2)*ln(-a*x+1)/a^6/(c-c/a^2/x^2)^(5/2)/x^5-23/16*(-a^2*x^
2+1)^(5/2)*ln(a*x+1)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.21, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{7\left(1-a^2\right.}{16a^6x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4) + (1 - a^2*x^2)^(5/2)
/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 - a*x)) + (1 - a^2*x^2)^(5/2)/(8*a^6
*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)^2) - (1 - a^2*x^2)^(5/2)/(a^6*(c - c
/(a^2*x^2))^(5/2)*x^5*(1 + a*x)) + (7*(1 - a^2*x^2)^(5/2)*Log[1 - a*x])/(16
*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) - (23*(1 - a^2*x^2)^(5/2)*Log[1 + a*x])/(
16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{-\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(1-ax)^2(1+ax)^3}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(1-a^2x^2\right)^{5/2}\int \left(\frac{1}{a^5}+\frac{1}{8a^5(-1+ax)^2}+\frac{7}{16a^5(-1+ax)}-\frac{1}{4a^5(1+ax)^3}+\frac{1}{a^5(1+ax)^2}-\frac{23}{16a^5(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(1-a^2x^2\right)^{5/2}}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1-ax)}+\frac{\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1+ax)^2}-\frac{(1}{a^6\left(c-\frac{}{a^2}\right.}\end{aligned}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 0.33

$$\frac{\left(1-a^2x^2\right)^{5/2}\left(2\left(8ax+\frac{1}{1-ax}-\frac{8}{ax+1}+\frac{1}{(ax+1)^2}\right)+7\log(1-ax)-23\log(ax+1)\right)}{16a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - a^2*x^2)^(5/2)*(2*(8*a*x + (1 - a*x)^(-1) + (1 + a*x)^(-2) - 8/(1 + a
*x)) + 7*Log[1 - a*x] - 23*Log[1 + a*x]))/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x
^5)
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,a^6x^6\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^7c^3x^7+a^6c^3x^6-3\,a^5c^3x^5-3\,a^4c^3x^4+3\,a^3c^3x^3+3\,a^2c^3x^2-ac^3x-c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^6*x^6*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^7*c^
3*x^7 + a^6*c^3*x^6 - 3*a^5*c^3*x^5 - 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 + 3*a^2
*c^3*x^2 - a*c^3*x - c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

maple [A] time = 0.06, size = 167, normalized size = 0.63

$$\frac{\sqrt{-a^2x^2+1}\ (ax-1)\left(16x^4a^4+7\ln(ax-1)\,x^3a^3-23a^3x^3\ln(ax+1)+16x^3a^3+7\ln(ax-1)\,x^2a^2-23\ln(ax+\right.}{16a^6x^5\left(\frac{c(a^2x^2-1)}{a^2x^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(a*x-1)*(16*x^4*a^4+7*ln(a*x-1)*x^3*a^3-23*a^3*x^3*
ln(a*x+1)+16*x^3*a^3+7*ln(a*x-1)*x^2*a^2-23*ln(a*x+1)*x^2*a^2-34*a^2*x^2-7*
ln(a*x-1)*x*a+23*a*x*ln(a*x+1)-18*a*x-7*ln(a*x-1)+23*ln(a*x+1)+12)/a^6/x^5/
(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}\left(ax+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/
2)*(a*x + 1)), x)
```

3.722 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=361

$$-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{3\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right.}$$

```
[Out] -(-a^2*x^2+1)^(7/2)/a^7/(c-c/a^2/x^2)^(7/2)/x^6+1/32*(-a^2*x^2+1)^(7/2)/a^8
/(c-c/a^2/x^2)^(7/2)/x^7/(-a*x+1)^2-5/16*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^
2)^(7/2)/x^7/(-a*x+1)+1/24*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(
a*x+1)^3-11/32*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(a*x+1)^2+3/2
*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(a*x+1)-19/32*(-a^2*x^2+1)^
(7/2)*ln(-a*x+1)/a^8/(c-c/a^2/x^2)^(7/2)/x^7+51/32*(-a^2*x^2+1)^(7/2)*ln(a*
x+1)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.24, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{3\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{11\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] -((1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6)) + (1 - a^2*x^2)^(7
/2)/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)^2) - (5*(1 - a^2*x^2)^(7/
2))/(16*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(2
4*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^3) - (11*(1 - a^2*x^2)^(7/2))/(
32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^2) + (3*(1 - a^2*x^2)^(7/2))/(
2*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) - (19*(1 - a^2*x^2)^(7/2)*Log[
1 - a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) + (51*(1 - a^2*x^2)^(7/2)*Lo
g[1 + a*x])/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{e^{-\tanh^{-1}(ax)}x^7}{\left(1-a^2x^2\right)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(1-ax)^3(1+ax)^4}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \left(-\frac{1}{a^7}-\frac{1}{16a^7(-1+ax)^3}-\frac{5}{16a^7(-1+ax)^2}-\frac{19}{32a^7(-1+ax)}-\frac{1}{8a^7(1+ax)^4}+\frac{11}{16a^7(1+ax)^3}-\frac{}{2a^7}\right.}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{\left(1-a^2x^2\right)^{7/2}}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)^2}-\frac{5\left(1-a^2x^2\right)^{7/2}}{16a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)}+\frac{}{24a^8}$$

Mathematica [A] time = 0.12, size = 145, normalized size = 0.40

$$\frac{\sqrt{1-a^2x^2}\left(96a^6x^6+96a^5x^5-366a^4x^4-222a^3x^3+338a^2x^2+122ax+57(ax-1)^2(ax+1)^3\log(1-ax)-153\right.}{96a^2x(ax-1)^2\sqrt{c-\frac{c}{a^2x^2}}\,(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-88 + 122*a*x + 338*a^2*x^2 - 222*a^3*x^3 - 366*a^4*x^4
 + 96*a^5*x^5 + 96*a^6*x^6 + 57*(-1 + a*x)^2*(1 + a*x)^3*Log[1 - a*x] - 153
*(-1 + a*x)^2*(1 + a*x)^3*Log[1 + a*x]))/(96*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-
1 + a*x)^2*(c + a*c*x)^3)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,a^8x^8\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^9c^4x^9+a^8c^4x^8-4\,a^7c^4x^7-4\,a^6c^4x^6+6\,a^5c^4x^5+6\,a^4c^4x^4-4\,a^3c^4x^3-4\,a^2c^4x^2+ac^4x+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*a^8*x^8*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^9*c^
4*x^9 + a^8*c^4*x^8 - 4*a^7*c^4*x^7 - 4*a^6*c^4*x^6 + 6*a^5*c^4*x^5 + 6*a^4
*c^4*x^4 - 4*a^3*c^4*x^3 - 4*a^2*c^4*x^2 + a*c^4*x + c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

maple [A] time = 0.06, size = 239, normalized size = 0.66

$$\frac{\sqrt{-a^2x^2+1}\ (ax-1)\left(96x^6a^6+57\ln(ax-1)\,x^5a^5-153\ln(ax+1)\,x^5a^5+96x^5a^5+57\ln(ax-1)\,x^4a^4-153\ln(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/96*(-a^2*x^2+1)^(1/2)*(a*x-1)*(96*x^6*a^6+57*ln(a*x-1)*x^5*a^5-153*ln(a*x
+1)*x^5*a^5+96*x^5*a^5+57*ln(a*x-1)*x^4*a^4-153*ln(a*x+1)*x^4*a^4-366*x^4*a
```

```
^4-114*ln(a*x-1)*x^3*a^3+306*a^3*x^3*ln(a*x+1)-222*x^3*a^3-114*ln(a*x-1)*x^
2*a^2+306*ln(a*x+1)*x^2*a^2+338*a^2*x^2+57*ln(a*x-1)*x*a-153*a*x*ln(a*x+1)+
122*a*x+57*ln(a*x-1)-153*ln(a*x+1)-88)/a^8/x^7/(c*(a^2*x^2-1)/a^2/x^2)^(7/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/
2)*(a*x + 1)), x)
```

3.723 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=455

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{28(ax+1)}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{8(ax+1)}+\frac{47a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{336(1-ax)(ax+1)}+\frac{11a^8x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{128(1-ax)^4(ax+1)^4}-\frac{2a^8x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}\sin^{-1}(a}{(1-ax)^{9/2}(ax+1)^{9/2}}$$

```
[Out] 11/128*a^8*(c-c/a^2/x^2)^(9/2)*x^9/(-a*x+1)^4/(a*x+1)^4+39/64*a^7*(c-c/a^2/
x^2)^(9/2)*x^8/(-a*x+1)^4/(a*x+1)^3-11/640*a^6*(c-c/a^2/x^2)^(9/2)*x^7/(-a*
x+1)^4/(a*x+1)^2+1/28*a*(c-c/a^2/x^2)^(9/2)*x^2/(a*x+1)-103/160*a^5*(c-c/a^
2/x^2)^(9/2)*x^6/(-a*x+1)^4/(a*x+1)+629/960*a^4*(c-c/a^2/x^2)^(9/2)*x^5/(-a
*x+1)^3/(a*x+1)-2/5*a^3*(c-c/a^2/x^2)^(9/2)*x^4/(-a*x+1)^2/(a*x+1)+47/336*a
^2*(c-c/a^2/x^2)^(9/2)*x^3/(-a*x+1)/(a*x+1)-1/8*(c-c/a^2/x^2)^(9/2)*x*(-a*x
+1)/(a*x+1)-2*a^8*(c-c/a^2/x^2)^(9/2)*x^9*arcsin(a*x)/(-a*x+1)^(9/2)/(a*x+1
)^(9/2)+245/128*a^8*(c-c/a^2/x^2)^(9/2)*x^9*arctanh((-a*x+1)^(1/2)*(a*x+1)^
(1/2))/(-a*x+1)^(9/2)/(a*x+1)^(9/2)
```

Rubi [A] time = 0.54, antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{11a^8x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{128(1-ax)^4(ax+1)^4}+\frac{39a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{64(1-ax)^4(ax+1)^3}-\frac{11a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{640(1-ax)^4(ax+1)^2}-\frac{103a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{160(1-ax)^4(ax+1)}+\frac{629a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{960(1-ax)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (11*a^8*(c - c/(a^2*x^2))^(9/2)*x^9)/(128*(1 - a*x)^4*(1 + a*x)^4) + (39*a^
7*(c - c/(a^2*x^2))^(9/2)*x^8)/(64*(1 - a*x)^4*(1 + a*x)^3) - (11*a^6*(c -
c/(a^2*x^2))^(9/2)*x^7)/(640*(1 - a*x)^4*(1 + a*x)^2) + (a*(c - c/(a^2*x^2)
)^(9/2)*x^2)/(28*(1 + a*x)) - (103*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(160*(1
 - a*x)^4*(1 + a*x)) + (629*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(960*(1 - a*x)
^3*(1 + a*x)) - (2*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)/(5*(1 - a*x)^2*(1 + a*x
)) + (47*a^2*(c - c/(a^2*x^2))^(9/2)*x^3)/(336*(1 - a*x)*(1 + a*x)) - ((c -
 c/(a^2*x^2))^(9/2)*x*(1 - a*x))/(8*(1 + a*x)) - (2*a^8*(c - c/(a^2*x^2))^(
9/2)*x^9*ArcSin[a*x])/((1 - a*x)^(9/2)*(1 + a*x)^(9/2)) + (245*a^8*(c - c/(
a^2*x^2))^(9/2)*x^9*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(128*(1 - a*x)^(9
/2)*(1 + a*x)^(9/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
```

```
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{9/2}(1+ax)^{9/2}}{x^9}dx}{(1-ax)^{9/2}(1+ax)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{11/2}(1+ax)^{7/2}}{x^9}dx}{(1-ax)^{9/2}(1+ax)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1-ax)}{8(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{9/2}(1+ax)^{5/2}\left(-2a-9a^2x\right)}{x^8}dx}{8(1-ax)^{9/2}(1+ax)^{9/2}}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1-ax)}{8(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^{7/2}(1+ax)^{5}\cdots}{x\cdots}}{56(1-ax)^{9/2}(1+ax\cdots}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}+\frac{47a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3}{336(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x(1-ax)}{8(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\cdots\right.\right.}{\cdots}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5(1-ax)^2(1+ax)}+\frac{47a^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^3}{336(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\cdots}{8(1+\cdots}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}+\frac{629a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{960(1-ax)^3(1+ax)}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5(1-ax)^2(1+ax)}+\frac{47a^2\left(c-\cdots\right.}{336(1-ax\cdots}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{103a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^6}{160(1-ax)^4(1+ax)}+\frac{629a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^5}{960(1-ax)^3(1+ax)}-\frac{2a^3\left(c-\cdots\right.}{5(1-ax\cdots}$$

$$=-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{103a^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^6}{160(1-ax)^4(1+ax)}+\frac{629a^4\cdots}{960(1\cdots}$$

$$=\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{28(1+ax)}-\frac{103a^5\left(c\cdots\right.}{160(1-\cdots}$$

$$=\frac{11a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c\cdots\right.}{2\cdots}$$

$$=\frac{11a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c\cdots\right.}{2\cdots}$$

$$=\frac{11a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c\cdots\right.}{2\cdots}$$

$$=\frac{11a^8\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9}{128(1-ax)^4(1+ax)^4}+\frac{39a^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^8}{64(1-ax)^4(1+ax)^3}-\frac{11a^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^7}{640(1-ax)^4(1+ax)^2}+\frac{a\left(c\cdots\right.}{2\cdots}$$

Mathematica [A] time = 0.19, size = 166, normalized size = 0.36

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}\left(-26880a^8x^8\log\left(\sqrt{a^2x^2-1}+ax\right)+25725a^8x^8\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(13440a^8x^8+4505\cdots\right.\right.}{13440a^8x^7\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^(2*ArcTanh[a*x]),x]

[Out] -1/13440*(c^4*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1680 - 3840*a*x - 4760*a^2*x^2 + 16896*a^3*x^3 + 770*a^4*x^4 - 31232*a^5*x^5 + 14595*a^6*x^6 + 45056*a^7*x^7 + 13440*a^8*x^8) + 25725*a^8*x^8*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 26880*a^8*x^8*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(a^8*x^7*Sqrt[-1 + a^2*x ^2])

fricas [A] time = 0.60, size = 482, normalized size = 1.06

$$\left[-\frac{53760\,a^7\sqrt{-c}\,c^4x^7\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-25725\,a^7\sqrt{-c}\,c^4x^7\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(13440\,a^8c\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")

[Out] [-1/26880*(53760*a^7*sqrt(-c)*c^4*x^7*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x ^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 25725*a^7*sqrt(-c)*c^4*x^7*log(-(a^2* c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(134 40*a^8*c^4*x^8 + 45056*a^7*c^4*x^7 + 14595*a^6*c^4*x^6 - 31232*a^5*c^4*x^5 + 770*a^4*c^4*x^4 + 16896*a^3*c^4*x^3 - 4760*a^2*c^4*x^2 - 3840*a*c^4*x + 1 680*c^4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7), -1/13440*(25725*a^7*c^ (9/2)*x^7*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 13440*a^7*c^(9/2)*x^7*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c* x^2 - c)/(a^2*x^2)) - c) + (13440*a^8*c^4*x^8 + 45056*a^7*c^4*x^7 + 14595*a ^6*c^4*x^6 - 31232*a^5*c^4*x^5 + 770*a^4*c^4*x^4 + 16896*a^3*c^4*x^3 - 4760 *a^2*c^4*x^2 - 3840*a*c^4*x + 1680*c^4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a ^8*x^7)]

giac [A] time = 101.60, size = 707, normalized size = 1.55

$$\frac{1}{6720}\left(\frac{25725\,c^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}-\frac{13440\,c^{\frac{9}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{6720\,\sqrt{a^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6720*(25725*c^(9/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x)/a^2 - 13440*c^(9/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))
*sgn(x)/(a*abs(a)) - 6720*sqrt(a^2*c*x^2 - c)*c^4*sgn(x)/a^2 + (14595*(sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 - c))^15*c^5*abs(a)*sgn(x) - 107520*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^14*a*c^(11/2)*sgn(x) + 76055*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^13*c^6*abs(a)*sgn(x) - 430080*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^12*a*c^(13/2)*sgn(x) + 64435*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^11*c^7*abs(a)*sgn(x) - 1111040*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^10*
a*c^(15/2)*sgn(x) + 110495*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*c^8*abs(
a)*sgn(x) - 1576960*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*c^(17/2)*sgn(
x) - 110495*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^9*abs(a)*sgn(x) - 141
2096*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(19/2)*sgn(x) - 64435*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^10*abs(a)*sgn(x) - 831488*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(21/2)*sgn(x) - 76055*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^11*abs(a)*sgn(x) - 252928*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))^2*a*c^(23/2)*sgn(x) - 14595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c)
)*c^12*abs(a)*sgn(x) - 45056*a*c^(25/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^8*a^2*abs(a)))*abs(a)
```

maple [B] time = 0.12, size = 965, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/40320*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/a^2*(5040*a^4*(c*(a^2*x^2-1)/a^2)^(
11/2)*(-c/a^2)^(1/2)-77175*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a
^2-c)/a^2/x)*x^8*c^6+23808*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^7*a^
11+17535*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^6*a^10-13056*(-c/a^2)^
(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^5*a^9+6510*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)
/a^2)^(11/2)*x^4*a^8-6912*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^3*a^7
+10920*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x^2*a^6-11520*(-c/a^2)^(1/
```

```
2)*(c*(a^2*x^2-1)/a^2)^(11/2)*x*a^5+58590*(-c/a^2)^(1/2)*c^(11/2)*ln(x*c^(1
/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^8*a+22050*(-c/a^2)^(1/2)*c^(11/2)*ln((c^(1
/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^8*a-23808*(-c/a^2)^(1/2)*
(c*(a^2*x^2-1)/a^2)^(9/2)*x^9*a^11*c-8575*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
)^(9/2)*x^8*a^10*c-8960*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(9/2)*x^8*a^
10*c+26784*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^9*a^9*c^2+10080*(-c/a
^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^9*a^9*c^2+11025*(-c/a^2)^(1/2)*(c
*(a^2*x^2-1)/a^2)^(7/2)*x^8*a^8*c^2-31248*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
)^(5/2)*x^9*a^7*c^3-11760*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^9*
a^7*c^3-15435*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^8*a^6*c^3+39060*(-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^9*a^5*c^4+14700*(-c/a^2)^(1/2)*((a
*x-1)*(a*x+1)*c/a^2)^(3/2)*x^9*a^5*c^4+25725*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/
a^2)^(3/2)*x^8*a^4*c^4-58590*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^9*a
^3*c^5-22050*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^9*a^3*c^5-77175
*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^8*a^2*c^5)/(c*(a^2*x^2-1)/a^2)^
(9/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(a^2x^2-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(9/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
[Out] -int(((c - c/(a^2*x^2))^(9/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [C] time = 50.94, size = 1408, normalized size = 3.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(9/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**4*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
 (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqr
t(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**4*Piecewise((-a*sqrt(c)
*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*
asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + 2*c**4*Piecewi
se((I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2)))
 - I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1),
 (-a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True)
)/a**2 - 6*c**4*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(
3*c), True))/a**3 + 6*c**4*Piecewise((2*a**3*sqrt(c)*sqrt(a**2*x**2 - 1)/(1
5*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - sqrt(c)*sqrt(a**2*x**2 - 1
)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt(c)*sqrt(-a**2*x**2 + 1)/(
15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3) - I*sqrt(c)*sqrt(-a**2*x
**2 + 1)/(5*a*x**5), True))/a**5 - 2*c**4*Piecewise((I*a**5*sqrt(c)*acosh(1
/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*x**2))) + I*a**2*sqrt(c
)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/(24*x**5*sqrt(-1 + 1/(a*
*2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*sqrt(c)/(16*x*sqrt(1 - 1/
(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a**2*x**2))) - 5*sqrt(c)/
(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2*x**7*sqrt(1 - 1/(a**2*x
**2))), True))/a**6 - 2*c**4*Piecewise((8*a**5*sqrt(c)*sqrt(a**2*x**2 - 1)/
(105*x) + 4*a**3*sqrt(c)*sqrt(a**2*x**2 - 1)/(105*x**3) + a*sqrt(c)*sqrt(a*
*2*x**2 - 1)/(35*x**5) - sqrt(c)*sqrt(a**2*x**2 - 1)/(7*a*x**7), Abs(a**2*x
**2) > 1), (8*I*a**5*sqrt(c)*sqrt(-a**2*x**2 + 1)/(105*x) + 4*I*a**3*sqrt(c
)*sqrt(-a**2*x**2 + 1)/(105*x**3) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(35*x*
*5) - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(7*a*x**7), True))/a**7 + c**4*Piecewi
se((5*I*a**7*sqrt(c)*acosh(1/(a*x))/128 - 5*I*a**6*sqrt(c)/(128*x*sqrt(-1 +
 1/(a**2*x**2))) + 5*I*a**4*sqrt(c)/(384*x**3*sqrt(-1 + 1/(a**2*x**2))) + I
*a**2*sqrt(c)/(192*x**5*sqrt(-1 + 1/(a**2*x**2))) + 7*I*sqrt(c)/(48*x**7*sq
rt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(8*a**2*x**9*sqrt(-1 + 1/(a**2*x**2))),
 1/Abs(a**2*x**2) > 1), (-5*a**7*sqrt(c)*asin(1/(a*x))/128 + 5*a**6*sqrt(c)
/(128*x*sqrt(1 - 1/(a**2*x**2))) - 5*a**4*sqrt(c)/(384*x**3*sqrt(1 - 1/(a**
2*x**2))) - a**2*sqrt(c)/(192*x**5*sqrt(1 - 1/(a**2*x**2))) - 7*sqrt(c)/(48
*x**7*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(8*a**2*x**9*sqrt(1 - 1/(a**2*x**2
))), True))/a**8
```

3.724 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=375

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{15(ax+1)}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(ax+1)}+\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}-\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{2a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\sin^{-1}(ax)}{(1-ax)^{7/2}(ax+1)^{7/2}}$$

```
[Out] -7/16*a^6*(c-c/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(a*x+1)^3-3/8*a^5*(c-c/a^2/x^2
)^(7/2)*x^6/(-a*x+1)^3/(a*x+1)^2+1/15*a*(c-c/a^2/x^2)^(7/2)*x^2/(a*x+1)+19/
16*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)-2/3*a^3*(c-c/a^2/x^2)^(7/
2)*x^4/(-a*x+1)^2/(a*x+1)+23/120*a^2*(c-c/a^2/x^2)^(7/2)*x^3/(-a*x+1)/(a*x+
1)-1/6*(c-c/a^2/x^2)^(7/2)*x*(-a*x+1)/(a*x+1)+2*a^6*(c-c/a^2/x^2)^(7/2)*x^7
*arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)-25/16*a^6*(c-c/a^2/x^2)^(7/2)*x^7
*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1)^(7/2)
```

Rubi [A] time = 0.47, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}-\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{8(1-ax)^3(ax+1)^2}+\frac{19a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3(1-ax)^2(ax+1)}+\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}+\frac{ax^2\left(\right.}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-7*a^6*(c - c/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) - (3*a^5*
(c - c/(a^2*x^2))^(7/2)*x^6)/(8*(1 - a*x)^3*(1 + a*x)^2) + (a*(c - c/(a^2*x
^2))^(7/2)*x^2)/(15*(1 + a*x)) + (19*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(16*(
1 - a*x)^3*(1 + a*x)) - (2*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a*x)^2*
(1 + a*x)) + (23*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(120*(1 - a*x)*(1 + a*x))
 - ((c - c/(a^2*x^2))^(7/2)*x*(1 - a*x))/(6*(1 + a*x)) + (2*a^6*(c - c/(a^2
*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) - (25*a^6*(
c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16*(1 - a
*x)^(7/2)*(1 + a*x)^(7/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}\,dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{9/2}(1+ax)^{5/2}}{x^7}\,dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{7/2}(1+ax)^{3/2}\left(-2a-7a^2x\right)}{x^6}\,dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{5/2}(1+ax)^{3}}{x}}{30(1-ax)^{7/2}(1+ax}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}+\frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right.\right.}{}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)}+\frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(1+}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}+\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)}+\frac{23a^2\left(c-\right.}{120(1-ax)(}$$

$$=-\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}+\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}-\frac{2a^3\left(c-\right.}{3(1-ax)^2}$$

$$=-\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}-\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}+\frac{19a^4\left(c-\right.}{16(1-ax}$$

$$=-\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}-\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}+\frac{19a^4\left(c-\right.}{16(1-ax}$$

$$=-\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}-\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}+\frac{19a^4\left(c-\right.}{16(1-ax}$$

$$=-\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}-\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}+\frac{19a^4\left(c-\right.}{16(1-ax}$$

Mathematica [A] time = 0.16, size = 150, normalized size = 0.40

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(-480a^6x^6\log\left(\sqrt{a^2x^2-1}+ax\right)+375a^6x^6\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(240a^6x^6+736a^5x^5+10\right.\right.}{240a^6x^5\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/240*(c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 + 96*a*x + 70*a^
2*x^2 - 352*a^3*x^3 + 105*a^4*x^4 + 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^
6*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]])
)/(a^6*x^5*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 1.22, size = 438, normalized size = 1.17

$$\left[-\frac{960\,a^5\sqrt{-c}\,c^3x^5\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-375\,a^5\sqrt{-c}\,c^3x^5\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(240\,a^6c^3x^6+7\right.}{480\,a^6x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2
+ 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(240*a^6*c
^3*x^6 + 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x
^2 + 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), -1/24
0*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
a^2*c*x^2 - c)) - 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 + 736*a^5*c^3*x^5 +
105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 + 96*a*c^3*x - 40*c^3)*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.07, size = 795, normalized size = 2.12

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}} x \left(-2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\, x^7a^9c + 2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{9}{2}}\sqrt{-\frac{c}{a^2}}\, x^5a^9 - 375\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\, x^6a^8c +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(-2016*(c*(a^2*x^2-1)/a^2)^(7/2
)*(-c/a^2)^(1/2)*x^7*a^9*c+2016*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^
5*a^9-375*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*x^6*a^8*c+480*(-c/a^2)^(
1/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a^8*c-105*(c*(a^2*x^2-1)/a^2)^(9/2)*
(-c/a^2)^(1/2)*x^4*a^8+2352*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*x^7*a^
7*c^2-560*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7*c^2+224*(c*(
a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^3*a^7+525*(c*(a^2*x^2-1)/a^2)^(5/2)*
(-c/a^2)^(1/2)*x^6*a^6*c^2-2940*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^
7*a^5*c^3+700*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^7*a^5*c^3-630*
(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^2*a^6-875*(c*(a^2*x^2-1)/a^2)^(3
/2)*(-c/a^2)^(1/2)*x^6*a^4*c^3+672*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)
*x*a^5+4410*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^7*a^3*c^4-1050*(-c/a
^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^7*a^3*c^4-280*a^4*(c*(a^2*x^2-1)/
a^2)^(9/2)*(-c/a^2)^(1/2)+2625*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^6
*a^2*c^4-4410*(-c/a^2)^(1/2)*c^(9/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2)
)*x^6*a+1050*(-c/a^2)^(1/2)*c^(9/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/
2)+c*x)/c^(1/2))*x^6*a+2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-c)/a^2/x)*x^6*c^5)/(c*(a^2*x^2-1)/a^2)^(7/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(a^2\,x^2-1\right)}{\left(a\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(7/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(7/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [C] time = 24.01, size = 1059, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
 (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqr
t(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**3*Piecewise((-a*sqrt(c)
*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*
asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c**3*Piecewise
((I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
 I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (
-a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/
a**2 - 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*
c), True))/a**3 + c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*
sqrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a
**2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*
x**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(
a**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2
*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 + 2*c**3*Piecewise((2*a**3*sqrt
(c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) -
sqrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt
(c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3
) - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 - c**3*Piecewise
((I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2
*x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/
(24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a*
*2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*s
qrt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a
**2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2
*x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

3.725 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=293

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)}+\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}+\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{2a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\sin^{-1}(ax)}{(1-ax)^{5/2}(ax+1)^{5/2}}$$

```
[Out] 7/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(a*x+1)^2+1/6*a*(c-c/a^2/x^2)^(5
/2)*x^2/(a*x+1)-2*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+1)+7/24*a^2*(
c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)/(a*x+1)-1/4*(c-c/a^2/x^2)^(5/2)*x*(-a*x+1)/
(a*x+1)-2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5/2)/(a*x+1)^(5
/2)+9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(
-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

Rubi [A] time = 0.43, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^2(ax+1)}+\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)}-\frac{2a^4x^5\left(c\right.}{(1-}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (7*a^4*(c - c/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) + (a*(c - c
/(a^2*x^2))^(5/2)*x^2)/(6*(1 + a*x)) - (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/
((1 - a*x)^2*(1 + a*x)) + (7*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(24*(1 - a*x)
*(1 + a*x)) - ((c - c/(a^2*x^2))^(5/2)*x*(1 - a*x))/(4*(1 + a*x)) - (2*a^4*
(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)^(5/2))
+ (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/
(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{7/2}(1+ax)^{3/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{5/2}\sqrt{1+ax}\left(-2a-5a^2x\right)}{x^4}dx}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{3/2}\sqrt{1+ax}\left(-\right.}{x^3}}{12(1-ax)^{5/2}(1+ax)^{5/}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/}\right.}{2}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} - \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1}{4(1+ax)}$$

$$= \frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5}}{24(1-ax)(1+}$$

$$= \frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5}}{24(1-ax)(1+}$$

$$= \frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5}}{24(1-ax)(1+}$$

$$= \frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} + \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5}}{24(1-ax)(1+}$$

Mathematica [A] time = 0.14, size = 134, normalized size = 0.46

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(-48a^4x^4\log\left(\sqrt{a^2x^2-1}+ax\right)+27a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(24a^4x^4+64a^3x^3-3a^2x^2\right.\right.}{24a^4x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/24*(c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 - 16*a*x - 3*a^2*x^
2 + 64*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 48
*a^4*x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(a^4*x^3*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 1.65, size = 394, normalized size = 1.34

$$\left[-\frac{96\,a^{3}\sqrt{-c}\,c^{2}x^{3}\arctan\left(\frac{a^{2}\sqrt{-c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)-27\,a^{3}\sqrt{-c}\,c^{2}x^{3}\log\left(-\frac{a^{2}cx^{2}+2\,a\sqrt{-c}\,x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)+2\left(24\,a^{4}c^{2}x^{4}+64\right.}{48\,a^{4}x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) - 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 + 2
*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*c^2*x
^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)))/(a^4*x^3), -1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqr
t((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 24*a^3*c^(5/2)*x^3*log(2*a^
2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*
c^2*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*
x^2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

giac [A] time = 13.72, size = 416, normalized size = 1.42

$$\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^{2}c}\,x-\sqrt{a^{2}cx^{2}-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^{2}}-\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^{2}c}\,x+\sqrt{a^{2}cx^{2}-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^{2}cx^{2}-c}\,c^{2}\mathrm{sgn}}{a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn
(x)/a^2 - 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/
(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) + 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4*
abs(a)*sgn(x) + 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(x
) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) + 160*(sqr
```

```
t(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) + 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

maple [B] time = 0.05, size = 625, normalized size = 2.13

$$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}} x\left(-80\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}} x^5a^7c+80\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}} x^3a^7-48\sqrt{-\frac{c}{a^2}}\,\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}} x^4a^6c-27\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1
)/a^2)^(5/2)*x^5*a^7*c+80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^3*a^7-
48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^6*c-27*(-c/a^2)^(1/2)
*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^6*c+60*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a
^2)^(3/2)*x^5*a^5*c^2+75*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^2*a^6+1
00*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^5*a^5*c^2-80*(-c/a^2)^(1/2)*(
c*(a^2*x^2-1)/a^2)^(7/2)*x*a^5+45*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*
x^4*a^4*c^2-90*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c^3-150
*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c^3+30*a^4*(c*(a^2*x^2-1)
/a^2)^(7/2)*(-c/a^2)^(1/2)+150*(-c/a^2)^(1/2)*c^(7/2)*ln(x*c^(1/2)+(c*(a^2*
x^2-1)/a^2)^(1/2))*x^4*a+90*(-c/a^2)^(1/2)*c^(7/2)*ln((c^(1/2)*((a*x-1)*(a*
x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^4*a-135*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
)^(1/2)*x^4*a^2*c^3-135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-
c)/a^2/x)*x^4*c^4)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(5/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(5/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(5/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [C] time = 13.61, size = 500, normalized size = 1.71

$$-c^2\left(\begin{cases}\frac{\sqrt{c}\,\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\,\log(ax)}{a}+\frac{i\sqrt{c}\,\log\left(a^2x^2\right)}{2a}+\frac{\sqrt{c}\,\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right|>1\\ \frac{i\sqrt{c}\,\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\,\log\left(a^2x^2\right)}{2a}-\frac{i\sqrt{c}\,\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)+\frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}\,x}{\sqrt{a^2x^2-1}}+\sqrt{c}\,\operatorname{acosh}(ax)+\\ \frac{ia\sqrt{c}\,x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\,\operatorname{asin}(ax)-\end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*s
qrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
 (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqr
t(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**2*Piecewise((-a*sqrt(c)
*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*
asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - 2*c**2*Piecewi
se((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 + c**
2*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1
+ 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(
c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sq
rt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt
(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**
2*x**2))), True))/a**4
```

3.726 $\int e^{-2\tanh^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^{3/2} dx$

Optimal. Leaf size=212

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}+\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tan}{2(1-ax)^3}$$

```
[Out] a*(c-c/a^2/x^2)^(3/2)*x^2/(a*x+1)+5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1)/
(a*x+1)-1/2*(c-c/a^2/x^2)^(3/2)*x*(-a*x+1)/(a*x+1)+2*a^2*(c-c/a^2/x^2)^(3/2
)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)-1/2*a^2*(c-c/a^2/x^2)^(3/2)*
x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

Rubi [A] time = 0.38, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}-\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tan}{2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 + a*x) + (5*a^2*(c - c/(a^2*x^2))^(3/2)*
x^3)/(2*(1 - a*x)*(1 + a*x)) - ((c - c/(a^2*x^2))^(3/2)*x*(1 - a*x))/(2*(1
+ a*x)) + (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/2)*
(1 + a*x)^(3/2)) - (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x]*S
qrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{5/2}\sqrt{1+ax}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{3/2}\left(-2a-3a^2x\right)}{x^2\sqrt{1+ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{\sqrt{1-ax}\left(a^2+5a^3x\right)}{x\sqrt{1+ax}}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}+\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}+\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/}\right.}{2a(1-}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}+\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}+\frac{\left(a^2\left(c-\frac{c}{a^2x^2}\right.\right.}{2(1-}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}+\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{\left(a^3\left(c-\frac{c}{a^2x^2}\right.\right.}{}$$

$$=\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}+\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right.}{(1-ax)^{3}}$$

Mathematica [A] time = 0.11, size = 115, normalized size = 0.54

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(2a^2x^2+4ax-1\right)-4a^2x^2\log\left(\sqrt{a^2x^2-1}+ax\right)+a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2a^2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/2*(c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 + 4*a*x + 2*a^2*x^2)
+ a^2*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*
x^2]]))/(a^2*x*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.64, size = 317, normalized size = 1.50

$$\left[-\frac{8\,a\sqrt{-c}\,cx\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-a\sqrt{-c}\,cx\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(2\,a^2cx^2+4\,acx-c\right)\sqrt{\frac{a^2cx^2}{a^2x}}}{4\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x
^2))/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(2*a^2*c*x^2 + 4*a*c*x - c)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), -1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 2*a*c^(3/2)*x*log(2*a^
2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c
*x^2 + 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

giac [A] time = 4.42, size = 265, normalized size = 1.25

$$\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}-\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}\,c\,\mathrm{sgn}(x)}{a^2}-\frac{(\sqrt{a^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] (c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
- 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a)
) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^3*c^2*abs(a)*sgn(x) + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/2
)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) + 4*a*c^
(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a)))
*abs(a)
```

maple [B] time = 0.05, size = 454, normalized size = 2.14

$$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}} x \left(12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}} x^3a^5c-12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}} x\,a^5-4\sqrt{-\frac{c}{a^2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}} x^2a^4c+\sqrt{-\frac{c}{a^2}}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(3/2)*x^3*a^5*c-12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^5-4*(-c
/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^2*a^4*c+(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^2*a^4*c+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*
x^3*a^3*c^2+3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)-18*(-c/a^2)^(1/2
)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^3*c^2+18*(-c/a^2)^(1/2)*c^(5/2)*ln(x*c^(1
/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^2*a-6*(-c/a^2)^(1/2)*c^(5/2)*ln((c^(1/2)*(
(a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a-3*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(1/2)*x^2*a^2*c^2-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/
2)*a^2-c)/a^2/x)*x^2*c^3)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(3/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(3/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(3/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [C] time = 9.25, size = 376, normalized size = 1.77

$$-c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)+\frac{2c\left(\begin{cases}-\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}(ax)+\cdots\\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}(ax)-\cdots\end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt
(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I
*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c
)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c*Piecewise((-a*sqrt(c)*x/sqr
t(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)),
Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a
*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c*Piecewise((I*a*sqr
t(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c
)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c
)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

3.727 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=118

$$-x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(
a*x+1)^(1/2)-x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-
a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6159, 6129, 102, 157, 41, 216, 92, 208}

$$-x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$=-\sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$=-\sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+a}}}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$=-\sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\left(2a\sqrt{c}\right.}{}$$
$$=-\sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)+\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[-1 + a^2*x^2] + ArcTan[1/Sqrt[-1 + a^2*x^2]
] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

fricas [A] time = 0.69, size = 270, normalized size = 2.29

$$\left[-\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\sqrt{-c}\,\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},-\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\,\text{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(-c)*log(-(a^
2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, -(
a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*
c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sq
rt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 0.04, size = 198, normalized size = 1.68

$$-\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^{2}}}\,a^{2}\sqrt{-\frac{c}{a^{2}}}-2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^{2}}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^{2}}}-\sqrt{-\frac{c}{a^{2}}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{2}-c\ln\left(\frac{2\ldots}{}\right.\right.}{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{2}\sqrt{-\frac{c}{a^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)-2*c^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))
*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^{2}x^{2}-1\right)\sqrt{c-\frac{c}{a^{2}x^{2}}}}{(ax+1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\right)dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x*sqrt(c - c/
(a**2*x**2))/(a*x + 1), x)
```

3.728 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=111

$$\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{ax+1}\,\sqrt{1-ax}\,\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (-a*x+1)^2/a^2/x/(c-c/a^2/x^2)^(1/2)+2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^2)
^(1/2)+2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6159, 6129, 78, 50, 41, 216}

$$\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{ax+1}\,\sqrt{1-ax}\,\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (1 - a*x)^2/(a^2*Sqrt[c - c/(a^2*x^2)]*x) + (2*(1 - a*x)*(1 + a*x))/(a^2*Sq
rt[c - c/(a^2*x^2)]*x) + (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*S
qrt[c - c/(a^2*x^2)]*x)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{x\sqrt{1-ax}}{(1+ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$
$$= \frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{2\sqrt{1-ax}\,\sqrt{1+ax}\,\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.62

$$\frac{-a^2x^2+2\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)-2ax+3}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (3 - 2*a*x - a^2*x^2 + 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/
(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

fricas [A] time = 0.60, size = 214, normalized size = 1.93

$$\left[\frac{(ax+1)\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(a^2x^2+3\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx+ac},\,-\frac{2\,(ax+1)\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)\cdots}{a^2cx+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] [((a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) - (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
*c*x + a*c), -(2*(a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^2*c*x + a*c)]
```

giac [A] time = 0.21, size = 97, normalized size = 0.87

$$-\frac{(ax+1)a^2\sqrt{c-\frac{2\,c}{ax+1}}+\frac{4\,a^2c\arctan\left(\frac{\sqrt{c-\frac{2\,c}{ax+1}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\,a^2\sqrt{c-\frac{2\,c}{ax+1}}}{a^3c\operatorname{sgn}\left(-\frac{1}{ax+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((a*x + 1)*a^2*sqrt(c - 2*c/(a*x + 1)) + 4*a^2*c*arctan(sqrt(c - 2*c/(a*x
+ 1))/sqrt(-c))/sqrt(-c) + 2*a^2*sqrt(c - 2*c/(a*x + 1)))/(a^3*c*sgn(-1/(a*
x + 1) + 1))
```

maple [A] time = 0.04, size = 177, normalized size = 1.59

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\left(\sqrt{c}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x\,a^2-2\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)xac+2a\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a\sqrt{c}-2\ln\cdots\right)}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,c^{\frac{3}{2}}a\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2)^(1/2)*(c^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2-2*ln(x*
c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a*c+2*a*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*
c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2)-2*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a
^2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2-1}{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*sqrt(c - c/(a^2*x^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a^2\,x^2-1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax\sqrt{c-\frac{c}{a^2x^2}}+\sqrt{c-\frac{c}{a^2x^2}}}\,dx-\int \left(-\frac{1}{ax\sqrt{c-\frac{c}{a^2x^2}}+\sqrt{c-\frac{c}{a^2x^2}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(c - c/(a**2*x**2)) + sqrt(c - c/(a**2*x**2))), x) -
 Integral(-1/(a*x*sqrt(c - c/(a**2*x**2)) + sqrt(c - c/(a**2*x**2))), x)
```

3.729 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^{3/2}(1-ax)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] 1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x-2/3*(-a*x+1)^2*(a*x+1)*(2*a*x+5)/a
^4/(c-c/a^2/x^2)^(3/2)/x^3-2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^4/(
c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.37, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6159, 6129, 98, 143, 41, 216}

$$-\frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^{3/2}(1-ax)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) - (2*(1 - a*x)^2*(1 + a*x)*(5
 + 2*a*x))/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - (2*(1 - a*x)^(3/2)*(1 + a*
x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{\sqrt{1-ax}\,(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2-4ax)}{\sqrt{1-ax}\,(1+ax)^{3/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,d}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} - \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A] time = 0.08, size = 95, normalized size = 0.77

$$\frac{-3a^3x^3-11a^2x^2+6(ax+1)\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)+4ax+10}{3a^2cx(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (10 + 4*a*x - 11*a^2*x^2 - 3*a^3*x^3 + 6*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a
*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
```

fricas [A] time = 0.70, size = 280, normalized size = 2.26

$$\left[\frac{3\left(a^2x^2+2\,ax+1\right)\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(3\,a^3x^3+14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)},-\frac{6\left(a^2x^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas
")
```

```
[Out] [1/3*(3*(a^2*x^2 + 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2), -1/3*(6*
(a^2*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)]
```

giac [A] time = 0.22, size = 127, normalized size = 1.02

$$-\frac{6\,(ax+1)a^2\sqrt{c-\frac{2\,c}{ax+1}}+\frac{24\,a^2c\arctan\left(\frac{\sqrt{c-\frac{2\,c}{ax+1}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{a^2\left(c-\frac{2\,c}{ax+1}\right)^{\frac{3}{2}}c^2+15\,a^2\sqrt{c-\frac{2\,c}{ax+1}}\,c^3}{c^3}}{6\,a^3c^2\mathrm{sgn}\left(-\frac{1}{ax+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*(6*(a*x + 1)*a^2*sqrt(c - 2*c/(a*x + 1)) + 24*a^2*c*arctan(sqrt(c - 2*
c/(a*x + 1))/sqrt(-c))/sqrt(-c) + (a^2*(c - 2*c/(a*x + 1))^(3/2)*c^2 + 15*a
^2*sqrt(c - 2*c/(a*x + 1))*c^3)/c^3)/(a^3*c^2*sgn(-1/(a*x + 1) + 1))
```

maple [B] time = 0.05, size = 326, normalized size = 2.63

$$-\frac{\left(3c^{\frac{3}{2}}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\;x^3a^3+15x^2a^2c^{\frac{3}{2}}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\;-4c^{\frac{3}{2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\;x^2a^2-6\ln\left(x\sqrt{c}\;+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{\frac{(ax-1)(ax-}{a^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/3*(3*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^3+15*x^2*a^2*c^(3/2)*((
a*x-1)*(a*x+1)*c/a^2)^(1/2)-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2-6*l
n(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^
2*x^2-1)/a^2)^(1/2)*x*a^2*c-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a-6*ln(x*
c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^2*x^
2-1)/a^2)^(1/2)*a*c-12*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+2*(c*(a^2*x^2-
```

```
1)/a^2)^(1/2)*c^(3/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(1/2)/x^3/(c*(a^2*x^
2-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a^2*x^2))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a^2\,x^2-1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{acx\sqrt{c-\frac{c}{a^2x^2}}+c\sqrt{c-\frac{c}{a^2x^2}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{ax}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}}\,dx-\int\left(-\frac{1}{acx\sqrt{c-\frac{c}{a^2x^2}}+c\sqrt{c-\frac{c}{a^2x^2}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{ax}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] -Integral(a*x/(a*c*x*sqrt(c - c/(a**2*x**2)) + c*sqrt(c - c/(a**2*x**2)) -
c*sqrt(c - c/(a**2*x**2))/(a*x) - c*sqrt(c - c/(a**2*x**2))/(a**2*x**2)), x
) - Integral(-1/(a*c*x*sqrt(c - c/(a**2*x**2)) + c*sqrt(c - c/(a**2*x**2))
- c*sqrt(c - c/(a**2*x**2))/(a*x) - c*sqrt(c - c/(a**2*x**2))/(a**2*x**2)),
 x)
```

3.730 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1-ax)^{5/2}\sin^{-1}(ax)}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2}{5a^3x}$$

```
[Out] (-a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x+2/5*(-a*x+1)^3/a^3/(c-c/a^2/x^2)^(5/2)
/x^2-2/15*(-a*x+1)^3*(a*x+1)/a^4/(c-c/a^2/x^2)^(5/2)/x^3+2/15*(-a*x+1)^3*(a
*x+1)^2*(13*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5+2*(-a*x+1)^(5/2)*(a*x+1)^(5
/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.40, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$-\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(1-ax)^3}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1-a}{a^6x^5\left(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x) + (2*(1 - a*x)^3)/(5*a^3*(c - c
/(a^2*x^2))^(5/2)*x^2) - (2*(1 - a*x)^3*(1 + a*x))/(15*a^4*(c - c/(a^2*x^2)
)^(5/2)*x^3) + (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 13*a*x))/(15*a^6*(c - c/(a^
2*x^2))^(5/2)*x^5) + (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcSin[a*x])/(a^6*(
c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{3/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+2ax)}{\sqrt{1-ax}\,(1+ax)^{7/2}}\,dx}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2\left(6a+8a^2x\right)}{\sqrt{1-ax}\,(1+ax)^{5/2}}\,dx}{5a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \cdots}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{\cdots}}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28+\cdots}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28+\cdots}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{2(1-ax)^3(1+ax)^2(28+\cdots}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

Mathematica [A] time = 0.10, size = 105, normalized size = 0.54

$$\frac{-15a^4x^4-76a^3x^3-32a^2x^2+30(ax+1)^2\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)+82ax+56}{15a^2c^2x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (56 + 82*a*x - 32*a^2*x^2 - 76*a^3*x^3 - 15*a^4*x^4 + 30*(1 + a*x)^2*Sqrt[-
1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x^2
)]*x*(1 + a*x)^2)
```

fricas [A] time = 0.57, size = 352, normalized size = 1.81

$$\left[\frac{15\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(15\,a^5x^5+76\,a^4x^4+32\,a^3x^3-82\,a^2x^2\right.}{15\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas
")
```

```
[Out] [1/15*(15*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - (15*a^5*x^5 + 76*a^4*x^
4 + 32*a^3*x^3 - 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3), -1/15*(30*(a^4*x^4 + 2*a^3
*x^3 - 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^2*c*x^2 - c)) + (15*a^5*x^5 + 76*a^4*x^4 + 32*a^3*x^3 - 82*a^2*x
^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3
- 2*a^2*c^3*x - a*c^3)]
```

giac [A] time = 0.25, size = 208, normalized size = 1.07

$$-\frac{4\,\arctan\left(\frac{\sqrt{c-\frac{2\,c}{ax+1}}}{\sqrt{-c}}\right)}{a\sqrt{-c}\,c^2\mathrm{sgn}\left(-\frac{1}{ax+1}+1\right)}+\frac{8\,c-\frac{17\,c}{ax+1}}{4\left(\left(c-\frac{2\,c}{ax+1}\right)^{\frac{3}{2}}-\sqrt{c-\frac{2\,c}{ax+1}}\,c\right)ac^2\mathrm{sgn}\left(-\frac{1}{ax+1}+1\right)}-\frac{3\,a^4\left(c-\frac{2\,c}{ax+1}\right)^{\frac{5}{2}}c^{20}+35\,a^4\left(c-\frac{2}{ax+}\right.}{120\,a^5c^{25}\mathrm{sgn}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4*arctan(sqrt(c - 2*c/(a*x + 1))/sqrt(-c))/(a*sqrt(-c)*c^2*sgn(-1/(a*x + 1
) + 1)) + 1/4*(8*c - 17*c/(a*x + 1))/(((c - 2*c/(a*x + 1))^(3/2) - sqrt(c -
 2*c/(a*x + 1))*c)*a*c^2*sgn(-1/(a*x + 1) + 1)) - 1/120*(3*a^4*(c - 2*c/(a*
x + 1))^(5/2)*c^20 + 35*a^4*(c - 2*c/(a*x + 1))^(3/2)*c^21 + 345*a^4*sqrt(c
 - 2*c/(a*x + 1))*c^22)/(a^5*c^25*sgn(-1/(a*x + 1) + 1))
```

maple [B] time = 0.05, size = 462, normalized size = 2.38

$$-\frac{\left(15c^{\frac{5}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^5a^5+45x^4c^{\frac{5}{2}}a^4\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}+16c^{\frac{5}{2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}x^4a^4-60c^{\frac{5}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^3a^3+16\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/15*(15*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^5*a^5+45*x^4*c^(5/2)*a^4*
((a*x-1)*(a*x+1)*c/a^2)^(3/2)+16*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4-
60*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^3*a^3+16*c^(5/2)*(c*(a^2*x^2-1)/
a^2)^(3/2)*x^3*a^3-30*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x
+1)*c/a^2)^(3/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^4*c-90*c^(5/2)*((a*x-1)*(a*x
+1)*c/a^2)^(3/2)*x^2*a^2-24*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^2-30*ln
(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(c*(a^2
*x^2-1)/a^2)^(3/2)*a^3*c+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x*a-24*c^
(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2
)+6*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(3/2
)/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a^2*x^2))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a^2\,x^2-1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^2x\sqrt{c-\frac{c}{a^2x^2}}+c^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{ax}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4}}\,dx-\int\left(-\frac{}{ac^2x\sqrt{c-\frac{c}{a^2x^2}}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] -Integral(a*x/(a*c**2*x*sqrt(c - c/(a**2*x**2)) + c**2*sqrt(c - c/(a**2*x**
2)) - 2*c**2*sqrt(c - c/(a**2*x**2))/(a*x) - 2*c**2*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + c**2*sqrt(c - c/(a**2*x**2))/(a**3*x**3) + c**2*sqrt(c - c/(
a**2*x**2))/(a**4*x**4)), x) - Integral(-1/(a*c**2*x*sqrt(c - c/(a**2*x**2)
) + c**2*sqrt(c - c/(a**2*x**2)) - 2*c**2*sqrt(c - c/(a**2*x**2))/(a*x) - 2
*c**2*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + c**2*sqrt(c - c/(a**2*x**2))/(a
**3*x**3) + c**2*sqrt(c - c/(a**2*x**2))/(a**4*x**4)), x)
```

3.731 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=270

$$\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^{7/2}(1-ax)^{7/2}\sin^{-1}(ax)}{a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{82(}{105}$$

```
[Out] 1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x-10/3*(-a*x+1)^3/a^3/(c-c/a^2/x^2)^
(7/2)/x^2-12/7*(-a*x+1)^4/a^4/(c-c/a^2/x^2)^(7/2)/x^3-82/105*(-a*x+1)^4*(a*
x+1)/a^5/(c-c/a^2/x^2)^(7/2)/x^4-2/35*(-a*x+1)^4*(a*x+1)^2/a^6/(c-c/a^2/x^2
)^(7/2)/x^5-2/35*(-a*x+1)^4*(a*x+1)^3*(37*a*x+72)/a^8/(c-c/a^2/x^2)^(7/2)/x
^7-2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.43, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6159, 6129, 98, 150, 143, 41, 216}

$$-\frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{82(ax+1)(1-ax)^4}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{12(1-ax)^4}{7a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{10(1-ax}{3a^3x^2\left(c-\frac{c}{a^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(7/2)*x) - (10*(1 - a*x)^3)/(3*a^3*(c
- c/(a^2*x^2))^(7/2)*x^2) - (12*(1 - a*x)^4)/(7*a^4*(c - c/(a^2*x^2))^(7/2)
*x^3) - (82*(1 - a*x)^4*(1 + a*x))/(105*a^5*(c - c/(a^2*x^2))^(7/2)*x^4) -
(2*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^2*x^2))^(7/2)*x^5) - (2*(1 -
a*x)^4*(1 + a*x)^3*(72 + 37*a*x))/(35*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) - (2
*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])/(a^8*(c - c/(a^2*x^2))^(7/2)*
x^7)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
```

```
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{5/2}(1+ax)^{9/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+4ax)}{(1-ax)^{3/2}(1+ax)^{9/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4\left(-50a-14a^2x\right)}{\sqrt{1-ax}\,(1+ax)^{9/2}}\,dx}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int}{21a^6\left(c-\frac{c}{a^2x^2}\right.}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} -$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{2}{35}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{2}{35}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{2}{35}$$

$$= \frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} - \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} - \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4} - \frac{2}{35}$$

Mathematica [A] time = 0.12, size = 131, normalized size = 0.49

$$\frac{-105a^6x^6-562a^5x^5-74a^4x^4+1226a^3x^3+636a^2x^2+210(ax-1)(ax+1)^3\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)-6}{105a^2x(ax-1)\sqrt{c-\frac{c}{a^2x^2}}\,(acx+c)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (-432 - 654*a*x + 636*a^2*x^2 + 1226*a^3*x^3 - 74*a^4*x^4 - 562*a^5*x^5 - 1
05*a^6*x^6 + 210*(-1 + a*x)*(1 + a*x)^3*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-
1 + a^2*x^2]])/(105*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)*(c + a*c*x)^3)
```

fricas [A] time = 0.78, size = 496, normalized size = 1.84

$$\left[\frac{105\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)-\left(105\,a^7x^7+56\right.}{105\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas
")
```

```
[Out] [1/105*(105*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) - (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4 - 636*a^3
*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x
+ a*c^4), -1/105*(210*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2
+ 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2
))/(a^2*c*x^2 - c)) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^
4 - 636*a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2
*a^2*c^4*x + a*c^4)]
```

giac [A] time = 3.82, size = 251, normalized size = 0.93

$$-\frac{(ax+1)\sqrt{c-\frac{2c}{ax+1}}}{ac^4\mathrm{sgn}\left(-\frac{1}{ax+1}+1\right)}-\frac{4\,\arctan\left(\frac{\sqrt{c-\frac{2c}{ax+1}}}{\sqrt{-c}}\right)}{a\sqrt{-c}\,c^3\mathrm{sgn}\left(-\frac{1}{ax+1}+1\right)}+\frac{14\,c-\frac{27\,c}{ax+1}}{48\,a\left(c-\frac{2c}{ax+1}\right)^{\frac{3}{2}}c^3\mathrm{sgn}\left(-\frac{1}{ax+1}+1\right)}-\frac{15\,a^6\left(c-\frac{2c}{ax+1}\right)^{\frac{7}{2}}c^{42}+189\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] -(a*x + 1)*sqrt(c - 2*c/(a*x + 1))/(a*c^4*sgn(-1/(a*x + 1) + 1)) - 4*arctan
(sqrt(c - 2*c/(a*x + 1))/sqrt(-c))/(a*sqrt(-c)*c^3*sgn(-1/(a*x + 1) + 1)) +
 1/48*(14*c - 27*c/(a*x + 1))/(a*(c - 2*c/(a*x + 1))^(3/2)*c^3*sgn(-1/(a*x
+ 1) + 1)) - 1/3360*(15*a^6*(c - 2*c/(a*x + 1))^(7/2)*c^42 + 189*a^6*(c - 2
*c/(a*x + 1))^(5/2)*c^43 + 1330*a^6*(c - 2*c/(a*x + 1))^(3/2)*c^44 + 10710*
a^6*sqrt(c - 2*c/(a*x + 1))*c^45)/(a^7*c^49*sgn(-1/(a*x + 1) + 1))
```

maple [B] time = 0.06, size = 572, normalized size = 2.12

$$\left(-105c^{\frac{7}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}}x^7a^7+96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^6a^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}}+96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^5a^5+39\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/105*(-105*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7+96*(c*(a^2*x^2-1)
/a^2)^(5/2)*c^(7/2)*x^6*a^6-553*x^6*c^(7/2)*a^6*((a*x-1)*(a*x+1)*c/a^2)^(5/
2)+96*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^5*a^5+392*c^(7/2)*((a*x-1)*(a*x+1
)*c/a^2)^(5/2)*x^5*a^5-240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^4*a^4+1540*c
^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^4+210*(c*(a^2*x^2-1)/a^2)^(5/2)*
ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x*a^6
*c-240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^3*a^3-350*c^(7/2)*((a*x-1)*(a*x+
1)*c/a^2)^(5/2)*x^3*a^3+210*(c*(a^2*x^2-1)/a^2)^(5/2)*ln(x*c^(1/2)+(c*(a^2*
x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*a^5*c+180*(c*(a^2*x^2-1)/a
^2)^(5/2)*c^(7/2)*x^2*a^2-1470*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^2*a^
2+180*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x*a+42*c^(7/2)*((a*x-1)*(a*x+1)*c/a
^2)^(5/2)*x*a-30*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)+462*c^(7/2)*((a*x-1)*(a*
x+1)*c/a^2)^(5/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(5/2)/x^7/(c*(a^2*x^2-1)
/a^2/x^2)^(7/2)/a^8/c^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2x^2-1}{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)/((a*x + 1)^2*(c - c/(a^2*x^2))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{a^2\,x^2-1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}\,{\left(a\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ac^3x\sqrt{c-\frac{c}{a^2x^2}}+c^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{ax}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^3}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^5x^5}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^6x^6}}\, d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] -Integral(a*x/(a*c**3*x*sqrt(c - c/(a**2*x**2)) + c**3*sqrt(c - c/(a**2*x**
2)) - 3*c**3*sqrt(c - c/(a**2*x**2))/(a*x) - 3*c**3*sqrt(c - c/(a**2*x**2))
/(a**2*x**2) + 3*c**3*sqrt(c - c/(a**2*x**2))/(a**3*x**3) + 3*c**3*sqrt(c -
 c/(a**2*x**2))/(a**4*x**4) - c**3*sqrt(c - c/(a**2*x**2))/(a**5*x**5) - c*
*3*sqrt(c - c/(a**2*x**2))/(a**6*x**6)), x) - Integral(-1/(a*c**3*x*sqrt(c
- c/(a**2*x**2)) + c**3*sqrt(c - c/(a**2*x**2)) - 3*c**3*sqrt(c - c/(a**2*x
**2))/(a*x) - 3*c**3*sqrt(c - c/(a**2*x**2))/(a**2*x**2) + 3*c**3*sqrt(c -
c/(a**2*x**2))/(a**3*x**3) + 3*c**3*sqrt(c - c/(a**2*x**2))/(a**4*x**4) - c
**3*sqrt(c - c/(a**2*x**2))/(a**5*x**5) - c**3*sqrt(c - c/(a**2*x**2))/(a**
6*x**6)), x)
```

3.732 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=299

$$\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{7\left(1-a^2x^2\right)^{9/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{8\left(1-a^2x^2\right)^{9/2}}+\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{3a^8x^9\log(x)\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^5x^6\left(c\right.}{\left(1-a\right.}$$

```
[Out] -1/8*(c-c/a^2/x^2)^(9/2)*x/(-a^2*x^2+1)^(9/2)+3/7*a*(c-c/a^2/x^2)^(9/2)*x^2
/(-a^2*x^2+1)^(9/2)-8/5*a^3*(c-c/a^2/x^2)^(9/2)*x^4/(-a^2*x^2+1)^(9/2)+3/2*
a^4*(c-c/a^2/x^2)^(9/2)*x^5/(-a^2*x^2+1)^(9/2)+2*a^5*(c-c/a^2/x^2)^(9/2)*x^
6/(-a^2*x^2+1)^(9/2)-4*a^6*(c-c/a^2/x^2)^(9/2)*x^7/(-a^2*x^2+1)^(9/2)+a^9*(
c-c/a^2/x^2)^(9/2)*x^10/(-a^2*x^2+1)^(9/2)-3*a^8*(c-c/a^2/x^2)^(9/2)*x^9*ln
(x)/(-a^2*x^2+1)^(9/2)
```

Rubi [A] time = 0.20, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{a^9x^{10}\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}-\frac{4a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{2a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}-\frac{8a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{5\left(1-a^2x^2\right)^{9/2}}+\frac{3ax^2\left(c-\right.}{7\left(1-a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c - c/(a^2*x^2))^(9/2)*x)/(8*(1 - a^2*x^2)^(9/2)) + (3*a*(c - c/(a^2*x^2
))^(9/2)*x^2)/(7*(1 - a^2*x^2)^(9/2)) - (8*a^3*(c - c/(a^2*x^2))^(9/2)*x^4)
/(5*(1 - a^2*x^2)^(9/2)) + (3*a^4*(c - c/(a^2*x^2))^(9/2)*x^5)/(2*(1 - a^2*
x^2)^(9/2)) + (2*a^5*(c - c/(a^2*x^2))^(9/2)*x^6)/(1 - a^2*x^2)^(9/2) - (4*
a^6*(c - c/(a^2*x^2))^(9/2)*x^7)/(1 - a^2*x^2)^(9/2) + (a^9*(c - c/(a^2*x^2
))^(9/2)*x^10)/(1 - a^2*x^2)^(9/2) - (3*a^8*(c - c/(a^2*x^2))^(9/2)*x^9*Log
[x])/(1 - a^2*x^2)^(9/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}}{x^9}\,dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\frac{(1-ax)^6(1+ax)^3}{x^9}\,dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^9\right)\int\left(a^9+\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}-\frac{6a^4}{x^5}-\frac{6a^5}{x^4}+\frac{8a^6}{x^3}-\frac{3a^8}{x}\right)dx}{\left(1-a^2x^2\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}x}{8\left(1-a^2x^2\right)^{9/2}}+\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^2}{7\left(1-a^2x^2\right)^{9/2}}-\frac{8a^3\left(c-\frac{c}{a^2x^2}\right)^{9/2}x^4}{5\left(1-a^2x^2\right)^{9/2}}+\frac{3a^4\left(c-\frac{c}{a^2x^2}\right)^{9/2}}{2\left(1-a^2x^2\right)^{9/2}}$$

Mathematica [A] time = 0.09, size = 97, normalized size = 0.32

$$\frac{x^9\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(a^9x-3a^8\log(x)-\frac{4a^6}{x^2}+\frac{2a^5}{x^3}+\frac{3a^4}{2x^4}-\frac{8a^3}{5x^5}+\frac{3a}{7x^7}-\frac{1}{8x^8}\right)}{\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*x^9*(-1/8*1/x^8 + (3*a)/(7*x^7) - (8*a^3)/(5*x^5)
+ (3*a^4)/(2*x^4) + (2*a^5)/x^3 - (4*a^6)/x^2 + a^9*x - 3*a^8*Log[x]))/(1 -
 a^2*x^2)^(9/2)
```

fricas [A] time = 0.79, size = 544, normalized size = 1.82

$$\left[\frac{420\left(a^{9}c^{4}x^{9}-a^{7}c^{4}x^{7}\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{6}+a^{2}cx^{2}-cx^{4}-\left(ax^{5}-ax\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c}{a^{2}x^{4}-x^{2}}\right)-\left(280\,a^{9}c^{4}x^{9}-1120\,a^{6}c^{4}x^{6}+56\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/280*(420*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2
 - c*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - c)/(a^2*x^4 - x^2)) - (280*a^9*c^4*x^9 - 1120*a^6*c^4*x^6 + 560
*a^5*c^4*x^5 - (280*a^9 - 1120*a^6 + 560*a^5 + 420*a^4 - 448*a^3 + 120*a -
35)*c^4*x^8 + 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqr
t(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7), 1/28
0*(840*(a^9*c^4*x^9 - a^7*c^4*x^7)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3
 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^
2 + c)) - (280*a^9*c^4*x^9 - 1120*a^6*c^4*x^6 + 560*a^5*c^4*x^5 - (280*a^9
- 1120*a^6 + 560*a^5 + 420*a^4 - 448*a^3 + 120*a - 35)*c^4*x^8 + 420*a^4*c^
4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqrt(-a^2*x^2 + 1)*sqrt((a^
2*c*x^2 - c)/(a^2*x^2)))/(a^10*x^9 - a^8*x^7)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{9}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1)^3, x)
```

maple [A] time = 0.05, size = 102, normalized size = 0.34

$$\frac{\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{9}{2}} x\sqrt{-a^{2}x^{2}+1}\,\left(-280a^{9}x^{9}+840a^{8}\ln(x)x^{8}+1120x^{6}a^{6}-560x^{5}a^{5}-420x^{4}a^{4}+448x^{3}a^{3}-120ax+3\right.}{280\left(a^{2}x^{2}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/280*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a^2*x^2-1)^5*(-a^2*x^2+1)^(1/2)*(-28
0*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6-560*x^5*a^5-420*x^4*a^4+448*x^3*a^
3-120*a*x+35)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(9/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{9/2}\,\left(1-a^2\,x^2\right)^{3/2}}{(a\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(9/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(9/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(9/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

3.733 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=301

$$\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{5\left(1-a^2x^2\right)^{7/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6\left(1-a^2x^2\right)^{7/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}-\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{3a^6x^7\log(x)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{a^5x^6\left(c-\right.}{\left(1-a^2x\right.}$$

```
[Out] -1/6*(c-c/a^2/x^2)^(7/2)*x/(-a^2*x^2+1)^(7/2)+3/5*a*(c-c/a^2/x^2)^(7/2)*x^2
/(-a^2*x^2+1)^(7/2)-1/4*a^2*(c-c/a^2/x^2)^(7/2)*x^3/(-a^2*x^2+1)^(7/2)-5/3*
a^3*(c-c/a^2/x^2)^(7/2)*x^4/(-a^2*x^2+1)^(7/2)+5/2*a^4*(c-c/a^2/x^2)^(7/2)*
x^5/(-a^2*x^2+1)^(7/2)+a^5*(c-c/a^2/x^2)^(7/2)*x^6/(-a^2*x^2+1)^(7/2)-a^7*(
c-c/a^2/x^2)^(7/2)*x^8/(-a^2*x^2+1)^(7/2)+3*a^6*(c-c/a^2/x^2)^(7/2)*x^7*ln(
x)/(-a^2*x^2+1)^(7/2)
```

Rubi [A] time = 0.19, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$-\frac{a^7x^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{\left(1-a^2x^2\right)^{7/2}}+\frac{5a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{2\left(1-a^2x^2\right)^{7/2}}-\frac{5a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3\left(1-a^2x^2\right)^{7/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{4\left(1-a^2x^2\right)^{7/2}}+\frac{3ax^2\left(c-\right.}{5\left(1-a^2x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c - c/(a^2*x^2))^(7/2)*x)/(6*(1 - a^2*x^2)^(7/2)) + (3*a*(c - c/(a^2*x^2
))^(7/2)*x^2)/(5*(1 - a^2*x^2)^(7/2)) - (a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(
4*(1 - a^2*x^2)^(7/2)) - (5*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a^2*x^
2)^(7/2)) + (5*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(2*(1 - a^2*x^2)^(7/2)) + (
a^5*(c - c/(a^2*x^2))^(7/2)*x^6)/(1 - a^2*x^2)^(7/2) - (a^7*(c - c/(a^2*x^2
))^(7/2)*x^8)/(1 - a^2*x^2)^(7/2) + (3*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*Log[
x])/(1 - a^2*x^2)^(7/2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx &= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}\\ &= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^5(1+ax)^2}{x^7}dx}{\left(1-a^2x^2\right)^{7/2}}\\ &= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\left(-a^7+\frac{1}{x^7}-\frac{3a}{x^6}+\frac{a^2}{x^5}+\frac{5a^3}{x^4}-\frac{5a^4}{x^3}-\frac{a^5}{x^2}+\frac{3a^6}{x}\right)dx}{\left(1-a^2x^2\right)^{7/2}}\\ &= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}{6\left(1-a^2x^2\right)^{7/2}}+\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{5\left(1-a^2x^2\right)^{7/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{4\left(1-a^2x^2\right)^{7/2}}-\frac{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3\left(1-a^2x^2\right)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 98, normalized size = 0.33

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(60a^7x^7-180a^6x^6\log(x)-60a^5x^5-150a^4x^4+100a^3x^3+15a^2x^2-36ax+10\right)}{60a^6x^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(10 - 36*a*x + 15*a^2*x^2 + 100*a^3*x^3 - 150*a^
4*x^4 - 60*a^5*x^5 + 60*a^7*x^7 - 180*a^6*x^6*Log[x]))/(60*a^6*x^5*Sqrt[1 -
 a^2*x^2])
```

fricas [A] time = 0.71, size = 544, normalized size = 1.81

$$\left[\frac{90\left(a^7c^3x^7 - a^5c^3x^5\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right) - \left(60\,a^7c^3x^7 - 60\,a^5c^3x^5 - 150\,a^4c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/60*(90*(a^7*c^3*x^7 - a^5*c^3*x^5)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 -
 c*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) - c)/(a^2*x^4 - x^2)) - (60*a^7*c^3*x^7 - 60*a^5*c^3*x^5 - 150*a^4*
c^3*x^4 - (60*a^7 - 60*a^5 - 150*a^4 + 100*a^3 + 15*a^2 - 36*a + 10)*c^3*x^
6 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2 - 36*a*c^3*x + 10*c^3)*sqrt(-a^2*x^2 +
 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^8*x^7 - a^6*x^5), 1/60*(180*(a^7*c^
3*x^7 - a^5*c^3*x^5)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqrt(c
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (60*
a^7*c^3*x^7 - 60*a^5*c^3*x^5 - 150*a^4*c^3*x^4 - (60*a^7 - 60*a^5 - 150*a^4
 + 100*a^3 + 15*a^2 - 36*a + 10)*c^3*x^6 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2
 - 36*a*c^3*x + 10*c^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))
/(a^8*x^7 - a^6*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1)^3, x)
```

maple [A] time = 0.05, size = 102, normalized size = 0.34

$$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}} x\sqrt{-a^2x^2+1}\,\left(-60a^7x^7+180a^6\ln(x)x^6+60x^5a^5+150x^4a^4-100x^3a^3-15a^2x^2+36ax-10\right)}{60\left(a^2x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a^2*x^2-1)^4*(-a^2*x^2+1)^(1/2)*(-60*
a^7*x^7+180*a^6*ln(x)*x^6+60*x^5*a^5+150*x^4*a^4-100*x^3*a^3-15*a^2*x^2+36*
a*x-10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(1-a^2\,x^2\right)^{3/2}}{(a\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(7/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

3.734 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=218

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{3a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -1/4*(c-c/a^2/x^2)^(5/2)*x/(-a^2*x^2+1)^(5/2)+a*(c-c/a^2/x^2)^(5/2)*x^2/(-a
^2*x^2+1)^(5/2)-a^2*(c-c/a^2/x^2)^(5/2)*x^3/(-a^2*x^2+1)^(5/2)-2*a^3*(c-c/a
^2/x^2)^(5/2)*x^4/(-a^2*x^2+1)^(5/2)+a^5*(c-c/a^2/x^2)^(5/2)*x^6/(-a^2*x^2+
1)^(5/2)-3*a^4*(c-c/a^2/x^2)^(5/2)*x^5*ln(x)/(-a^2*x^2+1)^(5/2)
```

Rubi [A] time = 0.18, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 75}

$$\frac{a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4\left(1-a^2x^2\right)^{5/2}}-\frac{3a^4x^5\log(x)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c - c/(a^2*x^2))^(5/2)*x)/(4*(1 - a^2*x^2)^(5/2)) + (a*(c - c/(a^2*x^2))
^(5/2)*x^2)/(1 - a^2*x^2)^(5/2) - (a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(1 - a^
2*x^2)^(5/2) - (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)/(1 - a^2*x^2)^(5/2) + (a
^5*(c - c/(a^2*x^2))^(5/2)*x^6)/(1 - a^2*x^2)^(5/2) - (3*a^4*(c - c/(a^2*x^
2))^(5/2)*x^5*Log[x])/(1 - a^2*x^2)^(5/2)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx &= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}\\
&= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^4(1+ax)}{x^5}dx}{\left(1-a^2x^2\right)^{5/2}}\\
&= \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{\left(1-a^2x^2\right)^{5/2}}\\
&= -\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}{4\left(1-a^2x^2\right)^{5/2}}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{\left(1-a^2x^2\right)^{5/2}}-\frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{\left(1-a^2x^2\right)^{5/2}}-\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{\left(1-a^2x^2\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.41

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(4a^5x^5-5a^4x^4-12a^4x^4\log(x)-8a^3x^3-4a^2x^2+4ax-1\right)}{4a^4x^3\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(-1 + 4*a*x - 4*a^2*x^2 - 8*a^3*x^3 - 5*a^4*x^4
+ 4*a^5*x^5 - 12*a^4*x^4*Log[x]))/(4*a^4*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 480, normalized size = 2.20

$$\left[\frac{6\left(a^5c^2x^5-a^3c^2x^3\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-\left(4a^5c^2x^5-8a^3c^2x^3-\left(4a^5-8a^3\right.\right.}{4\left(a^6x^5-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/4*(6*(a^5*c^2*x^5 - a^3*c^2*x^3)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c
*x^4 - (a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) - c)/(a^2*x^4 - x^2)) - (4*a^5*c^2*x^5 - 8*a^3*c^2*x^3 - (4*a^5 - 8*a
^3 - 4*a^2 + 4*a - 1)*c^2*x^4 - 4*a^2*c^2*x^2 + 4*a*c^2*x - c^2)*sqrt(-a^2*
x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5 - a^4*x^3), 1/4*(12*(a^5
*c^2*x^5 - a^3*c^2*x^3)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (
4*a^5*c^2*x^5 - 8*a^3*c^2*x^3 - (4*a^5 - 8*a^3 - 4*a^2 + 4*a - 1)*c^2*x^4 -
 4*a^2*c^2*x^2 + 4*a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^6*x^5 - a^4*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1)^3, x)
```

maple [A] time = 0.05, size = 86, normalized size = 0.39

$$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}x\sqrt{-a^2x^2+1}\,\left(-4x^5a^5+12a^4\ln(x)x^4+8x^3a^3+4a^2x^2-4ax+1\right)}{4\left(a^2x^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/4*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/(a^2*x^2-1)^3*(-a^2*x^2+1)^(1/2)*(-4*x^
5*a^5+12*a^4*ln(x)*x^4+8*x^3*a^3+4*a^2*x^2-4*a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(1-a^2\,x^2\right)^{3/2}}{\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(5/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(5/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(
5/2)/(a*x + 1)**3, x)
```

3.735 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=146

$$\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -1/2*(c-c/a^2/x^2)^(3/2)*x/(-a^2*x^2+1)^(3/2)+3*a*(c-c/a^2/x^2)^(3/2)*x^2/(
-a^2*x^2+1)^(3/2)-a^3*(c-c/a^2/x^2)^(3/2)*x^4/(-a^2*x^2+1)^(3/2)+3*a^2*(c-c
/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$-\frac{a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}+\frac{3ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}-\frac{x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2x^3\log(x)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((c - c/(a^2*x^2))^(3/2)*x)/(2*(1 - a^2*x^2)^(3/2)) + (3*a*(c - c/(a^2*x^2
))^(3/2)*x^2)/(1 - a^2*x^2)^(3/2) - (a^3*(c - c/(a^2*x^2))^(3/2)*x^4)/(1 -
a^2*x^2)^(3/2) + (3*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*Log[x])/(1 - a^2*x^2)^(
3/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^3}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\left(-a^3+\frac{1}{x^3}-\frac{3a}{x^2}+\frac{3a^2}{x}\right)dx}{\left(1-a^2x^2\right)^{3/2}}$$
$$=-\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{2\left(1-a^2x^2\right)^{3/2}}+\frac{3a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{\left(1-a^2x^2\right)^{3/2}}-\frac{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^4}{\left(1-a^2x^2\right)^{3/2}}+\frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}{\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.44

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(2a^3x^3-6a^2x^2\log(x)-6ax+1\right)}{2a^2x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(1 - 6*a*x + 2*a^3*x^3 - 6*a^2*x^2*Log[x]))/(2*a^2
*x*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.77, size = 376, normalized size = 2.58

$$\left[\frac{3\left(a^3cx^3-acx\right)\sqrt{-c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\sqrt{-c}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-\left(2a^3cx^3-\left(2a^3-6a+1\right)cx^2-6acx+\right.}{2\left(a^4x^3-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(3*(a^3*c*x^3 - a*c*x)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - (
a*x^5 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) -
c)/(a^2*x^4 - x^2)) - (2*a^3*c*x^3 - (2*a^3 - 6*a + 1)*c*x^2 - 6*a*c*x + c)
*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x), 1/2
*(6*(a^3*c*x^3 - a*c*x)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a*x)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - (
2*a^3*c*x^3 - (2*a^3 - 6*a + 1)*c*x^2 - 6*a*c*x + c)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3 - a^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1)^3, x)
```

maple [A] time = 0.05, size = 70, normalized size = 0.48

$$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}} x\sqrt{-a^2x^2+1}\,\left(-2x^3a^3+6a^2\ln(x)x^2+6ax-1\right)}{2\left(a^2x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a^2*x^2-1)^2*(-a^2*x^2+1)^(1/2)*(-2*x^
3*a^3+6*a^2*ln(x)*x^2+6*a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(1-a^2\,x^2\right)^{3/2}}{{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(3/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(
3/2)/(a*x + 1)**3, x)
```

3.736 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=106

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] a*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-
a^2*x^2+1)^(1/2)-4*x*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 72}

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1
 - a^2*x^2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1-ax)^2}{x(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.42

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,(ax-4\log(ax+1)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(a*x + Log[x] - 4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*x^2 + 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

maple [A] time = 0.05, size = 60, normalized size = 0.57

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(ax+\ln(x)-4\ln\left(ax+1\right)\right)\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(a*x+ln(x)-4*ln(a*x+1))*(-a^2*x^2+1)^(1/2)
/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(a*x + 1)**3, x)
```

3.737 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-a^2x^2}\,\log(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] -(-a^2*x^2+1)^(1/2)/a/(c-c/a^2/x^2)^(1/2)+2*(-a^2*x^2+1)^(1/2)/a^2/x/(a*x+1
)/(c-c/a^2/x^2)^(1/2)+3*ln(a*x+1)*(-a^2*x^2+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1
/2)
```

Rubi [A] time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 77}

$$-\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-a^2x^2}\,\log(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*Sqrt[c - c/(a^2*x^2)])) + (2*Sqrt[1 - a^2*x^2])/(a^2
*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)) + (3*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(a
^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x(1-ax)}{(1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a}-\frac{2}{a(1+ax)^2}+\frac{3}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{\sqrt{1-a^2x^2}}{a\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-a^2x^2}}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}+\frac{3\sqrt{1-a^2x^2}\,\log(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.48

$$\frac{\sqrt{1-a^2x^2}\left(-ax+\frac{2}{ax+1}+3\log(ax+1)\right)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-(a*x) + 2/(1 + a*x) + 3*Log[1 + a*x]))/(a^2*Sqrt[c - c
/(a^2*x^2)]*x)
```

fricas [A] time = 1.73, size = 439, normalized size = 3.60

$$\left[-\frac{3\left(a^3x^3+a^2x^2-ax-1\right)\sqrt{-c}\log\left(\frac{a^6cx^6+4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2-4\,acx+\left(a^5x^5+4\,a^4x^4+6\,a^3x^3+4\,a^2x^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2}{a^4x^4+2\,a^3x^3-2\,ax-1}\right.}{2\left(a^4cx^3+a^3cx^2-a^2cx-ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(3*(a^3*x^3 + a^2*x^2 - a*x - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^
5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 - 4*a*c*x + (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3
+ 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) -
2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)) + 2*(a^3*x^3 + 3*a^2*x^2)*sqrt(-a^2
*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*c*x^3 + a^3*c*x^2 - a^2*c*x
 - a*c), (3*(a^3*x^3 + a^2*x^2 - a*x - 1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x +
 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 +
 2*a^2*c*x^2 - a*c*x - 2*c)) - (a^3*x^3 + 3*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*c*x^3 + a^3*c*x^2 - a^2*c*x - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*sqrt(c - c/(a^2*x^2))), x)
```

maple [A] time = 0.05, size = 78, normalized size = 0.64

$$\frac{\sqrt{-a^2x^2+1}\,\left(-a^2x^2+3ax\ln\left(ax+1\right)-ax+3\ln\left(ax+1\right)+2\right)}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,a^2\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-a^2*x^2+1)^(1/2)/a^2*(-a^2*x^2+3*a*x*ln
(a*x+1)-a*x+3*ln(a*x+1)+2)/(a*x+1)
```

maxima [C] time = 0.34, size = 53, normalized size = 0.43

$$-\frac{i\,a^2\sqrt{c}\,x^2+i\,a\sqrt{c}\,x-2i\,\sqrt{c}}{a^2cx+ac}+\frac{3i\,\log\left(ax+1\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] -(I*a^2*sqrt(c)*x^2 + I*a*sqrt(c)*x - 2*I*sqrt(c))/(a^2*c*x + a*c) + 3*I*lo
g(a*x + 1)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)
))*(a*x + 1)**3), x)
```

3.738 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{3\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] (-a^2*x^2+1)^(3/2)/a^3/(c-c/a^2/x^2)^(3/2)/x^2+1/2*(-a^2*x^2+1)^(3/2)/a^4/(
c-c/a^2/x^2)^(3/2)/x^3/(a*x+1)^2-3*(-a^2*x^2+1)^(3/2)/a^4/(c-c/a^2/x^2)^(3/
2)/x^3/(a*x+1)-3*(-a^2*x^2+1)^(3/2)*ln(a*x+1)/a^4/(c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4x^3(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{3\left(1-a^2x^2\right)^{3/2}\log(ax+1)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(a^3*(c - c/(a^2*x^2))^(3/2)*x^2) + (1 - a^2*x^2)^(3/2)
/(2*a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)^2) - (3*(1 - a^2*x^2)^(3/2))/
(a^4*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)) - (3*(1 - a^2*x^2)^(3/2)*Log[1
+ a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{x^3}{(1+ax)^3}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}\int \left(\frac{1}{a^3}-\frac{1}{a^3(1+ax)^3}+\frac{3}{a^3(1+ax)^2}-\frac{3}{a^3(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}+\frac{\left(1-a^2x^2\right)^{3/2}}{2a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1+ax)^2}-\frac{3\left(1-a^2x^2\right)^{3/2}}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1+ax)}-\frac{3\left(1-a^2x^2\right)}{a^4\left(c-\right.}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.51

$$-\frac{\sqrt{1-a^2x^2}\left(a^2x^2-1\right)\left(-\frac{3}{a^4(ax+1)}+\frac{1}{2a^4(ax+1)^2}-\frac{3\log(ax+1)}{a^4}+\frac{x}{a^3}\right)}{x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(-1 + a^2*x^2)*(x/a^3 + 1/(2*a^4*(1 + a*x)^2) - 3/(a^4
*(1 + a*x)) - (3*Log[1 + a*x])/a^4))/((c - c/(a^2*x^2))^(3/2)*x^3))
```

fricas [A] time = 0.66, size = 477, normalized size = 2.81

$$\left[-\frac{3\left(a^{4}x^{4}+2\,a^{3}x^{3}-2\,ax-1\right)\sqrt{-c}\,\log\left(\frac{a^{6}cx^{6}+4\,a^{5}cx^{5}+5\,a^{4}cx^{4}-4\,a^{2}cx^{2}-4\,acx+\left(a^{5}x^{5}+4\,a^{4}x^{4}+6\,a^{3}x^{3}+4\,a^{2}x^{2}\right)\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c}\,\sqrt{\frac{a^{2}cx^{2}\cdots}{a^{2}x^{2}}}}{a^{4}x^{4}+2\,a^{3}x^{3}-2\,ax-1}\right.}{2\left(a^{5}c^{2}x^{4}+2\,a^{4}c^{2}x^{3}-2\,a^{2}c^{2}x-ac^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*
c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 - 4*a*c*x + (a^5*x^5 + 4*a^4*x^4 + 6*a^3*
x^3 + 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)) + (2*a^4*x^4 + 9*a^3*x^3 + 6*a^
2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^2*x^4 + 2
*a^4*c^2*x^3 - 2*a^2*c^2*x - a*c^2), 1/2*(6*(a^4*x^4 + 2*a^3*x^3 - 2*a*x -
1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^
2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2 - a*c*x - 2*c)) - (2*a^4*x
^4 + 9*a^3*x^3 + 6*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)))/(a^5*c^2*x^4 + 2*a^4*c^2*x^3 - 2*a^2*c^2*x - a*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 106, normalized size = 0.62

$$\frac{\left(-2x^{3}a^{3}+6\ln\left(ax+1\right)x^{2}a^{2}-4a^{2}x^{2}+12ax\ln\left(ax+1\right)+4ax+6\ln\left(ax+1\right)+5\right)\left(ax-1\right)\sqrt{-a^{2}x^{2}+1}}{2\left(ax+1\right)a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(-2*x^3*a^3+6*ln(a*x+1)*x^2*a^2-4*a^2*x^2+12*a*x*ln(a*x+1)+4*a*x+6*ln(a
*x+1)+5)*(a*x-1)*(-a^2*x^2+1)^(1/2)/(a*x+1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)
^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**
(3/2)*(a*x + 1)**3), x)
```

3.739 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$\frac{31\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6x^5(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}\log(1-ax)}{16a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

```
[Out] -(-a^2*x^2+1)^(5/2)/a^5/(c-c/a^2/x^2)^(5/2)/x^4+1/6*(-a^2*x^2+1)^(5/2)/a^6/
(c-c/a^2/x^2)^(5/2)/x^5/(a*x+1)^3-9/8*(-a^2*x^2+1)^(5/2)/a^6/(c-c/a^2/x^2)^
(5/2)/x^5/(a*x+1)^2+31/8*(-a^2*x^2+1)^(5/2)/a^6/(c-c/a^2/x^2)^(5/2)/x^5/(a*
x+1)-1/16*(-a^2*x^2+1)^(5/2)*ln(-a*x+1)/a^6/(c-c/a^2/x^2)^(5/2)/x^5+49/16*(
-a^2*x^2+1)^(5/2)*ln(a*x+1)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.22, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{31\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6x^5(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6x^5(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{\left(1-a^2x^2\right)^{5/2}}{a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{(1-a}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] -((1 - a^2*x^2)^(5/2)/(a^5*(c - c/(a^2*x^2))^(5/2)*x^4)) + (1 - a^2*x^2)^(5
/2)/(6*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)^3) - (9*(1 - a^2*x^2)^(5/2
))/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)^2) + (31*(1 - a^2*x^2)^(5/2
))/(8*a^6*(c - c/(a^2*x^2))^(5/2)*x^5*(1 + a*x)) - ((1 - a^2*x^2)^(5/2)*Log
[1 - a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (49*(1 - a^2*x^2)^(5/2)*L
og[1 + a*x])/(16*a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{x^5}{(1-ax)(1+ax)^4}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(1-a^2x^2\right)^{5/2}\int \left(-\frac{1}{a^5}-\frac{1}{16a^5(-1+ax)}-\frac{1}{2a^5(1+ax)^4}+\frac{9}{4a^5(1+ax)^3}-\frac{31}{8a^5(1+ax)^2}+\frac{49}{16a^5(1+ax)}\right)dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$
$$= -\frac{\left(1-a^2x^2\right)^{5/2}}{a^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}+\frac{\left(1-a^2x^2\right)^{5/2}}{6a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1+ax)^3}-\frac{9\left(1-a^2x^2\right)^{5/2}}{8a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5(1+ax)^2}+\frac{31}{8a^6\left(c-\right.}$$

Mathematica [A] time = 0.09, size = 110, normalized size = 0.41

$$\frac{\sqrt{1-a^2x^2}\left(-48a^4x^4-144a^3x^3+42a^2x^2+270ax-3(ax+1)^3\log(1-ax)+147(ax+1)^3\log(ax+1)+140\right)}{48a^2c^2x(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(140 + 270*a*x + 42*a^2*x^2 - 144*a^3*x^3 - 48*a^4*x^4 -
 3*(1 + a*x)^3*Log[1 - a*x] + 147*(1 + a*x)^3*Log[1 + a*x]))/(48*a^2*c^2*Sq
rt[c - c/(a^2*x^2)]*x*(1 + a*x)^3)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,a^6x^6\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^7c^3x^7+3\,a^6c^3x^6+a^5c^3x^5-5\,a^4c^3x^4-5\,a^3c^3x^3+a^2c^3x^2+3\,ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^6*x^6*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^7*c
^3*x^7 + 3*a^6*c^3*x^6 + a^5*c^3*x^5 - 5*a^4*c^3*x^4 - 5*a^3*c^3*x^3 + a^2*
c^3*x^2 + 3*a*c^3*x + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(5/2)), x)
```

maple [A] time = 0.06, size = 176, normalized size = 0.66

$$-\frac{\left(48x^4a^4+3\ln(ax-1)\,x^3a^3-147a^3x^3\ln(ax+1)+144x^3a^3+9\ln(ax-1)\,x^2a^2-441\ln(ax+1)\,x^2a^2-42a^2\right.}{48\,(ax+1)\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] -1/48*(48*x^4*a^4+3*ln(a*x-1)*x^3*a^3-147*a^3*x^3*ln(a*x+1)+144*x^3*a^3+9*l
n(a*x-1)*x^2*a^2-441*ln(a*x+1)*x^2*a^2-42*a^2*x^2+9*ln(a*x-1)*x*a-441*a*x*l
n(a*x+1)-270*a*x+3*ln(a*x-1)-147*ln(a*x+1)-140)*(a*x-1)^2*(-a^2*x^2+1)^(1/2
)/(a*x+1)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**
(5/2)*(a*x + 1)**3), x)
```

3.740 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=359

$$\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{75\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{59\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(ax+1)^3\left(c-\frac{c}{a^2x^2}\right.}$$

```
[Out] (-a^2*x^2+1)^(7/2)/a^7/(c-c/a^2/x^2)^(7/2)/x^6+1/32*(-a^2*x^2+1)^(7/2)/a^8/
(c-c/a^2/x^2)^(7/2)/x^7/(-a*x+1)+1/16*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^
(7/2)/x^7/(a*x+1)^4-1/2*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(a*x
+1)^3+59/32*(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(a*x+1)^2-75/16*
(-a^2*x^2+1)^(7/2)/a^8/(c-c/a^2/x^2)^(7/2)/x^7/(a*x+1)+9/64*(-a^2*x^2+1)^(7
/2)*ln(-a*x+1)/a^8/(c-c/a^2/x^2)^(7/2)/x^7-201/64*(-a^2*x^2+1)^(7/2)*ln(a*x
+1)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.25, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 88}

$$\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{75\left(1-a^2x^2\right)^{7/2}}{16a^8x^7(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{59\left(1-a^2x^2\right)^{7/2}}{32a^8x^7(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{\left(1-a^2x^2\right)^{7/2}}{2a^8x^7(ax+1)^3\left(c-\frac{c}{a^2x^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (1 - a^2*x^2)^(7/2)/(a^7*(c - c/(a^2*x^2))^(7/2)*x^6) + (1 - a^2*x^2)^(7/2)
/(32*a^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 - a*x)) + (1 - a^2*x^2)^(7/2)/(16*a
^8*(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^4) - (1 - a^2*x^2)^(7/2)/(2*a^8*(c
 - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^3) + (59*(1 - a^2*x^2)^(7/2))/(32*a^8*(
c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)^2) - (75*(1 - a^2*x^2)^(7/2))/(16*a^8*
(c - c/(a^2*x^2))^(7/2)*x^7*(1 + a*x)) + (9*(1 - a^2*x^2)^(7/2)*Log[1 - a*x
])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) - (201*(1 - a^2*x^2)^(7/2)*Log[1 +
a*x])/(64*a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{e^{-3\tanh^{-1}(ax)}x^7}{\left(1-a^2x^2\right)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \frac{x^7}{(1-ax)^2(1+ax)^5}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}\int \left(\frac{1}{a^7}+\frac{1}{32a^7(-1+ax)^2}+\frac{9}{64a^7(-1+ax)}-\frac{1}{4a^7(1+ax)^5}+\frac{3}{2a^7(1+ax)^4}-\frac{59}{16a^7(1+ax)^3}+\frac{75}{16a^7(1+a\cdots}\right.}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(1-a^2x^2\right)^{7/2}}{a^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}+\frac{\left(1-a^2x^2\right)^{7/2}}{32a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1-ax)}+\frac{\left(1-a^2x^2\right)^{7/2}}{16a^8\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7(1+ax)^4}-\frac{(\cdots}{2a^8\left(c-\cdots\right.}$$

Mathematica [A] time = 0.12, size = 146, normalized size = 0.41

$$\frac{\sqrt{1-a^2x^2}\left(-2\left(32a^6x^6+96a^5x^5-87a^4x^4-309a^3x^3-59a^2x^2+207ax+104\right)-9(ax-1)(ax+1)^4\log(1-ax)\cdots\right.}{64a^2c^3x(ax-1)(ax+1)^4\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2*(104 + 207*a*x - 59*a^2*x^2 - 309*a^3*x^3 - 87*a^4*x
^4 + 96*a^5*x^5 + 32*a^6*x^6) - 9*(-1 + a*x)*(1 + a*x)^4*Log[1 - a*x] + 201
*(-1 + a*x)*(1 + a*x)^4*Log[1 + a*x]))/(64*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*
(-1 + a*x)*(1 + a*x)^4)
```

fricas [F] time = 1.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,a^8x^8\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^9c^4x^9+3\,a^8c^4x^8-8\,a^6c^4x^6-6\,a^5c^4x^5+6\,a^4c^4x^4+8\,a^3c^4x^3-3\,ac^4x-c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*a^8*x^8*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^9*c
^4*x^9 + 3*a^8*c^4*x^8 - 8*a^6*c^4*x^6 - 6*a^5*c^4*x^5 + 6*a^4*c^4*x^4 + 8*
a^3*c^4*x^3 - 3*a*c^4*x - c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(7/2)), x)
```

maple [A] time = 0.06, size = 248, normalized size = 0.69

$$-\frac{\sqrt{-a^2x^2+1}\ (ax-1)^2\left(64x^6a^6+9\ln(ax-1)\,x^5a^5-201\ln(ax+1)\,x^5a^5+192x^5a^5+27\ln(ax-1)\,x^4a^4-603\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/64*(-a^2*x^2+1)^(1/2)*(a*x-1)^2*(64*x^6*a^6+9*ln(a*x-1)*x^5*a^5-201*ln(a
*x+1)*x^5*a^5+192*x^5*a^5+27*ln(a*x-1)*x^4*a^4-603*ln(a*x+1)*x^4*a^4-174*x^
```

```
4*a^4+18*ln(a*x-1)*x^3*a^3-402*a^3*x^3*ln(a*x+1)-618*x^3*a^3-18*ln(a*x-1)*x
^2*a^2+402*ln(a*x+1)*x^2*a^2-118*a^2*x^2-27*ln(a*x-1)*x*a+603*a*x*ln(a*x+1)
+414*a*x-9*ln(a*x-1)+201*ln(a*x+1)+208)/(a*x+1)/a^8/x^7/(c*(a^2*x^2-1)/a^2/
x^2)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*(c - c/(a^2*x^2))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

3.741 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx$

Optimal. Leaf size=80

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{m\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-a^2x^2}}$$

```
[Out] x^(1+m)*(c-c/a^2/x^2)^(1/2)/m/(-a^2*x^2+1)^(1/2)+a*x^(2+m)*(c-c/a^2/x^2)^(1
/2)/(1+m)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6160, 6150, 43}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{m\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^(1 + m))/(m*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^
2*x^2)]*x^(2 + m))/((1 + m)*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{\tanh^{-1}(ax)}x^{-1+m}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int x^{-1+m}(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(x^{-1+m}+ax^m\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^{1+m}}{m\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^{2+m}}{(1+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.64

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax^{m+1}}{m+1}+\frac{x^m}{m}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(x^m/m + (a*x^(1 + m))/(1 + m)))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.04, size = 78, normalized size = 0.98

$$-\frac{\sqrt{-a^2x^2+1}\left(amx^2+(m+1)x\right)x^m\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\left(a^2m^2+a^2m\right)x^2-m^2-m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
[Out] -sqrt(-a^2*x^2 + 1)*(a*m*x^2 + (m + 1)*x)*x^m*sqrt((a^2*c*x^2 - c)/(a^2*x^2
))/((a^2*m^2 + a^2*m)*x^2 - m^2 - m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}\,x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^m/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 53, normalized size = 0.66

$$\frac{x^{1+m}\left(axm+m+1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{\left(1+m\right)m\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+m+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(1+m)/m/(-a^2*x^2+1)^(1/2
)
```

maxima [C] time = 0.39, size = 30, normalized size = 0.38

$$\frac{\sqrt{c}\,xx^m}{i\,m+i}-\frac{i\,\sqrt{c}\,x^m}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] sqrt(c)*x*x^m/(I*m + I) - I*sqrt(c)*x^m/(a*m)
```

mupad [B] time = 1.30, size = 45, normalized size = 0.56

$$\frac{x\,x^m\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(m+a\,m\,x+1\right)}{m\,\sqrt{1-a^2\,x^2}\,\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (x*x^m*(c - c/(a^2*x^2))^(1/2)*(m + a*m*x + 1))/(m*(1 - a^2*x^2)^(1/2)*(m +
 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{-c \left(-1 + \frac{1}{ax}\right) \left(1 + \frac{1}{ax}\right)} \, (ax+1)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x -
1)*(a*x + 1)), x)
```

3.742 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=74

$$\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

```
[Out] 1/2*x^3*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+1/3*a*x^4*(c-c/a^2/x^2)^(1/2
)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6160, 6150, 43}

$$\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)
]*x^4)/(3*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int x(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(x+ax^2\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{2\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.57

$$\frac{x^3(2ax+3)\sqrt{c-\frac{c}{a^2x^2}}}{6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^3*(3 + 2*a*x))/(6*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.60, size = 58, normalized size = 0.78

$$-\frac{\left(2\,ax^4+3\,x^3\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/6*(2*a*x^4 + 3*x^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^2/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 43, normalized size = 0.58

$$\frac{x^{3}\left(2ax+3\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{6\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/6*x^3*(2*a*x+3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

maxima [C] time = 0.39, size = 20, normalized size = 0.27

$$-\frac{1}{3}i\sqrt{c}\,x^{3}-\frac{i\sqrt{c}\,x^{2}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] -1/3*I*sqrt(c)*x^3 - 1/2*I*sqrt(c)*x^2/a
```

mupad [B] time = 1.00, size = 38, normalized size = 0.51

$$\frac{\sqrt{c-\frac{c}{a^{2}\,x^{2}}}\,\left(\frac{a\,x^{4}}{3}+\frac{x^{3}}{2}\right)}{\sqrt{1-a^{2}\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - c/(a^2*x^2))^(1/2)*((a*x^4)/3 + x^3/2))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x -
1)*(a*x + 1)), x)
```

3.743 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=71

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

```
[Out] x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+1/2*a*x^3*(c-c/a^2/x^2)^(1/2)/(-
a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6160, 6140}

$$\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}+\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (a*Sqrt[c - c/(a^2*x^2)]*x^
3)/(2*Sqrt[1 - a^2*x^2])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.58

$$\frac{x\left(\frac{ax^2}{2}+x\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(x + (a*x^2)/2))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.64, size = 57, normalized size = 0.80

$$-\frac{\sqrt{-a^2x^2+1}\left(ax^3+2\,x^2\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(a*x^3 + 2*x^2)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 42, normalized size = 0.59

$$\frac{x^{2}\left(ax+2\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}}{2\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/2*x^2*(a*x+2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

maxima [C] time = 0.39, size = 18, normalized size = 0.25

$$-\frac{1}{2}i\sqrt{c}\,x^{2}-\frac{i\sqrt{c}\,x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*I*sqrt(c)*x^2 - I*sqrt(c)*x/a
```

mupad [B] time = 0.97, size = 36, normalized size = 0.51

$$\frac{\sqrt{c-\frac{c}{a^{2}\,x^{2}}}\,\left(\frac{a\,x^{3}}{2}+x^{2}\right)}{\sqrt{1-a^{2}\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - c/(a^2*x^2))^(1/2)*((a*x^3)/2 + x^2))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*
(a*x + 1)), x)
```

3.744 $\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] a*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-
a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 43}

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{1+ax}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\left(a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.54

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,(ax+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(a*x + Log[x]))/Sqrt[1 - a^2*x^2]
```

fricas [B] time = 0.70, size = 320, normalized size = 4.71

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-2\left(a^2x^2-a^2x\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^3x^2-a\right)},-\frac{\left(a^2x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
```

```
x^4 - x^2)) - 2*(a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^3*x^2 - a), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)
*(a*x^3 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 +
1)*c*x^2 + c)) + (a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^3*x^2 - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c - \frac{c}{a^2x^2}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.04, size = 52, normalized size = 0.76

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,(ax+\ln(x))\,\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(a*x+ln(x))*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

maxima [C] time = 0.39, size = 17, normalized size = 0.25

$$-i\sqrt{c}\,x - \frac{i\sqrt{c}\,\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -I*sqrt(c)*x - I*sqrt(c)*log(x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/sqrt(-(a*x - 1)*(a
*x + 1)), x)
```

3.745 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=66

$$\frac{ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+a*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a^2
*x^2+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6160, 6150, 43}

$$\frac{ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)]*x*Log
[x])/Sqrt[1 - a^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1+ax}{x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^2}+\frac{a}{x}\right)dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.56

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax\log(x)-1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1 + a*x*Log[x]))/Sqrt[1 - a^2*x^2]
```

fricas [B] time = 0.88, size = 294, normalized size = 4.45

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2x^2+1}\,(x-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^2-1\right)},-\frac{\left(a^2x^2-1\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
x^4 - x^2)) - 2*sqrt(-a^2*x^2 + 1)*(x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))
/(a^2*x^2 - 1), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 +
a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 +
 c)) + sqrt(-a^2*x^2 + 1)*(x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^2
 - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.04, size = 52, normalized size = 0.79

$$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(a\ln(x)x-1)\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*ln(x)*x-1)*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

maxima [C] time = 0.38, size = 19, normalized size = 0.29

$$-i\sqrt{c}\,\log(x)+\frac{i\sqrt{c}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] -I*sqrt(c)*log(x) + I*sqrt(c)/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x*sqrt(-(a*x - 1)
*(a*x + 1))), x)
```

3.746 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

[Out] -1/2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/x/(-a^2*x^2+1)^(1/2)

Rubi [A] time = 0.21, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6160, 6150, 37}

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]

[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(2*x*Sqrt[1 - a^2*x^2])

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1+ax}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{2x\sqrt{1-a^2x^2}}$$

Mathematica [**A**] time = 0.02, size = 42, normalized size = 0.98

$$-\frac{(2ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*(1 + 2*a*x))/(x*Sqrt[1 - a^2*x^2])
```

fricas [**A**] time = 0.56, size = 63, normalized size = 1.47

$$-\frac{\sqrt{-a^2x^2+1}\left((2\,a+1)x^2-2\,ax-1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*((2*a + 1)*x^2 - 2*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^2*x^3 - x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.03, size = 43, normalized size = 1.00

$$-\frac{(2ax+1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/(-a^2*x^2+1)^(1/2)
```

maxima [C] time = 0.38, size = 20, normalized size = 0.47

$$\frac{i\sqrt{c}}{x}+\frac{i\sqrt{c}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] I*sqrt(c)/x + 1/2*I*sqrt(c)/(a*x^2)
```

mupad [B] time = 1.01, size = 35, normalized size = 0.81

$$-\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+\frac{1}{2}\right)}{x\,\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] -((c - c/(a^2*x^2))^(1/2)*(a*x + 1/2))/(x*(1 - a^2*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\sqrt{-(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**2*sqrt(-(a*x -
 1)*(a*x + 1))), x)
```

3.747 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=160

$$-\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -7/8*x*(c-c/a^2/x^2)^(1/2)/a^3-7/24*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3-1/6*x
*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3-1/4*x^2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^
2+7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 90, 80, 50, 41, 216}

$$-\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (-7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) - (7*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)
)/(24*a^3) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(6*a^3) - (Sqrt[c - c/(a
^2*x^2)]*x^2*(1 + a*x)^2)/(4*a^2) + (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])
/(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x^2(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(-1-2ax)(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{4a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2} + \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{12a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2} + \frac{(7\sqrt{\ldots}}{8\ldots}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(\ldots}{4a^2}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(\ldots}{4a^2}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(\ldots}{4a^2}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 0.58

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(21\log\left(\sqrt{a^2x^2-1}+ax\right)+\sqrt{a^2x^2-1}\left(6a^3x^3+16a^2x^2+21ax+32\right)\right)}{24a^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] -1/24*(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(32 + 21*a*x + 16*a^2*x^
2 + 6*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(a^3*Sqrt[-1 + a^2*x^2]
)
```

fricas [A] time = 0.54, size = 222, normalized size = 1.39

$$\left[-\frac{2\left(6\,a^4x^4+16\,a^3x^3+21\,a^2x^2+32\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-21\,\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},-\frac{\left(6\,a^4x^4+16\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/48*(2*(6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 + 32*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - 21*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*
x^2 - c)/(a^2*x^2)) - c))/a^4, -1/24*((6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2
+ 32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)
*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

giac [A] time = 3.09, size = 128, normalized size = 0.80

$$-\frac{1}{48}\left(2\,\sqrt{a^2cx^2-c}\left(\left(2\,x\left(\frac{3\,x\mathrm{sgn}(x)}{a^2}+\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x+\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\,\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-\right.\right.}{a^4|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 + 8*sgn(x)/a^3) + 21*sgn
(x)/a^4)*x + 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*
c*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) - 64*sqrt(-c)*
abs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

maple [A] time = 0.04, size = 196, normalized size = 1.22

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-6x\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4-16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-27\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x\,a^2c+27c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-48c^{\frac{3}{2}}\right.}{24\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,c\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4-16
*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2*c+27*c^(3
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln((c^(1/2)*((a*x-1)
*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))-48*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(
c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))*x^3/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx-\int \frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(x**3*sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x**4*sqrt
(c - c/(a**2*x**2))/(a*x - 1), x)
```

3.748 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=123

$$-\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -x*(c-c/a^2/x^2)^(1/2)/a^2-1/3*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2-1/3*x*(a*x
+1)^2*(c-c/a^2/x^2)^(1/2)/a^2+x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*x+1
)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6159, 6129, 80, 50, 41, 216}

$$-\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/a^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(3*a
^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(3*a^2) + (Sqrt[c - c/(a^2*x^2)
]*x*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{2\tanh^{-1}(ax)}x\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2} + \frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{3a\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int -}{a\sqrt{1-ax}\,\sqrt{}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int -}{a\sqrt{1-ax}\,\sqrt{1}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}}{a^2\sqrt{1-ax}\,\sqrt{1+}}$$

Mathematica [A] time = 0.07, size = 84, normalized size = 0.68

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\,\left(a^2x^2+3ax+5\right)+3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{3a^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] -1/3*(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 + 3*a*x + a^2*x^2) + 3
*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(a^2*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.66, size = 204, normalized size = 1.66

$$\left[-\frac{2\left(a^3x^3+3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3}, -\frac{\left(a^3x^3+3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2cx}{a^2}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/6*(2*(a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*
sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
 - c))/a^3, -1/3*((a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
a^2*c*x^2 - c)))/a^3]
```

giac [A] time = 0.19, size = 116, normalized size = 0.94

$$-\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\operatorname{sgn}(x)}{a^2}+\frac{3\operatorname{sgn}(x)}{a^3}\right)+\frac{5\operatorname{sgn}(x)}{a^4}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^3|a|}+\frac{\left(3a\sqrt{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 + 3*sgn(x)/a^3) + 5*sgn(x)/a^4
) - 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*ab
s(a)) + (3*a*sqrt(c)*log(abs(c)) - 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))
*abs(a)
```

maple [A] time = 0.04, size = 174, normalized size = 1.41

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x\,a^2c+3c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{c}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)-\right.}{3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-3*(c*(a
^2*x^2-1)/a^2)^(1/2)*x*a^2*c+3*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/
2))-6*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))-6*((a
*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/a^3/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))*x^2/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax-1}\,dx-\int \frac{ax^{3}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(x**2*sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x**3*sqrt
(c - c/(a**2*x**2))/(a*x - 1), x)
```

3.749 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=98

$$-\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] -3/2*x*(c-c/a^2/x^2)^(1/2)/a-1/2*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a+3/2*x*arcs
in(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6159, 6129, 50, 41, 216}

$$-\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) - (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(2
*a) + (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a
*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 0.79

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{1-a^2x^2}\,(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*x*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[
1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.58, size = 188, normalized size = 1.92

$$\left[-\frac{2\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},-\frac{\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}\,\text{arc}}{2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(c)*log(
2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2,
-1/2*((a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan
(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

giac [A] time = 0.22, size = 106, normalized size = 1.08

$$-\frac{1}{4}\left(2\,\sqrt{a^2cx^2-c}\left(\frac{x\text{sgn}(x)}{a^2}+\frac{4\,\text{sgn}(x)}{a^3}\right)-\frac{6\,\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\text{sgn}(x)}{a^2|a|}+\frac{\left(3\,a\sqrt{c}\,\log\left(|c|\right)-8\,\sqrt{-}\right.}{a^3|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 + 4*sgn(x)/a^3) - 6*sqrt(c)*log(a
bs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c
)*log(abs(c)) - 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

maple [A] time = 0.04, size = 147, normalized size = 1.50

$$\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\, x\left(-x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\, a^{2}+\sqrt{c}\, \ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)-4\sqrt{c}\, \ln\left(\frac{\sqrt{c}\, \sqrt{\frac{(ax-1)(ax+1)c}{a^{2}}}+cx}{\sqrt{c}}\right)-4\sqrt{\frac{(ax-1)(ax+1)c}{a^{2}}}\, a\right)}{2\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2+c^(1/
2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-4*c^(1/2)*ln((c^(1/2)*((a*x-1)*(
a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))-4*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a)/(c*(a^
2*x^2-1)/a^2)^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^{2}\sqrt{c-\frac{c}{a^{2}x^{2}}}\, x}{a^{2}x^{2}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))*x/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\sqrt{c-\frac{c}{a^{2}\,x^{2}}}\,(a\,x+1)^{2}}{a^{2}\,x^{2}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax-1}\, dx-\int \frac{ax^{2}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{ax-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(x*sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x**2*sqrt(c
- c/(a**2*x**2))/(a*x - 1), x)
```

3.750 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=118

$$-x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] -x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(
a*x+1)^(1/2)-x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-
a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6159, 6129, 102, 157, 41, 216, 92, 208}

$$-x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c}\right.}{}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.08, size = 81, normalized size = 0.69

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2
]] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.63, size = 270, normalized size = 2.29

$$\left[-\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-4\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\sqrt{-c}\,\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, -\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\,\text{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(-c)*log(-(a^
2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, -(
a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*
c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sq
rt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 0.04, size = 198, normalized size = 1.68

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\, x \left(2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right) a\sqrt{-\frac{c}{a^2}}+2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\, a^2\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\, a^2-c\ln\left(\frac{2\cdots}{\cdots}\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\, a^2\sqrt{-\frac{c}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*c^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/
a^2)^(1/2)+c*x)/c^(1/2))*a*(-c/a^2)^(1/2)+2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a
^2*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c - \frac{c}{a^{2}x^{2}}}}{ax - 1}\, dx - \int \frac{ax\sqrt{c - \frac{c}{a^{2}x^{2}}}}{ax - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x - 1), x) - Integral(a*x*sqrt(c - c/(
a**2*x**2))/(a*x - 1), x)
```

3.751 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=118

$$-\sqrt{c-\frac{c}{a^2x^2}}+\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)+a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*
x+1)^(1/2)-2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/
(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6159, 6129, 98, 157, 41, 216, 92, 208}

$$-\sqrt{c-\frac{c}{a^2x^2}}+\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
a*x]*Sqrt[1 + a*x]) - (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\sqrt{c-\frac{c}{a^2x^2}} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-2a-a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\sqrt{c-\frac{c}{a^2x^2}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+}}}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\sqrt{c-\frac{c}{a^2x^2}} + \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\right.}{\sqrt{1-ax}\,\sqrt{1+}}$$

$$= -\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right.}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.72

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\sqrt{a^2x^2-1}-ax\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2*
x^2]] - a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

fricas [A] time = 1.49, size = 255, normalized size = 2.16

$$\left[\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\,\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\,,2\,\sqrt{c}\,\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^
2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a
^2*x^2)) - 2*c)/x^2) - sqrt((a^2*c*x^2 - c)/(a^2*x^2)), 2*sqrt(c)*arctan(a*
sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*lo
g(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - sq
rt((a^2*c*x^2 - c)/(a^2*x^2))]
```

giac [A] time = 0.32, size = 128, normalized size = 1.08

$$-\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}-\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}+\frac{2c^{\frac{3}{2}}\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
 - sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) + 2
*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs
(a)
```

maple [B] time = 0.05, size = 305, normalized size = 2.58

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^2a^3c+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)xa-2\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\right.}{a\sqrt{\frac{c\left(a^2\right.}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*(-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
x^2*a^3*c+a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)+(-c/a^2)^(1/2)*c^(3/
2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a-2*(-c/a^2)^(1/2)*c^(3/2)*ln(
(c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x*a-2*(-c/a^2)^(1/2)*(
(a*x-1)*(a*x+1)*c/a^2)^(1/2)*x*a^2*c+2*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)*c*x+2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)
*x*c^2)/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\sqrt{ax+1}\,\sqrt{ax-1}\,\sqrt{c}}{ax}-\int\frac{\left(a\sqrt{c}\,x+2\sqrt{c}\right)\sqrt{ax+1}\,\sqrt{ax-1}}{a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] -sqrt(a*x + 1)*sqrt(a*x - 1)*sqrt(c)/(a*x) - integrate((a*sqrt(c)*x + 2*sqr
t(c))*sqrt(a*x + 1)*sqrt(a*x - 1)/(a^2*x^3 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^2}{x\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c - \frac{c}{a^{2} x^{2}}}}{a x^{2} - x}\, dx - \int \frac{a x \sqrt{c - \frac{c}{a^{2} x^{2}}}}{a x^{2} - x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**2 - x), x) - Integral(a*x*sqrt(c -
c/(a**2*x**2))/(a*x**2 - x), x)
```

3.752 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=111

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{2\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] -3/2*a*(c-c/a^2/x^2)^(1/2)-1/2*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x-3/2*a^2*x*arct
anh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1
)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6159, 6129, 94, 92, 208}

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{2\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/2 - (Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(2*x) -
(3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqr
t[1 - a*x]*Sqrt[1 + a*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x}+\frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x}+\frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x}-\frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\mathrm{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1}\right.}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x}-\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{2\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.71

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(3a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-(4ax+1)\sqrt{a^2x^2-1}\right)}{2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-((1 + 4*a*x)*Sqrt[-1 + a^2*x^2]) + 3*a^2*x^2*ArcTa
n[1/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.60, size = 177, normalized size = 1.59

$$\left[\frac{3\,a\sqrt{-c}\,x\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},\ \frac{3\,a\sqrt{c}\,x\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(4\,ax+1)\sqrt{\frac{a^2c}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) - 2*(4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
c*x^2 - c)) - (4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

giac [B] time = 3.01, size = 195, normalized size = 1.76

$$-\left(3\,\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3ac\mathrm{sgn}(x)-4\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2c}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2c}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] -(3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
 ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) - 4*(sqrt(a^2*c)*x - s
```

```
qrt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - c))*a*c^2*sgn(x) - 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

maple [B] time = 0.05, size = 347, normalized size = 3.13

$$\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(-4\sqrt{-\frac{c}{a^{2}}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,x^{3}a^{3}c+4\sqrt{-\frac{c}{a^{2}}}\,\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}}x\,a^{3}+4\sqrt{-\frac{c}{a^{2}}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)x^{2}a-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^
(1/2)*x^3*a^3*c+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^3+4*(-c/a^2)
^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^2*a-4*(-c/a^2)^(1/
2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a-4*
(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^2*a^2*c+3*(-c/a^2)^(1/2)*(c*
(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2*c+a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/
2)+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)*x^2*c^2)/
(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^{2}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{\left(a^{2}x^{2}-1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))/((a^2*x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^{2}\,x^{2}}}\,\left(a\,x+1\right)^{2}}{x^{2}\,\left(a^{2}\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{ax^3 - x^2}\,dx - \int \frac{ax\sqrt{c - \frac{c}{a^2x^2}}}{ax^3 - x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**3 - x**2), x) - Integral(a*x*sqrt(c
 - c/(a**2*x**2))/(a*x**3 - x**2), x)
```

3.753 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=139

$$-a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -a^2*(c-c/a^2/x^2)^(1/2)-1/3*a*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x-1/3*(a*x+1)^2*
(c-c/a^2/x^2)^(1/2)/x^2-a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^
2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6159, 6129, 96, 94, 92, 208}

$$-a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] -(a^2*Sqrt[c - c/(a^2*x^2)]) - (a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(3*x) -
(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(3*x^2) - (a^3*Sqrt[c - c/(a^2*x^2)]*x*
ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^4\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{3\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} + \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} + \frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{\sqrt{1-ax}\,\sqrt{}}$$

$$= -a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} - \frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\mathrm{Su}}{}$$

$$= -a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} - \frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh}{\sqrt{1-a}}$$

Mathematica [A] time = 0.08, size = 71, normalized size = 0.51

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-5a^2x^2+\frac{3a^3x^3\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{\sqrt{a^2x^2-1}}-3ax-1\right)}{3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1 - 3*a*x - 5*a^2*x^2 + (3*a^3*x^3*ArcTan[1/Sqrt[-
1 + a^2*x^2]])/Sqrt[-1 + a^2*x^2]))/(3*x^2)
```

fricas [A] time = 0.76, size = 201, normalized size = 1.45

$$\left[\frac{3\,a^2\sqrt{-c}\,x^2\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(5\,a^2x^2+3\,ax+1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2},\ \frac{3\,a^2\sqrt{c}\,x^2\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(\right.}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) - 2*(5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, 1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)))/x^2]
```

giac [A] time = 11.14, size = 231, normalized size = 1.66

$$-\frac{2}{3}\left(3\,a\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^5 ac\mathrm{sgn}(x)-3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] -2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) - 3*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) - 12*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))*a*c^3*sgn(x) - 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

maple [B] time = 0.05, size = 378, normalized size = 2.72

$$\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a\left(-6\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^4a^3c+6\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^2a^3+6\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)x^3a-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

```
[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(1/2)*x^4*a^3*c+6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^3+6*(-
c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^3*a-6*(-c/a^
2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^
3*a-6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^2*c+3*(-c/a^2)^(1/
2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^2*c+3*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)
^(3/2)*x*a^2+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)
*x^3*c^2+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2
*x^2-1)/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2 a^2 \sqrt{c}\, x^2+\sqrt{c}\right) \sqrt{a x+1}\, \sqrt{a x-1}}{3\, a x^3}-\int \frac{\left(a \sqrt{c}\, x+2\, \sqrt{c}\right) \sqrt{a x+1}\, \sqrt{a x-1}}{a^2 x^5-x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="maxi
ma")
```

```
[Out] -1/3*(2*a^2*sqrt(c)*x^2 + sqrt(c))*sqrt(a*x + 1)*sqrt(a*x - 1)/(a*x^3) - in
tegrate((a*sqrt(c)*x + 2*sqrt(c))*sqrt(a*x + 1)*sqrt(a*x - 1)/(a^2*x^5 - x^
3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^2\, x^2}}\, {\left(a\, x+1\right)}^2}{x^3\, \left(a^2\, x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c-\frac{c}{a^2 x^2}}}{a x^4-x^3}\, dx-\int \frac{a x \sqrt{c-\frac{c}{a^2 x^2}}}{a x^4-x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x**3,x)

[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**4 - x**3), x) - Integral(a*x*sqrt(c
 - c/(a**2*x**2))/(a*x**4 - x**3), x)
```

3.754 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}$$

```
[Out] -4/3*a^3*(c-c/a^2/x^2)^(1/2)-1/4*(c-c/a^2/x^2)^(1/2)/x^3-2/3*a*(c-c/a^2/x^2
)^(1/2)/x^2-7/8*a^2*(c-c/a^2/x^2)^(1/2)/x-7/8*a^4*x*arctanh((-a*x+1)^(1/2)*
(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 98, 151, 12, 92, 208}

$$-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/3 - Sqrt[c - c/(a^2*x^2)]/(4*x^3) - (2*a*Sqr
t[c - c/(a^2*x^2)])/(3*x^2) - (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) - (7*a^4*
Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a
*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^5\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-8a-7a^2x}{x^4\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2+16a^3x}{x^3\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-32a^3-21a^4x}{x^2\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{24\sqrt{1-a}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{8\sqrt{1-}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x}{8\sqrt{1}}$$

Mathematica [A] time = 0.08, size = 95, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(21a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\sqrt{a^2x^2-1}\left(32a^3x^3+21a^2x^2+16ax+6\right)\right)}{24x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*
a^3*x^3)) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2
*x^2])
```

fricas [A] time = 0.76, size = 217, normalized size = 1.39

$$\left[\frac{21\,a^3\sqrt{-c}\,x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(32\,a^3x^3+21\,a^2x^2+16\,ax+6\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3},\frac{21\,a^3\sqrt{c}\,x^3\arctan\left(\frac{a\sqrt{\cdots}}{\cdots}\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, 1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(c
)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (32*a^3*x^3 + 21*a^2
*x^2 + 16*a*x + 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

giac [B] time = 6.42, size = 316, normalized size = 2.03

$$-\frac{1}{12}\left(21\,a^2\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^7a^2c\,\mathrm{sgn}(x)+45\left(\sqrt{a^2c}\,x-\sqrt{a\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] -1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) - 96*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^3*a^2*c^3*sgn(x) - 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))
^2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c
^4*sgn(x) - 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^2 + c)^4)*abs(a)
```

maple [B] time = 0.05, size = 410, normalized size = 2.63

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\, a^2\left(-48\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\, x^5a^3c+48\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}} x^3a^3+48\sqrt{-\frac{c}{a^2}}\, c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

```
[Out] 1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(1/2)*x^5*a^3*c+48*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^3*a^3
+48*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^4*a-48
*(-c/a^2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1
/2))*x^4*a-48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^4*a^2*c+21*(-c
/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^4*a^2*c+27*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(3/2)*x^2*a^2+21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-c)/a^2/x)*x^4*c^2+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x+6*(c*
(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(
1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{\left(a^2x^2-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*x + 1)^2*sqrt(c - c/(a^2*x^2))/((a^2*x^2 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^2}{x^4\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{ax^5 - x^4}\, dx - \int \frac{ax\sqrt{c - \frac{c}{a^2x^2}}}{ax^5 - x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**5 - x**4), x) - Integral(a*x*sqrt(c
 - c/(a**2*x**2))/(a*x**5 - x**4), x)
```

3.755 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$-\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}-\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}$$

```
[Out] -6/5*a^4*(c-c/a^2/x^2)^(1/2)-1/5*(c-c/a^2/x^2)^(1/2)/x^4-1/2*a*(c-c/a^2/x^2
)^(1/2)/x^3-3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2-3/4*a^3*(c-c/a^2/x^2)^(1/2)/x-3
/4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)
^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 98, 151, 12, 92, 208}

$$-\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (-6*a^4*Sqrt[c - c/(a^2*x^2)])/5 - Sqrt[c - c/(a^2*x^2)]/(5*x^4) - (a*Sqrt[
c - c/(a^2*x^2)])/(2*x^3) - (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) - (3*a^3*
Sqrt[c - c/(a^2*x^2)])/(4*x) - (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[
1 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^6\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-10a-9a^2x}{x^5\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{36a^2+30a^3x}{x^4\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-90a^3-72a^4x}{x^3\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{}{x^2}}{120\sqrt{1-ax}}$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} -$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} +$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} -$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} -$$

Mathematica [A] time = 0.09, size = 103, normalized size = 0.57

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(15a^5x^5\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-\sqrt{a^2x^2-1}\left(24a^4x^4+15a^3x^3+12a^2x^2+10ax+4\right)\right)}{20x^4\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-(Sqrt[-1 + a^2*x^2]*(4 + 10*a*x + 12*a^2*x^2 + 15*
a^3*x^3 + 24*a^4*x^4)) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*
Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.89, size = 233, normalized size = 1.29

$$\left[\frac{15\,a^4\sqrt{-c}\,x^4\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4},\ \frac{15\,a^4\sqrt{c}\,x^4\,a\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="fric
as")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, 1/20*(15*a^4*sqrt(c)*x^4*ar
ctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (24*a^4
*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
))/x^4]
```

giac [B] time = 5.49, size = 362, normalized size = 2.00

$$-\frac{1}{10}\left(15\,a^3\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^9a^3c\mathrm{sgn}(x)+70\left(\sqrt{a^2c}\,x-\sqrt{a\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="giac
")
```

```
[Out] -1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) - 40*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) - 200*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^3*c^4*sgn(x) - 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*
```

```
c^5*sgn(x) - 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2 + c)^5)*abs(a)
```

maple [B] time = 0.06, size = 447, normalized size = 2.47

$$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\, a^2 \left(-40\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\, x^6a^4c + 40\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\, x^4a^4 + 15\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\, x^5a^3c + 40\sqrt{-\frac{c}{a^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^5,x)
```

```
[Out] 1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(c*(a^2*x^2-1)/a^2)^(1/2)*(
-c/a^2)^(1/2)*x^6*a^4*c+40*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^4*a^4
+15*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c+40*(-c/a^2)^(1/2)*c^
(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^5*a^2-40*(-c/a^2)^(1/2)*c^(
3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^5*a^2-40*(-c
/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c+25*(-c/a^2)^(1/2)*(c*(a
^2*x^2-1)/a^2)^(3/2)*x^3*a^3+15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1
/2)*a^2-c)/a^2/x)*x^5*a*c^2+16*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2
*a^2+10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x+4*(c*(a^2*x^2-1)/a^2)^
(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(8\,a^4\sqrt{c}\,x^4 + 4\,a^2\sqrt{c}\,x^2 + 3\,\sqrt{c}\right)\sqrt{ax+1}\,\sqrt{ax-1}}{15\,ax^5} - \int \frac{\left(a\sqrt{c}\,x + 2\,\sqrt{c}\right)\sqrt{ax+1}\,\sqrt{ax-1}}{a^2x^7 - x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="maxi
ma")
```

```
[Out] -1/15*(8*a^4*sqrt(c)*x^4 + 4*a^2*sqrt(c)*x^2 + 3*sqrt(c))*sqrt(a*x + 1)*sqr
t(a*x - 1)/(a*x^5) - integrate((a*sqrt(c)*x + 2*sqrt(c))*sqrt(a*x + 1)*sqrt
(a*x - 1)/(a^2*x^7 - x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c - \frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)^2}{x^5\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{ax^6 - x^5}\,dx - \int \frac{ax\sqrt{c - \frac{c}{a^2x^2}}}{ax^6 - x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] -Integral(sqrt(c - c/(a**2*x**2))/(a*x**6 - x**5), x) - Integral(a*x*sqrt(c
 - c/(a**2*x**2))/(a*x**6 - x**5), x)
```

3.756 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=188

$$-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-a^2x^2}}-\frac{ax^5\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] -4*x^2*(c-c/a^2/x^2)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)-2*x^3*(c-c/a^2/x^2)^(1/2)
/a/(-a^2*x^2+1)^(1/2)-x^4*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/4*a*x^5*
(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-4*x*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a
^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{ax^5\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a^2*Sqrt[1 - a^2*x^2]) - (2*Sqrt[c - c/(a^2
*x^2)]*x^3)/(a*Sqrt[1 - a^2*x^2]) - (Sqrt[c - c/(a^2*x^2)]*x^4)/Sqrt[1 - a^
2*x^2] - (a*Sqrt[c - c/(a^2*x^2)]*x^5)/(4*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c -
c/(a^2*x^2)]*x*Log[1 - a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{3\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x^2(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(-\frac{4}{a^2}-\frac{4x}{a}-3x^2-ax^3-\frac{4}{a^2(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{a\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^5}{4\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^3\sqrt{1-}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.38

$$
\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(1-ax)}{a^3}-\frac{4x}{a^2}-\frac{ax^4}{4}-\frac{2x^2}{a}-x^3\right)}{\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((-4*x)/a^2 - (2*x^2)/a - x^3 - (a*x^4)/4 - (4*Log
[1 - a*x])/a^3))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.97, size = 419, normalized size = 2.23

$$
\left[\frac{8\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx-\left(a^5x^5-4\,a^4x^4+6\,a^3x^3-4\,a^2x^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\left(a^5x^5+4\,\right.}{4\left(a^6x^2-a^4\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(8*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 + 4*a*c*x - (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2)*sqrt
(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 - 2
*a^3*x^3 + 2*a*x - 1)) + (a^5*x^5 + 4*a^4*x^4 + 8*a^3*x^3 + 16*a^2*x^2)*sqr
t(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^2 - a^4), 1/4*(16*(
a^2*x^2 - 1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c
)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 - 2*a^2*c*x^2 - a*c*x + 2*c))
+ (a^5*x^5 + 4*a^4*x^4 + 8*a^3*x^3 + 16*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a
^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^2 - a^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.05, size = 85, normalized size = 0.45

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\sqrt{-a^2x^2+1}\,\left(x^4a^4+4x^3a^3+8a^2x^2+16ax+16\ln\left(ax-1\right)\right)}{4\left(a^2x^2-1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(x^4*a^4+4*x^3*a^3+8
*a^2*x^2+16*a*x+16*ln(a*x-1))/(a^2*x^2-1)/a^3
```

maxima [C] time = 0.46, size = 196, normalized size = 1.04

$$\frac{1}{4}a^3\left(\frac{i a^2\sqrt{c}\,x^4+2i\sqrt{c}\,x^2}{a^5}+\frac{2i\sqrt{c}\,\log\left(ax+1\right)}{a^7}+\frac{2i\sqrt{c}\,\log\left(ax-1\right)}{a^7}\right)+\frac{1}{2}a^2\left(\frac{2\left(i a^2\sqrt{c}\,x^3+3i\sqrt{c}\,x\right)}{a^5}-\frac{3i\sqrt{c}\,\log}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/4*a^3*((I*a^2*sqrt(c)*x^4 + 2*I*sqrt(c)*x^2)/a^5 + 2*I*sqrt(c)*log(a*x +
1)/a^7 + 2*I*sqrt(c)*log(a*x - 1)/a^7) + 1/2*a^2*(2*(I*a^2*sqrt(c)*x^3 + 3*
I*sqrt(c)*x)/a^5 - 3*I*sqrt(c)*log(a*x + 1)/a^6 + 3*I*sqrt(c)*log(a*x - 1)/
a^6) - 3/2*a*(-I*sqrt(c)*x^2/a^3 - I*sqrt(c)*log(a*x + 1)/a^5 - I*sqrt(c)*l
og(a*x - 1)/a^5) + I*sqrt(c)*x/a^3 - 1/2*I*sqrt(c)*log(a*x + 1)/a^4 + 1/2*I
*sqrt(c)*log(a*x - 1)/a^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{- c \left(-1 + \frac{1}{a x}\right) \left(1 + \frac{1}{a x}\right)} \left(a x + 1\right)^{3}}{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1
)*(a*x + 1))**(3/2), x)
```

3.757 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=153

$$-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}-\frac{3x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

```
[Out] -4*x^2*(c-c/a^2/x^2)^(1/2)/a/(-a^2*x^2+1)^(1/2)-3/2*x^3*(c-c/a^2/x^2)^(1/2)
/(-a^2*x^2+1)^(1/2)-1/3*a*x^4*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-4*x*ln
(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 77}

$$-\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}-\frac{3x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - c/(a^2*x
^2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)]*x^4)/(3*Sqrt[1 -
 a^2*x^2]) - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/(a^2*Sqrt[1 - a^2*x^2
])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{3\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{x(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\left(-\frac{4}{a}-3x-ax^2-\frac{4}{a(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a\sqrt{1-a^2x^2}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{2\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.42

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(1-ax)}{a^2}-\frac{ax^3}{3}-\frac{4x}{a}-\frac{3x^2}{2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((-4*x)/a - (3*x^2)/2 - (a*x^3)/3 - (4*Log[1 - a*x
])/a^2))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.94, size = 405, normalized size = 2.65

$$\left[\frac{12\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx-\left(a^5x^5-4\,a^4x^4+6\,a^3x^3-4\,a^2x^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\left(2\,a^4x^4+\right.}{6\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/6*(12*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4
- 4*a^2*c*x^2 + 4*a*c*x - (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2)*sqr
t(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 -
2*a^3*x^3 + 2*a*x - 1)) + (2*a^4*x^4 + 9*a^3*x^3 + 24*a^2*x^2)*sqrt(-a^2*x^
2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*x^2 - a^3), 1/6*(24*(a^2*x^2 -
 1)*sqrt(c)*arctan((a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a
^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 - 2*a^2*c*x^2 - a*c*x + 2*c)) + (2*a^4*
x^4 + 9*a^3*x^3 + 24*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)))/(a^5*x^2 - a^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 \sqrt{c - \frac{c}{a^2x^2}}\, x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.04, size = 78, normalized size = 0.51

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\, x\sqrt{-a^2x^2+1}\, \left(2x^3a^3+9a^2x^2+24ax+24\ln\left(ax-1\right)\right)}{6\left(a^2x^2-1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3+9*a^2*x^2
+24*a*x+24*ln(a*x-1))/(a^2*x^2-1)/a^2
```

maxima [C] time = 0.46, size = 172, normalized size = 1.12

$$\frac{1}{6}a^3\left(\frac{2\left(i a^2\sqrt{c}\,x^3+3i\sqrt{c}\,x\right)}{a^5}-\frac{3i\sqrt{c}\,\log\left(ax+1\right)}{a^6}+\frac{3i\sqrt{c}\,\log\left(ax-1\right)}{a^6}\right)-\frac{3}{2}a^2\left(-\frac{i\sqrt{c}\,x^2}{a^3}-\frac{i\sqrt{c}\,\log\left(ax+1\right)}{a^5}-\frac{i}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm
="maxima")
```

```
[Out] 1/6*a^3*(2*(I*a^2*sqrt(c)*x^3 + 3*I*sqrt(c)*x)/a^5 - 3*I*sqrt(c)*log(a*x +
1)/a^6 + 3*I*sqrt(c)*log(a*x - 1)/a^6) - 3/2*a^2*(-I*sqrt(c)*x^2/a^3 - I*sq
rt(c)*log(a*x + 1)/a^5 - I*sqrt(c)*log(a*x - 1)/a^5) - 3/2*a*(-2*I*sqrt(c)*
x/a^3 + I*sqrt(c)*log(a*x + 1)/a^4 - I*sqrt(c)*log(a*x - 1)/a^4) + 1/2*I*sq
rt(c)*log(a*x + 1)/a^3 + 1/2*I*sqrt(c)*log(a*x - 1)/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{- c \left(-1 + \frac{1}{a x}\right) \left(1 + \frac{1}{a x}\right)} \left(a x + 1\right)^{3}}{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1
)*(a*x + 1))**(3/2), x)
```

3.758 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=114

$$-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

```
[Out] -3*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/2*a*x^3*(c-c/a^2/x^2)^(1/2)
/(-a^2*x^2+1)^(1/2)-4*x*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6160, 6140, 43}

$$-\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] - (a*Sqrt[c - c/(a^2*x^2)]
*x^3)/(2*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/(a*S
qrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\left(-3-ax+\frac{4}{1-ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{2\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.48

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.42, size = 387, normalized size = 3.39

$$\left[\frac{4\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx-\left(a^5x^5-4a^4x^4+6a^3x^3-4a^2x^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4-2a^3x^3+2ax-1}\right)+\left(a^3x^3+6\right.}{2\left(a^4x^2-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/2*(4*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 + 4*a*c*x - (a^5*x^5 - 4*a^4*x^4 + 6*a^3*x^3 - 4*a^2*x^2)*sqrt
(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 - 2
*a^3*x^3 + 2*a*x - 1)) + (a^3*x^3 + 6*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2
*c*x^2 - c)/(a^2*x^2)))/(a^4*x^2 - a^2), 1/2*(8*(a^2*x^2 - 1)*sqrt(c)*arcta
n((a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^3*c*x^3 - 2*a^2*c*x^2 - a*c*x + 2*c)) + (a^3*x^3 + 6*a^2*x^2)*sq
rt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^2 - a^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}\,x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))*x/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.04, size = 69, normalized size = 0.61

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\sqrt{-a^2x^2+1}\,\left(a^2x^2+6ax+8\ln\left(ax-1\right)\right)}{2\left(a^2x^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(a^2*x^2+6*a*x+8*ln(
a*x-1))/(a^2*x^2-1)/a
```

maxima [C] time = 0.47, size = 149, normalized size = 1.31

$$-\frac{1}{2}a^3\left(-\frac{i\sqrt{c}\,x^2}{a^3}-\frac{i\sqrt{c}\,\log\left(ax+1\right)}{a^5}-\frac{i\sqrt{c}\,\log\left(ax-1\right)}{a^5}\right)-\frac{3}{2}a^2\left(-\frac{2i\sqrt{c}\,x}{a^3}+\frac{i\sqrt{c}\,\log\left(ax+1\right)}{a^4}-\frac{i\sqrt{c}\,\log\left(ax-1\right)}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="
maxima")
```

```
[Out] -1/2*a^3*(-I*sqrt(c)*x^2/a^3 - I*sqrt(c)*log(a*x + 1)/a^5 - I*sqrt(c)*log(a
*x - 1)/a^5) - 3/2*a^2*(-2*I*sqrt(c)*x/a^3 + I*sqrt(c)*log(a*x + 1)/a^4 - I
*sqrt(c)*log(a*x - 1)/a^4) - 3/2*a*(-I*sqrt(c)*log(a*x + 1)/a^3 - I*sqrt(c)
*log(a*x - 1)/a^3) - 1/2*I*sqrt(c)*log(a*x + 1)/a^2 + 1/2*I*sqrt(c)*log(a*x
 - 1)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(
a*x + 1))**(3/2), x)
```

3.759 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=108

$$-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -a*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+x*ln(x)*(c-c/a^2/x^2)^(1/2)/(
-a^2*x^2+1)^(1/2)-4*x*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 72}

$$-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqr
t[1 - a^2*x^2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1+ax)^2}{x(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.44

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,(-ax-4\log(1-ax)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/Sqrt[1 - a^2*x
^2]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
x^2 - 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.04, size = 61, normalized size = 0.56

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-ax+\ln(x)-4\ln(ax-1)\right)\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a*x+ln(x)-4*ln(a*x-1))*(-a^2*x^2+1)^(1/2
)/(a^2*x^2-1)
```

maxima [C] time = 0.47, size = 148, normalized size = 1.37

$$-\frac{1}{2}a^3\left(-\frac{2i\sqrt{c}\,x}{a^3}+\frac{i\sqrt{c}\,\log(ax+1)}{a^4}-\frac{i\sqrt{c}\,\log(ax-1)}{a^4}\right)-\frac{3}{2}a^2\left(-\frac{i\sqrt{c}\,\log(ax+1)}{a^3}-\frac{i\sqrt{c}\,\log(ax-1)}{a^3}\right)-\frac{3}{2}a\left(\frac{i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*a^3*(-2*I*sqrt(c)*x/a^3 + I*sqrt(c)*log(a*x + 1)/a^4 - I*sqrt(c)*log(a
*x - 1)/a^4) - 3/2*a^2*(-I*sqrt(c)*log(a*x + 1)/a^3 - I*sqrt(c)*log(a*x - 1
)/a^3) - 3/2*a*(I*sqrt(c)*log(a*x + 1)/a^2 - I*sqrt(c)*log(a*x - 1)/a^2) +
1/2*I*sqrt(c)*log(a*x + 1)/a + 1/2*I*sqrt(c)*log(a*x - 1)/a - I*sqrt(c)*log
(x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(-(a*x - 1)*(a*
x + 1))**(3/2), x)
```

3.760 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4ax\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+3*a*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a
^2*x^2+1)^(1/2)-4*a*x*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4ax\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - c/(a^2*x^2)]*x*L
og[x])/Sqrt[1 - a^2*x^2] - (4*a*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[
1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
```

```
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^2}{x^2(1-ax)}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^2}+\frac{3a}{x}-\frac{4a^2}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1-ax)}{\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.46

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(3ax\log(x)-4ax\log(1-ax)-1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1 + 3*a*x*Log[x] - 4*a*x*Log[1 - a*x]))/Sqrt[1 - a
^2*x^2]
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^3-2\,ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="
fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
x^3 - 2*a*x^2 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="
giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x), x)
```

maple [A] time = 0.05, size = 63, normalized size = 0.59

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\left(3a\ln(x)x-4\ln\left(ax-1\right)xa-1\right)\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(3*a*ln(x)*x-4*ln(a*x-1)*x*a-1)*(-a^2*x^2+1)
^(1/2)/(a^2*x^2-1)
```

maxima [C] time = 0.52, size = 144, normalized size = 1.35

$$-\frac{1}{2}a^3\left(-\frac{i\sqrt{c}\,\log\left(ax+1\right)}{a^3}-\frac{i\sqrt{c}\,\log\left(ax-1\right)}{a^3}\right)-\frac{3}{2}a^2\left(\frac{i\sqrt{c}\,\log\left(ax+1\right)}{a^2}-\frac{i\sqrt{c}\,\log\left(ax-1\right)}{a^2}\right)-\frac{3}{2}a\left(-\frac{i\sqrt{c}\,\log\left(ax\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="
maxima")
```

```
[Out] -1/2*a^3*(-I*sqrt(c)*log(a*x + 1)/a^3 - I*sqrt(c)*log(a*x - 1)/a^3) - 3/2*a
^2*(I*sqrt(c)*log(a*x + 1)/a^2 - I*sqrt(c)*log(a*x - 1)/a^2) - 3/2*a*(-I*sq
rt(c)*log(a*x + 1)/a - I*sqrt(c)*log(a*x - 1)/a + 2*I*sqrt(c)*log(x)/a) - 1
/2*I*sqrt(c)*log(a*x + 1) + 1/2*I*sqrt(c)*log(a*x - 1) + I*sqrt(c)/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^3}{x\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)^3}{x\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x*(-(a*x - 1)*
(a*x + 1))**(3/2)), x)
```

3.761 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=148

$$-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}+\frac{4a^2x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^2x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -3*a*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/2*(c-c/a^2/x^2)^(1/2)/x/(-a^2
*x^2+1)^(1/2)+4*a^2*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-4*a^2*x*
ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}+\frac{4a^2x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^2x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(2*x
*Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x
^2] - (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^2}{x^3(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^3}+\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}} + \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1-a}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.43

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(1-ax)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/2*1/x^2 - (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1
 - a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.18, size = 492, normalized size = 3.32

$$\left[\frac{4\left(a^3x^3-ax\right)\sqrt{-c}\,\log\left(-\frac{4a^5cx^5-\left(2a^6-4a^5+6a^4-4a^3+a^2\right)cx^6-\left(4a^4+4a^3-6a^2+4a-1\right)cx^4+5a^2cx^2-4acx-\left(4a^4x^4-6a^3x^3-\left(4a^4-6a^3+4a^2\right.\right.}{a^4x^6-2a^3x^5+2ax^3-x^2}\right.}{2\left(a^2x^3-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(4*(a^3*x^3 - a*x)*sqrt(-c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*a^4
 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*c*x
^2 - 4*a*c*x - (4*a^4*x^4 - 6*a^3*x^3 - (4*a^4 - 6*a^3 + 4*a^2 - a)*x^5 + 4
*a^2*x^2 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
 + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2)) - sqrt(-a^2*x^2 + 1)*((6*a + 1
)*x^2 - 6*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^3 - x), 1/2*(8*(
a^3*x^3 - a*x)*sqrt(c)*arctan((2*a^2*x^2 - (2*a^3 - 2*a^2 + a)*x^3 - a*x)*s
qrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2
*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) - sqrt(-a^2*x^2 +
 1)*((6*a + 1)*x^2 - 6*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^3 -
 x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

maple [A] time = 0.05, size = 78, normalized size = 0.53

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{-a^2x^2+1}\,\left(8a^2\ln(x)x^2-8\ln\left(ax-1\right)x^2a^2-6ax-1\right)}{2x\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-a^2*x^2+1)^(1/2)*(8*a^2*ln(x)*x^2-8*
ln(a*x-1)*x^2*a^2-6*a*x-1)/(a^2*x^2-1)
```

maxima [C] time = 0.45, size = 159, normalized size = 1.07

$$-\frac{1}{2}a^3\left(\frac{i\sqrt{c}\log\left(ax+1\right)}{a^2}-\frac{i\sqrt{c}\log\left(ax-1\right)}{a^2}\right)-\frac{3}{2}a^2\left(-\frac{i\sqrt{c}\log\left(ax+1\right)}{a}-\frac{i\sqrt{c}\log\left(ax-1\right)}{a}+\frac{2i\sqrt{c}\log(x)}{a}\right)+\frac{1}{2}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm
="maxima")
```

```
[Out] -1/2*a^3*(I*sqrt(c)*log(a*x + 1)/a^2 - I*sqrt(c)*log(a*x - 1)/a^2) - 3/2*a^
2*(-I*sqrt(c)*log(a*x + 1)/a - I*sqrt(c)*log(a*x - 1)/a + 2*I*sqrt(c)*log(x
)/a) + 1/2*I*a*sqrt(c)*log(a*x + 1) + 1/2*I*a*sqrt(c)*log(a*x - 1) - I*a*sq
rt(c)*log(x) - 3/2*(I*sqrt(c)*log(a*x + 1) - I*sqrt(c)*log(a*x - 1) - 2*I*s
qrt(c)/(a*x))*a + 1/2*I*sqrt(c)/(a*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^3}{x^2\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{- c \left(-1 + \frac{1}{a x}\right) \left(1 + \frac{1}{a x}\right)} \left(a x + 1\right)^{3}}{x^{2} \left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x**2*(-(a*x -
1)*(a*x + 1))**(3/2)), x)
```

3.762 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=187

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{4a^3x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^3x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -4*a^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/3*(c-c/a^2/x^2)^(1/2)/x^2/(
-a^2*x^2+1)^(1/2)-3/2*a*(c-c/a^2/x^2)^(1/2)/x/(-a^2*x^2+1)^(1/2)+4*a^3*x*ln
(x)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-4*a^3*x*ln(-a*x+1)*(c-c/a^2/x^2)
^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{4a^3x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^3x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (-4*a^2*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(3
*x^2*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - c/(a^2*x^2)])/(2*x*Sqrt[1 - a^2*x^2
]) + (4*a^3*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^3*Sqrt
[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^4}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^2}{x^4(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^4}+\frac{3a}{x^3}+\frac{4a^2}{x^2}+\frac{4a^3}{x}-\frac{4a^4}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}} + \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a^3\sqrt{c}}{}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.40

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(4a^3\log(x)-4a^3\log(1-ax)-\frac{4a^2}{x}-\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/3*1/x^3 - (3*a)/(2*x^2) - (4*a^2)/x + 4*a^3*Lo
g[x] - 4*a^3*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 3.92, size = 530, normalized size = 2.83

$$\left[\frac{12\left(a^4x^4-a^2x^2\right)\sqrt{-c}\,\log\left(-\frac{4a^5cx^5-\left(2a^6-4a^5+6a^4-4a^3+a^2\right)cx^6-\left(4a^4+4a^3-6a^2+4a-1\right)cx^4+5a^2cx^2-4acx-\left(4a^4x^4-6a^3x^3-\left(4a^4-6a^3+\right.\right.}{a^4x^6-2a^3x^5+2ax^3-x^2}\right.}{6\left(a^2x^4-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm
="fricas")
```

```
[Out] [1/6*(12*(a^4*x^4 - a^2*x^2)*sqrt(-c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 +
6*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^
2*c*x^2 - 4*a*c*x - (4*a^4*x^4 - 6*a^3*x^3 - (4*a^4 - 6*a^3 + 4*a^2 - a)*x^
5 + 4*a^2*x^2 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*
x^2)) + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2)) + (24*a^2*x^2 - (24*a^2 +
 9*a + 2)*x^3 + 9*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^2*x^4 - x^2), 1/6*(24*(a^4*x^4 - a^2*x^2)*sqrt(c)*arctan((2*a^2*x^2
- (2*a^3 - 2*a^2 + a)*x^3 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2
 - 2*a*c*x + c)) + (24*a^2*x^2 - (24*a^2 + 9*a + 2)*x^3 + 9*a*x + 2)*sqrt(-
a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^4 - x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

maple [A] time = 0.05, size = 86, normalized size = 0.46

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{-a^2x^2+1}\,\left(24a^3\ln(x)x^3-24\ln(ax-1)\,x^3a^3-24a^2x^2-9ax-2\right)}{6x^2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

```
[Out] -1/6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*(-a^2*x^2+1)^(1/2)*(24*a^3*ln(x)*x^3
-24*ln(a*x-1)*x^3*a^3-24*a^2*x^2-9*a*x-2)/(a^2*x^2-1)
```

maxima [C] time = 0.46, size = 184, normalized size = 0.98

$$-\frac{1}{2}a^3\left(-\frac{i\sqrt{c}\,\log(ax+1)}{a}-\frac{i\sqrt{c}\,\log(ax-1)}{a}+\frac{2i\sqrt{c}\,\log(x)}{a}\right)-\frac{1}{2}i\,a^2\sqrt{c}\,\log(ax+1)+\frac{1}{2}i\,a^2\sqrt{c}\,\log(ax-1)-\frac{3}{2}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm
="maxima")
```

```
[Out] -1/2*a^3*(-I*sqrt(c)*log(a*x + 1)/a - I*sqrt(c)*log(a*x - 1)/a + 2*I*sqrt(c
)*log(x)/a) - 1/2*I*a^2*sqrt(c)*log(a*x + 1) + 1/2*I*a^2*sqrt(c)*log(a*x -
1) - 3/2*(I*sqrt(c)*log(a*x + 1) - I*sqrt(c)*log(a*x - 1) - 2*I*sqrt(c)/(a*
x))*a^2 - 3/2*(-I*a*sqrt(c)*log(a*x + 1) - I*a*sqrt(c)*log(a*x - 1) + 2*I*a
*sqrt(c)*log(x) - I*sqrt(c)/(a*x^2))*a + 1/3*(3*I*a^2*sqrt(c)*x^2 + I*sqrt(
c))/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^3}{x^3\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{- c \left(-1 + \frac{1}{a x}\right) \left(1 + \frac{1}{a x}\right)} \left(a x + 1\right)^{3}}{x^{3} \left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x**3*(-(a*x -
1)*(a*x + 1))**(3/2)), x)
```

3.763 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=222

$$-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{4a^4x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-\cdots}}{\sqrt{1-a^2x\cdots}}$$

```
[Out] -4*a^3*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/4*(c-c/a^2/x^2)^(1/2)/x^3/(
-a^2*x^2+1)^(1/2)-a*(c-c/a^2/x^2)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)-2*a^2*(c-c/a
^2/x^2)^(1/2)/x/(-a^2*x^2+1)^(1/2)+4*a^4*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a^2*
x^2+1)^(1/2)-4*a^4*x*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{4a^4x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-\cdots}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(4
*x^3*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)])/(x^2*Sqrt[1 - a^2*x^2])
 - (2*a^2*Sqrt[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - c/
(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - c/(a^2*x^2)]*x*Log
[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^5}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^2}{x^5(1-ax)}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^5}+\frac{3a}{x^4}+\frac{4a^2}{x^3}+\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-a^2x}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.36

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(4a^4\log(x)-4a^4\log(1-ax)-\frac{4a^3}{x}-\frac{2a^2}{x^2}-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/4*1/x^4 - a/x^3 - (2*a^2)/x^2 - (4*a^3)/x + 4*
a^4*Log[x] - 4*a^4*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.97, size = 556, normalized size = 2.50

$$\left[\frac{8\left(a^5x^5-a^3x^3\right)\sqrt{-c}\,\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx-\left(4\,a^4x^4-6\,a^3x^3-\left(4\,a^4-6\,a^3+4\right.\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{4\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm
="fricas")
```

```
[Out] [1/4*(8*(a^5*x^5 - a^3*x^3)*sqrt(-c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6
*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2
*c*x^2 - 4*a*c*x - (4*a^4*x^4 - 6*a^3*x^3 - (4*a^4 - 6*a^3 + 4*a^2 - a)*x^5
 + 4*a^2*x^2 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2)) + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2)) + (16*a^3*x^3 - (16*a^3 +
8*a^2 + 4*a + 1)*x^4 + 8*a^2*x^2 + 4*a*x + 1)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)))/(a^2*x^5 - x^3), 1/4*(16*(a^5*x^5 - a^3*x^3)*sqrt(c)
*arctan((2*a^2*x^2 - (2*a^3 - 2*a^2 + a)*x^3 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt
(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a
^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) + (16*a^3*x^3 - (16*a^3 + 8*a^2 + 4*a +
 1)*x^4 + 8*a^2*x^2 + 4*a*x + 1)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a
^2*x^2)))/(a^2*x^5 - x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x^4), x)
```

maple [A] time = 0.05, size = 94, normalized size = 0.42

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{-a^2x^2+1}\,\left(16a^4\ln(x)x^4-16\ln\left(ax-1\right)x^4a^4-16x^3a^3-8a^2x^2-4ax-1\right)}{4x^3\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

```
[Out] -1/4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*(-a^2*x^2+1)^(1/2)*(16*a^4*ln(x)*x^4
-16*ln(a*x-1)*x^4*a^4-16*x^3*a^3-8*a^2*x^2-4*a*x-1)/(a^2*x^2-1)
```

maxima [C] time = 0.46, size = 209, normalized size = 0.94

$$\frac{1}{2}i\,a^3\sqrt{c}\,\log\left(ax+1\right)+\frac{1}{2}i\,a^3\sqrt{c}\,\log\left(ax-1\right)-i\,a^3\sqrt{c}\,\log(x)-\frac{1}{2}\left(i\sqrt{c}\,\log\left(ax+1\right)-i\sqrt{c}\,\log\left(ax-1\right)-\frac{2i\sqrt{c}}{ax}\right)a^3-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm
="maxima")
```

```
[Out] 1/2*I*a^3*sqrt(c)*log(a*x + 1) + 1/2*I*a^3*sqrt(c)*log(a*x - 1) - I*a^3*sqr
t(c)*log(x) - 1/2*(I*sqrt(c)*log(a*x + 1) - I*sqrt(c)*log(a*x - 1) - 2*I*sq
rt(c)/(a*x))*a^3 - 3/2*(-I*a*sqrt(c)*log(a*x + 1) - I*a*sqrt(c)*log(a*x - 1
) + 2*I*a*sqrt(c)*log(x) - I*sqrt(c)/(a*x^2))*a^2 - 1/2*(3*I*a^2*sqrt(c)*lo
g(a*x + 1) - 3*I*a^2*sqrt(c)*log(a*x - 1) - 2*(3*I*a^2*sqrt(c)*x^2 + I*sqrt
(c))/(a*x^3))*a + 1/4*(2*I*a^2*sqrt(c)*x^2 + I*sqrt(c))/(a*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^3}{x^4\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x^4*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x^4*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\left(ax+1\right)^{3}}{x^{4}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x**4*(-(a*x -
1)*(a*x + 1))**(3/2)), x)
```

3.764 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=263

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{4a^5x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^5x\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] -4*a^4*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/5*(c-c/a^2/x^2)^(1/2)/x^4/(
-a^2*x^2+1)^(1/2)-3/4*a*(c-c/a^2/x^2)^(1/2)/x^3/(-a^2*x^2+1)^(1/2)-4/3*a^2*
(c-c/a^2/x^2)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)-2*a^3*(c-c/a^2/x^2)^(1/2)/x/(-a^
2*x^2+1)^(1/2)+4*a^5*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-4*a^5*x
*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}+\frac{4a^5x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^5x\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (-4*a^4*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(5
*x^4*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - c/(a^2*x^2)])/(4*x^3*Sqrt[1 - a^2*x
^2]) - (4*a^2*Sqrt[c - c/(a^2*x^2)])/(3*x^2*Sqrt[1 - a^2*x^2]) - (2*a^3*Sqr
t[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) + (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] - (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*Log[1 - a*x])/Sq
rt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^6}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^2}{x^6(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^6}+\frac{3a}{x^5}+\frac{4a^2}{x^4}+\frac{4a^3}{x^3}+\frac{4a^4}{x^2}+\frac{4a^5}{x}-\frac{4a^6}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}} - \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}} - \frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.35

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(4a^5\log(x)-4a^5\log(1-ax)-\frac{4a^4}{x}-\frac{2a^3}{x^2}-\frac{4a^2}{3x^3}-\frac{3a}{4x^4}-\frac{1}{5x^5}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/5*1/x^5 - (3*a)/(4*x^4) - (4*a^2)/(3*x^3) - (2
*a^3)/x^2 - (4*a^4)/x + 4*a^5*Log[x] - 4*a^5*Log[1 - a*x]))/Sqrt[1 - a^2*x^
2]
```

fricas [A] time = 0.94, size = 582, normalized size = 2.21

$$\left[\frac{120\left(a^6x^6-a^4x^4\right)\sqrt{-c}\,\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx-\left(4\,a^4x^4-6\,a^3x^3-\left(4\,a^4-6\right.\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm
="fricas")
```

```
[Out] [1/60*(120*(a^6*x^6 - a^4*x^4)*sqrt(-c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5
+ 6*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*
a^2*c*x^2 - 4*a*c*x - (4*a^4*x^4 - 6*a^3*x^3 - (4*a^4 - 6*a^3 + 4*a^2 - a)*
x^5 + 4*a^2*x^2 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2)) + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2)) + (240*a^4*x^4 + 120*a^
3*x^3 - (240*a^4 + 120*a^3 + 80*a^2 + 45*a + 12)*x^5 + 80*a^2*x^2 + 45*a*x
+ 12)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^6 - x^4),
1/60*(240*(a^6*x^6 - a^4*x^4)*sqrt(c)*arctan((2*a^2*x^2 - (2*a^3 - 2*a^2 +
a)*x^3 - a*x)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2
*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) +
(240*a^4*x^4 + 120*a^3*x^3 - (240*a^4 + 120*a^3 + 80*a^2 + 45*a + 12)*x^5 +
 80*a^2*x^2 + 45*a*x + 12)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^2*x^6 - x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm
="giac")
```

```
[Out] integrate((a*x + 1)^3*sqrt(c - c/(a^2*x^2))/((-a^2*x^2 + 1)^(3/2)*x^5), x)
```

maple [A] time = 0.05, size = 102, normalized size = 0.39

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{-a^2x^2+1}\,\left(240a^5\ln(x)x^5-240\ln(ax-1)\,x^5a^5-240x^4a^4-120x^3a^3-80a^2x^2-45ax-12\right)}{60x^4\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x)
```

```
[Out] -1/60*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*(-a^2*x^2+1)^(1/2)*(240*a^5*ln(x)*x
^5-240*ln(a*x-1)*x^5*a^5-240*x^4*a^4-120*x^3*a^3-80*a^2*x^2-45*a*x-12)/(a^2
*x^2-1)
```

maxima [C] time = 0.46, size = 240, normalized size = 0.91

$$-\frac{1}{2}i\,a^4\sqrt{c}\,\log\left(ax+1\right)+\frac{1}{2}i\,a^4\sqrt{c}\,\log\left(ax-1\right)-\frac{1}{2}\left(-i\,a\sqrt{c}\,\log\left(ax+1\right)-i\,a\sqrt{c}\,\log\left(ax-1\right)+2i\,a\sqrt{c}\,\log(x)-\frac{i\sqrt{c}}{ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm
="maxima")
```

```
[Out] -1/2*I*a^4*sqrt(c)*log(a*x + 1) + 1/2*I*a^4*sqrt(c)*log(a*x - 1) - 1/2*(-I*
a*sqrt(c)*log(a*x + 1) - I*a*sqrt(c)*log(a*x - 1) + 2*I*a*sqrt(c)*log(x) -
I*sqrt(c)/(a*x^2))*a^3 - 1/2*(3*I*a^2*sqrt(c)*log(a*x + 1) - 3*I*a^2*sqrt(c
)*log(a*x - 1) - 2*(3*I*a^2*sqrt(c)*x^2 + I*sqrt(c))/(a*x^3))*a^2 - 3/4*(-2
*I*a^3*sqrt(c)*log(a*x + 1) - 2*I*a^3*sqrt(c)*log(a*x - 1) + 4*I*a^3*sqrt(c
)*log(x) - (2*I*a^2*sqrt(c)*x^2 + I*sqrt(c))/(a*x^4))*a + 1/15*I*(15*a^4*sq
rt(c)*x^4 + 5*a^2*sqrt(c)*x^2 + 3*sqrt(c))/(a*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(a\,x+1\right)}^3}{x^5\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x^5*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1)^3)/(x^5*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\left(ax+1\right)^{3}}{x^{5}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)**3/(x**5*(-(a*x -
1)*(a*x + 1))**(3/2)), x)
```

3.765 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx$

Optimal. Leaf size=81

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{m\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-a^2x^2}}$$

```
[Out] x^(1+m)*(c-c/a^2/x^2)^(1/2)/m/(-a^2*x^2+1)^(1/2)-a*x^(2+m)*(c-c/a^2/x^2)^(1
/2)/(1+m)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 43}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{m\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^(1 + m))/(m*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^
2*x^2)]*x^(2 + m))/((1 + m)*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-\tanh^{-1}(ax)}x^{-1+m}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int x^{-1+m}(1-ax)\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(x^{-1+m}-ax^m\right)\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^{1+m}}{m\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^{2+m}}{(1+m)\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.64

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{x^m}{m}-\frac{ax^{m+1}}{m+1}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcTanh[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(x^m/m - (a*x^(1 + m))/(1 + m)))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.83, size = 78, normalized size = 0.96

$$\frac{\sqrt{-a^2x^2+1}\left(amx^2-(m+1)x\right)x^m\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{\left(a^2m^2+a^2m\right)x^2-m^2-m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
fricas")
[Out] sqrt(-a^2*x^2 + 1)*(a*m*x^2 - (m + 1)*x)*x^m*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
)/((a^2*m^2 + a^2*m)*x^2 - m^2 - m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{a^2x^2}}\,x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))*x^m/(a*x + 1), x)
```

maple [A] time = 0.03, size = 69, normalized size = 0.85

$$\frac{x^{1+m}\left(axm-m-1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{-a^2x^2+1}}{\left(1+m\right)m\left(ax-1\right)\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x-m-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)/(1+m)/
m/(a*x-1)/(a*x+1)
```

maxima [C] time = 0.39, size = 55, normalized size = 0.68

$$\frac{\left(i\,a\sqrt{c}\,mx+\sqrt{c}\left(-i\,m-i\right)\right)(ax+1)(ax-1)x^m}{\left(m^2+m\right)a^3x^2-\left(m^2+m\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] (I*a*sqrt(c)*m*x + sqrt(c)*(-I*m - I))*(a*x + 1)*(a*x - 1)*x^m/((m^2 + m)*a
^3*x^2 - (m^2 + m)*a)
```

mupad [B] time = 1.15, size = 78, normalized size = 0.96

$$-\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{x^m\,x^2\,\sqrt{1-a^2\,x^2}}{a\,(m+1)}-\frac{x\,x^m\,\sqrt{1-a^2\,x^2}}{a^2\,m}\right)}{\frac{1}{a^2}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] -((c - c/(a^2*x^2))^(1/2)*((x^m*x^2*(1 - a^2*x^2)^(1/2))/(a*(m + 1)) - (x*x
^m*(1 - a^2*x^2)^(1/2))/(a^2*m)))/(1/a^2 - x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x
)))/(a*x + 1), x)
```

3.766 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=74

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}$$

```
[Out] 1/2*x^3*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/3*a*x^4*(c-c/a^2/x^2)^(1/2
)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 43}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)
]*x^4)/(3*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int x(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\left(x-ax^2\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{2\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.57

$$-\frac{x^3(2ax-3)\sqrt{c-\frac{c}{a^2x^2}}}{6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] -1/6*(Sqrt[c - c/(a^2*x^2)]*x^3*(-3 + 2*a*x))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 2.44, size = 58, normalized size = 0.78

$$\frac{\left(2\,ax^4-3\,x^3\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*a*x^4 - 3*x^3)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a
^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1), x)
```

maple [A] time = 0.03, size = 57, normalized size = 0.77

$$\frac{x^{3}\left(2ax-3\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{-a^{2}x^{2}+1}}{6\left(ax-1\right)\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/6*x^3*(2*a*x-3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/
(a*x+1)
```

maxima [C] time = 0.38, size = 43, normalized size = 0.58

$$\frac{\left(2i\,a\sqrt{c}\,x^{3}-3i\,\sqrt{c}\,x^{2}\right)(ax+1)(ax-1)}{6\left(a^{3}x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/6*(2*I*a*sqrt(c)*x^3 - 3*I*sqrt(c)*x^2)*(a*x + 1)*(a*x - 1)/(a^3*x^2 - a)
```

mupad [B] time = 0.97, size = 67, normalized size = 0.91

$$-\frac{\sqrt{c-\frac{c}{a^{2}\,x^{2}}}\,\left(\frac{x^{4}\,\sqrt{1-a^{2}\,x^{2}}}{3\,a}-\frac{x^{3}\,\sqrt{1-a^{2}\,x^{2}}}{2\,a^{2}}\right)}{\frac{1}{a^{2}}-x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] -((c - c/(a^2*x^2))^(1/2)*((x^4*(1 - a^2*x^2)^(1/2))/(3*a) - (x^3*(1 - a^2*
x^2)^(1/2))/(2*a^2)))/(1/a^2 - x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x
)))/(a*x + 1), x)
```

3.767 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=71

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

[Out] x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/2*a*x^3*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6160, 6140}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcTanh[a*x],x]

[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] - (a*Sqrt[c - c/(a^2*x^2)]*x^3)/(2*Sqrt[1 - a^2*x^2])

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.58

$$\frac{x\left(x-\frac{ax^2}{2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(x - (a*x^2)/2))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.16, size = 57, normalized size = 0.80

$$\frac{\sqrt{-a^2x^2+1}\left(ax^3-2\,x^2\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(a*x^3 - 2*x^2)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{a^2x^2}}\,x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1), x)
```

maple [A] time = 0.03, size = 56, normalized size = 0.79

$$\frac{x^{2}\left(a x-2\right)\sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\,\sqrt{-a^{2} x^{2}+1}}{2\left(a x-1\right)\left(a x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*x^2*(a*x-2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(a
*x+1)
```

maxima [C] time = 0.39, size = 51, normalized size = 0.72

$$\frac{\left(i\,a^{2}\sqrt{c}\,x^{2}-2 i\,a\sqrt{c}\,x+2 i\,\sqrt{c}\right)(a x+1)(a x-1)}{2\left(a^{4} x^{2}-a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 1/2*(I*a^2*sqrt(c)*x^2 - 2*I*a*sqrt(c)*x + 2*I*sqrt(c))*(a*x + 1)*(a*x - 1)
/(a^4*x^2 - a^2)
```

mupad [B] time = 0.93, size = 67, normalized size = 0.94

$$-\frac{\sqrt{c-\frac{c}{a^{2}\,x^{2}}}\,\left(\frac{x^{3}\,\sqrt{1-a^{2}\,x^{2}}}{2\,a}-\frac{x^{2}\,\sqrt{1-a^{2}\,x^{2}}}{a^{2}}\right)}{\frac{1}{a^{2}}-x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] -((c - c/(a^2*x^2))^(1/2)*((x^3*(1 - a^2*x^2)^(1/2))/(2*a) - (x^2*(1 - a^2*
x^2)^(1/2))/a^2))/(1/a^2 - x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\left(a x-1\right)\left(a x+1\right)}\,\sqrt{-c\left(-1+\frac{1}{a x}\right)\left(1+\frac{1}{a x}\right)}}{a x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))
/(a*x + 1), x)
```

3.768 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] -a*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+x*ln(x)*(c-c/a^2/x^2)^(1/2)/(
-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 43}

$$\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcTanh[a*x],x]
```

```
[Out] -((a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2]) + (Sqrt[c - c/(a^2*x^2)]
*x*Log[x])/Sqrt[1 - a^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1-ax}{x}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(-a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.55

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,(\log(x)-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-(a*x) + Log[x]))/Sqrt[1 - a^2*x^2]
```

fricas [B] time = 0.85, size = 321, normalized size = 4.65

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)+2\left(a^2x^2-a^2x\right)\sqrt{-a^2x^2+1}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^3x^2-a\right)},\ -\frac{\left(a^2x \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
```

```
x^4 - x^2)) + 2*(a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^3*x^2 - a), -((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)
*(a*x^3 + a*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 +
1)*c*x^2 + c)) - (a^2*x^2 - a^2*x)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^3*x^2 - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

maple [A] time = 0.04, size = 53, normalized size = 0.77

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-ax+\ln(x)\right)\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a*x+ln(x))*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(
a*x + 1), x)
```

3.769 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-a*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a^2
*x^2+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 43}

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - c/(a^2*x^2)]*x*Log
[x])/Sqrt[1 - a^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1-ax}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^2}-\frac{a}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.57

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax\log(x)+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcTanh[a*x]*x),x]
[Out] -((Sqrt[c - c/(a^2*x^2)]*(1 + a*x*Log[x]))/Sqrt[1 - a^2*x^2])
```

fricas [B] time = 1.11, size = 295, normalized size = 4.40

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\left(ax^5-ax\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2x^2+1}\,(x-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^2-1\right)}, \frac{\left(a^2x^2-1\right)\sqrt{\ }}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(-c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - (a*x^5 -
a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c)/(a^2*
x^4 - x^2)) - 2*sqrt(-a^2*x^2 + 1)*(x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))
/(a^2*x^2 - 1), ((a^2*x^2 - 1)*sqrt(c)*arctan(sqrt(-a^2*x^2 + 1)*(a*x^3 + a
*x)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^4 - (a^2 + 1)*c*x^2 +
c)) - sqrt(-a^2*x^2 + 1)*(x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^2
- 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(x)]sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [A] time = 0.05, size = 51, normalized size = 0.76

$$\frac{\sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{-a^{2} x^{2}+1}\, \left(a \ln \left(x \right) x +1\right)}{a^{2} x^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)*(a*ln(x)*x+1)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{c-\frac{c}{a^{2} x^{2}}}}{(a x+1) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{1-a^2\,x^2}}{x\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x\left(ax+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(
x*(a*x + 1)), x)
```

3.770 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=44

$$-\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

```
[Out] -1/2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/x/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 37}

$$-\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*x*Sqrt[1 - a^2*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1-ax}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{2x\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$\frac{x\left(\frac{a}{x}-\frac{1}{2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 + a/x)*x)/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.05, size = 63, normalized size = 1.43

$$\frac{\sqrt{-a^2x^2+1}\left((2\,a-1)x^2-2\,ax+1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*((2*a - 1)*x^2 - 2*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^2*x^3 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

maple [A] time = 0.03, size = 57, normalized size = 1.30

$$-\frac{(2ax-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{-a^2x^2+1}}{2\,(ax-1)\,(ax+1)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -1/2*(2*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(a*
x+1)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

mupad [B] time = 0.96, size = 64, normalized size = 1.45

$$-\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{\sqrt{1-a^2\,x^2}}{2\,a^2}-\frac{x\,\sqrt{1-a^2\,x^2}}{a}\right)}{\frac{x}{a^2}-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(1/2))/(x^2*(a*x + 1)),x)
```

```
[Out] -((c - c/(a^2*x^2))^(1/2)*((1 - a^2*x^2)^(1/2)/(2*a^2) - (x*(1 - a^2*x^2)^(
1/2))/a))/(x/a^2 - x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x^{2}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(
x**2*(a*x + 1)), x)
```

3.771 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=163

$$-\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}+\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{8a^3\sqrt{ax+1}\sqrt{1-ax}}$$

```
[Out] 7/8*x*(c-c/a^2/x^2)^(1/2)/a^3+7/24*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3+1/6*x
*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3-1/4*x^2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/
a^2+7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 90, 80, 50, 41, 216}

$$-\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}+\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{8a^3\sqrt{ax+1}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) + (7*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))
/(24*a^3) + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(6*a^3) - (Sqrt[c - c/(a^
2*x^2)]*x^2*(1 - a*x)^2)/(4*a^2) + (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/
(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{x^2(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$=-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1-ax)^{3/2}(-1+2ax)}{\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}+\frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{12a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$=\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}+\frac{\left(7\sqrt{c-\right.}{8a^2}$$

$$=\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1}{4a^2}$$

$$=\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1}{4a^2}$$

$$=\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1}{4a^2}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 0.57

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(21\log\left(\sqrt{a^2x^2-1}+ax\right)+\sqrt{a^2x^2-1}\left(6a^3x^3-16a^2x^2+21ax-32\right)\right)}{24a^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/24*(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(-32 + 21*a*x - 16*a^2*x
^2 + 6*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(a^3*Sqrt[-1 + a^2*x^2
])
```

fricas [A] time = 3.20, size = 222, normalized size = 1.36

$$\left[-\frac{2\left(6\,a^4x^4-16\,a^3x^3+21\,a^2x^2-32\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-21\,\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},-\frac{\left(6\,a^4x^4-16\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] [-1/48*(2*(6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 - 32*a*x)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)) - 21*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*
x^2 - c)/(a^2*x^2)) - c))/a^4, -1/24*((6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2
- 32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)
*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

giac [A] time = 0.27, size = 128, normalized size = 0.79

$$-\frac{1}{48}\left(2\,\sqrt{a^2cx^2-c}\left(\left(2\,x\left(\frac{3\,x\mathrm{sgn}(x)}{a^2}-\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x-\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\,\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2\right.}\right.}{a^4|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] -1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 - 8*sgn(x)/a^3) + 21*sgn
(x)/a^4)*x - 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*
c*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) + 64*sqrt(-c)*
abs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

maple [A] time = 0.04, size = 196, normalized size = 1.20

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-6x\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4+16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-27\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x\,a^2c+27c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-48c\right.}{24\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,c\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4+16
*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2*c+27*c^(3
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln((c^(1/2)*((a*x-1)
*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))+48*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(
c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x^3*(c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\right)dx-\int \frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**3*sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x**4*sqr
t(c - c/(a**2*x**2))/(a*x + 1), x)
```

3.772 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=126

$$-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{a^2\sqrt{ax+1}\,\sqrt{1-ax}}$$

```
[Out] -x*(c-c/a^2/x^2)^(1/2)/a^2-1/3*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2-1/3*x*(-a
*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^2-x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*x
+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6159, 6129, 80, 50, 41, 216}

$$-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{a^2\sqrt{ax+1}\,\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/a^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(3*a
^2) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(3*a^2) - (Sqrt[c - c/(a^2*x^2)
]*x*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}x\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} - \frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{3a\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{a\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{a\sqrt{1-ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\sin}{a^2\sqrt{1-ax}\,\sqrt{1}}$$

Mathematica [A] time = 0.07, size = 84, normalized size = 0.67

$$-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\,\left(a^2x^2-3ax+5\right)-3\log\left(\sqrt{a^2x^2-1}\,+ax\right)\right)}{3a^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/3*(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 - 3*a*x + a^2*x^2) - 3
*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(a^2*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 2.05, size = 204, normalized size = 1.62

$$\left[-\frac{2\left(a^3x^3-3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\,-3\,\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\,-c\right)}{6\,a^3},-\frac{\left(a^3x^3-3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2c}{a}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] [-1/6*(2*(a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*
sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
 - c))/a^3, -1/3*((a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2)) + 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(
a^2*c*x^2 - c)))/a^3]
```

giac [A] time = 0.24, size = 117, normalized size = 0.93

$$-\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\operatorname{sgn}(x)}{a^2}-\frac{3\operatorname{sgn}(x)}{a^3}\right)+\frac{5\operatorname{sgn}(x)}{a^4}\right)+\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^3|a|}-\frac{\left(3a\sqrt{c}\ \mathrm{lo}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] -1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 - 3*sgn(x)/a^3) + 5*sgn(x)/a^4
) + 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*ab
s(a)) - (3*a*sqrt(c)*log(abs(c)) + 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))
*abs(a)
```

maple [A] time = 0.04, size = 173, normalized size = 1.37

$$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,x\,a^2c+3c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)+6\cdots\right.}{3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((c*(a^2*x^2-1)/a^2)^(3/2)*a^3-3*(c*(a
^2*x^2-1)/a^2)^(1/2)*x*a^2*c+3*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/
2))-6*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))+6*((a
*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/a^3/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}{\left(a\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x^2*(c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\right)dx-\int \frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**2*sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x**3*sqr
t(c - c/(a**2*x**2))/(a*x + 1), x)
```

3.773 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=99

$$\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] 3/2*x*(c-c/a^2/x^2)^(1/2)/a+1/2*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a+3/2*x*arcs
in(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6159, 6129, 50, 41, 216}

$$\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(2*
a) + (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a*
x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a}+\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 100, normalized size = 1.01

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{ax+1}\left(a^2x^2-5ax+4\right)-6\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.75, size = 188, normalized size = 1.90

$$\left[-\frac{2\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},-\frac{\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}\,\text{arcta}}{2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2,
-1/2*((a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

giac [A] time = 0.18, size = 106, normalized size = 1.07

$$-\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\text{sgn}(x)}{a^2}-\frac{4\,\text{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\text{sgn}(x)}{a^2|a|}+\frac{\left(3\,a\sqrt{c}\,\log\left(|c|\right)+8\,\sqrt{-c}\right.}{a^3|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 - 4*sgn(x)/a^3) - 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c)*log(abs(c)) + 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

maple [A] time = 0.04, size = 147, normalized size = 1.48

$$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\, x \left(-x\sqrt{\frac{c(a^2x^2-1)}{a^2}}\, a^2+\sqrt{c}\, \ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-4\sqrt{c}\, \ln\left(\frac{\sqrt{c}\, \sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)+4\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\, a\right)}{2\sqrt{\frac{c(a^2x^2-1)}{a^2}}\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2+c^(1/
2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-4*c^(1/2)*ln((c^(1/2)*((a*x-1)*(
a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))+4*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a)/(c*(a^
2*x^2-1)/a^2)^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(a^2x^2-1)\sqrt{c-\frac{c}{a^2x^2}}\, x}{(ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}{(a\,x+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x*(c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\right) dx-\int \frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x*sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x**2*sqrt(c
 - c/(a**2*x**2))/(a*x + 1), x)
```

3.774 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=118

$$-x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] -x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(
a*x+1)^(1/2)-x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-
a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6159, 6129, 102, 157, 41, 216, 92, 208}

$$-x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[
1 - a*x]*Sqrt[1 + a*x]) - (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1-}}}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2a\right.}{}$$

$$= -\sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right.}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)+\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-Sqrt[-1 + a^2*x^2] + ArcTan[1/Sqrt[-1 + a^2*x^2]
] + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

fricas [A] time = 0.74, size = 270, normalized size = 2.29

$$\left[-\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\sqrt{-c}\,\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, -\frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-
c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(-c)*log(-(a^
2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, -(
a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*
c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sq
rt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 0.04, size = 198, normalized size = 1.68

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}-2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-c\ln\left(\frac{2\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-2c}{a^2x}\right)\right)}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)-2*c^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))
*a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\right)dx-\int\frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x + 1), x) - Integral(a*x*sqrt(c - c/
(a**2*x**2))/(a*x + 1), x)
```

3.775 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=118

$$-\sqrt{c-\frac{c}{a^2x^2}}+\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)+a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*
x+1)^(1/2)+2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/
(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6159, 6129, 98, 157, 41, 216, 92, 208}

$$-\sqrt{c-\frac{c}{a^2x^2}}+\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
a*x]*Sqrt[1 + a*x]) + (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sq
rt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{2a-a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+a}}}{\sqrt{1-ax}\,\sqrt{1+ax}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}} + \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\right.}{\sqrt{1-ax}\,\sqrt{1+}}$$
$$= -\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right.}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.70

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+ax\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2
*x^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 1.57, size = 255, normalized size = 2.16

$$\left[\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-\sqrt{\frac{a^2cx^2-c}{a^2x^2}},-2\sqrt{c}\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="fricas
")
```

```
[Out] [sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^
2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a
^2*x^2)) - 2*c)/x^2) - sqrt((a^2*c*x^2 - c)/(a^2*x^2)), -2*sqrt(c)*arctan(a
*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*l
og(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) - s
qrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

giac [A] time = 0.34, size = 126, normalized size = 1.07

$$\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}+\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}-\frac{2c^{\frac{3}{2}}\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac")
```

```
[Out] (4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
+ sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) - 2*
c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs(
a)
```

maple [B] time = 0.04, size = 307, normalized size = 2.60

$$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,x^2a^3c-a^3\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+2\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(\frac{\sqrt{c}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)xa-\sqrt{-\frac{c}{a^2}}\,c\right.}{a\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
x^2*a^3*c-a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)+2*(-c/a^2)^(1/2)*c^(
3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x*a-(-c/a^2)^(
1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a-2*(-c/a^2)^(1/2)*(
(a*x-1)*(a*x+1)*c/a^2)^(1/2)*x*a^2*c+2*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)*c*x+2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)
*x*c^2)/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxima
")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}{x\,{\left(a\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x*(a*x + 1)^2),x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^2+x}\right)\, dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^2+x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x,x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**2 + x), x) - Integral(a*x*sqrt(c -
 c/(a**2*x**2))/(a*x**2 + x), x)
```

3.776 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=112

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{2\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] 3/2*a*(c-c/a^2/x^2)^(1/2)-1/2*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x-3/2*a^2*x*arct
anh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1
)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6159, 6129, 94, 92, 208}

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}-\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}-\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{2\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/2 - (Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(2*x) - (
3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqrt
[1 - a*x]*Sqrt[1 + a*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} - \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} + \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} - \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-\right.}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.70

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((4ax-1)\sqrt{a^2x^2-1}+3a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((-1 + 4*a*x)*Sqrt[-1 + a^2*x^2] + 3*a^2*x^2*ArcTan[
1/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.51, size = 176, normalized size = 1.57

$$\left[\frac{3\,a\sqrt{-c}\,x\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},\ \frac{3\,a\sqrt{c}\,x\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) + 2*(4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*
c*x^2 - c)) + (4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

giac [B] time = 0.38, size = 195, normalized size = 1.74

$$-\left(3\,\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3ac\,\mathrm{sgn}(x)+4\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2 \cdots}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2 \cdots}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="giac
")
```

```
[Out] -(3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
 ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) + 4*(sqrt(a^2*c)*x - s
```

```
qrt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - c))*a*c^2*sgn(x) + 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

maple [B] time = 0.05, size = 348, normalized size = 3.11

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^3a^3c+4\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x\,a^3+4\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)x^2a-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)
^(1/2)*x^3*a^3*c+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^3+4*(-c/a^2
)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^2*a-4*(-c/a^2)^(1
/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a+4
*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^2*a^2*c-3*(-c/a^2)^(1/2)*(c
*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2*c-a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1
/2)-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)*x^2*c^2)
/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}{x^2\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x^2*(a*x + 1)^2),x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x^2*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^3+x^2}\right)dx-\int\frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**3 + x**2), x) - Integral(a*x*sqrt(
c - c/(a**2*x**2))/(a*x**3 + x**2), x)
```

3.777 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=140

$$-a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}-\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -a^2*(c-c/a^2/x^2)^(1/2)+1/3*a*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x-1/3*(-a*x+1)^
2*(c-c/a^2/x^2)^(1/2)/x^2+a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/
a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6159, 6129, 96, 94, 92, 208}

$$-a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}-\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -(a^2*Sqrt[c - c/(a^2*x^2)]) + (a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(3*x) -
(Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(3*x^2) + (a^3*Sqrt[c - c/(a^2*x^2)]*x*
ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^4\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{3\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} + \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} - \frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \cdots}{\sqrt{1-ax}\,\sqrt{\cdots}}$$

$$= -a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} + \frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\mathrm{Su}\cdots}{\cdots}$$

$$= -a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} + \frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh\cdots}{\sqrt{1-a\cdots}}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.61

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(5a^2x^2-3ax+1\right)+3a^3x^3\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{3x^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -1/3*(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 - 3*a*x + 5*a^2*x^2) + 3
*a^3*x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(x^2*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.72, size = 200, normalized size = 1.43

$$\left[\frac{3\,a^2\sqrt{-c}\,x^2\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(5\,a^2x^2-3\,ax+1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2}, -\frac{3\,a^2\sqrt{c}\,x^2\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\cdots}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fricas")

[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^2, -1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^2]

giac [A] time = 2.78, size = 231, normalized size = 1.65

$$\frac{2}{3}\left(3\,a\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^5 ac\mathrm{sgn}(x)+3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="giac")

[Out] 2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) + 3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) + 12*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a*c^3*sgn(x) + 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^3)*abs(a)

maple [B] time = 0.05, size = 378, normalized size = 2.70

$$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a\left(-6\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,x^4a^3c+6\sqrt{-\frac{c}{a^2}}\,\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}x^2a^3+6\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)x^3a-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)

[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^4*a^3*c+6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^3+6*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^3*a-6*(-c/a^

```
2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^
3*a+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^2*c-3*(-c/a^2)^(1/
2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^2*c-3*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)
^(3/2)*x*a^2-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)
*x^3*c^2+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2
*x^2-1)/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}{x^3\,\left(a\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x^3*(a*x + 1)^2),x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x^3*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^4+x^3}\right)dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^4+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**4 + x**3), x) - Integral(a*x*sqrt(
c - c/(a**2*x**2))/(a*x**4 + x**3), x)
```

3.778 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}+\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}$$

```
[Out] 4/3*a^3*(c-c/a^2/x^2)^(1/2)-1/4*(c-c/a^2/x^2)^(1/2)/x^3+2/3*a*(c-c/a^2/x^2)
^(1/2)/x^2-7/8*a^2*(c-c/a^2/x^2)^(1/2)/x-7/8*a^4*x*arctanh((-a*x+1)^(1/2)*(
a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 98, 151, 12, 92, 208}

$$\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}-\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/3 - Sqrt[c - c/(a^2*x^2)]/(4*x^3) + (2*a*Sqrt
[c - c/(a^2*x^2)])/(3*x^2) - (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) - (7*a^4*S
qrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a*
x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^5\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{8a-7a^2x}{x^4\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2-16a^3x}{x^3\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{32a^3-21a^4x}{x^2\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{24\sqrt{1-ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{8\sqrt{1-}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\,\mathrm{t}}{8\sqrt{1}}$$

Mathematica [A] time = 0.08, size = 94, normalized size = 0.60

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(21a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(32a^3x^3-21a^2x^2+16ax-6\right)\right)}{24x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-6 + 16*a*x - 21*a^2*x^2 + 32*a
^3*x^3) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2*x
^2])
```

fricas [A] time = 1.58, size = 216, normalized size = 1.38

$$\left[\frac{21\,a^3\sqrt{-c}\,x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(32\,a^3x^3-21\,a^2x^2+16\,ax-6\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3}, \frac{21\,a^3\sqrt{c}\,x^3\arctan\left(\frac{a\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fric
as")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, 1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(c
)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (32*a^3*x^3 - 21*a^2
*x^2 + 16*a*x - 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

giac [B] time = 6.17, size = 316, normalized size = 2.03

$$-\frac{1}{12}\left(21\,a^2\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^7a^2c\mathrm{sgn}(x)+45\left(\sqrt{a^2c}\,x-\sqrt{a\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="giac
")
```

```
[Out] -1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) + 96*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^3*a^2*c^3*sgn(x) + 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))
^2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c
^4*sgn(x) + 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^2 + c)^4)*abs(a)
```

maple [B] time = 0.05, size = 410, normalized size = 2.63

$$-\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\, a^{2}\left(-48\sqrt{-\frac{c}{a^{2}}}\,\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\, x^{5}a^{3}c+48\sqrt{-\frac{c}{a^{2}}}\,\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} x^{3}a^{3}+48\sqrt{-\frac{c}{a^{2}}}\, c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(-c/a^2)^(1/2)*(c*(a^2*x^2
-1)/a^2)^(1/2)*x^5*a^3*c+48*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^3*a^
3+48*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^4*a-4
8*(-c/a^2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(
1/2))*x^4*a+48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^4*a^2*c-21*(-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^4*a^2*c-27*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^2*a^2-21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-c)/a^2/x)*x^4*c^2+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x-6*(c
*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^
(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^{2}x^{2}-1\right)\sqrt{c-\frac{c}{a^{2}x^{2}}}}{\left(ax+1\right)^{2}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^{2}\,x^{2}}}\,\left(a^{2}\,x^{2}-1\right)}{x^{4}\,\left(a\,x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x^4*(a*x + 1)^2),x)
```

[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x^4*(a*x + 1)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^5+x^4}\right)dx-\int\frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^5+x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)

[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**5 + x**4), x) - Integral(a*x*sqrt(
c - c/(a**2*x**2))/(a*x**5 + x**4), x)

3.779 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$-\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}-\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c ...}}{4...}$$

```
[Out] -6/5*a^4*(c-c/a^2/x^2)^(1/2)-1/5*(c-c/a^2/x^2)^(1/2)/x^4+1/2*a*(c-c/a^2/x^2
)^(1/2)/x^3-3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2+3/4*a^3*(c-c/a^2/x^2)^(1/2)/x+3
/4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)
^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6159, 6129, 98, 151, 12, 92, 208}

$$-\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{a...}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] (-6*a^4*Sqrt[c - c/(a^2*x^2)])/5 - Sqrt[c - c/(a^2*x^2)]/(5*x^4) + (a*Sqrt[
c - c/(a^2*x^2)])/(2*x^3) - (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) + (3*a^3*
Sqrt[c - c/(a^2*x^2)])/(4*x) + (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[
1 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^6\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{10a-9a^2x}{x^5\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{36a^2-30a^3x}{x^4\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{90a^3-72a^4x}{x^3\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{120\sqrt{1-a}}$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} -$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} -$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} +$$

$$= -\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} +$$

Mathematica [A] time = 0.09, size = 102, normalized size = 0.56

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(15a^5x^5\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(24a^4x^4-15a^3x^3+12a^2x^2-10ax+4\right)\right)}{20x^4\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] -1/20*(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 - 10*a*x + 12*a^2*x^2 -
 15*a^3*x^3 + 24*a^4*x^4) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(x^4*
Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.57, size = 232, normalized size = 1.28

$$\left[\frac{15\,a^4\sqrt{-c}\,x^4\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4},-\frac{15\,a^4\sqrt{c}\,x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="fric
as")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, -1/20*(15*a^4*sqrt(c)*x^4*a
rctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (24*a^
4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/x^4]
```

giac [B] time = 7.93, size = 362, normalized size = 2.00

$$\frac{1}{10}\left(15\,a^3\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^9a^3c\,\mathrm{sgn}(x)+70\left(\sqrt{a^2c}\,x-\sqrt{a^2c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="giac
")
```

```
[Out] 1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) + 40*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) + 200*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^3*a^3*c^4*sgn(x) + 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2
*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*c
```

```
^5*sgn(x) + 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^2 + c)^5)*abs(a)
```

maple [B] time = 0.06, size = 447, normalized size = 2.47

$$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\, a^2\left(-40\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,x^6a^4c+40\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,x^4a^4-15\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,x^5a^3c+40\sqrt{-\frac{c}{a^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x)
```

```
[Out] 1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(c*(a^2*x^2-1)/a^2)^(1/2)*(
-c/a^2)^(1/2)*x^6*a^4*c+40*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^4*a^4
-15*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c+40*(-c/a^2)^(1/2)*c^
(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^5*a^2-40*(-c/a^2)^(1/2)*c^(
3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^5*a^2+40*(-c
/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c-25*(-c/a^2)^(1/2)*(c*(a
^2*x^2-1)/a^2)^(3/2)*x^3*a^3-15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1
/2)*a^2-c)/a^2/x)*x^5*a*c^2+16*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2
*a^2-10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x+4*(c*(a^2*x^2-1)/a^2)^
(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="maxi
ma")
```

```
[Out] -integrate((a^2*x^2 - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^2*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a^2\,x^2-1\right)}{x^5\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x^5*(a*x + 1)^2),x)
```

```
[Out] -int(((c - c/(a^2*x^2))^(1/2)*(a^2*x^2 - 1))/(x^5*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax^6+x^5}\right)dx-\int \frac{ax\sqrt{c-\frac{c}{a^2x^2}}}{ax^6+x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**5,x)
```

```
[Out] -Integral(-sqrt(c - c/(a**2*x**2))/(a*x**6 + x**5), x) - Integral(a*x*sqrt(
c - c/(a**2*x**2))/(a*x**6 + x**5), x)
```

3.780 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=187

$$-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-a^2x^2}}+\frac{ax^5\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{2x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] -4*x^2*(c-c/a^2/x^2)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)+2*x^3*(c-c/a^2/x^2)^(1/2)
/a/(-a^2*x^2+1)^(1/2)-x^4*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+1/4*a*x^5*
(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+4*x*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^
3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$\frac{ax^5\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{2x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a^2*Sqrt[1 - a^2*x^2]) + (2*Sqrt[c - c/(a^2
*x^2)]*x^3)/(a*Sqrt[1 - a^2*x^2]) - (Sqrt[c - c/(a^2*x^2)]*x^4)/Sqrt[1 - a^
2*x^2] + (a*Sqrt[c - c/(a^2*x^2)]*x^5)/(4*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c -
c/(a^2*x^2)]*x*Log[1 + a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x^2(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a^2\sqrt{1-a^2x^2}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{a\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^5}{4\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1}}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.37

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4\log(ax+1)}{a^3}-\frac{4x}{a^2}+\frac{ax^4}{4}+\frac{2x^2}{a}-x^3\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((-4*x)/a^2 + (2*x^2)/a - x^3 + (a*x^4)/4 + (4*Log
[1 + a*x])/a^3))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.67, size = 419, normalized size = 2.24

$$\left[\frac{8\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx+\left(a^5x^5+4a^4x^4+6a^3x^3+4a^2x^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\left(a^5x^5-4\right.}{4\left(a^6x^2-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(8*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 - 4*a*c*x + (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3 + 4*a^2*x^2)*sqrt
(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 + 2
*a^3*x^3 - 2*a*x - 1)) - (a^5*x^5 - 4*a^4*x^4 + 8*a^3*x^3 - 16*a^2*x^2)*sqr
t(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^2 - a^4), -1/4*(16*
(a^2*x^2 - 1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(
c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2 - a*c*x - 2*c))
 + (a^5*x^5 - 4*a^4*x^4 + 8*a^3*x^3 - 16*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((
a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^2 - a^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1)^3, x)
```

maple [A] time = 0.04, size = 85, normalized size = 0.45

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\sqrt{-a^2x^2+1}\,\left(x^4a^4-4x^3a^3+8a^2x^2-16ax+16\ln\left(ax+1\right)\right)}{4\left(a^2x^2-1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(x^4*a^4-4*x^3*a^3+
8*a^2*x^2-16*a*x+16*ln(a*x+1))/(a^2*x^2-1)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \sqrt{- c \left(-1 + \frac{1}{a x}\right) \left(1 + \frac{1}{a x}\right)}}{\left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(
a*x)))/(a*x + 1)**3, x)
```

3.781 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=152

$$\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{a^2\sqrt{1-a^2x^2}}+\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}-\frac{3x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

```
[Out] 4*x^2*(c-c/a^2/x^2)^(1/2)/a/(-a^2*x^2+1)^(1/2)-3/2*x^3*(c-c/a^2/x^2)^(1/2)/
(-a^2*x^2+1)^(1/2)+1/3*a*x^4*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-4*x*ln(
a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 77}

$$\frac{ax^4\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-a^2x^2}}-\frac{3x^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}+\frac{4x^2\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x^2)/(a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - c/(a^2*x^
2)]*x^3)/(2*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)]*x^4)/(3*Sqrt[1 -
a^2*x^2]) - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/(a^2*Sqrt[1 - a^2*x^2]
)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-3\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a\sqrt{1-a^2x^2}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{2\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1+ax)}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.42

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{4\log(ax+1)}{a^2}+\frac{ax^3}{3}+\frac{4x}{a}-\frac{3x^2}{2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((4*x)/a - (3*x^2)/2 + (a*x^3)/3 - (4*Log[1 + a*x]
)/a^2))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.69, size = 407, normalized size = 2.68

$$\left[\frac{12\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^6cx^6+4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2-4\,acx-\left(a^5x^5+4\,a^4x^4+6\,a^3x^3+4\,a^2x^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{a^4x^4+2\,a^3x^3-2\,ax-1}\right)-\left(2\,a^4x^4-\right.}{6\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm ="fricas")

[Out] [1/6*(12*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 - 4*a*c*x - (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3 + 4*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)) - (2*a^4*x^4 - 9*a^3*x^3 + 24*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*x^2 - a^3), 1/6*(24*(a^2*x^2 - 1)*sqrt(c)*arctan((a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2 - a*c*x - 2*c)) - (2*a^4*x^4 - 9*a^3*x^3 + 24*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*x^2 - a^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm ="giac")

[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1)^3, x)

maple [A] time = 0.05, size = 78, normalized size = 0.51

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\sqrt{-a^2x^2+1}\,\left(-2x^3a^3+9a^2x^2-24ax+24\ln\left(ax+1\right)\right)}{6\left(a^2x^2-1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)

[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(-2*x^3*a^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))/(a^2*x^2-1)/a^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(1-a^2\,x^2\right)^{3/2}}{\left(a\,x+1\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \sqrt{- c \left(-1 + \frac{1}{a x}\right) \left(1 + \frac{1}{a x}\right)}}{\left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(
a*x)))/(a*x + 1)**3, x)
```

3.782 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=113

$$-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{a\sqrt{1-a^2x^2}}+\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}$$

```
[Out] -3*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+1/2*a*x^3*(c-c/a^2/x^2)^(1/2)
/(-a^2*x^2+1)^(1/2)+4*x*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6160, 6140, 43}

$$\frac{ax^3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (a*Sqrt[c - c/(a^2*x^2)]
*x^3)/(2*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/(a*S
qrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}} \\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}} \\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(-3+ax+\frac{4}{1+ax}\right)dx}{\sqrt{1-a^2x^2}} \\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{2\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1+ax)}{a\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.48

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcTanh[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/Sqrt[1 -
a^2*x^2]
```

fricas [A] time = 0.60, size = 387, normalized size = 3.42

$$\left[\frac{4\left(a^2x^2-1\right)\sqrt{-c}\,\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx+\left(a^5x^5+4a^4x^4+6a^3x^3+4a^2x^2\right)\sqrt{-a^2x^2+1}\,\sqrt{-c}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\left(a^3x^3-6\right.}{2\left(a^4x^2-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/2*(4*(a^2*x^2 - 1)*sqrt(-c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 - 4*a*c*x + (a^5*x^5 + 4*a^4*x^4 + 6*a^3*x^3 + 4*a^2*x^2)*sqrt
(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/(a^4*x^4 + 2
*a^3*x^3 - 2*a*x - 1)) - (a^3*x^3 - 6*a^2*x^2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2
*c*x^2 - c)/(a^2*x^2)))/(a^4*x^2 - a^2), -1/2*(8*(a^2*x^2 - 1)*sqrt(c)*arct
an((a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a
^2*x^2))/(a^3*c*x^3 + 2*a^2*c*x^2 - a*c*x - 2*c)) + (a^3*x^3 - 6*a^2*x^2)*s
qrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^2 - a^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}\,x}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1)^3, x)
```

maple [A] time = 0.04, size = 69, normalized size = 0.61

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\sqrt{-a^2x^2+1}\,\left(a^2x^2-6ax+8\ln\left(ax+1\right)\right)}{2\left(a^2x^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-a^2*x^2+1)^(1/2)*(a^2*x^2-6*a*x+8*ln
(a*x+1))/(a^2*x^2-1)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}\,x}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int((x*(c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \sqrt{- c \left(-1 + \frac{1}{a x}\right) \left(1 + \frac{1}{a x}\right)}}{\left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x
)))/(a*x + 1)**3, x)
```

3.783 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=106

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] a*x^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-
a^2*x^2+1)^(1/2)-4*x*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 72}

$$\frac{ax^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a^2*x^2)]*x^2)/Sqrt[1 - a^2*x^2] + (Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1
 - a^2*x^2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1-ax)^2}{x(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.42

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,(ax-4\log(ax+1)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcTanh[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(a*x + Log[x] - 4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fr
icas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*x^2 + 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

maple [A] time = 0.04, size = 60, normalized size = 0.57

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(ax+\ln(x)-4\ln(ax+1)\right)\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(a*x+ln(x)-4*ln(a*x+1))*(-a^2*x^2+1)^(1/2)
/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(1-a^2\,x^2\right)^{3/2}}{(a\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(a*x + 1)**3, x)
```

3.784 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=106

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{3ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4ax\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-3*a*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a
^2*x^2+1)^(1/2)+4*a*x*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{3ax\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4ax\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - c/(a^2*x^2)]*x*L
og[x])/Sqrt[1 - a^2*x^2] + (4*a*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[
1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
```

```
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^2}{x^2(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}} + \frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1+ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.45

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(-3ax\log(x)+4ax\log(ax+1)-1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1 - 3*a*x*Log[x] + 4*a*x*Log[1 + a*x]))/Sqrt[1 - a
^2*x^2]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2x^3+2\,ax^2+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*x^3 + 2*a*x^2 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="
giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x), x)
```

maple [A] time = 0.05, size = 62, normalized size = 0.58

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{-a^2x^2+1}\,\left(3a\ln(x)x-4ax\ln\left(ax+1\right)+1\right)}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(-a^2*x^2+1)^(1/2)*(3*a*ln(x)*x-4*a*x*ln(a*x+
1)+1)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm="
maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(1-a^2\,x^2\right)}^{3/2}}{x\,{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x*(a*x + 1)^3),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(x*(a*x + 1)**3), x)
```

3.785 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=147

$$\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}+\frac{4a^2x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^2x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] 3*a*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/2*(c-c/a^2/x^2)^(1/2)/x/(-a^2*
x^2+1)^(1/2)+4*a^2*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-4*a^2*x*l
n(a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}+\frac{4a^2x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^2x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(2*x*
Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^
2] - (4*a^2*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^2}{x^3(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}} + \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\log(1+a}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.43

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1
 + a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.62, size = 485, normalized size = 3.30

$$\left[\frac{4\left(a^3x^3-ax\right)\sqrt{-c}\,\log\left(\frac{4a^5cx^5+\left(2a^6+4a^5+6a^4+4a^3+a^2\right)cx^6+\left(4a^4-4a^3-6a^2-4a-1\right)cx^4-5a^2cx^2-4acx-\left(4a^4x^4+6a^3x^3-\left(4a^4+6a^3+4a^2+\right.\right.}{a^4x^6+2a^3x^5-2ax^3-x^2}\right.}{2\left(a^2x^3-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(4*(a^3*x^3 - a*x)*sqrt(-c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*a^4
+ 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*c*x^
2 - 4*a*c*x - (4*a^4*x^4 + 6*a^3*x^3 - (4*a^4 + 6*a^3 + 4*a^2 + a)*x^5 + 4*
a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
- c)/(a^4*x^6 + 2*a^3*x^5 - 2*a*x^3 - x^2)) + sqrt(-a^2*x^2 + 1)*((6*a - 1)
*x^2 - 6*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^3 - x), -1/2*(8*(
a^3*x^3 - a*x)*sqrt(c)*arctan(-(2*a^2*x^2 + (2*a^3 + 2*a^2 + a)*x^3 + a*x)*
sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (
2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) - sqrt(-a^2*x^2
+ 1)*((6*a - 1)*x^2 - 6*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^3
- x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^2), x)
```

maple [A] time = 0.05, size = 78, normalized size = 0.53

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{-a^2x^2+1}\,\left(8a^2\ln(x)x^2-8\ln\left(ax+1\right)x^2a^2+6ax-1\right)}{2x\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-a^2*x^2+1)^(1/2)*(8*a^2*ln(x)*x^2-8*
ln(a*x+1)*x^2*a^2+6*a*x-1)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(1-a^2\,x^2\right)}^{3/2}}{x^2\,{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^2*(a*x + 1)^3),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^2*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x^{2}\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(x**2*(a*x + 1)**3), x)
```

3.786 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=186

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}-\frac{4a^3x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4a^3x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] -4*a^2*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/3*(c-c/a^2/x^2)^(1/2)/x^2/(
-a^2*x^2+1)^(1/2)+3/2*a*(c-c/a^2/x^2)^(1/2)/x/(-a^2*x^2+1)^(1/2)-4*a^3*x*ln
(x)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+4*a^3*x*ln(a*x+1)*(c-c/a^2/x^2)^
(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}-\frac{4a^3x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4a^3x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (-4*a^2*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(3
*x^2*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - c/(a^2*x^2)])/(2*x*Sqrt[1 - a^2*x^2
]) - (4*a^3*Sqrt[c - c/(a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] + (4*a^3*Sqrt
[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^4}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^2}{x^4(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{2x\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\log(x)}{\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{}}{}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 0.39

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^3\log(x)+4a^3\log(ax+1)-\frac{4a^2}{x}+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Lo
g[x] + 4*a^3*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.79, size = 523, normalized size = 2.81

$$\left[\frac{12\left(a^4x^4-a^2x^2\right)\sqrt{-c}\,\log\left(\frac{4a^5cx^5+\left(2a^6+4a^5+6a^4+4a^3+a^2\right)cx^6+\left(4a^4-4a^3-6a^2-4a-1\right)cx^4-5a^2cx^2-4acx+\left(4a^4x^4+6a^3x^3-\left(4a^4+6a^3+4\right.\right.}{a^4x^6+2a^3x^5-2ax^3-x^2}\right.}{6\left(a^2x^4-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm
="fricas")
```

```
[Out] [1/6*(12*(a^4*x^4 - a^2*x^2)*sqrt(-c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6
*a^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2
*c*x^2 - 4*a*c*x + (4*a^4*x^4 + 6*a^3*x^3 - (4*a^4 + 6*a^3 + 4*a^2 + a)*x^5
 + 4*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x
^2)) - c)/(a^4*x^6 + 2*a^3*x^5 - 2*a*x^3 - x^2)) + (24*a^2*x^2 - (24*a^2 -
9*a + 2)*x^3 - 9*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
))/(a^2*x^4 - x^2), 1/6*(24*(a^4*x^4 - a^2*x^2)*sqrt(c)*arctan(-(2*a^2*x^2
+ (2*a^3 + 2*a^2 + a)*x^3 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x^2
 - 2*a*c*x - c)) + (24*a^2*x^2 - (24*a^2 - 9*a + 2)*x^3 - 9*a*x + 2)*sqrt(-
a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^4 - x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^3), x)
```

maple [A] time = 0.05, size = 86, normalized size = 0.46

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{-a^2x^2+1}\,\left(24a^3\ln(x)x^3-24a^3x^3\ln\left(ax+1\right)+24a^2x^2-9ax+2\right)}{6x^2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*(-a^2*x^2+1)^(1/2)*(24*a^3*ln(x)*x^3-
24*a^3*x^3*ln(a*x+1)+24*a^2*x^2-9*a*x+2)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(1-a^2\,x^2\right)}^{3/2}}{x^3\,{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^3*(a*x + 1)^3),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^3*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x^{3}\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(x**3*(a*x + 1)**3), x)
```

3.787 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=220

$$-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{4a^4x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}$$

```
[Out] 4*a^3*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/4*(c-c/a^2/x^2)^(1/2)/x^3/(-
a^2*x^2+1)^(1/2)+a*(c-c/a^2/x^2)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)-2*a^2*(c-c/a^
2/x^2)^(1/2)/x/(-a^2*x^2+1)^(1/2)+4*a^4*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a^2*x
^2+1)^(1/2)-4*a^4*x*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{4a^4x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{4a^4x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(4*
x^3*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - c/(a^2*x^2)])/(x^2*Sqrt[1 - a^2*x^2])
- (2*a^2*Sqrt[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - c/(
a^2*x^2)]*x*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - c/(a^2*x^2)]*x*Log[
1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^5}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^2}{x^5(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-a^2x}}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.35

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(4a^4\log(x)-4a^4\log(ax+1)+\frac{4a^3}{x}-\frac{2a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*
a^4*Log[x] - 4*a^4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.68, size = 551, normalized size = 2.50

$$\left[\frac{8\left(a^5x^5-a^3x^3\right)\sqrt{-c}\,\log\left(\frac{4a^5cx^5+\left(2a^6+4a^5+6a^4+4a^3+a^2\right)cx^6+\left(4a^4-4a^3-6a^2-4a-1\right)cx^4-5a^2cx^2-4acx-\left(4a^4x^4+6a^3x^3-\left(4a^4+6a^3+4a\right.\right.}{a^4x^6+2a^3x^5-2ax^3-x^2}\right.}{4\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm
="fricas")
```

```
[Out] [1/4*(8*(a^5*x^5 - a^3*x^3)*sqrt(-c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*
a^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*
c*x^2 - 4*a*c*x - (4*a^4*x^4 + 6*a^3*x^3 - (4*a^4 + 6*a^3 + 4*a^2 + a)*x^5
+ 4*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c)/(a^4*x^6 + 2*a^3*x^5 - 2*a*x^3 - x^2)) - (16*a^3*x^3 - (16*a^3 - 8
*a^2 + 4*a - 1)*x^4 - 8*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)))/(a^2*x^5 - x^3), -1/4*(16*(a^5*x^5 - a^3*x^3)*sqrt(c)
*arctan(-(2*a^2*x^2 + (2*a^3 + 2*a^2 + a)*x^3 + a*x)*sqrt(-a^2*x^2 + 1)*sqr
t(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2*a^3*c*x^3 - (2*a^3 + a^2)*c*x^4 + (
a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) + (16*a^3*x^3 - (16*a^3 - 8*a^2 + 4*a
- 1)*x^4 - 8*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^2*x^5 - x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^4), x)
```

maple [A] time = 0.06, size = 94, normalized size = 0.43

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{-a^2x^2+1}\,\left(16a^4\ln(x)x^4-16\ln\left(ax+1\right)x^4a^4+16x^3a^3-8a^2x^2+4ax-1\right)}{4x^3\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] -1/4*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*(-a^2*x^2+1)^(1/2)*(16*a^4*ln(x)*x^4
-16*ln(a*x+1)*x^4*a^4+16*x^3*a^3-8*a^2*x^2+4*a*x-1)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{\left(ax+1\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(1-a^2\,x^2\right)}^{3/2}}{x^4\,{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^4*(a*x + 1)^3),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^4*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x^{4}\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(x**4*(a*x + 1)**3), x)
```

3.788 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=262

$$-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{4a^5x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4a^5x\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-}}{\sqrt{1-a^2}}$$

```
[Out] -4*a^4*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)-1/5*(c-c/a^2/x^2)^(1/2)/x^4/(
-a^2*x^2+1)^(1/2)+3/4*a*(c-c/a^2/x^2)^(1/2)/x^3/(-a^2*x^2+1)^(1/2)-4/3*a^2*
(c-c/a^2/x^2)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)+2*a^3*(c-c/a^2/x^2)^(1/2)/x/(-a^
2*x^2+1)^(1/2)-4*a^5*x*ln(x)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)+4*a^5*x
*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6160, 6150, 88}

$$-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}-\frac{4a^5x\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}+\frac{4a^5x\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (-4*a^4*Sqrt[c - c/(a^2*x^2)])/Sqrt[1 - a^2*x^2] - Sqrt[c - c/(a^2*x^2)]/(5
*x^4*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - c/(a^2*x^2)])/(4*x^3*Sqrt[1 - a^2*x
^2]) - (4*a^2*Sqrt[c - c/(a^2*x^2)])/(3*x^2*Sqrt[1 - a^2*x^2]) + (2*a^3*Sqr
t[c - c/(a^2*x^2)])/(x*Sqrt[1 - a^2*x^2]) - (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*
Log[x])/Sqrt[1 - a^2*x^2] + (4*a^5*Sqrt[c - c/(a^2*x^2)]*x*Log[1 + a*x])/Sq
rt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx &= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^6}\,dx}{\sqrt{1-a^2x^2}} \\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^2}{x^6(1+ax)}\,dx}{\sqrt{1-a^2x^2}} \\
&= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \left(\frac{1}{x^6}-\frac{3a}{x^5}+\frac{4a^2}{x^4}-\frac{4a^3}{x^3}+\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{1+ax}\right)dx}{\sqrt{1-a^2x^2}} \\
&= -\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-\frac{c}{a^2x^2}}}{4x^3\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2\sqrt{1-a^2x^2}}+\frac{2a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.33

$$
\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^5\log(x)+4a^5\log(ax+1)-\frac{240a^4x^4-120a^3x^3+80a^2x^2-45ax+12}{60x^5}\right)}{\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-1/60*(12 - 45*a*x + 80*a^2*x^2 - 120*a^3*x^3 + 2
40*a^4*x^4)/x^5 - 4*a^5*Log[x] + 4*a^5*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.05, size = 575, normalized size = 2.19

$$\left[\frac{120\left(a^{6}x^{6}-a^{4}x^{4}\right)\sqrt{-c}\,\log\left(\frac{4\,a^{5}cx^{5}+\left(2\,a^{6}+4\,a^{5}+6\,a^{4}+4\,a^{3}+a^{2}\right)cx^{6}+\left(4\,a^{4}-4\,a^{3}-6\,a^{2}-4\,a-1\right)cx^{4}-5\,a^{2}cx^{2}-4\,acx+\left(4\,a^{4}x^{4}+6\,a^{3}x^{3}-\left(4\,a^{4}+6\,a^{3}\right.\right.}{a^{4}x^{6}+2\,a^{3}x^{5}-2\,ax^{3}-x^{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm
="fricas")
```

```
[Out] [1/60*(120*(a^6*x^6 - a^4*x^4)*sqrt(-c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 +
 6*a^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a
^2*c*x^2 - 4*a*c*x + (4*a^4*x^4 + 6*a^3*x^3 - (4*a^4 + 6*a^3 + 4*a^2 + a)*x
^5 + 4*a^2*x^2 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(-c)*sqrt((a^2*c*x^2 - c)/(a^2
*x^2)) - c)/(a^4*x^6 + 2*a^3*x^5 - 2*a*x^3 - x^2)) + (240*a^4*x^4 - 120*a^3
*x^3 - (240*a^4 - 120*a^3 + 80*a^2 - 45*a + 12)*x^5 + 80*a^2*x^2 - 45*a*x +
 12)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x^6 - x^4), 1
/60*(240*(a^6*x^6 - a^4*x^4)*sqrt(c)*arctan(-(2*a^2*x^2 + (2*a^3 + 2*a^2 +
a)*x^3 + a*x)*sqrt(-a^2*x^2 + 1)*sqrt(c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(2
*a^3*c*x^3 - (2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) +
(240*a^4*x^4 - 120*a^3*x^3 - (240*a^4 - 120*a^3 + 80*a^2 - 45*a + 12)*x^5 +
 80*a^2*x^2 - 45*a*x + 12)*sqrt(-a^2*x^2 + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/(a^2*x^6 - x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^{2}x^{2}}}}{\left(ax+1\right)^{3}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm
="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^5), x)
```

maple [A] time = 0.05, size = 102, normalized size = 0.39

$$\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\sqrt{-a^{2}x^{2}+1}\,\left(240a^{5}\ln(x)x^{5}-240\ln\left(ax+1\right)x^{5}a^{5}+240x^{4}a^{4}-120x^{3}a^{3}+80a^{2}x^{2}-45ax+12\right)}{60x^{4}\left(a^{2}x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] 1/60*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*(-a^2*x^2+1)^(1/2)*(240*a^5*ln(x)*x^
5-240*ln(a*x+1)*x^5*a^5+240*x^4*a^4-120*x^3*a^3+80*a^2*x^2-45*a*x+12)/(a^2*
x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorithm
="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt(c - c/(a^2*x^2))/((a*x + 1)^3*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(1-a^2\,x^2\right)^{3/2}}{x^5\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^5*(a*x + 1)^3),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(1 - a^2*x^2)^(3/2))/(x^5*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x^5\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))
)/(x**5*(a*x + 1)**3), x)
```

3.789 $\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=53

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1(1-2p,-2p;2-2p;ax)}{1-2p}$$

```
[Out] (c-c/a^2/x^2)^p*x*hypergeom([-2*p, 1-2*p],[2-2*p],a*x)/(1-2*p)/((-a^2*x^2+1
)^p)
```

Rubi [A] time = 0.12, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6160, 6150, 64}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1(1-2p,-2p;2-2p;ax)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, a*x])/((1
- 2*p)*(1 - a^2*x^2)^p)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2p\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx=\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{-2p\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1-ax)^{2p}\,dx$$
$$=\frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p}\,{}_2F_1(1-2p,-2p;2-2p;ax)}{1-2p}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1(1-2p,-2p;2-2p;ax)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, a*x])/((1
- 2*p)*(1 - a^2*x^2)^p)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a^2*c*x^2 - c)/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2x^2}\right)^p \mathrm{e}^{-2p \operatorname{arctanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x)
```

```
[Out] int((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c - \frac{c}{a^2x^2}\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arctanh(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{-2\,p\,\mathrm{atanh}(a\,x)}\,\left(c - \frac{c}{a^2\,x^2}\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*p*atanh(a*x))*(c - c/(a^2*x^2))^p,x)
```

```
[Out] int(exp(-2*p*atanh(a*x))*(c - c/(a^2*x^2))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)\right)^p e^{-2p \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/exp(2*p*atanh(a*x)),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(-2*p*atanh(a*x)), x)
```

3.790 $\int e^{2p\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=54

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1(1-2p,-2p;2-2p;-ax)}{1-2p}$$

```
[Out] (c-c/a^2/x^2)^p*x*hypergeom([-2*p, 1-2*p],[2-2*p],-a*x)/(1-2*p)/((-a^2*x^2+
1)^p)
```

Rubi [A] time = 0.12, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6160, 6150, 64}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1(1-2p,-2p;2-2p;-ax)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, -(a*x)])/(
(1 - 2*p)*(1 - a^2*x^2)^p)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{2p\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{2p\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx$$
$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1+ax)^{2p} dx$$
$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p} {}_2F_1(1-2p,-2p;2-2p;-ax)}{1-2p}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.00

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1(1-2p,-2p;2-2p;-ax)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, -(a*x)])/(
(1 - 2*p)*(1 - a^2*x^2)^p)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\frac{ax+1}{ax-1}\right)^p\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^p*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int e^{2p\,\mathrm{arctanh}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x)
[Out] int(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{2\,p\,\mathrm{atanh}(a\,x)}\left(c-\frac{c}{a^2\,x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*atanh(a*x))*(c - c/(a^2*x^2))^p,x)
[Out] int(exp(2*p*atanh(a*x))*(c - c/(a^2*x^2))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{2p\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*atanh(a*x))*(c-c/a**2/x**2)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(2*p*atanh(a*x)), x)
```

3.791 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=331

$$-\frac{c^2(ax+1)^{\frac{n-4}{2}}(1-ax)^{3-\frac{n}{2}}}{3a^4x^3}-\frac{c^2(n+10)(ax+1)^{\frac{n-4}{2}}(1-ax)^{3-\frac{n}{2}}}{6a^3x^2}-\frac{c^2\left(n^2+5n+14\right)(ax+1)^{\frac{n-4}{2}}(1-ax)^{3-\frac{n}{2}}}{6a^2x}-\frac{c^2n\left(10\right.}{}$$

```
[Out] -4*c^2*(-a*x+1)^(3-1/2*n)*(a*x+1)^(-2+1/2*n)/a/(4-n)-1/3*c^2*(-a*x+1)^(3-1/
2*n)*(a*x+1)^(-2+1/2*n)/a^4/x^3-1/6*c^2*(10+n)*(-a*x+1)^(3-1/2*n)*(a*x+1)^(
-2+1/2*n)/a^3/x^2-1/6*c^2*(n^2+5*n+14)*(-a*x+1)^(3-1/2*n)*(a*x+1)^(-2+1/2*n
)/a^2/x-1/3*c^2*n*(-n^2+10)*(-a*x+1)^(2-1/2*n)*(a*x+1)^(-2+1/2*n)*hypergeom
([1, -2+1/2*n],[-1+1/2*n],(a*x+1)/(-a*x+1))/a/(4-n)+2^(-1+1/2*n)*c^2*n*(-a*
x+1)^(3-1/2*n)*hypergeom([3-1/2*n, 2-1/2*n],[4-1/2*n],-1/2*a*x+1/2)/a/(n^2-
10*n+24)
```

Rubi [C] time = 0.13, antiderivative size = 71, normalized size of antiderivative = 0.21, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6157, 6150, 136}

$$\frac{c^2 2^{3-\frac{n}{2}}(ax+1)^{\frac{n+6}{2}}F_1\left(\frac{n+6}{2};\frac{n-4}{2},4;\frac{n+8}{2};\frac{1}{2}(ax+1),ax+1\right)}{a(n+6)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (2^(3 - n/2)*c^2*(1 + a*x)^((6 + n)/2)*AppellF1[(6 + n)/2, (-4 + n)/2, 4, (
8 + n)/2, (1 + a*x)/2, 1 + a*x])/(a*(6 + n))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=\frac{c^2\int\frac{e^{n\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}$$
$$=\frac{c^2\int\frac{(1-ax)^{2-\frac{n}{2}}(1+ax)^{2+\frac{n}{2}}}{x^4}\,dx}{a^4}$$
$$=\frac{2^{3-\frac{n}{2}}c^2(1+ax)^{\frac{6+n}{2}}F_1\left(\frac{6+n}{2};\frac{1}{2}(-4+n),4;\frac{8+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{a(6+n)}$$

Mathematica [A] time = 0.85, size = 229, normalized size = 0.69

$$-\frac{c^2e^{n\tanh^{-1}(ax)}\left(a^3\left(n^2-10\right)nx^3e^{2\tanh^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\tanh^{-1}(ax)}\right)+a^3\left(n^3+2n^2-10n-20\right)x^3\,{}_2F_1\left(1,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -1/6*(c^2*E^(n*ArcTanh[a*x])*(4 + 2*n + 2*a*n*x + a*n^2*x - 24*a^2*x^2 - 12
*a^2*n*x^2 + 2*a^2*n^2*x^2 + a^2*n^3*x^2 - 2*a^3*n*x^3 - a^3*n^2*x^3 + a^3*
E^(2*ArcTanh[a*x])*n*(-10 + n^2)*x^3*Hypergeometric2F1[1, 1 + n/2, 2 + n/2,
 E^(2*ArcTanh[a*x])] + a^3*(-20 - 10*n + 2*n^2 + n^3)*x^3*Hypergeometric2F1
[1, n/2, 1 + n/2, E^(2*ArcTanh[a*x])] - 24*a^3*E^(2*ArcTanh[a*x])*x^3*Hyper
geometric2F1[2, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])]))/(a^4*(2 + n)*x^3)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n)/
(a^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2 x^2}\right)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} \left(c - \frac{c}{a^2 x^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2 x^2}\right)^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int e^{n \operatorname{atanh}(a\,x)} \left(c - \frac{c}{a^2\,x^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2))^2,x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2 \left(\int a^4 e^{n \operatorname{atanh}(ax)}\, dx + \int \frac{e^{n \operatorname{atanh}(ax)}}{x^4}\, dx + \int \left(-\frac{2a^2 e^{n \operatorname{atanh}(ax)}}{x^2}\right) dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2)**2,x)
```

```
[Out] c**2*(Integral(a**4*exp(n*atanh(a*x)), x) + Integral(exp(n*atanh(a*x))/x**4
, x) + Integral(-2*a**2*exp(n*atanh(a*x))/x**2, x))/a**4
```

3.792 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=137

$$\frac{4c(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{a(2-n)}-\frac{c2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(2-n)}$$

```
[Out] 4*c*(-a*x+1)^(1-1/2*n)*(a*x+1)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],
(-a*x+1)/(a*x+1))/a/(2-n)-2^(1+1/2*n)*c*(-a*x+1)^(1-1/2*n)*hypergeom([-1/2*
n, 1-1/2*n],[2-1/2*n],-1/2*a*x+1/2)/a/(2-n)
```

Rubi [C] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 0.51, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6157, 6150, 136}

$$-\frac{c2^{2-\frac{n}{2}}(ax+1)^{\frac{n+4}{2}}F_1\left(\frac{n+4}{2};\frac{n-2}{2},2;\frac{n+6}{2};\frac{1}{2}(ax+1),ax+1\right)}{a(n+4)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] -((2^(2 - n/2)*c*(1 + a*x)^((4 + n)/2)*AppellF1[(4 + n)/2, (-2 + n)/2, 2, (
6 + n)/2, (1 + a*x)/2, 1 + a*x])/(a*(4 + n)))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\frac{c\int \frac{e^{n\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}\,dx}{a^2}$$
$$= -\frac{c\int \frac{(1-ax)^{1-\frac{n}{2}}(1+ax)^{1+\frac{n}{2}}}{x^2}\,dx}{a^2}$$
$$= -\frac{2^{2-\frac{n}{2}}c(1+ax)^{\frac{4+n}{2}}F_1\left(\frac{4+n}{2};\frac{1}{2}(-2+n),2;\frac{6+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{a(4+n)}$$

Mathematica [A] time = 0.22, size = 126, normalized size = 0.92

$$\frac{ce^{n\tanh^{-1}(ax)}\left(anxe^{2\tanh^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\tanh^{-1}(ax)}\right)+a(n+2)x\,{}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\tanh^{-1}(ax)}\right)+4axe^{2\tanh}\right.}{a^2(n+2)x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*E^(n*ArcTanh[a*x])*(2 + n + a*E^(2*ArcTanh[a*x])*n*x*Hypergeometric2F1[1
, 1 + n/2, 2 + n/2, E^(2*ArcTanh[a*x])] + a*(2 + n)*x*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcTanh[a*x])] + 4*a*E^(2*ArcTanh[a*x])*x*Hypergeometric
2F1[2, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*x])]))/(a^2*(2 + n)*x)
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}\left(c - \frac{c}{a^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x)
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2x^2}\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2),x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{atanh}(a\,x)}\,\left(c - \frac{c}{a^2\,x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2)),x)
[Out] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int a^2e^{n\,\mathrm{atanh}\,(ax)}\,dx + \int\left(-\frac{e^{n\,\mathrm{atanh}\,(ax)}}{x^2}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2*exp(n*atanh(a*x)), x) + Integral(-exp(n*atanh(a*x))/x**2,
x))/a**2
```

3.793 $\int \frac{e^{n \tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac}+\frac{x(ax+1)^{n/2}(1-ax)^{-n/2}}{c}-\frac{(1-n)(ax+1)^{n/2}(1-ax)^{-n/2}}{acn}$$

```
[Out] -(1-n)*(a*x+1)^(1/2*n)/a/c/n/((-a*x+1)^(1/2*n))+x*(a*x+1)^(1/2*n)/c/((-a*x+
1)^(1/2*n))-2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],-1/2*a*x+1/2)/
a/c/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.17, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6157, 6150, 90, 79, 69}

$$-\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac}+\frac{x(ax+1)^{n/2}(1-ax)^{-n/2}}{c}-\frac{(1-n)(ax+1)^{n/2}(1-ax)^{-n/2}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(((1 - n)*(1 + a*x)^(n/2))/(a*c*n*(1 - a*x)^(n/2))) + (x*(1 + a*x)^(n/2))/
(c*(1 - a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2, (
1 - a*x)/2])/(a*c*(1 - a*x)^(n/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{a^2\int \frac{e^{n\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\ &= -\frac{a^2\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}\\ &= \frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{c}+\frac{\int(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-1-anx)\,dx}{c}\\ &= -\frac{(1-n)(1-ax)^{-n/2}(1+ax)^{n/2}}{acn}+\frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{c}-\frac{n\int(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{c}\\ &= -\frac{(1-n)(1-ax)^{-n/2}(1+ax)^{n/2}}{acn}+\frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{c}-\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\right.}{ac}\end{aligned}$$

Mathematica [A] time = 0.11, size = 82, normalized size = 0.66

$$\frac{(1-ax)^{-n/2}\left((ax+1)^{n/2}(anx+n-1)-2^{\frac{n}{2}+1}n\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] ((1 + a*x)^(n/2)*(-1 + n + a*n*x) - 2^(1 + n/2)*n*Hypergeometric2F1[-1/2*n,
 -1/2*n, 1 - n/2, (1 - a*x)/2])/(a*c*n*(1 - a*x)^(n/2))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{c-\frac{c}{a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2)),x)
[Out] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2 e^{n\,\mathrm{atanh}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2),x)
[Out] a**2*Integral(x**2*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

3.794 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=373

$$-\frac{a^2x^3(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}-1}}{c^2}-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(\frac{2-n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac^2(2-n)}-\frac{(n+3)\left(2-n^2\right)(ax+1)^{n/2}(1}{ac^2\left(4-n^2\right)}$$

```
[Out] (1-n)*(3+n)*(-a*x+1)^(-1-1/2*n)*(a*x+1)^(-1+1/2*n)/a/c^2/(2-n)+(3+n)*x*(-a*
x+1)^(-1-1/2*n)*(a*x+1)^(-1+1/2*n)/c^2-a^2*x^3*(-a*x+1)^(-1-1/2*n)*(a*x+1)^
(-1+1/2*n)/c^2+(-a*x+1)^(1-1/2*n)*(a*x+1)^(-1+1/2*n)/a/c^2/(2-n)-(a*x+1)^(-
1+1/2*n)/a/c^2/((-a*x+1)^(1/2*n))-(3+n)*(-n^2+2)*(-a*x+1)^(-1-1/2*n)*(a*x+1
)^(1/2*n)/a/c^2/(-n^2+4)-(3+n)*(-n^2+2)*(a*x+1)^(1/2*n)/a/c^2/n/(-n^2+4)/((
-a*x+1)^(1/2*n))-2^(1/2*n)*n*(-a*x+1)^(1-1/2*n)*hypergeom([1-1/2*n, 1-1/2*n
],[2-1/2*n],-1/2*a*x+1/2)/a/c^2/(2-n)
```

Rubi [A] time = 0.42, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {6157, 6150, 100, 159, 89, 79, 69, 90, 45, 37}

$$-\frac{a^2x^3(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}-1}}{c^2}-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(\frac{2-n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{ac^2(2-n)}-\frac{(n+3)\left(2-n^2\right)(ax+1)^{n/2}(1}{ac^2\left(4-n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] ((1 - n)*(3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a*c^2*(2 - n
)) + ((3 + n)*x*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/c^2 - (a^2*x^3
*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/c^2 + ((1 - a*x)^(1 - n/2)*(1
 + a*x)^((-2 + n)/2))/(a*c^2*(2 - n)) - (1 + a*x)^((-2 + n)/2)/(a*c^2*(1 -
a*x)^(n/2)) - ((3 + n)*(2 - n^2)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^(n/2))/(a*c
^2*(4 - n^2)) - ((3 + n)*(2 - n^2)*(1 + a*x)^(n/2))/(a*c^2*n*(4 - n^2)*(1 -
 a*x)^(n/2)) - (2^(n/2)*n*(1 - a*x)^(1 - n/2)*Hypergeometric2F1[(2 - n)/2,
1 - n/2, 2 - n/2, (1 - a*x)/2])/(a*c^2*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \frac{a^4\int \frac{e^{n\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^2}\,dx}{c^2}$$

$$= \frac{a^4\int x^4(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2}$$

$$= -\frac{a^2x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2\int x^2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}(-3-anx)\,dx}{c^2}$$

$$= -\frac{a^2x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{\left(a^2n\right)\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2} + \frac{\left(a^2(3+n)\right)\int x$$

$$= \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{(1-ax)^{-n/2}(1+ax)}{ac^2}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2x^3(1-a}{}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2x^3(1-a}{}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2} - \frac{a^2x^3(1-a}{}$$

Mathematica [A] time = 0.16, size = 178, normalized size = 0.48

$$-\frac{(1-ax)^{-\frac{n}{2}-1}\left((ax+1)^{n/2}\left(n^2\left(1-2a^2x^2\right)+6a^2x^2+n\left(-4a^3x^3+4a^2x^2+6ax-4\right)+n^3(ax-1)^2(ax+1)-6\right)-2}{ac^2(n-2)n(n+2)(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*((1 + a*x)^(n/2)*(-6 + 6*a^2*x^2 + n^3*(-1 + a*x)^2
*(1 + a*x) + n^2*(1 - 2*a^2*x^2) + n*(-4 + 6*a*x + 4*a^2*x^2 - 4*a^3*x^3))
- 2^(n/2)*n^2*(2 + n)*(-1 + a*x)^2*(1 + a*x)*Hypergeometric2F1[1 - n/2, 1 -
 n/2, 2 - n/2, (1 - a*x)/2]))/(a*c^2*(-2 + n)*n*(2 + n)*(1 + a*x)))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral(a^4*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2
 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{\left(c-\frac{c}{a^2\,x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2))^2,x)
[Out] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4 \int \frac{x^4 e^{n\,\mathrm{atanh}\,(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2)**2,x)
[Out] a**4*Integral(x**4*exp(n*atanh(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

3.795 $\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=430

$$-\frac{a^2\left(3-n^2\right)x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{n-3}{2};\frac{n-1}{2};\frac{ax+1}{1-ax}\right)}{(3-n)\left(1-a^2x^2\right)^{3/2}}+\frac{a^2 2^{\frac{n-1}{2}}nx^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}(1-ax)^{\frac{5-n}{2}}\,{}_2F\cdots}{(3-n)(5-n)\left(1-\cdots\right.}$$

```
[Out] -1/2*(c-c/a^2/x^2)^(3/2)*x*(-a*x+1)^(5/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)/(-a^2*
x^2+1)^(3/2)-1/2*a*(4+n)*(c-c/a^2/x^2)^(3/2)*x^2*(-a*x+1)^(5/2-1/2*n)*(a*x+
1)^(-3/2+1/2*n)/(-a^2*x^2+1)^(3/2)-3*a^2*(c-c/a^2/x^2)^(3/2)*x^3*(-a*x+1)^(
5/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)/(3-n)/(-a^2*x^2+1)^(3/2)-a^2*(-n^2+3)*(c-c/
a^2/x^2)^(3/2)*x^3*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*hypergeom([1,
-3/2+1/2*n],[-1/2+1/2*n],(a*x+1)/(-a*x+1))/(3-n)/(-a^2*x^2+1)^(3/2)+2^(-1/2
+1/2*n)*a^2*n*(c-c/a^2/x^2)^(3/2)*x^3*(-a*x+1)^(5/2-1/2*n)*hypergeom([5/2-1
/2*n, 3/2-1/2*n],[7/2-1/2*n],-1/2*a*x+1/2)/(3-n)/(5-n)/(-a^2*x^2+1)^(3/2)
```

Rubi [C] time = 0.21, antiderivative size = 103, normalized size of antiderivative = 0.24, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6160, 6150, 136}

$$-\frac{a^2 2^{\frac{5}{2}-\frac{n}{2}}x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}(ax+1)^{\frac{n+5}{2}}F_1\left(\frac{n+5}{2};\frac{n-3}{2},3;\frac{n+7}{2};\frac{1}{2}(ax+1),ax+1\right)}{(n+5)\left(1-a^2x^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -((2^(5/2 - n/2)*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*(1 + a*x)^((5 + n)/2)*Appe
llF1[(5 + n)/2, (-3 + n)/2, 3, (7 + n)/2, (1 + a*x)/2, 1 + a*x])/((5 + n)*(
1 - a^2*x^2)^(3/2)))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{n\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$=\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{\frac{3}{2}-\frac{n}{2}}(1+ax)^{\frac{3}{2}+\frac{n}{2}}}{x^3}dx}{\left(1-a^2x^2\right)^{3/2}}$$

$$=-\frac{2^{\frac{5}{2}-\frac{n}{2}}a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3(1+ax)^{\frac{5+n}{2}}F_1\left(\frac{5+n}{2};\frac{1}{2}(-3+n),3;\frac{7+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{(5+n)\left(1-a^2x^2\right)^{3/2}}$$

Mathematica [A] time = 1.69, size = 190, normalized size = 0.44

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}e^{n\tanh^{-1}(ax)}\operatorname{csch}\left(\frac{1}{2}\tanh^{-1}(ax)\right)\operatorname{sech}\left(\frac{1}{2}\tanh^{-1}(ax)\right)\left(-(n+1)\operatorname{csch}\left(\frac{1}{2}\tanh^{-1}(ax)\right)\operatorname{sech}\left(\frac{1}{2}\tanh^{-1}(ax\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (c*E^(n*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)]*x*Csch[ArcTanh[a*x]/2]*Sech[Arc
Tanh[a*x]/2]*(8*a*E^ArcTanh[a*x]*n*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n
)/2, -E^(2*ArcTanh[a*x])] - 4*a*E^ArcTanh[a*x]*(-3 + n^2)*x*Hypergeometric2
F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcTanh[a*x])] - (1 + n)*(a*x*(n + a*x) +
(1 - a^2*x^2)*Cosh[2*ArcTanh[a*x]])*Csch[ArcTanh[a*x]/2]*Sech[ArcTanh[a*x]/
2]))/(8*(1 + n)*(-1 + a^2*x^2))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^{2}cx^{2}-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*x^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n \,\mathrm{arctanh}\left(a x\right)} \left(c-\frac{c}{a^{2} x^{2}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c-\frac{c}{a^{2} x^{2}}\right)^{\frac{3}{2}} \left(\frac{a x+1}{a x-1}\right)^{\frac{1}{2}\, n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int e^{n\,\mathrm{atanh}(a\,x)}\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2))^(3/2),x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.796 $\int e^{n\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=272

$$\frac{2^{\frac{n+1}{2}}nx\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{\left(n^2-4n+3\right)\sqrt{1-a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\right.}{(1-n)\sqrt{1-a^2x^2}}$$

```
[Out] -x*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(c-c/a^2/x^2)^(1/2)/(1-n)/(-a^
2*x^2+1)^(1/2)+2*x*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*hypergeom([1,
-1/2+1/2*n],[1/2+1/2*n],(a*x+1)/(-a*x+1))*(c-c/a^2/x^2)^(1/2)/(1-n)/(-a^2*x
^2+1)^(1/2)+2^(1/2+1/2*n)*n*x*(-a*x+1)^(3/2-1/2*n)*hypergeom([3/2-1/2*n, 1/
2-1/2*n],[5/2-1/2*n],-1/2*a*x+1/2)*(c-c/a^2/x^2)^(1/2)/(n^2-4*n+3)/(-a^2*x^
2+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 302, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6160, 6150, 105, 69, 131}

$$\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1}{2}(-n-1)}\,{}_2F_1\left(1,\frac{1}{2}(-n-1);\frac{1-n}{2};\frac{1-ax}{ax+1}\right)}{(n+1)\sqrt{1-a^2x^2}}-\frac{2^{\frac{n+3}{2}}x\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^{\frac{1}{2}(-n-1)}\,{}_2F_1\left(\frac{1}{2}(-n-\right.}{(n+1)\sqrt{1-a^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (2*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((1 + n)/2)*Hyp
ergeometric2F1[1, (-1 - n)/2, (1 - n)/2, (1 - a*x)/(1 + a*x)])/((1 + n)*Sqr
t[1 - a^2*x^2]) - (2^((3 + n)/2)*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^((-1 - n
)/2)*Hypergeometric2F1[(-1 - n)/2, (-1 - n)/2, (1 - n)/2, (1 - a*x)/2])/((1
 + n)*Sqrt[1 - a^2*x^2]) + (2^((3 + n)/2)*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)
^((1 - n)/2)*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/
2])/((1 - n)*Sqrt[1 - a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}}}{\sqrt{1-a^2x^2}}$$

$$= \frac{2^{\frac{3+n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{(1-n)\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)}{}$$

$$= \frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1+n}{2}}\,{}_2F_1\left(1,\frac{1}{2}(-1-n);\frac{1-n}{2};\frac{1-ax}{1+ax}\right)}{(1+n)\sqrt{1-a^2x^2}} - \frac{2^{\frac{3+n}{2}}\sqrt{c-}}{}$$

Mathematica [A] time = 0.17, size = 208, normalized size = 0.76

$$\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^{\frac{1}{2}(-n-1)}\left((n-1)(ax+1)^{\frac{n+1}{2}}\,{}_2F_1\left(1,-\frac{n}{2}-\frac{1}{2};\frac{1}{2}-\frac{n}{2};\frac{1-ax}{ax+1}\right)+2^{\frac{n+1}{2}}\left((n+1)(ax-1)\,{}_2F_1\left(-\frac{n}{2}-\frac{1}{2},\right.\right.\right.}{\left(n^2-1\right)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (2*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^((-1 - n)/2)*((-1 + n)*(1 + a*x)^((1 +
 n)/2)*Hypergeometric2F1[1, -1/2 - n/2, 1/2 - n/2, (1 - a*x)/(1 + a*x)] + 2
^((1 + n)/2)*(-((-1 + n)*Hypergeometric2F1[-1/2 - n/2, -1/2 - n/2, 1/2 - n/
2, 1/2 - (a*x)/2]) + (1 + n)*(-1 + a*x)*Hypergeometric2F1[-1/2 - n/2, 1/2 -
 n/2, 3/2 - n/2, 1/2 - (a*x)/2])))/((-1 + n^2)*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int {\mathrm e}^{n \,\mathrm{arctanh}\left(a x \right)} \sqrt{c -\frac{c}{a^{2} x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{a^{2} x^{2}}} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2} \, n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{e}}^{n\,\mathrm{atanh}\left(a\,x\right)}\,\sqrt{c-\frac{c}{a^2\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2))^(1/2),x)
[Out] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}e^{n\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*exp(n*atanh(a*x)), x)
```

3.797 $\int \frac{e^{n \tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}} dx$

Optimal. Leaf size=182

$$-\frac{2^{\frac{n+3}{2}} n\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(1-n^2\right)x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{a^2(n+1)x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] -(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/a^2/(1+n)/x/(c
-c/a^2/x^2)^(1/2)-2^(3/2+1/2*n)*n*(-a*x+1)^(1/2-1/2*n)*hypergeom([1/2-1/2*n
, -1/2-1/2*n],[3/2-1/2*n],-1/2*a*x+1/2)*(-a^2*x^2+1)^(1/2)/a^2/(-n^2+1)/x/(
c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6160, 6150, 79, 69}

$$-\frac{2^{\frac{n+3}{2}} n\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(1-n^2\right)x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{a^2(n+1)x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^2*(1 +
 n)*Sqrt[c - c/(a^2*x^2)]*x)) - (2^((3 + n)/2)*n*(1 - a*x)^((1 - n)/2)*Sqrt
[1 - a^2*x^2]*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)
/2])/(a^2*(1 - n^2)*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= \frac{\sqrt{1-a^2x^2}\int x(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{a^2(1+n)\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{\left(n\sqrt{1-a^2x^2}\right)\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1+n}{2}}\,dx}{a(1+n)\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{a^2(1+n)\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{2^{\frac{3+n}{2}}n(1-ax)^{\frac{1-n}{2}}\sqrt{1-a^2x^2}\;{}_2F_1\left(\frac{1}{2}(-1-n),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}\right.}{a^2\left(1-n^2\right)\sqrt{c-\frac{c}{a^2x^2}}\,x}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 130, normalized size = 0.71

$$
\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}n\,{}_2F_1\left(-\frac{n}{2}-\frac{1}{2},\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)-(n-1)(ax+1)^{\frac{n+1}{2}}\right)}{a^2(n-1)(n+1)x\sqrt{c-\frac{c}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)*(1 + a*x)^((1 + n)/2))
 + 2^((3 + n)/2)*n*Hypergeometric2F1[-1/2 - n/2, 1/2 - n/2, 3/2 - n/2, 1/2
- (a*x)/2]))/(a^2*(-1 + n)*(1 + n)*Sqrt[c - c/(a^2*x^2)]*x)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2))^(1/2),x)
[Out] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(exp(n*atanh(a*x))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x)
```

3.798 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{\left(1-a^2x^2\right)^{3/2}(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2^{\frac{n-1}{2}}n\left(1-a^2x^2\right)^{3/2}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{3-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a^4(3-n)x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right.}{}$$

```
[Out] -(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1)^(3/2)/a^2/(c-c/a^2
/x^2)^(3/2)/x+(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(2+2*n+n^2-a*n*(3+
2*n)*x)*(-a^2*x^2+1)^(3/2)/a^4/(-n^2+1)/(c-c/a^2/x^2)^(3/2)/x^3-2^(-1/2+1/2
*n)*n*(-a*x+1)^(3/2-1/2*n)*(-a^2*x^2+1)^(3/2)*hypergeom([3/2-1/2*n, 3/2-1/2
*n],[5/2-1/2*n],-1/2*a*x+1/2)/a^4/(3-n)/(c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.29, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6160, 6150, 100, 145, 69}

$$-\frac{2^{\frac{n-1}{2}}n\left(1-a^2x^2\right)^{3/2}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{3-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a^4(3-n)x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{\left(1-a^2x^2\right)^{3/2}(ax+1)^{\frac{n-1}{2}}\left(-a(2n+3)nx+n^2+2n\right.}{a^4\left(1-n^2\right)x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -(((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(3/2))/(a^2*
(c - c/(a^2*x^2))^(3/2)*x)) + ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2
)*(2 + 2*n + n^2 - a*n*(3 + 2*n)*x)*(1 - a^2*x^2)^(3/2))/(a^4*(1 - n^2)*(c
- c/(a^2*x^2))^(3/2)*x^3) - (2^((-1 + n)/2)*n*(1 - a*x)^((3 - n)/2)*(1 - a^
2*x^2)^(3/2)*Hypergeometric2F1[(3 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2
])/(a^4*(3 - n)*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\left(1-a^2x^2\right)^{3/2}\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= \frac{\left(1-a^2x^2\right)^{3/2}\int x^3(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(1-a^2x^2\right)^{3/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} - \frac{\left(1-a^2x^2\right)^{3/2}\int x(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}(-}{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(1-a^2x^2\right)^{3/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(2+2n+n^2-\right.}{a^4\left(1-n^2\right)\left(c-\frac{c}{a^2x^2}\right)^{3}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(1-a^2x^2\right)^{3/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(2+2n+n^2-\right.}{a^4\left(1-n^2\right)\left(c-\frac{c}{a^2x^2}\right)^{3}}$$

Mathematica [A] time = 0.25, size = 186, normalized size = 0.70

$$\frac{\left(1-a^2x^2\right)^{3/2}(1-ax)^{\frac{1}{2}(-n-1)}\left(-4a^4x^2(ax+1)^{\frac{n-1}{2}}+\frac{a^2 2^{\frac{n+3}{2}}n(ax-1)^2\,{}_2F_1\left(\frac{3}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2};\frac{5}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{n-3}+\frac{4a^2\left(n^2(2ax-1)+n(3ax-2)-2\right)(ax+}{n^2-1}\right.}{4a^6x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 - a^2*x^2)^(3/2)*(-4*a^4*x^2*(1 + a*x)^((-1 + n)
/2) + (4*a^2*(1 + a*x)^((-1 + n)/2)*(-2 + n^2*(-1 + 2*a*x) + n*(-2 + 3*a*x)
))/(-1 + n^2) + (2^((3 + n)/2)*a^2*n*(-1 + a*x)^2*Hypergeometric2F1[3/2 - n
/2, 3/2 - n/2, 5/2 - n/2, 1/2 - (a*x)/2])/(-3 + n)))/(4*a^6*(c - c/(a^2*x^2
))^(3/2)*x^3)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^4x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^4c^2x^4-2\,a^2c^2x^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(a^4*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^(3/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2))^(3/2),x)
```

```
[Out] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2), x)
```

3.799 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}} \, dx$

Optimal. Leaf size=1039

$$-\frac{(ax+1)^{\frac{n-3}{2}}\left(1-a^2x^2\right)^{5/2}(1-ax)^{\frac{1}{2}(-n-3)}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{(n+4)(ax+1)^{\frac{n-3}{2}}\left(1-a^2x^2\right)^{5/2}(1-ax)^{\frac{1}{2}(-n-3)}}{a^3(n+3)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{3(n+4)(ax+1)^{\frac{n-3}{2}}}{a^5(n+3)}$$

```
[Out] (4+n)*(-a*x+1)^(-3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(5/2)/a^3/(3+
n)/(c-c/a^2/x^2)^(5/2)/x^2-(-a*x+1)^(-3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2
*x^2+1)^(5/2)/a^2/(c-c/a^2/x^2)^(5/2)/x+n*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-3
/2+1/2*n)*(-a^2*x^2+1)^(5/2)/a^6/(1+n)/(c-c/a^2/x^2)^(5/2)/x^5-3*(2-n)*(4+n
)*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(5/2)/a^6/(-n^2+9
)/(c-c/a^2/x^2)^(5/2)/x^5-3*(4+n)*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-3/2+1/2*n
)*(-a^2*x^2+1)^(5/2)/a^5/(3+n)/(c-c/a^2/x^2)^(5/2)/x^4-2*n*(-a*x+1)^(1/2-1/
2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(5/2)/a^6/(-n^2+1)/(c-c/a^2/x^2)^(5/
2)/x^5+2*n*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(5/2)/a^6
/(n^3-3*n^2-n+3)/(c-c/a^2/x^2)^(5/2)/x^5-3*n*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^
(-1/2+1/2*n)*(-a^2*x^2+1)^(5/2)/a^6/(1+n)/(c-c/a^2/x^2)^(5/2)/x^5+3*(4+n)*(
-n^2+2*n+1)*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1)^(5/2)/a
^6/(-n^3-n^2+9*n+9)/(c-c/a^2/x^2)^(5/2)/x^5+3*n*(-a*x+1)^(1/2-1/2*n)*(a*x+1
)^(-1/2+1/2*n)*(-a^2*x^2+1)^(5/2)/a^6/(-n^2+1)/(c-c/a^2/x^2)^(5/2)/x^5-3*(4
+n)*(-n^2+2*n+1)*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1)^(5/
2)/a^6/(n^4-10*n^2+9)/(c-c/a^2/x^2)^(5/2)/x^5+3*n*(-a*x+1)^(-1/2-1/2*n)*(a*
x+1)^(1/2+1/2*n)*(-a^2*x^2+1)^(5/2)/a^6/(1+n)/(c-c/a^2/x^2)^(5/2)/x^5-2^(3/
2+1/2*n)*n*(-a*x+1)^(-1/2-1/2*n)*(-a^2*x^2+1)^(5/2)*hypergeom([-1/2-1/2*n,
-1/2-1/2*n],[1/2-1/2*n],-1/2*a*x+1/2)/a^6/(1+n)/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.77, antiderivative size = 1039, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6160, 6150, 100, 159, 128, 45, 37, 69, 94, 90, 79}

$$-\frac{(ax+1)^{\frac{n-3}{2}}\left(1-a^2x^2\right)^{5/2}(1-ax)^{\frac{1}{2}(-n-3)}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{(n+4)(ax+1)^{\frac{n-3}{2}}\left(1-a^2x^2\right)^{5/2}(1-ax)^{\frac{1}{2}(-n-3)}}{a^3(n+3)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{3(n+4)(ax+1)^{\frac{n-3}{2}}}{a^5(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((4 + n)*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))
/(a^3*(3 + n)*(c - c/(a^2*x^2))^(5/2)*x^2) - ((1 - a*x)^((-3 - n)/2)*(1 + a
```

```
*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^2*(c - c/(a^2*x^2))^(5/2)*x) + (n*
(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1
+ n)*(c - c/(a^2*x^2))^(5/2)*x^5) - (3*(2 - n)*(4 + n)*(1 - a*x)^((-1 - n)/
2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(9 - n^2)*(c - c/(a^2*x
^2))^(5/2)*x^5) - (3*(4 + n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*
(1 - a^2*x^2)^(5/2))/(a^5*(3 + n)*(c - c/(a^2*x^2))^(5/2)*x^4) - (2*n*(1 -
a*x)^((1 - n)/2)*(1 + a*x)^((-3 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1 - n^2)
*(c - c/(a^2*x^2))^(5/2)*x^5) + (2*n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 +
 n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1 + n)*(3 - 4*n + n^2)*(c - c/(a^2*x^2))^
(5/2)*x^5) - (3*n*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^
2)^(5/2))/(a^6*(1 + n)*(c - c/(a^2*x^2))^(5/2)*x^5) + (3*(4 + n)*(1 + 2*n -
 n^2)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(5/2))/(a
^6*(3 - n)*(1 + n)*(3 + n)*(c - c/(a^2*x^2))^(5/2)*x^5) + (3*n*(1 - a*x)^((
1 - n)/2)*(1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(1 - n^2)*(c - c
/(a^2*x^2))^(5/2)*x^5) - (3*(4 + n)*(1 + 2*n - n^2)*(1 - a*x)^((1 - n)/2)*(
1 + a*x)^((-1 + n)/2)*(1 - a^2*x^2)^(5/2))/(a^6*(9 - 10*n^2 + n^4)*(c - c/(
a^2*x^2))^(5/2)*x^5) + (3*n*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((1 + n)/2)*(1
 - a^2*x^2)^(5/2))/(a^6*(1 + n)*(c - c/(a^2*x^2))^(5/2)*x^5) - (2^((3 + n)/
2)*n*(1 - a*x)^((-1 - n)/2)*(1 - a^2*x^2)^(5/2)*Hypergeometric2F1[(-1 - n)/
2, (-1 - n)/2, (1 - n)/2, (1 - a*x)/2])/(a^6*(1 + n)*(c - c/(a^2*x^2))^(5/2
)*x^5)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 128

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[
m, 0] && ILtQ[n, 0]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\left(1-a^2x^2\right)^{5/2}\int \frac{e^{n\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{\left(1-a^2x^2\right)^{5/2}\int x^5(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{\left(1-a^2x^2\right)^{5/2}\int x^3(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{\left(n\left(1-a^2x^2\right)^{5/2}\right)\int x^3(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-}}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x\right.}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x\right.}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x\right.}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x\right.}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}$$

$$= \frac{(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x^2\right)^{5/2}}{a^3(3+n)\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\left(1-a^2x\right.}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}$$

Mathematica [A] time = 6.32, size = 227, normalized size = 0.22

$$\frac{\left(a^2x^2-1\right)^2\left(-\frac{4\left(a^2x^2-1\right)\left(\frac{2ne^{(n+1)\tanh^{-1}(ax)}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-e^{2\tanh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}-(n+1)e^{n\tanh^{-1}(ax)}\right)}{n+1}-\frac{e^{n\tanh^{-1}(ax)}\left(3\left(n^2-1\right)\sqrt{1-a^2x^2}\cosh\left(3\tanh^{-1}(a\right.\right.}{}\right.}{4a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((-1 + a^2*x^2)^2*((-8*E^(n*ArcTanh[a*x])*(-1 + a*n*x))/(-1 + n^2) - (E^(n*
ArcTanh[a*x])*(-9 + n^2 + 10*a*n*x - 2*a*n^3*x - 2*a*n*(-1 + n^2)*x*Cosh[2*
ArcTanh[a*x]] + 3*(-1 + n^2)*Sqrt[1 - a^2*x^2]*Cosh[3*ArcTanh[a*x]]))/(9 -
10*n^2 + n^4) - (4*(-1 + a^2*x^2)*(-(E^(n*ArcTanh[a*x])*(1 + n)) + (2*E^((1
 + n)*ArcTanh[a*x])*n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E^(2*ArcT
anh[a*x])])/Sqrt[1 - a^2*x^2]))/(1 + n)))/(4*a^6*(c - c/(a^2*x^2))^(5/2)*x^
5)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^6x^6\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(a^6*x^6*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^(5/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)}}{\left(c-\frac{c}{a^{2} x^{2}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x)
[Out] int(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{\left(c-\frac{c}{a^{2} x^{2}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n \operatorname{atanh}(a\, x)}}{\left(c-\frac{c}{a^{2}\, x^{2}}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2))^(5/2),x)
[Out] int(exp(n*atanh(a*x))/(c - c/(a^2*x^2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(c-c/a**2/x**2)**(5/2),x)
[Out] Timed out
```

3.800 $\int e^{n \tanh^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^p \, dx$

Optimal. Leaf size=72

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p F_1\left(1-2p;\frac{1}{2}(n-2p),-\frac{n}{2}-p;2-2p;ax,-ax\right)}{1-2p}$$

```
[Out] (c-c/a^2/x^2)^p*x*AppellF1(1-2*p,1/2*n-p,-1/2*n-p,2-2*p,a*x,-a*x)/(1-2*p)/(
(-a^2*x^2+1)^p)
```

Rubi [A] time = 0.13, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6160, 6150, 133}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p F_1\left(1-2p;\frac{1}{2}(n-2p),-\frac{n}{2}-p;2-2p;ax,-ax\right)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*AppellF1[1 - 2*p, (n - 2*p)/2, -n/2 - p, 2 - 2*p, a*
x, -(a*x)])/((1 - 2*p)*(1 - a^2*x^2)^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
```

```
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx &= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{n\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx\\ &= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1-ax)^{-\frac{n}{2}+p}(1+ax)^{\frac{n}{2}+p} dx\\ &= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p} F_1\left(1-2p;\frac{1}{2}(n-2p),-\frac{n}{2}-p;2-2p;ax,-ax\right)}{1-2p}\end{aligned}$$

Mathematica [F] time = 0.39, size = 0, normalized size = 0.00

$$\int e^{n\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
[Out] Integrate[E^(n*ArcTanh[a*x])*(c - c/(a^2*x^2))^p, x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} \left(c - \frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x)
[Out] int(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2x^2}\right)^p \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{atanh}(a\,x)} \left(c - \frac{c}{a^2\,x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2))^p,x)
[Out] int(exp(n*atanh(a*x))*(c - c/(a^2*x^2))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)\right)^p e^{n \operatorname{atanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(c-c/a**2/x**2)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(n*atanh(a*x)), x)
```

3.801 $\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=339

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(\frac{1}{2}(1-2p),2-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{6a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2}{3-}$$

```
[Out] 2*a*(c-c/a^2/x^2)^p*x^2/(1-p)/(-a*x+1)/(a*x+1)+(c-c/a^2/x^2)^p*x*hypergeom(
[2-p, 1/2-p],[3/2-p],a^2*x^2)/(1-2*p)/((-a*x+1)^p)/((a*x+1)^p)+6*a^2*(c-c/a
^2/x^2)^p*x^3*hypergeom([2-p, 3/2-p],[5/2-p],a^2*x^2)/(3-2*p)/((-a*x+1)^p)/
((a*x+1)^p)+a^4*(c-c/a^2/x^2)^p*x^5*hypergeom([2-p, 5/2-p],[7/2-p],a^2*x^2)
/(5-2*p)/((-a*x+1)^p)/((a*x+1)^p)+2*a^3*(c-c/a^2/x^2)^p*x^4*hypergeom([2-p,
 2-p],[3-p],a^2*x^2)/(2-p)/((-a*x+1)^p)/((a*x+1)^p)
```

Rubi [A] time = 0.34, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6159, 6129, 127, 95, 125, 364}

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(\frac{1}{2}(1-2p),2-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{6a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2}{3-}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] (2*a*(c - c/(a^2*x^2))^p*x^2)/((1 - p)*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*
x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 2 - p, (3 - 2*p)/2, a^2*x^2])/((1
- 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (6*a^2*(c - c/(a^2*x^2))^p*x^3*Hypergeome
tric2F1[(3 - 2*p)/2, 2 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1 - a*x)^p*(
1 + a*x)^p) + (a^4*(c - c/(a^2*x^2))^p*x^5*Hypergeometric2F1[(5 - 2*p)/2, 2
 - p, (7 - 2*p)/2, a^2*x^2])/((5 - 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (2*a^3*(
c - c/(a^2*x^2))^p*x^4*Hypergeometric2F1[2 - p, 2 - p, 3 - p, a^2*x^2])/((2
 - p)*(1 - a*x)^p*(1 + a*x)^p)
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b
*x)^(m - n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d,
 0] && IGtQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int e^{4\tanh^{-1}(ax)}x^{-2p}(1-ax)^p(1+ax)^p\,dx$$
$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-2+p}(1+ax)^{2+p}\,dx$$
$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int \left(4ax^{1-2p}(1-ax)^{-2+p}(1+ax)^{-2+p}+6a\right.$$
$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-2+p}(1+ax)^{-2+p}\,dx+\left(4a\right.$$
$$= \frac{2a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{(1-p)(1-ax)(1+ax)}+\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}\left(1-a^2x\right.$$
$$= \frac{2a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{(1-p)(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^p x(1-ax)^{-p}(1+ax)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),2-p\right.}{1-2p}$$

Mathematica [C] time = 0.21, size = 217, normalized size = 0.64

$$-\frac{x(1-ax)^{-p}\left(-\left(a^2x^2-1\right)^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(-4(ax+1)(ax-1)^p\left(1-a^2x^2\right)^p F_1(1-2p;1-p,-p;2-2p;ax,-ax)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -(((c - c/(a^2*x^2))^p*x*(-4*(-1 + a*x)^p*(1 + a*x)*(1 - a^2*x^2)^p*AppellF
1[1 - 2*p, 1 - p, -p, 2 - 2*p, a*x, -(a*x)] + 4*(-1 + a*x)^p*(1 + a*x)^(2*p
)*(1 - a^2*x^2)^p*Hypergeometric2F1[1 - 2*p, 2 - p, 2 - 2*p, (2*a*x)/(1 + a
*x)] + (1 - a*x)^p*(1 + a*x)*(-1 + a^2*x^2)^p*Hypergeometric2F1[1/2 - p, -p
, 3/2 - p, a^2*x^2]))/((-1 + 2*p)*(1 - a*x)^p*(1 + a*x)*(-(-1 + a^2*x^2)^2)
^p))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a^2*x^2 - 2*a
*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4\left(c-\frac{c}{a^2x^2}\right)^p}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^4*(c - c/(a^2*x^2))^p/(a^2*x^2 - 1)^2, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4\left(c-\frac{c}{a^2x^2}\right)^p}{\left(-a^2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x)
```

```
[Out] int((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4\left(c-\frac{c}{a^2x^2}\right)^p}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^4*(c - c/(a^2*x^2))^p/(a^2*x^2 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^p\,(a\,x+1)^4}{\left(a^2\,x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^p*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] int(((c - c/(a^2*x^2))^p*(a*x + 1)^4)/(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p(ax+1)^2}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(c-c/a**2/x**2)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

3.802 $\int e^{3\tanh^{-1}(ax)} \left(c-\frac{c}{a^2x^2}\right)^p dx$

Optimal. Leaf size=217

$$\frac{a(5-2p)x^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(1-p,\frac{3}{2}-p;2-p;a^2x^2\right)}{2(1-p)}+\frac{3a^2x^3\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(\frac{1}{2}(3-2p),\right.}{3-2p}$$

```
[Out] 3*a^2*(c-c/a^2/x^2)^p*x^3*hypergeom([3/2-p, 3/2-p],[5/2-p],a^2*x^2)/(3-2*p)
/((-a^2*x^2+1)^p)+1/2*a*(5-2*p)*(c-c/a^2/x^2)^p*x^2*hypergeom([1-p, 3/2-p],
[2-p],a^2*x^2)/(1-p)/((-a^2*x^2+1)^p)+(c-c/a^2/x^2)^p*x/(1-2*p)/(-a^2*x^2+1
)^(1/2)-a*(c-c/a^2/x^2)^p*x^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6160, 6148, 1809, 1808, 364, 807}

$$\frac{3a^2x^3\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(\frac{1}{2}(3-2p),\frac{3}{2}-p;\frac{1}{2}(5-2p);a^2x^2\right)}{3-2p}+\frac{a(5-2p)x^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(1\right.}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) - (a*(c - c/(a^2*x^2)
)^p*x^2)/Sqrt[1 - a^2*x^2] + (3*a^2*(c - c/(a^2*x^2))^p*x^3*Hypergeometric2
F1[(3 - 2*p)/2, 3/2 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1 - a^2*x^2)^p)
 + (a*(5 - 2*p)*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p, 3/2 - p, 2
 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1808

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> With
[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/c^q, Int[(c*x)^(m + q)*(a + b*x^2
)^p, x], x] + Dist[1/c^q, Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[c^q*Pq - Co
eff[Pq, x, q]*(c*x)^q, x], x], x] /; EqQ[q, 1] || EqQ[m + q + 2*p + 1, 0]]
/; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] &&  !(IGtQ[m, 0] && ILtQ[p + 1
/2, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{3\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p} dx$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-a^2-a^3\right.}{a^2}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(-a^2-a^3(5-2p)x\right)\left(1-\right.}{a^2}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x}{(1-2p)\sqrt{1-a^2x^2}} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}} + \frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^p x^3\left(1-a^2x^2\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(3-\right.}{3-2p}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x}{(1-2p)\sqrt{1-a^2x^2}} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}} + \frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^p x^3\left(1-a^2x^2\right)^{-p}\ {}_2F_1\left(\frac{1}{2}(3-\right.}{3-2p}$$

Mathematica [A] time = 0.16, size = 175, normalized size = 0.81

$$\frac{1}{2}x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(-\frac{3ax\,{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;a^2x^2\right)}{p-1} + \frac{6a^2x^2\,{}_2F_1\left(\frac{3}{2}-p,\frac{3}{2}-p;\frac{5}{2}-p;a^2x^2\right)}{3-2p} + \frac{2\left(1-\right.}{1-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*((2*(1 - a^2*x^2)^(-1/2 + p))/(1 - 2*p) - (3*a*x*Hyp
ergeometric2F1[1 - p, 3/2 - p, 2 - p, a^2*x^2])/(-1 + p) + (6*a^2*x^2*Hyper
geometric2F1[3/2 - p, 3/2 - p, 5/2 - p, a^2*x^2])/(3 - 2*p) + (a^3*x^3*Hype
rgeometric2F1[3/2 - p, 2 - p, 3 - p, a^2*x^2])/(2 - p)))/(2*(1 - a^2*x^2)^p
)
```

fricas [F] time = 1.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{a^2x^2-2\,ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a^2*x^
2 - 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^p/(-a^2*x^2 + 1)^(3/2), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(c-\frac{c}{a^2x^2}\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(c-c/a^2/x^2)^p,x, algorithm="maxima
")
```

```
[Out] integrate((a*x + 1)^3*(c - c/(a^2*x^2))^p/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^p\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - c/(a^2*x^2))^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{p}(ax+1)^{3}}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(c-c/a**2/x**2)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x
 + 1))**(3/2), x)
```

3.803 $\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=217

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{ax^2(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1}{1-p}$$

```
[Out] (c-c/a^2/x^2)^p*x*hypergeom([1-p, 1/2-p],[3/2-p],a^2*x^2)/(1-2*p)/((-a*x+1)
^p)/((a*x+1)^p)+a^2*(c-c/a^2/x^2)^p*x^3*hypergeom([1-p, 3/2-p],[5/2-p],a^2*
x^2)/(3-2*p)/((-a*x+1)^p)/((a*x+1)^p)+a*(c-c/a^2/x^2)^p*x^2*hypergeom([1-p,
 1-p],[2-p],a^2*x^2)/(1-p)/((-a*x+1)^p)/((a*x+1)^p)
```

Rubi [A] time = 0.26, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6159, 6129, 127, 125, 364}

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F}{3-2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1 - p, (3 - 2*p)/2, a
^2*x^2])/((1 - 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (a^2*(c - c/(a^2*x^2))^p*x^3
*Hypergeometric2F1[(3 - 2*p)/2, 1 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1
 - a*x)^p*(1 + a*x)^p) + (a*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p
, 1 - p, 2 - p, a^2*x^2])/((1 - p)*(1 - a*x)^p*(1 + a*x)^p)
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b
*x)^(m - n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d,
 0] && IGtQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int e^{2\tanh^{-1}(ax)}x^{-2p}(1-ax)^p(1+ax)^p\,dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-1+p}(1+ax)^{1+p}\,dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int \left(2ax^{1-2p}(1-ax)^{-1+p}(1+ax)^{-1+p}+a^2x^2\right.$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx+\left(2a\left(c\right.\right.$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-1+p}\,dx+\left(2a\left(c-\frac{c}{a^2x^2}\right)\right.$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x(1-ax)^{-p}(1+ax)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{a^2\left(c-\right.}{}$$

Mathematica [C] time = 0.11, size = 142, normalized size = 0.65

$$\frac{x(1-ax)^{-p}\left(-\left(a^2x^2-1\right)^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left((1-ax)^p\left(a^2x^2-1\right)^p {}_2F_1\left(\frac{1}{2}-p,-p;\frac{3}{2}-p;a^2x^2\right)-2(ax-1)^p\left(1-a^2x\right.\right.}{2p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*(-2*(-1 + a*x)^p*(1 - a^2*x^2)^p*AppellF1[1 - 2*p, 1
 - p, -p, 2 - 2*p, a*x, -(a*x)] + (1 - a*x)^p*(-1 + a^2*x^2)^p*Hypergeometr
ic2F1[1/2 - p, -p, 3/2 - p, a^2*x^2]))/((-1 + 2*p)*(1 - a*x)^p*(-(-1 + a^2*
x^2)^2)^p)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax+1)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-(a*x + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2*(c - c/(a^2*x^2))^p/(a^2*x^2 - 1), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^p}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(c-\frac{c}{a^2x^2}\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^p,x, algorithm="maxima")
[Out] -integrate((a*x + 1)^2*(c - c/(a^2*x^2))^p/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(c-\frac{c}{a^2\,x^2}\right)^p\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - c/(a^2*x^2))^p*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] int(-((c - c/(a^2*x^2))^p*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 11.35, size = 697, normalized size = 3.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(c-c/a**2/x**2)**p,x)
[Out] -a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 + a*a**(-2*p)*c**p*p*x**3
*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 3/2)*hyper((1 - p, 3/2 - p), (5/2
 - p,), a**2*x**2)/(2*gamma(p - 1/2)*gamma(p + 1)) - a**(-2*p)*c**p*p*x**2*
x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,),
 a**2*x**2)/(2*gamma(2 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a
 - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2)
- 0**p*atanh(1/(a*x))/a**2 + a*a**(-2*p)*c**p*p*x**3*x**(-2*p)*exp(I*pi*p)*
gamma(p)*gamma(p - 3/2)*hyper((1 - p, 3/2 - p), (5/2 - p,), a**2*x**2)/(2*g
amma(p - 1/2)*gamma(p + 1)) - a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*g
amma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2
- p)*gamma(p + 1)), True)) - Piecewise((0**p*log(a**2*x**2 - 1)/(2*a) - 0**
p*acoth(a*x)/a - a*a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gam
ma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(
p + 1)) + a**(-2*p)*c**p*p*x*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 1/2)*
```

```
hyper((1 - p, 1/2 - p), (3/2 - p,), a**2*x**2)/(2*gamma(p + 1/2)*gamma(p +
1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a) - 0**p*atanh(a*x)
/a - a*a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hy
per((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(p + 1)) + a*
*(-2*p)*c**p*p*x*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 1/2)*hyper((1 - p
, 1/2 - p), (3/2 - p,), a**2*x**2)/(2*gamma(p + 1/2)*gamma(p + 1)), True))
```

3.804 $\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=137

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{ax^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(\frac{1}{2}-p,1-p;2\right.}{2(1-p)}$$

```
[Out] (c-c/a^2/x^2)^p*x*hypergeom([1/2-p, 1/2-p],[3/2-p],a^2*x^2)/(1-2*p)/((-a^2*
x^2+1)^p)+1/2*a*(c-c/a^2/x^2)^p*x^2*hypergeom([1-p, 1/2-p],[2-p],a^2*x^2)/(
1-p)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6160, 6148, 808, 364}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{ax^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(\frac{1}{2}-p,1-p;2\right.}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^p,x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1/2 - p, (3 - 2*p)/2,
 a^2*x^2])/((1 - 2*p)*(1 - a^2*x^2)^p) + (a*(c - c/(a^2*x^2))^p*x^2*Hyperge
ometric2F1[1/2 - p, 1 - p, 2 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
```

```
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx + \left(a\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1\right.\right.$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p} {}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x}{}$$

Mathematica [A] time = 0.04, size = 112, normalized size = 0.82

$$-\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(2(p-1)\,{}_2F_1\left(\frac{1}{2}-p,\frac{1}{2}-p;\frac{3}{2}-p;a^2x^2\right)+a(2p-1)x\,{}_2F_1\left(\frac{1}{2}-p,1-p;2-p;a^2x^2\right)\right)}{2(p-1)(2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -1/2*((c - c/(a^2*x^2))^p*x*(2*(-1 + p)*Hypergeometric2F1[1/2 - p, 1/2 - p,
 3/2 - p, a^2*x^2] + a*(-1 + 2*p)*x*Hypergeometric2F1[1/2 - p, 1 - p, 2 - p
, a^2*x^2]))/((-1 + p)*(-1 + 2*p)*(1 - a^2*x^2)^p)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^p/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x)
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(c-c/a^2/x^2)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^p/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^p\,(a\,x+1)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - c/(a^2*x^2))^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)

[Out] int(((c - c/(a^2*x^2))^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)

sympy [C] time = 14.93, size = 178, normalized size = 1.30

$$\frac{ac^{p}x^{2}\Gamma\left(p+\frac{1}{2}\right){}_{3}F_{2}\left(\begin{matrix}\frac{1}{2},1,1\\2,p+1\end{matrix}\middle|a^{2}x^{2}e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(p+1\right)}+\frac{c^{p}x\Gamma\left(p+\frac{1}{2}\right){}_{3}F_{2}\left(\begin{matrix}-\frac{1}{2},1,-p\\\frac{1}{2},\frac{1}{2}\end{matrix}\middle|\frac{e^{2i\pi}}{a^{2}x^{2}}\right)}{\sqrt{\pi}\,\Gamma\left(p+1\right)}+\frac{c^{p}x\Gamma\left(p+\frac{1}{2}\right){}_{3}F_{2}\left(\begin{matrix}\frac{1}{2},\frac{1}{2},1\\\frac{3}{2},p+1\end{matrix}\middle|a^{2}x^{2}e^{2i\pi}\right)}{\sqrt{\pi}\,\Gamma\left(p+1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(c-c/a**2/x**2)**p,x)

[Out] a*c**p*x**2*gamma(p + 1/2)*hyper((1/2, 1, 1), (2, p + 1), a**2*x**2*exp_pol
ar(2*I*pi))/(2*sqrt(pi)*gamma(p + 1)) + c**p*x*gamma(p + 1/2)*hyper((-1/2,
1, -p), (1/2, 1/2), exp_polar(2*I*pi)/(a**2*x**2))/(sqrt(pi)*gamma(p + 1))
+ c**p*x*gamma(p + 1/2)*hyper((1/2, 1/2, 1), (3/2, p + 1), a**2*x**2*exp_po
lar(2*I*pi))/(sqrt(pi)*gamma(p + 1)) - c**p*meijerg(((-1, p), (1,)), ((-1,
0), (-1/2,)), exp_polar(I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a*gamma(-p)*ga
mma(p + 1))

3.805 $\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=137

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}-\frac{ax^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}-p,1-p;2-\right.}{2(1-p)}$$

```
[Out] (c-c/a^2/x^2)^p*x*hypergeom([1/2-p, 1/2-p],[3/2-p],a^2*x^2)/(1-2*p)/((-a^2*
x^2+1)^p)-1/2*a*(c-c/a^2/x^2)^p*x^2*hypergeom([1-p, 1/2-p],[2-p],a^2*x^2)/(
1-p)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6160, 6149, 808, 364}

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}-\frac{ax^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}-p,1-p;2-\right.}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^ArcTanh[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1/2 - p, (3 - 2*p)/2,
 a^2*x^2])/((1 - 2*p)*(1 - a^2*x^2)^p) - (a*(c - c/(a^2*x^2))^p*x^2*Hyperge
ometric2F1[1/2 - p, 1 - p, 2 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
```

```
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] &&  !IntegerQ[p] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{-\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-\frac{1}{2}+p} dx - \left(a\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1\right.\right.$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x\left(1-a^2x^2\right)^{-p} {}_2F_1\left(\frac{1}{2}(1-2p),\frac{1}{2}-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^p}{}$$

Mathematica [A] time = 0.03, size = 112, normalized size = 0.82

$$\frac{x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(a(2p-1)x\,{}_2F_1\left(\frac{1}{2}-p,1-p;2-p;a^2x^2\right)-2(p-1)\,{}_2F_1\left(\frac{1}{2}-p,\frac{1}{2}-p;\frac{3}{2}-p;a^2x^2\right)\right)}{2(p-1)(2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^ArcTanh[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*(-2*(-1 + p)*Hypergeometric2F1[1/2 - p, 1/2 - p, 3/2
 - p, a^2*x^2] + a*(-1 + 2*p)*x*Hypergeometric2F1[1/2 - p, 1 - p, 2 - p, a^
2*x^2]))/(2*(-1 + p)*(-1 + 2*p)*(1 - a^2*x^2)^p)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^p/(a*x + 1), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^p\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
[Out] int((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(c-\frac{c}{a^2x^2}\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*x^2 + 1)*(c - c/(a^2*x^2))^p/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^p\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{p}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p/(a
*x + 1), x)
```

3.806 $\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=218

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}-\frac{ax^2(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\right.}{1-p}$$

```
[Out] (c-c/a^2/x^2)^p*x*hypergeom([1-p, 1/2-p],[3/2-p],a^2*x^2)/(1-2*p)/((-a*x+1)
^p)/((a*x+1)^p)+a^2*(c-c/a^2/x^2)^p*x^3*hypergeom([1-p, 3/2-p],[5/2-p],a^2*
x^2)/(3-2*p)/((-a*x+1)^p)/((a*x+1)^p)-a*(c-c/a^2/x^2)^p*x^2*hypergeom([1-p,
 1-p],[2-p],a^2*x^2)/(1-p)/((-a*x+1)^p)/((a*x+1)^p)
```

Rubi [A] time = 0.26, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6159, 6129, 127, 125, 364}

$$\frac{x(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p}+\frac{a^2x^3(1-ax)^{-p}(ax+1)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\,{}_2F_1}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*Hypergeometric2F1[(1 - 2*p)/2, 1 - p, (3 - 2*p)/2, a
^2*x^2])/((1 - 2*p)*(1 - a*x)^p*(1 + a*x)^p) + (a^2*(c - c/(a^2*x^2))^p*x^3
*Hypergeometric2F1[(3 - 2*p)/2, 1 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1
 - a*x)^p*(1 + a*x)^p) - (a*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p
, 1 - p, 2 - p, a^2*x^2])/((1 - p)*(1 - a*x)^p*(1 + a*x)^p)
```

Rule 125

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0] && GtQ[a, 0] && GtQ[c, 0
]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^n*(c + d*x)^n*(f*x)^p, (a + b
*x)^(m - n), x], x] /; FreeQ[{a, b, c, d, f, m, n, p}, x] && EqQ[b*c + a*d,
 0] && IGtQ[m - n, 0] && NeQ[m + n + p + 2, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int e^{-2\tanh^{-1}(ax)}x^{-2p}(1-ax)^p(1+ax)^p\,dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{1+p}(1+ax)^{-1+p}\,dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int \left(-2ax^{1-2p}(1-ax)^{-1+p}(1+ax)^{-1+p} + \right.$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx - \left(2a\right.$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}(1-ax)^{-p}(1+ax)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-1+p}\,dx - \left(2a\left(c-\frac{c}{a^2x}\right.\right.$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x(1-ax)^{-p}(1+ax)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),1-p;\frac{1}{2}(3-2p);a^2x^2\right)}{1-2p} + \frac{a^2\left(c\right.}{}$$

Mathematica [C] time = 0.11, size = 142, normalized size = 0.65

$$\frac{x(1-ax)^{-p}\left(-\left(a^2x^2-1\right)^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left((1-ax)^p\left(a^2x^2-1\right)^p\,{}_2F_1\left(\frac{1}{2}-p,-p;\frac{3}{2}-p;a^2x^2\right)-2(ax-1)^p\left(1-a^2x^2\right)\right.}{2p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*(-2*(-1 + a*x)^p*(1 - a^2*x^2)^p*AppellF1[1 - 2*p, -
p, 1 - p, 2 - 2*p, a*x, -(a*x)] + (1 - a*x)^p*(-1 + a^2*x^2)^p*Hypergeometr
ic2F1[1/2 - p, -p, 3/2 - p, a^2*x^2]))/((-1 + 2*p)*(1 - a*x)^p*(-(-1 + a^2*
x^2)^2)^p)
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax-1)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a*x - 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*(c - c/(a^2*x^2))^p/(a*x + 1)^2, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^p\left(-a^2x^2+1\right)}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

[Out] int((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2x^2-1\right)\left(c-\frac{c}{a^2x^2}\right)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")

[Out] -integrate((a^2*x^2 - 1)*(c - c/(a^2*x^2))^p/(a*x + 1)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^p\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((c - c/(a^2*x^2))^p*(a^2*x^2 - 1))/(a*x + 1)^2,x)

[Out] -int(((c - c/(a^2*x^2))^p*(a^2*x^2 - 1))/(a*x + 1)^2, x)

sympy [C] time = 11.77, size = 695, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a**2/x**2)**p/(a*x+1)**2*(-a**2*x**2+1),x)

[Out] -a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 + a*a**(-2*p)*c**p*p*x**3*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 3/2)*hyper((1 - p, 3/2 - p), (5/2 - p,), a**2*x**2)/(2*gamma(p - 1/2)*gamma(p + 1)) + a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))/a**2 + a*a**(-2*p)*c**p*p*x**3*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 3/2)*hyper((1 - p, 3/2 - p), (5/2 - p,), a**2*x**2)/(2*gamma(p - 1/2)*gamma(p + 1)) + a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(p + 1)), True)) + Piecewise((0**p*log(a**2*x**2 - 1)/(2*a) + 0**p*acoth(a*x)/a - a*a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(

```
p + 1)) - a**(-2*p)*c**p*p*x*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 1/2)*
hyper((1 - p, 1/2 - p), (3/2 - p,), a**2*x**2)/(2*gamma(p + 1/2)*gamma(p +
1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a) + 0**p*atanh(a*x)
/a - a*a**(-2*p)*c**p*p*x**2*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hy
per((1 - p, 1 - p), (2 - p,), a**2*x**2)/(2*gamma(2 - p)*gamma(p + 1)) - a*
*(-2*p)*c**p*p*x*x**(-2*p)*exp(I*pi*p)*gamma(p)*gamma(p - 1/2)*hyper((1 - p
, 1/2 - p), (3/2 - p,), a**2*x**2)/(2*gamma(p + 1/2)*gamma(p + 1)), True))
```

3.807 $\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=216

$$-\frac{a(5-2p)x^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;a^2x^2\right)}{2(1-p)}+\frac{3a^2x^3\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(\frac{1}{2}(3-2\right.}{3-2p}$$

```
[Out] 3*a^2*(c-c/a^2/x^2)^p*x^3*hypergeom([3/2-p, 3/2-p],[5/2-p],a^2*x^2)/(3-2*p)
/((-a^2*x^2+1)^p)-1/2*a*(5-2*p)*(c-c/a^2/x^2)^p*x^2*hypergeom([1-p, 3/2-p],
[2-p],a^2*x^2)/(1-p)/((-a^2*x^2+1)^p)+(c-c/a^2/x^2)^p*x/(1-2*p)/(-a^2*x^2+1
)^(1/2)+a*(c-c/a^2/x^2)^p*x^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6160, 6149, 1809, 1808, 364, 807}

$$\frac{3a^2x^3\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(\frac{1}{2}(3-2p),\frac{3}{2}-p;\frac{1}{2}(5-2p);a^2x^2\right)}{3-2p}-\frac{a(5-2p)x^2\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(\right.}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) + (a*(c - c/(a^2*x^2)
)^p*x^2)/Sqrt[1 - a^2*x^2] + (3*a^2*(c - c/(a^2*x^2))^p*x^3*Hypergeometric2
F1[(3 - 2*p)/2, 3/2 - p, (5 - 2*p)/2, a^2*x^2])/((3 - 2*p)*(1 - a^2*x^2)^p)
 - (a*(5 - 2*p)*(c - c/(a^2*x^2))^p*x^2*Hypergeometric2F1[1 - p, 3/2 - p, 2
 - p, a^2*x^2])/(2*(1 - p)*(1 - a^2*x^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1808

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> With
[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/c^q, Int[(c*x)^(m + q)*(a + b*x^2
)^p, x], x] + Dist[1/c^q, Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[c^q*Pq - Co
eff[Pq, x, q]*(c*x)^q, x], x], x] /; EqQ[q, 1] || EqQ[m + q + 2*p + 1, 0]]
/; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && !(IGtQ[m, 0] && ILtQ[p + 1
/2, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 6160

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(x^(2*p)*(c + d/x^2)^p)/(1 + (c*x^2)/d)^p, Int[(u*(1 + (c*x^2)/d
)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ
[c + a^2*d, 0] && !IntegerQ[p] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int e^{-3\tanh^{-1}(ax)}x^{-2p}\left(1-a^2x^2\right)^p dx$$

$$= \left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}(1-ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p} dx$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-a^2+a\right.}{a^2}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^p x^{2p}\left(1-a^2x^2\right)^{-p}\right)\int x^{-2p}\left(-a^2+a^3(5-2p)x\right)\left(1-\right.}{a^2}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x}{(1-2p)\sqrt{1-a^2x^2}} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}} + \frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^p x^3\left(1-a^2x^2\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(\right.}{3-2p}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^p x}{(1-2p)\sqrt{1-a^2x^2}} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^p x^2}{\sqrt{1-a^2x^2}} + \frac{3a^2\left(c-\frac{c}{a^2x^2}\right)^p x^3\left(1-a^2x^2\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(\right.}{3-2p}$$

Mathematica [A] time = 0.14, size = 173, normalized size = 0.80

$$\frac{1}{2}x\left(1-a^2x^2\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{3ax\,{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;a^2x^2\right)}{p-1}+\frac{6a^2x^2\,{}_2F_1\left(\frac{3}{2}-p,\frac{3}{2}-p;\frac{5}{2}-p;a^2x^2\right)}{3-2p}+\frac{2\left(1-\right.}{1-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*x*((2*(1 - a^2*x^2)^(-1/2 + p))/(1 - 2*p) + (3*a*x*Hyp
ergeometric2F1[1 - p, 3/2 - p, 2 - p, a^2*x^2])/(-1 + p) + (6*a^2*x^2*Hyper
geometric2F1[3/2 - p, 3/2 - p, 5/2 - p, a^2*x^2])/(3 - 2*p) + (a^3*x^3*Hype
rgeometric2F1[3/2 - p, 2 - p, 3 - p, a^2*x^2])/(-2 + p)))/(2*(1 - a^2*x^2)^
p)
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(ax-1)\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*((a^2*c*x^2 - c)/(a^2*x^2))^p/(a^2*x
^2 + 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^p/(a*x + 1)^3, x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^p\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(c-\frac{c}{a^2x^2}\right)^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(c - c/(a^2*x^2))^p/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^p\,\left(1-a^2\,x^2\right)^{3/2}}{\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^p*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - c/(a^2*x^2))^p*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{p}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p
/(a*x + 1)**3, x)
```

3.808 $\int e^{\tanh^{-1}(x)}x\sqrt{1+x}\,\sin(x)\,dx$

Optimal. Leaf size=240

$$-2\sqrt{2\pi}\,\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-3\sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-3\sqrt{\frac{\pi}{2}}\,\sin(1)S\left(\sqrt{\right.$$

```
[Out] -(1-x)^(3/2)*cos(x)-3/2*cos(1)*FresnelC(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/
2)*Pi^(1/2)+5/4*cos(1)*FresnelS(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/2)*Pi^(1
/2)-5/4*FresnelC(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)-3/2*
FresnelS(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)+3*cos(x)*(1-
x)^(1/2)-3/2*sin(x)*(1-x)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354, 3386}

$$-2\sqrt{2\pi}\,\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-3\sqrt{\frac{\pi}{2}}\,\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*Sqrt[1 + x]*Sin[x],x]
```

```
[Out] 3*Sqrt[1 - x]*Cos[x] - (1 - x)^(3/2)*Cos[x] - 3*Sqrt[Pi/2]*Cos[1]*FresnelC[
Sqrt[2/Pi]*Sqrt[1 - x]] - (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 -
 x]])/2 + 2*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] + (3*Sqrt[Pi
/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1])/2 - 2*Sqrt[2*Pi]*FresnelC[Sqrt
[2/Pi]*Sqrt[1 - x]]*Sin[1] - 3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]]*
Sin[1] - (3*Sqrt[1 - x]*Sin[x])/2
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}x\sqrt{1+x}\,\sin(x)\,dx = \int \frac{x(1+x)\sin(x)}{\sqrt{1-x}}\,dx$$
$$= -\left(2\,\text{Subst}\left(\int \left(-2+x^2\right)\left(-1+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \left(2\sin\left(1-x^2\right) - 3x^2\sin\left(1-x^2\right) + x^4\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1-x}\right.\right.$$
$$= -\left(2\,\text{Subst}\left(\int x^4\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)\right) - 4\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x, \sqrt{\right.$$
$$= 3\sqrt{1-x}\,\cos(x) - (1-x)^{3/2}\cos(x) - 3\,\text{Subst}\left(\int \cos\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right) + 3$$
$$= 3\sqrt{1-x}\,\cos(x) - (1-x)^{3/2}\cos(x) + 2\sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) - 2\sqrt{2\pi}\,C\left(\sqrt{\right.$$
$$= 3\sqrt{1-x}\,\cos(x) - (1-x)^{3/2}\cos(x) - 3\sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) + 2\sqrt{2\pi}\,\cos($$
$$= 3\sqrt{1-x}\,\cos(x) - (1-x)^{3/2}\cos(x) - 3\sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\,\cos($$

Mathematica [C] time = 9.27, size = 184, normalized size = 0.77

$$-\frac{\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{x+1}\,\left((\cos(x+1) - i\sin(x+1))\left((2+2i)\left(2x^2+(2-3i)x-(4-3i)\right)(\cos(1)+i\sin(1)) - (6+5i)\sqrt{\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*x*Sqrt[1 + x]*Sin[x],x]
```

```
[Out] ((-1/16 + I/16)*Sqrt[1 + x]*((-6 + 5*I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erfi[((1 +
I)*Sqrt[-1 + x])/Sqrt[2]]*(Cos[1] + I*Sin[1]) + (2 + 2*I)*((-4 - 3*I) + (2
+ 3*I)*x + 2*x^2)*(Cos[x] + I*Sin[x]) + ((2 + 2*I)*((-4 + 3*I) + (2 - 3*I)*
x + 2*x^2)*(Cos[1] + I*Sin[1]) - (6 + 5*I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erf[((1
+ I)*Sqrt[-1 + x])/Sqrt[2]]*(Cos[x] + I*Sin[x]))*(Cos[1 + x] - I*Sin[1 + x]
)))/Sqrt[1 - x^2]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\,\sqrt{x+1}\,x\sin(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(x + 1)*x*sin(x)/(x - 1), x)
```

giac [C] time = 0.20, size = 124, normalized size = 0.52

$$-\left(\frac{11}{16}i-\frac{1}{16}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{11}{16}i+\frac{1}{16}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] -(11/16*I - 1/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1))
*e^I + (11/16*I + 1/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x
+ 1))*e^(-I) - 1/4*I*(-2*I*(-x + 1)^(3/2) + (4*I - 3)*sqrt(-x + 1))*e^(I*x)
 - 1/4*I*(-2*I*(-x + 1)^(3/2) + (4*I + 3)*sqrt(-x + 1))*e^(-I*x) + 1/2*sqrt
(-x + 1)*e^(I*x) + 1/2*sqrt(-x + 1)*e^(-I*x) + 1.79526793396000
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(1+x)^{\frac{3}{2}}\,x\sin(x)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

```
[Out] int((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

maxima [C] time = 0.44, size = 639, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*((((2*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*
x + I)) - 1))*cos(1) + 2*(sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf
(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + (2*(sqrt(pi)*(e
rf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1) + (-2*I
*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x + I)) - 1)
)*sin(1))*sin(1/2*arctan2(x - 1, 0)))*(x - 1)^2 + (((-I*cos(1) - sin(1))*ga
mma(5/2, I*x - I) + (I*cos(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2*arcta
n2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(5/2, I*x - I) + (cos(1) + I*sin(
1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*x^2 - ((3*((I*cos(1)
```

```
+ sin(1))*gamma(3/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I*x + I))*
cos(3/2*arctan2(x - 1, 0)) + ((3*cos(1) - 3*I*sin(1))*gamma(3/2, I*x - I) +
 (3*cos(1) + 3*I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*
abs(x - 1) + 2*((-I*cos(1) - sin(1))*gamma(5/2, I*x - I) + (I*cos(1) - sin(
1))*gamma(5/2, -I*x + I))*cos(5/2*arctan2(x - 1, 0)) - ((2*cos(1) - 2*I*sin
(1))*gamma(5/2, I*x - I) + (2*cos(1) + 2*I*sin(1))*gamma(5/2, -I*x + I))*si
n(5/2*arctan2(x - 1, 0)))*x - (3*((-I*cos(1) - sin(1))*gamma(3/2, I*x - I)
+ (I*cos(1) - sin(1))*gamma(3/2, -I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((
3*cos(1) - 3*I*sin(1))*gamma(3/2, I*x - I) + (3*cos(1) + 3*I*sin(1))*gamma(
3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*abs(x - 1) + ((-I*cos(1) - sin(
1))*gamma(5/2, I*x - I) + (I*cos(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2
*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(5/2, I*x - I) + (cos(1) +
I*sin(1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*sqrt(-x + 1)/((
x - 1)^2*sqrt(abs(x - 1)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sin(x)\,(x+1)^{3/2}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x)*(x + 1)^(3/2))/(1 - x^2)^(1/2),x)
```

```
[Out] int((x*sin(x)*(x + 1)^(3/2))/(1 - x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(-x**2+1)**(1/2)*x*sin(x),x)
```

```
[Out] Timed out
```

3.809 $\int e^{\tanh^{-1}(x)}\sqrt{1+x}\,\sin(x)\,dx$

Optimal. Leaf size=141

$$-2\sqrt{2\pi}\,\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+2\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\right.$$

```
[Out] -1/2*cos(1)*FresnelC(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/2)*Pi^(1/2)-1/2*Fre
snelS(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)+2*cos(1)*Fresne
lS(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/2)*Pi^(1/2)-2*FresnelC(2^(1/2)/Pi^(1/
2)*(1-x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)+cos(x)*(1-x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354}

$$-2\sqrt{2\pi}\,\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+2\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 + x]*Sin[x],x]
```

```
[Out] Sqrt[1 - x]*Cos[x] - Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]] + 2
*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] - 2*Sqrt[2*Pi]*FresnelC
[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x
]]*Sin[1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}\sqrt{1+x}\,\sin(x)\,dx = \int \frac{(1+x)\sin(x)}{\sqrt{1-x}}\,dx$$
$$= 2\,\text{Subst}\left(\int \left(-2+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-2\sin\left(1-x^2\right)+x^2\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1-x}\right)$$
$$= 2\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right) - 4\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right.$$
$$= \sqrt{1-x}\,\cos(x) + (4\cos(1))\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{1-x}\right) - (4\sin(1))\,\text{Subst}\left(\int\right.$$
$$= \sqrt{1-x}\,\cos(x) + 2\sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) - 2\sqrt{2\pi}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)\sin(1) - c$$
$$= \sqrt{1-x}\,\cos(x) - \sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) + 2\sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) - 2$$

Mathematica [C] time = 8.42, size = 134, normalized size = 0.95

$$\left(\frac{1}{8}+\frac{i}{8}\right)\left(\frac{e^{-ix}\sqrt{1-x^2}\left((4+i)\sqrt{2\pi}\,e^{i(x+1)}\text{erfi}\left(\frac{(1+i)\sqrt{x-1}}{\sqrt{2}}\right)+(2-2i)\sqrt{x-1}\left(1+e^{2ix}\right)\right)}{\sqrt{x-1}\sqrt{x+1}}-(4-i)e^{-i}\sqrt{2\pi}\,\text{erfi}\left(\left(\frac{1}{2}+\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 + x]*Sin[x],x]
```

```
[Out] (1/8 + I/8)*(((-4 + I)*Sqrt[2*Pi]*Erfi[(1/2 + I/2)*Sqrt[2 - 2*x]])/E^I + (S
qrt[1 - x^2]*((2 - 2*I)*(1 + E^((2*I)*x))*Sqrt[-1 + x] + (4 + I)*E^(I*(1 +
x))*Sqrt[2*Pi]*Erfi[((1 + I)*Sqrt[-1 + x])/Sqrt[2]]))/(E^(I*x)*Sqrt[-1 + x]
*Sqrt[1 + x]))
```

fricas [F] time = 2.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\sqrt{x+1}\,\sin(x)}{x-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(x + 1)*sin(x)/(x - 1), x)
```

giac [C] time = 0.18, size = 74, normalized size = 0.52

$$-\left(\frac{5}{8}i+\frac{3}{8}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{5}{8}i-\frac{3}{8}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}+\frac{1}{2}\sqrt{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="giac")
```

```
[Out] -(5/8*I + 3/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1))*e^
I + (5/8*I - 3/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x + 1))*
e^(-I) + 1/2*sqrt(-x + 1)*e^(I*x) + 1/2*sqrt(-x + 1)*e^(-I*x) + 1.992845037
43000
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(1+x)^{\frac{3}{2}}\sin(x)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x)
```

```
[Out] int((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x)
```

maxima [C] time = 0.41, size = 349, normalized size = 2.48

$$-\frac{\left(\left(\left((-i\cos(1)-\sin(1))\Gamma\left(\frac{3}{2},ix-i\right)+(i\cos(1)-\sin(1))\Gamma\left(\frac{3}{2},-ix+i\right)\right)\cos\left(\frac{3}{2}\arctan(x-1,0)\right)-\left((\cos(1)-i\,\mathrm{s}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*((((-I*cos(1) - sin(1))*gamma(3/2, I*x - I) + (I*cos(1) - sin(1))*gamm
a(3/2, -I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(3
/2, I*x - I) + (cos(1) + I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x
- 1, 0)))*x + (((2*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - 2*I*sqrt(pi)*(erf(
sqrt(-I*x + I)) - 1))*cos(1) + 2*(sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(
pi)*(erf(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + (2*(sqr
t(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1)
 + (-2*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x +
I)) - 1))*sin(1))*sin(1/2*arctan2(x - 1, 0)))*abs(x - 1) + ((I*cos(1) + sin
(1))*gamma(3/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I*x + I))*cos(3
/2*arctan2(x - 1, 0)) + ((cos(1) - I*sin(1))*gamma(3/2, I*x - I) + (cos(1)
+ I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*sqrt(-x + 1)*
sqrt(abs(x - 1))/(x - 1)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x)\,(x+1)^{3/2}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*(x + 1)^(3/2))/(1 - x^2)^(1/2),x)
```

```
[Out] int((sin(x)*(x + 1)^(3/2))/(1 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}\sin(x)}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(-x**2+1)**(1/2)*sin(x),x)
```

```
[Out] Integral((x + 1)**(3/2)*sin(x)/sqrt(-(x - 1)*(x + 1)), x)
```

3.810 $\int e^{\tanh^{-1}(x)}\sqrt{1-x}\,x\sin(x)\,dx$

Optimal. Leaf size=163

$$\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\right.$$

```
[Out] -(1+x)^(3/2)*cos(x)-1/2*cos(1)*FresnelC(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*2^(1/
2)*Pi^(1/2)-3/4*cos(1)*FresnelS(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*2^(1/2)*Pi^(1
/2)+3/4*FresnelC(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)-1/2*
FresnelS(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)+cos(x)*(1+x)
^(1/2)+3/2*sin(x)*(1+x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6129, 6742, 3385, 3354, 3352, 3351, 3386, 3353}

$$\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\frac{3}{2}\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 - x]*x*Sin[x],x]
```

```
[Out] Sqrt[1 + x]*Cos[x] - (1 + x)^(3/2)*Cos[x] - Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt
[2/Pi]*Sqrt[1 + x]] - (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]
)/2 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1])/2 - Sqrt[Pi/2]
*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1] + (3*Sqrt[1 + x]*Sin[x])/2
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}\sqrt{1-x}\,x\sin(x)\,dx = \int x\sqrt{1+x}\,\sin(x)\,dx$$
$$= -\left(2\,\text{Subst}\left(\int x^2\left(-1+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \left(-x^2\sin\left(1-x^2\right)+x^4\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1+x}\right)\right)$$
$$= 2\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right) - 2\,\text{Subst}\left(\int x^4\sin\left(1-x^2\right)\,dx, x, \sqrt{\right.$$
$$= \sqrt{1+x}\,\cos(x) - (1+x)^{3/2}\cos(x) + 3\,\text{Subst}\left(\int x^2\cos\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right) -$$
$$= \sqrt{1+x}\,\cos(x) - (1+x)^{3/2}\cos(x) + \frac{3}{2}\sqrt{1+x}\,\sin(x) + \frac{3}{2}\,\text{Subst}\left(\int \sin\left(1-x^2\right)\right.$$
$$= \sqrt{1+x}\,\cos(x) - (1+x)^{3/2}\cos(x) - \sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right) - \sqrt{\frac{\pi}{2}}\,S\left(\sqrt{\frac{2}{\pi}}\right.$$
$$= \sqrt{1+x}\,\cos(x) - (1+x)^{3/2}\cos(x) - \sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\,\cos(1$$

Mathematica [C] time = 8.68, size = 168, normalized size = 1.03

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)e^{-i(x+1)}\sqrt{1-x}\,\left(e^{i}\left((3-2i)\sqrt{2\pi}\,e^{i(x+1)}\sqrt{-x-1}\,\text{erfi}\left(\frac{(1+i)\sqrt{-x-1}}{\sqrt{2}}\right)+(2+2i)\left(e^{2ix}(-3+2ix)+2ix+3\right)(x\right.\right.}{\sqrt{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 - x]*x*Sin[x],x]
```

```
[Out] ((1/16 + I/16)*Sqrt[1 - x]*((-3 - 2*I)*E^(I*x)*Sqrt[2*Pi]*Sqrt[-1 - x]*Erf[
((1 + I)*Sqrt[-1 - x])/Sqrt[2]] + E^I*((2 + 2*I)*(3 + E^((2*I)*x)*(-3 + (2*
I)*x) + (2*I)*x)*(1 + x) + (3 - 2*I)*E^(I*(1 + x))*Sqrt[2*Pi]*Sqrt[-1 - x]*
Erfi[((1 + I)*Sqrt[-1 - x])/Sqrt[2]])))/(E^(I*(1 + x))*Sqrt[1 - x^2])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\,x\sqrt{-x+1}\,\sin(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*x*sqrt(-x + 1)*sin(x)/(x - 1), x)
```

giac [C] time = 0.19, size = 108, normalized size = 0.66

$$\left(\frac{1}{16}i+\frac{5}{16}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{i}-\left(\frac{1}{16}i-\frac{5}{16}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{(-i)}+\frac{1}{4}i\left(2i\,(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] (1/16*I + 5/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*e^
I - (1/16*I - 5/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1))
*e^(-I) + 1/4*I*(2*I*(x + 1)^(3/2) - (4*I + 3)*sqrt(x + 1))*e^(I*x) + 1/4*I
*(2*I*(x + 1)^(3/2) - (4*I - 3)*sqrt(x + 1))*e^(-I*x) - 1/2*sqrt(x + 1)*e^(
I*x) - 1/2*sqrt(x + 1)*e^(-I*x) - 0.537182832596000
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(1+x)\sqrt{1-x}\,x\sin(x)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x)
```

```
[Out] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x)
```

maxima [C] time = 0.50, size = 909, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(((I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x - I
)) - 1))*cos(1) - (sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-
I*x - I)) - 1))*sin(1))*abs(x + 1)*cos(1/2*arctan2(x + 1, 0)) + ((sqrt(pi)*
(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) + (I*
sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*s
in(1))*abs(x + 1)*sin(1/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1))*gamma(
3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x + (I*cos(1) -
sin(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*co
s(3/2*arctan2(x + 1, 0)) + (((cos(1) + I*sin(1))*gamma(3/2, I*x + I) + (cos
(1) - I*sin(1))*gamma(3/2, -I*x - I))*x + (cos(1) + I*sin(1))*gamma(3/2, I*
x + I) + (cos(1) - I*sin(1))*gamma(3/2, -I*x - I))*sin(3/2*arctan2(x + 1, 0
)))*sqrt(abs(x + 1))/(x + 1)^(3/2) - 1/2*(((-I*sqrt(pi)*(erf(sqrt(I*x + I))
```

```
 - 1) + I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) + (sqrt(pi)*(erf(sqrt(
I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*cos(
1/2*arctan2(x + 1, 0)) - ((sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(er
f(sqrt(-I*x - I)) - 1))*cos(1) - (-I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + I*
sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*sin(1/2*arctan2(x + 1
, 0)) - 2*(((I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*
gamma(3/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I*cos
(1) - sin(1))*gamma(3/2, -I*x - I))*abs(x + 1)*cos(3/2*arctan2(x + 1, 0)) -
 (((2*cos(1) + 2*I*sin(1))*gamma(3/2, I*x + I) + (2*cos(1) - 2*I*sin(1))*ga
mma(3/2, -I*x - I))*x + (2*cos(1) + 2*I*sin(1))*gamma(3/2, I*x + I) + (2*co
s(1) - 2*I*sin(1))*gamma(3/2, -I*x - I))*abs(x + 1)*sin(3/2*arctan2(x + 1,
0)) + (((I*cos(1) - sin(1))*gamma(5/2, I*x + I) + (-I*cos(1) - sin(1))*gamm
a(5/2, -I*x - I))*x^2 - 2*((-I*cos(1) + sin(1))*gamma(5/2, I*x + I) + (I*co
s(1) + sin(1))*gamma(5/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(5/2, I*x
 + I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*cos(5/2*arctan2(x + 1, 0
)) + (((cos(1) + I*sin(1))*gamma(5/2, I*x + I) + (cos(1) - I*sin(1))*gamma(
5/2, -I*x - I))*x^2 + ((2*cos(1) + 2*I*sin(1))*gamma(5/2, I*x + I) + (2*cos
(1) - 2*I*sin(1))*gamma(5/2, -I*x - I))*x + (cos(1) + I*sin(1))*gamma(5/2,
I*x + I) + (cos(1) - I*sin(1))*gamma(5/2, -I*x - I))*sin(5/2*arctan2(x + 1,
 0)))/((x + 1)^(3/2)*sqrt(abs(x + 1)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(x)\,\sqrt{1-x}\,\,(x+1)}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x)*(1 - x)^(1/2)*(x + 1))/(1 - x^2)^(1/2),x)
[Out] int((x*sin(x)*(1 - x)^(1/2)*(x + 1))/(1 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{1-x}\,\,(x+1)\sin(x)}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(1/2)*x*sin(x),x)
[Out] Integral(x*sqrt(1 - x)*(x + 1)*sin(x)/sqrt(-(x - 1)*(x + 1)), x)
```

3.811 $\int e^{\tanh^{-1}(x)}\sqrt{1-x}\,\sin(x)\,dx$

Optimal. Leaf size=72

$$\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{x+1}\cos(x)$$

```
[Out] 1/2*cos(1)*FresnelC(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*2^(1/2)*Pi^(1/2)+1/2*Fres
nelS(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)-cos(x)*(1+x)^(1/
2)
```

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6129, 3296, 3306, 3305, 3351, 3304, 3352}

$$\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{x+1}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*Sqrt[1 - x]*Sin[x],x]
```

```
[Out] -(Sqrt[1 + x]*Cos[x]) + Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]]
+ Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(x)}\sqrt{1-x}\,\sin(x)\,dx &= \int \sqrt{1+x}\,\sin(x)\,dx\\ &= -\sqrt{1+x}\,\cos(x)+\frac{1}{2}\int \frac{\cos(x)}{\sqrt{1+x}}\,dx\\ &= -\sqrt{1+x}\,\cos(x)+\frac{1}{2}\cos(1)\int \frac{\cos(1+x)}{\sqrt{1+x}}\,dx+\frac{1}{2}\sin(1)\int \frac{\sin(1+x)}{\sqrt{1+x}}\,dx\\ &= -\sqrt{1+x}\,\cos(x)+\cos(1)\,\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{1+x}\right)+\sin(1)\,\text{Subst}\left(\int \text{si}\right.\\ &= -\sqrt{1+x}\,\cos(x)+\sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\,\sqrt{1+x}\right)+\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{1+x}\right)\sin(1)\end{aligned}$$

Mathematica [C] time = 0.02, size = 77, normalized size = 1.07

$$-\frac{e^{-i}\sqrt{x+1}\,\Gamma\left(\frac{3}{2},-i(x+1)\right)}{2\sqrt{-i(x+1)}}-\frac{e^{i}\sqrt{x+1}\,\Gamma\left(\frac{3}{2},i(x+1)\right)}{2\sqrt{i(x+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*Sqrt[1 - x]*Sin[x],x]
```

```
[Out] -1/2*(Sqrt[1 + x]*Gamma[3/2, (-I)*(1 + x)])/(E^I*Sqrt[(-I)*(1 + x)]) - (E^I
*Sqrt[1 + x]*Gamma[3/2, I*(1 + x)])/(2*Sqrt[I*(1 + x)])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\,\sqrt{-x+1}\,\sin(x)}{x-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(-x + 1)*sin(x)/(x - 1), x)
```

giac [C] time = 0.18, size = 66, normalized size = 0.92

$$\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{i}-\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{(-i)}-\frac{1}{2}\sqrt{x+1}\,e^{(ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x, algorithm="giac")
```

```
[Out] (1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*e^I
- (1/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1))*e^(
-I) - 1/2*sqrt(x + 1)*e^(I*x) - 1/2*sqrt(x + 1)*e^(-I*x) - 0.33960572912500
0
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(1+x)\sqrt{1-x}\,\sin(x)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x)
```

```
[Out] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x)
```

maxima [C] time = 0.44, size = 498, normalized size = 6.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(1/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(((-I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*x -
I)) - 1))*cos(1) + (sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(
-I*x - I)) - 1))*sin(1))*cos(1/2*arctan2(x + 1, 0)) - ((sqrt(pi)*(erf(sqrt(
I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) - (-I*sqrt(pi)*
(erf(sqrt(I*x + I)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*si
n(1/2*arctan2(x + 1, 0)))*sqrt(x + 1)/sqrt(abs(x + 1)) - 1/2*(((I*sqrt(pi)*
(erf(sqrt(I*x + I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) - (
sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin
(1))*abs(x + 1)*cos(1/2*arctan2(x + 1, 0)) + ((sqrt(pi)*(erf(sqrt(I*x + I))
 - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) + (I*sqrt(pi)*(erf(sqrt(
I*x + I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*abs(x + 1)*s
in(1/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I
*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(3/2,
I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*cos(3/2*arctan2(x + 1
, 0)) + (((cos(1) + I*sin(1))*gamma(3/2, I*x + I) + (cos(1) - I*sin(1))*gam
ma(3/2, -I*x - I))*x + (cos(1) + I*sin(1))*gamma(3/2, I*x + I) + (cos(1) -
I*sin(1))*gamma(3/2, -I*x - I))*sin(3/2*arctan2(x + 1, 0)))*sqrt(abs(x + 1)
)/(x + 1)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x)\sqrt{1-x}\ (x+1)}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*(1 - x)^(1/2)*(x + 1))/(1 - x^2)^(1/2),x)
```

```
[Out] int((sin(x)*(1 - x)^(1/2)*(x + 1))/(1 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}\ (x+1)\sin(x)}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(1/2)*sin(x),x)
```

```
[Out] Integral(sqrt(1 - x)*(x + 1)*sin(x)/sqrt(-(x - 1)*(x + 1)), x)
```

3.812 $\int e^{\tanh^{-1}(x)}x(1+x)^{3/2}\sin(x)\,dx$

Optimal. Leaf size=335

$$-4\sqrt{2\pi}\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{15}{2}\sqrt{\frac{\pi}{2}}\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-4\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{15}{4}\sqrt{\frac{\pi}{2}}\cos(1)C$$

```
[Out] -5*(1-x)^(3/2)*cos(x)+(1-x)^(5/2)*cos(x)+5/2*(1-x)^(3/2)*sin(x)-17/8*cos(1)
*FresnelC(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/2)*Pi^(1/2)+1/4*cos(1)*Fresnel
S(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/2)*Pi^(1/2)-1/4*FresnelC(2^(1/2)/Pi^(1
/2)*(1-x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)-17/8*FresnelS(2^(1/2)/Pi^(1/2)*(1-
x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)+17/4*cos(x)*(1-x)^(1/2)-15/2*sin(x)*(1-x)
^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354, 3386}

$$-4\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{15}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-4\sqrt{2\pi}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*x*(1 + x)^(3/2)*Sin[x],x]
```

```
[Out] (17*Sqrt[1 - x]*Cos[x])/4 - 5*(1 - x)^(3/2)*Cos[x] + (1 - x)^(5/2)*Cos[x] +
 (15*Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]])/4 - 4*Sqrt[2*Pi]*C
os[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]] - (15*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqr
t[2/Pi]*Sqrt[1 - x]])/2 + 4*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 -
x]] + (15*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1])/2 - 4*Sqrt[2*
Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1] + (15*Sqrt[Pi/2]*FresnelS[Sqrt[
2/Pi]*Sqrt[1 - x]]*Sin[1])/4 - 4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]
]*Sin[1] - (15*Sqrt[1 - x]*Sin[x])/2 + (5*(1 - x)^(3/2)*Sin[x])/2
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}x(1+x)^{3/2}\sin(x)\,dx = \int \frac{x(1+x)^2\sin(x)}{\sqrt{1-x}}\,dx$$
$$= 2\,\text{Subst}\left(\int \left(-2+x^2\right)^2\left(-1+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-4\sin\left(1-x^2\right)+8x^2\sin\left(1-x^2\right)-5x^4\sin\left(1-x^2\right)+x^6\sin\left(1-\right.\right.\right.$$
$$= 2\,\text{Subst}\left(\int x^6\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)-8\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x, \sqrt{1-\right.$$
$$= 8\sqrt{1-x}\,\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)-5\,\text{Subst}\left(\int x^4\cos\left(1-\right.\right.$$
$$= 8\sqrt{1-x}\,\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)+4\sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\right.$$
$$= \frac{17}{4}\sqrt{1-x}\,\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)-4\sqrt{2\pi}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\right.$$
$$= \frac{17}{4}\sqrt{1-x}\,\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)-4\sqrt{2\pi}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\right.$$
$$= \frac{17}{4}\sqrt{1-x}\,\cos(x)-5(1-x)^{3/2}\cos(x)+(1-x)^{5/2}\cos(x)+\frac{15}{4}\sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\sqrt{\right.$$

Mathematica [C] time = 9.77, size = 200, normalized size = 0.60

$$\frac{\left(\frac{1}{32}+\frac{i}{32}\right)\sqrt{x+1}\left((\cos(x+1)-i\sin(x+1))\left((17+2i)\sqrt{2\pi}\sqrt{x-1}\,\text{erf}\left(\frac{(1+i)\sqrt{x-1}}{\sqrt{2}}\right)(\sin(x)-i\cos(x))+(2+2i)\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*x*(1 + x)^(3/2)*Sin[x],x]
```

```
[Out] ((1/32 + I/32)*Sqrt[1 + x]*((-2 - 17*I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erfi[((1 +
I)*Sqrt[-1 + x])/Sqrt[2]]*(Cos[1] + I*Sin[1]) - (2 - 2*I)*((-1 - 20*I) - (1
1 - 10*I)*x + (8 + 10*I)*x^2 + 4*x^3)*(Cos[x] + I*Sin[x]) + ((2 + 2*I)*((-2
0 - I) + (10 - 11*I)*x + (10 + 8*I)*x^2 + (4*I)*x^3)*(Cos[1] + I*Sin[1]) +
(17 + 2*I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erf[((1 + I)*Sqrt[-1 + x])/Sqrt[2]]*((-I
)*Cos[x] + Sin[x]))*(Cos[1 + x] - I*Sin[1 + x])))/Sqrt[1 - x^2]
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(x^2+x\right)\sqrt{-x^2+1}\,\sqrt{x+1}\,\sin(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-(x^2 + x)*sqrt(-x^2 + 1)*sqrt(x + 1)*sin(x)/(x - 1), x)
```

giac [C] time = 0.24, size = 202, normalized size = 0.60

$$-\left(\frac{19}{32}i-\frac{15}{32}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{19}{32}i+\frac{15}{32}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] -(19/32*I - 15/32)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1)
)*e^I + (19/32*I + 15/32)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-
x + 1))*e^(-I) - 1/8*I*(4*I*(x - 1)^2*sqrt(-x + 1) - (12*I - 10)*(-x + 1)^(
3/2) - (3*I + 18)*sqrt(-x + 1))*e^(I*x) - 1/2*I*(-2*I*(-x + 1)^(3/2) + (4*I
 - 3)*sqrt(-x + 1))*e^(I*x) - 1/8*I*(4*I*(x - 1)^2*sqrt(-x + 1) - (12*I + 1
0)*(-x + 1)^(3/2) - (3*I - 18)*sqrt(-x + 1))*e^(-I*x) - 1/2*I*(-2*I*(-x + 1
)^(3/2) + (4*I + 3)*sqrt(-x + 1))*e^(-I*x) + 1/2*sqrt(-x + 1)*e^(I*x) + 1/2
*sqrt(-x + 1)*e^(-I*x) + 3.25954715712000
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(1+x)^{\frac{5}{2}}\, x \sin(x)}{\sqrt{-x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

```
[Out] int((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

maxima [C] time = 0.47, size = 1010, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*((((-I*cos(1) - sin(1))*gamma(7/2, I*x - I) + (I*cos(1) - sin(1))*gamm
a(7/2, -I*x + I))*cos(7/2*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(7
/2, I*x - I) + (cos(1) + I*sin(1))*gamma(7/2, -I*x + I))*sin(7/2*arctan2(x
- 1, 0)))*x^3 + (((4*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - 4*I*sqrt(pi)*(er
```

```
f(sqrt(-I*x + I)) - 1))*cos(1) + 4*(sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqr
t(pi)*(erf(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + (4*(s
qrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(
1) + (-4*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + 4*I*sqrt(pi)*(erf(sqrt(-I*x
+ I)) - 1))*sin(1))*sin(1/2*arctan2(x - 1, 0)))*(x - 1)^2*abs(x - 1) - (8*(
(-I*cos(1) - sin(1))*gamma(3/2, I*x - I) + (I*cos(1) - sin(1))*gamma(3/2, -
I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((8*cos(1) - 8*I*sin(1))*gamma(3/2,
I*x - I) + (8*cos(1) + 8*I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x
- 1, 0)))*(x - 1)^2 - ((5*((I*cos(1) + sin(1))*gamma(5/2, I*x - I) + (-I*co
s(1) + sin(1))*gamma(5/2, -I*x + I))*cos(5/2*arctan2(x - 1, 0)) + ((5*cos(1
) - 5*I*sin(1))*gamma(5/2, I*x - I) + (5*cos(1) + 5*I*sin(1))*gamma(5/2, -I
*x + I))*sin(5/2*arctan2(x - 1, 0)))*abs(x - 1) + 3*((-I*cos(1) - sin(1))*g
amma(7/2, I*x - I) + (I*cos(1) - sin(1))*gamma(7/2, -I*x + I))*cos(7/2*arct
an2(x - 1, 0)) - ((3*cos(1) - 3*I*sin(1))*gamma(7/2, I*x - I) + (3*cos(1) +
 3*I*sin(1))*gamma(7/2, -I*x + I))*sin(7/2*arctan2(x - 1, 0)))*x^2 - ((8*((
I*cos(1) + sin(1))*gamma(3/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I
*x + I))*cos(3/2*arctan2(x - 1, 0)) + ((8*cos(1) - 8*I*sin(1))*gamma(3/2, I
*x - I) + (8*cos(1) + 8*I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x -
 1, 0)))*(x - 1)^2 + (10*((-I*cos(1) - sin(1))*gamma(5/2, I*x - I) + (I*cos
(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2*arctan2(x - 1, 0)) - ((10*cos(1
) - 10*I*sin(1))*gamma(5/2, I*x - I) + (10*cos(1) + 10*I*sin(1))*gamma(5/2,
 -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*abs(x - 1) + 3*((I*cos(1) + sin(1))
*gamma(7/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(7/2, -I*x + I))*cos(7/2*a
rctan2(x - 1, 0)) + ((3*cos(1) - 3*I*sin(1))*gamma(7/2, I*x - I) + (3*cos(1
) + 3*I*sin(1))*gamma(7/2, -I*x + I))*sin(7/2*arctan2(x - 1, 0)))*x - (5*((
I*cos(1) + sin(1))*gamma(5/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(5/2, -I
*x + I))*cos(5/2*arctan2(x - 1, 0)) + ((5*cos(1) - 5*I*sin(1))*gamma(5/2, I
*x - I) + (5*cos(1) + 5*I*sin(1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x -
 1, 0)))*abs(x - 1) + ((I*cos(1) + sin(1))*gamma(7/2, I*x - I) + (-I*cos(1)
 + sin(1))*gamma(7/2, -I*x + I))*cos(7/2*arctan2(x - 1, 0)) + ((cos(1) - I*
sin(1))*gamma(7/2, I*x - I) + (cos(1) + I*sin(1))*gamma(7/2, -I*x + I))*sin
(7/2*arctan2(x - 1, 0)))*sqrt(-x + 1)*sqrt(abs(x - 1))/(x - 1)^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\ \sin(x)\,(x+1)^{5/2}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x)*(x + 1)^(5/2))/(1 - x^2)^(1/2),x)
```

```
[Out] int((x*sin(x)*(x + 1)^(5/2))/(1 - x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(-x**2+1)**(1/2)*x*sin(x),x)
```

```
[Out] Timed out
```

3.813 $\int e^{\tanh^{-1}(x)}(1+x)^{3/2}\sin(x)\,dx$

Optimal. Leaf size=236

$$-4\sqrt{2\pi}\,\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2\sqrt{2\pi}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2\sqrt{2\pi}\,\sin(1)S\left(\sqrt{\right.$$

```
[Out] -(1-x)^(3/2)*cos(x)+13/4*cos(1)*FresnelS(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1
/2)*Pi^(1/2)-13/4*FresnelC(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*sin(1)*2^(1/2)*Pi^
(1/2)-2*cos(1)*FresnelC(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/2)*Pi^(1/2)-2*Fr
esnelS(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)+4*cos(x)*(1-x)
^(1/2)-3/2*sin(x)*(1-x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354, 3386}

$$-4\sqrt{2\pi}\,\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-2\sqrt{2\pi}\,\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 + x)^(3/2)*Sin[x],x]
```

```
[Out] 4*Sqrt[1 - x]*Cos[x] - (1 - x)^(3/2)*Cos[x] - 2*Sqrt[2*Pi]*Cos[1]*FresnelC[
Sqrt[2/Pi]*Sqrt[1 - x]] - (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 -
 x]])/2 + 4*Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] + (3*Sqrt[Pi
/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]]*Sin[1])/2 - 4*Sqrt[2*Pi]*FresnelC[Sqrt
[2/Pi]*Sqrt[1 - x]]*Sin[1] - 2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]]*
Sin[1] - (3*Sqrt[1 - x]*Sin[x])/2
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}(1+x)^{3/2}\sin(x)\,dx = \int \frac{(1+x)^2\sin(x)}{\sqrt{1-x}}\,dx$$
$$= -\left(2\,\text{Subst}\left(\int \left(-2+x^2\right)^2 \sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \left(4\sin\left(1-x^2\right)-4x^2\sin\left(1-x^2\right)+x^4\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1-x}\right.\right.$$
$$= -\left(2\,\text{Subst}\left(\int x^4\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)\right) - 8\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x, \right.$$
$$= 4\sqrt{1-x}\,\cos(x) - (1-x)^{3/2}\cos(x) + 3\,\text{Subst}\left(\int x^2\cos\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)$$
$$= 4\sqrt{1-x}\,\cos(x) - (1-x)^{3/2}\cos(x) + 4\sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) - 4\sqrt{2\pi}\,C\left(\right.$$
$$= 4\sqrt{1-x}\,\cos(x) - (1-x)^{3/2}\cos(x) - 2\sqrt{2\pi}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) + 4\sqrt{2\pi}\,\cos$$
$$= 4\sqrt{1-x}\,\cos(x) - (1-x)^{3/2}\cos(x) - 2\sqrt{2\pi}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\,\cos$$

Mathematica [C] time = 8.69, size = 178, normalized size = 0.75

$$\frac{i\sqrt{1-x^2}\left(-(\cos(x+1)-i\sin(x+1))\left((21+5i)\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{(1+i)\sqrt{x-1}}{\sqrt{2}}\right)(\sin(x)-i\cos(x))+2\sqrt{x-1}\,(2ix+(3+6i))(\right.\right.}{8\sqrt{x-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 + x)^(3/2)*Sin[x],x]
```

```
[Out] ((I/8)*Sqrt[1 - x^2]*((5 + 21*I)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[-1 + x])/Sqr
t[2]]*((-I)*Cos[1] + Sin[1]) + 2*Sqrt[-1 + x]*((6 + 3*I) + 2*x)*((-I)*Cos[x
] + Sin[x]) - (2*((3 + 6*I) + (2*I)*x)*Sqrt[-1 + x]*(Cos[1] + I*Sin[1]) + (
21 + 5*I)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[-1 + x])/Sqrt[2]]*((-I)*Cos[x] + Sin
[x]))*(Cos[1 + x] - I*Sin[1 + x])))/(Sqrt[-1 + x]*Sqrt[1 + x])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\,(x+1)^{\frac{3}{2}}\sin(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="fricas")

[Out] integral(-sqrt(-x^2 + 1)*(x + 1)^(3/2)*sin(x)/(x - 1), x)

giac [C] time = 0.22, size = 122, normalized size = 0.52

$$-\left(\frac{21}{16}i+\frac{5}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{21}{16}i-\frac{5}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="giac")

[Out] -(21/16*I + 5/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1)) *e^I + (21/16*I - 5/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x + 1))*e^(-I) - 1/4*I*(-2*I*(-x + 1)^(3/2) + (4*I - 3)*sqrt(-x + 1))*e^(I*x) - 1/4*I*(-2*I*(-x + 1)^(3/2) + (4*I + 3)*sqrt(-x + 1))*e^(-I*x) + sqrt(-x + 1)*e^(I*x) + sqrt(-x + 1)*e^(-I*x) + 3.78811297138000

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(1+x)^{\frac{5}{2}}\sin(x)}{\sqrt{-x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x)

[Out] int((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x)

maxima [C] time = 0.44, size = 639, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(5/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="maxima")

[Out] -1/2*((((4*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - 4*I*sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1) + 4*(sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + (4*(sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1) + (-4*I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + 4*I*sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*sin(1))*sin(1/2*arctan2(x - 1, 0)))*(x - 1)^2 + (((-I*cos(1) - sin(1))*gamma(5/2, I*x - I) + (I*cos(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(5/2, I*x - I) + (cos(1) + I*sin(1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*x^2 - ((4*((I*cos(1)

```
+ sin(1))*gamma(3/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I*x + I))*
cos(3/2*arctan2(x - 1, 0)) + ((4*cos(1) - 4*I*sin(1))*gamma(3/2, I*x - I) +
 (4*cos(1) + 4*I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*
abs(x - 1) + 2*((-I*cos(1) - sin(1))*gamma(5/2, I*x - I) + (I*cos(1) - sin(
1))*gamma(5/2, -I*x + I))*cos(5/2*arctan2(x - 1, 0)) - ((2*cos(1) - 2*I*sin
(1))*gamma(5/2, I*x - I) + (2*cos(1) + 2*I*sin(1))*gamma(5/2, -I*x + I))*si
n(5/2*arctan2(x - 1, 0)))*x - (4*((-I*cos(1) - sin(1))*gamma(3/2, I*x - I)
+ (I*cos(1) - sin(1))*gamma(3/2, -I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((
4*cos(1) - 4*I*sin(1))*gamma(3/2, I*x - I) + (4*cos(1) + 4*I*sin(1))*gamma(
3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*abs(x - 1) + ((-I*cos(1) - sin(
1))*gamma(5/2, I*x - I) + (I*cos(1) - sin(1))*gamma(5/2, -I*x + I))*cos(5/2
*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(5/2, I*x - I) + (cos(1) +
I*sin(1))*gamma(5/2, -I*x + I))*sin(5/2*arctan2(x - 1, 0)))*sqrt(-x + 1)/((
x - 1)^2*sqrt(abs(x - 1)))
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(x)\,(x+1)^{5/2}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*(x + 1)^(5/2))/(1 - x^2)^(1/2),x)
```

```
[Out] int((sin(x)*(x + 1)^(5/2))/(1 - x^2)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(-x**2+1)**(1/2)*sin(x),x)
```

```
[Out] Timed out
```

3.814 $\int e^{\tanh^{-1}(x)}(1-x)^{3/2}x\sin(x)\,dx$

Optimal. Leaf size=193

$$\frac{9}{2}\sqrt{\frac{\pi}{2}}\ \sin(1)C\left(\sqrt{\frac{2}{\pi}}\ \sqrt{x+1}\right)+\frac{7}{4}\sqrt{\frac{\pi}{2}}\ \cos(1)C\left(\sqrt{\frac{2}{\pi}}\ \sqrt{x+1}\right)+\frac{7}{4}\sqrt{\frac{\pi}{2}}\ \sin(1)S\left(\sqrt{\frac{2}{\pi}}\ \sqrt{x+1}\right)-\frac{9}{2}\sqrt{\frac{\pi}{2}}\ \cos(1)S\left(\right.$$

```
[Out] -3*(1+x)^(3/2)*cos(x)+(1+x)^(5/2)*cos(x)-5/2*(1+x)^(3/2)*sin(x)+7/8*cos(1)*
FresnelC(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*2^(1/2)*Pi^(1/2)-9/4*cos(1)*FresnelS
(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*2^(1/2)*Pi^(1/2)+9/4*FresnelC(2^(1/2)/Pi^(1/
2)*(1+x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)+7/8*FresnelS(2^(1/2)/Pi^(1/2)*(1+x)
^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)-7/4*cos(x)*(1+x)^(1/2)+9/2*sin(x)*(1+x)^(1/
2)
```

Rubi [A] time = 0.40, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6129, 6742, 3385, 3354, 3352, 3351, 3386, 3353}

$$\frac{9}{2}\sqrt{\frac{\pi}{2}}\ \sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\ \sqrt{x+1}\right)+\frac{7}{4}\sqrt{\frac{\pi}{2}}\ \cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\ \sqrt{x+1}\right)+\frac{7}{4}\sqrt{\frac{\pi}{2}}\ \sin(1)S\left(\sqrt{\frac{2}{\pi}}\ \sqrt{x+1}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 - x)^(3/2)*x*Sin[x],x]
```

```
[Out] (-7*Sqrt[1 + x]*Cos[x])/4 - 3*(1 + x)^(3/2)*Cos[x] + (1 + x)^(5/2)*Cos[x] +
 (7*Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]])/4 - (9*Sqrt[Pi/2]*C
os[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]])/2 + (9*Sqrt[Pi/2]*FresnelC[Sqrt[2/P
i]*Sqrt[1 + x]]*Sin[1])/2 + (7*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*
Sin[1])/4 + (9*Sqrt[1 + x]*Sin[x])/2 - (5*(1 + x)^(3/2)*Sin[x])/2
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}(1-x)^{3/2}x\sin(x)\,dx = \int (1-x)x\sqrt{1+x}\,\sin(x)\,dx$$
$$= 2\,\text{Subst}\left(\int x^2\left(-2+x^2\right)\left(-1+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(2x^2\sin\left(1-x^2\right)-3x^4\sin\left(1-x^2\right)+x^6\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1+\right.$$
$$= 2\,\text{Subst}\left(\int x^6\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right)+4\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)\,dx, x,\right.$$
$$= 2\sqrt{1+x}\,\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)-2\,\text{Subst}\left(\int \cos\left(1-\right.\right.$$
$$= 2\sqrt{1+x}\,\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)+\frac{9}{2}\sqrt{1+x}\,\sin(x)-\frac{5}{2}$$
$$= -\frac{7}{4}\sqrt{1+x}\,\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)-\sqrt{2\pi}\,\cos(1)C\left(\sqrt{\right.$$
$$= -\frac{7}{4}\sqrt{1+x}\,\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)-\sqrt{2\pi}\,\cos(1)C\left(\sqrt{\right.$$
$$= -\frac{7}{4}\sqrt{1+x}\,\cos(x)-3(1+x)^{3/2}\cos(x)+(1+x)^{5/2}\cos(x)+\frac{15}{4}\sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\right.$$

Mathematica [C] time = 8.59, size = 193, normalized size = 1.00

$$\left(\frac{1}{32}+\frac{i}{32}\right)\sqrt{1-x}\,\left((\cos(1)-i\sin(1))\left((2+2i)\left(-4ix^3+10x^2+(2+19i)x-(8-15i)\right)(\cos(x+1)+i\sin(x+1))\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 - x)^(3/2)*x*Sin[x],x]
```

```
[Out] ((1/32 + I/32)*Sqrt[1 - x]*(((2 + 2*I)*((8 + 15*I) - (2 - 19*I)*x - 10*x^2
- (4*I)*x^3))/E^(I*x) + (18 + 7*I)*E^I*Sqrt[2*Pi]*Sqrt[-1 - x]*Erfi[((1 + I
)*Sqrt[-1 - x])/Sqrt[2]] + (Cos[1] - I*Sin[1])*((-18 + 7*I)*Sqrt[2*Pi]*Sqrt
[-1 - x]*Erf[((1 + I)*Sqrt[-1 - x])/Sqrt[2]] + (2 + 2*I)*((-8 + 15*I) + (2
+ 19*I)*x + 10*x^2 - (4*I)*x^3)*(Cos[1 + x] + I*Sin[1 + x]))))/Sqrt[1 - x^2
]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^2+1}\,x\sqrt{-x+1}\,\sin(x), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*x*sqrt(-x + 1)*sin(x), x)
```

giac [C] time = 0.22, size = 122, normalized size = 0.63

$$\left(\frac{25}{32}i+\frac{11}{32}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{i}-\left(\frac{25}{32}i-\frac{11}{32}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{(-i)}-\frac{1}{8}i\left(4i\,(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] (25/32*I + 11/32)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*
e^I - (25/32*I - 11/32)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x +
 1))*e^(-I) - 1/8*I*(4*I*(x + 1)^(5/2) - (12*I + 10)*(x + 1)^(3/2) - (3*I -
 18)*sqrt(x + 1))*e^(I*x) - 1/8*I*(4*I*(x + 1)^(5/2) - (12*I - 10)*(x + 1)^
(3/2) - (3*I + 18)*sqrt(x + 1))*e^(-I*x) - 1/2*sqrt(x + 1)*e^(I*x) - 1/2*sq
rt(x + 1)*e^(-I*x) - 0.330988710799000
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(1+x)\,(1-x)^{\frac{3}{2}}\,x\sin(x)}{\sqrt{-x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x)
```

```
[Out] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x)
```

maxima [C] time = 0.57, size = 1488, normalized size = 7.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*x*sin(x),x, algorithm="maxima")
```

```
[Out] 1/2*(((-2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x
 - I)) - 1))*cos(1) + 2*(sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(
sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*cos(1/2*arctan2(x + 1, 0)) - (2*(sq
rt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1
) - (-2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x -
 I)) - 1))*sin(1))*(x + 1)^2*sin(1/2*arctan2(x + 1, 0)) - 3*(((I*cos(1) - s
in(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x +
 (I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2,
```

```
-I*x - I))*abs(x + 1)*cos(3/2*arctan2(x + 1, 0)) - (((3*cos(1) + 3*I*sin(1)
)*gamma(3/2, I*x + I) + (3*cos(1) - 3*I*sin(1))*gamma(3/2, -I*x - I))*x + (
3*cos(1) + 3*I*sin(1))*gamma(3/2, I*x + I) + (3*cos(1) - 3*I*sin(1))*gamma(
3/2, -I*x - I))*abs(x + 1)*sin(3/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1
))*gamma(5/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*x^2 - 2
*((-I*cos(1) + sin(1))*gamma(5/2, I*x + I) + (I*cos(1) + sin(1))*gamma(5/2,
 -I*x - I))*x + (I*cos(1) - sin(1))*gamma(5/2, I*x + I) + (-I*cos(1) - sin(
1))*gamma(5/2, -I*x - I))*cos(5/2*arctan2(x + 1, 0)) + (((cos(1) + I*sin(1)
)*gamma(5/2, I*x + I) + (cos(1) - I*sin(1))*gamma(5/2, -I*x - I))*x^2 + ((2
*cos(1) + 2*I*sin(1))*gamma(5/2, I*x + I) + (2*cos(1) - 2*I*sin(1))*gamma(5
/2, -I*x - I))*x + (cos(1) + I*sin(1))*gamma(5/2, I*x + I) + (cos(1) - I*si
n(1))*gamma(5/2, -I*x - I))*sin(5/2*arctan2(x + 1, 0)))/((x + 1)^(3/2)*sqrt
(abs(x + 1))) + 1/2*(((2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - 2*I*sqrt(pi)
*(erf(sqrt(-I*x - I)) - 1))*cos(1) - 2*(sqrt(pi)*(erf(sqrt(I*x + I)) - 1) +
 sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*abs(x + 1)*cos(1/2*a
rctan2(x + 1, 0)) + (2*(sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(s
qrt(-I*x - I)) - 1))*cos(1) + (2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - 2*I*
sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^2*abs(x + 1)*sin(1/2*ar
ctan2(x + 1, 0)) - 5*(((-I*cos(1) + sin(1))*gamma(3/2, I*x + I) + (I*cos(1)
 + sin(1))*gamma(3/2, -I*x - I))*x + (-I*cos(1) + sin(1))*gamma(3/2, I*x +
I) + (I*cos(1) + sin(1))*gamma(3/2, -I*x - I))*(x + 1)^2*cos(3/2*arctan2(x
+ 1, 0)) + (((5*cos(1) + 5*I*sin(1))*gamma(3/2, I*x + I) + (5*cos(1) - 5*I*
sin(1))*gamma(3/2, -I*x - I))*x + (5*cos(1) + 5*I*sin(1))*gamma(3/2, I*x +
I) + (5*cos(1) - 5*I*sin(1))*gamma(3/2, -I*x - I))*(x + 1)^2*sin(3/2*arctan
2(x + 1, 0)) - 4*(((I*cos(1) - sin(1))*gamma(5/2, I*x + I) + (-I*cos(1) - s
in(1))*gamma(5/2, -I*x - I))*x^2 + 2*((I*cos(1) - sin(1))*gamma(5/2, I*x +
I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*x + (I*cos(1) - sin(1))*gam
ma(5/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*abs(x + 1)*co
s(5/2*arctan2(x + 1, 0)) - (((4*cos(1) + 4*I*sin(1))*gamma(5/2, I*x + I) +
(4*cos(1) - 4*I*sin(1))*gamma(5/2, -I*x - I))*x^2 + ((8*cos(1) + 8*I*sin(1)
)*gamma(5/2, I*x + I) + (8*cos(1) - 8*I*sin(1))*gamma(5/2, -I*x - I))*x + (
4*cos(1) + 4*I*sin(1))*gamma(5/2, I*x + I) + (4*cos(1) - 4*I*sin(1))*gamma(
5/2, -I*x - I))*abs(x + 1)*sin(5/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1
))*gamma(7/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(7/2, -I*x - I))*x^3 - 3
*((-I*cos(1) + sin(1))*gamma(7/2, I*x + I) + (I*cos(1) + sin(1))*gamma(7/2,
 -I*x - I))*x^2 - 3*((-I*cos(1) + sin(1))*gamma(7/2, I*x + I) + (I*cos(1) +
 sin(1))*gamma(7/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(7/2, I*x + I)
+ (-I*cos(1) - sin(1))*gamma(7/2, -I*x - I))*cos(7/2*arctan2(x + 1, 0)) + (
((cos(1) + I*sin(1))*gamma(7/2, I*x + I) + (cos(1) - I*sin(1))*gamma(7/2, -
I*x - I))*x^3 + ((3*cos(1) + 3*I*sin(1))*gamma(7/2, I*x + I) + (3*cos(1) -
3*I*sin(1))*gamma(7/2, -I*x - I))*x^2 + ((3*cos(1) + 3*I*sin(1))*gamma(7/2,
 I*x + I) + (3*cos(1) - 3*I*sin(1))*gamma(7/2, -I*x - I))*x + (cos(1) + I*s
in(1))*gamma(7/2, I*x + I) + (cos(1) - I*sin(1))*gamma(7/2, -I*x - I))*sin(
7/2*arctan2(x + 1, 0)))*sqrt(abs(x + 1))/(x + 1)^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(x)\,(1-x)^{3/2}\,(x+1)}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x)*(1 - x)^(3/2)*(x + 1))/(1 - x^2)^(1/2),x)
[Out] int((x*sin(x)*(1 - x)^(3/2)*(x + 1))/(1 - x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(3/2)*x*sin(x),x)
[Out] Timed out
```

3.815 $\int e^{\tanh^{-1}(x)}(1-x)^{3/2}\sin(x)\,dx$

Optimal. Leaf size=157

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{2\pi}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{2\pi}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\cos(1)S\left(\sqrt{\right.$$

```
[Out] (1+x)^(3/2)*cos(x)+3/4*cos(1)*FresnelS(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*2^(1/2
)*Pi^(1/2)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/
2)+cos(1)*FresnelC(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*2^(1/2)*Pi^(1/2)+FresnelS(
2^(1/2)/Pi^(1/2)*(1+x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)-2*cos(x)*(1+x)^(1/2)-
3/2*sin(x)*(1+x)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6129, 6742, 3385, 3354, 3352, 3351, 3386, 3353}

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{2\pi}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\sqrt{2\pi}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)+\frac{3}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[x]*(1 - x)^(3/2)*Sin[x],x]
```

```
[Out] -2*Sqrt[1 + x]*Cos[x] + (1 + x)^(3/2)*Cos[x] + Sqrt[2*Pi]*Cos[1]*FresnelC[S
qrt[2/Pi]*Sqrt[1 + x]] + (3*Sqrt[Pi/2]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 +
x]])/2 - (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1])/2 + Sqrt[2*
Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]]*Sin[1] - (3*Sqrt[1 + x]*Sin[x])/2
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\tanh^{-1}(x)}(1-x)^{3/2}\sin(x)\,dx = \int (1-x)\sqrt{1+x}\,\sin(x)\,dx$$
$$= 2\,\text{Subst}\left(\int x^2\left(-2+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(-2x^2\sin\left(1-x^2\right)+x^4\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1+x}\right)$$
$$= 2\,\text{Subst}\left(\int x^4\sin\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right) - 4\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)\,dx, x, \sqrt{\right.$$
$$= -2\sqrt{1+x}\,\cos(x) + (1+x)^{3/2}\cos(x) + 2\,\text{Subst}\left(\int \cos\left(1-x^2\right)\,dx, x, \sqrt{1+x}\right)$$
$$= -2\sqrt{1+x}\,\cos(x) + (1+x)^{3/2}\cos(x) - \frac{3}{2}\sqrt{1+x}\,\sin(x) - \frac{3}{2}\,\text{Subst}\left(\int \sin\left(1-\right.\right.$$
$$= -2\sqrt{1+x}\,\cos(x) + (1+x)^{3/2}\cos(x) + \sqrt{2\pi}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right) + \sqrt{2\pi}\,S\left(\right.$$
$$= -2\sqrt{1+x}\,\cos(x) + (1+x)^{3/2}\cos(x) + \sqrt{2\pi}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right) + \frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{c}$$

Mathematica [C] time = 7.20, size = 176, normalized size = 1.12

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)e^{-ix}\sqrt{1-x^2}\left(-(3+4i)\sqrt{2\pi}\,e^{ix}\text{erf}\left(\frac{(1+i)\sqrt{-x-1}}{\sqrt{2}}\right)(\cos(1)-i\sin(1)) + (4+3i)\sqrt{2\pi}\,e^{ix}\text{erfi}\left(\frac{(1+i)\sqrt{-x-1}}{\sqrt{2}}\right)(\text{si}\right.}{\sqrt{-x-1}\,\sqrt{1-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[x]*(1 - x)^(3/2)*Sin[x],x]
```

```
[Out] ((1/16 + I/16)*Sqrt[1 - x^2]*((2 + 2*I)*Sqrt[-1 - x]*((-3 + 2*I) + E^((2*I)
*x)*((3 + 2*I) - (2*I)*x) - (2*I)*x) - (3 + 4*I)*E^(I*x)*Sqrt[2*Pi]*Erf[((1
 + I)*Sqrt[-1 - x])/Sqrt[2]]*(Cos[1] - I*Sin[1]) + (4 + 3*I)*E^(I*x)*Sqrt[2
*Pi]*Erfi[((1 + I)*Sqrt[-1 - x])/Sqrt[2]]*((-I)*Cos[1] + Sin[1])))/(E^(I*x)
*Sqrt[-1 - x]*Sqrt[1 - x])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^2+1}\,\sqrt{-x+1}\,\sin(x), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*sqrt(-x + 1)*sin(x), x)
```

giac [C] time = 0.19, size = 86, normalized size = 0.55

$$\left(\frac{1}{16}i-\frac{7}{16}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{i}-\left(\frac{1}{16}i+\frac{7}{16}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{(-i)}-\frac{1}{4}i\left(2i\,(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x, algorithm="giac")
```

```
[Out] (1/16*I - 7/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*e^
I - (1/16*I + 7/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1))
*e^(-I) - 1/4*I*(2*I*(x + 1)^(3/2) - (4*I + 3)*sqrt(x + 1))*e^(I*x) - 1/4*I
*(2*I*(x + 1)^(3/2) - (4*I - 3)*sqrt(x + 1))*e^(-I*x) + 0.197577103470000
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(1+x)\,(1-x)^{\frac{3}{2}}\sin(x)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x)
```

```
[Out] int((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x)
```

maxima [C] time = 0.50, size = 912, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+1)^(1/2)*(1-x)^(3/2)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/2*(((2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*x
- I)) - 1))*cos(1) - 2*(sqrt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(s
qrt(-I*x - I)) - 1))*sin(1))*abs(x + 1)*cos(1/2*arctan2(x + 1, 0)) + (2*(sq
rt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1
) + (2*I*sqrt(pi)*(erf(sqrt(I*x + I)) - 1) - 2*I*sqrt(pi)*(erf(sqrt(-I*x -
I)) - 1))*sin(1))*abs(x + 1)*sin(1/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin
(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x + (
I*cos(1) - sin(1))*gamma(3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I
*x - I))*cos(3/2*arctan2(x + 1, 0)) + (((cos(1) + I*sin(1))*gamma(3/2, I*x
+ I) + (cos(1) - I*sin(1))*gamma(3/2, -I*x - I))*x + (cos(1) + I*sin(1))*ga
mma(3/2, I*x + I) + (cos(1) - I*sin(1))*gamma(3/2, -I*x - I))*sin(3/2*arcta
n2(x + 1, 0)))*sqrt(abs(x + 1))/(x + 1)^(3/2) + 1/2*(((-2*I*sqrt(pi)*(erf(s
qrt(I*x + I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) + 2*(sq
```

```
rt(pi)*(erf(sqrt(I*x + I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1
))*(x + 1)^2*cos(1/2*arctan2(x + 1, 0)) - (2*(sqrt(pi)*(erf(sqrt(I*x + I))
- 1) + sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*cos(1) - (-2*I*sqrt(pi)*(erf(sqr
t(I*x + I)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(-I*x - I)) - 1))*sin(1))*(x + 1)^
2*sin(1/2*arctan2(x + 1, 0)) - 3*(((I*cos(1) - sin(1))*gamma(3/2, I*x + I)
+ (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*x + (I*cos(1) - sin(1))*gamma(
3/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(3/2, -I*x - I))*abs(x + 1)*cos(3
/2*arctan2(x + 1, 0)) - (((3*cos(1) + 3*I*sin(1))*gamma(3/2, I*x + I) + (3*
cos(1) - 3*I*sin(1))*gamma(3/2, -I*x - I))*x + (3*cos(1) + 3*I*sin(1))*gamm
a(3/2, I*x + I) + (3*cos(1) - 3*I*sin(1))*gamma(3/2, -I*x - I))*abs(x + 1)*
sin(3/2*arctan2(x + 1, 0)) + (((I*cos(1) - sin(1))*gamma(5/2, I*x + I) + (-
I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*x^2 - 2*((-I*cos(1) + sin(1))*gamm
a(5/2, I*x + I) + (I*cos(1) + sin(1))*gamma(5/2, -I*x - I))*x + (I*cos(1) -
 sin(1))*gamma(5/2, I*x + I) + (-I*cos(1) - sin(1))*gamma(5/2, -I*x - I))*c
os(5/2*arctan2(x + 1, 0)) + (((cos(1) + I*sin(1))*gamma(5/2, I*x + I) + (co
s(1) - I*sin(1))*gamma(5/2, -I*x - I))*x^2 + ((2*cos(1) + 2*I*sin(1))*gamma
(5/2, I*x + I) + (2*cos(1) - 2*I*sin(1))*gamma(5/2, -I*x - I))*x + (cos(1)
+ I*sin(1))*gamma(5/2, I*x + I) + (cos(1) - I*sin(1))*gamma(5/2, -I*x - I))
*sin(5/2*arctan2(x + 1, 0)))/((x + 1)^(3/2)*sqrt(abs(x + 1)))
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x)\,(1-x)^{3/2}\,(x+1)}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*(1 - x)^(3/2)*(x + 1))/(1 - x^2)^(1/2),x)
```

```
[Out] int((sin(x)*(1 - x)^(3/2)*(x + 1))/(1 - x^2)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+1)**(1/2)*(1-x)**(3/2)*sin(x),x)
```

```
[Out] Timed out
```

3.816 $\int \frac{e^{\tanh^{-1}(x)} x \sin(x)}{\sqrt{1+x}} \, dx$

Optimal. Leaf size=140

$$-\sqrt{2\pi}\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1\right.$$

```
[Out] -1/2*cos(1)*FresnelC(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/2)*Pi^(1/2)-1/2*Fre
snelS(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)+cos(1)*FresnelS
(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/2)*Pi^(1/2)-FresnelC(2^(1/2)/Pi^(1/2)*(
1-x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)+cos(x)*(1-x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6129, 6742, 3353, 3352, 3351, 3385, 3354}

$$-\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\cos(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{\frac{\pi}{2}}\sin(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)+\sqrt{2\pi}\ \text{c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*x*Sin[x])/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 - x]*Cos[x] - Sqrt[Pi/2]*Cos[1]*FresnelC[Sqrt[2/Pi]*Sqrt[1 - x]] + S
qrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] - Sqrt[2*Pi]*FresnelC[Sqr
t[2/Pi]*Sqrt[1 - x]]*Sin[1] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]]*S
in[1]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3385

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cos[c + d*x^n])/(d*n), x] + Dist[(e^n*(m - n + 1)
)/(d*n), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(x)}x\sin(x)}{\sqrt{1+x}}\,dx = \int \frac{x\sin(x)}{\sqrt{1-x}}\,dx$$

$$= 2\,\text{Subst}\left(\int \left(-1+x^2\right)\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(-\sin\left(1-x^2\right)+x^2\sin\left(1-x^2\right)\right)\,dx, x, \sqrt{1-x}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right)\right) + 2\,\text{Subst}\left(\int x^2\sin\left(1-x^2\right)\,dx, x, \sqrt{1-x}\right.$$

$$= \sqrt{1-x}\,\cos(x) + (2\cos(1))\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{1-x}\right) - (2\sin(1))\,\text{Subst}\left(\int \text{co}\right.$$

$$= \sqrt{1-x}\,\cos(x) + \sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) - \sqrt{2\pi}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)\sin(1) - \cos(1)\,\text{S}$$

$$= \sqrt{1-x}\,\cos(x) - \sqrt{\frac{\pi}{2}}\,\cos(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) + \sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right) - \sqrt{2\pi}\,C$$

Mathematica [C] time = 8.17, size = 162, normalized size = 1.16

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{x+1}\left((\cos(x+1)-i\sin(x+1))\left((1+2i)\sqrt{2\pi}\sqrt{x-1}\operatorname{erf}\left(\frac{(1+i)\sqrt{x-1}}{\sqrt{2}}\right)(\sin(x)-i\cos(x))-(2-2i)(x-1\right.\right.}{\sqrt{1-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[x]*x*Sin[x])/Sqrt[1 + x],x]
```

```
[Out] ((1/8 + I/8)*Sqrt[1 + x]*((-2 - I)*Sqrt[2*Pi]*Sqrt[-1 + x]*Erfi[((1 + I)*Sq
rt[-1 + x])/Sqrt[2]]*(Cos[1] + I*Sin[1]) - (2 - 2*I)*(-1 + x)*(Cos[x] + I*S
in[x]) + ((-2 + 2*I)*(-1 + x)*(Cos[1] + I*Sin[1]) + (1 + 2*I)*Sqrt[2*Pi]*Sq
rt[-1 + x]*Erf[((1 + I)*Sqrt[-1 + x])/Sqrt[2]]*((-I)*Cos[x] + Sin[x]))*(Cos
[1 + x] - I*Sin[1 + x])))/Sqrt[1 - x^2]
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\sqrt{x+1}\,x\sin(x)}{x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(x + 1)*x*sin(x)/(x^2 - 1), x)
```

giac [C] time = 0.52, size = 74, normalized size = 0.53

$$-\left(\frac{3}{8}i+\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{3}{8}i-\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}+\frac{1}{2}\sqrt{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="giac")
```

```
[Out] -(3/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1))*e^
I + (3/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x + 1))*
e^(-I) + 1/2*sqrt(-x + 1)*e^(I*x) + 1/2*sqrt(-x + 1)*e^(-I*x) + 1.166225383
28000
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x}\,x\sin(x)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

```
[Out] int((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x)
```

maxima [C] time = 0.41, size = 347, normalized size = 2.48

$$-\frac{\left(\left(\left((-i\cos(1)-\sin(1))\Gamma\left(\frac{3}{2},ix-i\right)+(i\cos(1)-\sin(1))\Gamma\left(\frac{3}{2},-ix+i\right)\right)\cos\left(\frac{3}{2}\arctan(x-1,0)\right)-\left((\cos(1)-i\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*((((-I*cos(1) - sin(1))*gamma(3/2, I*x - I) + (I*cos(1) - sin(1))*gamm
a(3/2, -I*x + I))*cos(3/2*arctan2(x - 1, 0)) - ((cos(1) - I*sin(1))*gamma(3
/2, I*x - I) + (cos(1) + I*sin(1))*gamma(3/2, -I*x + I))*sin(3/2*arctan2(x
- 1, 0)))*x + (((I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - I*sqrt(pi)*(erf(sqrt
(-I*x + I)) - 1))*cos(1) + (sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(e
rf(sqrt(-I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + ((sqrt(pi)*(e
rf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1) + (-I*s
qrt(pi)*(erf(sqrt(I*x - I)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*si
n(1))*sin(1/2*arctan2(x - 1, 0)))*abs(x - 1) + ((I*cos(1) + sin(1))*gamma(3
/2, I*x - I) + (-I*cos(1) + sin(1))*gamma(3/2, -I*x + I))*cos(3/2*arctan2(x
 - 1, 0)) + ((cos(1) - I*sin(1))*gamma(3/2, I*x - I) + (cos(1) + I*sin(1))*
gamma(3/2, -I*x + I))*sin(3/2*arctan2(x - 1, 0)))*sqrt(-x + 1)*sqrt(abs(x -
 1))/(x - 1)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(x)\,\sqrt{x+1}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x)*(x + 1)^(1/2))/(1 - x^2)^(1/2),x)
```

```
[Out] int((x*sin(x)*(x + 1)^(1/2))/(1 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{x+1}\,\sin(x)}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2)*x*sin(x),x)
```

```
[Out] Integral(x*sqrt(x + 1)*sin(x)/sqrt(-(x - 1)*(x + 1)), x)
```

3.817 $\int \frac{e^{\tanh^{-1}(x)}\sin(x)}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=62

$$\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{2\pi}\sin(1)C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)$$

```
[Out] cos(1)*FresnelS(2^(1/2)/Pi^(1/2)*(1-x)^(1/2))*2^(1/2)*Pi^(1/2)-FresnelC(2^(
1/2)/Pi^(1/2)*(1-x)^(1/2))*sin(1)*2^(1/2)*Pi^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6129, 3306, 3305, 3351, 3304, 3352}

$$\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{2\pi}\sin(1)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[x]*Sin[x])/Sqrt[1 + x],x]
```

```
[Out] Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 - x]] - Sqrt[2*Pi]*FresnelC[Sq
rt[2/Pi]*Sqrt[1 - x]]*Sin[1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(x)}\sin(x)}{\sqrt{1+x}}\,dx &= \int \frac{\sin(x)}{\sqrt{1-x}}\,dx\\ &= -\left(\cos(1)\int \frac{\sin(1-x)}{\sqrt{1-x}}\,dx\right)+\sin(1)\int \frac{\cos(1-x)}{\sqrt{1-x}}\,dx\\ &= (2\cos(1))\,\text{Subst}\left(\int \sin\left(x^2\right)\,dx,x,\sqrt{1-x}\right)-(2\sin(1))\,\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,\sqrt{1-x}\right)\\ &= \sqrt{2\pi}\,\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)-\sqrt{2\pi}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right)\sin(1)\end{aligned}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 1.13

$$-\frac{e^{-i}\left(e^{2i}\sqrt{-i(x-1)}\,\Gamma\left(\frac{1}{2},-i(x-1)\right)+\sqrt{i(x-1)}\,\Gamma\left(\frac{1}{2},i(x-1)\right)\right)}{2\sqrt{1-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[x]*Sin[x])/Sqrt[1 + x],x]
```

```
[Out] -1/2*(E^(2*I)*Sqrt[(-I)*(-1 + x)]*Gamma[1/2, (-I)*(-1 + x)] + Sqrt[I*(-1 +
x)]*Gamma[1/2, I*(-1 + x)])/(E^I*Sqrt[1 - x])
```

fricas [F] time = 2.00, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^2+1}\,\sqrt{x+1}\,\sin(x)}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*sqrt(x + 1)*sin(x)/(x^2 - 1), x)
```

giac [C] time = 0.18, size = 48, normalized size = 0.77

$$-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{i}+\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{-x+1}\right)e^{(-i)}+0.82661$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="giac")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(-x + 1))*e^
I + (1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(-x + 1))*
e^(-I) + 0.826619654151000
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x}\,\sin(x)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x)
```

```
[Out] int((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x)
```

maxima [C] time = 0.38, size = 168, normalized size = 2.71

$$-\frac{\left(\left(\left(i\sqrt{\pi}\left(\text{erf}\left(\sqrt{i\,x-i}\right)-1\right)-i\sqrt{\pi}\left(\text{erf}\left(\sqrt{-i\,x+i}\right)-1\right)\right)\cos(1)+\left(\sqrt{\pi}\left(\text{erf}\left(\sqrt{i\,x-i}\right)-1\right)+\sqrt{\pi}\left(\text{erf}\left(\sqrt{-i\,x+\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(((I*sqrt(pi)*(erf(sqrt(I*x - I)) - 1) - I*sqrt(pi)*(erf(sqrt(-I*x + I
)) - 1))*cos(1) + (sqrt(pi)*(erf(sqrt(I*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-
```

```
I*x + I)) - 1))*sin(1))*cos(1/2*arctan2(x - 1, 0)) + ((sqrt(pi)*(erf(sqrt(I
*x - I)) - 1) + sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*cos(1) + (-I*sqrt(pi)*(
erf(sqrt(I*x - I)) - 1) + I*sqrt(pi)*(erf(sqrt(-I*x + I)) - 1))*sin(1))*sin
(1/2*arctan2(x - 1, 0)))*sqrt(-x + 1)/sqrt(abs(x - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(x)\sqrt{x+1}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*(x + 1)^(1/2))/(1 - x^2)^(1/2),x)
```

```
[Out] int((sin(x)*(x + 1)^(1/2))/(1 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\,\sin(x)}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2)*sin(x),x)
```

```
[Out] Integral(sqrt(x + 1)*sin(x)/sqrt(-(x - 1)*(x + 1)), x)
```

3.818 $\int e^{\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}\left(18a^2+2(1-6a)bx-10a+7\right)}{24b^4}-\frac{\left(-8a^3+12a^2-12a+3\right)\sqrt{-a-bx+1}\,\sqrt{a+bx+}}{8b^4}$$

```
[Out] 1/8*(-8*a^3+12*a^2-12*a+3)*arcsin(b*x+a)/b^4-1/4*x^2*(b*x+a+1)^(3/2)*(-b*x-
a+1)^(1/2)/b^2-1/24*(b*x+a+1)^(3/2)*(7-10*a+18*a^2+2*(1-6*a)*b*x)*(-b*x-a+1
)^(1/2)/b^4-1/8*(-8*a^3+12*a^2-12*a+3)*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b^4
```

Rubi [A] time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6163, 100, 147, 50, 53, 619, 216}

$$-\frac{\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}\left(18a^2+2(1-6a)bx-10a+7\right)}{24b^4}-\frac{\left(-8a^3+12a^2-12a+3\right)\sqrt{-a-bx+1}\,\sqrt{a+bx+}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]*x^3,x]
```

```
[Out] -((3 - 12*a + 12*a^2 - 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4)
- (x^2*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(4*b^2) - (Sqrt[1 - a - b*x]*
(1 + a + b*x)^(3/2)*(7 - 10*a + 18*a^2 + 2*(1 - 6*a)*b*x))/(24*b^4) + ((3 -
 12*a + 12*a^2 - 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx$$
$$= -\frac{x^2\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{4b^2} - \frac{\int \frac{x\sqrt{1+a+bx}\left(-2\left(1-a^2\right)-(1-6a)bx\right)}{\sqrt{1-a-bx}}\,dx}{4b^2}$$
$$= -\frac{x^2\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}\left(7-10a+18a^2+2(1-6a)bx\right.}{24b^4}$$
$$= -\frac{\left(3-12a+12a^2-8a^3\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{8b^4} - \frac{x^2\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-}}{}$$
$$= -\frac{\left(3-12a+12a^2-8a^3\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{8b^4} - \frac{x^2\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-}}{}$$
$$= -\frac{\left(3-12a+12a^2-8a^3\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{8b^4} - \frac{x^2\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-}}{}$$
$$= -\frac{\left(3-12a+12a^2-8a^3\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{8b^4} - \frac{x^2\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{4b^2} - \frac{\sqrt{1-}}{}$$

Mathematica [A] time = 0.39, size = 149, normalized size = 0.96

$$-\frac{\left(8a^3-12a^2+12a-3\right)\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{-b}\,\sqrt{-a-bx+1}}{\sqrt{2}\,\sqrt{b}}\right)}{4b^{9/2}} - \frac{\sqrt{-a^2-2abx-b^2x^2+1}\,\left(-6a^3+a^2(6bx+44)-a\left(6b^2x^2\right.\right.}{24b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]*x^3,x]
```

```
[Out] -1/24*(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(16 - 6*a^3 + 9*b*x + 8*b^2*x^2 +
6*b^3*x^3 + a^2*(44 + 6*b*x) - a*(39 + 20*b*x + 6*b^2*x^2)))/b^4 - ((-3 + 1
2*a - 12*a^2 + 8*a^3)*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2
]*Sqrt[b])])/(4*b^(9/2))
```

fricas [A] time = 0.60, size = 144, normalized size = 0.92

$$\frac{3\left(8\,a^3-12\,a^2+12\,a-3\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)-\left(6\,b^3x^3-2\,(3\,a-4)b^2x^2-6\,a^3+\left(6\,a^2-20\,a+9\right)bx\right.}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^3 - 12*a^2 + 12*a - 3)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (6*b^3*x^3 - 2*(3*a - 4)*b^2*
x^2 - 6*a^3 + (6*a^2 - 20*a + 9)*b*x + 44*a^2 - 39*a + 16)*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1))/b^4
```

giac [A] time = 0.23, size = 139, normalized size = 0.89

$$-\frac{1}{24}\sqrt{-(bx+a)^2+1}\left(\left(2x\left(\frac{3x}{b}-\frac{3ab^5-4b^5}{b^7}\right)+\frac{6a^2b^4-20ab^4+9b^4}{b^7}\right)x-\frac{6a^3b^3-44a^2b^3+39ab^3-16b^3}{b^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-(b*x + a)^2 + 1)*((2*x*(3*x/b - (3*a*b^5 - 4*b^5)/b^7) + (6*a^2
*b^4 - 20*a*b^4 + 9*b^4)/b^7)*x - (6*a^3*b^3 - 44*a^2*b^3 + 39*a*b^3 - 16*b
^3)/b^7) + 1/8*(8*a^3 - 12*a^2 + 12*a - 3)*arcsin(-b*x - a)*sgn(b)/(b^3*abs
(b))
```

maple [B] time = 0.04, size = 487, normalized size = 3.12

$$-\frac{x^3\sqrt{-b^2x^2-2abx-a^2+1}}{4b}+\frac{a\,x^2\sqrt{-b^2x^2-2abx-a^2+1}}{4b^2}-\frac{a^2x\sqrt{-b^2x^2-2abx-a^2+1}}{4b^3}+\frac{a^3\sqrt{-b^2x^2-2abx-}}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3,x)
```

```
[Out] -1/4*x^3/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/4*a/b^2*x^2*(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)-1/4*a^2/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/4*a^3/b^4*(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a^2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)
/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+13/8*a/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-
3/8/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/8/b^3/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/3*x^2/b^2*(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)+5/6*a/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-11/6*a^2/b^4*(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)-a^3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2))-3/2*a/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/
b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-2/3/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

maxima [B] time = 0.43, size = 540, normalized size = 3.46

$$-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,x^3}{4\,b}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(a+1)x^2}{3\,b^2}+\frac{7\sqrt{-b^2x^2-2\,abx-a^2+1}\,ax^2}{12\,b^2}+\frac{5\,(a+1)a^3\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x^3/b - 1/3*sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)*(a + 1)*x^2/b^2 + 7/12*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x^2
/b^2 + 5/2*(a + 1)*a^3*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))
/b^4 - 35/8*a^4*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 +
5/6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a + 1)*a*x/b^3 - 35/24*sqrt(-b^2*x^
2 - 2*a*b*x - a^2 + 1)*a^2*x/b^3 - 3/2*(a^2 - 1)*(a + 1)*a*arcsin(-(b^2*x +
 a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 + 15/4*(a^2 - 1)*a^2*arcsin(-(b^2*
x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 - 5/2*sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*(a + 1)*a^2/b^4 + 35/8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/b^4
 + 3/8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)*x/b^3 - 3/8*(a^2 - 1)^2
*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 + 2/3*sqrt(-b^2*x
^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)*(a + 1)/b^4 - 55/24*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*(a^2 - 1)*a/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(a+b\,x+1)}{\sqrt{1-(a+b\,x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x + 1))/(1 - (a + b*x)^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x + 1))/(1 - (a + b*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(a+bx+1)}{\sqrt{-(a+bx-1)\,(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + 1)/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

3.819 $\int e^{\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=130

$$-\frac{\left(2a^2-2a+1\right)\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{2b^3}+\frac{\left(2a^2-2a+1\right)\sin^{-1}(a+bx)}{2b^3}-\frac{(1-4a)\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{6b^3}$$

```
[Out] 1/2*(2*a^2-2*a+1)*arcsin(b*x+a)/b^3-1/6*(1-4*a)*(b*x+a+1)^(3/2)*(-b*x-a+1)^
(1/2)/b^3-1/3*x*(b*x+a+1)^(3/2)*(-b*x-a+1)^(1/2)/b^2-1/2*(2*a^2-2*a+1)*(-b*
x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b^3
```

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6163, 90, 80, 50, 53, 619, 216}

$$-\frac{\left(2a^2-2a+1\right)\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{2b^3}+\frac{\left(2a^2-2a+1\right)\sin^{-1}(a+bx)}{2b^3}-\frac{x\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{3b^2}-(1-$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]*x^2,x]
```

```
[Out] -((1 - 2*a + 2*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) - ((1 - 4*
a)*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(6*b^3) - (x*Sqrt[1 - a - b*x]*(1
 + a + b*x)^(3/2))/(3*b^2) + ((1 - 2*a + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{\tanh^{-1}(a+bx)}x^2\,dx = \int \frac{x^2\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx$$

$$= -\frac{x\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{3b^2} - \frac{\int \frac{\sqrt{1+a+bx}\left(-1+a^2-(1-4a)bx\right)}{\sqrt{1-a-bx}}\,dx}{3b^2}$$

$$= -\frac{(1-4a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{6b^3} - \frac{x\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{3b^2} + \frac{\left(1-2a+2a^2\right)\int \frac{\sqrt{}}{\sqrt{}}}{2b^2}$$

$$= -\frac{\left(1-2a+2a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} - \frac{(1-4a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{6b^3} - \frac{x\sqrt{1-}}{}$$

$$= -\frac{\left(1-2a+2a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} - \frac{(1-4a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{6b^3} - \frac{x\sqrt{1-}}{}$$

$$= -\frac{\left(1-2a+2a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} - \frac{(1-4a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{6b^3} - \frac{x\sqrt{1-}}{}$$

$$= -\frac{\left(1-2a+2a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} - \frac{(1-4a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{6b^3} - \frac{x\sqrt{1-}}{}$$

Mathematica [A] time = 0.18, size = 159, normalized size = 1.22

$$\frac{-\sqrt{b}\sqrt{-a^2-2abx-b^2x^2+1}\left(2a^2-a(2bx+9)+2b^2x^2+3bx+4\right)+6\left(2a^2+1\right)\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{-b}\,\sqrt{-a-bx+1}}{\sqrt{2}\,\sqrt{b}}\right)+}{6b^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]*x^2,x]
```

```
[Out] (-(Sqrt[b]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(4 + 2*a^2 + 3*b*x + 2*b^2*x^2
 - a*(9 + 2*b*x))) + 6*(1 + 2*a^2)*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a -
b*x])/(Sqrt[2]*Sqrt[b])] + 12*a*Sqrt[-b]*ArcSinh[(Sqrt[b]*Sqrt[1 - a - b*x]
)/(Sqrt[2]*Sqrt[-b])])/(6*b^(7/2))
```

fricas [A] time = 1.16, size = 116, normalized size = 0.89

$$-\frac{3\left(2\,a^2-2\,a+1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\left(2\,b^2x^2-(2\,a-3)bx+2\,a^2-9\,a+4\right)\sqrt{-b^2x^2-2\,abx-a^2}}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(2*a^2 - 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x +
a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (2*b^2*x^2 - (2*a - 3)*b*x + 2*a^2 - 9*
a + 4)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b^3
```

giac [A] time = 0.21, size = 97, normalized size = 0.75

$$-\frac{1}{6}\sqrt{-(bx+a)^2+1}\left(x\left(\frac{2x}{b}-\frac{2ab^3-3b^3}{b^5}\right)+\frac{2a^2b^2-9ab^2+4b^2}{b^5}\right)-\frac{\left(2a^2-2a+1\right)\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-(b*x + a)^2 + 1)*(x*(2*x/b - (2*a*b^3 - 3*b^3)/b^5) + (2*a^2*b^2
 - 9*a*b^2 + 4*b^2)/b^5) - 1/2*(2*a^2 - 2*a + 1)*arcsin(-b*x - a)*sgn(b)/(b
^2*abs(b))
```

maple [B] time = 0.04, size = 315, normalized size = 2.42

$$-\frac{x^2\sqrt{-b^2x^2-2abx-a^2+1}}{3b}+\frac{ax\sqrt{-b^2x^2-2abx-a^2+1}}{3b^2}-\frac{a^2\sqrt{-b^2x^2-2abx-a^2+1}}{3b^3}-\frac{a\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+}}\right.}{b^2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2,x)
```

```
[Out] -1/3*x^2/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/3*a/b^2*x*(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)-1/3*a^2/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/b^2/(b^2)^(1/2)*arct
an((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-2/3/b^3*(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2)-1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a/b^3*(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)+a^2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b
)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
```

maxima [B] time = 0.42, size = 355, normalized size = 2.73

$$-\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,x^2}{3b}-\frac{3(a+1)a^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-\left(a^2-1\right)b^2}}\right)}{2b^3}+\frac{5a^3\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-\left(a^2-1\right)b^2}}\right)}{2b^3}-\frac{\sqrt{-b^2x^2-2\,ab}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x^2/b - 3/2*(a + 1)*a^2*arcsin(-(b^
2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + 5/2*a^3*arcsin(-(b^2*x + a*
b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 - 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*(a + 1)*x/b^2 + 5/6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x/b^2 + 1/2*(a
^2 - 1)*(a + 1)*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 -
3/2*(a^2 - 1)*a*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 +
3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a + 1)*a/b^3 - 5/2*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1)*a^2/b^3 + 2/3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 -
1)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,x+1\right)}{\sqrt{1-{\left(a+b\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x + 1))/(1 - (a + b*x)^2)^(1/2),x)
[Out] int((x^2*(a + b*x + 1))/(1 - (a + b*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + bx + 1\right)}{\sqrt{- \left(a + bx - 1\right) \left(a + bx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x**2,x)
[Out] Integral(x**2*(a + b*x + 1)/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

3.820 $\int e^{\tanh^{-1}(a+bx)} x \, dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{2b^2}-\frac{(1-2a)\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{2b^2}+\frac{(1-2a)\sin^{-1}(a+bx)}{2b^2}$$

```
[Out] 1/2*(1-2*a)*arcsin(b*x+a)/b^2-1/2*(b*x+a+1)^(3/2)*(-b*x-a+1)^(1/2)/b^2-1/2*
(1-2*a)*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6163, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{2b^2}-\frac{(1-2a)\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{2b^2}+\frac{(1-2a)\sin^{-1}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]*x,x]
```

```
[Out] -((1 - 2*a)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) - (Sqrt[1 - a - b*
x]*(1 + a + b*x)^(3/2))/(2*b^2) + ((1 - 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{\tanh^{-1}(a+bx)}x\,dx = \int \frac{x\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx$$

$$= -\frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} + \frac{(1-2a)\int \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx}{2b}$$

$$= -\frac{(1-2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} - \frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} + \frac{(1-2a)\int \frac{1}{\sqrt{1-a-bx}\,\sqrt{1+a}}$$

$$= -\frac{(1-2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} - \frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} + \frac{(1-2a)\int \frac{1}{\sqrt{(1-a)(1+a)-2}}}{2b}$$

$$= -\frac{(1-2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} - \frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} - \frac{(1-2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.}{4b}$$

$$= -\frac{(1-2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} - \frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} + \frac{(1-2a)\sin^{-1}(a+bx)}{2b^2}$$

Mathematica [A] time = 0.09, size = 130, normalized size = 1.55

$$\frac{\sqrt{b}\sqrt{-a^2-2abx-b^2x^2+1}\,(a-bx-2)+2\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)+4a\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{-b}}\right)}{2b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]*x,x]
```

```
[Out] (Sqrt[b]*(-2 + a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + 2*Sqrt[-b]*ArcS
inh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])] + 4*a*Sqrt[-b]*ArcSinh[
(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(2*b^(5/2))
```

fricas [A] time = 0.48, size = 92, normalized size = 1.10

$$\frac{(2\,a-1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx-a+2)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2
 + 2*a*b*x + a^2 - 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - a + 2))/
b^2
```

giac [A] time = 0.39, size = 59, normalized size = 0.70

$$-\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b}-\frac{ab-2\,b}{b^3}\right)+\frac{(2\,a-1)\arcsin\left(-bx-a\right)\mathrm{sgn}(b)}{2\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-(b*x + a)^2 + 1)*(x/b - (a*b - 2*b)/b^3) + 1/2*(2*a - 1)*arcsin(
-b*x - a)*sgn(b)/(b*abs(b))
```

maple [B] time = 0.04, size = 178, normalized size = 2.12

$$-\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b}+\frac{a\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2}+\frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2b\sqrt{b^2}}-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{b^2}-\frac{a}{\ }$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x,x)

[Out] -1/2*x/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2*a/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))

maxima [B] time = 0.41, size = 209, normalized size = 2.49

$$\frac{(a+1)a \arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^2} - \frac{3\,a^2 \arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^2} - \frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,x}{2\,b} + \frac{(a^2-1) \arcsin\left(-\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x,x, algorithm="maxima")

[Out] (a + 1)*a*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^2 - 3/2*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^2 - 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b + 1/2*(a^2 - 1)*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^2 - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a + 1)/b^2 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/b^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,x+1)}{\sqrt{1-(a+b\,x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x + 1))/(1 - (a + b*x)^2)^(1/2),x)

[Out] int((x*(a + b*x + 1))/(1 - (a + b*x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+bx+1)}{\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x,x)

[Out] Integral(x*(a + b*x + 1)/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)

3.821 $\int e^{\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=39

$$\frac{\sin^{-1}(a+bx)}{b}-\frac{\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{b}$$

```
[Out] arcsin(b*x+a)/b-(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6161, 50, 53, 619, 216}

$$\frac{\sin^{-1}(a+bx)}{b}-\frac{\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x],x]
```

```
[Out] -((Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/b) + ArcSin[a + b*x]/b
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(a+bx)}\,dx &= \int \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \int \frac{1}{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b^2} \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \frac{\sin^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.72

$$\frac{\sin^{-1}(a+bx)-\sqrt{1-(a+bx)^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a + b*x],x]
```

```
[Out] (-Sqrt[1 - (a + b*x)^2] + ArcSin[a + b*x])/b
```

fricas [B] time = 0.65, size = 76, normalized size = 1.95

$$-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}+\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1) + arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

giac [A] time = 0.21, size = 36, normalized size = 0.92

$$-\frac{\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{|b|}-\frac{\sqrt{-(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(-b*x - a)*sgn(b)/abs(b) - sqrt(-(b*x + a)^2 + 1)/b
```

maple [A] time = 0.04, size = 71, normalized size = 1.82

$$-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{b}+\frac{\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b
)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 65, normalized size = 1.67

$$-\frac{\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-\left(a^2-1\right)b^2}}\right)}{b}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b - sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)/b
```

mupad [B] time = 1.42, size = 101, normalized size = 2.59

$$\frac{\mathrm{asin}\left(a+b\,x\right)}{b}-\frac{\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}}{b}-\frac{a\,\ln\left(\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}-\frac{x\,b^2+a\,b}{\sqrt{-b^2}}\right)}{\sqrt{-b^2}}+\frac{a\,\mathrm{asin}\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)/(1 - (a + b*x)^2)^(1/2),x)
```

```
[Out] asin(a + b*x)/b - (1 - b^2*x^2 - 2*a*b*x - a^2)^(1/2)/b - (a*log((1 - b^2*x
^2 - 2*a*b*x - a^2)^(1/2) - (a*b + b^2*x)/(-b^2)^(1/2)))/(-b^2)^(1/2) + (a*
asin(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+1}{\sqrt{-\left(a+bx-1\right)\left(a+bx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral((a + b*x + 1)/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

3.822 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=64

$$\sin^{-1}(a+bx)-\frac{2(a+1)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{\sqrt{1-a^2}}$$

```
[Out] arcsin(b*x+a)-2*(1+a)*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x
-a+1)^(1/2))/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6163, 105, 53, 619, 216, 93, 208}

$$\sin^{-1}(a+bx)-\frac{2(a+1)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/x,x]
```

```
[Out] ArcSin[a + b*x] - (2*(1 + a)*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[
1 + a]*Sqrt[1 - a - b*x])])/Sqrt[1 - a^2]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}}{x}\,dx = \int \frac{\sqrt{1+a+bx}}{x\sqrt{1-a-bx}}\,dx$$
$$= -\left((-1-a)\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx\right) + b\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx$$
$$= (2(1+a))\,\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right) + b\int \frac{1}{\sqrt{(1-a)(1+a)-2abx}}$$
$$= -\frac{2(1+a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{\sqrt{1-a^2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b}$$
$$= \sin^{-1}(a+bx) - \frac{2(1+a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{\sqrt{1-a^2}}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 1.66

$$\frac{2\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}} - \frac{2\sqrt{-a-1}\,\tanh^{-1}\left(\frac{\sqrt{-a-1}\sqrt{-a-bx+1}}{\sqrt{a-1}\sqrt{a+bx+1}}\right)}{\sqrt{a-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/x,x]
```

```
[Out] (2*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/Sqrt[b
] - (2*Sqrt[-1 - a]*ArcTanh[(Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*
Sqrt[1 + a + b*x])])/Sqrt[-1 + a]
```

fricas [B] time = 0.70, size = 306, normalized size = 4.78

$$\left[\frac{1}{2}\sqrt{-\frac{a+1}{a-1}}\,\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\left(a^3-a\right)bx-4\,a^2-2\sqrt{-b^2x^2-2\,abx-a^2+1}\left(a^3+\left(a^2-a\right)bx-a^2-\right.}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-(a + 1)/(a - 1))*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*
b*x - 4*a^2 - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^3 + (a^2 - a)*b*x - a
```

```
^2 - a + 1)*sqrt(-(a + 1)/(a - 1)) + 2)/x^2) - arctan(sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)), sqrt((a + 1)/(a - 1
))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt((a + 1)
/(a - 1))/((a + 1)*b^2*x^2 + a^3 + 2*(a^2 + a)*b*x + a^2 - a - 1)) - arctan
(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)
)]
```

giac [A] time = 0.23, size = 79, normalized size = 1.23

$$-\frac{b\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{|b|}+\frac{2\,(ab+b)\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -b*arcsin(-b*x - a)*sgn(b)/abs(b) + 2*(a*b + b)*arctan((((sqrt(-(b*x + a)^2
+ 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1)*abs(b))
```

maple [B] time = 0.04, size = 168, normalized size = 2.62

$$\frac{b\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{\sqrt{b^2}}-\frac{\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{x}\right)}{\sqrt{-a^2+1}}-\frac{\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{x}\right)}{\sqrt{-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x,x)
```

```
[Out] b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/
(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2))/x)-1/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2))/x)*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [B] time = 1.39, size = 132, normalized size = 2.06

$$\operatorname{asin}\left(a+b\,x\right)-\frac{\ln\left(\frac{\sqrt{1-a^2}\,\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}}{x}-\frac{a^2-1}{x}-a\,b\right)}{\sqrt{1-a^2}}-\frac{a\,\ln\left(\frac{\sqrt{1-a^2}\,\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}}{x}-\frac{a^2-1}{x}-a\,b\right)}{\sqrt{1-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)/(x*(1 - (a + b*x)^2)^(1/2)),x)
```

```
[Out] asin(a + b*x) - log(((1 - a^2)^(1/2)*(1 - b^2*x^2 - 2*a*b*x - a^2)^(1/2))/x
 - (a^2 - 1)/x - a*b)/(1 - a^2)^(1/2) - (a*log(((1 - a^2)^(1/2)*(1 - b^2*x^
2 - 2*a*b*x - a^2)^(1/2))/x - (a^2 - 1)/x - a*b))/(1 - a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+bx+1}{x\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] Integral((a + b*x + 1)/(x*sqrt(-(a + b*x - 1)*(a + b*x + 1))), x)
```

3.823 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=98

$$-\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{(1-a)x}$$

```
[Out] -2*b*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(1/2))/(1-a
)/(-a^2+1)^(1/2)-(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1-a)/x
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6163, 94, 93, 208}

$$-\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{(1-a)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/x^2,x]
```

```
[Out] -((Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/((1 - a)*x)) - (2*b*ArcTanh[(Sqrt[1
 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)*Sqrt[1
- a^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{\sqrt{1+a+bx}}{x^2\sqrt{1-a-bx}}\,dx \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{(1-a)x}+\frac{b\int \frac{1}{x\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{1-a} \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{(1-a)x}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx,x,\frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1-a} \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{(1-a)x}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{1-a-bx}}\right)}{(1-a)\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.90

$$\frac{\sqrt{-((a+bx-1)(a+bx+1))}}{(a-1)x}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{-a-1}\,\sqrt{-a-bx+1}}{\sqrt{a-1}\,\sqrt{a+bx+1}}\right)}{\sqrt{-a-1}\,(a-1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/x^2,x]
```

```
[Out] Sqrt[-((-1 + a + b*x)*(1 + a + b*x))]/((-1 + a)*x) - (2*b*ArcTanh[(Sqrt[-1
- a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqrt[1 + a + b*x])])/(Sqrt[-1 - a]*(-
1 + a)^(3/2))
```

fricas [A] time = 1.12, size = 282, normalized size = 2.88

$$\left[-\frac{\sqrt{-a^2+1}\,bx\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\,\left(a^3-a\right)bx-2\,\sqrt{-b^2x^2-2\,abx-a^2+1}\,\left(abx+a^2-1\right)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)-2\,\sqrt{-b^2x^2-2\,abx-a^2+\cdots}}{2\,\left(a^3-a^2-a+1\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + 1)*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*
x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) -
 4*a^2 + 2)/x^2) - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/((a^3 -
a^2 - a + 1)*x), -(sqrt(a^2 - 1)*b*x*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*
b*x - 2*a^2 + 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/((a^3 - a
^2 - a + 1)*x)]
```

giac [B] time = 0.25, size = 190, normalized size = 1.94

$$-\frac{2\,b^2\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}\,(a|b|-|b|)}+\frac{2\left(ab^2-\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)b^2}{b^2x+ab}\right)}{\left(a^2|b|-a|b|\right)\left(\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)^2a}{\left(b^2x+ab\right)^2}+a-\frac{2\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)}{b^2x+ab}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqr
t(a^2 - 1))/(sqrt(a^2 - 1)*(a*abs(b) - abs(b))) + 2*(a*b^2 - (sqrt(-(b*x +
a)^2 + 1)*abs(b) + b)*b^2/(b^2*x + a*b))/((a^2*abs(b) - a*abs(b))*((sqrt(-(
b*x + a)^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-(b*x + a)^2
+ 1)*abs(b) + b)/(b^2*x + a*b)))
```

maple [B] time = 0.04, size = 265, normalized size = 2.70

$$-\frac{b\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{x}\right)}{\sqrt{-a^2+1}}-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{\left(-a^2+1\right)x}-\frac{ab\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a\cdots}}{x}\right)}{\left(-a^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2,x)
```

```
[Out] -b/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2))/x)-1/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a*b/(-a^2+1)^(3
/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x
)-a/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a^2*b/(-a^2+1)^(3/2)*ln((-2*a
^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [B] time = 1.69, size = 241, normalized size = 2.46

$$\frac{\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}}{x\,\left(a^2-1\right)}-\frac{b\,\ln\left(\frac{\sqrt{1-a^2}\,\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}}{x}-\frac{a^2-1}{x}-a\,b\right)}{\sqrt{1-a^2}}+\frac{a^2\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a-1}{\sqrt{1-a^2}\,\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}}\right)}{{\left(1-a^2\right)}^{3/2}}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)/(x^2*(1 - (a + b*x)^2)^(1/2)),x)
```

```
[Out] (1 - b^2*x^2 - 2*a*b*x - a^2)^(1/2)/(x*(a^2 - 1)) - (b*log(((1 - a^2)^(1/2)
*(1 - b^2*x^2 - 2*a*b*x - a^2)^(1/2))/x - (a^2 - 1)/x - a*b))/(1 - a^2)^(1/
2) + (a^2*b*atanh((a^2 + a*b*x - 1)/((1 - a^2)^(1/2)*(1 - b^2*x^2 - 2*a*b*x
 - a^2)^(1/2))))/(1 - a^2)^(3/2) + (a*(1 - b^2*x^2 - 2*a*b*x - a^2)^(1/2))/
(x*(a^2 - 1)) + (a*b*atanh((a^2 + a*b*x - 1)/((1 - a^2)^(1/2)*(1 - b^2*x^2
- 2*a*b*x - a^2)^(1/2))))/(1 - a^2)^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+1}{x^2\sqrt{-(a+bx-1)\,(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*x + 1)/(x**2*sqrt(-(a + b*x - 1)*(a + b*x + 1))), x)
```

3.824 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=162

$$-\frac{(2a+1)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2(a+1)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{2\left(1-a^2\right)x^2}-\frac{(2a+1)b\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2(1-a)^2(a+1)x}$$

```
[Out] -(1+2*a)*b^2*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(1/
2))/(1-a)^2/(1+a)/(-a^2+1)^(1/2)-1/2*(b*x+a+1)^(3/2)*(-b*x-a+1)^(1/2)/(-a^2
+1)/x^2-1/2*(1+2*a)*b*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1-a)^2/(1+a)/x
```

Rubi [A] time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6163, 96, 94, 93, 208}

$$-\frac{(2a+1)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2(a+1)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{2\left(1-a^2\right)x^2}-\frac{(2a+1)b\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2(1-a)^2(a+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/x^3,x]
```

```
[Out] -((1 + 2*a)*b*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*(1 - a)^2*(1 + a)*x)
- (Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(2*(1 - a^2)*x^2) - ((1 + 2*a)*b^
2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])
/((1 - a)^2*(1 + a)*Sqrt[1 - a^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{1+a+bx}}{x^3\sqrt{1-a-bx}}\,dx$$

$$= -\frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2\left(1-a^2\right)x^2} + \frac{((1+2a)b)\int \frac{\sqrt{1+a+bx}}{x^2\sqrt{1-a-bx}}\,dx}{2\left(1-a^2\right)}$$

$$= -\frac{(1+2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2(1-a)^2(1+a)x} - \frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2\left(1-a^2\right)x^2} + \frac{\left((1+2a)b^2\right)\int \frac{}{x\sqrt{1-a-b}}}{2(1-a)^2(1+}$$

$$= -\frac{(1+2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2(1-a)^2(1+a)x} - \frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2\left(1-a^2\right)x^2} + \frac{\left((1+2a)b^2\right)\text{Subst}\left(\int\right.}{(1-}$$

$$= -\frac{(1+2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2(1-a)^2(1+a)x} - \frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2\left(1-a^2\right)x^2} - \frac{(1+2a)b^2\tanh^{-1}\left(\frac{\sqrt{1}}{\sqrt{1}}\right.}{(1-a)^2(1+a)\sqrt{}}$$

Mathematica [A] time = 0.13, size = 123, normalized size = 0.76

$$\frac{\left(a^2-abx-2bx-1\right)\sqrt{-a^2-2abx-b^2x^2+1}}{2(a-1)^2(a+1)x^2}-\frac{(2a+1)b^2\tanh^{-1}\left(\frac{\sqrt{-a-1}\sqrt{-a-bx+1}}{\sqrt{a-1}\sqrt{a+bx+1}}\right)}{(-a-1)^{3/2}(a-1)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/x^3,x]
```

```
[Out] ((-1 + a^2 - 2*b*x - a*b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/(2*(-1 + a)^
2*(1 + a)*x^2) - ((1 + 2*a)*b^2*ArcTanh[(Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(S
qrt[-1 + a]*Sqrt[1 + a + b*x])])/((-1 - a)^(3/2)*(-1 + a)^(5/2))
```

fricas [A] time = 0.72, size = 359, normalized size = 2.22

$$\left[-\frac{\sqrt{-a^2+1}\,(2\,a+1)b^2x^2\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\left(a^3-a\right)bx+2\,\sqrt{-b^2x^2-2\,abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)-2\left(a^4-\left(a^3+2\right.\right.}{4\left(a^5-a^4-2\,a^3+2\,a^2+a-1\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 + 1)*(2*a + 1)*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 +
4*(a^3 - a)*b*x + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt(-a^2 + 1) - 4*a^2 + 2)/x^2) - 2*(a^4 - (a^3 + 2*a^2 - a - 2)*b*x - 2*a^2
 + 1)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 - a^4 - 2*a^3 + 2*a^2 + a -
 1)*x^2), 1/2*(sqrt(a^2 - 1)*(2*a + 1)*b^2*x^2*arctan(sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*
(a^3 - a)*b*x - 2*a^2 + 1)) + (a^4 - (a^3 + 2*a^2 - a - 2)*b*x - 2*a^2 + 1)
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 - a^4 - 2*a^3 + 2*a^2 + a - 1)*x
^2)]
```

giac [B] time = 0.39, size = 622, normalized size = 3.84

$$\frac{\left(2\,ab^3+b^3\right)\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^3|b|-a^2|b|-a|b|+|b|\right)\sqrt{a^2-1}}-\frac{\frac{2\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)^2a^4b^3}{\left(b^2x+ab\right)^2}+2\,a^4b^3-\frac{5\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a^3b^3}{b^2x+ab}+\frac{2\left(\sqrt{-(bx+a)^2+1}\right.}{\left(b^2x+ab\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (2*a*b^3 + b^3)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b)
 - 1)/sqrt(a^2 - 1))/((a^3*abs(b) - a^2*abs(b) - a*abs(b) + abs(b))*sqrt(a^
2 - 1)) - (2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^4*b^3/(b^2*x + a*b)^2
+ 2*a^4*b^3 - 5*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^3*b^3/(b^2*x + a*b) +
 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^3*b^3/(b^2*x + a*b)^2 - 3*(sqrt(
-(b*x + a)^2 + 1)*abs(b) + b)^3*a^3*b^3/(b^2*x + a*b)^3 + 2*a^3*b^3 - 6*(sq
rt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^3/(b^2*x + a*b) + 3*(sqrt(-(b*x + a)
^2 + 1)*abs(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 - 2*(sqrt(-(b*x + a)^2 + 1)*a
bs(b) + b)^3*a^2*b^3/(b^2*x + a*b)^3 - a^2*b^3 + 2*(sqrt(-(b*x + a)^2 + 1)*
abs(b) + b)*a*b^3/(b^2*x + a*b) + 4*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a
*b^3/(b^2*x + a*b)^2 + 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x
 + a*b)^3 - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((
a^5*abs(b) - a^4*abs(b) - a^3*abs(b) + a^2*abs(b))*((sqrt(-(b*x + a)^2 + 1)
*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b
)/(b^2*x + a*b))^2)
```

maple [B] time = 0.04, size = 453, normalized size = 2.80

$$-\frac{b\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}{\left(-a^{2}+1\right)x}-\frac{3a\,b^{2}\ln\left(\frac{-2a^{2}+2-2abx+2\sqrt{-a^{2}+1}\,\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}{x}\right)}{2\left(-a^{2}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}{2\left(-a^{2}+1\right)x^{2}}-\frac{3ab\sqrt{-}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^3,x)
```

```
[Out] -b/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*a*b^2/(-a^2+1)^(3/2)*ln((-
2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(-a
^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*a*b/(-a^2+1)^2/x*(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2)-3/2*a^2*b^2/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1
)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2*b^2/(-a^2+1)^(3/2)*ln((-2*a^
2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2*a/(-a^2
+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*a^2*b/(-a^2+1)^2/x*(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2)-3/2*a^3*b^2/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1
)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x+1}{x^3\,\sqrt{1-{\left(a+b\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)/(x^3*(1 - (a + b*x)^2)^(1/2)),x)
```

```
[Out] int((a + b*x + 1)/(x^3*(1 - (a + b*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+1}{x^3\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*x + 1)/(x**3*sqrt(-(a + b*x - 1)*(a + b*x + 1))), x)
```

3.825 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=213

$$-\frac{\left(2a^2+2a+1\right)b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\left(1-a^2\right)^{5/2}}-\frac{(a+4)(2a+1)b^2\sqrt{-a-bx+1}\sqrt{a+bx+1}}{6(1-a)^3(a+1)^2x}-\frac{\sqrt{-a-bx+1}\sqrt{a+b}}{3(1-a)x^3}$$

```
[Out] -(2*a^2+2*a+1)*b^3*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+
1)^(1/2))/(1-a)/(-a^2+1)^(5/2)-1/3*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1-a)/x
^3-1/6*(3+2*a)*b*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1-a)^2/(1+a)/x^2-1/6*(4+
a)*(1+2*a)*b^2*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1-a)^3/(1+a)^2/x
```

Rubi [A] time = 0.18, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 99, 151, 12, 93, 208}

$$-\frac{\left(2a^2+2a+1\right)b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\left(1-a^2\right)^{5/2}}-\frac{(a+4)(2a+1)b^2\sqrt{-a-bx+1}\sqrt{a+bx+1}}{6(1-a)^3(a+1)^2x}-\frac{(2a+3)b\sqrt{-a-bx+}}{6(1-a)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/x^4,x]
```

```
[Out] -(Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(3*(1 - a)*x^3) - ((3 + 2*a)*b*Sqrt[
1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)^2*(1 + a)*x^2) - ((4 + a)*(1 + 2
*a)*b^2*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)^3*(1 + a)^2*x) - ((
1 + 2*a + 2*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*S
qrt[1 - a - b*x])])/((1 - a)*(1 - a^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{1+a+bx}}{x^4\sqrt{1-a-bx}}\,dx$$
$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1-a)x^3} + \frac{\int \frac{(3+2a)b+2b^2x}{x^3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{3(1-a)}$$
$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{\int \frac{-(4+a)(1+2a)b^2-(3+2a)b^3x}{x^2\sqrt{1-a-bx}\,\sqrt{1+a+bx}}}{6(1-a)^2(1+a)}$$
$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{(4+a)(1+2a)b^2\sqrt{1-a}}{6(1-a)^3(1}$$
$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{(4+a)(1+2a)b^2\sqrt{1-a}}{6(1-a)^3(1}$$
$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{(4+a)(1+2a)b^2\sqrt{1-a}}{6(1-a)^3(1}$$
$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1-a)x^3} - \frac{(3+2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)^2(1+a)x^2} - \frac{(4+a)(1+2a)b^2\sqrt{1-a}}{6(1-a)^3(1}$$

Mathematica [A] time = 0.27, size = 193, normalized size = 0.91

$$\frac{\frac{3(2a^2+2a+1)b^2x^2\left(\sqrt{-a-1}\,\sqrt{a-1}\,\sqrt{-((a+bx-1)(a+bx+1))}-2bx\tanh^{-1}\left(\frac{\sqrt{-a-1}\,\sqrt{-a-bx+1}}{\sqrt{a-1}\,\sqrt{a+bx+1}}\right)\right)}{\sqrt{-a-1}\,(a-1)^{3/2}} - (4a+1)bx\sqrt{-a-bx+1}\,(a+bx+1)^{3/2} +}{6\left(a^2-1\right)^2 x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/x^4,x]
```

```
[Out] (2*(-1 + a)*(1 + a)*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2) - (1 + 4*a)*b*x*S
qrt[1 - a - b*x]*(1 + a + b*x)^(3/2) + (3*(1 + 2*a + 2*a^2)*b^2*x^2*(Sqrt[-
1 - a]*Sqrt[-1 + a]*Sqrt[-((-1 + a + b*x)*(1 + a + b*x))] - 2*b*x*ArcTanh[(
Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqrt[1 + a + b*x])]))/(Sqrt[-
1 - a]*(-1 + a)^(3/2)))/(6*(-1 + a^2)^2*x^3)
```

fricas [A] time = 1.76, size = 488, normalized size = 2.29

$$\left[-\frac{3\left(2\,a^2+2\,a+1\right)\sqrt{-a^2+1}\,b^3x^3\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\left(a^3-a\right)bx-2\,\sqrt{-b^2x^2-2\,abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)-2\left(2\,a\right.}{12\left(a^7-a^6-3\,a^5+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2 + 2*a + 1)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^2
 + 2*a^4 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x +
a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) - 2*(2*a^6 + (2*a^4 + 9*a^3 + 2*a
^2 - 9*a - 4)*b^2*x^2 - 6*a^4 - (2*a^5 + 3*a^4 - 4*a^3 - 6*a^2 + 2*a + 3)*b
*x + 6*a^2 - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^7 - a^6 - 3*a^5 + 3
*a^4 + 3*a^3 - 3*a^2 - a + 1)*x^3), -1/6*(3*(2*a^2 + 2*a + 1)*sqrt(a^2 - 1)
*b^3*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a
^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - (2*a^6 +
 (2*a^4 + 9*a^3 + 2*a^2 - 9*a - 4)*b^2*x^2 - 6*a^4 - (2*a^5 + 3*a^4 - 4*a^3
 - 6*a^2 + 2*a + 3)*b*x + 6*a^2 - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((
a^7 - a^6 - 3*a^5 + 3*a^4 + 3*a^3 - 3*a^2 - a + 1)*x^3)]
```

giac [B] time = 1.05, size = 1376, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -(2*a^2*b^4 + 2*a*b^4 + b^4)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/
(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^5*abs(b) - a^4*abs(b) - 2*a^3*abs(b)
+ 2*a^2*abs(b) + a*abs(b) - abs(b))*sqrt(a^2 - 1)) + 1/3*(12*(sqrt(-(b*x +
a)^2 + 1)*abs(b) + b)^2*a^7*b^4/(b^2*x + a*b)^2 + 6*(sqrt(-(b*x + a)^2 + 1)
*abs(b) + b)^4*a^7*b^4/(b^2*x + a*b)^4 + 6*a^7*b^4 - 24*(sqrt(-(b*x + a)^2
+ 1)*abs(b) + b)*a^6*b^4/(b^2*x + a*b) + 24*(sqrt(-(b*x + a)^2 + 1)*abs(b)
+ b)^2*a^6*b^4/(b^2*x + a*b)^2 - 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a
^6*b^4/(b^2*x + a*b)^3 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^6*b^4/(
b^2*x + a*b)^4 - 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^6*b^4/(b^2*x +
a*b)^5 + 12*a^6*b^4 - 57*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^5*b^4/(b^2*x
 + a*b) + 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^5*b^4/(b^2*x + a*b)^2
- 72*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^5*b^4/(b^2*x + a*b)^3 + 30*(sq
rt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^5*b^4/(b^2*x + a*b)^4 - 15*(sqrt(-(b*x
 + a)^2 + 1)*abs(b) + b)^5*a^5*b^4/(b^2*x + a*b)^5 - 2*a^5*b^4 + 84*(sqrt(-
(b*x + a)^2 + 1)*abs(b) + b)^2*a^4*b^4/(b^2*x + a*b)^2 - 12*(sqrt(-(b*x + a
```

```
)^2 + 1)*abs(b) + b)^3*a^4*b^4/(b^2*x + a*b)^3 + 51*(sqrt(-(b*x + a)^2 + 1)
*abs(b) + b)^4*a^4*b^4/(b^2*x + a*b)^4 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b)
+ b)^5*a^4*b^4/(b^2*x + a*b)^5 - 3*a^4*b^4 + 12*(sqrt(-(b*x + a)^2 + 1)*abs
(b) + b)*a^3*b^4/(b^2*x + a*b) - 30*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a
^3*b^4/(b^2*x + a*b)^3 - 18*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^3*b^4/(
b^2*x + a*b)^4 + 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^3*b^4/(b^2*x + a
*b)^5 + 2*a^3*b^4 - 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^4/(b^2*x +
a*b) - 18*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^2*b^4/(b^2*x + a*b)^2 - 4
*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^2*b^4/(b^2*x + a*b)^3 - 18*(sqrt(-
(b*x + a)^2 + 1)*abs(b) + b)^4*a^2*b^4/(b^2*x + a*b)^4 - 6*(sqrt(-(b*x + a)
^2 + 1)*abs(b) + b)^5*a^2*b^4/(b^2*x + a*b)^5 + 12*(sqrt(-(b*x + a)^2 + 1)*
abs(b) + b)^2*a*b^4/(b^2*x + a*b)^2 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b
)^3*a*b^4/(b^2*x + a*b)^3 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a*b^4/
(b^2*x + a*b)^4 - 8*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*b^4/(b^2*x + a*b)
^3)/((a^8*abs(b) - a^7*abs(b) - 2*a^6*abs(b) + 2*a^5*abs(b) + a^4*abs(b) -
a^3*abs(b))*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a -
2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b))^3)
```

maple [B] time = 0.04, size = 683, normalized size = 3.21

$$-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{3\left(-a^2+1\right)x^3}-\frac{5ab\sqrt{-b^2x^2-2abx-a^2+1}}{6\left(-a^2+1\right)^2x^2}-\frac{5a^2b^2\sqrt{-b^2x^2-2abx-a^2+1}}{2\left(-a^2+1\right)^3x}-\frac{5a^3b^3\ln\left(\frac{-2a^2+2-2abx+\cdots}{\cdots}\right)}{2\left(-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^4,x)
```

```
[Out] -1/3/(-a^2+1)/x^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-5/6*a*b/(-a^2+1)^2/x^2*(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)-5/2*a^2*b^2/(-a^2+1)^3/x*(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)-5/2*a^3*b^3/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(
-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3/2*a*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a
*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-2/3*b^2/(-a^2+1)^2
/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/3*a/(-a^2+1)/x^3*(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)-5/6*a^2*b/(-a^2+1)^2/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-5/2*a^3*b^
2/(-a^2+1)^3/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-5/2*a^4*b^3/(-a^2+1)^(7/2)*ln
((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3*a^
2*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))/x)-13/6*a*b^2/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1
/2*b/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2*b^3/(-a^2+1)^(3/2)*ln(
(-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,x+1}{x^4\,\sqrt{1-(a+b\,x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)/(x^4*(1 - (a + b*x)^2)^(1/2)),x)
```

```
[Out] int((a + b*x + 1)/(x^4*(1 - (a + b*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+1}{x^4\sqrt{-(a+bx-1)\,(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x**4,x)
```

```
[Out] Integral((a + b*x + 1)/(x**4*sqrt(-(a + b*x - 1)*(a + b*x + 1))), x)
```

3.826 $\int e^{2\tanh^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=83

$$-\frac{2(1-a)^4\log(-a-bx+1)}{b^5}-\frac{2(1-a)^3x}{b^4}-\frac{(1-a)^2x^2}{b^3}-\frac{2(1-a)x^3}{3b^2}-\frac{x^4}{2b}-\frac{x^5}{5}$$

```
[Out] -2*(1-a)^3*x/b^4-(1-a)^2*x^2/b^3-2/3*(1-a)*x^3/b^2-1/2*x^4/b-1/5*x^5-2*(1-a
)^4*ln(-b*x-a+1)/b^5
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2(1-a)x^3}{3b^2}-\frac{(1-a)^2x^2}{b^3}-\frac{2(1-a)^3x}{b^4}-\frac{2(1-a)^4\log(-a-bx+1)}{b^5}-\frac{x^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])*x^4,x]
```

```
[Out] (-2*(1 - a)^3*x)/b^4 - ((1 - a)^2*x^2)/b^3 - (2*(1 - a)*x^3)/(3*b^2) - x^4/
(2*b) - x^5/5 - (2*(1 - a)^4*Log[1 - a - b*x])/b^5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{2\tanh^{-1}(a+bx)}x^4\,dx = \int \frac{x^4(1+a+bx)}{1-a-bx}\,dx$$
$$= \int \left(\frac{2(-1+a)^3}{b^4} - \frac{2(-1+a)^2x}{b^3} + \frac{2(-1+a)x^2}{b^2} - \frac{2x^3}{b} - x^4 - \frac{2(-1+a)^4}{b^4(-1+a+bx)}\right)dx$$
$$= -\frac{2(1-a)^3x}{b^4} - \frac{(1-a)^2x^2}{b^3} - \frac{2(1-a)x^3}{3b^2} - \frac{x^4}{2b} - \frac{x^5}{5} - \frac{2(1-a)^4\log(1-a-bx)}{b^5}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.90

$$-\frac{2(a-1)^4\log(-a-bx+1)}{b^5} + \frac{2(a-1)^3x}{b^4} - \frac{(a-1)^2x^2}{b^3} + \frac{2(a-1)x^3}{3b^2} - \frac{x^4}{2b} - \frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])*x^4,x]
```

```
[Out] (2*(-1 + a)^3*x)/b^4 - ((-1 + a)^2*x^2)/b^3 + (2*(-1 + a)*x^3)/(3*b^2) - x^
4/(2*b) - x^5/5 - (2*(-1 + a)^4*Log[1 - a - b*x])/b^5
```

fricas [A] time = 0.61, size = 93, normalized size = 1.12

$$-\frac{6\,b^5x^5 + 15\,b^4x^4 - 20\,(a-1)b^3x^3 + 30\left(a^2 - 2\,a + 1\right)b^2x^2 - 60\left(a^3 - 3\,a^2 + 3\,a - 1\right)bx + 60\left(a^4 - 4\,a^3 + 6\,a^2 - 4\,a\right.}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^4,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 + 15*b^4*x^4 - 20*(a - 1)*b^3*x^3 + 30*(a^2 - 2*a + 1)*b^2
*x^2 - 60*(a^3 - 3*a^2 + 3*a - 1)*b*x + 60*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*
log(b*x + a - 1))/b^5
```

giac [A] time = 0.18, size = 122, normalized size = 1.47

$$-\frac{2\left(a^4 - 4\,a^3 + 6\,a^2 - 4\,a + 1\right)\log\left(|bx + a - 1|\right)}{b^5} - \frac{6\,b^5x^5 + 15\,b^4x^4 - 20\,ab^3x^3 + 30\,a^2b^2x^2 + 20\,b^3x^3 - 60\,a^3bx - 6}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^4,x, algorithm="giac")
```

```
[Out] -2*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(abs(b*x + a - 1))/b^5 - 1/30*(6*b^5*
x^5 + 15*b^4*x^4 - 20*a*b^3*x^3 + 30*a^2*b^2*x^2 + 20*b^3*x^3 - 60*a^3*b*x
- 60*a*b^2*x^2 + 180*a^2*b*x + 30*b^2*x^2 - 180*a*b*x + 60*b*x)/b^5
```

maple [B] time = 0.03, size = 161, normalized size = 1.94

$$-\frac{x^5}{5}-\frac{x^4}{2b}+\frac{2x^3a}{3b^2}-\frac{2x^3}{3b^2}-\frac{x^2a^2}{b^3}+\frac{2x^2a}{b^3}+\frac{2x\,a^3}{b^4}-\frac{x^2}{b^3}-\frac{6a^2x}{b^4}+\frac{6ax}{b^4}-\frac{2x}{b^4}-\frac{2\ln\left(bx+a-1\right)a^4}{b^5}+\frac{8\ln\left(bx+a-1\right)a^3}{b^5}-\frac{12\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)*x^4,x)
```

```
[Out] -1/5*x^5-1/2*x^4/b+2/3/b^2*x^3*a-2/3/b^2*x^3-1/b^3*x^2*a^2+2/b^3*x^2*a+2/b^
4*x*a^3-1/b^3*x^2-6/b^4*a^2*x+6/b^4*a*x-2/b^4*x-2/b^5*ln(b*x+a-1)*a^4+8/b^5
*ln(b*x+a-1)*a^3-12/b^5*ln(b*x+a-1)*a^2+8/b^5*ln(b*x+a-1)*a-2/b^5*ln(b*x+a-
1)
```

maxima [A] time = 0.31, size = 94, normalized size = 1.13

$$-\frac{6\,b^4x^5+15\,b^3x^4-20\,(a-1)b^2x^3+30\left(a^2-2\,a+1\right)bx^2-60\left(a^3-3\,a^2+3\,a-1\right)x}{30\,b^4}-\frac{2\left(a^4-4\,a^3+6\,a^2-4\,a+\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^4,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 + 15*b^3*x^4 - 20*(a - 1)*b^2*x^3 + 30*(a^2 - 2*a + 1)*b*x
^2 - 60*(a^3 - 3*a^2 + 3*a - 1)*x)/b^4 - 2*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*
log(b*x + a - 1)/b^5
```

mupad [B] time = 0.09, size = 142, normalized size = 1.71

$$x^4\left(\frac{a-1}{4\,b}-\frac{a+1}{4\,b}\right)-\frac{x^5}{5}-\frac{\ln\left(a+b\,x-1\right)\,\left(2\,a^4-8\,a^3+12\,a^2-8\,a+2\right)}{b^5}+\frac{x^2\,\left(\frac{a-1}{b}-\frac{a+1}{b}\right)\,{\left(a-1\right)}^2}{2\,b^2}-\frac{x^3\,\left(\frac{a-1}{b}-\frac{a+1}{b}\right.}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(a + b*x + 1)^2)/((a + b*x)^2 - 1),x)
```

```
[Out] x^4*((a - 1)/(4*b) - (a + 1)/(4*b)) - x^5/5 - (log(a + b*x - 1)*(12*a^2 - 8
*a - 8*a^3 + 2*a^4 + 2))/b^5 + (x^2*((a - 1)/b - (a + 1)/b)*(a - 1)^2)/(2*b
^2) - (x^3*((a - 1)/b - (a + 1)/b)*(a - 1))/(3*b) - (x*((a - 1)/b - (a + 1)
/b)*(a - 1)^3)/b^3
```

sympy [A] time = 0.34, size = 102, normalized size = 1.23

$$-\frac{x^5}{5}-x^3\left(-\frac{2a}{3b^2}+\frac{2}{3b^2}\right)-x^2\left(\frac{a^2}{b^3}-\frac{2a}{b^3}+\frac{1}{b^3}\right)-x\left(-\frac{2a^3}{b^4}+\frac{6a^2}{b^4}-\frac{6a}{b^4}+\frac{2}{b^4}\right)-\frac{x^4}{2b}-\frac{2\left(a-1\right)^4\log\left(a+bx-1\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)*x**4,x)
```

```
[Out] -x**5/5 - x**3*(-2*a/(3*b**2) + 2/(3*b**2)) - x**2*(a**2/b**3 - 2*a/b**3 +
b**(-3)) - x*(-2*a**3/b**4 + 6*a**2/b**4 - 6*a/b**4 + 2/b**4) - x**4/(2*b)
- 2*(a - 1)**4*log(a + b*x - 1)/b**5
```

3.827 $\int e^{2\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=66

$$-\frac{2(1-a)^3\log(-a-bx+1)}{b^4}-\frac{2(1-a)^2x}{b^3}-\frac{(1-a)x^2}{b^2}-\frac{2x^3}{3b}-\frac{x^4}{4}$$

```
[Out] -2*(1-a)^2*x/b^3-(1-a)*x^2/b^2-2/3*x^3/b-1/4*x^4-2*(1-a)^3*ln(-b*x-a+1)/b^4
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{(1-a)x^2}{b^2}-\frac{2(1-a)^2x}{b^3}-\frac{2(1-a)^3\log(-a-bx+1)}{b^4}-\frac{2x^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] (-2*(1 - a)^2*x)/b^3 - ((1 - a)*x^2)/b^2 - (2*x^3)/(3*b) - x^4/4 - (2*(1 -
a)^3*Log[1 - a - b*x])/b^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{2\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1+a+bx)}{1-a-bx}\,dx$$
$$= \int \left(-\frac{2(-1+a)^2}{b^3} + \frac{2(-1+a)x}{b^2} - \frac{2x^2}{b} - x^3 + \frac{2(-1+a)^3}{b^3(-1+a+bx)}\right)dx$$
$$= -\frac{2(1-a)^2x}{b^3} - \frac{(1-a)x^2}{b^2} - \frac{2x^3}{3b} - \frac{x^4}{4} - \frac{2(1-a)^3\log(1-a-bx)}{b^4}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 1.00

$$-\frac{2(1-a)^3\log(-a-bx+1)}{b^4} - \frac{2(1-a)^2x}{b^3} - \frac{(1-a)x^2}{b^2} - \frac{2x^3}{3b} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] (-2*(1 - a)^2*x)/b^3 - ((1 - a)*x^2)/b^2 - (2*x^3)/(3*b) - x^4/4 - (2*(1 - a)^3*Log[1 - a - b*x])/b^4
```

fricas [A] time = 0.70, size = 67, normalized size = 1.02

$$-\frac{3\,b^4x^4 + 8\,b^3x^3 - 12\,(a-1)b^2x^2 + 24\left(a^2 - 2\,a + 1\right)bx - 24\left(a^3 - 3\,a^2 + 3\,a - 1\right)\log\left(bx + a - 1\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 + 8*b^3*x^3 - 12*(a - 1)*b^2*x^2 + 24*(a^2 - 2*a + 1)*b*x - 24*(a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1))/b^4
```

giac [A] time = 0.15, size = 82, normalized size = 1.24

$$\frac{2\left(a^3 - 3\,a^2 + 3\,a - 1\right)\log\left(|bx + a - 1|\right)}{b^4} - \frac{3\,b^4x^4 + 8\,b^3x^3 - 12\,ab^2x^2 + 24\,a^2bx + 12\,b^2x^2 - 48\,abx + 24\,bx}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] 2*(a^3 - 3*a^2 + 3*a - 1)*log(abs(b*x + a - 1))/b^4 - 1/12*(3*b^4*x^4 + 8*b^3*x^3 - 12*a*b^2*x^2 + 24*a^2*b*x + 12*b^2*x^2 - 48*a*b*x + 24*b*x)/b^4
```

maple [A] time = 0.03, size = 108, normalized size = 1.64

$$-\frac{x^4}{4}-\frac{2x^3}{3b}+\frac{x^2a}{b^2}-\frac{x^2}{b^2}-\frac{2a^2x}{b^3}+\frac{4ax}{b^3}-\frac{2x}{b^3}+\frac{2\ln(bx+a-1)\,a^3}{b^4}-\frac{6\ln(bx+a-1)\,a^2}{b^4}+\frac{6\ln(bx+a-1)\,a}{b^4}-\frac{2\ln(bx+a}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)*x^3,x)
```

```
[Out] -1/4*x^4-2/3*x^3/b+1/b^2*x^2*a-1/b^2*x^2-2/b^3*a^2*x+4*a*x/b^3-2/b^3*x+2/b^
4*ln(b*x+a-1)*a^3-6/b^4*ln(b*x+a-1)*a^2+6/b^4*ln(b*x+a-1)*a-2/b^4*ln(b*x+a-
1)
```

maxima [A] time = 0.30, size = 68, normalized size = 1.03

$$-\frac{3\,b^3x^4+8\,b^2x^3-12\,(a-1)bx^2+24\left(a^2-2\,a+1\right)x}{12\,b^3}+\frac{2\left(a^3-3\,a^2+3\,a-1\right)\log\left(bx+a-1\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^3*x^4 + 8*b^2*x^3 - 12*(a - 1)*b*x^2 + 24*(a^2 - 2*a + 1)*x)/b^3
 + 2*(a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4
```

mupad [B] time = 0.89, size = 106, normalized size = 1.61

$$x^3\left(\frac{a-1}{3\,b}-\frac{a+1}{3\,b}\right)-\frac{x^4}{4}+\frac{\ln\left(a+b\,x-1\right)\,\left(2\,a^3-6\,a^2+6\,a-2\right)}{b^4}-\frac{x^2\,\left(\frac{a-1}{b}-\frac{a+1}{b}\right)\,\left(a-1\right)}{2\,b}+\frac{x\,\left(\frac{a-1}{b}-\frac{a+1}{b}\right)\,\left(a-1\right)^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a + b*x + 1)^2)/((a + b*x)^2 - 1),x)
```

```
[Out] x^3*((a - 1)/(3*b) - (a + 1)/(3*b)) - x^4/4 + (log(a + b*x - 1)*(6*a - 6*a^
2 + 2*a^3 - 2))/b^4 - (x^2*((a - 1)/b - (a + 1)/b)*(a - 1))/(2*b) + (x*((a
- 1)/b - (a + 1)/b)*(a - 1)^2)/b^2
```

sympy [A] time = 0.26, size = 66, normalized size = 1.00

$$-\frac{x^4}{4}-x^2\left(-\frac{a}{b^2}+\frac{1}{b^2}\right)-x\left(\frac{2a^2}{b^3}-\frac{4a}{b^3}+\frac{2}{b^3}\right)-\frac{2x^3}{3b}+\frac{2\,(a-1)^3\log\left(a+bx-1\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)*x**3,x)
```

```
[Out] -x**4/4 - x**2*(-a/b**2 + b**(-2)) - x*(2*a**2/b**3 - 4*a/b**3 + 2/b**3) -
2*x**3/(3*b) + 2*(a - 1)**3*log(a + b*x - 1)/b**4
```

3.828 $\int e^{2\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=49

$$-\frac{2(1-a)^2\log(-a-bx+1)}{b^3}-\frac{2(1-a)x}{b^2}-\frac{x^2}{b}-\frac{x^3}{3}$$

```
[Out] -2*(1-a)*x/b^2-x^2/b-1/3*x^3-2*(1-a)^2*ln(-b*x-a+1)/b^3
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2(1-a)x}{b^2}-\frac{2(1-a)^2\log(-a-bx+1)}{b^3}-\frac{x^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] (-2*(1 - a)*x)/b^2 - x^2/b - x^3/3 - (2*(1 - a)^2*Log[1 - a - b*x])/b^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{2\tanh^{-1}(a+bx)}x^2\,dx=\int\frac{x^2(1+a+bx)}{1-a-bx}\,dx$$
$$=\int\left(\frac{2(-1+a)}{b^2}-\frac{2x}{b}-x^2-\frac{2(-1+a)^2}{b^2(-1+a+bx)}\right)dx$$
$$=-\frac{2(1-a)x}{b^2}-\frac{x^2}{b}-\frac{x^3}{3}-\frac{2(1-a)^2\log(1-a-bx)}{b^3}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.90

$$-\frac{bx\left(-6a+b^2x^2+3bx+6\right)+6(a-1)^2\log(-a-bx+1)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] -1/3*(b*x*(6 - 6*a + 3*b*x + b^2*x^2) + 6*(-1 + a)^2*Log[1 - a - b*x])/b^3
```

fricas [A] time = 0.97, size = 45, normalized size = 0.92

$$-\frac{b^3x^3+3\,b^2x^2-6\,(a-1)bx+6\left(a^2-2\,a+1\right)\log\left(bx+a-1\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + 3*b^2*x^2 - 6*(a - 1)*b*x + 6*(a^2 - 2*a + 1)*log(b*x + a -
 1))/b^3
```

giac [A] time = 0.15, size = 52, normalized size = 1.06

$$-\frac{2\left(a^2-2\,a+1\right)\log\left(|bx+a-1|\right)}{b^3}-\frac{b^3x^3+3\,b^2x^2-6\,abx+6\,bx}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] -2*(a^2 - 2*a + 1)*log(abs(b*x + a - 1))/b^3 - 1/3*(b^3*x^3 + 3*b^2*x^2 - 6
*a*b*x + 6*b*x)/b^3
```

maple [A] time = 0.03, size = 68, normalized size = 1.39

$$-\frac{x^3}{3}-\frac{x^2}{b}+\frac{2ax}{b^2}-\frac{2x}{b^2}-\frac{2\ln\left(bx+a-1\right)a^2}{b^3}+\frac{4\ln\left(bx+a-1\right)a}{b^3}-\frac{2\ln\left(bx+a-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)*x^2,x)
```

```
[Out] -1/3*x^3-x^2/b+2/b^2*a*x-2*x/b^2-2/b^3*ln(b*x+a-1)*a^2+4/b^3*ln(b*x+a-1)*a-
2/b^3*ln(b*x+a-1)
```

maxima [A] time = 0.31, size = 46, normalized size = 0.94

$$-\frac{b^2x^3+3\,bx^2-6\,(a-1)x}{3\,b^2}-\frac{2\left(a^2-2\,a+1\right)\log\left(bx+a-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b^2*x^3 + 3*b*x^2 - 6*(a - 1)*x)/b^2 - 2*(a^2 - 2*a + 1)*log(b*x + a
- 1)/b^3
```

mupad [B] time = 0.06, size = 74, normalized size = 1.51

$$x^2\left(\frac{a-1}{2\,b}-\frac{a+1}{2\,b}\right)-\frac{x^3}{3}-\frac{\ln\left(a+b\,x-1\right)\,\left(2\,a^2-4\,a+2\right)}{b^3}-\frac{x\,\left(\frac{a-1}{b}-\frac{a+1}{b}\right)\,\left(a-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a + b*x + 1)^2)/((a + b*x)^2 - 1),x)
```

```
[Out] x^2*((a - 1)/(2*b) - (a + 1)/(2*b)) - x^3/3 - (log(a + b*x - 1)*(2*a^2 - 4*
a + 2))/b^3 - (x*((a - 1)/b - (a + 1)/b)*(a - 1))/b
```

sympy [A] time = 0.21, size = 42, normalized size = 0.86

$$-\frac{x^3}{3}-x\left(-\frac{2a}{b^2}+\frac{2}{b^2}\right)-\frac{x^2}{b}-\frac{2\,(a-1)^2\log\left(a+bx-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)*x**2,x)
```

```
[Out] -x**3/3 - x*(-2*a/b**2 + 2/b**2) - x**2/b - 2*(a - 1)**2*log(a + b*x - 1)/b
**3
```

3.829 $\int e^{2\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=34

$$-\frac{2(1-a)\log(-a-bx+1)}{b^2}-\frac{2x}{b}-\frac{x^2}{2}$$

```
[Out] -2*x/b-1/2*x^2-2*(1-a)*ln(-b*x-a+1)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6163, 77}

$$-\frac{2(1-a)\log(-a-bx+1)}{b^2}-\frac{2x}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])*x,x]
```

```
[Out] (-2*x)/b - x^2/2 - (2*(1 - a)*Log[1 - a - b*x])/b^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{2\tanh^{-1}(a+bx)}x\,dx = \int \frac{x(1+a+bx)}{1-a-bx}\,dx$$
$$= \int \left(-\frac{2}{b} - x + \frac{2(-1+a)}{b(-1+a+bx)}\right)dx$$
$$= -\frac{2x}{b} - \frac{x^2}{2} - \frac{2(1-a)\log(1-a-bx)}{b^2}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$-\frac{2(1-a)\log(-a-bx+1)}{b^2} - \frac{2x}{b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])*x,x]
[Out] (-2*x)/b - x^2/2 - (2*(1 - a)*Log[1 - a - b*x])/b^2
```

fricas [A] time = 0.57, size = 29, normalized size = 0.85

$$-\frac{b^2x^2 + 4\,bx - 4\,(a-1)\log\left(bx + a - 1\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x,x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 + 4*b*x - 4*(a - 1)*log(b*x + a - 1))/b^2
```

giac [A] time = 0.15, size = 34, normalized size = 1.00

$$\frac{2\,(a-1)\log\left(|bx + a - 1|\right)}{b^2} - \frac{b^2x^2 + 4\,bx}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x,x, algorithm="giac")
[Out] 2*(a - 1)*log(abs(b*x + a - 1))/b^2 - 1/2*(b^2*x^2 + 4*b*x)/b^2
```

maple [A] time = 0.03, size = 38, normalized size = 1.12

$$-\frac{x^2}{2} - \frac{2x}{b} + \frac{2\ln\left(bx + a - 1\right)a}{b^2} - \frac{2\ln\left(bx + a - 1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)*x,x)
```

```
[Out] -1/2*x^2-2*x/b+2/b^2*ln(b*x+a-1)*a-2/b^2*ln(b*x+a-1)
```

maxima [A] time = 0.30, size = 30, normalized size = 0.88

$$-\frac{bx^2 + 4x}{2b} + \frac{2(a-1)\log(bx + a - 1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)*x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + 4*x)/b + 2*(a - 1)*log(b*x + a - 1)/b^2
```

mupad [B] time = 0.87, size = 40, normalized size = 1.18

$$x\left(\frac{a-1}{b} - \frac{a+1}{b}\right) - \frac{x^2}{2} + \frac{\ln(a + bx - 1)(2a - 2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a + b*x + 1)^2)/((a + b*x)^2 - 1),x)
```

```
[Out] x*((a - 1)/b - (a + 1)/b) - x^2/2 + (log(a + b*x - 1)*(2*a - 2))/b^2
```

sympy [A] time = 0.16, size = 26, normalized size = 0.76

$$-\frac{x^2}{2} - \frac{2x}{b} + \frac{2(a-1)\log(a + bx - 1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)*x,x)
```

```
[Out] -x**2/2 - 2*x/b + 2*(a - 1)*log(a + b*x - 1)/b**2
```

3.830 $\int e^{2 \tanh^{-1}(a+bx)} dx$

Optimal. Leaf size=19

$$-\frac{2 \log(-a-bx+1)}{b}-x$$

[Out] -x-2*ln(-b*x-a+1)/b

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6161, 43}

$$-\frac{2 \log(-a-bx+1)}{b}-x$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a + b*x]),x]

[Out] -x - (2*Log[1 - a - b*x])/b

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{2 \tanh^{-1}(a+bx)} dx &= \int \frac{1+a+bx}{1-a-bx} dx \\
&= \int \left(-1-\frac{2}{-1+a+bx}\right) dx \\
&= -x-\frac{2 \log(1-a-bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{2\log(-a-bx+1)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x]),x]
[Out] -x - (2*Log[1 - a - b*x])/b
```

fricas [A] time = 1.12, size = 18, normalized size = 0.95

$$-\frac{bx+2\log(bx+a-1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2),x, algorithm="fricas")
[Out] -(b*x + 2*log(b*x + a - 1))/b
```

giac [A] time = 0.24, size = 17, normalized size = 0.89

$$-x-\frac{2\log(|bx+a-1|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2),x, algorithm="giac")
[Out] -x - 2*log(abs(b*x + a - 1))/b
```

maple [A] time = 0.02, size = 17, normalized size = 0.89

$$-x-\frac{2\ln(bx+a-1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2),x)
[Out] -x-2/b*ln(b*x+a-1)
```

maxima [A] time = 0.32, size = 16, normalized size = 0.84

$$-x-\frac{2\log(bx+a-1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -x - 2*log(b*x + a - 1)/b
```

mupad [B] time = 0.04, size = 16, normalized size = 0.84

$$-x-\frac{2\ \ln\left(a+b\,x-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + 1)^2/((a + b*x)^2 - 1),x)
```

```
[Out] - x - (2*log(a + b*x - 1))/b
```

sympy [A] time = 0.12, size = 14, normalized size = 0.74

$$-x-\frac{2\log\left(a+bx-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2),x)
```

```
[Out] -x - 2*log(a + b*x - 1)/b
```

3.831 $\int \frac{e^{2\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=33

$$\frac{(a+1)\log(x)}{1-a}-\frac{2\log(-a-bx+1)}{1-a}$$

```
[Out] (1+a)*ln(x)/(1-a)-2*ln(-b*x-a+1)/(1-a)
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 72}

$$\frac{(a+1)\log(x)}{1-a}-\frac{2\log(-a-bx+1)}{1-a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])/x,x]
```

```
[Out] ((1 + a)*Log[x])/(1 - a) - (2*Log[1 - a - b*x])/(1 - a)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(a+bx)}}{x}\,dx &= \int \frac{1+a+bx}{x(1-a-bx)}\,dx\\
&= \int \left(\frac{-1-a}{(-1+a)x}+\frac{2b}{(-1+a)(-1+a+bx)}\right)dx\\
&= \frac{(1+a)\log(x)}{1-a}-\frac{2\log(1-a-bx)}{1-a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.79

$$\frac{2\log(-a-bx+1)-(a+1)\log(x)}{a-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])/x,x]
[Out] (-((1 + a)*Log[x]) + 2*Log[1 - a - b*x])/(-1 + a)
```

fricas [A] time = 0.46, size = 23, normalized size = 0.70

$$-\frac{(a+1)\log(x)-2\,\log(bx+a-1)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x,x, algorithm="fricas")
[Out] -((a + 1)*log(x) - 2*log(b*x + a - 1))/(a - 1)
```

giac [A] time = 0.19, size = 34, normalized size = 1.03

$$\frac{2\,b\log(|bx+a-1|)}{ab-b}-\frac{(a+1)\log(|x|)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x,x, algorithm="giac")
[Out] 2*b*log(abs(b*x + a - 1))/(a*b - b) - (a + 1)*log(abs(x))/(a - 1)
```

maple [A] time = 0.03, size = 35, normalized size = 1.06

$$-\frac{\ln(x)}{a-1}-\frac{\ln(x)a}{a-1}+\frac{2\ln(bx+a-1)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)/x,x)
[Out] -1/(a-1)*ln(x)-1/(a-1)*ln(x)*a+2/(a-1)*ln(b*x+a-1)
```

maxima [A] time = 0.31, size = 27, normalized size = 0.82

$$-\frac{(a+1)\log(x)}{a-1}+\frac{2\,\log(bx+a-1)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] -(a + 1)*log(x)/(a - 1) + 2*log(b*x + a - 1)/(a - 1)
```

mupad [B] time = 0.12, size = 28, normalized size = 0.85

$$\frac{2\,\ln\left(a+b\,x-1\right)}{a-1}-\frac{2\,\ln(x)}{a-1}-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + 1)^2/(x*((a + b*x)^2 - 1)),x)
```

```
[Out] (2*log(a + b*x - 1))/(a - 1) - (2*log(x))/(a - 1) - log(x)
```

sympy [B] time = 0.46, size = 88, normalized size = 2.67

$$-\frac{(a+1)\log\left(x+\frac{a^2-\frac{a^2(a+1)}{a-1}+\frac{2a(a+1)}{a-1}-1-\frac{a+1}{a-1}}{ab+3b}\right)}{a-1}+\frac{2\log\left(x+\frac{a^2+\frac{2a^2}{a-1}-\frac{4a}{a-1}-1+\frac{2}{a-1}}{ab+3b}\right)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)/x,x)
```

```
[Out] -(a + 1)*log(x + (a**2 - a**2*(a + 1)/(a - 1) + 2*a*(a + 1)/(a - 1) - 1 - (
a + 1)/(a - 1))/(a*b + 3*b))/(a - 1) + 2*log(x + (a**2 + 2*a**2/(a - 1) - 4
*a/(a - 1) - 1 + 2/(a - 1))/(a*b + 3*b))/(a - 1)
```

3.832 $\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=48

$$\frac{2b\log(x)}{(1-a)^2}-\frac{2b\log(-a-bx+1)}{(1-a)^2}-\frac{a+1}{(1-a)x}$$

```
[Out] (-1-a)/(1-a)/x+2*b*ln(x)/(1-a)^2-2*b*ln(-b*x-a+1)/(1-a)^2
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$\frac{2b\log(x)}{(1-a)^2}-\frac{2b\log(-a-bx+1)}{(1-a)^2}-\frac{a+1}{(1-a)x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] -((1 + a)/((1 - a)*x)) + (2*b*Log[x])/(1 - a)^2 - (2*b*Log[1 - a - b*x])/(1
 - a)^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^2}\,dx = \int \frac{1+a+bx}{x^2(1-a-bx)}\,dx$$
$$= \int \left(\frac{-1-a}{(-1+a)x^2} + \frac{2b}{(-1+a)^2 x} - \frac{2b^2}{(-1+a)^2(-1+a+bx)}\right)dx$$
$$= -\frac{1+a}{(1-a)x} + \frac{2b\log(x)}{(1-a)^2} - \frac{2b\log(1-a-bx)}{(1-a)^2}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.71

$$\frac{a^2 - 2bx\log(-a-bx+1) + 2bx\log(x) - 1}{(a-1)^2 x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])/x^2,x]
[Out] (-1 + a^2 + 2*b*x*Log[x] - 2*b*x*Log[1 - a - b*x])/((-1 + a)^2*x)
```

fricas [A] time = 1.18, size = 39, normalized size = 0.81

$$-\frac{2\,bx\log\left(bx+a-1\right) - 2\,bx\log(x) - a^2 + 1}{\left(a^2 - 2\,a + 1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^2,x, algorithm="fricas")
[Out] -(2*b*x*log(b*x + a - 1) - 2*b*x*log(x) - a^2 + 1)/((a^2 - 2*a + 1)*x)
```

giac [A] time = 0.14, size = 57, normalized size = 1.19

$$-\frac{2\,b^2\log\left(|bx+a-1|\right)}{a^2b - 2\,ab + b} + \frac{2\,b\log\left(|x|\right)}{a^2 - 2\,a + 1} + \frac{a^2 - 1}{(a-1)^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^2,x, algorithm="giac")
[Out] -2*b^2*log(abs(b*x + a - 1))/(a^2*b - 2*a*b + b) + 2*b*log(abs(x))/(a^2 - 2
*a + 1) + (a^2 - 1)/((a - 1)^2*x)
```

maple [A] time = 0.03, size = 46, normalized size = 0.96

$$\frac{1}{(a-1)\,x}+\frac{a}{(a-1)\,x}+\frac{2b\ln(x)}{(a-1)^2}-\frac{2b\ln(bx+a-1)}{(a-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)/x^2,x)
```

```
[Out] 1/(a-1)/x+1/(a-1)/x*a+2*b/(a-1)^2*ln(x)-2*b/(a-1)^2*ln(b*x+a-1)
```

maxima [A] time = 0.31, size = 48, normalized size = 1.00

$$-\frac{2\,b\log(bx+a-1)}{a^2-2\,a+1}+\frac{2\,b\log(x)}{a^2-2\,a+1}+\frac{a+1}{(a-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*b*log(b*x + a - 1)/(a^2 - 2*a + 1) + 2*b*log(x)/(a^2 - 2*a + 1) + (a + 1
)/((a - 1)*x)
```

mupad [B] time = 0.10, size = 47, normalized size = 0.98

$$\frac{a+1}{x\,(a-1)}-\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,x+\frac{a^2-2\,a+1}{a-1}}{a-1}\right)}{(a-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + 1)^2/(x^2*((a + b*x)^2 - 1)),x)
```

```
[Out] (a + 1)/(x*(a - 1)) - (4*b*atanh((2*b*x + (a^2 - 2*a + 1)/(a - 1))/(a - 1))
)/(a - 1)^2
```

sympy [B] time = 0.35, size = 144, normalized size = 3.00

$$\frac{2b\log\left(x+\frac{-\frac{2a^3b}{(a-1)^2}+\frac{6a^2b}{(a-1)^2}+2ab-\frac{6ab}{(a-1)^2}-2b+\frac{2b}{(a-1)^2}}{4b^2}\right)}{(a-1)^2}-\frac{2b\log\left(x+\frac{\frac{2a^3b}{(a-1)^2}-\frac{6a^2b}{(a-1)^2}+2ab+\frac{6ab}{(a-1)^2}-2b-\frac{2b}{(a-1)^2}}{4b^2}\right)}{(a-1)^2}-\frac{-a-1}{x\,(a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)/x**2,x)
```

```
[Out] 2*b*log(x + (-2*a**3*b/(a - 1)**2 + 6*a**2*b/(a - 1)**2 + 2*a*b - 6*a*b/(a
- 1)**2 - 2*b + 2*b/(a - 1)**2)/(4*b**2))/(a - 1)**2 - 2*b*log(x + (2*a**3*
b/(a - 1)**2 - 6*a**2*b/(a - 1)**2 + 2*a*b + 6*a*b/(a - 1)**2 - 2*b - 2*b/(
a - 1)**2)/(4*b**2))/(a - 1)**2 - (-a - 1)/(x*(a - 1))
```

3.833 $\int \frac{e^{2 \tanh^{-1}(a+bx)}}{x^3} dx$

Optimal. Leaf size=67

$$\frac{2b^2 \log(x)}{(1-a)^3} - \frac{2b^2 \log(-a-bx+1)}{(1-a)^3} - \frac{2b}{(1-a)^2 x} - \frac{a+1}{2(1-a)x^2}$$

```
[Out] 1/2*(-1-a)/(1-a)/x^2-2*b/(1-a)^2/x+2*b^2*ln(x)/(1-a)^3-2*b^2*ln(-b*x-a+1)/(
1-a)^3
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$\frac{2b^2 \log(x)}{(1-a)^3} - \frac{2b^2 \log(-a-bx+1)}{(1-a)^3} - \frac{2b}{(1-a)^2 x} - \frac{a+1}{2(1-a)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] -(1 + a)/(2*(1 - a)*x^2) - (2*b)/((1 - a)^2*x) + (2*b^2*Log[x])/(1 - a)^3 -
 (2*b^2*Log[1 - a - b*x])/(1 - a)^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{1+a+bx}{x^3(1-a-bx)}\,dx$$
$$= \int \left(\frac{-1-a}{(-1+a)x^3} + \frac{2b}{(-1+a)^2x^2} - \frac{2b^2}{(-1+a)^3x} + \frac{2b^3}{(-1+a)^3(-1+a+bx)}\right) dx$$
$$= -\frac{1+a}{2(1-a)x^2} - \frac{2b}{(1-a)^2x} + \frac{2b^2\log(x)}{(1-a)^3} - \frac{2b^2\log(1-a-bx)}{(1-a)^3}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.81

$$\frac{(a-1)\left(a^2-4bx-1\right)+4b^2x^2\log(-a-bx+1)-4b^2x^2\log(x)}{2(a-1)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] ((-1 + a)*(-1 + a^2 - 4*b*x) - 4*b^2*x^2*Log[x] + 4*b^2*x^2*Log[1 - a - b*x
])/(2*(-1 + a)^3*x^2)
```

fricas [A] time = 1.13, size = 65, normalized size = 0.97

$$\frac{4\,b^2x^2\log\left(bx+a-1\right)-4\,b^2x^2\log(x)+a^3-4\,(a-1)bx-a^2-a+1}{2\left(a^3-3\,a^2+3\,a-1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b^2*x^2*log(b*x + a - 1) - 4*b^2*x^2*log(x) + a^3 - 4*(a - 1)*b*x -
a^2 - a + 1)/((a^3 - 3*a^2 + 3*a - 1)*x^2)
```

giac [A] time = 0.18, size = 91, normalized size = 1.36

$$\frac{2\,b^3\log\left(|bx+a-1|\right)}{a^3b-3\,a^2b+3\,ab-b} - \frac{2\,b^2\log\left(|x|\right)}{a^3-3\,a^2+3\,a-1} + \frac{a^3-a^2-4\,(ab-b)x-a+1}{2\,(a-1)^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(abs(b*x + a - 1))/(a^3*b - 3*a^2*b + 3*a*b - b) - 2*b^2*log(abs(x
))/(a^3 - 3*a^2 + 3*a - 1) + 1/2*(a^3 - a^2 - 4*(a*b - b)*x - a + 1)/((a -
1)^3*x^2)
```

maple [A] time = 0.03, size = 63, normalized size = 0.94

$$\frac{1}{2\left(a-1\right)x^{2}}+\frac{a}{2\left(a-1\right)x^{2}}-\frac{2b}{\left(a-1\right)^{2}x}-\frac{2b^{2}\ln(x)}{\left(a-1\right)^{3}}+\frac{2b^{2}\ln\left(bx+a-1\right)}{\left(a-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)/x^3,x)
```

```
[Out] 1/2/(a-1)/x^2+1/2/(a-1)/x^2*a-2*b/(a-1)^2/x-2/(a-1)^3*b^2*ln(x)+2/(a-1)^3*b
^2*ln(b*x+a-1)
```

maxima [A] time = 0.31, size = 74, normalized size = 1.10

$$\frac{2\,b^{2}\log\left(bx+a-1\right)}{a^{3}-3\,a^{2}+3\,a-1}-\frac{2\,b^{2}\log(x)}{a^{3}-3\,a^{2}+3\,a-1}+\frac{a^{2}-4\,bx-1}{2\left(a^{2}-2\,a+1\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] 2*b^2*log(b*x + a - 1)/(a^3 - 3*a^2 + 3*a - 1) - 2*b^2*log(x)/(a^3 - 3*a^2
+ 3*a - 1) + 1/2*(a^2 - 4*b*x - 1)/((a^2 - 2*a + 1)*x^2)
```

mupad [B] time = 0.93, size = 66, normalized size = 0.99

$$\frac{\frac{a+1}{2\,(a-1)}-\frac{2\,b\,x}{(a-1)^{2}}}{x^{2}}+\frac{4\,b^{2}\,\mathrm{atanh}\left(\frac{a^{3}-3\,a^{2}+3\,a-1}{(a-1)^{3}}+\frac{2\,b\,x}{a-1}\right)}{\left(a-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + 1)^2/(x^3*((a + b*x)^2 - 1)),x)
```

```
[Out] ((a + 1)/(2*(a - 1)) - (2*b*x)/(a - 1)^2)/x^2 + (4*b^2*atanh((3*a - 3*a^2 +
 a^3 - 1)/(a - 1)^3 + (2*b*x)/(a - 1)))/(a - 1)^3
```

sympy [B] time = 0.45, size = 209, normalized size = 3.12

$$-\frac{2b^{2}\log\left(x+\frac{-\frac{2a^{4}b^{2}}{(a-1)^{3}}+\frac{8a^{3}b^{2}}{(a-1)^{3}}-\frac{12a^{2}b^{2}}{(a-1)^{3}}+2ab^{2}+\frac{8ab^{2}}{(a-1)^{3}}-2b^{2}-\frac{2b^{2}}{(a-1)^{3}}}{4b^{3}}\right)}{\left(a-1\right)^{3}}+\frac{2b^{2}\log\left(x+\frac{\frac{2a^{4}b^{2}}{(a-1)^{3}}-\frac{8a^{3}b^{2}}{(a-1)^{3}}+\frac{12a^{2}b^{2}}{(a-1)^{3}}+2ab^{2}-\frac{8ab^{2}}{(a-1)^{3}}-2b^{2}+\frac{2b^{2}}{(a-1)^{3}}}{4b^{3}}\right)}{\left(a-1\right)^{3}}-\frac{-a}{x^{2}(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)/x**3,x)
```

```
[Out] -2*b**2*log(x + (-2*a**4*b**2/(a - 1)**3 + 8*a**3*b**2/(a - 1)**3 - 12*a**2
*b**2/(a - 1)**3 + 2*a*b**2 + 8*a*b**2/(a - 1)**3 - 2*b**2 - 2*b**2/(a - 1)
**3)/(4*b**3))/(a - 1)**3 + 2*b**2*log(x + (2*a**4*b**2/(a - 1)**3 - 8*a**3
*b**2/(a - 1)**3 + 12*a**2*b**2/(a - 1)**3 + 2*a*b**2 - 8*a*b**2/(a - 1)**3
 - 2*b**2 + 2*b**2/(a - 1)**3)/(4*b**3))/(a - 1)**3 - (-a**2 + 4*b*x + 1)/(
x**2*(2*a**2 - 4*a + 2))
```

3.834 $\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^4} dx$

Optimal. Leaf size=82

$$\frac{2b^3 \log(x)}{(1-a)^4} - \frac{2b^3 \log(-a-bx+1)}{(1-a)^4} - \frac{2b^2}{(1-a)^3 x} - \frac{b}{(1-a)^2 x^2} - \frac{a+1}{3(1-a)x^3}$$

```
[Out] 1/3*(-1-a)/(1-a)/x^3-b/(1-a)^2/x^2-2*b^2/(1-a)^3/x+2*b^3*ln(x)/(1-a)^4-2*b^
3*ln(-b*x-a+1)/(1-a)^4
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2b^2}{(1-a)^3 x} + \frac{2b^3 \log(x)}{(1-a)^4} - \frac{2b^3 \log(-a-bx+1)}{(1-a)^4} - \frac{b}{(1-a)^2 x^2} - \frac{a+1}{3(1-a)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a + b*x])/x^4,x]
```

```
[Out] -(1 + a)/(3*(1 - a)*x^3) - b/((1 - a)^2*x^2) - (2*b^2)/((1 - a)^3*x) + (2*b
^3*Log[x])/(1 - a)^4 - (2*b^3*Log[1 - a - b*x])/(1 - a)^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{1+a+bx}{x^4(1-a-bx)}\,dx$$
$$= \int \left(\frac{-1-a}{(-1+a)x^4} + \frac{2b}{(-1+a)^2x^3} - \frac{2b^2}{(-1+a)^3x^2} + \frac{2b^3}{(-1+a)^4x} - \frac{2b^4}{(-1+a)^4(-1+a+bx)}\right)dx$$
$$= -\frac{1+a}{3(1-a)x^3} - \frac{b}{(1-a)^2x^2} - \frac{2b^2}{(1-a)^3x} + \frac{2b^3\log(x)}{(1-a)^4} - \frac{2b^3\log(1-a-bx)}{(1-a)^4}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.91

$$\frac{(a-1)\left(a^3-a^2-3abx-a+6b^2x^2+3bx+1\right)-6b^3x^3\log(-a-bx+1)+6b^3x^3\log(x)}{3(a-1)^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a + b*x])/x^4,x]
```

```
[Out] ((-1 + a)*(1 - a - a^2 + a^3 + 3*b*x - 3*a*b*x + 6*b^2*x^2) + 6*b^3*x^3*Log
[x] - 6*b^3*x^3*Log[1 - a - b*x])/(3*(-1 + a)^4*x^3)
```

fricas [A] time = 1.41, size = 88, normalized size = 1.07

$$-\frac{6\,b^3x^3\log\left(bx+a-1\right)-6\,b^3x^3\log(x)-6\,(a-1)b^2x^2-a^4+2\,a^3+3\left(a^2-2\,a+1\right)bx-2\,a+1}{3\left(a^4-4\,a^3+6\,a^2-4\,a+1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*b^3*x^3*log(b*x + a - 1) - 6*b^3*x^3*log(x) - 6*(a - 1)*b^2*x^2 - a
^4 + 2*a^3 + 3*(a^2 - 2*a + 1)*b*x - 2*a + 1)/((a^4 - 4*a^3 + 6*a^2 - 4*a +
 1)*x^3)
```

giac [A] time = 0.17, size = 120, normalized size = 1.46

$$-\frac{2\,b^4\log\left(|bx+a-1|\right)}{a^4b-4\,a^3b+6\,a^2b-4\,ab+b}+\frac{2\,b^3\log\left(|x|\right)}{a^4-4\,a^3+6\,a^2-4\,a+1}+\frac{a^4-2\,a^3+6\left(ab^2-b^2\right)x^2-3\left(a^2b-2\,ab+b\right)x+2}{3\,(a-1)^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^4,x, algorithm="giac")
```

```
[Out] -2*b^4*log(abs(b*x + a - 1))/(a^4*b - 4*a^3*b + 6*a^2*b - 4*a*b + b) + 2*b^
3*log(abs(x))/(a^4 - 4*a^3 + 6*a^2 - 4*a + 1) + 1/3*(a^4 - 2*a^3 + 6*(a*b^2
 - b^2)*x^2 - 3*(a^2*b - 2*a*b + b)*x + 2*a - 1)/((a - 1)^4*x^3)
```

maple [A] time = 0.04, size = 76, normalized size = 0.93

$$\frac{1}{3(a-1)x^3}+\frac{a}{3(a-1)x^3}-\frac{b}{(a-1)^2x^2}+\frac{2b^3\ln(x)}{(a-1)^4}+\frac{2b^2}{(a-1)^3x}-\frac{2b^3\ln(bx+a-1)}{(a-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^2/(1-(b*x+a)^2)/x^4,x)
```

```
[Out] 1/3/(a-1)/x^3+1/3/(a-1)/x^3*a-b/(a-1)^2/x^2+2/(a-1)^4*b^3*ln(x)+2/(a-1)^3*b
^2/x-2/(a-1)^4*b^3*ln(b*x+a-1)
```

maxima [A] time = 0.32, size = 108, normalized size = 1.32

$$-\frac{2\,b^3\log(bx+a-1)}{a^4-4\,a^3+6\,a^2-4\,a+1}+\frac{2\,b^3\log(x)}{a^4-4\,a^3+6\,a^2-4\,a+1}+\frac{6\,b^2x^2+a^3-3\,(a-1)bx-a^2-a+1}{3\left(a^3-3\,a^2+3\,a-1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^2/(1-(b*x+a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] -2*b^3*log(b*x + a - 1)/(a^4 - 4*a^3 + 6*a^2 - 4*a + 1) + 2*b^3*log(x)/(a^4
 - 4*a^3 + 6*a^2 - 4*a + 1) + 1/3*(6*b^2*x^2 + a^3 - 3*(a - 1)*b*x - a^2 -
a + 1)/((a^3 - 3*a^2 + 3*a - 1)*x^3)
```

mupad [B] time = 0.90, size = 84, normalized size = 1.02

$$\frac{\frac{a+1}{3\,(a-1)}+\frac{2\,b^2\,x^2}{(a-1)^3}-\frac{b\,x}{(a-1)^2}}{x^3}-\frac{4\,b^3\,\mathrm{atanh}\left(\frac{a^4-4\,a^3+6\,a^2-4\,a+1}{(a-1)^4}+\frac{2\,b\,x}{a-1}\right)}{(a-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + 1)^2/(x^4*((a + b*x)^2 - 1)),x)
```

```
[Out] ((a + 1)/(3*(a - 1)) + (2*b^2*x^2)/(a - 1)^3 - (b*x)/(a - 1)^2)/x^3 - (4*b^
3*atanh((6*a^2 - 4*a - 4*a^3 + a^4 + 1)/(a - 1)^4 + (2*b*x)/(a - 1)))/(a -
1)^4
```

sympy [B] time = 0.56, size = 260, normalized size = 3.17

$$\frac{2b^3\log\left(x+\frac{-\frac{2a^5b^3}{(a-1)^4}+\frac{10a^4b^3}{(a-1)^4}-\frac{20a^3b^3}{(a-1)^4}+\frac{20a^2b^3}{(a-1)^4}+2ab^3-\frac{10ab^3}{(a-1)^4}-2b^3+\frac{2b^3}{(a-1)^4}}{4b^4}\right)}{(a-1)^4}-\frac{2b^3\log\left(x+\frac{\frac{2a^5b^3}{(a-1)^4}-\frac{10a^4b^3}{(a-1)^4}+\frac{20a^3b^3}{(a-1)^4}-\frac{20a^2b^3}{(a-1)^4}+2ab^3+\frac{10ab^3}{(a-1)^4}-2b^3-}{4b^4}\right.}{(a-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**2/(1-(b*x+a)**2)/x**4,x)
```

```
[Out] 2*b**3*log(x + (-2*a**5*b**3/(a - 1)**4 + 10*a**4*b**3/(a - 1)**4 - 20*a**3
*b**3/(a - 1)**4 + 20*a**2*b**3/(a - 1)**4 + 2*a*b**3 - 10*a*b**3/(a - 1)**
4 - 2*b**3 + 2*b**3/(a - 1)**4)/(4*b**4))/(a - 1)**4 - 2*b**3*log(x + (2*a*
*5*b**3/(a - 1)**4 - 10*a**4*b**3/(a - 1)**4 + 20*a**3*b**3/(a - 1)**4 - 20
*a**2*b**3/(a - 1)**4 + 2*a*b**3 + 10*a*b**3/(a - 1)**4 - 2*b**3 - 2*b**3/(
a - 1)**4)/(4*b**4))/(a - 1)**4 - (-a**3 + a**2 + a - 6*b**2*x**2 + x*(3*a*
b - 3*b) - 1)/(x**3*(3*a**3 - 9*a**2 + 9*a - 3))
```

3.835 $\int e^{3\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=187

$$\frac{\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}\left(22a^2+2(11-10a)bx-54a+29\right)}{8b^4}+\frac{3\left(-8a^3+36a^2-44a+17\right)\sqrt{-a-bx+1}\,\sqrt{a+}}{8b^4}$$

```
[Out] -3/8*(-8*a^3+36*a^2-44*a+17)*arcsin(b*x+a)/b^4+2*x^3*(b*x+a+1)^(3/2)/b/(-b*
x-a+1)^(1/2)+9/4*x^2*(b*x+a+1)^(3/2)*(-b*x-a+1)^(1/2)/b^2+1/8*(b*x+a+1)^(3/
2)*(29-54*a+22*a^2+2*(11-10*a)*b*x)*(-b*x-a+1)^(1/2)/b^4+3/8*(-8*a^3+36*a^2
-44*a+17)*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b^4
```

Rubi [A] time = 0.18, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6163, 97, 153, 147, 50, 53, 619, 216}

$$\frac{\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}\left(22a^2+2(11-10a)bx-54a+29\right)}{8b^4}+\frac{3\left(-8a^3+36a^2-44a+17\right)\sqrt{-a-bx+1}\,\sqrt{a+}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] (3*(17 - 44*a + 36*a^2 - 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4
) + (2*x^3*(1 + a + b*x)^(3/2))/(b*Sqrt[1 - a - b*x]) + (9*x^2*Sqrt[1 - a -
 b*x]*(1 + a + b*x)^(3/2))/(4*b^2) + (Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2)
*(29 - 54*a + 22*a^2 + 2*(11 - 10*a)*b*x))/(8*b^4) - (3*(17 - 44*a + 36*a^2
 - 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{3\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1+a+bx)^{3/2}}{(1-a-bx)^{3/2}}\,dx$$

$$= \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - \frac{2\int \frac{x^2\sqrt{1+a+bx}\left(3(1+a)+\frac{9bx}{2}\right)}{\sqrt{1-a-bx}}\,dx}{b}$$

$$= \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{4b^2} + \frac{\int \frac{x\sqrt{1+a+bx}\left(-9(1-a)(1+a)b-\frac{3}{2}(11-10a)b^2\right.}{\sqrt{1-a-bx}}}{2b^3}$$

$$= \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{4b^2} + \frac{\sqrt{1-a-bx}\,(1+a+bx)^{3/2}\left(29-5\right.}{8b^4}$$

$$= \frac{3\left(17-44a+36a^2-8a^3\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{8b^4} + \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-}}{}$$

$$= \frac{3\left(17-44a+36a^2-8a^3\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{8b^4} + \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-}}{}$$

$$= \frac{3\left(17-44a+36a^2-8a^3\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{8b^4} + \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-}}{}$$

$$= \frac{3\left(17-44a+36a^2-8a^3\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{8b^4} + \frac{2x^3(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{9x^2\sqrt{1-a-}}{}$$

Mathematica [A] time = 0.29, size = 203, normalized size = 1.09

$$\frac{24a\left(2a^2+11\right)\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{-b}\,\sqrt{-a-bx+1}}{\sqrt{2}\,\sqrt{b}}\right)+6\left(36a^2+17\right)\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{-a-bx+1}}{\sqrt{2}\,\sqrt{-b}}\right)-\frac{\sqrt{b}\,\sqrt{a+bx+1}\left(-2a^4+78a^3+a^2(2\right.}{}}{8b^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] (-((Sqrt[b]*Sqrt[1 + a + b*x]*(-80 + 78*a^3 - 2*a^4 + 29*b*x + 11*b^2*x^2 +
 6*b^3*x^3 + 2*b^4*x^4 + a^2*(-233 + 22*b*x) + a*(237 - 54*b*x - 10*b^2*x^2
)))/Sqrt[1 - a - b*x]) + 24*a*(11 + 2*a^2)*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[
1 - a - b*x])/(Sqrt[2]*Sqrt[b])] + 6*(17 + 36*a^2)*Sqrt[-b]*ArcSinh[(Sqrt[b
]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(8*b^(9/2))
```

fricas [A] time = 0.73, size = 192, normalized size = 1.03

$$-\frac{3\left(8a^4-44a^3+\left(8a^3-36a^2+44a-17\right)bx+80a^2-61a+17\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)-\left(2b^4x^4+\right.}{8\left(b^5x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*(8*a^4 - 44*a^3 + (8*a^3 - 36*a^2 + 44*a - 17)*b*x + 80*a^2 - 61*a
+ 17)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*
x + a^2 - 1)) - (2*b^4*x^4 + 6*b^3*x^3 - (10*a - 11)*b^2*x^2 - 2*a^4 + 78*a
^3 + (22*a^2 - 54*a + 29)*b*x - 233*a^2 + 237*a - 80)*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1))/(b^5*x + (a - 1)*b^4)
```

giac [A] time = 0.52, size = 193, normalized size = 1.03

$$\frac{1}{8}\sqrt{-(bx+a)^2+1}\left(\left(\left(2x\left(\frac{x}{b}-\frac{ab^{11}-4b^{11}}{b^{13}}\right)+\frac{2a^2b^{10}-20ab^{10}+19b^{10}}{b^{13}}\right)x-\frac{2a^3b^9-44a^2b^9+93ab^9-48b^9}{b^{13}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-(b*x + a)^2 + 1)*((2*x*(x/b - (a*b^11 - 4*b^11)/b^13) + (2*a^2*b^
10 - 20*a*b^10 + 19*b^10)/b^13)*x - (2*a^3*b^9 - 44*a^2*b^9 + 93*a*b^9 - 48
*b^9)/b^13) - 3/8*(8*a^3 - 36*a^2 + 44*a - 17)*arcsin(-b*x - a)*sgn(b)/(b^3
*abs(b)) - 8*(a^3 - 3*a^2 + 3*a - 1)/(b^3*((sqrt(-(b*x + a)^2 + 1)*abs(b) +
 b)/(b^2*x + a*b) - 1)*abs(b))
```

maple [B] time = 0.07, size = 756, normalized size = 4.04

$$-\frac{3x^2a^2}{2b^2\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{a\,x^3}{2b\sqrt{-b^2x^2-2abx-a^2+1}}-\frac{27a^2\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2b^3\sqrt{b^2}}+\frac{265a^2x}{8b^3\sqrt{-b^2x^2-2ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^3,x)
```

```
[Out] -3/2*x^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2+1/2/b*a*x^3/(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2)-27/2*a^2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2))+265/8/b^3*a^2*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+53/
8/b^2*a*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/4/b^3*a^4*x/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)+3*a^3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2))+33/2*a/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2))-53/2*a/b^3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-25/2*
a^3/b^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-x^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
-1/2*a^2/b^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-19/2*a^4/b^4/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)-1/4*a*x^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-17/8/b*x^3/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)+51/8/b^3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+155/8/b^4*a^
3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-157/8/b^4*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
-1/4*b*x^5/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/4/b^4*a^5/(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)-5*x^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-51/8/b^3/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+10/b^4/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2)
```

maxima [B] time = 0.45, size = 2349, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*x^5/sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1) + 315/4*a^6*x/((a^2*b^2 - (a^
2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b) + 3/4*a*x^4/sqrt(-b^2*x^2
 - 2*a*b*x - a^2 + 1) - 945/8*(a^2 - 1)*a^4*x/((a^2*b^2 - (a^2 - 1)*b^2)*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b) - 21/8*a^2*x^3/(sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)*b) + 105/8*(a^2 - 1)*a^5/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x
^2 - 2*a*b*x - a^2 + 1)*b^2) - 105*(a*b^2 + b^2)*a^5*x/((a^2*b^2 - (a^2 - 1
)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^3) + 45*(a^2*b + 2*a*b + b)*a^4
*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) + 169
/4*(a^2 - 1)^2*a^2*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*b) - 6*(a^3 + 3*a^2 + 3*a + 1)*a^3*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b) + 105/8*a^3*x^2/(sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)*b^2) + 5/8*(a^2 - 1)*x^3/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b)
 - (a*b^2 + b^2)*x^4/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) - 14*(a^2 - 1
)^2*a^3/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2)
 + 265/2*(a*b^2 + b^2)*(a^2 - 1)*a^3*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*b^3) - 93/2*(a^2*b + 2*a*b + b)*(a^2 - 1)*a^2*x/((
a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) - 15/8*(a^
2 - 1)^3*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b)
 + 5*(a^3 + 3*a^2 + 3*a + 1)*(a^2 - 1)*a*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(
```

```
-b^2*x^2 - 2*a*b*x - a^2 + 1)*b) - 49/8*(a^2 - 1)*a*x^2/(sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)*b^2) + 7/2*(a*b^2 + b^2)*a*x^3/(sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*b^3) - 3/2*(a^2*b + 2*a*b + b)*x^3/(sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*b^2) + 315/8*a^4*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/
b^4 - 35/2*(a*b^2 + b^2)*(a^2 - 1)*a^4/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2
*x^2 - 2*a*b*x - a^2 + 1)*b^4) + 15/2*(a^2*b + 2*a*b + b)*(a^2 - 1)*a^3/((a
^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^3) + 15/8*(a^2
 - 1)^3*a/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2
) - (a^3 + 3*a^2 + 3*a + 1)*(a^2 - 1)*a^2/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) - 61/2*(a*b^2 + b^2)*(a^2 - 1)^2*a*x/((a^
2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^3) + 9/2*(a^2*b
 + 2*a*b + b)*(a^2 - 1)^2*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*
b*x - a^2 + 1)*b^2) - 35/2*(a*b^2 + b^2)*a^2*x^2/(sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*b^4) + 15/2*(a^2*b + 2*a*b + b)*a*x^2/(sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*b^3) - (a^3 + 3*a^2 + 3*a + 1)*x^2/(sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*b^2) - 105/4*(a^2 - 1)*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 -
 1)*b^2))/b^4 + 29/2*(a*b^2 + b^2)*(a^2 - 1)^2*a^2/((a^2*b^2 - (a^2 - 1)*b^
2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^4) + 105/4*(a^2 - 1)*a^3/(sqrt(-b^2
*x^2 - 2*a*b*x - a^2 + 1)*b^4) - 9/2*(a^2*b + 2*a*b + b)*(a^2 - 1)^2*a/((a^
2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^3) + 4*(a*b^2 +
 b^2)*(a^2 - 1)*x^2/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^4) - 105/2*(a*b^2
 + b^2)*a^3*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^6 + 45/2
*(a^2*b + 2*a*b + b)*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2
))/b^5 + 15/8*(a^2 - 1)^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^
2))/b^4 - 3*(a^3 + 3*a^2 + 3*a + 1)*a*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 -
(a^2 - 1)*b^2))/b^4 - 49/4*(a^2 - 1)^2*a/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*b^4) + 45/2*(a*b^2 + b^2)*(a^2 - 1)*a*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2
- (a^2 - 1)*b^2))/b^6 - 9/2*(a^2*b + 2*a*b + b)*(a^2 - 1)*arcsin(-(b^2*x +
a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^5 - 35*(a*b^2 + b^2)*(a^2 - 1)*a^2/(s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^6) + 15*(a^2*b + 2*a*b + b)*(a^2 - 1)*a
/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^5) - 2*(a^3 + 3*a^2 + 3*a + 1)*(a^2
- 1)/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^4) + 8*(a*b^2 + b^2)*(a^2 - 1)^2
/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 (a + b x + 1)^3}{\left(1 - (a + b x)^2\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x + 1)^3)/(1 - (a + b*x)^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*x + 1)^3)/(1 - (a + b*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a+bx+1)^3}{(-(a+bx-1)(a+bx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + 1)**3/(-(a + b*x - 1)*(a + b*x + 1))**(3/2), x)
```

3.836 $\int e^{3\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=168

$$\frac{\left(6a^2-18a+11\right)\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{6b^3}+\frac{\left(6a^2-18a+11\right)\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{2b^3}-\frac{\left(6a^2-18a+11\right)\mathrm{s}}{2b^3}$$

```
[Out] -1/2*(6*a^2-18*a+11)*arcsin(b*x+a)/b^3+(1-a)^2*(b*x+a+1)^(5/2)/b^3/(-b*x-a+
1)^(1/2)+1/6*(6*a^2-18*a+11)*(b*x+a+1)^(3/2)*(-b*x-a+1)^(1/2)/b^3+1/3*(b*x+
a+1)^(5/2)*(-b*x-a+1)^(1/2)/b^3+1/2*(6*a^2-18*a+11)*(-b*x-a+1)^(1/2)*(b*x+a
+1)^(1/2)/b^3
```

Rubi [A] time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 89, 80, 50, 53, 619, 216}

$$\frac{\left(6a^2-18a+11\right)\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{6b^3}+\frac{\left(6a^2-18a+11\right)\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{2b^3}-\frac{\left(6a^2-18a+11\right)\mathrm{s}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] ((11 - 18*a + 6*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) + ((11 -
18*a + 6*a^2)*Sqrt[1 - a - b*x]*(1 + a + b*x)^(3/2))/(6*b^3) + ((1 - a)^2*(
1 + a + b*x)^(5/2))/(b^3*Sqrt[1 - a - b*x]) + (Sqrt[1 - a - b*x]*(1 + a + b
*x)^(5/2))/(3*b^3) - ((11 - 18*a + 6*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{3\tanh^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1+a+bx)^{3/2}}{(1-a-bx)^{3/2}}\,dx$$

$$= \frac{(1-a)^2(1+a+bx)^{5/2}}{b^3\sqrt{1-a-bx}} - \frac{\int \frac{(1+a+bx)^{3/2}\left((3-2a)(1-a)b+b^2x\right)}{\sqrt{1-a-bx}}\,dx}{b^3}$$

$$= \frac{(1-a)^2(1+a+bx)^{5/2}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\,(1+a+bx)^{5/2}}{3b^3} - \frac{\left(11-18a+6a^2\right)\int \frac{(1+a+bx)^{3/2}}{\sqrt{1-a-bx}}\,dx}{3b^2}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{6b^3} + \frac{(1-a)^2(1+a+bx)^{5/2}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\,(1}{3b^3}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} + \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\,(1+a+bx)^{3}}{6b^3}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} + \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\,(1+a+bx)^{3}}{6b^3}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} + \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\,(1+a+bx)^{3}}{6b^3}$$

$$= \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} + \frac{\left(11-18a+6a^2\right)\sqrt{1-a-bx}\,(1+a+bx)^{3}}{6b^3}$$

Mathematica [A] time = 0.22, size = 170, normalized size = 1.01

$$\frac{6\left(6a^2+11\right)\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{-a-bx+1}}{\sqrt{2}\,\sqrt{-b}}\right) - \frac{\sqrt{b}\,\sqrt{a+bx+1}\left(2a^3-53a^2+a(103-16bx)+2b^3x^3+7b^2x^2+19bx-52\right)}{\sqrt{-a-bx+1}} + 108a\sqrt{-b}\,\sinh^{-1}}{6b^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] (-((Sqrt[b]*Sqrt[1 + a + b*x]*(-52 - 53*a^2 + 2*a^3 + 19*b*x + 7*b^2*x^2 +
2*b^3*x^3 + a*(103 - 16*b*x)))/Sqrt[1 - a - b*x]) + 108*a*Sqrt[-b]*ArcSinh[
(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])] + 6*(11 + 6*a^2)*Sqrt[-b]*A
rcSinh[(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(6*b^(7/2))
```

fricas [A] time = 0.74, size = 159, normalized size = 0.95

$$\frac{3\left(6a^3+\left(6a^2-18a+11\right)bx-24a^2+29a-11\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)+\left(2b^3x^3+7b^2x^2+2a^3-(16\right.}{6\left(b^4x+(a-1)b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(6*a^3 + (6*a^2 - 18*a + 11)*b*x - 24*a^2 + 29*a - 11)*arctan(sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (2*
b^3*x^3 + 7*b^2*x^2 + 2*a^3 - (16*a - 19)*b*x - 53*a^2 + 103*a - 52)*sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1))/(b^4*x + (a - 1)*b^3)
```

giac [A] time = 0.30, size = 148, normalized size = 0.88

$$\frac{1}{6}\sqrt{-(bx+a)^2+1}\left(x\left(\frac{2x}{b}-\frac{2ab^6-9b^6}{b^8}\right)+\frac{2a^2b^5-27ab^5+28b^5}{b^8}\right)+\frac{\left(6a^2-18a+11\right)\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-(b*x + a)^2 + 1)*(x*(2*x/b - (2*a*b^6 - 9*b^6)/b^8) + (2*a^2*b^5
- 27*a*b^5 + 28*b^5)/b^8) + 1/2*(6*a^2 - 18*a + 11)*arcsin(-b*x - a)*sgn(b)
/(b^2*abs(b)) + 8*(a^2 - 2*a + 1)/(b^2*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)
/(b^2*x + a*b) - 1)*abs(b))
```

maple [B] time = 0.05, size = 552, normalized size = 3.29

$$-\frac{3a^2\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{b^2\sqrt{b^2}}+\frac{23a^2x}{2b^2\sqrt{-b^2x^2-2abx-a^2+1}}-\frac{3x^3}{2\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{3a\,x^2}{2b\sqrt{-b^2x^2-2abx-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^2,x)
```

```
[Out] -3*a^2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2))+23/2*a^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3/2*x^3/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)+3/2/b*a*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/3/b^2*a^3*x/(
-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-53/3/b^2*a*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-2
5/3/b^3*a^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-13/3/b*x^2/(-b^2*x^2-2*a*b*x-a^2
```

```
+1)^(1/2)+9*a/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2))-17/2*a/b^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+17/2*a^3/b^3/(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)-1/3*b*x^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/3*a*x^3
/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/3/b^3*a^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+
11/2*x/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-11/2/b^2/(b^2)^(1/2)*arctan((b^2)
^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+26/3/b^3/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)
```

maxima [B] time = 0.43, size = 1645, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -35*a^5*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) -
1/3*b*x^4/sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1) + 265/6*(a^2 - 1)*a^3*x/((a^2*
b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) + 7/6*a*x^3/sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1) - 35/6*(a^2 - 1)*a^4/((a^2*b^2 - (a^2 - 1)*b^2
)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b) - 61/6*(a^2 - 1)^2*a*x/((a^2*b^2 -
(a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) + 2*(a^3 + 3*a^2 + 3*a +
 1)*a^2*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) +
45*(a*b^2 + b^2)*a^4*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*b^2) - 18*(a^2*b + 2*a*b + b)*a^3*x/((a^2*b^2 - (a^2 - 1)*b^2)*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b) - 35/6*a^2*x^2/(sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)*b) + 29/6*(a^2 - 1)^2*a^2/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*b) - (a^3 + 3*a^2 + 3*a + 1)*(a^2 - 1)*x/((a^2*b^2
 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) - 93/2*(a*b^2 + b^2)*
(a^2 - 1)*a^2*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*b^2) + 15*(a^2*b + 2*a*b + b)*(a^2 - 1)*a*x/((a^2*b^2 - (a^2 - 1)*b^2)*s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b) + 4/3*(a^2 - 1)*x^2/(sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)*b) - 3/2*(a*b^2 + b^2)*x^3/(sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1)*b^2) - 35/2*a^3*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/
b^3 + 15/2*(a*b^2 + b^2)*(a^2 - 1)*a^3/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2
*x^2 - 2*a*b*x - a^2 + 1)*b^3) - 3*(a^2*b + 2*a*b + b)*(a^2 - 1)*a^2/((a^2*
b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) + (a^3 + 3*a^2
 + 3*a + 1)*(a^2 - 1)*a/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)*b) + 9/2*(a*b^2 + b^2)*(a^2 - 1)^2*x/((a^2*b^2 - (a^2 - 1)*b^2)*
sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) + 15/2*(a*b^2 + b^2)*a*x^2/(sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1)*b^3) - 3*(a^2*b + 2*a*b + b)*x^2/(sqrt(-b^2*x^2
 - 2*a*b*x - a^2 + 1)*b^2) + 15/2*(a^2 - 1)*a*arcsin(-(b^2*x + a*b)/sqrt(a^
2*b^2 - (a^2 - 1)*b^2))/b^3 - 9/2*(a*b^2 + b^2)*(a^2 - 1)^2*a/((a^2*b^2 - (
a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^3) - 35/3*(a^2 - 1)*a^2/
(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^3) + 45/2*(a*b^2 + b^2)*a^2*arcsin(-(
b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^5 - 9*(a^2*b + 2*a*b + b)*a*a
```

```
rcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 + (a^3 + 3*a^2 + 3*
a + 1)*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + 8/3*(a^2
- 1)^2/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^3) - 9/2*(a*b^2 + b^2)*(a^2 -
1)*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^5 + 15*(a*b^2 + b
^2)*(a^2 - 1)*a/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^5) - 6*(a^2*b + 2*a*b
 + b)*(a^2 - 1)/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(a+b\,x+1\right)}^3}{{\left(1-{\left(a+b\,x\right)}^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x + 1)^3)/(1 - (a + b*x)^2)^(3/2),x)
[Out] int((x^2*(a + b*x + 1)^3)/(1 - (a + b*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + bx + 1\right)^{3}}{\left(- \left(a + bx - 1\right) \left(a + bx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)*x**2,x)
[Out] Integral(x**2*(a + b*x + 1)**3/(-(a + b*x - 1)*(a + b*x + 1))**(3/2), x)
```

3.837 $\int e^{3\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=121

$$\frac{(1-a)(a+bx+1)^{5/2}}{b^2\sqrt{-a-bx+1}}+\frac{(3-2a)\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{2b^2}+\frac{3(3-2a)\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{2b^2}-\frac{3(3-2a)\sin$$

```
[Out] -3/2*(3-2*a)*arcsin(b*x+a)/b^2+(1-a)*(b*x+a+1)^(5/2)/b^2/(-b*x-a+1)^(1/2)+1
/2*(3-2*a)*(b*x+a+1)^(3/2)*(-b*x-a+1)^(1/2)/b^2+3/2*(3-2*a)*(-b*x-a+1)^(1/2
)*(b*x+a+1)^(1/2)/b^2
```

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 78, 50, 53, 619, 216}

$$\frac{(1-a)(a+bx+1)^{5/2}}{b^2\sqrt{-a-bx+1}}+\frac{(3-2a)\sqrt{-a-bx+1}\,(a+bx+1)^{3/2}}{2b^2}+\frac{3(3-2a)\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{2b^2}-\frac{3(3-2a)\sin$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])*x,x]
```

```
[Out] (3*(3 - 2*a)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) + ((3 - 2*a)*Sqrt
[1 - a - b*x]*(1 + a + b*x)^(3/2))/(2*b^2) + ((1 - a)*(1 + a + b*x)^(5/2))/
(b^2*Sqrt[1 - a - b*x]) - (3*(3 - 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{3\tanh^{-1}(a+bx)}x\,dx = \int \frac{x(1+a+bx)^{3/2}}{(1-a-bx)^{3/2}}\,dx$$

$$= \frac{(1-a)(1+a+bx)^{5/2}}{b^2\sqrt{1-a-bx}} - \frac{(3-2a)\int \frac{(1+a+bx)^{3/2}}{\sqrt{1-a-bx}}\,dx}{b}$$

$$= \frac{(3-2a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a+bx)^{5/2}}{b^2\sqrt{1-a-bx}} - \frac{(3(3-2a))\int \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx}{2b}$$

$$= \frac{3(3-2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} + \frac{(3-2a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a}{b^2\sqrt{1-a}}$$

$$= \frac{3(3-2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} + \frac{(3-2a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a}{b^2\sqrt{1-a}}$$

$$= \frac{3(3-2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} + \frac{(3-2a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a}{b^2\sqrt{1-a}}$$

$$= \frac{3(3-2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} + \frac{(3-2a)\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}{2b^2} + \frac{(1-a)(1+a}{b^2\sqrt{1-a}}$$

Mathematica [A] time = 0.16, size = 142, normalized size = 1.17

$$\frac{\frac{\sqrt{b}\sqrt{a+bx+1}\left(a^2-15a-b^2x^2-5bx+14\right)}{\sqrt{-a-bx+1}}+12a\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)+18\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{-b}}\right)}{2b^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])*x,x]
```

```
[Out] ((Sqrt[b]*Sqrt[1 + a + b*x]*(14 - 15*a + a^2 - 5*b*x - b^2*x^2))/Sqrt[1 - a
 - b*x] + 12*a*Sqrt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[
b])] + 18*Sqrt[-b]*ArcSinh[(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])
/(2*b^(5/2))
```

fricas [A] time = 0.73, size = 131, normalized size = 1.08

$$-\frac{3\left((2a-3)bx+2a^2-5a+3\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)-\left(b^2x^2-a^2+5bx+15a-14\right)\sqrt{-b^2x^2-2ab}}{2\left(b^3x+(a-1)b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] -1/2*(3*((2*a - 3)*b*x + 2*a^2 - 5*a + 3)*arctan(sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (b^2*x^2 - a^2 + 5*b*x
+ 15*a - 14)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/(b^3*x + (a - 1)*b^2)
```

giac [A] time = 0.25, size = 109, normalized size = 0.90

$$\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b}-\frac{ab^2-6\,b^2}{b^4}\right)-\frac{3\,(2\,a-3)\arcsin\left(-bx-a\right)\mathrm{sgn}(b)}{2\,b|b|}-\frac{8\,(a-1)}{b\left(\frac{\sqrt{-(bx+a)^2+1}\,|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-(b*x + a)^2 + 1)*(x/b - (a*b^2 - 6*b^2)/b^4) - 3/2*(2*a - 3)*arcs
in(-b*x - a)*sgn(b)/(b*abs(b)) - 8*(a - 1)/(b*((sqrt(-(b*x + a)^2 + 1)*abs(
b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

maple [B] time = 0.05, size = 381, normalized size = 3.15

$$-\frac{10ax}{b\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{3a\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{b\sqrt{b^2}}+\frac{a^2x}{2b\sqrt{-b^2x^2-2abx-a^2+1}}-\frac{3x^2}{\sqrt{-b^2x^2-2abx-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x,x)
```

```
[Out] -10*a/b/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+3*a/b/(b^2)^(1/2)*arctan((b^2)^(1/
2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/b*a^2*x/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)-3*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+7/b^2/(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)-9/2/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))-7*a^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2/b^2*a/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2)+9/2/b*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2*b*x^3/(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)-1/2*a*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2/b^2*a^
3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

maxima [B] time = 0.43, size = 1137, normalized size = 9.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 15*a^4*b*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) -
 31/2*(a^2 - 1)*a^2*b*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)) - 1/2*b*x^3/sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1) + 5/2*(a^2 - 1)*
a^3/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) + 6*(a^2
*b + 2*a*b + b)*a^2*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)) - 18*(a*b^2 + b^2)*a^3*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2
 - 2*a*b*x - a^2 + 1)*b) + 3/2*(a^2 - 1)^2*b*x/((a^2*b^2 - (a^2 - 1)*b^2)*s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) - (a^3 + 3*a^2 + 3*a + 1)*a*b*x/((a^2*b^
2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) + 5/2*a*x^2/sqrt(-b^
2*x^2 - 2*a*b*x - a^2 + 1) - 3/2*(a^2 - 1)^2*a/((a^2*b^2 - (a^2 - 1)*b^2)*s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) - (a^3 + 3*a^2 + 3*a + 1)*a^2/((a^2*b^2
- (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) - 3*(a^2*b + 2*a*b + b
)*(a^2 - 1)*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
) + 15*(a*b^2 + b^2)*(a^2 - 1)*a*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2
 - 2*a*b*x - a^2 + 1)*b) + 15/2*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a
^2 - 1)*b^2))/b^2 - 3*(a*b^2 + b^2)*(a^2 - 1)*a^2/((a^2*b^2 - (a^2 - 1)*b^2
)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) + 3*(a^2*b + 2*a*b + b)*(a^2 - 1)
*a/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b) - 3*(a*
b^2 + b^2)*x^2/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) - 3/2*(a^2 - 1)*arc
sin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^2 + 5*(a^2 - 1)*a/(sqrt
(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) - 9*(a*b^2 + b^2)*a*arcsin(-(b^2*x + a*
b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 + 3*(a^2*b + 2*a*b + b)*arcsin(-(b^2*
x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + (a^3 + 3*a^2 + 3*a + 1)/(sqrt
(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2) - 6*(a*b^2 + b^2)*(a^2 - 1)/(sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,x+1)^3}{\left(1-(a+b\,x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + 1)^3)/(1 - (a + b*x)^2)^(3/2),x)
```

```
[Out] int((x*(a + b*x + 1)^3)/(1 - (a + b*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+bx+1)^3}{\left(-(a+bx-1)\,(a+bx+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)*x,x)
[Out] Integral(x*(a + b*x + 1)**3/(-(a + b*x - 1)*(a + b*x + 1))**(3/2), x)
```

3.838 $\int e^{3\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=68

$$\frac{2(a+bx+1)^{3/2}}{b\sqrt{-a-bx+1}}+\frac{3\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{b}-\frac{3\sin^{-1}(a+bx)}{b}$$

```
[Out] -3*arcsin(b*x+a)/b+2*(b*x+a+1)^(3/2)/b/(-b*x-a+1)^(1/2)+3*(-b*x-a+1)^(1/2)*
(b*x+a+1)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6161, 47, 50, 53, 619, 216}

$$\frac{2(a+bx+1)^{3/2}}{b\sqrt{-a-bx+1}}+\frac{3\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{b}-\frac{3\sin^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] (3*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/b + (2*(1 + a + b*x)^(3/2))/(b*Sqrt
[1 - a - b*x]) - (3*ArcSin[a + b*x])/b
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(a+bx)}\,dx &= \int \frac{(1+a+bx)^{3/2}}{(1-a-bx)^{3/2}}\,dx\\
&= \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - 3\int \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\,dx\\
&= \frac{3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - 3\int \frac{1}{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx\\
&= \frac{3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - 3\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx\\
&= \frac{3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b^2}\\
&= \frac{3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \frac{2(1+a+bx)^{3/2}}{b\sqrt{1-a-bx}} - \frac{3\sin^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.63

$$\frac{\left(1-\frac{4}{a+bx-1}\right)\sqrt{1-(a+bx)^2}}{b} - \frac{3\sin^{-1}(a+bx)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] ((1 - 4/(-1 + a + b*x))*Sqrt[1 - (a + b*x)^2])/b - (3*ArcSin[a + b*x])/b
```

fricas [A] time = 0.58, size = 100, normalized size = 1.47

$$\frac{3\,(bx+a-1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a-5)}{b^2x+(a-1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (3*(b*x + a - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x
^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a - 5)
)/(b^2*x + (a - 1)*b)
```

giac [A] time = 0.31, size = 75, normalized size = 1.10

$$\frac{3\,\arcsin\left(-bx-a\right)\mathrm{sgn}(b)}{|b|}+\frac{\sqrt{-(bx+a)^2+1}}{b}+\frac{8}{\left(\frac{\sqrt{-(bx+a)^2+1}\,|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3*arcsin(-b*x - a)*sgn(b)/abs(b) + sqrt(-(b*x + a)^2 + 1)/b + 8/(((sqrt(-(b
*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

maple [B] time = 0.04, size = 388, normalized size = 5.71

$$\frac{2\,(1+a)^3\left(-2b^2x-2ab\right)}{\left(-4b^2\left(-a^2+1\right)-4a^2b^2\right)\sqrt{-b^2x^2-2abx-a^2+1}}-\frac{3\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{\sqrt{b^2}}+\frac{3a}{b\sqrt{-b^2x^2-2abx-a^2+1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] 2*(1+a)^3*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)-3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1
```

```
)^(1/2))+3/b*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a^2/(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)*x-3/b*a^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-4/b*a^2/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)-a^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-1/b*a^4/(-b^2*x^2-2*a*b*x
-a^2+1)^(1/2)-b*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-5*a*x/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)+3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+5/b/(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)
```

maxima [B] time = 0.42, size = 753, normalized size = 11.07

$$-\frac{6\,a^3b^2x}{\left(a^2b^2-\left(a^2-1\right)b^2\right)\sqrt{-b^2x^2-2\,abx-a^2+1}}+\frac{5\left(a^2-1\right)ab^2x}{\left(a^2b^2-\left(a^2-1\right)b^2\right)\sqrt{-b^2x^2-2\,abx-a^2+1}}-\frac{\left(a^2\right.}{\left(a^2b^2-\left(a^2-1\right)b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -6*a^3*b^2*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))
 + 5*(a^2 - 1)*a*b^2*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)) - (a^2 - 1)*a^2*b/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a
*b*x - a^2 + 1)) + 6*(a*b^2 + b^2)*a^2*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1)) - 3*(a^2*b + 2*a*b + b)*a*b*x/((a^2*b^2 - (a^2
 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) + (a^3 + 3*a^2 + 3*a + 1)*b^
2*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) - b*x^2/
sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1) - 3*(a^2*b + 2*a*b + b)*a^2/((a^2*b^2 -
(a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) + (a^3 + 3*a^2 + 3*a + 1
)*a*b/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)) - 3*(a
*b^2 + b^2)*(a^2 - 1)*x/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)) - 3*a*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b +
3*(a*b^2 + b^2)*(a^2 - 1)*a/((a^2*b^2 - (a^2 - 1)*b^2)*sqrt(-b^2*x^2 - 2*a*
b*x - a^2 + 1)*b) - 2*(a^2 - 1)/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b) + 3*
(a*b^2 + b^2)*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + 3*
(a^2*b + 2*a*b + b)/(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x+1)^3}{\left(1-(a+b\,x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)^3/(1 - (a + b*x)^2)^(3/2),x)
```

```
[Out] int((a + b*x + 1)^3/(1 - (a + b*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+1)^3}{(-(a+bx-1)(a+bx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2),x)
[Out] Integral((a + b*x + 1)**3/(-(a + b*x - 1)*(a + b*x + 1))**(3/2), x)
```

3.839 $\int \frac{e^{3\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=107

$$-\frac{2(a+1)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}+\frac{4\sqrt{a+bx+1}}{(1-a)\sqrt{-a-bx+1}}-\sin^{-1}(a+bx)$$

```
[Out] -arcsin(b*x+a)-2*(1+a)^2*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-
b*x-a+1)^(1/2))/(1-a)/(-a^2+1)^(1/2)+4*(b*x+a+1)^(1/2)/(1-a)/(-b*x-a+1)^(1/
2)
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6163, 98, 157, 53, 619, 216, 93, 208}

$$-\frac{2(a+1)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}+\frac{4\sqrt{a+bx+1}}{(1-a)\sqrt{-a-bx+1}}-\sin^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])/x,x]
```

```
[Out] (4*Sqrt[1 + a + b*x])/((1 - a)*Sqrt[1 - a - b*x]) - ArcSin[a + b*x] - (2*(1
 + a)^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b
*x])])/((1 - a)*Sqrt[1 - a^2])
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(a+bx)}}{x}\,dx = \int \frac{(1+a+bx)^{3/2}}{x(1-a-bx)^{3/2}}\,dx$$

$$= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} - \frac{2\int \frac{-\frac{1}{2}(1+a)^2b+\frac{1}{2}(1-a)b^2x}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{(1-a)b}$$

$$= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} + \frac{(1+a)^2\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{1-a} - b\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx$$

$$= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} + \frac{\left(2(1+a)^2\right)\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1-a} - b\int \frac{}{\sqrt{(1-a)(1-}}$$

$$= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} - \frac{2(1+a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1-a)\sqrt{1-a^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b\right.}{2b}$$

$$= \frac{4\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} - \sin^{-1}(a+bx) - \frac{2(1+a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{(1-a)\sqrt{1-a^2}}$$

Mathematica [A] time = 0.59, size = 153, normalized size = 1.43

$$\frac{2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{-b}} - \frac{2\left(2\sqrt{a-1}\sqrt{a+bx+1} + (-a-1)^{3/2}\sqrt{-a-bx+1}\tanh^{-1}\left(\frac{\sqrt{-a-1}\sqrt{-a-bx+1}}{\sqrt{a-1}\sqrt{a+bx+1}}\right)\right)}{(a-1)^{3/2}\sqrt{-a-bx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])/x,x]
[Out] (2*Sqrt[b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/Sqrt[-b
] - (2*(2*Sqrt[-1 + a]*Sqrt[1 + a + b*x] + (-1 - a)^(3/2)*Sqrt[1 - a - b*x]
*ArcTanh[(Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqrt[1 + a + b*x])]
))/((-1 + a)^(3/2)*Sqrt[1 - a - b*x])
```

fricas [B] time = 0.70, size = 439, normalized size = 4.10

$$\left[\frac{\left((a+1)bx + a^2 - 1\right)\sqrt{-\frac{a+1}{a-1}}\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-4a^2+2\sqrt{-b^2x^2-2abx-a^2+1}\left(a^3+\left(a^2-a\right)bx-a^2-a+1\right)\sqrt{-\frac{a+1}{a-1}}+2}{x^2}\right) + 2\left(\right.}{2\left((a-1)bx + a^2 - 2a + 1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(((a + 1)*b*x + a^2 - 1)*sqrt(-(a + 1)/(a - 1))*log(((2*a^2 - 1)*b^2*x
^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*(a^3 + (a^2 - a)*b*x - a^2 - a + 1)*sqrt(-(a + 1)/(a - 1)) + 2)/x^2) + 2*(
(a - 1)*b*x + a^2 - 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x
 + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
)/((a - 1)*b*x + a^2 - 2*a + 1), -(((a + 1)*b*x + a^2 - 1)*sqrt((a + 1)/(a
- 1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt((a +
 1)/(a - 1))/((a + 1)*b^2*x^2 + a^3 + 2*(a^2 + a)*b*x + a^2 - a - 1)) - ((a
 - 1)*b*x + a^2 - 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x +
 a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 4*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/
((a - 1)*b*x + a^2 - 2*a + 1)]
```

giac [A] time = 0.32, size = 139, normalized size = 1.30

$$\frac{b\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{|b|}-\frac{2\left(a^{2}b+2\,ab+b\right)\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^{2}+1}\,|b|+b\right)a}{b^{2}x+ab}-1}{\sqrt{a^{2}-1}}\right)}{\sqrt{a^{2}-1}\,(a|b|-|b|)}-\frac{8\,b}{(a|b|-|b|)\left(\frac{\sqrt{-(bx+a)^{2}+1}\,|b|+b}{b^{2}x+ab}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] b*arcsin(-b*x - a)*sgn(b)/abs(b) - 2*(a^2*b + 2*a*b + b)*arctan(((sqrt(-(b*
x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1
)*(a*abs(b) - abs(b))) - 8*b/((a*abs(b) - abs(b))*((sqrt(-(b*x + a)^2 + 1)*
abs(b) + b)/(b^2*x + a*b) - 1))
```

maple [B] time = 0.04, size = 1019, normalized size = 9.52

$$-\frac{3bax}{\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}-\frac{2b\,a^{2}x}{\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}+\frac{6b\left(-2b^{2}x-2ab\right)}{\left(-4b^{2}\left(-a^{2}+1\right)-4a^{2}b^{2}\right)\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}+\frac{}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x,x)
```

```
[Out] -3*b*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-2*b*a^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)*x+6*b*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)+3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)+2*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-2*a^3/(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)-3*a^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(-a^2+1)^(3/2)*ln((-2*a^2+2-
2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+a*b/(-a^2+1)/(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+a^4*b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
)*x+6*a^2*b*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)+12*a*b*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)+3*a^3*b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+3*a^
2*b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-b/(b^2)^(1/2)*arctan((b^2)^(1
/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+b*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)+4/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^3+4*a^2/(-a^2+1)/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)+3/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+a^5/(-a^2+
1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a^4/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2)-1/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2))/x)*a^3-3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1
/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2-3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*
a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x+1)^3}{x\,\left(1-(a+b\,x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)^3/(x*(1 - (a + b*x)^2)^(3/2)),x)
```

```
[Out] int((a + b*x + 1)^3/(x*(1 - (a + b*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+1)^3}{x\,(-(a+bx-1)\,(a+bx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)/x,x)
[Out] Integral((a + b*x + 1)**3/(x*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)), x)
```

3.840 $\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=134

$$-\frac{6(a+1)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2\sqrt{1-a^2}}-\frac{(a+bx+1)^{3/2}}{(1-a)x\sqrt{-a-bx+1}}+\frac{6b\sqrt{a+bx+1}}{(1-a)^2\sqrt{-a-bx+1}}$$

```
[Out] -6*(1+a)*b*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(1/2)
)/(1-a)^2/(-a^2+1)^(1/2)-(b*x+a+1)^(3/2)/(1-a)/x/(-b*x-a+1)^(1/2)+6*b*(b*x+
a+1)^(1/2)/(1-a)^2/(-b*x-a+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 94, 93, 208}

$$-\frac{6(a+1)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^2\sqrt{1-a^2}}-\frac{(a+bx+1)^{3/2}}{(1-a)x\sqrt{-a-bx+1}}+\frac{6b\sqrt{a+bx+1}}{(1-a)^2\sqrt{-a-bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] (6*b*Sqrt[1 + a + b*x])/((1 - a)^2*Sqrt[1 - a - b*x]) - (1 + a + b*x)^(3/2)
/((1 - a)*x*Sqrt[1 - a - b*x]) - (6*(1 + a)*b*ArcTanh[(Sqrt[1 - a]*Sqrt[1 +
 a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)^2*Sqrt[1 - a^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{(1+a+bx)^{3/2}}{x^2(1-a-bx)^{3/2}}\,dx \\
&= -\frac{(1+a+bx)^{3/2}}{(1-a)x\sqrt{1-a-bx}} + \frac{(3b)\int \frac{\sqrt{1+a+bx}}{x(1-a-bx)^{3/2}}\,dx}{1-a} \\
&= \frac{6b\sqrt{1+a+bx}}{(1-a)^2\sqrt{1-a-bx}} - \frac{(1+a+bx)^{3/2}}{(1-a)x\sqrt{1-a-bx}} + \frac{(3(1+a)b)\int \frac{1}{x\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{(1-a)^2} \\
&= \frac{6b\sqrt{1+a+bx}}{(1-a)^2\sqrt{1-a-bx}} - \frac{(1+a+bx)^{3/2}}{(1-a)x\sqrt{1-a-bx}} + \frac{(6(1+a)b)\operatorname{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1-}}{\sqrt{1-}}\right)}{(1-a)^2} \\
&= \frac{6b\sqrt{1+a+bx}}{(1-a)^2\sqrt{1-a-bx}} - \frac{(1+a+bx)^{3/2}}{(1-a)x\sqrt{1-a-bx}} - \frac{6(1+a)b\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{1-a-bx}}\right)}{(1-a)^2\sqrt{1-a^2}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 106, normalized size = 0.79

$$
\frac{\sqrt{a+bx+1}\,\left(a^2+abx+5bx-1\right)}{(a-1)^2x\sqrt{-a-bx+1}} - \frac{6\sqrt{-a-1}\,b\tanh^{-1}\left(\frac{\sqrt{-a-1}\,\sqrt{-a-bx+1}}{\sqrt{a-1}\,\sqrt{a+bx+1}}\right)}{(a-1)^{5/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] (Sqrt[1 + a + b*x]*(-1 + a^2 + 5*b*x + a*b*x))/((-1 + a)^2*x*Sqrt[1 - a - b
*x]) - (6*Sqrt[-1 - a]*b*ArcTanh[(Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1
+ a]*Sqrt[1 + a + b*x])])/(-1 + a)^(5/2)
```

fricas [A] time = 0.80, size = 370, normalized size = 2.76

$$\left[\frac{3\left(b^2x^2+(a-1)bx\right)\sqrt{-\frac{a+1}{a-1}}\,\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\left(a^3-a\right)bx-4\,a^2-2\,\sqrt{-b^2x^2-2\,abx-a^2+1}\left(a^3+\left(a^2-a\right)bx-a^2-a+1\right)\sqrt{-\frac{a+1}{a-1}}+2}{x^2}\right)-2\,\sqrt{\cdots}}{2\left(\left(a^2-2\,a+1\right)bx^2+\left(a^3-3\,a^2+3\,a-1\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + (a - 1)*b*x)*sqrt(-(a + 1)/(a - 1))*log(((2*a^2 - 1)*b^2
*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(a^3 + (a^2 - a)*b*x - a^2 - a + 1)*sqrt(-(a + 1)/(a - 1)) + 2)/x^2) - 2
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((a + 5)*b*x + a^2 - 1))/((a^2 - 2*a +
1)*b*x^2 + (a^3 - 3*a^2 + 3*a - 1)*x), (3*(b^2*x^2 + (a - 1)*b*x)*sqrt((a +
 1)/(a - 1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt((a + 1)/(a - 1))/((a + 1)*b^2*x^2 + a^3 + 2*(a^2 + a)*b*x + a^2 - a - 1)
) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((a + 5)*b*x + a^2 - 1))/((a^2 - 2*a
 + 1)*b*x^2 + (a^3 - 3*a^2 + 3*a - 1)*x)]
```

giac [B] time = 0.23, size = 498, normalized size = 3.72

$$\frac{6\left(ab^2+b^2\right)\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^2|b|-2\,a|b|+|b|\right)\sqrt{a^2-1}}-\frac{2\left(\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a^2b^2}{b^2x+ab}-\frac{4\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)^2a^2b^2}{\left(b^2x+ab\right)^2}-5\,a^2b^2+\frac{10\left(\sqrt{-(bx+a)^2+\cdots}\right.}{b^2x+\cdots}\right.}{\left(a^3|b|-2\,a^2|b|+a|b|\right)\left(\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a}{b^2x+ab}-\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)\cdots}{\left(b^2x+ab\right)^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 6*(a*b^2 + b^2)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b)
 - 1)/sqrt(a^2 - 1))/((a^2*abs(b) - 2*a*abs(b) + abs(b))*sqrt(a^2 - 1)) - 2
*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^2/(b^2*x + a*b) - 4*(sqrt(-(b*x
 + a)^2 + 1)*abs(b) + b)^2*a^2*b^2/(b^2*x + a*b)^2 - 5*a^2*b^2 + 10*(sqrt(-
(b*x + a)^2 + 1)*abs(b) + b)*a*b^2/(b^2*x + a*b) - (sqrt(-(b*x + a)^2 + 1)*
abs(b) + b)^2*a*b^2/(b^2*x + a*b)^2 - a*b^2 + (sqrt(-(b*x + a)^2 + 1)*abs(b
) + b)*b^2/(b^2*x + a*b) - (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*b^2/(b^2*x
 + a*b)^2)/((a^3*abs(b) - 2*a^2*abs(b) + a*abs(b))*((sqrt(-(b*x + a)^2 + 1)
*abs(b) + b)*a/(b^2*x + a*b) - (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a/(b^2
```

```
*x + a*b)^2 + (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a/(b^2*x + a*b)^3 - a +
 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 2*(sqrt(-(b*x + a)^2
 + 1)*abs(b) + b)^2/(b^2*x + a*b)^2))
```

maple [B] time = 0.05, size = 1520, normalized size = 11.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^2,x)
```

```
[Out] 9*a^2*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a*b/(-a^2+1)^2/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2)+12*a^3*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+5
*b*a^4/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+12*b*a^3/(-a^2+1)/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)-3*b/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/
2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2+8*b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)*a+12*b*a^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-b^2*a/(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)*x-3*a*b/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2
+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+12*a^4*b/(-a^2+1)^2/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)-3/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-3/(-a^
2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a-1/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)*a^3+2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x*b^2+3*a^6*b/(-a^2+
1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a^4*b/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a
*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-9*a^3*b/(-a^2+1)^(
5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/
x)-9*a^2*b/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2))/x)+6*b^2*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-6*a*b/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*
(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+9*a^5*b/(-a^2+1)^2/(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)+b/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+6*a*b^2*(-2*b^2*x
-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a^5*b^
2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-1/(-a^2+1)/x/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)-b*a^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*b/(-a^2+1)/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)-3*b/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/
2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+9*a^4*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)*x+9*a^3*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+3*a^
2*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+5*a^3*b^2/(-a^2+1)/(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)*x+12*a^2*b^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/
2)*x+9*a*b^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x+1)^3}{x^2\,\left(1-(a+b\,x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)^3/(x^2*(1 - (a + b*x)^2)^(3/2)),x)
```

```
[Out] int((a + b*x + 1)^3/(x^2*(1 - (a + b*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+1)^3}{x^2\,(-(a+bx-1)\,(a+bx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x + 1)**3/(x**2*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)), x)
```

3.841 $\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=202

$$-\frac{3(2a+3)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^3\sqrt{1-a^2}}-\frac{(a+bx+1)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{-a-bx+1}}+\frac{3(2a+3)b^2\sqrt{a+bx+1}}{(1-a)^3(a+1)\sqrt{-a-bx+1}}-\frac{(2a+3)b(}{2(1-a)^2(a+}$$

```
[Out] -3*(3+2*a)*b^2*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(
1/2))/(1-a)^3/(-a^2+1)^(1/2)-1/2*(3+2*a)*b*(b*x+a+1)^(3/2)/(1-a)^2/(1+a)/x/
(-b*x-a+1)^(1/2)-1/2*(b*x+a+1)^(5/2)/(-a^2+1)/x^2/(-b*x-a+1)^(1/2)+3*(3+2*a
)*b^2*(b*x+a+1)^(1/2)/(1-a)^3/(1+a)/(-b*x-a+1)^(1/2)
```

Rubi [**A**] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6163, 96, 94, 93, 208}

$$-\frac{3(2a+3)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)^3\sqrt{1-a^2}}-\frac{(a+bx+1)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{-a-bx+1}}+\frac{3(2a+3)b^2\sqrt{a+bx+1}}{(1-a)^3(a+1)\sqrt{-a-bx+1}}-\frac{(2a+3)b(}{2(1-a)^2(a+}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] (3*(3 + 2*a)*b^2*Sqrt[1 + a + b*x])/((1 - a)^3*(1 + a)*Sqrt[1 - a - b*x]) -
 ((3 + 2*a)*b*(1 + a + b*x)^(3/2))/(2*(1 - a)^2*(1 + a)*x*Sqrt[1 - a - b*x]
) - (1 + a + b*x)^(5/2)/(2*(1 - a^2)*x^2*Sqrt[1 - a - b*x]) - (3*(3 + 2*a)*
b^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])
])/((1 - a)^3*Sqrt[1 - a^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1+a+bx)^{3/2}}{x^3(1-a-bx)^{3/2}}\,dx$$

$$= -\frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} + \frac{((3+2a)b)\int \frac{(1+a+bx)^{3/2}}{x^2(1-a-bx)^{3/2}}\,dx}{2\left(1-a^2\right)}$$

$$= -\frac{(3+2a)b(1+a+bx)^{3/2}}{2(1-a)^2(1+a)x\sqrt{1-a-bx}} - \frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} + \frac{\left(3(3+2a)b^2\right)\int \frac{\sqrt{1+a+bx}}{x(1-a-bx)^{3/2}}\,d}{2(1-a)^2(1+a)}$$

$$= \frac{3(3+2a)b^2\sqrt{1+a+bx}}{(1-a)^3(1+a)\sqrt{1-a-bx}} - \frac{(3+2a)b(1+a+bx)^{3/2}}{2(1-a)^2(1+a)x\sqrt{1-a-bx}} - \frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} +$$

$$= \frac{3(3+2a)b^2\sqrt{1+a+bx}}{(1-a)^3(1+a)\sqrt{1-a-bx}} - \frac{(3+2a)b(1+a+bx)^{3/2}}{2(1-a)^2(1+a)x\sqrt{1-a-bx}} - \frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} +$$

$$= \frac{3(3+2a)b^2\sqrt{1+a+bx}}{(1-a)^3(1+a)\sqrt{1-a-bx}} - \frac{(3+2a)b(1+a+bx)^{3/2}}{2(1-a)^2(1+a)x\sqrt{1-a-bx}} - \frac{(1+a+bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1-a-bx}} -$$

Mathematica [A] time = 0.17, size = 141, normalized size = 0.70

$$\frac{\sqrt{a+bx+1}\left(a^3-a^2-a\left(b^2x^2+5bx+1\right)-14b^2x^2+5bx+1\right)}{2(a-1)^3x^2\sqrt{-a-bx+1}} - \frac{3(2a+3)b^2\tanh^{-1}\left(\frac{\sqrt{-a-1}\sqrt{-a-bx+1}}{\sqrt{a-1}\sqrt{a+bx+1}}\right)}{\sqrt{-a-1}\,(a-1)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] (Sqrt[1 + a + b*x]*(1 - a^2 + a^3 + 5*b*x - 14*b^2*x^2 - a*(1 + 5*b*x + b^2
*x^2)))/(2*(-1 + a)^3*x^2*Sqrt[1 - a - b*x]) - (3*(3 + 2*a)*b^2*ArcTanh[(Sq
rt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqrt[1 + a + b*x])])/(Sqrt[-1 -
 a]*(-1 + a)^(7/2))
```

fricas [A] time = 1.55, size = 523, normalized size = 2.59

$$\left[-\frac{3\left((2a+3)b^3x^3+\left(2a^2+a-3\right)b^2x^2\right)\sqrt{-a^2+1}\,\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-2\sqrt{-b^2x^2-2abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2-}}{x^2}\right.}{4\left(\left(a^5-3a^4+2a^3+2a^2-3a+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(3*((2*a + 3)*b^3*x^3 + (2*a^2 + a - 3)*b^2*x^2)*sqrt(-a^2 + 1)*log((
(2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 2*(a^5 - (a^
3 + 14*a^2 - a - 14)*b^2*x^2 - a^4 - 2*a^3 - 5*(a^3 - a^2 - a + 1)*b*x + 2*
a^2 + a - 1)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 - 3*a^4 + 2*a^3 + 2*
a^2 - 3*a + 1)*b*x^3 + (a^6 - 4*a^5 + 5*a^4 - 5*a^2 + 4*a - 1)*x^2), -1/2*(
3*((2*a + 3)*b^3*x^3 + (2*a^2 + a - 3)*b^2*x^2)*sqrt(a^2 - 1)*arctan(sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2
*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^5 - (a^3 + 14*a^2 - a - 14)
*b^2*x^2 - a^4 - 2*a^3 - 5*(a^3 - a^2 - a + 1)*b*x + 2*a^2 + a - 1)*sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 - 3*a^4 + 2*a^3 + 2*a^2 - 3*a + 1)*b*x^3
 + (a^6 - 4*a^5 + 5*a^4 - 5*a^2 + 4*a - 1)*x^2)]
```

giac [B] time = 0.45, size = 691, normalized size = 3.42

$$-\frac{8\,b^3}{\left(a^3|b| - 3\,a^2|b| + 3\,a|b| - |b|\right)\left(\frac{\sqrt{-(bx+a)^2+1}\,|b|+b}{b^2x+ab} - 1\right)} - \frac{3\left(2\,ab^3 + 3\,b^3\right)\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a}{b^2x+ab} - 1}{\sqrt{a^2-1}}\right)}{\left(a^3|b| - 3\,a^2|b| + 3\,a|b| - |b|\right)\sqrt{a^2-1}} + \frac{2\left(\sqrt{-(bx+a)^2+1}\,|b| - \ldots\right.}{\left(b^2x+ab\right)^2 \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -8*b^3/((a^3*abs(b) - 3*a^2*abs(b) + 3*a*abs(b) - abs(b))*((sqrt(-(b*x + a)
^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)) - 3*(2*a*b^3 + 3*b^3)*arctan(((sqrt
(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^3*ab
s(b) - 3*a^2*abs(b) + 3*a*abs(b) - abs(b))*sqrt(a^2 - 1)) + (2*(sqrt(-(b*x
+ a)^2 + 1)*abs(b) + b)^2*a^4*b^3/(b^2*x + a*b)^2 + 2*a^4*b^3 - 5*(sqrt(-(b
*x + a)^2 + 1)*abs(b) + b)*a^3*b^3/(b^2*x + a*b) + 6*(sqrt(-(b*x + a)^2 + 1
)*abs(b) + b)^2*a^3*b^3/(b^2*x + a*b)^2 - 3*(sqrt(-(b*x + a)^2 + 1)*abs(b)
+ b)^3*a^3*b^3/(b^2*x + a*b)^3 + 6*a^3*b^3 - 18*(sqrt(-(b*x + a)^2 + 1)*abs
(b) + b)*a^2*b^3/(b^2*x + a*b) + 3*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^
2*b^3/(b^2*x + a*b)^2 - 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^2*b^3/(b^
2*x + a*b)^3 - a^2*b^3 + 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a*b^3/(b^2*x
 + a*b) + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a*b^3/(b^2*x + a*b)^2 +
2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(sqrt(-(b
*x + a)^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^5*abs(b) - 3*a^4*abs(
```

```
b) + 3*a^3*abs(b) - a^2*abs(b))*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a/(b
^2*x + a*b)^2 + a - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b))^2)
```

maple [B] time = 0.05, size = 2194, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^3,x)

[Out] -45/2*a^4*b^2/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2))/x)-45/2*a^3*b^2/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*a*b*x
+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(-a^2+1)/x^2/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)*a^3-3/2/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/
2)*a^2-3/2/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+30*a^5*b^2/(-a^2+1
)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+45/2*a^3*b^2/(-a^2+1)^3/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)+2*b^3*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)-15/2*a^2*b^2/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*a*b*x+2*(-
a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+27/2*a*b^2/(-a^2+1)^2/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)-3*b/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+39*
a^3*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9*b^2*a/(-a^2+1)/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2)+9*b^2*a^3/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+15
*b^2*a^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+15/2*a^7*b^2/(-a^2+1)^3/(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2)+45/2*a^6*b^2/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)-15/2*a^5*b^2/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+15/2*a^2*b^2/(-a^2+1)^3/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)+30*a^4*b^2/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-27/2*a*
b^2/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2))/x)+6/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x*b^3+31/2*a^5*b
^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+75/2*a^4*b^2/(-a^2+1)^2/(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2)+29*a^2*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/
2)-45/2*a^2*b^2/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2))/x)-21/2*a^3*b^2/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b
*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3*a*b^2/(-a^2+1)^(3/
2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
-3*b/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-6*b/(-a^2+1)/x/(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)*a+15/2*a^6*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)*x+45/2*a^5*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+45/2*a^4*b^3
/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-5/2*a^4*b/(-a^2+1)^2/x/(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)-15/2*a^3*b/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)-15/2*a^2*b/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+57/2*a^2*b^3/(-a
^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+15*b^3*a/(-a^2+1)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)*x+9*b^3*a^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3/2*b^
2/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))/x)+3/2*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*b^2/(-a^
```

```
2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)+13/2*a*b^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+31/2*a^4*b^
3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+75/2*a^3*b^3/(-a^2+1)^2/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)*x-5/2*a*b/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2)+15/2*a^3*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3*b^2/(-a^2+1
)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
))/x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x+1)^3}{x^3\,\left(1-(a+b\,x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)^3/(x^3*(1 - (a + b*x)^2)^(3/2)),x)
```

```
[Out] int((a + b*x + 1)^3/(x^3*(1 - (a + b*x)^2)^(3/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+1)^3}{x^3\,(-(a+bx-1)\,(a+bx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x + 1)**3/(x**3*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)), x)
```

3.842 $\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=260

$$\frac{(2a^2+51a+52)\,b^3\sqrt{a+bx+1}}{6(1-a)^4(a+1)\sqrt{-a-bx+1}}-\frac{(6a^2+18a+11)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(1-a)^4(a+1)\sqrt{1-a^2}}-\frac{(16a+19)b^2\sqrt{a+bx+1}}{6(1-a)^3(a+1)x\sqrt{-a-bx+1}}$$

```
[Out] -(6*a^2+18*a+11)*b^3*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-
a+1)^(1/2))/(1-a)^4/(1+a)/(-a^2+1)^(1/2)+1/6*(2*a^2+51*a+52)*b^3*(b*x+a+1)^
(1/2)/(1-a)^4/(1+a)/(-b*x-a+1)^(1/2)-1/3*(1+a)*(b*x+a+1)^(1/2)/(1-a)/x^3/(-
b*x-a+1)^(1/2)-7/6*b*(b*x+a+1)^(1/2)/(1-a)^2/x^2/(-b*x-a+1)^(1/2)-1/6*(19+1
6*a)*b^2*(b*x+a+1)^(1/2)/(1-a)^3/(1+a)/x/(-b*x-a+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 98, 151, 152, 12, 93, 208}

$$\frac{(2a^2+51a+52)\,b^3\sqrt{a+bx+1}}{6(1-a)^4(a+1)\sqrt{-a-bx+1}}-\frac{(6a^2+18a+11)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(1-a)^4(a+1)\sqrt{1-a^2}}-\frac{(16a+19)b^2\sqrt{a+bx+1}}{6(1-a)^3(a+1)x\sqrt{-a-bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a + b*x])/x^4,x]
```

```
[Out] ((52 + 51*a + 2*a^2)*b^3*Sqrt[1 + a + b*x])/(6*(1 - a)^4*(1 + a)*Sqrt[1 - a
 - b*x]) - ((1 + a)*Sqrt[1 + a + b*x])/(3*(1 - a)*x^3*Sqrt[1 - a - b*x]) -
(7*b*Sqrt[1 + a + b*x])/(6*(1 - a)^2*x^2*Sqrt[1 - a - b*x]) - ((19 + 16*a)*
b^2*Sqrt[1 + a + b*x])/(6*(1 - a)^3*(1 + a)*x*Sqrt[1 - a - b*x]) - ((11 + 1
8*a + 6*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[
1 - a - b*x])])/((1 - a)^4*(1 + a)*Sqrt[1 - a^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
```

```
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{(1+a+bx)^{3/2}}{x^4(1-a-bx)^{3/2}}\,dx$$

$$= -\frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{\int \frac{-7(1+a)b-6b^2x}{x^3(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx}{3(1-a)}$$

$$= -\frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} + \frac{\int \frac{(1+a)(19+16a)b^2+14(1+a)b^3x}{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx}{6(1-a)^2(1+a)}$$

$$= -\frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{(19+16a)b^2\sqrt{1+a+bx}}{6(1-a)^3(1+a)x\sqrt{1-a-bx}} - \frac{\int -}{}$$

$$= \frac{\left(52+51a+2a^2\right)b^3\sqrt{1+a+bx}}{6(1-a)^4(1+a)\sqrt{1-a-bx}} - \frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{}{6(}$$

$$= \frac{\left(52+51a+2a^2\right)b^3\sqrt{1+a+bx}}{6(1-a)^4(1+a)\sqrt{1-a-bx}} - \frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{}{6(}$$

$$= \frac{\left(52+51a+2a^2\right)b^3\sqrt{1+a+bx}}{6(1-a)^4(1+a)\sqrt{1-a-bx}} - \frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{}{6(}$$

$$= \frac{\left(52+51a+2a^2\right)b^3\sqrt{1+a+bx}}{6(1-a)^4(1+a)\sqrt{1-a-bx}} - \frac{(1+a)\sqrt{1+a+bx}}{3(1-a)x^3\sqrt{1-a-bx}} - \frac{7b\sqrt{1+a+bx}}{6(1-a)^2x^2\sqrt{1-a-bx}} - \frac{}{6(}$$

Mathematica [A] time = 0.31, size = 199, normalized size = 0.77

$$-\frac{-\left(6a^2+18a+11\right)b^2x^2\left(\sqrt{a-1}\sqrt{a+bx+1}\left(a^2+abx+5bx-1\right)-6\sqrt{-a-1}\,bx\sqrt{-a-bx+1}\tanh^{-1}\left(\frac{\sqrt{-a-1}}{\sqrt{a-1}}\right.\right.}{6(a-1)^{5/2}\left(a^2-1\right)^2x^3\sqrt{-a-bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a + b*x])/x^4,x]
```

```
[Out] -1/6*(-2*(-1 + a)^(7/2)*(1 + a)*(1 + a + b*x)^(5/2) + (-1 + a)^(5/2)*(3 + 4
*a)*b*x*(1 + a + b*x)^(5/2) - (11 + 18*a + 6*a^2)*b^2*x^2*(Sqrt[-1 + a]*Sqr
```

```
t[1 + a + b*x]*(-1 + a^2 + 5*b*x + a*b*x) - 6*Sqrt[-1 - a]*b*x*Sqrt[1 - a -
 b*x]*ArcTanh[(Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqrt[1 + a + b
*x])]))/((-1 + a)^(5/2)*(-1 + a^2)^2*x^3*Sqrt[1 - a - b*x])
```

fricas [A] time = 0.88, size = 704, normalized size = 2.71

$$\left[-\frac{3\left(\left(6\,a^2+18\,a+11\right)b^4x^4+\left(6\,a^3+12\,a^2-7\,a-11\right)b^3x^3\right)\sqrt{-a^2+1}\,\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\left(a^3-a\right)bx+2\,\sqrt{-b^2x^2-2\,abx-}}{x^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((6*a^2 + 18*a + 11)*b^4*x^4 + (6*a^3 + 12*a^2 - 7*a - 11)*b^3*x^
3)*sqrt(-a^2 + 1)*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x + 2*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 +
 2)/x^2) + 2*(2*a^7 + (2*a^4 + 51*a^3 + 50*a^2 - 51*a - 52)*b^3*x^3 - 2*a^6
 - 6*a^5 + (16*a^4 + 3*a^3 - 35*a^2 - 3*a + 19)*b^2*x^2 + 6*a^4 + 6*a^3 - 7
*(a^5 - a^4 - 2*a^3 + 2*a^2 + a - 1)*b*x - 6*a^2 - 2*a + 2)*sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1))/((a^7 - 3*a^6 + a^5 + 5*a^4 - 5*a^3 - a^2 + 3*a - 1)*b
*x^4 + (a^8 - 4*a^7 + 4*a^6 + 4*a^5 - 10*a^4 + 4*a^3 + 4*a^2 - 4*a + 1)*x^3
), 1/6*(3*((6*a^2 + 18*a + 11)*b^4*x^4 + (6*a^3 + 12*a^2 - 7*a - 11)*b^3*x^
3)*sqrt(a^2 - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1
)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) -
(2*a^7 + (2*a^4 + 51*a^3 + 50*a^2 - 51*a - 52)*b^3*x^3 - 2*a^6 - 6*a^5 + (1
6*a^4 + 3*a^3 - 35*a^2 - 3*a + 19)*b^2*x^2 + 6*a^4 + 6*a^3 - 7*(a^5 - a^4 -
 2*a^3 + 2*a^2 + a - 1)*b*x - 6*a^2 - 2*a + 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1))/((a^7 - 3*a^6 + a^5 + 5*a^4 - 5*a^3 - a^2 + 3*a - 1)*b*x^4 + (a^8 -
 4*a^7 + 4*a^6 + 4*a^5 - 10*a^4 + 4*a^3 + 4*a^2 - 4*a + 1)*x^3)]
```

giac [B] time = 0.28, size = 1521, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 8*b^4/((a^4*abs(b) - 4*a^3*abs(b) + 6*a^2*abs(b) - 4*a*abs(b) + abs(b))*((s
qrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)) + (6*a^2*b^4 + 18*a*b
^4 + 11*b^4)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) -
1)/sqrt(a^2 - 1))/((a^5*abs(b) - 3*a^4*abs(b) + 2*a^3*abs(b) + 2*a^2*abs(b)
 - 3*a*abs(b) + abs(b))*sqrt(a^2 - 1)) - 1/3*(12*(sqrt(-(b*x + a)^2 + 1)*ab
s(b) + b)^2*a^7*b^4/(b^2*x + a*b)^2 + 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)
```

```
^4*a^7*b^4/(b^2*x + a*b)^4 + 6*a^7*b^4 - 24*(sqrt(-(b*x + a)^2 + 1)*abs(b)
+ b)*a^6*b^4/(b^2*x + a*b) + 72*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^6*b
^4/(b^2*x + a*b)^2 - 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^6*b^4/(b^2*
x + a*b)^3 + 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^6*b^4/(b^2*x + a*b)
^4 - 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^6*b^4/(b^2*x + a*b)^5 + 36*
a^6*b^4 - 171*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^5*b^4/(b^2*x + a*b) + 8
4*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^5*b^4/(b^2*x + a*b)^2 - 216*(sqrt
(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^5*b^4/(b^2*x + a*b)^3 + 54*(sqrt(-(b*x +
 a)^2 + 1)*abs(b) + b)^4*a^5*b^4/(b^2*x + a*b)^4 - 45*(sqrt(-(b*x + a)^2 +
1)*abs(b) + b)^5*a^5*b^4/(b^2*x + a*b)^5 + 22*a^5*b^4 - 120*(sqrt(-(b*x + a
)^2 + 1)*abs(b) + b)*a^4*b^4/(b^2*x + a*b) + 252*(sqrt(-(b*x + a)^2 + 1)*ab
s(b) + b)^2*a^4*b^4/(b^2*x + a*b)^2 - 156*(sqrt(-(b*x + a)^2 + 1)*abs(b) +
b)^3*a^4*b^4/(b^2*x + a*b)^3 + 153*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^
4*b^4/(b^2*x + a*b)^4 - 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^4*b^4/(b
^2*x + a*b)^5 - 9*a^4*b^4 + 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^3*b^4/
(b^2*x + a*b) + 192*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^3*b^4/(b^2*x +
a*b)^2 - 90*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a^3*b^4/(b^2*x + a*b)^3 +
 78*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a^3*b^4/(b^2*x + a*b)^4 + 18*(sqr
t(-(b*x + a)^2 + 1)*abs(b) + b)^5*a^3*b^4/(b^2*x + a*b)^5 + 2*a^3*b^4 - 6*(
sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^4/(b^2*x + a*b) - 54*(sqrt(-(b*x +
 a)^2 + 1)*abs(b) + b)^2*a^2*b^4/(b^2*x + a*b)^2 - 100*(sqrt(-(b*x + a)^2 +
 1)*abs(b) + b)^3*a^2*b^4/(b^2*x + a*b)^3 - 54*(sqrt(-(b*x + a)^2 + 1)*abs(
b) + b)^4*a^2*b^4/(b^2*x + a*b)^4 - 6*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^5
*a^2*b^4/(b^2*x + a*b)^5 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a*b^4/(
b^2*x + a*b)^2 + 36*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*a*b^4/(b^2*x + a*
b)^3 + 12*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^4*a*b^4/(b^2*x + a*b)^4 - 8*(
sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^3*b^4/(b^2*x + a*b)^3)/((a^8*abs(b) - 3*
a^7*abs(b) + 2*a^6*abs(b) + 2*a^5*abs(b) - 3*a^4*abs(b) + a^3*abs(b))*((sqr
t(-(b*x + a)^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-(b*x + a
)^2 + 1)*abs(b) + b)/(b^2*x + a*b))^3)
```

maple [B] time = 0.05, size = 2947, normalized size = 11.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^4,x)
```

```
[Out] 8/3*b^4/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+35/2*a^3*b^3/(-a^2+1)^4
/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+70*a^5*b^3/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)+260/3*a^3*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+15/2*a*b^
3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-35/2*a^3*b^3/(-a^2+1)^(9/2)*ln(
(-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-15/2*
a*b^3/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))/x)-105/2*a^4*b^3/(-a^2+1)^(9/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^
```

2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/(-a^2+1)/x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-1/(-a^2+1)/x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+70*a^6*b^3/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+105/2*a^4*b^3/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+35/2*a^8*b^3/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-45*a^2*b^3/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-4/3*b^2/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+62/3*b^3*a/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+56*b^3*a^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+7*b^3*a^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*b/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-30*a^4*b^3/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+125/3*a^6*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+205/2*a^5*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+110*a^4*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+45*a^2*b^3/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/3/(-a^2+1)/x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^3+105/2*a^7*b^3/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-35/2*a^6*b^3/(-a^2+1)^(9/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-105/2*a^5*b^3/(-a^2+1)^(9/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+41*b^3*a^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+187/6*b^3*a^4/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*b^2/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-27/2*a^2*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-135/2*a^3*b^3/(-a^2+1)^(7/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-18*a*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+6*b^4/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+6*b^3/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a-23/2*b^2/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+55/2*b^4/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x*a-3/2*b/(-a^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-53/6*b^2/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^3-19*b^2/(-a^2+1)^2/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^2-35/2*a^4*b^2/(-a^2+1)^3/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-35/2*a^3*b^2/(-a^2+1)^3/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+35/2*a^7*b^4/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+105/2*a^6*b^4/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+105/2*a^5*b^4/(-a^2+1)^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+125/3*a^5*b^4/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+205/2*a^4*b^4/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+80*a^3*b^4/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+7*b^4*a/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-7/6*a*b/(-a^2+1)^2/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-35/6*a^2*b^2/(-a^2+1)^3/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+56*b^4*a^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3*b^2*a/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+187/6*b^4*a^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+115/6*a^2*b^4/(-a^2+1)^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-7/6*a^4*b/(-a^2+1)^2/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-7/2*a^3*b/(-a^2+1)^2/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-7/2*a^2*b/(-a^2+1)^2/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-35/6*a^5*b^2/(-a^2+1)^3/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/2*b^3/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-9/2*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+b^3/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/

```
2)-b^3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2))/x)-1/3/(-a^2+1)/x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*b/(-a
^2+1)/x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+35/2*a^4*b^4/(-a^2+1)^4/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2)*x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)^3/(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x+1)^3}{x^4\,\left(1-(a+b\,x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + 1)^3/(x^4*(1 - (a + b*x)^2)^(3/2)),x)
```

```
[Out] int((a + b*x + 1)^3/(x^4*(1 - (a + b*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+1)^3}{x^4\,(-(a+bx-1)\,(a+bx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)**3/(1-(b*x+a)**2)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x + 1)**3/(x**4*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)), x)
```

3.843 $\int e^{-\tanh^{-1}(a+bx)} x^3\,dx$

Optimal. Leaf size=156

$$-\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}\left(18a^2-2(6a+1)bx+10a+7\right)}{24b^4}-\frac{\left(8a^3+12a^2+12a+3\right)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{8b^4}$$

```
[Out] -1/8*(8*a^3+12*a^2+12*a+3)*arcsin(b*x+a)/b^4-1/4*x^2*(-b*x-a+1)^(3/2)*(b*x+
a+1)^(1/2)/b^2-1/24*(-b*x-a+1)^(3/2)*(7+10*a+18*a^2-2*(1+6*a)*b*x)*(b*x+a+1
)^(1/2)/b^4-1/8*(8*a^3+12*a^2+12*a+3)*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b^4
```

Rubi [A] time = 0.17, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 100, 147, 50, 53, 619, 216}

$$-\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}\left(18a^2-2(6a+1)bx+10a+7\right)}{24b^4}-\frac{\left(8a^3+12a^2+12a+3\right)\sqrt{-a-bx+1}\sqrt{a+bx+1}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^ArcTanh[a + b*x],x]
```

```
[Out] -((3 + 12*a + 12*a^2 + 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4)
- (x^2*(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(4*b^2) - ((1 - a - b*x)^(3/2
)*Sqrt[1 + a + b*x]*(7 + 10*a + 18*a^2 - 2*(1 + 6*a)*b*x))/(24*b^4) - ((3 +
 12*a + 12*a^2 + 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx$$
$$= -\frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{\int \frac{x\sqrt{1-a-bx}\left(-2\left(1-a^2\right)+(1+6a)bx\right)}{\sqrt{1+a+bx}}\,dx}{4b^2}$$
$$= -\frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}\left(7+10a+18a^2-2(1+6a)\right.}{24b^4}$$
$$= -\frac{\left(3+12a+12a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-}{}$$
$$= -\frac{\left(3+12a+12a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-}{}$$
$$= -\frac{\left(3+12a+12a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-}{}$$
$$= -\frac{\left(3+12a+12a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} - \frac{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{(1-}{}$$

Mathematica [A] time = 0.44, size = 160, normalized size = 1.03

$$\frac{\frac{6\left(8a^3+12a^2+12a+3\right)\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{-b}\,\sqrt{-a-bx+1}}{\sqrt{2}\,\sqrt{b}}\right)}{\sqrt{-b}} + \frac{\sqrt{a+bx+1}\left(6a^4+38a^3+5a^2(6bx-1)+a\left(-18b^2x^2+50bx-23\right)-6b^4x^4+14b^3x^3-17b^2x^2+25bx-16\right)}{\sqrt{-a-bx+1}}}{24b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^ArcTanh[a + b*x],x]
```

```
[Out] ((Sqrt[1 + a + b*x]*(-16 + 38*a^3 + 6*a^4 + 25*b*x - 17*b^2*x^2 + 14*b^3*x^
3 - 6*b^4*x^4 + 5*a^2*(-1 + 6*b*x) + a*(-23 + 50*b*x - 18*b^2*x^2)))/Sqrt[1
 - a - b*x] + (6*(3 + 12*a + 12*a^2 + 8*a^3)*Sqrt[b]*ArcSinh[(Sqrt[-b]*Sqrt
[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/Sqrt[-b])/(24*b^4)
```

fricas [A] time = 0.68, size = 143, normalized size = 0.92

$$\frac{3\left(8\,a^3+12\,a^2+12\,a+3\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\left(6\,b^3x^3-2\,(3\,a+4)b^2x^2-6\,a^3+\left(6\,a^2+20\,a+9\right)bx\right.}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^3 + 12*a^2 + 12*a + 3)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (6*b^3*x^3 - 2*(3*a + 4)*b^2*
x^2 - 6*a^3 + (6*a^2 + 20*a + 9)*b*x - 44*a^2 - 39*a - 16)*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1))/b^4
```

giac [A] time = 0.22, size = 148, normalized size = 0.95

$$\frac{1}{24}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(\left(2\,x\left(\frac{3\,x}{b}-\frac{3\,ab^5+4\,b^5}{b^7}\right)+\frac{6\,a^2b^4+20\,ab^4+9\,b^4}{b^7}\right)x-\frac{6\,a^3b^3+44\,a^2b^3+39\,ab^3+}{b^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((2*x*(3*x/b - (3*a*b^5 + 4*b^5)/b^
7) + (6*a^2*b^4 + 20*a*b^4 + 9*b^4)/b^7)*x - (6*a^3*b^3 + 44*a^2*b^3 + 39*a
*b^3 + 16*b^3)/b^7) + 1/8*(8*a^3 + 12*a^2 + 12*a + 3)*arcsin(-b*x - a)*sgn(
b)/(b^3*abs(b))
```

maple [B] time = 0.05, size = 809, normalized size = 5.19

$$\frac{3a^2x\sqrt{-b^2x^2-2abx-a^2+1}}{2b^3}+\frac{3a^2\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2b^3\sqrt{b^2}}+\frac{3a\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2b^3\sqrt{b^2}}-\frac{\arctan\left(\frac{\sqrt{b^2}\,(}{\sqrt{-\left(x+\frac{1+a}{b}\right.}}\right.}{b^3\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] 3/2*a^2/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a^2/b^3/(b^2)^(1/2)*arctan
((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/2*a/b^3/(b^2)^(1/2)*
arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/b^3/(b^2)^(1/2
)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(
1/2))*a^3-3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b
)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a^2-3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x
+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a+3/2*a/b^3*x*(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2
)+1/3/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+3/2*a^2/b^4*(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)-1/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^3-3/b^4*(-(x+(1
+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^2-3/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1
+a)/b))^(1/2)*a-1/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(
```

```
1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+5/8*a/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)-1/4/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+3/4/b^4*a*(-b^2*x^2-2*a*b*x-a^
2+1)^(3/2)+3/2*a^3/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+5/8/b^3*x*(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2)+5/8/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2))
```

maxima [B] time = 0.42, size = 338, normalized size = 2.17

$$\frac{3\sqrt{-b^2x^2-2\,abx-a^2+1}\,a^2x}{2\,b^3}-\frac{a^3\arcsin\left(bx+a\right)}{b^4}+\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,a^3}{2\,b^4}-\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}x}{4\,b^3}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2*x/b^3 - a^3*arcsin(b*x + a)/b^4
+ 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/b^4 - 1/4*(-b^2*x^2 - 2*a*b*x
- a^2 + 1)^(3/2)*x/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x/b^3 - 3
/2*a^2*arcsin(b*x + a)/b^4 + 3/4*(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a/b^4
 - 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2/b^4 + 5/8*sqrt(-b^2*x^2 - 2*a
*b*x - a^2 + 1)*x/b^3 - 3/2*a*arcsin(b*x + a)/b^4 + 1/3*(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)^(3/2)/b^4 - 19/8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/b^4 - 3/8
*arcsin(b*x + a)/b^4 - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{1-{\left(a+b\,x\right)}^2}}{a+b\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - (a + b*x)^2)^(1/2))/(a + b*x + 1),x)
```

```
[Out] int((x^3*(1 - (a + b*x)^2)^(1/2))/(a + b*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt{-\left(a+bx-1\right)\left(a+bx+1\right)}}{a+bx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a+1)*(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a + b*x - 1)*(a + b*x + 1))/(a + b*x + 1), x)
```

3.844 $\int e^{-\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=130

$$\frac{\left(2a^2+2a+1\right)\sqrt{a+bx+1}\,\sqrt{-a-bx+1}}{2b^3}+\frac{\left(2a^2+2a+1\right)\sin^{-1}(a+bx)}{2b^3}+\frac{(4a+1)\sqrt{a+bx+1}\,(-a-bx+1)^{3/2}}{6b^3}$$

```
[Out] 1/2*(2*a^2+2*a+1)*arcsin(b*x+a)/b^3+1/6*(1+4*a)*(-b*x-a+1)^(3/2)*(b*x+a+1)^
(1/2)/b^3-1/3*x*(-b*x-a+1)^(3/2)*(b*x+a+1)^(1/2)/b^2+1/2*(2*a^2+2*a+1)*(-b*
x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b^3
```

Rubi [A] time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 90, 80, 50, 53, 619, 216}

$$\frac{\left(2a^2+2a+1\right)\sqrt{a+bx+1}\,\sqrt{-a-bx+1}}{2b^3}+\frac{\left(2a^2+2a+1\right)\sin^{-1}(a+bx)}{2b^3}-\frac{x\sqrt{a+bx+1}\,(-a-bx+1)^{3/2}}{3b^2}+\frac{(4a+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^ArcTanh[a + b*x],x]
```

```
[Out] ((1 + 2*a + 2*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) + ((1 + 4*a
)*(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(6*b^3) - (x*(1 - a - b*x)^(3/2)*S
qrt[1 + a + b*x])/(3*b^2) + ((1 + 2*a + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-\tanh^{-1}(a+bx)}x^2\,dx = \int \frac{x^2\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx$$

$$= -\frac{x(1-a-bx)^{3/2}\sqrt{1+a+bx}}{3b^2} - \frac{\int \frac{\sqrt{1-a-bx}\left(-1+a^2+(1+4a)bx\right)}{\sqrt{1+a+bx}}\,dx}{3b^2}$$

$$= \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-a-bx)^{3/2}\sqrt{1+a+bx}}{3b^2} + \frac{\left(1+2a+2a^2\right)\int \frac{\sqrt{}}{\sqrt{}}}{2b^2}$$

$$= \frac{\left(1+2a+2a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} + \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-}{}$$

$$= \frac{\left(1+2a+2a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} + \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-}{}$$

$$= \frac{\left(1+2a+2a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} + \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-}{}$$

$$= \frac{\left(1+2a+2a^2\right)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^3} + \frac{(1+4a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3} - \frac{x(1-}{}$$

Mathematica [A] time = 0.16, size = 126, normalized size = 0.97

$$\frac{\left(2a^2+2a+1\right)\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{-b}\,\sqrt{-a-bx+1}}{\sqrt{2}\,\sqrt{b}}\right)}{b^{7/2}} - \frac{\sqrt{a+bx+1}\,\left(2a^3+7a^2+a(8bx-5)+2b^3x^3-5b^2x^2+7bx-4\right)}{6b^3\sqrt{-a-bx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^ArcTanh[a + b*x],x]
```

```
[Out] -1/6*(Sqrt[1 + a + b*x]*(-4 + 7*a^2 + 2*a^3 + 7*b*x - 5*b^2*x^2 + 2*b^3*x^3
 + a*(-5 + 8*b*x)))/(b^3*Sqrt[1 - a - b*x]) + ((1 + 2*a + 2*a^2)*Sqrt[-b]*A
rcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/b^(7/2)
```

fricas [A] time = 0.48, size = 117, normalized size = 0.90

$$-\frac{3\left(2\,a^2+2\,a+1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right) - \left(2\,b^2x^2-(2\,a+3)bx+2\,a^2+9\,a+4\right)\sqrt{-b^2x^2-2\,abx-a^2}}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(2*a^2 + 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x +
a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (2*b^2*x^2 - (2*a + 3)*b*x + 2*a^2 + 9*
a + 4)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b^3
```

giac [A] time = 0.84, size = 106, normalized size = 0.82

$$\frac{1}{6}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(x\left(\frac{2\,x}{b}-\frac{2\,ab^3+3\,b^3}{b^5}\right)+\frac{2\,a^2b^2+9\,ab^2+4\,b^2}{b^5}\right)-\frac{\left(2\,a^2+2\,a+1\right)\arcsin\left(-bx-a\right)\mathrm{sgn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x*(2*x/b - (2*a*b^3 + 3*b^3)/b^5) +
 (2*a^2*b^2 + 9*a*b^2 + 4*b^2)/b^5) - 1/2*(2*a^2 + 2*a + 1)*arcsin(-b*x - a
)*sgn(b)/(b^2*abs(b))
```

maple [B] time = 0.04, size = 535, normalized size = 4.12

$$-\frac{\left(-b^2x^2-2abx-a^2+1\right)^{\frac{3}{2}}}{3b^3}-\frac{ax\sqrt{-b^2x^2-2abx-a^2+1}}{b^2}-\frac{a^2\sqrt{-b^2x^2-2abx-a^2+1}}{b^3}-\frac{a\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right.}{b^2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/3/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-a/b^2*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)-a^2/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/b^2/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)-1/2*a/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2/b^2/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/b^3*(-(x+(1+a)/b)
^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^2+2/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b)
)^(1/2)*a+1/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)+1/b^2/(b^2)^(1/2
)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(
1/2))*a^2+2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b
)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a+1/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(
1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

maxima [A] time = 0.43, size = 174, normalized size = 1.34

$$-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,ax}{b^2}+\frac{a^2\arcsin\left(bx+a\right)}{b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,x}{2\,b^2}+\frac{a\arcsin\left(bx+a\right)}{b^3}-\frac{\left(-b^2x^2-2\,abx\right.}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x/b^2 + a^2*arcsin(b*x + a)/b^3 - 1/2
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 + a*arcsin(b*x + a)/b^3 - 1/3*(-b
^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*a/b^3 + 1/2*arcsin(b*x + a)/b^3 + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b^
3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{1-{\left(a+b\,x\right)}^2}}{a+b\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - (a + b*x)^2)^(1/2))/(a + b*x + 1),x)
```

```
[Out] int((x^2*(1 - (a + b*x)^2)^(1/2))/(a + b*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{-\left(a+bx-1\right)\left(a+bx+1\right)}}{a+bx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a+1)*(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a + b*x - 1)*(a + b*x + 1))/(a + b*x + 1), x)
```

3.845 $\int e^{-\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{a+bx+1}\,(-a-bx+1)^{3/2}}{2b^2}-\frac{(2a+1)\sqrt{a+bx+1}\,\sqrt{-a-bx+1}}{2b^2}-\frac{(2a+1)\sin^{-1}(a+bx)}{2b^2}$$

```
[Out] -1/2*(1+2*a)*arcsin(b*x+a)/b^2-1/2*(-b*x-a+1)^(3/2)*(b*x+a+1)^(1/2)/b^2-1/2
*(1+2*a)*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{a+bx+1}\,(-a-bx+1)^{3/2}}{2b^2}-\frac{(2a+1)\sqrt{a+bx+1}\,\sqrt{-a-bx+1}}{2b^2}-\frac{(2a+1)\sin^{-1}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^ArcTanh[a + b*x],x]
```

```
[Out] -((1 + 2*a)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) - ((1 - a - b*x)^(
3/2)*Sqrt[1 + a + b*x])/(2*b^2) - ((1 + 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-\tanh^{-1}(a+bx)}x\,dx = \int \frac{x\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx$$

$$= -\frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2} - \frac{(1+2a)\int \frac{\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx}{2b}$$

$$= -\frac{(1+2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2} - \frac{(1+2a)\int \frac{1}{\sqrt{1-a-bx}\,\sqrt{1}}}{2b}$$

$$= -\frac{(1+2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2} - \frac{(1+2a)\int \frac{1}{\sqrt{(1-a)(1+a)}}}{2b}$$

$$= -\frac{(1+2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2} + \frac{(1+2a)\,\text{Subst}\Big(\int \frac{}{\sqrt{1}}}{}$$

$$= -\frac{(1+2a)\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2b^2} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2} - \frac{(1+2a)\sin^{-1}(a+bx}{2b^2}$$

Mathematica [A] time = 0.11, size = 99, normalized size = 1.18

$$\frac{\sqrt{a+bx+1}\left(a^2+a-b^2x^2+3bx-2\right)}{2b^2\sqrt{-a-bx+1}}+\frac{(2a+1)\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)}{(-b)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^ArcTanh[a + b*x],x]
```

```
[Out] (Sqrt[1 + a + b*x]*(-2 + a + a^2 + 3*b*x - b^2*x^2))/(2*b^2*Sqrt[1 - a - b*
x]) + ((1 + 2*a)*Sqrt[b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt
[b])])/(-b)^(5/2)
```

fricas [A] time = 0.65, size = 91, normalized size = 1.08

$$\frac{(2a+1)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)+\sqrt{-b^2x^2-2abx-a^2+1}(bx-a-2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2
 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - a - 2))/
b^2
```

giac [A] time = 0.18, size = 68, normalized size = 0.81

$$\frac{1}{2}\sqrt{-b^2x^2-2abx-a^2+1}\left(\frac{x}{b}-\frac{ab+2b}{b^3}\right)+\frac{(2a+1)\arcsin(-bx-a)\operatorname{sgn}(b)}{2b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x/b - (a*b + 2*b)/b^3) + 1/2*(2*a +
 1)*arcsin(-b*x - a)*sgn(b)/(b*abs(b))
```

maple [B] time = 0.04, size = 302, normalized size = 3.60

$$\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b}+\frac{a\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2}+\frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2b\sqrt{b^2}}-\frac{\sqrt{-\left(x+\frac{1+a}{b}\right)^2b^2+2b\left(x+\frac{1}{}\right.}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*x/b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2*a/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2)+1/2/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2))-1/b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)-1/b/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
-1/b^2*a*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)-1/b*a/(b^2)^(1/2)*arcta
n((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

maxima [A] time = 0.41, size = 107, normalized size = 1.27

$$\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,x}{2\,b}-\frac{a\arcsin\left(bx+a\right)}{b^2}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,a}{2\,b^2}-\frac{\arcsin\left(bx+a\right)}{2\,b^2}-\frac{\sqrt{-b^2x^2-2\,abx-a}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b - a*arcsin(b*x + a)/b^2 - 1/2*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/b^2 - 1/2*arcsin(b*x + a)/b^2 - sqrt(-b^
2*x^2 - 2*a*b*x - a^2 + 1)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{1-{\left(a+b\,x\right)}^2}}{a+b\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - (a + b*x)^2)^(1/2))/(a + b*x + 1),x)
```

```
[Out] int((x*(1 - (a + b*x)^2)^(1/2))/(a + b*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\left(a+bx-1\right)\left(a+bx+1\right)}}{a+bx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)*(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a + b*x - 1)*(a + b*x + 1))/(a + b*x + 1), x)
```

3.846 $\int e^{-\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{b}+\frac{\sin^{-1}(a+bx)}{b}$$

```
[Out] arcsin(b*x+a)/b+(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6161, 50, 53, 619, 216}

$$\frac{\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{b}+\frac{\sin^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/b + ArcSin[a + b*x]/b
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(a+bx)}\,dx &= \int \frac{\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx \\
&= \frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \int \frac{1}{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx \\
&= \frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx \\
&= \frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b^2} \\
&= \frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \frac{\sin^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.68

$$\frac{\sqrt{1-(a+bx)^2}+\sin^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[1 - (a + b*x)^2] + ArcSin[a + b*x])/b
```

fricas [B] time = 0.59, size = 77, normalized size = 2.03

$$\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}-\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")

[Out] (sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1) - arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b

giac [A] time = 0.18, size = 44, normalized size = 1.16

$$-\frac{\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{|b|}+\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="giac")

[Out] -arcsin(-b*x - a)*sgn(b)/abs(b) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b

maple [B] time = 0.03, size = 95, normalized size = 2.50

$$\frac{\sqrt{-\left(x+\frac{1+a}{b}\right)^2 b^2+2b\left(x+\frac{1+a}{b}\right)}}{b}+\frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{1+a}{b}-\frac{1}{b}\right)}{\sqrt{-\left(x+\frac{1+a}{b}\right)^2 b^2+2b\left(x+\frac{1+a}{b}\right)}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x)

[Out] 1/b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)+1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))

maxima [A] time = 0.44, size = 37, normalized size = 0.97

$$\frac{\arcsin\left(bx+a\right)}{b}+\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")

[Out] arcsin(b*x + a)/b + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-(a+b\,x)^2}}{a+b\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(1/2)/(a + b*x + 1),x)
```

```
[Out] int((1 - (a + b*x)^2)^(1/2)/(a + b*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(a+bx-1)(a+bx+1)}}{a+bx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(a + b*x + 1), x)
```

3.847 $\int \frac{e^{-\tanh^{-1}(a+bx)}}{x} \, dx$

Optimal. Leaf size=68

$$-\frac{2(1-a)\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{\sqrt{1-a^2}}-\sin^{-1}(a+bx)$$

```
[Out] -arcsin(b*x+a)-2*(1-a)*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*
x-a+1)^(1/2))/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 105, 53, 619, 216, 93, 208}

$$-\frac{2(1-a)\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{\sqrt{1-a^2}}-\sin^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a + b*x]*x),x]
```

```
[Out] -ArcSin[a + b*x] - (2*(1 - a)*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt
[1 + a]*Sqrt[1 - a - b*x])])/Sqrt[1 - a^2]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(a+bx)}}{x} dx = \int \frac{\sqrt{1-a-bx}}{x\sqrt{1+a+bx}} dx$$
$$= -\left((-1+a)\int \frac{1}{x\sqrt{1-a-bx}\sqrt{1+a+bx}} dx\right) - b\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}} dx$$
$$= (2(1-a))\,\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2} dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right) - b\int \frac{1}{\sqrt{(1-a)(1+a)-2abx}$$
$$= -\frac{2(1-a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{\sqrt{1-a^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}} dx, x, -2ab-2b^2x\right)}{2b}$$
$$= -\sin^{-1}(a+bx) - \frac{2(1-a)\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{1-a-bx}}\right)}{\sqrt{1-a^2}}$$

Mathematica [A] time = 0.04, size = 106, normalized size = 1.56

$$\frac{2\sqrt{-b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{-b}}\right)}{\sqrt{b}} - \frac{2\sqrt{a-1}\tanh^{-1}\left(\frac{\sqrt{-a-1}\sqrt{-a-bx+1}}{\sqrt{a-1}\sqrt{a+bx+1}}\right)}{\sqrt{-a-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a + b*x]*x),x]
```

```
[Out] (2*Sqrt[-b]*ArcSinh[(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/Sqrt[b
] - (2*Sqrt[-1 + a]*ArcTanh[(Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*
Sqrt[1 + a + b*x])])/Sqrt[-1 - a]
```

fricas [B] time = 0.61, size = 303, normalized size = 4.46

$$\left[\frac{1}{2}\sqrt{-\frac{a-1}{a+1}}\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-4a^2+2\sqrt{-b^2x^2-2abx-a^2+1}\left(a^3+\left(a^2+a\right)bx+a^2-\right.}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-(a - 1)/(a + 1))*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*
b*x - 4*a^2 + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^3 + (a^2 + a)*b*x + a
```

```
^2 - a - 1)*sqrt(-(a - 1)/(a + 1)) + 2)/x^2) + arctan(sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)), -sqrt((a - 1)/(a +
1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt((a - 1
)/(a + 1))/((a - 1)*b^2*x^2 + a^3 + 2*(a^2 - a)*b*x - a^2 - a + 1)) + arcta
n(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1
))]
```

giac [A] time = 0.41, size = 89, normalized size = 1.31

$$\frac{b \arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{|b|}-\frac{2\,(ab-b)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] b*arcsin(-b*x - a)*sgn(b)/abs(b) - 2*(a*b - b)*arctan(((sqrt(-b^2*x^2 - 2*a
*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 -
 1)*abs(b))
```

maple [B] time = 0.04, size = 249, normalized size = 3.66

$$\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{1+a}-\frac{ab\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{(1+a)\sqrt{b^2}}-\frac{\sqrt{-a^2+1}\,\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{x}\right)}{1+a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x,x)
```

```
[Out] 1/(1+a)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(1+a)*a*b/(b^2)^(1/2)*arctan((b^2)
^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/(1+a)*(-a^2+1)^(1/2)*ln((-
2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/(1+a)
*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)-1/(1+a)*b/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-(a+b\,x)^2}}{x\,(a+b\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(1/2)/(x*(a + b*x + 1)),x)
```

```
[Out] int((1 - (a + b*x)^2)^(1/2)/(x*(a + b*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(a+bx-1)(a+bx+1)}}{x(a+bx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(x*(a + b*x + 1)), x)
```

3.848 $\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=94

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{(a+1)x}$$

```
[Out] 2*b*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(1/2))/(1+a)
/(-a^2+1)^(1/2)-(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1+a)/x
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 94, 93, 208}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)\sqrt{1-a^2}}-\frac{\sqrt{-a-bx+1}\sqrt{a+bx+1}}{(a+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a + b*x]*x^2),x]
```

```
[Out] -((Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/((1 + a)*x)) + (2*b*ArcTanh[(Sqrt[1
 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 + a)*Sqrt[1
- a^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{\sqrt{1-a-bx}}{x^2\sqrt{1+a+bx}}\,dx \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{(1+a)x} - \frac{b\int \frac{1}{x\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{1+a} \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{(1+a)x} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1+a} \\
&= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{(1+a)x} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{1-a-bx}}\right)}{(1+a)\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 0.95

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{-a-1}\,\sqrt{-a-bx+1}}{\sqrt{a-1}\,\sqrt{a+bx+1}}\right)}{(-a-1)^{3/2}\sqrt{a-1}} - \frac{\sqrt{-((a+bx-1)(a+bx+1))}}{(a+1)x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a + b*x]*x^2),x]
```

```
[Out] -(Sqrt[-((-1 + a + b*x)*(1 + a + b*x))]/((1 + a)*x)) + (2*b*ArcTanh[(Sqrt[-
1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqrt[1 + a + b*x])])/((-1 - a)^(3/2
)*Sqrt[-1 + a])
```

fricas [A] time = 0.58, size = 277, normalized size = 2.95

$$\left[-\frac{\sqrt{-a^2+1}\,bx\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\,\left(a^3-a\right)bx-2\,\sqrt{-b^2x^2-2\,abx-a^2+1}\,\left(abx+a^2-1\right)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)+2\,\sqrt{-b^2x^2-2\,abx-a^2}}{2\left(a^3+a^2-a-1\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + 1)*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*
x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) -
 4*a^2 + 2)/x^2) + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/((a^3 +
a^2 - a - 1)*x), -(sqrt(a^2 - 1)*b*x*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*
b*x - 2*a^2 + 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/((a^3 + a
^2 - a - 1)*x)]
```

giac [B] time = 0.27, size = 223, normalized size = 2.37

$$-\frac{2\,b^2\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}\,\left(a|b|+|b|\right)}-\frac{2\left(ab^2-\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)b^2}{b^2x+ab}\right)}{\left(a^2|b|+a|b|\right)\left(\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)^2a}{\left(b^2x+ab\right)^2}+a-\frac{2\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)}{b^2x+ab}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a
*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1)*(a*abs(b) + abs(b))) - 2*(a*b^2 - (s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*b^2/(b^2*x + a*b))/((a^2*abs(
b) + a*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x
+ a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a
*b)))
```

maple [B] time = 0.05, size = 565, normalized size = 6.01

$$-\frac{b\sqrt{-b^2x^2-2abx-a^2+1}}{(1+a)^2}+\frac{b^2a\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{(1+a)^2\sqrt{b^2}}+\frac{b\sqrt{-a^2+1}\;\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\;\sqrt{-b^2x^2-2abx-a^2+}}{x}\right.}{(1+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^2,x)
```

```
[Out] -1/(1+a)^2*b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(1+a)^2*b^2*a/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/(1+a)^2*b*(-a^2+1
)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
))/x)-1/(1+a)/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-2/(1+a)*a*b/(-a^2+1
)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(1+a)*a^2*b^2/(-a^2+1)/(b^2)^(1/2)*arcta
n((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/(1+a)*a*b/(-a^2+1)^
(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
/x)-1/(1+a)*b^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-1/(1+a)*b^2/(-a^2
+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+
1/(1+a)^2*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)+1/(1+a)^2*b^2/(b^2)^
(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b
))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(bx+a)^2+1}}{(bx+a+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(b*x + a)^2 + 1)/((b*x + a + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-(a+b\,x)^2}}{x^2\,(a+b\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(1/2)/(x^2*(a + b*x + 1)),x)
```

```
[Out] int((1 - (a + b*x)^2)^(1/2)/(x^2*(a + b*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(a+bx-1)(a+bx+1)}}{x^2(a+bx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2)/x**2,x)
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(x**2*(a + b*x + 1)), x)
```

3.849 $\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=162

$$-\frac{(1-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)(a+1)^2\sqrt{1-a^2}}-\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}}{2\left(1-a^2\right)x^2}+\frac{(1-2a)b\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2(1-a)(a+1)^2x}$$

```
[Out] -(1-2*a)*b^2*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(1/
2))/(1-a)/(1+a)^2/(-a^2+1)^(1/2)-1/2*(-b*x-a+1)^(3/2)*(b*x+a+1)^(1/2)/(-a^2
+1)/x^2+1/2*(1-2*a)*b*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1-a)/(1+a)^2/x
```

Rubi [A] time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6163, 96, 94, 93, 208}

$$-\frac{(1-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(1-a)(a+1)^2\sqrt{1-a^2}}-\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}}{2\left(1-a^2\right)x^2}+\frac{(1-2a)b\sqrt{-a-bx+1}\sqrt{a+bx+1}}{2(1-a)(a+1)^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a + b*x]*x^3),x]
```

```
[Out] ((1 - 2*a)*b*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*(1 - a)*(1 + a)^2*x) -
 ((1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x^2) - ((1 - 2*a)*b^2
*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/
((1 - a)*(1 + a)^2*Sqrt[1 - a^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{1-a-bx}}{x^3\sqrt{1+a+bx}}\,dx$$

$$= -\frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2\left(1-a^2\right)x^2} - \frac{((1-2a)b)\int \frac{\sqrt{1-a-bx}}{x^2\sqrt{1+a+bx}}\,dx}{2\left(1-a^2\right)}$$

$$= \frac{(1-2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2(1-a)(1+a)^2x} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2\left(1-a^2\right)x^2} + \frac{\left((1-2a)b^2\right)\int \frac{}{x\sqrt{1-a-b}}}{2(1-a)(1+}$$

$$= \frac{(1-2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2(1-a)(1+a)^2x} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2\left(1-a^2\right)x^2} + \frac{\left((1-2a)b^2\right)\text{Subst}\left(\int\right.}{(1-}$$

$$= \frac{(1-2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{2(1-a)(1+a)^2x} - \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2\left(1-a^2\right)x^2} - \frac{(1-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-}}{\sqrt{1-}}\right.}{(1-a)(1+a)^2\sqrt{1}}$$

Mathematica [A] time = 0.13, size = 122, normalized size = 0.75

$$\frac{(2a-1)b^2\tanh^{-1}\left(\frac{\sqrt{-a-1}\sqrt{-a-bx+1}}{\sqrt{a-1}\sqrt{a+bx+1}}\right)}{(-a-1)^{5/2}(a-1)^{3/2}}-\frac{\left(a^2-abx+2bx-1\right)\sqrt{-a^2-2abx-b^2x^2+1}}{2(a-1)(a+1)^2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a + b*x]*x^3),x]
```

```
[Out] -1/2*((-1 + a^2 + 2*b*x - a*b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/((-1 +
a)*(1 + a)^2*x^2) + ((-1 + 2*a)*b^2*ArcTanh[(Sqrt[-1 - a]*Sqrt[1 - a - b*x]
)/(Sqrt[-1 + a]*Sqrt[1 + a + b*x])])/((-1 - a)^(5/2)*(-1 + a)^(3/2))
```

fricas [A] time = 0.66, size = 356, normalized size = 2.20

$$\left[-\frac{\sqrt{-a^2+1}\,(2\,a-1)b^2x^2\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\,\left(a^3-a\right)bx+2\,\sqrt{-b^2x^2-2\,abx-a^2+1}\,\left(abx+a^2-1\right)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)+2\left(a^4-\left(a^3-2\right.\right.}{4\left(a^5+a^4-2\,a^3-2\,a^2+a+1\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 + 1)*(2*a - 1)*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 +
4*(a^3 - a)*b*x + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 2*(a^4 - (a^3 - 2*a^2 - a + 2)*b*x - 2*a^2
 + 1)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 + a^4 - 2*a^3 - 2*a^2 + a +
 1)*x^2), 1/2*(sqrt(a^2 - 1)*(2*a - 1)*b^2*x^2*arctan(sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*
(a^3 - a)*b*x - 2*a^2 + 1)) - (a^4 - (a^3 - 2*a^2 - a + 2)*b*x - 2*a^2 + 1)
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 + a^4 - 2*a^3 - 2*a^2 + a + 1)*x
^2)]
```

giac [B] time = 0.29, size = 750, normalized size = 4.63

$$\frac{\left(2\,ab^3-b^3\right)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^3|b|+a^2|b|-a|b|-|b|\right)\sqrt{a^2-1}}+\frac{\frac{2\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)^2a^4b^3}{\left(b^2x+ab\right)^2}+2\,a^4b^3-\frac{5\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)a^3b^3}{b^2x+ab}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (2*a*b^3 - b^3)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(
b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^3*abs(b) + a^2*abs(b) - a*abs(b) - abs
(b))*sqrt(a^2 - 1)) + (2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*
a^4*b^3/(b^2*x + a*b)^2 + 2*a^4*b^3 - 5*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*abs(b) + b)*a^3*b^3/(b^2*x + a*b) - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
abs(b) + b)^2*a^3*b^3/(b^2*x + a*b)^2 - 3*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*abs(b) + b)^3*a^3*b^3/(b^2*x + a*b)^3 - 2*a^3*b^3 + 6*(sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)*abs(b) + b)*a^2*b^3/(b^2*x + a*b) + 3*(sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)*abs(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 + 2*(sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^2*b^3/(b^2*x + a*b)^3 - a^2*b^3 + 2*(sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a*b^3/(b^2*x + a*b) - 4*(sqrt(
-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a*b^3/(b^2*x + a*b)^2 + 2*(sqrt
(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^5*ab
s(b) + a^4*abs(b) - a^3*abs(b) - a^2*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)/(b^2*x + a*b))^2)
```

maple [B] time = 0.05, size = 1116, normalized size = 6.89

$$\frac{b^{2}\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}{\left(1+a\right)^{3}}-\frac{b^{3}a\arctan\left(\frac{\sqrt{b^{2}}\,\left(x+\frac{a}{b}\right)}{\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}\right)}{\left(1+a\right)^{3}\sqrt{b^{2}}}-\frac{b^{2}\sqrt{-a^{2}+1}\,\ln\left(\frac{-2a^{2}+2-2abx+2\sqrt{-a^{2}+1}\,\sqrt{-b^{2}x^{2}-2abx-a^{2}+}}{x}\right.}{\left(1+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^3,x)
```

```
[Out] 1/(1+a)^3*b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(1+a)^3*b^3*a/(b^2)^(1/2)*ar
ctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/(1+a)^3*b^2*(-a^
2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2))/x)+1/(1+a)^2*b/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+2/(1+a)^2*b^
2*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(1+a)^2*b^3*a^2/(-a^2+1)/(b^2
)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/(1+a)^
2*b^2*a/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2))/x)+1/(1+a)^2*b^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*
x+1/(1+a)^2*b^3/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2))-1/(1+a)^3*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/
2)-1/(1+a)^3*b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/
b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))-1/2/(1+a)/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a
^2+1)^(3/2)-1/2/(1+a)*a*b/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-1/(1+
a)*a^2*b^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2/(1+a)*a^3*b^3/(-a^
```

```
2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
))+1/2/(1+a)*a^2*b^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(
-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(1+a)*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)*x-1/2/(1+a)*a*b^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2
)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/2/(1+a)*b^2/(-a^2+1)*(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)+1/2/(1+a)*b^3/(-a^2+1)*a/(b^2)^(1/2)*arctan((b^2)^(1/2
)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)*b^2/(-a^2+1)^(1/2)*ln((
-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(bx+a)^2+1}}{(bx+a+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(-(b*x + a)^2 + 1)/((b*x + a + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-(a+b\,x)^2}}{x^3\,(a+b\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(1/2)/(x^3*(a + b*x + 1)),x)
[Out] int((1 - (a + b*x)^2)^(1/2)/(x^3*(a + b*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(a+bx-1)(a+bx+1)}}{x^3(a+bx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2)/x**3,x)
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(x**3*(a + b*x + 1)), x)
```

3.850 $\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^4} \, dx$

Optimal. Leaf size=210

$$\frac{(2a^2-2a+1)\, b^3 \tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(a+1)\left(1-a^2\right)^{5/2}} - \frac{(1-2a)(4-a)b^2\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{6(1-a)^2(a+1)^3x} - \frac{\sqrt{-a-bx+1}\,\sqrt{a+bx}}{3(a+1)x^3}$$

```
[Out] (2*a^2-2*a+1)*b^3*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1
)^(1/2))/(1+a)/(-a^2+1)^(5/2)-1/3*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1+a)/x^
3+1/6*(3-2*a)*b*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1-a)/(1+a)^2/x^2-1/6*(1-2
*a)*(4-a)*b^2*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/(1-a)^2/(1+a)^3/x
```

Rubi [A] time = 0.17, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6163, 99, 151, 12, 93, 208}

$$\frac{(2a^2-2a+1)\, b^3 \tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(a+1)\left(1-a^2\right)^{5/2}} - \frac{(1-2a)(4-a)b^2\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{6(1-a)^2(a+1)^3x} + \frac{(3-2a)b\sqrt{-a-bx+1}}{6(1-a)(a+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a + b*x]*x^4),x]
```

```
[Out] -(Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(3*(1 + a)*x^3) + ((3 - 2*a)*b*Sqrt[
1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)*(1 + a)^2*x^2) - ((1 - 2*a)*(4 -
 a)*b^2*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(6*(1 - a)^2*(1 + a)^3*x) + ((
1 - 2*a + 2*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*S
qrt[1 - a - b*x])])/((1 + a)*(1 - a^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{1-a-bx}}{x^4\sqrt{1+a+bx}}\,dx$$

$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{\int \frac{-(3-2a)b+2b^2x}{x^3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{3(1+a)}$$

$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{\int \frac{-(1-2a)(4-a)b^2+(3-2a)b^3}{x^2\sqrt{1-a-bx}\,\sqrt{1+a+bx}}}{6(1-a)(1+a)^2}$$

$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{(1-2a)(4-a)b^2\sqrt{1-}}{6(1-a)^2(1}$$

$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{(1-2a)(4-a)b^2\sqrt{1-}}{6(1-a)^2(1}$$

$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{(1-2a)(4-a)b^2\sqrt{1-}}{6(1-a)^2(1}$$

$$= -\frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{3(1+a)x^3} + \frac{(3-2a)b\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{6(1-a)(1+a)^2x^2} - \frac{(1-2a)(4-a)b^2\sqrt{1-}}{6(1-a)^2(1}$$

Mathematica [A] time = 0.24, size = 194, normalized size = 0.92

$$\frac{\frac{3\left(2a^2-2a+1\right)b^2x^2\left(\sqrt{-a-1}\,\sqrt{a-1}\,\sqrt{-((a+bx-1)(a+bx+1))}+2bx\tanh^{-1}\left(\frac{\sqrt{-a-1}\,\sqrt{-a-bx+1}}{\sqrt{a-1}\,\sqrt{a+bx+1}}\right)\right)}{(-a-1)^{3/2}\sqrt{a-1}} + (1-4a)bx(-a-bx+1)^{3/2}\sqrt{a+bx+1} -}{6\left(a^2-1\right)^2x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a + b*x]*x^4),x]
```

```
[Out] (-2*(1 - a)*(1 + a)*(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x] + (1 - 4*a)*b*x*(
1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x] + (3*(1 - 2*a + 2*a^2)*b^2*x^2*(Sqrt[-
1 - a]*Sqrt[-1 + a]*Sqrt[-((-1 + a + b*x)*(1 + a + b*x))] + 2*b*x*ArcTanh[(
Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqrt[1 + a + b*x])]))/((-1 -
a)^(3/2)*Sqrt[-1 + a]))/(6*(-1 + a^2)^2*x^3)
```

fricas [A] time = 0.72, size = 483, normalized size = 2.30

$$\left[-\frac{3\left(2a^2-2a+1\right)\sqrt{-a^2+1}\,b^3x^3\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-2\sqrt{-b^2x^2-2abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+2\left(2a\right.}{12\left(a^7+a^6-3a^5-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2 - 2*a + 1)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^2
 + 2*a^4 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x +
a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 2*(2*a^6 + (2*a^4 - 9*a^3 + 2*a
^2 + 9*a - 4)*b^2*x^2 - 6*a^4 - (2*a^5 - 3*a^4 - 4*a^3 + 6*a^2 + 2*a - 3)*b
*x + 6*a^2 - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^7 + a^6 - 3*a^5 - 3
*a^4 + 3*a^3 + 3*a^2 - a - 1)*x^3), -1/6*(3*(2*a^2 - 2*a + 1)*sqrt(a^2 - 1)
*b^3*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a
^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (2*a^6 +
 (2*a^4 - 9*a^3 + 2*a^2 + 9*a - 4)*b^2*x^2 - 6*a^4 - (2*a^5 - 3*a^4 - 4*a^3
 + 6*a^2 + 2*a - 3)*b*x + 6*a^2 - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((
a^7 + a^6 - 3*a^5 - 3*a^4 + 3*a^3 + 3*a^2 - a - 1)*x^3)]
```

giac [B] time = 0.25, size = 1659, normalized size = 7.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -(2*a^2*b^4 - 2*a*b^4 + b^4)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*ab
s(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^5*abs(b) + a^4*abs(b) - 2
*a^3*abs(b) - 2*a^2*abs(b) + a*abs(b) + abs(b))*sqrt(a^2 - 1)) - 1/3*(12*(s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^7*b^4/(b^2*x + a*b)^2 + 6
*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^7*b^4/(b^2*x + a*b)^4
+ 6*a^7*b^4 - 24*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^6*b^4/(b
^2*x + a*b) - 24*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^6*b^4/
(b^2*x + a*b)^2 - 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^6*
b^4/(b^2*x + a*b)^3 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*
a^6*b^4/(b^2*x + a*b)^4 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b
)^5*a^6*b^4/(b^2*x + a*b)^5 - 12*a^6*b^4 + 57*(sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1)*abs(b) + b)*a^5*b^4/(b^2*x + a*b) + 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1)*abs(b) + b)^2*a^5*b^4/(b^2*x + a*b)^2 + 72*(sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)*abs(b) + b)^3*a^5*b^4/(b^2*x + a*b)^3 + 30*(sqrt(-b^2*x^2 - 2*a*
b*x - a^2 + 1)*abs(b) + b)^4*a^5*b^4/(b^2*x + a*b)^4 + 15*(sqrt(-b^2*x^2 -
```

```
2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^5*b^4/(b^2*x + a*b)^5 - 2*a^5*b^4 - 84*(
sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^4*b^4/(b^2*x + a*b)^2 -
12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^4*b^4/(b^2*x + a*b)^
3 - 51*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^4*b^4/(b^2*x + a
*b)^4 + 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^4*b^4/(b^2*x
 + a*b)^5 + 3*a^4*b^4 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*
a^3*b^4/(b^2*x + a*b) + 30*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^
3*a^3*b^4/(b^2*x + a*b)^3 - 18*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) +
 b)^4*a^3*b^4/(b^2*x + a*b)^4 - 6*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b
) + b)^5*a^3*b^4/(b^2*x + a*b)^5 + 2*a^3*b^4 - 6*(sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*abs(b) + b)*a^2*b^4/(b^2*x + a*b) + 18*(sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*abs(b) + b)^2*a^2*b^4/(b^2*x + a*b)^2 - 4*(sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)*abs(b) + b)^3*a^2*b^4/(b^2*x + a*b)^3 + 18*(sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)*abs(b) + b)^4*a^2*b^4/(b^2*x + a*b)^4 - 6*(sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^2*b^4/(b^2*x + a*b)^5 + 12*(sqrt(-b^2*x
^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a*b^4/(b^2*x + a*b)^2 - 12*(sqrt(-b^2
*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a*b^4/(b^2*x + a*b)^3 + 12*(sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a*b^4/(b^2*x + a*b)^4 - 8*(sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*b^4/(b^2*x + a*b)^3)/((a^8*abs(b
) + a^7*abs(b) - 2*a^6*abs(b) - 2*a^5*abs(b) + a^4*abs(b) + a^3*abs(b))*((s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(
sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b))^3)
```

maple [B] time = 0.05, size = 1711, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^4,x)
```

```
[Out] 1/(1+a)^4*b^4/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^
2*b^2+2*b*(x+(1+a)/b))^(1/2))+1/(1+a)^4*b^3*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a
*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+1/2/(1+a)^2*b^3/(-
a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/2/(1+a)^2*b^3/(-a^2+1)^(1/2)*ln((-2
*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/3/(1+a
)/(-a^2+1)/x^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+1/(1+a)^3*b^4*a^2/(-a^2+1)/(b
^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/2/(1
+a)*a*b/(-a^2+1)^2/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-1/2/(1+a)*a^2*b^2/(-a
^2+1)^3/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+1/2/(1+a)*a^4*b^4/(-a^2+1)^3/(b^2)
^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/2/(1+a)
*a^2*b^4/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-1/2/(1+a)*a^2*b^4/(-a^
2+1)^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2
))-1/(1+a)^3*b^4/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2))-1/(1+a)^3*b^2/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/
2)-2/(1+a)^3*b^3*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(1+a)^3*b^3*a/
```

```
(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2))/x)-1/(1+a)^3*b^4/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+1/2/(1
+a)*a^2*b^4/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2))+1/2/(1+a)^2*b^2*a/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^
(3/2)-1/2/(1+a)^2*b^4*a^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)
/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^2*b^4*a/(-a^2+1)^2*(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2)*x+1/2/(1+a)^2*b^4*a/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/2/(1+a)^2*b^4/(-a^2+1)*a/(b^
2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/(1+a)
^4*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)-1/(1+a)^4*b^3*(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2)+1/(1+a)^4*b^4*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^2*b/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*
x-a^2+1)^(3/2)+1/(1+a)^2*b^3*a^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-
1/2/(1+a)^2*b^3*a^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/(1+a)*a^3*b^3/(-a^2+1)^3*(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2)+1/2/(1+a)*a^3*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a
^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(1+a)*a*b^3/(-a^2+1)^2*(
-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/2/(1+a)*a*b^3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2
*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(bx+a)^2+1}}{(bx+a+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(-(b*x + a)^2 + 1)/((b*x + a + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{1-(a+b\,x)^2}}{x^4\,(a+b\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(1/2)/(x^4*(a + b*x + 1)),x)
[Out] int((1 - (a + b*x)^2)^(1/2)/(x^4*(a + b*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(a+bx-1)(a+bx+1)}}{x^4(a+bx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)*(1-(b*x+a)**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a + b*x - 1)*(a + b*x + 1))/(x**4*(a + b*x + 1)), x)
```

3.851 $\int e^{-2\tanh^{-1}(a+bx)}x^4\,dx$

Optimal. Leaf size=71

$$\frac{2(a+1)^4\log(a+bx+1)}{b^5}-\frac{2(a+1)^3x}{b^4}+\frac{(a+1)^2x^2}{b^3}-\frac{2(a+1)x^3}{3b^2}+\frac{x^4}{2b}-\frac{x^5}{5}$$

```
[Out] -2*(1+a)^3*x/b^4+(1+a)^2*x^2/b^3-2/3*(1+a)*x^3/b^2+1/2*x^4/b-1/5*x^5+2*(1+a
)^4*ln(b*x+a+1)/b^5
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2(a+1)x^3}{3b^2}+\frac{(a+1)^2x^2}{b^3}-\frac{2(a+1)^3x}{b^4}+\frac{2(a+1)^4\log(a+bx+1)}{b^5}+\frac{x^4}{2b}-\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*(1 + a)^3*x)/b^4 + ((1 + a)^2*x^2)/b^3 - (2*(1 + a)*x^3)/(3*b^2) + x^4/
(2*b) - x^5/5 + (2*(1 + a)^4*Log[1 + a + b*x])/b^5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(a+bx)}x^4\,dx = \int \frac{x^4(1-a-bx)}{1+a+bx}\,dx$$
$$= \int \left(-\frac{2(1+a)^3}{b^4} + \frac{2(1+a)^2x}{b^3} - \frac{2(1+a)x^2}{b^2} + \frac{2x^3}{b} - x^4 + \frac{2(1+a)^4}{b^4(1+a+bx)}\right) dx$$
$$= -\frac{2(1+a)^3x}{b^4} + \frac{(1+a)^2x^2}{b^3} - \frac{2(1+a)x^3}{3b^2} + \frac{x^4}{2b} - \frac{x^5}{5} + \frac{2(1+a)^4\log(1+a+bx)}{b^5}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 1.00

$$\frac{2(a+1)^4\log(a+bx+1)}{b^5} - \frac{2(a+1)^3x}{b^4} + \frac{(a+1)^2x^2}{b^3} - \frac{2(a+1)x^3}{3b^2} + \frac{x^4}{2b} - \frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*(1 + a)^3*x)/b^4 + ((1 + a)^2*x^2)/b^3 - (2*(1 + a)*x^3)/(3*b^2) + x^4/
(2*b) - x^5/5 + (2*(1 + a)^4*Log[1 + a + b*x])/b^5
```

fricas [A] time = 0.51, size = 93, normalized size = 1.31

$$-\frac{6\,b^5x^5 - 15\,b^4x^4 + 20\,(a+1)b^3x^3 - 30\left(a^2+2\,a+1\right)b^2x^2 + 60\left(a^3+3\,a^2+3\,a+1\right)bx - 60\left(a^4+4\,a^3+6\,a^2+\right.}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/30*(6*b^5*x^5 - 15*b^4*x^4 + 20*(a + 1)*b^3*x^3 - 30*(a^2 + 2*a + 1)*b^2
*x^2 + 60*(a^3 + 3*a^2 + 3*a + 1)*b*x - 60*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*
log(b*x + a + 1))/b^5
```

giac [B] time = 0.31, size = 202, normalized size = 2.85

$$\frac{(bx+a+1)^5\left(\frac{15\,(2\,ab+3\,b)}{(bx+a+1)b} - \frac{20\left(3\,a^2b^2+10\,ab^2+7\,b^2\right)}{(bx+a+1)^2b^2} + \frac{60\left(a^3b^3+6\,a^2b^3+9\,ab^3+4\,b^3\right)}{(bx+a+1)^3b^3} - \frac{30\left(a^4b^4+12\,a^3b^4+30\,a^2b^4+28\,ab^4+9\,b^4\right)}{(bx+a+1)^4b^4} - 6\right)}{30\,b^5} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/30*(b*x + a + 1)^5*(15*(2*a*b + 3*b)/((b*x + a + 1)*b) - 20*(3*a^2*b^2 +
10*a*b^2 + 7*b^2)/((b*x + a + 1)^2*b^2) + 60*(a^3*b^3 + 6*a^2*b^3 + 9*a*b^3
```

```
 + 4*b^3)/((b*x + a + 1)^3*b^3) - 30*(a^4*b^4 + 12*a^3*b^4 + 30*a^2*b^4 + 2
8*a*b^4 + 9*b^4)/((b*x + a + 1)^4*b^4) - 6)/b^5 - 2*(a^4 + 4*a^3 + 6*a^2 +
4*a + 1)*log(abs(b*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b^5
```

maple [B] time = 0.03, size = 159, normalized size = 2.24

$$-\frac{x^5}{5}+\frac{x^4}{2b}-\frac{2x^3a}{3b^2}-\frac{2x^3}{3b^2}+\frac{x^2a^2}{b^3}+\frac{2x^2a}{b^3}-\frac{2x\,a^3}{b^4}+\frac{x^2}{b^3}-\frac{6a^2x}{b^4}-\frac{6ax}{b^4}-\frac{2x}{b^4}+\frac{2\ln\left(bx+a+1\right)a^4}{b^5}+\frac{8\ln\left(bx+a+1\right)a^3}{b^5}+\frac{12\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a+1)^2*(1-(b*x+a)^2),x)
```

```
[Out] -1/5*x^5+1/2*x^4/b-2/3/b^2*x^3*a-2/3/b^2*x^3+1/b^3*x^2*a^2+2/b^3*x^2*a-2/b^
4*x*a^3+1/b^3*x^2-6/b^4*a^2*x-6/b^4*a*x-2/b^4*x+2/b^5*ln(b*x+a+1)*a^4+8/b^5
*ln(b*x+a+1)*a^3+12/b^5*ln(b*x+a+1)*a^2+8/b^5*ln(b*x+a+1)*a+2/b^5*ln(b*x+a+
1)
```

maxima [A] time = 0.35, size = 94, normalized size = 1.32

$$-\frac{6\,b^4x^5-15\,b^3x^4+20\,(a+1)b^2x^3-30\left(a^2+2\,a+1\right)bx^2+60\left(a^3+3\,a^2+3\,a+1\right)x}{30\,b^4}+\frac{2\left(a^4+4\,a^3+6\,a^2+4\,a+\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/30*(6*b^4*x^5 - 15*b^3*x^4 + 20*(a + 1)*b^2*x^3 - 30*(a^2 + 2*a + 1)*b*x
^2 + 60*(a^3 + 3*a^2 + 3*a + 1)*x)/b^4 + 2*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*
log(b*x + a + 1)/b^5
```

mupad [B] time = 0.06, size = 141, normalized size = 1.99

$$\frac{\ln\left(a+b\,x+1\right)\,\left(2\,a^4+8\,a^3+12\,a^2+8\,a+2\right)}{b^5}-\frac{x^5}{5}-x^4\,\left(\frac{a-1}{4\,b}-\frac{a+1}{4\,b}\right)-\frac{x^2\,\left(\frac{a-1}{b}-\frac{a+1}{b}\right)\,{\left(a+1\right)}^2}{2\,b^2}+\frac{x^3\,\left(\frac{a-1}{b}-\frac{a+1}{b}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*((a + b*x)^2 - 1))/(a + b*x + 1)^2,x)
```

```
[Out] (log(a + b*x + 1)*(8*a + 12*a^2 + 8*a^3 + 2*a^4 + 2))/b^5 - x^5/5 - x^4*((a
 - 1)/(4*b) - (a + 1)/(4*b)) - (x^2*((a - 1)/b - (a + 1)/b)*(a + 1)^2)/(2*b
^2) + (x^3*((a - 1)/b - (a + 1)/b)*(a + 1))/(3*b) + (x*((a - 1)/b - (a + 1)
/b)*(a + 1)^3)/b^3
```

sympy [A] time = 0.28, size = 102, normalized size = 1.44

$$-\frac{x^5}{5}-x^3\left(\frac{2a}{3b^2}+\frac{2}{3b^2}\right)-x^2\left(-\frac{a^2}{b^3}-\frac{2a}{b^3}-\frac{1}{b^3}\right)-x\left(\frac{2a^3}{b^4}+\frac{6a^2}{b^4}+\frac{6a}{b^4}+\frac{2}{b^4}\right)+\frac{x^4}{2b}+\frac{2\,(a+1)^4\log\,(a+bx+1)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x**5/5 - x**3*(2*a/(3*b**2) + 2/(3*b**2)) - x**2*(-a**2/b**3 - 2*a/b**3 -
1/b**3) - x*(2*a**3/b**4 + 6*a**2/b**4 + 6*a/b**4 + 2/b**4) + x**4/(2*b) +
2*(a + 1)**4*log(a + b*x + 1)/b**5
```

3.852 $\int e^{-2\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=57

$$-\frac{2(a+1)^3\log(a+bx+1)}{b^4}+\frac{2(a+1)^2x}{b^3}-\frac{(a+1)x^2}{b^2}+\frac{2x^3}{3b}-\frac{x^4}{4}$$

```
[Out] 2*(1+a)^2*x/b^3-(1+a)*x^2/b^2+2/3*x^3/b-1/4*x^4-2*(1+a)^3*ln(b*x+a+1)/b^4
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{(a+1)x^2}{b^2}+\frac{2(a+1)^2x}{b^3}-\frac{2(a+1)^3\log(a+bx+1)}{b^4}+\frac{2x^3}{3b}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (2*(1 + a)^2*x)/b^3 - ((1 + a)*x^2)/b^2 + (2*x^3)/(3*b) - x^4/4 - (2*(1 + a
)^3*Log[1 + a + b*x])/b^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1-a-bx)}{1+a+bx}\,dx$$
$$= \int \left(\frac{2(1+a)^2}{b^3} - \frac{2(1+a)x}{b^2} + \frac{2x^2}{b} - x^3 - \frac{2(1+a)^3}{b^3(1+a+bx)}\right)dx$$
$$= \frac{2(1+a)^2x}{b^3} - \frac{(1+a)x^2}{b^2} + \frac{2x^3}{3b} - \frac{x^4}{4} - \frac{2(1+a)^3\log(1+a+bx)}{b^4}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.00

$$-\frac{2(a+1)^3\log(a+bx+1)}{b^4} + \frac{2(a+1)^2x}{b^3} - \frac{(a+1)x^2}{b^2} + \frac{2x^3}{3b} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (2*(1 + a)^2*x)/b^3 - ((1 + a)*x^2)/b^2 + (2*x^3)/(3*b) - x^4/4 - (2*(1 + a
)^3*Log[1 + a + b*x])/b^4
```

fricas [A] time = 2.26, size = 67, normalized size = 1.18

$$-\frac{3\,b^4x^4 - 8\,b^3x^3 + 12\,(a+1)b^2x^2 - 24\left(a^2+2\,a+1\right)bx + 24\left(a^3+3\,a^2+3\,a+1\right)\log\left(bx+a+1\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*x^4 - 8*b^3*x^3 + 12*(a + 1)*b^2*x^2 - 24*(a^2 + 2*a + 1)*b*x
+ 24*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1))/b^4
```

giac [B] time = 0.26, size = 148, normalized size = 2.60

$$\frac{(bx+a+1)^4\left(\frac{4\,(3\,ab+5\,b)}{(bx+a+1)b} - \frac{18\left(a^2b^2+4\,ab^2+3\,b^2\right)}{(bx+a+1)^2b^2} + \frac{12\left(a^3b^3+9\,a^2b^3+15\,ab^3+7\,b^3\right)}{(bx+a+1)^3b^3} - 3\right)}{12\,b^4} + \frac{2\left(a^3+3\,a^2+3\,a+1\right)\log\left(\frac{|bx+a+1|}{(bx+a+1)^2}\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/12*(b*x + a + 1)^4*(4*(3*a*b + 5*b)/((b*x + a + 1)*b) - 18*(a^2*b^2 + 4*a
*b^2 + 3*b^2)/((b*x + a + 1)^2*b^2) + 12*(a^3*b^3 + 9*a^2*b^3 + 15*a*b^3 +
```

```
7*b^3)/((b*x + a + 1)^3*b^3) - 3)/b^4 + 2*(a^3 + 3*a^2 + 3*a + 1)*log(abs(b
*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b^4
```

maple [B] time = 0.03, size = 109, normalized size = 1.91

$$-\frac{x^4}{4}+\frac{2x^3}{3b}-\frac{x^2a}{b^2}-\frac{x^2}{b^2}+\frac{2a^2x}{b^3}+\frac{4ax}{b^3}+\frac{2x}{b^3}-\frac{2\ln(bx+a+1)\,a^3}{b^4}-\frac{6\ln(bx+a+1)\,a^2}{b^4}-\frac{6\ln(bx+a+1)\,a}{b^4}-\frac{2\ln(bx+a+1)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a+1)^2*(1-(b*x+a)^2),x)
```

```
[Out] -1/4*x^4+2/3*x^3/b-1/b^2*x^2*a-1/b^2*x^2+2/b^3*a^2*x+4*a*x/b^3+2/b^3*x-2/b^
4*ln(b*x+a+1)*a^3-6/b^4*ln(b*x+a+1)*a^2-6/b^4*ln(b*x+a+1)*a-2/b^4*ln(b*x+a+
1)
```

maxima [A] time = 0.31, size = 68, normalized size = 1.19

$$-\frac{3\,b^3x^4-8\,b^2x^3+12\,(a+1)bx^2-24\left(a^2+2\,a+1\right)x}{12\,b^3}-\frac{2\left(a^3+3\,a^2+3\,a+1\right)\log(bx+a+1)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^3*x^4 - 8*b^2*x^3 + 12*(a + 1)*b*x^2 - 24*(a^2 + 2*a + 1)*x)/b^3
 - 2*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)/b^4
```

mupad [B] time = 0.05, size = 109, normalized size = 1.91

$$\frac{x^2\left(\frac{a-1}{b}-\frac{a+1}{b}\right)(a+1)}{2\,b}-\frac{x^4}{4}-\frac{\ln(a+b\,x+1)\left(2\,a^3+6\,a^2+6\,a+2\right)}{b^4}-x^3\left(\frac{a-1}{3\,b}-\frac{a+1}{3\,b}\right)-\frac{x\left(\frac{a-1}{b}-\frac{a+1}{b}\right)(a+1)^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*((a + b*x)^2 - 1))/(a + b*x + 1)^2,x)
```

```
[Out] (x^2*((a - 1)/b - (a + 1)/b)*(a + 1))/(2*b) - x^4/4 - (log(a + b*x + 1)*(6*
a + 6*a^2 + 2*a^3 + 2))/b^4 - x^3*((a - 1)/(3*b) - (a + 1)/(3*b)) - (x*((a
- 1)/b - (a + 1)/b)*(a + 1)^2)/b^2
```

sympy [A] time = 0.23, size = 68, normalized size = 1.19

$$-\frac{x^4}{4}-x^2\left(\frac{a}{b^2}+\frac{1}{b^2}\right)-x\left(-\frac{2a^2}{b^3}-\frac{4a}{b^3}-\frac{2}{b^3}\right)+\frac{2x^3}{3b}-\frac{2\,(a+1)^3\log(a+bx+1)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x**4/4 - x**2*(a/b**2 + b**(-2)) - x*(-2*a**2/b**3 - 4*a/b**3 - 2/b**3) +
2*x**3/(3*b) - 2*(a + 1)**3*log(a + b*x + 1)/b**4
```

3.853 $\int e^{-2\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=41

$$\frac{2(a+1)^2\log(a+bx+1)}{b^3}-\frac{2(a+1)x}{b^2}+\frac{x^2}{b}-\frac{x^3}{3}$$

```
[Out] -2*(1+a)*x/b^2+x^2/b-1/3*x^3+2*(1+a)^2*ln(b*x+a+1)/b^3
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2(a+1)x}{b^2}+\frac{2(a+1)^2\log(a+bx+1)}{b^3}+\frac{x^2}{b}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*(1 + a)*x)/b^2 + x^2/b - x^3/3 + (2*(1 + a)^2*Log[1 + a + b*x])/b^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(a+bx)}x^2\,dx = \int \frac{x^2(1-a-bx)}{1+a+bx}\,dx$$
$$= \int \left(-\frac{2(1+a)}{b^2} + \frac{2x}{b} - x^2 + \frac{2(1+a)^2}{b^2(1+a+bx)}\right) dx$$
$$= -\frac{2(1+a)x}{b^2} + \frac{x^2}{b} - \frac{x^3}{3} + \frac{2(1+a)^2\log(1+a+bx)}{b^3}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{2(a+1)^2\log(a+bx+1)}{b^3} - \frac{2(a+1)x}{b^2} + \frac{x^2}{b} - \frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*ArcTanh[a + b*x]),x]
[Out] (-2*(1 + a)*x)/b^2 + x^2/b - x^3/3 + (2*(1 + a)^2*Log[1 + a + b*x])/b^3
```

fricas [A] time = 0.67, size = 45, normalized size = 1.10

$$-\frac{b^3x^3 - 3\,b^2x^2 + 6\,(a+1)bx - 6\left(a^2+2\,a+1\right)\log\left(bx+a+1\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
[Out] -1/3*(b^3*x^3 - 3*b^2*x^2 + 6*(a + 1)*b*x - 6*(a^2 + 2*a + 1)*log(b*x + a +
 1))/b^3
```

giac [B] time = 0.16, size = 102, normalized size = 2.49

$$\frac{(bx+a+1)^3\left(\frac{3\,(ab+2\,b)}{(bx+a+1)b} - \frac{3\left(a^2b^2+6\,ab^2+5\,b^2\right)}{(bx+a+1)^2b^2} - 1\right)}{3\,b^3} - \frac{2\left(a^2+2\,a+1\right)\log\left(\frac{|bx+a+1|}{(bx+a+1)^2|b|}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
[Out] 1/3*(b*x + a + 1)^3*(3*(a*b + 2*b)/((b*x + a + 1)*b) - 3*(a^2*b^2 + 6*a*b^2
 + 5*b^2)/((b*x + a + 1)^2*b^2) - 1)/b^3 - 2*(a^2 + 2*a + 1)*log(abs(b*x +
a + 1)/((b*x + a + 1)^2*abs(b)))/b^3
```

maple [A] time = 0.03, size = 67, normalized size = 1.63

$$-\frac{x^3}{3}+\frac{x^2}{b}-\frac{2ax}{b^2}-\frac{2x}{b^2}+\frac{2\ln\left(bx+a+1\right)a^2}{b^3}+\frac{4\ln\left(bx+a+1\right)a}{b^3}+\frac{2\ln\left(bx+a+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a+1)^2*(1-(b*x+a)^2),x)
```

```
[Out] -1/3*x^3+x^2/b-2/b^2*a*x-2*x/b^2+2/b^3*ln(b*x+a+1)*a^2+4/b^3*ln(b*x+a+1)*a+
2/b^3*ln(b*x+a+1)
```

maxima [A] time = 0.32, size = 46, normalized size = 1.12

$$-\frac{b^2x^3-3\,bx^2+6\,(a+1)x}{3\,b^2}+\frac{2\left(a^2+2\,a+1\right)\log\left(bx+a+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(b^2*x^3 - 3*b*x^2 + 6*(a + 1)*x)/b^2 + 2*(a^2 + 2*a + 1)*log(b*x + a
+ 1)/b^3
```

mupad [B] time = 0.06, size = 73, normalized size = 1.78

$$\frac{\ln\left(a+b\,x+1\right)\,\left(2\,a^2+4\,a+2\right)}{b^3}-\frac{x^3}{3}-x^2\,\left(\frac{a-1}{2\,b}-\frac{a+1}{2\,b}\right)+\frac{x\,\left(\frac{a-1}{b}-\frac{a+1}{b}\right)\,\left(a+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*((a + b*x)^2 - 1))/(a + b*x + 1)^2,x)
```

```
[Out] (log(a + b*x + 1)*(4*a + 2*a^2 + 2))/b^3 - x^3/3 - x^2*((a - 1)/(2*b) - (a
+ 1)/(2*b)) + (x*((a - 1)/b - (a + 1)/b)*(a + 1))/b
```

sympy [A] time = 0.20, size = 41, normalized size = 1.00

$$-\frac{x^3}{3}-x\left(\frac{2a}{b^2}+\frac{2}{b^2}\right)+\frac{x^2}{b}+\frac{2\left(a+1\right)^2\log\left(a+bx+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x**3/3 - x*(2*a/b**2 + 2/b**2) + x**2/b + 2*(a + 1)**2*log(a + b*x + 1)/b*
*3
```

3.854 $\int e^{-2\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=29

$$-\frac{2(a+1)\log(a+bx+1)}{b^2}+\frac{2x}{b}-\frac{x^2}{2}$$

```
[Out] 2*x/b-1/2*x^2-2*(1+a)*ln(b*x+a+1)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6163, 77}

$$-\frac{2(a+1)\log(a+bx+1)}{b^2}+\frac{2x}{b}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(2*ArcTanh[a + b*x]),x]
```

```
[Out] (2*x)/b - x^2/2 - (2*(1 + a)*Log[1 + a + b*x])/b^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(a+bx)}x\,dx = \int \frac{x(1-a-bx)}{1+a+bx}\,dx$$
$$= \int \left(\frac{2}{b} - x - \frac{2(1+a)}{b(1+a+bx)}\right) dx$$
$$= \frac{2x}{b} - \frac{x^2}{2} - \frac{2(1+a)\log(1+a+bx)}{b^2}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{2(a+1)\log(a+bx+1)}{b^2} + \frac{2x}{b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(2*ArcTanh[a + b*x]),x]
[Out] (2*x)/b - x^2/2 - (2*(1 + a)*Log[1 + a + b*x])/b^2
```

fricas [A] time = 0.48, size = 29, normalized size = 1.00

$$-\frac{b^2x^2 - 4\,bx + 4\,(a+1)\log\left(bx + a + 1\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 - 4*b*x + 4*(a + 1)*log(b*x + a + 1))/b^2
```

giac [B] time = 0.17, size = 69, normalized size = 2.38

$$\frac{\frac{(bx+a+1)^2\left(\frac{2\,(ab+3\,b)}{(bx+a+1)b}-1\right)}{b} + \frac{4\,(a+1)\log\left(\frac{|bx+a+1|}{(bx+a+1)^2|b|}\right)}{b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
[Out] 1/2*((b*x + a + 1)^2*(2*(a*b + 3*b)/((b*x + a + 1)*b) - 1)/b + 4*(a + 1)*lo
g(abs(b*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b)/b
```

maple [A] time = 0.03, size = 38, normalized size = 1.31

$$-\frac{x^2}{2} + \frac{2x}{b} - \frac{2\ln\left(bx + a + 1\right)a}{b^2} - \frac{2\ln\left(bx + a + 1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a+1)^2*(1-(b*x+a)^2),x)
```

```
[Out] -1/2*x^2+2*x/b-2/b^2*ln(b*x+a+1)*a-2/b^2*ln(b*x+a+1)
```

maxima [A] time = 0.32, size = 30, normalized size = 1.03

$$-\frac{bx^2 - 4x}{2b} - \frac{2(a+1)\log(bx+a+1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 - 4*x)/b - 2*(a + 1)*log(b*x + a + 1)/b^2
```

mupad [B] time = 0.89, size = 42, normalized size = 1.45

$$-\frac{x^2}{2} - x\left(\frac{a-1}{b} - \frac{a+1}{b}\right) - \frac{\ln(a+bx+1)\,(2a+2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*((a + b*x)^2 - 1))/(a + b*x + 1)^2,x)
```

```
[Out] - x^2/2 - x*((a - 1)/b - (a + 1)/b) - (log(a + b*x + 1)*(2*a + 2))/b^2
```

sympy [A] time = 0.14, size = 26, normalized size = 0.90

$$-\frac{x^2}{2} + \frac{2x}{b} - \frac{2(a+1)\log(a+bx+1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x**2/2 + 2*x/b - 2*(a + 1)*log(a + b*x + 1)/b**2
```

3.855 $\int e^{-2\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=16

$$\frac{2\log(a+bx+1)}{b}-x$$

[Out] -x+2*ln(b*x+a+1)/b

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6161, 43}

$$\frac{2\log(a+bx+1)}{b}-x$$

Antiderivative was successfully verified.

[In] Int[E^(-2*ArcTanh[a + b*x]),x]

[Out] -x + (2*Log[1 + a + b*x])/b

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(a+bx)}\,dx &= \int \frac{1-a-bx}{1+a+bx}\,dx\\ &= \int \left(-1+\frac{2}{1+a+bx}\right)dx\\ &= -x+\frac{2\log(1+a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2\log(a+bx+1)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-2*ArcTanh[a + b*x]),x]
[Out] -x + (2*Log[1 + a + b*x])/b
```

fricas [A] time = 1.31, size = 18, normalized size = 1.12

$$-\frac{bx-2\log(bx+a+1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="fricas")
[Out] -(b*x - 2*log(b*x + a + 1))/b
```

giac [B] time = 0.53, size = 38, normalized size = 2.38

$$-\frac{bx+a+1}{b}-\frac{2\log\left(\frac{|bx+a+1|}{(bx+a+1)^2|b|}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="giac")
[Out] -(b*x + a + 1)/b - 2*log(abs(b*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b
```

maple [A] time = 0.02, size = 17, normalized size = 1.06

$$-x+\frac{2\ln(bx+a+1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2),x)
[Out] -x+2*ln(b*x+a+1)/b
```

maxima [A] time = 0.31, size = 16, normalized size = 1.00

$$-x+\frac{2\log(bx+a+1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -x + 2*log(b*x + a + 1)/b
```

mupad [B] time = 0.04, size = 16, normalized size = 1.00

$$\frac{2\,\ln\left(a+b\,x+1\right)}{b}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a + b*x)^2 - 1)/(a + b*x + 1)^2,x)
```

```
[Out] (2*log(a + b*x + 1))/b - x
```

sympy [A] time = 0.11, size = 12, normalized size = 0.75

$$-x+\frac{2\log\left(a+bx+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2),x)
```

```
[Out] -x + 2*log(a + b*x + 1)/b
```

3.856 $\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=28

$$\frac{(1-a)\log(x)}{a+1}-\frac{2\log(a+bx+1)}{a+1}$$

```
[Out] (1-a)*ln(x)/(1+a)-2*ln(b*x+a+1)/(1+a)
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 72}

$$\frac{(1-a)\log(x)}{a+1}-\frac{2\log(a+bx+1)}{a+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a + b*x])*x),x]
```

```
[Out] ((1 - a)*Log[x])/(1 + a) - (2*Log[1 + a + b*x])/(1 + a)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x}\,dx &= \int \frac{1-a-bx}{x(1+a+bx)}\,dx\\ &= \int \left(\frac{1-a}{(1+a)x}-\frac{2b}{(1+a)(1+a+bx)}\right)dx\\ &= \frac{(1-a)\log(x)}{1+a}-\frac{2\log(1+a+bx)}{1+a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.82

$$\frac{-2\log(a+bx+1)-a\log(x)+\log(x)}{a+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a + b*x])*x),x]
```

```
[Out] (Log[x] - a*Log[x] - 2*Log[1 + a + b*x])/(1 + a)
```

fricas [A] time = 0.59, size = 23, normalized size = 0.82

$$-\frac{(a-1)\log(x)+2\,\log(bx+a+1)}{a+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] -((a - 1)*log(x) + 2*log(b*x + a + 1))/(a + 1)
```

giac [B] time = 0.32, size = 66, normalized size = 2.36

$$-b\left(\frac{(a-1)\log\left(\left|-\frac{a}{bx+a+1}-\frac{1}{bx+a+1}+1\right|\right)}{ab+b}-\frac{\log\left(\frac{|bx+a+1|}{(bx+a+1)^2|b|}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] -b*((a - 1)*log(abs(-a/(b*x + a + 1) - 1/(b*x + a + 1) + 1))/(a*b + b) - lo
g(abs(b*x + a + 1)/((b*x + a + 1)^2*abs(b)))/b)
```

maple [A] time = 0.03, size = 34, normalized size = 1.21

$$\frac{\ln(x)}{1+a}-\frac{\ln(x)a}{1+a}-\frac{2\ln(bx+a+1)}{1+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x,x)
```

```
[Out] 1/(1+a)*ln(x)-1/(1+a)*ln(x)*a-2*ln(b*x+a+1)/(1+a)
```

maxima [A] time = 0.31, size = 27, normalized size = 0.96

$$-\frac{(a-1)\log(x)}{a+1}-\frac{2\,\log(bx+a+1)}{a+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] -(a - 1)*log(x)/(a + 1) - 2*log(b*x + a + 1)/(a + 1)
```

mupad [B] time = 0.10, size = 28, normalized size = 1.00

$$\frac{2\,\ln(x)}{a+1}-\ln(x)-\frac{2\,\ln\left(a+b\,x+1\right)}{a+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a + b*x)^2 - 1)/(x*(a + b*x + 1)^2),x)
```

```
[Out] (2*log(x))/(a + 1) - log(x) - (2*log(a + b*x + 1))/(a + 1)
```

sympy [B] time = 0.46, size = 90, normalized size = 3.21

$$-\frac{(a-1)\log\left(x+\frac{-\frac{a^2(a-1)}{a+1}+a^2-\frac{2a(a-1)}{a+1}-\frac{a-1}{a+1}-1}{ab-3b}\right)}{a+1}-\frac{2\log\left(x+\frac{a^2-\frac{2a^2}{a+1}-\frac{4a}{a+1}-1-\frac{2}{a+1}}{ab-3b}\right)}{a+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2)/x,x)
```

```
[Out] -(a - 1)*log(x + (-a**2*(a - 1)/(a + 1) + a**2 - 2*a*(a - 1)/(a + 1) - (a -
 1)/(a + 1) - 1)/(a*b - 3*b))/(a + 1) - 2*log(x + (a**2 - 2*a**2/(a + 1) -
4*a/(a + 1) - 1 - 2/(a + 1))/(a*b - 3*b))/(a + 1)
```

3.857 $\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=41

$$-\frac{2b\log(x)}{(a+1)^2}+\frac{2b\log(a+bx+1)}{(a+1)^2}-\frac{1-a}{(a+1)x}$$

[Out] (-1+a)/(1+a)/x-2*b*ln(x)/(1+a)^2+2*b*ln(b*x+a+1)/(1+a)^2

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2b\log(x)}{(a+1)^2}+\frac{2b\log(a+bx+1)}{(a+1)^2}-\frac{1-a}{(a+1)x}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcTanh[a + b*x])*x^2),x]

[Out] -((1 - a)/((1 + a)*x)) - (2*b*Log[x])/(1 + a)^2 + (2*b*Log[1 + a + b*x])/(1 + a)^2

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^2}\,dx = \int \frac{1-a-bx}{x^2(1+a+bx)}\,dx$$
$$= \int \left(\frac{1-a}{(1+a)x^2} - \frac{2b}{(1+a)^2x} + \frac{2b^2}{(1+a)^2(1+a+bx)}\right)dx$$
$$= -\frac{1-a}{(1+a)x} - \frac{2b\log(x)}{(1+a)^2} + \frac{2b\log(1+a+bx)}{(1+a)^2}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.76

$$\frac{a^2+2bx\log(a+bx+1)-2bx\log(x)-1}{(a+1)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a + b*x])*x^2),x]
[Out] (-1 + a^2 - 2*b*x*Log[x] + 2*b*x*Log[1 + a + b*x])/((1 + a)^2*x)
```

fricas [A] time = 2.38, size = 36, normalized size = 0.88

$$\frac{2\,bx\log\left(bx+a+1\right)-2\,bx\log(x)+a^2-1}{\left(a^2+2\,a+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^2,x, algorithm="fricas")
[Out] (2*b*x*log(b*x + a + 1) - 2*b*x*log(x) + a^2 - 1)/((a^2 + 2*a + 1)*x)
```

giac [B] time = 0.19, size = 80, normalized size = 1.95

$$-\frac{2\,b^2\log\left(\left|-\frac{a}{bx+a+1}-\frac{1}{bx+a+1}+1\right|\right)}{a^2b+2\,ab+b}-\frac{ab-b}{(a+1)^2\left(\frac{a}{bx+a+1}+\frac{1}{bx+a+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^2,x, algorithm="giac")
[Out] -2*b^2*log(abs(-a/(b*x + a + 1) - 1/(b*x + a + 1) + 1))/(a^2*b + 2*a*b + b)
 - (a*b - b)/((a + 1)^2*(a/(b*x + a + 1) + 1/(b*x + a + 1) - 1))
```

maple [A] time = 0.03, size = 47, normalized size = 1.15

$$-\frac{1}{(1+a)\,x}+\frac{a}{(1+a)\,x}-\frac{2b\ln(x)}{(1+a)^2}+\frac{2b\ln(bx+a+1)}{(1+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^2,x)
```

```
[Out] -1/(1+a)/x+1/(1+a)/x*a-2*b*ln(x)/(1+a)^2+2*b*ln(b*x+a+1)/(1+a)^2
```

maxima [A] time = 0.31, size = 48, normalized size = 1.17

$$\frac{2\,b\log(bx+a+1)}{a^2+2\,a+1}-\frac{2\,b\log(x)}{a^2+2\,a+1}+\frac{a-1}{(a+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*log(b*x + a + 1)/(a^2 + 2*a + 1) - 2*b*log(x)/(a^2 + 2*a + 1) + (a - 1)
/((a + 1)*x)
```

mupad [B] time = 0.92, size = 47, normalized size = 1.15

$$\frac{a-1}{x\,(a+1)}+\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,x+\frac{a^2+2\,a+1}{a+1}}{a+1}\right)}{(a+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a + b*x)^2 - 1)/(x^2*(a + b*x + 1)^2),x)
```

```
[Out] (a - 1)/(x*(a + 1)) + (4*b*atanh((2*b*x + (2*a + a^2 + 1)/(a + 1))/(a + 1))
)/(a + 1)^2
```

sympy [B] time = 0.35, size = 143, normalized size = 3.49

$$-\frac{2b\log\left(x+\frac{-\frac{2a^3b}{(a+1)^2}-\frac{6a^2b}{(a+1)^2}+2ab-\frac{6ab}{(a+1)^2}+2b-\frac{2b}{(a+1)^2}}{4b^2}\right)}{(a+1)^2}+\frac{2b\log\left(x+\frac{\frac{2a^3b}{(a+1)^2}+\frac{6a^2b}{(a+1)^2}+2ab+\frac{6ab}{(a+1)^2}+2b+\frac{2b}{(a+1)^2}}{4b^2}\right)}{(a+1)^2}-\frac{1-a}{x\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2)/x**2,x)
```

```
[Out] -2*b*log(x + (-2*a**3*b/(a + 1)**2 - 6*a**2*b/(a + 1)**2 + 2*a*b - 6*a*b/(a
 + 1)**2 + 2*b - 2*b/(a + 1)**2)/(4*b**2))/(a + 1)**2 + 2*b*log(x + (2*a**3
*b/(a + 1)**2 + 6*a**2*b/(a + 1)**2 + 2*a*b + 6*a*b/(a + 1)**2 + 2*b + 2*b/
(a + 1)**2)/(4*b**2))/(a + 1)**2 - (1 - a)/(x*(a + 1))
```

3.858 $\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=58

$$\frac{2b^2\log(x)}{(a+1)^3}-\frac{2b^2\log(a+bx+1)}{(a+1)^3}+\frac{2b}{(a+1)^2x}-\frac{1-a}{2(a+1)x^2}$$

```
[Out] 1/2*(-1+a)/(1+a)/x^2+2*b/(1+a)^2/x+2*b^2*ln(x)/(1+a)^3-2*b^2*ln(b*x+a+1)/(1
+a)^3
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$\frac{2b^2\log(x)}{(a+1)^3}-\frac{2b^2\log(a+bx+1)}{(a+1)^3}+\frac{2b}{(a+1)^2x}-\frac{1-a}{2(a+1)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a + b*x])*x^3),x]
```

```
[Out] -(1 - a)/(2*(1 + a)*x^2) + (2*b)/((1 + a)^2*x) + (2*b^2*Log[x])/(1 + a)^3 -
 (2*b^2*Log[1 + a + b*x])/(1 + a)^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{1-a-bx}{x^3(1+a+bx)}\,dx$$
$$= \int \left(\frac{1-a}{(1+a)x^3} - \frac{2b}{(1+a)^2x^2} + \frac{2b^2}{(1+a)^3x} - \frac{2b^3}{(1+a)^3(1+a+bx)}\right)dx$$
$$= -\frac{1-a}{2(1+a)x^2} + \frac{2b}{(1+a)^2x} + \frac{2b^2\log(x)}{(1+a)^3} - \frac{2b^2\log(1+a+bx)}{(1+a)^3}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.88

$$\frac{(a+1)\left(a^2+4bx-1\right)-4b^2x^2\log(a+bx+1)+4b^2x^2\log(x)}{2(a+1)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a + b*x])*x^3),x]
```

```
[Out] ((1 + a)*(-1 + a^2 + 4*b*x) + 4*b^2*x^2*Log[x] - 4*b^2*x^2*Log[1 + a + b*x]
)/(2*(1 + a)^3*x^2)
```

fricas [A] time = 0.60, size = 65, normalized size = 1.12

$$-\frac{4\,b^2x^2\log\left(bx+a+1\right)-4\,b^2x^2\log(x)-a^3-4\,(a+1)bx-a^2+a+1}{2\left(a^3+3\,a^2+3\,a+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b^2*x^2*log(b*x + a + 1) - 4*b^2*x^2*log(x) - a^3 - 4*(a + 1)*b*x -
 a^2 + a + 1)/((a^3 + 3*a^2 + 3*a + 1)*x^2)
```

giac [B] time = 0.18, size = 121, normalized size = 2.09

$$\frac{2\,b^3\log\left(\left|-\frac{a}{bx+a+1}-\frac{1}{bx+a+1}+1\right|\right)}{a^3b+3\,a^2b+3\,ab+b}-\frac{\frac{ab^2-5\,b^2}{a+1}-\frac{2\left(ab^3-3\,b^3\right)}{(bx+a+1)b}}{2\,(a+1)^2\left(\frac{a}{bx+a+1}+\frac{1}{bx+a+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 2*b^3*log(abs(-a/(b*x + a + 1) - 1/(b*x + a + 1) + 1))/(a^3*b + 3*a^2*b + 3
*a*b + b) - 1/2*((a*b^2 - 5*b^2)/(a + 1) - 2*(a*b^3 - 3*b^3)/((b*x + a + 1)
*b))/((a + 1)^2*(a/(b*x + a + 1) + 1/(b*x + a + 1) - 1)^2)
```

maple [A] time = 0.04, size = 63, normalized size = 1.09

$$-\frac{1}{2\left(1+a\right)x^{2}}+\frac{a}{2\left(1+a\right)x^{2}}+\frac{2b}{\left(1+a\right)^{2}x}+\frac{2b^{2}\ln\left(x\right)}{\left(1+a\right)^{3}}-\frac{2b^{2}\ln\left(bx+a+1\right)}{\left(1+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^3,x)
```

```
[Out] -1/2/(1+a)/x^2+1/2/(1+a)/x^2*a+2*b/(1+a)^2/x+2*b^2*ln(x)/(1+a)^3-2*b^2*ln(b
*x+a+1)/(1+a)^3
```

maxima [A] time = 0.31, size = 74, normalized size = 1.28

$$-\frac{2\,b^{2}\log\left(bx+a+1\right)}{a^{3}+3\,a^{2}+3\,a+1}+\frac{2\,b^{2}\log\left(x\right)}{a^{3}+3\,a^{2}+3\,a+1}+\frac{a^{2}+4\,bx-1}{2\left(a^{2}+2\,a+1\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*b^2*log(b*x + a + 1)/(a^3 + 3*a^2 + 3*a + 1) + 2*b^2*log(x)/(a^3 + 3*a^2
 + 3*a + 1) + 1/2*(a^2 + 4*b*x - 1)/((a^2 + 2*a + 1)*x^2)
```

mupad [B] time = 0.91, size = 66, normalized size = 1.14

$$\frac{\frac{a-1}{2\,(a+1)}+\frac{2\,b\,x}{(a+1)^{2}}}{x^{2}}-\frac{4\,b^{2}\,\mathrm{atanh}\left(\frac{a^{3}+3\,a^{2}+3\,a+1}{(a+1)^{3}}+\frac{2\,b\,x}{a+1}\right)}{(a+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a + b*x)^2 - 1)/(x^3*(a + b*x + 1)^2),x)
```

```
[Out] ((a - 1)/(2*(a + 1)) + (2*b*x)/(a + 1)^2)/x^2 - (4*b^2*atanh((3*a + 3*a^2 +
 a^3 + 1)/(a + 1)^3 + (2*b*x)/(a + 1)))/(a + 1)^3
```

sympy [B] time = 0.45, size = 209, normalized size = 3.60

$$\frac{2b^{2}\log\left(x+\frac{-\frac{2a^{4}b^{2}}{(a+1)^{3}}-\frac{8a^{3}b^{2}}{(a+1)^{3}}-\frac{12a^{2}b^{2}}{(a+1)^{3}}+2ab^{2}-\frac{8ab^{2}}{(a+1)^{3}}+2b^{2}-\frac{2b^{2}}{(a+1)^{3}}}{4b^{3}}\right)}{(a+1)^{3}}-\frac{2b^{2}\log\left(x+\frac{\frac{2a^{4}b^{2}}{(a+1)^{3}}+\frac{8a^{3}b^{2}}{(a+1)^{3}}+\frac{12a^{2}b^{2}}{(a+1)^{3}}+2ab^{2}+\frac{8ab^{2}}{(a+1)^{3}}+2b^{2}+\frac{2b^{2}}{(a+1)^{3}}}{4b^{3}}\right)}{(a+1)^{3}}-\frac{-a}{x^{2}(2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2)/x**3,x)
```

```
[Out] 2*b**2*log(x + (-2*a**4*b**2/(a + 1)**3 - 8*a**3*b**2/(a + 1)**3 - 12*a**2*
b**2/(a + 1)**3 + 2*a*b**2 - 8*a*b**2/(a + 1)**3 + 2*b**2 - 2*b**2/(a + 1)*
*3)/(4*b**3))/(a + 1)**3 - 2*b**2*log(x + (2*a**4*b**2/(a + 1)**3 + 8*a**3*
b**2/(a + 1)**3 + 12*a**2*b**2/(a + 1)**3 + 2*a*b**2 + 8*a*b**2/(a + 1)**3
+ 2*b**2 + 2*b**2/(a + 1)**3)/(4*b**3))/(a + 1)**3 - (-a**2 - 4*b*x + 1)/(x
**2*(2*a**2 + 4*a + 2))
```

3.859 $\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{2b^3\log(x)}{(a+1)^4}+\frac{2b^3\log(a+bx+1)}{(a+1)^4}-\frac{2b^2}{(a+1)^3x}+\frac{b}{(a+1)^2x^2}-\frac{1-a}{3(a+1)x^3}$$

```
[Out] 1/3*(-1+a)/(1+a)/x^3+b/(1+a)^2/x^2-2*b^2/(1+a)^3/x-2*b^3*ln(x)/(1+a)^4+2*b^
3*ln(b*x+a+1)/(1+a)^4
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 77}

$$-\frac{2b^2}{(a+1)^3x}-\frac{2b^3\log(x)}{(a+1)^4}+\frac{2b^3\log(a+bx+1)}{(a+1)^4}+\frac{b}{(a+1)^2x^2}-\frac{1-a}{3(a+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a + b*x])*x^4),x]
```

```
[Out] -(1 - a)/(3*(1 + a)*x^3) + b/((1 + a)^2*x^2) - (2*b^2)/((1 + a)^3*x) - (2*b
^3*Log[x])/(1 + a)^4 + (2*b^3*Log[1 + a + b*x])/(1 + a)^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{1-a-bx}{x^4(1+a+bx)}\,dx$$
$$= \int \left(\frac{1-a}{(1+a)x^4} - \frac{2b}{(1+a)^2x^3} + \frac{2b^2}{(1+a)^3x^2} - \frac{2b^3}{(1+a)^4x} + \frac{2b^4}{(1+a)^4(1+a+bx)}\right)dx$$
$$= -\frac{1-a}{3(1+a)x^3} + \frac{b}{(1+a)^2x^2} - \frac{2b^2}{(1+a)^3x} - \frac{2b^3\log(x)}{(1+a)^4} + \frac{2b^3\log(1+a+bx)}{(1+a)^4}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 1.00

$$-\frac{2b^3\log(x)}{(a+1)^4} + \frac{2b^3\log(a+bx+1)}{(a+1)^4} - \frac{2b^2}{(a+1)^3x} + \frac{b}{(a+1)^2x^2} - \frac{1-a}{3(a+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a + b*x])*x^4),x]
```

```
[Out] -1/3*(1 - a)/((1 + a)*x^3) + b/((1 + a)^2*x^2) - (2*b^2)/((1 + a)^3*x) - (2
*b^3*Log[x])/(1 + a)^4 + (2*b^3*Log[1 + a + b*x])/(1 + a)^4
```

fricas [A] time = 0.58, size = 86, normalized size = 1.23

$$\frac{6\,b^3x^3\log\left(bx+a+1\right) - 6\,b^3x^3\log(x) - 6\,(a+1)b^2x^2 + a^4 + 2\,a^3 + 3\left(a^2+2\,a+1\right)bx - 2\,a - 1}{3\left(a^4+4\,a^3+6\,a^2+4\,a+1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*b^3*x^3*log(b*x + a + 1) - 6*b^3*x^3*log(x) - 6*(a + 1)*b^2*x^2 + a^
4 + 2*a^3 + 3*(a^2 + 2*a + 1)*b*x - 2*a - 1)/((a^4 + 4*a^3 + 6*a^2 + 4*a +
1)*x^3)
```

giac [B] time = 0.44, size = 159, normalized size = 2.27

$$-\frac{2\,b^4\log\left(\left|-\frac{a}{bx+a+1} - \frac{1}{bx+a+1} + 1\right|\right)}{a^4b + 4\,a^3b + 6\,a^2b + 4\,ab + b} - \frac{\frac{ab^3-10\,b^3}{a+1} - \frac{3\left(ab^4-8\,b^4\right)}{(bx+a+1)b} + \frac{3\left(a^2b^5-4\,ab^5-5\,b^5\right)}{(bx+a+1)^2b^2}}{3\,(a+1)^3\left(\frac{a}{bx+a+1} + \frac{1}{bx+a+1} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^4,x, algorithm="giac")
```

```
[Out] -2*b^4*log(abs(-a/(b*x + a + 1) - 1/(b*x + a + 1) + 1))/(a^4*b + 4*a^3*b +
6*a^2*b + 4*a*b + b) - 1/3*((a*b^3 - 10*b^3)/(a + 1) - 3*(a*b^4 - 8*b^4)/((
b*x + a + 1)*b) + 3*(a^2*b^5 - 4*a*b^5 - 5*b^5)/((b*x + a + 1)^2*b^2))/((a
+ 1)^3*(a/(b*x + a + 1) + 1/(b*x + a + 1) - 1)^3)
```

maple [A] time = 0.03, size = 75, normalized size = 1.07

$$-\frac{1}{3\left(1+a\right)x^{3}}+\frac{a}{3\left(1+a\right)x^{3}}+\frac{b}{\left(1+a\right)^{2}x^{2}}-\frac{2b^{3}\ln(x)}{\left(1+a\right)^{4}}-\frac{2b^{2}}{\left(1+a\right)^{3}x}+\frac{2b^{3}\ln\left(bx+a+1\right)}{\left(1+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^4,x)
```

```
[Out] -1/3/(1+a)/x^3+1/3/(1+a)/x^3*a+b/(1+a)^2/x^2-2*b^3*ln(x)/(1+a)^4-2*b^2/(1+a
)^3/x+2*b^3*ln(b*x+a+1)/(1+a)^4
```

maxima [A] time = 0.31, size = 108, normalized size = 1.54

$$\frac{2\,b^{3}\log\left(bx+a+1\right)}{a^{4}+4\,a^{3}+6\,a^{2}+4\,a+1}-\frac{2\,b^{3}\log(x)}{a^{4}+4\,a^{3}+6\,a^{2}+4\,a+1}-\frac{6\,b^{2}x^{2}-a^{3}-3\,(a+1)bx-a^{2}+a+1}{3\left(a^{3}+3\,a^{2}+3\,a+1\right)x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^2*(1-(b*x+a)^2)/x^4,x, algorithm="maxima")
```

```
[Out] 2*b^3*log(b*x + a + 1)/(a^4 + 4*a^3 + 6*a^2 + 4*a + 1) - 2*b^3*log(x)/(a^4
+ 4*a^3 + 6*a^2 + 4*a + 1) - 1/3*(6*b^2*x^2 - a^3 - 3*(a + 1)*b*x - a^2 + a
 + 1)/((a^3 + 3*a^2 + 3*a + 1)*x^3)
```

mupad [B] time = 0.10, size = 83, normalized size = 1.19

$$\frac{\frac{a-1}{3\,(a+1)}-\frac{2\,b^{2}\,x^{2}}{(a+1)^{3}}+\frac{b\,x}{(a+1)^{2}}}{x^{3}}+\frac{4\,b^{3}\,\mathrm{atanh}\left(\frac{a^{4}+4\,a^{3}+6\,a^{2}+4\,a+1}{(a+1)^{4}}+\frac{2\,b\,x}{a+1}\right)}{(a+1)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a + b*x)^2 - 1)/(x^4*(a + b*x + 1)^2),x)
```

```
[Out] ((a - 1)/(3*(a + 1)) - (2*b^2*x^2)/(a + 1)^3 + (b*x)/(a + 1)^2)/x^3 + (4*b^
3*atanh((4*a + 6*a^2 + 4*a^3 + a^4 + 1)/(a + 1)^4 + (2*b*x)/(a + 1)))/(a +
1)^4
```

sympy [B] time = 0.53, size = 262, normalized size = 3.74

$$-\frac{2b^3\log\left(x+\frac{-\frac{2a^5b^3}{(a+1)^4}-\frac{10a^4b^3}{(a+1)^4}-\frac{20a^3b^3}{(a+1)^4}-\frac{20a^2b^3}{(a+1)^4}+2ab^3-\frac{10ab^3}{(a+1)^4}+2b^3-\frac{2b^3}{(a+1)^4}}{4b^4}\right)}{(a+1)^4}+\frac{2b^3\log\left(x+\frac{\frac{2a^5b^3}{(a+1)^4}+\frac{10a^4b^3}{(a+1)^4}+\frac{20a^3b^3}{(a+1)^4}+\frac{20a^2b^3}{(a+1)^4}+2ab^3+\frac{10ab^3}{(a+1)^4}+}{4b^4}\right.}{(a+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**2*(1-(b*x+a)**2)/x**4,x)
```

```
[Out] -2*b**3*log(x + (-2*a**5*b**3/(a + 1)**4 - 10*a**4*b**3/(a + 1)**4 - 20*a**
3*b**3/(a + 1)**4 - 20*a**2*b**3/(a + 1)**4 + 2*a*b**3 - 10*a*b**3/(a + 1)*
*4 + 2*b**3 - 2*b**3/(a + 1)**4)/(4*b**4))/(a + 1)**4 + 2*b**3*log(x + (2*a
**5*b**3/(a + 1)**4 + 10*a**4*b**3/(a + 1)**4 + 20*a**3*b**3/(a + 1)**4 + 2
0*a**2*b**3/(a + 1)**4 + 2*a*b**3 + 10*a*b**3/(a + 1)**4 + 2*b**3 + 2*b**3/
(a + 1)**4)/(4*b**4))/(a + 1)**4 - (-a**3 - a**2 + a + 6*b**2*x**2 + x*(-3*
a*b - 3*b) + 1)/(x**3*(3*a**3 + 9*a**2 + 9*a + 3))
```

3.860 $\int e^{-3\tanh^{-1}(a+bx)} x^3\,dx$

Optimal. Leaf size=187

$$\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}\left(22a^2-2(10a+11)bx+54a+29\right)}{8b^4}+\frac{3\left(8a^3+36a^2+44a+17\right)\sqrt{-a-bx+1}\sqrt{a+b}}{8b^4}$$

```
[Out] 3/8*(8*a^3+36*a^2+44*a+17)*arcsin(b*x+a)/b^4-2*x^3*(-b*x-a+1)^(3/2)/b/(b*x+
a+1)^(1/2)+9/4*x^2*(-b*x-a+1)^(3/2)*(b*x+a+1)^(1/2)/b^2+1/8*(-b*x-a+1)^(3/2
)*(29+54*a+22*a^2-2*(11+10*a)*b*x)*(b*x+a+1)^(1/2)/b^4+3/8*(8*a^3+36*a^2+44
*a+17)*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b^4
```

Rubi [A] time = 0.19, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6163, 97, 153, 147, 50, 53, 619, 216}

$$\frac{(-a-bx+1)^{3/2}\sqrt{a+bx+1}\left(22a^2-2(10a+11)bx+54a+29\right)}{8b^4}+\frac{3\left(8a^3+36a^2+44a+17\right)\sqrt{-a-bx+1}\sqrt{a+b}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*x^3*(1 - a - b*x)^(3/2))/(b*Sqrt[1 + a + b*x]) + (3*(17 + 44*a + 36*a^2
 + 8*a^3)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(8*b^4) + (9*x^2*(1 - a - b*
x)^(3/2)*Sqrt[1 + a + b*x])/(4*b^2) + ((1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x
]*(29 + 54*a + 22*a^2 - 2*(11 + 10*a)*b*x))/(8*b^4) + (3*(17 + 44*a + 36*a^
2 + 8*a^3)*ArcSin[a + b*x])/(8*b^4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(a+bx)}x^3\,dx = \int \frac{x^3(1-a-bx)^{3/2}}{(1+a+bx)^{3/2}}\,dx$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{2\int \frac{x^2\left(3(1-a)-\frac{9bx}{2}\right)\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx}{b}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{9x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} - \frac{\int \frac{x\sqrt{1-a-bx}\left(9(1-a)(1+a)b-\frac{3}{2}(11+10a)b^2\right.}{\sqrt{1+a+bx}}}{2b^3}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{9x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}{4b^2} + \frac{(1-a-bx)^{3/2}\sqrt{1+a+bx}\,(29+}{8b^4}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{3\left(17+44a+36a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{9x^2(1-a-}{}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{3\left(17+44a+36a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{9x^2(1-a-}{}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{3\left(17+44a+36a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{9x^2(1-a-}{}$$

$$= -\frac{2x^3(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} + \frac{3\left(17+44a+36a^2+8a^3\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{8b^4} + \frac{9x^2(1-a-}{}$$

Mathematica [A] time = 0.21, size = 231, normalized size = 1.24

$$\frac{6\left(8a^3+36a^2+44a+17\right)\sqrt{b}\sqrt{-a^2-2abx-b^2x^2+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{-b}}\right)+\sqrt{-b}\left(-2a^5-2a^4(bx+38)-5a^3(\right.}{8(-b)^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[-b]*(80 - 2*a^5 - 51*b*x - 40*b^2*x^2 + 17*b^3*x^3 - 8*b^4*x^4 + 2*b^
5*x^5 - 2*a^4*(38 + b*x) - 5*a^3*(31 + 20*b*x) - a^2*(4 + 265*b*x + 12*b^2*
x^2) + a*(157 - 212*b*x - 53*b^2*x^2 + 4*b^3*x^3 + 2*b^4*x^4)) + 6*(17 + 44
*a + 36*a^2 + 8*a^3)*Sqrt[b]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*ArcSinh[(Sqr
t[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(8*(-b)^(9/2)*Sqrt[-((-1 + a +
 b*x)*(1 + a + b*x))])
```

fricas [A] time = 0.82, size = 191, normalized size = 1.02

$$-\frac{3\left(8\,a^4+44\,a^3+\left(8\,a^3+36\,a^2+44\,a+17\right)bx+80\,a^2+61\,a+17\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\left(2\,b^4x^4-\right.}{8\left(b^5x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(3*(8*a^4 + 44*a^3 + (8*a^3 + 36*a^2 + 44*a + 17)*b*x + 80*a^2 + 61*a
+ 17)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*
x + a^2 - 1)) + (2*b^4*x^4 - 6*b^3*x^3 + (10*a + 11)*b^2*x^2 - 2*a^4 - 78*a
^3 - (22*a^2 + 54*a + 29)*b*x - 233*a^2 - 237*a - 80)*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1))/(b^5*x + (a + 1)*b^4)
```

giac [A] time = 0.23, size = 211, normalized size = 1.13

$$-\frac{1}{8}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(\left(2\,x\left(\frac{x}{b}-\frac{ab^{11}+4\,b^{11}}{b^{13}}\right)+\frac{2\,a^2b^{10}+20\,ab^{10}+19\,b^{10}}{b^{13}}\right)x-\frac{2\,a^3b^9+44\,a^2b^9+93\,ab^9}{b^{13}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((2*x*(x/b - (a*b^11 + 4*b^11)/b^13
) + (2*a^2*b^10 + 20*a*b^10 + 19*b^10)/b^13)*x - (2*a^3*b^9 + 44*a^2*b^9 +
93*a*b^9 + 48*b^9)/b^13) - 3/8*(8*a^3 + 36*a^2 + 44*a + 17)*arcsin(-b*x - a
)*sgn(b)/(b^3*abs(b)) - 8*(a^3 + 3*a^2 + 3*a + 1)/(b^3*((sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)*abs(b))
```

maple [B] time = 0.05, size = 1271, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] 27/2/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x*a^2+33/2/b^3*a*(-(x+(
1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+3/b^7/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*
b^2+2*b*(x+(1+a)/b))^(5/2)*a^2+3/b^7/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*
(x+(1+a)/b))^(5/2)*a+2/b^6/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b
))^(5/2)*a^3+9/b^6/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)
*a^2+12/b^6/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)*a+3/b^
3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x*a^3+1/b^7/(x+1/b+a/b)^3*(-(x
+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)*a^3+33/2/b^3/(b^2)^(1/2)*arctan((b^2
)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a+27/2/
b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*
(x+(1+a)/b))^(1/2))*a^2+3/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b
)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a^3+1/b^7/(x+1/b+a/b)^3*(-(x+
(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+5/b^6/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b
^2+2*b*(x+(1+a)/b))^(5/2)+2/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)*
a^3+9/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)*a^2+3/b^4*(-(x+(1+a)/b
)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^4+6/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b
))^(1/2)*x+11/b^4*a*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+27/2/b^4*(-(
x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^3+33/2/b^4*(-(x+(1+a)/b)^2*b^2+2*
b*(x+(1+a)/b))^(1/2)*a^2+6/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a
+6/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2
*b*(x+(1+a)/b))^(1/2))+1/4/b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+1/4/b^4*a*(
-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+3/8*a/b^4*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/8/
b^3*x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/8/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)
*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+4/b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(
1+a)/b))^(3/2)
```

maxima [C] time = 0.43, size = 985, normalized size = 5.27

$$-\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}a^3}{b^6x^2+2\,ab^5x+a^2b^4+2\,b^5x+2\,ab^4+b^4}-\frac{3\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}a^2}{b^6x^2+2\,ab^5x+a^2b^4+2\,b^5x+2\,ab^4+b^4}+\frac{3\left(-b^2x^2-2\,abx-a^2+1\right.}{2\left(b^5x+ab^4+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a^3/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 +
2*b^5*x + 2*a*b^4 + b^4) - 3*(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a^2/(b^6*
x^2 + 2*a*b^5*x + a^2*b^4 + 2*b^5*x + 2*a*b^4 + b^4) + 3/2*(-b^2*x^2 - 2*a*
b*x - a^2 + 1)^(3/2)*a^2/(b^5*x + a*b^4 + b^4) + 6*sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)*a^3/(b^5*x + a*b^4 + b^4) - 3*(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/
2)*a/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 + 2*b^5*x + 2*a*b^4 + b^4) + 3*(-b^2*x^
2 - 2*a*b*x - a^2 + 1)^(3/2)*a/(b^5*x + a*b^4 + b^4) + 18*sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)*a^2/(b^5*x + a*b^4 + b^4) - (-b^2*x^2 - 2*a*b*x - a^2 + 1
)^(3/2)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 + 2*b^5*x + 2*a*b^4 + b^4) + 3/2*(-b
^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/(b^5*x + a*b^4 + b^4) + 18*sqrt(-b^2*x^2
```

```
- 2*a*b*x - a^2 + 1)*a/(b^5*x + a*b^4 + b^4) + 3*a^3*arcsin(b*x + a)/b^4 +
6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^5*x + a*b^4 + b^4) + 1/4*(-b^2*x^2
- 2*a*b*x - a^2 + 1)^(3/2)*x/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 4*b*x
 + 4*a + 3)*a*x/b^3 + 27/2*a^2*arcsin(b*x + a)/b^4 - 3/4*(-b^2*x^2 - 2*a*b*
x - a^2 + 1)^(3/2)*a/b^4 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 4*b*x + 4*a +
 3)*a^2/b^4 + 9/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2/b^4 - 3/2*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 4*b*x + 4*a + 3)*x/b^3 + 3/8*sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)*x/b^3 + 3/2*I*a*arcsin(b*x + a + 2)/b^4 + 18*a*arcsin(b*x + a)/
b^4 - (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/b^4 - 9/2*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 4*b*x + 4*a + 3)*a/b^4 + 75/8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a
/b^4 + 3/2*I*arcsin(b*x + a + 2)/b^4 + 63/8*arcsin(b*x + a)/b^4 - 3*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 4*b*x + 4*a + 3)/b^4 + 9/2*sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\left(1-(a+b\,x)^2\right)^{3/2}}{(a+b\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - (a + b*x)^2)^(3/2))/(a + b*x + 1)^3,x)
[Out] int((x^3*(1 - (a + b*x)^2)^(3/2))/(a + b*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-(a+bx-1)(a+bx+1)\right)^{\frac{3}{2}}}{(a+bx+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2),x)
[Out] Integral(x**3*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)/(a + b*x + 1)**3, x)
```

3.861 $\int e^{-3\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=167

$$-\frac{(6a^2+18a+11)\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{6b^3}-\frac{(6a^2+18a+11)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^3}-\frac{(6a^2+18a+11)\,\mathrm{s}}{2b^3}$$

```
[Out] -1/2*(6*a^2+18*a+11)*arcsin(b*x+a)/b^3-(1+a)^2*(-b*x-a+1)^(5/2)/b^3/(b*x+a+
1)^(1/2)-1/6*(6*a^2+18*a+11)*(-b*x-a+1)^(3/2)*(b*x+a+1)^(1/2)/b^3-1/3*(-b*x
-a+1)^(5/2)*(b*x+a+1)^(1/2)/b^3-1/2*(6*a^2+18*a+11)*(-b*x-a+1)^(1/2)*(b*x+a
+1)^(1/2)/b^3
```

Rubi [A] time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 89, 80, 50, 53, 619, 216}

$$-\frac{(6a^2+18a+11)\sqrt{a+bx+1}(-a-bx+1)^{3/2}}{6b^3}-\frac{(6a^2+18a+11)\sqrt{a+bx+1}\sqrt{-a-bx+1}}{2b^3}-\frac{(6a^2+18a+11)\,\mathrm{s}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] -(((1 + a)^2*(1 - a - b*x)^(5/2))/(b^3*Sqrt[1 + a + b*x])) - ((11 + 18*a +
6*a^2)*Sqrt[1 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^3) - ((11 + 18*a + 6*a^2)*
(1 - a - b*x)^(3/2)*Sqrt[1 + a + b*x])/(6*b^3) - ((1 - a - b*x)^(5/2)*Sqrt[
1 + a + b*x])/(3*b^3) - ((11 + 18*a + 6*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(a+bx)}x^2\,dx=\int\frac{x^2(1-a-bx)^{3/2}}{(1+a+bx)^{3/2}}\,dx$$
$$=-\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}}+\frac{\int\frac{(1-a-bx)^{3/2}\left(-(1+a)(3+2a)b+b^2x\right)}{\sqrt{1+a+bx}}\,dx}{b^3}$$
$$=-\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}}-\frac{(1-a-bx)^{5/2}\sqrt{1+a+bx}}{3b^3}-\frac{\left(11+18a+6a^2\right)\int\frac{(1-a-bx)^{3/2}}{\sqrt{1+a+bx}}\,dx}{3b^2}$$
$$=-\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}}-\frac{\left(11+18a+6a^2\right)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{6b^3}-\frac{(1-a-bx)^{5/2}\ldots}{3b^3}$$
$$=-\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}}-\frac{\left(11+18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3}-\frac{\left(11+18a+6a^2\right)\ldots}{}$$
$$=-\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}}-\frac{\left(11+18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3}-\frac{\left(11+18a+6a^2\right)\ldots}{}$$
$$=-\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}}-\frac{\left(11+18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3}-\frac{\left(11+18a+6a^2\right)\ldots}{}$$
$$=-\frac{(1+a)^2(1-a-bx)^{5/2}}{b^3\sqrt{1+a+bx}}-\frac{\left(11+18a+6a^2\right)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^3}-\frac{\left(11+18a+6a^2\right)\ldots}{}$$

Mathematica [A] time = 0.17, size = 190, normalized size = 1.14

$$-\frac{6\left(6a^2+18a+11\right)\sqrt{b}\sqrt{-a^2-2abx-b^2x^2+1}\,\sinh^{-1}\left(\frac{\sqrt{-b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{-b}\left(2a^4+a^3(2bx+51)+a^2(69bx+\ldots\right.}{6(-b)^{7/2}\sqrt{-((a+bx-1)(a+bx+1)\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] -1/6*(Sqrt[-b]*(-52 + 2*a^4 + 33*b*x + 26*b^2*x^2 - 9*b^3*x^3 + 2*b^4*x^4 +
 a^3*(51 + 2*b*x) + a^2*(50 + 69*b*x) + a*(-51 + 106*b*x + 9*b^2*x^2 + 2*b^
3*x^3)) + 6*(11 + 18*a + 6*a^2)*Sqrt[b]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*A
rcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/((-b)^(7/2)*Sqrt[-(
(-1 + a + b*x)*(1 + a + b*x))])
```

fricas [A] time = 0.66, size = 159, normalized size = 0.95

$$\frac{3\left(6\,a^{3}+\left(6\,a^{2}+18\,a+11\right)bx+24\,a^{2}+29\,a+11\right)\arctan\left(\frac{\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}\,(bx+a)}{b^{2}x^{2}+2\,abx+a^{2}-1}\right)-\left(2\,b^{3}x^{3}-7\,b^{2}x^{2}+2\,a^{3}+(1\right.}{6\left(b^{4}x+(a+1)b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(6*a^3 + (6*a^2 + 18*a + 11)*b*x + 24*a^2 + 29*a + 11)*arctan(sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (2*
b^3*x^3 - 7*b^2*x^2 + 2*a^3 + (16*a + 19)*b*x + 53*a^2 + 103*a + 52)*sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1))/(b^4*x + (a + 1)*b^3)
```

giac [A] time = 0.21, size = 166, normalized size = 0.99

$$-\frac{1}{6}\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}\left(x\left(\frac{2\,x}{b}-\frac{2\,ab^{6}+9\,b^{6}}{b^{8}}\right)+\frac{2\,a^{2}b^{5}+27\,ab^{5}+28\,b^{5}}{b^{8}}\right)+\frac{\left(6\,a^{2}+18\,a+11\right)\arcsin\left(-bx\right.}{2\,b^{2}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x*(2*x/b - (2*a*b^6 + 9*b^6)/b^8)
+ (2*a^2*b^5 + 27*a*b^5 + 28*b^5)/b^8) + 1/2*(6*a^2 + 18*a + 11)*arcsin(-b*
x - a)*sgn(b)/(b^2*abs(b)) + 8*(a^2 + 2*a + 1)/(b^2*((sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)*abs(b))
```

maple [B] time = 0.05, size = 830, normalized size = 4.97

$$-\frac{3\arctan\left(\frac{\sqrt{b^{2}}\left(x+\frac{1+a}{b}-\frac{1}{b}\right)}{\sqrt{-\left(x+\frac{1+a}{b}\right)^{2}b^{2}+2b\left(x+\frac{1+a}{b}\right)}}\right)a^{2}}{b^{2}\sqrt{b^{2}}}-\frac{9\arctan\left(\frac{\sqrt{b^{2}}\left(x+\frac{1+a}{b}-\frac{1}{b}\right)}{\sqrt{-\left(x+\frac{1+a}{b}\right)^{2}b^{2}+2b\left(x+\frac{1+a}{b}\right)}}\right)a}{b^{2}\sqrt{b^{2}}}-\frac{3\sqrt{-\left(x+\frac{1+a}{b}\right)^{2}b^{2}+2b\left(x+\frac{1+a}{b}\right)}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] -3/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2
*b*(x+(1+a)/b))^(1/2))*a^2-9/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-
1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))*a-3/b^2*(-(x+(1+a)/b)^2*b^
```

```
2+2*b*(x+(1+a)/b))^(1/2)*x*a^2-9/b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(
1/2)*x*a-1/b^6/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)*a^2
-2/b^6/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)*a-2/b^5/(x+
1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)*a^2-6/b^5/(x+1/b+a/b)
^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)*a-11/3/b^3*(-(x+(1+a)/b)^2*b^
2+2*b*(x+(1+a)/b))^(3/2)-11/2/b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2
)*x-1/b^6/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-4/b^5/(x
+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-2/b^3*(-(x+(1+a)/b)^
2*b^2+2*b*(x+(1+a)/b))^(3/2)*a^2-6/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))
^(3/2)*a-3/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^3-9/b^3*(-(x+(1
+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^2-11/2/b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x
+(1+a)/b))^(1/2)*a-11/2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/
(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

maxima [C] time = 0.42, size = 622, normalized size = 3.72

$$\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}a^2}{b^5x^2+2\,ab^4x+a^2b^3+2\,b^4x+2\,ab^3+b^3}+\frac{2\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}a}{b^5x^2+2\,ab^4x+a^2b^3+2\,b^4x+2\,ab^3+b^3}-\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}}{b^4x+ab^3+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a^2/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 + 2
*b^4*x + 2*a*b^3 + b^3) + 2*(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3 + 2*b^4*x + 2*a*b^3 + b^3) - (-b^2*x^2 - 2*a*b*x - a
^2 + 1)^(3/2)*a/(b^4*x + a*b^3 + b^3) - 6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*a^2/(b^4*x + a*b^3 + b^3) + (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3 + 2*b^4*x + 2*a*b^3 + b^3) - (-b^2*x^2 - 2*a*b*x - a
^2 + 1)^(3/2)/(b^4*x + a*b^3 + b^3) - 12*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*a/(b^4*x + a*b^3 + b^3) - 3*a^2*arcsin(b*x + a)/b^3 - 6*sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)/(b^4*x + a*b^3 + b^3) + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 4*b*x + 4*a + 3)*x/b^2 - 9*a*arcsin(b*x + a)/b^3 + 1/3*(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)^(3/2)/b^3 + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 4*b*x + 4*a + 3)
*a/b^3 - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/b^3 - 1/2*I*arcsin(b*x + a
+ 2)/b^3 - 6*arcsin(b*x + a)/b^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 4*b*x + 4
*a + 3)/b^3 - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(1-(a+b\,x)^2\right)^{3/2}}{(a+b\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - (a + b*x)^2)^(3/2))/(a + b*x + 1)^3,x)
```

```
[Out] int((x^2*(1 - (a + b*x)^2)^(3/2))/(a + b*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(-\left(a+bx-1\right)\left(a+bx+1\right)\right)^{\frac{3}{2}}}{\left(a+bx+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral(x**2*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)/(a + b*x + 1)**3, x)
```

3.862 $\int e^{-3\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=119

$$\frac{(a+1)(-a-bx+1)^{5/2}}{b^2\sqrt{a+bx+1}}+\frac{(2a+3)\sqrt{a+bx+1}\,(-a-bx+1)^{3/2}}{2b^2}+\frac{3(2a+3)\sqrt{a+bx+1}\,\sqrt{-a-bx+1}}{2b^2}+\frac{3(2a+3)\sin^{-1}(a+bx)}{2b^2}$$

```
[Out] 3/2*(3+2*a)*arcsin(b*x+a)/b^2+(1+a)*(-b*x-a+1)^(5/2)/b^2/(b*x+a+1)^(1/2)+1/
2*(3+2*a)*(-b*x-a+1)^(3/2)*(b*x+a+1)^(1/2)/b^2+3/2*(3+2*a)*(-b*x-a+1)^(1/2)
*(b*x+a+1)^(1/2)/b^2
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 78, 50, 53, 619, 216}

$$\frac{(a+1)(-a-bx+1)^{5/2}}{b^2\sqrt{a+bx+1}}+\frac{(2a+3)\sqrt{a+bx+1}\,(-a-bx+1)^{3/2}}{2b^2}+\frac{3(2a+3)\sqrt{a+bx+1}\,\sqrt{-a-bx+1}}{2b^2}+\frac{3(2a+3)\sin^{-1}(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] ((1 + a)*(1 - a - b*x)^(5/2))/(b^2*Sqrt[1 + a + b*x]) + (3*(3 + 2*a)*Sqrt[1
 - a - b*x]*Sqrt[1 + a + b*x])/(2*b^2) + ((3 + 2*a)*(1 - a - b*x)^(3/2)*Sqr
t[1 + a + b*x])/(2*b^2) + (3*(3 + 2*a)*ArcSin[a + b*x])/(2*b^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(a+bx)}x\,dx = \int \frac{x(1-a-bx)^{3/2}}{(1+a+bx)^{3/2}}\,dx$$
$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{(3+2a)\int \frac{(1-a-bx)^{3/2}}{\sqrt{1+a+bx}}\,dx}{b}$$
$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+a+bx}}{2b^2} + \frac{(3(3+2a))\int \frac{\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx}{2b}$$
$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{3(3+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+}}{2b^2}$$
$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{3(3+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+}}{2b^2}$$

$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{3(3+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+}}{2b^2}$$
$$= \frac{(1+a)(1-a-bx)^{5/2}}{b^2\sqrt{1+a+bx}} + \frac{3(3+2a)\sqrt{1-a-bx}\sqrt{1+a+bx}}{2b^2} + \frac{(3+2a)(1-a-bx)^{3/2}\sqrt{1+}}{2b^2}$$

Mathematica [A] time = 0.13, size = 157, normalized size = 1.32

$$\frac{6(2a+3)\sqrt{b}\sqrt{-a^2-2abx-b^2x^2+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{-a-bx+1}}{\sqrt{2}\sqrt{-b}}\right)+\sqrt{-b}\left(-a^3-a^2(bx+14)+a\left(b^2x^2-20bx+1\right)+b^3\right.}{2(-b)^{5/2}\sqrt{-((a+bx-1)(a+bx+1))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(3*ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[-b]*(14 - a^3 - 9*b*x - 6*b^2*x^2 + b^3*x^3 - a^2*(14 + b*x) + a*(1 -
 20*b*x + b^2*x^2)) + 6*(3 + 2*a)*Sqrt[b]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]
*ArcSinh[(Sqrt[b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[-b])])/(2*(-b)^(5/2)*Sqr
t[-((-1 + a + b*x)*(1 + a + b*x))])
```

fricas [A] time = 1.26, size = 130, normalized size = 1.09

$$-\frac{3\left((2a+3)bx+2a^2+5a+3\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)+\left(b^2x^2-a^2-5bx-15a-14\right)\sqrt{-b^2x^2-2abx}}{2\left(b^3x+(a+1)b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*((2*a + 3)*b*x + 2*a^2 + 5*a + 3)*arctan(sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (b^2*x^2 - a^2 - 5*b*x
- 15*a - 14)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/(b^3*x + (a + 1)*b^2)
```

giac [A] time = 0.24, size = 127, normalized size = 1.07

$$-\frac{1}{2}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(\frac{x}{b}-\frac{ab^2+6\,b^2}{b^4}\right)-\frac{3\,(2\,a+3)\arcsin\left(-bx-a\right)\mathrm{sgn}(b)}{2\,b|b|}-\frac{8\,(a+1)}{b\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b}{b^2x+ab}+1\right)|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x/b - (a*b^2 + 6*b^2)/b^4) - 3/2*(
2*a + 3)*arcsin(-b*x - a)*sgn(b)/(b*abs(b)) - 8*(a + 1)/(b*((sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)*abs(b))
```

maple [B] time = 0.04, size = 543, normalized size = 4.56

$$\frac{\left(-\left(x+\frac{1+a}{b}\right)^2b^2+2b\left(x+\frac{1+a}{b}\right)\right)^{\frac{5}{2}}}{b^5\left(x+\frac{1}{b}+\frac{a}{b}\right)^3}+\frac{3\left(-\left(x+\frac{1+a}{b}\right)^2b^2+2b\left(x+\frac{1+a}{b}\right)\right)^{\frac{5}{2}}}{b^4\left(x+\frac{1}{b}+\frac{a}{b}\right)^2}+\frac{3\left(-\left(x+\frac{1+a}{b}\right)^2b^2+2b\left(x+\frac{1+a}{b}\right)\right)^{\frac{3}{2}}}{b^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] 1/b^5/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+3/b^4/(x+1/b
+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+3/b^2*(-(x+(1+a)/b)^2*b^
2+2*b*(x+(1+a)/b))^(3/2)+9/2/b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x
+9/2/b^2*a*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)+9/2/b/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
+1/b^5*a/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+2/b^4*a/(
x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+2/b^2*a*(-(x+(1+a)/
b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+3/b*a*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^
(1/2)*x+3/b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a^2+3/b*a/(b^2)^(1
/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))
^(1/2))
```

maxima [B] time = 0.41, size = 299, normalized size = 2.51

$$-\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}a}{b^4x^2+2\,ab^3x+a^2b^2+2\,b^3x+2\,ab^2+b^2}-\frac{\left(-b^2x^2-2\,abx-a^2+1\right)^{\frac{3}{2}}}{b^4x^2+2\,ab^3x+a^2b^2+2\,b^3x+2\,ab^2+b^2}+\frac{\left(-b^2x^2-2\,abx-a^2+1\right)}{2\left(b^3x+ab^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)*a/(b^4*x^2 + 2*a*b^3*x + a^2*b^2 + 2*
b^3*x + 2*a*b^2 + b^2) - (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/(b^4*x^2 + 2*
a*b^3*x + a^2*b^2 + 2*b^3*x + 2*a*b^2 + b^2) + 1/2*(-b^2*x^2 - 2*a*b*x - a^
2 + 1)^(3/2)/(b^3*x + a*b^2 + b^2) + 6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a
/(b^3*x + a*b^2 + b^2) + 3*a*arcsin(b*x + a)/b^2 + 6*sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)/(b^3*x + a*b^2 + b^2) + 9/2*arcsin(b*x + a)/b^2 + 3/2*sqrt(-b^
2*x^2 - 2*a*b*x - a^2 + 1)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(1-(a+b\,x)^2\right)^{3/2}}{(a+b\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - (a + b*x)^2)^(3/2))/(a + b*x + 1)^3,x)
```

```
[Out] int((x*(1 - (a + b*x)^2)^(3/2))/(a + b*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(a+bx-1\right)\left(a+bx+1\right)\right)^{\frac{3}{2}}}{\left(a+bx+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral(x*(-(a + b*x - 1)*(a + b*x + 1))**(3/2)/(a + b*x + 1)**3, x)
```

3.863 $\int e^{-3\tanh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2(-a-bx+1)^{3/2}}{b\sqrt{a+bx+1}}-\frac{3\sqrt{a+bx+1}\,\sqrt{-a-bx+1}}{b}-\frac{3\sin^{-1}(a+bx)}{b}$$

```
[Out] -3*arcsin(b*x+a)/b-2*(-b*x-a+1)^(3/2)/b/(b*x+a+1)^(1/2)-3*(-b*x-a+1)^(1/2)*
(b*x+a+1)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6161, 47, 50, 53, 619, 216}

$$-\frac{2(-a-bx+1)^{3/2}}{b\sqrt{a+bx+1}}-\frac{3\sqrt{a+bx+1}\,\sqrt{-a-bx+1}}{b}-\frac{3\sin^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-3*ArcTanh[a + b*x]),x]
```

```
[Out] (-2*(1 - a - b*x)^(3/2))/(b*Sqrt[1 + a + b*x]) - (3*Sqrt[1 - a - b*x]*Sqrt[
1 + a + b*x])/b - (3*ArcSin[a + b*x])/b
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
```

```
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\tanh^{-1}(a+bx)}\,dx &= \int \frac{(1-a-bx)^{3/2}}{(1+a+bx)^{3/2}}\,dx\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - 3\int \frac{\sqrt{1-a-bx}}{\sqrt{1+a+bx}}\,dx\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - \frac{3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} - 3\int \frac{1}{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - \frac{3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} - 3\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - \frac{3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b^2}\\
&= -\frac{2(1-a-bx)^{3/2}}{b\sqrt{1+a+bx}} - \frac{3\sqrt{1-a-bx}\,\sqrt{1+a+bx}}{b} - \frac{3\sin^{-1}(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.63

$$
\frac{\sqrt{1-(a+bx)^2}\left(-\frac{4}{a+bx+1}-1\right)}{b} - \frac{3\sin^{-1}(a+bx)}{b}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-3*ArcTanh[a + b*x]),x]
```

```
[Out] (Sqrt[1 - (a + b*x)^2]*(-1 - 4/(1 + a + b*x)))/b - (3*ArcSin[a + b*x])/b
```

fricas [A] time = 0.63, size = 101, normalized size = 1.49

$$\frac{3\,(bx+a+1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a+5)}{b^2x+(a+1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (3*(b*x + a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x
^2 + 2*a*b*x + a^2 - 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a + 5)
)/(b^2*x + (a + 1)*b)
```

giac [A] time = 0.21, size = 94, normalized size = 1.38

$$\frac{3\,\arcsin\left(-bx-a\right)\mathrm{sgn}\left(b\right)}{|b|}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b}+\frac{8}{\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b}{b^2x+ab}+1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3*arcsin(-b*x - a)*sgn(b)/abs(b) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/b + 8
/(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)*abs(b
))
```

maple [B] time = 0.04, size = 264, normalized size = 3.88

$$-\frac{\left(-\left(x+\frac{1+a}{b}\right)^2b^2+2b\left(x+\frac{1+a}{b}\right)\right)^{\frac{5}{2}}}{b^4\left(x+\frac{1}{b}+\frac{a}{b}\right)^3}-\frac{2\left(-\left(x+\frac{1+a}{b}\right)^2b^2+2b\left(x+\frac{1+a}{b}\right)\right)^{\frac{5}{2}}}{b^3\left(x+\frac{1}{b}+\frac{a}{b}\right)^2}-\frac{2\left(-\left(x+\frac{1+a}{b}\right)^2b^2+2b\left(x+\frac{1+a}{b}\right)\right)^{\frac{3}{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x)
```

```
[Out] -1/b^4/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-2/b^3/(x+1/
b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-2/b*(-(x+(1+a)/b)^2*b^2
+2*b*(x+(1+a)/b))^(3/2)-3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-3/b*
(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-3/(b^2)^(1/2)*arctan((b^2)^(1/
2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

maxima [A] time = 0.41, size = 104, normalized size = 1.53

$$\frac{\left(-b^2x^2 - 2\,abx - a^2 + 1\right)^{\frac{3}{2}}}{b^3x^2 + 2\,ab^2x + a^2b + 2\,b^2x + 2\,ab + b} - \frac{3\,\arcsin\left(bx + a\right)}{b} - \frac{6\,\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}}{b^2x + ab + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (-b^2*x^2 - 2*a*b*x - a^2 + 1)^(3/2)/(b^3*x^2 + 2*a*b^2*x + a^2*b + 2*b^2*x
 + 2*a*b + b) - 3*arcsin(b*x + a)/b - 6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/
(b^2*x + a*b + b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-{\left(a+b\,x\right)}^2\right)}^{3/2}}{{\left(a+b\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(3/2)/(a + b*x + 1)^3,x)
```

```
[Out] int((1 - (a + b*x)^2)^(3/2)/(a + b*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(a + bx - 1\right)\left(a + bx + 1\right)\right)^{\frac{3}{2}}}{\left(a + bx + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((-(a + b*x - 1)*(a + b*x + 1))**(3/2)/(a + b*x + 1)**3, x)
```

3.864 $\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=103

$$-\frac{2(1-a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(a+1)\sqrt{1-a^2}}+\frac{4\sqrt{-a-bx+1}}{(a+1)\sqrt{a+bx+1}}+\sin^{-1}(a+bx)$$

```
[Out] arcsin(b*x+a)-2*(1-a)^2*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b
*x-a+1)^(1/2))/(1+a)/(-a^2+1)^(1/2)+4*(-b*x-a+1)^(1/2)/(1+a)/(b*x+a+1)^(1/2
)
```

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6163, 98, 157, 53, 619, 216, 93, 208}

$$-\frac{2(1-a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(a+1)\sqrt{1-a^2}}+\frac{4\sqrt{-a-bx+1}}{(a+1)\sqrt{a+bx+1}}+\sin^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a + b*x])*x),x]
```

```
[Out] (4*Sqrt[1 - a - b*x])/((1 + a)*Sqrt[1 + a + b*x]) + ArcSin[a + b*x] - (2*(1
 - a)^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b
*x])])/((1 + a)*Sqrt[1 - a^2])
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x}\,dx = \int \frac{(1-a-bx)^{3/2}}{x(1+a+bx)^{3/2}}\,dx$$

$$= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} + \frac{2\int \frac{\frac{1}{2}(1-a)^2b+\frac{1}{2}(1+a)b^2x}{x\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{(1+a)b}$$

$$= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} + \frac{(1-a)^2\int \frac{1}{x\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{1+a} + b\int \frac{1}{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx$$

$$= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} + \frac{\left(2(1-a)^2\right)\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1+a} + b\int \frac{}{\sqrt{(1-a)}}$$

$$= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} - \frac{2(1-a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{1-a-bx}}\right)}{(1+a)\sqrt{1-a^2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-\right.}{2b}$$

$$= \frac{4\sqrt{1-a-bx}}{(1+a)\sqrt{1+a+bx}} + \sin^{-1}(a+bx) - \frac{2(1-a)^2\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{1-a-bx}}\right)}{(1+a)\sqrt{1-a^2}}$$

Mathematica [A] time = 0.44, size = 137, normalized size = 1.33

$$-\frac{4(a+bx-1)}{(a+1)\sqrt{-((a+bx-1)(a+bx+1))}}+\frac{2\sqrt{-b}\,\sinh^{-1}\left(\frac{\sqrt{-b}\,\sqrt{-a-bx+1}}{\sqrt{2}\,\sqrt{b}}\right)}{\sqrt{b}}-\frac{2(a-1)^{3/2}\tanh^{-1}\left(\frac{\sqrt{-a-1}\,\sqrt{-a-bx+1}}{\sqrt{a-1}\,\sqrt{a+bx+1}}\right)}{(-a-1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a + b*x])*x),x]
```

```
[Out] (-4*(-1 + a + b*x))/((1 + a)*Sqrt[-((-1 + a + b*x)*(1 + a + b*x))]) + (2*Sq
rt[-b]*ArcSinh[(Sqrt[-b]*Sqrt[1 - a - b*x])/(Sqrt[2]*Sqrt[b])])/Sqrt[b] - (
2*(-1 + a)^(3/2)*ArcTanh[(Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqr
t[1 + a + b*x])])/(-1 - a)^(3/2)
```

fricas [B] time = 0.46, size = 438, normalized size = 4.25

$$\left[\frac{\left((a-1)bx+a^2-1\right)\sqrt{-\frac{a-1}{a+1}}\,\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-4a^2-2\sqrt{-b^2x^2-2abx-a^2+1}\left(a^3+\left(a^2+a\right)bx+a^2-a-1\right)\sqrt{-\frac{a-1}{a+1}}+2}{x^2}\right)-2}{2\left((a+1)bx+a^2+2a+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(((a - 1)*b*x + a^2 - 1)*sqrt(-(a - 1)/(a + 1))*log(((2*a^2 - 1)*b^2*x
^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*(a^3 + (a^2 + a)*b*x + a^2 - a - 1)*sqrt(-(a - 1)/(a + 1)) + 2)/x^2) - 2*(
(a + 1)*b*x + a^2 + 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x
 + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 8*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
)/((a + 1)*b*x + a^2 + 2*a + 1), (((a - 1)*b*x + a^2 - 1)*sqrt((a - 1)/(a +
 1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt((a -
1)/(a + 1))/((a - 1)*b^2*x^2 + a^3 + 2*(a^2 - a)*b*x - a^2 - a + 1)) - ((a
+ 1)*b*x + a^2 + 2*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x +
a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 4*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/(
(a + 1)*b*x + a^2 + 2*a + 1)]
```

giac [A] time = 0.22, size = 154, normalized size = 1.50

$$-\frac{b\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{|b|}+\frac{2\left(a^2b-2\,ab+b\right)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}\left(a|b|+|b|\right)}-\frac{8\,b}{\left(a|b|+|b|\right)\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b}{b^2x+ab}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -b*arcsin(-b*x - a)*sgn(b)/abs(b) + 2*(a^2*b - 2*a*b + b)*arctan(((sqrt(-b^
2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/
(sqrt(a^2 - 1)*(a*abs(b) + abs(b))) - 8*b/((a*abs(b) + abs(b))*((sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1))
```

maple [B] time = 0.05, size = 1062, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x,x)
```

```
[Out] 1/3/(1+a)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-1/2/(1+a)^3*a*b*(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)*x-3/2/(1+a)^3*a^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2/(1+a)^3
*a*b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
+1/(1+a)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(1+a)^3*a^3*b/(b^2)^(1/2)*arcta
n((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/(1+a)^3*(-a^2+1)^(1
```

```
/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x
)*a^2-1/(1+a)^3*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*
x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/3/(1+a)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b
))^(3/2)-1/2/(1+a)^3*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-1/2/(1+
a)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-1/2/(1+a)^3*b/(b^2)^(1/2)
*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1
/2))+1/(1+a)/b^3/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+2
/(1+a)/b^2/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+2/(1+a)
*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+3/(1+a)*b*(-(x+(1+a)/b)^2*b^2+2
*b*(x+(1+a)/b))^(1/2)*x+3/(1+a)*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*
a+3/(1+a)*b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*
b^2+2*b*(x+(1+a)/b))^(1/2))-1/(1+a)^2/b^2/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2
+2*b*(x+(1+a)/b))^(5/2)-1/(1+a)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2
)-3/2/(1+a)^2*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-3/2/(1+a)^2*(-
(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-3/2/(1+a)^2*b/(b^2)^(1/2)*arctan
((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(bx+a)^2+1\right)^{\frac{3}{2}}}{(bx+a+1)^3 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
[Out] integrate((-(b*x + a)^2 + 1)^(3/2)/((b*x + a + 1)^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-(a+b\,x)^2\right)^{3/2}}{x\,(a+b\,x+1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(3/2)/(x*(a + b*x + 1)^3),x)
[Out] int((1 - (a + b*x)^2)^(3/2)/(x*(a + b*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(a+bx-1\right)\left(a+bx+1\right)\right)^{\frac{3}{2}}}{x\left(a+bx+1\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2)/x,x)
[Out] Integral((-(a + b*x - 1)*(a + b*x + 1))**(3/2)/(x*(a + b*x + 1)**3), x)
```

3.865 $\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=130

$$\frac{6(1-a)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)^2\sqrt{1-a^2}}-\frac{(-a-bx+1)^{3/2}}{(a+1)x\sqrt{a+bx+1}}-\frac{6b\sqrt{-a-bx+1}}{(a+1)^2\sqrt{a+bx+1}}$$

```
[Out] 6*(1-a)*b*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(1/2))
/(1+a)^2/(-a^2+1)^(1/2)-(-b*x-a+1)^(3/2)/(1+a)/x/(b*x+a+1)^(1/2)-6*b*(-b*x-
a+1)^(1/2)/(1+a)^2/(b*x+a+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 94, 93, 208}

$$\frac{6(1-a)b\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)^2\sqrt{1-a^2}}-\frac{(-a-bx+1)^{3/2}}{(a+1)x\sqrt{a+bx+1}}-\frac{6b\sqrt{-a-bx+1}}{(a+1)^2\sqrt{a+bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a + b*x])*x^2),x]
```

```
[Out] (-6*b*Sqrt[1 - a - b*x])/((1 + a)^2*Sqrt[1 + a + b*x]) - (1 - a - b*x)^(3/2
)/((1 + a)*x*Sqrt[1 + a + b*x]) + (6*(1 - a)*b*ArcTanh[(Sqrt[1 - a]*Sqrt[1
+ a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 + a)^2*Sqrt[1 - a^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^2}\,dx = \int \frac{(1-a-bx)^{3/2}}{x^2(1+a+bx)^{3/2}}\,dx$$

$$= -\frac{(1-a-bx)^{3/2}}{(1+a)x\sqrt{1+a+bx}} - \frac{(3b)\int \frac{\sqrt{1-a-bx}}{x(1+a+bx)^{3/2}}\,dx}{1+a}$$

$$= -\frac{6b\sqrt{1-a-bx}}{(1+a)^2\sqrt{1+a+bx}} - \frac{(1-a-bx)^{3/2}}{(1+a)x\sqrt{1+a+bx}} - \frac{(3(1-a)b)\int \frac{1}{x\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{(1+a)^2}$$

$$= -\frac{6b\sqrt{1-a-bx}}{(1+a)^2\sqrt{1+a+bx}} - \frac{(1-a-bx)^{3/2}}{(1+a)x\sqrt{1+a+bx}} - \frac{(6(1-a)b)\,\mathrm{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \ldots\right)}{(1+a)^2}$$

$$= -\frac{6b\sqrt{1-a-bx}}{(1+a)^2\sqrt{1+a+bx}} - \frac{(1-a-bx)^{3/2}}{(1+a)x\sqrt{1+a+bx}} + \frac{6(1-a)b\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{1-a-bx}}\right)}{(1+a)^2\sqrt{1-a^2}}$$

Mathematica [A] time = 0.10, size = 106, normalized size = 0.82

$$\frac{\sqrt{-a-bx+1}\,\left(a^2+abx-5bx-1\right)}{(a+1)^2x\sqrt{a+bx+1}} + \frac{6\sqrt{a-1}\,b\tanh^{-1}\left(\frac{\sqrt{-a-1}\,\sqrt{-a-bx+1}}{\sqrt{a-1}\,\sqrt{a+bx+1}}\right)}{(-a-1)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a + b*x])*x^2),x]
```

```
[Out] (Sqrt[1 - a - b*x]*(-1 + a^2 - 5*b*x + a*b*x))/((1 + a)^2*x*Sqrt[1 + a + b*
x]) + (6*Sqrt[-1 + a]*b*ArcTanh[(Sqrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 +
 a]*Sqrt[1 + a + b*x])])/(-1 - a)^(5/2)
```

fricas [A] time = 0.70, size = 369, normalized size = 2.84

$$\left[\frac{3\left(b^{2}x^{2}+(a+1)bx\right)\sqrt{-\frac{a-1}{a+1}}\,\log\left(\frac{\left(2\,a^{2}-1\right)b^{2}x^{2}+2\,a^{4}+4\left(a^{3}-a\right)bx-4\,a^{2}-2\,\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}\left(a^{3}+\left(a^{2}+a\right)bx+a^{2}-a-1\right)\sqrt{-\frac{a-1}{a+1}}+2}{x^{2}}\right)+2}{2\left(\left(a^{2}+2\,a+1\right)bx^{2}+\left(a^{3}+3\,a^{2}+3\,a+1\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + (a + 1)*b*x)*sqrt(-(a - 1)/(a + 1))*log(((2*a^2 - 1)*b^2
*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(a^3 + (a^2 + a)*b*x + a^2 - a - 1)*sqrt(-(a - 1)/(a + 1)) + 2)/x^2) + 2
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((a - 5)*b*x + a^2 - 1))/((a^2 + 2*a +
1)*b*x^2 + (a^3 + 3*a^2 + 3*a + 1)*x), (3*(b^2*x^2 + (a + 1)*b*x)*sqrt((a -
 1)/(a + 1))*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt((a - 1)/(a + 1))/((a - 1)*b^2*x^2 + a^3 + 2*(a^2 - a)*b*x - a^2 - a + 1)
) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*((a - 5)*b*x + a^2 - 1))/((a^2 + 2*a
 + 1)*b*x^2 + (a^3 + 3*a^2 + 3*a + 1)*x)]
```

giac [B] time = 0.21, size = 604, normalized size = 4.65

$$\frac{6\left(ab^{2}-b^{2}\right)\arctan\left(\frac{\frac{\left(\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}\,|b|+b\right)a}{b^{2}x+ab}-1}{\sqrt{a^{2}-1}}\right)}{\left(a^{2}|b|+2\,a|b|+|b|\right)\sqrt{a^{2}-1}}+\frac{2\left(\frac{\left(\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}\,|b|+b\right)a^{2}b^{2}}{b^{2}x+ab}+\frac{4\left(\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}\,|b|+b\right)^{2}a^{2}b^{2}}{\left(b^{2}x+ab\right)^{2}}+5\,a^{2}b\cdots\right.}{\left(a^{3}|b|+2\,a^{2}|b|+a|b|\right)\left(\frac{\left(\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}\,|b|+b\right)a}{b^{2}x+ab}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 6*(a*b^2 - b^2)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(
b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^2*abs(b) + 2*a*abs(b) + abs(b))*sqrt(a
^2 - 1)) + 2*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^2*b^2/(b^2*
x + a*b) + 4*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^2*b^2/(b^2
*x + a*b)^2 + 5*a^2*b^2 - 10*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b
)*a*b^2/(b^2*x + a*b) - (sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a
*b^2/(b^2*x + a*b)^2 - a*b^2 + (sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) +
 b)*b^2/(b^2*x + a*b) + (sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*b
^2/(b^2*x + a*b)^2)/((a^3*abs(b) + 2*a^2*abs(b) + a*abs(b))*((sqrt(-b^2*x^2
 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) + (sqrt(-b^2*x^2 - 2*a*b*
```

```
x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + (sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)^3*a/(b^2*x + a*b)^3 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
abs(b) + b)^2/(b^2*x + a*b)^2))
```

maple [B] time = 0.06, size = 1710, normalized size = 13.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^2,x)
```

```
[Out] -3/(1+a)^3*a^4*b^2/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2))+9/2/(1+a)^3*a^2*b^2/(-a^2+1)/(b^2)^(1/2)*arctan((b^
2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/2/(1+a)^3*a^2*b^2/(-a^2+
1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3/(1+a)^2*b^2/(b^2)^(1/2)*arctan((b^2)^
(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+2/(1+a)^3
/b/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+3/(1+a)^3*b^2*(
-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+3/(1+a)^3*b*(-(x+(1+a)/b)^2*b^2
+2*b*(x+(1+a)/b))^(1/2)*a-1/(1+a)^2/b^2/(x+1/b+a/b)^3*(-(x+(1+a)/b)^2*b^2+2
*b*(x+(1+a)/b))^(5/2)-2/(1+a)^2/b/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+
(1+a)/b))^(5/2)-3/(1+a)^2*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-
3/(1+a)^2*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a+3/2/(1+a)^4*b^2*(-
(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+3/2/(1+a)^4*b*(-(x+(1+a)/b)^2*b^
2+2*b*(x+(1+a)/b))^(1/2)*a+3/2/(1+a)^4*b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(
x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))-1/(1+a)^3/(-a^2+
1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+9/2/(1+a)^4*b*a^2*(-b^2*x^2-2*a*b*x-a^2
+1)^(1/2)+3/(1+a)^4*b*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+3/(1+a)^3*b^2/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+1/(1+a)^4*
b*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)-1/(1+a)^4*b*(-b^2*x^2-2*a*b*x-
a^2+1)^(3/2)-3/(1+a)^4*b*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-2/(1+a)^2*b*(-(x+(1
+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+2/(1+a)^3*b*(-(x+(1+a)/b)^2*b^2+2*b*(x+
(1+a)/b))^(3/2)+3/2/(1+a)^4*b^2*a*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+9/2/(1+a
)^4*b^2*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2))-3/(1+a)^4*b^2*a^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2))-3/(1+a)^4*b*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2
+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2-2/(1+a)^3*a*b/(-a^2+1)*(-b
^2*x^2-2*a*b*x-a^2+1)^(3/2)+9/2/(1+a)^3*a^3*b/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2)-9/2/(1+a)^3*a*b/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/(1+a)^
3*a^3*b/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2))/x)+3/(1+a)^3*a*b/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-
a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/(1+a)^3*b^2/(-a^2+1)*(-b^
2*x^2-2*a*b*x-a^2+1)^(3/2)*x-3/2/(1+a)^3*b^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2
```

```
+1)^(1/2)*x-3/2/(1+a)^3*b^2/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)
/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(bx+a)^2+1\right)^{\frac{3}{2}}}{(bx+a+1)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((-(b*x + a)^2 + 1)^(3/2)/((b*x + a + 1)^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-(a+b\,x)^2\right)^{3/2}}{x^2\,(a+b\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(3/2)/(x^2*(a + b*x + 1)^3),x)
[Out] int((1 - (a + b*x)^2)^(3/2)/(x^2*(a + b*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(a+bx-1)(a+bx+1)\right)^{\frac{3}{2}}}{x^2\,(a+bx+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2)/x**2,x)
[Out] Integral((-(a + b*x - 1)*(a + b*x + 1))**(3/2)/(x**2*(a + b*x + 1)**3), x)
```

3.866 $\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=200

$$-\frac{3(3-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)^3\sqrt{1-a^2}}-\frac{(-a-bx+1)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{a+bx+1}}+\frac{3(3-2a)b^2\sqrt{-a-bx+1}}{(1-a)(a+1)^3\sqrt{a+bx+1}}+\frac{(3-2a)b(-a-}{2(1-a)(a+1)^2x}$$

```
[Out] -3*(3-2*a)*b^2*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(
1/2))/(1+a)^3/(-a^2+1)^(1/2)+1/2*(3-2*a)*b*(-b*x-a+1)^(3/2)/(1-a)/(1+a)^2/x
/(b*x+a+1)^(1/2)-1/2*(-b*x-a+1)^(5/2)/(-a^2+1)/x^2/(b*x+a+1)^(1/2)+3*(3-2*a
)*b^2*(-b*x-a+1)^(1/2)/(1-a)/(1+a)^3/(b*x+a+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6163, 96, 94, 93, 208}

$$-\frac{3(3-2a)b^2\tanh^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{-a-bx+1}}\right)}{(a+1)^3\sqrt{1-a^2}}-\frac{(-a-bx+1)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{a+bx+1}}+\frac{3(3-2a)b^2\sqrt{-a-bx+1}}{(1-a)(a+1)^3\sqrt{a+bx+1}}+\frac{(3-2a)b(-a-}{2(1-a)(a+1)^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a + b*x])*x^3),x]
```

```
[Out] (3*(3 - 2*a)*b^2*Sqrt[1 - a - b*x])/((1 - a)*(1 + a)^3*Sqrt[1 + a + b*x]) +
 ((3 - 2*a)*b*(1 - a - b*x)^(3/2))/(2*(1 - a)*(1 + a)^2*x*Sqrt[1 + a + b*x]
) - (1 - a - b*x)^(5/2)/(2*(1 - a^2)*x^2*Sqrt[1 + a + b*x]) - (3*(3 - 2*a)*
b^2*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])
])/((1 + a)^3*Sqrt[1 - a^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1-a-bx)^{3/2}}{x^3(1+a+bx)^{3/2}}\,dx$$

$$= -\frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} - \frac{((3-2a)b)\int \frac{(1-a-bx)^{3/2}}{x^2(1+a+bx)^{3/2}}\,dx}{2\left(1-a^2\right)}$$

$$= \frac{(3-2a)b(1-a-bx)^{3/2}}{2(1-a)(1+a)^2x\sqrt{1+a+bx}} - \frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} + \frac{\left(3(3-2a)b^2\right)\int \frac{\sqrt{1-a-bx}}{x(1+a+bx)^{3/2}}\,dx}{2(1-a)(1+a)^2}$$

$$= \frac{3(3-2a)b^2\sqrt{1-a-bx}}{(1-a)(1+a)^3\sqrt{1+a+bx}} + \frac{(3-2a)b(1-a-bx)^{3/2}}{2(1-a)(1+a)^2x\sqrt{1+a+bx}} - \frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} +$$

$$= \frac{3(3-2a)b^2\sqrt{1-a-bx}}{(1-a)(1+a)^3\sqrt{1+a+bx}} + \frac{(3-2a)b(1-a-bx)^{3/2}}{2(1-a)(1+a)^2x\sqrt{1+a+bx}} - \frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} +$$

$$= \frac{3(3-2a)b^2\sqrt{1-a-bx}}{(1-a)(1+a)^3\sqrt{1+a+bx}} + \frac{(3-2a)b(1-a-bx)^{3/2}}{2(1-a)(1+a)^2x\sqrt{1+a+bx}} - \frac{(1-a-bx)^{5/2}}{2\left(1-a^2\right)x^2\sqrt{1+a+bx}} -$$

Mathematica [A] time = 0.16, size = 139, normalized size = 0.70

$$\frac{\sqrt{-a-bx+1}\left(a^3+a^2-a\left(b^2x^2-5bx+1\right)+14b^2x^2+5bx-1\right)}{2(a+1)^3x^2\sqrt{a+bx+1}}+\frac{3(2a-3)b^2\tanh^{-1}\left(\frac{\sqrt{-a-1}\,\sqrt{-a-bx+1}}{\sqrt{a-1}\,\sqrt{a+bx+1}}\right)}{(-a-1)^{7/2}\sqrt{a-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a + b*x])*x^3),x]
```

```
[Out] (Sqrt[1 - a - b*x]*(-1 + a^2 + a^3 + 5*b*x + 14*b^2*x^2 - a*(1 - 5*b*x + b^
2*x^2)))/(2*(1 + a)^3*x^2*Sqrt[1 + a + b*x]) + (3*(-3 + 2*a)*b^2*ArcTanh[(S
qrt[-1 - a]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqrt[1 + a + b*x])])/((-1 - a)
^(7/2)*Sqrt[-1 + a])
```

fricas [A] time = 0.66, size = 520, normalized size = 2.60

$$\left[-\frac{3\left((2a-3)b^3x^3+\left(2a^2-a-3\right)b^2x^2\right)\sqrt{-a^2+1}\,\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx-2\sqrt{-b^2x^2-2abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}}{x^2}\right.}{4\left(\left(a^5+3a^4+2a^3-2a^2-3a-1\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(3*((2*a - 3)*b^3*x^3 + (2*a^2 - a - 3)*b^2*x^2)*sqrt(-a^2 + 1)*log((
(2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x -
 a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) - 2*(a^5 - (a^
3 - 14*a^2 - a + 14)*b^2*x^2 + a^4 - 2*a^3 + 5*(a^3 + a^2 - a - 1)*b*x - 2*
a^2 + a + 1)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 + 3*a^4 + 2*a^3 - 2*
a^2 - 3*a - 1)*b*x^3 + (a^6 + 4*a^5 + 5*a^4 - 5*a^2 - 4*a - 1)*x^2), -1/2*(
3*((2*a - 3)*b^3*x^3 + (2*a^2 - a - 3)*b^2*x^2)*sqrt(a^2 - 1)*arctan(sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2
*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - (a^5 - (a^3 - 14*a^2 - a + 14)
*b^2*x^2 + a^4 - 2*a^3 + 5*(a^3 + a^2 - a - 1)*b*x - 2*a^2 + a + 1)*sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1))/((a^5 + 3*a^4 + 2*a^3 - 2*a^2 - 3*a - 1)*b*x^3
 + (a^6 + 4*a^5 + 5*a^4 - 5*a^2 - 4*a - 1)*x^2)]
```

giac [B] time = 0.44, size = 822, normalized size = 4.11

$$-\frac{8\,b^3}{\left(a^3|b|+3\,a^2|b|+3\,a|b|+|b|\right)\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b}{b^2x+ab}+1\right)}-\frac{3\left(2\,ab^3-3\,b^3\right)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^3|b|+3\,a^2|b|+3\,a|b|+|b|\right)\sqrt{a^2-1}}-\frac{2\left(\sqrt{-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -8*b^3/((a^3*abs(b) + 3*a^2*abs(b) + 3*a*abs(b) + abs(b))*((sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)) - 3*(2*a*b^3 - 3*b^3)*a
rctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)
/sqrt(a^2 - 1))/((a^3*abs(b) + 3*a^2*abs(b) + 3*a*abs(b) + abs(b))*sqrt(a^2
 - 1)) - (2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^4*b^3/(b^2*
x + a*b)^2 + 2*a^4*b^3 - 5*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*
a^3*b^3/(b^2*x + a*b) - 6*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2
*a^3*b^3/(b^2*x + a*b)^2 - 3*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b
)^3*a^3*b^3/(b^2*x + a*b)^3 - 6*a^3*b^3 + 18*(sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1)*abs(b) + b)*a^2*b^3/(b^2*x + a*b) + 3*(sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*abs(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 + 6*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)^3*a^2*b^3/(b^2*x + a*b)^3 - a^2*b^3 + 2*(sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*abs(b) + b)*a*b^3/(b^2*x + a*b) - 12*(sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1)*abs(b) + b)^2*a*b^3/(b^2*x + a*b)^2 + 2*(sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(sqrt(-b^2*x^2
```

```
- 2*a*b*x - a^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^5*abs(b) + 3*a^
4*abs(b) + 3*a^3*abs(b) + a^2*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
abs(b) + b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*abs(b) + b)/(b^2*x + a*b))^2)
```

maple [B] time = 0.06, size = 2848, normalized size = 14.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^3,x)
```

```
[Out] 3/(1+a)^3*b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^
2*b^2+2*b*(x+(1+a)/b))^(1/2))-9/4/(1+a)^3*a^4*b^2/(-a^2+1)^2*(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)+9/4/(1+a)^3*a^2*b^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2)+3/2/(1+a)^3*a^4*b^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2
)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3/2/(1+a)^3*a^2*b^2/(-a^2+1)^(3/2)*ln(
(-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+9/4/(
1+a)^3*b^2/(-a^2+1)*a^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2/(1+a)^3*b^2/(-a^
2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2))/x)*a^2+27/2/(1+a)^4*b^2*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/(
1+a)^4*b^2*a^3/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2))/x)-9/(1+a)^4*b^2*a/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a
*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+3/(1+a)^4*b^3/(-a^
2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x+9/2/(1+a)^4*b^3/(-a^2+1)*(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2)*x+9/2/(1+a)^4*b^3/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2
)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+6/(1+a)^5*b^3*a^3/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+6/(1+a)^5*b^2*(-a^2
+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1
/2))/x)*a^2+3/2/(1+a)^3*b^2/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^
(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+1/(1+a)^3/b/(x+1/b+a/b)^3*(-(x+(1+
a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+3/(1+a)^3*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x
+(1+a)/b))^(1/2)*x+3/(1+a)^3*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)
*a-9/2/(1+a)^4*b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a
)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))-1/2/(1+a)^3/(-a^2+1)/x^2*(-b^2*x^2-2*a*b
*x-a^2+1)^(5/2)-1/2/(1+a)^3*b^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-3/2
/(1+a)^3*b^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-9/2/(1+a)^4*b^2*(-(x+(
1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-6/(1+a)^5*b^2*(-a^2+1)^(1/2)*ln((-2*
a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3/(1+a)^5
*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-3/(1+a)^5*b^2*(-(x+(1+a)/
b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a-3/(1+a)^5*b^3/(b^2)^(1/2)*arctan((b^2)^(1
/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))-9/2/(1+a)^4
*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-9/(1+a)^5*b^2*a^2*(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)-9/2/(1+a)^4*b^3*a^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)*x-27/2/(1+a)^4*b^3*a^2/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+
```

```
a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+9/(1+a)^4*b^3*a^4/(-a^2+1)/(b^2)^(1/2)
*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^3*a*b
/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)-3/4/(1+a)^3*a^3*b^3/(-a^2+1)^2
*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-9/4/(1+a)^3*a^3*b^3/(-a^2+1)^2/(b^2)^(1/2
)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/2/(1+a)^3*a^
5*b^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2))+1/2/(1+a)^3*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x+
3/4/(1+a)^3*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+3/4/(1+a)^3*a
*b^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))+3/4/(1+a)^3*b^3/(-a^2+1)*a*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+9/4
/(1+a)^3*b^3/(-a^2+1)*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2))-3/2/(1+a)^3*b^3/(-a^2+1)*a^3/(b^2)^(1/2)*arctan((b^2)^(
1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-2/(1+a)^5*b^2*(-(x+(1+a)/b)^2*
b^2+2*b*(x+(1+a)/b))^(3/2)+2/(1+a)^5*b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+6/(
1+a)^5*b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+2/(1+a)^3/(x+1/b+a/b)^2*(-(x+(1+a
)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)+2/(1+a)^3*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+
(1+a)/b))^(3/2)-3/(1+a)^4/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b)
)^(5/2)-3/(1+a)^4*b^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)-3/(1+a)^5*
b^3*a*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-9/(1+a)^5*b^3*a/(b^2)^(1/2)*arctan((
b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/(1+a)^4*b/(-a^2+1)/x*(
-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+6/(1+a)^4*b^2*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^
2+1)^(3/2)-27/2/(1+a)^4*b^2*a^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+1/(
1+a)^3*a^2*b^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(bx+a)^2+1\right)^{\frac{3}{2}}}{(bx+a+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((-(b*x + a)^2 + 1)^(3/2)/((b*x + a + 1)^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-(a+b\,x)^2\right)^{3/2}}{x^3\,(a+b\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(3/2)/(x^3*(a + b*x + 1)^3),x)
[Out] int((1 - (a + b*x)^2)^(3/2)/(x^3*(a + b*x + 1)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2)/x**3,x)
[Out] Timed out
```

3.867 $\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=257

$$-\frac{(2a^2-51a+52)\,b^3\sqrt{-a-bx+1}}{6(1-a)(a+1)^4\sqrt{a+bx+1}}+\frac{(6a^2-18a+11)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(1-a)(a+1)^4\sqrt{1-a^2}}-\frac{(19-16a)b^2\sqrt{-a-bx+1}}{6(1-a)(a+1)^3x\sqrt{a+bx+1}}$$

```
[Out] (6*a^2-18*a+11)*b^3*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a
+1)^(1/2))/(1-a)/(1+a)^4/(-a^2+1)^(1/2)-1/6*(2*a^2-51*a+52)*b^3*(-b*x-a+1)^
(1/2)/(1-a)/(1+a)^4/(b*x+a+1)^(1/2)-1/3*(1-a)*(-b*x-a+1)^(1/2)/(1+a)/x^3/(b
*x+a+1)^(1/2)+7/6*b*(-b*x-a+1)^(1/2)/(1+a)^2/x^2/(b*x+a+1)^(1/2)-1/6*(19-16
*a)*b^2*(-b*x-a+1)^(1/2)/(1-a)/(1+a)^3/x/(b*x+a+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6163, 98, 151, 152, 12, 93, 208}

$$-\frac{(2a^2-51a+52)\,b^3\sqrt{-a-bx+1}}{6(1-a)(a+1)^4\sqrt{a+bx+1}}+\frac{(6a^2-18a+11)\,b^3\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(1-a)(a+1)^4\sqrt{1-a^2}}-\frac{(19-16a)b^2\sqrt{-a-bx+1}}{6(1-a)(a+1)^3x\sqrt{a+bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a + b*x])*x^4),x]
```

```
[Out] -((52 - 51*a + 2*a^2)*b^3*Sqrt[1 - a - b*x])/(6*(1 - a)*(1 + a)^4*Sqrt[1 +
a + b*x]) - ((1 - a)*Sqrt[1 - a - b*x])/(3*(1 + a)*x^3*Sqrt[1 + a + b*x]) +
 (7*b*Sqrt[1 - a - b*x])/(6*(1 + a)^2*x^2*Sqrt[1 + a + b*x]) - ((19 - 16*a)
*b^2*Sqrt[1 - a - b*x])/(6*(1 - a)*(1 + a)^3*x*Sqrt[1 + a + b*x]) + ((11 -
18*a + 6*a^2)*b^3*ArcTanh[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt
[1 - a - b*x])])/((1 - a)*(1 + a)^4*Sqrt[1 - a^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
```

```
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{(1-a-bx)^{3/2}}{x^4(1+a+bx)^{3/2}}\,dx$$

$$= -\frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} - \frac{\int \frac{7(1-a)b-6b^2x}{x^3\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}\,dx}{3(1+a)}$$

$$= -\frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} + \frac{\int \frac{(19-16a)(1-a)b^2-14(1-a)b^3x}{x^2\sqrt{1-a-bx}\,(1+a+bx)^{3/2}}\,dx}{6(1-a)(1+a)^2}$$

$$= -\frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} - \frac{(19-16a)b^2\sqrt{1-a-bx}}{6(1-a)(1+a)^3x\sqrt{1+a+bx}} -$$

$$= -\frac{\left(52-51a+2a^2\right)b^3\sqrt{1-a-bx}}{6(1-a)(1+a)^4\sqrt{1+a+bx}} - \frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} -$$

$$= -\frac{\left(52-51a+2a^2\right)b^3\sqrt{1-a-bx}}{6(1-a)(1+a)^4\sqrt{1+a+bx}} - \frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} -$$

$$= -\frac{\left(52-51a+2a^2\right)b^3\sqrt{1-a-bx}}{6(1-a)(1+a)^4\sqrt{1+a+bx}} - \frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} -$$

$$= -\frac{\left(52-51a+2a^2\right)b^3\sqrt{1-a-bx}}{6(1-a)(1+a)^4\sqrt{1+a+bx}} - \frac{(1-a)\sqrt{1-a-bx}}{3(1+a)x^3\sqrt{1+a+bx}} + \frac{7b\sqrt{1-a-bx}}{6(1+a)^2x^2\sqrt{1+a+bx}} -$$

Mathematica [A] time = 0.32, size = 194, normalized size = 0.75

$$\frac{\frac{\left(6a^2-18a+11\right)b^2x^2\left(\sqrt{-a-1}\,\sqrt{-a-bx+1}\,\left(a^2+abx-5bx-1\right)+6\sqrt{a-1}\,bx\sqrt{a+bx+1}\,\tanh^{-1}\left(\frac{\sqrt{-a-1}\,\sqrt{-a-bx+1}}{\sqrt{a-1}\,\sqrt{a+bx+1}}\right)\right)}{(-a-1)^{5/2}} - 2(1-a)(a+1)(-a-bx+1)^{5}}{6\left(a^2-1\right)^2x^3\sqrt{a+bx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a + b*x])*x^4),x]
```

```
[Out] (-2*(1 - a)*(1 + a)*(1 - a - b*x)^(5/2) + (3 - 4*a)*b*x*(1 - a - b*x)^(5/2)
 + ((11 - 18*a + 6*a^2)*b^2*x^2*(Sqrt[-1 - a]*Sqrt[1 - a - b*x]*(-1 + a^2 -
 5*b*x + a*b*x) + 6*Sqrt[-1 + a]*b*x*Sqrt[1 + a + b*x]*ArcTanh[(Sqrt[-1 - a
]*Sqrt[1 - a - b*x])/(Sqrt[-1 + a]*Sqrt[1 + a + b*x])]))/(-1 - a)^(5/2))/(6
*(-1 + a^2)^2*x^3*Sqrt[1 + a + b*x])
```

fricas [A] time = 0.76, size = 697, normalized size = 2.71

$$\left[-\frac{3\left(\left(6\,a^2-18\,a+11\right)b^4x^4+\left(6\,a^3-12\,a^2-7\,a+11\right)b^3x^3\right)\sqrt{-a^2+1}\,\log\left(\frac{\left(2\,a^2-1\right)b^2x^2+2\,a^4+4\left(a^3-a\right)bx+2\,\sqrt{-b^2x^2-2\,abx-}}{x^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((6*a^2 - 18*a + 11)*b^4*x^4 + (6*a^3 - 12*a^2 - 7*a + 11)*b^3*x^
3)*sqrt(-a^2 + 1)*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x + 2*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 +
 2)/x^2) - 2*(2*a^7 + (2*a^4 - 51*a^3 + 50*a^2 + 51*a - 52)*b^3*x^3 + 2*a^6
 - 6*a^5 - (16*a^4 - 3*a^3 - 35*a^2 + 3*a + 19)*b^2*x^2 - 6*a^4 + 6*a^3 + 7
*(a^5 + a^4 - 2*a^3 - 2*a^2 + a + 1)*b*x + 6*a^2 - 2*a - 2)*sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1))/((a^7 + 3*a^6 + a^5 - 5*a^4 - 5*a^3 + a^2 + 3*a + 1)*b
*x^4 + (a^8 + 4*a^7 + 4*a^6 - 4*a^5 - 10*a^4 - 4*a^3 + 4*a^2 + 4*a + 1)*x^3
), 1/6*(3*((6*a^2 - 18*a + 11)*b^4*x^4 + (6*a^3 - 12*a^2 - 7*a + 11)*b^3*x^
3)*sqrt(a^2 - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1
)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) +
(2*a^7 + (2*a^4 - 51*a^3 + 50*a^2 + 51*a - 52)*b^3*x^3 + 2*a^6 - 6*a^5 - (1
6*a^4 - 3*a^3 - 35*a^2 + 3*a + 19)*b^2*x^2 - 6*a^4 + 6*a^3 + 7*(a^5 + a^4 -
 2*a^3 - 2*a^2 + a + 1)*b*x + 6*a^2 - 2*a - 2)*sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1))/((a^7 + 3*a^6 + a^5 - 5*a^4 - 5*a^3 + a^2 + 3*a + 1)*b*x^4 + (a^8 +
 4*a^7 + 4*a^6 - 4*a^5 - 10*a^4 - 4*a^3 + 4*a^2 + 4*a + 1)*x^3)]
```

giac [B] time = 0.26, size = 1839, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 8*b^4/((a^4*abs(b) + 4*a^3*abs(b) + 6*a^2*abs(b) + 4*a*abs(b) + abs(b))*((s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a*b) + 1)) + (6*a^2*
b^4 - 18*a*b^4 + 11*b^4)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b)
 + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^5*abs(b) + 3*a^4*abs(b) + 2*a
```

```
^3*abs(b) - 2*a^2*abs(b) - 3*a*abs(b) - abs(b))*sqrt(a^2 - 1)) + 1/3*(12*(s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^7*b^4/(b^2*x + a*b)^2 + 6
*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^7*b^4/(b^2*x + a*b)^4
+ 6*a^7*b^4 - 24*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^6*b^4/(b
^2*x + a*b) - 72*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^6*b^4/
(b^2*x + a*b)^2 - 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^6*
b^4/(b^2*x + a*b)^3 - 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*
a^6*b^4/(b^2*x + a*b)^4 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b
)^5*a^6*b^4/(b^2*x + a*b)^5 - 36*a^6*b^4 + 171*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)*a^5*b^4/(b^2*x + a*b) + 84*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)^2*a^5*b^4/(b^2*x + a*b)^2 + 216*(sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)*abs(b) + b)^3*a^5*b^4/(b^2*x + a*b)^3 + 54*(sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 1)*abs(b) + b)^4*a^5*b^4/(b^2*x + a*b)^4 + 45*(sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^5*b^4/(b^2*x + a*b)^5 + 22*a^5*b^4 - 1
20*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^4*b^4/(b^2*x + a*b) -
252*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^4*b^4/(b^2*x + a*b)
^2 - 156*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^4*b^4/(b^2*x +
 a*b)^3 - 153*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^4*b^4/(b^
2*x + a*b)^4 - 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^4*b^4
/(b^2*x + a*b)^5 + 9*a^4*b^4 - 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b
) + b)*a^3*b^4/(b^2*x + a*b) + 192*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(
b) + b)^2*a^3*b^4/(b^2*x + a*b)^2 + 90*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
abs(b) + b)^3*a^3*b^4/(b^2*x + a*b)^3 + 78*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*abs(b) + b)^4*a^3*b^4/(b^2*x + a*b)^4 - 18*(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*abs(b) + b)^5*a^3*b^4/(b^2*x + a*b)^5 + 2*a^3*b^4 - 6*(sqrt(-b^2*x^
2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a^2*b^4/(b^2*x + a*b) + 54*(sqrt(-b^2*x^
2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a^2*b^4/(b^2*x + a*b)^2 - 100*(sqrt(-b
^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a^2*b^4/(b^2*x + a*b)^3 + 54*(sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a^2*b^4/(b^2*x + a*b)^4 - 6*(
sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^5*a^2*b^4/(b^2*x + a*b)^5 +
12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a*b^4/(b^2*x + a*b)^2
- 36*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a*b^4/(b^2*x + a*b)^
3 + 12*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^4*a*b^4/(b^2*x + a*b
)^4 - 8*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*b^4/(b^2*x + a*b)
^3)/((a^8*abs(b) + 3*a^7*abs(b) + 2*a^6*abs(b) - 2*a^5*abs(b) - 3*a^4*abs(b
) - a^3*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x
 + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x +
a*b))^3)
```

maple [B] time = 0.07, size = 4212, normalized size = 16.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^4,x)
```

[Out] -3/(1+a)^4*b^4/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+1/(1+a)^3*b^4/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+1/(1+a)^3*b^4/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+5/(1+a)^6*b^4*a*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+15/(1+a)^6*b^4*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-10/(1+a)^6*b^4*a^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-10/(1+a)^6*b^3*(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2+1/2/(1+a)^3*a^5*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-1/2/(1+a)^3*a^3*b^3/(-a^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+3/2/(1+a)^3*a^3*b^3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-3/2/(1+a)^3*a*b^3/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+2/3/(1+a)^3*b^4/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x+3/2/(1+a)^4*b/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)-3/(1+a)^4*b^3*a^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+27/4/(1+a)^4*b^3*a^4/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-27/4/(1+a)^4*b^3*a^2/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-9/2/(1+a)^4*b^3*a^4/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-6/(1+a)^5*b^2/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)-12/(1+a)^5*b^3*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+27/(1+a)^5*b^3*a^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-27/(1+a)^5*b^3*a/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-18/(1+a)^5*b^3*a^3/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+18/(1+a)^5*b^3*a/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)-6/(1+a)^5*b^4/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x-9/(1+a)^5*b^4/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-9/(1+a)^5*b^4/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+2/3/(1+a)^3*b^2/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+1/3/(1+a)^3*a^3*b^3/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-3/4/(1+a)^3*a^5*b^3/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/4/(1+a)^3*a^3*b^3/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+7/6/(1+a)^3*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)-9/4/(1+a)^3*a^3*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+5/2/(1+a)^3*a*b^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-27/4/(1+a)^4*b^3/(-a^2+1)*a^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/2/(1+a)^4*b^3/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)*a^2+9/2/(1+a)^4*b^3*a^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+9/(1+a)^5*b^4*a^2/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+27/(1+a)^5*b^4*a^2/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-18/(1+a)^5*b^4*a^4/(-a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-1/6/(1+a)^3*a*b/(-a^2+1)^2/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+1/6/(1+a)^3*a^2*b^2/(-a^2+1)^3/x*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)-1/4/(1+a)^3*a^4*b^4/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-3/4/(1+a)^3*a^4*b^4/(-a^2+1)^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/(1+a)^3*a^6*b^4/(-a^2+1)^3/(b^2)^

```
(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/6/(1+a)^
3*a^2*b^4/(-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x+1/4/(1+a)^3*a^2*b^4/(
-a^2+1)^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-9/4/(1+a)^4*b^4/(-a^2+1)*a*(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)*x-27/4/(1+a)^4*b^4/(-a^2+1)*a/(b^2)^(1/2)*arctan(
(b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+9/2/(1+a)^4*b^4/(-a^2+1
)*a^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
)+27/4/(1+a)^4*b^4*a^3/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2))-9/2/(1+a)^4*b^4*a^5/(-a^2+1)^2/(b^2)^(1/2)*ar
ctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-3/2/(1+a)^4*b^4*a/
(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)*x-9/4/(1+a)^4*b^4*a/(-a^2+1)^2*(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-9/4/(1+a)^4*b^4*a/(-a^2+1)^2/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+5/(1+a)^6*b^4*(-(x+
(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+5/(1+a)^6*b^3*(-(x+(1+a)/b)^2*b^2+2
*b*(x+(1+a)/b))^(1/2)*a+5/(1+a)^6*b^4/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+(1+
a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+10/(1+a)^6*b^3*(-a^2+
1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/
2))/x)+15/(1+a)^6*b^3*a^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-2/(1+a)^4*b/(x+1/b
+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-3/(1+a)^4*b^4*(-(x+(1+a)
/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x-3/(1+a)^4*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x
+(1+a)/b))^(1/2)*a-1/3/(1+a)^3/(-a^2+1)/x^3*(-b^2*x^2-2*a*b*x-a^2+1)^(5/2)+
4/(1+a)^5*b/(x+1/b+a/b)^2*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-2/(1+a
)^4*b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)-1/(1+a)^4/(x+1/b+a/b)^3*
(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(5/2)-10/(1+a)^6*b^3*(-b^2*x^2-2*a*b*x
-a^2+1)^(1/2)-10/3/(1+a)^6*b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+10/3/(1+a)^6*
b^3*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(3/2)+4/(1+a)^5*b^3*(-(x+(1+a)/b)^
2*b^2+2*b*(x+(1+a)/b))^(3/2)+1/4/(1+a)^3*a^2*b^4/(-a^2+1)^3/(b^2)^(1/2)*arc
tan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-3/4/(1+a)^3*a^2*b^4
/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-9/4/(1+a)^3*a^2*b^4/(-a^2+1)^2
/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+3/2
/(1+a)^3*a^4*b^4/(-a^2+1)^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2))-3/2/(1+a)^4*b^2*a/(-a^2+1)^2/x*(-b^2*x^2-2*a*b*x-a^
2+1)^(5/2)+9/4/(1+a)^4*b^4*a^3/(-a^2+1)^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+
6/(1+a)^5*b^4*(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*x+6/(1+a)^5*b^3*(-
(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2)*a+6/(1+a)^5*b^4/(b^2)^(1/2)*arctan
((b^2)^(1/2)*(x+(1+a)/b-1/b)/(-(x+(1+a)/b)^2*b^2+2*b*(x+(1+a)/b))^(1/2))+3/
2/(1+a)^4*b^3/(-a^2+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)+9/2/(1+a)^4*b^3/(-a^2
+1)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-9/2/(1+a)^4*b^3/(-a^2+1)^(1/2)*ln((-2*a^
2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(bx+a)^2+1\right)^{\frac{3}{2}}}{(bx+a+1)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)^3*(1-(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((-(b*x + a)^2 + 1)^(3/2)/((b*x + a + 1)^3*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-(a+b\,x)^2\right)^{3/2}}{x^4\,(a+b\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (a + b*x)^2)^(3/2)/(x^4*(a + b*x + 1)^3),x)
[Out] int((1 - (a + b*x)^2)^(3/2)/(x^4*(a + b*x + 1)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a+1)**3*(1-(b*x+a)**2)**(3/2)/x**4,x)
[Out] Timed out
```

3.868 $\int \frac{e^{\tanh^{-1}(1+bx)}}{2+bx}\,dx$

Optimal. Leaf size=10

$$\frac{\sin^{-1}(bx+1)}{b}$$

```
[Out] arcsin(b*x+1)/b
```

Rubi [A] time = 0.04, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6163, 53, 619, 216}

$$\frac{\sin^{-1}(bx+1)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[1 + b*x]/(2 + b*x),x]
```

```
[Out] ArcSin[1 + b*x]/b
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(1+bx)}}{2+bx}\,dx = \int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx$$
$$= \int \frac{1}{\sqrt{-2bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2b-2b^2x\right)}{2b^2}$$
$$= \frac{\sin^{-1}(1+bx)}{b}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 3.70

$$-\frac{2\sqrt{-bx}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}\sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[1 + b*x]/(2 + b*x),x]
[Out] (-2*Sqrt[-(b*x)]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(b^(3/2)*Sqrt[x])
```

fricas [B] time = 0.64, size = 28, normalized size = 2.80

$$-\frac{2\arctan\left(\frac{\sqrt{-b^2x^2-2bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(b*x+1)^2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-b^2*x^2 - 2*b*x)/(b*x))/b
```

giac [A] time = 0.19, size = 15, normalized size = 1.50

$$-\frac{\arcsin\left(-bx-1\right)\operatorname{sgn}(b)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(b*x+1)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(-b*x - 1)*sgn(b)/abs(b)
```

maple [B] time = 0.03, size = 34, normalized size = 3.40

$$\frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{1}{b}\right)}{\sqrt{-b^2x^2-2bx}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(b*x+1)^2+1)^(1/2),x)
```

```
[Out] 1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+1/b)/(-b^2*x^2-2*b*x)^(1/2))
```

maxima [A] time = 0.40, size = 18, normalized size = 1.80

$$-\frac{\arcsin\left(-\frac{b^2x+b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(b*x+1)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(b^2*x + b)/b)/b
```

mupad [B] time = 0.97, size = 10, normalized size = 1.00

$$\frac{\mathrm{asin}\left(b\,x+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - (b*x + 1)^2)^(1/2),x)
```

```
[Out] asin(b*x + 1)/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-(bx+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(b*x+1)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - (b*x + 1)**2), x)
```

3.869 $\int \frac{e^{\tanh^{-1}(a+bx)}x^3}{1-a^2-2abx-b^2x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{3\left(2a^2-2a+1\right)\sin^{-1}(a+bx)}{2b^4}+\frac{\sqrt{-a-bx+1}\,\sqrt{a+bx+1}\,((3-2a)bx+(1-2a)(4-a))}{2b^4}+\frac{(1-a)x^2\sqrt{a+bx+1}}{b^2\sqrt{-a-bx+1}}$$

```
[Out] -3/2*(2*a^2-2*a+1)*arcsin(b*x+a)/b^4+(1-a)*x^2*(b*x+a+1)^(1/2)/b^2/(-b*x-a+
1)^(1/2)+1/2*((1-2*a)*(4-a)+(3-2*a)*b*x)*(-b*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b
^4
```

Rubi [A] time = 0.17, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6164, 98, 147, 53, 619, 216}

$$-\frac{3\left(2a^2-2a+1\right)\sin^{-1}(a+bx)}{2b^4}+\frac{(1-a)x^2\sqrt{a+bx+1}}{b^2\sqrt{-a-bx+1}}+\frac{\sqrt{-a-bx+1}\,\sqrt{a+bx+1}\,((3-2a)bx+(1-2a)(4-a))}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a + b*x]*x^3)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] ((1 - a)*x^2*Sqrt[1 + a + b*x])/(b^2*Sqrt[1 - a - b*x]) + (Sqrt[1 - a - b*x
]*Sqrt[1 + a + b*x]*((1 - 2*a)*(4 - a) + (3 - 2*a)*b*x))/(2*b^4) - (3*(1 -
2*a + 2*a^2)*ArcSin[a + b*x])/(2*b^4)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}x^3}{1-a^2-2abx-b^2x^2}\,dx = \int \frac{x^3}{(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx$$

$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} - \frac{\int \frac{x\left(2\left(1-a^2\right)+(3-2a)bx\right)}{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{b^2}$$

$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}\,((1-2a)(4-a)+(3-2a)bx)}{2b^4} - \cdots$$

$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}\,((1-2a)(4-a)+(3-2a)bx)}{2b^4} - \cdots$$

$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}\,((1-2a)(4-a)+(3-2a)bx)}{2b^4} + \cdots$$

$$= \frac{(1-a)x^2\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\,\sqrt{1+a+bx}\,((1-2a)(4-a)+(3-2a)bx)}{2b^4} - \cdots$$

Mathematica [**A**] time = 0.37, size = 90, normalized size = 0.83

$$-\frac{3\left(2a^2-2a+1\right)\sin^{-1}(a+bx)-\frac{\sqrt{-a^2-2abx-b^2x^2+1}\left(2a^3-11a^2+a(13-4bx)+b^2x^2+bx-4\right)}{a+bx-1}}{2b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a + b*x]*x^3)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] -1/2*(-((Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(-4 - 11*a^2 + 2*a^3 + b*x + b^2
*x^2 + a*(13 - 4*b*x)))/(-1 + a + b*x)) + 3*(1 - 2*a + 2*a^2)*ArcSin[a + b*
x])/b^4
```

fricas [**A**] time = 0.67, size = 150, normalized size = 1.38

$$\frac{3\left(2\,a^3+\left(2\,a^2-2\,a+1\right)bx-4\,a^2+3\,a-1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\left(b^2x^2+2\,a^3-(4\,a-1)bx-11\,a^2+\cdots\right.}{2\left(b^5x+(a-1)b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="fricas")
```

```
[Out] 1/2*(3*(2*a^3 + (2*a^2 - 2*a + 1)*b*x - 4*a^2 + 3*a - 1)*arctan(sqrt(-b^2*x
^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (b^2*x^2
 + 2*a^3 - (4*a - 1)*b*x - 11*a^2 + 13*a - 4)*sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1))/(b^5*x + (a - 1)*b^4)
```

giac [A] time = 0.24, size = 125, normalized size = 1.15

$$\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b^3}-\frac{5\,ab^6-2\,b^6}{b^{10}}\right)+\frac{3\left(2\,a^2-2\,a+1\right)\arcsin\left(-bx-a\right)\mathrm{sgn}\left(b\right)}{2\,b^3|b|}-\frac{2\left(a^3-3\,a^2+3\,a-1\right)}{b^3\left(\frac{\sqrt{-(bx+a)^2+1}\,|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="giac")
```

```
[Out] 1/2*sqrt(-(b*x + a)^2 + 1)*(x/b^3 - (5*a*b^6 - 2*b^6)/b^10) + 3/2*(2*a^2 -
2*a + 1)*arcsin(-b*x - a)*sgn(b)/(b^3*abs(b)) - 2*(a^3 - 3*a^2 + 3*a - 1)/(
b^3*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

maple [B] time = 0.04, size = 499, normalized size = 4.58

$$-\frac{x^3}{2b\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{3a\,x^2}{2b^2\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{15a^2x}{2b^3\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{9a^3}{2b^4\sqrt{-b^2x^2-2abx-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] -1/2/b*x^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2/b^2*a*x^2/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)+15/2/b^3*a^2*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+9/2/b^4*a^3/(-b^2
*x^2-2*a*b*x-a^2+1)^(1/2)-3*a^2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-9/2/b^4*a/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/
2/b^3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2/b^3/(b^2)^(1/2)*arctan((b^2)^(1/
2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))-x^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)
^(1/2)-5*a/b^3*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a^2/b^4/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)-a^3/b^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-a^4/b^4/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)+3*a/b^3/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2))+2/b^4/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

maxima [B] time = 0.60, size = 576, normalized size = 5.28

$$\left(\frac{2\sqrt{-b^2x^2-2\,abx-a^2+1}\,a^3}{b^6x+ab^5-b^5}-\frac{3\sqrt{-b^2x^2-2\,abx-a^2+1}\,a^2b}{b^7x+ab^6+b^6}-\frac{3\sqrt{-b^2x^2-2\,abx-a^2+1}\,a^2b}{b^7x+ab^6-b^6}+\frac{3\sqrt{-b^2x^2-2\,abx-a^2+1}\,a^2}{b^6x+ab^5+b^5}-\frac{3\sqrt{-b^2x^2-2\,abx-a^2+1}\,a^2}{b^6x+ab^5-b^5}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^3/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*(2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/(b^6*x + a*b^5 - b^5) - 3*sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2*b/(b^7*x + a*b^6 + b^6) - 3*sqrt(-b^2*x
^2 - 2*a*b*x - a^2 + 1)*a^2*b/(b^7*x + a*b^6 - b^6) + 3*sqrt(-b^2*x^2 - 2*a
*b*x - a^2 + 1)*a^2/(b^6*x + a*b^5 + b^5) - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1)*a^2/(b^6*x + a*b^5 - b^5) + 6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(b
^6*x + a*b^5 - b^5) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b/(b^7*x + a*b^6 +
 b^6) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b/(b^7*x + a*b^6 - b^6) + sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^6*x + a*b^5 + b^5) - sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)/(b^6*x + a*b^5 - b^5) - 6*a^2*arcsin(b*x + a)/b^5 + sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*x/b^4 + 6*a*arcsin(b*x + a)/b^5 - 5*sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*a/b^5 - 3*arcsin(b*x + a)/b^5 + 2*sqrt(-b^2*x^2 - 2*a*
b*x - a^2 + 1)/b^5)*b^2/sqrt(a^2*b^2 - (a^2 - 1)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\left(a+b\,x+1\right)}{\sqrt{1-{\left(a+b\,x\right)}^2}\,\left(a^2+2\,a\,b\,x+b^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a + b*x + 1))/((1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
- 1)),x)
```

```
[Out] -int((x^3*(a + b*x + 1))/((1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
- 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{a\sqrt{-a^2-2abx-b^2x^2+1}+bx\sqrt{-a^2-2abx-b^2x^2+1}-\sqrt{-a^2-2abx-b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x**3/(-b**2*x**2-2*a*b*x-a**2+1),
x)
```

```
[Out] -Integral(x**3/(a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x*sqrt(-a**2 -
2*a*b*x - b**2*x**2 + 1) - sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

3.870 $\int \frac{e^{\tanh^{-1}(a+bx)}x^2}{1-a^2-2abx-b^2x^2}\,dx$

Optimal. Leaf size=78

$$\frac{(1-a)^2\sqrt{a+bx+1}}{b^3\sqrt{-a-bx+1}}+\frac{\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{b^3}-\frac{(1-2a)\sin^{-1}(a+bx)}{b^3}$$

```
[Out] -(1-2*a)*arcsin(b*x+a)/b^3+(1-a)^2*(b*x+a+1)^(1/2)/b^3/(-b*x-a+1)^(1/2)+(-b
*x-a+1)^(1/2)*(b*x+a+1)^(1/2)/b^3
```

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6164, 89, 80, 53, 619, 216}

$$\frac{(1-a)^2\sqrt{a+bx+1}}{b^3\sqrt{-a-bx+1}}+\frac{\sqrt{-a-bx+1}\,\sqrt{a+bx+1}}{b^3}-\frac{(1-2a)\sin^{-1}(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a + b*x]*x^2)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] ((1 - a)^2*Sqrt[1 + a + b*x])/(b^3*Sqrt[1 - a - b*x]) + (Sqrt[1 - a - b*x]*
Sqrt[1 + a + b*x])/b^3 - ((1 - 2*a)*ArcSin[a + b*x])/b^3
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
```

```
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}x^2}{1-a^2-2abx-b^2x^2}\,dx = \int \frac{x^2}{(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx$$

$$= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} - \frac{\int \frac{(1-a)b+b^2x}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{b^3}$$

$$= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b^3} - \frac{(1-2a)\int \frac{1}{\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{b^2}$$

$$= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b^3} - \frac{(1-2a)\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,d}{b^2}$$

$$= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b^3} + \frac{(1-2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x,\right.}{2b^4}$$

$$= \frac{(1-a)^2\sqrt{1+a+bx}}{b^3\sqrt{1-a-bx}} + \frac{\sqrt{1-a-bx}\sqrt{1+a+bx}}{b^3} - \frac{(1-2a)\sin^{-1}(a+bx)}{b^3}$$

Mathematica [A] time = 0.21, size = 64, normalized size = 0.82

$$-\frac{\frac{(a^2-3a-bx+2)\sqrt{-a^2-2abx-b^2x^2+1}}{a+bx-1} - (2a-1)\sin^{-1}(a+bx)}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a + b*x]*x^2)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] -((((2 - 3*a + a^2 - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/(-1 + a + b*x)
 - (-1 + 2*a)*ArcSin[a + b*x])/b^3)
```

fricas [A] time = 0.83, size = 120, normalized size = 1.54

$$-\frac{\left((2a-1)bx+2a^2-3a+1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)+\sqrt{-b^2x^2-2abx-a^2+1}\left(a^2-bx-3a+2\right)}{b^4x+(a-1)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="fricas")
```

```
[Out] -(((2*a - 1)*b*x + 2*a^2 - 3*a + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1)*(a^2 - b*x - 3*a + 2))/(b^4*x + (a - 1)*b^3)
```

giac [A] time = 0.25, size = 94, normalized size = 1.21

$$-\frac{(2a-1)\arcsin(-bx-a)\operatorname{sgn}(b)}{b^2|b|}+\frac{\sqrt{-(bx+a)^2+1}}{b^3}+\frac{2\left(a^2-2a+1\right)}{b^2\left(\frac{\sqrt{-(bx+a)^2+1}\,|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="giac")
```

```
[Out] -(2*a - 1)*arcsin(-b*x - a)*sgn(b)/(b^2*abs(b)) + sqrt(-(b*x + a)^2 + 1)/b^
3 + 2*(a^2 - 2*a + 1)/(b^2*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*
b) - 1)*abs(b))
```

maple [B] time = 0.04, size = 325, normalized size = 4.17

$$-\frac{x^2}{b\sqrt{-b^2x^2-2abx-a^2+1}}-\frac{4ax}{b^2\sqrt{-b^2x^2-2abx-a^2+1}}-\frac{2a^2}{b^3\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{2a\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2}}\right.}{b^2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] -1/b*x^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-4/b^2*a*x/(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2)-2/b^3*a^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+2*a/b^2/(b^2)^(1/2)*arctan((
b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+2/b^3/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)+x/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/b^3/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)+a^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x+a^3/b^3/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)-1/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+a/b)/(-b^2*x^2-2*a
*b*x-a^2+1)^(1/2))
```

maxima [B] time = 0.52, size = 329, normalized size = 4.22

$$-\frac{\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,a^2}{b^5x+ab^4-b^4}-\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,ab}{b^6x+ab^5+b^5}-\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,ab}{b^6x+ab^5-b^5}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,a}{b^5x+ab^4+b^4}-\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,a}{b^5x+ab^4-b^4}+\cdots\right.}{\sqrt{a^2b^2-\left(a^2-1\right)b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="maxima")
```

```
[Out] -(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^2/(b^5*x + a*b^4 - b^4) - sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*a*b/(b^6*x + a*b^5 + b^5) - sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)*a*b/(b^6*x + a*b^5 - b^5) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*a/(b^5*x + a*b^4 + b^4) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(b^5*x + a*
b^4 - b^4) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^5*x + a*b^4 - b^4) - 2*a
*arcsin(b*x + a)/b^4 + arcsin(b*x + a)/b^4 - sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)/b^4)*b^2/sqrt(a^2*b^2 - (a^2 - 1)*b^2)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\left(a+b\,x+1\right)}{\sqrt{1-{\left(a+b\,x\right)}^2}\,\left(a^2+2\,a\,b\,x+b^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a + b*x + 1))/((1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
- 1)),x)
```

```
[Out] -int((x^2*(a + b*x + 1))/((1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
- 1)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}}{a \sqrt{-a^{2} - 2abx - b^{2}x^{2} + 1} + bx \sqrt{-a^{2} - 2abx - b^{2}x^{2} + 1} - \sqrt{-a^{2} - 2abx - b^{2}x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x**2/(-b**2*x**2-2*a*b*x-a**2+1),
x)
```

```
[Out] -Integral(x**2/(a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x*sqrt(-a**2 -
2*a*b*x - b**2*x**2 + 1) - sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

3.871 $\int \frac{e^{\tanh^{-1}(a+bx)}x}{1-a^2-2abx-b^2x^2}\,dx$

Optimal. Leaf size=44

$$\frac{(1-a)\sqrt{a+bx+1}}{b^2\sqrt{-a-bx+1}}-\frac{\sin^{-1}(a+bx)}{b^2}$$

```
[Out] -arcsin(b*x+a)/b^2+(1-a)*(b*x+a+1)^(1/2)/b^2/(-b*x-a+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {6164, 78, 53, 619, 216}

$$\frac{(1-a)\sqrt{a+bx+1}}{b^2\sqrt{-a-bx+1}}-\frac{\sin^{-1}(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a + b*x]*x)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] ((1 - a)*Sqrt[1 + a + b*x])/(b^2*Sqrt[1 - a - b*x]) - ArcSin[a + b*x]/b^2
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(a+bx)}x}{1-a^2-2abx-b^2x^2}\,dx &= \int \frac{x}{(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx\\
&= \frac{(1-a)\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} - \frac{\int \frac{1}{\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{b}\\
&= \frac{(1-a)\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} - \frac{\int \frac{1}{\sqrt{(1-a)(1+a)-2abx-b^2x^2}}\,dx}{b}\\
&= \frac{(1-a)\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2ab-2b^2x\right)}{2b^3}\\
&= \frac{(1-a)\sqrt{1+a+bx}}{b^2\sqrt{1-a-bx}} - \frac{\sin^{-1}(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 49, normalized size = 1.11

$$-\frac{\sin^{-1}(a+bx)-\frac{(a-1)\sqrt{-a^2-2abx-b^2x^2+1}}{a+bx-1}}{b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTanh[a + b*x]*x)/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] -((-(((-1 + a)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/(-1 + a + b*x)) + ArcSin[
a + b*x])/b^2)
```

fricas [B] time = 0.57, size = 98, normalized size = 2.23

$$\frac{(bx+a-1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}\,(a-1)}{b^3x+(a-1)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="fricas")
```

```
[Out] ((b*x + a - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2
 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a - 1))/(b^3*x
 + (a - 1)*b^2)
```

giac [A] time = 0.28, size = 66, normalized size = 1.50

$$\frac{\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{b|b|}-\frac{2\,(a-1)}{b\left(\frac{\sqrt{-(bx+a)^2+1}\,|b|+b}{b^2x+ab}-1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="giac")
```

```
[Out] arcsin(-b*x - a)*sgn(b)/(b*abs(b)) - 2*(a - 1)/(b*((sqrt(-(b*x + a)^2 + 1)*
abs(b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

maple [B] time = 0.04, size = 160, normalized size = 3.64

$$\frac{x}{b\sqrt{-b^2x^2-2abx-a^2+1}}-\frac{\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{b\sqrt{b^2}}+\frac{1}{b^2\sqrt{-b^2x^2-2abx-a^2+1}}-\frac{ax}{b\sqrt{-b^2x^2-2abx-a^2+1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] 1/b*x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+
a/b)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a
/b/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x-a^2/b^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
```

maxima [B] time = 0.46, size = 120, normalized size = 2.73

$$\frac{b^2\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,a}{b^4x+ab^3-b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}}{b^4x+ab^3-b^3}-\frac{\arcsin(bx+a)}{b^3}\right)}{\sqrt{a^2b^2-\left(a^2-1\right)b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)*x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="maxima")
```

```
[Out] b^2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a/(b^4*x + a*b^3 - b^3) - sqrt(-b^2
*x^2 - 2*a*b*x - a^2 + 1)/(b^4*x + a*b^3 - b^3) - arcsin(b*x + a)/b^3)/sqrt
(a^2*b^2 - (a^2 - 1)*b^2)
```

mupad [B] time = 1.65, size = 229, normalized size = 5.20

$$\frac{\left(\frac{a^2\,x}{b}+\frac{a\,\left(a^2-1\right)}{b^2}\right)\,\sqrt{1-{\left(a+b\,x\right)}^2}}{a^2+2\,a\,b\,x+b^2\,x^2-1}+\frac{b\,\ln\left(\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}-\frac{x\,b^2+a\,b}{\sqrt{-b^2}}\right)}{{\left(-b^2\right)}^{3/2}}+\frac{\left(\frac{a^2-1}{b^2}+\frac{a\,x}{b}\right)\,\sqrt{1-{\left(a+b\,x\right)}^2}}{a^2+2\,a\,b\,x+b^2\,x^2-1}+\frac{\cdots}{b\,\left(b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a + b*x + 1))/((1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x -
1)),x)
```

```
[Out] (((a^2*x)/b + (a*(a^2 - 1))/b^2)*(1 - (a + b*x)^2)^(1/2))/(a^2 + b^2*x^2 +
2*a*b*x - 1) + (b*log((1 - b^2*x^2 - 2*a*b*x - a^2)^(1/2) - (a*b + b^2*x)/(
-b^2)^(1/2)))/(-b^2)^(3/2) + (((a^2 - 1)/b^2 + (a*x)/b)*(1 - (a + b*x)^2)^(
1/2))/(a^2 + b^2*x^2 + 2*a*b*x - 1) + (x*(b^2*(a^2 - 1) - 2*a^2*b^2) - a*b*
(a^2 - 1))/(b*(b^2*(a^2 - 1) - a^2*b^2)*(1 - b^2*x^2 - 2*a*b*x - a^2)^(1/2)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{a\sqrt{-a^2-2abx-b^2x^2+1}+bx\sqrt{-a^2-2abx-b^2x^2+1}-\sqrt{-a^2-2abx-b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)*x/(-b**2*x**2-2*a*b*x-a**2+1),x)
```

```
[Out] -Integral(x/(a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x*sqrt(-a**2 - 2*a
*b*x - b**2*x**2 + 1) - sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

3.872 $\int \frac{e^{\tanh^{-1}(a+bx)}}{1-a^2-2abx-b^2x^2} dx$

Optimal. Leaf size=27

$$\frac{\sqrt{a+bx+1}}{b\sqrt{-a-bx+1}}$$

```
[Out] (b*x+a+1)^(1/2)/b/(-b*x-a+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {6164, 37}

$$\frac{\sqrt{a+bx+1}}{b\sqrt{-a-bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] Sqrt[1 + a + b*x]/(b*Sqrt[1 - a - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(a+bx)}}{1-a^2-2abx-b^2x^2} dx &= \int \frac{1}{(1-a-bx)^{3/2}\sqrt{1+a+bx}} dx\\ &= \frac{\sqrt{1+a+bx}}{b\sqrt{1-a-bx}}\end{aligned}$$

Mathematica [C] time = 0.08, size = 12, normalized size = 0.44

$$\frac{e^{\tanh^{-1}(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a + b*x]/(1 - a^2 - 2*a*b*x - b^2*x^2),x]
```

```
[Out] E^ArcTanh[a + b*x]/b
```

fricas [A] time = 0.53, size = 37, normalized size = 1.37

$$-\frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}}{b^2x + (a-1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/(-b^2*x^2-2*a*b*x-a^2+1),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)/(b^2*x + (a - 1)*b)
```

giac [A] time = 0.40, size = 40, normalized size = 1.48

$$\frac{2}{\left(\frac{\sqrt{-(bx+a)^2+1}\,|b|+b}{b^2x+ab} - 1\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/(-b^2*x^2-2*a*b*x-a^2+1),x, algorit
hm="giac")
```

```
[Out] 2/(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1)*abs(b))
```

maple [A] time = 0.03, size = 42, normalized size = 1.56

$$-\frac{(bx+a-1)\,(bx+a+1)^2}{b\left(-b^2x^2 - 2abx - a^2 + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] -(b*x+a-1)*(b*x+a+1)^2/b/(-b^2*x^2-2*a*b*x-a^2+1)^(3/2)
```

maxima [B] time = 0.43, size = 65, normalized size = 2.41

$$-\frac{\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}\,b^2}{\sqrt{a^2b^2 - \left(a^2 - 1\right)b^2}\left(b^3x + ab^2 - b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/(-b^2*x^2-2*a*b*x-a^2+1),x, algorit
hm="maxima")
```

```
[Out] -sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*b^2/(sqrt(a^2*b^2 - (a^2 - 1)*b^2)*(b^3
*x + a*b^2 - b^2))
```

mupad [B] time = 0.18, size = 26, normalized size = 0.96

$$-\frac{\sqrt{1-(a+b\,x)^2}}{b\,\left(a+b\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + 1)/((1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x - 1)),
x)
```

```
[Out] -(1 - (a + b*x)^2)^(1/2)/(b*(a + b*x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a\sqrt{-a^2 - 2abx - b^2x^2 + 1} + bx\sqrt{-a^2 - 2abx - b^2x^2 + 1} - \sqrt{-a^2 - 2abx - b^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/(-b**2*x**2-2*a*b*x-a**2+1),x)
```

```
[Out] -Integral(1/(a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x*sqrt(-a**2 - 2*a
*b*x - b**2*x**2 + 1) - sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

3.873 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x(1-a^2-2abx-b^2x^2)}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{a+bx+1}}{(1-a)\sqrt{-a-bx+1}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}$$

```
[Out] -2*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(1/2))/(1-a)/
(-a^2+1)^(1/2)+(b*x+a+1)^(1/2)/(1-a)/(-b*x-a+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6164, 96, 93, 208}

$$\frac{\sqrt{a+bx+1}}{(1-a)\sqrt{-a-bx+1}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(1-a)\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/(x*(1 - a^2 - 2*a*b*x - b^2*x^2)),x]
```

```
[Out] Sqrt[1 + a + b*x]/((1 - a)*Sqrt[1 - a - b*x]) - (2*ArcTanh[(Sqrt[1 - a]*Sqr
t[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)*Sqrt[1 - a^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(a+bx)}}{x\left(1-a^2-2abx-b^2x^2\right)}\,dx &= \int \frac{1}{x(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx \\
&= \frac{\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} + \frac{\int \frac{1}{x\sqrt{1-a-bx}\,\sqrt{1+a+bx}}\,dx}{1-a} \\
&= \frac{\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1-a-(-1+a)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{1-a-bx}}\right)}{1-a} \\
&= \frac{\sqrt{1+a+bx}}{(1-a)\sqrt{1-a-bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{1-a-bx}}\right)}{(1-a)\sqrt{1-a^2}}
\end{aligned}
$$

Mathematica [A] time = 0.25, size = 118, normalized size = 1.27

$$
-\frac{-\frac{\sqrt{-a^2-2abx-b^2x^2+1}}{a+bx-1} - \frac{\log\left(\sqrt{1-a^2}\,\sqrt{-a^2-2abx-b^2x^2+1}-a^2-abx+1\right)}{\sqrt{1-a^2}} + \frac{\log(x)}{\sqrt{1-a^2}}}{a-1}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/(x*(1 - a^2 - 2*a*b*x - b^2*x^2)),x]
```

```
[Out] -((-(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]/(-1 + a + b*x)) + Log[x]/Sqrt[1 - a^
2] - Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]]
/Sqrt[1 - a^2])/(-1 + a))
```

fricas [A] time = 0.62, size = 316, normalized size = 3.40

$$\left[-\frac{\sqrt{-a^2+1}\,(bx+a-1)\log\left(\frac{(2\,a^2-1)b^2x^2+2\,a^4+4\,(a^3-a)bx-2\,\sqrt{-b^2x^2-2\,abx-a^2+1}\,(abx+a^2-1)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)-2\,\sqrt{-b^2x^2-2\,a}}{2\left(a^4-2\,a^3+\left(a^3-a^2-a+1\right)bx+2\,a-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + 1)*(b*x + a - 1)*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a
^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-
a^2 + 1) - 4*a^2 + 2)/x^2) - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)
)/(a^4 - 2*a^3 + (a^3 - a^2 - a + 1)*b*x + 2*a - 1), -(sqrt(a^2 - 1)*(b*x +
 a - 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^
2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - sqrt(-b^2
*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1))/(a^4 - 2*a^3 + (a^3 - a^2 - a + 1)*b*x
 + 2*a - 1)]
```

giac [A] time = 0.37, size = 112, normalized size = 1.20

$$-\frac{2\,b\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}\,(a|b|-|b|)}-\frac{2\,b}{(a|b|-|b|)\left(\frac{\sqrt{-(bx+a)^2+1}\,|b|+b}{b^2x+ab}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="giac")
```

```
[Out] -2*b*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - 1)/sqrt(
a^2 - 1))/(sqrt(a^2 - 1)*(a*abs(b) - abs(b))) - 2*b/((a*abs(b) - abs(b))*((
sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^2*x + a*b) - 1))
```

maple [B] time = 0.04, size = 391, normalized size = 4.20

$$\frac{2b\left(-2b^2x-2ab\right)}{\left(-4b^2\left(-a^2+1\right)-4a^2b^2\right)\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{1}{\left(-a^2+1\right)\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{abx}{\left(-a^2+1\right)\sqrt{-b^2x^2-2ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] 2*b*(-2*b^2*x-2*a*b)/(-4*b^2*(-a^2+1)-4*a^2*b^2)/(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2)+1/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+a*b/(-a^2+1)/(-b^2*x^2-2*a*b
*x-a^2+1)^(1/2)*x+a^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-1/(-a^2+1)^(3
/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x
)+1/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+a^2*b/(-a^2+1)/(-b^2*x^2-2*a*
b*x-a^2+1)^(1/2)*x+1/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a^3-1/(-a^2+1)
^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
)/x)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx + a + 1}{\left(b^2x^2 + 2\,abx + a^2 - 1\right)\sqrt{-(bx + a)^2 + 1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x/(-b^2*x^2-2*a*b*x-a^2+1),x, algor
ithm="maxima")
```

```
[Out] -integrate((b*x + a + 1)/((b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-(b*x + a)^2 +
 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a + b\,x + 1}{x\,\sqrt{1 - (a + b\,x)^2}\,\left(a^2 + 2\,a\,b\,x + b^2\,x^2 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + 1)/(x*(1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x - 1)
),x)
```

```
[Out] int(-(a + b*x + 1)/(x*(1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x - 1)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax\sqrt{-a^2 - 2abx - b^2x^2 + 1} + bx^2\sqrt{-a^2 - 2abx - b^2x^2 + 1} - x\sqrt{-a^2 - 2abx - b^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x/(-b**2*x**2-2*a*b*x-a**2+1),x)
```

```
[Out] -Integral(1/(a*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x**2*sqrt(-a**2
- 2*a*b*x - b**2*x**2 + 1) - x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)), x)
```

3.874 $\int \frac{e^{\tanh^{-1}(a+bx)}}{x^2(1-a^2-2abx-b^2x^2)}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{a+bx+1}}{(1-a^2)\,x\sqrt{-a-bx+1}}-\frac{2(2a+1)b\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(1-a)^2(a+1)\sqrt{1-a^2}}+\frac{(a+2)b\sqrt{a+bx+1}}{(1-a)^2(a+1)\sqrt{-a-bx+1}}$$

```
[Out] -2*(1+2*a)*b*arctanh((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(-b*x-a+1)^(1/
2))/(1-a)^2/(1+a)/(-a^2+1)^(1/2)+(2+a)*b*(b*x+a+1)^(1/2)/(1-a)^2/(1+a)/(-b*
x-a+1)^(1/2)-(b*x+a+1)^(1/2)/(-a^2+1)/x/(-b*x-a+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6164, 103, 152, 12, 93, 208}

$$-\frac{\sqrt{a+bx+1}}{(1-a^2)\,x\sqrt{-a-bx+1}}-\frac{2(2a+1)b\tanh^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{-a-bx+1}}\right)}{(1-a)^2(a+1)\sqrt{1-a^2}}+\frac{(a+2)b\sqrt{a+bx+1}}{(1-a)^2(a+1)\sqrt{-a-bx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a + b*x]/(x^2*(1 - a^2 - 2*a*b*x - b^2*x^2)),x]
```

```
[Out] ((2 + a)*b*Sqrt[1 + a + b*x])/((1 - a)^2*(1 + a)*Sqrt[1 - a - b*x]) - Sqrt[
1 + a + b*x]/((1 - a^2)*x*Sqrt[1 - a - b*x]) - (2*(1 + 2*a)*b*ArcTanh[(Sqrt
[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[1 - a - b*x])])/((1 - a)^2*(1
+ a)*Sqrt[1 - a^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6164

```
Int[E^(ArcTanh[(a_) + (b_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> Dist[(c/(1 - a^2))^p, Int[u*(1 - a - b*x)^(p - n
/2)*(1 + a + b*x)^(p + n/2), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && E
qQ[b*d - 2*a*e, 0] && EqQ[b^2*c + e*(1 - a^2), 0] && (IntegerQ[p] || GtQ[c/
(1 - a^2), 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(a+bx)}}{x^2\left(1-a^2-2abx-b^2x^2\right)}\,dx = \int \frac{1}{x^2(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx$$

$$= -\frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}} - \frac{\int \frac{-(1+2a)b-b^2x}{x(1-a-bx)^{3/2}\sqrt{1+a+bx}}\,dx}{1-a^2}$$

$$= \frac{(2+a)b\sqrt{1+a+bx}}{(1-a)^2(1+a)\sqrt{1-a-bx}} - \frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}} + \frac{\int \frac{(1+2a)b^2}{x\sqrt{1-a-bx}\sqrt{1+a+bx}}\,dx}{(1-a)^2(1+a)b}$$

$$= \frac{(2+a)b\sqrt{1+a+bx}}{(1-a)^2(1+a)\sqrt{1-a-bx}} - \frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}} + \frac{((1+2a)b)\int \frac{1}{x\sqrt{1-a-bx}}}{(1-a)^2(1+}$$

$$= \frac{(2+a)b\sqrt{1+a+bx}}{(1-a)^2(1+a)\sqrt{1-a-bx}} - \frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}} + \frac{(2(1+2a)b)\,\mathrm{Subst}\left(\int\right.}{(1-}$$

$$= \frac{(2+a)b\sqrt{1+a+bx}}{(1-a)^2(1+a)\sqrt{1-a-bx}} - \frac{\sqrt{1+a+bx}}{\left(1-a^2\right)x\sqrt{1-a-bx}} - \frac{2(1+2a)b\tanh^{-1}\left(\frac{\sqrt{1}}{\sqrt{1}}\right.}{(1-a)^2(1+a)\sqrt{}}$$

Mathematica [A] time = 0.32, size = 149, normalized size = 0.99

$$-\frac{\sqrt{-a^2-2abx-b^2x^2+1}\left(\frac{b}{a+bx-1}+\frac{1}{ax+x}\right)+\frac{(2a+1)b\log\left(\sqrt{1-a^2}\sqrt{-a^2-2abx-b^2x^2+1}-a^2-abx+1\right)}{(a+1)\sqrt{1-a^2}}-\frac{(2a+1)b\log(x)}{(a+1)\sqrt{1-a^2}}}{(a-1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a + b*x]/(x^2*(1 - a^2 - 2*a*b*x - b^2*x^2)),x]
```

```
[Out] -((Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*((x + a*x)^(-1) + b/(-1 + a + b*x)) -
((1 + 2*a)*b*Log[x])/((1 + a)*Sqrt[1 - a^2]) + ((1 + 2*a)*b*Log[1 - a^2 - a
*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]])/((1 + a)*Sqrt[1 -
a^2]))/(-1 + a)^2)
```

fricas [A] time = 0.72, size = 459, normalized size = 3.06

$$\left[-\frac{\left((2a+1)b^2x^2+\left(2a^2-a-1\right)bx\right)\sqrt{-a^2+1}\,\log\left(\frac{\left(2a^2-1\right)b^2x^2+2a^4+4\left(a^3-a\right)bx+2\sqrt{-b^2x^2-2abx-a^2+1}\left(abx+a^2-1\right)\sqrt{-a^2+1}-4}{x^2}\right.}{2\left(\left(a^5-a^4-2a^3+2a^2+a-1\right)bx^2+\left(a^6-2a^5-a^4+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="fricas")
```

```
[Out] [-1/2*(((2*a + 1)*b^2*x^2 + (2*a^2 - a - 1)*b*x)*sqrt(-a^2 + 1)*log(((2*a^2
 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 2*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1)*(a^3 + (a^3 + 2*a^2 - a - 2)*b*x - a^2 - a + 1))/((a^5 -
 a^4 - 2*a^3 + 2*a^2 + a - 1)*b*x^2 + (a^6 - 2*a^5 - a^4 + 4*a^3 - a^2 - 2*
a + 1)*x), (((2*a + 1)*b^2*x^2 + (2*a^2 - a - 1)*b*x)*sqrt(a^2 - 1)*arctan(
sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 -
1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)*(a^3 + (a^3 + 2*a^2 - a - 2)*b*x - a^2 - a + 1))/((a^5 - a^4 - 2
*a^3 + 2*a^2 + a - 1)*b*x^2 + (a^6 - 2*a^5 - a^4 + 4*a^3 - a^2 - 2*a + 1)*x
)]
```

giac [B] time = 0.20, size = 519, normalized size = 3.46

$$\frac{2\left(2\,ab^2+b^2\right)\arctan\left(\frac{\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^3|b|-a^2|b|-a|b|+|b|\right)\sqrt{a^2-1}}+\frac{2\left(\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)^2a^3b^2}{\left(b^2x+ab\right)^2}+a^3b^2-\frac{2\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a^2b^2}{b^2x+ab}+\frac{\left(\sqrt{-(bx+a)^2+1}\right.}{\left(b^2x+a\right.}\right.}{\left(a^4|b|-a^3|b|-a^2|b|+a|b|\right)\left(\frac{\left(\sqrt{-(bx+a)^2+1}\,|b|+b\right)a}{b^2x+ab}-\frac{\left(\sqrt{-(bx+a)^2}\right.}{\left(b^2x\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="giac")
```

```
[Out] 2*(2*a*b^2 + b^2)*arctan(((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*
b) - 1)/sqrt(a^2 - 1))/((a^3*abs(b) - a^2*abs(b) - a*abs(b) + abs(b))*sqrt(
a^2 - 1)) + 2*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^3*b^2/(b^2*x + a*b)^
2 + a^3*b^2 - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a^2*b^2/(b^2*x + a*b) +
 (sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2*a^2*b^2/(b^2*x + a*b)^2 + a^2*b^2 -
3*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a*b^2/(b^2*x + a*b) + a*b^2 - (sqrt(-
(b*x + a)^2 + 1)*abs(b) + b)*b^2/(b^2*x + a*b) + (sqrt(-(b*x + a)^2 + 1)*ab
s(b) + b)^2*b^2/(b^2*x + a*b)^2)/((a^4*abs(b) - a^3*abs(b) - a^2*abs(b) + a
*abs(b))*((sqrt(-(b*x + a)^2 + 1)*abs(b) + b)*a/(b^2*x + a*b) - (sqrt(-(b*x
 + a)^2 + 1)*abs(b) + b)^2*a/(b^2*x + a*b)^2 + (sqrt(-(b*x + a)^2 + 1)*abs(
b) + b)^3*a/(b^2*x + a*b)^3 - a + 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)/(b^
2*x + a*b) - 2*(sqrt(-(b*x + a)^2 + 1)*abs(b) + b)^2/(b^2*x + a*b)^2))
```

maple [B] time = 0.04, size = 671, normalized size = 4.47

$$\frac{b}{\left(-a^{2}+1\right)\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}+\frac{3a\,b^{2}x}{\left(-a^{2}+1\right)\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}+\frac{3b\,a^{2}}{\left(-a^{2}+1\right)\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2/(-b^2*x^2-2*a*b*x-a^2+1),x)
```

```
[Out] b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a*b^2/(-a^2+1)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2)*x+3*b*a^2/(-a^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-b/(-a^2+1)
^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
)/x)-1/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a*b/(-a^2+1)^2/(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)+3*a^2*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x
+3*a^3*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a*b/(-a^2+1)^(5/2)*ln(
(-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)+2/(-a
^2+1)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*x*b^2+2*b/(-a^2+1)/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)*a-1/(-a^2+1)/x/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)*a+3*a^2*b/(-a^2+1
)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3*a^3*b^2/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a
^2+1)^(1/2)*x+3*a^4*b/(-a^2+1)^2/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3*a^2*b/(-a
^2+1)^(5/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)^
(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx+a+1}{\left(b^{2}x^{2}+2\,abx+a^{2}-1\right)\sqrt{-(bx+a)^{2}+1}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)^2)^(1/2)/x^2/(-b^2*x^2-2*a*b*x-a^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*x + a + 1)/((b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-(b*x + a)^2 +
 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x+1}{x^{2}\,\sqrt{1-(a+b\,x)^{2}}\,\left(a^{2}+2\,a\,b\,x+b^{2}\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + 1)/(x^2*(1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x -
1)),x)
```

```
[Out] int(-(a + b*x + 1)/(x^2*(1 - (a + b*x)^2)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x -
1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax^2\sqrt{-a^2-2abx-b^2x^2+1}+bx^3\sqrt{-a^2-2abx-b^2x^2+1}-x^2\sqrt{-a^2-2abx-b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+1)/(1-(b*x+a)**2)**(1/2)/x**2/(-b**2*x**2-2*a*b*x-a**2+1),
x)
```

```
[Out] -Integral(1/(a*x**2*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1) + b*x**3*sqrt(-a*
*2 - 2*a*b*x - b**2*x**2 + 1) - x**2*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1))
, x)
```

3.875 $\int e^{n \tanh^{-1}(a+bx)} x^m \, dx$

Optimal. Leaf size=109

$$\frac{x^{m+1}(-a-bx+1)^{-n/2}(a+bx+1)^{n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\left(\frac{bx}{a+1}+1\right)^{-n/2}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;\frac{bx}{1-a},-\frac{bx}{a+1}\right)}{m+1}$$

```
[Out] x^(1+m)*(b*x+a+1)^(1/2*n)*(1-b*x/(1-a))^(1/2*n)*AppellF1(1+m,1/2*n,-1/2*n,2
+m,b*x/(1-a),-b*x/(1+a))/(1+m)/((-b*x-a+1)^(1/2*n))/((1+b*x/(1+a))^(1/2*n))
```

Rubi [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6163, 135, 133}

$$\frac{x^{m+1}(-a-bx+1)^{-n/2}(a+bx+1)^{n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\left(\frac{bx}{a+1}+1\right)^{-n/2}F_1\left(m+1;\frac{n}{2},-\frac{n}{2};m+2;\frac{bx}{1-a},-\frac{bx}{a+1}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(1 + a + b*x)^(n/2)*(1 - (b*x)/(1 - a))^(n/2)*AppellF1[1 + m, n/
2, -n/2, 2 + m, (b*x)/(1 - a), -((b*x)/(1 + a))])/((1 + m)*(1 - a - b*x)^(n
/2)*(1 + (b*x)/(1 + a))^(n/2))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
```

```
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\tanh^{-1}(a+bx)}x^m\,dx &= \int x^m(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx\\ &= \left((1-a-bx)^{-n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\right)\int x^m(1+a+bx)^{n/2}\left(1-\frac{bx}{1-a}\right)^{-n/2}\,dx\\ &= \left((1-a-bx)^{-n/2}(1+a+bx)^{n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\left(1+\frac{bx}{1+a}\right)^{-n/2}\right)\int x^m\left(1-\frac{bx}{1-a}\right)^{-n/2}\left(1+\right.\\ &= \frac{x^{1+m}(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\left(1-\frac{bx}{1-a}\right)^{n/2}\left(1+\frac{bx}{1+a}\right)^{-n/2}F_1\left(1+m;\frac{n}{2},-\frac{n}{2};2+m;\frac{bx}{1-a}\right.}{1+m}\end{aligned}$$

Mathematica [F] time = 0.84, size = 0, normalized size = 0.00

$$\int e^{n\tanh^{-1}(a+bx)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])*x^m,x]
[Out] Integrate[E^(n*ArcTanh[a + b*x])*x^m, x]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^m\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^m,x, algorithm="fricas")
[Out] integral(x^m*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^m,x, algorithm="giac")
[Out] integrate(x^m*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(bx+a)} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))*x^m,x)
[Out] int(exp(n*arctanh(b*x+a))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{bx + a + 1}{bx + a - 1}\right)^{\frac{1}{2}n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \, e^{n \operatorname{atanh}(a+b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*atanh(a + b*x)),x)
[Out] int(x^m*exp(n*atanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{n \operatorname{atanh}(a+bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))*x**m,x)
[Out] Integral(x**m*exp(n*atanh(a + b*x)), x)
```

3.876 $\int e^{n\tanh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=206

$$-\frac{(a+bx+1)^{\frac{n+2}{2}}\left(18a^2-2bx(6a-n)-10an+n^2+6\right)(-a-bx+1)^{1-\frac{n}{2}}}{24b^4}+\frac{2^{\frac{n}{2}-2}\left(24a^3-36a^2n+12a\left(n^2+2\right)-n\right.}{}$$

```
[Out] -1/4*x^2*(-b*x-a+1)^(1-1/2*n)*(b*x+a+1)^(1+1/2*n)/b^2-1/24*(-b*x-a+1)^(1-1/
2*n)*(b*x+a+1)^(1+1/2*n)*(6+18*a^2-10*a*n+n^2-2*b*(6*a-n)*x)/b^4+1/3*2^(-2+
1/2*n)*(24*a^3-36*a^2*n+12*a*(n^2+2)-n*(n^2+8))*(-b*x-a+1)^(1-1/2*n)*hyperg
eom([-1/2*n, 1-1/2*n],[2-1/2*n],-1/2*b*x-1/2*a+1/2)/b^4/(2-n)
```

Rubi [A] time = 0.18, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 100, 147, 69}

$$\frac{2^{\frac{n}{2}-2}\left(-36a^2n+24a^3+12a\left(n^2+2\right)-n\left(n^2+8\right)\right)(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{3b^4(2-n)}-\frac{(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] -(x^2*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(4*b^2) - ((1 - a
 - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2)*(6 + 18*a^2 - 10*a*n + n^2 - 2*b
*(6*a - n)*x))/(24*b^4) + (2^(-2 + n/2)*(24*a^3 - 36*a^2*n + 12*a*(2 + n^2)
 - n*(8 + n^2))*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2
- n/2, (1 - a - b*x)/2])/(3*b^4*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{n\tanh^{-1}(a+bx)}x^3\,dx = \int x^3(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx$$

$$= -\frac{x^2(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{4b^2} - \frac{\int x(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\left(-2\left(1-a^2\right)+b(\right.}{4b^2}$$

$$= -\frac{x^2(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{4b^2} - \frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}\left(6+18a^2-10an+\right.}{24b^4}$$

$$= -\frac{x^2(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{4b^2} - \frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}\left(6+18a^2-10an+\right.}{24b^4}$$

Mathematica [A] time = 0.25, size = 220, normalized size = 1.07

$$\frac{(-a-bx+1)^{1-\frac{n}{2}}\left(b^2(n-2)x^2(a+bx+1)^{\frac{n}{2}+1}-2^{\frac{n}{2}+3}(n-6a)\,{}_2F_1\left(-\frac{n}{2}-2,1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)-(a+1)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])*x^3,x]
```

```
[Out] ((1 - a - b*x)^(1 - n/2)*(b^2*(-2 + n)*x^2*(1 + a + b*x)^(1 + n/2) - 2^(3 +
 n/2)*(-6*a + n)*Hypergeometric2F1[-2 - n/2, 1 - n/2, 2 - n/2, (1 - a - b*x
)/2] - 2^(3 + n/2)*(1 + a)*(1 + 5*a - n)*Hypergeometric2F1[-1 - n/2, 1 - n/
2, 2 - n/2, (1 - a - b*x)/2] + 2^(1 + n/2)*(1 + a)^2*(2 + 4*a - n)*Hypergeo
metric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - a - b*x)/2]))/(4*b^4*(2 - n))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(bx+a)} x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))*x^3,x)
```

```
[Out] int(exp(n*arctanh(b*x+a))*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^3,x, algorithm="maxima")
[Out] integrate(x^3*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{e}^{n\,\mathrm{atanh}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(n*atanh(a + b*x)),x)
[Out] int(x^3*exp(n*atanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 e^{n\,\mathrm{atanh}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))*x**3,x)
[Out] Integral(x**3*exp(n*atanh(a + b*x)), x)
```

3.877 $\int e^{n\tanh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=170

$$-\frac{2^{n/2}\left(6a^2-6an+n^2+2\right)(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{3b^3(2-n)}+\frac{(4a-n)(a+bx+1)^{\frac{n+2}{2}}(-a-}{6b^3}$$

```
[Out] 1/6*(4*a-n)*(-b*x-a+1)^(1-1/2*n)*(b*x+a+1)^(1+1/2*n)/b^3-1/3*x*(-b*x-a+1)^(
1-1/2*n)*(b*x+a+1)^(1+1/2*n)/b^2-1/3*2^(1/2*n)*(6*a^2-6*a*n+n^2+2)*(-b*x-a+
1)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],-1/2*b*x-1/2*a+1/2)/b^3/
(2-n)
```

Rubi [A] time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 90, 80, 69}

$$-\frac{2^{n/2}\left(6a^2-6an+n^2+2\right)(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{3b^3(2-n)}+\frac{(4a-n)(a+bx+1)^{\frac{n+2}{2}}(-a-}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] ((4*a - n)*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(6*b^3) - (x*
(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(3*b^2) - (2^(n/2)*(2 +
6*a^2 - 6*a*n + n^2)*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/
2, 2 - n/2, (1 - a - b*x)/2])/(3*b^3*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{n\tanh^{-1}(a+bx)}x^2\,dx = \int x^2(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx$$

$$= -\frac{x(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{3b^2} - \frac{\int (1-a-bx)^{-n/2}(1+a+bx)^{n/2}\left(-1+a^2+b(4a-n\right.}{3b^2}$$

$$= \frac{(4a-n)(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{6b^3} - \frac{x(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{3b^2} + \frac{\left(2+6a^2-\right.}{}$$

$$= \frac{(4a-n)(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{6b^3} - \frac{x(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{3b^2} - \frac{2^{n/2}\left(2+6a\right.}{}$$

Mathematica [A] time = 0.11, size = 127, normalized size = 0.75

$$\frac{(-a-bx+1)^{1-\frac{n}{2}}\left(\frac{2^{\frac{n}{2}+1}\left(6a^2-6an+n^2+2\right){}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{n-2}+(4a-n)(a+bx+1)^{\frac{n}{2}+1}-2bx(a+bx+1)^{\frac{n}{2}+1}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])*x^2,x]
```

```
[Out] ((1 - a - b*x)^(1 - n/2)*((4*a - n)*(1 + a + b*x)^(1 + n/2) - 2*b*x*(1 + a
+ b*x)^(1 + n/2) + (2^(1 + n/2)*(2 + 6*a^2 - 6*a*n + n^2)*Hypergeometric2F1
[1 - n/2, -1/2*n, 2 - n/2, (1 - a - b*x)/2])/(-2 + n)))/(6*b^3)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^2,x, algorithm="fricas")
[Out] integral(x^2*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^2,x, algorithm="giac")
[Out] integrate(x^2*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arctanh}(bx+a)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))*x^2,x)
[Out] int(exp(n*arctanh(b*x+a))*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{e}^{n\,\mathrm{atanh}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(n*atanh(a + b*x)),x)
[Out] int(x^2*exp(n*atanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{n \operatorname{atanh}(a+bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))*x**2,x)
[Out] Integral(x**2*exp(n*atanh(a + b*x)), x)
```

3.878 $\int e^{n\tanh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=114

$$\frac{2^{n/2}(2a-n)(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{b^2(2-n)}-\frac{(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n+2}{2}}}{2b^2}$$

```
[Out] -1/2*(-b*x-a+1)^(1-1/2*n)*(b*x+a+1)^(1+1/2*n)/b^2+2^(1/2*n)*(2*a-n)*(-b*x-a
+1)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],-1/2*b*x-1/2*a+1/2)/b^2
/(2-n)
```

Rubi [A] time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6163, 80, 69}

$$\frac{2^{n/2}(2a-n)(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{b^2(2-n)}-\frac{(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n+2}{2}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])*x,x]
```

```
[Out] -((1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(2*b^2) + (2^(n/2)*(2*
a - n)*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (1
 - a - b*x)/2])/(b^2*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int e^{n\tanh^{-1}(a+bx)}x\,dx = \int x(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx$$
$$= -\frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{2b^2} - \frac{(2a-n)\int(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx}{2b}$$
$$= -\frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{2b^2} + \frac{2^{n/2}(2a-n)(1-a-bx)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-\right.}{b^2(2-n)}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 0.84

$$\frac{(-a-bx+1)^{1-\frac{n}{2}}\left(\frac{b2^{\frac{n}{2}+1}(n-2a)\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{n-2}-b(a+bx+1)^{\frac{n}{2}+1}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])*x,x]
```

```
[Out] ((1 - a - b*x)^(1 - n/2)*(-(b*(1 + a + b*x)^(1 + n/2)) + (2^(1 + n/2)*b*(-2
*a + n)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - a - b*x)/2])/(-2 +
 n)))/(2*b^3)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x,x, algorithm="fricas")
```

```
[Out] integral(x*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x,x, algorithm="giac")
[Out] integrate(x*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(bx+a)} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))*x,x)
[Out] int(exp(n*arctanh(b*x+a))*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{bx + a + 1}{bx + a - 1}\right)^{\frac{1}{2} n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))*x,x, algorithm="maxima")
[Out] integrate(x*((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \, e^{n \operatorname{atanh}(a+b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(n*atanh(a + b*x)),x)
[Out] int(x*exp(n*atanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x e^{n \operatorname{atanh}(a+bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))*x,x)
[Out] Integral(x*exp(n*atanh(a + b*x)), x)
```

3.879 $\int e^{n \tanh^{-1}(a+bx)} \, dx$

Optimal. Leaf size=71

$$-\frac{2^{\frac{n}{2}+1}(-a-bx+1)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{b(2-n)}$$

```
[Out] -2^(1+1/2*n)*(-b*x-a+1)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],-1/
2*b*x-1/2*a+1/2)/b/(2-n)
```

Rubi [A] time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6161, 69}

$$-\frac{2^{\frac{n}{2}+1}(-a-bx+1)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)}{b(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x]),x]
```

```
[Out] -((2^(1 + n/2)*(1 - a - b*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 -
 n/2, (1 - a - b*x)/2])/(b*(2 - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6161

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 + a*c
 + b*c*x)^(n/2)/(1 - a*c - b*c*x)^(n/2), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\tanh^{-1}(a+bx)}\,dx=\int(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,dx$$
$$=-\frac{2^{1+\frac{n}{2}}(1-a-bx)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-a-bx)\right)}{b(2-n)}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.70

$$\frac{4e^{(n+2)\tanh^{-1}(a+bx)}\,{}_2F_1\left(2,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\tanh^{-1}(a+bx)}\right)}{b(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a + b*x]),x]
```

```
[Out] (4*E^((2 + n)*ArcTanh[a + b*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(
2*ArcTanh[a + b*x])])/(b*(2 + n))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arctanh}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a)),x)
```

```
[Out] int(exp(n*arctanh(b*x+a)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{atanh}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a + b*x)),x)
```

```
[Out] int(exp(n*atanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{atanh}{(a+bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a)),x)
```

```
[Out] Integral(exp(n*atanh(a + b*x)), x)
```

3.880 $\int \frac{e^{n\tanh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=135

$$\frac{2(-a-bx+1)^{-n/2}(a+bx+1)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{n}-\frac{2^{\frac{n}{2}+1}(-a-bx+1)^{-n/2}\,{}_2F_1\Bigl(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(-a}{n}$$

```
[Out] 2*(b*x+a+1)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(1+a)*(-b*x-a+1)/(1-a)/
(b*x+a+1))/n/((-b*x-a+1)^(1/2*n))-2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1
-1/2*n],-1/2*b*x-1/2*a+1/2)/n/((-b*x-a+1)^(1/2*n))
```

Rubi [A] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6163, 105, 69, 131}

$$\frac{2(-a-bx+1)^{-n/2}(a+bx+1)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{n}-\frac{2^{\frac{n}{2}+1}(-a-bx+1)^{-n/2}\,{}_2F_1\Bigl(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(-a}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])/x,x]
```

```
[Out] (2*(1 + a + b*x)^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, ((1 + a)*(1 - a
- b*x))/((1 - a)*(1 + a + b*x))])/(n*(1 - a - b*x)^(n/2)) - (2^(1 + n/2)*Hy
pergeometric2F1[-n/2, -n/2, 1 - n/2, (1 - a - b*x)/2])/(n*(1 - a - b*x)^(n/
2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(a+bx)}}{x}\,dx = \int \frac{(1-a-bx)^{-n/2}(1+a+bx)^{n/2}}{x}\,dx$$
$$= -\left((-1+a)\int \frac{(1-a-bx)^{-1-\frac{n}{2}}(1+a+bx)^{n/2}}{x}\,dx\right) - b\int (1-a-bx)^{-1-\frac{n}{2}}(1+a+bx)^{n/2}\,dx$$
$$= \frac{2(1-a-bx)^{-n/2}(1+a+bx)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{(1+a)(1-a-bx)}{(1-a)(1+a+bx)}\right)}{n} - \frac{2^{1+\frac{n}{2}}(1-a-bx)^{-n/2}\,{}_2F_1}{}$$

Mathematica [A] time = 0.04, size = 111, normalized size = 0.82

$$\frac{2(-a-bx+1)^{-n/2}\left((a+bx+1)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{(a+1)(a+bx-1)}{(a-1)(a+bx+1)}\right)-2^{n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(-a-bx+1)\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])/x,x]
```

```
[Out] (2*((1 + a + b*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, ((1 + a)*(-1
+ a + b*x))/((-1 + a)*(1 + a + b*x))] - 2^(n/2)*Hypergeometric2F1[-1/2*n, -
1/2*n, 1 - n/2, (1 - a - b*x)/2]))/(n*(1 - a - b*x)^(n/2))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}\,n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(bx+a)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))/x,x)
[Out] int(exp(n*arctanh(b*x+a))/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}\,n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a+b\,x)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a + b*x))/x,x)
[Out] int(exp(n*atanh(a + b*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(a+bx)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))/x,x)
[Out] Integral(exp(n*atanh(a + b*x))/x, x)
```

3.881 $\int \frac{e^{n \tanh^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=92

$$-\frac{4b(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n-2}{2}} {}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{(1-a)^2(2-n)}$$

```
[Out] -4*b*(-b*x-a+1)^(1-1/2*n)*(b*x+a+1)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/
2*n],(1+a)*(-b*x-a+1)/(1-a)/(b*x+a+1))/(1-a)^2/(2-n)
```

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6163, 131}

$$-\frac{4b(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n-2}{2}} {}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(a+1)(-a-bx+1)}{(1-a)(a+bx+1)}\right)}{(1-a)^2(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] (-4*b*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((-2 + n)/2)*Hypergeometric2F1[
2, 1 - n/2, 2 - n/2, ((1 + a)*(1 - a - b*x))/((1 - a)*(1 + a + b*x))])/((1
- a)^2*(2 - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(a+bx)}}{x^2}\,dx = \int \frac{(1-a-bx)^{-n/2}(1+a+bx)^{n/2}}{x^2}\,dx$$
$$= -\frac{4b(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(1+a)(1-a-bx)}{(1-a)(1+a+bx)}\right)}{(1-a)^2(2-n)}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.90

$$\frac{4b(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n}{2}-1}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(a+1)(a+bx-1)}{(a-1)(a+bx+1)}\right)}{(a-1)^2(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])/x^2,x]
```

```
[Out] (4*b*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^(-1 + n/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, ((1 + a)*(-1 + a + b*x))/((-1 + a)*(1 + a + b*x))])/((-1
+ a)^2*(-2 + n))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] integral(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^2, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(bx+a)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))/x^2,x)
[Out] int(exp(n*arctanh(b*x+a))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^2,x, algorithm="maxima")
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{atanh}(a+b\,x)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a + b*x))/x^2,x)
[Out] int(exp(n*atanh(a + b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}{\left(a+bx \right)}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))/x**2,x)
[Out] Integral(exp(n*atanh(a + b*x))/x**2, x)
```

3.882 $\int \frac{e^{n\tanh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=152

$$-\frac{(a+bx+1)^{\frac{n+2}{2}}(-a-bx+1)^{1-\frac{n}{2}}}{2\left(1-a^2\right)x^2}-\frac{2b^2(2a+n)(a+bx+1)^{\frac{n-2}{2}}(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(a+1)(-a-bx+1}{(1-a)(a+bx+1)}\right.}{(1-a)^3(a+1)(2-n)}$$

```
[Out] -1/2*(-b*x-a+1)^(1-1/2*n)*(b*x+a+1)^(1+1/2*n)/(-a^2+1)/x^2-2*b^2*(2*a+n)*(-
b*x-a+1)^(1-1/2*n)*(b*x+a+1)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(1
+a)*(-b*x-a+1)/(1-a)/(b*x+a+1))/(1-a)^3/(1+a)/(2-n)
```

Rubi [A] time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6163, 96, 131}

$$-\frac{(a+bx+1)^{\frac{n+2}{2}}(-a-bx+1)^{1-\frac{n}{2}}}{2\left(1-a^2\right)x^2}-\frac{2b^2(2a+n)(a+bx+1)^{\frac{n-2}{2}}(-a-bx+1)^{1-\frac{n}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(a+1)(-a-bx+1}{(1-a)(a+bx+1)}\right.}{(1-a)^3(a+1)(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] -((1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((2 + n)/2))/(2*(1 - a^2)*x^2) - (2
*b^2*(2*a + n)*(1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^((-2 + n)/2)*Hypergeom
etric2F1[2, 1 - n/2, 2 - n/2, ((1 + a)*(1 - a - b*x))/((1 - a)*(1 + a + b*x
))])/((1 - a)^3*(1 + a)*(2 - n))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
```

```
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6163

```
Int[E^(ArcTanh[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.)
, x_Symbol] :> Int[((d + e*x)^m*(1 + a*c + b*c*x)^(n/2))/(1 - a*c - b*c*x)^
(n/2), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{(1-a-bx)^{-n/2}(1+a+bx)^{n/2}}{x^3}\,dx$$

$$= -\frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{2\left(1-a^2\right)x^2} + \frac{(b(2a+n))\int \frac{(1-a-bx)^{-n/2}(1+a+bx)^{n/2}}{x^2}\,dx}{2\left(1-a^2\right)}$$

$$= -\frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{2\left(1-a^2\right)x^2} - \frac{2b^2(2a+n)(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\right.}{(1-a)^3(1+a)(2-n)}$$

Mathematica [A] time = 0.06, size = 123, normalized size = 0.81

$$\frac{(-a-bx+1)^{1-\frac{n}{2}}(a+bx+1)^{\frac{n}{2}-1}\left((a-1)^2(n-2)(a+bx+1)^2-4b^2x^2(2a+n)\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{(a+1)(a+bx-1)}{(a-1)(a+bx+1)}\right)\right)}{2(a-1)^3(a+1)(n-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a + b*x])/x^3,x]
```

```
[Out] ((1 - a - b*x)^(1 - n/2)*(1 + a + b*x)^(-1 + n/2)*((-1 + a)^2*(-2 + n)*(1 +
 a + b*x)^2 - 4*b^2*(2*a + n)*x^2*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, ((
1 + a)*(-1 + a + b*x))/((-1 + a)*(1 + a + b*x))]))/(2*(-1 + a)^3*(1 + a)*(-
2 + n)*x^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^3,x, algorithm="fricas")
[Out] integral(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^3,x, algorithm="giac")
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(bx+a)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(b*x+a))/x^3,x)
[Out] int(exp(n*arctanh(b*x+a))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bx+a+1}{bx+a-1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(b*x+a))/x^3,x, algorithm="maxima")
[Out] integrate(((b*x + a + 1)/(b*x + a - 1))^(1/2*n)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a+b\,x)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a + b*x))/x^3,x)
```

```
[Out] int(exp(n*atanh(a + b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(a+bx)}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(b*x+a))/x**3,x)
```

```
[Out] Integral(exp(n*atanh(a + b*x))/x**3, x)
```

3.883 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=127

$$-\frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a}+\frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2}+\frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{35}{128}c^4x\sqrt{1-a^2x^2}+\frac{35c^4\sin^{-1}}{128a}$$

```
[Out] 35/192*c^4*x*(-a^2*x^2+1)^(3/2)+7/48*c^4*x*(-a^2*x^2+1)^(5/2)+1/8*c^4*x*(-a
^2*x^2+1)^(7/2)-1/9*c^4*(-a^2*x^2+1)^(9/2)/a+35/128*c^4*arcsin(a*x)/a+35/12
8*c^4*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6138, 641, 195, 216}

$$-\frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a}+\frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2}+\frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{35}{128}c^4x\sqrt{1-a^2x^2}+\frac{35c^4\sin^{-1}}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] (35*c^4*x*Sqrt[1 - a^2*x^2])/128 + (35*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (7*
c^4*x*(1 - a^2*x^2)^(5/2))/48 + (c^4*x*(1 - a^2*x^2)^(7/2))/8 - (c^4*(1 - a
^2*x^2)^(9/2))/(9*a) + (35*c^4*ArcSin[a*x])/(128*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\int (1+ax)\left(1-a^2x^2\right)^{7/2}\,dx$$
$$= -\frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + c^4\int \left(1-a^2x^2\right)^{7/2}\,dx$$
$$= \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} - \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{8}\left(7c^4\right)\int \left(1-a^2x^2\right)^{5/2}\,dx$$
$$= \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} - \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{48}\left(35c^4\right)\int \left(1-a^2\right.$$
$$= \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} - \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a}$$
$$= \frac{35}{128}c^4x\sqrt{1-a^2x^2} + \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)$$
$$= \frac{35}{128}c^4x\sqrt{1-a^2x^2} + \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)$$

Mathematica [A] time = 0.16, size = 107, normalized size = 0.84

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(128a^8x^8+144a^7x^7-512a^6x^6-600a^5x^5+768a^4x^4+978a^3x^3-512a^2x^2-837ax+128\right)+630\right.}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/1152*(c^4*(Sqrt[1 - a^2*x^2]*(128 - 837*a*x - 512*a^2*x^2 + 978*a^3*x^3
+ 768*a^4*x^4 - 600*a^5*x^5 - 512*a^6*x^6 + 144*a^7*x^7 + 128*a^8*x^8) + 63
0*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/a
```

fricas [A] time = 0.67, size = 136, normalized size = 1.07

$$-\frac{630\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(128\,a^8c^4x^8+144\,a^7c^4x^7-512\,a^6c^4x^6-600\,a^5c^4x^5+768\,a^4c^4x^4+978\,a^3c^4x^3-\right.}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas"
)
```

```
[Out] -1/1152*(630*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (128*a^8*c^4*x^8
+ 144*a^7*c^4*x^7 - 512*a^6*c^4*x^6 - 600*a^5*c^4*x^5 + 768*a^4*c^4*x^4 + 9
78*a^3*c^4*x^3 - 512*a^2*c^4*x^2 - 837*a*c^4*x + 128*c^4)*sqrt(-a^2*x^2 + 1
))/a
```

giac [A] time = 0.20, size = 127, normalized size = 1.00

$$\frac{35\,c^4\arcsin\left(ax\right)\operatorname{sgn}(a)}{128\,|a|}-\frac{1}{1152}\,\sqrt{-a^2x^2+1}\left(\frac{128\,c^4}{a}-\left(837\,c^4+2\left(256\,ac^4-\left(489\,a^2c^4+4\left(96\,a^3c^4-\left(75\,a^4c^4+\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 35/128*c^4*arcsin(a*x)*sgn(a)/abs(a) - 1/1152*sqrt(-a^2*x^2 + 1)*(128*c^4/a
 - (837*c^4 + 2*(256*a*c^4 - (489*a^2*c^4 + 4*(96*a^3*c^4 - (75*a^4*c^4 + 2
*(32*a^5*c^4 - (8*a^7*c^4*x + 9*a^6*c^4)*x)*x)*x)*x)*x)*x)*x)
```

maple [B] time = 0.09, size = 229, normalized size = 1.80

$$\frac{93c^4x\sqrt{-a^2x^2+1}}{128}-\frac{c^4\sqrt{-a^2x^2+1}}{9a}-\frac{c^4a^7x^8\sqrt{-a^2x^2+1}}{9}+\frac{4c^4a^5x^6\sqrt{-a^2x^2+1}}{9}-\frac{2c^4a^3x^4\sqrt{-a^2x^2+1}}{3}+\frac{4c^4a\,x^2\sqrt{-}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] 93/128*c^4*x*(-a^2*x^2+1)^(1/2)-1/9*c^4*(-a^2*x^2+1)^(1/2)/a-1/9*c^4*a^7*x^
8*(-a^2*x^2+1)^(1/2)+4/9*c^4*a^5*x^6*(-a^2*x^2+1)^(1/2)-2/3*c^4*a^3*x^4*(-a
^2*x^2+1)^(1/2)+4/9*c^4*a*x^2*(-a^2*x^2+1)^(1/2)-1/8*c^4*a^6*x^7*(-a^2*x^2+
1)^(1/2)+25/48*c^4*a^4*x^5*(-a^2*x^2+1)^(1/2)-163/192*c^4*a^2*x^3*(-a^2*x^2
+1)^(1/2)+35/128*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 210, normalized size = 1.65

$$-\frac{1}{9}\sqrt{-a^2x^2+1}\,a^7c^4x^8-\frac{1}{8}\sqrt{-a^2x^2+1}\,a^6c^4x^7+\frac{4}{9}\sqrt{-a^2x^2+1}\,a^5c^4x^6+\frac{25}{48}\sqrt{-a^2x^2+1}\,a^4c^4x^5-\frac{2}{3}\sqrt{-a^2x^2+1}\,a^3c^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima"
)
```

```
[Out] -1/9*sqrt(-a^2*x^2 + 1)*a^7*c^4*x^8 - 1/8*sqrt(-a^2*x^2 + 1)*a^6*c^4*x^7 +
4/9*sqrt(-a^2*x^2 + 1)*a^5*c^4*x^6 + 25/48*sqrt(-a^2*x^2 + 1)*a^4*c^4*x^5 -
 2/3*sqrt(-a^2*x^2 + 1)*a^3*c^4*x^4 - 163/192*sqrt(-a^2*x^2 + 1)*a^2*c^4*x^
3 + 4/9*sqrt(-a^2*x^2 + 1)*a*c^4*x^2 + 93/128*sqrt(-a^2*x^2 + 1)*c^4*x + 35
/128*c^4*arcsin(a*x)/a - 1/9*sqrt(-a^2*x^2 + 1)*c^4/a
```

mupad [B] time = 0.09, size = 118, normalized size = 0.93

$$\frac{35\,c^4\,x\,\sqrt{1-a^2\,x^2}}{128}+\frac{35\,c^4\,x\,{\left(1-a^2\,x^2\right)}^{3/2}}{192}+\frac{7\,c^4\,x\,{\left(1-a^2\,x^2\right)}^{5/2}}{48}+\frac{c^4\,x\,{\left(1-a^2\,x^2\right)}^{7/2}}{8}-\frac{c^4\,{\left(1-a^2\,x^2\right)}^{9/2}}{9\,a}-\frac{35\,c^4\,\mathrm{asinh}\left(\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (35*c^4*x*(1 - a^2*x^2)^(1/2))/128 + (35*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (
7*c^4*x*(1 - a^2*x^2)^(5/2))/48 + (c^4*x*(1 - a^2*x^2)^(7/2))/8 - (c^4*(1 -
 a^2*x^2)^(9/2))/(9*a) - (35*c^4*asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2))/(128*a
^2)
```

sympy [A] time = 40.90, size = 452, normalized size = 3.56

$$\begin{cases} \frac{-\frac{c^4(-a^2x^2+1)^{\frac{3}{2}}}{3}+c^4\left(\begin{cases}\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\mathrm{asin}\,(ax)}{2} & \text{for}\ ax>-1\wedge ax<1\end{cases}\right)-3c^4\left(\begin{cases}-\frac{ax\left(-2a^2x^2+1\right)\sqrt{-a^2x^2+1}}{8}+\frac{\mathrm{asin}\,(ax)}{8} & \text{for}\ ax>-\end{cases}\right.}{} \\ c^4x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] Piecewise(((-c**4*(-a**2*x**2 + 1)**(3/2)/3 + c**4*Piecewise((a*x*sqrt(-a**
2*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) - 3*c**4*Piecewise((-
a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 + asin(a*x)/8, (a*x > -1) & (
a*x < 1))) - 3*c**4*Piecewise(((-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)
**(3/2)/3, (a*x > -1) & (a*x < 1))) + 3*c**4*Piecewise((-a**3*x**3*(-a**2*x
**2 + 1)**(3/2)/6 - a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/16 + asin(a
*x)/16, (a*x > -1) & (a*x < 1))) + 3*c**4*Piecewise((-(-a**2*x**2 + 1)**(7/
2)/7 + 2*(-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)**(3/2)/3, (a*x > -1)
```

```
& (a*x < 1))) - c**4*Piecewise((-a**3*x**3*(-a**2*x**2 + 1)**(3/2)/6 - a*x*
(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/32 - a*x*sqrt(-a**2*x**2 + 1)*(-16*
a**6*x**6 + 24*a**4*x**4 - 10*a**2*x**2 + 1)/128 + 5*asin(a*x)/128, (a*x >
-1) & (a*x < 1))) - c**4*Piecewise(((-a**2*x**2 + 1)**(9/2)/9 - 3*(-a**2*x*
*2 + 1)**(7/2)/7 + 3*(-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)**(3/2)/3,
 (a*x > -1) & (a*x < 1))))/a, Ne(a, 0)), (c**4*x, True))
```

3.884 $\int e^{\tanh^{-1}(ax)} \left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=105

$$-\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{16a}$$

```
[Out] 5/24*c^3*x*(-a^2*x^2+1)^(3/2)+1/6*c^3*x*(-a^2*x^2+1)^(5/2)-1/7*c^3*(-a^2*x^
2+1)^(7/2)/a+5/16*c^3*arcsin(a*x)/a+5/16*c^3*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6138, 641, 195, 216}

$$-\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/16 + (5*c^3*x*(1 - a^2*x^2)^(3/2))/24 + (c^3*x*
(1 - a^2*x^2)^(5/2))/6 - (c^3*(1 - a^2*x^2)^(7/2))/(7*a) + (5*c^3*ArcSin[a*
x])/(16*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = c^3\int (1+ax)\left(1-a^2x^2\right)^{5/2}\,dx$$
$$= -\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a} + c^3\int \left(1-a^2x^2\right)^{5/2}\,dx$$
$$= \frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2} - \frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a} + \frac{1}{6}\left(5c^3\right)\int \left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2} + \frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2} - \frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a} + \frac{1}{8}\left(5c^3\right)\int \sqrt{1-a^2x^2}$$
$$= \frac{5}{16}c^3x\sqrt{1-a^2x^2} + \frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2} + \frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2} - \frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a} +$$
$$= \frac{5}{16}c^3x\sqrt{1-a^2x^2} + \frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2} + \frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2} - \frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a} +$$

Mathematica [A] time = 0.13, size = 91, normalized size = 0.87

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(48a^6x^6+56a^5x^5-144a^4x^4-182a^3x^3+144a^2x^2+231ax-48\right)-210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{336a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(-48 + 231*a*x + 144*a^2*x^2 - 182*a^3*x^3 - 144*a^
4*x^4 + 56*a^5*x^5 + 48*a^6*x^6) - 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(336
*a)
```

fricas [A] time = 0.65, size = 115, normalized size = 1.10

$$-\frac{210\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(48\,a^6c^3x^6+56\,a^5c^3x^5-144\,a^4c^3x^4-182\,a^3c^3x^3+144\,a^2c^3x^2+231\,ac^3x-48\,c\right.}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/336*(210*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (48*a^6*c^3*x^6 +
56*a^5*c^3*x^5 - 144*a^4*c^3*x^4 - 182*a^3*c^3*x^3 + 144*a^2*c^3*x^2 + 231*
a*c^3*x - 48*c^3)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.19, size = 103, normalized size = 0.98

$$\frac{5\,c^3\arcsin\left(ax\right)\mathrm{sgn}(a)}{16\,|a|}-\frac{1}{336}\,\sqrt{-a^2x^2+1}\left(\frac{48\,c^3}{a}-\left(231\,c^3+2\left(72\,ac^3-\left(91\,a^2c^3+4\left(18\,a^3c^3-\left(6\,a^5c^3x+7\,a^4c^3\right)\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 5/16*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/336*sqrt(-a^2*x^2 + 1)*(48*c^3/a - (
231*c^3 + 2*(72*a*c^3 - (91*a^2*c^3 + 4*(18*a^3*c^3 - (6*a^5*c^3*x + 7*a^4*
c^3)*x)*x)*x)*x)*x)
```

maple [B] time = 0.06, size = 183, normalized size = 1.74

$$\frac{c^3a^5x^6\sqrt{-a^2x^2+1}}{7}-\frac{3c^3a^3x^4\sqrt{-a^2x^2+1}}{7}+\frac{3c^3a\,x^2\sqrt{-a^2x^2+1}}{7}-\frac{c^3\sqrt{-a^2x^2+1}}{7a}+\frac{c^3a^4x^5\sqrt{-a^2x^2+1}}{6}-\frac{13c^3a^2x^3\sqrt{-}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/7*c^3*a^5*x^6*(-a^2*x^2+1)^(1/2)-3/7*c^3*a^3*x^4*(-a^2*x^2+1)^(1/2)+3/7*c
^3*a*x^2*(-a^2*x^2+1)^(1/2)-1/7*c^3*(-a^2*x^2+1)^(1/2)/a+1/6*c^3*a^4*x^5*(-
a^2*x^2+1)^(1/2)-13/24*c^3*a^2*x^3*(-a^2*x^2+1)^(1/2)+11/16*c^3*x*(-a^2*x^2
+1)^(1/2)+5/16*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 164, normalized size = 1.56

$$\frac{1}{7}\sqrt{-a^2x^2+1}\,a^5c^3x^6+\frac{1}{6}\sqrt{-a^2x^2+1}\,a^4c^3x^5-\frac{3}{7}\sqrt{-a^2x^2+1}\,a^3c^3x^4-\frac{13}{24}\sqrt{-a^2x^2+1}\,a^2c^3x^3+\frac{3}{7}\sqrt{-a^2x^2+1}\,ac^3x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

[Out] 1/7*sqrt(-a^2*x^2 + 1)*a^5*c^3*x^6 + 1/6*sqrt(-a^2*x^2 + 1)*a^4*c^3*x^5 - 3/7*sqrt(-a^2*x^2 + 1)*a^3*c^3*x^4 - 13/24*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^3 + 3/7*sqrt(-a^2*x^2 + 1)*a*c^3*x^2 + 11/16*sqrt(-a^2*x^2 + 1)*c^3*x + 5/16*c^3*arcsin(a*x)/a - 1/7*sqrt(-a^2*x^2 + 1)*c^3/a

mupad [B] time = 0.90, size = 100, normalized size = 0.95

$$\frac{5\,c^3\,x\,\sqrt{1-a^2\,x^2}}{16}+\frac{5\,c^3\,x\,{\left(1-a^2\,x^2\right)}^{3/2}}{24}+\frac{c^3\,x\,{\left(1-a^2\,x^2\right)}^{5/2}}{6}-\frac{c^3\,{\left(1-a^2\,x^2\right)}^{7/2}}{7\,a}-\frac{5\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - a^2*c*x^2)^3*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)

[Out] (5*c^3*x*(1 - a^2*x^2)^(1/2))/16 + (5*c^3*x*(1 - a^2*x^2)^(3/2))/24 + (c^3*x*(1 - a^2*x^2)^(5/2))/6 - (c^3*(1 - a^2*x^2)^(7/2))/(7*a) - (5*c^3*asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2))/(16*a^2)

sympy [A] time = 24.98, size = 267, normalized size = 2.54

$$\begin{cases} \frac{-\frac{c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}+c^3\left(\begin{cases}\frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\mathrm{asin}\,(ax)}{2} & \text{for } ax>-1 \wedge ax<1\end{cases}\right)-2c^3\left(\begin{cases}-\frac{ax\left(-2a^2x^2+1\right)\sqrt{-a^2x^2+1}}{8}+\frac{\mathrm{asin}\,(ax)}{8} & \text{for } ax> \end{cases}\right.}{} \\ c^3x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**3,x)

[Out] Piecewise(((-c**3*(-a**2*x**2 + 1)**(3/2)/3 + c**3*Piecewise((a*x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) - 2*c**3*Piecewise((-a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 + asin(a*x)/8, (a*x > -1) & (a*x < 1))) - 2*c**3*Piecewise(((-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)**(3/2)/3, (a*x > -1) & (a*x < 1))) + c**3*Piecewise((-a**3*x**3*(-a**2*x**2 + 1)**(3/2)/6 - a*x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/16 + asin(a*x)/16, (a*x > -1) & (a*x < 1))) + c**3*Piecewise((-(-a**2*x**2 + 1)**(7/2)/7 + 2*(-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)**(3/2)/3, (a*x > -1) & (a*x < 1))))/a, Ne(a, 0)), (c**3*x, True))

3.885 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=83

$$-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{3c^2\sin^{-1}(ax)}{8a}$$

```
[Out] 1/4*c^2*x*(-a^2*x^2+1)^(3/2)-1/5*c^2*(-a^2*x^2+1)^(5/2)/a+3/8*c^2*arcsin(a*
x)/a+3/8*c^2*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6138, 641, 195, 216}

$$-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{3c^2\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] (3*c^2*x*Sqrt[1 - a^2*x^2])/8 + (c^2*x*(1 - a^2*x^2)^(3/2))/4 - (c^2*(1 - a
^2*x^2)^(5/2))/(5*a) + (3*c^2*ArcSin[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= c^2\int (1+ax)\left(1-a^2x^2\right)^{3/2}\,dx\\
&= -\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+c^2\int \left(1-a^2x^2\right)^{3/2}\,dx\\
&= \frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}\left(3c^2\right)\int \sqrt{1-a^2x^2}\,dx\\
&= \frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{8}\left(3c^2\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}-\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{3c^2\sin^{-1}(ax)}{8a}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.90

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(8a^4x^4+10a^3x^3-16a^2x^2-25ax+8\right)+30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/40*(c^2*(Sqrt[1 - a^2*x^2]*(8 - 25*a*x - 16*a^2*x^2 + 10*a^3*x^3 + 8*a^4
*x^4) + 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/a
```

fricas [A] time = 0.48, size = 92, normalized size = 1.11

$$-\frac{30\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(8\,a^4c^2x^4+10\,a^3c^2x^3-16\,a^2c^2x^2-25\,ac^2x+8\,c^2\right)\sqrt{-a^2x^2+1}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

[Out] -1/40*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (8*a^4*c^2*x^4 + 10*a^3*c^2*x^3 - 16*a^2*c^2*x^2 - 25*a*c^2*x + 8*c^2)*sqrt(-a^2*x^2 + 1))/a

giac [A] time = 0.21, size = 78, normalized size = 0.94

$$\frac{3\,c^2 \arcsin\left(ax\right) \operatorname{sgn}(a)}{8\,|a|} + \frac{1}{40}\,\sqrt{-a^2x^2+1}\left(\left(25\,c^2+2\left(8\,ac^2-\left(4\,a^3c^2x+5\,a^2c^2\right)x\right)x\right)x-\frac{8\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] 3/8*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/40*sqrt(-a^2*x^2 + 1)*((25*c^2 + 2*(8*a*c^2 - (4*a^3*c^2*x + 5*a^2*c^2)*x)*x)*x - 8*c^2/a)

maple [A] time = 0.04, size = 137, normalized size = 1.65

$$-\frac{c^2a^3x^4\sqrt{-a^2x^2+1}}{5}+\frac{2c^2a\,x^2\sqrt{-a^2x^2+1}}{5}-\frac{c^2\sqrt{-a^2x^2+1}}{5a}-\frac{c^2a^2x^3\sqrt{-a^2x^2+1}}{4}+\frac{5c^2x\sqrt{-a^2x^2+1}}{8}+\frac{3c^2\arctan\left(\cdots\right.}{8\sqrt{a^2}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^2,x)

[Out] -1/5*c^2*a^3*x^4*(-a^2*x^2+1)^(1/2)+2/5*c^2*a*x^2*(-a^2*x^2+1)^(1/2)-1/5*c^2*(-a^2*x^2+1)^(1/2)/a-1/4*c^2*a^2*x^3*(-a^2*x^2+1)^(1/2)+5/8*c^2*x*(-a^2*x^2+1)^(1/2)+3/8*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))

maxima [A] time = 0.41, size = 118, normalized size = 1.42

$$-\frac{1}{5}\sqrt{-a^2x^2+1}\,a^3c^2x^4-\frac{1}{4}\sqrt{-a^2x^2+1}\,a^2c^2x^3+\frac{2}{5}\sqrt{-a^2x^2+1}\,ac^2x^2+\frac{5}{8}\sqrt{-a^2x^2+1}\,c^2x+\frac{3\,c^2\arcsin\left(ax\right)}{8\,a}-\frac{\sqrt{-a^2x^2\cdots}}{5\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] -1/5*sqrt(-a^2*x^2 + 1)*a^3*c^2*x^4 - 1/4*sqrt(-a^2*x^2 + 1)*a^2*c^2*x^3 + 2/5*sqrt(-a^2*x^2 + 1)*a*c^2*x^2 + 5/8*sqrt(-a^2*x^2 + 1)*c^2*x + 3/8*c^2*arcsin(a*x)/a - 1/5*sqrt(-a^2*x^2 + 1)*c^2/a

mupad [B] time = 0.91, size = 82, normalized size = 0.99

$$\frac{3\,c^2\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{c^2\,x\,{\left(1-a^2\,x^2\right)}^{3/2}}{4}-\frac{c^2\,{\left(1-a^2\,x^2\right)}^{5/2}}{5\,a}-\frac{3\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (3*c^2*x*(1 - a^2*x^2)^(1/2))/8 + (c^2*x*(1 - a^2*x^2)^(3/2))/4 - (c^2*(1 -
 a^2*x^2)^(5/2))/(5*a) - (3*c^2*asinh(x*(-a^2)^(1/2))*(-a^2)^(1/2))/(8*a^2)
```

sympy [A] time = 20.15, size = 143, normalized size = 1.72

$$\begin{cases} \frac{-\frac{c^2(-a^2x^2+1)^{\frac{3}{2}}}{3}+c^2\left(\begin{cases} \frac{ax\sqrt{-a^2x^2+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for } ax>-1 \wedge ax<1 \end{cases}\right)-c^2\left(\begin{cases} -\frac{ax(-2a^2x^2+1)\sqrt{-a^2x^2+1}}{8}+\frac{\operatorname{asin}(ax)}{8} & \text{for } ax>-\end{cases}\right.}{a} \\ c^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise(((-c**2*(-a**2*x**2 + 1)**(3/2)/3 + c**2*Piecewise((a*x*sqrt(-a**
2*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))) - c**2*Piecewise((-a*
x*(-2*a**2*x**2 + 1)*sqrt(-a**2*x**2 + 1)/8 + asin(a*x)/8, (a*x > -1) & (a*
x < 1))) - c**2*Piecewise(((-a**2*x**2 + 1)**(5/2)/5 - (-a**2*x**2 + 1)**(3
/2)/3, (a*x > -1) & (a*x < 1))))/a, Ne(a, 0)), (c**2*x, True))
```

3.886 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=55

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

```
[Out] -1/3*c*(-a^2*x^2+1)^(3/2)/a+1/2*c*arcsin(a*x)/a+1/2*c*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6138, 641, 195, 216}

$$-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 - (c*(1 - a^2*x^2)^(3/2))/(3*a) + (c*ArcSin[a*x])
/(2*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= c\int(1+ax)\sqrt{1-a^2x^2}\,dx\\ &= -\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+c\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}-\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{c\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 1.04

$$\frac{c\left(\sqrt{1-a^2x^2}\left(2a^2x^2+3ax-2\right)-6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*(Sqrt[1 - a^2*x^2]*(-2 + 3*a*x + 2*a^2*x^2) - 6*ArcSin[Sqrt[1 - a*x]/Sqr
t[2]]))/(6*a)
```

fricas [A] time = 0.54, size = 63, normalized size = 1.15

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^2cx^2+3\,acx-2\,c\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^2*c*x^2 + 3*a*c*x -
 2*c)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.21, size = 46, normalized size = 0.84

$$\frac{c \arcsin(ax)\operatorname{sgn}(a)}{2|a|} + \frac{1}{6}\sqrt{-a^2x^2+1}\left((2\,acx + 3\,c)x - \frac{2\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*c*arcsin(a*x)*sgn(a)/abs(a) + 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c*x + 3*c)*x
 - 2*c/a)
```

maple [A] time = 0.04, size = 83, normalized size = 1.51

$$\frac{ca\,x^2\sqrt{-a^2x^2+1}}{3} - \frac{c\sqrt{-a^2x^2+1}}{3a} + \frac{cx\sqrt{-a^2x^2+1}}{2} + \frac{c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*c*a*x^2*(-a^2*x^2+1)^(1/2)-1/3*c*(-a^2*x^2+1)^(1/2)/a+1/2*c*x*(-a^2*x^2
+1)^(1/2)+1/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.40, size = 64, normalized size = 1.16

$$\frac{1}{3}\sqrt{-a^2x^2+1}\,acx^2 + \frac{1}{2}\sqrt{-a^2x^2+1}\,cx + \frac{c\arcsin(ax)}{2\,a} - \frac{\sqrt{-a^2x^2+1}\,c}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-a^2*x^2 + 1)*a*c*x^2 + 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/2*c*arcsin(
a*x)/a - 1/3*sqrt(-a^2*x^2 + 1)*c/a
```

mupad [B] time = 0.88, size = 80, normalized size = 1.45

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{a\,c}{3\,\sqrt{-a^2}} + \frac{c\,x\,\sqrt{-a^2}}{2} - \frac{a^3\,c\,x^2}{3\,\sqrt{-a^2}}\right)}{\sqrt{-a^2}} + \frac{c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

[Out] ((1 - a^2*x^2)^(1/2)*((a*c)/(3*(-a^2)^(1/2)) + (c*x*(-a^2)^(1/2))/2 - (a^3*c*x^2)/(3*(-a^2)^(1/2))))/(-a^2)^(1/2) + (c*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2))

sympy [A] time = 4.82, size = 53, normalized size = 0.96

$$\begin{cases} \frac{-\frac{c\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{3}+c\left(\begin{cases} \frac{ax\sqrt{-a^{2}x^{2}+1}}{2}+\frac{\operatorname{asin}(ax)}{2} & \text{for } ax>-1 \wedge ax<1 \end{cases}\right)}{a} & \text{for } a\neq 0 \\ cx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c),x)

[Out] Piecewise(((-c*(-a**2*x**2 + 1)**(3/2)/3 + c*Piecewise((a*x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/2, (a*x > -1) & (a*x < 1))))/a, Ne(a, 0)), (c*x, True))

3.887 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx$

Optimal. Leaf size=101

$$-\frac{3\sin^{-1}(ax)}{2a^5c}+\frac{x^3(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{(9ax+16)\sqrt{1-a^2x^2}}{6a^5c}+\frac{4x^2\sqrt{1-a^2x^2}}{3a^3c}$$

```
[Out] -3/2*arcsin(a*x)/a^5/c+x^3*(a*x+1)/a^2/c/(-a^2*x^2+1)^(1/2)+4/3*x^2*(-a^2*x
^2+1)^(1/2)/a^3/c+1/6*(9*a*x+16)*(-a^2*x^2+1)^(1/2)/a^5/c
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 819, 833, 780, 216}

$$\frac{x^3(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{4x^2\sqrt{1-a^2x^2}}{3a^3c}+\frac{(9ax+16)\sqrt{1-a^2x^2}}{6a^5c}-\frac{3\sin^{-1}(ax)}{2a^5c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] (x^3*(1 + a*x))/(a^2*c*Sqrt[1 - a^2*x^2]) + (4*x^2*Sqrt[1 - a^2*x^2])/(3*a^
3*c) + ((16 + 9*a*x)*Sqrt[1 - a^2*x^2])/(6*a^5*c) - (3*ArcSin[a*x])/(2*a^5*
c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
```

```
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= \frac{x^3(1+ax)}{a^2c\sqrt{1-a^2x^2}} - \frac{\int \frac{x^2(3+4ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2c}\\
&= \frac{x^3(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{4x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{\int \frac{x\left(-8a-9a^2x\right)}{\sqrt{1-a^2x^2}}\,dx}{3a^4c}\\
&= \frac{x^3(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{4x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{(16+9ax)\sqrt{1-a^2x^2}}{6a^5c} - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^4c}\\
&= \frac{x^3(1+ax)}{a^2c\sqrt{1-a^2x^2}} + \frac{4x^2\sqrt{1-a^2x^2}}{3a^3c} + \frac{(16+9ax)\sqrt{1-a^2x^2}}{6a^5c} - \frac{3\sin^{-1}(ax)}{2a^5c}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.73

$$-\frac{2a^4x^4+3a^3x^3+8a^2x^2+9\sqrt{1-a^2x^2}\sin^{-1}(ax)-9ax-16}{6a^5c\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] -1/6*(-16 - 9*a*x + 8*a^2*x^2 + 3*a^3*x^3 + 2*a^4*x^4 + 9*Sqrt[1 - a^2*x^2]
*ArcSin[a*x])/(a^5*c*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 86, normalized size = 0.85

$$\frac{16\,ax+18\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3x^3+a^2x^2+7\,ax-16\right)\sqrt{-a^2x^2+1}-16}{6\left(a^6cx-a^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] 1/6*(16*a*x + 18*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3*
x^3 + a^2*x^2 + 7*a*x - 16)*sqrt(-a^2*x^2 + 1) - 16)/(a^6*c*x - a^5*c)
```

giac [A] time = 0.21, size = 102, normalized size = 1.01

$$\frac{1}{6}\sqrt{-a^2x^2+1}\left(x\left(\frac{2\,x}{a^3c}+\frac{3}{a^4c}\right)+\frac{10}{a^5c}\right)-\frac{3\arcsin\left(ax\right)\operatorname{sgn}(a)}{2\,a^4c|a|}+\frac{2}{a^4c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] 1/6*sqrt(-a^2*x^2 + 1)*(x*(2*x/(a^3*c) + 3/(a^4*c)) + 10/(a^5*c)) - 3/2*arc
sin(a*x)*sgn(a)/(a^4*c*abs(a)) + 2/(a^4*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/
(a^2*x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 143, normalized size = 1.42

$$\frac{x^2\sqrt{-a^2x^2+1}}{3a^3c}+\frac{5\sqrt{-a^2x^2+1}}{3a^5c}+\frac{x\sqrt{-a^2x^2+1}}{2a^4c}-\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2c\,a^4\sqrt{a^2}}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{c\,a^6\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*x^2*(-a^2*x^2+1)^(1/2)/a^3/c+5/3*(-a^2*x^2+1)^(1/2)/a^5/c+1/2*x*(-a^2*x
^2+1)^(1/2)/a^4/c-3/2/c/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(
1/2))-1/c/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [B] time = 0.54, size = 204, normalized size = 2.02

$$-\frac{1}{6}a\left(\frac{3\sqrt{-a^2x^2+1}\,c}{a^7c^2x+a^6c^2}+\frac{3\sqrt{-a^2x^2+1}\,c}{a^7c^2x-a^6c^2}-\frac{3\sqrt{-a^2x^2+1}}{a^7cx+a^6c}+\frac{3\sqrt{-a^2x^2+1}}{a^7cx-a^6c}-\frac{2\sqrt{-a^2x^2+1}\,x^2}{a^4c}-\frac{3\sqrt{-a^2x^2+1}\,x}{a^5c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/6*a*(3*sqrt(-a^2*x^2 + 1)*c/(a^7*c^2*x + a^6*c^2) + 3*sqrt(-a^2*x^2 + 1)
*c/(a^7*c^2*x - a^6*c^2) - 3*sqrt(-a^2*x^2 + 1)/(a^7*c*x + a^6*c) + 3*sqrt(
-a^2*x^2 + 1)/(a^7*c*x - a^6*c) - 2*sqrt(-a^2*x^2 + 1)*x^2/(a^4*c) - 3*sqrt
(-a^2*x^2 + 1)*x/(a^5*c) + 9*arcsin(a*x)/(a^6*c) - 10*sqrt(-a^2*x^2 + 1)/(a
^6*c))
```

mupad [B] time = 0.89, size = 140, normalized size = 1.39

$$\frac{5\sqrt{1-a^2\,x^2}}{3\,a^5\,c}-\frac{\sqrt{1-a^2\,x^2}}{\sqrt{-a^2}\,\left(a^3\,c\,\sqrt{-a^2}-a^4\,c\,x\,\sqrt{-a^2}\right)}+\frac{x\,\sqrt{1-a^2\,x^2}}{2\,a^4\,c}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^4\,c\,\sqrt{-a^2}}+\frac{x^2\,\sqrt{1-a^2\,x^2}}{3\,a^3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (5*(1 - a^2*x^2)^(1/2))/(3*a^5*c) - (1 - a^2*x^2)^(1/2)/((-a^2)^(1/2)*(a^3*
c*(-a^2)^(1/2) - a^4*c*x*(-a^2)^(1/2))) + (x*(1 - a^2*x^2)^(1/2))/(2*a^4*c)
 - (3*asinh(x*(-a^2)^(1/2)))/(2*a^4*c*(-a^2)^(1/2)) + (x^2*(1 - a^2*x^2)^(1
/2))/(3*a^3*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**4/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a*x**5/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1))
, x))/c
```

3.888 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$-\frac{3\sin^{-1}(ax)}{2a^4c}+\frac{x^2(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{(3ax+4)\sqrt{1-a^2x^2}}{2a^4c}$$

```
[Out] -3/2*arcsin(a*x)/a^4/c+x^2*(a*x+1)/a^2/c/(-a^2*x^2+1)^(1/2)+1/2*(3*a*x+4)*(
-a^2*x^2+1)^(1/2)/a^4/c
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 780, 216}

$$\frac{x^2(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{(3ax+4)\sqrt{1-a^2x^2}}{2a^4c}-\frac{3\sin^{-1}(ax)}{2a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] (x^2*(1 + a*x))/(a^2*c*Sqrt[1 - a^2*x^2]) + ((4 + 3*a*x)*Sqrt[1 - a^2*x^2])
/(2*a^4*c) - (3*ArcSin[a*x])/(2*a^4*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
```

```
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^3}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^3(1+ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= \frac{x^2(1+ax)}{a^2c\sqrt{1-a^2x^2}}-\frac{\int \frac{x(2+3ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2c}\\
&= \frac{x^2(1+ax)}{a^2c\sqrt{1-a^2x^2}}+\frac{(4+3ax)\sqrt{1-a^2x^2}}{2a^4c}-\frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^3c}\\
&= \frac{x^2(1+ax)}{a^2c\sqrt{1-a^2x^2}}+\frac{(4+3ax)\sqrt{1-a^2x^2}}{2a^4c}-\frac{3\sin^{-1}(ax)}{2a^4c}\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.88

$$-\frac{a^3x^3+2a^2x^2+3\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-3ax-4}{2a^4c\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] -1/2*(-4 - 3*a*x + 2*a^2*x^2 + a^3*x^3 + 3*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(
a^4*c*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.50, size = 77, normalized size = 1.04

$$\frac{4\,ax+6\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(a^2x^2+ax-4\right)\sqrt{-a^2x^2+1}-4}{2\left(a^5cx-a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] 1/2*(4*a*x + 6*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a^2*x^2
+ a*x - 4)*sqrt(-a^2*x^2 + 1) - 4)/(a^5*c*x - a^4*c)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 119, normalized size = 1.61

$$\frac{x\sqrt{-a^2x^2+1}}{2a^3c}-\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2c\,a^3\sqrt{a^2}}+\frac{\sqrt{-a^2x^2+1}}{a^4c}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{c\,a^5\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c),x)
```

```
[Out] 1/2*x*(-a^2*x^2+1)^(1/2)/a^3/c-3/2/c/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(
-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/a^4/c-1/c/a^5/(x-1/a)*(-a^2*(x-1/a)^2
-2*a*(x-1/a))^(1/2)
```

maxima [B] time = 0.49, size = 178, normalized size = 2.41

$$-\frac{1}{2}a\left(\frac{\sqrt{-a^2x^2+1}\,c}{a^6c^2x+a^5c^2}+\frac{\sqrt{-a^2x^2+1}\,c}{a^6c^2x-a^5c^2}-\frac{\sqrt{-a^2x^2+1}}{a^6cx+a^5c}+\frac{\sqrt{-a^2x^2+1}}{a^6cx-a^5c}-\frac{\sqrt{-a^2x^2+1}\,x}{a^4c}+\frac{3\arcsin(ax)}{a^5c}-\frac{2\sqrt{-a^2x}}{a^5c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*(sqrt(-a^2*x^2 + 1)*c/(a^6*c^2*x + a^5*c^2) + sqrt(-a^2*x^2 + 1)*c/(
a^6*c^2*x - a^5*c^2) - sqrt(-a^2*x^2 + 1)/(a^6*c*x + a^5*c) + sqrt(-a^2*x^2
```

```
 + 1)/(a^6*c*x - a^5*c) - sqrt(-a^2*x^2 + 1)*x/(a^4*c) + 3*arcsin(a*x)/(a^5
*c) - 2*sqrt(-a^2*x^2 + 1)/(a^5*c))
```

mupad [B] time = 0.93, size = 128, normalized size = 1.73

$$\frac{\sqrt{1-a^2\,x^2}}{a^3\,c\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^3\,c\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{1}{a^2\,c\,\sqrt{-a^2}}-\frac{x\,\sqrt{-a^2}}{2\,a^3\,c}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(a^3*c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (3*asinh(x*(-a^2)^(1/2)))/(2*a^3*c*(-a^2)^(1/2)) - ((1 - a^2*x^2)^(1/2)*(
1/(a^2*c*(-a^2)^(1/2)) - (x*(-a^2)^(1/2))/(2*a^3*c)))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**3/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a*x**4/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1))
, x))/c
```

3.889 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx$

Optimal. Leaf size=60

$$-\frac{\sin^{-1}(ax)}{a^3c}+\frac{ax+1}{a^3c\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{a^3c}$$

```
[Out] -arcsin(a*x)/a^3/c+(a*x+1)/a^3/c/(-a^2*x^2+1)^(1/2)+(-a^2*x^2+1)^(1/2)/a^3/
c
```

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 797, 641, 216, 637}

$$\frac{ax+1}{a^3c\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}}{a^3c}-\frac{\sin^{-1}(ax)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)/(a^3*c*Sqrt[1 - a^2*x^2]) + Sqrt[1 - a^2*x^2]/(a^3*c) - ArcSin[a*
x]/(a^3*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
```

```
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^2(1+ax)}{(1-a^2x^2)^{3/2}}\,dx}{c} \\
&= \frac{\int \frac{1+ax}{(1-a^2x^2)^{3/2}}\,dx}{a^2c} - \frac{\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{a^2c} \\
&= \frac{1+ax}{a^3c\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{a^3c} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2c} \\
&= \frac{1+ax}{a^3c\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}}{a^3c} - \frac{\sin^{-1}(ax)}{a^3c}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.90

$$\frac{-a^2x^2-\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+ax+2}{a^3c\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (2 + a*x - a^2*x^2 - Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(a^3*c*Sqrt[1 - a^2*x^2
])
```

fricas [A] time = 0.57, size = 69, normalized size = 1.15

$$\frac{2\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,(ax-2)-2}{a^4cx-a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] (2*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(a*x - 2) - 2)/(a^4*c*x - a^3*c)
```

giac [A] time = 0.50, size = 78, normalized size = 1.30

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a^2c|a|}+\frac{\sqrt{-a^2x^2+1}}{a^3c}+\frac{2}{a^2c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*c*abs(a)) + sqrt(-a^2*x^2 + 1)/(a^3*c) + 2/(a^2*c*
((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 98, normalized size = 1.63

$$\frac{\sqrt{-a^2x^2+1}}{a^3c}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c\,a^2\sqrt{a^2}}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{c\,a^4\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)/a^3/c-1/c/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2
+1)^(1/2))-1/c/a^4/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [B] time = 0.46, size = 157, normalized size = 2.62

$$-\frac{1}{2}a\left(\frac{\sqrt{-a^2x^2+1}\,c}{a^5c^2x+a^4c^2}+\frac{\sqrt{-a^2x^2+1}\,c}{a^5c^2x-a^4c^2}-\frac{\sqrt{-a^2x^2+1}}{a^5cx+a^4c}+\frac{\sqrt{-a^2x^2+1}}{a^5cx-a^4c}+\frac{2\arcsin(ax)}{a^4c}-\frac{2\sqrt{-a^2x^2+1}}{a^4c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*(sqrt(-a^2*x^2 + 1)*c/(a^5*c^2*x + a^4*c^2) + sqrt(-a^2*x^2 + 1)*c/(
a^5*c^2*x - a^4*c^2) - sqrt(-a^2*x^2 + 1)/(a^5*c*x + a^4*c) + sqrt(-a^2*x^2
```

```
 + 1)/(a^5*c*x - a^4*c) + 2*arcsin(a*x)/(a^4*c) - 2*sqrt(-a^2*x^2 + 1)/(a^4
*c))
```

mupad [B] time = 0.07, size = 93, normalized size = 1.55

$$\frac{\sqrt{1-a^2\,x^2}}{a^3\,c}-\frac{\sqrt{1-a^2\,x^2}}{\left(a\,c\,\sqrt{-a^2}-a^2\,c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,c\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(a^3*c) - (1 - a^2*x^2)^(1/2)/((a*c*(-a^2)^(1/2) - a^2*
c*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - asinh(x*(-a^2)^(1/2))/(a^2*c*(-a^2)^(1/2)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**2/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a*x**3/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1))
, x))/c
```

3.890 $\int \frac{e^{\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx$

Optimal. Leaf size=39

$$\frac{ax+1}{a^2c\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^2c}$$

```
[Out] -arcsin(a*x)/a^2/c+(a*x+1)/a^2/c/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6148, 778, 216}

$$\frac{ax+1}{a^2c\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)/(a^2*c*Sqrt[1 - a^2*x^2]) - ArcSin[a*x]/(a^2*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx = \frac{\int \frac{x(1+ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c}$$
$$= \frac{1+ax}{a^2c\sqrt{1-a^2x^2}} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{ac}$$
$$= \frac{1+ax}{a^2c\sqrt{1-a^2x^2}} - \frac{\sin^{-1}(ax)}{a^2c}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.15

$$\frac{\frac{ax}{\sqrt{1-a^2x^2}} + \frac{1}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2),x]
```

```
[Out] (1/Sqrt[1 - a^2*x^2] + (a*x)/Sqrt[1 - a^2*x^2] - ArcSin[a*x])/(a^2*c)
```

fricas [A] time = 0.75, size = 64, normalized size = 1.64

$$\frac{ax + 2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - \sqrt{-a^2x^2+1} - 1}{a^3cx - a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] (a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 +
 1) - 1)/(a^3*c*x - a^2*c)
```

giac [A] time = 0.22, size = 59, normalized size = 1.51

$$-\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{ac|a|} + \frac{2}{ac\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a*c*abs(a)) + 2/(a*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/
(a^2*x) - 1)*abs(a))
```

maple [B] time = 0.04, size = 79, normalized size = 2.03

$$-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{ca\sqrt{a^2}}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{c\,a^3\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-1/c/a^3/(x-1/a)
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)
```

maxima [B] time = 0.44, size = 137, normalized size = 3.51

$$-\frac{1}{2}\,a\left(\frac{\sqrt{-a^2x^2+1}\,c}{a^4c^2x+a^3c^2}+\frac{\sqrt{-a^2x^2+1}\,c}{a^4c^2x-a^3c^2}-\frac{\sqrt{-a^2x^2+1}}{a^4cx+a^3c}+\frac{\sqrt{-a^2x^2+1}}{a^4cx-a^3c}+\frac{2\arcsin\left(ax\right)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] -1/2*a*(sqrt(-a^2*x^2 + 1)*c/(a^4*c^2*x + a^3*c^2) + sqrt(-a^2*x^2 + 1)*c/(
a^4*c^2*x - a^3*c^2) - sqrt(-a^2*x^2 + 1)/(a^4*c*x + a^3*c) + sqrt(-a^2*x^2
 + 1)/(a^4*c*x - a^3*c) + 2*arcsin(a*x)/(a^3*c))
```

mupad [B] time = 0.90, size = 64, normalized size = 1.64

$$\frac{1}{a^2\,c\,\sqrt{1-a^2\,x^2}}+\frac{x}{a\,c\,\sqrt{1-a^2\,x^2}}+\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}{a^3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] 1/(a^2*c*(1 - a^2*x^2)^(1/2)) + x/(a*c*(1 - a^2*x^2)^(1/2)) + (asinh(x*(-a^
2)^(1/2))*(-a^2)^(1/2))/(a^3*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^2}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) +
Integral(a*x**2/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x
))/c
```

3.891 $\int \frac{e^{\tanh^{-1}(ax)}}{c-a^2cx^2} dx$

Optimal. Leaf size=13

$$\frac{e^{\tanh^{-1}(ax)}}{ac}$$

```
[Out] (a*x+1)/(-a^2*x^2+1)^(1/2)/a/c
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6137}

$$\frac{e^{\tanh^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] E^ArcTanh[a*x]/(a*c)
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{c-a^2cx^2} dx = \frac{e^{\tanh^{-1}(ax)}}{ac}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 2.00

$$\frac{\sqrt{ax+1}}{ac\sqrt{1-ax}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] Sqrt[1 + a*x]/(a*c*Sqrt[1 - a*x])
```

fricas [A]   time = 0.49, size = 33, normalized size = 2.54

$$\frac{ax - \sqrt{-a^2x^2 + 1} - 1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] (a*x - sqrt(-a^2*x^2 + 1) - 1)/(a^2*c*x - a*c)
```

giac [A]   time = 0.21, size = 37, normalized size = 2.85

$$\frac{2}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A]   time = 0.03, size = 25, normalized size = 1.92

$$\frac{ax + 1}{\sqrt{-a^2x^2 + 1}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x)
```

```
[Out] (a*x+1)/(-a^2*x^2+1)^(1/2)/a/c
```

maxima [B]   time = 0.42, size = 125, normalized size = 9.62

$$-\frac{1}{2}\,a\left(\frac{\sqrt{-a^2x^2 + 1}\,c}{a^3c^2x + a^2c^2} + \frac{\sqrt{-a^2x^2 + 1}\,c}{a^3c^2x - a^2c^2} - \frac{\sqrt{-a^2x^2 + 1}}{a^3cx + a^2c} + \frac{\sqrt{-a^2x^2 + 1}}{a^3cx - a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*(sqrt(-a^2*x^2 + 1)*c/(a^3*c^2*x + a^2*c^2) + sqrt(-a^2*x^2 + 1)*c/(
a^3*c^2*x - a^2*c^2) - sqrt(-a^2*x^2 + 1)/(a^3*c*x + a^2*c) + sqrt(-a^2*x^2
 + 1)/(a^3*c*x - a^2*c))
```

mupad [B] time = 0.87, size = 47, normalized size = 3.62

$$\frac{\sqrt{1-a^2\,x^2}}{c\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a*x/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
+ Integral(1/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/
c
```

3.892 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)}\,dx$

Optimal. Leaf size=44

$$\frac{ax+1}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

[Out] -arctanh((-a^2*x^2+1)^(1/2))/c+(a*x+1)/c/(-a^2*x^2+1)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 12, 266, 63, 208}

$$\frac{ax+1}{c\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)),x]

[Out] (1 + a*x)/(c*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)}\,dx = \frac{\int \frac{1+ax}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{c}$$

$$= \frac{1+ax}{c\sqrt{1-a^2x^2}} + \frac{\int \frac{a^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2c}$$

$$= \frac{1+ax}{c\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c}$$

$$= \frac{1+ax}{c\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c}$$

$$= \frac{1+ax}{c\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2c}$$

$$= \frac{1+ax}{c\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.25

$$-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c} - \frac{a^3(-x)-a^2}{a^2c\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)),x]
[Out] -((-a^2 - a^3*x)/(a^2*c*Sqrt[1 - a^2*x^2])) - ArcTanh[Sqrt[1 - a^2*x^2]]/c
```

fricas [A] time = 0.70, size = 55, normalized size = 1.25

$$\frac{ax + (ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1} - 1}{acx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] (a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1) - 1)/
(a*c*x - c)
```

giac [A] time = 0.19, size = 80, normalized size = 1.82

$$-\frac{a \log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c|a|}+\frac{2\,a}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a))
 + 2*a/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

maple [A] time = 0.04, size = 60, normalized size = 1.36

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c*(arctanh(1/(-a^2*x^2+1)^(1/2))+1/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a)
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [B] time = 0.89, size = 67, normalized size = 1.52

$$\frac{a\sqrt{1-a^2\,x^2}}{c\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{\operatorname{atanh}\left(\sqrt{1-a^2\,x^2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a*(1 - a^2*x^2)^(1/2))/(c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- atanh((1 - a^2*x^2)^(1/2))/c
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) +
Integral(1/(-a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/
c
```

3.893 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{ax+1}{cx\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{cx}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

```
[Out] -a*arctanh((-a^2*x^2+1)^(1/2))/c+(a*x+1)/c/x/(-a^2*x^2+1)^(1/2)-2*(-a^2*x^2
+1)^(1/2)/c/x
```

Rubi [A] time = 0.12, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 807, 266, 63, 208}

$$\frac{ax+1}{cx\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-a^2x^2}}{cx}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] (1 + a*x)/(c*x*Sqrt[1 - a^2*x^2]) - (2*Sqrt[1 - a^2*x^2])/(c*x) - (a*ArcTan
h[Sqrt[1 - a^2*x^2]])/c
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)}\,dx &= \frac{\int \frac{1+ax}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{c}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}} + \frac{\int \frac{2a^2+a^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2c}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}} - \frac{2\sqrt{1-a^2x^2}}{cx} + \frac{a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}} - \frac{2\sqrt{1-a^2x^2}}{cx} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}} - \frac{2\sqrt{1-a^2x^2}}{cx} - \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{ac}\\
&= \frac{1+ax}{cx\sqrt{1-a^2x^2}} - \frac{2\sqrt{1-a^2x^2}}{cx} - \frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.96

$$
\frac{2a^2x^2 - ax\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) + ax - 1}{cx\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] (-1 + a*x + 2*a^2*x^2 - a*x*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(
c*x*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.60, size = 78, normalized size = 1.11

$$
\frac{a^2x^2 - ax + \left(a^2x^2 - ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \sqrt{-a^2x^2+1}\,(2\,ax-1)}{acx^2 - cx}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] (a^2*x^2 - a*x + (a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^
2*x^2 + 1)*(2*a*x - 1))/(a*c*x^2 - c*x)
```

giac [B] time = 0.63, size = 159, normalized size = 2.27

$$-\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{c|a|}-\frac{\left(a^2-\frac{5\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{x}\right)a^2x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{2\,cx|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a
)) - 1/2*(a^2 - 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/x)*a^2*x/((sqrt(-a^2*x^2
+ 1)*abs(a) + a)*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) -
1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(c*x*abs(a))
```

maple [A] time = 0.04, size = 75, normalized size = 1.07

$$-\frac{a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\sqrt{-a^2x^2+1}}{x}+\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c*(a*arctanh(1/(-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/x+1/(x-1/a)*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(1/2))
```

maxima [A] time = 0.32, size = 104, normalized size = 1.49

$$-\frac{\frac{a^2\log\left(\sqrt{-a^2x^2+1}+1\right)}{c}-\frac{a^2\log\left(\sqrt{-a^2x^2+1}-1\right)}{c}-\frac{2\,a^2}{\sqrt{-a^2x^2+1}\,c}}{2\,a}+\frac{2\,a^2x^2-1}{\sqrt{ax+1}\,\sqrt{-ax+1}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/2*(a^2*log(sqrt(-a^2*x^2 + 1) + 1)/c - a^2*log(sqrt(-a^2*x^2 + 1) - 1)/c
 - 2*a^2/(sqrt(-a^2*x^2 + 1)*c))/a + (2*a^2*x^2 - 1)/(sqrt(a*x + 1)*sqrt(-a
*x + 1)*c*x)
```

mupad [B] time = 0.90, size = 90, normalized size = 1.29

$$\frac{a^2\sqrt{1-a^2x^2}}{c\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{\sqrt{1-a^2x^2}}{c\,x}-\frac{a\,\mathrm{atanh}\left(\sqrt{1-a^2x^2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^2*(1 - a^2*x^2)^(1/2))/(c*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2)
) - (1 - a^2*x^2)^(1/2)/(c*x) - (a*atanh((1 - a^2*x^2)^(1/2)))/c
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{-a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a/(-a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x)
+ Integral(1/(-a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)),
 x))/c
```

3.894 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)}\,dx$

Optimal. Leaf size=99

$$-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{3\sqrt{1-a^2x^2}}{2cx^2}+\frac{ax+1}{cx^2\sqrt{1-a^2x^2}}-\frac{3a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

```
[Out] -3/2*a^2*arctanh((-a^2*x^2+1)^(1/2))/c+(a*x+1)/c/x^2/(-a^2*x^2+1)^(1/2)-3/2
*(-a^2*x^2+1)^(1/2)/c/x^2-2*a*(-a^2*x^2+1)^(1/2)/c/x
```

Rubi [A] time = 0.14, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{2a\sqrt{1-a^2x^2}}{cx}-\frac{3\sqrt{1-a^2x^2}}{2cx^2}+\frac{ax+1}{cx^2\sqrt{1-a^2x^2}}-\frac{3a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)),x]
```

```
[Out] (1 + a*x)/(c*x^2*Sqrt[1 - a^2*x^2]) - (3*Sqrt[1 - a^2*x^2])/(2*c*x^2) - (2*
a*Sqrt[1 - a^2*x^2])/(c*x) - (3*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)}\,dx = \frac{\int \frac{1+ax}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c}$$

$$= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3a^2+2a^3x}{x^3\sqrt{1-a^2x^2}}\,dx}{a^2c}$$

$$= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{2cx^2} - \frac{\int \frac{-4a^3-3a^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{2a^2c}$$

$$= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} + \frac{\left(3a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{2c}$$

$$= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{4c}$$

$$= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{2c}$$

$$= \frac{1+ax}{cx^2\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{2cx^2} - \frac{2a\sqrt{1-a^2x^2}}{cx} - \frac{3a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.84

$$-\frac{-4a^3x^3-3a^2x^2+3a^2x^2\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2ax+1}{2cx^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)),x]
```

```
[Out] -1/2*(1 + 2*a*x - 3*a^2*x^2 - 4*a^3*x^3 + 3*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTa
nh[Sqrt[1 - a^2*x^2]])/(c*x^2*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.57, size = 99, normalized size = 1.00

$$\frac{2\,a^3x^3-2\,a^2x^2+3\left(a^3x^3-a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(4\,a^2x^2-ax-1\right)\sqrt{-a^2x^2+1}}{2\left(acx^3-cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*x^3 - 2*a^2*x^2 + 3*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (4*a^2*x^2 - a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*c*x^3 - c*x^2)
```

giac [B] time = 0.22, size = 224, normalized size = 2.26

$$-\frac{\left(a^3+\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x}-\frac{20\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{3\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,c|a|}-\frac{\frac{4\left(\sqrt{-a^2x^2+1}\,|a|+a\right)ac|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}\cdots\right.}{ax\cdots}}{8\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -1/8*(a^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 20*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c*((sqrt(
-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 3/2*a^3*log(1/2*abs(-2*sqr
t(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/(c*abs(a)) - 1/8*(4*(sqrt(-a^2*
x^2 + 1)*abs(a) + a)*a*c*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*c*abs
(a)/(a*x^2))/(a^2*c^2)
```

maple [A] time = 0.04, size = 97, normalized size = 0.98

$$-\frac{\frac{3a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{a\sqrt{-a^2x^2+1}}{x}+\frac{a\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}+\frac{\sqrt{-a^2x^2+1}}{2x^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c*(3/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+a*(-a^2*x^2+1)^(1/2)/x+a/(x-1/a
)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ax+1}{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [B] time = 0.07, size = 117, normalized size = 1.18

$$-\frac{\sqrt{1-a^2\,x^2}}{2\,c\,x^2}-\frac{a\,\sqrt{1-a^2\,x^2}}{c\,x}-\frac{a^3\,\sqrt{1-a^2\,x^2}}{\left(\frac{c\,\sqrt{-a^2}}{a}-c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+\frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/(2*c) - (1 - a^2*x^2)^(1/2)/(2*c*x^2)
 - (a*(1 - a^2*x^2)^(1/2))/(c*x) - (a^3*(1 - a^2*x^2)^(1/2))/(((c*(-a^2)^(1
/2))/a - c*x*(-a^2)^(1/2))*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{-a^{2}x^{4}\sqrt{-a^{2}x^{2}+1}+x^{2}\sqrt{-a^{2}x^{2}+1}}\, dx+\int \frac{1}{-a^{2}x^{5}\sqrt{-a^{2}x^{2}+1}+x^{3}\sqrt{-a^{2}x^{2}+1}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a/(-a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)),
x) + Integral(1/(-a**2*x**5*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1
)), x))/c
```

3.895 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)}\,dx$

Optimal. Leaf size=128

$$-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{3a\sqrt{1-a^2x^2}}{2cx^2}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}+\frac{ax+1}{cx^3\sqrt{1-a^2x^2}}-\frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

```
[Out] -3/2*a^3*arctanh((-a^2*x^2+1)^(1/2))/c+(a*x+1)/c/x^3/(-a^2*x^2+1)^(1/2)-4/3
*(-a^2*x^2+1)^(1/2)/c/x^3-3/2*a*(-a^2*x^2+1)^(1/2)/c/x^2-8/3*a^2*(-a^2*x^2+
1)^(1/2)/c/x
```

Rubi [A] time = 0.17, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{8a^2\sqrt{1-a^2x^2}}{3cx}-\frac{3a\sqrt{1-a^2x^2}}{2cx^2}-\frac{4\sqrt{1-a^2x^2}}{3cx^3}+\frac{ax+1}{cx^3\sqrt{1-a^2x^2}}-\frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)),x]
```

```
[Out] (1 + a*x)/(c*x^3*Sqrt[1 - a^2*x^2]) - (4*Sqrt[1 - a^2*x^2])/(3*c*x^3) - (3*
a*Sqrt[1 - a^2*x^2])/(2*c*x^2) - (8*a^2*Sqrt[1 - a^2*x^2])/(3*c*x) - (3*a^3
*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)}\,dx = \frac{\int \frac{1+ax}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{c}$$

$$= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}} + \frac{\int \frac{4a^2+3a^3x}{x^4\sqrt{1-a^2x^2}}\,dx}{a^2c}$$

$$= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}} - \frac{4\sqrt{1-a^2x^2}}{3cx^3} - \frac{\int \frac{-9a^3-8a^4x}{x^3\sqrt{1-a^2x^2}}\,dx}{3a^2c}$$

$$= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}} - \frac{4\sqrt{1-a^2x^2}}{3cx^3} - \frac{3a\sqrt{1-a^2x^2}}{2cx^2} + \frac{\int \frac{16a^4+9a^5x}{x^2\sqrt{1-a^2x^2}}\,dx}{6a^2c}$$

$$= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}} - \frac{4\sqrt{1-a^2x^2}}{3cx^3} - \frac{3a\sqrt{1-a^2x^2}}{2cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} + \frac{\left(3a^3\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{2c}$$

$$= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}} - \frac{4\sqrt{1-a^2x^2}}{3cx^3} - \frac{3a\sqrt{1-a^2x^2}}{2cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} + \frac{\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\right.}{4c}$$

$$= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}} - \frac{4\sqrt{1-a^2x^2}}{3cx^3} - \frac{3a\sqrt{1-a^2x^2}}{2cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,\right.}{2c}$$

$$= \frac{1+ax}{cx^3\sqrt{1-a^2x^2}} - \frac{4\sqrt{1-a^2x^2}}{3cx^3} - \frac{3a\sqrt{1-a^2x^2}}{2cx^2} - \frac{8a^2\sqrt{1-a^2x^2}}{3cx} - \frac{3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right.}{2c}$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.71

$$-\frac{-16a^4x^4-9a^3x^3+8a^2x^2+9a^3x^3\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ax+2}{6cx^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)),x]
```

```
[Out] -1/6*(2 + 3*a*x + 8*a^2*x^2 - 9*a^3*x^3 - 16*a^4*x^4 + 9*a^3*x^3*Sqrt[1 - a
^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(c*x^3*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.64, size = 107, normalized size = 0.84

$$\frac{6\,a^4x^4-6\,a^3x^3+9\left(a^4x^4-a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(16\,a^3x^3-7\,a^2x^2-ax-2\right)\sqrt{-a^2x^2+1}}{6\left(acx^4-cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*a^4*x^4 - 6*a^3*x^3 + 9*(a^4*x^4 - a^3*x^3)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (16*a^3*x^3 - 7*a^2*x^2 - a*x - 2)*sqrt(-a^2*x^2 + 1))/(a*c*x^4 -
 c*x^3)
```

giac [B] time = 0.21, size = 283, normalized size = 2.21

$$-\frac{\left(a^4+\frac{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2}{x}+\frac{18\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{x^2}-\frac{69\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^2x^3}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{3\,a^4\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,c|a|}-\frac{21\left(\sqrt{-a^2x^2+1}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -1/24*(a^4 + 2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2/x + 18*(sqrt(-a^2*x^2 +
1)*abs(a) + a)^2/x^2 - 69*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^2*x^3))*a^6*
x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(
a^2*x) - 1)*abs(a)) - 3/2*a^4*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*
a)/(a^2*abs(x)))/(c*abs(a)) - 1/24*(21*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*
c^2/x + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2*c^2/x^2 + (sqrt(-a^2*x^2 +
1)*abs(a) + a)^3*c^2/x^3)/(a^2*c^3*abs(a))
```

maple [A] time = 0.05, size = 140, normalized size = 1.09

$$-\frac{a^3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{5a^2\sqrt{-a^2x^2+1}}{3x}+\frac{\sqrt{-a^2x^2+1}}{3x^3}+\frac{a^2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}-a\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c*(a^3*arctanh(1/(-a^2*x^2+1)^(1/2))+5/3*a^2*(-a^2*x^2+1)^(1/2)/x+1/3*(-
a^2*x^2+1)^(1/2)/x^3+a^2/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-a*(-1/2
*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))))
```

maxima [A] time = 0.33, size = 148, normalized size = 1.16

$$-\frac{\frac{3\,a^4\log\left(\sqrt{-a^2x^2+1}+1\right)}{c}-\frac{3\,a^4\log\left(\sqrt{-a^2x^2+1}-1\right)}{c}+\frac{2\left(3\left(a^2x^2-1\right)a^4+2\,a^4\right)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}c-\sqrt{-a^2x^2+1}\,c}}{4\,a}+\frac{8\,a^4x^4-4\,a^2x^2-1}{3\sqrt{ax+1}\sqrt{-ax+1}\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -1/4*(3*a^4*log(sqrt(-a^2*x^2 + 1) + 1)/c - 3*a^4*log(sqrt(-a^2*x^2 + 1) -
1)/c + 2*(3*(a^2*x^2 - 1)*a^4 + 2*a^4)/((-a^2*x^2 + 1)^(3/2)*c - sqrt(-a^2*
x^2 + 1)*c))/a + 1/3*(8*a^4*x^4 - 4*a^2*x^2 - 1)/(sqrt(a*x + 1)*sqrt(-a*x +
 1)*c*x^3)
```

mupad [B] time = 0.90, size = 140, normalized size = 1.09

$$-\frac{\sqrt{1-a^2\,x^2}}{3\,c\,x^3}-\frac{a\,\sqrt{1-a^2\,x^2}}{2\,c\,x^2}-\frac{5\,a^2\,\sqrt{1-a^2\,x^2}}{3\,c\,x}-\frac{a^4\,\sqrt{1-a^2\,x^2}}{\left(\frac{c\,\sqrt{-a^2}}{a}-c\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+\frac{a^3\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^3*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/(2*c) - (1 - a^2*x^2)^(1/2)/(3*c*x^3)
 - (a*(1 - a^2*x^2)^(1/2))/(2*c*x^2) - (5*a^2*(1 - a^2*x^2)^(1/2))/(3*c*x)
- (a^4*(1 - a^2*x^2)^(1/2))/((((c*(-a^2)^(1/2))/a - c*x*(-a^2)^(1/2))*(-a^2)
^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{-a^2x^5\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^2x^6\sqrt{-a^2x^2+1}+x^4\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(a/(-a**2*x**5*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)),
x) + Integral(1/(-a**2*x**6*sqrt(-a**2*x**2 + 1) + x**4*sqrt(-a**2*x**2 + 1
)), x))/c
```

3.896 $\int \frac{e^{\tanh^{-1}(ax)}x^6}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=137

$$\frac{5\sin^{-1}(ax)}{2a^7c^2}+\frac{x^5(ax+1)}{3a^2c^2(1-a^2x^2)^{3/2}}-\frac{(15ax+32)\sqrt{1-a^2x^2}}{6a^7c^2}-\frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{x^3(6ax+5)}{3a^4c^2\sqrt{1-a^2x^2}}$$

```
[Out] 1/3*x^5*(a*x+1)/a^2/c^2/(-a^2*x^2+1)^(3/2)+5/2*arcsin(a*x)/a^7/c^2-1/3*x^3*
(6*a*x+5)/a^4/c^2/(-a^2*x^2+1)^(1/2)-8/3*x^2*(-a^2*x^2+1)^(1/2)/a^5/c^2-1/6
*(15*a*x+32)*(-a^2*x^2+1)^(1/2)/a^7/c^2
```

Rubi [A] time = 0.15, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 819, 833, 780, 216}

$$\frac{x^5(ax+1)}{3a^2c^2(1-a^2x^2)^{3/2}}-\frac{x^3(6ax+5)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{(15ax+32)\sqrt{1-a^2x^2}}{6a^7c^2}+\frac{5\sin^{-1}(ax)}{2a^7c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^5*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (x^3*(5 + 6*a*x))/(3*a^4*
c^2*Sqrt[1 - a^2*x^2]) - (8*x^2*Sqrt[1 - a^2*x^2])/(3*a^5*c^2) - ((32 + 15*
a*x)*Sqrt[1 - a^2*x^2])/(6*a^7*c^2) + (5*ArcSin[a*x])/(2*a^7*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
```

```
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{x^6(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$

$$= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{x^4(5+6ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$

$$= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^3(5+6ax)}{3a^4c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{x^2(15+24ax)}{\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}$$

$$= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^3(5+6ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{\int \frac{x\left(-48a-45a^2x\right)}{\sqrt{1-a^2x^2}}\,dx}{9a^6c^2}$$

$$= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^3(5+6ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{(32+15ax)\sqrt{1-a^2x^2}}{6a^7c^2} + \frac{5\int \frac{1}{\sqrt{1-}}}{2a^6}$$

$$= \frac{x^5(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^3(5+6ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8x^2\sqrt{1-a^2x^2}}{3a^5c^2} - \frac{(32+15ax)\sqrt{1-a^2x^2}}{6a^7c^2} + \frac{5\sin^{-1}(}{2a^7c^2}$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.68

$$\frac{2a^5x^5+a^4x^4+11a^3x^3-31a^2x^2+15(ax-1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-17ax+32}{6a^7c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (32 - 17*a*x - 31*a^2*x^2 + 11*a^3*x^3 + a^4*x^4 + 2*a^5*x^5 + 15*(-1 + a*x
)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(6*a^7*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.56, size = 159, normalized size = 1.16

$$-\frac{32\,a^3x^3-32\,a^2x^2-32\,ax+30\left(a^3x^3-a^2x^2-ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^5x^5+a^4x^4+11\,a^3x^3-31\,a^2x^2\right.}{6\left(a^{10}c^2x^3-a^9c^2x^2-a^8c^2x+a^7c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/6*(32*a^3*x^3 - 32*a^2*x^2 - 32*a*x + 30*(a^3*x^3 - a^2*x^2 - a*x + 1)*a
rctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^5*x^5 + a^4*x^4 + 11*a^3*x^3 -
 31*a^2*x^2 - 17*a*x + 32)*sqrt(-a^2*x^2 + 1) + 32)/(a^10*c^2*x^3 - a^9*c^2
*x^2 - a^8*c^2*x + a^7*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^6}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^6/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 225, normalized size = 1.64

$$-\frac{x^2\sqrt{-a^2x^2+1}}{3a^5c^2}-\frac{8\sqrt{-a^2x^2+1}}{3c^2a^7}-\frac{x\sqrt{-a^2x^2+1}}{2c^2a^6}+\frac{5\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2c^2a^6\sqrt{a^2}}+\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{6c^2a^9\left(x-\frac{1}{a}\right)^2}+\frac{31\sqrt{-a^2\left(\right.}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3*x^2*(-a^2*x^2+1)^(1/2)/a^5/c^2-8/3/c^2/a^7*(-a^2*x^2+1)^(1/2)-1/2/c^2/
a^6*x*(-a^2*x^2+1)^(1/2)+5/2/c^2/a^6/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*x^2+1)^(1/2))+1/6/c^2/a^9/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+31/
12/c^2/a^8/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/4/c^2/a^8/(x+1/a)*(
-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^6}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*x^6/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.08, size = 241, normalized size = 1.76

$$\frac{\sqrt{1-a^2\,x^2}}{6\,\left(a^9\,c^2\,x^2-2\,a^8\,c^2\,x+a^7\,c^2\right)}+\frac{\sqrt{1-a^2\,x^2}}{4\,\left(a^5\,c^2\,\sqrt{-a^2}+a^6\,c^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+\frac{31\,\sqrt{1-a^2\,x^2}}{12\,\left(a^5\,c^2\,\sqrt{-a^2}-a^6\,c^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a*x + 1))/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(6*(a^7*c^2 - 2*a^8*c^2*x + a^9*c^2*x^2)) + (1 - a^2*x^
2)^(1/2)/(4*(a^5*c^2*(-a^2)^(1/2) + a^6*c^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) +
 (31*(1 - a^2*x^2)^(1/2))/(12*(a^5*c^2*(-a^2)^(1/2) - a^6*c^2*x*(-a^2)^(1/2
))*(-a^2)^(1/2)) - (8*(1 - a^2*x^2)^(1/2))/(3*a^7*c^2) - (x*(1 - a^2*x^2)^(
1/2))/(2*a^6*c^2) + (5*asinh(x*(-a^2)^(1/2)))/(2*a^6*c^2*(-a^2)^(1/2)) - (x
^2*(1 - a^2*x^2)^(1/2))/(3*a^5*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^6}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^7}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**6/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**6/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**7/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

3.897 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=110

$$\frac{5\sin^{-1}(ax)}{2a^6c^2}+\frac{x^4(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{(15ax+16)\sqrt{1-a^2x^2}}{6a^6c^2}-\frac{x^2(5ax+4)}{3a^4c^2\sqrt{1-a^2x^2}}$$

```
[Out] 1/3*x^4*(a*x+1)/a^2/c^2/(-a^2*x^2+1)^(3/2)+5/2*arcsin(a*x)/a^6/c^2-1/3*x^2*
(5*a*x+4)/a^4/c^2/(-a^2*x^2+1)^(1/2)-1/6*(15*a*x+16)*(-a^2*x^2+1)^(1/2)/a^6
/c^2
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 780, 216}

$$\frac{x^4(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x^2(5ax+4)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{(15ax+16)\sqrt{1-a^2x^2}}{6a^6c^2}+\frac{5\sin^{-1}(ax)}{2a^6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^4*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (x^2*(4 + 5*a*x))/(3*a^4*
c^2*Sqrt[1 - a^2*x^2]) - ((16 + 15*a*x)*Sqrt[1 - a^2*x^2])/(6*a^6*c^2) + (5
*ArcSin[a*x])/(2*a^6*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
```

```
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{x^5(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{x^4(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{x^3(4+5ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}\\
&= \frac{x^4(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^2(4+5ax)}{3a^4c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{x(8+15ax)}{\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}\\
&= \frac{x^4(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^2(4+5ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{(16+15ax)\sqrt{1-a^2x^2}}{6a^6c^2} + \frac{5\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^5c^2}\\
&= \frac{x^4(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x^2(4+5ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{(16+15ax)\sqrt{1-a^2x^2}}{6a^6c^2} + \frac{5\sin^{-1}(ax)}{2a^6c^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 86, normalized size = 0.78

$$
\frac{3a^4x^4+3a^3x^3-23a^2x^2+15(ax-1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-ax+16}{6a^6c^2(ax-1)\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (16 - a*x - 23*a^2*x^2 + 3*a^3*x^3 + 3*a^4*x^4 + 15*(-1 + a*x)*Sqrt[1 - a^2
*x^2]*ArcSin[a*x])/(6*a^6*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.73, size = 152, normalized size = 1.38

$$-\frac{16\,a^{3}x^{3}-16\,a^{2}x^{2}-16\,ax+30\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)+\left(3\,a^{4}x^{4}+3\,a^{3}x^{3}-23\,a^{2}x^{2}-ax+\right.}{6\left(a^{9}c^{2}x^{3}-a^{8}c^{2}x^{2}-a^{7}c^{2}x+a^{6}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/6*(16*a^3*x^3 - 16*a^2*x^2 - 16*a*x + 30*(a^3*x^3 - a^2*x^2 - a*x + 1)*a
rctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^4*x^4 + 3*a^3*x^3 - 23*a^2*x^2
 - a*x + 16)*sqrt(-a^2*x^2 + 1) + 16)/(a^9*c^2*x^3 - a^8*c^2*x^2 - a^7*c^2*
x + a^6*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 202, normalized size = 1.84

$$-\frac{x\sqrt{-a^{2}x^{2}+1}}{2c^{2}a^{5}}+\frac{5\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{2c^{2}a^{5}\sqrt{a^{2}}}-\frac{\sqrt{-a^{2}x^{2}+1}}{c^{2}a^{6}}+\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{6c^{2}a^{8}\left(x-\frac{1}{a}\right)^{2}}+\frac{25\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\right.}}{12c^{2}a^{7}\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/c^2/a^5*x*(-a^2*x^2+1)^(1/2)+5/2/c^2/a^5/(a^2)^(1/2)*arctan((a^2)^(1/2
)*x/(-a^2*x^2+1)^(1/2))-1/c^2/a^6*(-a^2*x^2+1)^(1/2)+1/6/c^2/a^8/(x-1/a)^2*
```

```
(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+25/12/c^2/a^7/(x-1/a)*(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(1/2)+1/4/c^2/a^7/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^6}{\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx-\frac{\frac{3\,\sqrt{-a^2x^2+1}}{c^2}-\frac{6\,a^2x^2-5}{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] a*integrate(x^6/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(-a*
x + 1)), x) - 1/3*(3*sqrt(-a^2*x^2 + 1)/c^2 - (6*a^2*x^2 - 5)/((-a^2*x^2 +
1)^(3/2)*c^2))/a^6
```

mupad [B] time = 0.90, size = 218, normalized size = 1.98

$$\frac{\sqrt{1-a^2\,x^2}}{6\,\left(a^8\,c^2\,x^2-2\,a^7\,c^2\,x+a^6\,c^2\right)}-\frac{\sqrt{1-a^2\,x^2}}{4\,\left(a^4\,c^2\,\sqrt{-a^2}+a^5\,c^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+\frac{25\,\sqrt{1-a^2\,x^2}}{12\,\left(a^4\,c^2\,\sqrt{-a^2}-a^5\,c^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a*x + 1))/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(6*(a^6*c^2 - 2*a^7*c^2*x + a^8*c^2*x^2)) - (1 - a^2*x^
2)^(1/2)/(4*(a^4*c^2*(-a^2)^(1/2) + a^5*c^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) +
 (25*(1 - a^2*x^2)^(1/2))/(12*(a^4*c^2*(-a^2)^(1/2) - a^5*c^2*x*(-a^2)^(1/2
))*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a^6*c^2) - (x*(1 - a^2*x^2)^(1/2))/
(2*a^5*c^2) + (5*asinh(x*(-a^2)^(1/2)))/(2*a^5*c^2*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^6}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**5/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**6/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

3.898 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=99

$$\frac{\sin^{-1}(ax)}{a^5c^2}+\frac{x^3(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{8\sqrt{1-a^2x^2}}{3a^5c^2}-\frac{x(4ax+3)}{3a^4c^2\sqrt{1-a^2x^2}}$$

```
[Out] 1/3*x^3*(a*x+1)/a^2/c^2/(-a^2*x^2+1)^(3/2)+arcsin(a*x)/a^5/c^2-1/3*x*(4*a*x
+3)/a^4/c^2/(-a^2*x^2+1)^(1/2)-8/3*(-a^2*x^2+1)^(1/2)/a^5/c^2
```

Rubi [A] time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 641, 216}

$$\frac{x^3(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x(4ax+3)}{3a^4c^2\sqrt{1-a^2x^2}}-\frac{8\sqrt{1-a^2x^2}}{3a^5c^2}+\frac{\sin^{-1}(ax)}{a^5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^3*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (x*(3 + 4*a*x))/(3*a^4*c^
2*Sqrt[1 - a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*a^5*c^2) + ArcSin[a*x]/(a^5
*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
```

```
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{x^3(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{x^2(3+4ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}\\
&= \frac{x^3(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3a^4c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3+8ax}{\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}\\
&= \frac{x^3(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3a^5c^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^4c^2}\\
&= \frac{x^3(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x(3+4ax)}{3a^4c^2\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3a^5c^2} + \frac{\sin^{-1}(ax)}{a^5c^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.79

$$\frac{3a^3x^3-7a^2x^2+3(ax-1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-5ax+8}{3a^5c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[
a*x])/(3*a^5*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.68, size = 144, normalized size = 1.45

$$-\frac{8\,a^3x^3 - 8\,a^2x^2 - 8\,ax + 6\left(a^3x^3 - a^2x^2 - ax + 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(3\,a^3x^3 - 7\,a^2x^2 - 5\,ax + 8\right)\sqrt{-a^2x^2 +}}{3\left(a^8c^2x^3 - a^7c^2x^2 - a^6c^2x + a^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(8*a^3*x^3 - 8*a^2*x^2 - 8*a*x + 6*(a^3*x^3 - a^2*x^2 - a*x + 1)*arcta
n((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (3*a^3*x^3 - 7*a^2*x^2 - 5*a*x + 8)*sqr
t(-a^2*x^2 + 1) + 8)/(a^8*c^2*x^3 - a^7*c^2*x^2 - a^6*c^2*x + a^5*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^4}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^4/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

maple [B] time = 0.05, size = 180, normalized size = 1.82

$$-\frac{\sqrt{-a^2x^2+1}}{a^5c^2}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^2a^4\sqrt{a^2}}+\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{6c^2a^7\left(x-\frac{1}{a}\right)^2}+\frac{19\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{12c^2a^6\left(x-\frac{1}{a}\right)}-\frac{\sqrt{-a^2\left(x+\right.}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a^5/c^2+1/c^2/a^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^
2*x^2+1)^(1/2))+1/6/c^2/a^7/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+19
/12/c^2/a^6/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/4/c^2/a^6/(x+1/a)*
(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^4}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*x^4/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.07, size = 196, normalized size = 1.98

$$\frac{\sqrt{1-a^2\,x^2}}{6\,\left(a^7\,c^2\,x^2-2\,a^6\,c^2\,x+a^5\,c^2\right)}+\frac{\sqrt{1-a^2\,x^2}}{4\,\left(a^3\,c^2\,\sqrt{-a^2}\,+a^4\,c^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+\frac{19\,\sqrt{1-a^2\,x^2}}{12\,\left(a^3\,c^2\,\sqrt{-a^2}\,-a^4\,c^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(6*(a^5*c^2 - 2*a^6*c^2*x + a^7*c^2*x^2)) + (1 - a^2*x^
2)^(1/2)/(4*(a^3*c^2*(-a^2)^(1/2) + a^4*c^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) +
 (19*(1 - a^2*x^2)^(1/2))/(12*(a^3*c^2*(-a^2)^(1/2) - a^4*c^2*x*(-a^2)^(1/2
))*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(a^5*c^2) + asinh(x*(-a^2)^(1/2))/(a
^4*c^2*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**5/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

3.899 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=74

$$\frac{\sin^{-1}(ax)}{a^4c^2}+\frac{x^2(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3ax+2}{3a^4c^2\sqrt{1-a^2x^2}}$$

```
[Out] 1/3*x^2*(a*x+1)/a^2/c^2/(-a^2*x^2+1)^(3/2)+arcsin(a*x)/a^4/c^2+1/3*(-3*a*x-
2)/a^4/c^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 778, 216}

$$\frac{x^2(ax+1)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{3ax+2}{3a^4c^2\sqrt{1-a^2x^2}}+\frac{\sin^{-1}(ax)}{a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^2*(1 + a*x))/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - (2 + 3*a*x)/(3*a^4*c^2*Sq
rt[1 - a^2*x^2]) + ArcSin[a*x]/(a^4*c^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
```

```
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{x^3(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\
&= \frac{x^2(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{x(2+3ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}\\
&= \frac{x^2(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2+3ax}{3a^4c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3c^2}\\
&= \frac{x^2(1+ax)}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{2+3ax}{3a^4c^2\sqrt{1-a^2x^2}} + \frac{\sin^{-1}(ax)}{a^4c^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.93

$$\frac{-4a^2x^2+3(ax-1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+ax+2}{3a^4c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2 + a*x - 4*a^2*x^2 + 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(3*a^4*c
^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [B] time = 0.63, size = 137, normalized size = 1.85

$$-\frac{2\,a^{3}x^{3}-2\,a^{2}x^{2}-2\,ax+6\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\arctan\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{ax}\right)-\left(4\,a^{2}x^{2}-ax-2\right)\sqrt{-a^{2}x^{2}+1}+2}{3\left(a^{7}c^{2}x^{3}-a^{6}c^{2}x^{2}-a^{5}c^{2}x+a^{4}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(2*a^3*x^3 - 2*a^2*x^2 - 2*a*x + 6*(a^3*x^3 - a^2*x^2 - a*x + 1)*arcta
n((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (4*a^2*x^2 - a*x - 2)*sqrt(-a^2*x^2 + 1
) + 2)/(a^7*c^2*x^3 - a^6*c^2*x^2 - a^5*c^2*x + a^4*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.04, size = 160, normalized size = 2.16

$$\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x}{\sqrt{-a^{2}x^{2}+1}}\right)}{c^{2}a^{3}\sqrt{a^{2}}}+\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{6c^{2}a^{6}\left(x-\frac{1}{a}\right)^{2}}+\frac{13\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{12c^{2}a^{5}\left(x-\frac{1}{a}\right)}+\frac{\sqrt{-a^{2}\left(x+\frac{1}{a}\right)^{2}+2a\left(x+\frac{1}{a}\right)}}{4c^{2}a^{5}\left(x+\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/c^2/a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/6/c^2/a^6/
(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+13/12/c^2/a^5/(x-1/a)*(-a^2*(x
-1/a)^2-2*a*(x-1/a))^(1/2)+1/4/c^2/a^5/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{x^{4}}{\left(a^{4}c^{2}x^{4}-2\,a^{2}c^{2}x^{2}+c^{2}\right)\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx+\frac{3\,a^{2}x^{2}-2}{3\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}a^{4}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] a*integrate(x^4/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(-a*
x + 1)), x) + 1/3*(3*a^2*x^2 - 2)/((-a^2*x^2 + 1)^(3/2)*a^4*c^2)
```

mupad [B] time = 0.07, size = 176, normalized size = 2.38

$$\frac{\sqrt{1-a^2\,x^2}}{6\,\left(a^6\,c^2\,x^2-2\,a^5\,c^2\,x+a^4\,c^2\right)}-\frac{\sqrt{1-a^2\,x^2}}{4\,\left(a^2\,c^2\,\sqrt{-a^2}+a^3\,c^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+\frac{13\,\sqrt{1-a^2\,x^2}}{12\,\left(a^2\,c^2\,\sqrt{-a^2}-a^3\,c^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(6*(a^4*c^2 - 2*a^5*c^2*x + a^6*c^2*x^2)) - (1 - a^2*x^
2)^(1/2)/(4*(a^2*c^2*(-a^2)^(1/2) + a^3*c^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) +
 (13*(1 - a^2*x^2)^(1/2))/(12*(a^2*c^2*(-a^2)^(1/2) - a^3*c^2*x*(-a^2)^(1/2
))*(-a^2)^(1/2)) + asinh(x*(-a^2)^(1/2))/(a^3*c^2*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**4/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

3.900 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=57

$$\frac{x^2(ax+1)}{3ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{2}{3a^3c^2\sqrt{1-a^2x^2}}$$

```
[Out] 1/3*x^2*(a*x+1)/a/c^2/(-a^2*x^2+1)^(3/2)-2/3/a^3/c^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 796, 12, 261}

$$\frac{x^2(ax+1)}{3ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{2}{3a^3c^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^2*(1 + a*x))/(3*a*c^2*(1 - a^2*x^2)^(3/2)) - 2/(3*a^3*c^2*Sqrt[1 - a^2*x
^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^2}\,dx=\frac{\int \frac{x^2(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$
$$=\frac{x^2(1+ax)}{3ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{\int \frac{2ax}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$
$$=\frac{x^2(1+ax)}{3ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{2\int \frac{x}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3ac^2}$$
$$=\frac{x^2(1+ax)}{3ac^2\left(1-a^2x^2\right)^{3/2}}-\frac{2}{3a^3c^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.79

$$\frac{-a^2x^2-2ax+2}{3a^3c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2 - 2*a*x - a^2*x^2)/(3*a^3*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.50, size = 91, normalized size = 1.60

$$-\frac{2\,a^3x^3-2\,a^2x^2-2\,ax-\left(a^2x^2+2\,ax-2\right)\sqrt{-a^2x^2+1}+2}{3\left(a^6c^2x^3-a^5c^2x^2-a^4c^2x+a^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(2*a^3*x^3 - 2*a^2*x^2 - 2*a*x - (a^2*x^2 + 2*a*x - 2)*sqrt(-a^2*x^2 +
 1) + 2)/(a^6*c^2*x^3 - a^5*c^2*x^2 - a^4*c^2*x + a^3*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^2}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^2/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 41, normalized size = 0.72

$$-\frac{a^2x^2+2ax-2}{3\,(ax-1)\,c^2\sqrt{-a^2x^2+1}\;a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3*(a^2*x^2+2*a*x-2)/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^2}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*x^2/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.93, size = 102, normalized size = 1.79

$$-\frac{12\,a^7\,c^2\,{\left(1-a^2\,x^2\right)}^{3/2}-36\,a^7\,c^2\,{\left(1-a^2\,x^2\right)}^{5/2}+12\,a^8\,c^2\,x\,{\left(1-a^2\,x^2\right)}^{3/2}-12\,a^8\,c^2\,x\,{\left(1-a^2\,x^2\right)}^{5/2}}{36\,a^{10}\,c^4\,{\left(a^2\,x^2-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] -(12*a^7*c^2*(1 - a^2*x^2)^(3/2) - 36*a^7*c^2*(1 - a^2*x^2)^(5/2) + 12*a^8*
c^2*x*(1 - a^2*x^2)^(3/2) - 12*a^8*c^2*x*(1 - a^2*x^2)^(5/2))/(36*a^10*c^4*
(a^2*x^2 - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**2/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**3/(a**4*x**4*sqrt(-a**2*
x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c
**2
```

3.901 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=55

$$\frac{ax+1}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x}{3ac^2\sqrt{1-a^2x^2}}$$

```
[Out] 1/3*(a*x+1)/a^2/c^2/(-a^2*x^2+1)^(3/2)-1/3*x/a/c^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6148, 778, 191}

$$\frac{ax+1}{3a^2c^2\left(1-a^2x^2\right)^{3/2}}-\frac{x}{3ac^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (1 + a*x)/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - x/(3*a*c^2*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{x(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$

$$= \frac{1+ax}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3ac^2}$$

$$= \frac{1+ax}{3a^2c^2\left(1-a^2x^2\right)^{3/2}} - \frac{x}{3ac^2\sqrt{1-a^2x^2}}$$

Mathematica [**A**] time = 0.02, size = 44, normalized size = 0.80

$$\frac{-a^2x^2+ax-1}{3a^2c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-1 + a*x - a^2*x^2)/(3*a^2*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [**A**] time = 0.52, size = 89, normalized size = 1.62

$$\frac{a^3x^3-a^2x^2-ax+\left(a^2x^2-ax+1\right)\sqrt{-a^2x^2+1}+1}{3\left(a^5c^2x^3-a^4c^2x^2-a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] 1/3*(a^3*x^3 - a^2*x^2 - a*x + (a^2*x^2 - a*x + 1)*sqrt(-a^2*x^2 + 1) + 1)/
(a^5*c^2*x^3 - a^4*c^2*x^2 - a^3*c^2*x + a^2*c^2)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*x/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 41, normalized size = 0.75

$$-\frac{a^{2} x^{2}-a x+1}{3 \left(a x-1\right) c^{2} \sqrt{-a^{2} x^{2}+1}\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3*(a^2*x^2-a*x+1)/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a \int \frac{x^{2}}{\left(a^{4} c^{2} x^{4}-2 \, a^{2} c^{2} x^{2}+c^{2}\right) \sqrt{a x+1} \sqrt{-a x+1}}\, dx + \frac{1}{3 \left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} a^{2} c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] a*integrate(x^2/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(-a*
x + 1)), x) + 1/3/((-a^2*x^2 + 1)^(3/2)*a^2*c^2)
```

mupad [B] time = 0.92, size = 63, normalized size = 1.15

$$\frac{1}{3\,a^2\,c^2\,{\left(1-a^2\,x^2\right)}^{3/2}}-\frac{x}{3\,a\,c^2\,\sqrt{1-a^2\,x^2}}+\frac{x}{3\,a\,c^2\,{\left(1-a^2\,x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] 1/(3*a^2*c^2*(1 - a^2*x^2)^(3/2)) - x/(3*a*c^2*(1 - a^2*x^2)^(1/2)) + x/(3*
a*c^2*(1 - a^2*x^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^{4} x^{4} \sqrt{- a^{2} x^{2} + 1} - 2 a^{2} x^{2} \sqrt{- a^{2} x^{2} + 1} + \sqrt{- a^{2} x^{2} + 1}}\, dx + \int \frac{a x^{2}}{a^{4} x^{4} \sqrt{- a^{2} x^{2} + 1} - 2 a^{2} x^{2} \sqrt{- a^{2} x^{2} + 1} + \sqrt{- a^{2} x^{2} + 1}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 +
 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x**2/(a**4*x**4*sqrt(-a**2*x**
2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.902 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=52

$$\frac{2x}{3c^2\sqrt{1-a^2x^2}}+\frac{ax+1}{3ac^2\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] 1/3*(a*x+1)/a/c^2/(-a^2*x^2+1)^(3/2)+2/3*x/c^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6138, 639, 191}

$$\frac{2x}{3c^2\sqrt{1-a^2x^2}}+\frac{ax+1}{3ac^2\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (1 + a*x)/(3*a*c^2*(1 - a^2*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1+ax}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$
$$= \frac{1+ax}{3ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c^2}$$
$$= \frac{1+ax}{3ac^2\left(1-a^2x^2\right)^{3/2}} + \frac{2x}{3c^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 0.87

$$\frac{2a^2x^2-2ax-1}{3ac^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-1 - 2*a*x + 2*a^2*x^2)/(3*a*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [B] time = 0.55, size = 89, normalized size = 1.71

$$\frac{a^3x^3-a^2x^2-ax-\left(2\,a^2x^2-2\,ax-1\right)\sqrt{-a^2x^2+1}\,+1}{3\left(a^4c^2x^3-a^3c^2x^2-a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/3*(a^3*x^3 - a^2*x^2 - a*x - (2*a^2*x^2 - 2*a*x - 1)*sqrt(-a^2*x^2 + 1) +
 1)/(a^4*c^2*x^3 - a^3*c^2*x^2 - a^2*c^2*x + a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 42, normalized size = 0.81

$$\frac{2a^2x^2 - 2ax - 1}{3\left(ax - 1\right)c^2\sqrt{-a^2x^2+1}\; a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/3*(2*a^2*x^2-2*a*x-1)/(a*x-1)/c^2/(-a^2*x^2+1)^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(a^2cx^2 - c\right)^2\sqrt{-a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.08, size = 48, normalized size = 0.92

$$\frac{\sqrt{1-a^2\,x^2}\;\left(-2\,a^2\,x^2+2\,a\,x+1\right)}{3\,a\,c^2\,{\left(a\,x-1\right)}^2\;\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(2*a*x - 2*a^2*x^2 + 1))/(3*a*c^2*(a*x - 1)^2*(a*x + 1
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^4x^4\sqrt{-a^2x^2+1} - 2a^2x^2\sqrt{-a^2x^2+1} + \sqrt{-a^2x^2+1}}\, dx + \int \frac{1}{a^4x^4\sqrt{-a^2x^2+1} - 2a^2x^2\sqrt{-a^2x^2+1} + \sqrt{-a^2x^2+1}}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a*x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2
 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**4*sqrt(-a**2*x**2 +
 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.903 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=74

$$\frac{ax+1}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{2ax+3}{3c^2\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

```
[Out] 1/3*(a*x+1)/c^2/(-a^2*x^2+1)^(3/2)-arctanh((-a^2*x^2+1)^(1/2))/c^2+1/3*(2*a
*x+3)/c^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 12, 266, 63, 208}

$$\frac{ax+1}{3c^2\left(1-a^2x^2\right)^{3/2}}+\frac{2ax+3}{3c^2\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] (1 + a*x)/(3*c^2*(1 - a^2*x^2)^(3/2)) + (3 + 2*a*x)/(3*c^2*Sqrt[1 - a^2*x^2
]) - ArcTanh[Sqrt[1 - a^2*x^2]]/c^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1+ax}{x\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$

$$= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{3a^2+2a^3x}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$

$$= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+2ax}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{3a^4}{x\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}$$

$$= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+2ax}{3c^2\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+2ax}{3c^2\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c^2}$$

$$= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+2ax}{3c^2\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2c^2}$$

$$= \frac{1+ax}{3c^2\left(1-a^2x^2\right)^{3/2}} + \frac{3+2ax}{3c^2\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.04

$$\frac{2a^2x^2-3(ax-1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ax-4}{3c^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-4 + a*x + 2*a^2*x^2 - 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2
*x^2]])/(3*c^2*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.74, size = 127, normalized size = 1.72

$$\frac{4\,a^3x^3-4\,a^2x^2-4\,ax+3\left(a^3x^3-a^2x^2-ax+1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(2\,a^2x^2+ax-4\right)\sqrt{-a^2x^2+1}+4}{3\left(a^3c^2x^3-a^2c^2x^2-ac^2x+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] 1/3*(4*a^3*x^3 - 4*a^2*x^2 - 4*a*x + 3*(a^3*x^3 - a^2*x^2 - a*x + 1)*log((s
qrt(-a^2*x^2 + 1) - 1)/x) - (2*a^2*x^2 + a*x - 4)*sqrt(-a^2*x^2 + 1) + 4)/(
a^3*c^2*x^3 - a^2*c^2*x^2 - a*c^2*x + c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x), x)
```

maple [B] time = 0.04, size = 182, normalized size = 2.46

$$\frac{-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}}{2a}-\frac{3\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{4a\left(x-\frac{1}{a}\right)}+\frac{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}{4a\left(x+\frac{1}{a}\right)}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/c^2*(-arctanh(1/(-a^2*x^2+1)^(1/2))+1/2/a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^
2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))-3/4/a/
(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/4/a/(x+1/a)*(-a^2*(x+1/a)^2+2*
a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [B] time = 0.91, size = 173, normalized size = 2.34

$$\frac{a^2\sqrt{1-a^2x^2}}{6\left(a^4c^2x^2-2a^3c^2x+a^2c^2\right)}-\frac{a\sqrt{1-a^2x^2}}{4\sqrt{-a^2}\left(c^2x\sqrt{-a^2}+\frac{c^2\sqrt{-a^2}}{a}\right)}+\frac{11\,a\sqrt{1-a^2x^2}}{12\sqrt{-a^2}\left(c^2x\sqrt{-a^2}-\frac{c^2\sqrt{-a^2}}{a}\right)}+\frac{\operatorname{atan}\left(\sqrt{1-a}\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (atan((1 - a^2*x^2)^(1/2)*1i)*1i)/c^2 + (a^2*(1 - a^2*x^2)^(1/2))/(6*(a^2*c
^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) - (a*(1 - a^2*x^2)^(1/2))/(4*(-a^2)^(1/2)*
(c^2*x*(-a^2)^(1/2) + (c^2*(-a^2)^(1/2))/a)) + (11*a*(1 - a^2*x^2)^(1/2))/(
12*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^5\sqrt{-a^2x^2+1}-2a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 +
 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**5*sqrt(-a**2*x**2 + 1
) - 2*a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/c**2
```

3.904 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=105

$$\frac{ax+1}{3c^2x\left(1-a^2x^2\right)^{3/2}}-\frac{8\sqrt{1-a^2x^2}}{3c^2x}+\frac{3ax+4}{3c^2x\sqrt{1-a^2x^2}}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

```
[Out] 1/3*(a*x+1)/c^2/x/(-a^2*x^2+1)^(3/2)-a*arctanh((-a^2*x^2+1)^(1/2))/c^2+1/3*
(3*a*x+4)/c^2/x/(-a^2*x^2+1)^(1/2)-8/3*(-a^2*x^2+1)^(1/2)/c^2/x
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 807, 266, 63, 208}

$$\frac{ax+1}{3c^2x\left(1-a^2x^2\right)^{3/2}}-\frac{8\sqrt{1-a^2x^2}}{3c^2x}+\frac{3ax+4}{3c^2x\sqrt{1-a^2x^2}}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] (1 + a*x)/(3*c^2*x*(1 - a^2*x^2)^(3/2)) + (4 + 3*a*x)/(3*c^2*x*Sqrt[1 - a^2
*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*c^2*x) - (a*ArcTanh[Sqrt[1 - a^2*x^2]])/c
^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1+ax}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$

$$= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{4a^2+3a^3x}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$

$$= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}} + \frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}} + \frac{\int \frac{8a^4+3a^5x}{x^2\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}$$

$$= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}} + \frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x} + \frac{a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^2}$$

$$= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}} + \frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c^2}$$

$$= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}} + \frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{ac^2}$$

$$= \frac{1+ax}{3c^2x\left(1-a^2x^2\right)^{3/2}} + \frac{4+3ax}{3c^2x\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x} - \frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^2}$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.87

$$\frac{8a^3x^3-5a^2x^2-3ax(ax-1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-7ax+3}{3c^2x(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] (3 - 7*a*x - 5*a^2*x^2 + 8*a^3*x^3 - 3*a*x*(-1 + a*x)*Sqrt[1 - a^2*x^2]*Arc
Tanh[Sqrt[1 - a^2*x^2]])/(3*c^2*x*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.64, size = 153, normalized size = 1.46

$$\frac{4\,a^4x^4-4\,a^3x^3-4\,a^2x^2+4\,ax+3\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(8\,a^3x^3-5\,a^2x^2-7\,ax+3\right)\sqrt{-}}{3\left(a^3c^2x^4-a^2c^2x^3-ac^2x^2+c^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] 1/3*(4*a^4*x^4 - 4*a^3*x^3 - 4*a^2*x^2 + 4*a*x + 3*(a^4*x^4 - a^3*x^3 - a^2
*x^2 + a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (8*a^3*x^3 - 5*a^2*x^2 - 7*a*
x + 3)*sqrt(-a^2*x^2 + 1))/(a^3*c^2*x^4 - a^2*c^2*x^3 - a*c^2*x^2 + c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.05, size = 150, normalized size = 1.43

$$\frac{-a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{\sqrt{-a^2x^2+1}}{x}+\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{6a\left(x-\frac{1}{a}\right)^2}-\frac{17\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{12\left(x-\frac{1}{a}\right)}-\frac{\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}}{4\left(x+\frac{1}{a}\right)}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/c^2*(-a*arctanh(1/(-a^2*x^2+1)^(1/2))-(-a^2*x^2+1)^(1/2)/x+1/6/a/(x-1/a)^
2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-17/12/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(1/2)-1/4/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [A] time = 0.34, size = 144, normalized size = 1.37

$$-\frac{\frac{3\,a^2\log\left(\sqrt{-a^2x^2+1}+1\right)}{c^2}-\frac{3\,a^2\log\left(\sqrt{-a^2x^2+1}-1\right)}{c^2}+\frac{2\left(3\left(a^2x^2-1\right)a^2-a^2\right)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}}{6\,a}+\frac{8\,a^4x^4-12\,a^2x^2+3}{3\left(a^2c^2x^3-c^2x\right)\sqrt{ax+1}\,\sqrt{-ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] -1/6*(3*a^2*log(sqrt(-a^2*x^2 + 1) + 1)/c^2 - 3*a^2*log(sqrt(-a^2*x^2 + 1)
- 1)/c^2 + 2*(3*(a^2*x^2 - 1)*a^2 - a^2)/((-a^2*x^2 + 1)^(3/2)*c^2))/a + 1/
3*(8*a^4*x^4 - 12*a^2*x^2 + 3)/((a^2*c^2*x^3 - c^2*x)*sqrt(a*x + 1)*sqrt(-a
*x + 1))
```

mupad [B] time = 0.90, size = 198, normalized size = 1.89

$$\frac{a^3\sqrt{1-a^2x^2}}{6\left(a^4c^2x^2-2a^3c^2x+a^2c^2\right)}-\frac{\sqrt{1-a^2x^2}}{c^2x}+\frac{a^2\sqrt{1-a^2x^2}}{4\sqrt{-a^2}\left(c^2x\sqrt{-a^2}+\frac{c^2\sqrt{-a^2}}{a}\right)}+\frac{17a^2\sqrt{1-a^2x^2}}{12\sqrt{-a^2}\left(c^2x\sqrt{-a^2}-\frac{c^2\sqrt{-a^2}}{a}\right)}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^3*(1 - a^2*x^2)^(1/2))/(6*(a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) - (1 -
a^2*x^2)^(1/2)/(c^2*x) + (a*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/c^2 + (a^2*(1
- a^2*x^2)^(1/2))/(4*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) + (c^2*(-a^2)^(1/2))/
a)) + (17*a^2*(1 - a^2*x^2)^(1/2))/(12*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (
c^2*(-a^2)^(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{a^4x^5\sqrt{-a^2x^2+1}-2a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^6\sqrt{-a^2x^2+1}-2a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a/(a**4*x**5*sqrt(-a**2*x**2 + 1) - 2*a**2*x**3*sqrt(-a**2*x**2 +
 1) + x*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**6*sqrt(-a**2*x**2 +
 1) - 2*a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x))/c*
*2
```

3.905 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^2} dx$

Optimal. Leaf size=134

$$-\frac{8a\sqrt{1-a^2x^2}}{3c^2x}-\frac{5\sqrt{1-a^2x^2}}{2c^2x^2}+\frac{4ax+5}{3c^2x^2\sqrt{1-a^2x^2}}+\frac{ax+1}{3c^2x^2\left(1-a^2x^2\right)^{3/2}}-\frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

```
[Out] 1/3*(a*x+1)/c^2/x^2/(-a^2*x^2+1)^(3/2)-5/2*a^2*arctanh((-a^2*x^2+1)^(1/2))/
c^2+1/3*(4*a*x+5)/c^2/x^2/(-a^2*x^2+1)^(1/2)-5/2*(-a^2*x^2+1)^(1/2)/c^2/x^2
-8/3*a*(-a^2*x^2+1)^(1/2)/c^2/x
```

Rubi [A] time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{8a\sqrt{1-a^2x^2}}{3c^2x}-\frac{5\sqrt{1-a^2x^2}}{2c^2x^2}+\frac{4ax+5}{3c^2x^2\sqrt{1-a^2x^2}}+\frac{ax+1}{3c^2x^2\left(1-a^2x^2\right)^{3/2}}-\frac{5a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^2),x]
```

```
[Out] (1 + a*x)/(3*c^2*x^2*(1 - a^2*x^2)^(3/2)) + (5 + 4*a*x)/(3*c^2*x^2*Sqrt[1 -
 a^2*x^2]) - (5*Sqrt[1 - a^2*x^2])/(2*c^2*x^2) - (8*a*Sqrt[1 - a^2*x^2])/(3
*c^2*x) - (5*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/(2*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1+ax}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{5a^2+4a^3x}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} + \frac{\int \frac{15a^4+8a^5x}{x^3\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{\int \frac{-16a^5-15a^6x}{x^2\sqrt{1-a^2x^2}}\,dx}{6a^4c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{8a\sqrt{1-a^2x^2}}{3c^2x} + \frac{\left(5a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}}{2c^2}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{8a\sqrt{1-a^2x^2}}{3c^2x} + \frac{\left(5a^2\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{8a\sqrt{1-a^2x^2}}{3c^2x} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-}\right.}{}$$

$$= \frac{1+ax}{3c^2x^2\left(1-a^2x^2\right)^{3/2}} + \frac{5+4ax}{3c^2x^2\sqrt{1-a^2x^2}} - \frac{5\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{8a\sqrt{1-a^2x^2}}{3c^2x} - \frac{5a^2\tanh^{-1}\left(\sqrt{1\right.}{2c^2}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 0.77

$$\frac{16a^4x^4-a^3x^3-23a^2x^2-15a^2x^2(ax-1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ax+3}{6c^2x^2(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^2),x]
```

```
[Out] (3 + 3*a*x - 23*a^2*x^2 - a^3*x^3 + 16*a^4*x^4 - 15*a^2*x^2*(-1 + a*x)*Sqrt
[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(6*c^2*x^2*(-1 + a*x)*Sqrt[1 - a^
2*x^2])
```

fricas [A] time = 0.74, size = 171, normalized size = 1.28

$$\frac{14\,a^5x^5 - 14\,a^4x^4 - 14\,a^3x^3 + 14\,a^2x^2 + 15\left(a^5x^5 - a^4x^4 - a^3x^3 + a^2x^2\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(16\,a^4x^4 - a^3x^3 - 23\,a\right.}{6\left(a^3c^2x^5 - a^2c^2x^4 - ac^2x^3 + c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] 1/6*(14*a^5*x^5 - 14*a^4*x^4 - 14*a^3*x^3 + 14*a^2*x^2 + 15*(a^5*x^5 - a^4*
x^4 - a^3*x^3 + a^2*x^2)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (16*a^4*x^4 - a^
3*x^3 - 23*a^2*x^2 + 3*a*x + 3)*sqrt(-a^2*x^2 + 1))/(a^3*c^2*x^5 - a^2*c^2*
x^4 - a*c^2*x^3 + c^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.05, size = 214, normalized size = 1.60

$$\frac{-\frac{5a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2} - \frac{a\sqrt{-a^2x^2+1}}{x} + \frac{a\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2} - \frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}\right)}{2} - \frac{7a\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{4\left(x-\frac{1}{a}\right)} - \frac{\sqrt{-a^2x^2+1}}{2x^2} + \frac{a\sqrt{}}{}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/c^2*(-5/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-a*(-a^2*x^2+1)^(1/2)/x+1/2*a*
(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/
```

```
a)^2-2*a*(x-1/a))^(1/2))-7/4*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1
/2*(-a^2*x^2+1)^(1/2)/x^2+1/4*a/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [B] time = 0.07, size = 221, normalized size = 1.65

$$\frac{a^4\sqrt{1-a^2x^2}}{6\left(a^4c^2x^2-2a^3c^2x+a^2c^2\right)}-\frac{\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{a\sqrt{1-a^2x^2}}{c^2x}-\frac{a^3\sqrt{1-a^2x^2}}{4\sqrt{-a^2}\left(c^2x\sqrt{-a^2}+\frac{c^2\sqrt{-a^2}}{a}\right)}+\frac{23a^3\sqrt{1-a}}{12\sqrt{-a^2}\left(c^2x\sqrt{-a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*5i)/(2*c^2) + (a^4*(1 - a^2*x^2)^(1/2))/(
6*(a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) - (1 - a^2*x^2)^(1/2)/(2*c^2*x^2)
- (a*(1 - a^2*x^2)^(1/2))/(c^2*x) - (a^3*(1 - a^2*x^2)^(1/2))/(4*(-a^2)^(1/
2)*(c^2*x*(-a^2)^(1/2) + (c^2*(-a^2)^(1/2))/a)) + (23*a^3*(1 - a^2*x^2)^(1/
2))/(12*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) - (c^2*(-a^2)^(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{a^4x^6\sqrt{-a^2x^2+1}-2a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^7\sqrt{-a^2x^2+1}-2a^2x^5\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a/(a**4*x**6*sqrt(-a**2*x**2 + 1) - 2*a**2*x**4*sqrt(-a**2*x**2 +
 1) + x**2*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**7*sqrt(-a**2*x**
2 + 1) - 2*a**2*x**5*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)), x))
/c**2
```

3.906 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{16a^2\sqrt{1-a^2x^2}}{3c^2x}-\frac{5a\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{8\sqrt{1-a^2x^2}}{3c^2x^3}+\frac{5ax+6}{3c^2x^3\sqrt{1-a^2x^2}}+\frac{ax+1}{3c^2x^3\left(1-a^2x^2\right)^{3/2}}-\frac{5a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

```
[Out] 1/3*(a*x+1)/c^2/x^3/(-a^2*x^2+1)^(3/2)-5/2*a^3*arctanh((-a^2*x^2+1)^(1/2))/
c^2+1/3*(5*a*x+6)/c^2/x^3/(-a^2*x^2+1)^(1/2)-8/3*(-a^2*x^2+1)^(1/2)/c^2/x^3
-5/2*a*(-a^2*x^2+1)^(1/2)/c^2/x^2-16/3*a^2*(-a^2*x^2+1)^(1/2)/c^2/x
```

Rubi [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{16a^2\sqrt{1-a^2x^2}}{3c^2x}-\frac{5a\sqrt{1-a^2x^2}}{2c^2x^2}-\frac{8\sqrt{1-a^2x^2}}{3c^2x^3}+\frac{5ax+6}{3c^2x^3\sqrt{1-a^2x^2}}+\frac{ax+1}{3c^2x^3\left(1-a^2x^2\right)^{3/2}}-\frac{5a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)^2),x]
```

```
[Out] (1 + a*x)/(3*c^2*x^3*(1 - a^2*x^2)^(3/2)) + (6 + 5*a*x)/(3*c^2*x^3*Sqrt[1 -
 a^2*x^2]) - (8*Sqrt[1 - a^2*x^2])/(3*c^2*x^3) - (5*a*Sqrt[1 - a^2*x^2])/(2
*c^2*x^2) - (16*a^2*Sqrt[1 - a^2*x^2])/(3*c^2*x) - (5*a^3*ArcTanh[Sqrt[1 -
a^2*x^2]])/(2*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1+ax}{x^4\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{6a^2+5a^3x}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{3a^2c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} + \frac{\int \frac{24a^4+15a^5x}{x^4\sqrt{1-a^2x^2}}\,dx}{3a^4c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{\int \frac{-45a^5-48a^6x}{x^3\sqrt{1-a^2x^2}}\,dx}{9a^4c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} + \frac{\int \frac{96a^6+45a^7x}{x^2\sqrt{1-a^2x^2}}\,dx}{18a^4c^2}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{16a^2\sqrt{1-a^2x^2}}{3c^2x}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{16a^2\sqrt{1-a^2x^2}}{3c^2x}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{16a^2\sqrt{1-a^2x^2}}{3c^2x}$$

$$= \frac{1+ax}{3c^2x^3\left(1-a^2x^2\right)^{3/2}} + \frac{6+5ax}{3c^2x^3\sqrt{1-a^2x^2}} - \frac{8\sqrt{1-a^2x^2}}{3c^2x^3} - \frac{5a\sqrt{1-a^2x^2}}{2c^2x^2} - \frac{16a^2\sqrt{1-a^2x^2}}{3c^2x}$$

Mathematica [A] time = 0.05, size = 110, normalized size = 0.68

$$\frac{32a^5x^5-17a^4x^4-31a^3x^3+11a^2x^2-15a^3x^3(ax-1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+ax+2}{6c^2x^3(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)^2),x]

[Out] (2 + a*x + 11*a^2*x^2 - 31*a^3*x^3 - 17*a^4*x^4 + 32*a^5*x^5 - 15*a^3*x^3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(6*c^2*x^3*(-1 + a*x)*Sqrt[1 - a^2*x^2])

fricas [A] time = 0.84, size = 178, normalized size = 1.11

$$\frac{14\,a^6x^6-14\,a^5x^5-14\,a^4x^4+14\,a^3x^3+15\left(a^6x^6-a^5x^5-a^4x^4+a^3x^3\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(32\,a^5x^5-17\,a^4x^4-\right.}{6\left(a^3c^2x^6-a^2c^2x^5-ac^2x^4+c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] 1/6*(14*a^6*x^6 - 14*a^5*x^5 - 14*a^4*x^4 + 14*a^3*x^3 + 15*(a^6*x^6 - a^5*x^5 - a^4*x^4 + a^3*x^3)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (32*a^5*x^5 - 17*a^4*x^4 - 31*a^3*x^3 + 11*a^2*x^2 + a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^3*c^2*x^6 - a^2*c^2*x^5 - a*c^2*x^4 + c^2*x^3)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)*x^4), x)

maple [A] time = 0.05, size = 260, normalized size = 1.61

$$\frac{-2a^3\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{8a^2\sqrt{-a^2x^2+1}}{3x}+\frac{a^2\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}\right)}{2}-\frac{\sqrt{-a^2x^2+1}}{3x^3}-\frac{9a^2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(\right.}}{4\left(x-\frac{1}{a}\right)}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^2,x)

```
[Out] 1/c^2*(-2*a^3*arctanh(1/(-a^2*x^2+1)^(1/2))-8/3*a^2*(-a^2*x^2+1)^(1/2)/x+1/
2*a^2*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2))-1/3*(-a^2*x^2+1)^(1/2)/x^3-9/4*a^2/(x-1/a)*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+a*(-1/2*(-a^2*x^2+1)^(1/2)/x^2-1/2*a^2*ar
ctanh(1/(-a^2*x^2+1)^(1/2)))-1/4*a^2/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(
1/2))
```

maxima [A] time = 0.34, size = 191, normalized size = 1.19

$$-\frac{\frac{15\,a^4\log\left(\sqrt{-a^2x^2+1}+1\right)}{c^2}-\frac{15\,a^4\log\left(\sqrt{-a^2x^2+1}-1\right)}{c^2}-\frac{2\left(15\left(a^2x^2-1\right)^2a^4+10\left(a^2x^2-1\right)a^4-2\,a^4\right)}{\left(-a^2x^2+1\right)^{\frac{5}{2}}c^2-\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}}{12\,a}+\frac{16\,a^6x^6-24\,a^4x^4+6\,a^2x^2+1}{3\left(a^2c^2x^5-c^2x^3\right)\sqrt{ax+1}\sqrt{-ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] -1/12*(15*a^4*log(sqrt(-a^2*x^2 + 1) + 1)/c^2 - 15*a^4*log(sqrt(-a^2*x^2 +
1) - 1)/c^2 - 2*(15*(a^2*x^2 - 1)^2*a^4 + 10*(a^2*x^2 - 1)*a^4 - 2*a^4)/((-
a^2*x^2 + 1)^(5/2)*c^2 - (-a^2*x^2 + 1)^(3/2)*c^2))/a + 1/3*(16*a^6*x^6 - 2
4*a^4*x^4 + 6*a^2*x^2 + 1)/((a^2*c^2*x^5 - c^2*x^3)*sqrt(a*x + 1)*sqrt(-a*x
 + 1))
```

mupad [B] time = 0.91, size = 244, normalized size = 1.52

$$\frac{a^5\,\sqrt{1-a^2\,x^2}}{6\,\left(a^4\,c^2\,x^2-2\,a^3\,c^2\,x+a^2\,c^2\right)}-\frac{\sqrt{1-a^2\,x^2}}{3\,c^2\,x^3}-\frac{a\,\sqrt{1-a^2\,x^2}}{2\,c^2\,x^2}-\frac{8\,a^2\,\sqrt{1-a^2\,x^2}}{3\,c^2\,x}+\frac{a^4\,\sqrt{1-a^2\,x^2}}{4\,\sqrt{-a^2}\,\left(c^2\,x\,\sqrt{-a^2}+\frac{c^2\,\sqrt{-a^2}}{a}\right)}+\frac{}{12\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^3*atan((1 - a^2*x^2)^(1/2)*1i)*5i)/(2*c^2) + (a^5*(1 - a^2*x^2)^(1/2))/(
6*(a^2*c^2 - 2*a^3*c^2*x + a^4*c^2*x^2)) - (1 - a^2*x^2)^(1/2)/(3*c^2*x^3)
- (a*(1 - a^2*x^2)^(1/2))/(2*c^2*x^2) - (8*a^2*(1 - a^2*x^2)^(1/2))/(3*c^2*
x) + (a^4*(1 - a^2*x^2)^(1/2))/(4*(-a^2)^(1/2)*(c^2*x*(-a^2)^(1/2) + (c^2*(
-a^2)^(1/2))/a)) + (29*a^4*(1 - a^2*x^2)^(1/2))/(12*(-a^2)^(1/2)*(c^2*x*(-a
^2)^(1/2) - (c^2*(-a^2)^(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{a^4x^7\sqrt{-a^2x^2+1}-2a^2x^5\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^4x^8\sqrt{-a^2x^2+1}-2a^2x^6\sqrt{-a^2x^2+1}+x^4\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(a/(a**4*x**7*sqrt(-a**2*x**2 + 1) - 2*a**2*x**5*sqrt(-a**2*x**2 +
 1) + x**3*sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**8*sqrt(-a**2*x**
2 + 1) - 2*a**2*x**6*sqrt(-a**2*x**2 + 1) + x**4*sqrt(-a**2*x**2 + 1)), x))
/c**2
```

3.907 $\int \frac{e^{\tanh^{-1}(ax)}x^7}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=143

$$-\frac{7\sin^{-1}(ax)}{2a^8c^3}+\frac{x^6(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}+\frac{(35ax+32)\sqrt{1-a^2x^2}}{10a^8c^3}+\frac{x^2(35ax+24)}{15a^6c^3\sqrt{1-a^2x^2}}-\frac{x^4(7ax+6)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] 1/5*x^6*(a*x+1)/a^2/c^3/(-a^2*x^2+1)^(5/2)-1/15*x^4*(7*a*x+6)/a^4/c^3/(-a^2
*x^2+1)^(3/2)-7/2*arcsin(a*x)/a^8/c^3+1/15*x^2*(35*a*x+24)/a^6/c^3/(-a^2*x^
2+1)^(1/2)+1/10*(35*a*x+32)*(-a^2*x^2+1)^(1/2)/a^8/c^3
```

Rubi [A] time = 0.18, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 780, 216}

$$\frac{x^6(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}-\frac{x^4(7ax+6)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{x^2(35ax+24)}{15a^6c^3\sqrt{1-a^2x^2}}+\frac{(35ax+32)\sqrt{1-a^2x^2}}{10a^8c^3}-\frac{7\sin^{-1}(ax)}{2a^8c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^7)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^6*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (x^4*(6 + 7*a*x))/(15*a^4
*c^3*(1 - a^2*x^2)^(3/2)) + (x^2*(24 + 35*a*x))/(15*a^6*c^3*Sqrt[1 - a^2*x^
2]) + ((32 + 35*a*x)*Sqrt[1 - a^2*x^2])/(10*a^8*c^3) - (7*ArcSin[a*x])/(2*a
^8*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
 + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
 + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^7}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^7(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{x^5(6+7ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^4(6+7ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{x^3(24+35ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3}$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^4(6+7ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x^2(24+35ax)}{15a^6c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{x(48+105ax)}{\sqrt{1-a^2x^2}}\,dx}{15a^6c^3}$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^4(6+7ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x^2(24+35ax)}{15a^6c^3\sqrt{1-a^2x^2}} + \frac{(32+35ax)\sqrt{1-a^2x^2}}{10a^8c^3} -$$

$$= \frac{x^6(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^4(6+7ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x^2(24+35ax)}{15a^6c^3\sqrt{1-a^2x^2}} + \frac{(32+35ax)\sqrt{1-a^2x^2}}{10a^8c^3} -$$

Mathematica [A] time = 0.08, size = 116, normalized size = 0.81

$$\frac{-15a^6x^6-15a^5x^5+176a^4x^4+4a^3x^3-249a^2x^2-105(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+9ax+96}{30a^8c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^7)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (96 + 9*a*x - 249*a^2*x^2 + 4*a^3*x^3 + 176*a^4*x^4 - 15*a^5*x^5 - 15*a^6*x
^6 - 105*(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(30*a^8*c^3*
(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.74, size = 221, normalized size = 1.55

$$\frac{96\,a^5x^5-96\,a^4x^4-192\,a^3x^3+192\,a^2x^2+96\,ax+210\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}}{ax}\right.}{30\left(a^{13}c^3x^5-a^{12}c^3x^4-2\,a^{11}c^3x^3+2\,a^{10}c^3x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^7/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/30*(96*a^5*x^5 - 96*a^4*x^4 - 192*a^3*x^3 + 192*a^2*x^2 + 96*a*x + 210*(a
^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*arctan((sqrt(-a^2*x^2 +
 1) - 1)/(a*x)) + (15*a^6*x^6 + 15*a^5*x^5 - 176*a^4*x^4 - 4*a^3*x^3 + 249*
a^2*x^2 - 9*a*x - 96)*sqrt(-a^2*x^2 + 1) - 96)/(a^13*c^3*x^5 - a^12*c^3*x^4
 - 2*a^11*c^3*x^3 + 2*a^10*c^3*x^2 + a^9*c^3*x - a^8*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^7/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 283, normalized size = 1.98

$$\frac{x\sqrt{-a^2x^2+1}}{2c^3a^7}-\frac{7\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2c^3a^7\sqrt{a^2}}+\frac{\sqrt{-a^2x^2+1}}{c^3a^8}-\frac{7\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{15c^3a^{10}\left(x-\frac{1}{a}\right)^2}-\frac{773\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\right.}}{240c^3a^9\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^7/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/2/c^3/a^7*x*(-a^2*x^2+1)^(1/2)-7/2/c^3/a^7/(a^2)^(1/2)*arctan((a^2)^(1/2)
*x/(-a^2*x^2+1)^(1/2))+1/c^3/a^8*(-a^2*x^2+1)^(1/2)-7/15/c^3/a^10/(x-1/a)^2
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-773/240/c^3/a^9/(x-1/a)*(-a^2*(x-1/a)^2
-2*a*(x-1/a))^(1/2)-1/20/c^3/a^11/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1
/2)+1/24/c^3/a^10/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-31/48/c^3/a^
9/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\int \frac{x^8}{\left(a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3\right)\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx+\frac{\frac{5\,\sqrt{-a^2x^2+1}}{c^3}+\frac{5\,a^2x^2+15\left(a^2x^2-1\right)^2-4}{\left(-a^2x^2+1\right)^{\frac{5}{2}}c^3}}{5\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^7/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -a*integrate(x^8/((a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(
a*x + 1)*sqrt(-a*x + 1)), x) + 1/5*(5*sqrt(-a^2*x^2 + 1)/c^3 + (5*a^2*x^2 +
 15*(a^2*x^2 - 1)^2 - 4)/((-a^2*x^2 + 1)^(5/2)*c^3))/a^8
```

mupad [B] time = 0.10, size = 352, normalized size = 2.46

$$\frac{\sqrt{1-a^2\,x^2}}{24\,\left(a^{10}\,c^3\,x^2+2\,a^9\,c^3\,x+a^8\,c^3\right)}-\frac{7\,\sqrt{1-a^2\,x^2}}{15\,\left(a^{10}\,c^3\,x^2-2\,a^9\,c^3\,x+a^8\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{20\,\sqrt{-a^2}\,\left(a^6\,c^3\,\sqrt{-a^2}+3\,a^8\,c^3\,x^2\,\sqrt{-a^2}-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a*x + 1))/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(24*(a^8*c^3 + 2*a^9*c^3*x + a^10*c^3*x^2)) - (7*(1 - a
^2*x^2)^(1/2))/(15*(a^8*c^3 - 2*a^9*c^3*x + a^10*c^3*x^2)) - (1 - a^2*x^2)^
(1/2)/(20*(-a^2)^(1/2)*(a^6*c^3*(-a^2)^(1/2) + 3*a^8*c^3*x^2*(-a^2)^(1/2) -
 a^9*c^3*x^3*(-a^2)^(1/2) - 3*a^7*c^3*x*(-a^2)^(1/2))) + (31*(1 - a^2*x^2)^
(1/2))/(48*(a^6*c^3*(-a^2)^(1/2) + a^7*c^3*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
(773*(1 - a^2*x^2)^(1/2))/(240*(a^6*c^3*(-a^2)^(1/2) - a^7*c^3*x*(-a^2)^(1/
2))*(-a^2)^(1/2)) + (1 - a^2*x^2)^(1/2)/(a^8*c^3) + (x*(1 - a^2*x^2)^(1/2))
/(2*a^7*c^3) - (7*asinh(x*(-a^2)^(1/2)))/(2*a^7*c^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^7}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^8}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**7/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**7/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**8/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.908 $\int \frac{e^{\tanh^{-1}(ax)}x^6}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=133

$$-\frac{\sin^{-1}(ax)}{a^7c^3}+\frac{x^5(ax+1)}{5a^2c^3(1-a^2x^2)^{5/2}}+\frac{16\sqrt{1-a^2x^2}}{5a^7c^3}+\frac{x(8ax+5)}{5a^6c^3\sqrt{1-a^2x^2}}-\frac{x^3(6ax+5)}{15a^4c^3(1-a^2x^2)^{3/2}}$$

```
[Out] 1/5*x^5*(a*x+1)/a^2/c^3/(-a^2*x^2+1)^(5/2)-1/15*x^3*(6*a*x+5)/a^4/c^3/(-a^2
*x^2+1)^(3/2)-arcsin(a*x)/a^7/c^3+1/5*x*(8*a*x+5)/a^6/c^3/(-a^2*x^2+1)^(1/2
)+16/5*(-a^2*x^2+1)^(1/2)/a^7/c^3
```

Rubi [A] time = 0.15, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 641, 216}

$$\frac{x^5(ax+1)}{5a^2c^3(1-a^2x^2)^{5/2}}-\frac{x^3(6ax+5)}{15a^4c^3(1-a^2x^2)^{3/2}}+\frac{x(8ax+5)}{5a^6c^3\sqrt{1-a^2x^2}}+\frac{16\sqrt{1-a^2x^2}}{5a^7c^3}-\frac{\sin^{-1}(ax)}{a^7c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^5*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (x^3*(5 + 6*a*x))/(15*a^4
*c^3*(1 - a^2*x^2)^(3/2)) + (x*(5 + 8*a*x))/(5*a^6*c^3*Sqrt[1 - a^2*x^2]) +
 (16*Sqrt[1 - a^2*x^2])/(5*a^7*c^3) - ArcSin[a*x]/(a^7*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
```

```
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^6(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{x^5(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{x^4(5+6ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{x^5(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^3(5+6ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{x^2(15+24ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3}$$

$$= \frac{x^5(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^3(5+6ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x(5+8ax)}{5a^6c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{15+48ax}{\sqrt{1-a^2x^2}}\,dx}{15a^6c^3}$$

$$= \frac{x^5(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^3(5+6ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x(5+8ax)}{5a^6c^3\sqrt{1-a^2x^2}} + \frac{16\sqrt{1-a^2x^2}}{5a^7c^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,d}{a^6c^3}$$

$$= \frac{x^5(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^3(5+6ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x(5+8ax)}{5a^6c^3\sqrt{1-a^2x^2}} + \frac{16\sqrt{1-a^2x^2}}{5a^7c^3} - \frac{\sin^{-1}(ax)}{a^7c^3}$$

Mathematica [A] time = 0.07, size = 108, normalized size = 0.81

$$\frac{-15a^5x^5+38a^4x^4+52a^3x^3-87a^2x^2-15(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-33ax+48}{15a^7c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (48 - 33*a*x - 87*a^2*x^2 + 52*a^3*x^3 + 38*a^4*x^4 - 15*a^5*x^5 - 15*(-1 +
 a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(15*a^7*c^3*(-1 + a*x)^2*(
1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 213, normalized size = 1.60

$$\frac{48\,a^5x^5 - 48\,a^4x^4 - 96\,a^3x^3 + 96\,a^2x^2 + 48\,ax + 30\left(a^5x^5 - a^4x^4 - 2\,a^3x^3 + 2\,a^2x^2 + ax - 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right.}{15\left(a^{12}c^3x^5 - a^{11}c^3x^4 - 2\,a^{10}c^3x^3 + 2\,a^9c^3x^2 + a^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/15*(48*a^5*x^5 - 48*a^4*x^4 - 96*a^3*x^3 + 96*a^2*x^2 + 48*a*x + 30*(a^5*
x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*arctan((sqrt(-a^2*x^2 + 1)
 - 1)/(a*x)) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x
- 48)*sqrt(-a^2*x^2 + 1) - 48)/(a^12*c^3*x^5 - a^11*c^3*x^4 - 2*a^10*c^3*x^
3 + 2*a^9*c^3*x^2 + a^8*c^3*x - a^7*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)x^6}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^6/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

maple [B] time = 0.05, size = 262, normalized size = 1.97

$$\frac{\sqrt{-a^2x^2+1}}{a^7c^3} - \frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^3a^6\sqrt{a^2}} - \frac{23\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{60c^3a^9\left(x-\frac{1}{a}\right)^2} - \frac{493\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{240c^3a^8\left(x-\frac{1}{a}\right)} - \frac{\sqrt{-a^2\left(x\right.}}{20c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^3,x)
```

```
[Out] (-a^2*x^2+1)^(1/2)/a^7/c^3-1/c^3/a^6/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*x^2+1)^(1/2))-23/60/c^3/a^9/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-4
93/240/c^3/a^8/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/20/c^3/a^10/(x-
1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/24/c^3/a^9/(x+1/a)^2*(-a^2*(x+1
/a)^2+2*a*(x+1/a))^(1/2)+25/48/c^3/a^8/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)x^6}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)*x^6/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.92, size = 379, normalized size = 2.85

$$\frac{a^6\sqrt{1-a^2\,x^2}}{30\,\left(a^{15}\,c^3\,x^2-2\,a^{14}\,c^3\,x+a^{13}\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{24\,\left(a^9\,c^3\,x^2+2\,a^8\,c^3\,x+a^7\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{20\,\sqrt{-a^2}\,\left(a^5\,c^3\,\sqrt{-a^2}+3\,a^7\,c^3\,x^2\,\sqrt{-a^2}-a^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a*x + 1))/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^6*(1 - a^2*x^2)^(1/2))/(30*(a^13*c^3 - 2*a^14*c^3*x + a^15*c^3*x^2)) - (
1 - a^2*x^2)^(1/2)/(24*(a^7*c^3 + 2*a^8*c^3*x + a^9*c^3*x^2)) - (1 - a^2*x^
2)^(1/2)/(20*(-a^2)^(1/2)*(a^5*c^3*(-a^2)^(1/2) + 3*a^7*c^3*x^2*(-a^2)^(1/2
) - a^8*c^3*x^3*(-a^2)^(1/2) - 3*a^6*c^3*x*(-a^2)^(1/2))) - (5*(1 - a^2*x^2
)^(1/2))/(12*(a^7*c^3 - 2*a^8*c^3*x + a^9*c^3*x^2)) - (25*(1 - a^2*x^2)^(1/
2))/(48*(a^5*c^3*(-a^2)^(1/2) + a^6*c^3*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (49
3*(1 - a^2*x^2)^(1/2))/(240*(a^5*c^3*(-a^2)^(1/2) - a^6*c^3*x*(-a^2)^(1/2))
*(-a^2)^(1/2)) + (1 - a^2*x^2)^(1/2)/(a^7*c^3) - asinh(x*(-a^2)^(1/2))/(a^6
*c^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^6}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^7}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**6/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**6/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**7/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.909 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=108

$$-\frac{\sin^{-1}(ax)}{a^6c^3}+\frac{x^4(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}+\frac{15ax+8}{15a^6c^3\sqrt{1-a^2x^2}}-\frac{x^2(5ax+4)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] 1/5*x^4*(a*x+1)/a^2/c^3/(-a^2*x^2+1)^(5/2)-1/15*x^2*(5*a*x+4)/a^4/c^3/(-a^2
*x^2+1)^(3/2)-arcsin(a*x)/a^6/c^3+1/15*(15*a*x+8)/a^6/c^3/(-a^2*x^2+1)^(1/2
)
```

Rubi [A] time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 778, 216}

$$\frac{x^4(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}-\frac{x^2(5ax+4)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{15ax+8}{15a^6c^3\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^4*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (x^2*(4 + 5*a*x))/(15*a^4
*c^3*(1 - a^2*x^2)^(3/2)) + (8 + 15*a*x)/(15*a^6*c^3*Sqrt[1 - a^2*x^2]) - A
rcSin[a*x]/(a^6*c^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
```

```
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{x^5(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3} \\
&= \frac{x^4(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{x^3(4+5ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3} \\
&= \frac{x^4(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^2(4+5ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{x(8+15ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3} \\
&= \frac{x^4(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^2(4+5ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{8+15ax}{15a^6c^3\sqrt{1-a^2x^2}} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^5c^3} \\
&= \frac{x^4(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{x^2(4+5ax)}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{8+15ax}{15a^6c^3\sqrt{1-a^2x^2}} - \frac{\sin^{-1}(ax)}{a^6c^3}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 100, normalized size = 0.93

$$
\frac{23a^4x^4-8a^3x^3-27a^2x^2-15(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)+7ax+8}{15a^6c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8 + 7*a*x - 27*a^2*x^2 - 8*a^3*x^3 + 23*a^4*x^4 - 15*(-1 + a*x)^2*(1 + a*x
)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(15*a^6*c^3*(-1 + a*x)^2*(1 + a*x)*Sqrt[1
- a^2*x^2])
```

fricas [B] time = 0.72, size = 206, normalized size = 1.91

$$\frac{8\,a^5x^5-8\,a^4x^4-16\,a^3x^3+16\,a^2x^2+8\,ax+30\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-(2}{15\left(a^{11}c^3x^5-a^{10}c^3x^4-2\,a^9c^3x^3+2\,a^8c^3x^2+a^7c^3x-a^6c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/15*(8*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 16*a^2*x^2 + 8*a*x + 30*(a^5*x^5
 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) -
1)/(a*x)) - (23*a^4*x^4 - 8*a^3*x^3 - 27*a^2*x^2 + 7*a*x + 8)*sqrt(-a^2*x^2
 + 1) - 8)/(a^11*c^3*x^5 - a^10*c^3*x^4 - 2*a^9*c^3*x^3 + 2*a^8*c^3*x^2 + a
^7*c^3*x - a^6*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 243, normalized size = 2.25

$$-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c^3a^5\sqrt{a^2}}-\frac{3\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{10c^3a^8\left(x-\frac{1}{a}\right)^2}-\frac{91\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{80c^3a^7\left(x-\frac{1}{a}\right)}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right.}}{20c^3a^9\left(x-\frac{1}{a}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/c^3/a^5/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-3/10/c^3/a^
8/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-91/80/c^3/a^7/(x-1/a)*(-a^2*
```

```
(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/20/c^3/a^9/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-
1/a))^(1/2)+1/24/c^3/a^8/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-19/48
/c^3/a^7/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\int \frac{x^6}{\left(a^6c^3x^6 - 3\,a^4c^3x^4 + 3\,a^2c^3x^2 - c^3\right)\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx + \frac{10\,a^2x^2 + 15\left(a^2x^2 - 1\right)^2 - 7}{15\left(-a^2x^2 + 1\right)^{\frac{5}{2}}a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -a*integrate(x^6/((a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(
a*x + 1)*sqrt(-a*x + 1)), x) + 1/15*(10*a^2*x^2 + 15*(a^2*x^2 - 1)^2 - 7)/(
(-a^2*x^2 + 1)^(5/2)*a^6*c^3)
```

mupad [B] time = 0.93, size = 312, normalized size = 2.89

$$\frac{\sqrt{1-a^2\,x^2}}{24\,\left(a^8\,c^3\,x^2+2\,a^7\,c^3\,x+a^6\,c^3\right)}-\frac{3\,\sqrt{1-a^2\,x^2}}{10\,\left(a^8\,c^3\,x^2-2\,a^7\,c^3\,x+a^6\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{20\,\sqrt{-a^2}\,\left(a^4\,c^3\,\sqrt{-a^2}+3\,a^6\,c^3\,x^2\,\sqrt{-a^2}-a^7\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a*x + 1))/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(24*(a^6*c^3 + 2*a^7*c^3*x + a^8*c^3*x^2)) - (3*(1 - a^
2*x^2)^(1/2))/(10*(a^6*c^3 - 2*a^7*c^3*x + a^8*c^3*x^2)) - (1 - a^2*x^2)^(1
/2)/(20*(-a^2)^(1/2)*(a^4*c^3*(-a^2)^(1/2) + 3*a^6*c^3*x^2*(-a^2)^(1/2) - a
^7*c^3*x^3*(-a^2)^(1/2) - 3*a^5*c^3*x*(-a^2)^(1/2))) + (19*(1 - a^2*x^2)^(1
/2))/(48*(a^4*c^3*(-a^2)^(1/2) + a^5*c^3*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (9
1*(1 - a^2*x^2)^(1/2))/(80*(a^4*c^3*(-a^2)^(1/2) - a^5*c^3*x*(-a^2)^(1/2))*
(-a^2)^(1/2)) - asinh(x*(-a^2)^(1/2))/(a^5*c^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^5}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx + \int \frac{ax^6}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**5/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**6/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.910 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=81

$$\frac{x^4(ax+1)}{5ac^3(1-a^2x^2)^{5/2}}+\frac{4}{5a^5c^3\sqrt{1-a^2x^2}}-\frac{4}{15a^5c^3(1-a^2x^2)^{3/2}}$$

```
[Out] 1/5*x^4*(a*x+1)/a/c^3/(-a^2*x^2+1)^(5/2)-4/15/a^5/c^3/(-a^2*x^2+1)^(3/2)+4/
5/a^5/c^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 805, 266, 43}

$$\frac{x^4(ax+1)}{5ac^3(1-a^2x^2)^{5/2}}+\frac{4}{5a^5c^3\sqrt{1-a^2x^2}}-\frac{4}{15a^5c^3(1-a^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^4*(1 + a*x))/(5*a*c^3*(1 - a^2*x^2)^(5/2)) - 4/(15*a^5*c^3*(1 - a^2*x^2)
^(3/2)) + 4/(5*a^5*c^3*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{x^4(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}\\
&= \frac{x^4(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{4\int \frac{x^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5ac^3}\\
&= \frac{x^4(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{2\,\mathrm{Subst}\left(\int \frac{x}{\left(1-a^2x\right)^{5/2}}\,dx,x,x^2\right)}{5ac^3}\\
&= \frac{x^4(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{2\,\mathrm{Subst}\left(\int \left(\frac{1}{a^2\left(1-a^2x\right)^{5/2}}-\frac{1}{a^2\left(1-a^2x\right)^{3/2}}\right)dx,x,x^2\right)}{5ac^3}\\
&= \frac{x^4(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}}-\frac{4}{15a^5c^3\left(1-a^2x^2\right)^{3/2}}+\frac{4}{5a^5c^3\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 68, normalized size = 0.84

$$\frac{3a^4x^4+12a^3x^3-12a^2x^2-8ax+8}{15a^5c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8 - 8*a*x - 12*a^2*x^2 + 12*a^3*x^3 + 3*a^4*x^4)/(15*a^5*c^3*(-1 + a*x)^2*
(1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [B] time = 0.75, size = 146, normalized size = 1.80

$$\frac{8 a^5 x^5 - 8 a^4 x^4 - 16 a^3 x^3 + 16 a^2 x^2 + 8 ax - \left(3 a^4 x^4 + 12 a^3 x^3 - 12 a^2 x^2 - 8 ax + 8\right)\sqrt{-a^2 x^2 + 1} - 8}{15 \left(a^{10} c^3 x^5 - a^9 c^3 x^4 - 2 a^8 c^3 x^3 + 2 a^7 c^3 x^2 + a^6 c^3 x - a^5 c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/15*(8*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 16*a^2*x^2 + 8*a*x - (3*a^4*x^4
+ 12*a^3*x^3 - 12*a^2*x^2 - 8*a*x + 8)*sqrt(-a^2*x^2 + 1) - 8)/(a^10*c^3*x^
5 - a^9*c^3*x^4 - 2*a^8*c^3*x^3 + 2*a^7*c^3*x^2 + a^6*c^3*x - a^5*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)x^4}{\left(a^2 c x^2 - c\right)^3 \sqrt{-a^2 x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^4/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 58, normalized size = 0.72

$$-\frac{3x^4 a^4 + 12x^3 a^3 - 12a^2 x^2 - 8ax + 8}{15\,(ax-1)\, c^3 \left(-a^2 x^2 + 1\right)^{\frac{3}{2}} a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15*(3*a^4*x^4+12*a^3*x^3-12*a^2*x^2-8*a*x+8)/(a*x-1)/c^3/(-a^2*x^2+1)^(3
/2)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)x^4}{\left(a^2 c x^2 - c\right)^3 \sqrt{-a^2 x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)*x^4/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.92, size = 335, normalized size = 4.14

$$\frac{a^4\sqrt{1-a^2x^2}}{30\left(a^{11}c^3x^2-2a^{10}c^3x+a^9c^3\right)}-\frac{\sqrt{1-a^2x^2}}{24\left(a^7c^3x^2+2a^6c^3x+a^5c^3\right)}-\frac{\sqrt{1-a^2x^2}}{20\sqrt{-a^2}\left(a^3c^3\sqrt{-a^2}+3a^5c^3x^2\sqrt{-a^2}-a^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^4*(1 - a^2*x^2)^(1/2))/(30*(a^9*c^3 - 2*a^10*c^3*x + a^11*c^3*x^2)) - (1
 - a^2*x^2)^(1/2)/(24*(a^5*c^3 + 2*a^6*c^3*x + a^7*c^3*x^2)) - (1 - a^2*x^2
)^(1/2)/(20*(-a^2)^(1/2)*(a^3*c^3*(-a^2)^(1/2) + 3*a^5*c^3*x^2*(-a^2)^(1/2)
 - a^6*c^3*x^3*(-a^2)^(1/2) - 3*a^4*c^3*x*(-a^2)^(1/2))) - (1 - a^2*x^2)^(1
/2)/(4*(a^5*c^3 - 2*a^6*c^3*x + a^7*c^3*x^2)) - (13*(1 - a^2*x^2)^(1/2))/(4
8*(a^3*c^3*(-a^2)^(1/2) + a^4*c^3*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (113*(1 -
 a^2*x^2)^(1/2))/(240*(a^3*c^3*(-a^2)^(1/2) - a^4*c^3*x*(-a^2)^(1/2))*(-a^2
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^5}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**4/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**5/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.911 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=74

$$\frac{ax^5}{5c^3\left(1-a^2x^2\right)^{5/2}}-\frac{1}{3a^4c^3\left(1-a^2x^2\right)^{3/2}}+\frac{1}{5a^4c^3\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] 1/5/a^4/c^3/(-a^2*x^2+1)^(5/2)+1/5*a*x^5/c^3/(-a^2*x^2+1)^(5/2)-1/3/a^4/c^3
/(-a^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 819, 778, 191}

$$\frac{x^2(ax+1)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}+\frac{x}{5a^3c^3\sqrt{1-a^2x^2}}-\frac{3ax+2}{15a^4c^3\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^2*(1 + a*x))/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - (2 + 3*a*x)/(15*a^4*c^3*(
1 - a^2*x^2)^(3/2)) + x/(5*a^3*c^3*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
```

```
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{x^3(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}\\
&= \frac{x^2(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{x(2+3ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}\\
&= \frac{x^2(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{2+3ax}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{5a^3c^3}\\
&= \frac{x^2(1+ax)}{5a^2c^3\left(1-a^2x^2\right)^{5/2}} - \frac{2+3ax}{15a^4c^3\left(1-a^2x^2\right)^{3/2}} + \frac{x}{5a^3c^3\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.92

$$\frac{3a^4x^4-3a^3x^3+3a^2x^2+2ax-2}{15a^4c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-2 + 2*a*x + 3*a^2*x^2 - 3*a^3*x^3 + 3*a^4*x^4)/(15*a^4*c^3*(-1 + a*x)^2*(
1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [B]   time = 0.55, size = 145, normalized size = 1.96

$$-\frac{2\,a^5x^5-2\,a^4x^4-4\,a^3x^3+4\,a^2x^2+2\,ax+\left(3\,a^4x^4-3\,a^3x^3+3\,a^2x^2+2\,ax-2\right)\sqrt{-a^2x^2+1}-2}{15\left(a^9c^3x^5-a^8c^3x^4-2\,a^7c^3x^3+2\,a^6c^3x^2+a^5c^3x-a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] -1/15*(2*a^5*x^5 - 2*a^4*x^4 - 4*a^3*x^3 + 4*a^2*x^2 + 2*a*x + (3*a^4*x^4 -
 3*a^3*x^3 + 3*a^2*x^2 + 2*a*x - 2)*sqrt(-a^2*x^2 + 1) - 2)/(a^9*c^3*x^5 -
a^8*c^3*x^4 - 2*a^7*c^3*x^3 + 2*a^6*c^3*x^2 + a^5*c^3*x - a^4*c^3)
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A]   time = 0.03, size = 58, normalized size = 0.78

$$-\frac{3x^4a^4-3x^3a^3+3a^2x^2+2ax-2}{15\left(ax-1\right)c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15*(3*a^4*x^4-3*a^3*x^3+3*a^2*x^2+2*a*x-2)/(a*x-1)/c^3/(-a^2*x^2+1)^(3/2
)/a^4
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-a\int\frac{x^4}{\left(a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3\right)\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx+\frac{5\,a^2x^2-2}{15\left(-a^2x^2+1\right)^{\frac{5}{2}}a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -a*integrate(x^4/((a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(
a*x + 1)*sqrt(-a*x + 1)), x) + 1/15*(5*a^2*x^2 - 2)/((-a^2*x^2 + 1)^(5/2)*a
^4*c^3)
```

mupad [B] time = 0.08, size = 287, normalized size = 3.88

$$\frac{\sqrt{1-a^2\,x^2}}{24\,\left(a^6\,c^3\,x^2+2\,a^5\,c^3\,x+a^4\,c^3\right)}-\frac{2\,\sqrt{1-a^2\,x^2}}{15\,\left(a^6\,c^3\,x^2-2\,a^5\,c^3\,x+a^4\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{20\,\sqrt{-a^2}\,\left(a^2\,c^3\,\sqrt{-a^2}+3\,a^4\,c^3\,x^2\,\sqrt{-a^2}-a^5\,c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(24*(a^4*c^3 + 2*a^5*c^3*x + a^6*c^3*x^2)) - (2*(1 - a^
2*x^2)^(1/2))/(15*(a^4*c^3 - 2*a^5*c^3*x + a^6*c^3*x^2)) - (1 - a^2*x^2)^(1
/2)/(20*(-a^2)^(1/2)*(a^2*c^3*(-a^2)^(1/2) + 3*a^4*c^3*x^2*(-a^2)^(1/2) - a
^5*c^3*x^3*(-a^2)^(1/2) - 3*a^3*c^3*x*(-a^2)^(1/2))) + (7*(1 - a^2*x^2)^(1/
2))/(48*(a^2*c^3*(-a^2)^(1/2) + a^3*c^3*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (13
*(1 - a^2*x^2)^(1/2))/(240*(a^2*c^3*(-a^2)^(1/2) - a^3*c^3*x*(-a^2)^(1/2))*
(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^3}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^4}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**3/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**4/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.912 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=88

$$\frac{x^2(ax+1)}{5ac^3(1-a^2x^2)^{5/2}}-\frac{2x}{15a^2c^3\sqrt{1-a^2x^2}}-\frac{2(1-ax)}{15a^3c^3(1-a^2x^2)^{3/2}}$$

```
[Out] 1/5*x^2*(a*x+1)/a/c^3/(-a^2*x^2+1)^(5/2)-2/15*(-a*x+1)/a^3/c^3/(-a^2*x^2+1)
^(3/2)-2/15*x/a^2/c^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 796, 778, 191}

$$\frac{x^2(ax+1)}{5ac^3(1-a^2x^2)^{5/2}}-\frac{2x}{15a^2c^3\sqrt{1-a^2x^2}}-\frac{2(1-ax)}{15a^3c^3(1-a^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^2*(1 + a*x))/(5*a*c^3*(1 - a^2*x^2)^(5/2)) - (2*(1 - a*x))/(15*a^3*c^3*(
1 - a^2*x^2)^(3/2)) - (2*x)/(15*a^2*c^3*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 796

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(x^2*(a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^2(1+ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{x^2(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{\int \frac{x\left(2a-2a^2x\right)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{x^2(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{2(1-ax)}{15a^3c^3\left(1-a^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^2c^3}$$

$$= \frac{x^2(1+ax)}{5ac^3\left(1-a^2x^2\right)^{5/2}} - \frac{2(1-ax)}{15a^3c^3\left(1-a^2x^2\right)^{3/2}} - \frac{2x}{15a^2c^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 0.77

$$\frac{-2a^4x^4+2a^3x^3+3a^2x^2+2ax-2}{15a^3c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-2 + 2*a*x + 3*a^2*x^2 + 2*a^3*x^3 - 2*a^4*x^4)/(15*a^3*c^3*(-1 + a*x)^2*(
1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.80, size = 146, normalized size = 1.66

$$-\frac{2\,a^5x^5-2\,a^4x^4-4\,a^3x^3+4\,a^2x^2+2\,ax-\left(2\,a^4x^4-2\,a^3x^3-3\,a^2x^2-2\,ax+2\right)\sqrt{-a^2x^2+1}-2}{15\left(a^8c^3x^5-a^7c^3x^4-2\,a^6c^3x^3+2\,a^5c^3x^2+a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] -1/15*(2*a^5*x^5 - 2*a^4*x^4 - 4*a^3*x^3 + 4*a^2*x^2 + 2*a*x - (2*a^4*x^4 -
 2*a^3*x^3 - 3*a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1) - 2)/(a^8*c^3*x^5 -
a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^5*c^3*x^2 + a^4*c^3*x - a^3*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)x^2}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^2/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 58, normalized size = 0.66

$$\frac{2x^4a^4-2x^3a^3-3a^2x^2-2ax+2}{15\left(ax-1\right)c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/15*(2*a^4*x^4-2*a^3*x^3-3*a^2*x^2-2*a*x+2)/(a*x-1)/c^3/(-a^2*x^2+1)^(3/2)
/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)x^2}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)*x^2/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.91, size = 329, normalized size = 3.74

$$\frac{9\sqrt{1-a^2\,x^2}}{80\,\left(a\,c^3\,\sqrt{-a^2}-a^2\,c^3\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{24\,\left(a^5\,c^3\,x^2+2\,a^4\,c^3\,x+a^3\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{48\,\left(a\,c^3\,\sqrt{-a^2}+a^2\,c^3\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (9*(1 - a^2*x^2)^(1/2))/(80*(a*c^3*(-a^2)^(1/2) - a^2*c^3*x*(-a^2)^(1/2))*(
-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(24*(a^3*c^3 + 2*a^4*c^3*x + a^5*c^3*x^2
)) - (1 - a^2*x^2)^(1/2)/(48*(a*c^3*(-a^2)^(1/2) + a^2*c^3*x*(-a^2)^(1/2))*
(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/(12*(a^3*c^3 - 2*a^4*c^3*x + a^5*c^3*x^
2)) + (a^2*(1 - a^2*x^2)^(1/2))/(30*(a^5*c^3 - 2*a^6*c^3*x + a^7*c^3*x^2))
- (1 - a^2*x^2)^(1/2)/(20*(-a^2)^(1/2)*(a*c^3*(-a^2)^(1/2) + 3*a^3*c^3*x^2*
(-a^2)^(1/2) - a^4*c^3*x^3*(-a^2)^(1/2) - 3*a^2*c^3*x*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^3}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**2/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x**3/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.913 $\int \frac{e^{\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=80

$$-\frac{2x}{15ac^3\sqrt{1-a^2x^2}}-\frac{x}{15ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] 1/5*(a*x+1)/a^2/c^3/(-a^2*x^2+1)^(5/2)-1/15*x/a/c^3/(-a^2*x^2+1)^(3/2)-2/15
*x/a/c^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6148, 778, 192, 191}

$$-\frac{2x}{15ac^3\sqrt{1-a^2x^2}}-\frac{x}{15ac^3\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5a^2c^3\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1 + a*x)/(5*a^2*c^3*(1 - a^2*x^2)^(5/2)) - x/(15*a*c^3*(1 - a^2*x^2)^(3/2)
) - (2*x)/(15*a*c^3*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{x(1+ax)}{(1-a^2x^2)^{7/2}}\,dx}{c^3}$$

$$= \frac{1+ax}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{\int \frac{1}{(1-a^2x^2)^{5/2}}\,dx}{5ac^3}$$

$$= \frac{1+ax}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{x}{15ac^3(1-a^2x^2)^{3/2}} - \frac{2\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{15ac^3}$$

$$= \frac{1+ax}{5a^2c^3(1-a^2x^2)^{5/2}} - \frac{x}{15ac^3(1-a^2x^2)^{3/2}} - \frac{2x}{15ac^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 0.85

$$\frac{-2a^4x^4+2a^3x^3+3a^2x^2-3ax+3}{15a^2c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (3 - 3*a*x + 3*a^2*x^2 + 2*a^3*x^3 - 2*a^4*x^4)/(15*a^2*c^3*(-1 + a*x)^2*(1
 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [B] time = 0.68, size = 145, normalized size = 1.81

$$\frac{3\,a^5x^5-3\,a^4x^4-6\,a^3x^3+6\,a^2x^2+3\,ax+\left(2\,a^4x^4-2\,a^3x^3-3\,a^2x^2+3\,ax-3\right)\sqrt{-a^2x^2+1}-3}{15\left(a^7c^3x^5-a^6c^3x^4-2\,a^5c^3x^3+2\,a^4c^3x^2+a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] 1/15*(3*a^5*x^5 - 3*a^4*x^4 - 6*a^3*x^3 + 6*a^2*x^2 + 3*a*x + (2*a^4*x^4 -
2*a^3*x^3 - 3*a^2*x^2 + 3*a*x - 3)*sqrt(-a^2*x^2 + 1) - 3)/(a^7*c^3*x^5 - a
^6*c^3*x^4 - 2*a^5*c^3*x^3 + 2*a^4*c^3*x^2 + a^3*c^3*x - a^2*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)x}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] integrate(-(a*x + 1)*x/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 58, normalized size = 0.72

$$\frac{2x^4a^4-2x^3a^3-3a^2x^2+3ax-3}{15\,(ax-1)\,c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/15*(2*a^4*x^4-2*a^3*x^3-3*a^2*x^2+3*a*x-3)/(a*x-1)/c^3/(-a^2*x^2+1)^(3/2)
/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\int \frac{x^2}{\left(a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3\right)\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx+\frac{1}{5\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] -a*integrate(x^2/((a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(
a*x + 1)*sqrt(-a*x + 1)), x) + 1/5/((-a^2*x^2 + 1)^(5/2)*a^2*c^3)
```

mupad [B] time = 0.07, size = 271, normalized size = 3.39

$$\frac{\sqrt{1-a^2\,x^2}}{30\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{\sqrt{1-a^2\,x^2}}{24\,\left(a^4\,c^3\,x^2+2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{5\,\sqrt{1-a^2\,x^2}}{48\,\left(c^3\,\sqrt{-a^2}+a\,c^3\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}+\frac{}{240\,\left(c^3\,\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/(30*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (1 - a^2*x
^2)^(1/2)/(24*(a^2*c^3 + 2*a^3*c^3*x + a^4*c^3*x^2)) - (5*(1 - a^2*x^2)^(1/
2))/(48*(c^3*(-a^2)^(1/2) + a*c^3*x*(-a^2)^(1/2))*(-a^2)^(1/2)) + (7*(1 - a
^2*x^2)^(1/2))/(240*(c^3*(-a^2)^(1/2) - a*c^3*x*(-a^2)^(1/2))*(-a^2)^(1/2))
 - (1 - a^2*x^2)^(1/2)/(20*(-a^2)^(1/2)*(c^3*(-a^2)^(1/2) + 3*a^2*c^3*x^2*(
-a^2)^(1/2) - a^3*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{ax^2}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2
+ 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integ
ral(a*x**2/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 +
 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.914 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=74

$$\frac{8x}{15c^3\sqrt{1-a^2x^2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5ac^3\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] 1/5*(a*x+1)/a/c^3/(-a^2*x^2+1)^(5/2)+4/15*x/c^3/(-a^2*x^2+1)^(3/2)+8/15*x/c
^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6138, 639, 192, 191}

$$\frac{8x}{15c^3\sqrt{1-a^2x^2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5ac^3\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1 + a*x)/(5*a*c^3*(1 - a^2*x^2)^(5/2)) + (4*x)/(15*c^3*(1 - a^2*x^2)^(3/2)
) + (8*x)/(15*c^3*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx=\frac{\int \frac{1+ax}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$
$$=\frac{1+ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{4\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^3}$$
$$=\frac{1+ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^3}$$
$$=\frac{1+ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}+\frac{8x}{15c^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.80

$$\frac{8a^4x^4-8a^3x^3-12a^2x^2+12ax+3}{15ac^3(1-ax)^{5/2}(ax+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (3 + 12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4)/(15*a*c^3*(1 - a*x)^(5/2)
*(1 + a*x)^(3/2))
```

fricas [B] time = 0.56, size = 144, normalized size = 1.95

$$\frac{3\,a^5x^5-3\,a^4x^4-6\,a^3x^3+6\,a^2x^2+3\,ax-\left(8\,a^4x^4-8\,a^3x^3-12\,a^2x^2+12\,ax+3\right)\sqrt{-a^2x^2+1}-3}{15\left(a^6c^3x^5-a^5c^3x^4-2\,a^4c^3x^3+2\,a^3c^3x^2+a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/15*(3*a^5*x^5 - 3*a^4*x^4 - 6*a^3*x^3 + 6*a^2*x^2 + 3*a*x - (8*a^4*x^4 -
8*a^3*x^3 - 12*a^2*x^2 + 12*a*x + 3)*sqrt(-a^2*x^2 + 1) - 3)/(a^6*c^3*x^5 -
 a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 + a^2*c^3*x - a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 58, normalized size = 0.78

$$-\frac{8x^4a^4-8x^3a^3-12a^2x^2+12ax+3}{15\,(ax-1)\,c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15*(8*a^4*x^4-8*a^3*x^3-12*a^2*x^2+12*a*x+3)/(a*x-1)/c^3/(-a^2*x^2+1)^(3
/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.90, size = 279, normalized size = 3.77

$$\frac{7\,a\,\sqrt{1-a^2\,x^2}}{60\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{a\,\sqrt{1-a^2\,x^2}}{24\,\left(a^4\,c^3\,x^2+2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{11\,\sqrt{1-a^2\,x^2}}{48\,\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-a^2}+\frac{c^3\,\sqrt{-a^2}}{a}\right)}+\frac{73}{240\,\sqrt{-a^2}\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (7*a*(1 - a^2*x^2)^(1/2))/(60*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) - (a*(
1 - a^2*x^2)^(1/2))/(24*(a^2*c^3 + 2*a^3*c^3*x + a^4*c^3*x^2)) + (11*(1 - a
^2*x^2)^(1/2))/(48*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) + (c^3*(-a^2)^(1/2))/a)
) + (73*(1 - a^2*x^2)^(1/2))/(240*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(
-a^2)^(1/2))/a)) + (1 - a^2*x^2)^(1/2)/(20*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/
2) - (c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)^(
1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(a*x/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**
2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Int
egral(1/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1)
 - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.915 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=101

$$\frac{ax+1}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{8ax+15}{15c^3\sqrt{1-a^2x^2}}+\frac{4ax+5}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

```
[Out] 1/5*(a*x+1)/c^3/(-a^2*x^2+1)^(5/2)+1/15*(4*a*x+5)/c^3/(-a^2*x^2+1)^(3/2)-ar
ctanh((-a^2*x^2+1)^(1/2))/c^3+1/15*(8*a*x+15)/c^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 12, 266, 63, 208}

$$\frac{ax+1}{5c^3\left(1-a^2x^2\right)^{5/2}}+\frac{8ax+15}{15c^3\sqrt{1-a^2x^2}}+\frac{4ax+5}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^3),x]
```

```
[Out] (1 + a*x)/(5*c^3*(1 - a^2*x^2)^(5/2)) + (5 + 4*a*x)/(15*c^3*(1 - a^2*x^2)^(
3/2)) + (15 + 8*a*x)/(15*c^3*Sqrt[1 - a^2*x^2]) - ArcTanh[Sqrt[1 - a^2*x^2]
]/c^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1+ax}{x\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{5a^2+4a^3x}{x\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{15a^4+8a^5x}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3}$$

$$= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{15+8ax}{15c^3\sqrt{1-a^2x^2}} + \frac{\int \frac{15a^6}{x\sqrt{1-a^2x^2}}\,dx}{15a^6c^3}$$

$$= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{15+8ax}{15c^3\sqrt{1-a^2x^2}} + \frac{\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx}{c^3}$$

$$= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{15+8ax}{15c^3\sqrt{1-a^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right)}{2c^3}$$

$$= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{15+8ax}{15c^3\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a^2c^3}$$

$$= \frac{1+ax}{5c^3\left(1-a^2x^2\right)^{5/2}} + \frac{5+4ax}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{15+8ax}{15c^3\sqrt{1-a^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Mathematica [A] time = 0.05, size = 108, normalized size = 1.07

$$\frac{8a^4x^4+7a^3x^3-27a^2x^2-15(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-8ax+23}{15c^3(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^3),x]
```

```
[Out] (23 - 8*a*x - 27*a^2*x^2 + 7*a^3*x^3 + 8*a^4*x^4 - 15*(-1 + a*x)^2*(1 + a*x
)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(15*c^3*(-1 + a*x)^2*(1 + a
*x)*Sqrt[1 - a^2*x^2])
```

fricas [B] time = 0.61, size = 198, normalized size = 1.96

$$\frac{23\,a^5x^5 - 23\,a^4x^4 - 46\,a^3x^3 + 46\,a^2x^2 + 23\,ax + 15\left(a^5x^5 - a^4x^4 - 2\,a^3x^3 + 2\,a^2x^2 + ax - 1\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - (\ldots}{15\left(a^5c^3x^5 - a^4c^3x^4 - 2\,a^3c^3x^3 + 2\,a^2c^3x^2 + ac^3x - c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] 1/15*(23*a^5*x^5 - 23*a^4*x^4 - 46*a^3*x^3 + 46*a^2*x^2 + 23*a*x + 15*(a^5*
x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log((sqrt(-a^2*x^2 + 1) -
1)/x) - (8*a^4*x^4 + 7*a^3*x^3 - 27*a^2*x^2 - 8*a*x + 23)*sqrt(-a^2*x^2 + 1
) - 23)/(a^5*c^3*x^5 - a^4*c^3*x^4 - 2*a^3*c^3*x^3 + 2*a^2*c^3*x^2 + a*c^3*
x - c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x), x)
```

maple [B] time = 0.05, size = 384, normalized size = 3.80

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2} - \frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}}{2a} + \frac{11\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{16a\left(x-\frac{1}{a}\right)} + \frac{\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{5a\left(x-\frac{1}{a}\right)^3} - \frac{2a\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right.\ldots}}{3a\left(x\ldots\right.}\ldots\right.}{\ldots}}{4a^2\ldots}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/c^3*(arctanh(1/(-a^2*x^2+1)^(1/2))-1/2/a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^
2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))+11/16/
a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/4/a^2*(1/5/a/(x-1/a)^3*(-a^2
*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x
-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)))+1/8/a*(-1/3/a
/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-1/3/(x+1/a)*(-a^2*(x+1/a)^2+2
*a*(x+1/a))^(1/2))-5/16/a/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [B] time = 0.09, size = 306, normalized size = 3.03

$$\frac{a^2\sqrt{1-a^2x^2}}{5\left(a^4c^3x^2-2a^3c^3x+a^2c^3\right)}+\frac{a^2\sqrt{1-a^2x^2}}{24\left(a^4c^3x^2+2a^3c^3x+a^2c^3\right)}-\frac{17a\sqrt{1-a^2x^2}}{48\sqrt{-a^2}\left(c^3x\sqrt{-a^2}+\frac{c^3\sqrt{-a^2}}{a}\right)}+\frac{71a}{80\sqrt{-a^2}\left(c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (atan((1 - a^2*x^2)^(1/2)*1i)*1i)/c^3 + (a^2*(1 - a^2*x^2)^(1/2))/(5*(a^2*c
^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (a^2*(1 - a^2*x^2)^(1/2))/(24*(a^2*c^3 +
 2*a^3*c^3*x + a^4*c^3*x^2)) - (17*a*(1 - a^2*x^2)^(1/2))/(48*(-a^2)^(1/2)*
(c^3*x*(-a^2)^(1/2) + (c^3*(-a^2)^(1/2))/a)) + (71*a*(1 - a^2*x^2)^(1/2))/(
80*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a)) + (a*(1 - a^2*
x^2)^(1/2))/(20*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a +
 a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^6x^7\sqrt{-a^2x^2+1}+3a^4x^5\sqrt{-a^2x^2+1}-3a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(a/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2
+ 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integ
ral(1/(-a**6*x**7*sqrt(-a**2*x**2 + 1) + 3*a**4*x**5*sqrt(-a**2*x**2 + 1) -
 3*a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.916 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=135

$$\frac{ax+1}{5c^3x(1-a^2x^2)^{5/2}}-\frac{16\sqrt{1-a^2x^2}}{5c^3x}+\frac{5ax+8}{5c^3x\sqrt{1-a^2x^2}}+\frac{5ax+6}{15c^3x(1-a^2x^2)^{3/2}}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

```
[Out] 1/5*(a*x+1)/c^3/x/(-a^2*x^2+1)^(5/2)+1/15*(5*a*x+6)/c^3/x/(-a^2*x^2+1)^(3/2
)-a*arctanh((-a^2*x^2+1)^(1/2))/c^3+1/5*(5*a*x+8)/c^3/x/(-a^2*x^2+1)^(1/2)-
16/5*(-a^2*x^2+1)^(1/2)/c^3/x
```

Rubi [A] time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 823, 807, 266, 63, 208}

$$\frac{ax+1}{5c^3x(1-a^2x^2)^{5/2}}-\frac{16\sqrt{1-a^2x^2}}{5c^3x}+\frac{5ax+8}{5c^3x\sqrt{1-a^2x^2}}+\frac{5ax+6}{15c^3x(1-a^2x^2)^{3/2}}-\frac{a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^3),x]
```

```
[Out] (1 + a*x)/(5*c^3*x*(1 - a^2*x^2)^(5/2)) + (6 + 5*a*x)/(15*c^3*x*(1 - a^2*x^
2)^(3/2)) + (8 + 5*a*x)/(5*c^3*x*Sqrt[1 - a^2*x^2]) - (16*Sqrt[1 - a^2*x^2]
)/(5*c^3*x) - (a*ArcTanh[Sqrt[1 - a^2*x^2]])/c^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1+ax}{x^2\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{6a^2+5a^3x}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}} + \frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{24a^4+15a^5x}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3}$$

$$= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}} + \frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}} + \frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}} + \frac{\int \frac{48a^6+15a^7x}{x^2\sqrt{1-a^2x^2}}\,dx}{15a^6c^3}$$

$$= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}} + \frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}} + \frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}} - \frac{16\sqrt{1-a^2x^2}}{5c^3x} + \frac{a\int \frac{1}{x\sqrt{1-a^2}}}{c^3}$$

$$= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}} + \frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}} + \frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}} - \frac{16\sqrt{1-a^2x^2}}{5c^3x} + \frac{a\,\mathrm{Subst}\Big(}{}$$

$$= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}} + \frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}} + \frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}} - \frac{16\sqrt{1-a^2x^2}}{5c^3x} - \frac{\mathrm{Subst}\Big(\int}{}$$

$$= \frac{1+ax}{5c^3x\left(1-a^2x^2\right)^{5/2}} + \frac{6+5ax}{15c^3x\left(1-a^2x^2\right)^{3/2}} + \frac{8+5ax}{5c^3x\sqrt{1-a^2x^2}} - \frac{16\sqrt{1-a^2x^2}}{5c^3x} - \frac{a\tanh^{-1}\Big(}{}$$

Mathematica [A] time = 0.06, size = 121, normalized size = 0.90

$$\frac{48a^5x^5-33a^4x^4-87a^3x^3+52a^2x^2-15ax(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+38ax-15}{15c^3x(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^3),x]
```

[Out] (-15 + 38*a*x + 52*a^2*x^2 - 87*a^3*x^3 - 33*a^4*x^4 + 48*a^5*x^5 - 15*a*x*
(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 - a^2*x^2]])/(15*c^
3*x*(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2])

fricas [A] time = 0.65, size = 223, normalized size = 1.65

$$\frac{23\,a^{6}x^{6}-23\,a^{5}x^{5}-46\,a^{4}x^{4}+46\,a^{3}x^{3}+23\,a^{2}x^{2}-23\,ax+15\left(a^{6}x^{6}-a^{5}x^{5}-2\,a^{4}x^{4}+2\,a^{3}x^{3}+a^{2}x^{2}-ax\right)\log\left(\frac{\sqrt{-}\cdots}{}\right.}{15\left(a^{5}c^{3}x^{6}-a^{4}c^{3}x^{5}-2\,a^{3}c^{3}x^{4}+2\,a^{2}c^{3}x^{3}+ac\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")

[Out] 1/15*(23*a^6*x^6 - 23*a^5*x^5 - 46*a^4*x^4 + 46*a^3*x^3 + 23*a^2*x^2 - 23*a
x + 15(a^6*x^6 - a^5*x^5 - 2*a^4*x^4 + 2*a^3*x^3 + a^2*x^2 - a*x)*log((sq
rt(-a^2*x^2 + 1) - 1)/x) - (48*a^5*x^5 - 33*a^4*x^4 - 87*a^3*x^3 + 52*a^2*x
^2 + 38*a*x - 15)*sqrt(-a^2*x^2 + 1))/(a^5*c^3*x^6 - a^4*c^3*x^5 - 2*a^3*c^
3*x^4 + 2*a^2*c^3*x^3 + a*c^3*x^2 - c^3*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ax+1}{\left(a^{2}cx^{2}-c\right)^{3}\sqrt{-a^{2}x^{2}+1}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")

[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x^2), x)

maple [B] time = 0.05, size = 314, normalized size = 2.33

$$-\frac{a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)+\frac{\sqrt{-a^{2}x^{2}+1}}{x}-\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{4a\left(x-\frac{1}{a}\right)^{2}}+\frac{27\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{16\left(x-\frac{1}{a}\right)}+\frac{\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{5a\left(x-\frac{1}{a}\right)^{3}}-\frac{2a\left(\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-\cdots}}{3a\left(x-\frac{1}{a}\cdots\right.}\right.}{\cdots}}{4a}\cdots}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^3,x)

```
[Out] -1/c^3*(a*arctanh(1/(-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/x-1/4/a/(x-1/a)^
2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+27/16/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1
/a))^(1/2)+1/4/a*(1/5/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-2/5*a*
(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-1/
a)^2-2*a*(x-1/a))^(1/2)))+1/24/a/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/
2)+23/48/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [A] time = 0.36, size = 178, normalized size = 1.32

$$-\frac{\frac{15\,a^2\log\left(\sqrt{-a^2x^2+1}+1\right)}{c^3}-\frac{15\,a^2\log\left(\sqrt{-a^2x^2+1}-1\right)}{c^3}-\frac{2\left(15\left(a^2x^2-1\right)^2a^2-5\left(a^2x^2-1\right)a^2+3\,a^2\right)}{\left(-a^2x^2+1\right)^{\frac{5}{2}}c^3}}{30\,a}+\frac{16\,a^6x^6-40\,a^4x^4+30\,a^2x^2}{5\left(a^4c^3x^5-2\,a^2c^3x^3+c^3x\right)\sqrt{ax+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -1/30*(15*a^2*log(sqrt(-a^2*x^2 + 1) + 1)/c^3 - 15*a^2*log(sqrt(-a^2*x^2 +
1) - 1)/c^3 - 2*(15*(a^2*x^2 - 1)^2*a^2 - 5*(a^2*x^2 - 1)*a^2 + 3*a^2)/((-a
^2*x^2 + 1)^(5/2)*c^3))/a + 1/5*(16*a^6*x^6 - 40*a^4*x^4 + 30*a^2*x^2 - 5)/
((a^4*c^3*x^5 - 2*a^2*c^3*x^3 + c^3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1))
```

mupad [B] time = 0.09, size = 335, normalized size = 2.48

$$\frac{17\,a^3\,\sqrt{1-a^2\,x^2}}{60\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{a^3\,\sqrt{1-a^2\,x^2}}{24\,\left(a^4\,c^3\,x^2+2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{c^3\,x}+\frac{23\,a^2\,\sqrt{1-a^2\,x^2}}{48\,\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-a^2}+\frac{c^3\,\sqrt{-a^2}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (17*a^3*(1 - a^2*x^2)^(1/2))/(60*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) - (
a^3*(1 - a^2*x^2)^(1/2))/(24*(a^2*c^3 + 2*a^3*c^3*x + a^4*c^3*x^2)) - (1 -
a^2*x^2)^(1/2)/(c^3*x) + (a*atan((1 - a^2*x^2)^(1/2)*1i)*1i)/c^3 + (23*a^2*
(1 - a^2*x^2)^(1/2))/(48*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) + (c^3*(-a^2)^(1/
2))/a)) + (413*a^2*(1 - a^2*x^2)^(1/2))/(240*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/
2) - (c^3*(-a^2)^(1/2))/a)) + (a^2*(1 - a^2*x^2)^(1/2))/(20*(-a^2)^(1/2)*(3
*c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a
*c^3*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{-a^6x^7\sqrt{-a^2x^2+1}+3a^4x^5\sqrt{-a^2x^2+1}-3a^2x^3\sqrt{-a^2x^2+1}+x\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^6x^8\sqrt{-a^2x^2+1}+3a^4x^6\sqrt{-a^2x^2+1}-3a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(a/(-a**6*x**7*sqrt(-a**2*x**2 + 1) + 3*a**4*x**5*sqrt(-a**2*x**2
+ 1) - 3*a**2*x**3*sqrt(-a**2*x**2 + 1) + x*sqrt(-a**2*x**2 + 1)), x) + Int
egral(1/(-a**6*x**8*sqrt(-a**2*x**2 + 1) + 3*a**4*x**6*sqrt(-a**2*x**2 + 1)
 - 3*a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.917 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^3} dx$

Optimal. Leaf size=164

$$-\frac{16a\sqrt{1-a^2x^2}}{5c^3x}-\frac{7\sqrt{1-a^2x^2}}{2c^3x^2}+\frac{24ax+35}{15c^3x^2\sqrt{1-a^2x^2}}+\frac{6ax+7}{15c^3x^2\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5c^3x^2\left(1-a^2x^2\right)^{5/2}}-\frac{7a^2\tanh^{-1}\left(\sqrt{1-\right.}}{2c^3}$$

```
[Out] 1/5*(a*x+1)/c^3/x^2/(-a^2*x^2+1)^(5/2)+1/15*(6*a*x+7)/c^3/x^2/(-a^2*x^2+1)^
(3/2)-7/2*a^2*arctanh((-a^2*x^2+1)^(1/2))/c^3+1/15*(24*a*x+35)/c^3/x^2/(-a^
2*x^2+1)^(1/2)-7/2*(-a^2*x^2+1)^(1/2)/c^3/x^2-16/5*a*(-a^2*x^2+1)^(1/2)/c^3
/x
```

Rubi [A] time = 0.19, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 823, 835, 807, 266, 63, 208}

$$-\frac{16a\sqrt{1-a^2x^2}}{5c^3x}-\frac{7\sqrt{1-a^2x^2}}{2c^3x^2}+\frac{24ax+35}{15c^3x^2\sqrt{1-a^2x^2}}+\frac{6ax+7}{15c^3x^2\left(1-a^2x^2\right)^{3/2}}+\frac{ax+1}{5c^3x^2\left(1-a^2x^2\right)^{5/2}}-\frac{7a^2\tanh^{-1}\left(\sqrt{1-\right.}}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^3),x]
```

```
[Out] (1 + a*x)/(5*c^3*x^2*(1 - a^2*x^2)^(5/2)) + (7 + 6*a*x)/(15*c^3*x^2*(1 - a^
2*x^2)^(3/2)) + (35 + 24*a*x)/(15*c^3*x^2*Sqrt[1 - a^2*x^2]) - (7*Sqrt[1 -
a^2*x^2])/(2*c^3*x^2) - (16*a*Sqrt[1 - a^2*x^2])/(5*c^3*x) - (7*a^2*ArcTanh
[Sqrt[1 - a^2*x^2]])/(2*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1+ax}{x^3\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{\int \frac{7a^2+6a^3x}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{5a^2c^3}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{\int \frac{35a^4+24a^5x}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{15a^4c^3}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} + \frac{\int \frac{105a^6+48a^7x}{x^3\sqrt{1-a^2x^2}}\,dx}{15a^6c^3}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{\int \frac{-96a^7}{x^2\sqrt{}}}{30}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{16a\sqrt{1}}{5c}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{16a\sqrt{1}}{5c}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{16a\sqrt{1}}{5c}$$

$$= \frac{1+ax}{5c^3x^2\left(1-a^2x^2\right)^{5/2}} + \frac{7+6ax}{15c^3x^2\left(1-a^2x^2\right)^{3/2}} + \frac{35+24ax}{15c^3x^2\sqrt{1-a^2x^2}} - \frac{7\sqrt{1-a^2x^2}}{2c^3x^2} - \frac{16a\sqrt{1}}{5c}$$

Mathematica [A] time = 0.06, size = 133, normalized size = 0.81

$$\frac{96a^6x^6+9a^5x^5-249a^4x^4+4a^3x^3+176a^2x^2-105a^2x^2(ax-1)^2(ax+1)\sqrt{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-15ax}{30c^3x^2(ax-1)^2(ax+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-15 - 15*a*x + 176*a^2*x^2 + 4*a^3*x^3 - 249*a^4*x^4 + 9*a^5*x^5 + 96*a^6*
x^6 - 105*a^2*x^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2]*ArcTanh[Sqrt[1 -
 a^2*x^2]])/(30*c^3*x^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.75, size = 241, normalized size = 1.47

$$\frac{116\,a^7x^7-116\,a^6x^6-232\,a^5x^5+232\,a^4x^4+116\,a^3x^3-116\,a^2x^2+105\left(a^7x^7-a^6x^6-2\,a^5x^5+2\,a^4x^4+a^3x^3-a^2\right.}{30\left(a^5c^3x^7-a^4c^3x^6-2\,a^3c^3x^5+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] 1/30*(116*a^7*x^7 - 116*a^6*x^6 - 232*a^5*x^5 + 232*a^4*x^4 + 116*a^3*x^3 -
 116*a^2*x^2 + 105*(a^7*x^7 - a^6*x^6 - 2*a^5*x^5 + 2*a^4*x^4 + a^3*x^3 - a
^2*x^2)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (96*a^6*x^6 + 9*a^5*x^5 - 249*a^4
*x^4 + 4*a^3*x^3 + 176*a^2*x^2 - 15*a*x - 15)*sqrt(-a^2*x^2 + 1))/(a^5*c^3*
x^7 - a^4*c^3*x^6 - 2*a^3*c^3*x^5 + 2*a^2*c^3*x^4 + a*c^3*x^3 - c^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [B] time = 0.06, size = 326, normalized size = 1.99

$$-\frac{\frac{7a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}+\frac{a\sqrt{-a^2x^2+1}}{x}-\frac{11a\left(\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3a\left(x-\frac{1}{a}\right)^2}-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{3\left(x-\frac{1}{a}\right)}\right)}{10}+\frac{39a\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{16\left(x-\frac{1}{a}\right)}+\frac{\sqrt{-a^2x^2+1}}{2x^2}+}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/c^3*(7/2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))+a*(-a^2*x^2+1)^(1/2)/x-11/10*
a*(1/3/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/3/(x-1/a)*(-a^2*(x-
1/a)^2-2*a*(x-1/a))^(1/2))+39/16*a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/
2)+1/2*(-a^2*x^2+1)^(1/2)/x^2+1/20/a/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))
^(1/2)+1/8*a*(-1/3/a/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)-1/3/(x+1/
a)*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))-9/16*a/(x+1/a)*(-a^2*(x+1/a)^2+2*a*(
x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [B] time = 0.92, size = 357, normalized size = 2.18

$$\frac{11\,a^4\,\sqrt{1-a^2\,x^2}}{30\,\left(a^4\,c^3\,x^2-2\,a^3\,c^3\,x+a^2\,c^3\right)}+\frac{a^4\,\sqrt{1-a^2\,x^2}}{24\,\left(a^4\,c^3\,x^2+2\,a^3\,c^3\,x+a^2\,c^3\right)}-\frac{\sqrt{1-a^2\,x^2}}{2\,c^3\,x^2}-\frac{a\,\sqrt{1-a^2\,x^2}}{c^3\,x}-\frac{29\,a^3\,\sqrt{1-}}{48\,\sqrt{-a^2}\,\left(c^3\,x\,\sqrt{-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*7i)/(2*c^3) + (11*a^4*(1 - a^2*x^2)^(1/2)
)/(30*(a^2*c^3 - 2*a^3*c^3*x + a^4*c^3*x^2)) + (a^4*(1 - a^2*x^2)^(1/2))/(2
4*(a^2*c^3 + 2*a^3*c^3*x + a^4*c^3*x^2)) - (1 - a^2*x^2)^(1/2)/(2*c^3*x^2)
- (a*(1 - a^2*x^2)^(1/2))/(c^3*x) - (29*a^3*(1 - a^2*x^2)^(1/2))/(48*(-a^2)
^(1/2)*(c^3*x*(-a^2)^(1/2) + (c^3*(-a^2)^(1/2))/a)) + (673*a^3*(1 - a^2*x^2
)^(1/2))/(240*(-a^2)^(1/2)*(c^3*x*(-a^2)^(1/2) - (c^3*(-a^2)^(1/2))/a)) + (
a^3*(1 - a^2*x^2)^(1/2))/(20*(-a^2)^(1/2)*(3*c^3*x*(-a^2)^(1/2) - (c^3*(-a^
2)^(1/2))/a + a^2*c^3*x^3*(-a^2)^(1/2) - 3*a*c^3*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{-a^6x^8\sqrt{-a^2x^2+1}+3a^4x^6\sqrt{-a^2x^2+1}-3a^2x^4\sqrt{-a^2x^2+1}+x^2\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{-a^6x^9\sqrt{-a^2x^2+1}+3a^4x^7\sqrt{-a^2x^2+1}-3a^2x^5\sqrt{-a^2x^2+1}+x^3\sqrt{-a^2}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(a/(-a**6*x**8*sqrt(-a**2*x**2 + 1) + 3*a**4*x**6*sqrt(-a**2*x**2
+ 1) - 3*a**2*x**4*sqrt(-a**2*x**2 + 1) + x**2*sqrt(-a**2*x**2 + 1)), x) +
Integral(1/(-a**6*x**9*sqrt(-a**2*x**2 + 1) + 3*a**4*x**7*sqrt(-a**2*x**2 +
 1) - 3*a**2*x**5*sqrt(-a**2*x**2 + 1) + x**3*sqrt(-a**2*x**2 + 1)), x))/c*
*3
```

3.918 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=96

$$\frac{16x}{35c^4\sqrt{1-a^2x^2}}+\frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}}+\frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{ax+1}{7ac^4\left(1-a^2x^2\right)^{7/2}}$$

```
[Out] 1/7*(a*x+1)/a/c^4/(-a^2*x^2+1)^(7/2)+6/35*x/c^4/(-a^2*x^2+1)^(5/2)+8/35*x/c
^4/(-a^2*x^2+1)^(3/2)+16/35*x/c^4/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6138, 639, 192, 191}

$$\frac{16x}{35c^4\sqrt{1-a^2x^2}}+\frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}}+\frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}}+\frac{ax+1}{7ac^4\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (1 + a*x)/(7*a*c^4*(1 - a^2*x^2)^(7/2)) + (6*x)/(35*c^4*(1 - a^2*x^2)^(5/2)
) + (8*x)/(35*c^4*(1 - a^2*x^2)^(3/2)) + (16*x)/(35*c^4*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{1+ax}{\left(1-a^2x^2\right)^{9/2}}\,dx}{c^4}$$

$$= \frac{1+ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6\int \frac{1}{\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^4}$$

$$= \frac{1+ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{24\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^4}$$

$$= \frac{1+ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}} + \frac{16\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{35c^4}$$

$$= \frac{1+ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}} + \frac{16x}{35c^4\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.78

$$\frac{-16a^6x^6+16a^5x^5+40a^4x^4-40a^3x^3-30a^2x^2+30ax+5}{35ac^4(1-ax)^{7/2}(ax+1)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (5 + 30*a*x - 30*a^2*x^2 - 40*a^3*x^3 + 40*a^4*x^4 + 16*a^5*x^5 - 16*a^6*x^
6)/(35*a*c^4*(1 - a*x)^(7/2)*(1 + a*x)^(5/2))
```

fricas [B] time = 0.74, size = 198, normalized size = 2.06

$$\frac{5\,a^7x^7-5\,a^6x^6-15\,a^5x^5+15\,a^4x^4+15\,a^3x^3-15\,a^2x^2-5\,ax-\left(16\,a^6x^6-16\,a^5x^5-40\,a^4x^4+40\,a^3x^3+30\,a^2x^2\right.}{35\left(a^8c^4x^7-a^7c^4x^6-3\,a^6c^4x^5+3\,a^5c^4x^4+3\,a^4c^4x^3-3\,a^3c^4x^2-a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas"
)
```

```
[Out] 1/35*(5*a^7*x^7 - 5*a^6*x^6 - 15*a^5*x^5 + 15*a^4*x^4 + 15*a^3*x^3 - 15*a^2
*x^2 - 5*a*x - (16*a^6*x^6 - 16*a^5*x^5 - 40*a^4*x^4 + 40*a^3*x^3 + 30*a^2*
x^2 - 30*a*x - 5)*sqrt(-a^2*x^2 + 1) + 5)/(a^8*c^4*x^7 - a^7*c^4*x^6 - 3*a^
6*c^4*x^5 + 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 - 3*a^3*c^4*x^2 - a^2*c^4*x + a*c
^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^4\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^4*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 74, normalized size = 0.77

$$\frac{16x^6a^6-16x^5a^5-40x^4a^4+40x^3a^3+30a^2x^2-30ax-5}{35\left(ax-1\right)c^4\left(-a^2x^2+1\right)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/35*(16*a^6*x^6-16*a^5*x^5-40*a^4*x^4+40*a^3*x^3+30*a^2*x^2-30*a*x-5)/(a*x
-1)/c^4/(-a^2*x^2+1)^(5/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(a^2cx^2-c\right)^4\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)/((a^2*c*x^2 - c)^4*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.93, size = 477, normalized size = 4.97

$$\frac{7\,a\,\sqrt{1-a^2\,x^2}}{80\,\left(a^4\,c^4\,x^2-2\,a^3\,c^4\,x+a^2\,c^4\right)}-\frac{a\,\sqrt{1-a^2\,x^2}}{20\,\left(a^4\,c^4\,x^2+2\,a^3\,c^4\,x+a^2\,c^4\right)}+\frac{a^3\,\sqrt{1-a^2\,x^2}}{140\,\left(a^6\,c^4\,x^2-2\,a^5\,c^4\,x+a^4\,c^4\right)}+\frac{}{56\,\left(a^6\,c^4\,x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^4*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (7*a*(1 - a^2*x^2)^(1/2))/(80*(a^2*c^4 - 2*a^3*c^4*x + a^4*c^4*x^2)) - (a*(
1 - a^2*x^2)^(1/2))/(20*(a^2*c^4 + 2*a^3*c^4*x + a^4*c^4*x^2)) + (a^3*(1 -
a^2*x^2)^(1/2))/(140*(a^4*c^4 - 2*a^5*c^4*x + a^6*c^4*x^2)) + (a*(1 - a^2*x
^2)^(1/2))/(56*(a^2*c^4 - 4*a^3*c^4*x + 6*a^4*c^4*x^2 - 4*a^5*c^4*x^3 + a^6
*c^4*x^4)) + (33*(1 - a^2*x^2)^(1/2))/(160*(-a^2)^(1/2)*(c^4*x*(-a^2)^(1/2)
 + (c^4*(-a^2)^(1/2))/a)) + (281*(1 - a^2*x^2)^(1/2))/(1120*(-a^2)^(1/2)*(c
^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a)) + (1 - a^2*x^2)^(1/2)/(80*(-a^2)
^(1/2)*(3*c^4*x*(-a^2)^(1/2) + (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1
/2) + 3*a*c^4*x^2*(-a^2)^(1/2))) + (27*(1 - a^2*x^2)^(1/2))/(560*(-a^2)^(1/
2)*(3*c^4*x*(-a^2)^(1/2) - (c^4*(-a^2)^(1/2))/a + a^2*c^4*x^3*(-a^2)^(1/2)
- 3*a*c^4*x^2*(-a^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{a^8x^8\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+1}-4a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{1}{a^8x^8\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+}}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (Integral(a*x/(a**8*x**8*sqrt(-a**2*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x**2
 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 + 1)
 + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**8*x**8*sqrt(-a**2*x**2 + 1) -
 4*a**6*x**6*sqrt(-a**2*x**2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a*
*2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.919 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^5}\,dx$

Optimal. Leaf size=118

$$\frac{128x}{315c^5\sqrt{1-a^2x^2}}+\frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}}+\frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}}+\frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}}+\frac{ax+1}{9ac^5\left(1-a^2x^2\right)^{9/2}}$$

```
[Out] 1/9*(a*x+1)/a/c^5/(-a^2*x^2+1)^(9/2)+8/63*x/c^5/(-a^2*x^2+1)^(7/2)+16/105*x
/c^5/(-a^2*x^2+1)^(5/2)+64/315*x/c^5/(-a^2*x^2+1)^(3/2)+128/315*x/c^5/(-a^2
*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6138, 639, 192, 191}

$$\frac{128x}{315c^5\sqrt{1-a^2x^2}}+\frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}}+\frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}}+\frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}}+\frac{ax+1}{9ac^5\left(1-a^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^5,x]
```

```
[Out] (1 + a*x)/(9*a*c^5*(1 - a^2*x^2)^(9/2)) + (8*x)/(63*c^5*(1 - a^2*x^2)^(7/2)
) + (16*x)/(105*c^5*(1 - a^2*x^2)^(5/2)) + (64*x)/(315*c^5*(1 - a^2*x^2)^(3
/2)) + (128*x)/(315*c^5*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
```

```
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^5}\,dx = \frac{\int \frac{1+ax}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^5}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8\int \frac{1}{\left(1-a^2x^2\right)^{9/2}}\,dx}{9c^5}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16\int \frac{1}{\left(1-a^2x^2\right)^{7/2}}\,dx}{21c^5}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx}{105c^5}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{128\int}{}$$

$$= \frac{1+ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{}{315c^5}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 0.77

$$\frac{128a^8x^8-128a^7x^7-448a^6x^6+448a^5x^5+560a^4x^4-560a^3x^3-280a^2x^2+280ax+35}{315ac^5(1-ax)^{9/2}(ax+1)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^5,x]
```

```
[Out] (35 + 280*a*x - 280*a^2*x^2 - 560*a^3*x^3 + 560*a^4*x^4 + 448*a^5*x^5 - 448
*a^6*x^6 - 128*a^7*x^7 + 128*a^8*x^8)/(315*a*c^5*(1 - a*x)^(9/2)*(1 + a*x)^
(7/2))
```

fricas [B] time = 0.79, size = 252, normalized size = 2.14

$$\frac{35\,a^9x^9-35\,a^8x^8-140\,a^7x^7+140\,a^6x^6+210\,a^5x^5-210\,a^4x^4-140\,a^3x^3+140\,a^2x^2+35\,ax-\left(128\,a^8x^8-128\right.}{315\left(a^{10}c^5x^9-a^9c^5x^8-4\,a^8c^5x^7+4\,a^7c^5x^6+6\,a^6c^5x^5-6\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="fricas"
)
```

```
[Out] 1/315*(35*a^9*x^9 - 35*a^8*x^8 - 140*a^7*x^7 + 140*a^6*x^6 + 210*a^5*x^5 -
210*a^4*x^4 - 140*a^3*x^3 + 140*a^2*x^2 + 35*a*x - (128*a^8*x^8 - 128*a^7*x
^7 - 448*a^6*x^6 + 448*a^5*x^5 + 560*a^4*x^4 - 560*a^3*x^3 - 280*a^2*x^2 +
280*a*x + 35)*sqrt(-a^2*x^2 + 1) - 35)/(a^10*c^5*x^9 - a^9*c^5*x^8 - 4*a^8*
c^5*x^7 + 4*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 4
*a^3*c^5*x^2 + a^2*c^5*x - a*c^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ax+1}{\left(a^2cx^2-c\right)^5\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)/((a^2*c*x^2 - c)^5*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 90, normalized size = 0.76

$$-\frac{128x^8a^8-128a^7x^7-448x^6a^6+448x^5a^5+560x^4a^4-560x^3a^3-280a^2x^2+280ax+35}{315\,(ax-1)\,c^5\left(-a^2x^2+1\right)^{\frac{7}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x)
```

```
[Out] -1/315*(128*a^8*x^8-128*a^7*x^7-448*a^6*x^6+448*a^5*x^5+560*a^4*x^4-560*a^3
*x^3-280*a^2*x^2+280*a*x+35)/(a*x-1)/c^5/(-a^2*x^2+1)^(7/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax+1}{\left(a^2cx^2-c\right)^5\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)/((a^2*c*x^2 - c)^5*sqrt(-a^2*x^2 + 1)), x)
```

mupad [B] time = 0.99, size = 783, normalized size = 6.64

$$\frac{751\,a\,\sqrt{1-a^2\,x^2}}{10080\,\left(a^4\,c^5\,x^2-2\,a^3\,c^5\,x+a^2\,c^5\right)}-\frac{19\,a\,\sqrt{1-a^2\,x^2}}{384\,\left(a^4\,c^5\,x^2+2\,a^3\,c^5\,x+a^2\,c^5\right)}+\frac{}{144\,\sqrt{-a^2}\,\left(5\,c^5\,x\,\sqrt{-a^2}-\frac{c^5\,\sqrt{-a^2}}{a}+10\,a^2\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^5*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] (751*a*(1 - a^2*x^2)^(1/2))/(10080*(a^2*c^5 - 2*a^3*c^5*x + a^4*c^5*x^2)) -
 (19*a*(1 - a^2*x^2)^(1/2))/(384*(a^2*c^5 + 2*a^3*c^5*x + a^4*c^5*x^2)) + (
1 - a^2*x^2)^(1/2)/(144*(-a^2)^(1/2)*(5*c^5*x*(-a^2)^(1/2) - (c^5*(-a^2)^(1
/2))/a + 10*a^2*c^5*x^3*(-a^2)^(1/2) - 5*a^3*c^5*x^4*(-a^2)^(1/2) + a^4*c^5
*x^5*(-a^2)^(1/2) - 10*a*c^5*x^2*(-a^2)^(1/2))) + (a^3*(1 - a^2*x^2)^(1/2))
/(140*(a^4*c^5 - 2*a^5*c^5*x + a^6*c^5*x^2)) - (a^3*(1 - a^2*x^2)^(1/2))/(5
60*(a^4*c^5 + 2*a^5*c^5*x + a^6*c^5*x^2)) + (a*(1 - a^2*x^2)^(1/2))/(56*(a^
2*c^5 - 4*a^3*c^5*x + 6*a^4*c^5*x^2 - 4*a^5*c^5*x^3 + a^6*c^5*x^4)) - (a*(1
 - a^2*x^2)^(1/2))/(224*(a^2*c^5 + 4*a^3*c^5*x + 6*a^4*c^5*x^2 + 4*a^5*c^5*
x^3 + a^6*c^5*x^4)) + (5053*(1 - a^2*x^2)^(1/2))/(26880*(-a^2)^(1/2)*(c^5*x
*(-a^2)^(1/2) + (c^5*(-a^2)^(1/2))/a)) + (17609*(1 - a^2*x^2)^(1/2))/(80640
*(-a^2)^(1/2)*(c^5*x*(-a^2)^(1/2) - (c^5*(-a^2)^(1/2))/a)) + (41*(1 - a^2*x
^2)^(1/2))/(2240*(-a^2)^(1/2)*(3*c^5*x*(-a^2)^(1/2) + (c^5*(-a^2)^(1/2))/a
+ a^2*c^5*x^3*(-a^2)^(1/2) + 3*a*c^5*x^2*(-a^2)^(1/2))) + (149*(1 - a^2*x^2
)^(1/2))/(3360*(-a^2)^(1/2)*(3*c^5*x*(-a^2)^(1/2) - (c^5*(-a^2)^(1/2))/a +
a^2*c^5*x^3*(-a^2)^(1/2) - 3*a*c^5*x^2*(-a^2)^(1/2))) + (a^2*(1 - a^2*x^2)^
(1/2))/(252*(a^3*c^5 - 4*a^4*c^5*x + 6*a^5*c^5*x^2 - 4*a^6*c^5*x^3 + a^7*c^
5*x^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{-a^{10}x^{10}\sqrt{-a^2x^2+1}+5a^8x^8\sqrt{-a^2x^2+1}-10a^6x^6\sqrt{-a^2x^2+1}+10a^4x^4\sqrt{-a^2x^2+1}-5a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{}{-a^{10}x^{10}\sqrt{-a^2x^2+1}+5a^8x^8}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**5,x)
```

```
[Out] (Integral(a*x/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a**2*x
**2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2*x**2
 + 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Inte
gral(1/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a**2*x**2 + 1
) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2*x**2 + 1) -
 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**5
```

3.920 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{\log(1-ax)}{a^5}-\frac{x}{a^4}-\frac{x^2}{2a^3}-\frac{x^3}{3a^2}-\frac{x^4}{4a}$$

```
[Out] -x/a^4-1/2*x^2/a^3-1/3*x^3/a^2-1/4*x^4/a-ln(-a*x+1)/a^5
```

Rubi [A] time = 0.10, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 43}

$$-\frac{x^3}{3a^2}-\frac{x^2}{2a^3}-\frac{x}{a^4}-\frac{\log(1-ax)}{a^5}-\frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a^4) - x^2/(2*a^3) - x^3/(3*a^2) - x^4/(4*a) - Log[1 - a*x]/a^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4}{1-ax}\,dx$$
$$= \int \left(-\frac{1}{a^4} - \frac{x}{a^3} - \frac{x^2}{a^2} - \frac{x^3}{a} - \frac{1}{a^4(-1+ax)}\right) dx$$
$$= -\frac{x}{a^4} - \frac{x^2}{2a^3} - \frac{x^3}{3a^2} - \frac{x^4}{4a} - \frac{\log(1-ax)}{a^5}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 1.00

$$-\frac{\log(1-ax)}{a^5} - \frac{x}{a^4} - \frac{x^2}{2a^3} - \frac{x^3}{3a^2} - \frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/Sqrt[1 - a^2*x^2],x]
[Out] -(x/a^4) - x^2/(2*a^3) - x^3/(3*a^2) - x^4/(4*a) - Log[1 - a*x]/a^5
```

fricas [A] time = 0.49, size = 42, normalized size = 0.86

$$-\frac{3\,a^4x^4 + 4\,a^3x^3 + 6\,a^2x^2 + 12\,ax + 12\,\log\left(ax-1\right)}{12\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^4,x, algorithm="fricas")
[Out] -1/12*(3*a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 12*a*x + 12*log(a*x - 1))/a^5
```

giac [A] time = 0.26, size = 44, normalized size = 0.90

$$-\frac{3\,a^3x^4 + 4\,a^2x^3 + 6\,ax^2 + 12\,x}{12\,a^4} - \frac{\log\left(|ax-1|\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^4,x, algorithm="giac")
[Out] -1/12*(3*a^3*x^4 + 4*a^2*x^3 + 6*a*x^2 + 12*x)/a^4 - log(abs(a*x - 1))/a^5
```

maple [A] time = 0.03, size = 43, normalized size = 0.88

$$-\frac{x^4}{4a} - \frac{x^3}{3a^2} - \frac{x^2}{2a^3} - \frac{x}{a^4} - \frac{\ln\left(ax-1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x^4,x)
```

```
[Out] -1/4*x^4/a-1/3*x^3/a^2-1/2*x^2/a^3-x/a^4-1/a^5*ln(a*x-1)
```

maxima [A] time = 0.31, size = 43, normalized size = 0.88

$$-\frac{3 a^3 x^4 + 4 a^2 x^3 + 6 a x^2 + 12 x}{12 a^4} - \frac{\log(ax-1)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^4,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^4 + 4*a^2*x^3 + 6*a*x^2 + 12*x)/a^4 - log(a*x - 1)/a^5
```

mupad [B] time = 0.04, size = 42, normalized size = 0.86

$$-\frac{\ln(a x-1)}{a^5} - \frac{x}{a^4} - \frac{x^4}{4 a} - \frac{x^3}{3 a^2} - \frac{x^2}{2 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(a*x + 1))/(a^2*x^2 - 1),x)
```

```
[Out] - log(a*x - 1)/a^5 - x/a^4 - x^4/(4*a) - x^3/(3*a^2) - x^2/(2*a^3)
```

sympy [A] time = 18.39, size = 39, normalized size = 0.80

$$-\frac{x^4}{4a} - \frac{x^3}{3a^2} - \frac{x^2}{2a^3} - \frac{x}{a^4} - \frac{\log(ax-1)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x**4,x)
```

```
[Out] -x**4/(4*a) - x**3/(3*a**2) - x**2/(2*a**3) - x/a**4 - log(a*x - 1)/a**5
```

3.921 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{\log(1-ax)}{a^4}-\frac{x}{a^3}-\frac{x^2}{2a^2}-\frac{x^3}{3a}$$

```
[Out] -x/a^3-1/2*x^2/a^2-1/3*x^3/a-ln(-a*x+1)/a^4
```

Rubi [A] time = 0.10, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 43}

$$-\frac{x^2}{2a^2}-\frac{x}{a^3}-\frac{\log(1-ax)}{a^4}-\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a^3) - x^2/(2*a^2) - x^3/(3*a) - Log[1 - a*x]/a^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3}{1-ax}\,dx$$
$$= \int \left(-\frac{1}{a^3} - \frac{x}{a^2} - \frac{x^2}{a} - \frac{1}{a^3(-1+ax)}\right)dx$$
$$= -\frac{x}{a^3} - \frac{x^2}{2a^2} - \frac{x^3}{3a} - \frac{\log(1-ax)}{a^4}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$-\frac{\log(1-ax)}{a^4} - \frac{x}{a^3} - \frac{x^2}{2a^2} - \frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/Sqrt[1 - a^2*x^2],x]
[Out] -(x/a^3) - x^2/(2*a^2) - x^3/(3*a) - Log[1 - a*x]/a^4
```

fricas [A] time = 0.59, size = 34, normalized size = 0.87

$$-\frac{2\,a^3x^3 + 3\,a^2x^2 + 6\,ax + 6\,\log(ax-1)}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^3,x, algorithm="fricas")
[Out] -1/6*(2*a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^4
```

giac [A] time = 0.16, size = 36, normalized size = 0.92

$$-\frac{2\,a^2x^3 + 3\,ax^2 + 6\,x}{6\,a^3} - \frac{\log(|ax-1|)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^3,x, algorithm="giac")
[Out] -1/6*(2*a^2*x^3 + 3*a*x^2 + 6*x)/a^3 - log(abs(a*x - 1))/a^4
```

maple [A] time = 0.03, size = 35, normalized size = 0.90

$$-\frac{x^3}{3a} - \frac{x^2}{2a^2} - \frac{x}{a^3} - \frac{\ln(ax-1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x^3,x)
```

```
[Out] -1/3*x^3/a-1/2*x^2/a^2-x/a^3-1/a^4*ln(a*x-1)
```

maxima [A] time = 0.32, size = 35, normalized size = 0.90

$$-\frac{2\,a^2x^3+3\,ax^2+6\,x}{6\,a^3}-\frac{\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*x^3 + 3*a*x^2 + 6*x)/a^3 - log(a*x - 1)/a^4
```

mupad [B] time = 0.04, size = 34, normalized size = 0.87

$$-\frac{\ln\left(a\,x-1\right)}{a^4}-\frac{x}{a^3}-\frac{x^3}{3\,a}-\frac{x^2}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a*x + 1))/(a^2*x^2 - 1),x)
```

```
[Out] - log(a*x - 1)/a^4 - x/a^3 - x^3/(3*a) - x^2/(2*a^2)
```

sympy [A] time = 0.10, size = 31, normalized size = 0.79

$$-\frac{x^3}{3a}-\frac{x^2}{2a^2}-\frac{x}{a^3}-\frac{\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x**3,x)
```

```
[Out] -x**3/(3*a) - x**2/(2*a**2) - x/a**3 - log(a*x - 1)/a**4
```

3.922 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\log(1-ax)}{a^3}-\frac{x}{a^2}-\frac{x^2}{2a}$$

```
[Out] -x/a^2-1/2*x^2/a-ln(-a*x+1)/a^3
```

Rubi [A] time = 0.09, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 43}

$$-\frac{x}{a^2}-\frac{\log(1-ax)}{a^3}-\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x/a^2) - x^2/(2*a) - Log[1 - a*x]/a^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2}{1-ax}\,dx$$
$$= \int \left(-\frac{1}{a^2} - \frac{x}{a} - \frac{1}{a^2(-1+ax)}\right)dx$$
$$= -\frac{x}{a^2} - \frac{x^2}{2a} - \frac{\log(1-ax)}{a^3}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{\log(1-ax)}{a^3} - \frac{x}{a^2} - \frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/Sqrt[1 - a^2*x^2],x]
[Out] -(x/a^2) - x^2/(2*a) - Log[1 - a*x]/a^3
```

fricas [A] time = 0.54, size = 25, normalized size = 0.86

$$-\frac{a^2x^2 + 2\,ax + 2\,\log\left(ax-1\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^2,x, algorithm="fricas")
[Out] -1/2*(a^2*x^2 + 2*a*x + 2*log(a*x - 1))/a^3
```

giac [A] time = 0.20, size = 27, normalized size = 0.93

$$-\frac{ax^2 + 2\,x}{2\,a^2} - \frac{\log\left(|ax-1|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^2,x, algorithm="giac")
[Out] -1/2*(a*x^2 + 2*x)/a^2 - log(abs(a*x - 1))/a^3
```

maple [A] time = 0.03, size = 27, normalized size = 0.93

$$-\frac{x^2}{2a} - \frac{x}{a^2} - \frac{\ln\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x^2,x)
```

```
[Out] -1/2*x^2/a-x/a^2-1/a^3*ln(a*x-1)
```

maxima [A] time = 0.32, size = 26, normalized size = 0.90

$$-\frac{ax^2+2\,x}{2\,a^2}-\frac{\log\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2 + 2*x)/a^2 - log(a*x - 1)/a^3
```

mupad [B] time = 0.87, size = 23, normalized size = 0.79

$$-\frac{\ln\left(a\,x-1\right)+a\,x+\frac{a^2\,x^2}{2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1))/(a^2*x^2 - 1),x)
```

```
[Out] -(log(a*x - 1) + a*x + (a^2*x^2)/2)/a^3
```

sympy [A] time = 0.09, size = 22, normalized size = 0.76

$$-\frac{x^2}{2a}-\frac{x}{a^2}-\frac{\log\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x**2,x)
```

```
[Out] -x**2/(2*a) - x/a**2 - log(a*x - 1)/a**3
```

3.923 $\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\log(1-ax)}{a^2}-\frac{x}{a}$$

[Out] -x/a-ln(-a*x+1)/a^2

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6150, 43}

$$-\frac{\log(1-ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcTanh[a*x]*x)/Sqrt[1 - a^2*x^2],x]

[Out] -(x/a) - Log[1 - a*x]/a^2

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx &= \int \frac{x}{1-ax}\,dx\\ &= \int \left(-\frac{1}{a}-\frac{1}{a(-1+ax)}\right)dx\\ &= -\frac{x}{a}-\frac{\log(1-ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{\log(1-ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/Sqrt[1 - a^2*x^2],x]
[Out] -(x/a) - Log[1 - a*x]/a^2
```

fricas [A] time = 0.54, size = 15, normalized size = 0.79

$$-\frac{ax+\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x,x, algorithm="fricas")
[Out] -(a*x + log(a*x - 1))/a^2
```

giac [A] time = 0.15, size = 19, normalized size = 1.00

$$-\frac{x}{a}-\frac{\log(|ax-1|)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x,x, algorithm="giac")
[Out] -x/a - log(abs(a*x - 1))/a^2
```

maple [A] time = 0.03, size = 19, normalized size = 1.00

$$-\frac{x}{a}-\frac{\ln(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x,x)
[Out] -x/a-1/a^2*ln(a*x-1)
```

maxima [A] time = 0.36, size = 18, normalized size = 0.95

$$-\frac{x}{a}-\frac{\log(ax-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x,x, algorithm="maxima")
```

```
[Out] -x/a - log(a*x - 1)/a^2
```

mupad [B] time = 0.86, size = 15, normalized size = 0.79

$$-\frac{\ln\left(a\,x-1\right)+a\,x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x + 1))/(a^2*x^2 - 1),x)
```

```
[Out] -(log(a*x - 1) + a*x)/a^2
```

sympy [A] time = 0.08, size = 14, normalized size = 0.74

$$-\frac{x}{a}-\frac{\log\left(ax-1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x,x)
```

```
[Out] -x/a - log(a*x - 1)/a**2
```

3.924 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(1-ax)}{a}$$

[Out] -ln(-a*x+1)/a

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6140, 31}

$$-\frac{\log(1-ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]

[Out] -(Log[1 - a*x]/a)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx &= \int \frac{1}{1-ax}\,dx\\ &= -\frac{\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(1-ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(Log[1 - a*x]/a)
```

fricas [A] time = 0.59, size = 11, normalized size = 0.92

$$-\frac{\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -log(a*x - 1)/a
```

giac [A] time = 0.20, size = 12, normalized size = 1.00

$$-\frac{\log\left(|ax-1|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -log(abs(a*x - 1))/a
```

maple [A] time = 0.03, size = 12, normalized size = 1.00

$$-\frac{\ln\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1),x)
```

```
[Out] -1/a*ln(a*x-1)
```

maxima [A] time = 0.31, size = 11, normalized size = 0.92

$$-\frac{\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -log(a*x - 1)/a
```

mupad [B] time = 0.02, size = 11, normalized size = 0.92

$$-\frac{\ln\left(a\,x-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(a^2*x^2 - 1),x)
```

```
[Out] -log(a*x - 1)/a
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$-\frac{\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1),x)
```

```
[Out] -log(a*x - 1)/a
```

3.925 $\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=12

$$\log(x)-\log(1-ax)$$

```
[Out] ln(x)-ln(-a*x+1)
```

Rubi [A] time = 0.08, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6150, 36, 29, 31}

$$\log(x)-\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Log[x] - Log[1 - a*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{x(1-ax)}\,dx$$
$$= a\int \frac{1}{1-ax}\,dx + \int \frac{1}{x}\,dx$$
$$= \log(x) - \log(1-ax)$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\log(x) - \log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
[Out] Log[x] - Log[1 - a*x]
```

fricas [A] time = 0.54, size = 11, normalized size = 0.92

$$-\log(ax-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x,x, algorithm="fricas")
[Out] -log(a*x - 1) + log(x)
```

giac [A] time = 0.19, size = 13, normalized size = 1.08

$$-\log(|ax-1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x,x, algorithm="giac")
[Out] -log(abs(a*x - 1)) + log(abs(x))
```

maple [A] time = 0.03, size = 12, normalized size = 1.00

$$\ln(x) - \ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)/x,x)
```

```
[Out] ln(x)-ln(a*x-1)
```

maxima [A] time = 0.31, size = 11, normalized size = 0.92

$$-\log(ax-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x,x, algorithm="maxima")
[Out] -log(a*x - 1) + log(x)
```

mupad [B] time = 0.05, size = 9, normalized size = 0.75

$$2\,\mathrm{atanh}\left(2\,a\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(x*(a^2*x^2 - 1)),x)
[Out] 2*atanh(2*a*x - 1)
```

sympy [A] time = 0.11, size = 8, normalized size = 0.67

$$\log(x)-\log\left(x-\frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)/x,x)
[Out] log(x) - log(x - 1/a)
```

3.926 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=20

$$a\log(x)-a\log(1-ax)-\frac{1}{x}$$

```
[Out] -1/x+a*ln(x)-a*ln(-a*x+1)
```

Rubi [A] time = 0.09, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 44}

$$a\log(x)-a\log(1-ax)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -x^(-1) + a*Log[x] - a*Log[1 - a*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{1-a^2x^2}}\,dx &= \int \frac{1}{x^2(1-ax)}\,dx\\
&= \int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx\\
&= -\frac{1}{x}+a\log(x)-a\log(1-ax)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$
a\log(x)-a\log(1-ax)-\frac{1}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
[Out] -x^(-1) + a*Log[x] - a*Log[1 - a*x]
```

fricas [A] time = 0.58, size = 22, normalized size = 1.10

$$
-\frac{ax\log\left(ax-1\right)-ax\log(x)+1}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^2,x, algorithm="fricas")
[Out] -(a*x*log(a*x - 1) - a*x*log(x) + 1)/x
```

giac [A] time = 0.19, size = 21, normalized size = 1.05

$$
-a\log\left(|ax-1|\right)+a\log\left(|x|\right)-\frac{1}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^2,x, algorithm="giac")
[Out] -a*log(abs(a*x - 1)) + a*log(abs(x)) - 1/x
```

maple [A] time = 0.03, size = 20, normalized size = 1.00

$$
-\frac{1}{x}+a\ln(x)-a\ln\left(ax-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)/x^2,x)
```

```
[Out] -1/x+a*ln(x)-a*ln(a*x-1)
```

maxima [A] time = 0.31, size = 19, normalized size = 0.95

$$-a\log\left(ax-1\right)+a\log(x)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^2,x, algorithm="maxima")
```

```
[Out] -a*log(a*x - 1) + a*log(x) - 1/x
```

mupad [B] time = 0.87, size = 16, normalized size = 0.80

$$2\,a\,\mathrm{atanh}\left(2\,a\,x-1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] 2*a*atanh(2*a*x - 1) - 1/x
```

sympy [A] time = 0.14, size = 15, normalized size = 0.75

$$-a\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)/x**2,x)
```

```
[Out] -a*(-log(x) + log(x - 1/a)) - 1/x
```

3.927 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=32

$$a^2\log(x) - a^2\log(1-ax) - \frac{a}{x} - \frac{1}{2x^2}$$

```
[Out] -1/2/x^2-a/x+a^2*ln(x)-a^2*ln(-a*x+1)
```

Rubi [A] time = 0.09, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 44}

$$a^2\log(x) - a^2\log(1-ax) - \frac{a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -1/(2*x^2) - a/x + a^2*Log[x] - a^2*Log[1 - a*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{x^3(1-ax)}\,dx$$
$$= \int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{a^2}{x}-\frac{a^3}{-1+ax}\right)dx$$
$$= -\frac{1}{2x^2}-\frac{a}{x}+a^2\log(x)-a^2\log(1-ax)$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$a^2\log(x)-a^2\log(1-ax)-\frac{a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
[Out] -1/2*1/x^2 - a/x + a^2*Log[x] - a^2*Log[1 - a*x]
```

fricas [A] time = 0.44, size = 35, normalized size = 1.09

$$-\frac{2\,a^2x^2\log\left(ax-1\right)-2\,a^2x^2\log(x)+2\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^3,x, algorithm="fricas")
[Out] -1/2*(2*a^2*x^2*log(a*x - 1) - 2*a^2*x^2*log(x) + 2*a*x + 1)/x^2
```

giac [A] time = 0.14, size = 31, normalized size = 0.97

$$-a^2\log\left(|ax-1|\right)+a^2\log\left(|x|\right)-\frac{2\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^3,x, algorithm="giac")
[Out] -a^2*log(abs(a*x - 1)) + a^2*log(abs(x)) - 1/2*(2*a*x + 1)/x^2
```

maple [A] time = 0.03, size = 30, normalized size = 0.94

$$-\frac{1}{2x^2}-\frac{a}{x}+a^2\ln(x)-a^2\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)/x^3,x)
```

```
[Out] -1/2/x^2-a/x+a^2*ln(x)-a^2*ln(a*x-1)
```

maxima [A] time = 0.31, size = 29, normalized size = 0.91

$$-a^2 \log (ax - 1) + a^2 \log(x) - \frac{2\,ax + 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^3,x, algorithm="maxima")
```

```
[Out] -a^2*log(a*x - 1) + a^2*log(x) - 1/2*(2*a*x + 1)/x^2
```

mupad [B] time = 0.05, size = 23, normalized size = 0.72

$$2\,a^2\,\mathrm{atanh}\,(2\,a\,x - 1) - \frac{a\,x + \frac{1}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] 2*a^2*atanh(2*a*x - 1) - (a*x + 1/2)/x^2
```

sympy [A] time = 0.17, size = 26, normalized size = 0.81

$$-a^2 \left(-\log (x) + \log \left(x - \frac{1}{a}\right)\right) - \frac{2ax + 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)/x**3,x)
```

```
[Out] -a**2*(-log(x) + log(x - 1/a)) - (2*a*x + 1)/(2*x**2)
```

3.928 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=42

$$a^3\log(x)-a^3\log(1-ax)-\frac{a^2}{x}-\frac{a}{2x^2}-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3-1/2*a/x^2-a^2/x+a^3*ln(x)-a^3*ln(-a*x+1)
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 44}

$$-\frac{a^2}{x}+a^3\log(x)-a^3\log(1-ax)-\frac{a}{2x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -1/(3*x^3) - a/(2*x^2) - a^2/x + a^3*Log[x] - a^3*Log[1 - a*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{x^4(1-ax)}\,dx$$
$$= \int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{a^2}{x^2}+\frac{a^3}{x}-\frac{a^4}{-1+ax}\right)dx$$
$$= -\frac{1}{3x^3}-\frac{a}{2x^2}-\frac{a^2}{x}+a^3\log(x)-a^3\log(1-ax)$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$a^3\log(x)-a^3\log(1-ax)-\frac{a^2}{x}-\frac{a}{2x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*Sqrt[1 - a^2*x^2]),x]
[Out] -1/3*1/x^3 - a/(2*x^2) - a^2/x + a^3*Log[x] - a^3*Log[1 - a*x]
```

fricas [A] time = 0.76, size = 43, normalized size = 1.02

$$-\frac{6\,a^3x^3\log(ax-1)-6\,a^3x^3\log(x)+6\,a^2x^2+3\,ax+2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^4,x, algorithm="fricas")
[Out] -1/6*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 2)/x^
3
```

giac [A] time = 0.17, size = 39, normalized size = 0.93

$$-a^3\log(|ax-1|)+a^3\log(|x|)-\frac{6\,a^2x^2+3\,ax+2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^4,x, algorithm="giac")
[Out] -a^3*log(abs(a*x - 1)) + a^3*log(abs(x)) - 1/6*(6*a^2*x^2 + 3*a*x + 2)/x^3
```

maple [A] time = 0.03, size = 38, normalized size = 0.90

$$-\frac{1}{3x^3}-\frac{a}{2x^2}-\frac{a^2}{x}+a^3\ln(x)-a^3\ln(ax-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)/x^4,x)
```

```
[Out] -1/3/x^3-1/2*a/x^2-a^2/x+a^3*ln(x)-a^3*ln(a*x-1)
```

maxima [A] time = 0.31, size = 37, normalized size = 0.88

$$-a^3\log\left(ax-1\right)+a^3\log(x)-\frac{6\,a^2x^2+3\,ax+2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)/x^4,x, algorithm="maxima")
```

```
[Out] -a^3*log(a*x - 1) + a^3*log(x) - 1/6*(6*a^2*x^2 + 3*a*x + 2)/x^3
```

mupad [B] time = 0.05, size = 31, normalized size = 0.74

$$2\,a^3\,\mathrm{atanh}\left(2\,a\,x-1\right)-\frac{a^2\,x^2+\frac{a\,x}{2}+\frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] 2*a^3*atanh(2*a*x - 1) - ((a*x)/2 + a^2*x^2 + 1/3)/x^3
```

sympy [A] time = 0.17, size = 34, normalized size = 0.81

$$-a^3\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)-\frac{6a^2x^2+3ax+2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)/x**4,x)
```

```
[Out] -a**3*(-log(x) + log(x - 1/a)) - (6*a**2*x**2 + 3*a*x + 2)/(6*x**3)
```

3.929 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{1}{2a^5(1-ax)}+\frac{7\log(1-ax)}{4a^5}+\frac{\log(ax+1)}{4a^5}+\frac{x}{a^4}+\frac{x^2}{2a^3}$$

```
[Out] x/a^4+1/2*x^2/a^3+1/2/a^5/(-a*x+1)+7/4*ln(-a*x+1)/a^5+1/4*ln(a*x+1)/a^5
```

Rubi [A] time = 0.12, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{x^2}{2a^3}+\frac{x}{a^4}+\frac{1}{2a^5(1-ax)}+\frac{7\log(1-ax)}{4a^5}+\frac{\log(ax+1)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] x/a^4 + x^2/(2*a^3) + 1/(2*a^5*(1 - a*x)) + (7*Log[1 - a*x])/(4*a^5) + Log[
1 + a*x]/(4*a^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^{3/2}}\,dx = \int \frac{x^4}{(1-ax)^2(1+ax)}\,dx$$
$$= \int \left(\frac{1}{a^4}+\frac{x}{a^3}+\frac{1}{2a^4(-1+ax)^2}+\frac{7}{4a^4(-1+ax)}+\frac{1}{4a^4(1+ax)}\right)dx$$
$$= \frac{x}{a^4}+\frac{x^2}{2a^3}+\frac{1}{2a^5(1-ax)}+\frac{7\log(1-ax)}{4a^5}+\frac{\log(1+ax)}{4a^5}$$

Mathematica [**A**] time = 0.05, size = 45, normalized size = 0.78

$$\frac{2\left(a^2x^2+2ax+\frac{1}{1-ax}\right)+7\log(1-ax)+\log(ax+1)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*(2*a*x + a^2*x^2 + (1 - a*x)^(-1)) + 7*Log[1 - a*x] + Log[1 + a*x])/(4*a
^5)
```

fricas [**A**] time = 0.61, size = 62, normalized size = 1.07

$$\frac{2\,a^3x^3+2\,a^2x^2-4\,ax+(ax-1)\log\left(ax+1\right)+7\,(ax-1)\log\left(ax-1\right)-2}{4\left(a^6x-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^4,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3*x^3 + 2*a^2*x^2 - 4*a*x + (a*x - 1)*log(a*x + 1) + 7*(a*x - 1)*l
og(a*x - 1) - 2)/(a^6*x - a^5)
```

giac [**A**] time = 0.15, size = 56, normalized size = 0.97

$$\frac{\log\left(|ax+1|\right)}{4\,a^5}+\frac{7\,\log\left(|ax-1|\right)}{4\,a^5}+\frac{a^3x^2+2\,a^2x}{2\,a^6}-\frac{1}{2\,(ax-1)a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^4,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a*x + 1))/a^5 + 7/4*log(abs(a*x - 1))/a^5 + 1/2*(a^3*x^2 + 2*a^
2*x)/a^6 - 1/2/((a*x - 1)*a^5)
```

maple [A] time = 0.03, size = 49, normalized size = 0.84

$$\frac{x^2}{2a^3}+\frac{x}{a^4}-\frac{1}{2a^5\,(ax-1)}+\frac{7\ln\,(ax-1)}{4a^5}+\frac{\ln\,(ax+1)}{4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x^4,x)
```

```
[Out] 1/2*x^2/a^3+x/a^4-1/2/a^5/(a*x-1)+7/4/a^5*ln(a*x-1)+1/4*ln(a*x+1)/a^5
```

maxima [A] time = 0.31, size = 52, normalized size = 0.90

$$-\frac{1}{2\left(a^6x-a^5\right)}+\frac{ax^2+2\,x}{2\,a^4}+\frac{\log\,(ax+1)}{4\,a^5}+\frac{7\,\log\,(ax-1)}{4\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^4,x, algorithm="maxima")
```

```
[Out] -1/2/(a^6*x - a^5) + 1/2*(a*x^2 + 2*x)/a^4 + 1/4*log(a*x + 1)/a^5 + 7/4*log
(a*x - 1)/a^5
```

mupad [B] time = 0.91, size = 54, normalized size = 0.93

$$\frac{7\,\ln\,(a\,x-1)}{4\,a^5}+\frac{\ln\,(a\,x+1)}{4\,a^5}-\frac{1}{2\,a\,\left(a^5\,x-a^4\right)}+\frac{x}{a^4}+\frac{x^2}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/(a^2*x^2 - 1)^2,x)
```

```
[Out] (7*log(a*x - 1))/(4*a^5) + log(a*x + 1)/(4*a^5) - 1/(2*a*(a^5*x - a^4)) + x
/a^4 + x^2/(2*a^3)
```

sympy [A] time = 0.26, size = 48, normalized size = 0.83

$$-\frac{1}{2a^6x-2a^5}+\frac{x^2}{2a^3}+\frac{x}{a^4}+\frac{\frac{7\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x**4,x)
```

```
[Out] -1/(2*a**6*x - 2*a**5) + x**2/(2*a**3) + x/a**4 + (7*log(x - 1/a)/4 + log(x
 + 1/a)/4)/a**5
```

3.930 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}+\frac{x}{a^3}$$

```
[Out] x/a^3+1/2/a^4/(-a*x+1)+5/4*ln(-a*x+1)/a^4-1/4*ln(a*x+1)/a^4
```

Rubi [A] time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{x}{a^3}+\frac{1}{2a^4(1-ax)}+\frac{5\log(1-ax)}{4a^4}-\frac{\log(ax+1)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] x/a^3 + 1/(2*a^4*(1 - a*x)) + (5*Log[1 - a*x])/(4*a^4) - Log[1 + a*x]/(4*a^
4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx = \int \frac{x^3}{(1-ax)^2(1+ax)}\,dx$$
$$= \int \left(\frac{1}{a^3} + \frac{1}{2a^3(-1+ax)^2} + \frac{5}{4a^3(-1+ax)} - \frac{1}{4a^3(1+ax)}\right)dx$$
$$= \frac{x}{a^3} + \frac{1}{2a^4(1-ax)} + \frac{5\log(1-ax)}{4a^4} - \frac{\log(1+ax)}{4a^4}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.81

$$\frac{4ax + \frac{2}{1-ax} + 5\log(1-ax) - \log(ax+1)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(1 - a^2*x^2)^(3/2),x]
[Out] (4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1 + a*x])/(4*a^4)
```

fricas [A] time = 0.57, size = 55, normalized size = 1.15

$$\frac{4\,a^2x^2 - 4\,ax - (ax-1)\log(ax+1) + 5\,(ax-1)\log(ax-1) - 2}{4\left(a^5x - a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^3,x, algorithm="fricas")
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)/(a^5*x - a^4)
```

giac [A] time = 0.19, size = 42, normalized size = 0.88

$$\frac{x}{a^3} - \frac{\log(|ax+1|)}{4\,a^4} + \frac{5\,\log(|ax-1|)}{4\,a^4} - \frac{1}{2\,(ax-1)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^3,x, algorithm="giac")
[Out] x/a^3 - 1/4*log(abs(a*x + 1))/a^4 + 5/4*log(abs(a*x - 1))/a^4 - 1/2/((a*x -
 1)*a^4)
```

maple [A] time = 0.03, size = 41, normalized size = 0.85

$$\frac{x}{a^3} - \frac{1}{2a^4(ax-1)} + \frac{5\ln(ax-1)}{4a^4} - \frac{\ln(ax+1)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x^3,x)
```

```
[Out] x/a^3-1/2/a^4/(a*x-1)+5/4/a^4*ln(a*x-1)-1/4*ln(a*x+1)/a^4
```

maxima [A] time = 0.31, size = 43, normalized size = 0.90

$$-\frac{1}{2\left(a^5x - a^4\right)} + \frac{x}{a^3} - \frac{\log(ax+1)}{4a^4} + \frac{5\log(ax-1)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a^5*x - a^4) + x/a^3 - 1/4*log(a*x + 1)/a^4 + 5/4*log(a*x - 1)/a^4
```

mupad [B] time = 0.07, size = 46, normalized size = 0.96

$$\frac{5\ln(ax-1)}{4a^4} - \frac{\ln(ax+1)}{4a^4} - \frac{1}{2a\left(a^4x - a^3\right)} + \frac{x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/(a^2*x^2 - 1)^2,x)
```

```
[Out] (5*log(a*x - 1))/(4*a^4) - log(a*x + 1)/(4*a^4) - 1/(2*a*(a^4*x - a^3)) + x
/a^3
```

sympy [A] time = 0.25, size = 39, normalized size = 0.81

$$-\frac{1}{2a^5x - 2a^4} + \frac{x}{a^3} + \frac{\frac{5\log\left(x-\frac{1}{a}\right)}{4} - \frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x**3,x)
```

```
[Out] -1/(2*a**5*x - 2*a**4) + x/a**3 + (5*log(x - 1/a)/4 - log(x + 1/a)/4)/a**4
```

3.931 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}$$

```
[Out] 1/2/a^3/(-a*x+1)+3/4*ln(-a*x+1)/a^3+1/4*ln(a*x+1)/a^3
```

Rubi [A] time = 0.11, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 1/(2*a^3*(1 - a*x)) + (3*Log[1 - a*x])/(4*a^3) + Log[1 + a*x]/(4*a^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{3/2}}\,dx = \int \frac{x^2}{(1-ax)^2(1+ax)}\,dx$$
$$= \int \left(\frac{1}{2a^2(-1+ax)^2} + \frac{3}{4a^2(-1+ax)} + \frac{1}{4a^2(1+ax)}\right)dx$$
$$= \frac{1}{2a^3(1-ax)} + \frac{3\log(1-ax)}{4a^3} + \frac{\log(1+ax)}{4a^3}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.77

$$\frac{\frac{2}{1-ax} + 3\log(1-ax) + \log(ax+1)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(1 - a^2*x^2)^(3/2),x]
[Out] (2/(1 - a*x) + 3*Log[1 - a*x] + Log[1 + a*x])/(4*a^3)
```

fricas [A] time = 0.75, size = 42, normalized size = 0.98

$$\frac{(ax-1)\log(ax+1) + 3(ax-1)\log(ax-1) - 2}{4\left(a^4x - a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^2,x, algorithm="fricas")
[Out] 1/4*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^4*x - a^3)
```

giac [A] time = 0.17, size = 37, normalized size = 0.86

$$\frac{\log(|ax+1|)}{4\,a^3} + \frac{3\,\log(|ax-1|)}{4\,a^3} - \frac{1}{2\,(ax-1)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^2,x, algorithm="giac")
[Out] 1/4*log(abs(a*x + 1))/a^3 + 3/4*log(abs(a*x - 1))/a^3 - 1/2/((a*x - 1)*a^3)
```

maple [A] time = 0.03, size = 36, normalized size = 0.84

$$-\frac{1}{2a^3(ax-1)} + \frac{3\ln(ax-1)}{4a^3} + \frac{\ln(ax+1)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x^2,x)
```

```
[Out] -1/2/a^3/(a*x-1)+3/4/a^3*ln(a*x-1)+1/4/a^3*ln(a*x+1)
```

maxima [A] time = 0.31, size = 38, normalized size = 0.88

$$-\frac{1}{2\left(a^{4}x-a^{3}\right)}+\frac{\log\left(ax+1\right)}{4\,a^{3}}+\frac{3\,\log\left(ax-1\right)}{4\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a^4*x - a^3) + 1/4*log(a*x + 1)/a^3 + 3/4*log(a*x - 1)/a^3
```

mupad [B] time = 0.95, size = 39, normalized size = 0.91

$$\frac{3\,\ln\left(a\,x-1\right)}{4\,a^{3}}+\frac{\ln\left(a\,x+1\right)}{4\,a^{3}}-\frac{1}{2\,\left(a^{4}\,x-a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/(a^2*x^2 - 1)^2,x)
```

```
[Out] (3*log(a*x - 1))/(4*a^3) + log(a*x + 1)/(4*a^3) - 1/(2*(a^4*x - a^3))
```

sympy [A] time = 0.21, size = 34, normalized size = 0.79

$$-\frac{1}{2a^{4}x-2a^{3}}+\frac{\frac{3\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x**2,x)
```

```
[Out] -1/(2*a**4*x - 2*a**3) + (3*log(x - 1/a)/4 + log(x + 1/a)/4)/a**3
```

3.932 $\int \frac{e^{\tanh^{-1}(ax)}x}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2a^2(1-ax)}-\frac{\tanh^{-1}(ax)}{2a^2}$$

```
[Out] 1/2/a^2/(-a*x+1)-1/2*arctanh(a*x)/a^2
```

Rubi [A] time = 0.08, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6150, 77, 207}

$$\frac{1}{2a^2(1-ax)}-\frac{\tanh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 1/(2*a^2*(1 - a*x)) - ArcTanh[a*x]/(2*a^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{3/2}}\,dx &= \int \frac{x}{(1-ax)^2(1+ax)}\,dx \\
&= \int \left(\frac{1}{2a(-1+ax)^2}+\frac{1}{2a\left(-1+a^2x^2\right)}\right)dx \\
&= \frac{1}{2a^2(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{2a} \\
&= \frac{1}{2a^2(1-ax)}-\frac{\tanh^{-1}(ax)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.81

$$\frac{\frac{1}{1-ax}-\tanh^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(1 - a^2*x^2)^(3/2),x]
[Out] ((1 - a*x)^(-1) - ArcTanh[a*x])/(2*a^2)
```

fricas [A] time = 0.51, size = 42, normalized size = 1.56

$$-\frac{(ax-1)\log(ax+1)-(ax-1)\log(ax-1)+2}{4\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x,x, algorithm="fricas")
[Out] -1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) + 2)/(a^3*x - a^2)
```

giac [A] time = 0.17, size = 37, normalized size = 1.37

$$-\frac{\log(|ax+1|)}{4a^2}+\frac{\log(|ax-1|)}{4a^2}-\frac{1}{2(ax-1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x,x, algorithm="giac")
```

```
[Out] -1/4*log(abs(a*x + 1))/a^2 + 1/4*log(abs(a*x - 1))/a^2 - 1/2/((a*x - 1)*a^2
)
```

maple [A] time = 0.03, size = 36, normalized size = 1.33

$$-\frac{1}{2a^2\left(ax-1\right)}+\frac{\ln\left(ax-1\right)}{4a^2}-\frac{\ln\left(ax+1\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x,x)
[Out] -1/2/a^2/(a*x-1)+1/4/a^2*ln(a*x-1)-1/4/a^2*ln(a*x+1)
```

maxima [A] time = 0.34, size = 38, normalized size = 1.41

$$-\frac{1}{2\left(a^3x-a^2\right)}-\frac{\log\left(ax+1\right)}{4\,a^2}+\frac{\log\left(ax-1\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x,x, algorithm="maxima")
[Out] -1/2/(a^3*x - a^2) - 1/4*log(a*x + 1)/a^2 + 1/4*log(a*x - 1)/a^2
```

mupad [B] time = 0.05, size = 22, normalized size = 0.81

$$-\frac{1}{2\,a^2\,\left(a\,x-1\right)}-\frac{\mathrm{atanh}\left(a\,x\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/(a^2*x^2 - 1)^2,x)
[Out] - 1/(2*a^2*(a*x - 1)) - atanh(a*x)/(2*a^2)
```

sympy [A] time = 0.18, size = 32, normalized size = 1.19

$$-\frac{1}{2a^3x-2a^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x,x)
[Out] -1/(2*a**3*x - 2*a**2) + (log(x - 1/a)/4 - log(x + 1/a)/4)/a**2
```

3.933 $\int \frac{e^{\tanh^{-1}(ax)}}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}$$

```
[Out] 1/2/a/(-a*x+1)+1/2*arctanh(a*x)/a
```

Rubi [A] time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6140, 44, 207}

$$\frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx = \int \frac{1}{(1-ax)^2(1+ax)}\,dx$$
$$= \int \left(\frac{1}{2(-1+ax)^2} - \frac{1}{2\left(-1+a^2x^2\right)}\right)dx$$
$$= \frac{1}{2a(1-ax)} - \frac{1}{2}\int \frac{1}{-1+a^2x^2}\,dx$$
$$= \frac{1}{2a(1-ax)} + \frac{\tanh^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.74

$$\frac{\frac{1}{1-ax} + \tanh^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^(-1) + ArcTanh[a*x])/(2*a)
```

fricas [A] time = 0.58, size = 40, normalized size = 1.48

$$\frac{(ax-1)\log(ax+1) - (ax-1)\log(ax-1) - 2}{4\left(a^2x - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*x - a)
```

giac [A] time = 0.49, size = 37, normalized size = 1.37

$$\frac{\log(|ax+1|)}{4a} - \frac{\log(|ax-1|)}{4a} - \frac{1}{2(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a*x + 1))/a - 1/4*log(abs(a*x - 1))/a - 1/2/((a*x - 1)*a)
```

maple [A] time = 0.03, size = 36, normalized size = 1.33

$$-\frac{1}{2a\left(ax-1\right)}-\frac{\ln\left(ax-1\right)}{4a}+\frac{\ln\left(ax+1\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/2/a/(a*x-1)-1/4/a*ln(a*x-1)+1/4*ln(a*x+1)/a
```

maxima [A] time = 0.32, size = 36, normalized size = 1.33

$$\frac{\log\left(ax+1\right)}{4\,a}-\frac{\log\left(ax-1\right)}{4\,a}-\frac{1}{2\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*log(a*x + 1)/a - 1/4*log(a*x - 1)/a - 1/2/(a^2*x - a)
```

mupad [B] time = 0.92, size = 22, normalized size = 0.81

$$\frac{\mathrm{atanh}\left(a\,x\right)}{2\,a}-\frac{1}{2\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(a^2*x^2 - 1)^2,x)
```

```
[Out] atanh(a*x)/(2*a) - 1/(2*a*(a*x - 1))
```

sympy [A] time = 0.18, size = 29, normalized size = 1.07

$$-\frac{1}{2a^2x-2a}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2,x)
```

```
[Out] -1/(2*a**2*x - 2*a) + (-log(x - 1/a)/4 + log(x + 1/a)/4)/a
```

3.934 $\int \frac{e^{\tanh^{-1}(ax)}}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2(1-ax)}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)$$

```
[Out] 1/2/(-a*x+1)+ln(x)-3/4*ln(-a*x+1)-1/4*ln(a*x+1)
```

Rubi [A] time = 0.11, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 72}

$$\frac{1}{2(1-ax)}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] 1/(2*(1 - a*x)) + Log[x] - (3*Log[1 - a*x])/4 - Log[1 + a*x]/4
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x(1-ax)^2(1+ax)}\,dx$$
$$= \int \left(\frac{1}{x} + \frac{a}{2(-1+ax)^2} - \frac{3a}{4(-1+ax)} - \frac{a}{4(1+ax)}\right) dx$$
$$= \frac{1}{2(1-ax)} + \log(x) - \frac{3}{4}\log(1-ax) - \frac{1}{4}\log(1+ax)$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.89

$$\frac{1}{2-2ax} - \frac{3}{4}\log(1-ax) - \frac{1}{4}\log(ax+1) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
[Out] (2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a*x])/4 - Log[1 + a*x]/4
```

fricas [A] time = 0.67, size = 45, normalized size = 1.25

$$-\frac{(ax-1)\log(ax+1) + 3(ax-1)\log(ax-1) - 4(ax-1)\log(x) + 2}{4(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x,x, algorithm="fricas")
[Out] -1/4*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 4*(a*x - 1)*log(x
) + 2)/(a*x - 1)
```

giac [A] time = 0.19, size = 31, normalized size = 0.86

$$-\frac{1}{2(ax-1)} - \frac{1}{4}\log(|ax+1|) - \frac{3}{4}\log(|ax-1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x,x, algorithm="giac")
[Out] -1/2/(a*x - 1) - 1/4*log(abs(a*x + 1)) - 3/4*log(abs(a*x - 1)) + log(abs(x)
)
```

maple [A] time = 0.03, size = 29, normalized size = 0.81

$$\ln(x) - \frac{1}{2\,(ax-1)} - \frac{3\ln\,(ax-1)}{4} - \frac{\ln\,(ax+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2/x,x)
```

```
[Out] ln(x)-1/2/(a*x-1)-3/4*ln(a*x-1)-1/4*ln(a*x+1)
```

maxima [A] time = 0.34, size = 28, normalized size = 0.78

$$-\frac{1}{2\,(ax-1)} - \frac{1}{4}\,\log\,(ax+1) - \frac{3}{4}\,\log\,(ax-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x,x, algorithm="maxima")
```

```
[Out] -1/2/(a*x - 1) - 1/4*log(a*x + 1) - 3/4*log(a*x - 1) + log(x)
```

mupad [B] time = 0.05, size = 30, normalized size = 0.83

$$\ln(x) - \frac{3\,\ln\,(1-a\,x)}{4} - \frac{\ln\,(a\,x+1)}{4} - \frac{1}{2\,\,(a\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(a^2*x^2 - 1)^2),x)
```

```
[Out] log(x) - (3*log(1 - a*x))/4 - log(a*x + 1)/4 - 1/(2*(a*x - 1))
```

sympy [A] time = 0.26, size = 29, normalized size = 0.81

$$\log\,(x) - \frac{3\log\left(x-\frac{1}{a}\right)}{4} - \frac{\log\left(x+\frac{1}{a}\right)}{4} - \frac{1}{2ax-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2/x,x)
```

```
[Out] log(x) - 3*log(x - 1/a)/4 - log(x + 1/a)/4 - 1/(2*a*x - 2)
```

3.935 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{a}{2(1-ax)} + a\log(x) - \frac{5}{4}a\log(1-ax) + \frac{1}{4}a\log(ax+1) - \frac{1}{x}$$

```
[Out] -1/x+1/2*a/(-a*x+1)+a*ln(x)-5/4*a*ln(-a*x+1)+1/4*a*ln(a*x+1)
```

Rubi [A] time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{a}{2(1-ax)} + a\log(x) - \frac{5}{4}a\log(1-ax) + \frac{1}{4}a\log(ax+1) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -x^(-1) + a/(2*(1 - a*x)) + a*Log[x] - (5*a*Log[1 - a*x])/4 + (a*Log[1 + a*
x])/4
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^2(1-ax)^2(1+ax)}\,dx$$
$$= \int \left(\frac{1}{x^2}+\frac{a}{x}+\frac{a^2}{2(-1+ax)^2}-\frac{5a^2}{4(-1+ax)}+\frac{a^2}{4(1+ax)}\right)dx$$
$$= -\frac{1}{x}+\frac{a}{2(1-ax)}+a\log(x)-\frac{5}{4}a\log(1-ax)+\frac{1}{4}a\log(1+ax)$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$\frac{a}{2(1-ax)}+a\log(x)-\frac{5}{4}a\log(1-ax)+\frac{1}{4}a\log(ax+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -x^(-1) + a/(2*(1 - a*x)) + a*Log[x] - (5*a*Log[1 - a*x])/4 + (a*Log[1 + a*
x])/4
```

fricas [A] time = 0.64, size = 75, normalized size = 1.63

$$-\frac{6\,ax-\left(a^2x^2-ax\right)\log\left(ax+1\right)+5\left(a^2x^2-ax\right)\log\left(ax-1\right)-4\left(a^2x^2-ax\right)\log(x)-4}{4\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*x - (a^2*x^2 - a*x)*log(a*x + 1) + 5*(a^2*x^2 - a*x)*log(a*x - 1)
 - 4*(a^2*x^2 - a*x)*log(x) - 4)/(a*x^2 - x)
```

giac [A] time = 0.20, size = 44, normalized size = 0.96

$$\frac{1}{4}\,a\log\left(|ax+1|\right)-\frac{5}{4}\,a\log\left(|ax-1|\right)+a\log\left(|x|\right)-\frac{3\,ax-2}{2\,(ax-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/4*a*log(abs(a*x + 1)) - 5/4*a*log(abs(a*x - 1)) + a*log(abs(x)) - 1/2*(3*
a*x - 2)/((a*x - 1)*x)
```

maple [A] time = 0.03, size = 39, normalized size = 0.85

$$-\frac{1}{x}+a\ln(x)-\frac{a}{2\,(ax-1)}-\frac{5a\ln\,(ax-1)}{4}+\frac{a\ln\,(ax+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2/x^2,x)
```

```
[Out] -1/x+a*ln(x)-1/2*a/(a*x-1)-5/4*a*ln(a*x-1)+1/4*a*ln(a*x+1)
```

maxima [A] time = 0.32, size = 42, normalized size = 0.91

$$\frac{1}{4}\,a\log\,(ax+1)-\frac{5}{4}\,a\log\,(ax-1)+a\log(x)-\frac{3\,ax-2}{2\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*a*log(a*x + 1) - 5/4*a*log(a*x - 1) + a*log(x) - 1/2*(3*a*x - 2)/(a*x^2
 - x)
```

mupad [B] time = 0.89, size = 40, normalized size = 0.87

$$a\,\ln(x)-\frac{5\,a\,\ln\,(a\,x-1)}{4}+\frac{a\,\ln\,(a\,x+1)}{4}+\frac{\frac{3\,a\,x}{2}-1}{x-a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(a^2*x^2 - 1)^2),x)
```

```
[Out] a*log(x) - (5*a*log(a*x - 1))/4 + (a*log(a*x + 1))/4 + ((3*a*x)/2 - 1)/(x -
 a*x^2)
```

sympy [A] time = 0.37, size = 42, normalized size = 0.91

$$a\log\,(x)-\frac{5a\log\left(x-\frac{1}{a}\right)}{4}+\frac{a\log\left(x+\frac{1}{a}\right)}{4}+\frac{-3ax+2}{2ax^2-2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2/x**2,x)
```

```
[Out] a*log(x) - 5*a*log(x - 1/a)/4 + a*log(x + 1/a)/4 + (-3*a*x + 2)/(2*a*x**2 -
 2*x)
```

3.936 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{a^2}{2(1-ax)} + 2a^2\log(x) - \frac{7}{4}a^2\log(1-ax) - \frac{1}{4}a^2\log(ax+1) - \frac{a}{x} - \frac{1}{2x^2}$$

```
[Out] -1/2/x^2-a/x+1/2*a^2/(-a*x+1)+2*a^2*ln(x)-7/4*a^2*ln(-a*x+1)-1/4*a^2*ln(a*x
+1)
```

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{a^2}{2(1-ax)} + 2a^2\log(x) - \frac{7}{4}a^2\log(1-ax) - \frac{1}{4}a^2\log(ax+1) - \frac{a}{x} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -1/(2*x^2) - a/x + a^2/(2*(1 - a*x)) + 2*a^2*Log[x] - (7*a^2*Log[1 - a*x])/
4 - (a^2*Log[1 + a*x])/4
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^3(1-ax)^2(1+ax)}\,dx$$
$$= \int \left(\frac{1}{x^3} + \frac{a}{x^2} + \frac{2a^2}{x} + \frac{a^3}{2(-1+ax)^2} - \frac{7a^3}{4(-1+ax)} - \frac{a^3}{4(1+ax)}\right) dx$$
$$= -\frac{1}{2x^2} - \frac{a}{x} + \frac{a^2}{2(1-ax)} + 2a^2\log(x) - \frac{7}{4}a^2\log(1-ax) - \frac{1}{4}a^2\log(1+ax)$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.94

$$\frac{1}{4}\left(\frac{2a^2}{1-ax} + 8a^2\log(x) - 7a^2\log(1-ax) - a^2\log(ax+1) - \frac{4a}{x} - \frac{2}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8*a^2*Log[x] - 7*a^2*Log[1 - a*x] -
 a^2*Log[1 + a*x])/4
```

fricas [A] time = 0.62, size = 96, normalized size = 1.52

$$-\frac{6\,a^2x^2 - 2\,ax + \left(a^3x^3 - a^2x^2\right)\log\left(ax+1\right) + 7\left(a^3x^3 - a^2x^2\right)\log\left(ax-1\right) - 8\left(a^3x^3 - a^2x^2\right)\log(x) - 2}{4\left(ax^3 - x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^2*x^2 - 2*a*x + (a^3*x^3 - a^2*x^2)*log(a*x + 1) + 7*(a^3*x^3 - a
^2*x^2)*log(a*x - 1) - 8*(a^3*x^3 - a^2*x^2)*log(x) - 2)/(a*x^3 - x^2)
```

giac [A] time = 0.19, size = 59, normalized size = 0.94

$$-\frac{1}{4}\,a^2\log\left(|ax+1|\right) - \frac{7}{4}\,a^2\log\left(|ax-1|\right) + 2\,a^2\log\left(|x|\right) - \frac{3\,a^2x^2 - ax - 1}{2\,(ax-1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/4*a^2*log(abs(a*x + 1)) - 7/4*a^2*log(abs(a*x - 1)) + 2*a^2*log(abs(x))
- 1/2*(3*a^2*x^2 - a*x - 1)/((a*x - 1)*x^2)
```

maple [A] time = 0.04, size = 54, normalized size = 0.86

$$-\frac{1}{2x^2}-\frac{a}{x}+2a^2\ln(x)-\frac{a^2}{2\,(ax-1)}-\frac{7a^2\ln\left(ax-1\right)}{4}-\frac{a^2\ln\left(ax+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2/x^3,x)
```

```
[Out] -1/2/x^2-a/x+2*a^2*ln(x)-1/2*a^2/(a*x-1)-7/4*a^2*ln(a*x-1)-1/4*a^2*ln(a*x+1
)
```

maxima [A] time = 0.31, size = 59, normalized size = 0.94

$$-\frac{1}{4}\,a^2\log\left(ax+1\right)-\frac{7}{4}\,a^2\log\left(ax-1\right)+2\,a^2\log(x)-\frac{3\,a^2x^2-ax-1}{2\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*log(a*x + 1) - 7/4*a^2*log(a*x - 1) + 2*a^2*log(x) - 1/2*(3*a^2*x^
2 - a*x - 1)/(a*x^3 - x^2)
```

mupad [B] time = 0.91, size = 58, normalized size = 0.92

$$2\,a^2\,\ln(x)-\frac{7\,a^2\,\ln\left(a\,x-1\right)}{4}-\frac{a^2\,\ln\left(a\,x+1\right)}{4}+\frac{-\frac{3\,a^2\,x^2}{2}+\frac{a\,x}{2}+\frac{1}{2}}{a\,x^3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] 2*a^2*log(x) - (7*a^2*log(a*x - 1))/4 - (a^2*log(a*x + 1))/4 + ((a*x)/2 - (
3*a^2*x^2)/2 + 1/2)/(a*x^3 - x^2)
```

sympy [A] time = 0.40, size = 58, normalized size = 0.92

$$2a^2\log\left(x\right)-\frac{7a^2\log\left(x-\frac{1}{a}\right)}{4}-\frac{a^2\log\left(x+\frac{1}{a}\right)}{4}+\frac{-3a^2x^2+ax+1}{2ax^3-2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2/x**3,x)
```

```
[Out] 2*a**2*log(x) - 7*a**2*log(x - 1/a)/4 - a**2*log(x + 1/a)/4 + (-3*a**2*x**2
 + a*x + 1)/(2*a*x**3 - 2*x**2)
```

3.937 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=73

$$\frac{a^3}{2(1-ax)} + 2a^3\log(x) - \frac{9}{4}a^3\log(1-ax) + \frac{1}{4}a^3\log(ax+1) - \frac{2a^2}{x} - \frac{a}{2x^2} - \frac{1}{3x^3}$$

```
[Out] -1/3/x^3-1/2*a/x^2-2*a^2/x+1/2*a^3/(-a*x+1)+2*a^3*ln(x)-9/4*a^3*ln(-a*x+1)+
1/4*a^3*ln(a*x+1)
```

Rubi [A] time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{a^3}{2(1-ax)} - \frac{2a^2}{x} + 2a^3\log(x) - \frac{9}{4}a^3\log(1-ax) + \frac{1}{4}a^3\log(ax+1) - \frac{a}{2x^2} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -1/(3*x^3) - a/(2*x^2) - (2*a^2)/x + a^3/(2*(1 - a*x)) + 2*a^3*Log[x] - (9*
a^3*Log[1 - a*x])/4 + (a^3*Log[1 + a*x])/4
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx = \int \frac{1}{x^4(1-ax)^2(1+ax)}\,dx$$
$$= \int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{2a^2}{x^2}+\frac{2a^3}{x}+\frac{a^4}{2(-1+ax)^2}-\frac{9a^4}{4(-1+ax)}+\frac{a^4}{4(1+ax)}\right)dx$$
$$= -\frac{1}{3x^3}-\frac{a}{2x^2}-\frac{2a^2}{x}+\frac{a^3}{2(1-ax)}+2a^3\log(x)-\frac{9}{4}a^3\log(1-ax)+\frac{1}{4}a^3\log(1+ax)$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.92

$$\frac{1}{12}\left(\frac{6a^3}{1-ax}+24a^3\log(x)-27a^3\log(1-ax)+3a^3\log(ax+1)-\frac{24a^2}{x}-\frac{6a}{x^2}-\frac{4}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-4/x^3 - (6*a)/x^2 - (24*a^2)/x + (6*a^3)/(1 - a*x) + 24*a^3*Log[x] - 27*a
^3*Log[1 - a*x] + 3*a^3*Log[1 + a*x])/12
```

fricas [A] time = 0.59, size = 105, normalized size = 1.44

$$-\frac{30\,a^3x^3-18\,a^2x^2-2\,ax-3\left(a^4x^4-a^3x^3\right)\log\left(ax+1\right)+27\left(a^4x^4-a^3x^3\right)\log\left(ax-1\right)-24\left(a^4x^4-a^3x^3\right)\log(x)}{12\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(30*a^3*x^3 - 18*a^2*x^2 - 2*a*x - 3*(a^4*x^4 - a^3*x^3)*log(a*x + 1)
 + 27*(a^4*x^4 - a^3*x^3)*log(a*x - 1) - 24*(a^4*x^4 - a^3*x^3)*log(x) - 4)
/(a*x^4 - x^3)
```

giac [A] time = 0.15, size = 67, normalized size = 0.92

$$\frac{1}{4}\,a^3\log\left(|ax+1|\right)-\frac{9}{4}\,a^3\log\left(|ax-1|\right)+2\,a^3\log\left(|x|\right)-\frac{15\,a^3x^3-9\,a^2x^2-ax-2}{6\,(ax-1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/4*a^3*log(abs(a*x + 1)) - 9/4*a^3*log(abs(a*x - 1)) + 2*a^3*log(abs(x)) -
 1/6*(15*a^3*x^3 - 9*a^2*x^2 - a*x - 2)/((a*x - 1)*x^3)
```

maple [A] time = 0.04, size = 62, normalized size = 0.85

$$-\frac{1}{3x^3}-\frac{a}{2x^2}-\frac{2a^2}{x}+2a^3\ln(x)-\frac{a^3}{2\,(ax-1)}-\frac{9a^3\ln\,(ax-1)}{4}+\frac{a^3\ln\,(ax+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2/x^4,x)
```

```
[Out] -1/3/x^3-1/2*a/x^2-2*a^2/x+2*a^3*ln(x)-1/2*a^3/(a*x-1)-9/4*a^3*ln(a*x-1)+1/
4*a^3*ln(a*x+1)
```

maxima [A] time = 0.32, size = 67, normalized size = 0.92

$$\frac{1}{4}\,a^3\log\,(ax+1)-\frac{9}{4}\,a^3\log\,(ax-1)+2\,a^3\log(x)-\frac{15\,a^3x^3-9\,a^2x^2-ax-2}{6\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*log(a*x + 1) - 9/4*a^3*log(a*x - 1) + 2*a^3*log(x) - 1/6*(15*a^3*x^
3 - 9*a^2*x^2 - a*x - 2)/(a*x^4 - x^3)
```

mupad [B] time = 0.91, size = 66, normalized size = 0.90

$$2\,a^3\,\ln(x)-\frac{9\,a^3\,\ln\,(a\,x-1)}{4}+\frac{a^3\,\ln\,(a\,x+1)}{4}+\frac{-\frac{5\,a^3\,x^3}{2}+\frac{3\,a^2\,x^2}{2}+\frac{a\,x}{6}+\frac{1}{3}}{a\,x^4-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(a^2*x^2 - 1)^2),x)
```

```
[Out] 2*a^3*log(x) - (9*a^3*log(a*x - 1))/4 + (a^3*log(a*x + 1))/4 + ((a*x)/6 + (
3*a^2*x^2)/2 - (5*a^3*x^3)/2 + 1/3)/(a*x^4 - x^3)
```

sympy [A] time = 0.43, size = 66, normalized size = 0.90

$$2a^3\log\,(x)-\frac{9a^3\log\left(x-\frac{1}{a}\right)}{4}+\frac{a^3\log\left(x+\frac{1}{a}\right)}{4}+\frac{-15a^3x^3+9a^2x^2+ax+2}{6ax^4-6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2/x**4,x)
```

```
[Out] 2*a**3*log(x) - 9*a**3*log(x - 1/a)/4 + a**3*log(x + 1/a)/4 + (-15*a**3*x**
3 + 9*a**2*x**2 + a*x + 2)/(6*a*x**4 - 6*x**3)
```

3.938 $\int \frac{e^{\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{5}{4a^7(1-ax)}-\frac{1}{8a^7(ax+1)}+\frac{1}{8a^7(1-ax)^2}-\frac{39\log(1-ax)}{16a^7}-\frac{9\log(ax+1)}{16a^7}-\frac{x}{a^6}-\frac{x^2}{2a^5}$$

```
[Out] -x/a^6-1/2*x^2/a^5+1/8/a^7/(-a*x+1)^2-5/4/a^7/(-a*x+1)-1/8/a^7/(a*x+1)-39/1
6*ln(-a*x+1)/a^7-9/16*ln(a*x+1)/a^7
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$-\frac{x^2}{2a^5}-\frac{x}{a^6}-\frac{5}{4a^7(1-ax)}-\frac{1}{8a^7(ax+1)}+\frac{1}{8a^7(1-ax)^2}-\frac{39\log(1-ax)}{16a^7}-\frac{9\log(ax+1)}{16a^7}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^6)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -(x/a^6) - x^2/(2*a^5) + 1/(8*a^7*(1 - a*x)^2) - 5/(4*a^7*(1 - a*x)) - 1/(8
*a^7*(1 + a*x)) - (39*Log[1 - a*x])/(16*a^7) - (9*Log[1 + a*x])/(16*a^7)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{x^6}{(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(-\frac{1}{a^6} - \frac{x}{a^5} - \frac{1}{4a^6(-1+ax)^3} - \frac{5}{4a^6(-1+ax)^2} - \frac{39}{16a^6(-1+ax)} + \frac{1}{8a^6(1+ax)^2} - \frac{9}{16a^6(1\ldots}\right.$$
$$= -\frac{x}{a^6} - \frac{x^2}{2a^5} + \frac{1}{8a^7(1-ax)^2} - \frac{5}{4a^7(1-ax)} - \frac{1}{8a^7(1+ax)} - \frac{39\log(1-ax)}{16a^7} - \frac{9\log(1+ax)}{16a^7}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 0.74

$$\frac{2\left(-4a^2x^2-8ax+\frac{10}{ax-1}-\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-39\log(1-ax)-9\log(ax+1)}{16a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^6)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (2*(-8*a*x - 4*a^2*x^2 + (-1 + a*x)^(-2) + 10/(-1 + a*x) - (1 + a*x)^(-1))
- 39*Log[1 - a*x] - 9*Log[1 + a*x])/(16*a^7)
```

fricas [A] time = 0.66, size = 125, normalized size = 1.42

$$-\frac{8\,a^5x^5+8\,a^4x^4-24\,a^3x^3-26\,a^2x^2+10\,ax+9\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+39\left(a^3x^3-a^2x^2-ax+1\right)\ldots}{16\left(a^{10}x^3-a^9x^2-a^8x+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^6,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a^5*x^5 + 8*a^4*x^4 - 24*a^3*x^3 - 26*a^2*x^2 + 10*a*x + 9*(a^3*x^
3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 39*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(
a*x - 1) + 20)/(a^10*x^3 - a^9*x^2 - a^8*x + a^7)
```

giac [A] time = 0.19, size = 77, normalized size = 0.88

$$-\frac{9\,\log\left(|ax+1|\right)}{16\,a^7}-\frac{39\,\log\left(|ax-1|\right)}{16\,a^7}-\frac{a^5x^2+2\,a^4x}{2\,a^{10}}+\frac{9\,a^2x^2+3\,ax-10}{8\,(ax+1)(ax-1)^2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^6,x, algorithm="giac")
```

```
[Out] -9/16*log(abs(a*x + 1))/a^7 - 39/16*log(abs(a*x - 1))/a^7 - 1/2*(a^5*x^2 +
2*a^4*x)/a^10 + 1/8*(9*a^2*x^2 + 3*a*x - 10)/((a*x + 1)*(a*x - 1)^2*a^7)
```

maple [A] time = 0.04, size = 74, normalized size = 0.84

$$-\frac{x^2}{2a^5}-\frac{x}{a^6}+\frac{1}{8a^7\left(ax-1\right)^2}+\frac{5}{4a^7\left(ax-1\right)}-\frac{39\ln\left(ax-1\right)}{16a^7}-\frac{1}{8a^7\left(ax+1\right)}-\frac{9\ln\left(ax+1\right)}{16a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^6,x)
```

```
[Out] -1/2*x^2/a^5-x/a^6+1/8/a^7/(a*x-1)^2+5/4/a^7/(a*x-1)-39/16/a^7*ln(a*x-1)-1/
8/a^7/(a*x+1)-9/16*ln(a*x+1)/a^7
```

maxima [A] time = 0.32, size = 80, normalized size = 0.91

$$\frac{9\,a^2x^2+3\,ax-10}{8\,\left(a^{10}x^3-a^9x^2-a^8x+a^7\right)}-\frac{ax^2+2\,x}{2\,a^6}-\frac{9\,\log\left(ax+1\right)}{16\,a^7}-\frac{39\,\log\left(ax-1\right)}{16\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^6,x, algorithm="maxima")
```

```
[Out] 1/8*(9*a^2*x^2 + 3*a*x - 10)/(a^10*x^3 - a^9*x^2 - a^8*x + a^7) - 1/2*(a*x^
2 + 2*x)/a^6 - 9/16*log(a*x + 1)/a^7 - 39/16*log(a*x - 1)/a^7
```

mupad [B] time = 0.08, size = 82, normalized size = 0.93

$$-\frac{39\,\ln\left(a\,x-1\right)}{16\,a^7}-\frac{9\,\ln\left(a\,x+1\right)}{16\,a^7}-\frac{x}{a^6}-\frac{\frac{3\,x}{8}+\frac{9\,a\,x^2}{8}-\frac{5}{4\,a}}{-a^9\,x^3+a^8\,x^2+a^7\,x-a^6}-\frac{x^2}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6*(a*x + 1))/(a^2*x^2 - 1)^3,x)
```

```
[Out] - (39*log(a*x - 1))/(16*a^7) - (9*log(a*x + 1))/(16*a^7) - x/a^6 - ((3*x)/8
 + (9*a*x^2)/8 - 5/(4*a))/(a^7*x - a^6 + a^8*x^2 - a^9*x^3) - x^2/(2*a^5)
```

sympy [A] time = 0.44, size = 82, normalized size = 0.93

$$-\frac{-9a^2x^2-3ax+10}{8a^{10}x^3-8a^9x^2-8a^8x+8a^7}-\frac{x^2}{2a^5}-\frac{x}{a^6}-\frac{3\left(\frac{13\log\left(x-\frac{1}{a}\right)}{16}+\frac{3\log\left(x+\frac{1}{a}\right)}{16}\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**6,x)
```

```
[Out] -(-9*a**2*x**2 - 3*a*x + 10)/(8*a**10*x**3 - 8*a**9*x**2 - 8*a**8*x + 8*a**
7) - x**2/(2*a**5) - x/a**6 - 3*(13*log(x - 1/a)/16 + 3*log(x + 1/a)/16)/a*
*7
```

3.939 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{a^6(1-ax)}+\frac{1}{8a^6(ax+1)}+\frac{1}{8a^6(1-ax)^2}-\frac{23\log(1-ax)}{16a^6}+\frac{7\log(ax+1)}{16a^6}-\frac{x}{a^5}$$

```
[Out] -x/a^5+1/8/a^6/(-a*x+1)^2-1/a^6/(-a*x+1)+1/8/a^6/(a*x+1)-23/16*ln(-a*x+1)/a
^6+7/16*ln(a*x+1)/a^6
```

Rubi [A] time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$-\frac{x}{a^5}-\frac{1}{a^6(1-ax)}+\frac{1}{8a^6(ax+1)}+\frac{1}{8a^6(1-ax)^2}-\frac{23\log(1-ax)}{16a^6}+\frac{7\log(ax+1)}{16a^6}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -(x/a^5) + 1/(8*a^6*(1 - a*x)^2) - 1/(a^6*(1 - a*x)) + 1/(8*a^6*(1 + a*x))
- (23*Log[1 - a*x])/(16*a^6) + (7*Log[1 + a*x])/(16*a^6)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{x^5}{(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(-\frac{1}{a^5} - \frac{1}{4a^5(-1+ax)^3} - \frac{1}{a^5(-1+ax)^2} - \frac{23}{16a^5(-1+ax)} - \frac{1}{8a^5(1+ax)^2} + \frac{7}{16a^5(1+ax)}\right.$$
$$= -\frac{x}{a^5} + \frac{1}{8a^6(1-ax)^2} - \frac{1}{a^6(1-ax)} + \frac{1}{8a^6(1+ax)} - \frac{23\log(1-ax)}{16a^6} + \frac{7\log(1+ax)}{16a^6}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.72

$$\frac{2\left(-8ax+\frac{8}{ax-1}+\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-23\log(1-ax)+7\log(ax+1)}{16a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (2*(-8*a*x + (-1 + a*x)^(-2) + 8/(-1 + a*x) + (1 + a*x)^(-1)) - 23*Log[1 -
a*x] + 7*Log[1 + a*x])/(16*a^6)
```

fricas [A] time = 0.54, size = 117, normalized size = 1.54

$$-\frac{16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+23\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax\right.}{16\left(a^9x^3-a^8x^2-a^7x+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) +
12)/(a^9*x^3 - a^8*x^2 - a^7*x + a^6)
```

giac [A] time = 0.17, size = 64, normalized size = 0.84

$$-\frac{x}{a^5}+\frac{7\,\log\left(|ax+1|\right)}{16\,a^6}-\frac{23\,\log\left(|ax-1|\right)}{16\,a^6}+\frac{9\,a^2x^2-ax-6}{8\,(ax+1)(ax-1)^2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^5,x, algorithm="giac")
```

```
[Out] -x/a^5 + 7/16*log(abs(a*x + 1))/a^6 - 23/16*log(abs(a*x - 1))/a^6 + 1/8*(9*
a^2*x^2 - a*x - 6)/((a*x + 1)*(a*x - 1)^2*a^6)
```

maple [A] time = 0.04, size = 65, normalized size = 0.86

$$-\frac{x}{a^5}-\frac{23\ln(ax-1)}{16a^6}+\frac{1}{8a^6(ax-1)^2}+\frac{1}{a^6(ax-1)}+\frac{1}{8a^6(ax+1)}+\frac{7\ln(ax+1)}{16a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^5,x)
```

```
[Out] -x/a^5-23/16/a^6*ln(a*x-1)+1/8/a^6/(a*x-1)^2+1/a^6/(a*x-1)+1/8/a^6/(a*x+1)+
7/16*ln(a*x+1)/a^6
```

maxima [A] time = 0.33, size = 72, normalized size = 0.95

$$\frac{9\,a^2x^2-ax-6}{8\left(a^9x^3-a^8x^2-a^7x+a^6\right)}-\frac{x}{a^5}+\frac{7\,\log(ax+1)}{16\,a^6}-\frac{23\,\log(ax-1)}{16\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(9*a^2*x^2 - a*x - 6)/(a^9*x^3 - a^8*x^2 - a^7*x + a^6) - x/a^5 + 7/16*
log(a*x + 1)/a^6 - 23/16*log(a*x - 1)/a^6
```

mupad [B] time = 0.93, size = 73, normalized size = 0.96

$$\frac{7\,\ln(a\,x+1)}{16\,a^6}-\frac{23\,\ln(a\,x-1)}{16\,a^6}-\frac{x}{a^5}+\frac{\frac{x}{8}-\frac{9\,a\,x^2}{8}+\frac{3}{4\,a}}{-a^8\,x^3+a^7\,x^2+a^6\,x-a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5*(a*x + 1))/(a^2*x^2 - 1)^3,x)
```

```
[Out] (7*log(a*x + 1))/(16*a^6) - (23*log(a*x - 1))/(16*a^6) - x/a^5 + (x/8 - (9*
a*x^2)/8 + 3/(4*a))/(a^6*x - a^5 + a^7*x^2 - a^8*x^3)
```

sympy [A] time = 0.43, size = 71, normalized size = 0.93

$$-\frac{-9a^2x^2+ax+6}{8a^9x^3-8a^8x^2-8a^7x+8a^6}-\frac{x}{a^5}-\frac{\frac{23\log\left(x-\frac{1}{a}\right)}{16}-\frac{7\log\left(x+\frac{1}{a}\right)}{16}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**5,x)

[Out] -(-9*a**2*x**2 + a*x + 6)/(8*a**9*x**3 - 8*a**8*x**2 - 8*a**7*x + 8*a**6) -
 x/a**5 - (23*log(x - 1/a)/16 - 7*log(x + 1/a)/16)/a**6
```

3.940 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{3}{4a^5(1-ax)}-\frac{1}{8a^5(ax+1)}+\frac{1}{8a^5(1-ax)^2}-\frac{11\log(1-ax)}{16a^5}-\frac{5\log(ax+1)}{16a^5}$$

```
[Out] 1/8/a^5/(-a*x+1)^2-3/4/a^5/(-a*x+1)-1/8/a^5/(a*x+1)-11/16*ln(-a*x+1)/a^5-5/
16*ln(a*x+1)/a^5
```

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$-\frac{3}{4a^5(1-ax)}-\frac{1}{8a^5(ax+1)}+\frac{1}{8a^5(1-ax)^2}-\frac{11\log(1-ax)}{16a^5}-\frac{5\log(ax+1)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a^5*(1 - a*x)^2) - 3/(4*a^5*(1 - a*x)) - 1/(8*a^5*(1 + a*x)) - (11*Log
[1 - a*x])/(16*a^5) - (5*Log[1 + a*x])/(16*a^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{x^4}{(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(-\frac{1}{4a^4(-1+ax)^3} - \frac{3}{4a^4(-1+ax)^2} - \frac{11}{16a^4(-1+ax)} + \frac{1}{8a^4(1+ax)^2} - \frac{5}{16a^4(1+ax)}\right) dx$$
$$= \frac{1}{8a^5(1-ax)^2} - \frac{3}{4a^5(1-ax)} - \frac{1}{8a^5(1+ax)} - \frac{11\log(1-ax)}{16a^5} - \frac{5\log(1+ax)}{16a^5}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 0.76

$$\frac{\frac{2\left(5a^2x^2+3ax-6\right)}{(ax-1)^2(ax+1)} - 11\log(1-ax) - 5\log(ax+1)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] ((2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*(1 + a*x)) - 11*Log[1 - a*x] -
5*Log[1 + a*x])/(16*a^5)
```

fricas [A] time = 0.50, size = 101, normalized size = 1.40

$$\frac{10\,a^2x^2 + 6\,ax - 5\left(a^3x^3 - a^2x^2 - ax + 1\right)\log\left(ax+1\right) - 11\left(a^3x^3 - a^2x^2 - ax + 1\right)\log\left(ax-1\right) - 12}{16\left(a^8x^3 - a^7x^2 - a^6x + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^4,x, algorithm="fricas")
```

```
[Out] 1/16*(10*a^2*x^2 + 6*a*x - 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) - 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 12)/(a^8*x^3 - a^7*x^2 - a^6
*x + a^5)
```

giac [A] time = 0.27, size = 58, normalized size = 0.81

$$-\frac{5\,\log\left(|ax+1|\right)}{16\,a^5} - \frac{11\,\log\left(|ax-1|\right)}{16\,a^5} + \frac{5\,a^2x^2 + 3\,ax - 6}{8\,(ax+1)(ax-1)^2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^4,x, algorithm="giac")
```

```
[Out] -5/16*log(abs(a*x + 1))/a^5 - 11/16*log(abs(a*x - 1))/a^5 + 1/8*(5*a^2*x^2
+ 3*a*x - 6)/((a*x + 1)*(a*x - 1)^2*a^5)
```

maple [A] time = 0.04, size = 60, normalized size = 0.83

$$\frac{1}{8a^5(ax-1)^2}+\frac{3}{4a^5(ax-1)}-\frac{11\ln(ax-1)}{16a^5}-\frac{1}{8a^5(ax+1)}-\frac{5\ln(ax+1)}{16a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^4,x)
```

```
[Out] 1/8/a^5/(a*x-1)^2+3/4/a^5/(a*x-1)-11/16/a^5*ln(a*x-1)-1/8/a^5/(a*x+1)-5/16*
ln(a*x+1)/a^5
```

maxima [A] time = 0.32, size = 66, normalized size = 0.92

$$\frac{5\,a^2x^2+3\,ax-6}{8\left(a^8x^3-a^7x^2-a^6x+a^5\right)}-\frac{5\,\log(ax+1)}{16\,a^5}-\frac{11\,\log(ax-1)}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^4,x, algorithm="maxima")
```

```
[Out] 1/8*(5*a^2*x^2 + 3*a*x - 6)/(a^8*x^3 - a^7*x^2 - a^6*x + a^5) - 5/16*log(a*
x + 1)/a^5 - 11/16*log(a*x - 1)/a^5
```

mupad [B] time = 0.18, size = 67, normalized size = 0.93

$$-\frac{11\,\ln(a\,x-1)}{16\,a^5}-\frac{5\,\ln(a\,x+1)}{16\,a^5}-\frac{\frac{3\,x}{8\,a^4}-\frac{3}{4\,a^5}+\frac{5\,x^2}{8\,a^3}}{-a^3\,x^3+a^2\,x^2+a\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(a*x + 1))/(a^2*x^2 - 1)^3,x)
```

```
[Out] - (11*log(a*x - 1))/(16*a^5) - (5*log(a*x + 1))/(16*a^5) - ((3*x)/(8*a^4) -
 3/(4*a^5) + (5*x^2)/(8*a^3))/(a*x + a^2*x^2 - a^3*x^3 - 1)
```

sympy [A] time = 0.38, size = 68, normalized size = 0.94

$$-\frac{-5a^2x^2-3ax+6}{8a^8x^3-8a^7x^2-8a^6x+8a^5}-\frac{\frac{11\log\left(x-\frac{1}{a}\right)}{16}+\frac{5\log\left(x+\frac{1}{a}\right)}{16}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**4,x)

[Out] -(-5*a**2*x**2 - 3*a*x + 6)/(8*a**8*x**3 - 8*a**7*x**2 - 8*a**6*x + 8*a**5)
 - (11*log(x - 1/a)/16 + 5*log(x + 1/a)/16)/a**5
```

3.941 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(ax+1)}+\frac{1}{8a^4(1-ax)^2}+\frac{3\tanh^{-1}(ax)}{8a^4}$$

```
[Out] 1/8/a^4/(-a*x+1)^2-1/2/a^4/(-a*x+1)+1/8/a^4/(a*x+1)+3/8*arctanh(a*x)/a^4
```

Rubi [A] time = 0.12, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6150, 88, 207}

$$-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(ax+1)}+\frac{1}{8a^4(1-ax)^2}+\frac{3\tanh^{-1}(ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a^4*(1 - a*x)^2) - 1/(2*a^4*(1 - a*x)) + 1/(8*a^4*(1 + a*x)) + (3*ArcT
anh[a*x])/(8*a^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{x^3}{(1-ax)^3(1+ax)^2}\,dx \\
&= \int \left(-\frac{1}{4a^3(-1+ax)^3}-\frac{1}{2a^3(-1+ax)^2}-\frac{1}{8a^3(1+ax)^2}-\frac{3}{8a^3\left(-1+a^2x^2\right)}\right)dx \\
&= \frac{1}{8a^4(1-ax)^2}-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(1+ax)}-\frac{3\int \frac{1}{-1+a^2x^2}\,dx}{8a^3} \\
&= \frac{1}{8a^4(1-ax)^2}-\frac{1}{2a^4(1-ax)}+\frac{1}{8a^4(1+ax)}+\frac{3\tanh^{-1}(ax)}{8a^4}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.95

$$\frac{5a^2x^2-ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)-2}{8a^4(ax-1)^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-2 - a*x + 5*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTanh[a*x])/(8*a^4*(-1 +
 a*x)^2*(1 + a*x))
```

fricas [B] time = 0.74, size = 101, normalized size = 1.80

$$\frac{10\,a^2x^2-2\,ax+3\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)-3\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-1\right)-4}{16\left(a^7x^3-a^6x^2-a^5x+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(10*a^2*x^2 - 2*a*x + 3*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) - 3
*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 4)/(a^7*x^3 - a^6*x^2 - a^5*x
 + a^4)
```

giac [A] time = 0.19, size = 58, normalized size = 1.04

$$\frac{3\,\log\left(|ax+1|\right)}{16\,a^4}-\frac{3\,\log\left(|ax-1|\right)}{16\,a^4}+\frac{5\,a^2x^2-ax-2}{8\,(ax+1)(ax-1)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(a*x + 1))/a^4 - 3/16*log(abs(a*x - 1))/a^4 + 1/8*(5*a^2*x^2 -
a*x - 2)/((a*x + 1)*(a*x - 1)^2*a^4)
```

maple [A] time = 0.04, size = 60, normalized size = 1.07

$$\frac{1}{8a^4\left(ax-1\right)^2}+\frac{1}{2a^4\left(ax-1\right)}-\frac{3\ln\left(ax-1\right)}{16a^4}+\frac{1}{8a^4\left(ax+1\right)}+\frac{3\ln\left(ax+1\right)}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^3,x)
```

```
[Out] 1/8/a^4/(a*x-1)^2+1/2/a^4/(a*x-1)-3/16/a^4*ln(a*x-1)+1/8/a^4/(a*x+1)+3/16*l
n(a*x+1)/a^4
```

maxima [A] time = 0.31, size = 66, normalized size = 1.18

$$\frac{5\,a^2x^2-ax-2}{8\left(a^7x^3-a^6x^2-a^5x+a^4\right)}+\frac{3\,\log\left(ax+1\right)}{16\,a^4}-\frac{3\,\log\left(ax-1\right)}{16\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(5*a^2*x^2 - a*x - 2)/(a^7*x^3 - a^6*x^2 - a^5*x + a^4) + 3/16*log(a*x
+ 1)/a^4 - 3/16*log(a*x - 1)/a^4
```

mupad [B] time = 0.06, size = 53, normalized size = 0.95

$$\frac{\frac{x}{8\,a^3}+\frac{1}{4\,a^4}-\frac{5\,x^2}{8\,a^2}}{-a^3\,x^3+a^2\,x^2+a\,x-1}+\frac{3\,\mathrm{atanh}\left(a\,x\right)}{8\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a*x + 1))/(a^2*x^2 - 1)^3,x)
```

```
[Out] (x/(8*a^3) + 1/(4*a^4) - (5*x^2)/(8*a^2))/(a*x + a^2*x^2 - a^3*x^3 - 1) + (
3*atanh(a*x))/(8*a^4)
```

sympy [A] time = 0.31, size = 66, normalized size = 1.18

$$-\frac{-5a^2x^2+ax+2}{8a^7x^3-8a^6x^2-8a^5x+8a^4}-\frac{\frac{3\log\left(x-\frac{1}{a}\right)}{16}-\frac{3\log\left(x+\frac{1}{a}\right)}{16}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**3,x)
```

```
[Out] -(-5*a**2*x**2 + a*x + 2)/(8*a**7*x**3 - 8*a**6*x**2 - 8*a**5*x + 8*a**4) -
 (3*log(x - 1/a)/16 - 3*log(x + 1/a)/16)/a**4
```

3.942 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{4a^3(1-ax)}-\frac{1}{8a^3(ax+1)}+\frac{1}{8a^3(1-ax)^2}-\frac{\tanh^{-1}(ax)}{8a^3}$$

```
[Out] 1/8/a^3/(-a*x+1)^2-1/4/a^3/(-a*x+1)-1/8/a^3/(a*x+1)-1/8*arctanh(a*x)/a^3
```

Rubi [A] time = 0.12, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6150, 88, 207}

$$-\frac{1}{4a^3(1-ax)}-\frac{1}{8a^3(ax+1)}+\frac{1}{8a^3(1-ax)^2}-\frac{\tanh^{-1}(ax)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a^3*(1 - a*x)^2) - 1/(4*a^3*(1 - a*x)) - 1/(8*a^3*(1 + a*x)) - ArcTanh
[a*x]/(8*a^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{5/2}}\,dx &= \int \frac{x^2}{(1-ax)^3(1+ax)^2}\,dx\\
&= \int \left(-\frac{1}{4a^2(-1+ax)^3}-\frac{1}{4a^2(-1+ax)^2}+\frac{1}{8a^2(1+ax)^2}+\frac{1}{8a^2\left(-1+a^2x^2\right)}\right)dx\\
&= \frac{1}{8a^3(1-ax)^2}-\frac{1}{4a^3(1-ax)}-\frac{1}{8a^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{8a^2}\\
&= \frac{1}{8a^3(1-ax)^2}-\frac{1}{4a^3(1-ax)}-\frac{1}{8a^3(1+ax)}-\frac{\tanh^{-1}(ax)}{8a^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.93

$$\frac{a^2x^2+3ax-(ax-1)^2(ax+1)\tanh^{-1}(ax)-2}{8a^3(ax-1)^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-2 + 3*a*x + a^2*x^2 - (-1 + a*x)^2*(1 + a*x)*ArcTanh[a*x])/(8*a^3*(-1 + a
*x)^2*(1 + a*x))
```

fricas [B] time = 0.72, size = 100, normalized size = 1.79

$$\frac{2\,a^2x^2+6\,ax-\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-1\right)-4}{16\left(a^6x^3-a^5x^2-a^4x+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*a^2*x^2 + 6*a*x - (a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + (a^3
*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 4)/(a^6*x^3 - a^5*x^2 - a^4*x + a^
3)
```

giac [A] time = 0.16, size = 57, normalized size = 1.02

$$-\frac{\log\left(|ax+1|\right)}{16\,a^3}+\frac{\log\left(|ax-1|\right)}{16\,a^3}+\frac{a^2x^2+3\,ax-2}{8\,(ax+1)(ax-1)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^2,x, algorithm="giac")
```

```
[Out] -1/16*log(abs(a*x + 1))/a^3 + 1/16*log(abs(a*x - 1))/a^3 + 1/8*(a^2*x^2 + 3
*a*x - 2)/((a*x + 1)*(a*x - 1)^2*a^3)
```

maple [A] time = 0.04, size = 60, normalized size = 1.07

$$\frac{1}{8a^3(ax-1)^2}+\frac{1}{4a^3(ax-1)}+\frac{\ln(ax-1)}{16a^3}-\frac{1}{8a^3(ax+1)}-\frac{\ln(ax+1)}{16a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x^2,x)
```

```
[Out] 1/8/a^3/(a*x-1)^2+1/4/a^3/(a*x-1)+1/16/a^3*ln(a*x-1)-1/8/a^3/(a*x+1)-1/16/a
^3*ln(a*x+1)
```

maxima [A] time = 0.32, size = 65, normalized size = 1.16

$$\frac{a^2x^2+3\,ax-2}{8\left(a^6x^3-a^5x^2-a^4x+a^3\right)}-\frac{\log(ax+1)}{16\,a^3}+\frac{\log(ax-1)}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*x^2 + 3*a*x - 2)/(a^6*x^3 - a^5*x^2 - a^4*x + a^3) - 1/16*log(a*x
+ 1)/a^3 + 1/16*log(a*x - 1)/a^3
```

mupad [B] time = 0.06, size = 54, normalized size = 0.96

$$-\frac{\frac{3\,x}{8\,a^2}-\frac{1}{4\,a^3}+\frac{x^2}{8\,a}}{-a^3\,x^3+a^2\,x^2+a\,x-1}-\frac{\operatorname{atanh}(a\,x)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1))/(a^2*x^2 - 1)^3,x)
```

```
[Out] - ((3*x)/(8*a^2) - 1/(4*a^3) + x^2/(8*a))/(a*x + a^2*x^2 - a^3*x^3 - 1) - a
tanh(a*x)/(8*a^3)
```

sympy [A] time = 0.29, size = 63, normalized size = 1.12

$$-\frac{-a^2x^2-3ax+2}{8a^6x^3-8a^5x^2-8a^4x+8a^3}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16}+\frac{\log\left(x+\frac{1}{a}\right)}{16}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**2,x)
```

```
[Out] -(-a**2*x**2 - 3*a*x + 2)/(8*a**6*x**3 - 8*a**5*x**2 - 8*a**4*x + 8*a**3) -
 (-log(x - 1/a)/16 + log(x + 1/a)/16)/a**3
```

3.943 $\int \frac{e^{\tanh^{-1}(ax)}x}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{1}{8a^2(ax+1)}+\frac{1}{8a^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{8a^2}$$

```
[Out] 1/8/a^2/(-a*x+1)^2+1/8/a^2/(a*x+1)-1/8*arctanh(a*x)/a^2
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6150, 77, 207}

$$\frac{1}{8a^2(ax+1)}+\frac{1}{8a^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a^2*(1 - a*x)^2) + 1/(8*a^2*(1 + a*x)) - ArcTanh[a*x]/(8*a^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{x}{(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(-\frac{1}{4a(-1+ax)^3}-\frac{1}{8a(1+ax)^2}+\frac{1}{8a\left(-1+a^2x^2\right)}\right)dx$$
$$= \frac{1}{8a^2(1-ax)^2}+\frac{1}{8a^2(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{8a}$$
$$= \frac{1}{8a^2(1-ax)^2}+\frac{1}{8a^2(1+ax)}-\frac{\tanh^{-1}(ax)}{8a^2}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.68

$$\frac{\frac{1}{ax+1}+\frac{1}{(ax-1)^2}-\tanh^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] ((-1 + a*x)^(-2) + (1 + a*x)^(-1) - ArcTanh[a*x])/(8*a^2)
```

fricas [B] time = 0.53, size = 100, normalized size = 2.44

$$\frac{2\,a^2x^2-2\,ax-\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-1\right)+4}{16\left(a^5x^3-a^4x^2-a^3x+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x,x, algorithm="fricas")
```

```
[Out] 1/16*(2*a^2*x^2 - 2*a*x - (a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + (a^3
*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) + 4)/(a^5*x^3 - a^4*x^2 - a^3*x + a^
2)
```

giac [A] time = 0.16, size = 57, normalized size = 1.39

$$-\frac{\log\left(|ax+1|\right)}{16\,a^2}+\frac{\log\left(|ax-1|\right)}{16\,a^2}+\frac{a^2x^2-ax+2}{8\,(ax+1)(ax-1)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x,x, algorithm="giac")
```

```
[Out] -1/16*log(abs(a*x + 1))/a^2 + 1/16*log(abs(a*x - 1))/a^2 + 1/8*(a^2*x^2 - a
*x + 2)/((a*x + 1)*(a*x - 1)^2*a^2)
```

maple [A] time = 0.04, size = 48, normalized size = 1.17

$$\frac{1}{8a^2\,(ax-1)^2}+\frac{\ln(ax-1)}{16a^2}+\frac{1}{8a^2\,(ax+1)}-\frac{\ln(ax+1)}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3*x,x)
```

```
[Out] 1/8/a^2/(a*x-1)^2+1/16/a^2*ln(a*x-1)+1/8/a^2/(a*x+1)-1/16/a^2*ln(a*x+1)
```

maxima [A] time = 0.32, size = 65, normalized size = 1.59

$$\frac{a^2x^2-ax+2}{8\left(a^5x^3-a^4x^2-a^3x+a^2\right)}-\frac{\log(ax+1)}{16\,a^2}+\frac{\log(ax-1)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*x^2 - a*x + 2)/(a^5*x^3 - a^4*x^2 - a^3*x + a^2) - 1/16*log(a*x +
1)/a^2 + 1/16*log(a*x - 1)/a^2
```

mupad [B] time = 0.93, size = 51, normalized size = 1.24

$$-\frac{\frac{1}{4\,a^2}-\frac{x}{8\,a}+\frac{x^2}{8}}{-a^3\,x^3+a^2\,x^2+a\,x-1}-\frac{\mathrm{atanh}\left(a\,x\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x + 1))/(a^2*x^2 - 1)^3,x)
```

```
[Out] - (1/(4*a^2) - x/(8*a) + x^2/8)/(a*x + a^2*x^2 - a^3*x^3 - 1) - atanh(a*x)/
(8*a^2)
```

sympy [A] time = 0.29, size = 61, normalized size = 1.49

$$-\frac{-a^2x^2+ax-2}{8a^5x^3-8a^4x^2-8a^3x+8a^2}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16}+\frac{\log\left(x+\frac{1}{a}\right)}{16}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x,x)
```

```
[Out] -(-a**2*x**2 + a*x - 2)/(8*a**5*x**3 - 8*a**4*x**2 - 8*a**3*x + 8*a**2) - (
-log(x - 1/a)/16 + log(x + 1/a)/16)/a**2
```

3.944 $\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=56

$$\frac{1}{4a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{1}{8a(1-ax)^2}+\frac{3\tanh^{-1}(ax)}{8a}$$

```
[Out] 1/8/a/(-a*x+1)^2+1/4/a/(-a*x+1)-1/8/a/(a*x+1)+3/8*arctanh(a*x)/a
```

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6140, 44, 207}

$$\frac{1}{4a(1-ax)}-\frac{1}{8a(ax+1)}+\frac{1}{8a(1-ax)^2}+\frac{3\tanh^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] 1/(8*a*(1 - a*x)^2) + 1/(4*a*(1 - a*x)) - 1/(8*a*(1 + a*x)) + (3*ArcTanh[a*
x])/(8*a)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{1}{(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(-\frac{1}{4(-1+ax)^3} + \frac{1}{4(-1+ax)^2} + \frac{1}{8(1+ax)^2} - \frac{3}{8\left(-1+a^2x^2\right)}\right) dx$$
$$= \frac{1}{8a(1-ax)^2} + \frac{1}{4a(1-ax)} - \frac{1}{8a(1+ax)} - \frac{3}{8}\int \frac{1}{-1+a^2x^2}\,dx$$
$$= \frac{1}{8a(1-ax)^2} + \frac{1}{4a(1-ax)} - \frac{1}{8a(1+ax)} + \frac{3\tanh^{-1}(ax)}{8a}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.95

$$\frac{-3a^2x^2 + 3ax + 3(ax-1)^2(ax+1)\tanh^{-1}(ax) + 2}{8a(ax-1)^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTanh[a*x])/(8*a*(-1 +
a*x)^2*(1 + a*x))
```

fricas [B] time = 0.60, size = 99, normalized size = 1.77

$$-\frac{6\,a^2x^2 - 6\,ax - 3\left(a^3x^3 - a^2x^2 - ax + 1\right)\log\left(ax+1\right) + 3\left(a^3x^3 - a^2x^2 - ax + 1\right)\log\left(ax-1\right) - 4}{16\left(a^4x^3 - a^3x^2 - a^2x + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a^2*x^2 - 6*a*x - 3*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 3
*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 4)/(a^4*x^3 - a^3*x^2 - a^2*x
 + a)
```

giac [A] time = 0.33, size = 58, normalized size = 1.04

$$\frac{3\,\log\left(|ax+1|\right)}{16\,a} - \frac{3\,\log\left(|ax-1|\right)}{16\,a} - \frac{3\,a^2x^2 - 3\,ax - 2}{8\,(ax+1)(ax-1)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(a*x + 1))/a - 3/16*log(abs(a*x - 1))/a - 1/8*(3*a^2*x^2 - 3*a*
x - 2)/((a*x + 1)*(a*x - 1)^2*a)
```

maple [A] time = 0.03, size = 60, normalized size = 1.07

$$\frac{1}{8a(ax-1)^2}-\frac{1}{4a(ax-1)}-\frac{3\ln(ax-1)}{16a}-\frac{1}{8a(ax+1)}+\frac{3\ln(ax+1)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/8/a/(a*x-1)^2-1/4/a/(a*x-1)-3/16/a*ln(a*x-1)-1/8/a/(a*x+1)+3/16*ln(a*x+1)
/a
```

maxima [A] time = 0.31, size = 64, normalized size = 1.14

$$-\frac{3\,a^2x^2-3\,ax-2}{8\left(a^4x^3-a^3x^2-a^2x+a\right)}+\frac{3\,\log(ax+1)}{16\,a}-\frac{3\,\log(ax-1)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*a^2*x^2 - 3*a*x - 2)/(a^4*x^3 - a^3*x^2 - a^2*x + a) + 3/16*log(a*x
 + 1)/a - 3/16*log(a*x - 1)/a
```

mupad [B] time = 0.06, size = 49, normalized size = 0.88

$$\frac{3\,\mathrm{atanh}(a\,x)}{8\,a}-\frac{\frac{3\,x}{8}-\frac{3\,a\,x^2}{8}+\frac{1}{4\,a}}{-a^3\,x^3+a^2\,x^2+a\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(a^2*x^2 - 1)^3,x)
```

```
[Out] (3*atanh(a*x))/(8*a) - ((3*x)/8 - (3*a*x^2)/8 + 1/(4*a))/(a*x + a^2*x^2 - a
^3*x^3 - 1)
```

sympy [A] time = 0.30, size = 65, normalized size = 1.16

$$-\frac{3a^2x^2-3ax-2}{8a^4x^3-8a^3x^2-8a^2x+8a}-\frac{\frac{3\log\left(x-\frac{1}{a}\right)}{16}-\frac{3\log\left(x+\frac{1}{a}\right)}{16}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3,x)
```

```
[Out] -(3*a**2*x**2 - 3*a*x - 2)/(8*a**4*x**3 - 8*a**3*x**2 - 8*a**2*x + 8*a) - (
3*log(x - 1/a)/16 - 3*log(x + 1/a)/16)/a
```

3.945 $\int \frac{e^{\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=59

$$\frac{1}{2(1-ax)}+\frac{1}{8(ax+1)}+\frac{1}{8(1-ax)^2}-\frac{11}{16}\log(1-ax)-\frac{5}{16}\log(ax+1)+\log(x)$$

```
[Out] 1/8/(-a*x+1)^2+1/2/(-a*x+1)+1/8/(a*x+1)+ln(x)-11/16*ln(-a*x+1)-5/16*ln(a*x+
1)
```

Rubi [A] time = 0.12, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{1}{2(1-ax)}+\frac{1}{8(ax+1)}+\frac{1}{8(1-ax)^2}-\frac{11}{16}\log(1-ax)-\frac{5}{16}\log(ax+1)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] 1/(8*(1 - a*x)^2) + 1/(2*(1 - a*x)) + 1/(8*(1 + a*x)) + Log[x] - (11*Log[1
- a*x])/16 - (5*Log[1 + a*x])/16
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(\frac{1}{x} - \frac{a}{4(-1+ax)^3} + \frac{a}{2(-1+ax)^2} - \frac{11a}{16(-1+ax)} - \frac{a}{8(1+ax)^2} - \frac{5a}{16(1+ax)}\right)dx$$
$$= \frac{1}{8(1-ax)^2} + \frac{1}{2(1-ax)} + \frac{1}{8(1+ax)} + \log(x) - \frac{11}{16}\log(1-ax) - \frac{5}{16}\log(1+ax)$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.92

$$\frac{1}{16}\left(\frac{8}{1-ax} + \frac{2}{ax+1} + \frac{2}{(ax-1)^2} - 11\log(1-ax) - 5\log(ax+1) + 16\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] (8/(1 - a*x) + 2/(-1 + a*x)^2 + 2/(1 + a*x) + 16*Log[x] - 11*Log[1 - a*x] -
 5*Log[1 + a*x])/16
```

fricas [B] time = 0.59, size = 122, normalized size = 2.07

$$-\frac{6\,a^2x^2 + 2\,ax + 5\left(a^3x^3 - a^2x^2 - ax + 1\right)\log\left(ax+1\right) + 11\left(a^3x^3 - a^2x^2 - ax + 1\right)\log\left(ax-1\right) - 16\left(a^3x^3 - a^2x^2\right.}{16\left(a^3x^3 - a^2x^2 - ax + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a^2*x^2 + 2*a*x + 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 16*(a^3*x^3 - a^2*x^2 - a*x
+ 1)*log(x) - 12)/(a^3*x^3 - a^2*x^2 - a*x + 1)
```

giac [A] time = 0.18, size = 51, normalized size = 0.86

$$-\frac{3\,a^2x^2 + ax - 6}{8\,(ax+1)(ax-1)^2} - \frac{5}{16}\,\log\left(|ax+1|\right) - \frac{11}{16}\,\log\left(|ax-1|\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^2*x^2 + a*x - 6)/((a*x + 1)*(a*x - 1)^2) - 5/16*log(abs(a*x + 1))
 - 11/16*log(abs(a*x - 1)) + log(abs(x))
```

maple [A] time = 0.04, size = 47, normalized size = 0.80

$$\ln(x)+\frac{1}{8\left(ax-1\right)^{2}}-\frac{1}{2\left(ax-1\right)}-\frac{11\ln\left(ax-1\right)}{16}+\frac{1}{8ax+8}-\frac{5\ln\left(ax+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3/x,x)
```

```
[Out] ln(x)+1/8/(a*x-1)^2-1/2/(a*x-1)-11/16*ln(a*x-1)+1/8/(a*x+1)-5/16*ln(a*x+1)
```

maxima [A] time = 0.31, size = 57, normalized size = 0.97

$$-\frac{3\,a^{2}x^{2}+ax-6}{8\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)}-\frac{5}{16}\,\log\left(ax+1\right)-\frac{11}{16}\,\log\left(ax-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x,x, algorithm="maxima")
```

```
[Out] -1/8*(3*a^2*x^2 + a*x - 6)/(a^3*x^3 - a^2*x^2 - a*x + 1) - 5/16*log(a*x + 1
) - 11/16*log(a*x - 1) + log(x)
```

mupad [B] time = 0.07, size = 57, normalized size = 0.97

$$\ln(x)-\frac{11\,\ln\left(1-a\,x\right)}{16}-\frac{5\,\ln\left(a\,x+1\right)}{16}+\frac{\frac{3\,a^{2}\,x^{2}}{8}+\frac{a\,x}{8}-\frac{3}{4}}{-a^{3}\,x^{3}+a^{2}\,x^{2}+a\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(x*(a^2*x^2 - 1)^3),x)
```

```
[Out] log(x) - (11*log(1 - a*x))/16 - (5*log(a*x + 1))/16 + ((a*x)/8 + (3*a^2*x^2
)/8 - 3/4)/(a*x + a^2*x^2 - a^3*x^3 - 1)
```

sympy [A] time = 0.39, size = 60, normalized size = 1.02

$$-\frac{3a^{2}x^{2}+ax-6}{8a^{3}x^{3}-8a^{2}x^{2}-8ax+8}+\log\left(x\right)-\frac{11\log\left(x-\frac{1}{a}\right)}{16}-\frac{5\log\left(x+\frac{1}{a}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3/x,x)
```

```
[Out] -(3*a**2*x**2 + a*x - 6)/(8*a**3*x**3 - 8*a**2*x**2 - 8*a*x + 8) + log(x) -
 11*log(x - 1/a)/16 - 5*log(x + 1/a)/16
```

3.946 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{3a}{4(1-ax)}-\frac{a}{8(ax+1)}+\frac{a}{8(1-ax)^2}+a\log(x)-\frac{23}{16}a\log(1-ax)+\frac{7}{16}a\log(ax+1)-\frac{1}{x}$$

```
[Out] -1/x+1/8*a/(-a*x+1)^2+3/4*a/(-a*x+1)-1/8*a/(a*x+1)+a*ln(x)-23/16*a*ln(-a*x+
1)+7/16*a*ln(a*x+1)
```

Rubi [A] time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{3a}{4(1-ax)}-\frac{a}{8(ax+1)}+\frac{a}{8(1-ax)^2}+a\log(x)-\frac{23}{16}a\log(1-ax)+\frac{7}{16}a\log(ax+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] -x^(-1) + a/(8*(1 - a*x)^2) + (3*a)/(4*(1 - a*x)) - a/(8*(1 + a*x)) + a*Log
[x] - (23*a*Log[1 - a*x])/16 + (7*a*Log[1 + a*x])/16
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^2(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{4(-1+ax)^3}+\frac{3a^2}{4(-1+ax)^2}-\frac{23a^2}{16(-1+ax)}+\frac{a^2}{8(1+ax)^2}+\frac{7a^2}{16(1+ax)}\right)dx$$
$$= -\frac{1}{x}+\frac{a}{8(1-ax)^2}+\frac{3a}{4(1-ax)}-\frac{a}{8(1+ax)}+a\log(x)-\frac{23}{16}a\log(1-ax)+\frac{7}{16}a\log(1+ax$$

Mathematica [A] time = 0.06, size = 65, normalized size = 0.92

$$\frac{1}{16}\left(\frac{12a}{1-ax}-\frac{2a}{ax+1}+\frac{2a}{(ax-1)^2}+16a\log(x)-23a\log(1-ax)+7a\log(ax+1)-\frac{16}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] (-16/x + (12*a)/(1 - a*x) + (2*a)/(-1 + a*x)^2 - (2*a)/(1 + a*x) + 16*a*Log
[x] - 23*a*Log[1 - a*x] + 7*a*Log[1 + a*x])/16
```

fricas [B] time = 0.65, size = 150, normalized size = 2.11

$$-\frac{30\,a^3x^3-22\,a^2x^2-28\,ax-7\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(ax+1\right)+23\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(ax-1\right)}{16\left(a^3x^4-a^2x^3-ax^2+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/16*(30*a^3*x^3 - 22*a^2*x^2 - 28*a*x - 7*(a^4*x^4 - a^3*x^3 - a^2*x^2 +
a*x)*log(a*x + 1) + 23*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(a*x - 1) - 1
6*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(x) + 16)/(a^3*x^4 - a^2*x^3 - a*x
^2 + x)
```

giac [A] time = 0.16, size = 67, normalized size = 0.94

$$\frac{7}{16}\,a\log\left(|ax+1|\right)-\frac{23}{16}\,a\log\left(|ax-1|\right)+a\log\left(|x|\right)-\frac{15\,a^3x^3-11\,a^2x^2-14\,ax+8}{8\,(ax+1)(ax-1)^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^2,x, algorithm="giac")
```

```
[Out] 7/16*a*log(abs(a*x + 1)) - 23/16*a*log(abs(a*x - 1)) + a*log(abs(x)) - 1/8*
(15*a^3*x^3 - 11*a^2*x^2 - 14*a*x + 8)/((a*x + 1)*(a*x - 1)^2*x)
```

maple [A] time = 0.04, size = 59, normalized size = 0.83

$$-\frac{1}{x}+a\ln(x)+\frac{a}{8\left(ax-1\right)^{2}}-\frac{3a}{4\left(ax-1\right)}-\frac{23a\ln\left(ax-1\right)}{16}-\frac{a}{8\left(ax+1\right)}+\frac{7a\ln\left(ax+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3/x^2,x)
```

```
[Out] -1/x+a*ln(x)+1/8*a/(a*x-1)^2-3/4*a/(a*x-1)-23/16*a*ln(a*x-1)-1/8*a/(a*x+1)+
7/16*a*ln(a*x+1)
```

maxima [A] time = 0.31, size = 72, normalized size = 1.01

$$\frac{7}{16}\,a\log\left(ax+1\right)-\frac{23}{16}\,a\log\left(ax-1\right)+a\log(x)-\frac{15\,a^{3}x^{3}-11\,a^{2}x^{2}-14\,ax+8}{8\left(a^{3}x^{4}-a^{2}x^{3}-ax^{2}+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^2,x, algorithm="maxima")
```

```
[Out] 7/16*a*log(a*x + 1) - 23/16*a*log(a*x - 1) + a*log(x) - 1/8*(15*a^3*x^3 - 1
1*a^2*x^2 - 14*a*x + 8)/(a^3*x^4 - a^2*x^3 - a*x^2 + x)
```

mupad [B] time = 0.94, size = 71, normalized size = 1.00

$$\frac{-\frac{15\,a^3\,x^3}{8}+\frac{11\,a^2\,x^2}{8}+\frac{7\,a\,x}{4}-1}{a^3\,x^4-a^2\,x^3-a\,x^2+x}+a\,\ln(x)-\frac{23\,a\,\ln\left(a\,x-1\right)}{16}+\frac{7\,a\,\ln\left(a\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(x^2*(a^2*x^2 - 1)^3),x)
```

```
[Out] ((7*a*x)/4 + (11*a^2*x^2)/8 - (15*a^3*x^3)/8 - 1)/(x - a*x^2 - a^2*x^3 + a^
3*x^4) + a*log(x) - (23*a*log(a*x - 1))/16 + (7*a*log(a*x + 1))/16
```

sympy [A] time = 0.52, size = 78, normalized size = 1.10

$$a\log\left(x\right)-\frac{23a\log\left(x-\frac{1}{a}\right)}{16}+\frac{7a\log\left(x+\frac{1}{a}\right)}{16}-\frac{15a^{3}x^{3}-11a^{2}x^{2}-14ax+8}{8a^{3}x^{4}-8a^{2}x^{3}-8ax^{2}+8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3/x**2,x)
```

```
[Out] a*log(x) - 23*a*log(x - 1/a)/16 + 7*a*log(x + 1/a)/16 - (15*a**3*x**3 - 11*
a**2*x**2 - 14*a*x + 8)/(8*a**3*x**4 - 8*a**2*x**3 - 8*a*x**2 + 8*x)
```

3.947 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{a^2}{1-ax}+\frac{a^2}{8(ax+1)}+\frac{a^2}{8(1-ax)^2}+3a^2\log(x)-\frac{39}{16}a^2\log(1-ax)-\frac{9}{16}a^2\log(ax+1)-\frac{a}{x}-\frac{1}{2x^2}$$

```
[Out] -1/2/x^2-a/x+1/8*a^2/(-a*x+1)^2+a^2/(-a*x+1)+1/8*a^2/(a*x+1)+3*a^2*ln(x)-39
/16*a^2*ln(-a*x+1)-9/16*a^2*ln(a*x+1)
```

Rubi [A] time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{a^2}{1-ax}+\frac{a^2}{8(ax+1)}+\frac{a^2}{8(1-ax)^2}+3a^2\log(x)-\frac{39}{16}a^2\log(1-ax)-\frac{9}{16}a^2\log(ax+1)-\frac{a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] -1/(2*x^2) - a/x + a^2/(8*(1 - a*x)^2) + a^2/(1 - a*x) + a^2/(8*(1 + a*x))
+ 3*a^2*Log[x] - (39*a^2*Log[1 - a*x])/16 - (9*a^2*Log[1 + a*x])/16
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^3(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{3a^2}{x}-\frac{a^3}{4(-1+ax)^3}+\frac{a^3}{(-1+ax)^2}-\frac{39a^3}{16(-1+ax)}-\frac{a^3}{8(1+ax)^2}-\frac{9a^3}{16(1+ax}\right.$$
$$= -\frac{1}{2x^2}-\frac{a}{x}+\frac{a^2}{8(1-ax)^2}+\frac{a^2}{1-ax}+\frac{a^2}{8(1+ax)}+3a^2\log(x)-\frac{39}{16}a^2\log(1-ax)-\frac{9}{16}a^2\log$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.93

$$\frac{1}{16}\left(\frac{16a^2}{1-ax}+\frac{2a^2}{ax+1}+\frac{2a^2}{(ax-1)^2}+48a^2\log(x)-39a^2\log(1-ax)-9a^2\log(ax+1)-\frac{16a}{x}-\frac{8}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] (-8/x^2 - (16*a)/x + (16*a^2)/(1 - a*x) + (2*a^2)/(-1 + a*x)^2 + (2*a^2)/(1
 + a*x) + 48*a^2*Log[x] - 39*a^2*Log[1 - a*x] - 9*a^2*Log[1 + a*x])/16
```

fricas [B] time = 0.69, size = 172, normalized size = 1.93

$$-\frac{30\,a^4x^4-6\,a^3x^3-44\,a^2x^2+8\,ax+9\left(a^5x^5-a^4x^4-a^3x^3+a^2x^2\right)\log\left(ax+1\right)+39\left(a^5x^5-a^4x^4-a^3x^3+a^2x^2\right)}{16\left(a^3x^5-a^2x^4-ax^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(30*a^4*x^4 - 6*a^3*x^3 - 44*a^2*x^2 + 8*a*x + 9*(a^5*x^5 - a^4*x^4 -
 a^3*x^3 + a^2*x^2)*log(a*x + 1) + 39*(a^5*x^5 - a^4*x^4 - a^3*x^3 + a^2*x^
2)*log(a*x - 1) - 48*(a^5*x^5 - a^4*x^4 - a^3*x^3 + a^2*x^2)*log(x) + 8)/(a
^3*x^5 - a^2*x^4 - a*x^3 + x^2)
```

giac [A] time = 0.40, size = 82, normalized size = 0.92

$$-\frac{9}{16}\,a^2\log\left(|ax+1|\right)-\frac{39}{16}\,a^2\log\left(|ax-1|\right)+3\,a^2\log\left(|x|\right)-\frac{15\,a^4x^4-3\,a^3x^3-22\,a^2x^2+4\,ax+4}{8\,(ax+1)(ax-1)^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^3,x, algorithm="giac")
```

```
[Out] -9/16*a^2*log(abs(a*x + 1)) - 39/16*a^2*log(abs(a*x - 1)) + 3*a^2*log(abs(x
)) - 1/8*(15*a^4*x^4 - 3*a^3*x^3 - 22*a^2*x^2 + 4*a*x + 4)/((a*x + 1)*(a*x
- 1)^2*x^2)
```

maple [A] time = 0.04, size = 78, normalized size = 0.88

$$-\frac{1}{2x^2}-\frac{a}{x}+3a^2\ln(x)-\frac{a^2}{ax-1}+\frac{a^2}{8\,(ax-1)^2}-\frac{39a^2\ln(ax-1)}{16}+\frac{a^2}{8ax+8}-\frac{9a^2\ln(ax+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3/x^3,x)
```

```
[Out] -1/2/x^2-a/x+3*a^2*ln(x)-a^2/(a*x-1)+1/8*a^2/(a*x-1)^2-39/16*a^2*ln(a*x-1)+
1/8*a^2/(a*x+1)-9/16*a^2*ln(a*x+1)
```

maxima [A] time = 0.33, size = 89, normalized size = 1.00

$$-\frac{9}{16}\,a^2\log(ax+1)-\frac{39}{16}\,a^2\log(ax-1)+3\,a^2\log(x)-\frac{15\,a^4x^4-3\,a^3x^3-22\,a^2x^2+4\,ax+4}{8\left(a^3x^5-a^2x^4-ax^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^3,x, algorithm="maxima")
```

```
[Out] -9/16*a^2*log(a*x + 1) - 39/16*a^2*log(a*x - 1) + 3*a^2*log(x) - 1/8*(15*a^
4*x^4 - 3*a^3*x^3 - 22*a^2*x^2 + 4*a*x + 4)/(a^3*x^5 - a^2*x^4 - a*x^3 + x^
2)
```

mupad [B] time = 0.96, size = 89, normalized size = 1.00

$$3\,a^2\,\ln(x)-\frac{39\,a^2\,\ln(a\,x-1)}{16}-\frac{9\,a^2\,\ln(a\,x+1)}{16}+\frac{\frac{15\,a^4\,x^4}{8}-\frac{3\,a^3\,x^3}{8}-\frac{11\,a^2\,x^2}{4}+\frac{a\,x}{2}+\frac{1}{2}}{-a^3\,x^5+a^2\,x^4+a\,x^3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(x^3*(a^2*x^2 - 1)^3),x)
```

```
[Out] 3*a^2*log(x) - (39*a^2*log(a*x - 1))/16 - (9*a^2*log(a*x + 1))/16 + ((a*x)/
2 - (11*a^2*x^2)/4 - (3*a^3*x^3)/8 + (15*a^4*x^4)/8 + 1/2)/(a*x^3 - x^2 + a
^2*x^4 - a^3*x^5)
```

sympy [A] time = 0.56, size = 95, normalized size = 1.07

$$3a^2\log(x)-\frac{39a^2\log\left(x-\frac{1}{a}\right)}{16}-\frac{9a^2\log\left(x+\frac{1}{a}\right)}{16}-\frac{15a^4x^4-3a^3x^3-22a^2x^2+4ax+4}{8a^3x^5-8a^2x^4-8ax^3+8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3/x**3,x)
```

```
[Out] 3*a**2*log(x) - 39*a**2*log(x - 1/a)/16 - 9*a**2*log(x + 1/a)/16 - (15*a**4
*x**4 - 3*a**3*x**3 - 22*a**2*x**2 + 4*a*x + 4)/(8*a**3*x**5 - 8*a**2*x**4
- 8*a*x**3 + 8*x**2)
```

3.948 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{5a^3}{4(1-ax)}-\frac{a^3}{8(ax+1)}+\frac{a^3}{8(1-ax)^2}+3a^3\log(x)-\frac{59}{16}a^3\log(1-ax)+\frac{11}{16}a^3\log(ax+1)-\frac{3a^2}{x}-\frac{a}{2x^2}-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3-1/2*a/x^2-3*a^2/x+1/8*a^3/(-a*x+1)^2+5/4*a^3/(-a*x+1)-1/8*a^3/(a*x
+1)+3*a^3*ln(x)-59/16*a^3*ln(-a*x+1)+11/16*a^3*ln(a*x+1)
```

Rubi [A] time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$\frac{5a^3}{4(1-ax)}-\frac{a^3}{8(ax+1)}+\frac{a^3}{8(1-ax)^2}-\frac{3a^2}{x}+3a^3\log(x)-\frac{59}{16}a^3\log(1-ax)+\frac{11}{16}a^3\log(ax+1)-\frac{a}{2x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] -1/(3*x^3) - a/(2*x^2) - (3*a^2)/x + a^3/(8*(1 - a*x)^2) + (5*a^3)/(4*(1 -
a*x)) - a^3/(8*(1 + a*x)) + 3*a^3*Log[x] - (59*a^3*Log[1 - a*x])/16 + (11*a
^3*Log[1 + a*x])/16
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(1-a^2x^2\right)^{5/2}}\,dx = \int \frac{1}{x^4(1-ax)^3(1+ax)^2}\,dx$$
$$= \int \left(\frac{1}{x^4} + \frac{a}{x^3} + \frac{3a^2}{x^2} + \frac{3a^3}{x} - \frac{a^4}{4(-1+ax)^3} + \frac{5a^4}{4(-1+ax)^2} - \frac{59a^4}{16(-1+ax)} + \frac{a^4}{8(1+ax)^2} + \frac{}{1}\right.$$
$$= -\frac{1}{3x^3} - \frac{a}{2x^2} - \frac{3a^2}{x} + \frac{a^3}{8(1-ax)^2} + \frac{5a^3}{4(1-ax)} - \frac{a^3}{8(1+ax)} + 3a^3\log(x) - \frac{59}{16}a^3\log(1-ax$$

Mathematica [A] time = 0.09, size = 91, normalized size = 0.89

$$\frac{1}{48}\left(\frac{60a^3}{1-ax} - \frac{6a^3}{ax+1} + \frac{6a^3}{(ax-1)^2} + 144a^3\log(x) - 177a^3\log(1-ax) + 33a^3\log(ax+1) - \frac{144a^2}{x} - \frac{24a}{x^2} - \frac{16}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(1 - a^2*x^2)^(5/2)),x]
```

```
[Out] (-16/x^3 - (24*a)/x^2 - (144*a^2)/x + (60*a^3)/(1 - a*x) + (6*a^3)/(-1 + a*
x)^2 - (6*a^3)/(1 + a*x) + 144*a^3*Log[x] - 177*a^3*Log[1 - a*x] + 33*a^3*L
og[1 + a*x])/48
```

fricas [B] time = 0.69, size = 180, normalized size = 1.76

$$-\frac{210\,a^5x^5 - 138\,a^4x^4 - 212\,a^3x^3 + 104\,a^2x^2 + 8\,ax - 33\left(a^6x^6 - a^5x^5 - a^4x^4 + a^3x^3\right)\log(ax+1) + 177\left(a^6x^6 - a^5\right.}{48\left(a^3x^6 - a^2x^5 - ax^4 + x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(210*a^5*x^5 - 138*a^4*x^4 - 212*a^3*x^3 + 104*a^2*x^2 + 8*a*x - 33*(
a^6*x^6 - a^5*x^5 - a^4*x^4 + a^3*x^3)*log(a*x + 1) + 177*(a^6*x^6 - a^5*x^
5 - a^4*x^4 + a^3*x^3)*log(a*x - 1) - 144*(a^6*x^6 - a^5*x^5 - a^4*x^4 + a^
3*x^3)*log(x) + 16)/(a^3*x^6 - a^2*x^5 - a*x^4 + x^3)
```

giac [A] time = 0.26, size = 90, normalized size = 0.88

$$\frac{11}{16}\,a^3\log\left(|ax+1|\right) - \frac{59}{16}\,a^3\log\left(|ax-1|\right) + 3\,a^3\log\left(|x|\right) - \frac{105\,a^5x^5 - 69\,a^4x^4 - 106\,a^3x^3 + 52\,a^2x^2 + 4\,ax + 8}{24\,(ax+1)(ax-1)^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^4,x, algorithm="giac")
```

```
[Out] 11/16*a^3*log(abs(a*x + 1)) - 59/16*a^3*log(abs(a*x - 1)) + 3*a^3*log(abs(x
)) - 1/24*(105*a^5*x^5 - 69*a^4*x^4 - 106*a^3*x^3 + 52*a^2*x^2 + 4*a*x + 8)
/((a*x + 1)*(a*x - 1)^2*x^3)
```

maple [A] time = 0.04, size = 86, normalized size = 0.84

$$-\frac{1}{3x^3}-\frac{a}{2x^2}-\frac{3a^2}{x}+3a^3\ln(x)+\frac{a^3}{8(ax-1)^2}-\frac{5a^3}{4(ax-1)}-\frac{59a^3\ln(ax-1)}{16}-\frac{a^3}{8(ax+1)}+\frac{11a^3\ln(ax+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^3/x^4,x)
```

```
[Out] -1/3/x^3-1/2*a/x^2-3*a^2/x+3*a^3*ln(x)+1/8*a^3/(a*x-1)^2-5/4*a^3/(a*x-1)-59
/16*a^3*ln(a*x-1)-1/8*a^3/(a*x+1)+11/16*a^3*ln(a*x+1)
```

maxima [A] time = 0.31, size = 97, normalized size = 0.95

$$\frac{11}{16}a^3\log(ax+1)-\frac{59}{16}a^3\log(ax-1)+3a^3\log(x)-\frac{105a^5x^5-69a^4x^4-106a^3x^3+52a^2x^2+4ax+8}{24\left(a^3x^6-a^2x^5-ax^4+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^3/x^4,x, algorithm="maxima")
```

```
[Out] 11/16*a^3*log(a*x + 1) - 59/16*a^3*log(a*x - 1) + 3*a^3*log(x) - 1/24*(105*
a^5*x^5 - 69*a^4*x^4 - 106*a^3*x^3 + 52*a^2*x^2 + 4*a*x + 8)/(a^3*x^6 - a^2
*x^5 - a*x^4 + x^3)
```

mupad [B] time = 0.94, size = 97, normalized size = 0.95

$$3a^3\ln(x)-\frac{59a^3\ln(ax-1)}{16}+\frac{11a^3\ln(ax+1)}{16}+\frac{\frac{35a^5x^5}{8}-\frac{23a^4x^4}{8}-\frac{53a^3x^3}{12}+\frac{13a^2x^2}{6}+\frac{ax}{6}+\frac{1}{3}}{-a^3x^6+a^2x^5+ax^4-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)/(x^4*(a^2*x^2 - 1)^3),x)
```

```
[Out] 3*a^3*log(x) - (59*a^3*log(a*x - 1))/16 + (11*a^3*log(a*x + 1))/16 + ((a*x)
/6 + (13*a^2*x^2)/6 - (53*a^3*x^3)/12 - (23*a^4*x^4)/8 + (35*a^5*x^5)/8 + 1
/3)/(a*x^4 - x^3 + a^2*x^5 - a^3*x^6)
```

sympy [A] time = 0.58, size = 104, normalized size = 1.02

$$3a^3\log(x)-\frac{59a^3\log\left(x-\frac{1}{a}\right)}{16}+\frac{11a^3\log\left(x+\frac{1}{a}\right)}{16}-\frac{105a^5x^5-69a^4x^4-106a^3x^3+52a^2x^2+4ax+8}{24a^3x^6-24a^2x^5-24ax^4+24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**3/x**4,x)
```

```
[Out] 3*a**3*log(x) - 59*a**3*log(x - 1/a)/16 + 11*a**3*log(x + 1/a)/16 - (105*a*
*5*x**5 - 69*a**4*x**4 - 106*a**3*x**3 + 52*a**2*x**2 + 4*a*x + 8)/(24*a**3
*x**6 - 24*a**2*x**5 - 24*a*x**4 + 24*x**3)
```

3.949 $\int e^{\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

```
[Out] 1/3*x^3*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+1/4*a*x^4*(-a^2*c*x^2+c)^(1
/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 43}

$$\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2]) + (a*x^4*Sqrt[c - a^2*c*x^2
])/(4*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int x^2(1+ax)\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(x^2+ax^3\right)\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}+\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.57

$$\frac{x^3(3ax+4)\sqrt{c-a^2cx^2}}{12\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^3*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.67, size = 50, normalized size = 0.68

$$-\frac{\sqrt{-a^2cx^2+c}\left(3\,ax^4+4\,x^3\right)\sqrt{-a^2x^2+1}}{12\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] -1/12*sqrt(-a^2*c*x^2 + c)*(3*a*x^4 + 4*x^3)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 -
1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 37, normalized size = 0.50

$$\frac{x^{3} \left(3 a x +4\right) \sqrt{-a^{2} c \,x^{2}+c}}{12 \sqrt{-a^{2} x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/12*x^3*(3*a*x+4)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 1.05, size = 38, normalized size = 0.51

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x^4}{4}+\frac{x^3}{3}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*((a*x^4)/4 + x^3/3))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{- c \left(a x - 1\right) \left(a x + 1\right)} \left(a x + 1\right)}{\sqrt{- \left(a x - 1\right) \left(a x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x +
 1)), x)
```

3.950 $\int e^{\tanh^{-1}(ax)} x \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

```
[Out] 1/2*x^2*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+1/3*a*x^3*(-a^2*c*x^2+c)^(1
/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6153, 6150, 43}

$$\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}+\frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(2*Sqrt[1 - a^2*x^2]) + (a*x^3*Sqrt[c - a^2*c*x^2
])/(3*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int x(1+ax)\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(x+ax^2\right)\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.57

$$\frac{x^2(2ax+3)\sqrt{c-a^2cx^2}}{6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*(3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.56, size = 50, normalized size = 0.68

$$-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\left(2\,ax^3+3\,x^2\right)}{6\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/6*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(2*a*x^3 + 3*x^2)/(a^2*x^2 - 1
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 37, normalized size = 0.50

$$\frac{x^{2} \left(2 a x +3\right) \sqrt{-a^{2} c \,x^{2}+c}}{6 \sqrt{-a^{2} x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*x^2*(2*a*x+3)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.98, size = 38, normalized size = 0.51

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x^3}{3}+\frac{x^2}{2}\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*((a*x^3)/3 + x^2/2))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{- c \left(a x - 1\right) \left(a x + 1\right)} \left(a x + 1\right)}{\sqrt{- \left(a x - 1\right) \left(a x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)
), x)
```

3.951 $\int e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

```
[Out] x*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+1/2*a*x^2*(-a^2*c*x^2+c)^(1/2)/(-
a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6143, 6140}

$$\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (a*x^2*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - a^2*x^2])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int (1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}} + \frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.58

$$\frac{\left(\frac{ax^2}{2}+x\right)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((x + (a*x^2)/2)*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.89, size = 47, normalized size = 0.68

$$-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\left(ax^2+2\,x\right)}{2\,\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x^2 + 2*x)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 34, normalized size = 0.49

$$\frac{x(ax+2)\sqrt{-a^2c\,x^2+c}}{2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.95, size = 34, normalized size = 0.49

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x^2}{2}+x\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(x + (a*x^2)/2))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(ax-1)(ax+1)}\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)),
 x)
```

3.952 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=65

$$\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

```
[Out] a*x*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a^
2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 43}

$$\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (Sqrt[c - a^2*c*x^2]*Log[x])/
Sqrt[1 - a^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}\\ &= \frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.55

$$\frac{\sqrt{c-a^2cx^2}\,(ax+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x]))/Sqrt[1 - a^2*x^2]
```

fricas [B] time = 0.87, size = 262, normalized size = 4.03

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\left(x^4-1\right)\sqrt{c}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(ax-a)}{2\left(a^2x^2-1\right)},\ \left(a^2x^2-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="f
ricas")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - sqrt(-a^2*
c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^4 - 1)*sqrt(c) - c)/(a^2*x^4 - x^2)) - 2*s
qrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - a))/(a^2*x^2 - 1), ((a^2*x^2
```

```
- 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^2 + 1)*sqrt
(-c)/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1)*(a*x - a))/(a^2*x^2 - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.05, size = 48, normalized size = 0.74

$$\frac{(-ax-\ln(x))\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (-a*x-ln(x))*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)
```

maxima [A] time = 0.34, size = 56, normalized size = 0.86

$$a\sqrt{c}\,x-\frac{1}{2}\,(-1)^{-2\,a^2cx^2+2\,c}\,\sqrt{c}\,\log\left(-2\,a^2c+\frac{2\,c}{x^2}\right)-\frac{1}{2}\,\sqrt{c}\,\log\left(x^2-\frac{1}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="m
axima")
```

```
[Out] a*sqrt(c)*x - 1/2*(-1)^(-2*a^2*c*x^2 + 2*c)*sqrt(c)*log(-2*a^2*c + 2*c/x^2)
 - 1/2*sqrt(c)*log(x^2 - 1/a^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(ax-1)(ax+1)}\,(ax+1)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1
))), x)
```

3.953 $\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=68

$$\frac{a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/x/(-a^2*x^2+1)^(1/2)+a*ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a
^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 43}

$$\frac{a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) + (a*Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}+\frac{a}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.59

$$\frac{\sqrt{c-a^2cx^2}\left(a\log(x)-\frac{1}{x}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) + a*Log[x]))/Sqrt[1 - a^2*x^2]
```

fricas [B] time = 0.75, size = 264, normalized size = 3.88

$$\left[\frac{\left(a^3x^3-ax\right)\sqrt{c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\left(x^4-1\right)\sqrt{c}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(x-1)}{2\left(a^2x^3-x\right)}, \frac{\left(a^3x^3-a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"fricas")
[Out] [1/2*((a^3*x^3 - a*x)*sqrt(c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^4 - 1)*sqrt(c) - c)/(a^2*x^4 - x^2)) - 2
```

```
*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x - 1))/(a^2*x^3 - x), ((a^3*x^3
- a*x)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^2 + 1)*sq
rt(-c)/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) - sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*
x^2 + 1)*(x - 1))/(a^2*x^3 - x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.04, size = 50, normalized size = 0.74

$$\frac{(-a\ln(x)x+1)\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}}{\left(a^2x^2-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] (-a*ln(x)*x+1)*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)/x
```

maxima [A] time = 0.35, size = 60, normalized size = 0.88

$$-\frac{1}{2}\left((-1)^{-2\,a^2cx^2+2\,c}\sqrt{c}\,\log\left(-2\,a^2c+\frac{2\,c}{x^2}\right)+\sqrt{c}\,\log\left(x^2-\frac{1}{a^2}\right)\right)a-\frac{\sqrt{c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm=
"maxima")
```

```
[Out] -1/2*((-1)^(-2*a^2*c*x^2 + 2*c)*sqrt(c)*log(-2*a^2*c + 2*c/x^2) + sqrt(c)*l
og(x^2 - 1/a^2))*a - sqrt(c)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\,(a\,x+1)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,\,(ax+1)}{x^2\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x
+ 1))), x)
```

3.954 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{2c(ax+1)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{c(ax+1)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

```
[Out] 2/3*c*(a*x+1)^3*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-1/4*c*(a*x+1)^4*(
-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 43}

$$\frac{2c(ax+1)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{c(ax+1)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*c*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) - (c*(1 + a*x
)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(1-ax)(1+ax)^2dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int\left(2(1+ax)^2-(1+ax)^3\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{2c(1+ax)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{c(1+ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.64

$$-\frac{cx\left(3a^3x^3+4a^2x^2-6ax-12\right)\sqrt{c-a^2cx^2}}{12\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/12*(c*x*Sqrt[c - a^2*c*x^2]*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))/Sqrt[
1 - a^2*x^2]
```

fricas [A] time = 0.81, size = 68, normalized size = 0.76

$$\frac{\left(3\,a^3cx^4+4\,a^2cx^3-6\,acx^2-12\,cx\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{12\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/12*(3*a^3*c*x^4 + 4*a^2*c*x^3 - 6*a*c*x^2 - 12*c*x)*sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 65, normalized size = 0.73

$$\frac{x\left(3x^{3}a^{3}+4a^{2}x^{2}-6ax-12\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{12\,(ax-1)\,(ax+1)\,\sqrt{-a^{2}x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/12*x*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*(-a^2*c*x^2+c)^(3/2)/(a*x-1)/(a*x+1)/
(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.33, size = 66, normalized size = 0.74

$$-\frac{1}{3}\,a^{2}c^{\frac{3}{2}}x^{3}+c^{\frac{3}{2}}x+\frac{1}{4}\left(a^{2}c^{\frac{3}{2}}x^{4}+2\,c^{\frac{3}{2}}x^{2}-\frac{4\,\sqrt{a^{4}cx^{4}-2\,a^{2}cx^{2}+c}\,c}{a^{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/3*a^2*c^(3/2)*x^3 + c^(3/2)*x + 1/4*(a^2*c^(3/2)*x^4 + 2*c^(3/2)*x^2 - 4
*sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*c/a^2)*a
```

mupad [B] time = 1.06, size = 55, normalized size = 0.62

$$\frac{\sqrt{c-a^{2}\,c\,x^{2}}\,\left(-\frac{c\,a^{3}\,x^{4}}{4}-\frac{c\,a^{2}\,x^{3}}{3}+\frac{c\,a\,x^{2}}{2}+c\,x\right)}{\sqrt{1-a^{2}\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(c*x - (a^2*c*x^3)/3 - (a^3*c*x^4)/4 + (a*c*x^2)/2))
/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1
)), x)
```

3.955 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=136

$$\frac{c^2(ax+1)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{4c^2(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{c^2(ax+1)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}$$

```
[Out] c^2*(a*x+1)^4*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-4/5*c^2*(a*x+1)^5*(
-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+1/6*c^2*(a*x+1)^6*(-a^2*c*x^2+c)^(
1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 43}

$$\frac{c^2(ax+1)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{4c^2(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{c^2(ax+1)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*(1 + a*x)^4*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (4*c^2*(1 + a
*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) + (c^2*(1 + a*x)^6*Sqrt[
c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)^2(1+ax)^3\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int\left(4(1+ax)^3-4(1+ax)^4+(1+ax)^5\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{c^2(1+ax)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4c^2(1+ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{c^2(1+ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.44

$$\frac{c^2(ax+1)^4\left(5a^2x^2-14ax+11\right)\sqrt{c-a^2cx^2}}{30a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*(1 + a*x)^4*(11 - 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a*Sqrt[
1 - a^2*x^2])
```

fricas [A] time = 0.87, size = 98, normalized size = 0.72

$$-\frac{\left(5\,a^5c^2x^6+6\,a^4c^2x^5-15\,a^3c^2x^4-20\,a^2c^2x^3+15\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{30\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] -1/30*(5*a^5*c^2*x^6 + 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 + 15
*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 81, normalized size = 0.60

$$\frac{x\left(5x^5a^5+6x^4a^4-15x^3a^3-20a^2x^2+15ax+30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{30\left(ax-1\right)^2\left(ax+1\right)^2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5+6*a^4*x^4-15*a^3*x^3-20*a^2*x^2+15*a*x+30)*(-a^2*c*x^2+c)
^(5/2)/(a*x-1)^2/(a*x+1)^2/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.34, size = 111, normalized size = 0.82

$$\frac{1}{5}\,a^4c^{\frac{5}{2}}x^5-\frac{2}{3}\,a^2c^{\frac{5}{2}}x^3+c^{\frac{5}{2}}x-\frac{1}{6}\left(\sqrt{a^4cx^4-2\,a^2cx^2+c}\,a^2c^2x^4-2\,a^2c^{\frac{5}{2}}x^4-4\,c^{\frac{5}{2}}x^2+\frac{7\,\sqrt{a^4cx^4-2\,a^2cx^2+c}\,c^2}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] 1/5*a^4*c^(5/2)*x^5 - 2/3*a^2*c^(5/2)*x^3 + c^(5/2)*x - 1/6*(sqrt(a^4*c*x^4
 - 2*a^2*c*x^2 + c)*a^2*c^2*x^4 - 2*a^2*c^(5/2)*x^4 - 4*c^(5/2)*x^2 + 7*sqr
t(a^4*c*x^4 - 2*a^2*c*x^2 + c)*c^2/a^2)*a
```

mupad [B] time = 1.05, size = 85, normalized size = 0.62

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a^5\,c^2\,x^6}{6}+\frac{a^4\,c^2\,x^5}{5}-\frac{a^3\,c^2\,x^4}{2}-\frac{2\,a^2\,c^2\,x^3}{3}+\frac{a\,c^2\,x^2}{2}+c^2\,x\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(c^2*x + (a*c^2*x^2)/2 - (2*a^2*c^2*x^3)/3 - (a^3*c^
2*x^4)/2 + (a^4*c^2*x^5)/5 + (a^5*c^2*x^6)/6))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1)(ax+1))^{\frac{5}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1
)), x)
```

3.956 $\int e^{\tanh^{-1}(ax)} \left(c - a^2cx^2\right)^{7/2} \, dx$

Optimal. Leaf size=183

$$-\frac{c^3(ax+1)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{6c^3(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{2c^3(ax+1)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{8c^3(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

```
[Out] 8/5*c^3*(a*x+1)^5*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-2*c^3*(a*x+1)^6
*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+6/7*c^3*(a*x+1)^7*(-a^2*c*x^2+c)
^(1/2)/a/(-a^2*x^2+1)^(1/2)-1/8*c^3*(a*x+1)^8*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*
x^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 43}

$$-\frac{c^3(ax+1)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{6c^3(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{2c^3(ax+1)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{8c^3(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (8*c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) - (2*c^3*(1
 + a*x)^6*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) + (6*c^3*(1 + a*x)^7*S
qrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (c^3*(1 + a*x)^8*Sqrt[c - a^2
*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
```

```
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int(1-ax)^3(1+ax)^4\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int\left(8(1+ax)^4-12(1+ax)^5+6(1+ax)^6-(1+ax)^7\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{8c^3(1+ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{2c^3(1+ax)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{6c^3(1+ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.37

$$-\frac{c^3(ax+1)^5\left(35a^3x^3-135a^2x^2+185ax-93\right)\sqrt{c-a^2cx^2}}{280a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/280*(c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(-93 + 185*a*x - 135*a^2*x^2 +
35*a^3*x^3))/(a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.69, size = 120, normalized size = 0.66

$$\frac{\left(35\,a^7c^3x^8+40\,a^6c^3x^7-140\,a^5c^3x^6-168\,a^4c^3x^5+210\,a^3c^3x^4+280\,a^2c^3x^3-140\,ac^3x^2-280\,c^3x\right)\sqrt{-a^2cx^2+c}}{280\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

```
[Out] 1/280*(35*a^7*c^3*x^8 + 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 - 168*a^4*c^3*x^5
+ 210*a^3*c^3*x^4 + 280*a^2*c^3*x^3 - 140*a*c^3*x^2 - 280*c^3*x)*sqrt(-a^2*
c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} c x^{2}+c\right)^{\frac{7}{2}} (a x+1)}{\sqrt{-a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 97, normalized size = 0.53

$$\frac{x \left(35 a^{7} x^{7}+40 x^{6} a^{6}-140 x^{5} a^{5}-168 x^{4} a^{4}+210 x^{3} a^{3}+280 a^{2} x^{2}-140 a x -280\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}{280 \left(a x -1\right)^{3} \left(a x +1\right)^{3} \sqrt{-a^{2} x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/280*x*(35*a^7*x^7+40*a^6*x^6-140*a^5*x^5-168*a^4*x^4+210*a^3*x^3+280*a^2*
x^2-140*a*x-280)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^3/(a*x+1)^3/(-a^2*x^2+1)^(1/2
)
```

maxima [A] time = 0.41, size = 154, normalized size = 0.84

$$-\frac{1}{7}\, a^{6} c^{\frac{7}{2}} x^{7}+\frac{3}{5}\, a^{4} c^{\frac{7}{2}} x^{5}-a^{2} c^{\frac{7}{2}} x^{3}+c^{\frac{7}{2}} x+\frac{1}{8} \left(\sqrt{a^{4} c x^{4}-2\, a^{2} c x^{2}+c}\, a^{4} c^{3} x^{6}-3\, \sqrt{a^{4} c x^{4}-2\, a^{2} c x^{2}+c}\, a^{2} c^{3} x^{4}+3\, a^{2} c^{\frac{7}{2}} x^{4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

```
[Out] -1/7*a^6*c^(7/2)*x^7 + 3/5*a^4*c^(7/2)*x^5 - a^2*c^(7/2)*x^3 + c^(7/2)*x +
1/8*(sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*a^4*c^3*x^6 - 3*sqrt(a^4*c*x^4 - 2*a
^2*c*x^2 + c)*a^2*c^3*x^4 + 3*a^2*c^(7/2)*x^4 + 6*c^(7/2)*x^2 - 10*sqrt(a^4
*c*x^4 - 2*a^2*c*x^2 + c)*c^3/a^2)*a
```

mupad [B] time = 1.06, size = 107, normalized size = 0.58

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(-\frac{a^7\,c^3\,x^8}{8}-\frac{a^6\,c^3\,x^7}{7}+\frac{a^5\,c^3\,x^6}{2}+\frac{3\,a^4\,c^3\,x^5}{5}-\frac{3\,a^3\,c^3\,x^4}{4}-a^2\,c^3\,x^3+\frac{a\,c^3\,x^2}{2}+c^3\,x\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(c^3*x + (a*c^3*x^2)/2 - a^2*c^3*x^3 - (3*a^3*c^3*x^
4)/4 + (3*a^4*c^3*x^5)/5 + (a^5*c^3*x^6)/2 - (a^6*c^3*x^7)/7 - (a^7*c^3*x^8
)/8))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1)(ax+1))^{\frac{7}{2}}(ax+1)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1
)), x)
```

3.957 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=194

$$-\frac{x^4\sqrt{1-a^2x^2}}{4a\sqrt{c-a^2cx^2}}-\frac{x^3\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a^5\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^4\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^3\sqrt{c-a^2cx^2}}$$

```
[Out] -x*(-a^2*x^2+1)^(1/2)/a^4/(-a^2*c*x^2+c)^(1/2)-1/2*x^2*(-a^2*x^2+1)^(1/2)/a
^3/(-a^2*c*x^2+c)^(1/2)-1/3*x^3*(-a^2*x^2+1)^(1/2)/a^2/(-a^2*c*x^2+c)^(1/2)
-1/4*x^4*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)-ln(-a*x+1)*(-a^2*x^2+1)^
(1/2)/a^5/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 43}

$$-\frac{x^4\sqrt{1-a^2x^2}}{4a\sqrt{c-a^2cx^2}}-\frac{x^3\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^3\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^4\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a^5\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^4*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*x^2]
)/(2*a^3*Sqrt[c - a^2*c*x^2]) - (x^3*Sqrt[1 - a^2*x^2])/(3*a^2*Sqrt[c - a^2
*c*x^2]) - (x^4*Sqrt[1 - a^2*x^2])/(4*a*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^
2*x^2]*Log[1 - a*x])/(a^5*Sqrt[c - a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^4}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^4}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^4}-\frac{x}{a^3}-\frac{x^2}{a^2}-\frac{x^3}{a}-\frac{1}{a^4(-1+ax)}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{a^4\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^3\sqrt{c-a^2cx^2}}-\frac{x^3\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}}-\frac{x^4\sqrt{1-a^2x^2}}{4a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-}{a^5\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.37

$$-\frac{\sqrt{1-a^2x^2}\left(ax\left(3a^3x^3+4a^2x^2+6ax+12\right)+12\log(1-ax)\right)}{12a^5\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/12*(Sqrt[1 - a^2*x^2]*(a*x*(12 + 6*a*x + 4*a^2*x^2 + 3*a^3*x^3) + 12*Log
[1 - a*x]))/(a^5*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.68, size = 387, normalized size = 1.99

$$\left[\frac{6\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx+\left(a^4x^4-4a^3x^3+6a^2x^2-4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4-2a^3x^3+2ax-1}\right)+\left(3a^4x^4+4\right.}{12\left(a^7cx^2-a^5c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(6*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
 - 1)) + (3*a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 12*a*x)*sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1))/(a^7*c*x^2 - a^5*c), -1/12*(12*(a^2*x^2 - 1)*sqrt(-c)*a
rctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c
)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (3*a^4*x^4 + 4*a^3*x^3
 + 6*a^2*x^2 + 12*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^7*c*x^2
- a^5*c)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 83, normalized size = 0.43

$$\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(3 x^{4} a^{4}+4 x^{3} a^{3}+6 a^{2} x^{2}+12 a x +12 \ln \left(a x -1\right)\right)}{12 \left(a^{2} x^{2}-1\right) c \,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/12*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*x^4*a^4+4*x^3*a^3+6*a^2*x
^2+12*a*x+12*ln(a*x-1))/(a^2*x^2-1)/c/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^{4}}{\sqrt{-a^{2}cx^{2}+c}\,\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^4/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,(a\,x+1)}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
[Out] int((x^4*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**4*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

3.958 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=155

$$-\frac{x^2\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}}-\frac{x^3\sqrt{1-a^2x^2}}{3a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^4\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^3\sqrt{c-a^2cx^2}}$$

```
[Out] -x*(-a^2*x^2+1)^(1/2)/a^3/(-a^2*c*x^2+c)^(1/2)-1/2*x^2*(-a^2*x^2+1)^(1/2)/a
^2/(-a^2*c*x^2+c)^(1/2)-1/3*x^3*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)-l
n(-a*x+1)*(-a^2*x^2+1)^(1/2)/a^4/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 43}

$$-\frac{x^3\sqrt{1-a^2x^2}}{3a\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^4\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^3*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*x^2]
)/(2*a^2*Sqrt[c - a^2*c*x^2]) - (x^3*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a^2*c
*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a^4*Sqrt[c - a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^3}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} \\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^3}{1-ax}\,dx}{\sqrt{c-a^2cx^2}} \\
&= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^3}-\frac{x}{a^2}-\frac{x^2}{a}-\frac{1}{a^3(-1+ax)}\right)dx}{\sqrt{c-a^2cx^2}} \\
&= -\frac{x\sqrt{1-a^2x^2}}{a^3\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}}-\frac{x^3\sqrt{1-a^2x^2}}{3a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^4\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.41

$$
-\frac{\sqrt{1-a^2x^2}\left(ax\left(2a^2x^2+3ax+6\right)+6\log(1-ax)\right)}{6a^4\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/6*(Sqrt[1 - a^2*x^2]*(a*x*(6 + 3*a*x + 2*a^2*x^2) + 6*Log[1 - a*x]))/(a^
4*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.81, size = 371, normalized size = 2.39

$$
\left[\frac{3\left(a^2x^2-1\right)\sqrt{c}\,\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx+\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\left(2\,a^3x^3+3\right.}{6\left(a^6cx^2-a^4c\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
- 1)) + (2*a^3*x^3 + 3*a^2*x^2 + 6*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
+ 1))/(a^6*c*x^2 - a^4*c), -1/6*(6*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*
c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2
*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (2*a^3*x^3 + 3*a^2*x^2 + 6*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*c*x^2 - a^4*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^3}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^3/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.04, size = 75, normalized size = 0.48

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(2x^3a^3+3a^2x^2+6ax+6\ln\left(ax-1\right)\right)}{6\left(a^2x^2-1\right)c\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3+3*a^2*x^2+6*a*x+6*
ln(a*x-1))/(a^2*x^2-1)/c/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{6}\,a\left(\frac{2\left(a^2x^3+3\,x\right)}{a^4}-\frac{3\,\log(ax+1)}{a^5}+\frac{3\,\log(ax-1)}{a^5}\right)}{\sqrt{c}}+\frac{\log\left(x^2-\frac{1}{a^2}\right)}{2\,a^4\sqrt{c}}+\frac{\sqrt{a^4cx^4-2\,a^2cx^2+c}}{2\,a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] a*integrate(-x^4/((a*x + 1)*(a*x - 1)), x)/sqrt(c) + 1/2*log(x^2 - 1/a^2)/(
a^4*sqrt(c)) + 1/2*sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)/(a^4*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a\,x+1\right)}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
[Out] int((x^3*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**3*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

3.959 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=116

$$-\frac{x^2\sqrt{1-a^2x^2}}{2a\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a^3\sqrt{c-a^2cx^2}}$$

```
[Out] -x*(-a^2*x^2+1)^(1/2)/a^2/(-a^2*c*x^2+c)^(1/2)-1/2*x^2*(-a^2*x^2+1)^(1/2)/a
/(-a^2*c*x^2+c)^(1/2)-ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/a^3/(-a^2*c*x^2+c)^(1/2
)
```

Rubi [A] time = 0.20, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 43}

$$-\frac{x^2\sqrt{1-a^2x^2}}{2a\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^2*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*x^2]
)/(2*a*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a^3*Sqrt[c
- a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^2}-\frac{x}{a}-\frac{1}{a^2(-1+ax)}\right)dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{x\sqrt{1-a^2x^2}}{a^2\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a^3\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.47

$$-\frac{\sqrt{1-a^2x^2}\,(ax(ax+2)+2\log(1-ax))}{2a^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/2*(Sqrt[1 - a^2*x^2]*(a*x*(2 + a*x) + 2*Log[1 - a*x]))/(a^3*Sqrt[c - a^2
*c*x^2])
```

fricas [A] time = 0.90, size = 352, normalized size = 3.03

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{c}\,\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx+\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\sqrt{-a^2cx^2+c}}{2\left(a^5cx^2-a^3c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/2*((a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*
a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)) + sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x)*sqrt(-a^2*x^2 + 1))/(a^5*c*x^2
 - a^3*c), -1/2*(2*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*
x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2
*c*x^2 + 2*a*c*x)) - sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x)*sqrt(-a^2*x^2 +
 1))/(a^5*c*x^2 - a^3*c)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 66, normalized size = 0.57

$$\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a^{2} x^{2}+2 a x +2 \ln \left(a x -1\right)\right)}{2 \left(a^{2} x^{2}-1\right) c \,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^2*x^2+2*a*x+2*ln(a*x-1))/(
a^2*x^2-1)/c/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^2}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^2/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a\,x+1\right)}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a x + 1\right)}{\sqrt{- \left(a x - 1\right) \left(a x + 1\right)} \sqrt{- c \left(a x - 1\right) \left(a x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

3.960 $\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{x\sqrt{1-a^2x^2}}{a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a^2\sqrt{c-a^2cx^2}}$$

```
[Out] -x*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)-ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/
a^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6153, 6150, 43}

$$-\frac{x\sqrt{1-a^2x^2}}{a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a^2*c*x^2])) - (Sqrt[1 - a^2*x^2]*Log[1
 - a*x])/(a^2*Sqrt[c - a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{x}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a}-\frac{1}{a(-1+ax)}\right)dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{x\sqrt{1-a^2x^2}}{a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a^2\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.58

$$-\frac{\sqrt{1-a^2x^2}\,(ax+\log(1-ax))}{a^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/Sqrt[c - a^2*c*x^2],x]
[Out] -((Sqrt[1 - a^2*x^2]*(a*x + Log[1 - a*x]))/(a^2*Sqrt[c - a^2*c*x^2]))
```

fricas [B] time = 0.67, size = 331, normalized size = 4.30

$$\left[\frac{2\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\,ax+\left(a^2x^2-1\right)\sqrt{c}\,\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx+\left(a^4x^4-4a^3x^3+6a^2x^2-4ax\right)\sqrt{-a^2cx^2+c}}{a^4x^4-2a^3x^3+2ax-1}\right.}{2\left(a^4cx^2-a^2c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
[Out] [1/2*(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^2*x^2 - 1)*sqrt(c)
*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 + 4*a*c*x + (a^4*
```

```
x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1
)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)))/(a^4*c*x^2 - a^2*c), (
sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^2*x^2 - 1)*sqrt(-c)*arctan
(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^
4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)))/(a^4*c*x^2 - a^2*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)*x/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.04, size = 55, normalized size = 0.71

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax+\ln\left(ax-1\right)\right)}{\left(a^2x^2-1\right)c\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x+ln(a*x-1))/(a^2*x^2-1)/c/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\frac{1}{2}\,a\left(\frac{2\,x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)}{\sqrt{c}}+\frac{\log\left(x^2-\frac{1}{a^2}\right)}{2\,a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] a*integrate(-x^2/((a*x + 1)*(a*x - 1)), x)/sqrt(c) + 1/2*log(x^2 - 1/a^2)/(
a^2*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a\,x+1\right)}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(ax+1)}{\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1
))), x)
```

3.961 $\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] -ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 31}

$$-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a*Sqrt[c - a^2*c*x^2]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(a*Sqrt[c - a^2*c*x^2]))
```

fricas [B] time = 0.57, size = 227, normalized size = 5.54

$$\left[\frac{\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx+\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)}{2\,a\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,(a^2x^2}{a^4cx^4-2\,a}\right.}{ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/2*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 + 4*a*c*x + (
a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1))/(a*sqrt(c)), -sqrt(
-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sq
rt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x))/(a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.03, size = 51, normalized size = 1.24

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\ln\left(ax-1\right)}{\left(a^2x^2-1\right)ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)/c/a*ln(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a\,x+1}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))
), x)
```

3.962 $\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] ln(x)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2)-ln(-a*x+1)*(-a^2*x^2+1)^(1/2)
/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6153, 6150, 36, 29, 31}

$$\frac{\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Log[x])/Sqrt[c - a^2*c*x^2] - (Sqrt[1 - a^2*x^2]*Log[1 -
 a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x(1-ax)}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x}\,dx}{\sqrt{c-a^2cx^2}} + \frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{1}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\log(1-ax)}{\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.59

$$
\frac{\sqrt{1-a^2x^2}\,(\log(x)-\log(1-ax))}{\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*Sqrt[c - a^2*c*x^2]),x]
[Out] (Sqrt[1 - a^2*x^2]*(Log[x] - Log[1 - a*x]))/Sqrt[c - a^2*c*x^2]
```

fricas [A] time = 0.82, size = 302, normalized size = 4.25

$$
\left[\frac{\log\left(-\frac{4a^5cx^5-\left(2a^6-4a^5+6a^4-4a^3+a^2\right)cx^6-\left(4a^4+4a^3-6a^2+4a-1\right)cx^4+5a^2cx^2-4acx+\left(4a^3x^3-\left(4a^3-6a^2+4a-1\right)x^4-6a^2x^2+4ax-1\right)\sqrt{-a^2cx^2}}{a^4x^6-2a^3x^5+2ax^3-x^2}\right.}{2\sqrt{c}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a
^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*c*x^2 - 4*a*c*x + (4*a^3*x^3 -
(4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*c*x^2 + c)
*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 - 2*a^3*x^5 + 2*a*x^3 - x^2))/sqr
t(c), -sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a^2 - 2
*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2
 - 2*a + 1)*c*x^2 - 2*a*c*x + c))/c]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.04, size = 53, normalized size = 0.75

$$\frac{(-\ln(x)+\ln(ax-1))\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}}{c\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-ln(x)+ln(a*x-1))*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
[Out] int((a*x + 1)/(x*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x\sqrt{-(ax-1)(ax+1)}\,\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral((a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1
))), x)
```

3.963 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] -(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2)+a*ln(x)*(-a^2*x^2+1)^(1/2)/(-a^2
*c*x^2+c)^(1/2)-a*ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 44}

$$-\frac{\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(x*Sqrt[c - a^2*c*x^2])) + (a*Sqrt[1 - a^2*x^2]*Log[x])
/Sqrt[c - a^2*c*x^2] - (a*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/Sqrt[c - a^2*c*x^
2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
```

```
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^2\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^2(1-ax)}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}\\
&= -\frac{\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.47

$$
\frac{\sqrt{1-a^2x^2}\left(a\log(x)-a\log(1-ax)-\frac{1}{x}\right)}{\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-x^(-1) + a*Log[x] - a*Log[1 - a*x]))/Sqrt[c - a^2*c*x^
2]
```

fricas [A] time = 0.83, size = 416, normalized size = 3.89

$$
\left[\frac{\left(a^3x^3-ax\right)\sqrt{c}\,\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3-6\,a^2+4\,a-1\right)x^4-6\,a^2\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{2\left(a^2cx^3-cx\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/2*((a^3*x^3 - a*x)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*a^4 -
4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*c*x^2
- 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 + 4*a*x
- 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 - 2*a^3*
x^5 + 2*a*x^3 - x^2)) - 2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x - 1))/
(a^2*c*x^3 - c*x), -((a^3*x^3 - a*x)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1)*((2*a^2 - 2*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^
3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) + sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x - 1))/(a^2*c*x^3 - c*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.04, size = 62, normalized size = 0.58

$$\frac{\left(-a\ln(x)x+\ln\left(ax-1\right)xa+1\right)\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}}{\left(a^2x^2-1\right)cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a*ln(x)*x+ln(a*x-1)*x*a+1)*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2
*x^2-1)/c/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{x^2\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/(x^2*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x^2\sqrt{-(ax-1)(ax+1)}\,\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

3.964 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=148

$$-\frac{a\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(1/2)-a*(-a^2*x^2+1)^(1/2)/x/(-a
^2*c*x^2+c)^(1/2)+a^2*ln(x)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2)-a^2*ln(
-a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 44}

$$-\frac{a\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*x^2*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(x*S
qrt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2]*Log[x])/Sqrt[c - a^2*c*x^2] -
(a^2*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^3\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^3(1-ax)}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{a^2}{x}-\frac{a^3}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}\\
&= -\frac{\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.42

$$
\frac{\sqrt{1-a^2x^2}\left(a^2\log(x)-a^2\log(1-ax)-\frac{a}{x}-\frac{1}{2x^2}\right)}{\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-1/2*1/x^2 - a/x + a^2*Log[x] - a^2*Log[1 - a*x]))/Sqrt
[c - a^2*c*x^2]
```

fricas [A] time = 0.82, size = 453, normalized size = 3.06

$$
\left[\frac{\left(a^4x^4-a^2x^2\right)\sqrt{c}\,\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3-6\,a^2+4\,a-1\right)x^4-6\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{2\left(a^2cx^4-cx^2\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/2*((a^4*x^4 - a^2*x^2)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*a^
4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*c*
x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 + 4*
a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 - 2*
a^3*x^5 + 2*a*x^3 - x^2)) - sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a +
 1)*x^2 - 2*a*x - 1))/(a^2*c*x^4 - c*x^2), -1/2*(2*(a^4*x^4 - a^2*x^2)*sqrt
(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a^2 - 2*a + 1)*x^2
 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)
*c*x^2 - 2*a*c*x + c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a + 1)
*x^2 - 2*a*x - 1))/(a^2*c*x^4 - c*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.04, size = 76, normalized size = 0.51

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(2a^2\ln(x)x^2-2\ln\left(ax-1\right)x^2a^2-2ax-1\right)}{2\left(a^2x^2-1\right)c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*a^2*ln(x)*x^2-2*ln(a*x-1)
*x^2*a^2-2*a*x-1)/(a^2*x^2-1)/c/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(-1)^{-2\,a^2cx^2+2\,c}\,a^2\log\left(-2\,a^2c+\frac{2\,c}{x^2}\right)}{2\,\sqrt{c}}+\frac{\frac{1}{2}\left(a\log\left(ax+1\right)-a\log\left(ax-1\right)-\frac{2}{x}\right)a}{\sqrt{c}}-\frac{\sqrt{a^4cx^4-2\,a^2cx^2+c}}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -1/2*(-1)^(-2*a^2*c*x^2 + 2*c)*a^2*log(-2*a^2*c + 2*c/x^2)/sqrt(c) + a*inte
grate(-1/((a*x + 1)*(a*x - 1)*x^2), x)/sqrt(c) - 1/2*sqrt(a^4*c*x^4 - 2*a^2
*c*x^2 + c)/(c*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{x^3\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/(x^3*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x^3\sqrt{-(ax-1)(ax+1)}\,\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

3.965 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{a^2\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{3x^3\sqrt{c-a^2cx^2}}+\frac{a^3\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^3\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] -1/3*(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(1/2)-1/2*a*(-a^2*x^2+1)^(1/2)/x
^2/(-a^2*c*x^2+c)^(1/2)-a^2*(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(1/2)+a^3*l
n(x)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2)-a^3*ln(-a*x+1)*(-a^2*x^2+1)^(1
/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 44}

$$-\frac{a^2\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{3x^3\sqrt{c-a^2cx^2}}+\frac{a^3\sqrt{1-a^2x^2}\,\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^3\sqrt{1-a^2x^2}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*x^3*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2*x
^2*Sqrt[c - a^2*c*x^2]) - (a^2*Sqrt[1 - a^2*x^2])/(x*Sqrt[c - a^2*c*x^2]) +
 (a^3*Sqrt[1 - a^2*x^2]*Log[x])/Sqrt[c - a^2*c*x^2] - (a^3*Sqrt[1 - a^2*x^2
]*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^4\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^4(1-ax)}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{a^2}{x^2}+\frac{a^3}{x}-\frac{a^4}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{3x^3\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}+\frac{a^3\sqrt{1-a^2x^2}\log(x)}{\sqrt{c-a^2cx^2}}-\frac{a^3\sqrt{1-a^2x^2}}{\sqrt{c-}}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.39

$$\frac{\sqrt{1-a^2x^2}\left(a^3\log(x)-a^3\log(1-ax)-\frac{a^2}{x}-\frac{a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-1/3*1/x^3 - a/(2*x^2) - a^2/x + a^3*Log[x] - a^3*Log[1
 - a*x]))/Sqrt[c - a^2*c*x^2]
```

fricas [A] time = 0.89, size = 482, normalized size = 2.58

$$\left[\frac{3\left(a^5x^5-a^3x^3\right)\sqrt{c}\,\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3-6\,a^2+4\,a-1\right)x^4\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{6\left(a^2cx^5-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(a^5*x^5 - a^3*x^3)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*
a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*
c*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 +
4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 -
2*a^3*x^5 + 2*a*x^3 - x^2)) + sqrt(-a^2*c*x^2 + c)*(6*a^2*x^2 - (6*a^2 + 3*
a + 2)*x^3 + 3*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*c*x^5 - c*x^3), -1/6*(6*(a
^5*x^5 - a^3*x^3)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*
((2*a^2 - 2*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c
*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) - sqrt(-a^2*c*x^2 + c)*(6*a^2*
x^2 - (6*a^2 + 3*a + 2)*x^3 + 3*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*c*x^5 - c
*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^4), x)
```

maple [A] time = 0.04, size = 84, normalized size = 0.45

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(6a^3\ln(x)x^3-6\ln\left(ax-1\right)x^3a^3-6a^2x^2-3ax-2\right)}{6\left(a^2x^2-1\right)c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/6*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(6*a^3*ln(x)*x^3-6*ln(a*x-1)
*x^3*a^3-6*a^2*x^2-3*a*x-2)/(a^2*x^2-1)/c/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{x^4\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/(x^4*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x^4\sqrt{-(ax-1)(ax+1)}\,\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(x**4*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

3.966 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=261

$$\frac{\sqrt{1-a^2x^2}}{2a^6c(1-ax)\sqrt{c-a^2cx^2}}+\frac{9\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^6c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^6c\sqrt{c-a^2cx^2}}+\frac{2x\sqrt{1-a^2x^2}}{a^5c\sqrt{c-a^2cx^2}}+\frac{x^2\sqrt{1-a^2x^2}}{2a^4c\sqrt{c-a^2cx^2}}+\cdots$$

```
[Out] 2*x*(-a^2*x^2+1)^(1/2)/a^5/c/(-a^2*c*x^2+c)^(1/2)+1/2*x^2*(-a^2*x^2+1)^(1/2
)/a^4/c/(-a^2*c*x^2+c)^(1/2)+1/3*x^3*(-a^2*x^2+1)^(1/2)/a^3/c/(-a^2*c*x^2+c
)^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/a^6/c/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+9/4*ln(-
a*x+1)*(-a^2*x^2+1)^(1/2)/a^6/c/(-a^2*c*x^2+c)^(1/2)-1/4*ln(a*x+1)*(-a^2*x^
2+1)^(1/2)/a^6/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{x^3\sqrt{1-a^2x^2}}{3a^3c\sqrt{c-a^2cx^2}}+\frac{x^2\sqrt{1-a^2x^2}}{2a^4c\sqrt{c-a^2cx^2}}+\frac{2x\sqrt{1-a^2x^2}}{a^5c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^6c(1-ax)\sqrt{c-a^2cx^2}}+\frac{9\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^6c\sqrt{c-a^2cx^2}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(a^5*c*Sqrt[c - a^2*c*x^2]) + (x^2*Sqrt[1 - a^2*x^2
])/(2*a^4*c*Sqrt[c - a^2*c*x^2]) + (x^3*Sqrt[1 - a^2*x^2])/(3*a^3*c*Sqrt[c
- a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*a^6*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) +
 (9*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*a^6*c*Sqrt[c - a^2*c*x^2]) - (Sqrt[1
 - a^2*x^2]*Log[1 + a*x])/(4*a^6*c*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^5}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{2}{a^5}+\frac{x}{a^4}+\frac{x^2}{a^3}+\frac{1}{2a^5(-1+ax)^2}+\frac{9}{4a^5(-1+ax)}-\frac{1}{4a^5(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{2x\sqrt{1-a^2x^2}}{a^5c\sqrt{c-a^2cx^2}}+\frac{x^2\sqrt{1-a^2x^2}}{2a^4c\sqrt{c-a^2cx^2}}+\frac{x^3\sqrt{1-a^2x^2}}{3a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^6c(1-ax)\sqrt{c-a^2cx^2}}+\frac{9\sqrt{1}}{4}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.33

$$\frac{\sqrt{1-a^2x^2}\left(4a^3x^3+6a^2x^2+24ax+\frac{6}{1-ax}+27\log(1-ax)-3\log(ax+1)\right)}{12a^6c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^(3/2),x]
[Out] (Sqrt[1 - a^2*x^2]*(24*a*x + 6*a^2*x^2 + 4*a^3*x^3 + 6/(1 - a*x) + 27*Log[1
 - a*x] - 3*Log[1 + a*x]))/(12*a^6*c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^5}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^5/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 119, normalized size = 0.46

$$-\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(4x^{4}a^{4}+2x^{3}a^{3}+18a^{2}x^{2}+27\ln\left(ax-1\right)xa-3ax\ln\left(ax+1\right)-24ax-27\ln\left(ax-\right.\right.}{12\left(a^{2}x^{2}-1\right)c^{2}a^{6}\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/12*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*x^4*a^4+2*x^3*a^3+18*a^2
*x^2+27*ln(a*x-1)*x*a-3*a*x*ln(a*x+1)-24*a*x-27*ln(a*x-1)+3*ln(a*x+1)-6)/(a
^2*x^2-1)/c^2/a^6/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\int-\frac{x^{6}}{\left(a^{2}c^{\frac{3}{2}}x^{2}-c^{\frac{3}{2}}\right)(ax+1)(ax-1)}\,dx-\frac{1}{2\left(a^{8}c^{\frac{3}{2}}x^{2}-a^{6}c^{\frac{3}{2}}\right)}+\frac{\log\left(-a^{2}cx^{2}+c\right)}{a^{6}c^{\frac{3}{2}}}-\frac{\sqrt{a^{4}cx^{4}-2\,a^{2}cx^{2}+c}}{2\,a^{6}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -a*integrate(-x^6/((a^2*c^(3/2)*x^2 - c^(3/2))*(a*x + 1)*(a*x - 1)), x) - 1
/2/(a^8*c^(3/2)*x^2 - a^6*c^(3/2)) + log(-a^2*c*x^2 + c)/(a^6*c^(3/2)) - 1/
2*sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)/(a^6*c^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a\,x+1\right)}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
[Out] int((x^5*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**5*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.967 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=218

$$\frac{\sqrt{1-a^2x^2}}{2a^5c(1-ax)\sqrt{c-a^2cx^2}}+\frac{7\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^5c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^5c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-a^2x^2}}{a^4c\sqrt{c-a^2cx^2}}+\frac{x^2\sqrt{1-a^2x^2}}{2a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] x*(-a^2*x^2+1)^(1/2)/a^4/c/(-a^2*c*x^2+c)^(1/2)+1/2*x^2*(-a^2*x^2+1)^(1/2)/
a^3/c/(-a^2*c*x^2+c)^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/a^5/c/(-a*x+1)/(-a^2*c*x^
2+c)^(1/2)+7/4*ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/a^5/c/(-a^2*c*x^2+c)^(1/2)+1/4
*ln(a*x+1)*(-a^2*x^2+1)^(1/2)/a^5/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{x^2\sqrt{1-a^2x^2}}{2a^3c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-a^2x^2}}{a^4c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^5c(1-ax)\sqrt{c-a^2cx^2}}+\frac{7\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^5c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^5c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(a^4*c*Sqrt[c - a^2*c*x^2]) + (x^2*Sqrt[1 - a^2*x^2])
/(2*a^3*c*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*a^5*c*(1 - a*x)*Sqrt[
c - a^2*c*x^2]) + (7*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*a^5*c*Sqrt[c - a^2*
c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*a^5*c*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^4}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{a^4}+\frac{x}{a^3}+\frac{1}{2a^4(-1+ax)^2}+\frac{7}{4a^4(-1+ax)}+\frac{1}{4a^4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{x\sqrt{1-a^2x^2}}{a^4c\sqrt{c-a^2cx^2}}+\frac{x^2\sqrt{1-a^2x^2}}{2a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^5c(1-ax)\sqrt{c-a^2cx^2}}+\frac{7\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^5c\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 0.35

$$\frac{\sqrt{1-a^2x^2}\left(2\left(a^2x^2+2ax+\frac{1}{1-ax}\right)+7\log(1-ax)+\log(ax+1)\right)}{4a^5c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
[Out] (Sqrt[1 - a^2*x^2]*(2*(2*a*x + a^2*x^2 + (1 - a*x)^(-1)) + 7*Log[1 - a*x] +
 Log[1 + a*x]))/(4*a^5*c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^4}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^4/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^4}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 110, normalized size = 0.50

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(2x^3a^3+2a^2x^2+7\ln\left(ax-1\right)xa+ax\ln\left(ax+1\right)-4ax-7\ln\left(ax-1\right)-\ln\left(ax+1\right)\right.}{4\left(a^2x^2-1\right)c^2a^5\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3+2*a^2*x^2+7*ln(a*
x-1)*x*a+a*x*ln(a*x+1)-4*a*x-7*ln(a*x-1)-ln(a*x+1)-2)/(a^2*x^2-1)/c^2/a^5/(
a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^4}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a\,x+1\right)}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^4*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.968 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{\sqrt{1-a^2x^2}}{2a^4c(1-ax)\sqrt{c-a^2cx^2}}+\frac{5\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^4c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-a^2x^2}}{a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] x*(-a^2*x^2+1)^(1/2)/a^3/c/(-a^2*c*x^2+c)^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/a^4/
c/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+5/4*ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/a^4/c/(-a
^2*c*x^2+c)^(1/2)-1/4*ln(a*x+1)*(-a^2*x^2+1)^(1/2)/a^4/c/(-a^2*c*x^2+c)^(1/
2)
```

Rubi [A] time = 0.23, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{x\sqrt{1-a^2x^2}}{a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^4c(1-ax)\sqrt{c-a^2cx^2}}+\frac{5\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(a^3*c*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*a^
4*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*
a^4*c*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*a^4*c*Sqrt
[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{x\sqrt{1-a^2x^2}}{a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2a^4c(1-ax)\sqrt{c-a^2cx^2}}+\frac{5\sqrt{1-a^2x^2}\log(1-ax)}{4a^4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1}{4a^4c\sqrt{c-a^2c}}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.40

$$\frac{\sqrt{1-a^2x^2}\left(4ax+\frac{2}{1-ax}+5\log(1-ax)-\log(ax+1)\right)}{4a^4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1 + a*x]))/(
4*a^4*c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^3}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^3/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 101, normalized size = 0.57

$$-\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(4 a^{2} x^{2}+5 \ln \left(a x -1\right) x a -a x \ln \left(a x +1\right)-4 a x -5 \ln \left(a x -1\right)+\ln \left(a x +1\right)-2\right)}{4 \left(a^{2} x^{2}-1\right) c^{2} a^{4} \left(a x -1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a^2*x^2+5*ln(a*x-1)*x*a-a
*x*ln(a*x+1)-4*a*x-5*ln(a*x-1)+ln(a*x+1)-2)/(a^2*x^2-1)/c^2/a^4/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a \int -\frac{x^{4}}{\left(a^{2} c^{\frac{3}{2}} x^{2}-c^{\frac{3}{2}}\right) (a x +1) (a x -1)}\, dx -\frac{1}{2 \left(a^{6} c^{\frac{3}{2}} x^{2}-a^{4} c^{\frac{3}{2}}\right)}+\frac{\log \left(-a^{2} c x^{2}+c\right)}{2\, a^{4} c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -a*integrate(-x^4/((a^2*c^(3/2)*x^2 - c^(3/2))*(a*x + 1)*(a*x - 1)), x) - 1
/2/(a^6*c^(3/2)*x^2 - a^4*c^(3/2)) + 1/2*log(-a^2*c*x^2 + c)/(a^4*c^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a\,x+1\right)}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
[Out] int((x^3*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**3*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.969 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)/a^3/c/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+3/4*ln(-a*x+1)*(
-a^2*x^2+1)^(1/2)/a^3/c/(-a^2*c*x^2+c)^(1/2)+1/4*ln(a*x+1)*(-a^2*x^2+1)^(1/
2)/a^3/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a^3*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2
*x^2]*Log[1 - a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[
1 + a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2a^2(-1+ax)^2}+\frac{3}{4a^2(-1+ax)}+\frac{1}{4a^2(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(1+ax)}{4a^3c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.55

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1/(2*a^3*(1 - a*x)) + (3*Log[1 - a*x])/(4*a^3) + Log[1
+ a*x]/(4*a^3)))/(c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^2}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 90, normalized size = 0.65

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(3\ln\left(ax-1\right)xa+ax\ln\left(ax+1\right)-3\ln\left(ax-1\right)-\ln\left(ax+1\right)-2\right)}{4\left(a^2x^2-1\right)c^2a^3\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x*a+a*x*ln(a*x+
1)-3*ln(a*x-1)-ln(a*x+1)-2)/(a^2*x^2-1)/c^2/a^3/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a\,x+1\right)}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (ax+1)}{\sqrt{-(ax-1)(ax+1)}\, (-c(ax-1)(ax+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.970 $\int \frac{e^{\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{1-a^2x^2}}{2a^2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2c\sqrt{c-a^2cx^2}}$$

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)/a^2/c/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/2*arctanh(a*x)
*(-a^2*x^2+1)^(1/2)/a^2/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6153, 6150, 77, 207}

$$\frac{\sqrt{1-a^2x^2}}{2a^2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a^2*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x
^2]*ArcTanh[a*x])/(2*a^2*c*Sqrt[c - a^2*c*x^2])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2a(-1+ax)^2}+\frac{1}{2a\left(-1+a^2x^2\right)}\right)dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}}{2a^2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\int \frac{1}{-1+a^2x^2}\,dx}{2ac\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}}{2a^2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2c\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.66

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2a^2(1-ax)}-\frac{\tanh^{-1}(ax)}{2a^2}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1/(2*a^2*(1 - a*x)) - ArcTanh[a*x]/(2*a^2)))/(c*Sqrt[c
- a^2*c*x^2])
```

fricas [A] time = 1.27, size = 348, normalized size = 3.82

$$\left[\frac{4\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,ax+\left(a^3x^3-a^2x^2-ax+1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2+4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{}}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right.}{8\left(a^5c^2x^3-a^4c^2x^2-a^3c^2x+a^2c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/8*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^3*x^3 - a^2*x^2 -
a*x + 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 + 4*(a^3*x^3 +
 a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4
*x^4 + 3*a^2*x^2 - 1)))/(a^5*c^2*x^3 - a^4*c^2*x^2 - a^3*c^2*x + a^2*c^2),
1/4*(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^3*x^3 - a^2*x^2 - a
*x + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c
)*x/(a^4*c*x^4 - c)))/(a^5*c^2*x^3 - a^4*c^2*x^2 - a^3*c^2*x + a^2*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)*x/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 88, normalized size = 0.97

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln(ax-1)\,xa-ax\ln(ax+1)-\ln(ax-1)+\ln(ax+1)-2\right)}{4\left(a^2x^2-1\right)c^2a^2(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a-a*x*ln(a*x+1)
-ln(a*x-1)+ln(a*x+1)-2)/(a^2*x^2-1)/c^2/a^2/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\int -\frac{x^2}{\left(a^2c^{\frac{3}{2}}x^2 - c^{\frac{3}{2}}\right)(ax+1)(ax-1)}\,dx - \frac{1}{2\left(a^4c^{\frac{3}{2}}x^2 - a^2c^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] -a*integrate(-x^2/((a^2*c^(3/2)*x^2 - c^(3/2))*(a*x + 1)*(a*x - 1)), x) - 1
/2/(a^4*c^(3/2)*x^2 - a^2*c^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a\,x+1)}{\left(c-a^2\,c\,x^2\right)^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**
(3/2)), x)
```

3.971 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{1-a^2x^2}}{2ac(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}$$

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)/a/c/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+1/2*arctanh(a*x)*(
-a^2*x^2+1)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{2ac(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2
]*ArcTanh[a*x])/(2*a*c*Sqrt[c - a^2*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}}{2ac(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\int \frac{1}{-1+a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}}{2ac(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.66

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a)))/(c*Sqrt[c - a^
2*c*x^2])
```

fricas [A] time = 0.76, size = 343, normalized size = 3.77

$$\left[\frac{4\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,ax+\left(a^3x^3-a^2x^2-ax+1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right.}{8\left(a^4c^2x^3-a^3c^2x^2-a^2c^2x+ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/8*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^3*x^3 - a^2*x^2 -
a*x + 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 +
 a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4
*x^4 + 3*a^2*x^2 - 1)))/(a^4*c^2*x^3 - a^3*c^2*x^2 - a^2*c^2*x + a*c^2), 1/
4*(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^3*x^3 - a^2*x^2 - a*x
 + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*
x/(a^4*c*x^4 - c)))/(a^4*c^2*x^3 - a^3*c^2*x^2 - a^2*c^2*x + a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 88, normalized size = 0.97

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)-\ln\left(ax-1\right)+\ln\left(ax+1\right)+2\right)}{4\left(a^2x^2-1\right)c^2a\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a-a*x*ln(a*x+1)-
ln(a*x-1)+ln(a*x+1)+2)/(a^2*x^2-1)/c^2/a/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3
/2)), x)
```

3.972 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4c\sqrt{c-a^2cx^2}}$$

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)/c/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+ln(x)*(-a^2*x^2+1)^(
1/2)/c/(-a^2*c*x^2+c)^(1/2)-3/4*ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/c/(-a^2*c*x^2
+c)^(1/2)-1/4*ln(a*x+1)*(-a^2*x^2+1)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 72}

$$\frac{\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*
Log[x])/(c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*c*S
qrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*c*Sqrt[c - a^2*c*
x^2])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x}+\frac{a}{2(-1+ax)^2}-\frac{3a}{4(-1+ax)}-\frac{a}{4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(}{4c\sqrt{c-a^2c}}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 0.39

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2-2ax}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a*x])/4 - Log[1
+ a*x]/4))/(c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{a^5c^2x^6-a^4c^2x^5-2\,a^3c^2x^4+2\,a^2c^2x^3+ac^2x^2-c^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^5*c^2*x^6 - a^4*c^2*x^
5 - 2*a^3*c^2*x^4 + 2*a^2*c^2*x^3 + a*c^2*x^2 - c^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.05, size = 96, normalized size = 0.58

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(4a\ln(x)x-3\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)-4\ln(x)+3\ln\left(ax-1\right)+\ln\left(ax+1\right)\right.}{4c^2\left(ax-1\right)\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a*ln(x)*x-3*ln(a*x-1)*x*a
-a*x*ln(a*x+1)-4*ln(x)+3*ln(a*x-1)+ln(a*x+1)-2)/c^2/(a*x-1)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{x\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/(x*(c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**
(3/2)), x)
```

3.973 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=206

$$\frac{a\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{5a\sqrt{1-a^2x^2}\,\log(1-ax)}{4c\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,\log(ax+1)}{4c\sqrt{c-a^2cx^2}}$$

```
[Out] -(-a^2*x^2+1)^(1/2)/c/x/(-a^2*c*x^2+c)^(1/2)+1/2*a*(-a^2*x^2+1)^(1/2)/c/(-a
*x+1)/(-a^2*c*x^2+c)^(1/2)+a*ln(x)*(-a^2*x^2+1)^(1/2)/c/(-a^2*c*x^2+c)^(1/2
)-5/4*a*ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)+1/4*a*ln(a*x+1
)*(-a^2*x^2+1)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{a\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{5a\sqrt{1-a^2x^2}\,\log(1-ax)}{4c\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,\log(ax+1)}{4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(c*x*Sqrt[c - a^2*c*x^2])) + (a*Sqrt[1 - a^2*x^2])/(2*c
*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[1 - a^2*x^2]*Log[x])/(c*Sqrt[c -
a^2*c*x^2]) - (5*a*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*c*Sqrt[c - a^2*c*x^2]
) + (a*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(4*c*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^2(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^2}+\frac{a}{x}+\frac{a^2}{2(-1+ax)^2}-\frac{5a^2}{4(-1+ax)}+\frac{a^2}{4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{5a\sqrt{1-a^2x^2}\,\log(1-}{4c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.37

$$\frac{\sqrt{1-a^2x^2}\left(\frac{2a}{1-ax}+4a\log(x)-5a\log(1-ax)+a\log(ax+1)-\frac{4}{x}\right)}{4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-4/x + (2*a)/(1 - a*x) + 4*a*Log[x] - 5*a*Log[1 - a*x]
+ a*Log[1 + a*x]))/(4*c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{a^5c^2x^7-a^4c^2x^6-2\,a^3c^2x^5+2\,a^2c^2x^4+ac^2x^3-c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^5*c^2*x^7 - a^4*c^2*x^
6 - 2*a^3*c^2*x^5 + 2*a^2*c^2*x^4 + a*c^2*x^3 - c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.05, size = 122, normalized size = 0.59

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(4a^2\ln(x)x^2-5\ln\left(ax-1\right)x^2a^2+\ln\left(ax+1\right)x^2a^2-4a\ln(x)x+5\ln\left(ax-1\right)xa-\right.}{4\left(a^2x^2-1\right)c^2\left(ax-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a^2*ln(x)*x^2-5*ln(a*x-1)
*x^2*a^2+ln(a*x+1)*x^2*a^2-4*a*ln(x)*x+5*ln(a*x-1)*x*a-a*x*ln(a*x+1)-6*a*x+
4)/(a^2*x^2-1)/c^2/(a*x-1)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{x^2\,\left(c-a^2\,c\,x^2\right)^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/(x^2*(c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x^2\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.974 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=255

$$\frac{a^2\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}+\frac{2a^2\sqrt{1-a^2x^2}\,\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{7a^2\sqrt{1-a^2x^2}\,\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1}}{}$$

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)/c/x^2/(-a^2*c*x^2+c)^(1/2)-a*(-a^2*x^2+1)^(1/2)/c/x
/(-a^2*c*x^2+c)^(1/2)+1/2*a^2*(-a^2*x^2+1)^(1/2)/c/(-a*x+1)/(-a^2*c*x^2+c)^
(1/2)+2*a^2*ln(x)*(-a^2*x^2+1)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)-7/4*a^2*ln(-a*x
+1)*(-a^2*x^2+1)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)-1/4*a^2*ln(a*x+1)*(-a^2*x^2+1
)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{a^2\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}+\frac{2a^2\sqrt{1-a^2x^2}\,\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{7a^2\sqrt{1-a^2x^2}\,\log(1-ax)}{4c\sqrt{c-a^2cx^2}}-\frac{a^2\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*c*x^2*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(c
*x*Sqrt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(2*c*(1 - a*x)*Sqrt[c - a
^2*c*x^2]) + (2*a^2*Sqrt[1 - a^2*x^2]*Log[x])/(c*Sqrt[c - a^2*c*x^2]) - (7*
a^2*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(4*c*Sqrt[c - a^2*c*x^2]) - (a^2*Sqrt[1
 - a^2*x^2]*Log[1 + a*x])/(4*c*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^3(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{2a^2}{x}+\frac{a^3}{2(-1+ax)^2}-\frac{7a^3}{4(-1+ax)}-\frac{a^3}{4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{2a^2\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-$$

Mathematica [A] time = 0.07, size = 91, normalized size = 0.36

$$\frac{\sqrt{1-a^2x^2}\left(\frac{2a^2}{1-ax}+8a^2\log(x)-7a^2\log(1-ax)-a^2\log(ax+1)-\frac{4a}{x}-\frac{2}{x^2}\right)}{4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8*a^2*Log[x] - 7
*a^2*Log[1 - a*x] - a^2*Log[1 + a*x]))/(4*c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{a^5c^2x^8-a^4c^2x^7-2\,a^3c^2x^6+2\,a^2c^2x^5+ac^2x^4-c^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^5*c^2*x^8 - a^4*c^2*x^
7 - 2*a^3*c^2*x^6 + 2*a^2*c^2*x^5 + a*c^2*x^4 - c^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.05, size = 142, normalized size = 0.56

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(8a^3\ln(x)x^3-7\ln\left(ax-1\right)x^3a^3-a^3x^3\ln\left(ax+1\right)-8a^2\ln(x)x^2+7\ln\left(ax-1\right)x^2a\right.}{4\left(a^2x^2-1\right)c^2\left(ax-1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(8*a^3*ln(x)*x^3-7*ln(a*x-1)
*x^3*a^3-a^3*x^3*ln(a*x+1)-8*a^2*ln(x)*x^2+7*ln(a*x-1)*x^2*a^2+ln(a*x+1)*x^
2*a^2-6*a^2*x^2+2*a*x+2)/(a^2*x^2-1)/c^2/(a*x-1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{x^3\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/(x^3*(c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x^{3}\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.975 $\int \frac{e^{\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=297

$$-\frac{2a^2\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{3cx^3\sqrt{c-a^2cx^2}}+\frac{a^3\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{2a^3\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{9a^3\sqrt{1-a^2x^2}}{4c\sqrt{c-}}$$

```
[Out] -1/3*(-a^2*x^2+1)^(1/2)/c/x^3/(-a^2*c*x^2+c)^(1/2)-1/2*a*(-a^2*x^2+1)^(1/2)
/c/x^2/(-a^2*c*x^2+c)^(1/2)-2*a^2*(-a^2*x^2+1)^(1/2)/c/x/(-a^2*c*x^2+c)^(1/
2)+1/2*a^3*(-a^2*x^2+1)^(1/2)/c/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+2*a^3*ln(x)*(
-a^2*x^2+1)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)-9/4*a^3*ln(-a*x+1)*(-a^2*x^2+1)^(1
/2)/c/(-a^2*c*x^2+c)^(1/2)+1/4*a^3*ln(a*x+1)*(-a^2*x^2+1)^(1/2)/c/(-a^2*c*x
^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{a^3\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}-\frac{2a^2\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{3cx^3\sqrt{c-a^2cx^2}}+\frac{2a^3\sqrt{1-a^2x^2}\log(x)}{c\sqrt{c-a^2cx^2}}-\frac{9a^3\sqrt{1-a^2x^2}}{4c\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(3*c*x^3*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2
*c*x^2*Sqrt[c - a^2*c*x^2]) - (2*a^2*Sqrt[1 - a^2*x^2])/(c*x*Sqrt[c - a^2*c
*x^2]) + (a^3*Sqrt[1 - a^2*x^2])/(2*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (2*a
^3*Sqrt[1 - a^2*x^2]*Log[x])/(c*Sqrt[c - a^2*c*x^2]) - (9*a^3*Sqrt[1 - a^2*
x^2]*Log[1 - a*x])/(4*c*Sqrt[c - a^2*c*x^2]) + (a^3*Sqrt[1 - a^2*x^2]*Log[1
 + a*x])/(4*c*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^4\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^4(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{2a^2}{x^2}+\frac{2a^3}{x}+\frac{a^4}{2(-1+ax)^2}-\frac{9a^4}{4(-1+ax)}+\frac{a^4}{4(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}}{3cx^3\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}-\frac{2a^2\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}}+\frac{a^3\sqrt{1-a^2x^2}}{2c(1-ax)\sqrt{c-a^2cx^2}}+\frac{2a^3}{}$$

Mathematica [A] time = 0.08, size = 99, normalized size = 0.33

$$\frac{\sqrt{1-a^2x^2}\left(\frac{6a^3}{1-ax}+24a^3\log(x)-27a^3\log(1-ax)+3a^3\log(ax+1)-\frac{24a^2}{x}-\frac{6a}{x^2}-\frac{4}{x^3}\right)}{12c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^4*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-4/x^3 - (6*a)/x^2 - (24*a^2)/x + (6*a^3)/(1 - a*x) + 2
4*a^3*Log[x] - 27*a^3*Log[1 - a*x] + 3*a^3*Log[1 + a*x]))/(12*c*Sqrt[c - a^
2*c*x^2])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{a^5c^2x^9-a^4c^2x^8-2\,a^3c^2x^7+2\,a^2c^2x^6+ac^2x^5-c^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^5*c^2*x^9 - a^4*c^2*x^
8 - 2*a^3*c^2*x^7 + 2*a^2*c^2*x^6 + a*c^2*x^5 - c^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^4), x)
```

maple [A] time = 0.05, size = 151, normalized size = 0.51

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(24a^4\ln(x)x^4-27\ln\left(ax-1\right)x^4a^4+3\ln\left(ax+1\right)x^4a^4-24a^3\ln(x)x^3+27\ln\left(ax-\right.\right.}{12\left(a^2x^2-1\right)c^2x^3\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/12*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(24*a^4*ln(x)*x^4-27*ln(a*x
-1)*x^4*a^4+3*ln(a*x+1)*x^4*a^4-24*a^3*ln(x)*x^3+27*ln(a*x-1)*x^3*a^3-3*a^3
*x^3*ln(a*x+1)-30*x^3*a^3+18*a^2*x^2+2*a*x+4)/(a^2*x^2-1)/c^2/x^3/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{x^4\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/(x^4*(c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x^{4}\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**4/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/(x**4*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.976 $\int \frac{e^{\tanh^{-1}(ax)}x^6}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=312

$$-\frac{5\sqrt{1-a^2x^2}}{4a^7c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^7c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^7c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{39\sqrt{1-a^2x^2}\,\log(1-ax)}{16a^7c^2\sqrt{c-a^2cx^2}}-\frac{9\sqrt{}}{}$$

```
[Out] -x*(-a^2*x^2+1)^(1/2)/a^6/c^2/(-a^2*c*x^2+c)^(1/2)-1/2*x^2*(-a^2*x^2+1)^(1/
2)/a^5/c^2/(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^2+1)^(1/2)/a^7/c^2/(-a*x+1)^2/(
-a^2*c*x^2+c)^(1/2)-5/4*(-a^2*x^2+1)^(1/2)/a^7/c^2/(-a*x+1)/(-a^2*c*x^2+c)^
(1/2)-1/8*(-a^2*x^2+1)^(1/2)/a^7/c^2/(a*x+1)/(-a^2*c*x^2+c)^(1/2)-39/16*ln(
-a*x+1)*(-a^2*x^2+1)^(1/2)/a^7/c^2/(-a^2*c*x^2+c)^(1/2)-9/16*ln(a*x+1)*(-a^
2*x^2+1)^(1/2)/a^7/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$-\frac{x^2\sqrt{1-a^2x^2}}{2a^5c^2\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}}{a^6c^2\sqrt{c-a^2cx^2}}-\frac{5\sqrt{1-a^2x^2}}{4a^7c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^7c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^7c^2(1-ax)^2\sqrt{c-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^6*c^2*Sqrt[c - a^2*c*x^2])) - (x^2*Sqrt[1 - a^2*
x^2])/(2*a^5*c^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a^7*c^2*(1 - a
*x)^2*Sqrt[c - a^2*c*x^2]) - (5*Sqrt[1 - a^2*x^2])/(4*a^7*c^2*(1 - a*x)*Sqr
t[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a^7*c^2*(1 + a*x)*Sqrt[c - a^2*c*x
^2]) - (39*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*a^7*c^2*Sqrt[c - a^2*c*x^2])
 - (9*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(16*a^7*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^6}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^6}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^6}-\frac{x}{a^5}-\frac{1}{4a^6(-1+ax)^3}-\frac{5}{4a^6(-1+ax)^2}-\frac{39}{16a^6(-1+ax)}+\frac{1}{8a^6(1+ax)^2}-\frac{9}{16a^6(1+ax)}\right) dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{x\sqrt{1-a^2x^2}}{a^6c^2\sqrt{c-a^2cx^2}}-\frac{x^2\sqrt{1-a^2x^2}}{2a^5c^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^7c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{5\sqrt{1-a^2x^2}}{4a^7c^2(1-ax)\sqrt{c-}}$$

Mathematica [A] time = 0.13, size = 97, normalized size = 0.31

$$\frac{\sqrt{1-a^2x^2}\left(2\left(-4a^2x^2-8ax+\frac{10}{ax-1}-\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-39\log(1-ax)-9\log(ax+1)\right)}{16a^7c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^6)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*(-8*a*x - 4*a^2*x^2 + (-1 + a*x)^(-2) + 10/(-1 + a*x)
 - (1 + a*x)^(-1)) - 39*Log[1 - a*x] - 9*Log[1 + a*x]))/(16*a^7*c^2*Sqrt[c
- a^2*c*x^2])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^6}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^6/(a^7*c^3*x^7 - a^6*c^3
*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^
3*x + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^6}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^6/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 190, normalized size = 0.61

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(8x^5a^5+8x^4a^4+39\ln\left(ax-1\right)x^3a^3+9a^3x^3\ln\left(ax+1\right)-24x^3a^3-39\ln\left(ax-1\right)x^2\right.}{16\left(a^2x^2-1\right)c^3a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5+8*x^4*a^4+39*ln(a
*x-1)*x^3*a^3+9*a^3*x^3*ln(a*x+1)-24*x^3*a^3-39*ln(a*x-1)*x^2*a^2-9*ln(a*x+
1)*x^2*a^2-26*a^2*x^2-39*ln(a*x-1)*x*a-9*a*x*ln(a*x+1)+10*a*x+39*ln(a*x-1)+
9*ln(a*x+1)+20)/(a^2*x^2-1)/c^3/a^7/(a*x-1)^2/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^6}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^6/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^6/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(a\,x+1\right)}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^6*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6} \left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**6/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**6*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

3.977 $\int \frac{e^{\tanh^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{\sqrt{1-a^2x^2}}{a^6c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{23\sqrt{1-a^2x^2}\,\log(1-ax)}{16a^6c^2\sqrt{c-a^2cx^2}}+\frac{7\sqrt{1}}{1}$$

```
[Out] -x*(-a^2*x^2+1)^(1/2)/a^5/c^2/(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^2+1)^(1/2)/a
^6/c^2/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)-(-a^2*x^2+1)^(1/2)/a^6/c^2/(-a*x+1)/
(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^2+1)^(1/2)/a^6/c^2/(a*x+1)/(-a^2*c*x^2+c)^
(1/2)-23/16*ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/a^6/c^2/(-a^2*c*x^2+c)^(1/2)+7/16
*ln(a*x+1)*(-a^2*x^2+1)^(1/2)/a^6/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$-\frac{x\sqrt{1-a^2x^2}}{a^5c^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{a^6c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{23\sqrt{1-a^2x^2}}{16a^6c^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*Sqrt[1 - a^2*x^2])/(a^5*c^2*Sqrt[c - a^2*c*x^2])) + Sqrt[1 - a^2*x^2]/
(8*a^6*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(a^6*c^2*(1
 - a*x)*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a^6*c^2*(1 + a*x)*Sqrt[
c - a^2*c*x^2]) - (23*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*a^6*c^2*Sqrt[c -
a^2*c*x^2]) + (7*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(16*a^6*c^2*Sqrt[c - a^2*c
*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^5}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^5}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{a^5}-\frac{1}{4a^5(-1+ax)^3}-\frac{1}{a^5(-1+ax)^2}-\frac{23}{16a^5(-1+ax)}-\frac{1}{8a^5(1+ax)^2}+\frac{7}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{a^5c^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^6c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{a^6c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-}}{8a^6c^2(1+ax}$$

Mathematica [A] time = 0.12, size = 87, normalized size = 0.32

$$\frac{\sqrt{1-a^2x^2}\left(2\left(-8ax+\frac{8}{ax-1}+\frac{1}{ax+1}+\frac{1}{(ax-1)^2}\right)-23\log(1-ax)+7\log(ax+1)\right)}{16a^6c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*(-8*a*x + (-1 + a*x)^(-2) + 8/(-1 + a*x) + (1 + a*x)^
(-1)) - 23*Log[1 - a*x] + 7*Log[1 + a*x]))/(16*a^6*c^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^5}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^5/(a^7*c^3*x^7 - a^6*c^3
*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^
3*x + c^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 182, normalized size = 0.68

$$\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(16x^{4}a^{4}+23\ln\left(ax-1\right)x^{3}a^{3}-7a^{3}x^{3}\ln\left(ax+1\right)-16x^{3}a^{3}-23\ln\left(ax-1\right)x^{2}a^{2}+7\ln\right.}{16\left(a^{2}x^{2}-1\right)c^{3}a^{6}\left(ax\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(16*x^4*a^4+23*ln(a*x-1)*x^3
*a^3-7*a^3*x^3*ln(a*x+1)-16*x^3*a^3-23*ln(a*x-1)*x^2*a^2+7*ln(a*x+1)*x^2*a^
2-34*a^2*x^2-23*ln(a*x-1)*x*a+7*a*x*ln(a*x+1)+18*a*x+23*ln(a*x-1)-7*ln(a*x+
1)+12)/(a^2*x^2-1)/c^3/a^6/(a*x-1)^2/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int-\frac{x^{6}}{\left(a^{4}c^{\frac{5}{2}}x^{4}-2\,a^{2}c^{\frac{5}{2}}x^{2}+c^{\frac{5}{2}}\right)(ax+1)(ax-1)}\,dx+\frac{1}{4\left(a^{10}c^{\frac{5}{2}}x^{4}-2\,a^{8}c^{\frac{5}{2}}x^{2}+a^{6}c^{\frac{5}{2}}\right)}+\frac{1}{a^{8}c^{\frac{5}{2}}x^{2}-a^{6}c^{\frac{5}{2}}}-\frac{\log\left(-a^{2}cx^{2}\right.}{2\,a^{6}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] a*integrate(-x^6/((a^4*c^(5/2)*x^4 - 2*a^2*c^(5/2)*x^2 + c^(5/2))*(a*x + 1)
*(a*x - 1)), x) + 1/4/(a^10*c^(5/2)*x^4 - 2*a^8*c^(5/2)*x^2 + a^6*c^(5/2))
+ 1/(a^8*c^(5/2)*x^2 - a^6*c^(5/2)) - 1/2*log(-a^2*c*x^2 + c)/(a^6*c^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,(a\,x+1)}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^5*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**5/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**5*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

3.978 $\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=232

$$-\frac{3\sqrt{1-a^2x^2}}{4a^5c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^5c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^5c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{11\sqrt{1-a^2x^2}\,\log(1-ax)}{16a^5c^2\sqrt{c-a^2cx^2}}-5\sqrt{\ }$$

```
[Out] 1/8*(-a^2*x^2+1)^(1/2)/a^5/c^2/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)-3/4*(-a^2*x^
2+1)^(1/2)/a^5/c^2/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/8*(-a^2*x^2+1)^(1/2)/a^5
/c^2/(a*x+1)/(-a^2*c*x^2+c)^(1/2)-11/16*ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/a^5/c
^2/(-a^2*c*x^2+c)^(1/2)-5/16*ln(a*x+1)*(-a^2*x^2+1)^(1/2)/a^5/c^2/(-a^2*c*x
^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$-\frac{3\sqrt{1-a^2x^2}}{4a^5c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^5c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^5c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{11\sqrt{1-a^2x^2}\,\log(1-ax)}{16a^5c^2\sqrt{c-a^2cx^2}}-5\sqrt{\ }$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^5*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 -
 a^2*x^2])/(4*a^5*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8
*a^5*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (11*Sqrt[1 - a^2*x^2]*Log[1 - a*x
])/(16*a^5*c^2*Sqrt[c - a^2*c*x^2]) - (5*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(1
6*a^5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^4}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^4}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a^4(-1+ax)^3}-\frac{3}{4a^4(-1+ax)^2}-\frac{11}{16a^4(-1+ax)}+\frac{1}{8a^4(1+ax)^2}-\frac{5}{16a^4(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{8a^5c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{4a^5c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^5c^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{11\sqrt{}}{1}$$

Mathematica [A] time = 0.10, size = 87, normalized size = 0.38

$$\frac{\sqrt{1-a^2x^2}\left(\frac{2\left(5a^2x^2+3ax-6\right)}{(ax-1)^2(ax+1)}-11\log(1-ax)-5\log(ax+1)\right)}{16a^5c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*(1 + a*x)) -
 11*Log[1 - a*x] - 5*Log[1 + a*x]))/(16*a^5*c^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^4}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^4/(a^7*c^3*x^7 - a^6*c^3
*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^
3*x + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^4}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 166, normalized size = 0.72

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(11\ln\left(ax-1\right)x^3a^3+5a^3x^3\ln\left(ax+1\right)-11\ln\left(ax-1\right)x^2a^2-5\ln\left(ax+1\right)x^2a^2-10\right.}{16\left(a^2x^2-1\right)c^3a^5\left(ax-1\right)^2\left(ax+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(11*ln(a*x-1)*x^3*a^3+5*a^3*
x^3*ln(a*x+1)-11*ln(a*x-1)*x^2*a^2-5*ln(a*x+1)*x^2*a^2-10*a^2*x^2-11*ln(a*x
-1)*x*a-5*a*x*ln(a*x+1)-6*a*x+11*ln(a*x-1)+5*ln(a*x+1)+12)/(a^2*x^2-1)/c^3/
a^5/(a*x-1)^2/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^4}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(a\,x+1)}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
[Out] int((x^4*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,(ax+1)}{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**4/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(x**4*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

3.979 $\int \frac{e^{\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{1-a^2x^2}}{2a^4c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^4c^2\sqrt{c-a^2cx^2}}$$

```
[Out] 1/8*(-a^2*x^2+1)^(1/2)/a^4/c^2/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)-1/2*(-a^2*x^
2+1)^(1/2)/a^4/c^2/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^2+1)^(1/2)/a^4
/c^2/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+3/8*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^4/c
^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6153, 6150, 88, 207}

$$-\frac{\sqrt{1-a^2x^2}}{2a^4c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^4c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^4*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^
2*x^2]/(2*a^4*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a^4
*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8
*a^4*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}} \\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^3}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}} \\
&= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a^3(-1+ax)^3}-\frac{1}{2a^3(-1+ax)^2}-\frac{1}{8a^3(1+ax)^2}-\frac{3}{8a^3\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}} \\
&= \frac{\sqrt{1-a^2x^2}}{8a^4c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2a^4c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\left(3\sqrt{\right.}}{} \\
&= \frac{\sqrt{1-a^2x^2}}{8a^4c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2a^4c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^4c^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{}}{}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.46

$$\frac{\sqrt{1-a^2x^2}\left(5a^2x^2-ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)-2\right)}{8a^4c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2 - a*x + 5*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTanh
[a*x]))/(8*a^4*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.69, size = 461, normalized size = 2.51

$$\left[\frac{3\left(a^{5}x^{5}-a^{4}x^{4}-2\,a^{3}x^{3}+2\,a^{2}x^{2}+ax-1\right)\sqrt{c}\,\log\left(-\frac{a^{6}cx^{6}+5\,a^{4}cx^{4}-5\,a^{2}cx^{2}-4\left(a^{3}x^{3}+ax\right)\sqrt{-a^{2}cx^{2}+c}\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{c}-c}{a^{6}x^{6}-3\,a^{4}x^{4}+3\,a^{2}x^{2}-1}\right)-4\left(2\,a^{3}\right.}{32\left(a^{9}c^{3}x^{5}-a^{8}c^{3}x^{4}-2\,a^{7}c^{3}x^{3}+2\,a^{6}c^{3}x^{2}+a^{5}c^{3}x-a^{4}c^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] [1/32*(3*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(c)*log(
-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2
 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)
) - 4*(2*a^3*x^3 + 3*a^2*x^2 - 3*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 +
1))/(a^9*c^3*x^5 - a^8*c^3*x^4 - 2*a^7*c^3*x^3 + 2*a^6*c^3*x^2 + a^5*c^3*x
- a^4*c^3), 1/16*(3*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*s
qrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*
c*x^4 - c)) - 2*(2*a^3*x^3 + 3*a^2*x^2 - 3*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-
a^2*x^2 + 1))/(a^9*c^3*x^5 - a^8*c^3*x^4 - 2*a^7*c^3*x^3 + 2*a^6*c^3*x^2 +
a^5*c^3*x - a^4*c^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 166, normalized size = 0.90

$$\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(3\ln\left(ax-1\right)x^{3}a^{3}-3a^{3}x^{3}\ln\left(ax+1\right)-3\ln\left(ax-1\right)x^{2}a^{2}+3\ln\left(ax+1\right)x^{2}a^{2}-10a^{2}\right.}{16\left(a^{2}x^{2}-1\right)c^{3}a^{4}\left(ax-1\right)^{2}\left(ax+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x^3*a^3-3*a^3*x
^3*ln(a*x+1)-3*ln(a*x-1)*x^2*a^2+3*ln(a*x+1)*x^2*a^2-10*a^2*x^2-3*ln(a*x-1)
*x*a+3*a*x*ln(a*x+1)+2*a*x+3*ln(a*x-1)-3*ln(a*x+1)+4)/(a^2*x^2-1)/c^3/a^4/(
a*x-1)^2/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int -\frac{x^4}{\left(a^4c^{\frac{5}{2}}x^4 - 2\,a^2c^{\frac{5}{2}}x^2 + c^{\frac{5}{2}}\right)(ax+1)(ax-1)}\,dx + \frac{1}{4\left(a^8c^{\frac{5}{2}}x^4 - 2\,a^6c^{\frac{5}{2}}x^2 + a^4c^{\frac{5}{2}}\right)} + \frac{1}{2\left(a^6c^{\frac{5}{2}}x^2 - a^4c^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] a*integrate(-x^4/((a^4*c^(5/2)*x^4 - 2*a^2*c^(5/2)*x^2 + c^(5/2))*(a*x + 1)
*(a*x - 1)), x) + 1/4/(a^8*c^(5/2)*x^4 - 2*a^6*c^(5/2)*x^2 + a^4*c^(5/2)) +
 1/2/(a^6*c^(5/2)*x^2 - a^4*c^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(a\,x+1)}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^3*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

3.980 $\int \frac{e^{\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{1-a^2x^2}}{4a^3c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^3c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^3c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^3c^2\sqrt{c-a^2cx^2}}$$

```
[Out] 1/8*(-a^2*x^2+1)^(1/2)/a^3/c^2/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)-1/4*(-a^2*x^
2+1)^(1/2)/a^3/c^2/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/8*(-a^2*x^2+1)^(1/2)/a^3
/c^2/(a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/8*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a^3/c
^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6153, 6150, 88, 207}

$$-\frac{\sqrt{1-a^2x^2}}{4a^3c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^3c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^3c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^3c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^3*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^
2*x^2]/(4*a^3*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a^3
*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a
^3*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a^2(-1+ax)^3}-\frac{1}{4a^2(-1+ax)^2}+\frac{1}{8a^2(1+ax)^2}+\frac{1}{8a^2\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{8a^3c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4a^3c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^3c^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1}}{8}$$

$$= \frac{\sqrt{1-a^2x^2}}{8a^3c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4a^3c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8a^3c^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1}}{8}$$

Mathematica [A] time = 0.06, size = 84, normalized size = 0.46

$$\frac{\sqrt{1-a^2x^2}\left(a^2x^2+3ax-(ax-1)^2(ax+1)\tanh^{-1}(ax)-2\right)}{8a^3c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2 + 3*a*x + a^2*x^2 - (-1 + a*x)^2*(1 + a*x)*ArcTanh[a
*x]))/(8*a^3*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.54, size = 457, normalized size = 2.48

$$\left[\frac{\left(a^5x^5 - a^4x^4 - 2\,a^3x^3 + 2\,a^2x^2 + ax - 1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2+4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right) - 4\left(2\,a^3x\right.}{32\left(a^8c^3x^5 - a^7c^3x^4 - 2\,a^6c^3x^3 + 2\,a^5c^3x^2 + a^4c^3x - a^3c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] [1/32*((a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(c)*log(-(
a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 + 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1))
- 4*(2*a^3*x^3 - a^2*x^2 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a
^8*c^3*x^5 - a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^5*c^3*x^2 + a^4*c^3*x - a^3*
c^3), -1/16*((a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(-c)
*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 -
 c)) + 2*(2*a^3*x^3 - a^2*x^2 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1
))/(a^8*c^3*x^5 - a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^5*c^3*x^2 + a^4*c^3*x -
 a^3*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 161, normalized size = 0.88

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln(ax-1)\,x^3a^3 - a^3x^3\ln(ax+1) - \ln(ax-1)\,x^2a^2 + \ln(ax+1)\,x^2a^2 + 2a^2x^2 - \right.}{16\left(a^2x^2-1\right)c^3a^3\,(ax-1)^2\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x^3*a^3-a^3*x^3*
ln(a*x+1)-ln(a*x-1)*x^2*a^2+ln(a*x+1)*x^2*a^2+2*a^2*x^2-ln(a*x-1)*x*a+a*x*l
n(a*x+1)+6*a*x+ln(a*x-1)-ln(a*x+1)-4)/(a^2*x^2-1)/c^3/a^3/(a*x-1)^2/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a\,x+1\right)}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

3.981 $\int \frac{e^{\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\sqrt{1-a^2x^2}}{8a^2c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^2c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^2c^2\sqrt{c-a^2cx^2}}$$

```
[Out] 1/8*(-a^2*x^2+1)^(1/2)/a^2/c^2/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^
2+1)^(1/2)/a^2/c^2/(a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/8*arctanh(a*x)*(-a^2*x^2+
1)^(1/2)/a^2/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6153, 6150, 77, 207}

$$\frac{\sqrt{1-a^2x^2}}{8a^2c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^2c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8a^2c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^2*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^
2*x^2]/(8*a^2*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*ArcTa
nh[a*x])/(8*a^2*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x}{(1-a^2x^2)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a(-1+ax)^3}-\frac{1}{8a(1+ax)^2}+\frac{1}{8a(-1+a^2x^2)}\right)dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}}{8a^2c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^2c^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\int \frac{1}{-1+a^2x^2}\,dx}{8ac^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}}{8a^2c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8a^2c^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2c^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.44

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{ax+1}+\frac{1}{(ax-1)^2}-\tanh^{-1}(ax)\right)}{8a^2c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((-1 + a*x)^(-2) + (1 + a*x)^(-1) - ArcTanh[a*x]))/(8*a^
2*c^2*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.67, size = 459, normalized size = 3.35

$$\left[\frac{\left(a^5x^5 - a^4x^4 - 2a^3x^3 + 2a^2x^2 + ax - 1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6 + 5a^4cx^4 - 5a^2cx^2 + 4\left(a^3x^3 + ax\right)\sqrt{-a^2cx^2 + c}\,\sqrt{-a^2x^2 + 1}\,\sqrt{c} - c}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\right) + 4\left(2a^3x\right.}{32\left(a^7c^3x^5 - a^6c^3x^4 - 2a^5c^3x^3 + 2a^4c^3x^2 + a^3c^3x - a^2c^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/32*((a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(c)*log(-(
a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 + 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1))
+ 4*(2*a^3*x^3 - 3*a^2*x^2 - a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/
(a^7*c^3*x^5 - a^6*c^3*x^4 - 2*a^5*c^3*x^3 + 2*a^4*c^3*x^2 + a^3*c^3*x - a^
2*c^3), -1/16*((a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(-
c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4
 - c)) - 2*(2*a^3*x^3 - 3*a^2*x^2 - a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
 + 1))/(a^7*c^3*x^5 - a^6*c^3*x^4 - 2*a^5*c^3*x^3 + 2*a^4*c^3*x^2 + a^3*c^3
*x - a^2*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x}{\left(-a^2cx^2 + c\right)^{\frac{5}{2}}\sqrt{-a^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)*x/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 161, normalized size = 1.18

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln(ax-1)x^3a^3 - a^3x^3\ln(ax+1) - \ln(ax-1)x^2a^2 + \ln(ax+1)x^2a^2 + 2a^2x^2 - \right.}{16\left(a^2x^2-1\right)c^3a^2(ax-1)^2(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x^3*a^3-a^3*x^3*
ln(a*x+1)-ln(a*x-1)*x^2*a^2+ln(a*x+1)*x^2*a^2+2*a^2*x^2-ln(a*x-1)*x*a+a*x*l
n(a*x+1)-2*a*x+ln(a*x-1)-ln(a*x+1)+4)/(a^2*x^2-1)/c^3/a^2/(a*x-1)^2/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int -\frac{x^2}{\left(a^4c^{\frac{5}{2}}x^4 - 2\,a^2c^{\frac{5}{2}}x^2 + c^{\frac{5}{2}}\right)(ax+1)(ax-1)}\,dx + \frac{1}{4\left(a^6c^{\frac{5}{2}}x^4 - 2\,a^4c^{\frac{5}{2}}x^2 + a^2c^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] a*integrate(-x^2/((a^4*c^(5/2)*x^4 - 2*a^2*c^(5/2)*x^2 + c^(5/2))*(a*x + 1)
*(a*x - 1)), x) + 1/4/(a^6*c^(5/2)*x^4 - 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a\,x+1\right)}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**
(5/2)), x)
```

3.982 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$\frac{\sqrt{1-a^2x^2}}{4ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

```
[Out] 1/8*(-a^2*x^2+1)^(1/2)/a/c^2/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)+1/4*(-a^2*x^2+
1)^(1/2)/a/c^2/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/8*(-a^2*x^2+1)^(1/2)/a/c^2/(
a*x+1)/(-a^2*c*x^2+c)^(1/2)+3/8*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/c^2/(-a^2
*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{4ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*
x^2]/(4*a*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^2*(
1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^2
*Sqrt[c - a^2*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4(-1+ax)^3}+\frac{1}{4(-1+ax)^2}+\frac{1}{8(1+ax)^2}-\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{4ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\left(3\sqrt{1-\right.}{8}$$

$$= \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{4ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-}}{8a}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 0.46

$$\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2+3ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)+2\right)}{8ac^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTan
h[a*x]))/(8*a*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.71, size = 455, normalized size = 2.47

$$\left[\frac{3\left(a^5x^5 - a^4x^4 - 2\,a^3x^3 + 2\,a^2x^2 + ax - 1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right) + 4\left(2\,a^{3}\right.}{32\left(a^6c^3x^5 - a^5c^3x^4 - 2\,a^4c^3x^3 + 2\,a^3c^3x^2 + a^2c^3x - ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/32*(3*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(c)*log(
-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2
 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)
) + 4*(2*a^3*x^3 + a^2*x^2 - 5*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)
)/(a^6*c^3*x^5 - a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 + a^2*c^3*x -
a*c^3), 1/16*(3*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*sqrt(
-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^
4 - c)) + 2*(2*a^3*x^3 + a^2*x^2 - 5*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1))/(a^6*c^3*x^5 - a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 + a^2*c^
3*x - a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 166, normalized size = 0.90

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(3\ln\left(ax-1\right)x^3a^3 - 3a^3x^3\ln\left(ax+1\right) - 3\ln\left(ax-1\right)x^2a^2 + 3\ln\left(ax+1\right)x^2a^2 + 6a^2x\right.}{16\left(a^2x^2-1\right)c^3a\left(ax-1\right)^2\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x^3*a^3-3*a^3*x
^3*ln(a*x+1)-3*ln(a*x-1)*x^2*a^2+3*ln(a*x+1)*x^2*a^2+6*a^2*x^2-3*ln(a*x-1)*
x*a+3*a*x*ln(a*x+1)-6*a*x+3*ln(a*x-1)-3*ln(a*x+1)-4)/(a^2*x^2-1)/c^3/a/(a*x
-1)^2/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\,\left(-c(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(5
/2)), x)
```

3.983 $\int \frac{e^{\tanh^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=252

$$\frac{\sqrt{1-a^2x^2}}{2c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c^2\sqrt{c-a^2cx^2}}-\frac{11\sqrt{1-a^2x^2}\log(1}{16c^2\sqrt{c-a^2cx}}$$

[Out] 1/8*(-a^2*x^2+1)^(1/2)/c^2/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)+1/2*(-a^2*x^2+1)^(1/2)/c^2/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^2+1)^(1/2)/c^2/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+ln(x)*(-a^2*x^2+1)^(1/2)/c^2/(-a^2*c*x^2+c)^(1/2)-11/16*ln(-a*x+1)*(-a^2*x^2+1)^(1/2)/c^2/(-a^2*c*x^2+c)^(1/2)-5/16*ln(a*x+1)*(-a^2*x^2+1)^(1/2)/c^2/(-a^2*c*x^2+c)^(1/2)

Rubi [A] time = 0.24, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{\sqrt{1-a^2x^2}}{2c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(x)}{c^2\sqrt{c-a^2cx^2}}-\frac{11\sqrt{1-a^2x^2}\log(1}{16c^2\sqrt{c-a^2cx}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]

[Out] Sqrt[1 - a^2*x^2]/(8*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(2*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[x])/(c^2*Sqrt[c - a^2*c*x^2]) - (11*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*c^2*Sqrt[c - a^2*c*x^2]) - (5*Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(16*c^2*Sqrt[c - a^2*c*x^2])

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x}-\frac{a}{4(-1+ax)^3}+\frac{a}{2(-1+ax)^2}-\frac{11a}{16(-1+ax)}-\frac{a}{8(1+ax)^2}-\frac{5a}{16(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{2c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8c^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2}}{c^2\sqrt{c-}}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.34

$$\frac{\sqrt{1-a^2x^2}\left(\frac{8}{1-ax}+\frac{2}{ax+1}+\frac{2}{(ax-1)^2}-11\log(1-ax)-5\log(ax+1)+16\log(x)\right)}{16c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
[Out] (Sqrt[1 - a^2*x^2]*(8/(1 - a*x) + 2/(-1 + a*x)^2 + 2/(1 + a*x) + 16*Log[x]
- 11*Log[1 - a*x] - 5*Log[1 + a*x]))/(16*c^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{a^7c^3x^8-a^6c^3x^7-3\,a^5c^3x^6+3\,a^4c^3x^5+3\,a^3c^3x^4-3\,a^2c^3x^3-ac^3x^2+c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^7*c^3*x^8 - a^6*c^3*x^7
 - 3*a^5*c^3*x^6 + 3*a^4*c^3*x^5 + 3*a^3*c^3*x^4 - 3*a^2*c^3*x^3 - a*c^3*x^
2 + c^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.05, size = 193, normalized size = 0.77

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(16a^3\ln(x)x^3-11\ln\left(ax-1\right)x^3a^3-5a^3x^3\ln\left(ax+1\right)-16a^2\ln(x)x^2+11\ln\left(ax-\right.\right.}{16\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(16*a^3*ln(x)*x^3-11*ln(a*x
-1)*x^3*a^3-5*a^3*x^3*ln(a*x+1)-16*a^2*ln(x)*x^2+11*ln(a*x-1)*x^2*a^2+5*ln(
a*x+1)*x^2*a^2-6*a^2*x^2-16*a*ln(x)*x+11*ln(a*x-1)*x*a+5*a*x*ln(a*x+1)-2*a*
x+16*ln(x)-11*ln(a*x-1)-5*ln(a*x+1)+12)/(a^2*x^2-1)/c^3/(a*x-1)^2/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{x\,{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
[Out] int((a*x + 1)/(x*(c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral((a*x + 1)/(x*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**
(5/2)), x)
```

3.984 $\int \frac{e^{\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=295

$$\frac{3a\sqrt{1-a^2x^2}}{4c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(x)}{c^2\sqrt{c-a^2cx^2}}-$$

```
[Out] -(-a^2*x^2+1)^(1/2)/c^2/x/(-a^2*c*x^2+c)^(1/2)+1/8*a*(-a^2*x^2+1)^(1/2)/c^2
/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)+3/4*a*(-a^2*x^2+1)^(1/2)/c^2/(-a*x+1)/(-a^
2*c*x^2+c)^(1/2)-1/8*a*(-a^2*x^2+1)^(1/2)/c^2/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+
a*ln(x)*(-a^2*x^2+1)^(1/2)/c^2/(-a^2*c*x^2+c)^(1/2)-23/16*a*ln(-a*x+1)*(-a^
2*x^2+1)^(1/2)/c^2/(-a^2*c*x^2+c)^(1/2)+7/16*a*ln(a*x+1)*(-a^2*x^2+1)^(1/2)
/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{3a\sqrt{1-a^2x^2}}{4c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\log(x)}{c^2\sqrt{c-a^2cx^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(c^2*x*Sqrt[c - a^2*c*x^2])) + (a*Sqrt[1 - a^2*x^2])/(8
*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + (3*a*Sqrt[1 - a^2*x^2])/(4*c^2*(1 -
 a*x)*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/(8*c^2*(1 + a*x)*Sqrt[c
- a^2*c*x^2]) + (a*Sqrt[1 - a^2*x^2]*Log[x])/(c^2*Sqrt[c - a^2*c*x^2]) - (2
3*a*Sqrt[1 - a^2*x^2]*Log[1 - a*x])/(16*c^2*Sqrt[c - a^2*c*x^2]) + (7*a*Sqr
t[1 - a^2*x^2]*Log[1 + a*x])/(16*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^2(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{4(-1+ax)^3}+\frac{3a^2}{4(-1+ax)^2}-\frac{23a^2}{16(-1+ax)}+\frac{a^2}{8(1+ax)^2}+\frac{7a^2}{16(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3a\sqrt{1-a^2x^2}}{4c^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a}}{8c^2(1+ax)\sqrt{c}}$$

Mathematica [A] time = 0.09, size = 97, normalized size = 0.33

$$\frac{\sqrt{1-a^2x^2}\left(\frac{12a}{1-ax}-\frac{2a}{ax+1}+\frac{2a}{(ax-1)^2}+16a\log(x)-23a\log(1-ax)+7a\log(ax+1)-\frac{16}{x}\right)}{16c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-16/x + (12*a)/(1 - a*x) + (2*a)/(-1 + a*x)^2 - (2*a)/(
1 + a*x) + 16*a*Log[x] - 23*a*Log[1 - a*x] + 7*a*Log[1 + a*x]))/(16*c^2*Sqr
t[c - a^2*c*x^2])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{a^7c^3x^9-a^6c^3x^8-3\,a^5c^3x^7+3\,a^4c^3x^6+3\,a^3c^3x^5-3\,a^2c^3x^4-ac^3x^3+c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^7*c^3*x^9 - a^6*c^3*x^8
 - 3*a^5*c^3*x^7 + 3*a^4*c^3*x^6 + 3*a^3*c^3*x^5 - 3*a^2*c^3*x^4 - a*c^3*x^
3 + c^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.05, size = 222, normalized size = 0.75

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(16a^4\ln(x)x^4-23\ln\left(ax-1\right)x^4a^4+7\ln\left(ax+1\right)x^4a^4-16a^3\ln(x)x^3+23\ln\left(ax-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(16*a^4*ln(x)*x^4-23*ln(a*x
-1)*x^4*a^4+7*ln(a*x+1)*x^4*a^4-16*a^3*ln(x)*x^3+23*ln(a*x-1)*x^3*a^3-7*a^3
*x^3*ln(a*x+1)-30*x^3*a^3-16*a^2*ln(x)*x^2+23*ln(a*x-1)*x^2*a^2-7*ln(a*x+1)
*x^2*a^2+22*a^2*x^2+16*a*ln(x)*x-23*ln(a*x-1)*x*a+7*a*x*ln(a*x+1)+28*a*x-16
)/(a^2*x^2-1)/c^3/(a*x-1)^2/(a*x+1)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{x^2\,{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/(x^2*(c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x^2\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

3.985 $\int \frac{e^{\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=345

$$\frac{a^2\sqrt{1-a^2x^2}}{c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}}+\frac{3a^2}{}$$

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)/c^2/x^2/(-a^2*c*x^2+c)^(1/2)-a*(-a^2*x^2+1)^(1/2)/c
^2/x/(-a^2*c*x^2+c)^(1/2)+1/8*a^2*(-a^2*x^2+1)^(1/2)/c^2/(-a*x+1)^2/(-a^2*c
*x^2+c)^(1/2)+a^2*(-a^2*x^2+1)^(1/2)/c^2/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+1/8*
a^2*(-a^2*x^2+1)^(1/2)/c^2/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+3*a^2*ln(x)*(-a^2*x
^2+1)^(1/2)/c^2/(-a^2*c*x^2+c)^(1/2)-39/16*a^2*ln(-a*x+1)*(-a^2*x^2+1)^(1/2
)/c^2/(-a^2*c*x^2+c)^(1/2)-9/16*a^2*ln(a*x+1)*(-a^2*x^2+1)^(1/2)/c^2/(-a^2*
c*x^2+c)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{a^2\sqrt{1-a^2x^2}}{c^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(ax+1)\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}}+\frac{3a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*c^2*x^2*Sqrt[c - a^2*c*x^2]) - (a*Sqrt[1 - a^2*x^2])/
(c^2*x*Sqrt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(8*c^2*(1 - a*x)^2*Sq
rt[c - a^2*c*x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(c^2*(1 - a*x)*Sqrt[c - a^2*c*
x^2]) + (a^2*Sqrt[1 - a^2*x^2])/(8*c^2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*
a^2*Sqrt[1 - a^2*x^2]*Log[x])/(c^2*Sqrt[c - a^2*c*x^2]) - (39*a^2*Sqrt[1 -
a^2*x^2]*Log[1 - a*x])/(16*c^2*Sqrt[c - a^2*c*x^2]) - (9*a^2*Sqrt[1 - a^2*x
^2]*Log[1 + a*x])/(16*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{x^3(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{3a^2}{x}-\frac{a^3}{4(-1+ax)^3}+\frac{a^3}{(-1+ax)^2}-\frac{39a^3}{16(-1+ax)}-\frac{a^3}{8(1+ax)^2}-\frac{9a^3}{16(1+ax)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}}-\frac{a\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{a^2\sqrt{1-a^2x^2}}{c^2(1-ax)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.12, size = 115, normalized size = 0.33

$$\frac{\sqrt{1-a^2x^2}\left(\frac{16a^2}{1-ax}+\frac{2a^2}{ax+1}+\frac{2a^2}{(ax-1)^2}+48a^2\log(x)-39a^2\log(1-ax)-9a^2\log(ax+1)-\frac{16a}{x}-\frac{8}{x^2}\right)}{16c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(x^3*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-8/x^2 - (16*a)/x + (16*a^2)/(1 - a*x) + (2*a^2)/(-1 +
a*x)^2 + (2*a^2)/(1 + a*x) + 48*a^2*Log[x] - 39*a^2*Log[1 - a*x] - 9*a^2*Lo
g[1 + a*x]))/(16*c^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{a^7c^3x^{10}-a^6c^3x^9-3\,a^5c^3x^8+3\,a^4c^3x^7+3\,a^3c^3x^6-3\,a^2c^3x^5-ac^3x^4+c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^7*c^3*x^10 - a^6*c^3*x^
9 - 3*a^5*c^3*x^8 + 3*a^4*c^3*x^7 + 3*a^3*c^3*x^6 - 3*a^2*c^3*x^5 - a*c^3*x
^4 + c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.05, size = 242, normalized size = 0.70

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(48a^5\ln(x)x^5-39\ln\left(ax-1\right)x^5a^5-9\ln\left(ax+1\right)x^5a^5-48a^4\ln(x)x^4+39\ln\left(ax-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(48*a^5*ln(x)*x^5-39*ln(a*x
-1)*x^5*a^5-9*ln(a*x+1)*x^5*a^5-48*a^4*ln(x)*x^4+39*ln(a*x-1)*x^4*a^4+9*ln(
a*x+1)*x^4*a^4-30*x^4*a^4-48*a^3*ln(x)*x^3+39*ln(a*x-1)*x^3*a^3+9*a^3*x^3*l
n(a*x+1)+6*x^3*a^3+48*a^2*ln(x)*x^2-39*ln(a*x-1)*x^2*a^2-9*ln(a*x+1)*x^2*a^
2+44*a^2*x^2-8*a*x-8)/(a^2*x^2-1)/c^3/(a*x-1)^2/(a*x+1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{x^3\,{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/(x^3*(c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{x^3\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

3.986 $\int \frac{e^{\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=277

$$\frac{3\sqrt{1-a^2x^2}}{16ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}+\frac{}{24a}$$

```
[Out] 1/24*(-a^2*x^2+1)^(1/2)/a/c^3/(-a*x+1)^3/(-a^2*c*x^2+c)^(1/2)+3/32*(-a^2*x^
2+1)^(1/2)/a/c^3/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)+3/16*(-a^2*x^2+1)^(1/2)/a/
c^3/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/32*(-a^2*x^2+1)^(1/2)/a/c^3/(a*x+1)^2/(
-a^2*c*x^2+c)^(1/2)-1/8*(-a^2*x^2+1)^(1/2)/a/c^3/(a*x+1)/(-a^2*c*x^2+c)^(1/
2)+5/16*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6143, 6140, 44, 207}

$$\frac{3\sqrt{1-a^2x^2}}{16ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}+\frac{}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(24*a*c^3*(1 - a*x)^3*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 -
a^2*x^2])/(32*a*c^3*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]
)/(16*a*c^3*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(32*a*c^3*(1
 + a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^3*(1 + a*x)*Sqrt[
c - a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a*c^3*Sqrt[c - a^2
*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^4(1+ax)^3}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{8(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{16\left(-1+a^2x^2\right)}\right)dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{24ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{16ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{}{32}$$

$$= \frac{\sqrt{1-a^2x^2}}{24ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{16ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{}{32}$$

Mathematica [A] time = 0.08, size = 103, normalized size = 0.37

$$\frac{\sqrt{1-a^2x^2}\left(-15a^4x^4+15a^3x^3+25a^2x^2-25ax+15(ax-1)^3(ax+1)^2\tanh^{-1}(ax)-8\right)}{48ac^3(ax-1)^3(ax+1)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-8 - 25*a*x + 25*a^2*x^2 + 15*a^3*x^3 - 15*a^4*x^4 + 15
*(-1 + a*x)^3*(1 + a*x)^2*ArcTanh[a*x]))/(48*a*c^3*(-1 + a*x)^3*(1 + a*x)^2
*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 1.20, size = 567, normalized size = 2.05

$$\left[\frac{15\left(a^7x^7 - a^6x^6 - 3\,a^5x^5 + 3\,a^4x^4 + 3\,a^3x^3 - 3\,a^2x^2 - ax + 1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6 + 5\,a^4cx^4 - 5\,a^2cx^2 - 4\left(a^3x^3 + ax\right)\sqrt{-a^2cx^2 + c}\,\sqrt{-a^2}}{a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1}\right.}{192\left(a^8c^4x^7 - a^7c^4x^6 - 3\,a^6c^4x^5 + 3\,a^5c^4x^4 + 3\,a^4c^4x^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fri
cas")
```

```
[Out] [1/192*(15*(a^7*x^7 - a^6*x^6 - 3*a^5*x^5 + 3*a^4*x^4 + 3*a^3*x^3 - 3*a^2*x
^2 - a*x + 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*
x^3 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 -
3*a^4*x^4 + 3*a^2*x^2 - 1)) + 4*(8*a^5*x^5 + 7*a^4*x^4 - 31*a^3*x^3 - 9*a^2
*x^2 + 33*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^8*c^4*x^7 - a^7*
c^4*x^6 - 3*a^6*c^4*x^5 + 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 - 3*a^3*c^4*x^2 - a
^2*c^4*x + a*c^4), 1/96*(15*(a^7*x^7 - a^6*x^6 - 3*a^5*x^5 + 3*a^4*x^4 + 3*
a^3*x^3 - 3*a^2*x^2 - a*x + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(
-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 - c)) + 2*(8*a^5*x^5 + 7*a^4*x^4 - 31
*a^3*x^3 - 9*a^2*x^2 + 33*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^
8*c^4*x^7 - a^7*c^4*x^6 - 3*a^6*c^4*x^5 + 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 - 3
*a^3*c^4*x^2 - a^2*c^4*x + a*c^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax + 1}{\left(-a^2cx^2 + c\right)^{\frac{7}{2}}\sqrt{-a^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 238, normalized size = 0.86

$$\frac{\sqrt{-a^2x^2 + 1}\,\sqrt{-c\left(a^2x^2 - 1\right)}\,\left(15\ln\left(ax - 1\right)x^5a^5 - 15\ln\left(ax + 1\right)x^5a^5 - 15\ln\left(ax - 1\right)x^4a^4 + 15\ln\left(ax + 1\right)x^4a^4 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/96*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x-1)*x^5*a^5-15*ln(
a*x+1)*x^5*a^5-15*ln(a*x-1)*x^4*a^4+15*ln(a*x+1)*x^4*a^4+30*x^4*a^4-30*ln(a
*x-1)*x^3*a^3+30*a^3*x^3*ln(a*x+1)-30*x^3*a^3+30*ln(a*x-1)*x^2*a^2-30*ln(a*
x+1)*x^2*a^2-50*a^2*x^2+15*ln(a*x-1)*x*a-15*a*x*ln(a*x+1)+50*a*x-15*ln(a*x-
1)+15*ln(a*x+1)+16)/(a^2*x^2-1)/c^4/a/(a*x-1)^3/(a*x+1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{\left(c-a^2\,c\,x^2\right)^{7/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(7/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((a*x + 1)/((c - a^2*c*x^2)^(7/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-(ax-1)(ax+1)}\,\left(-c(ax-1)(ax+1)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(7
/2)), x)
```

3.987 $\int e^{\tanh^{-1}(ax)} x^m \left(c - a^2 c x^2\right)^2 \, dx$

Optimal. Leaf size=80

$$\frac{c^2 x^{m+1} {}_2F_1\left(-\frac{3}{2}, \frac{m+1}{2}; \frac{m+3}{2}; a^2 x^2\right)}{m+1} + \frac{a c^2 x^{m+2} {}_2F_1\left(-\frac{3}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2 x^2\right)}{m+2}$$

```
[Out] c^2*x^(1+m)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a*c^2*x^
(2+m)*hypergeom([-3/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)
```

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6148, 808, 364}

$$\frac{c^2 x^{m+1} {}_2F_1\left(-\frac{3}{2}, \frac{m+1}{2}; \frac{m+3}{2}; a^2 x^2\right)}{m+1} + \frac{a c^2 x^{m+2} {}_2F_1\left(-\frac{3}{2}, \frac{m+2}{2}; \frac{m+4}{2}; a^2 x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^(1 + m)*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 +
 m) + (a*c^2*x^(2 + m)*Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2, a^2*x^
2])/(2 + m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
```

```
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx&=c^2\int x^m(1+ax)\left(1-a^2x^2\right)^{3/2}\,dx\\&=c^2\int x^m\left(1-a^2x^2\right)^{3/2}\,dx+\left(ac^2\right)\int x^{1+m}\left(1-a^2x^2\right)^{3/2}\,dx\\&=\frac{c^2x^{1+m}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ac^2x^{2+m}\,{}_2F_1\left(-\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}\end{aligned}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.02

$$c^2\left(\frac{x^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+1}{2}+1;a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+2}{2};\frac{m+2}{2}+1;a^2x^2\right)}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] c^2*((x^(1 + m)*Hypergeometric2F1[-3/2, (1 + m)/2, 1 + (1 + m)/2, a^2*x^2])
/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[-3/2, (2 + m)/2, 1 + (2 + m)/2, a
^2*x^2])/(2 + m))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a^3c^2x^3+a^2c^2x^2-ac^2x-c^2\right)\sqrt{-a^2x^2+1}\,x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] integral(-(a^3*c^2*x^3 + a^2*c^2*x^2 - a*c^2*x - c^2)*sqrt(-a^2*x^2 + 1)*x^
m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2cx^2-c\right)^2(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="gia c")

[Out] integrate((a^2*c*x^2 - c)^2*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)

maple [B] time = 0.37, size = 227, normalized size = 2.84

$$\frac{a^5c^2x^{6+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},3+\frac{m}{2}\right],\left[4+\frac{m}{2}\right],a^2x^2\right)}{6+m}-\frac{2a^3c^2x^{4+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},2+\frac{m}{2}\right],\left[3+\frac{m}{2}\right],a^2x^2\right)}{4+m}+\frac{a\,c^2x^{2+m}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^2,x)

[Out] a^5*c^2/(6+m)*x^(6+m)*hypergeom([1/2,3+1/2*m],[4+1/2*m],a^2*x^2)-2*a^3*c^2/(4+m)*x^(4+m)*hypergeom([1/2,2+1/2*m],[3+1/2*m],a^2*x^2)+a*c^2/(2+m)*x^(2+m)*hypergeom([1/2,1+1/2*m],[2+1/2*m],a^2*x^2)+c^2*a^4/(5+m)*x^(5+m)*hypergeom([1/2,5/2+1/2*m],[7/2+1/2*m],a^2*x^2)-2*c^2*a^2/(3+m)*x^(3+m)*hypergeom([1/2,3/2+1/2*m],[5/2+1/2*m],a^2*x^2)+c^2/(1+m)*x^(1+m)*hypergeom([1/2,1/2+1/2*m],[3/2+1/2*m],a^2*x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2-c\right)^2(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="max ima")

[Out] integrate((a^2*c*x^2 - c)^2*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(c-a^2\,c\,x^2\right)^2\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^m*(c - a^2*c*x^2)^2*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)

[Out] int((x^m*(c - a^2*c*x^2)^2*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)

sympy [C] time = 14.39, size = 223, normalized size = 2.79

$$-\frac{a^3c^2x^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}-\frac{a^2c^2x^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{ac^2x^2x^m\Gamma\left(\frac{m}{2}+1\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**3*c**2*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2 + 2), (m/2 + 3,), a**2
*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3)) - a**2*c**2*x**3*x**m*gamma(m/2
 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), a**2*x**2*exp_polar(2*I*pi))
/(2*gamma(m/2 + 5/2)) + a*c**2*x**2*x**m*gamma(m/2 + 1)*hyper((-1/2, m/2 +
1), (m/2 + 2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 2)) + c**2*x*x*
*m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_po
lar(2*I*pi))/(2*gamma(m/2 + 3/2))
```

3.988 $\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=76

$$\frac{cx^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{acx^{m+2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

```
[Out] c*x^(1+m)*hypergeom([-1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a*c*x^(2+m
)*hypergeom([-1/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6148, 808, 364}

$$\frac{cx^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{acx^{m+2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^(1 + m)*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m
) + (a*c*x^(2 + m)*Hypergeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/
(2 + m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
```

```
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)dx &= c\int x^m(1+ax)\sqrt{1-a^2x^2}\,dx\\ &= c\int x^m\sqrt{1-a^2x^2}\,dx+(ac)\int x^{1+m}\sqrt{1-a^2x^2}\,dx\\ &= \frac{cx^{1+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{acx^{2+m}\,{}_2F_1\left(-\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}\end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.95

$$cx^{m+1}\left(\frac{ax\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2}+1;\frac{m}{2}+2;a^2x^2\right)}{m+2}+\frac{{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2),x]
[Out] c*x^(1 + m)*((a*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, a^2*x^2])/(2 +
m) + Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2]/(1 + m))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2x^2+1}\,(acx+c)x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c),x, algorithm="frica
s")
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*c*x + c)*x^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2cx^2-c\right)(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] integrate(-(a^2*c*x^2 - c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

maple [B] time = 0.26, size = 143, normalized size = 1.88

$$-\frac{a^{3} c\, x^{4+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, 2+\frac{m}{2}\right], \left[3+\frac{m}{2}\right], a^{2} x^{2}\right)}{4+m}+\frac{a c\, x^{2+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1+\frac{m}{2}\right], \left[2+\frac{m}{2}\right], a^{2} x^{2}\right)}{2+m}-\frac{c\, a^{2} x^{3+m} \mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c),x)
```

```
[Out] -a^3*c/(4+m)*x^(4+m)*hypergeom([1/2,2+1/2*m],[3+1/2*m],a^2*x^2)+a*c/(2+m)*x
^(2+m)*hypergeom([1/2,1+1/2*m],[2+1/2*m],a^2*x^2)-c*a^2/(3+m)*x^(3+m)*hyper
geom([1/2,3/2+1/2*m],[5/2+1/2*m],a^2*x^2)+c/(1+m)*x^(1+m)*hypergeom([1/2,1/
2+1/2*m],[3/2+1/2*m],a^2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^{2} c x^{2}-c\right)(a x+1) x^{m}}{\sqrt{-a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -integrate((a^2*c*x^2 - c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m}\, \left(c-a^{2}\, c\, x^{2}\right)\, \left(a\, x+1\right)}{\sqrt{1-a^{2}\, x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 4.50, size = 104, normalized size = 1.37

$$\frac{acx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{cxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c),x)
```

```
[Out] a*c*x**2*x**m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*e
xp_polar(2*I*pi))/(2*gamma(m/2 + 2)) + c*x*x**m*gamma(m/2 + 1/2)*hyper((-1/
2, m/2 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3/
2))
```

3.989 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{c-a^2cx^2}\,dx$

Optimal. Leaf size=80

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c(m+1)}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c(m+2)}$$

```
[Out] x^(1+m)*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/c/(1+m)+a*x^(2+m)*h
ypergeom([3/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/c/(2+m)
```

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6148, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c(m+1)}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(c*(1 + m
)) + (a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(c
*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
```

```
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^m}{c-a^2cx^2}\,dx = \frac{\int \frac{x^m(1+ax)}{(1-a^2x^2)^{3/2}}\,dx}{c}$$
$$= \frac{\int \frac{x^m}{(1-a^2x^2)^{3/2}}\,dx}{c} + \frac{a\int \frac{x^{1+m}}{(1-a^2x^2)^{3/2}}\,dx}{c}$$
$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c(1+m)} + \frac{ax^{2+m}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c(2+m)}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 1.02

$$\frac{\frac{x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+1}{2}+1;a^2x^2\right)}{m+1} + \frac{ax^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+2}{2}+1;a^2x^2\right)}{m+2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] ((x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, 1 + (1 + m)/2, a^2*x^2])/(1 +
 m) + (a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2, 1 + (2 + m)/2, a^2*x^2
])/(2 + m))/c
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m}{a^3cx^3-a^2cx^2-acx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/(a^3*c*x^3 - a^2*c*x^2 - a*c*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)x^m}{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] integrate(-(a*x + 1)*x^m/((a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\, x^m}{\sqrt{-a^2x^2+1}\, \left(-a^2c\, x^2+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)x^m}{\left(a^2cx^2-c\right)\sqrt{-a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)*x^m/((a^2*c*x^2 - c)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\, (a\,x+1)}{\left(c-a^2\,c\,x^2\right)\,\sqrt{1-a^2\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/((c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^m*(a*x + 1))/((c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\, dx + \int \frac{axx^m}{-a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**m/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a*x*x**m/(-a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1
)), x))/c
```

3.990 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=80

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c^2(m+1)}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c^2(m+2)}$$

```
[Out] x^(1+m)*hypergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/c^2/(1+m)+a*x^(2+m)
*hypergeom([5/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/c^2/(2+m)
```

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6148, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c^2(m+1)}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(c^2*(1 +
 m)) + (a*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/
(c^2*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
```

```
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{x^m(1+ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\ &= \frac{\int \frac{x^m}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}+\frac{a\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2}\\ &= \frac{x^{1+m}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c^2(1+m)}+\frac{ax^{2+m}\,{}_2F_1\left(\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c^2(2+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 1.02

$$\frac{\frac{x^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+1}{2}+1;a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+2}{2}+1;a^2x^2\right)}{m+2}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] ((x^(1 + m)*Hypergeometric2F1[5/2, (1 + m)/2, 1 + (1 + m)/2, a^2*x^2])/(1 +
 m) + (a*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, 1 + (2 + m)/2, a^2*x^2
])/(2 + m))/c^2
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^m}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^m/(a^5*c^2*x^5 - a^4*c^2*x^4 - 2*a^3*c^2*x^3
 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*x^m/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^m}{\sqrt{-a^2x^2+1}\,\left(-a^2c\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\left(a^2cx^2-c\right)^2\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*x^m/((a^2*c*x^2 - c)^2*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(a\,x+1\right)}{\left(c-a^2\,c\,x^2\right)^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{axx^m}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(x**m/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a*x*x**m/(a**4*x**4*sqrt(-a**
2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))
/c**2
```

3.991 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=80

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c^3(m+1)}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c^3(m+2)}$$

```
[Out] x^(1+m)*hypergeom([7/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/c^3/(1+m)+a*x^(2+m)
*hypergeom([7/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/c^3/(2+m)
```

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6148, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c^3(m+1)}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c^3(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[7/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(c^3*(1 +
 m)) + (a*x^(2 + m)*Hypergeometric2F1[7/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/
(c^3*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
```

```
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{x^m(1+ax)}{(1-a^2x^2)^{7/2}}\,dx}{c^3}$$
$$= \frac{\int \frac{x^m}{(1-a^2x^2)^{7/2}}\,dx}{c^3} + \frac{a\int \frac{x^{1+m}}{(1-a^2x^2)^{7/2}}\,dx}{c^3}$$
$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c^3(1+m)} + \frac{ax^{2+m}\,{}_2F_1\left(\frac{7}{2},\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c^3(2+m)}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 1.02

$$\frac{\frac{x^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+1}{2}+1;a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+2}{2}+1;a^2x^2\right)}{m+2}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^3,x]
```

```
[Out] ((x^(1 + m)*Hypergeometric2F1[7/2, (1 + m)/2, 1 + (1 + m)/2, a^2*x^2])/(1 +
 m) + (a*x^(2 + m)*Hypergeometric2F1[7/2, (2 + m)/2, 1 + (2 + m)/2, a^2*x^2
])/(2 + m))/c^3
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/(a^7*c^3*x^7 - a^6*c^3*x^6 - 3*a^5*c^3*x^5
+ 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^3*x + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)x^m}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)*x^m/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^m}{\sqrt{-a^2x^2+1}\,\left(-a^2c\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)x^m}{\left(a^2cx^2-c\right)^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)*x^m/((a^2*c*x^2 - c)^3*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(a\,x+1\right)}{\left(c-a^2\,c\,x^2\right)^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{axx^m}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(x**m/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(a*x*x**m/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x
**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c*
*3
```

3.992 $\int e^{\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=82

$$\frac{a^5x^{m+6}}{m+6}+\frac{a^4x^{m+5}}{m+5}-\frac{2a^3x^{m+4}}{m+4}-\frac{2a^2x^{m+3}}{m+3}+\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1+m)/(1+m)+a*x^(2+m)/(2+m)-2*a^2*x^(3+m)/(3+m)-2*a^3*x^(4+m)/(4+m)+a^4*x
^(5+m)/(5+m)+a^5*x^(6+m)/(6+m)
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 88}

$$-\frac{2a^2x^{m+3}}{m+3}-\frac{2a^3x^{m+4}}{m+4}+\frac{a^4x^{m+5}}{m+5}+\frac{a^5x^{m+6}}{m+6}+\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(1 - a^2*x^2)^(5/2),x]
```

```
[Out] x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m) - (2*a^2*x^(3 + m))/(3 + m) - (2*
a^3*x^(4 + m))/(4 + m) + (a^4*x^(5 + m))/(5 + m) + (a^5*x^(6 + m))/(6 + m)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^{5/2}\,dx=\int x^m(1-ax)^2(1+ax)^3\,dx$$
$$=\int\left(x^m+ax^{1+m}-2a^2x^{2+m}-2a^3x^{3+m}+a^4x^{4+m}+a^5x^{5+m}\right)dx$$
$$=\frac{x^{1+m}}{1+m}+\frac{ax^{2+m}}{2+m}-\frac{2a^2x^{3+m}}{3+m}-\frac{2a^3x^{4+m}}{4+m}+\frac{a^4x^{5+m}}{5+m}+\frac{a^5x^{6+m}}{6+m}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.85

$$x^{m+1}\left(\frac{a^5x^5}{m+6}+\frac{a^4x^4}{m+5}-\frac{2a^3x^3}{m+4}-\frac{2a^2x^2}{m+3}+\frac{ax}{m+2}+\frac{1}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(1 - a^2*x^2)^(5/2),x]
```

```
[Out] x^(1 + m)*((1 + m)^(-1) + (a*x)/(2 + m) - (2*a^2*x^2)/(3 + m) - (2*a^3*x^3)
/(4 + m) + (a^4*x^4)/(5 + m) + (a^5*x^5)/(6 + m))
```

fricas [B] time = 0.67, size = 285, normalized size = 3.48

$$\frac{\left(\left(a^5m^5+15\,a^5m^4+85\,a^5m^3+225\,a^5m^2+274\,a^5m+120\,a^5\right)x^6+\left(a^4m^5+16\,a^4m^4+95\,a^4m^3+260\,a^4m^2+32\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] ((a^5*m^5 + 15*a^5*m^4 + 85*a^5*m^3 + 225*a^5*m^2 + 274*a^5*m + 120*a^5)*x^
6 + (a^4*m^5 + 16*a^4*m^4 + 95*a^4*m^3 + 260*a^4*m^2 + 324*a^4*m + 144*a^4)
*x^5 - 2*(a^3*m^5 + 17*a^3*m^4 + 107*a^3*m^3 + 307*a^3*m^2 + 396*a^3*m + 18
0*a^3)*x^4 - 2*(a^2*m^5 + 18*a^2*m^4 + 121*a^2*m^3 + 372*a^2*m^2 + 508*a^2*
m + 240*a^2)*x^3 + (a*m^5 + 19*a*m^4 + 137*a*m^3 + 461*a*m^2 + 702*a*m + 36
0*a)*x^2 + (m^5 + 20*m^4 + 155*m^3 + 580*m^2 + 1044*m + 720)*x)*x^m/(m^6 +
21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)
```

giac [B] time = 0.22, size = 460, normalized size = 5.61

$$\frac{a^5m^5x^6x^m+15\,a^5m^4x^6x^m+a^4m^5x^5x^m+85\,a^5m^3x^6x^m+16\,a^4m^4x^5x^m+225\,a^5m^2x^6x^m-2\,a^3m^5x^4x^m+95\,a^4m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^2*x^m,x, algorithm="giac")
```

```
[Out] (a^5*m^5*x^6*x^m + 15*a^5*m^4*x^6*x^m + a^4*m^5*x^5*x^m + 85*a^5*m^3*x^6*x^
m + 16*a^4*m^4*x^5*x^m + 225*a^5*m^2*x^6*x^m - 2*a^3*m^5*x^4*x^m + 95*a^4*m
^3*x^5*x^m + 274*a^5*m*x^6*x^m - 34*a^3*m^4*x^4*x^m + 260*a^4*m^2*x^5*x^m +
 120*a^5*x^6*x^m - 2*a^2*m^5*x^3*x^m - 214*a^3*m^3*x^4*x^m + 324*a^4*m*x^5*
x^m - 36*a^2*m^4*x^3*x^m - 614*a^3*m^2*x^4*x^m + 144*a^4*x^5*x^m + a*m^5*x^
2*x^m - 242*a^2*m^3*x^3*x^m - 792*a^3*m*x^4*x^m + 19*a*m^4*x^2*x^m - 744*a^
2*m^2*x^3*x^m - 360*a^3*x^4*x^m + m^5*x*x^m + 137*a*m^3*x^2*x^m - 1016*a^2*
m*x^3*x^m + 20*m^4*x*x^m + 461*a*m^2*x^2*x^m - 480*a^2*x^3*x^m + 155*m^3*x*
x^m + 702*a*m*x^2*x^m + 580*m^2*x*x^m + 360*a*x^2*x^m + 1044*m*x*x^m + 720*
x*x^m)/(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)
```

maple [B] time = 0.03, size = 338, normalized size = 4.12

$$\frac{x^{1+m}\left(a^5m^5x^5+15a^5m^4x^5+85a^5m^3x^5+a^4m^5x^4+225a^5m^2x^5+16a^4m^4x^4+274a^5m\,x^5+95a^4m^3x^4-2a^3m^5x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*(-a^2*x^2+1)^2*x^m,x)
```

```
[Out] x^(1+m)*(a^5*m^5*x^5+15*a^5*m^4*x^5+85*a^5*m^3*x^5+a^4*m^5*x^4+225*a^5*m^2*
x^5+16*a^4*m^4*x^4+274*a^5*m*x^5+95*a^4*m^3*x^4-2*a^3*m^5*x^3+120*a^5*x^5+2
60*a^4*m^2*x^4-34*a^3*m^4*x^3+324*a^4*m*x^4-214*a^3*m^3*x^3-2*a^2*m^5*x^2+1
44*a^4*x^4-614*a^3*m^2*x^3-36*a^2*m^4*x^2-792*a^3*m*x^3-242*a^2*m^3*x^2+a*m
^5*x-360*a^3*x^3-744*a^2*m^2*x^2+19*a*m^4*x-1016*a^2*m*x^2+137*a*m^3*x+m^5-
480*a^2*x^2+461*a*m^2*x+20*m^4+702*a*m*x+155*m^3+360*a*x+580*m^2+1044*m+720
)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

maxima [A] time = 0.32, size = 82, normalized size = 1.00

$$\frac{a^5x^{m+6}}{m+6}+\frac{a^4x^{m+5}}{m+5}-\frac{2\,a^3x^{m+4}}{m+4}-\frac{2\,a^2x^{m+3}}{m+3}+\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] a^5*x^(m + 6)/(m + 6) + a^4*x^(m + 5)/(m + 5) - 2*a^3*x^(m + 4)/(m + 4) - 2
*a^2*x^(m + 3)/(m + 3) + a*x^(m + 2)/(m + 2) + x^(m + 1)/(m + 1)
```

mupad [B] time = 1.16, size = 374, normalized size = 4.56

$$\frac{x\,x^m\,\left(m^5+20\,m^4+155\,m^3+580\,m^2+1044\,m+720\right)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}+\frac{a\,x^m\,x^2\,\left(m^5+19\,m^4+137\,m^3+461\,m^2+702\,m+\right.}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a^2*x^2 - 1)^2*(a*x + 1),x)
```

```
[Out] (x*x^m*(1044*m + 580*m^2 + 155*m^3 + 20*m^4 + m^5 + 720))/(1764*m + 1624*m^
2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (a*x^m*x^2*(702*m + 461*m^2 +
 137*m^3 + 19*m^4 + m^5 + 360))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21
*m^5 + m^6 + 720) + (a^5*x^m*x^6*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 +
 120))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (a^4*
x^m*x^5*(324*m + 260*m^2 + 95*m^3 + 16*m^4 + m^5 + 144))/(1764*m + 1624*m^2
 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) - (2*a^3*x^m*x^4*(396*m + 307*m^
2 + 107*m^3 + 17*m^4 + m^5 + 180))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 +
 21*m^5 + m^6 + 720) - (2*a^2*x^m*x^3*(508*m + 372*m^2 + 121*m^3 + 18*m^4 +
 m^5 + 240))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)
```

sympy [A] time = 1.38, size = 1760, normalized size = 21.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a**2*x**2+1)**2*x**m,x)
```

```
[Out] Piecewise((a**5*log(x) - a**4/x + a**3/x**2 + 2*a**2/(3*x**3) - a/(4*x**4)
- 1/(5*x**5), Eq(m, -6)), (a**5*x + a**4*log(x) + 2*a**3/x + a**2/x**2 - a/
(3*x**3) - 1/(4*x**4), Eq(m, -5)), (a**5*x**2/2 + a**4*x - 2*a**3*log(x) +
2*a**2/x - a/(2*x**2) - 1/(3*x**3), Eq(m, -4)), (a**5*x**3/3 + a**4*x**2/2
- 2*a**3*x - 2*a**2*log(x) - a/x - 1/(2*x**2), Eq(m, -3)), (a**5*x**4/4 + a
**4*x**3/3 - a**3*x**2 - 2*a**2*x + a*log(x) - 1/x, Eq(m, -2)), (a**5*x**5/
5 + a**4*x**4/4 - 2*a**3*x**3/3 - a**2*x**2 + a*x + log(x), Eq(m, -1)), (a*
*5*m**5*x**6*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*
m + 720) + 15*a**5*m**4*x**6*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1
624*m**2 + 1764*m + 720) + 85*a**5*m**3*x**6*x**m/(m**6 + 21*m**5 + 175*m**
4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 225*a**5*m**2*x**6*x**m/(m**6 +
21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 274*a**5*m*x**6
*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 1
20*a**5*x**6*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*
m + 720) + a**4*m**5*x**5*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624
*m**2 + 1764*m + 720) + 16*a**4*m**4*x**5*x**m/(m**6 + 21*m**5 + 175*m**4 +
 735*m**3 + 1624*m**2 + 1764*m + 720) + 95*a**4*m**3*x**5*x**m/(m**6 + 21*m
**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 260*a**4*m**2*x**5*
x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 32
4*a**4*m*x**5*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764
*m + 720) + 144*a**4*x**5*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624
*m**2 + 1764*m + 720) - 2*a**3*m**5*x**4*x**m/(m**6 + 21*m**5 + 175*m**4 +
735*m**3 + 1624*m**2 + 1764*m + 720) - 34*a**3*m**4*x**4*x**m/(m**6 + 21*m*
*5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 214*a**3*m**3*x**4*x
**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 614
```

```
*a**3*m**2*x**4*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 17
64*m + 720) - 792*a**3*m*x**4*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 +
1624*m**2 + 1764*m + 720) - 360*a**3*x**4*x**m/(m**6 + 21*m**5 + 175*m**4 +
 735*m**3 + 1624*m**2 + 1764*m + 720) - 2*a**2*m**5*x**3*x**m/(m**6 + 21*m*
*5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 36*a**2*m**4*x**3*x*
*m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 242*
a**2*m**3*x**3*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 176
4*m + 720) - 744*a**2*m**2*x**3*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3
+ 1624*m**2 + 1764*m + 720) - 1016*a**2*m*x**3*x**m/(m**6 + 21*m**5 + 175*m
**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) - 480*a**2*x**3*x**m/(m**6 + 21*
m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + a*m**5*x**2*x**m/(
m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 19*a*m**
4*x**2*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 72
0) + 137*a*m**3*x**2*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2
 + 1764*m + 720) + 461*a*m**2*x**2*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m*
*3 + 1624*m**2 + 1764*m + 720) + 702*a*m*x**2*x**m/(m**6 + 21*m**5 + 175*m*
*4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 360*a*x**2*x**m/(m**6 + 21*m**5
 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + m**5*x*x**m/(m**6 + 21
*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 20*m**4*x*x**m/(m
**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 155*m**3*
x*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) +
580*m**2*x*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m
+ 720) + 1044*m*x*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 +
1764*m + 720) + 720*x*x**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**
2 + 1764*m + 720), True))
```

3.993 $\int e^{\tanh^{-1}(ax)} x^m \left(1-a^2x^2\right)^{3/2} \, dx$

Optimal. Leaf size=54

$$-\frac{a^3x^{m+4}}{m+4}-\frac{a^2x^{m+3}}{m+3}+\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1+m)/(1+m)+a*x^(2+m)/(2+m)-a^2*x^(3+m)/(3+m)-a^3*x^(4+m)/(4+m)
```

Rubi [A] time = 0.10, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 75}

$$-\frac{a^2x^{m+3}}{m+3}-\frac{a^3x^{m+4}}{m+4}+\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(1 - a^2*x^2)^(3/2),x]
```

```
[Out] x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m) - (a^2*x^(3 + m))/(3 + m) - (a^3*
x^(4 + m))/(4 + m)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)} x^m \left(1-a^2x^2\right)^{3/2} \, dx &= \int x^m(1-ax)(1+ax)^2 \, dx\\ &= \int \left(x^m+ax^{1+m}-a^2x^{2+m}-a^3x^{3+m}\right) dx\\ &= \frac{x^{1+m}}{1+m}+\frac{ax^{2+m}}{2+m}-\frac{a^2x^{3+m}}{3+m}-\frac{a^3x^{4+m}}{4+m}\end{aligned}$$

Mathematica [A] time = 0.13, size = 54, normalized size = 1.00

$$\frac{x^{m+1}\left((2m+5)\left(\frac{a^2x^2}{m+3}+\frac{2ax}{m+2}+\frac{1}{m+1}\right)-(ax+1)^3\right)}{m+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(-(1 + a*x)^3 + (5 + 2*m)*((1 + m)^(-1) + (2*a*x)/(2 + m) + (a^2
*x^2)/(3 + m))))/(4 + m)
```

fricas [B] time = 0.50, size = 128, normalized size = 2.37

$$-\frac{\left(\left(a^3m^3+6\,a^3m^2+11\,a^3m+6\,a^3\right)x^4+\left(a^2m^3+7\,a^2m^2+14\,a^2m+8\,a^2\right)x^3-\left(am^3+8\,am^2+19\,am+12\,a\right)x^2-}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)*x^m,x, algorithm="fricas")
```

```
[Out] -((a^3*m^3 + 6*a^3*m^2 + 11*a^3*m + 6*a^3)*x^4 + (a^2*m^3 + 7*a^2*m^2 + 14*
a^2*m + 8*a^2)*x^3 - (a*m^3 + 8*a*m^2 + 19*a*m + 12*a)*x^2 - (m^3 + 9*m^2 +
 26*m + 24)*x)*x^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

giac [B] time = 0.18, size = 197, normalized size = 3.65

$$-\frac{a^3m^3x^4x^m+6\,a^3m^2x^4x^m+a^2m^3x^3x^m+11\,a^3mx^4x^m+7\,a^2m^2x^3x^m+6\,a^3x^4x^m-am^3x^2x^m+14\,a^2mx^3x^m-8\,a}{m^4+10\,m^3+35\,m^2+50\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)*x^m,x, algorithm="giac")
```

```
[Out] -(a^3*m^3*x^4*x^m + 6*a^3*m^2*x^4*x^m + a^2*m^3*x^3*x^m + 11*a^3*m*x^4*x^m
+ 7*a^2*m^2*x^3*x^m + 6*a^3*x^4*x^m - a*m^3*x^2*x^m + 14*a^2*m*x^3*x^m - 8*
a*m^2*x^2*x^m + 8*a^2*x^3*x^m - m^3*x*x^m - 19*a*m*x^2*x^m - 9*m^2*x*x^m -
12*a*x^2*x^m - 26*m*x*x^m - 24*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

maple [B] time = 0.03, size = 142, normalized size = 2.63

$$-\frac{x^{1+m}\left(a^3m^3x^3+6a^3m^2x^3+11a^3m\,x^3+a^2m^3x^2+6x^3a^3+7a^2m^2x^2+14a^2m\,x^2-a\,m^3x+8a^2x^2-8a\,m^2x-19a\right.}{(4+m)\,(3+m)\,(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*(-a^2*x^2+1)*x^m,x)
```

```
[Out] -x^(1+m)*(a^3*m^3*x^3+6*a^3*m^2*x^3+11*a^3*m*x^3+a^2*m^3*x^2+6*a^3*x^3+7*a^
2*m^2*x^2+14*a^2*m*x^2-a*m^3*x+8*a^2*x^2-8*a*m^2*x-19*a*m*x-m^3-12*a*x-9*m^
2-26*m-24)/(4+m)/(3+m)/(2+m)/(1+m)
```

maxima [A] time = 0.35, size = 54, normalized size = 1.00

$$-\frac{a^3x^{m+4}}{m+4} - \frac{a^2x^{m+3}}{m+3} + \frac{ax^{m+2}}{m+2} + \frac{x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)*x^m,x, algorithm="maxima")
```

```
[Out] -a^3*x^(m + 4)/(m + 4) - a^2*x^(m + 3)/(m + 3) + a*x^(m + 2)/(m + 2) + x^(m
 + 1)/(m + 1)
```

mupad [B] time = 1.00, size = 160, normalized size = 2.96

$$x^m \left(\frac{x\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24} + \frac{a\,x^2\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24} - \frac{a^3\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^m*(a^2*x^2 - 1)*(a*x + 1),x)
```

```
[Out] x^m*((x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) + (a
*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) - (a^3*
x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) - (a^2*x^
3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

sympy [A] time = 0.69, size = 585, normalized size = 10.83

$$\begin{cases} -a^3\log(x) + \frac{a^2}{x} - \frac{a}{2x^2} - \frac{1}{3x^3} \\ -a^3x - a^2\log(x) - \frac{a}{x} - \frac{1}{2x^2} \\ -\frac{a^3x^2}{2} - a^2x + a\log(x) - \frac{1}{x} \\ -\frac{a^3x^3}{3} - \frac{a^2x^2}{2} + ax + \log(x) \\ -\frac{a^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24} - \frac{6a^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24} - \frac{11a^3mx^4x^m}{m^4+10m^3+35m^2+50m+24} - \frac{6a^3x^4x^m}{m^4+10m^3+35m^2+50m+24} - \frac{a^2m^3x^3x^m}{m^4+10m^3+35m^2+5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a**2*x**2+1)*x**m,x)
```

```
[Out] Piecewise((-a**3*log(x) + a**2/x - a/(2*x**2) - 1/(3*x**3), Eq(m, -4)), (-a
**3*x - a**2*log(x) - a/x - 1/(2*x**2), Eq(m, -3)), (-a**3*x**2/2 - a**2*x
+ a*log(x) - 1/x, Eq(m, -2)), (-a**3*x**3/3 - a**2*x**2/2 + a*x + log(x), E
q(m, -1)), (-a**3*m**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 6
*a**3*m**2*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 11*a**3*m*x**
4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 6*a**3*x**4*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) - a**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35*m**
2 + 50*m + 24) - 7*a**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 2
4) - 14*a**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 8*a**2*x*
*3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + a*m**3*x**2*x**m/(m**4 + 1
0*m**3 + 35*m**2 + 50*m + 24) + 8*a*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**
2 + 50*m + 24) + 19*a*m*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
12*a*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + m**3*x*x**m/(m**4 +
 10*m**3 + 35*m**2 + 50*m + 24) + 9*m**2*x*x**m/(m**4 + 10*m**3 + 35*m**2 +
 50*m + 24) + 26*m*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*x*x**
m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24), True))
```

3.994 $\int e^{\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx$

Optimal. Leaf size=24

$$\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1+m)/(1+m)+a*x^(2+m)/(2+m)
```

Rubi [A] time = 0.08, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 43}

$$\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*Sqrt[1 - a^2*x^2],x]
```

```
[Out] x^(1 + m)/(1 + m) + (a*x^(2 + m))/(2 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx&=\int x^m(1+ax)\,dx\\&=\int\left(x^m+ax^{1+m}\right)dx\\&=\frac{x^{1+m}}{1+m}+\frac{ax^{2+m}}{2+m}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.83

$$x^{m+1}\left(\frac{ax}{m+2}+\frac{1}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*Sqrt[1 - a^2*x^2],x]
```

```
[Out] x^(1 + m)*((1 + m)^(-1) + (a*x)/(2 + m))
```

fricas [A] time = 0.61, size = 29, normalized size = 1.21

$$\frac{\left((am+a)x^2+(m+2)x\right)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*x^m,x, algorithm="fricas")
```

```
[Out] ((a*m + a)*x^2 + (m + 2)*x)*x^m/(m^2 + 3*m + 2)
```

giac [A] time = 0.24, size = 41, normalized size = 1.71

$$\frac{amx^2x^m+ax^2x^m+mxx^m+2\,xx^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*x^m,x, algorithm="giac")
```

```
[Out] (a*m*x^2*x^m + a*x^2*x^m + m*x*x^m + 2*x*x^m)/(m^2 + 3*m + 2)
```

maple [A] time = 0.02, size = 27, normalized size = 1.12

$$\frac{x^{1+m}\,(amx+ax+m+2)}{(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*x^m,x)
```

```
[Out] x^(1+m)*(a*m*x+a*x+m+2)/(2+m)/(1+m)
```

maxima [A] time = 0.30, size = 24, normalized size = 1.00

$$\frac{ax^{m+2}}{m+2}+\frac{x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*x^m,x, algorithm="maxima")
```

```
[Out] a*x^(m + 2)/(m + 2) + x^(m + 1)/(m + 1)
```

mupad [B] time = 1.10, size = 26, normalized size = 1.08

$$\frac{x^{m+1}\,\left(m+a\,x+a\,m\,x+2\right)}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x + 1),x)
```

```
[Out] (x^(m + 1)*(m + a*x + a*m*x + 2))/(3*m + m^2 + 2)
```

sympy [A] time = 0.25, size = 82, normalized size = 3.42

$$\begin{cases} a\log{\left(x \right)} - \frac{1}{x} & \text{for}\: m = -2 \\ ax + \log{\left(x \right)} & \text{for}\: m = -1 \\ \frac{amx^{2}x^{m}}{m^{2}+3m+2} + \frac{ax^{2}x^{m}}{m^{2}+3m+2} + \frac{mxx^{m}}{m^{2}+3m+2} + \frac{2xx^{m}}{m^{2}+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*x**m,x)
```

```
[Out] Piecewise((a*log(x) - 1/x, Eq(m, -2)), (a*x + log(x), Eq(m, -1)), (a*m*x**2
*x**m/(m**2 + 3*m + 2) + a*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m/(m**2 + 3*
m + 2) + 2*x*x**m/(m**2 + 3*m + 2), True))
```

3.995 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{m+1}$$

```
[Out] x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)/(1+m)
```

Rubi [A] time = 0.08, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6150, 64}

$$\frac{x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{1-a^2x^2}}\,dx &= \int \frac{x^m}{1-ax}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(1 + m)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral(-x^m/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^m,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)*x^m/(a^2*x^2 - 1), x)
```

maple [C] time = 0.26, size = 100, normalized size = 4.55

$$-\frac{\left(-a^2\right)^{-\frac{m}{2}}\left(-\frac{2x^m\left(-a^2\right)^{\frac{m}{2}}(-m-2)}{(2+m)m}-x^m\left(-a^2\right)^{\frac{m}{2}}\Phi\left(a^2x^2,1,\frac{m}{2}\right)\right)}{2a}+\frac{x^{1+m}\left(\frac{1}{2}+\frac{m}{2}\right)\Phi\left(a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)*x^m,x)
```

```
[Out] -1/2/a*(-a^2)^(-1/2*m)*(-2/(2+m)*x^m*(-a^2)^(1/2*m)*(-m-2)/m-x^m*(-a^2)^(1/
2*m)*LerchPhi(a^2*x^2,1,1/2*m))+1/(1+m)*x^(1+m)*(1/2+1/2*m)*LerchPhi(a^2*x^
2,1,1/2+1/2*m)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)*x^m,x, algorithm="maxima")
```

```
[Out] -integrate((a*x + 1)*x^m/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{x^m\,(a\,x+1)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(a*x + 1))/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^m*(a*x + 1))/(a^2*x^2 - 1), x)
```

sympy [B] time = 2.69, size = 44, normalized size = 2.00

$$\frac{mxx^m\Phi\left(ax,1,m+1\right)\Gamma\left(m+1\right)}{\Gamma\left(m+2\right)}+\frac{xx^m\Phi\left(ax,1,m+1\right)\Gamma\left(m+1\right)}{\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)*x**m,x)
```

```
[Out] m*x*x**m*lerchphi(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) + x*x**m*lerchph
i(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

3.996 $\int \frac{e^{\tanh^{-1}(ax)} x^m}{(1-a^2x^2)^{3/2}} dx$

Optimal. Leaf size=70

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(2,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

```
[Out] x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a*x^(2+m)*hyper
geom([2, 1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)
```

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6150, 82, 73, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(2,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) + (
a*x^(2 + m)*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m)
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(1-a^2x^2\right)^{3/2}}\,dx &= \int \frac{x^m}{(1-ax)^2(1+ax)}\,dx\\
&= a\int \frac{x^{1+m}}{(1-ax)^2(1+ax)^2}\,dx+\int \frac{x^m}{(1-ax)^2(1+ax)^2}\,dx\\
&= a\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{x^m}{\left(1-a^2x^2\right)^2}\,dx\\
&= \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(2,\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.96

$$x^{m+1}\left(\frac{ax\,{}_2F_1\left(2,\frac{m}{2}+1;\frac{m}{2}+2;a^2x^2\right)}{m+2}+\frac{{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] x^(1 + m)*((a*x*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, a^2*x^2])/(2 + m) +
Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, a^2*x^2]/(1 + m))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{a^3x^3-a^2x^2-ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^m,x, algorithm="fricas")
[Out] integral(x^m/(a^3*x^3 - a^2*x^2 - a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^m,x, algorithm="giac")
[Out] integrate((a*x + 1)*x^m/(a^2*x^2 - 1)^2, x)
```

maple [C] time = 0.26, size = 177, normalized size = 2.53

$$-\frac{\left(-a^2\right)^{-\frac{m}{2}}\left(\frac{x^m\left(-a^2\right)^{\frac{m}{2}}(-m-2)}{(2+m)\left(-a^2x^2+1\right)}+\frac{x^m\left(-a^2\right)^{\frac{m}{2}}m\Phi\left(a^2x^2,1,\frac{m}{2}\right)}{2}\right)}{2a}+\frac{\left(-a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(-\frac{2x^{1+m}\left(-a^2\right)^{\frac{1}{2}+\frac{m}{2}}(-1-m)}{(1+m)\left(-2a^2x^2+2\right)}+\frac{2x^{1+m}\left(-a^2\right)^{\frac{1}{2}+\frac{m}{2}}\left(-\frac{m^2}{4}+\frac{1}{4}\right)\Phi(\right.}{1+m}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^2*x^m,x)
[Out] -1/2/a*(-a^2)^(-1/2*m)*(1/(2+m)*x^m*(-a^2)^(1/2*m)*(-m-2)/(-a^2*x^2+1)+1/2*
x^m*(-a^2)^(1/2*m)*m*LerchPhi(a^2*x^2,1,1/2*m))+1/2*(-a^2)^(-1/2-1/2*m)*(-2
/(1+m)*x^(1+m)*(-a^2)^(1/2+1/2*m)*(-1-m)/(-2*a^2*x^2+2)+2/(1+m)*x^(1+m)*(-a
^2)^(1/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(a^2*x^2,1,1/2+1/2*m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^2*x^m,x, algorithm="maxima")
[Out] integrate((a*x + 1)*x^m/(a^2*x^2 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(a\,x+1\right)}{{\left(a^2\,x^2-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/(a^2*x^2 - 1)^2,x)
```

```
[Out] int((x^m*(a*x + 1))/(a^2*x^2 - 1)^2, x)
```

sympy [C] time = 29.32, size = 673, normalized size = 9.61

$$-\frac{a^2m^2x^3x^m\Phi\left(a^2x^2e^{2i\pi},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^2x^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)-8\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{a^2x^3x^m\Phi\left(a^2x^2e^{2i\pi},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^2x^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)-8\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+a\left(-\frac{a^2m^2x^4x^m\Phi\left(a^2x^2e^{2i\pi}\right.}{8a^2x^2\Gamma\left(\frac{m}{2}+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**2*x**m,x)
```

```
[Out] -a**2*m**2*x**3*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*ga
mma(m/2 + 1/2)/(8*a**2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) + a**2*x
**3*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/
2)/(8*a**2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) + a*(-a**2*m**2*x**4
*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(8*a
**2*x**2*gamma(m/2 + 2) - 8*gamma(m/2 + 2)) - 2*a**2*m*x**4*x**m*lerchphi(a
**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(8*a**2*x**2*gamma(m
/2 + 2) - 8*gamma(m/2 + 2)) + m**2*x**2*x**m*lerchphi(a**2*x**2*exp_polar(2
*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(8*a**2*x**2*gamma(m/2 + 2) - 8*gamma(m/
2 + 2)) + 2*m*x**2*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*g
amma(m/2 + 1)/(8*a**2*x**2*gamma(m/2 + 2) - 8*gamma(m/2 + 2)) - 2*m*x**2*x*
*m*gamma(m/2 + 1)/(8*a**2*x**2*gamma(m/2 + 2) - 8*gamma(m/2 + 2)) - 4*x**2*
x**m*gamma(m/2 + 1)/(8*a**2*x**2*gamma(m/2 + 2) - 8*gamma(m/2 + 2))) + m**2
*x*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(8*a**2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) - 2*m*x*x**m*gamma(m/
2 + 1/2)/(8*a**2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) - x*x**m*lerch
phi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**2*x**
2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2)) - 2*x*x**m*gamma(m/2 + 1/2)/(8*a**
2*x**2*gamma(m/2 + 3/2) - 8*gamma(m/2 + 3/2))
```

3.997 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=70

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(3,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

```
[Out] x^(1+m)*hypergeom([3, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)+a*x^(2+m)*hyper
geom([3, 1+1/2*m],[2+1/2*m],a^2*x^2)/(2+m)
```

Rubi [A] time = 0.12, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6150, 82, 73, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(3,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, a^2*x^2])/(1 + m) + (
a*x^(2 + m)*Hypergeometric2F1[3, (2 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m)
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^m}{(1-a^2x^2)^{5/2}}\,dx &= \int \frac{x^m}{(1-ax)^3(1+ax)^2}\,dx\\ &= a\int \frac{x^{1+m}}{(1-ax)^3(1+ax)^3}\,dx + \int \frac{x^m}{(1-ax)^3(1+ax)^3}\,dx\\ &= a\int \frac{x^{1+m}}{(1-a^2x^2)^3}\,dx + \int \frac{x^m}{(1-a^2x^2)^3}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{1+m} + \frac{ax^{2+m}\,{}_2F_1\left(3,\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{2+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.96

$$x^{m+1}\left(\frac{ax\,{}_2F_1\left(3,\frac{m}{2}+1;\frac{m}{2}+2;a^2x^2\right)}{m+2}+\frac{{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] x^(1 + m)*((a*x*Hypergeometric2F1[3, 1 + m/2, 2 + m/2, a^2*x^2])/(2 + m) +
Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, a^2*x^2]/(1 + m))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^m}{a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^m,x, algorithm="fricas")

[Out] integral(-x^m/(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)x^m}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^m,x, algorithm="giac")

[Out] integrate(-(a*x + 1)*x^m/(a^2*x^2 - 1)^3, x)

maple [C] time = 0.28, size = 224, normalized size = 3.20

$$\frac{\left(-a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{x^{1+m}\left(-a^2\right)^{\frac{1}{2}+\frac{m}{2}}\left(a^2m^2x^2-2a^2m\,x^2-3a^2x^2-m^2+4m+5\right)}{2(1+m)\left(-a^2x^2+1\right)^2}+\frac{4x^{1+m}\left(-a^2\right)^{\frac{1}{2}+\frac{m}{2}}\left(\frac{1}{16}m^3-\frac{3}{16}m^2-\frac{1}{16}m+\frac{3}{16}\right)\Phi\left(a^2x^2,1,\frac{1}{2}+\frac{m}{2}\right)}{1+m}\right)}{4}-\frac{\left(-a^2\right)^{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^3*x^m,x)

[Out] 1/4*(-a^2)^(-1/2-1/2*m)*(1/2/(1+m)*x^(1+m)*(-a^2)^(1/2+1/2*m)*(a^2*m^2*x^2-2*a^2*m*x^2-3*a^2*x^2-m^2+4*m+5)/(-a^2*x^2+1)^2+4/(1+m)*x^(1+m)*(-a^2)^(1/2+1/2*m)*(1/16*m^3-3/16*m^2-1/16*m+3/16)*LerchPhi(a^2*x^2,1,1/2+1/2*m))-1/4/a*(-a^2)^(-1/2*m)*(-1/2*x^m*(-a^2)^(1/2*m)*(a^2*m*x^2-m+2)/(-a^2*x^2+1)^2-1/4*x^m*(-a^2)^(1/2*m)*(-2+m)*m*LerchPhi(a^2*x^2,1,1/2*m))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)x^m}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^3*x^m,x, algorithm="maxima")

[Out] -integrate((a*x + 1)*x^m/(a^2*x^2 - 1)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^m\,\left(a\,x+1\right)}{\left(a^2\,x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^m*(a*x + 1))/(a^2*x^2 - 1)^3,x)

[Out] int(-(x^m*(a*x + 1))/(a^2*x^2 - 1)^3, x)

sympy [C] time = 50.31, size = 2152, normalized size = 30.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a**2*x**2+1)**3*x**m,x)

```
[Out] a**4*m**3*x**5*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gam
ma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2
) + 32*gamma(m/2 + 3/2)) - 3*a**4*m**2*x**5*x**m*lerchphi(a**2*x**2*exp_pol
ar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) -
 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) - a**4*m*x**5*x**m*le
rchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4
*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2
)) + 3*a**4*x**5*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*g
amma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3
/2) + 32*gamma(m/2 + 3/2)) - 2*a**2*m**3*x**3*x**m*lerchphi(a**2*x**2*exp_p
olar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2)
 - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 6*a**2*m**2*x**3*
x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(
32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/
2 + 3/2)) + 2*a**2*m**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2
+ 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 2*a**2*m*x*
*3*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma
(m/2 + 3/2)) - 4*a**2*m*x**3*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2
+ 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) - 6*a**2*x**3
*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m
/2 + 3/2)) - 6*a**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/
2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + a*(a**4*m**3*x*
*6*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(3
2*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2
)) - 4*a**4*m*x**6*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*g
amma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) +
32*gamma(m/2 + 2)) - 2*a**2*m**3*x**4*x**m*lerchphi(a**2*x**2*exp_polar(2*I
*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**
2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) + 2*a**2*m**2*x**4*x**m*gamma(m/2 + 1
)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2
```

```
 + 2)) + 8*a**2*m*x**4*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 +
1)*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2
) + 32*gamma(m/2 + 2)) - 8*a**2*x**4*x**m*gamma(m/2 + 1)/(32*a**4*x**4*gamm
a(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) + m**3*x**2*x
**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(32*a*
*4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) -
 2*m**2*x**2*x**m*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**
2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) - 4*m*x**2*x**m*lerchphi(a**2*x**2*ex
p_polar(2*I*pi), 1, m/2 + 1)*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) -
64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2)) + 4*m*x**2*x**m*gamma(m/2
+ 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a**2*x**2*gamma(m/2 + 2) + 32*gamma(
m/2 + 2)) + 16*x**2*x**m*gamma(m/2 + 1)/(32*a**4*x**4*gamma(m/2 + 2) - 64*a
**2*x**2*gamma(m/2 + 2) + 32*gamma(m/2 + 2))) + m**3*x*x**m*lerchphi(a**2*x
**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m
/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) - 3*m**2*x
*x**m*lerchphi(a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m
/2 + 3/2)) - 2*m**2*x*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2)
- 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) - m*x*x**m*lerchphi(
a**2*x**2*exp_polar(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*g
amma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 8*
m*x*x**m*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gam
ma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 3*x*x**m*lerchphi(a**2*x**2*exp_pola
r(2*I*pi), 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) -
64*a**2*x**2*gamma(m/2 + 3/2) + 32*gamma(m/2 + 3/2)) + 10*x*x**m*gamma(m/2
+ 1/2)/(32*a**4*x**4*gamma(m/2 + 3/2) - 64*a**2*x**2*gamma(m/2 + 3/2) + 32*
gamma(m/2 + 3/2))
```

3.998 $\int e^{\tanh^{-1}(ax)} x^m \left(c - a^2cx^2\right)^{5/2} dx$

Optimal. Leaf size=274

$$\frac{c^2x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ac^2x^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}-\frac{2a^2c^2x^{m+3}\sqrt{c-a^2cx^2}}{(m+3)\sqrt{1-a^2x^2}}+\frac{a^5c^2x^{m+6}\sqrt{c-a^2cx^2}}{(m+6)\sqrt{1-a^2x^2}}+\frac{a^4c^2x^{m+5}\sqrt{c-a^2cx^2}}{(m+5)\sqrt{1-a^2x^2}}-2$$

```
[Out] c^2*x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(1+m)/(-a^2*x^2+1)^(1/2)+a*c^2*x^(2+m)*(-a
^2*c*x^2+c)^(1/2)/(2+m)/(-a^2*x^2+1)^(1/2)-2*a^2*c^2*x^(3+m)*(-a^2*c*x^2+c)
^(1/2)/(3+m)/(-a^2*x^2+1)^(1/2)-2*a^3*c^2*x^(4+m)*(-a^2*c*x^2+c)^(1/2)/(4+m
)/(-a^2*x^2+1)^(1/2)+a^4*c^2*x^(5+m)*(-a^2*c*x^2+c)^(1/2)/(5+m)/(-a^2*x^2+1
)^(1/2)+a^5*c^2*x^(6+m)*(-a^2*c*x^2+c)^(1/2)/(6+m)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{c^2x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ac^2x^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}-\frac{2a^2c^2x^{m+3}\sqrt{c-a^2cx^2}}{(m+3)\sqrt{1-a^2x^2}}-\frac{2a^3c^2x^{m+4}\sqrt{c-a^2cx^2}}{(m+4)\sqrt{1-a^2x^2}}+\frac{a^4c^2x^{m+5}\sqrt{c-a^2cx^2}}{(m+5)\sqrt{1-a^2x^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*c^2*x^
(2 + m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) - (2*a^2*c^2*x^(3
+ m)*Sqrt[c - a^2*c*x^2])/((3 + m)*Sqrt[1 - a^2*x^2]) - (2*a^3*c^2*x^(4 + m
)*Sqrt[c - a^2*c*x^2])/((4 + m)*Sqrt[1 - a^2*x^2]) + (a^4*c^2*x^(5 + m)*Sqr
t[c - a^2*c*x^2])/((5 + m)*Sqrt[1 - a^2*x^2]) + (a^5*c^2*x^(6 + m)*Sqrt[c -
 a^2*c*x^2])/((6 + m)*Sqrt[1 - a^2*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^{5/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int x^m(1-ax)^2(1+ax)^3\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int\left(x^m+ax^{1+m}-2a^2x^{2+m}-2a^3x^{3+m}+a^4x^{4+m}+a^5x^{5+m}\right)\,d}{\sqrt{1-a^2x^2}}$$
$$=\frac{c^2x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}+\frac{ac^2x^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}-\frac{2a^2c^2x^{3+m}\sqrt{c-a^2cx^2}}{(3+m)\sqrt{1-a^2x^2}}-\frac{2a^3c^2x}{(4+}$$

Mathematica [A] time = 0.09, size = 102, normalized size = 0.37

$$\frac{c^2x^{m+1}\sqrt{c-a^2cx^2}\left(\frac{a^5x^5}{m+6}+\frac{a^4x^4}{m+5}-\frac{2a^3x^3}{m+4}-\frac{2a^2x^2}{m+3}+\frac{ax}{m+2}+\frac{1}{m+1}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*x^(1 + m)*Sqrt[c - a^2*c*x^2]*((1 + m)^(-1) + (a*x)/(2 + m) - (2*a^2*x
^2)/(3 + m) - (2*a^3*x^3)/(4 + m) + (a^4*x^4)/(5 + m) + (a^5*x^5)/(6 + m)))
/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.52, size = 476, normalized size = 1.74

$$\left(\left(a^5c^2m^5+15\,a^5c^2m^4+85\,a^5c^2m^3+225\,a^5c^2m^2+274\,a^5c^2m+120\,a^5c^2\right)x^6+\left(a^4c^2m^5+16\,a^4c^2m^4+95\,a^4c^2m\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] ((a^5*c^2*m^5 + 15*a^5*c^2*m^4 + 85*a^5*c^2*m^3 + 225*a^5*c^2*m^2 + 274*a^5
*c^2*m + 120*a^5*c^2)*x^6 + (a^4*c^2*m^5 + 16*a^4*c^2*m^4 + 95*a^4*c^2*m^3
+ 260*a^4*c^2*m^2 + 324*a^4*c^2*m + 144*a^4*c^2)*x^5 - 2*(a^3*c^2*m^5 + 17*
a^3*c^2*m^4 + 107*a^3*c^2*m^3 + 307*a^3*c^2*m^2 + 396*a^3*c^2*m + 180*a^3*c
^2)*x^4 - 2*(a^2*c^2*m^5 + 18*a^2*c^2*m^4 + 121*a^2*c^2*m^3 + 372*a^2*c^2*m
^2 + 508*a^2*c^2*m + 240*a^2*c^2)*x^3 + (a*c^2*m^5 + 19*a*c^2*m^4 + 137*a*c
^2*m^3 + 461*a*c^2*m^2 + 702*a*c^2*m + 360*a*c^2)*x^2 + (c^2*m^5 + 20*c^2*m
^4 + 155*c^2*m^3 + 580*c^2*m^2 + 1044*c^2*m + 720*c^2)*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1)*x^m/(m^6 + 21*m^5 + 175*m^4 + 735*m^3 - (a^2*m^6 + 2
1*a^2*m^5 + 175*a^2*m^4 + 735*a^2*m^3 + 1624*a^2*m^2 + 1764*a^2*m + 720*a^2
)*x^2 + 1624*m^2 + 1764*m + 720)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 377, normalized size = 1.38

$$\frac{x^{1+m}\left(a^5m^5x^5+15a^5m^4x^5+85a^5m^3x^5+a^4m^5x^4+225a^5m^2x^5+16a^4m^4x^4+274a^5m\,x^5+95a^4m^3x^4-2a^3m^5x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] x^(1+m)*(a^5*m^5*x^5+15*a^5*m^4*x^5+85*a^5*m^3*x^5+a^4*m^5*x^4+225*a^5*m^2*
x^5+16*a^4*m^4*x^4+274*a^5*m*x^5+95*a^4*m^3*x^4-2*a^3*m^5*x^3+120*a^5*x^5+2
60*a^4*m^2*x^4-34*a^3*m^4*x^3+324*a^4*m*x^4-214*a^3*m^3*x^3-2*a^2*m^5*x^2+1
44*a^4*x^4-614*a^3*m^2*x^3-36*a^2*m^4*x^2-792*a^3*m*x^3-242*a^2*m^3*x^2+a*m
^5*x-360*a^3*x^3-744*a^2*m^2*x^2+19*a*m^4*x-1016*a^2*m*x^2+137*a*m^3*x+m^5-
480*a^2*x^2+461*a*m^2*x+20*m^4+702*a*m*x+155*m^3+360*a*x+580*m^2+1044*m+720
)*(-a^2*c*x^2+c)^(5/2)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)/(a*x-1)^2/(a*x+1
)^2/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.35, size = 144, normalized size = 0.53

$$\frac{\left(\left(m^2+6\,m+8\right)a^4c^{\frac{5}{2}}x^6-2\left(m^2+8\,m+12\right)a^2c^{\frac{5}{2}}x^4+\left(m^2+10\,m+24\right)c^{\frac{5}{2}}x^2\right)ax^m}{m^3+12\,m^2+44\,m+48}+\frac{\left(\left(m^2+4\,m+3\right)a^4c^{\frac{5}{2}}x^5-2\,(\ldots}{m\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] ((m^2 + 6*m + 8)*a^4*c^(5/2)*x^6 - 2*(m^2 + 8*m + 12)*a^2*c^(5/2)*x^4 + (m^
2 + 10*m + 24)*c^(5/2)*x^2)*a*x^m/(m^3 + 12*m^2 + 44*m + 48) + ((m^2 + 4*m
+ 3)*a^4*c^(5/2)*x^5 - 2*(m^2 + 6*m + 5)*a^2*c^(5/2)*x^3 + (m^2 + 8*m + 15)
*c^(5/2)*x)*x^m/(m^3 + 9*m^2 + 23*m + 15)
```

mupad [B] time = 1.62, size = 468, normalized size = 1.71

$$x^m\left(\frac{c^2\,x\,\sqrt{c-a^2\,c\,x^2}\,\left(m^5+20\,m^4+155\,m^3+580\,m^2+1044\,m+720\right)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}+\frac{a\,c^2\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(m^5+19\,m^4+137\,m^3+461\,m^2+702\,m+360\right)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}+\frac{a^5\,c^2\,x^6\,\sqrt{c-\ldots}}{m^6+21\,\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(5/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (x^m*((c^2*x*(c - a^2*c*x^2)^(1/2)*(1044*m + 580*m^2 + 155*m^3 + 20*m^4 + m
^5 + 720))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (
a*c^2*x^2*(c - a^2*c*x^2)^(1/2)*(702*m + 461*m^2 + 137*m^3 + 19*m^4 + m^5 +
 360))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (a^5*
c^2*x^6*(c - a^2*c*x^2)^(1/2)*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 12
0))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (a^4*c^2
*x^5*(c - a^2*c*x^2)^(1/2)*(324*m + 260*m^2 + 95*m^3 + 16*m^4 + m^5 + 144))
/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) - (2*a^3*c^2*
x^4*(c - a^2*c*x^2)^(1/2)*(396*m + 307*m^2 + 107*m^3 + 17*m^4 + m^5 + 180))
/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) - (2*a^2*c^2*
x^3*(c - a^2*c*x^2)^(1/2)*(508*m + 372*m^2 + 121*m^3 + 18*m^4 + m^5 + 240))
/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)))/(1 - a^2*x^
2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.999 $\int e^{\tanh^{-1}(ax)} x^m \left(c-a^2cx^2\right)^{3/2} dx$

Optimal. Leaf size=174

$$\frac{cx^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{acx^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}-\frac{a^2cx^{m+3}\sqrt{c-a^2cx^2}}{(m+3)\sqrt{1-a^2x^2}}-\frac{a^3cx^{m+4}\sqrt{c-a^2cx^2}}{(m+4)\sqrt{1-a^2x^2}}$$

```
[Out] c*x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(1+m)/(-a^2*x^2+1)^(1/2)+a*c*x^(2+m)*(-a^2*c
*x^2+c)^(1/2)/(2+m)/(-a^2*x^2+1)^(1/2)-a^2*c*x^(3+m)*(-a^2*c*x^2+c)^(1/2)/(
3+m)/(-a^2*x^2+1)^(1/2)-a^3*c*x^(4+m)*(-a^2*c*x^2+c)^(1/2)/(4+m)/(-a^2*x^2+
1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 75}

$$\frac{cx^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{acx^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}-\frac{a^2cx^{m+3}\sqrt{c-a^2cx^2}}{(m+3)\sqrt{1-a^2x^2}}-\frac{a^3cx^{m+4}\sqrt{c-a^2cx^2}}{(m+4)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*c*x^(2 +
 m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) - (a^2*c*x^(3 + m)*Sqr
t[c - a^2*c*x^2])/((3 + m)*Sqrt[1 - a^2*x^2]) - (a^3*c*x^(4 + m)*Sqrt[c - a
^2*c*x^2])/((4 + m)*Sqrt[1 - a^2*x^2])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int x^m(1-ax)(1+ax)^2\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int\left(x^m+ax^{1+m}-a^2x^{2+m}-a^3x^{3+m}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{cx^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}+\frac{acx^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}-\frac{a^2cx^{3+m}\sqrt{c-a^2cx^2}}{(3+m)\sqrt{1-a^2x^2}}-\frac{a^3cx^{4+m}\sqrt{}}{(4+m)\sqrt{}}$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.48

$$\frac{cx^{m+1}\sqrt{c-a^2cx^2}\left((2m+5)\left(\frac{a^2x^2}{m+3}+\frac{2ax}{m+2}+\frac{1}{m+1}\right)-(ax+1)^3\right)}{(m+4)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*x^(1 + m)*Sqrt[c - a^2*c*x^2]*(-(1 + a*x)^3 + (5 + 2*m)*((1 + m)^(-1) +
(2*a*x)/(2 + m) + (a^2*x^2)/(3 + m))))/((4 + m)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.74, size = 211, normalized size = 1.21

$$-\frac{\sqrt{-a^2cx^2+c}\left(\left(a^3cm^3+6\,a^3cm^2+11\,a^3cm+6\,a^3c\right)x^4+\left(a^2cm^3+7\,a^2cm^2+14\,a^2cm+8\,a^2c\right)x^3-\left(acm^3+8\,a\right.\right.}{m^4+10\,m^3-\left(a^2m^4+10\,a^2m^3+35\,a^2m^2+50\,a^2m+24\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a^3*c*m^3 + 6*a^3*c*m^2 + 11*a^3*c*m + 6*a^3*c)*x^4
 + (a^2*c*m^3 + 7*a^2*c*m^2 + 14*a^2*c*m + 8*a^2*c)*x^3 - (a*c*m^3 + 8*a*c*
m^2 + 19*a*c*m + 12*a*c)*x^2 - (c*m^3 + 9*c*m^2 + 26*c*m + 24*c)*x)*sqrt(-a
^2*x^2 + 1)*x^m/(m^4 + 10*m^3 - (a^2*m^4 + 10*a^2*m^3 + 35*a^2*m^2 + 50*a^2
*m + 24*a^2)*x^2 + 35*m^2 + 50*m + 24)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 180, normalized size = 1.03

$$\frac{x^{1+m}\left(a^3m^3x^3+6a^3m^2x^3+11a^3m\,x^3+a^2m^3x^2+6x^3a^3+7a^2m^2x^2+14a^2m\,x^2-a\,m^3x+8a^2x^2-8a\,m^2x-19a\right.}{(4+m)(3+m)(2+m)(1+m)(ax-1)(ax+1)\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] x^(1+m)*(a^3*m^3*x^3+6*a^3*m^2*x^3+11*a^3*m*x^3+a^2*m^3*x^2+6*a^3*x^3+7*a^2
*m^2*x^2+14*a^2*m*x^2-a*m^3*x+8*a^2*x^2-8*a*m^2*x-19*a*m*x-m^3-12*a*x-9*m^2
-26*m-24)*(-a^2*c*x^2+c)^(3/2)/(4+m)/(3+m)/(2+m)/(1+m)/(a*x-1)/(a*x+1)/(-a^
2*x^2+1)^(1/2)
```

maxima [A] time = 0.37, size = 80, normalized size = 0.46

$$-\frac{\left(a^2c^{\frac{3}{2}}(m+2)x^4-c^{\frac{3}{2}}(m+4)x^2\right)ax^m}{m^2+6\,m+8}-\frac{\left(a^2c^{\frac{3}{2}}(m+1)x^3-c^{\frac{3}{2}}(m+3)x\right)x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -(a^2*c^(3/2)*(m + 2)*x^4 - c^(3/2)*(m + 4)*x^2)*a*x^m/(m^2 + 6*m + 8) - (a
^2*c^(3/2)*(m + 1)*x^3 - c^(3/2)*(m + 3)*x)*x^m/(m^2 + 4*m + 3)
```

mupad [B] time = 1.26, size = 228, normalized size = 1.31

$$\frac{x^m \left(\frac{c\,x\,\sqrt{c-a^2\,c\,x^2}\,\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{a\,c\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}-\frac{a^3\,c\,x^4\,\sqrt{c-a^2\,c\,x^2}\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}-\frac{a^2\,c\,x^3\,\sqrt{c-a^2\,c}}{m^4+10\,m^3}\right.}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(3/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (x^m*((c*x*(c - a^2*c*x^2)^(1/2)*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2
+ 10*m^3 + m^4 + 24) + (a*c*x^2*(c - a^2*c*x^2)^(1/2)*(19*m + 8*m^2 + m^3 +
 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) - (a^3*c*x^4*(c - a^2*c*x^2)^(1/2
)*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) - (a^2*c*x^
3*(c - a^2*c*x^2)^(1/2)*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 +
 m^4 + 24)))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**m*(-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)/sqrt(-(a*x - 1)*(a*
x + 1)), x)
```

3.1000 $\int e^{\tanh^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=82

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(1+m)/(-a^2*x^2+1)^(1/2)+a*x^(2+m)*(-a^2*c*x^2
+c)^(1/2)/(2+m)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 43}

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*x^(2 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\int e^{\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\sqrt{c-a^2cx^2}\int x^m(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\sqrt{c-a^2cx^2}\int\left(x^m+ax^{1+m}\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}+\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.60

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}\left(\frac{ax}{m+2}+\frac{1}{m+1}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*((1 + m)^(-1) + (a*x)/(2 + m))*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2
*x^2]
```

fricas [A] time = 0.73, size = 80, normalized size = 0.98

$$-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\left((am+a)x^2+(m+2)x\right)x^m}{\left(a^2m^2+3\,a^2m+2\,a^2\right)x^2-m^2-3\,m-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((a*m + a)*x^2 + (m + 2)*x)*x^m/((
a^2*m^2 + 3*a^2*m + 2*a^2)*x^2 - m^2 - 3*m - 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\,(ax + 1)x^m}{\sqrt{-a^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.03, size = 52, normalized size = 0.63

$$\frac{x^{1+m}\left(amx + ax + m + 2\right)\sqrt{-a^2c\,x^2 + c}}{\left(2 + m\right)\left(1 + m\right)\sqrt{-a^2x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+a*x+m+2)*(-a^2*c*x^2+c)^(1/2)/(2+m)/(1+m)/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.34, size = 30, normalized size = 0.37

$$\frac{a\sqrt{c}\,x^2x^m}{m + 2} + \frac{\sqrt{c}\,xx^m}{m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] a*sqrt(c)*x^2*x^m/(m + 2) + sqrt(c)*x*x^m/(m + 1)
```

mupad [B] time = 1.07, size = 51, normalized size = 0.62

$$\frac{x^{m+1}\sqrt{c - a^2\,c\,x^2}\ \left(m + a\,x + a\,m\,x + 2\right)}{\sqrt{1 - a^2\,x^2}\ \left(m^2 + 3\,m + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] (x^(m + 1)*(c - a^2*c*x^2)^(1/2)*(m + a*x + a*m*x + 2))/((1 - a^2*x^2)^(1/2
)*(3*m + m^2 + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{-c\,(ax-1)\,(ax+1)}\,\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/sqrt(-(a*x - 1)*(a*x +
 1)), x)
```

3.1001 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{(m+1)\sqrt{c-a^2cx^2}}$$

```
[Out] x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)*(-a^2*x^2+1)^(1/2)/(1+m)/(-a^2*c*x^2+
c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 64}

$$\frac{\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{(m+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/((1 +
 m)*Sqrt[c - a^2*c*x^2])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{1-ax}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{x^{1+m}\sqrt{1-a^2x^2}\;{}_2F_1(1,1+m;2+m;ax)}{(1+m)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 1.00

$$\frac{\sqrt{1-a^2x^2}\,x^{m+1}\;{}_2F_1(1,m+1;m+2;ax)}{(m+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/((1 +
 m)*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^m}{a^3cx^3-a^2cx^2-acx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^m/(a^3*c*x^3 - a^2*c*x^2
 - a*c*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^m/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^{m}}{\sqrt{-a^{2}x^{2}+1}\,\sqrt{-a^{2}c\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^{m}}{\sqrt{-a^{2}cx^{2}+c}\,\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^m/(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m}\,(a\,x+1)}{\sqrt{c-a^{2}\,c\,x^{2}}\,\sqrt{1-a^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x
+ 1))), x)
```

3.1002 $\int \frac{e^{\tanh^{-1}(ax)} x^m}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c(m+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(2,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/c/
(1+m)/(-a^2*c*x^2+c)^(1/2)+a*x^(2+m)*hypergeom([2, 1+1/2*m],[2+1/2*m],a^2*x
^2)*(-a^2*x^2+1)^(1/2)/c/(2+m)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6153, 6150, 82, 73, 364}

$$\frac{\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c(m+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(2,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c(m+2)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, a^2
*x^2])/(c*(1 + m)*Sqrt[c - a^2*c*x^2]) + (a*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hyp
ergeometric2F1[2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(c*(2 + m)*Sqrt[c - a^2*c
*x^2])
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{(1-ax)^2(1+ax)^2}\,dx}{c\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{(1-ax)^2(1+ax)^2}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{\left(1-a^2x^2\right)^2}\,dx}{c\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^2}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c(1+m)\sqrt{c-a^2cx^2}}+\frac{ax^{2+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(2,\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c(2+m)\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 107, normalized size = 0.80

$$\frac{\sqrt{1-a^2x^2}\left(\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+1}{2}+1;a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(2,\frac{m+2}{2};\frac{m+2}{2}+1;a^2x^2\right)}{m+2}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/
2, a^2*x^2])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[2, (2 + m)/2, 1 + (2
+ m)/2, a^2*x^2])/(2 + m)))/(c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^m}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^m/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^m/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^m}{\sqrt{-a^2x^2+1}\,\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^m/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(a\,x+1\right)}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\left(ax+1\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**m*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.1003 $\int \frac{e^{\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c^2(m+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(3,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c^2(m+2)\sqrt{c-a^2cx^2}}$$

```
[Out] x^(1+m)*hypergeom([3, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/c^
2/(1+m)/(-a^2*c*x^2+c)^(1/2)+a*x^(2+m)*hypergeom([3, 1+1/2*m],[2+1/2*m],a^2
*x^2)*(-a^2*x^2+1)^(1/2)/c^2/(2+m)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6153, 6150, 82, 73, 364}

$$\frac{\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{c^2(m+1)\sqrt{c-a^2cx^2}}+\frac{a\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(3,\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{c^2(m+2)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, a^2
*x^2])/(c^2*(1 + m)*Sqrt[c - a^2*c*x^2]) + (a*x^(2 + m)*Sqrt[1 - a^2*x^2]*H
ypergeometric2F1[3, (2 + m)/2, (4 + m)/2, a^2*x^2])/(c^2*(2 + m)*Sqrt[c - a
^2*c*x^2])
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 82

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^m}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{(1-ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{(1-ax)^3(1+ax)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{(1-ax)^3(1+ax)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^m}{\left(1-a^2x^2\right)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{c^2(1+m)\sqrt{c-a^2cx^2}}+\frac{ax^{2+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(3,\frac{2+m}{2};\frac{4+m}{2};a^2x^2\right)}{c^2(2+m)\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 107, normalized size = 0.80

$$\frac{\sqrt{1-a^2x^2}\left(\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+1}{2}+1;a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(3,\frac{m+2}{2};\frac{m+2}{2}+1;a^2x^2\right)}{m+2}\right)}{c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^m)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*((x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, 1 + (1 + m)/
2, a^2*x^2])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[3, (2 + m)/2, 1 + (2
+ m)/2, a^2*x^2])/(2 + m)))/(c^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^m}{a^7c^3x^7-a^6c^3x^6-3\,a^5c^3x^5+3\,a^4c^3x^4+3\,a^3c^3x^3-3\,a^2c^3x^2-ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^m/(a^7*c^3*x^7 - a^6*c^3
*x^6 - 3*a^5*c^3*x^5 + 3*a^4*c^3*x^4 + 3*a^3*c^3*x^3 - 3*a^2*c^3*x^2 - a*c^
3*x + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^m/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^m}{\sqrt{-a^2x^2+1}\,\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^m/((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,(a\,x+1)}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^m*(a*x + 1))/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**m*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(5/2)), x)
```

3.1004 $\int e^{\tanh^{-1}(ax)} x^m \left(c-a^2cx^2\right)^p \, dx$

Optimal. Leaf size=136

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\ {}_2F_1\left(\frac{m+1}{2},\frac{1}{2}-p;\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\ {}_2F_1\left(\frac{m+2}{2},\frac{1}{2}-p;\frac{m+4}{2}\right.}{m+2}$$

```
[Out] x^(1+m)*(-a^2*c*x^2+c)^p*hypergeom([1/2-p, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/
(1+m)/((-a^2*x^2+1)^p)+a*x^(2+m)*(-a^2*c*x^2+c)^p*hypergeom([1/2-p, 1+1/2*m
],[2+1/2*m],a^2*x^2)/(2+m)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6153, 6148, 808, 364}

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\ {}_2F_1\left(\frac{m+1}{2},\frac{1}{2}-p;\frac{m+3}{2};a^2x^2\right)}{m+1}+\frac{ax^{m+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\ {}_2F_1\left(\frac{m+2}{2},\frac{1}{2}-p;\frac{m+4}{2}\right.}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1/2 - p, (3 + m)/
2, a^2*x^2])/((1 + m)*(1 - a^2*x^2)^p) + (a*x^(2 + m)*(c - a^2*c*x^2)^p*Hyp
ergeometric2F1[(2 + m)/2, 1/2 - p, (4 + m)/2, a^2*x^2])/((2 + m)*(1 - a^2*x
^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
```

```
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^p\,dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx + \left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2c\right.\right.$$
$$= \frac{x^{1+m}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1+m}{2},\frac{1}{2}-p;\frac{3+m}{2};a^2x^2\right)}{1+m} + \frac{ax^{2+m}\left(1-a^2x^2\right.}{}$$

Mathematica [A] time = 0.04, size = 114, normalized size = 0.84

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{x^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}-p;\frac{m+1}{2}+1;a^2x^2\right)}{m+1}+\frac{ax^{m+2}\,{}_2F_1\left(\frac{m+2}{2},\frac{1}{2}-p;\frac{m+2}{2}+1;a^2x^2\right)}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^m*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*((x^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1/2 - p, 1 + (1
 + m)/2, a^2*x^2])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1/2
- p, 1 + (2 + m)/2, a^2*x^2])/(2 + m)))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^m/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^m/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^m\left(-a^2c\,x^2+c\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x^m}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^m/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\left(c-a^2\,c\,x^2\right)^p\,(a\,x+1)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 69.57, size = 381, normalized size = 2.80

$$-\frac{aa^{2p}c^{p}x^{2}x^{m}x^{2p}e^{i\pi p}\Gamma\left(p+\frac{1}{2}\right)\Gamma\left(-\frac{m}{2}-p-1\right){}_{3}F_{2}\left(\begin{matrix}\frac{1}{2},1,\frac{m}{2}+p+1\\p+1,\frac{m}{2}+p+2\end{matrix}\middle|a^{2}x^{2}e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(-\frac{m}{2}-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^{p}x^{2}x^{m}x^{2p}e^{i\pi p}\Gamma\left(p+\frac{1}{2}\right)\Gamma\left(-\frac{m}{2}\right.}{2\sqrt{\pi}\,\Gamma\left(-\frac{m}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**m*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a*a**(2*p)*c**p*x**2*x**m*x**(2*p)*exp(I*pi*p)*gamma(p + 1/2)*gamma(-m/2 -
 p - 1)*hyper((1/2, 1, m/2 + p + 1), (p + 1, m/2 + p + 2), a**2*x**2*exp_po
lar(2*I*pi))/(2*sqrt(pi)*gamma(-m/2 - p)*gamma(p + 1)) - a*a**(2*p)*c**p*x*
*2*x**m*x**(2*p)*exp(I*pi*p)*gamma(p + 1/2)*gamma(-m/2 - p - 1)*hyper((1, -
p, -m/2 - p - 1), (1/2, -m/2 - p), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-m/2 -
p)*gamma(p + 1)) - a**(2*p)*c**p*x*x**m*x**(2*p)*exp(I*pi*p)*gamma(p + 1/2)
*gamma(-m/2 - p - 1/2)*hyper((1/2, 1, m/2 + p + 1/2), (p + 1, m/2 + p + 3/2
), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(p + 1)*gamma(-m/2 - p + 1
/2)) - a**(2*p)*c**p*x*x**m*x**(2*p)*exp(I*pi*p)*gamma(p + 1/2)*gamma(-m/2
- p - 1/2)*hyper((1, -p, -m/2 - p - 1/2), (1/2, -m/2 - p + 1/2), 1/(a**2*x*
*2))/(2*sqrt(pi)*gamma(p + 1)*gamma(-m/2 - p + 1/2))
```

3.1005 $\int e^{\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=85

$$\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}$$

```
[Out] -(-a^2*x^2+1)^(1/2+p)/a^4/(1+2*p)+(-a^2*x^2+1)^(3/2+p)/a^4/(3+2*p)+1/5*a*x^
5*hypergeom([5/2, 1/2-p],[7/2],a^2*x^2)
```

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 266, 43, 364}

$$\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(1 - a^2*x^2)^p,x]
```

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^4*(1 + 2*p))) + (1 - a^2*x^2)^(3/2 + p)/(a^4*(
3 + 2*p)) + (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx &= \int x^3(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= a\int x^4\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= \frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\,\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{1}{2}+p}\,dx,x,x^2\right)\\ &= \frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\,\text{Subst}\left(\int\left(\frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{a^2}-\frac{\left(1-a^2x\right)^{\frac{1}{2}+p}}{a^2}\right)dx\right.\\ &= -\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^4(1+2p)}+\frac{\left(1-a^2x^2\right)^{\frac{3}{2}+p}}{a^4(3+2p)}+\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 77, normalized size = 0.91

$$\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\left(a^2(2p+1)x^2+2\right)}{a^4\left(4p^2+8p+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(1 - a^2*x^2)^p,x]
```

```
[Out] -(((1 - a^2*x^2)^(1/2 + p)*(2 + a^2*(1 + 2*p)*x^2))/(a^4*(3 + 8*p + 4*p^2))
) + (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^3}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*x^2+1)^p,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^3/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p x^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*x^2+1)^p,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p*x^3/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.35, size = 47, normalized size = 0.55

$$\frac{a\,x^5\,\mathrm{hypergeom}\left(\left[\frac{5}{2},\frac{1}{2}-p\right],\left[\frac{7}{2}\right],a^2x^2\right)}{5}+\frac{x^4\,\mathrm{hypergeom}\left(\left[2,\frac{1}{2}-p\right],[3],a^2x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*x^2+1)^p,x)
```

```
[Out] 1/5*a*x^5*hypergeom([5/2,1/2-p],[7/2],a^2*x^2)+1/4*x^4*hypergeom([2,1/2-p],
[3],a^2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^4e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx+\frac{\left(a^4\left(2\,p+1\right)x^4-a^2\left(2\,p-1\right)x^2-2\right)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}\,\left(4\,p^2+8\,p+3\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*x^2+1)^p,x, algorithm="maxim
a")
```

```
[Out] a*integrate(x^4*e^(p*log(a*x + 1) + p*log(-a*x + 1))/(sqrt(a*x + 1)*sqrt(-a
*x + 1)), x) + (a^4*(2*p + 1)*x^4 - a^2*(2*p - 1)*x^2 - 2)*(-a^2*x^2 + 1)^p
/(sqrt(-a^2*x^2 + 1)*(4*p^2 + 8*p + 3)*a^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\left(1-a^2x^2\right)^p(ax+1)}{\sqrt{1-a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - a^2*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*(1 - a^2*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 19.10, size = 258, normalized size = 3.04

$$-\frac{aa^{2p}x^5x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{5}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{5}{2}\\p+1,p+\frac{7}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^5x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{5}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-\\\frac{1}{2},-p\end{matrix}\right.}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a**2*x**2+1)**p,x)
```

```
[Out] -a*a**(2*p)*x**5*x**(2*p)*exp(I*pi*p)*gamma(-p - 5/2)*gamma(p + 1/2)*hyper(
(1/2, 1, p + 5/2), (p + 1, p + 7/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(p
i)*gamma(-p - 3/2)*gamma(p + 1)) - a*a**(2*p)*x**5*x**(2*p)*exp(I*pi*p)*gam
ma(-p - 5/2)*gamma(p + 1/2)*hyper((1, -p, -p - 5/2), (1/2, -p - 3/2), 1/(a*
*2*x**2))/(2*sqrt(pi)*gamma(-p - 3/2)*gamma(p + 1)) - meijerg(((-p - 1, 1),
 (-1,)), ((-p - 3/2, -p - 1), (0,)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p
+ 1/2)/(2*pi*a**4) - meijerg(((-1, -p - 2, 1), ()), ((-p - 2,), (-p - 3/2,
0)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a**4*gamma(-p)*gamma(p
 + 1))
```

3.1006 $\int e^{\tanh^{-1}(ax)} x^2 \left(1-a^2x^2\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3 \, {}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^3(2p+3)}$$

```
[Out] -(-a^2*x^2+1)^(1/2+p)/a^3/(1+2*p)+(-a^2*x^2+1)^(3/2+p)/a^3/(3+2*p)+1/3*x^3*
hypergeom([3/2, 1/2-p],[5/2],a^2*x^2)
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 364, 266, 43}

$$\frac{1}{3}x^3 \, {}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^3(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(1 - a^2*x^2)^p,x]
```

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^3*(1 + 2*p))) + (1 - a^2*x^2)^(3/2 + p)/(a^3*(
3 + 2*p)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx &= \int x^2(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= a\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{1}{2}a\,\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{1}{2}+p}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{1}{2}a\,\text{Subst}\left(\int\left(\frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{a^2}-\frac{\left(1-a^2x\right)^{\frac{1}{2}+p}}{a^2}\right)dx\right.\\
&= -\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^3(1+2p)}+\frac{\left(1-a^2x^2\right)^{\frac{3}{2}+p}}{a^3(3+2p)}+\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 76, normalized size = 0.90

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\left(a^2(2p+1)x^2+2\right)}{a^3\left(4p^2+8p+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(1 - a^2*x^2)^p,x]
```

```
[Out] -(((1 - a^2*x^2)^(1/2 + p)*(2 + a^2*(1 + 2*p)*x^2))/(a^3*(3 + 8*p + 4*p^2))
) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^2}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*x^2+1)^p,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^2/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*x^2+1)^p,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p*x^2/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.34, size = 47, normalized size = 0.56

$$\frac{a\,x^4 \operatorname{hypergeom}\left(\left[2,\frac{1}{2}-p\right],[3],a^2x^2\right)}{4}+\frac{x^3 \operatorname{hypergeom}\left(\left[\frac{3}{2},\frac{1}{2}-p\right],\left[\frac{5}{2}\right],a^2x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*x^2+1)^p,x)
```

```
[Out] 1/4*a*x^4*hypergeom([2,1/2-p],[3],a^2*x^2)+1/3*x^3*hypergeom([3/2,1/2-p],[5
/2],a^2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax+1)\left(-a^2x^2+1\right)^{p-\frac{1}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*x^2+1)^p,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(1-a^2 x^2\right)^p (a x+1)}{\sqrt{1-a^2 x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^2*(1 - a^2*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 13.51, size = 255, normalized size = 3.04

$$-\frac{a^{2p}x^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{3}{2}\\ p+1,p+\frac{5}{2}\end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}-\frac{a^{2p}x^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p-\\ \frac{1}{2},-p-\end{matrix}\right.}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a**2*x**2+1)**p,x)
[Out] -a**(2*p)*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyper((1
/2, 1, p + 3/2), (p + 1, p + 5/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)
*gamma(-p - 1/2)*gamma(p + 1)) - a**(2*p)*x**3*x**(2*p)*exp(I*pi*p)*gamma(-
p - 3/2)*gamma(p + 1/2)*hyper((1, -p, -p - 3/2), (1/2, -p - 1/2), 1/(a**2*x
**2))/(2*sqrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - meijerg(((-p - 1, 1), (-1
,)), ((-p - 3/2, -p - 1), (0,)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/
2)/(2*pi*a**3) - meijerg(((-1, -p - 2, 1), ()), ((-p - 2,), (-p - 3/2, 0)),
 exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a**3*gamma(-p)*gamma(p + 1
))
```

3.1007 $\int e^{\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=58

$$\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^2(2p+1)}$$

```
[Out] -(-a^2*x^2+1)^(1/2+p)/a^2/(1+2*p)+1/3*a*x^3*hypergeom([3/2, 1/2-p],[5/2],a^
2*x^2)
```

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6148, 764, 261, 364}

$$\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(1 - a^2*x^2)^p,x]
```

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^2*(1 + 2*p))) + (a*x^3*Hypergeometric2F1[3/2,
1/2 - p, 5/2, a^2*x^2])/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx &= \int x(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= a\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\int x\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= -\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^2(1+2p)}+\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.03

$$\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{2a^2\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x*(1 - a^2*x^2)^p,x]
```

```
[Out] -1/2*(1 - a^2*x^2)^(1/2 + p)/(a^2*(1/2 + p)) + (a*x^3*Hypergeometric2F1[3/2
, 1/2 - p, 5/2, a^2*x^2])/3
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*x^2+1)^p,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*x^2+1)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p*x/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.34, size = 47, normalized size = 0.81

$$\frac{a\,x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{2},\frac{1}{2}-p\right],\left[\frac{5}{2}\right],a^2x^2\right)}{3}+\frac{x^2\,\mathrm{hypergeom}\left(\left[1,\frac{1}{2}-p\right],[2],a^2x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*x^2+1)^p,x)
[Out] 1/3*a*x^3*hypergeom([3/2,1/2-p],[5/2],a^2*x^2)+1/2*x^2*hypergeom([1,1/2-p],
[2],a^2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^2 e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx-\frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}}{a^2\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*x^2+1)^p,x, algorithm="maxima"
)
[Out] a*integrate(x^2*e^(p*log(a*x + 1) + p*log(-a*x + 1))/(sqrt(a*x + 1)*sqrt(-a
*x + 1)), x) - (-a^2*x^2 + 1)^(p + 1/2)/(a^2*(2*p + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(1-a^2\,x^2\right)^p\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - a^2*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*(1 - a^2*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 12.10, size = 301, normalized size = 5.19

$$-\frac{aa^{2p}x^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{3}{2}\\p+1,p+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-\\\frac{1}{2},-p\end{matrix}\right.}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a**2*x**2+1)**p,x)
```

```
[Out] -a*a**(2*p)*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyper(
(1/2, 1, p + 3/2), (p + 1, p + 5/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(p
i)*gamma(-p - 1/2)*gamma(p + 1)) - a*a**(2*p)*x**3*x**(2*p)*exp(I*pi*p)*gam
ma(-p - 3/2)*gamma(p + 1/2)*hyper((1, -p, -p - 3/2), (1/2, -p - 1/2), 1/(a*
*2*x**2))/(2*sqrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - a**(2*p)*x**2*x**(2*p
)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper((1/2, 1), (p + 2,), a**2*x
**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(-p)*gamma(p + 1)) - a**(2*p)*x**2*
x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper((1, -p - 1), (1/2,)
, 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p)*gamma(p + 1))
```

3.1008 $\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=59

$$-\frac{2^{p+\frac{3}{2}}(1-ax)^{p+\frac{1}{2}}\,{}_2F_1\left(-p-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{a(2p+1)}$$

```
[Out] -2^(3/2+p)*(-a*x+1)^(1/2+p)*hypergeom([1/2+p, -1/2-p],[3/2+p],-1/2*a*x+1/2)
/a/(1+2*p)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6140, 69}

$$-\frac{2^{p+\frac{3}{2}}(1-ax)^{p+\frac{1}{2}}\,{}_2F_1\left(-p-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(1 - a^2*x^2)^p,x]
```

```
[Out] -((2^(3/2 + p)*(1 - a*x)^(1/2 + p)*Hypergeometric2F1[-1/2 - p, 1/2 + p, 3/2
 + p, (1 - a*x)/2])/(a*(1 + 2*p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx=\int (1-ax)^{-\frac{1}{2}+p}(1+ax)^{\frac{1}{2}+p}\,dx$$
$$=-\frac{2^{\frac{3}{2}+p}(1-ax)^{\frac{1}{2}+p}\,{}_2F_1\left(-\frac{1}{2}-p,\frac{1}{2}+p;\frac{3}{2}+p;\frac{1}{2}(1-ax)\right)}{a(1+2p)}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.00

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{1}{2}}\,{}_2F_1\left(-p-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{a\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(1 - a^2*x^2)^p,x]
[Out] -((2^(1/2 + p)*(1 - a*x)^(1/2 + p)*Hypergeometric2F1[-1/2 - p, 1/2 + p, 3/2
 + p, (1 - a*x)/2])/(a*(1/2 + p)))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p,x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.34, size = 44, normalized size = 0.75

$$\frac{a\,x^{2}\operatorname{hypergeom}\left(\left[1,\frac{1}{2}-p\right],[2],a^{2}x^{2}\right)}{2}+x\operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}-p\right],\left[\frac{3}{2}\right],a^{2}x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p,x)
```

```
[Out] 1/2*a*x^2*hypergeom([1,1/2-p],[2],a^2*x^2)+x*hypergeom([1/2,1/2-p],[3/2],a^
2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax+1)\left(-a^{2}x^{2}+1\right)^{p-\frac{1}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(1-a^{2}\,x^{2}\right)^{p}\,\left(a\,x+1\right)}{\sqrt{1-a^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((1 - a^2*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 11.47, size = 292, normalized size = 4.95

$$-\frac{aa^{2p}x^{2}x^{2p}e^{i\pi p}\Gamma\left(-p-1\right)\Gamma\left(p+\frac{1}{2}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},1\\p+2\end{matrix}\middle|a^{2}x^{2}e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^{2}x^{2p}e^{i\pi p}\Gamma\left(-p-1\right)\Gamma\left(p+\frac{1}{2}\right){}_{2}F_{1}\left(\begin{matrix}1,-p-1\\\frac{1}{2}\end{matrix}\middle|\frac{1}{a^{2}x^{2}}\right)}{2\sqrt{\pi}\,\Gamma\left(-p\right)\Gamma\left(p+1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*x**2+1)**p,x)
```

```
[Out] -a*a**(2*p)*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper((1
/2, 1), (p + 2,), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(-p)*gamma(
p + 1)) - a*a**(2*p)*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)
*hyper((1, -p - 1), (1/2,), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p)*gamma(p +
1)) - a**(2*p)*x*x**(2*p)*exp(I*pi*p)*gamma(-p - 1/2)*gamma(p + 1/2)*hyper(
(1/2, 1, p + 1/2), (p + 1, p + 3/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(p
i)*gamma(1/2 - p)*gamma(p + 1)) - a**(2*p)*x*x**(2*p)*exp(I*pi*p)*gamma(-p
- 1/2)*gamma(p + 1/2)*hyper((1, -p, -p - 1/2), (1/2, 1/2 - p), 1/(a**2*x**2
))/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1))
```

3.1009 $\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx$

Optimal. Leaf size=72

$$ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

```
[Out] a*x*hypergeom([1/2, 1/2-p],[3/2],a^2*x^2)-(-a^2*x^2+1)^(1/2+p)*hypergeom([1
, 1/2+p],[3/2+p],-a^2*x^2+1)/(1+2*p)
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 266, 65, 245}

$$ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x,x]
```

```
[Out] a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2] - ((1 - a^2*x^2)^(1/2 + p
)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx &= \int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx \\
&= a\int \left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx + \int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx \\
&= ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x}\,dx, x, x^2\right) \\
&= ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right) - \frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}\,{}_2F_1\left(1,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.03

$$ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right) - \frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x,x]
```

```
[Out] a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2] - ((1 - a^2*x^2)^(1/2 + p
)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(2*(1/2 + p))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^2-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^2 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [A] time = 0.38, size = 90, normalized size = 1.25

$$ax\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}-p\right],\left[\frac{3}{2}\right],a^2x^2\right)+\frac{\Gamma\left(\frac{3}{2}-p\right)a^2x^2\,\text{hypergeom}\left(\left[1,1,\frac{3}{2}-p\right],[2,2],a^2x^2\right)+\left(\Psi\left(\frac{1}{2}-p\right)+\gamma\right.}{2\Gamma\left(\frac{1}{2}-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x,x)
```

```
[Out] a*x*hypergeom([1/2,1/2-p],[3/2],a^2*x^2)+1/2*(GAMMA(3/2-p)*a^2*x^2*hypergeo
m([1,1,3/2-p],[2,2],a^2*x^2)+(Psi(1/2-p)+gamma+2*ln(x)+ln(-a^2))*GAMMA(1/2-
p))/GAMMA(1/2-p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^{p-\frac{1}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^p\,\left(a\,x+1\right)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^p*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((1 - a^2*x^2)^p*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [C] time = 30.72, size = 286, normalized size = 3.97

$$-\frac{aa^{2p}xx^{2p}e^{i\pi p}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{1}{2}\\p+1,p+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}xx^{2p}e^{i\pi p}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p-\\\frac{1}{2},\frac{1}{2}-p\end{matrix}\right.}{2\sqrt{\pi}\,\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*x**2+1)**p/x,x)
```

```
[Out] -a*a**(2*p)*x*x**(2*p)*exp(I*pi*p)*gamma(-p - 1/2)*gamma(p + 1/2)*hyper((1/
2, 1, p + 1/2), (p + 1, p + 3/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*
gamma(1/2 - p)*gamma(p + 1)) - a*a**(2*p)*x*x**(2*p)*exp(I*pi*p)*gamma(-p -
 1/2)*gamma(p + 1/2)*hyper((1, -p, -p - 1/2), (1/2, 1/2 - p), 1/(a**2*x**2)
)/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1)) - a**(2*p)*x**(2*p)*exp(I*pi*p)*
gamma(-p)*gamma(p + 1/2)*hyper((1/2, 1, p), (p + 1, p + 1), a**2*x**2*exp_p
olar(2*I*pi))/(2*sqrt(pi)*gamma(1 - p)*gamma(p + 1)) - a**(2*p)*x**(2*p)*ex
p(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1, -p, -p), (1/2, 1 - p), 1/(a**2
*x**2))/(2*sqrt(pi)*gamma(1 - p)*gamma(p + 1))
```

3.1010 $\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

```
[Out] -hypergeom([-1/2, 1/2-p],[1/2],a^2*x^2)/x-a*(-a^2*x^2+1)^(1/2+p)*hypergeom(
[1, 1/2+p],[3/2+p],-a^2*x^2+1)/(1+2*p)
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 364, 266, 65}

$$-\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x^2,x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) - (a*(1 - a^2*x^2)^(1/2
 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx &= \int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx \\
&= a\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx+\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx \\
&= -\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x}\,dx,x,x^2\right) \\
&= -\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a\left(1-a^2x^2\right)^{\frac{1}{2}+p}\,{}_2F_1\left(1,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.03

$$
-\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x^2,x]
```

[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) - (a*(1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(2*(1/2 + p))

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^3-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^2,x, algorithm="fricas")

[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^3 - x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^2,x, algorithm="giac")

[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*x^2), x)

maple [A] time = 0.36, size = 93, normalized size = 1.24

$$\frac{a\left(\Gamma\left(\frac{3}{2}-p\right)a^2x^2\,\text{hypergeom}\left(\left[1,1,\frac{3}{2}-p\right],[2,2],a^2x^2\right)+\left(\Psi\left(\frac{1}{2}-p\right)+\gamma+2\ln(x)+\ln\left(-a^2\right)\right)\Gamma\left(\frac{1}{2}-p\right)\right)}{2\Gamma\left(\frac{1}{2}-p\right)}-\text{hyp}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^2,x)

[Out] 1/2*a*(GAMMA(3/2-p)*a^2*x^2*hypergeom([1,1,3/2-p],[2,2],a^2*x^2)+(Psi(1/2-p)+gamma+2*ln(x)+ln(-a^2))*GAMMA(1/2-p))/GAMMA(1/2-p)-hypergeom([-1/2,1/2-p],[1/2],a^2*x^2)/x

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^{p-\frac{1}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^p\,\left(a\,x+1\right)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^p*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((1 - a^2*x^2)^p*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [C] time = 12.39, size = 280, normalized size = 3.73

$$-\frac{aa^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right)\Gamma\left(p+\frac{1}{2}\right){}_{3}F_{2}\left(\left.\begin{matrix}\frac{1}{2},1,p\\p+1,p+1\end{matrix}\right|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(1-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^{2p}e^{i\pi p}\Gamma\left(-p\right)\Gamma\left(p+\frac{1}{2}\right){}_{3}F_{2}\left(\left.\begin{matrix}1,-p,-p\\\frac{1}{2},1-p\end{matrix}\right|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\,\Gamma\left(1-p\right)\Gamma\left(p+1\right)}-\frac{a^{2p}x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*x**2+1)**p/x**2,x)
```

```
[Out] -a*a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1/2, 1, p)
, (p + 1, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(1 - p)*gam
ma(p + 1)) - a*a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper
((1, -p, -p), (1/2, 1 - p), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(1 - p)*gamma(p
 + 1)) - a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p)*gamma(p + 1/2)*hyper(
(1/2, 1, p - 1/2), (p + 1/2, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(p
i)*x*gamma(3/2 - p)*gamma(p + 1)) - a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1/2
 - p)*gamma(p + 1/2)*hyper((1, -p, 1/2 - p), (1/2, 3/2 - p), 1/(a**2*x**2))
/(2*sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1))
```

3.1011 $\int \frac{e^{\tanh^{-1}(ax)}(1-a^2x^2)^p}{x^3} dx$

Optimal. Leaf size=78

$$-\frac{a^2\left(1-a^2x^2\right)^{p+\frac{1}{2}}\ {}_2F_1\left(2,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{a\ {}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

```
[Out] -a*hypergeom([-1/2, 1/2-p],[1/2],a^2*x^2)/x-a^2*(-a^2*x^2+1)^(1/2+p)*hyperg
eom([2, 1/2+p],[3/2+p],-a^2*x^2+1)/(1+2*p)
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6148, 764, 266, 65, 364}

$$-\frac{a^2\left(1-a^2x^2\right)^{p+\frac{1}{2}}\ {}_2F_1\left(2,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{a\ {}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x^3,x]
```

```
[Out] -((a*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/x) - (a^2*(1 - a^2*x^2
)^(1/2 + p)*Hypergeometric2F1[2, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^3}\,dx &= \int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^3}\,dx \\
&= a\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx+\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^3}\,dx \\
&= -\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x^2}\,dx,x,x^2\right) \\
&= -\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a^2\left(1-a^2x^2\right)^{\frac{1}{2}+p}\,{}_2F_1\left(2,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 80, normalized size = 1.03

$$-\frac{a^2\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,{}_2F_1\left(2,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(1 - a^2*x^2)^p)/x^3,x]
```

```
[Out] -((a*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/x) - (a^2*(1 - a^2*x^2
)^(1/2 + p)*Hypergeometric2F1[2, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(2*(1/2 +
p))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^4-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^3,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^4 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.40, size = 112, normalized size = 1.44

$$-\frac{a\,\text{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{2}-p\right],\left[\frac{1}{2}\right],a^2x^2\right)}{x}-\frac{a^2\left(-\frac{\Gamma\left(\frac{5}{2}-p\right)a^2x^2\,\text{hypergeom}\left(\left[1,1,\frac{5}{2}-p\right],[2,3],a^2x^2\right)}{2}-\left(\Psi\left(\frac{3}{2}-p\right)+\gamma-1+2\ln(x\right.\right.}{2\Gamma\left(\frac{1}{2}-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^3,x)
```

```
[Out] -a*hypergeom([-1/2,1/2-p],[1/2],a^2*x^2)/x-1/2*a^2*(-1/2*GAMMA(5/2-p)*a^2*x
^2*hypergeom([1,1,5/2-p],[2,3],a^2*x^2)-(Psi(3/2-p)+gamma-1+2*ln(x)+ln(-a^2
))*GAMMA(3/2-p)+GAMMA(1/2-p)/x^2/a^2)/GAMMA(1/2-p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2x^2+1\right)^{p-\frac{1}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*x^2+1)^p/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)*(-a^2*x^2 + 1)^(p - 1/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^p\,\left(a\,x+1\right)}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^p*(a*x + 1))/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((1 - a^2*x^2)^p*(a*x + 1))/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [C] time = 56.40, size = 287, normalized size = 3.68

$$-\frac{aa^{2p}x^{2p}e^{i\pi p}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+\frac{1}{2}\right){}_{3}F_{2}\left(\begin{matrix}\frac{1}{2},1,p-\frac{1}{2}\\ p+\frac{1}{2},p+1\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,x\Gamma\left(\frac{3}{2}-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}x^{2p}e^{i\pi p}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+\frac{1}{2}\right){}_{3}F_{2}\left(\begin{matrix}1,-p,\frac{1}{2}-p\\ \frac{1}{2},\frac{3}{2}-p\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\,x\Gamma\left(\frac{3}{2}-p\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*x**2+1)**p/x**3,x)
```

```
[Out] -a*a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p)*gamma(p + 1/2)*hyper((1/2,
1, p - 1/2), (p + 1/2, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*x*g
amma(3/2 - p)*gamma(p + 1)) - a*a**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p
)*gamma(p + 1/2)*hyper((1, -p, 1/2 - p), (1/2, 3/2 - p), 1/(a**2*x**2))/(2*
sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1)) - a**(2*p)*x**(2*p)*exp(I*pi*p)*gam
ma(1 - p)*gamma(p + 1/2)*hyper((1/2, 1, p - 1), (p, p + 1), a**2*x**2*exp_p
olar(2*I*pi))/(2*sqrt(pi)*x**2*gamma(2 - p)*gamma(p + 1)) - a**(2*p)*x**(2*
p)*exp(I*pi*p)*gamma(1 - p)*gamma(p + 1/2)*hyper((1, -p, 1 - p), (1/2, 2 -
p), 1/(a**2*x**2))/(2*sqrt(pi)*x**2*gamma(2 - p)*gamma(p + 1))
```

3.1012 $\int e^{\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=134

$$\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(2p+1)}$$

```
[Out] (-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/a^4/(3+2*p)+1/5*a*x^5*(-a^2*c*x^2+c)^p*
hypergeom([5/2, 1/2-p],[7/2],a^2*x^2)/((-a^2*x^2+1)^p)-(-a^2*c*x^2+c)^p*(-a
^2*x^2+1)^(1/2)/a^4/(1+2*p)
```

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 266, 43, 364}

$$\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^3*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^4*(1 + 2*p))) + ((1 - a^2*x^2)^(
3/2)*(c - a^2*c*x^2)^p)/(a^4*(3 + 2*p)) + (a*x^5*(c - a^2*c*x^2)^p*Hypergeo
metric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/(5*(1 - a^2*x^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx$$

$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$

$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx\right.\right.$$

$$=\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-\right.\right.$$

$$=\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-\right.\right.$$

$$=-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(1+2p)}+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(3+2p)}+\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-\right.$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.78

$$\frac{\left(c-a^2cx^2\right)^p\left(a^5x^5\left(1-a^2x^2\right)^{-p}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)-\frac{5\sqrt{1-a^2x^2}\left(a^2(2p+1)x^2+2\right)}{4p^2+8p+3}\right)}{5a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^3*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*((-5*Sqrt[1 - a^2*x^2]*(2 + a^2*(1 + 2*p)*x^2))/(3 + 8*p
 + 4*p^2) + (a^5*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/(1 - a^
2*x^2)^p))/(5*a^4)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^3}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^3/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(-a^2cx^2+c\right)^p x^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^3/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\,x^3\left(-a^2c\,x^2+c\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$ac^{p}\int \frac{x^{4}e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx+\frac{\left(a^{4}c^{p}\left(2\,p+1\right)x^{4}-a^{2}c^{p}\left(2\,p-1\right)x^{2}-2\,c^{p}\right)\left(-a^{2}x^{2}+1\right)^{p}}{\sqrt{-a^{2}x^{2}+1}\left(4\,p^{2}+8\,p+3\right)a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="max
ima")
```

```
[Out] a*c^p*integrate(x^4*e^(p*log(a*x + 1) + p*log(-a*x + 1))/(sqrt(a*x + 1)*sqr
t(-a*x + 1)), x) + (a^4*c^p*(2*p + 1)*x^4 - a^2*c^p*(2*p - 1)*x^2 - 2*c^p)*
(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*(4*p^2 + 8*p + 3)*a^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3}\left(c-a^{2}\,c\,x^{2}\right)^{p}\,\left(a\,x+1\right)}{\sqrt{1-a^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*(c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 54.21, size = 272, normalized size = 2.03

$$-\frac{aa^{2p}c^{p}x^{5}x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{5}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_{3}F_{2}\left(\begin{matrix}\frac{1}{2},1,p+\frac{5}{2}\\ p+1,p+\frac{7}{2}\end{matrix}\middle|a^{2}x^{2}e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^{p}x^{5}x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{5}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_{3}F_{2}\left(\begin{matrix}1,-\\ \frac{1}{2},\end{matrix}\right.}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**3*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a*a**(2*p)*c**p*x**5*x**(2*p)*exp(I*pi*p)*gamma(-p - 5/2)*gamma(p + 1/2)*h
yper((1/2, 1, p + 5/2), (p + 1, p + 7/2), a**2*x**2*exp_polar(2*I*pi))/(2*s
qrt(pi)*gamma(-p - 3/2)*gamma(p + 1)) - a*a**(2*p)*c**p*x**5*x**(2*p)*exp(I
*pi*p)*gamma(-p - 5/2)*gamma(p + 1/2)*hyper((1, -p, -p - 5/2), (1/2, -p - 3
/2), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p - 3/2)*gamma(p + 1)) - c**p*meijer
g(((-p - 1, 1), (-1,)), ((-p - 3/2, -p - 1), (0,)), exp_polar(-I*pi)/(a**2*
```

```
x**2))*gamma(p + 1/2)/(2*pi*a**4) - c**p*meijerg(((-1, -p - 2, 1), ()), ((-
p - 2,), (-p - 3/2, 0)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a*
*4*gamma(-p)*gamma(p + 1))
```

3.1013 $\int e^{\tanh^{-1}(ax)} x^2 \left(c-a^2cx^2\right)^p \, dx$

Optimal. Leaf size=133

$$\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(2p+1)}$$

```
[Out] (-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/a^3/(3+2*p)+1/3*x^3*(-a^2*c*x^2+c)^p*hy
pergeom([3/2, 1/2-p],[5/2],a^2*x^2)/((-a^2*x^2+1)^p)-(-a^2*c*x^2+c)^p*(-a^2
*x^2+1)^(1/2)/a^3/(1+2*p)
```

Rubi [A] time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 364, 266, 43}

$$\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x^2*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^3*(1 + 2*p))) + ((1 - a^2*x^2)^(
3/2)*(c - a^2*c*x^2)^p)/(a^3*(3 + 2*p)) + (x^3*(c - a^2*c*x^2)^p*Hypergeome
tric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right.\right.\\
&= \frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2\right.\right.\\
&= \frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2\right.\right.\\
&= -\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(1+2p)}+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(3+2p)}+\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a\right.
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 102, normalized size = 0.77

$$\frac{1}{3}\left(c-a^2cx^2\right)^p\left(x^3\left(1-a^2x^2\right)^{-p}{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{3\sqrt{1-a^2x^2}\left(a^2(2p+1)x^2+2\right)}{a^3\left(4p^2+8p+3\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x^2*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*((-3*Sqrt[1 - a^2*x^2]*(2 + a^2*(1 + 2*p)*x^2))/(a^3*(3
+ 8*p + 4*p^2)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(1 -
a^2*x^2)^p))/3
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^px^2}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^2/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(-a^2cx^2+c\right)^px^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^2/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\,x^2\left(-a^2c\,x^2+c\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x^2/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(c-a^2\,c\,x^2\right)}^p\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 16.70, size = 269, normalized size = 2.02

$$-\frac{a^{2p}c^p x^3 x^{2p} e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{3}{2}\\ p+1,p+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}-\frac{a^{2p}c^p x^3 x^{2p} e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-\\ \frac{1}{2},-p\end{matrix}\right.}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a**(2*p)*c**p*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyp
er((1/2, 1, p + 3/2), (p + 1, p + 5/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqr
t(pi)*gamma(-p - 1/2)*gamma(p + 1)) - a**(2*p)*c**p*x**3*x**(2*p)*exp(I*pi*
p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyper((1, -p, -p - 3/2), (1/2, -p - 1/2),
 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - c**p*meijerg(((
-p - 1, 1), (-1,)), ((-p - 3/2, -p - 1), (0,)), exp_polar(-I*pi)/(a**2*x**2
))*gamma(p + 1/2)/(2*pi*a**3) - c**p*meijerg(((-1, -p - 2, 1), ()), ((-p -
2,), (-p - 3/2, 0)), exp_polar(-I*pi)/(a**2*x**2))*gamma(p + 1/2)/(2*a**3*g
amma(-p)*gamma(p + 1))
```

3.1014 $\int e^{\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=96

$$\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(2p+1)}$$

```
[Out] 1/3*a*x^3*(-a^2*c*x^2+c)^p*hypergeom([3/2, 1/2-p],[5/2],a^2*x^2)/((-a^2*x^2
+1)^p)-(-a^2*c*x^2+c)^p*(-a^2*x^2+1)^(1/2)/a^2/(1+2*p)
```

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6153, 6148, 764, 261, 364}

$$\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^2*(1 + 2*p))) + (a*x^3*(c - a^2*
c*x^2)^p*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx+\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right.\\ &= -\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(1+2p)}+\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.92

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{2a^2\left(p+\frac{1}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-1/2*(1 - a^2*x^2)^(1/2 + p)/(a^2*(1/2 + p)) + (a*x^3*H
ypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p x}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p*x/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x\left(-a^2c\,x^2+c\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$ac^p\int \frac{x^2e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx+\frac{\left(a^2c^px^2-c^p\right)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}\,a^2\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="maxim
a")
```

```
[Out] a*c^p*integrate(x^2*e^(p*log(a*x + 1) + p*log(-a*x + 1))/(sqrt(a*x + 1)*sqr
t(-a*x + 1)), x) + (a^2*c^p*x^2 - c^p)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)
*a^2*(2*p + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 17.59, size = 314, normalized size = 3.27

$$-\frac{aa^{2p}c^px^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p+\frac{3}{2}\\p+1,p+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^px^3x^{2p}e^{i\pi p}\Gamma\left(-p-\frac{3}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,\\\frac{1}{2},-\end{matrix}\right.}{2\sqrt{\pi}\,\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a*a**(2*p)*c**p*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*h
yper((1/2, 1, p + 3/2), (p + 1, p + 5/2), a**2*x**2*exp_polar(2*I*pi))/(2*s
qrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - a*a**(2*p)*c**p*x**3*x**(2*p)*exp(I
*pi*p)*gamma(-p - 3/2)*gamma(p + 1/2)*hyper((1, -p, -p - 3/2), (1/2, -p - 1
/2), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p - 1/2)*gamma(p + 1)) - a**(2*p)*c*
*p*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper((1/2, 1), (
p + 2,), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(-p)*gamma(p + 1)) -
 a**(2*p)*c**p*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyper
((1, -p - 1), (1/2,), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p)*gamma(p + 1))
```

3.1015 $\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=86

$$-\frac{2^{p+\frac{3}{2}}(1-ax)^{p+\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{a(2p+1)}$$

```
[Out] -2^(3/2+p)*(-a*x+1)^(1/2+p)*(-a^2*c*x^2+c)^p*hypergeom([1/2+p, -1/2-p],[3/2
+p],-1/2*a*x+1/2)/a/(1+2*p)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6143, 6140, 69}

$$-\frac{2^{p+\frac{3}{2}}(1-ax)^{p+\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTanh[a*x]*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(3/2 + p)*(1 - a*x)^(1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2
 - p, 1/2 + p, 3/2 + p, (1 - a*x)/2])/(a*(1 + 2*p)*(1 - a^2*x^2)^p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{-\frac{1}{2}+p}(1+ax)^{\frac{1}{2}+p}\,dx\\ &= -\frac{2^{\frac{3}{2}+p}(1-ax)^{\frac{1}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2}-p,\frac{1}{2}+p;\frac{3}{2}+p;\frac{1}{2}(1-ax)\right)}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 86, normalized size = 1.00

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\frac{1}{2}(1-ax)\right)}{a\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTanh[a*x]*(c - a^2*c*x^2)^p,x]
[Out] -((2^(1/2 + p)*(1 - a*x)^(1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2
 - p, 1/2 + p, 3/2 + p, (1 - a*x)/2])/(a*(1/2 + p)*(1 - a^2*x^2)^p))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas"
)
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2c\,x^2+c\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,(a\,x+1)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

sympy [C] time = 23.17, size = 306, normalized size = 3.56

$$-\frac{aa^{2p}c^px^2x^{2p}e^{i\pi p}\Gamma\left(-p-1\right)\Gamma\left(p+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1\\p+2\end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^px^2x^{2p}e^{i\pi p}\Gamma\left(-p-1\right)\Gamma\left(p+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1,-p-1\\\frac{1}{2}\end{matrix}\middle| \frac{}{a^2}\right.}{2\sqrt{\pi}\,\Gamma\left(-p\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] -a*a**(2*p)*c**p*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamma(p + 1/2)*hyp
er((1/2, 1), (p + 2,), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(-p)*g
amma(p + 1)) - a*a**(2*p)*c**p*x**2*x**(2*p)*exp(I*pi*p)*gamma(-p - 1)*gamm
a(p + 1/2)*hyper((1, -p - 1), (1/2,), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(-p)*
gamma(p + 1)) - a**(2*p)*c**p*x*x**(2*p)*exp(I*pi*p)*gamma(-p - 1/2)*gamma(
p + 1/2)*hyper((1/2, 1, p + 1/2), (p + 1, p + 3/2), a**2*x**2*exp_polar(2*I
*pi))/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1)) - a**(2*p)*c**p*x*x**(2*p)*e
xp(I*pi*p)*gamma(-p - 1/2)*gamma(p + 1/2)*hyper((1, -p, -p - 1/2), (1/2, 1/
2 - p), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1))
```

3.1016 $\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx$

Optimal. Leaf size=110

$$ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\,\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

```
[Out] a*x*(-a^2*c*x^2+c)^p*hypergeom([1/2, 1/2-p],[3/2],a^2*x^2)/((-a^2*x^2+1)^p)
-(-a^2*c*x^2+c)^p*hypergeom([1, 1/2+p],[3/2+p],-a^2*x^2+1)*(-a^2*x^2+1)^(1/
2)/(1+2*p)
```

Rubi [A] time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 266, 65, 245}

$$ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\,\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x,x]
```

```
[Out] (a*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2])/(1 -
a^2*x^2)^p - (Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1, 1/2
+ p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx + \left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int$$

$$= ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^{p}\right.$$

$$= ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F_1}{1+}$$

Mathematica [A] time = 0.03, size = 102, normalized size = 0.93

$$(1-a^2x^2)^{-p}(c-a^2cx^2)^p\left(ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{(1-a^2x^2)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2] - ((1
 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/
(2*(1/2 + p))))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(-a^2cx^2+c)^p}{ax^2-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^2 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)(-a^2cx^2+c)^p}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(-a^2c\,x^2+c\right)^p}{\sqrt{-a^2x^2+1}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1))/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [C] time = 17.33, size = 299, normalized size = 2.72

$$-\frac{aa^{2p}c^pxx^{2p}e^{i\pi p}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\left.\begin{matrix}\frac{1}{2},1,p+\frac{1}{2}\\p+1,p+\frac{3}{2}\end{matrix}\right|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^pxx^{2p}e^{i\pi p}\Gamma\left(-p-\frac{1}{2}\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-\\\frac{1}{2},\frac{1}{2}\end{matrix}\right.}{2\sqrt{\pi}\,\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**p/x,x)
```

```
[Out] -a*a**(2*p)*c**p*x*x**(2*p)*exp(I*pi*p)*gamma(-p - 1/2)*gamma(p + 1/2)*hype
r((1/2, 1, p + 1/2), (p + 1, p + 3/2), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt
(pi)*gamma(1/2 - p)*gamma(p + 1)) - a*a**(2*p)*c**p*x*x**(2*p)*exp(I*pi*p)*
gamma(-p - 1/2)*gamma(p + 1/2)*hyper((1, -p, -p - 1/2), (1/2, 1/2 - p), 1/(
a**2*x**2))/(2*sqrt(pi)*gamma(1/2 - p)*gamma(p + 1)) - a**(2*p)*c**p*x**(2*
p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1/2, 1, p), (p + 1, p + 1),
a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(1 - p)*gamma(p + 1)) - a**(2
*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1, -p, -p), (
1/2, 1 - p), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(1 - p)*gamma(p + 1))
```

3.1017 $\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=113

$$-\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

```
[Out] -(-a^2*c*x^2+c)^p*hypergeom([-1/2, 1/2-p],[1/2],a^2*x^2)/x/((-a^2*x^2+1)^p)
-a*(-a^2*c*x^2+c)^p*hypergeom([1, 1/2+p],[3/2+p],-a^2*x^2+1)*(-a^2*x^2+1)^(
1/2)/(1+2*p)
```

Rubi [A] time = 0.17, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 364, 266, 65}

$$-\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x^2,x]
```

```
[Out] -(((c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/(x*(1
- a^2*x^2)^p)) - (a*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1
, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^2}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx + \left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int$$

$$= -\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x} + \frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right.\right.$$

$$= -\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x} - \frac{a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F}{1+}$$

Mathematica [A] time = 0.04, size = 105, normalized size = 0.93

$$(1-a^2x^2)^{-p}(c-a^2cx^2)^p\left(-\frac{a(1-a^2x^2)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x^2,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) - (
a*(1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^
2])/(2*(1/2 + p))))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(-a^2cx^2+c)^p}{ax^3-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^3 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)(-a^2cx^2+c)^p}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(-a^2c\,x^2+c\right)^p}{\sqrt{-a^2x^2+1}\;x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1))/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [C] time = 12.52, size = 294, normalized size = 2.60

$$-\frac{aa^{2p}c^px^{2p}e^{i\pi p}\Gamma\left(-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}\frac{1}{2},1,p\\p+1,p+1\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,\Gamma\left(1-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^px^{2p}e^{i\pi p}\Gamma\left(-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\begin{matrix}1,-p,-p\\\frac{1}{2},1-p\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2\sqrt{\pi}\,\Gamma\left(1-p\right)\Gamma\left(p+1\right)}-\frac{a^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**p/x**2,x)
```

```
[Out] -a*a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p + 1/2)*hyper((1/2,
1, p), (p + 1, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*gamma(1 - p
)*gamma(p + 1)) - a*a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p +
1/2)*hyper((1, -p, -p), (1/2, 1 - p), 1/(a**2*x**2))/(2*sqrt(pi)*gamma(1 -
p)*gamma(p + 1)) - a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p)*gamma(
p + 1/2)*hyper((1/2, 1, p - 1/2), (p + 1/2, p + 1), a**2*x**2*exp_polar(2*I
*pi))/(2*sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1)) - a**(2*p)*c**p*x**(2*p)*e
xp(I*pi*p)*gamma(1/2 - p)*gamma(p + 1/2)*hyper((1, -p, 1/2 - p), (1/2, 3/2
- p), 1/(a**2*x**2))/(2*sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1))
```

3.1018 $\int \frac{e^{\tanh^{-1}(ax)}(c-a^2cx^2)^p}{x^3}\,dx$

Optimal. Leaf size=116

$$-\frac{a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a^2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p {}_2F_1\left(2,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

```
[Out] -a*(-a^2*c*x^2+c)^p*hypergeom([-1/2, 1/2-p],[1/2],a^2*x^2)/x/((-a^2*x^2+1)^
p)-a^2*(-a^2*c*x^2+c)^p*hypergeom([2, 1/2+p],[3/2+p],-a^2*x^2+1)*(-a^2*x^2+
1)^(1/2)/(1+2*p)
```

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6153, 6148, 764, 266, 65, 364}

$$-\frac{a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p {}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a^2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p {}_2F_1\left(2,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x^3,x]
```

```
[Out] -((a*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/(x*(
1 - a^2*x^2)^p)) - (a^2*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2
F1[2, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^3}\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^3}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^3}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^3}\,dx+\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int\\
&= -\frac{a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right.\right.\\
&= -\frac{a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{a^2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{1}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 108, normalized size = 0.93

$$(1-a^2x^2)^{-p}\left(c-a^2cx^2\right)^p\left(-\frac{a^2\left(1-a^2x^2\right)^{p+\frac{1}{2}}\ {}_2F_1\left(2,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*(c - a^2*c*x^2)^p)/x^3,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-((a*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/x)
 - (a^2*(1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[2, 1/2 + p, 3/2 + p, 1 -
a^2*x^2])/(2*(1/2 + p))))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax^4-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^4 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(-a^2c\,x^2+c\right)^p}{\sqrt{-a^2x^2+1}\;x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x)
```

```
[Out] int((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1))/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1))/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [C] time = 13.76, size = 301, normalized size = 2.59

$$-\frac{aa^{2p}c^px^{2p}e^{i\pi p}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\left.\begin{matrix}\frac{1}{2},1,p-\frac{1}{2}\\p+\frac{1}{2},p+1\end{matrix}\right|a^2x^2e^{2i\pi}\right)}{2\sqrt{\pi}\,x\Gamma\left(\frac{3}{2}-p\right)\Gamma\left(p+1\right)}-\frac{aa^{2p}c^px^{2p}e^{i\pi p}\Gamma\left(\frac{1}{2}-p\right)\Gamma\left(p+\frac{1}{2}\right){}_3F_2\left(\left.\begin{matrix}1,-p,\frac{1}{2}-p\\\frac{1}{2},\frac{3}{2}-p\end{matrix}\right|\right.}{2\sqrt{\pi}\,x\Gamma\left(\frac{3}{2}-p\right)\Gamma\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*(-a**2*c*x**2+c)**p/x**3,x)
```

```
[Out] -a*a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(1/2 - p)*gamma(p + 1/2)*hyper((
1/2, 1, p - 1/2), (p + 1/2, p + 1), a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi
)*x*gamma(3/2 - p)*gamma(p + 1)) - a*a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gam
ma(1/2 - p)*gamma(p + 1/2)*hyper((1, -p, 1/2 - p), (1/2, 3/2 - p), 1/(a**2*
x**2))/(2*sqrt(pi)*x*gamma(3/2 - p)*gamma(p + 1)) - a**(2*p)*c**p*x**(2*p)*
exp(I*pi*p)*gamma(1 - p)*gamma(p + 1/2)*hyper((1/2, 1, p - 1), (p, p + 1),
a**2*x**2*exp_polar(2*I*pi))/(2*sqrt(pi)*x**2*gamma(2 - p)*gamma(p + 1)) -
```

```
a**(2*p)*c**p*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*gamma(p + 1/2)*hyper((1, -p
, 1 - p), (1/2, 2 - p), 1/(a**2*x**2))/(2*sqrt(pi)*x**2*gamma(2 - p)*gamma(
p + 1))
```

3.1019 $\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=29

$$\frac{1}{7}a^2cx^7+\frac{1}{3}acx^6+\frac{cx^5}{5}$$

```
[Out] 1/5*c*x^5+1/3*a*c*x^6+1/7*a^2*c*x^7
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{7}a^2cx^7+\frac{1}{3}acx^6+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^5)/5 + (a*c*x^6)/3 + (a^2*c*x^7)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)\,dx&=c\int x^4(1+ax)^2\,dx\\&=c\int\left(x^4+2ax^5+a^2x^6\right)\,dx\\&=\frac{cx^5}{5}+\frac{1}{3}acx^6+\frac{1}{7}a^2cx^7\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.76

$$\frac{1}{105}cx^5\left(15a^2x^2+35ax+21\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2),x]
[Out] (c*x^5*(21 + 35*a*x + 15*a^2*x^2))/105
```

fricas [A] time = 0.53, size = 23, normalized size = 0.79

$$\frac{1}{7}a^2cx^7+\frac{1}{3}acx^6+\frac{1}{5}cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/7*a^2*c*x^7 + 1/3*a*c*x^6 + 1/5*c*x^5
```

giac [A] time = 0.16, size = 23, normalized size = 0.79

$$\frac{1}{7}a^2cx^7+\frac{1}{3}acx^6+\frac{1}{5}cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/7*a^2*c*x^7 + 1/3*a*c*x^6 + 1/5*c*x^5
```

maple [A] time = 0.02, size = 23, normalized size = 0.79

$$c\left(\frac{1}{7}a^2x^7+\frac{1}{3}x^6a+\frac{1}{5}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c),x)
[Out] c*(1/7*a^2*x^7+1/3*x^6*a+1/5*x^5)
```

maxima [A] time = 0.37, size = 23, normalized size = 0.79

$$\frac{1}{7}a^2cx^7+\frac{1}{3}acx^6+\frac{1}{5}cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/7*a^2*c*x^7 + 1/3*a*c*x^6 + 1/5*c*x^5
```

mupad [B] time = 0.89, size = 20, normalized size = 0.69

$$\frac{c\,x^5\,\left(15\,a^2\,x^2+35\,a\,x+21\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(c - a^2*c*x^2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c*x^5*(35*a*x + 15*a^2*x^2 + 21))/105
```

sympy [A] time = 0.07, size = 24, normalized size = 0.83

$$\frac{a^2cx^7}{7}+\frac{acx^6}{3}+\frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**7/7 + a*c*x**6/3 + c*x**5/5
```

3.1020 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=29

$$\frac{1}{6}a^2cx^6+\frac{2}{5}acx^5+\frac{cx^4}{4}$$

```
[Out] 1/4*c*x^4+2/5*a*c*x^5+1/6*a^2*c*x^6
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{6}a^2cx^6+\frac{2}{5}acx^5+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^4)/4 + (2*a*c*x^5)/5 + (a^2*c*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)\,dx&=c\int x^3(1+ax)^2\,dx\\&=c\int\left(x^3+2ax^4+a^2x^5\right)\,dx\\&=\frac{cx^4}{4}+\frac{2}{5}acx^5+\frac{1}{6}a^2cx^6\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.76

$$\frac{1}{60}cx^4\left(10a^2x^2+24ax+15\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2),x]
[Out] (c*x^4*(15 + 24*a*x + 10*a^2*x^2))/60
```

fricas [A] time = 0.63, size = 23, normalized size = 0.79

$$\frac{1}{6}\,a^2cx^6+\frac{2}{5}\,acx^5+\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/6*a^2*c*x^6 + 2/5*a*c*x^5 + 1/4*c*x^4
```

giac [A] time = 0.19, size = 23, normalized size = 0.79

$$\frac{1}{6}\,a^2cx^6+\frac{2}{5}\,acx^5+\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/6*a^2*c*x^6 + 2/5*a*c*x^5 + 1/4*c*x^4
```

maple [A] time = 0.02, size = 23, normalized size = 0.79

$$c\left(\frac{1}{6}a^2x^6+\frac{2}{5}a\,x^5+\frac{1}{4}x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c),x)
[Out] c*(1/6*a^2*x^6+2/5*a*x^5+1/4*x^4)
```

maxima [A] time = 0.33, size = 23, normalized size = 0.79

$$\frac{1}{6}\,a^2cx^6+\frac{2}{5}\,acx^5+\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c*x^6 + 2/5*a*c*x^5 + 1/4*c*x^4
```

mupad [B] time = 0.04, size = 20, normalized size = 0.69

$$\frac{c\,x^4\,\left(10\,a^2\,x^2+24\,a\,x+15\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - a^2*c*x^2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c*x^4*(24*a*x + 10*a^2*x^2 + 15))/60
```

sympy [A] time = 0.07, size = 26, normalized size = 0.90

$$\frac{a^2cx^6}{6}+\frac{2acx^5}{5}+\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**6/6 + 2*a*c*x**5/5 + c*x**4/4
```

3.1021 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=29

$$\frac{1}{5}a^2cx^5+\frac{1}{2}acx^4+\frac{cx^3}{3}$$

```
[Out] 1/3*c*x^3+1/2*a*c*x^4+1/5*a^2*c*x^5
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{5}a^2cx^5+\frac{1}{2}acx^4+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^3)/3 + (a*c*x^4)/2 + (a^2*c*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)\,dx&=c\int x^2(1+ax)^2\,dx\\&=c\int\left(x^2+2ax^3+a^2x^4\right)\,dx\\&=\frac{cx^3}{3}+\frac{1}{2}acx^4+\frac{1}{5}a^2cx^5\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.76

$$\frac{1}{30}cx^3\left(6a^2x^2+15ax+10\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^3*(10 + 15*a*x + 6*a^2*x^2))/30
```

fricas [A] time = 0.52, size = 23, normalized size = 0.79

$$\frac{1}{5}a^2cx^5+\frac{1}{2}acx^4+\frac{1}{3}cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/5*a^2*c*x^5 + 1/2*a*c*x^4 + 1/3*c*x^3
```

giac [A] time = 0.21, size = 23, normalized size = 0.79

$$\frac{1}{5}a^2cx^5+\frac{1}{2}acx^4+\frac{1}{3}cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/5*a^2*c*x^5 + 1/2*a*c*x^4 + 1/3*c*x^3
```

maple [A] time = 0.03, size = 23, normalized size = 0.79

$$c\left(\frac{1}{5}x^5a^2+\frac{1}{2}x^4a+\frac{1}{3}x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c),x)
```

```
[Out] c*(1/5*x^5*a^2+1/2*x^4*a+1/3*x^3)
```

maxima [A] time = 0.33, size = 23, normalized size = 0.79

$$\frac{1}{5}a^2cx^5+\frac{1}{2}acx^4+\frac{1}{3}cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c*x^5 + 1/2*a*c*x^4 + 1/3*c*x^3
```

mupad [B] time = 0.04, size = 20, normalized size = 0.69

$$\frac{c\,x^3\,\left(6\,a^2\,x^2+15\,a\,x+10\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - a^2*c*x^2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c*x^3*(15*a*x + 6*a^2*x^2 + 10))/30
```

sympy [A] time = 0.07, size = 24, normalized size = 0.83

$$\frac{a^2cx^5}{5} + \frac{acx^4}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**5/5 + a*c*x**4/2 + c*x**3/3
```

3.1022 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=29

$$\frac{1}{4}a^2cx^4+\frac{2}{3}acx^3+\frac{cx^2}{2}$$

```
[Out] 1/2*c*x^2+2/3*a*c*x^3+1/4*a^2*c*x^4
```

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6150, 43}

$$\frac{1}{4}a^2cx^4+\frac{2}{3}acx^3+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^2)/2 + (2*a*c*x^3)/3 + (a^2*c*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)\,dx &= c\int x(1+ax)^2\,dx\\
&= c\int\left(x+2ax^2+a^2x^3\right)\,dx\\
&= \frac{cx^2}{2}+\frac{2}{3}acx^3+\frac{1}{4}a^2cx^4
\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.76

$$\frac{1}{12}cx^2\left(3a^2x^2+8ax+6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2),x]
[Out] (c*x^2*(6 + 8*a*x + 3*a^2*x^2))/12
```

fricas [A] time = 0.57, size = 23, normalized size = 0.79

$$\frac{1}{4}\,a^2cx^4+\frac{2}{3}\,acx^3+\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/4*a^2*c*x^4 + 2/3*a*c*x^3 + 1/2*c*x^2
```

giac [A] time = 0.21, size = 23, normalized size = 0.79

$$\frac{1}{4}\,a^2cx^4+\frac{2}{3}\,acx^3+\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/4*a^2*c*x^4 + 2/3*a*c*x^3 + 1/2*c*x^2
```

maple [A] time = 0.02, size = 23, normalized size = 0.79

$$c\left(\frac{1}{4}a^2x^4+\frac{2}{3}x^3a+\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c),x)
[Out] c*(1/4*a^2*x^4+2/3*x^3*a+1/2*x^2)
```

maxima [A] time = 0.33, size = 23, normalized size = 0.79

$$\frac{1}{4}\,a^2cx^4+\frac{2}{3}\,acx^3+\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c*x^4 + 2/3*a*c*x^3 + 1/2*c*x^2
```

mupad [B] time = 0.03, size = 20, normalized size = 0.69

$$\frac{c\,x^2\,\left(3\,a^2\,x^2+8\,a\,x+6\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - a^2*c*x^2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c*x^2*(8*a*x + 3*a^2*x^2 + 6))/12
```

sympy [A] time = 0.07, size = 26, normalized size = 0.90

$$\frac{a^{2} c x^{4}}{4} + \frac{2 a c x^{3}}{3} + \frac{c x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**4/4 + 2*a*c*x**3/3 + c*x**2/2
```

3.1023 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=15

$$\frac{c(ax+1)^3}{3a}$$

```
[Out] 1/3*c*(a*x+1)^3/a
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6140, 32}

$$\frac{c(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (c*(1 + a*x)^3)/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= c\int (1+ax)^2\,dx\\ &= \frac{c(1+ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.27

$$c\left(\frac{a^2x^3}{3}+ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2),x]
[Out] c*(x + a*x^2 + (a^2*x^3)/3)
```

fricas [A] time = 0.67, size = 19, normalized size = 1.27

$$\frac{1}{3}\,a^2cx^3 + acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/3*a^2*c*x^3 + a*c*x^2 + c*x
```

giac [A] time = 0.18, size = 19, normalized size = 1.27

$$\frac{1}{3}\,a^2cx^3 + acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/3*a^2*c*x^3 + a*c*x^2 + c*x
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$\frac{c\,(ax+1)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c),x)
[Out] 1/3*c*(a*x+1)^3/a
```

maxima [A] time = 0.30, size = 19, normalized size = 1.27

$$\frac{1}{3}\,a^2cx^3 + acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] 1/3*a^2*c*x^3 + a*c*x^2 + c*x
```

mupad [B] time = 0.03, size = 17, normalized size = 1.13

$$\frac{c\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c*x*(3*a*x + a^2*x^2 + 3))/3
```

sympy [A] time = 0.07, size = 19, normalized size = 1.27

$$\frac{a^2cx^3}{3}+acx^2+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c),x)
```

```
[Out] a**2*c*x**3/3 + a*c*x**2 + c*x
```

3.1024 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)}{x}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}a^2cx^2+2acx+c\log(x)$$

```
[Out] 2*a*c*x+1/2*a^2*c*x^2+c*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{2}a^2cx^2+2acx+c\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x,x]
```

```
[Out] 2*a*c*x + (a^2*c*x^2)/2 + c*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x}\,dx &= c\int \frac{(1+ax)^2}{x}\,dx\\ &= c\int \left(2a+\frac{1}{x}+a^2x\right)dx\\ &= 2acx+\frac{1}{2}a^2cx^2+c\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$c\left(\frac{a^2x^2}{2}+2ax+\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x,x]
[Out] c*(2*a*x + (a^2*x^2)/2 + Log[x])
```

fricas [A] time = 1.02, size = 19, normalized size = 0.90

$$\frac{1}{2}\,a^2cx^2+2\,acx+c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x,x, algorithm="fricas")
[Out] 1/2*a^2*c*x^2 + 2*a*c*x + c*log(x)
```

giac [A] time = 0.29, size = 20, normalized size = 0.95

$$\frac{1}{2}\,a^2cx^2+2\,acx+c\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x,x, algorithm="giac")
[Out] 1/2*a^2*c*x^2 + 2*a*c*x + c*log(abs(x))
```

maple [A] time = 0.03, size = 20, normalized size = 0.95

$$2acx+\frac{a^2c\,x^2}{2}+c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x,x)
[Out] 2*a*c*x+1/2*a^2*c*x^2+c*ln(x)
```

maxima [A] time = 0.32, size = 19, normalized size = 0.90

$$\frac{1}{2}\,a^2cx^2+2\,acx+c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c*x^2 + 2*a*c*x + c*log(x)
```

mupad [B] time = 0.04, size = 19, normalized size = 0.90

$$\frac{c\,\left(2\,\ln(x)+4\,a\,x+a^2\,x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)),x)
```

```
[Out] (c*(2*log(x) + 4*a*x + a^2*x^2))/2
```

sympy [A] time = 0.10, size = 20, normalized size = 0.95

$$\frac{a^2cx^2}{2} + 2acx + c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)/x,x)
```

```
[Out] a**2*c*x**2/2 + 2*a*c*x + c*log(x)
```

3.1025 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)}{x^2}\,dx$

Optimal. Leaf size=19

$$a^2cx+2ac\log(x)-\frac{c}{x}$$

[Out] -c/x+a^2*c*x+2*a*c*ln(x)

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$a^2cx+2ac\log(x)-\frac{c}{x}$$

Antiderivative was successfully verified.

[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^2,x]

[Out] -(c/x) + a^2*c*x + 2*a*c*Log[x]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^2}\,dx &= c\int \frac{(1+ax)^2}{x^2}\,dx\\ &= c\int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx\\ &= -\frac{c}{x}+a^2cx+2ac\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.95

$$c\left(a^2x+2a\log(x)-\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^2,x]
[Out] c*(-x^(-1) + a^2*x + 2*a*Log[x])
```

fricas [A] time = 0.61, size = 23, normalized size = 1.21

$$\frac{a^2cx^2+2\,acx\log(x)-c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^2,x, algorithm="fricas")
[Out] (a^2*c*x^2 + 2*a*c*x*log(x) - c)/x
```

giac [A] time = 0.19, size = 20, normalized size = 1.05

$$a^2cx+2\,ac\log\left(|x|\right)-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^2,x, algorithm="giac")
[Out] a^2*c*x + 2*a*c*log(abs(x)) - c/x
```

maple [A] time = 0.03, size = 20, normalized size = 1.05

$$-\frac{c}{x}+a^2cx+2ac\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^2,x)
[Out] -c/x+a^2*c*x+2*a*c*ln(x)
```

maxima [A] time = 0.34, size = 19, normalized size = 1.00

$$a^2cx+2\,ac\log(x)-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^2,x, algorithm="maxima")
[Out] a^2*c*x + 2*a*c*log(x) - c/x
```

mupad [B] time = 0.89, size = 20, normalized size = 1.05

$$\frac{c\left(a^2 x^2 + 2\,a\,x\,\ln(x) - 1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)),x)
[Out] (c*(a^2*x^2 + 2*a*x*log(x) - 1))/x
```

sympy [A] time = 0.12, size = 17, normalized size = 0.89

$$a^2cx + 2ac\log\left(x\right) - \frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)/x**2,x)
[Out] a**2*c*x + 2*a*c*log(x) - c/x
```

3.1026 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^3}\,dx$

Optimal. Leaf size=23

$$a^2c\log(x)-\frac{2ac}{x}-\frac{c}{2x^2}$$

```
[Out] -1/2*c/x^2-2*a*c/x+a^2*c*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$a^2c\log(x)-\frac{2ac}{x}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^3,x]
```

```
[Out] -c/(2*x^2) - (2*a*c)/x + a^2*c*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^3}\,dx &= c\int \frac{(1+ax)^2}{x^3}\,dx\\
&= c\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{a^2}{x}\right)dx\\
&= -\frac{c}{2x^2}-\frac{2ac}{x}+a^2c\log(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.96

$$c\left(a^2\log(x)-\frac{2a}{x}-\frac{1}{2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^3,x]
[Out] c*(-1/2*1/x^2 - (2*a)/x + a^2*Log[x])
```

fricas [A] time = 1.01, size = 25, normalized size = 1.09

$$\frac{2\,a^2cx^2\log(x)-4\,acx-c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^3,x, algorithm="fricas")
[Out] 1/2*(2*a^2*c*x^2*log(x) - 4*a*c*x - c)/x^2
```

giac [A] time = 0.20, size = 21, normalized size = 0.91

$$a^2c\log(|x|)-\frac{4\,acx+c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^3,x, algorithm="giac")
[Out] a^2*c*log(abs(x)) - 1/2*(4*a*c*x + c)/x^2
```

maple [A] time = 0.03, size = 22, normalized size = 0.96

$$-\frac{c}{2x^2}-\frac{2ac}{x}+a^2c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^3,x)
[Out] -1/2*c/x^2-2*a*c/x+a^2*c*ln(x)
```

maxima [A] time = 0.33, size = 20, normalized size = 0.87

$$a^2c\log(x)-\frac{4\,acx+c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c*log(x) - 1/2*(4*a*c*x + c)/x^2
```

mupad [B] time = 0.05, size = 22, normalized size = 0.96

$$a^2\,c\,\ln(x) - \frac{\frac{c}{2} + 2\,a\,c\,x}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] a^2*c*log(x) - (c/2 + 2*a*c*x)/x^2
```

sympy [A] time = 0.15, size = 22, normalized size = 0.96

$$a^2c\log(x) + \frac{-4acx - c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)/x**3,x)
```

```
[Out] a**2*c*log(x) + (-4*a*c*x - c)/(2*x**2)
```

3.1027 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^4}\,dx$

Optimal. Leaf size=15

$$-\frac{c(ax+1)^3}{3x^3}$$

```
[Out] -1/3*c*(a*x+1)^3/x^3
```

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 37}

$$-\frac{c(ax+1)^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^4,x]
```

```
[Out] -(c*(1 + a*x)^3)/(3*x^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^4}\,dx &= c\int \frac{(1+ax)^2}{x^4}\,dx\\ &= -\frac{c(1+ax)^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{c(ax+1)^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2))/x^4,x]
[Out] -1/3*(c*(1 + a*x)^3)/x^3
```

fricas [A] time = 0.48, size = 21, normalized size = 1.40

$$-\frac{3\,a^2cx^2+3\,acx+c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^4,x, algorithm="fricas")
[Out] -1/3*(3*a^2*c*x^2 + 3*a*c*x + c)/x^3
```

giac [A] time = 0.35, size = 21, normalized size = 1.40

$$-\frac{3\,a^2cx^2+3\,acx+c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^4,x, algorithm="giac")
[Out] -1/3*(3*a^2*c*x^2 + 3*a*c*x + c)/x^3
```

maple [A] time = 0.03, size = 23, normalized size = 1.53

$$c\left(-\frac{a^2}{x}-\frac{1}{3x^3}-\frac{a}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^4,x)
[Out] c*(-a^2/x-1/3/x^3-a/x^2)
```

maxima [A] time = 0.31, size = 21, normalized size = 1.40

$$-\frac{3\,a^2cx^2+3\,acx+c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*c*x^2 + 3*a*c*x + c)/x^3
```

mupad [B] time = 0.04, size = 21, normalized size = 1.40

$$-\frac{c\,a^2\,x^2+c\,a\,x+\frac{c}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] -(c/3 + a^2*c*x^2 + a*c*x)/x^3
```

sympy [A] time = 0.16, size = 24, normalized size = 1.60

$$\frac{-3a^2cx^2-3acx-c}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)/x**4,x)
```

```
[Out] (-3*a**2*c*x**2 - 3*a*c*x - c)/(3*x**3)
```

3.1028 $\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=48

$$-\frac{1}{9}a^4c^2x^9-\frac{1}{4}a^3c^2x^8+\frac{1}{3}ac^2x^6+\frac{c^2x^5}{5}$$

```
[Out] 1/5*c^2*x^5+1/3*a*c^2*x^6-1/4*a^3*c^2*x^8-1/9*a^4*c^2*x^9
```

Rubi [A] time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 75}

$$-\frac{1}{9}a^4c^2x^9-\frac{1}{4}a^3c^2x^8+\frac{1}{3}ac^2x^6+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^5)/5 + (a*c^2*x^6)/3 - (a^3*c^2*x^8)/4 - (a^4*c^2*x^9)/9
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)^2\,dx&=c^2\int x^4(1-ax)(1+ax)^3\,dx\\&=c^2\int\left(x^4+2ax^5-2a^3x^7-a^4x^8\right)dx\\&=\frac{c^2x^5}{5}+\frac{1}{3}ac^2x^6-\frac{1}{4}a^3c^2x^8-\frac{1}{9}a^4c^2x^9\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.67

$$-\frac{1}{180}c^2x^5\left(20a^4x^4+45a^3x^3-60ax-36\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/180*(c^2*x^5*(-36 - 60*a*x + 45*a^3*x^3 + 20*a^4*x^4))
```

fricas [A] time = 0.52, size = 40, normalized size = 0.83

$$-\frac{1}{9}\,a^4c^2x^9-\frac{1}{4}\,a^3c^2x^8+\frac{1}{3}\,ac^2x^6+\frac{1}{5}\,c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/9*a^4*c^2*x^9 - 1/4*a^3*c^2*x^8 + 1/3*a*c^2*x^6 + 1/5*c^2*x^5
```

giac [A] time = 0.17, size = 40, normalized size = 0.83

$$-\frac{1}{9}\,a^4c^2x^9-\frac{1}{4}\,a^3c^2x^8+\frac{1}{3}\,ac^2x^6+\frac{1}{5}\,c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/9*a^4*c^2*x^9 - 1/4*a^3*c^2*x^8 + 1/3*a*c^2*x^6 + 1/5*c^2*x^5
```

maple [A] time = 0.02, size = 33, normalized size = 0.69

$$c^2\left(-\frac{1}{9}a^4x^9-\frac{1}{4}a^3x^8+\frac{1}{3}x^6a+\frac{1}{5}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^2,x)
```

```
[Out] c^2*(-1/9*a^4*x^9-1/4*a^3*x^8+1/3*x^6*a+1/5*x^5)
```

maxima [A] time = 0.35, size = 40, normalized size = 0.83

$$-\frac{1}{9}\,a^4c^2x^9-\frac{1}{4}\,a^3c^2x^8+\frac{1}{3}\,ac^2x^6+\frac{1}{5}\,c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/9*a^4*c^2*x^9 - 1/4*a^3*c^2*x^8 + 1/3*a*c^2*x^6 + 1/5*c^2*x^5
```

mupad [B] time = 0.06, size = 40, normalized size = 0.83

$$-\frac{a^4\,c^2\,x^9}{9}-\frac{a^3\,c^2\,x^8}{4}+\frac{a\,c^2\,x^6}{3}+\frac{c^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(c - a^2*c*x^2)^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c^2*x^5)/5 + (a*c^2*x^6)/3 - (a^3*c^2*x^8)/4 - (a^4*c^2*x^9)/9
```

sympy [A] time = 0.09, size = 41, normalized size = 0.85

$$-\frac{a^{4}c^{2}x^{9}}{9}-\frac{a^{3}c^{2}x^{8}}{4}+\frac{ac^{2}x^{6}}{3}+\frac{c^{2}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**4*c**2*x**9/9 - a**3*c**2*x**8/4 + a*c**2*x**6/3 + c**2*x**5/5
```

3.1029 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=48

$$-\frac{1}{8}a^4c^2x^8-\frac{2}{7}a^3c^2x^7+\frac{2}{5}ac^2x^5+\frac{c^2x^4}{4}$$

```
[Out] 1/4*c^2*x^4+2/5*a*c^2*x^5-2/7*a^3*c^2*x^7-1/8*a^4*c^2*x^8
```

Rubi [A] time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 75}

$$-\frac{1}{8}a^4c^2x^8-\frac{2}{7}a^3c^2x^7+\frac{2}{5}ac^2x^5+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^4)/4 + (2*a*c^2*x^5)/5 - (2*a^3*c^2*x^7)/7 - (a^4*c^2*x^8)/8
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^2\,dx&=c^2\int x^3(1-ax)(1+ax)^3\,dx\\&=c^2\int\left(x^3+2ax^4-2a^3x^6-a^4x^7\right)dx\\&=\frac{c^2x^4}{4}+\frac{2}{5}ac^2x^5-\frac{2}{7}a^3c^2x^7-\frac{1}{8}a^4c^2x^8\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.83

$$c^2\left(-\frac{1}{8}a^4x^8-\frac{2a^3x^7}{7}+\frac{2ax^5}{5}+\frac{x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^2,x]
[Out] c^2*(x^4/4 + (2*a*x^5)/5 - (2*a^3*x^7)/7 - (a^4*x^8)/8)
```

fricas [A] time = 0.54, size = 40, normalized size = 0.83

$$-\frac{1}{8}\,a^4c^2x^8-\frac{2}{7}\,a^3c^2x^7+\frac{2}{5}\,ac^2x^5+\frac{1}{4}\,c^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
[Out] -1/8*a^4*c^2*x^8 - 2/7*a^3*c^2*x^7 + 2/5*a*c^2*x^5 + 1/4*c^2*x^4
```

giac [A] time = 0.40, size = 40, normalized size = 0.83

$$-\frac{1}{8}\,a^4c^2x^8-\frac{2}{7}\,a^3c^2x^7+\frac{2}{5}\,ac^2x^5+\frac{1}{4}\,c^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] -1/8*a^4*c^2*x^8 - 2/7*a^3*c^2*x^7 + 2/5*a*c^2*x^5 + 1/4*c^2*x^4
```

maple [A] time = 0.02, size = 33, normalized size = 0.69

$$c^2\left(-\frac{1}{8}a^4x^8-\frac{2}{7}a^3x^7+\frac{2}{5}a\,x^5+\frac{1}{4}x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^2,x)
[Out] c^2*(-1/8*a^4*x^8-2/7*a^3*x^7+2/5*a*x^5+1/4*x^4)
```

maxima [A] time = 0.30, size = 40, normalized size = 0.83

$$-\frac{1}{8}\,a^4c^2x^8-\frac{2}{7}\,a^3c^2x^7+\frac{2}{5}\,ac^2x^5+\frac{1}{4}\,c^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/8*a^4*c^2*x^8 - 2/7*a^3*c^2*x^7 + 2/5*a*c^2*x^5 + 1/4*c^2*x^4
```

mupad [B] time = 0.05, size = 40, normalized size = 0.83

$$-\frac{a^4\,c^2\,x^8}{8}-\frac{2\,a^3\,c^2\,x^7}{7}+\frac{2\,a\,c^2\,x^5}{5}+\frac{c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - a^2*c*x^2)^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c^2*x^4)/4 + (2*a*c^2*x^5)/5 - (2*a^3*c^2*x^7)/7 - (a^4*c^2*x^8)/8
```

sympy [A] time = 0.09, size = 44, normalized size = 0.92

$$-\frac{a^4c^2x^8}{8}-\frac{2a^3c^2x^7}{7}+\frac{2ac^2x^5}{5}+\frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**4*c**2*x**8/8 - 2*a**3*c**2*x**7/7 + 2*a*c**2*x**5/5 + c**2*x**4/4
```

3.1030 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=48

$$-\frac{1}{7}a^4c^2x^7-\frac{1}{3}a^3c^2x^6+\frac{1}{2}ac^2x^4+\frac{c^2x^3}{3}$$

```
[Out] 1/3*c^2*x^3+1/2*a*c^2*x^4-1/3*a^3*c^2*x^6-1/7*a^4*c^2*x^7
```

Rubi [A] time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 75}

$$-\frac{1}{7}a^4c^2x^7-\frac{1}{3}a^3c^2x^6+\frac{1}{2}ac^2x^4+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^3)/3 + (a*c^2*x^4)/2 - (a^3*c^2*x^6)/3 - (a^4*c^2*x^7)/7
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^2\,dx&=c^2\int x^2(1-ax)(1+ax)^3\,dx\\&=c^2\int\left(x^2+2ax^3-2a^3x^5-a^4x^6\right)dx\\&=\frac{c^2x^3}{3}+\frac{1}{2}ac^2x^4-\frac{1}{3}a^3c^2x^6-\frac{1}{7}a^4c^2x^7\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.67

$$-\frac{1}{42}c^2x^3\left(6a^4x^4+14a^3x^3-21ax-14\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/42*(c^2*x^3*(-14 - 21*a*x + 14*a^3*x^3 + 6*a^4*x^4))
```

fricas [A] time = 0.66, size = 40, normalized size = 0.83

$$-\frac{1}{7}a^4c^2x^7-\frac{1}{3}a^3c^2x^6+\frac{1}{2}ac^2x^4+\frac{1}{3}c^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/7*a^4*c^2*x^7 - 1/3*a^3*c^2*x^6 + 1/2*a*c^2*x^4 + 1/3*c^2*x^3
```

giac [A] time = 0.16, size = 40, normalized size = 0.83

$$-\frac{1}{7}a^4c^2x^7-\frac{1}{3}a^3c^2x^6+\frac{1}{2}ac^2x^4+\frac{1}{3}c^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/7*a^4*c^2*x^7 - 1/3*a^3*c^2*x^6 + 1/2*a*c^2*x^4 + 1/3*c^2*x^3
```

maple [A] time = 0.03, size = 33, normalized size = 0.69

$$c^2\left(-\frac{1}{7}x^7a^4-\frac{1}{3}x^6a^3+\frac{1}{2}x^4a+\frac{1}{3}x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^2,x)
```

```
[Out] c^2*(-1/7*x^7*a^4-1/3*x^6*a^3+1/2*x^4*a+1/3*x^3)
```

maxima [A] time = 0.32, size = 40, normalized size = 0.83

$$-\frac{1}{7}a^4c^2x^7-\frac{1}{3}a^3c^2x^6+\frac{1}{2}ac^2x^4+\frac{1}{3}c^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/7*a^4*c^2*x^7 - 1/3*a^3*c^2*x^6 + 1/2*a*c^2*x^4 + 1/3*c^2*x^3
```

mupad [B] time = 0.05, size = 40, normalized size = 0.83

$$-\frac{a^4\,c^2\,x^7}{7}-\frac{a^3\,c^2\,x^6}{3}+\frac{a\,c^2\,x^4}{2}+\frac{c^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - a^2*c*x^2)^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c^2*x^3)/3 + (a*c^2*x^4)/2 - (a^3*c^2*x^6)/3 - (a^4*c^2*x^7)/7
```

sympy [A] time = 0.09, size = 41, normalized size = 0.85

$$-\frac{a^{4}c^{2}x^{7}}{7}-\frac{a^{3}c^{2}x^{6}}{3}+\frac{ac^{2}x^{4}}{2}+\frac{c^{2}x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**4*c**2*x**7/7 - a**3*c**2*x**6/3 + a*c**2*x**4/2 + c**2*x**3/3
```

3.1031 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6}a^4c^2x^6-\frac{2}{5}a^3c^2x^5+\frac{2}{3}ac^2x^3+\frac{c^2x^2}{2}$$

```
[Out] 1/2*c^2*x^2+2/3*a*c^2*x^3-2/5*a^3*c^2*x^5-1/6*a^4*c^2*x^6
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 75}

$$-\frac{1}{6}a^4c^2x^6-\frac{2}{5}a^3c^2x^5+\frac{2}{3}ac^2x^3+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^2)/2 + (2*a*c^2*x^3)/3 - (2*a^3*c^2*x^5)/5 - (a^4*c^2*x^6)/6
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^2\,dx&=c^2\int x(1-ax)(1+ax)^3\,dx\\&=c^2\int\left(x+2ax^2-2a^3x^4-a^4x^5\right)dx\\&=\frac{c^2x^2}{2}+\frac{2}{3}ac^2x^3-\frac{2}{5}a^3c^2x^5-\frac{1}{6}a^4c^2x^6\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.67

$$-\frac{1}{30}c^2x^2\left(5a^4x^4+12a^3x^3-20ax-15\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^2,x]
[Out] -1/30*(c^2*x^2*(-15 - 20*a*x + 12*a^3*x^3 + 5*a^4*x^4))
```

fricas [A] time = 0.55, size = 40, normalized size = 0.83

$$-\frac{1}{6}\,a^4c^2x^6-\frac{2}{5}\,a^3c^2x^5+\frac{2}{3}\,ac^2x^3+\frac{1}{2}\,c^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] -1/6*a^4*c^2*x^6 - 2/5*a^3*c^2*x^5 + 2/3*a*c^2*x^3 + 1/2*c^2*x^2
```

giac [A] time = 0.20, size = 40, normalized size = 0.83

$$-\frac{1}{6}\,a^4c^2x^6-\frac{2}{5}\,a^3c^2x^5+\frac{2}{3}\,ac^2x^3+\frac{1}{2}\,c^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] -1/6*a^4*c^2*x^6 - 2/5*a^3*c^2*x^5 + 2/3*a*c^2*x^3 + 1/2*c^2*x^2
```

maple [A] time = 0.02, size = 33, normalized size = 0.69

$$c^2\left(-\frac{1}{6}x^6a^4-\frac{2}{5}a^3x^5+\frac{2}{3}x^3a+\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^2,x)
[Out] c^2*(-1/6*x^6*a^4-2/5*a^3*x^5+2/3*x^3*a+1/2*x^2)
```

maxima [A] time = 0.34, size = 40, normalized size = 0.83

$$-\frac{1}{6}\,a^4c^2x^6-\frac{2}{5}\,a^3c^2x^5+\frac{2}{3}\,ac^2x^3+\frac{1}{2}\,c^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] -1/6*a^4*c^2*x^6 - 2/5*a^3*c^2*x^5 + 2/3*a*c^2*x^3 + 1/2*c^2*x^2
```

mupad [B] time = 0.05, size = 40, normalized size = 0.83

$$-\frac{a^4\,c^2\,x^6}{6}-\frac{2\,a^3\,c^2\,x^5}{5}+\frac{2\,a\,c^2\,x^3}{3}+\frac{c^2\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - a^2*c*x^2)^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] (c^2*x^2)/2 + (2*a*c^2*x^3)/3 - (2*a^3*c^2*x^5)/5 - (a^4*c^2*x^6)/6
```

sympy [A] time = 0.08, size = 44, normalized size = 0.92

$$-\frac{a^4c^2x^6}{6}-\frac{2a^3c^2x^5}{5}+\frac{2ac^2x^3}{3}+\frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**2,x)
[Out] -a**4*c**2*x**6/6 - 2*a**3*c**2*x**5/5 + 2*a*c**2*x**3/3 + c**2*x**2/2
```

3.1032 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=35

$$\frac{c^2(ax+1)^4}{2a}-\frac{c^2(ax+1)^5}{5a}$$

```
[Out] 1/2*c^2*(a*x+1)^4/a-1/5*c^2*(a*x+1)^5/a
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^2(ax+1)^4}{2a}-\frac{c^2(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(1 + a*x)^4)/(2*a) - (c^2*(1 + a*x)^5)/(5*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx&=c^2\int(1-ax)(1+ax)^3\,dx\\&=c^2\int\left(2(1+ax)^3-(1+ax)^4\right)\,dx\\&=\frac{c^2(1+ax)^4}{2a}-\frac{c^2(1+ax)^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.66

$$-\frac{c^2(ax+1)^4(2ax-3)}{10a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]

[Out] -1/10*(c^2*(1 + a*x)^4*(-3 + 2*a*x))/a

fricas [A] time = 0.71, size = 36, normalized size = 1.03

$$-\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")

[Out] -1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 + c^2*x

giac [A] time = 1.41, size = 36, normalized size = 1.03

$$-\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] -1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 + c^2*x

maple [A] time = 0.02, size = 28, normalized size = 0.80

$$c^2\left(-\frac{1}{5}a^4x^5 - \frac{1}{2}x^4a^3 + a\,x^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2,x)

[Out] c^2*(-1/5*a^4*x^5-1/2*x^4*a^3+a*x^2+x)

maxima [A] time = 0.32, size = 36, normalized size = 1.03

$$-\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 + c^2*x
```

mupad [B] time = 0.05, size = 36, normalized size = 1.03

$$-\frac{a^4\,c^2\,x^5}{5}-\frac{a^3\,c^2\,x^4}{2}+a\,c^2\,x^2+c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] c^2*x + a*c^2*x^2 - (a^3*c^2*x^4)/2 - (a^4*c^2*x^5)/5
```

sympy [A] time = 0.08, size = 36, normalized size = 1.03

$$-\frac{a^{4}c^{2}x^{5}}{5}-\frac{a^{3}c^{2}x^{4}}{2}+ac^{2}x^{2}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] -a**4*c**2*x**5/5 - a**3*c**2*x**4/2 + a*c**2*x**2 + c**2*x
```

3.1033 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{4}a^4c^2x^4-\frac{2}{3}a^3c^2x^3+2ac^2x+c^2\log(x)$$

```
[Out] 2*a*c^2*x-2/3*a^3*c^2*x^3-1/4*a^4*c^2*x^4+c^2*ln(x)
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 75}

$$-\frac{1}{4}a^4c^2x^4-\frac{2}{3}a^3c^2x^3+2ac^2x+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x,x]
```

```
[Out] 2*a*c^2*x - (2*a^3*c^2*x^3)/3 - (a^4*c^2*x^4)/4 + c^2*Log[x]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x}\,dx = c^2\int \frac{(1-ax)(1+ax)^3}{x}\,dx$$
$$= c^2\int \left(2a+\frac{1}{x}-2a^3x^2-a^4x^3\right)dx$$
$$= 2ac^2x-\frac{2}{3}a^3c^2x^3-\frac{1}{4}a^4c^2x^4+c^2\log(x)$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.82

$$-\frac{1}{12}c^2\left(3a^4x^4+8a^3x^3-24ax-12\log(x)+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x,x]
[Out] -1/12*(c^2*(3 - 24*a*x + 8*a^3*x^3 + 3*a^4*x^4 - 12*Log[x]))
```

fricas [A] time = 0.62, size = 36, normalized size = 0.90

$$-\frac{1}{4}\,a^4c^2x^4-\frac{2}{3}\,a^3c^2x^3+2\,ac^2x+c^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x,x, algorithm="fricas")
[Out] -1/4*a^4*c^2*x^4 - 2/3*a^3*c^2*x^3 + 2*a*c^2*x + c^2*log(x)
```

giac [A] time = 0.16, size = 37, normalized size = 0.92

$$-\frac{1}{4}\,a^4c^2x^4-\frac{2}{3}\,a^3c^2x^3+2\,ac^2x+c^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x,x, algorithm="giac")
[Out] -1/4*a^4*c^2*x^4 - 2/3*a^3*c^2*x^3 + 2*a*c^2*x + c^2*log(abs(x))
```

maple [A] time = 0.03, size = 37, normalized size = 0.92

$$2a\,c^2x-\frac{2a^3c^2x^3}{3}-\frac{a^4c^2x^4}{4}+c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x,x)
```

```
[Out] 2*a*c^2*x-2/3*a^3*c^2*x^3-1/4*a^4*c^2*x^4+c^2*ln(x)
```

maxima [A] time = 0.33, size = 36, normalized size = 0.90

$$-\frac{1}{4}a^4c^2x^4 - \frac{2}{3}a^3c^2x^3 + 2\,ac^2x + c^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x,x, algorithm="maxima")
```

```
[Out] -1/4*a^4*c^2*x^4 - 2/3*a^3*c^2*x^3 + 2*a*c^2*x + c^2*log(x)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.90

$$c^2\,\ln(x) - \frac{2\,a^3\,c^2\,x^3}{3} - \frac{a^4\,c^2\,x^4}{4} + 2\,a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^2*(a*x + 1)^2)/(x*(a^2*x^2 - 1)),x)
```

```
[Out] c^2*log(x) - (2*a^3*c^2*x^3)/3 - (a^4*c^2*x^4)/4 + 2*a*c^2*x
```

sympy [A] time = 0.12, size = 39, normalized size = 0.98

$$-\frac{a^4c^2x^4}{4} - \frac{2a^3c^2x^3}{3} + 2ac^2x + c^2\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x,x)
```

```
[Out] -a**4*c**2*x**4/4 - 2*a**3*c**2*x**3/3 + 2*a*c**2*x + c**2*log(x)
```

3.1034 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^2}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{3}a^4c^2x^3-a^3c^2x^2+2ac^2\log(x)-\frac{c^2}{x}$$

```
[Out] -c^2/x-a^3*c^2*x^2-1/3*a^4*c^2*x^3+2*a*c^2*ln(x)
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 75}

$$-\frac{1}{3}a^4c^2x^3-a^3c^2x^2+2ac^2\log(x)-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^2,x]
```

```
[Out] -(c^2/x) - a^3*c^2*x^2 - (a^4*c^2*x^3)/3 + 2*a*c^2*Log[x]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^2}\,dx = c^2\int \frac{(1-ax)(1+ax)^3}{x^2}\,dx$$
$$= c^2\int \left(\frac{1}{x^2}+\frac{2a}{x}-2a^3x-a^4x^2\right)dx$$
$$= -\frac{c^2}{x}-a^3c^2x^2-\frac{1}{3}a^4c^2x^3+2ac^2\log(x)$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{1}{3}a^4c^2x^3-a^3c^2x^2+2ac^2\log(x)-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^2,x]
```

```
[Out] -(c^2/x) - a^3*c^2*x^2 - (a^4*c^2*x^3)/3 + 2*a*c^2*Log[x]
```

fricas [A] time = 0.66, size = 41, normalized size = 1.00

$$-\frac{a^4c^2x^4+3\,a^3c^2x^3-6\,ac^2x\log(x)+3\,c^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^2,x, algorithm="fricas"
)
```

```
[Out] -1/3*(a^4*c^2*x^4 + 3*a^3*c^2*x^3 - 6*a*c^2*x*log(x) + 3*c^2)/x
```

giac [A] time = 0.22, size = 40, normalized size = 0.98

$$-\frac{1}{3}\,a^4c^2x^3-a^3c^2x^2+2\,ac^2\log\left(|x|\right)-\frac{c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^2,x, algorithm="giac")
```

```
[Out] -1/3*a^4*c^2*x^3 - a^3*c^2*x^2 + 2*a*c^2*log(abs(x)) - c^2/x
```

maple [A] time = 0.03, size = 40, normalized size = 0.98

$$-\frac{c^2}{x}-a^3c^2x^2-\frac{a^4c^2x^3}{3}+2a\,c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^2,x)
```

```
[Out] -c^2/x-a^3*c^2*x^2-1/3*a^4*c^2*x^3+2*a*c^2*ln(x)
```

maxima [A] time = 0.34, size = 39, normalized size = 0.95

$$-\frac{1}{3}\,a^4c^2x^3 - a^3c^2x^2 + 2\,ac^2\log(x) - \frac{c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^2,x, algorithm="maxima"
)
```

```
[Out] -1/3*a^4*c^2*x^3 - a^3*c^2*x^2 + 2*a*c^2*log(x) - c^2/x
```

mupad [B] time = 0.04, size = 39, normalized size = 0.95

$$2\,a\,c^2\,\ln(x) - \frac{c^2}{x} - a^3\,c^2\,x^2 - \frac{a^4\,c^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^2*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] 2*a*c^2*log(x) - c^2/x - a^3*c^2*x^2 - (a^4*c^2*x^3)/3
```

sympy [A] time = 0.14, size = 36, normalized size = 0.88

$$-\frac{a^4c^2x^3}{3} - a^3c^2x^2 + 2ac^2\log{(x)} - \frac{c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**2,x)
```

```
[Out] -a**4*c**2*x**3/3 - a**3*c**2*x**2 + 2*a*c**2*log(x) - c**2/x
```

3.1035 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^3}\,dx$

Optimal. Leaf size=17

$$-\frac{c^2(ax+1)^4}{2x^2}$$

```
[Out] -1/2*c^2*(a*x+1)^4/x^2
```

Rubi [A] time = 0.07, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 74}

$$-\frac{c^2(ax+1)^4}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^3,x]
```

```
[Out] -(c^2*(1 + a*x)^4)/(2*x^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^3}\,dx &= c^2\int \frac{(1-ax)(1+ax)^3}{x^3}\,dx\\ &= -\frac{c^2(1+ax)^4}{2x^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{c^2(ax+1)^4}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^3,x]
```

```
[Out] -1/2*(c^2*(1 + a*x)^4)/x^2
```

fricas [B] time = 0.51, size = 37, normalized size = 2.18

$$-\frac{a^4c^2x^4 + 4\,a^3c^2x^3 + 4\,ac^2x + c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^3,x, algorithm="fricas"
)
```

```
[Out] -1/2*(a^4*c^2*x^4 + 4*a^3*c^2*x^3 + 4*a*c^2*x + c^2)/x^2
```

giac [B] time = 0.56, size = 37, normalized size = 2.18

$$-\frac{1}{2}\,a^4c^2x^2 - 2\,a^3c^2x - \frac{4\,ac^2x + c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*a^4*c^2*x^2 - 2*a^3*c^2*x - 1/2*(4*a*c^2*x + c^2)/x^2
```

maple [A] time = 0.03, size = 31, normalized size = 1.82

$$c^2\left(-\frac{x^2a^4}{2} - 2a^3x - \frac{2a}{x} - \frac{1}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^3,x)
```

```
[Out] c^2*(-1/2*x^2*a^4-2*a^3*x-2*a/x-1/2/x^2)
```

maxima [B] time = 0.30, size = 37, normalized size = 2.18

$$-\frac{1}{2}\,a^4c^2x^2 - 2\,a^3c^2x - \frac{4\,ac^2x + c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^3,x, algorithm="maxima"
)
```

```
[Out] -1/2*a^4*c^2*x^2 - 2*a^3*c^2*x - 1/2*(4*a*c^2*x + c^2)/x^2
```

mupad [B] time = 0.04, size = 29, normalized size = 1.71

$$-\frac{c^2\,\left(a^4\,x^4+4\,a^3\,x^3+4\,a\,x+1\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^2*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] -(c^2*(4*a*x + 4*a^3*x^3 + a^4*x^4 + 1))/(2*x^2)
```

sympy [B] time = 0.14, size = 39, normalized size = 2.29

$$-\frac{a^4c^2x^2}{2}-2a^3c^2x-\frac{4ac^2x+c^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**3,x)
```

```
[Out] -a**4*c**2*x**2/2 - 2*a**3*c**2*x - (4*a*c**2*x + c**2)/(2*x**2)
```

3.1036 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^4}\,dx$

Optimal. Leaf size=39

$$a^4\left(-c^2\right)x-2a^3c^2\log(x)-\frac{ac^2}{x^2}-\frac{c^2}{3x^3}$$

[Out] -1/3*c^2/x^3-a*c^2/x^2-a^4*c^2*x-2*a^3*c^2*ln(x)

Rubi [A] time = 0.08, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 75}

$$a^4\left(-c^2\right)x-2a^3c^2\log(x)-\frac{ac^2}{x^2}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^4,x]

[Out] -c^2/(3*x^3) - (a*c^2)/x^2 - a^4*c^2*x - 2*a^3*c^2*Log[x]

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^4}\,dx = c^2\int \frac{(1-ax)(1+ax)^3}{x^4}\,dx$$
$$= c^2\int \left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{2a^3}{x}\right)dx$$
$$= -\frac{c^2}{3x^3}-\frac{ac^2}{x^2}-a^4c^2x-2a^3c^2\log(x)$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$a^4\left(-c^2\right)x-2a^3c^2\log(x)-\frac{ac^2}{x^2}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x]))*(c - a^2*c*x^2)^2)/x^4,x]
```

```
[Out] -1/3*c^2/x^3 - (a*c^2)/x^2 - a^4*c^2*x - 2*a^3*c^2*Log[x]
```

fricas [A] time = 0.88, size = 40, normalized size = 1.03

$$-\frac{3\,a^4c^2x^4+6\,a^3c^2x^3\log(x)+3\,ac^2x+c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^4,x, algorithm="fricas"
)
```

```
[Out] -1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/x^3
```

giac [A] time = 0.45, size = 37, normalized size = 0.95

$$-a^4c^2x-2\,a^3c^2\log\left(|x|\right)-\frac{3\,ac^2x+c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^4,x, algorithm="giac")
```

```
[Out] -a^4*c^2*x - 2*a^3*c^2*log(abs(x)) - 1/3*(3*a*c^2*x + c^2)/x^3
```

maple [A] time = 0.03, size = 38, normalized size = 0.97

$$-\frac{c^2}{3x^3}-\frac{a\,c^2}{x^2}-a^4c^2x-2a^3c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^4,x)
```

```
[Out] -1/3*c^2/x^3-a*c^2/x^2-a^4*c^2*x-2*a^3*c^2*ln(x)
```

maxima [A] time = 0.31, size = 36, normalized size = 0.92

$$-a^4c^2x - 2\,a^3c^2\log(x) - \frac{3\,ac^2x + c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^4,x, algorithm="maxima"
)
```

```
[Out] -a^4*c^2*x - 2*a^3*c^2*log(x) - 1/3*(3*a*c^2*x + c^2)/x^3
```

mupad [B] time = 0.04, size = 32, normalized size = 0.82

$$-\frac{c^2\,\left(3\,a\,x+3\,a^4\,x^4+6\,a^3\,x^3\,\ln(x)+1\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^2*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] -(c^2*(3*a*x + 3*a^4*x^4 + 6*a^3*x^3*log(x) + 1))/(3*x^3)
```

sympy [A] time = 0.19, size = 37, normalized size = 0.95

$$-a^4c^2x - 2a^3c^2\log{(x)} - \frac{3ac^2x + c^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**4,x)
```

```
[Out] -a**4*c**2*x - 2*a**3*c**2*log(x) - (3*a*c**2*x + c**2)/(3*x**3)
```

3.1037 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^5}\,dx$

Optimal. Leaf size=43

$$a^4\left(-c^2\right)\log(x)+\frac{2a^3c^2}{x}-\frac{2ac^2}{3x^3}-\frac{c^2}{4x^4}$$

```
[Out] -1/4*c^2/x^4-2/3*a*c^2/x^3+2*a^3*c^2/x-a^4*c^2*ln(x)
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 75}

$$\frac{2a^3c^2}{x}+a^4\left(-c^2\right)\log(x)-\frac{2ac^2}{3x^3}-\frac{c^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^5,x]
```

```
[Out] -c^2/(4*x^4) - (2*a*c^2)/(3*x^3) + (2*a^3*c^2)/x - a^4*c^2*Log[x]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^5}\,dx = c^2\int \frac{(1-ax)(1+ax)^3}{x^5}\,dx$$
$$= c^2\int \left(\frac{1}{x^5}+\frac{2a}{x^4}-\frac{2a^3}{x^2}-\frac{a^4}{x}\right)dx$$
$$= -\frac{c^2}{4x^4}-\frac{2ac^2}{3x^3}+\frac{2a^3c^2}{x}-a^4c^2\log(x)$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.81

$$c^2\left(a^4(-\log(x))+\frac{2a^3}{x}-\frac{2a}{3x^3}-\frac{1}{4x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^5,x]
[Out] c^2*(-1/4*1/x^4 - (2*a)/(3*x^3) + (2*a^3)/x - a^4*Log[x])
```

fricas [A] time = 0.67, size = 42, normalized size = 0.98

$$-\frac{12\,a^4c^2x^4\log(x)-24\,a^3c^2x^3+8\,ac^2x+3\,c^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^5,x, algorithm="fricas"
)
[Out] -1/12*(12*a^4*c^2*x^4*log(x) - 24*a^3*c^2*x^3 + 8*a*c^2*x + 3*c^2)/x^4
```

giac [A] time = 0.23, size = 41, normalized size = 0.95

$$-a^4c^2\log\left(|x|\right)+\frac{24\,a^3c^2x^3-8\,ac^2x-3\,c^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^5,x, algorithm="giac")
[Out] -a^4*c^2*log(abs(x)) + 1/12*(24*a^3*c^2*x^3 - 8*a*c^2*x - 3*c^2)/x^4
```

maple [A] time = 0.03, size = 40, normalized size = 0.93

$$-\frac{c^2}{4x^4}-\frac{2a\,c^2}{3x^3}+\frac{2a^3c^2}{x}-a^4c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^5,x)
```

```
[Out] -1/4*c^2/x^4-2/3*a*c^2/x^3+2*a^3*c^2/x-a^4*c^2*ln(x)
```

maxima [A] time = 0.31, size = 40, normalized size = 0.93

$$-a^4c^2\log(x) + \frac{24\,a^3c^2x^3 - 8\,ac^2x - 3\,c^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^5,x, algorithm="maxima"
)
```

```
[Out] -a^4*c^2*log(x) + 1/12*(24*a^3*c^2*x^3 - 8*a*c^2*x - 3*c^2)/x^4
```

mupad [B] time = 0.05, size = 32, normalized size = 0.74

$$-\frac{c^2\,\left(8\,a\,x-24\,a^3\,x^3+12\,a^4\,x^4\,\ln(x)+3\right)}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^2*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)),x)
```

```
[Out] -(c^2*(8*a*x - 24*a^3*x^3 + 12*a^4*x^4*log(x) + 3))/(12*x^4)
```

sympy [A] time = 0.22, size = 41, normalized size = 0.95

$$-a^4c^2\log{(x)} - \frac{-24a^3c^2x^3 + 8ac^2x + 3c^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**5,x)
```

```
[Out] -a**4*c**2*log(x) - (-24*a**3*c**2*x**3 + 8*a*c**2*x + 3*c**2)/(12*x**4)
```

3.1038 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^6}\,dx$

Optimal. Leaf size=42

$$\frac{a^4c^2}{x}+\frac{a^3c^2}{x^2}-\frac{ac^2}{2x^4}-\frac{c^2}{5x^5}$$

```
[Out] -1/5*c^2/x^5-1/2*a*c^2/x^4+a^3*c^2/x^2+a^4*c^2/x
```

Rubi [A] time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 75}

$$\frac{a^3c^2}{x^2}+\frac{a^4c^2}{x}-\frac{ac^2}{2x^4}-\frac{c^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^6,x]
```

```
[Out] -c^2/(5*x^5) - (a*c^2)/(2*x^4) + (a^3*c^2)/x^2 + (a^4*c^2)/x
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2}{x^6}\,dx = c^2\int \frac{(1-ax)(1+ax)^3}{x^6}\,dx$$
$$= c^2\int \left(\frac{1}{x^6}+\frac{2a}{x^5}-\frac{2a^3}{x^3}-\frac{a^4}{x^2}\right)dx$$
$$= -\frac{c^2}{5x^5}-\frac{ac^2}{2x^4}+\frac{a^3c^2}{x^2}+\frac{a^4c^2}{x}$$

Mathematica [**A**] time = 0.01, size = 23, normalized size = 0.55

$$\frac{c^2(ax+1)^4(3ax-2)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2)/x^6,x]
```

```
[Out] (c^2*(1 + a*x)^4*(-2 + 3*a*x))/(10*x^5)
```

fricas [**A**] time = 0.54, size = 40, normalized size = 0.95

$$\frac{10\,a^4c^2x^4+10\,a^3c^2x^3-5\,ac^2x-2\,c^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/10*(10*a^4*c^2*x^4 + 10*a^3*c^2*x^3 - 5*a*c^2*x - 2*c^2)/x^5
```

giac [**A**] time = 0.18, size = 40, normalized size = 0.95

$$\frac{10\,a^4c^2x^4+10\,a^3c^2x^3-5\,ac^2x-2\,c^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/10*(10*a^4*c^2*x^4 + 10*a^3*c^2*x^3 - 5*a*c^2*x - 2*c^2)/x^5
```

maple [**A**] time = 0.03, size = 31, normalized size = 0.74

$$c^2\left(\frac{a^4}{x}+\frac{a^3}{x^2}-\frac{a}{2x^4}-\frac{1}{5x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^6,x)
```

```
[Out] c^2*(a^4/x+a^3/x^2-1/2*a/x^4-1/5/x^5)
```

maxima [A] time = 0.30, size = 40, normalized size = 0.95

$$\frac{10 a^4 c^2 x^4 + 10 a^3 c^2 x^3 - 5 a c^2 x - 2 c^2}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^2/x^6,x, algorithm="maxima"
)
```

```
[Out] 1/10*(10*a^4*c^2*x^4 + 10*a^3*c^2*x^3 - 5*a*c^2*x - 2*c^2)/x^5
```

mupad [B] time = 0.04, size = 40, normalized size = 0.95

$$-\frac{-a^4 c^2 x^4 - a^3 c^2 x^3 + \frac{a c^2 x}{2} + \frac{c^2}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^2*(a*x + 1)^2)/(x^6*(a^2*x^2 - 1)),x)
```

```
[Out] -(c^2/5 - a^3*c^2*x^3 - a^4*c^2*x^4 + (a*c^2*x)/2)/x^5
```

sympy [A] time = 0.25, size = 42, normalized size = 1.00

$$-\frac{-10a^4c^2x^4 - 10a^3c^2x^3 + 5ac^2x + 2c^2}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**2/x**6,x)
```

```
[Out] -(-10*a**4*c**2*x**4 - 10*a**3*c**2*x**3 + 5*a*c**2*x + 2*c**2)/(10*x**5)
```

3.1039 $\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{1}{11}a^6c^3x^{11}+\frac{1}{5}a^5c^3x^{10}-\frac{1}{9}a^4c^3x^9-\frac{1}{2}a^3c^3x^8-\frac{1}{7}a^2c^3x^7+\frac{1}{3}ac^3x^6+\frac{c^3x^5}{5}$$

```
[Out] 1/5*c^3*x^5+1/3*a*c^3*x^6-1/7*a^2*c^3*x^7-1/2*a^3*c^3*x^8-1/9*a^4*c^3*x^9+1
/5*a^5*c^3*x^10+1/11*a^6*c^3*x^11
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{1}{11}a^6c^3x^{11}+\frac{1}{5}a^5c^3x^{10}-\frac{1}{9}a^4c^3x^9-\frac{1}{2}a^3c^3x^8-\frac{1}{7}a^2c^3x^7+\frac{1}{3}ac^3x^6+\frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*x^5)/5 + (a*c^3*x^6)/3 - (a^2*c^3*x^7)/7 - (a^3*c^3*x^8)/2 - (a^4*c^3*
x^9)/9 + (a^5*c^3*x^10)/5 + (a^6*c^3*x^11)/11
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^4\left(c-a^2cx^2\right)^3\,dx = c^3\int x^4(1-ax)^2(1+ax)^4\,dx$$
$$= c^3\int\left(x^4+2ax^5-a^2x^6-4a^3x^7-a^4x^8+2a^5x^9+a^6x^{10}\right)dx$$
$$= \frac{c^3x^5}{5}+\frac{1}{3}ac^3x^6-\frac{1}{7}a^2c^3x^7-\frac{1}{2}a^3c^3x^8-\frac{1}{9}a^4c^3x^9+\frac{1}{5}a^5c^3x^{10}+\frac{1}{11}a^6c^3x^{11}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.80

$$c^3\left(\frac{a^6x^{11}}{11}+\frac{a^5x^{10}}{5}-\frac{a^4x^9}{9}-\frac{a^3x^8}{2}-\frac{a^2x^7}{7}+\frac{ax^6}{3}+\frac{x^5}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^4*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*(x^5/5 + (a*x^6)/3 - (a^2*x^7)/7 - (a^3*x^8)/2 - (a^4*x^9)/9 + (a^5*x^1
0)/5 + (a^6*x^11)/11)
```

fricas [A] time = 0.72, size = 73, normalized size = 0.84

$$\frac{1}{11}\,a^6c^3x^{11}+\frac{1}{5}\,a^5c^3x^{10}-\frac{1}{9}\,a^4c^3x^9-\frac{1}{2}\,a^3c^3x^8-\frac{1}{7}\,a^2c^3x^7+\frac{1}{3}\,ac^3x^6+\frac{1}{5}\,c^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/11*a^6*c^3*x^11 + 1/5*a^5*c^3*x^10 - 1/9*a^4*c^3*x^9 - 1/2*a^3*c^3*x^8 -
1/7*a^2*c^3*x^7 + 1/3*a*c^3*x^6 + 1/5*c^3*x^5
```

giac [A] time = 0.23, size = 73, normalized size = 0.84

$$\frac{1}{11}\,a^6c^3x^{11}+\frac{1}{5}\,a^5c^3x^{10}-\frac{1}{9}\,a^4c^3x^9-\frac{1}{2}\,a^3c^3x^8-\frac{1}{7}\,a^2c^3x^7+\frac{1}{3}\,ac^3x^6+\frac{1}{5}\,c^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/11*a^6*c^3*x^11 + 1/5*a^5*c^3*x^10 - 1/9*a^4*c^3*x^9 - 1/2*a^3*c^3*x^8 -
1/7*a^2*c^3*x^7 + 1/3*a*c^3*x^6 + 1/5*c^3*x^5
```

maple [A] time = 0.02, size = 57, normalized size = 0.66

$$c^3\left(\frac{1}{11}a^6x^{11}+\frac{1}{5}a^5x^{10}-\frac{1}{9}a^4x^9-\frac{1}{2}a^3x^8-\frac{1}{7}a^2x^7+\frac{1}{3}x^6a+\frac{1}{5}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/11*a^6*x^11+1/5*a^5*x^10-1/9*a^4*x^9-1/2*a^3*x^8-1/7*a^2*x^7+1/3*x^6
*a+1/5*x^5)
```

maxima [A] time = 0.34, size = 73, normalized size = 0.84

$$\frac{1}{11}\,a^6c^3x^{11}+\frac{1}{5}\,a^5c^3x^{10}-\frac{1}{9}\,a^4c^3x^9-\frac{1}{2}\,a^3c^3x^8-\frac{1}{7}\,a^2c^3x^7+\frac{1}{3}\,ac^3x^6+\frac{1}{5}\,c^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/11*a^6*c^3*x^11 + 1/5*a^5*c^3*x^10 - 1/9*a^4*c^3*x^9 - 1/2*a^3*c^3*x^8 -
1/7*a^2*c^3*x^7 + 1/3*a*c^3*x^6 + 1/5*c^3*x^5
```

mupad [B] time = 0.89, size = 73, normalized size = 0.84

$$\frac{a^6\,c^3\,x^{11}}{11}+\frac{a^5\,c^3\,x^{10}}{5}-\frac{a^4\,c^3\,x^9}{9}-\frac{a^3\,c^3\,x^8}{2}-\frac{a^2\,c^3\,x^7}{7}+\frac{a\,c^3\,x^6}{3}+\frac{c^3\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(c - a^2*c*x^2)^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c^3*x^5)/5 + (a*c^3*x^6)/3 - (a^2*c^3*x^7)/7 - (a^3*c^3*x^8)/2 - (a^4*c^3*
x^9)/9 + (a^5*c^3*x^10)/5 + (a^6*c^3*x^11)/11
```

sympy [A] time = 0.10, size = 76, normalized size = 0.87

$$\frac{a^6c^3x^{11}}{11}+\frac{a^5c^3x^{10}}{5}-\frac{a^4c^3x^9}{9}-\frac{a^3c^3x^8}{2}-\frac{a^2c^3x^7}{7}+\frac{ac^3x^6}{3}+\frac{c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**11/11 + a**5*c**3*x**10/5 - a**4*c**3*x**9/9 - a**3*c**3*x**8/
2 - a**2*c**3*x**7/7 + a*c**3*x**6/3 + c**3*x**5/5
```

3.1040 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{1}{10}a^6c^3x^{10}+\frac{2}{9}a^5c^3x^9-\frac{1}{8}a^4c^3x^8-\frac{4}{7}a^3c^3x^7-\frac{1}{6}a^2c^3x^6+\frac{2}{5}ac^3x^5+\frac{c^3x^4}{4}$$

```
[Out] 1/4*c^3*x^4+2/5*a*c^3*x^5-1/6*a^2*c^3*x^6-4/7*a^3*c^3*x^7-1/8*a^4*c^3*x^8+2
/9*a^5*c^3*x^9+1/10*a^6*c^3*x^10
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{1}{10}a^6c^3x^{10}+\frac{2}{9}a^5c^3x^9-\frac{1}{8}a^4c^3x^8-\frac{4}{7}a^3c^3x^7-\frac{1}{6}a^2c^3x^6+\frac{2}{5}ac^3x^5+\frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*x^4)/4 + (2*a*c^3*x^5)/5 - (a^2*c^3*x^6)/6 - (4*a^3*c^3*x^7)/7 - (a^4*
c^3*x^8)/8 + (2*a^5*c^3*x^9)/9 + (a^6*c^3*x^10)/10
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^3\,dx = c^3\int x^3(1-ax)^2(1+ax)^4\,dx$$
$$= c^3\int\left(x^3+2ax^4-a^2x^5-4a^3x^6-a^4x^7+2a^5x^8+a^6x^9\right)dx$$
$$= \frac{c^3x^4}{4}+\frac{2}{5}ac^3x^5-\frac{1}{6}a^2c^3x^6-\frac{4}{7}a^3c^3x^7-\frac{1}{8}a^4c^3x^8+\frac{2}{9}a^5c^3x^9+\frac{1}{10}a^6c^3x^{10}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.80

$$c^3\left(\frac{a^6x^{10}}{10}+\frac{2a^5x^9}{9}-\frac{a^4x^8}{8}-\frac{4a^3x^7}{7}-\frac{a^2x^6}{6}+\frac{2ax^5}{5}+\frac{x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*(x^4/4 + (2*a*x^5)/5 - (a^2*x^6)/6 - (4*a^3*x^7)/7 - (a^4*x^8)/8 + (2*a
^5*x^9)/9 + (a^6*x^10)/10)
```

fricas [A] time = 0.59, size = 73, normalized size = 0.84

$$\frac{1}{10}\,a^6c^3x^{10}+\frac{2}{9}\,a^5c^3x^9-\frac{1}{8}\,a^4c^3x^8-\frac{4}{7}\,a^3c^3x^7-\frac{1}{6}\,a^2c^3x^6+\frac{2}{5}\,ac^3x^5+\frac{1}{4}\,c^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/10*a^6*c^3*x^10 + 2/9*a^5*c^3*x^9 - 1/8*a^4*c^3*x^8 - 4/7*a^3*c^3*x^7 - 1
/6*a^2*c^3*x^6 + 2/5*a*c^3*x^5 + 1/4*c^3*x^4
```

giac [A] time = 0.19, size = 73, normalized size = 0.84

$$\frac{1}{10}\,a^6c^3x^{10}+\frac{2}{9}\,a^5c^3x^9-\frac{1}{8}\,a^4c^3x^8-\frac{4}{7}\,a^3c^3x^7-\frac{1}{6}\,a^2c^3x^6+\frac{2}{5}\,ac^3x^5+\frac{1}{4}\,c^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/10*a^6*c^3*x^10 + 2/9*a^5*c^3*x^9 - 1/8*a^4*c^3*x^8 - 4/7*a^3*c^3*x^7 - 1
/6*a^2*c^3*x^6 + 2/5*a*c^3*x^5 + 1/4*c^3*x^4
```

maple [A] time = 0.02, size = 57, normalized size = 0.66

$$c^{3}\left(\frac{1}{10}a^{6}x^{10}+\frac{2}{9}a^{5}x^{9}-\frac{1}{8}a^{4}x^{8}-\frac{4}{7}a^{3}x^{7}-\frac{1}{6}a^{2}x^{6}+\frac{2}{5}a\,x^{5}+\frac{1}{4}x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/10*a^6*x^10+2/9*a^5*x^9-1/8*a^4*x^8-4/7*a^3*x^7-1/6*a^2*x^6+2/5*a*x^
5+1/4*x^4)
```

maxima [A] time = 0.36, size = 73, normalized size = 0.84

$$\frac{1}{10}\,a^{6}c^{3}x^{10}+\frac{2}{9}\,a^{5}c^{3}x^{9}-\frac{1}{8}\,a^{4}c^{3}x^{8}-\frac{4}{7}\,a^{3}c^{3}x^{7}-\frac{1}{6}\,a^{2}c^{3}x^{6}+\frac{2}{5}\,ac^{3}x^{5}+\frac{1}{4}\,c^{3}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/10*a^6*c^3*x^10 + 2/9*a^5*c^3*x^9 - 1/8*a^4*c^3*x^8 - 4/7*a^3*c^3*x^7 - 1
/6*a^2*c^3*x^6 + 2/5*a*c^3*x^5 + 1/4*c^3*x^4
```

mupad [B] time = 0.03, size = 73, normalized size = 0.84

$$\frac{a^{6}\,c^{3}\,x^{10}}{10}+\frac{2\,a^{5}\,c^{3}\,x^{9}}{9}-\frac{a^{4}\,c^{3}\,x^{8}}{8}-\frac{4\,a^{3}\,c^{3}\,x^{7}}{7}-\frac{a^{2}\,c^{3}\,x^{6}}{6}+\frac{2\,a\,c^{3}\,x^{5}}{5}+\frac{c^{3}\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - a^2*c*x^2)^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c^3*x^4)/4 + (2*a*c^3*x^5)/5 - (a^2*c^3*x^6)/6 - (4*a^3*c^3*x^7)/7 - (a^4*
c^3*x^8)/8 + (2*a^5*c^3*x^9)/9 + (a^6*c^3*x^10)/10
```

sympy [A] time = 0.10, size = 82, normalized size = 0.94

$$\frac{a^{6}c^{3}x^{10}}{10}+\frac{2a^{5}c^{3}x^{9}}{9}-\frac{a^{4}c^{3}x^{8}}{8}-\frac{4a^{3}c^{3}x^{7}}{7}-\frac{a^{2}c^{3}x^{6}}{6}+\frac{2ac^{3}x^{5}}{5}+\frac{c^{3}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**10/10 + 2*a**5*c**3*x**9/9 - a**4*c**3*x**8/8 - 4*a**3*c**3*x*
*7/7 - a**2*c**3*x**6/6 + 2*a*c**3*x**5/5 + c**3*x**4/4
```

3.1041 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=84

$$\frac{c^3(ax+1)^9}{9a^3}-\frac{3c^3(ax+1)^8}{4a^3}+\frac{13c^3(ax+1)^7}{7a^3}-\frac{2c^3(ax+1)^6}{a^3}+\frac{4c^3(ax+1)^5}{5a^3}$$

```
[Out] 4/5*c^3*(a*x+1)^5/a^3-2*c^3*(a*x+1)^6/a^3+13/7*c^3*(a*x+1)^7/a^3-3/4*c^3*(a
*x+1)^8/a^3+1/9*c^3*(a*x+1)^9/a^3
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{c^3(ax+1)^9}{9a^3}-\frac{3c^3(ax+1)^8}{4a^3}+\frac{13c^3(ax+1)^7}{7a^3}-\frac{2c^3(ax+1)^6}{a^3}+\frac{4c^3(ax+1)^5}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^3,x]
```

```
[Out] (4*c^3*(1 + a*x)^5)/(5*a^3) - (2*c^3*(1 + a*x)^6)/a^3 + (13*c^3*(1 + a*x)^7
)/(7*a^3) - (3*c^3*(1 + a*x)^8)/(4*a^3) + (c^3*(1 + a*x)^9)/(9*a^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^3\,dx=c^3\int x^2(1-ax)^2(1+ax)^4\,dx$$
$$=c^3\int\left(\frac{4(1+ax)^4}{a^2}-\frac{12(1+ax)^5}{a^2}+\frac{13(1+ax)^6}{a^2}-\frac{6(1+ax)^7}{a^2}+\frac{(1+ax)^8}{a^2}\right)dx$$
$$=\frac{4c^3(1+ax)^5}{5a^3}-\frac{2c^3(1+ax)^6}{a^3}+\frac{13c^3(1+ax)^7}{7a^3}-\frac{3c^3(1+ax)^8}{4a^3}+\frac{c^3(1+ax)^9}{9a^3}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.83

$$c^3\left(\frac{a^6x^9}{9}+\frac{a^5x^8}{4}-\frac{a^4x^7}{7}-\frac{2a^3x^6}{3}-\frac{a^2x^5}{5}+\frac{ax^4}{2}+\frac{x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*(x^3/3 + (a*x^4)/2 - (a^2*x^5)/5 - (2*a^3*x^6)/3 - (a^4*x^7)/7 + (a^5*x
^8)/4 + (a^6*x^9)/9)
```

fricas [A] time = 0.63, size = 73, normalized size = 0.87

$$\frac{1}{9}\,a^6c^3x^9+\frac{1}{4}\,a^5c^3x^8-\frac{1}{7}\,a^4c^3x^7-\frac{2}{3}\,a^3c^3x^6-\frac{1}{5}\,a^2c^3x^5+\frac{1}{2}\,ac^3x^4+\frac{1}{3}\,c^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] 1/9*a^6*c^3*x^9 + 1/4*a^5*c^3*x^8 - 1/7*a^4*c^3*x^7 - 2/3*a^3*c^3*x^6 - 1/5
*a^2*c^3*x^5 + 1/2*a*c^3*x^4 + 1/3*c^3*x^3
```

giac [A] time = 1.73, size = 73, normalized size = 0.87

$$\frac{1}{9}\,a^6c^3x^9+\frac{1}{4}\,a^5c^3x^8-\frac{1}{7}\,a^4c^3x^7-\frac{2}{3}\,a^3c^3x^6-\frac{1}{5}\,a^2c^3x^5+\frac{1}{2}\,ac^3x^4+\frac{1}{3}\,c^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*a^6*c^3*x^9 + 1/4*a^5*c^3*x^8 - 1/7*a^4*c^3*x^7 - 2/3*a^3*c^3*x^6 - 1/5
*a^2*c^3*x^5 + 1/2*a*c^3*x^4 + 1/3*c^3*x^3
```

maple [A] time = 0.02, size = 57, normalized size = 0.68

$$c^{3}\left(\frac{1}{9}x^{9}a^{6}+\frac{1}{4}a^{5}x^{8}-\frac{1}{7}x^{7}a^{4}-\frac{2}{3}x^{6}a^{3}-\frac{1}{5}x^{5}a^{2}+\frac{1}{2}x^{4}a+\frac{1}{3}x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/9*x^9*a^6+1/4*a^5*x^8-1/7*x^7*a^4-2/3*x^6*a^3-1/5*x^5*a^2+1/2*x^4*a+
1/3*x^3)
```

maxima [A] time = 0.31, size = 73, normalized size = 0.87

$$\frac{1}{9}a^{6}c^{3}x^{9}+\frac{1}{4}a^{5}c^{3}x^{8}-\frac{1}{7}a^{4}c^{3}x^{7}-\frac{2}{3}a^{3}c^{3}x^{6}-\frac{1}{5}a^{2}c^{3}x^{5}+\frac{1}{2}ac^{3}x^{4}+\frac{1}{3}c^{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] 1/9*a^6*c^3*x^9 + 1/4*a^5*c^3*x^8 - 1/7*a^4*c^3*x^7 - 2/3*a^3*c^3*x^6 - 1/5
*a^2*c^3*x^5 + 1/2*a*c^3*x^4 + 1/3*c^3*x^3
```

mupad [B] time = 0.03, size = 73, normalized size = 0.87

$$\frac{a^{6}\,c^{3}\,x^{9}}{9}+\frac{a^{5}\,c^{3}\,x^{8}}{4}-\frac{a^{4}\,c^{3}\,x^{7}}{7}-\frac{2\,a^{3}\,c^{3}\,x^{6}}{3}-\frac{a^{2}\,c^{3}\,x^{5}}{5}+\frac{a\,c^{3}\,x^{4}}{2}+\frac{c^{3}\,x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - a^2*c*x^2)^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c^3*x^3)/3 + (a*c^3*x^4)/2 - (a^2*c^3*x^5)/5 - (2*a^3*c^3*x^6)/3 - (a^4*c^
3*x^7)/7 + (a^5*c^3*x^8)/4 + (a^6*c^3*x^9)/9
```

sympy [A] time = 0.09, size = 78, normalized size = 0.93

$$\frac{a^{6}c^{3}x^{9}}{9}+\frac{a^{5}c^{3}x^{8}}{4}-\frac{a^{4}c^{3}x^{7}}{7}-\frac{2a^{3}c^{3}x^{6}}{3}-\frac{a^{2}c^{3}x^{5}}{5}+\frac{ac^{3}x^{4}}{2}+\frac{c^{3}x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**9/9 + a**5*c**3*x**8/4 - a**4*c**3*x**7/7 - 2*a**3*c**3*x**6/3
 - a**2*c**3*x**5/5 + a*c**3*x**4/2 + c**3*x**3/3
```

3.1042 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=69

$$\frac{c^3(ax+1)^8}{8a^2}-\frac{5c^3(ax+1)^7}{7a^2}+\frac{4c^3(ax+1)^6}{3a^2}-\frac{4c^3(ax+1)^5}{5a^2}$$

```
[Out] -4/5*c^3*(a*x+1)^5/a^2+4/3*c^3*(a*x+1)^6/a^2-5/7*c^3*(a*x+1)^7/a^2+1/8*c^3*
(a*x+1)^8/a^2
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 77}

$$\frac{c^3(ax+1)^8}{8a^2}-\frac{5c^3(ax+1)^7}{7a^2}+\frac{4c^3(ax+1)^6}{3a^2}-\frac{4c^3(ax+1)^5}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-4*c^3*(1 + a*x)^5)/(5*a^2) + (4*c^3*(1 + a*x)^6)/(3*a^2) - (5*c^3*(1 + a*
x)^7)/(7*a^2) + (c^3*(1 + a*x)^8)/(8*a^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^3\,dx=c^3\int x(1-ax)^2(1+ax)^4\,dx$$
$$=c^3\int\left(-\frac{4(1+ax)^4}{a}+\frac{8(1+ax)^5}{a}-\frac{5(1+ax)^6}{a}+\frac{(1+ax)^7}{a}\right)dx$$
$$=-\frac{4c^3(1+ax)^5}{5a^2}+\frac{4c^3(1+ax)^6}{3a^2}-\frac{5c^3(1+ax)^7}{7a^2}+\frac{c^3(1+ax)^8}{8a^2}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 1.01

$$c^3\left(\frac{a^6x^8}{8}+\frac{2a^5x^7}{7}-\frac{a^4x^6}{6}-\frac{4a^3x^5}{5}-\frac{a^2x^4}{4}+\frac{2ax^3}{3}+\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*(x^2/2 + (2*a*x^3)/3 - (a^2*x^4)/4 - (4*a^3*x^5)/5 - (a^4*x^6)/6 + (2*a
^5*x^7)/7 + (a^6*x^8)/8)
```

fricas [A] time = 0.49, size = 73, normalized size = 1.06

$$\frac{1}{8}a^6c^3x^8+\frac{2}{7}a^5c^3x^7-\frac{1}{6}a^4c^3x^6-\frac{4}{5}a^3c^3x^5-\frac{1}{4}a^2c^3x^4+\frac{2}{3}ac^3x^3+\frac{1}{2}c^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*a^6*c^3*x^8 + 2/7*a^5*c^3*x^7 - 1/6*a^4*c^3*x^6 - 4/5*a^3*c^3*x^5 - 1/4
*a^2*c^3*x^4 + 2/3*a*c^3*x^3 + 1/2*c^3*x^2
```

giac [A] time = 0.17, size = 73, normalized size = 1.06

$$\frac{1}{8}a^6c^3x^8+\frac{2}{7}a^5c^3x^7-\frac{1}{6}a^4c^3x^6-\frac{4}{5}a^3c^3x^5-\frac{1}{4}a^2c^3x^4+\frac{2}{3}ac^3x^3+\frac{1}{2}c^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*a^6*c^3*x^8 + 2/7*a^5*c^3*x^7 - 1/6*a^4*c^3*x^6 - 4/5*a^3*c^3*x^5 - 1/4
*a^2*c^3*x^4 + 2/3*a*c^3*x^3 + 1/2*c^3*x^2
```

maple [A] time = 0.03, size = 57, normalized size = 0.83

$$c^3\left(\frac{1}{8}x^8a^6+\frac{2}{7}a^5x^7-\frac{1}{6}x^6a^4-\frac{4}{5}a^3x^5-\frac{1}{4}a^2x^4+\frac{2}{3}x^3a+\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/8*x^8*a^6+2/7*a^5*x^7-1/6*x^6*a^4-4/5*a^3*x^5-1/4*a^2*x^4+2/3*x^3*a+
1/2*x^2)
```

maxima [A] time = 0.31, size = 73, normalized size = 1.06

$$\frac{1}{8}\,a^6c^3x^8+\frac{2}{7}\,a^5c^3x^7-\frac{1}{6}\,a^4c^3x^6-\frac{4}{5}\,a^3c^3x^5-\frac{1}{4}\,a^2c^3x^4+\frac{2}{3}\,ac^3x^3+\frac{1}{2}\,c^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^6*c^3*x^8 + 2/7*a^5*c^3*x^7 - 1/6*a^4*c^3*x^6 - 4/5*a^3*c^3*x^5 - 1/4
*a^2*c^3*x^4 + 2/3*a*c^3*x^3 + 1/2*c^3*x^2
```

mupad [B] time = 0.03, size = 73, normalized size = 1.06

$$\frac{a^6\,c^3\,x^8}{8}+\frac{2\,a^5\,c^3\,x^7}{7}-\frac{a^4\,c^3\,x^6}{6}-\frac{4\,a^3\,c^3\,x^5}{5}-\frac{a^2\,c^3\,x^4}{4}+\frac{2\,a\,c^3\,x^3}{3}+\frac{c^3\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - a^2*c*x^2)^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c^3*x^2)/2 + (2*a*c^3*x^3)/3 - (a^2*c^3*x^4)/4 - (4*a^3*c^3*x^5)/5 - (a^4*
c^3*x^6)/6 + (2*a^5*c^3*x^7)/7 + (a^6*c^3*x^8)/8
```

sympy [A] time = 0.10, size = 82, normalized size = 1.19

$$\frac{a^6c^3x^8}{8}+\frac{2a^5c^3x^7}{7}-\frac{a^4c^3x^6}{6}-\frac{4a^3c^3x^5}{5}-\frac{a^2c^3x^4}{4}+\frac{2ac^3x^3}{3}+\frac{c^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**8/8 + 2*a**5*c**3*x**7/7 - a**4*c**3*x**6/6 - 4*a**3*c**3*x**5
/5 - a**2*c**3*x**4/4 + 2*a*c**3*x**3/3 + c**3*x**2/2
```

3.1043 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=52

$$\frac{c^3(ax+1)^7}{7a}-\frac{2c^3(ax+1)^6}{3a}+\frac{4c^3(ax+1)^5}{5a}$$

```
[Out] 4/5*c^3*(a*x+1)^5/a-2/3*c^3*(a*x+1)^6/a+1/7*c^3*(a*x+1)^7/a
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^3(ax+1)^7}{7a}-\frac{2c^3(ax+1)^6}{3a}+\frac{4c^3(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (4*c^3*(1 + a*x)^5)/(5*a) - (2*c^3*(1 + a*x)^6)/(3*a) + (c^3*(1 + a*x)^7)/(
7*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx&=c^3\int(1-ax)^2(1+ax)^4\,dx\\&=c^3\int\left(4(1+ax)^4-4(1+ax)^5+(1+ax)^6\right)dx\\&=\frac{4c^3(1+ax)^5}{5a}-\frac{2c^3(1+ax)^6}{3a}+\frac{c^3(1+ax)^7}{7a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.60

$$\frac{c^3(ax+1)^5\left(15a^2x^2-40ax+29\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(1 + a*x)^5*(29 - 40*a*x + 15*a^2*x^2))/(105*a)
```

fricas [A] time = 0.73, size = 69, normalized size = 1.33

$$\frac{1}{7}a^6c^3x^7+\frac{1}{3}a^5c^3x^6-\frac{1}{5}a^4c^3x^5-a^3c^3x^4-\frac{1}{3}a^2c^3x^3+ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 + a*c^3*x^2 + c^3*x
```

giac [A] time = 0.18, size = 69, normalized size = 1.33

$$\frac{1}{7}a^6c^3x^7+\frac{1}{3}a^5c^3x^6-\frac{1}{5}a^4c^3x^5-a^3c^3x^4-\frac{1}{3}a^2c^3x^3+ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 + a*c^3*x^2 + c^3*x
```

maple [A] time = 0.03, size = 52, normalized size = 1.00

$$c^3\left(\frac{1}{7}x^7a^6+\frac{1}{3}x^6a^5-\frac{1}{5}a^4x^5-x^4a^3-\frac{1}{3}x^3a^2+a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(1/7*x^7*a^6+1/3*x^6*a^5-1/5*a^4*x^5-x^4*a^3-1/3*x^3*a^2+a*x^2+x)
```

maxima [A] time = 0.33, size = 69, normalized size = 1.33

$$\frac{1}{7}a^6c^3x^7+\frac{1}{3}a^5c^3x^6-\frac{1}{5}a^4c^3x^5-a^3c^3x^4-\frac{1}{3}a^2c^3x^3+ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 + a*c^3*x^2 + c^3*x
```

mupad [B] time = 0.04, size = 69, normalized size = 1.33

$$\frac{a^6\,c^3\,x^7}{7}+\frac{a^5\,c^3\,x^6}{3}-\frac{a^4\,c^3\,x^5}{5}-a^3\,c^3\,x^4-\frac{a^2\,c^3\,x^3}{3}+a\,c^3\,x^2+c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] c^3*x + a*c^3*x^2 - (a^2*c^3*x^3)/3 - a^3*c^3*x^4 - (a^4*c^3*x^5)/5 + (a^5*
c^3*x^6)/3 + (a^6*c^3*x^7)/7
```

sympy [A] time = 0.09, size = 70, normalized size = 1.35

$$\frac{a^{6}c^{3}x^{7}}{7}+\frac{a^{5}c^{3}x^{6}}{3}-\frac{a^{4}c^{3}x^{5}}{5}-a^{3}c^{3}x^{4}-\frac{a^{2}c^{3}x^{3}}{3}+ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] a**6*c**3*x**7/7 + a**5*c**3*x**6/3 - a**4*c**3*x**5/5 - a**3*c**3*x**4 - a
**2*c**3*x**3/3 + a*c**3*x**2 + c**3*x
```

3.1044 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^3}{x}\,dx$

Optimal. Leaf size=79

$$\frac{1}{6}a^6c^3x^6+\frac{2}{5}a^5c^3x^5-\frac{1}{4}a^4c^3x^4-\frac{4}{3}a^3c^3x^3-\frac{1}{2}a^2c^3x^2+2ac^3x+c^3\log(x)$$

```
[Out] 2*a*c^3*x-1/2*a^2*c^3*x^2-4/3*a^3*c^3*x^3-1/4*a^4*c^3*x^4+2/5*a^5*c^3*x^5+1
/6*a^6*c^3*x^6+c^3*ln(x)
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{1}{6}a^6c^3x^6+\frac{2}{5}a^5c^3x^5-\frac{1}{4}a^4c^3x^4-\frac{4}{3}a^3c^3x^3-\frac{1}{2}a^2c^3x^2+2ac^3x+c^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x,x]
```

```
[Out] 2*a*c^3*x - (a^2*c^3*x^2)/2 - (4*a^3*c^3*x^3)/3 - (a^4*c^3*x^4)/4 + (2*a^5*
c^3*x^5)/5 + (a^6*c^3*x^6)/6 + c^3*Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x}\,dx = c^3\int \frac{(1-ax)^2(1+ax)^4}{x}\,dx$$
$$= c^3\int \left(2a+\frac{1}{x}-a^2x-4a^3x^2-a^4x^3+2a^5x^4+a^6x^5\right)dx$$
$$= 2ac^3x-\frac{1}{2}a^2c^3x^2-\frac{4}{3}a^3c^3x^3-\frac{1}{4}a^4c^3x^4+\frac{2}{5}a^5c^3x^5+\frac{1}{6}a^6c^3x^6+c^3\log(x)$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.65

$$c^3\left(\frac{1}{60}ax\left(10a^5x^5+24a^4x^4-15a^3x^3-80a^2x^2-30ax+120\right)+\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x,x]
```

```
[Out] c^3*((a*x*(120 - 30*a*x - 80*a^2*x^2 - 15*a^3*x^3 + 24*a^4*x^4 + 10*a^5*x^5
))/60 + Log[x])
```

fricas [A] time = 1.36, size = 69, normalized size = 0.87

$$\frac{1}{6}\,a^6c^3x^6+\frac{2}{5}\,a^5c^3x^5-\frac{1}{4}\,a^4c^3x^4-\frac{4}{3}\,a^3c^3x^3-\frac{1}{2}\,a^2c^3x^2+2\,ac^3x+c^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x,x, algorithm="fricas")
```

```
[Out] 1/6*a^6*c^3*x^6 + 2/5*a^5*c^3*x^5 - 1/4*a^4*c^3*x^4 - 4/3*a^3*c^3*x^3 - 1/2
*a^2*c^3*x^2 + 2*a*c^3*x + c^3*log(x)
```

giac [A] time = 0.15, size = 70, normalized size = 0.89

$$\frac{1}{6}\,a^6c^3x^6+\frac{2}{5}\,a^5c^3x^5-\frac{1}{4}\,a^4c^3x^4-\frac{4}{3}\,a^3c^3x^3-\frac{1}{2}\,a^2c^3x^2+2\,ac^3x+c^3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x,x, algorithm="giac")
```

```
[Out] 1/6*a^6*c^3*x^6 + 2/5*a^5*c^3*x^5 - 1/4*a^4*c^3*x^4 - 4/3*a^3*c^3*x^3 - 1/2
*a^2*c^3*x^2 + 2*a*c^3*x + c^3*log(abs(x))
```

maple [A] time = 0.03, size = 70, normalized size = 0.89

$$2a\,c^3x-\frac{a^2c^3x^2}{2}-\frac{4a^3c^3x^3}{3}-\frac{a^4c^3x^4}{4}+\frac{2a^5c^3x^5}{5}+\frac{a^6c^3x^6}{6}+c^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x,x)
```

```
[Out] 2*a*c^3*x-1/2*a^2*c^3*x^2-4/3*a^3*c^3*x^3-1/4*a^4*c^3*x^4+2/5*a^5*c^3*x^5+1
/6*a^6*c^3*x^6+c^3*ln(x)
```

maxima [A] time = 0.30, size = 69, normalized size = 0.87

$$\frac{1}{6}\,a^6c^3x^6+\frac{2}{5}\,a^5c^3x^5-\frac{1}{4}\,a^4c^3x^4-\frac{4}{3}\,a^3c^3x^3-\frac{1}{2}\,a^2c^3x^2+2\,ac^3x+c^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x,x, algorithm="maxima")
```

```
[Out] 1/6*a^6*c^3*x^6 + 2/5*a^5*c^3*x^5 - 1/4*a^4*c^3*x^4 - 4/3*a^3*c^3*x^3 - 1/2
*a^2*c^3*x^2 + 2*a*c^3*x + c^3*log(x)
```

mupad [B] time = 0.04, size = 69, normalized size = 0.87

$$c^3\,\ln(x)-\frac{a^2\,c^3\,x^2}{2}-\frac{4\,a^3\,c^3\,x^3}{3}-\frac{a^4\,c^3\,x^4}{4}+\frac{2\,a^5\,c^3\,x^5}{5}+\frac{a^6\,c^3\,x^6}{6}+2\,a\,c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^3*(a*x + 1)^2)/(x*(a^2*x^2 - 1)),x)
```

```
[Out] c^3*log(x) - (a^2*c^3*x^2)/2 - (4*a^3*c^3*x^3)/3 - (a^4*c^3*x^4)/4 + (2*a^5
*c^3*x^5)/5 + (a^6*c^3*x^6)/6 + 2*a*c^3*x
```

sympy [A] time = 0.15, size = 76, normalized size = 0.96

$$\frac{a^6c^3x^6}{6}+\frac{2a^5c^3x^5}{5}-\frac{a^4c^3x^4}{4}-\frac{4a^3c^3x^3}{3}-\frac{a^2c^3x^2}{2}+2ac^3x+c^3\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3/x,x)
```

```
[Out] a**6*c**3*x**6/6 + 2*a**5*c**3*x**5/5 - a**4*c**3*x**4/4 - 4*a**3*c**3*x**3
/3 - a**2*c**3*x**2/2 + 2*a*c**3*x + c**3*log(x)
```

3.1045 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^3}{x^2}\,dx$

Optimal. Leaf size=76

$$\frac{1}{5}a^6c^3x^5+\frac{1}{2}a^5c^3x^4-\frac{1}{3}a^4c^3x^3-2a^3c^3x^2-a^2c^3x+2ac^3\log(x)-\frac{c^3}{x}$$

```
[Out] -c^3/x-a^2*c^3*x-2*a^3*c^3*x^2-1/3*a^4*c^3*x^3+1/2*a^5*c^3*x^4+1/5*a^6*c^3*
x^5+2*a*c^3*ln(x)
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{1}{5}a^6c^3x^5+\frac{1}{2}a^5c^3x^4-\frac{1}{3}a^4c^3x^3-2a^3c^3x^2-a^2c^3x+2ac^3\log(x)-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^2,x]
```

```
[Out] -(c^3/x) - a^2*c^3*x - 2*a^3*c^3*x^2 - (a^4*c^3*x^3)/3 + (a^5*c^3*x^4)/2 +
(a^6*c^3*x^5)/5 + 2*a*c^3*Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x^2}\,dx = c^3\int \frac{(1-ax)^2(1+ax)^4}{x^2}\,dx$$
$$= c^3\int \left(-a^2+\frac{1}{x^2}+\frac{2a}{x}-4a^3x-a^4x^2+2a^5x^3+a^6x^4\right)dx$$
$$= -\frac{c^3}{x}-a^2c^3x-2a^3c^3x^2-\frac{1}{3}a^4c^3x^3+\frac{1}{2}a^5c^3x^4+\frac{1}{5}a^6c^3x^5+2ac^3\log(x)$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.78

$$c^3\left(\frac{a^6x^5}{5}+\frac{a^5x^4}{2}-\frac{a^4x^3}{3}-2a^3x^2-a^2x+2a\log(x)-\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^2,x]
```

```
[Out] c^3*(-x^(-1) - a^2*x - 2*a^3*x^2 - (a^4*x^3)/3 + (a^5*x^4)/2 + (a^6*x^5)/5
+ 2*a*Log[x])
```

fricas [A] time = 0.68, size = 75, normalized size = 0.99

$$\frac{6\,a^6c^3x^6+15\,a^5c^3x^5-10\,a^4c^3x^4-60\,a^3c^3x^3-30\,a^2c^3x^2+60\,ac^3x\log(x)-30\,c^3}{30\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^2,x, algorithm="fricas"
)
```

```
[Out] 1/30*(6*a^6*c^3*x^6 + 15*a^5*c^3*x^5 - 10*a^4*c^3*x^4 - 60*a^3*c^3*x^3 - 30
*a^2*c^3*x^2 + 60*a*c^3*x*log(x) - 30*c^3)/x
```

giac [A] time = 0.20, size = 71, normalized size = 0.93

$$\frac{1}{5}\,a^6c^3x^5+\frac{1}{2}\,a^5c^3x^4-\frac{1}{3}\,a^4c^3x^3-2\,a^3c^3x^2-a^2c^3x+2\,ac^3\log\left(|x|\right)-\frac{c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/5*a^6*c^3*x^5 + 1/2*a^5*c^3*x^4 - 1/3*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - a^2*c
^3*x + 2*a*c^3*log(abs(x)) - c^3/x
```

maple [A] time = 0.03, size = 71, normalized size = 0.93

$$-\frac{c^{3}}{x}-a^{2}c^{3}x-2a^{3}c^{3}x^{2}-\frac{a^{4}c^{3}x^{3}}{3}+\frac{a^{5}c^{3}x^{4}}{2}+\frac{a^{6}c^{3}x^{5}}{5}+2a\,c^{3}\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^2,x)
```

```
[Out] -c^3/x-a^2*c^3*x-2*a^3*c^3*x^2-1/3*a^4*c^3*x^3+1/2*a^5*c^3*x^4+1/5*a^6*c^3*
x^5+2*a*c^3*ln(x)
```

maxima [A] time = 0.30, size = 70, normalized size = 0.92

$$\frac{1}{5}\,a^{6}c^{3}x^{5}+\frac{1}{2}\,a^{5}c^{3}x^{4}-\frac{1}{3}\,a^{4}c^{3}x^{3}-2\,a^{3}c^{3}x^{2}-a^{2}c^{3}x+2\,ac^{3}\log(x)-\frac{c^{3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^2,x, algorithm="maxima"
)
```

```
[Out] 1/5*a^6*c^3*x^5 + 1/2*a^5*c^3*x^4 - 1/3*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - a^2*c
^3*x + 2*a*c^3*log(x) - c^3/x
```

mupad [B] time = 0.04, size = 70, normalized size = 0.92

$$2\,a\,c^{3}\,\ln(x)-a^{2}\,c^{3}\,x-\frac{c^{3}}{x}-2\,a^{3}\,c^{3}\,x^{2}-\frac{a^{4}\,c^{3}\,x^{3}}{3}+\frac{a^{5}\,c^{3}\,x^{4}}{2}+\frac{a^{6}\,c^{3}\,x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^3*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] 2*a*c^3*log(x) - a^2*c^3*x - c^3/x - 2*a^3*c^3*x^2 - (a^4*c^3*x^3)/3 + (a^5
*c^3*x^4)/2 + (a^6*c^3*x^5)/5
```

sympy [A] time = 0.17, size = 70, normalized size = 0.92

$$\frac{a^{6}c^{3}x^{5}}{5}+\frac{a^{5}c^{3}x^{4}}{2}-\frac{a^{4}c^{3}x^{3}}{3}-2a^{3}c^{3}x^{2}-a^{2}c^{3}x+2ac^{3}\log\left(x\right)-\frac{c^{3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3/x**2,x)
```

```
[Out] a**6*c**3*x**5/5 + a**5*c**3*x**4/2 - a**4*c**3*x**3/3 - 2*a**3*c**3*x**2 -
 a**2*c**3*x + 2*a*c**3*log(x) - c**3/x
```

3.1046 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^3}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{1}{4}a^6c^3x^4+\frac{2}{3}a^5c^3x^3-\frac{1}{2}a^4c^3x^2-4a^3c^3x-a^2c^3\log(x)-\frac{2ac^3}{x}-\frac{c^3}{2x^2}$$

```
[Out] -1/2*c^3/x^2-2*a*c^3/x-4*a^3*c^3*x-1/2*a^4*c^3*x^2+2/3*a^5*c^3*x^3+1/4*a^6*
c^3*x^4-a^2*c^3*ln(x)
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{1}{4}a^6c^3x^4+\frac{2}{3}a^5c^3x^3-\frac{1}{2}a^4c^3x^2-4a^3c^3x-a^2c^3\log(x)-\frac{2ac^3}{x}-\frac{c^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^3,x]
```

```
[Out] -c^3/(2*x^2) - (2*a*c^3)/x - 4*a^3*c^3*x - (a^4*c^3*x^2)/2 + (2*a^5*c^3*x^3
)/3 + (a^6*c^3*x^4)/4 - a^2*c^3*Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x^3}\,dx = c^3\int \frac{(1-ax)^2(1+ax)^4}{x^3}\,dx$$
$$= c^3\int \left(-4a^3+\frac{1}{x^3}+\frac{2a}{x^2}-\frac{a^2}{x}-a^4x+2a^5x^2+a^6x^3\right)dx$$
$$= -\frac{c^3}{2x^2}-\frac{2ac^3}{x}-4a^3c^3x-\frac{1}{2}a^4c^3x^2+\frac{2}{3}a^5c^3x^3+\frac{1}{4}a^6c^3x^4-a^2c^3\log(x)$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.74

$$\frac{c^3\left(3a^6x^6+8a^5x^5-6a^4x^4-48a^3x^3-12a^2x^2\log(x)-24ax-6\right)}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^3,x]
```

```
[Out] (c^3*(-6 - 24*a*x - 48*a^3*x^3 - 6*a^4*x^4 + 8*a^5*x^5 + 3*a^6*x^6 - 12*a^2
*x^2*Log[x]))/(12*x^2)
```

fricas [A] time = 0.67, size = 75, normalized size = 0.96

$$\frac{3\,a^6c^3x^6+8\,a^5c^3x^5-6\,a^4c^3x^4-48\,a^3c^3x^3-12\,a^2c^3x^2\log(x)-24\,ac^3x-6\,c^3}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^3,x, algorithm="fricas"
)
```

```
[Out] 1/12*(3*a^6*c^3*x^6 + 8*a^5*c^3*x^5 - 6*a^4*c^3*x^4 - 48*a^3*c^3*x^3 - 12*a
^2*c^3*x^2*log(x) - 24*a*c^3*x - 6*c^3)/x^2
```

giac [A] time = 1.04, size = 70, normalized size = 0.90

$$\frac{1}{4}\,a^6c^3x^4+\frac{2}{3}\,a^5c^3x^3-\frac{1}{2}\,a^4c^3x^2-4\,a^3c^3x-a^2c^3\log\left(|x|\right)-\frac{4\,ac^3x+c^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/4*a^6*c^3*x^4 + 2/3*a^5*c^3*x^3 - 1/2*a^4*c^3*x^2 - 4*a^3*c^3*x - a^2*c^3
*log(abs(x)) - 1/2*(4*a*c^3*x + c^3)/x^2
```

maple [A] time = 0.03, size = 71, normalized size = 0.91

$$-\frac{c^3}{2x^2}-\frac{2a\,c^3}{x}-4a^3c^3x-\frac{a^4c^3x^2}{2}+\frac{2a^5c^3x^3}{3}+\frac{a^6c^3x^4}{4}-a^2c^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^3,x)
```

```
[Out] -1/2*c^3/x^2-2*a*c^3/x-4*a^3*c^3*x-1/2*a^4*c^3*x^2+2/3*a^5*c^3*x^3+1/4*a^6*
c^3*x^4-a^2*c^3*ln(x)
```

maxima [A] time = 0.31, size = 69, normalized size = 0.88

$$\frac{1}{4}\,a^6c^3x^4+\frac{2}{3}\,a^5c^3x^3-\frac{1}{2}\,a^4c^3x^2-4\,a^3c^3x-a^2c^3\log(x)-\frac{4\,ac^3x+c^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/4*a^6*c^3*x^4 + 2/3*a^5*c^3*x^3 - 1/2*a^4*c^3*x^2 - 4*a^3*c^3*x - a^2*c^3
*log(x) - 1/2*(4*a*c^3*x + c^3)/x^2
```

mupad [B] time = 0.04, size = 71, normalized size = 0.91

$$\frac{2\,a^5\,c^3\,x^3}{3}-4\,a^3\,c^3\,x-\frac{a^4\,c^3\,x^2}{2}-\frac{\frac{c^3}{2}+2\,a\,c^3\,x}{x^2}+\frac{a^6\,c^3\,x^4}{4}-a^2\,c^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^3*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] (2*a^5*c^3*x^3)/3 - 4*a^3*c^3*x - (a^4*c^3*x^2)/2 - (c^3/2 + 2*a*c^3*x)/x^2
 + (a^6*c^3*x^4)/4 - a^2*c^3*log(x)
```

sympy [A] time = 0.20, size = 75, normalized size = 0.96

$$\frac{a^6c^3x^4}{4}+\frac{2a^5c^3x^3}{3}-\frac{a^4c^3x^2}{2}-4a^3c^3x-a^2c^3\log{(x)}+\frac{-4ac^3x-c^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3/x**3,x)
```

```
[Out] a**6*c**3*x**4/4 + 2*a**5*c**3*x**3/3 - a**4*c**3*x**2/2 - 4*a**3*c**3*x -
a**2*c**3*log(x) + (-4*a*c**3*x - c**3)/(2*x**2)
```

3.1047 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x^4} dx$

Optimal. Leaf size=72

$$\frac{1}{3}a^6c^3x^3 + a^5c^3x^2 - a^4c^3x - 4a^3c^3\log(x) + \frac{a^2c^3}{x} - \frac{ac^3}{x^2} - \frac{c^3}{3x^3}$$

```
[Out] -1/3*c^3/x^3-a*c^3/x^2+a^2*c^3/x-a^4*c^3*x+a^5*c^3*x^2+1/3*a^6*c^3*x^3-4*a^
3*c^3*ln(x)
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{1}{3}a^6c^3x^3 + a^5c^3x^2 - a^4c^3x + \frac{a^2c^3}{x} - 4a^3c^3\log(x) - \frac{ac^3}{x^2} - \frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^4,x]
```

```
[Out] -c^3/(3*x^3) - (a*c^3)/x^2 + (a^2*c^3)/x - a^4*c^3*x + a^5*c^3*x^2 + (a^6*c
^3*x^3)/3 - 4*a^3*c^3*Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3}{x^4}\,dx = c^3\int \frac{(1-ax)^2(1+ax)^4}{x^4}\,dx$$
$$= c^3\int \left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{a^2}{x^2}-\frac{4a^3}{x}+2a^5x+a^6x^2\right)dx$$
$$= -\frac{c^3}{3x^3}-\frac{ac^3}{x^2}+\frac{a^2c^3}{x}-a^4c^3x+a^5c^3x^2+\frac{1}{3}a^6c^3x^3-4a^3c^3\log(x)$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.76

$$c^3\left(\frac{a^6x^3}{3}+a^5x^2-a^4x-4a^3\log(x)+\frac{a^2}{x}-\frac{a}{x^2}-\frac{1}{3x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3)/x^4,x]
```

```
[Out] c^3*(-1/3*1/x^3 - a/x^2 + a^2/x - a^4*x + a^5*x^2 + (a^6*x^3)/3 - 4*a^3*Log
[x])
```

fricas [A] time = 0.64, size = 74, normalized size = 1.03

$$\frac{a^6c^3x^6+3\,a^5c^3x^5-3\,a^4c^3x^4-12\,a^3c^3x^3\log(x)+3\,a^2c^3x^2-3\,ac^3x-c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/3*(a^6*c^3*x^6 + 3*a^5*c^3*x^5 - 3*a^4*c^3*x^4 - 12*a^3*c^3*x^3*log(x) +
3*a^2*c^3*x^2 - 3*a*c^3*x - c^3)/x^3
```

giac [A] time = 0.21, size = 71, normalized size = 0.99

$$\frac{1}{3}\,a^6c^3x^3+a^5c^3x^2-a^4c^3x-4\,a^3c^3\log\left(|x|\right)+\frac{3\,a^2c^3x^2-3\,ac^3x-c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/3*a^6*c^3*x^3 + a^5*c^3*x^2 - a^4*c^3*x - 4*a^3*c^3*log(abs(x)) + 1/3*(3*
a^2*c^3*x^2 - 3*a*c^3*x - c^3)/x^3
```

maple [A] time = 0.03, size = 69, normalized size = 0.96

$$-\frac{c^3}{3x^3}-\frac{a\,c^3}{x^2}+\frac{a^2c^3}{x}-a^4c^3x+a^5c^3x^2+\frac{a^6c^3x^3}{3}-4a^3c^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^4,x)
```

```
[Out] -1/3*c^3/x^3-a*c^3/x^2+a^2*c^3/x-a^4*c^3*x+a^5*c^3*x^2+1/3*a^6*c^3*x^3-4*a^
3*c^3*ln(x)
```

maxima [A] time = 0.31, size = 70, normalized size = 0.97

$$\frac{1}{3}\,a^6c^3x^3+a^5c^3x^2-a^4c^3x-4\,a^3c^3\log(x)+\frac{3\,a^2c^3x^2-3\,ac^3x-c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^3/x^4,x, algorithm="maxima"
)
```

```
[Out] 1/3*a^6*c^3*x^3 + a^5*c^3*x^2 - a^4*c^3*x - 4*a^3*c^3*log(x) + 1/3*(3*a^2*c
^3*x^2 - 3*a*c^3*x - c^3)/x^3
```

mupad [B] time = 0.04, size = 69, normalized size = 0.96

$$a^5\,c^3\,x^2-a^4\,c^3\,x-\frac{-a^2\,c^3\,x^2+a\,c^3\,x+\frac{c^3}{3}}{x^3}+\frac{a^6\,c^3\,x^3}{3}-4\,a^3\,c^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^3*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] a^5*c^3*x^2 - a^4*c^3*x - (c^3/3 - a^2*c^3*x^2 + a*c^3*x)/x^3 + (a^6*c^3*x^
3)/3 - 4*a^3*c^3*log(x)
```

sympy [A] time = 0.23, size = 70, normalized size = 0.97

$$\frac{a^6c^3x^3}{3}+a^5c^3x^2-a^4c^3x-4a^3c^3\log\left(x\right)+\frac{3a^2c^3x^2-3ac^3x-c^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**3/x**4,x)
```

```
[Out] a**6*c**3*x**3/3 + a**5*c**3*x**2 - a**4*c**3*x - 4*a**3*c**3*log(x) + (3*a
**2*c**3*x**2 - 3*a*c**3*x - c**3)/(3*x**3)
```

3.1048 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=69

$$-\frac{c^4(ax+1)^9}{9a}+\frac{3c^4(ax+1)^8}{4a}-\frac{12c^4(ax+1)^7}{7a}+\frac{4c^4(ax+1)^6}{3a}$$

```
[Out] 4/3*c^4*(a*x+1)^6/a-12/7*c^4*(a*x+1)^7/a+3/4*c^4*(a*x+1)^8/a-1/9*c^4*(a*x+1
)^9/a
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$-\frac{c^4(ax+1)^9}{9a}+\frac{3c^4(ax+1)^8}{4a}-\frac{12c^4(ax+1)^7}{7a}+\frac{4c^4(ax+1)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (4*c^4*(1 + a*x)^6)/(3*a) - (12*c^4*(1 + a*x)^7)/(7*a) + (3*c^4*(1 + a*x)^8
)/(4*a) - (c^4*(1 + a*x)^9)/(9*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= c^4\int(1-ax)^3(1+ax)^5\,dx\\ &= c^4\int\left(8(1+ax)^5-12(1+ax)^6+6(1+ax)^7-(1+ax)^8\right)dx\\ &= \frac{4c^4(1+ax)^6}{3a}-\frac{12c^4(1+ax)^7}{7a}+\frac{3c^4(1+ax)^8}{4a}-\frac{c^4(1+ax)^9}{9a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.57

$$-\frac{c^4(ax+1)^6\left(28a^3x^3-105a^2x^2+138ax-65\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/252*(c^4*(1 + a*x)^6*(-65 + 138*a*x - 105*a^2*x^2 + 28*a^3*x^3))/a
```

fricas [A] time = 0.58, size = 79, normalized size = 1.14

$$-\frac{1}{9}a^8c^4x^9-\frac{1}{4}a^7c^4x^8+\frac{2}{7}a^6c^4x^7+a^5c^4x^6-\frac{3}{2}a^3c^4x^4-\frac{2}{3}a^2c^4x^3+ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 + a*c^4*x^2 + c^4*x
```

giac [A] time = 0.31, size = 79, normalized size = 1.14

$$-\frac{1}{9}a^8c^4x^9-\frac{1}{4}a^7c^4x^8+\frac{2}{7}a^6c^4x^7+a^5c^4x^6-\frac{3}{2}a^3c^4x^4-\frac{2}{3}a^2c^4x^3+ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 + a*c^4*x^2 + c^4*x
```

maple [A] time = 0.02, size = 59, normalized size = 0.86

$$c^4\left(-\frac{1}{9}x^9a^8-\frac{1}{4}a^7x^8+\frac{2}{7}x^7a^6+x^6a^5-\frac{3}{2}x^4a^3-\frac{2}{3}x^3a^2+a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] c^4*(-1/9*x^9*a^8-1/4*a^7*x^8+2/7*x^7*a^6+x^6*a^5-3/2*x^4*a^3-2/3*x^3*a^2+a
*x^2+x)
```

maxima [A] time = 0.33, size = 79, normalized size = 1.14

$$-\frac{1}{9}\,a^{8}c^{4}x^{9}-\frac{1}{4}\,a^{7}c^{4}x^{8}+\frac{2}{7}\,a^{6}c^{4}x^{7}+a^{5}c^{4}x^{6}-\frac{3}{2}\,a^{3}c^{4}x^{4}-\frac{2}{3}\,a^{2}c^{4}x^{3}+ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 + a*c^4*x^2 + c^4*x
```

mupad [B] time = 0.04, size = 79, normalized size = 1.14

$$-\frac{a^{8}\,c^{4}\,x^{9}}{9}-\frac{a^{7}\,c^{4}\,x^{8}}{4}+\frac{2\,a^{6}\,c^{4}\,x^{7}}{7}+a^{5}\,c^{4}\,x^{6}-\frac{3\,a^{3}\,c^{4}\,x^{4}}{2}-\frac{2\,a^{2}\,c^{4}\,x^{3}}{3}+a\,c^{4}\,x^{2}+c^{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^4*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] c^4*x + a*c^4*x^2 - (2*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + a^5*c^4*x^6 + (
2*a^6*c^4*x^7)/7 - (a^7*c^4*x^8)/4 - (a^8*c^4*x^9)/9
```

sympy [A] time = 0.10, size = 87, normalized size = 1.26

$$-\frac{a^{8}c^{4}x^{9}}{9}-\frac{a^{7}c^{4}x^{8}}{4}+\frac{2a^{6}c^{4}x^{7}}{7}+a^{5}c^{4}x^{6}-\frac{3a^{3}c^{4}x^{4}}{2}-\frac{2a^{2}c^{4}x^{3}}{3}+ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] -a**8*c**4*x**9/9 - a**7*c**4*x**8/4 + 2*a**6*c**4*x**7/7 + a**5*c**4*x**6
- 3*a**3*c**4*x**4/2 - 2*a**2*c**4*x**3/3 + a*c**4*x**2 + c**4*x
```

3.1049 $\int \frac{e^{2\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx$

Optimal. Leaf size=63

$$\frac{1}{a^5c(1-ax)}+\frac{4\log(1-ax)}{a^5c}+\frac{3x}{a^4c}+\frac{x^2}{a^3c}+\frac{x^3}{3a^2c}$$

```
[Out] 3*x/a^4/c+x^2/a^3/c+1/3*x^3/a^2/c+1/a^5/c/(-a*x+1)+4*ln(-a*x+1)/a^5/c
```

Rubi [A] time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 43}

$$\frac{x^3}{3a^2c}+\frac{x^2}{a^3c}+\frac{3x}{a^4c}+\frac{1}{a^5c(1-ax)}+\frac{4\log(1-ax)}{a^5c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] (3*x)/(a^4*c) + x^2/(a^3*c) + x^3/(3*a^2*c) + 1/(a^5*c*(1 - a*x)) + (4*Log[
1 - a*x])/(a^5*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx = \frac{\int \frac{x^4}{(1-ax)^2}\,dx}{c}$$
$$= \frac{\int \left(\frac{3}{a^4}+\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{1}{a^4(-1+ax)^2}+\frac{4}{a^4(-1+ax)}\right)dx}{c}$$
$$= \frac{3x}{a^4c}+\frac{x^2}{a^3c}+\frac{x^3}{3a^2c}+\frac{1}{a^5c(1-ax)}+\frac{4\log(1-ax)}{a^5c}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.78

$$\frac{a^3x^3+3a^2x^2+9ax+\frac{3}{1-ax}+12\log(1-ax)}{3a^5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2),x]
[Out] (9*a*x + 3*a^2*x^2 + a^3*x^3 + 3/(1 - a*x) + 12*Log[1 - a*x])/(3*a^5*c)
```

fricas [A] time = 0.54, size = 59, normalized size = 0.94

$$\frac{a^4x^4+2\,a^3x^3+6\,a^2x^2-9\,ax+12\,(ax-1)\log\left(ax-1\right)-3}{3\left(a^6cx-a^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/3*(a^4*x^4 + 2*a^3*x^3 + 6*a^2*x^2 - 9*a*x + 12*(a*x - 1)*log(a*x - 1) -
3)/(a^6*c*x - a^5*c)
```

giac [A] time = 0.18, size = 70, normalized size = 1.11

$$\frac{4\,\log\left(|ax-1|\right)}{a^5c}-\frac{1}{(ax-1)a^5c}+\frac{a^4c^2x^3+3\,a^3c^2x^2+9\,a^2c^2x}{3\,a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 4*log(abs(a*x - 1))/(a^5*c) - 1/((a*x - 1)*a^5*c) + 1/3*(a^4*c^2*x^3 + 3*a^
3*c^2*x^2 + 9*a^2*c^2*x)/(a^6*c^3)
```

maple [A] time = 0.03, size = 61, normalized size = 0.97

$$\frac{x^3}{3a^2c}+\frac{x^2}{a^3c}+\frac{3x}{a^4c}-\frac{1}{c\,a^5\,(ax-1)}+\frac{4\ln(ax-1)}{c\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*x^3/a^2/c+x^2/a^3/c+3*x/a^4/c-1/c/a^5/(a*x-1)+4/c/a^5*ln(a*x-1)
```

maxima [A] time = 0.30, size = 57, normalized size = 0.90

$$-\frac{1}{a^6cx-a^5c}+\frac{a^2x^3+3\,ax^2+9\,x}{3\,a^4c}+\frac{4\,\log(ax-1)}{a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^6*c*x - a^5*c) + 1/3*(a^2*x^3 + 3*a*x^2 + 9*x)/(a^4*c) + 4*log(a*x -
1)/(a^5*c)
```

mupad [B] time = 0.05, size = 64, normalized size = 1.02

$$\frac{1}{a\,\left(a^4\,c-a^5\,c\,x\right)}+\frac{3\,x}{a^4\,c}+\frac{x^3}{3\,a^2\,c}+\frac{x^2}{a^3\,c}+\frac{4\,\ln(a\,x-1)}{a^5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(a*x + 1)^2)/((c - a^2*c*x^2)*(a^2*x^2 - 1)),x)
```

```
[Out] 1/(a*(a^4*c - a^5*c*x)) + (3*x)/(a^4*c) + x^3/(3*a^2*c) + x^2/(a^3*c) + (4*
log(a*x - 1))/(a^5*c)
```

sympy [A] time = 0.19, size = 53, normalized size = 0.84

$$-\frac{1}{a^6cx-a^5c}+\frac{x^3}{3a^2c}+\frac{x^2}{a^3c}+\frac{3x}{a^4c}+\frac{4\log(ax-1)}{a^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**6*c*x - a**5*c) + x**3/(3*a**2*c) + x**2/(a**3*c) + 3*x/(a**4*c) + 4
*log(a*x - 1)/(a**5*c)
```

3.1050 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{c-a^2cx^2}\,dx$

Optimal. Leaf size=53

$$\frac{1}{a^4c(1-ax)}+\frac{3\log(1-ax)}{a^4c}+\frac{2x}{a^3c}+\frac{x^2}{2a^2c}$$

```
[Out] 2*x/a^3/c+1/2*x^2/a^2/c+1/a^4/c/(-a*x+1)+3*ln(-a*x+1)/a^4/c
```

Rubi [A] time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 43}

$$\frac{x^2}{2a^2c}+\frac{2x}{a^3c}+\frac{1}{a^4c(1-ax)}+\frac{3\log(1-ax)}{a^4c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] (2*x)/(a^3*c) + x^2/(2*a^2*c) + 1/(a^4*c*(1 - a*x)) + (3*Log[1 - a*x])/(a^4
*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^3}{c-a^2cx^2}\,dx = \frac{\int \frac{x^3}{(1-ax)^2}\,dx}{c}$$
$$= \frac{\int \left(\frac{2}{a^3}+\frac{x}{a^2}+\frac{1}{a^3(-1+ax)^2}+\frac{3}{a^3(-1+ax)}\right)dx}{c}$$
$$= \frac{2x}{a^3c}+\frac{x^2}{2a^2c}+\frac{1}{a^4c(1-ax)}+\frac{3\log(1-ax)}{a^4c}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.77

$$\frac{a^2x^2+4ax+\frac{2}{1-ax}+6\log(1-ax)}{2a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2),x]
[Out] (4*a*x + a^2*x^2 + 2/(1 - a*x) + 6*Log[1 - a*x])/(2*a^4*c)
```

fricas [A] time = 0.58, size = 51, normalized size = 0.96

$$\frac{a^3x^3+3\,a^2x^2-4\,ax+6\,(ax-1)\log(ax-1)-2}{2\left(a^5cx-a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/2*(a^3*x^3 + 3*a^2*x^2 - 4*a*x + 6*(a*x - 1)*log(a*x - 1) - 2)/(a^5*c*x -
 a^4*c)
```

giac [A] time = 0.23, size = 53, normalized size = 1.00

$$\frac{3\,\log(|ax-1|)}{a^4c}+\frac{a^2cx^2+4\,acx}{2\,a^4c^2}-\frac{1}{(ax-1)a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 3*log(abs(a*x - 1))/(a^4*c) + 1/2*(a^2*c*x^2 + 4*a*c*x)/(a^4*c^2) - 1/((a*x
 - 1)*a^4*c)
```

maple [A] time = 0.03, size = 51, normalized size = 0.96

$$\frac{x^2}{2a^2c} + \frac{2x}{a^3c} - \frac{1}{c\,a^4\,(ax-1)} + \frac{3\ln(ax-1)}{c\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c),x)
```

```
[Out] 1/2*x^2/a^2/c+2*x/a^3/c-1/c/a^4/(a*x-1)+3/c/a^4*ln(a*x-1)
```

maxima [A] time = 0.31, size = 49, normalized size = 0.92

$$-\frac{1}{a^5cx - a^4c} + \frac{ax^2 + 4\,x}{2\,a^3c} + \frac{3\,\log(ax-1)}{a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^5*c*x - a^4*c) + 1/2*(a*x^2 + 4*x)/(a^3*c) + 3*log(a*x - 1)/(a^4*c)
```

mupad [B] time = 0.89, size = 54, normalized size = 1.02

$$\frac{1}{a\,\left(a^3\,c - a^4\,c\,x\right)} + \frac{2\,x}{a^3\,c} + \frac{x^2}{2\,a^2\,c} + \frac{3\,\ln(a\,x-1)}{a^4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a*x + 1)^2)/((c - a^2*c*x^2)*(a^2*x^2 - 1)),x)
```

```
[Out] 1/(a*(a^3*c - a^4*c*x)) + (2*x)/(a^3*c) + x^2/(2*a^2*c) + (3*log(a*x - 1))/
(a^4*c)
```

sympy [A] time = 0.17, size = 44, normalized size = 0.83

$$-\frac{1}{a^5cx - a^4c} + \frac{x^2}{2a^2c} + \frac{2x}{a^3c} + \frac{3\log(ax-1)}{a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**5*c*x - a**4*c) + x**2/(2*a**2*c) + 2*x/(a**3*c) + 3*log(a*x - 1)/(a
**4*c)
```

3.1051 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx$

Optimal. Leaf size=39

$$\frac{1}{a^3c(1-ax)}+\frac{2\log(1-ax)}{a^3c}+\frac{x}{a^2c}$$

```
[Out] x/a^2/c+1/a^3/c/(-a*x+1)+2*ln(-a*x+1)/a^3/c
```

Rubi [A] time = 0.10, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 43}

$$\frac{x}{a^2c}+\frac{1}{a^3c(1-ax)}+\frac{2\log(1-ax)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] x/(a^2*c) + 1/(a^3*c*(1 - a*x)) + (2*Log[1 - a*x])/(a^3*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx = \frac{\int \frac{x^2}{(1-ax)^2}\,dx}{c}$$
$$= \frac{\int \left(\frac{1}{a^2}+\frac{1}{a^2(-1+ax)^2}+\frac{2}{a^2(-1+ax)}\right)dx}{c}$$
$$= \frac{x}{a^2c}+\frac{1}{a^3c(1-ax)}+\frac{2\log(1-ax)}{a^3c}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.72

$$\frac{ax+\frac{1}{1-ax}+2\log(1-ax)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
[Out] (a*x + (1 - a*x)^(-1) + 2*Log[1 - a*x])/(a^3*c)
```

fricas [A] time = 0.63, size = 42, normalized size = 1.08

$$\frac{a^2x^2-ax+2\,(ax-1)\log\left(ax-1\right)-1}{a^4cx-a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^4*c*x - a^3*c)
```

giac [A] time = 0.88, size = 39, normalized size = 1.00

$$\frac{x}{a^2c}+\frac{2\,\log\left(|ax-1|\right)}{a^3c}-\frac{1}{(ax-1)a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] x/(a^2*c) + 2*log(abs(a*x - 1))/(a^3*c) - 1/((a*x - 1)*a^3*c)
```

maple [A] time = 0.03, size = 39, normalized size = 1.00

$$\frac{x}{a^2c}-\frac{1}{c\,a^3\,(ax-1)}+\frac{2\ln\left(ax-1\right)}{c\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c),x)
```

```
[Out] x/a^2/c-1/c/a^3/(a*x-1)+2/c/a^3*ln(a*x-1)
```

maxima [A] time = 0.32, size = 40, normalized size = 1.03

$$-\frac{1}{a^4cx - a^3c} + \frac{x}{a^2c} + \frac{2\,\log\left(ax - 1\right)}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^4*c*x - a^3*c) + x/(a^2*c) + 2*log(a*x - 1)/(a^3*c)
```

mupad [B] time = 0.05, size = 38, normalized size = 0.97

$$\frac{1}{a^3\,c - a^4\,c\,x} + \frac{x}{a^2\,c} + \frac{2\,\ln\left(a\,x - 1\right)}{a^3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1)^2)/((c - a^2*c*x^2)*(a^2*x^2 - 1)),x)
```

```
[Out] 1/(a^3*c - a^4*c*x) + x/(a^2*c) + (2*log(a*x - 1))/(a^3*c)
```

sympy [A] time = 0.16, size = 32, normalized size = 0.82

$$-\frac{1}{a^4cx - a^3c} + \frac{x}{a^2c} + \frac{2\log\left(ax - 1\right)}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c)
```

3.1052 $\int \frac{e^{2\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx$

Optimal. Leaf size=30

$$\frac{1}{a^2c(1-ax)}+\frac{\log(1-ax)}{a^2c}$$

```
[Out] 1/a^2/c/(-a*x+1)+ln(-a*x+1)/a^2/c
```

Rubi [A] time = 0.07, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{1}{a^2c(1-ax)}+\frac{\log(1-ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] 1/(a^2*c*(1 - a*x)) + Log[1 - a*x]/(a^2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx = \frac{\int \frac{x}{(1-ax)^2}\,dx}{c} = \frac{\int \left(\frac{1}{a(-1+ax)^2}+\frac{1}{a(-1+ax)}\right)dx}{c} = \frac{1}{a^2c(1-ax)}+\frac{\log(1-ax)}{a^2c}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.77

$$\frac{\frac{1}{1-ax}+\log(1-ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
[Out] ((1 - a*x)^(-1) + Log[1 - a*x])/(a^2*c)
```

fricas [A] time = 0.64, size = 30, normalized size = 1.00

$$\frac{(ax-1)\log(ax-1)-1}{a^3cx-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] ((a*x - 1)*log(a*x - 1) - 1)/(a^3*c*x - a^2*c)
```

giac [A] time = 0.23, size = 30, normalized size = 1.00

$$\frac{\log(|ax-1|)}{a^2c}-\frac{1}{(ax-1)a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] log(abs(a*x - 1))/(a^2*c) - 1/((a*x - 1)*a^2*c)
```

maple [A] time = 0.03, size = 30, normalized size = 1.00

$$-\frac{1}{c\,a^2\,(ax-1)}+\frac{\ln(ax-1)}{c\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c/a^2/(a*x-1)+1/c/a^2*ln(a*x-1)
```

maxima [A] time = 0.32, size = 31, normalized size = 1.03

$$-\frac{1}{a^3cx-a^2c}+\frac{\log(ax-1)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^3*c*x - a^2*c) + log(a*x - 1)/(a^2*c)
```

mupad [B] time = 0.04, size = 27, normalized size = 0.90

$$\frac{1}{a^2\,(c-a\,c\,x)}+\frac{\ln(a\,x-1)}{a^2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x + 1)^2)/((c - a^2*c*x^2)*(a^2*x^2 - 1)),x)
```

```
[Out] 1/(a^2*(c - a*c*x)) + log(a*x - 1)/(a^2*c)
```

sympy [A] time = 0.13, size = 24, normalized size = 0.80

$$-\frac{1}{a^3cx-a^2c}+\frac{\log(ax-1)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**3*c*x - a**2*c) + log(a*x - 1)/(a**2*c)
```

3.1053 $\int \frac{e^{2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{ac(1-ax)}$$

```
[Out] 1/a/c/(-a*x+1)
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 32}

$$\frac{1}{ac(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] 1/(a*c*(1 - a*x))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx &= \frac{\int \frac{1}{(1-ax)^2}\,dx}{c}\\ &= \frac{1}{ac(1-ax)}\end{aligned}$$

Mathematica [C] time = 0.01, size = 18, normalized size = 1.20

$$\frac{e^{2\tanh^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(2*ArcTanh[a*x])/(2*a*c)
```

fricas [A] time = 0.45, size = 15, normalized size = 1.00

$$-\frac{1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/(a^2*c*x - a*c)
```

giac [A] time = 0.17, size = 15, normalized size = 1.00

$$-\frac{1}{(ax - 1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/((a*x - 1)*a*c)
```

maple [A] time = 0.02, size = 16, normalized size = 1.07

$$-\frac{1}{ca\,(ax - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c/a/(a*x-1)
```

maxima [A] time = 0.34, size = 15, normalized size = 1.00

$$-\frac{1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^2*c*x - a*c)
```

mupad [B] time = 0.89, size = 13, normalized size = 0.87

$$\frac{1}{a\,\left(c-a\,c\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - a^2*c*x^2)*(a^2*x^2 - 1)),x)
```

```
[Out] 1/(a*(c - a*c*x))
```

sympy [A] time = 0.14, size = 12, normalized size = 0.80

$$-\frac{1}{a^{2}cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a**2*c*x - a*c)
```

3.1054 $\int \frac{e^{2\tanh^{-1}(ax)}}{x(c-a^2cx^2)}\,dx$

Optimal. Leaf size=31

$$\frac{1}{c(1-ax)}-\frac{\log(1-ax)}{c}+\frac{\log(x)}{c}$$

```
[Out] 1/c/(-a*x+1)+ln(x)/c-ln(-a*x+1)/c
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 44}

$$\frac{1}{c(1-ax)}-\frac{\log(1-ax)}{c}+\frac{\log(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)),x]
```

```
[Out] 1/(c*(1 - a*x)) + Log[x]/c - Log[1 - a*x]/c
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)}\,dx = \frac{\int \frac{1}{x(1-ax)^2}\,dx}{c}$$
$$= \frac{\int \left(\frac{1}{x}+\frac{a}{(-1+ax)^2}-\frac{a}{-1+ax}\right)dx}{c}$$
$$= \frac{1}{c(1-ax)}+\frac{\log(x)}{c}-\frac{\log(1-ax)}{c}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.77

$$\frac{\frac{1}{1-ax}-\log(1-ax)+\log(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)),x]
```

```
[Out] ((1 - a*x)^(-1) + Log[x] - Log[1 - a*x])/c
```

fricas [A] time = 0.65, size = 35, normalized size = 1.13

$$-\frac{(ax-1)\log(ax-1)-(ax-1)\log(x)+1}{acx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -((a*x - 1)*log(a*x - 1) - (a*x - 1)*log(x) + 1)/(a*c*x - c)
```

giac [A] time = 0.21, size = 32, normalized size = 1.03

$$-\frac{\log(|ax-1|)}{c}+\frac{\log(|x|)}{c}-\frac{1}{(ax-1)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -log(abs(a*x - 1))/c + log(abs(x))/c - 1/((a*x - 1)*c)
```

maple [A] time = 0.03, size = 31, normalized size = 1.00

$$\frac{\ln(x)}{c}-\frac{1}{c\,(ax-1)}-\frac{\ln(ax-1)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c),x)
```

```
[Out] ln(x)/c-1/c/(a*x-1)-1/c*ln(a*x-1)
```

maxima [A] time = 0.31, size = 30, normalized size = 0.97

$$-\frac{\log(ax-1)}{c}+\frac{\log(x)}{c}-\frac{1}{acx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -log(a*x - 1)/c + log(x)/c - 1/(a*c*x - c)
```

mupad [B] time = 0.91, size = 22, normalized size = 0.71

$$\frac{2\,\mathrm{atanh}\left(2\,a\,x-1\right)}{c}+\frac{1}{c-a\,c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x*(c - a^2*c*x^2)*(a^2*x^2 - 1)),x)
```

```
[Out] (2*atanh(2*a*x - 1))/c + 1/(c - a*c*x)
```

sympy [A] time = 0.22, size = 19, normalized size = 0.61

$$-\frac{1}{acx-c}+\frac{\log(x)-\log\left(x-\frac{1}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c),x)
```

```
[Out] -1/(a*c*x - c) + (log(x) - log(x - 1/a))/c
```

3.1055 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)}\,dx$

Optimal. Leaf size=43

$$\frac{a}{c(1-ax)}+\frac{2a\log(x)}{c}-\frac{2a\log(1-ax)}{c}-\frac{1}{cx}$$

```
[Out] -1/c/x+a/c/(-a*x+1)+2*a*ln(x)/c-2*a*ln(-a*x+1)/c
```

Rubi [A] time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 44}

$$\frac{a}{c(1-ax)}+\frac{2a\log(x)}{c}-\frac{2a\log(1-ax)}{c}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] -(1/(c*x)) + a/(c*(1 - a*x)) + (2*a*Log[x])/c - (2*a*Log[1 - a*x])/c
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)}\,dx = \frac{\int \frac{1}{x^2(1-ax)^2}\,dx}{c}$$
$$= \frac{\int \left(\frac{1}{x^2}+\frac{2a}{x}+\frac{a^2}{(-1+ax)^2}-\frac{2a^2}{-1+ax}\right)dx}{c}$$
$$= -\frac{1}{cx}+\frac{a}{c(1-ax)}+\frac{2a\log(x)}{c}-\frac{2a\log(1-ax)}{c}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.81

$$\frac{\frac{a}{1-ax}+2a\log(x)-2a\log(1-ax)-\frac{1}{x}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)),x]
[Out] (-x^(-1) + a/(1 - a*x) + 2*a*Log[x] - 2*a*Log[1 - a*x])/c
```

fricas [A] time = 0.71, size = 57, normalized size = 1.33

$$-\frac{2\,ax+2\left(a^2x^2-ax\right)\log\left(ax-1\right)-2\left(a^2x^2-ax\right)\log(x)-1}{acx^2-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -(2*a*x + 2*(a^2*x^2 - a*x)*log(a*x - 1) - 2*(a^2*x^2 - a*x)*log(x) - 1)/(a
*c*x^2 - c*x)
```

giac [A] time = 0.45, size = 45, normalized size = 1.05

$$-\frac{2\,a\log\left(|ax-1|\right)}{c}+\frac{2\,a\log\left(|x|\right)}{c}-\frac{2\,ax-1}{\left(ax^2-x\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] -2*a*log(abs(a*x - 1))/c + 2*a*log(abs(x))/c - (2*a*x - 1)/((a*x^2 - x)*c)
```

maple [A]   time = 0.03, size = 43, normalized size = 1.00

$$-\frac{1}{cx}+\frac{2a\ln(x)}{c}-\frac{a}{c\,(ax-1)}-\frac{2a\ln\,(ax-1)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c),x)
```

```
[Out] -1/c/x+2*a*ln(x)/c-1/c*a/(a*x-1)-2/c*a*ln(a*x-1)
```

maxima [A]   time = 0.31, size = 42, normalized size = 0.98

$$-\frac{2\,a\log\,(ax-1)}{c}+\frac{2\,a\log(x)}{c}-\frac{2\,ax-1}{acx^2-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -2*a*log(a*x - 1)/c + 2*a*log(x)/c - (2*a*x - 1)/(a*c*x^2 - c*x)
```

mupad [B]   time = 0.07, size = 34, normalized size = 0.79

$$\frac{2\,a\,x-1}{c\,x-a\,c\,x^2}+\frac{4\,a\,\mathrm{atanh}\,(2\,a\,x-1)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^2*(c - a^2*c*x^2)*(a^2*x^2 - 1)),x)
```

```
[Out] (2*a*x - 1)/(c*x - a*c*x^2) + (4*a*atanh(2*a*x - 1))/c
```

sympy [A]   time = 0.24, size = 31, normalized size = 0.72

$$\frac{2a\left(\log\,(x)-\log\left(x-\frac{1}{a}\right)\right)}{c}+\frac{-2ax+1}{acx^2-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c),x)
```

```
[Out] 2*a*(log(x) - log(x - 1/a))/c + (-2*a*x + 1)/(a*c*x**2 - c*x)
```

3.1056 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)}\,dx$

Optimal. Leaf size=60

$$\frac{a^2}{c(1-ax)}+\frac{3a^2\log(x)}{c}-\frac{3a^2\log(1-ax)}{c}-\frac{2a}{cx}-\frac{1}{2cx^2}$$

```
[Out] -1/2/c/x^2-2*a/c/x+a^2/c/(-a*x+1)+3*a^2*ln(x)/c-3*a^2*ln(-a*x+1)/c
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 44}

$$\frac{a^2}{c(1-ax)}+\frac{3a^2\log(x)}{c}-\frac{3a^2\log(1-ax)}{c}-\frac{2a}{cx}-\frac{1}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(2*c*x^2) - (2*a)/(c*x) + a^2/(c*(1 - a*x)) + (3*a^2*Log[x])/c - (3*a^2*
Log[1 - a*x])/c
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)}\,dx = \frac{\int \frac{1}{x^3(1-ax)^2}\,dx}{c}$$

$$= \frac{\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{3a^2}{x}+\frac{a^3}{(-1+ax)^2}-\frac{3a^3}{-1+ax}\right)dx}{c}$$

$$= -\frac{1}{2cx^2}-\frac{2a}{cx}+\frac{a^2}{c(1-ax)}+\frac{3a^2\log(x)}{c}-\frac{3a^2\log(1-ax)}{c}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.87

$$\frac{\frac{-6a^2x^2+3ax+1}{x^2(ax-1)}+6a^2\log(x)-6a^2\log(1-ax)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)),x]
```

```
[Out] ((1 + 3*a*x - 6*a^2*x^2)/(x^2*(-1 + a*x)) + 6*a^2*Log[x] - 6*a^2*Log[1 - a*
x])/(2*c)
```

fricas [A] time = 0.54, size = 75, normalized size = 1.25

$$-\frac{6\,a^2x^2-3\,ax+6\left(a^3x^3-a^2x^2\right)\log\left(ax-1\right)-6\left(a^3x^3-a^2x^2\right)\log(x)-1}{2\left(acx^3-cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(6*a^2*x^2 - 3*a*x + 6*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 6*(a^3*x^3 -
 a^2*x^2)*log(x) - 1)/(a*c*x^3 - c*x^2)
```

giac [A] time = 0.43, size = 56, normalized size = 0.93

$$-\frac{3\,a^2\log\left(|ax-1|\right)}{c}+\frac{3\,a^2\log\left(|x|\right)}{c}-\frac{6\,a^2x^2-3\,ax-1}{2\,(ax-1)cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c),x, algorithm="giac")
```

[Out] -3*a^2*log(abs(a*x - 1))/c + 3*a^2*log(abs(x))/c - 1/2*(6*a^2*x^2 - 3*a*x - 1)/((a*x - 1)*c*x^2)

maple [A] time = 0.04, size = 58, normalized size = 0.97

$$-\frac{1}{2c\,x^2}-\frac{2a}{cx}+\frac{3a^2\ln(x)}{c}-\frac{a^2}{c\,(ax-1)}-\frac{3a^2\ln\,(ax-1)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c),x)

[Out] -1/2/c/x^2-2*a/c/x+3*a^2*ln(x)/c-1/c*a^2/(a*x-1)-3/c*a^2*ln(a*x-1)

maxima [A] time = 0.31, size = 56, normalized size = 0.93

$$-\frac{3\,a^2\log\,(ax-1)}{c}+\frac{3\,a^2\log(x)}{c}-\frac{6\,a^2x^2-3\,ax-1}{2\left(acx^3-cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c),x, algorithm="maxima")

[Out] -3*a^2*log(a*x - 1)/c + 3*a^2*log(x)/c - 1/2*(6*a^2*x^2 - 3*a*x - 1)/(a*c*x^3 - c*x^2)

mupad [B] time = 0.08, size = 47, normalized size = 0.78

$$\frac{6\,a^2\,\mathrm{atanh}\,(2\,a\,x-1)}{c}-\frac{-3\,a^2\,x^2+\frac{3\,a\,x}{2}+\frac{1}{2}}{c\,x^2-a\,c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a*x + 1)^2/(x^3*(c - a^2*c*x^2)*(a^2*x^2 - 1)),x)

[Out] (6*a^2*atanh(2*a*x - 1))/c - ((3*a*x)/2 - 3*a^2*x^2 + 1/2)/(c*x^2 - a*c*x^3)

sympy [A] time = 0.27, size = 46, normalized size = 0.77

$$\frac{3a^2\left(\log\,(x)-\log\left(x-\frac{1}{a}\right)\right)}{c}+\frac{-6a^2x^2+3ax+1}{2acx^3-2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c),x)

[Out] 3*a**2*(log(x) - log(x - 1/a))/c + (-6*a**2*x**2 + 3*a*x + 1)/(2*a*c*x**3 - 2*c*x**2)

3.1057 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)}\,dx$

Optimal. Leaf size=71

$$\frac{a^3}{c(1-ax)}+\frac{4a^3\log(x)}{c}-\frac{4a^3\log(1-ax)}{c}-\frac{3a^2}{cx}-\frac{a}{cx^2}-\frac{1}{3cx^3}$$

```
[Out] -1/3/c/x^3-a/c/x^2-3*a^2/c/x+a^3/c/(-a*x+1)+4*a^3*ln(x)/c-4*a^3*ln(-a*x+1)/
c
```

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 44}

$$\frac{a^3}{c(1-ax)}-\frac{3a^2}{cx}+\frac{4a^3\log(x)}{c}-\frac{4a^3\log(1-ax)}{c}-\frac{a}{cx^2}-\frac{1}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^4*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(3*c*x^3) - a/(c*x^2) - (3*a^2)/(c*x) + a^3/(c*(1 - a*x)) + (4*a^3*Log[x
])/c - (4*a^3*Log[1 - a*x])/c
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)}\,dx = \frac{\int \frac{1}{x^4(1-ax)^2}\,dx}{c}$$
$$= \frac{\int \left(\frac{1}{x^4}+\frac{2a}{x^3}+\frac{3a^2}{x^2}+\frac{4a^3}{x}+\frac{a^4}{(-1+ax)^2}-\frac{4a^4}{-1+ax}\right)dx}{c}$$
$$= -\frac{1}{3cx^3}-\frac{a}{cx^2}-\frac{3a^2}{cx}+\frac{a^3}{c(1-ax)}+\frac{4a^3\log(x)}{c}-\frac{4a^3\log(1-ax)}{c}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.00

$$\frac{a^3}{c(1-ax)}+\frac{4a^3\log(x)}{c}-\frac{4a^3\log(1-ax)}{c}-\frac{3a^2}{cx}-\frac{a}{cx^2}-\frac{1}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^4*(c - a^2*c*x^2)),x]
```

```
[Out] -1/3*1/(c*x^3) - a/(c*x^2) - (3*a^2)/(c*x) + a^3/(c*(1 - a*x)) + (4*a^3*Log
[x])/c - (4*a^3*Log[1 - a*x])/c
```

fricas [A] time = 0.57, size = 83, normalized size = 1.17

$$-\frac{12\,a^3x^3-6\,a^2x^2-2\,ax+12\left(a^4x^4-a^3x^3\right)\log\left(ax-1\right)-12\left(a^4x^4-a^3x^3\right)\log(x)-1}{3\left(acx^4-cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/3*(12*a^3*x^3 - 6*a^2*x^2 - 2*a*x + 12*(a^4*x^4 - a^3*x^3)*log(a*x - 1)
- 12*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*c*x^4 - c*x^3)
```

giac [A] time = 0.31, size = 64, normalized size = 0.90

$$-\frac{4\,a^3\log\left(|ax-1|\right)}{c}+\frac{4\,a^3\log\left(|x|\right)}{c}-\frac{12\,a^3x^3-6\,a^2x^2-2\,ax-1}{3\,(ax-1)cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c),x, algorithm="giac")
```

[Out] -4*a^3*log(abs(a*x - 1))/c + 4*a^3*log(abs(x))/c - 1/3*(12*a^3*x^3 - 6*a^2*
x^2 - 2*a*x - 1)/((a*x - 1)*c*x^3)

maple [A] time = 0.04, size = 69, normalized size = 0.97

$$-\frac{1}{3c\,x^{3}}-\frac{a}{c\,x^{2}}-\frac{3a^{2}}{cx}+\frac{4a^{3}\ln(x)}{c}-\frac{a^{3}}{c\,(ax-1)}-\frac{4a^{3}\ln\left(ax-1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c),x)

[Out] -1/3/c/x^3-a/c/x^2-3*a^2/c/x+4*a^3*ln(x)/c-1/c*a^3/(a*x-1)-4/c*a^3*ln(a*x-1
)

maxima [A] time = 0.31, size = 64, normalized size = 0.90

$$-\frac{4\,a^{3}\log\left(ax-1\right)}{c}+\frac{4\,a^{3}\log(x)}{c}-\frac{12\,a^{3}x^{3}-6\,a^{2}x^{2}-2\,ax-1}{3\left(acx^{4}-cx^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c),x, algorithm="maxima")

[Out] -4*a^3*log(a*x - 1)/c + 4*a^3*log(x)/c - 1/3*(12*a^3*x^3 - 6*a^2*x^2 - 2*a*
x - 1)/(a*c*x^4 - c*x^3)

mupad [B] time = 0.93, size = 55, normalized size = 0.77

$$\frac{8\,a^{3}\,\mathrm{atanh}\left(2\,a\,x-1\right)}{c}-\frac{-4\,a^{3}\,x^{3}+2\,a^{2}\,x^{2}+\frac{2\,a\,x}{3}+\frac{1}{3}}{c\,x^{3}-a\,c\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a*x + 1)^2/(x^4*(c - a^2*c*x^2)*(a^2*x^2 - 1)),x)

[Out] (8*a^3*atanh(2*a*x - 1))/c - ((2*a*x)/3 + 2*a^2*x^2 - 4*a^3*x^3 + 1/3)/(c*x
^3 - a*c*x^4)

sympy [A] time = 0.29, size = 54, normalized size = 0.76

$$\frac{4a^{3}\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)}{c}+\frac{-12a^{3}x^{3}+6a^{2}x^{2}+2ax+1}{3acx^{4}-3cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**4/(-a**2*c*x**2+c),x)

[Out] 4*a**3*(log(x) - log(x - 1/a))/c + (-12*a**3*x**3 + 6*a**2*x**2 + 2*a*x + 1
)/(3*a*c*x**4 - 3*c*x**3)

3.1058 $\int \frac{e^{2\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{7}{4a^5c^2(1-ax)}+\frac{1}{4a^5c^2(1-ax)^2}-\frac{17\log(1-ax)}{8a^5c^2}+\frac{\log(ax+1)}{8a^5c^2}-\frac{x}{a^4c^2}$$

```
[Out] -x/a^4/c^2+1/4/a^5/c^2/(-a*x+1)^2-7/4/a^5/c^2/(-a*x+1)-17/8*ln(-a*x+1)/a^5/
c^2+1/8*ln(a*x+1)/a^5/c^2
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$-\frac{x}{a^4c^2}-\frac{7}{4a^5c^2(1-ax)}+\frac{1}{4a^5c^2(1-ax)^2}-\frac{17\log(1-ax)}{8a^5c^2}+\frac{\log(ax+1)}{8a^5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(x/(a^4*c^2)) + 1/(4*a^5*c^2*(1 - a*x)^2) - 7/(4*a^5*c^2*(1 - a*x)) - (17*
Log[1 - a*x])/(8*a^5*c^2) + Log[1 + a*x]/(8*a^5*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \left(-\frac{1}{a^4} - \frac{1}{2a^4(-1+ax)^3} - \frac{7}{4a^4(-1+ax)^2} - \frac{17}{8a^4(-1+ax)} + \frac{1}{8a^4(1+ax)}\right)dx}{c^2}$$
$$= -\frac{x}{a^4c^2} + \frac{1}{4a^5c^2(1-ax)^2} - \frac{7}{4a^5c^2(1-ax)} - \frac{17\log(1-ax)}{8a^5c^2} + \frac{\log(1+ax)}{8a^5c^2}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.86

$$\frac{-\frac{7}{4a^5(1-ax)} + \frac{1}{4a^5(1-ax)^2} - \frac{17\log(1-ax)}{8a^5} + \frac{\log(ax+1)}{8a^5} - \frac{x}{a^4}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-(x/a^4) + 1/(4*a^5*(1 - a*x)^2) - 7/(4*a^5*(1 - a*x)) - (17*Log[1 - a*x])
/(8*a^5) + Log[1 + a*x]/(8*a^5))/c^2
```

fricas [A] time = 0.61, size = 95, normalized size = 1.20

$$-\frac{8\,a^3x^3 - 16\,a^2x^2 - 6\,ax - (a^2x^2 - 2\,ax + 1)\log(ax+1) + 17\,(a^2x^2 - 2\,ax + 1)\log(ax-1) + 12}{8\,(a^7c^2x^2 - 2\,a^6c^2x + a^5c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
 17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^7*c^2*x^2 - 2*a^6*c^2*x + a
^5*c^2)
```

giac [A] time = 0.16, size = 61, normalized size = 0.77

$$-\frac{x}{a^4c^2} + \frac{\log(|ax+1|)}{8\,a^5c^2} - \frac{17\,\log(|ax-1|)}{8\,a^5c^2} + \frac{7\,ax - 6}{4\,(ax-1)^2a^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -x/(a^4*c^2) + 1/8*log(abs(a*x + 1))/(a^5*c^2) - 17/8*log(abs(a*x - 1))/(a^
5*c^2) + 1/4*(7*a*x - 6)/((a*x - 1)^2*a^5*c^2)
```

maple [A] time = 0.04, size = 69, normalized size = 0.87

$$-\frac{x}{a^4c^2}+\frac{7}{4c^2a^5(ax-1)}-\frac{17\ln(ax-1)}{8c^2a^5}+\frac{1}{4c^2a^5(ax-1)^2}+\frac{\ln(ax+1)}{8a^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -x/a^4/c^2+7/4/c^2/a^5/(a*x-1)-17/8/c^2/a^5*ln(a*x-1)+1/4/c^2/a^5/(a*x-1)^2
+1/8*ln(a*x+1)/a^5/c^2
```

maxima [A] time = 0.34, size = 75, normalized size = 0.95

$$\frac{7\,ax-6}{4\left(a^7c^2x^2-2\,a^6c^2x+a^5c^2\right)}-\frac{x}{a^4c^2}+\frac{\log(ax+1)}{8\,a^5c^2}-\frac{17\,\log(ax-1)}{8\,a^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*(7*a*x - 6)/(a^7*c^2*x^2 - 2*a^6*c^2*x + a^5*c^2) - x/(a^4*c^2) + 1/8*l
og(a*x + 1)/(a^5*c^2) - 17/8*log(a*x - 1)/(a^5*c^2)
```

mupad [B] time = 0.10, size = 77, normalized size = 0.97

$$\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^6\,c^2\,x^2-2\,a^5\,c^2\,x+a^4\,c^2}-\frac{x}{a^4\,c^2}-\frac{17\,\ln(a\,x-1)}{8\,a^5\,c^2}+\frac{\ln(a\,x+1)}{8\,a^5\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(a*x + 1)^2)/((c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] ((7*x)/4 - 3/(2*a))/(a^4*c^2 - 2*a^5*c^2*x + a^6*c^2*x^2) - x/(a^4*c^2) - (
17*log(a*x - 1))/(8*a^5*c^2) + log(a*x + 1)/(8*a^5*c^2)
```

sympy [A] time = 0.46, size = 71, normalized size = 0.90

$$-\frac{-7ax+6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}-\frac{x}{a^4c^2}-\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(-7*a*x + 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) - x/(a**4*c*
*2) - (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2)
```

3.1059 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{5}{4a^4c^2(1-ax)}+\frac{1}{4a^4c^2(1-ax)^2}-\frac{7\log(1-ax)}{8a^4c^2}-\frac{\log(ax+1)}{8a^4c^2}$$

```
[Out] 1/4/a^4/c^2/(-a*x+1)^2-5/4/a^4/c^2/(-a*x+1)-7/8*ln(-a*x+1)/a^4/c^2-1/8*ln(a
*x+1)/a^4/c^2
```

Rubi [A] time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$-\frac{5}{4a^4c^2(1-ax)}+\frac{1}{4a^4c^2(1-ax)^2}-\frac{7\log(1-ax)}{8a^4c^2}-\frac{\log(ax+1)}{8a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^4*c^2*(1 - a*x)^2) - 5/(4*a^4*c^2*(1 - a*x)) - (7*Log[1 - a*x])/(8*a
^4*c^2) - Log[1 + a*x]/(8*a^4*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{x^3}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \left(-\frac{1}{2a^3(-1+ax)^3}-\frac{5}{4a^3(-1+ax)^2}-\frac{7}{8a^3(-1+ax)}-\frac{1}{8a^3(1+ax)}\right)dx}{c^2}$$
$$= \frac{1}{4a^4c^2(1-ax)^2}-\frac{5}{4a^4c^2(1-ax)}-\frac{7\log(1-ax)}{8a^4c^2}-\frac{\log(1+ax)}{8a^4c^2}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.76

$$-\frac{-10ax+7(ax-1)^2\log(1-ax)+(ax-1)^2\log(ax+1)+8}{8a^4c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/8*(8 - 10*a*x + 7*(-1 + a*x)^2*Log[1 - a*x] + (-1 + a*x)^2*Log[1 + a*x])
/(a^4*c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.50, size = 79, normalized size = 1.13

$$\frac{10\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)-7\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)-8}{8\left(a^6c^2x^2-2\,a^5c^2x+a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/8*(10*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) - 7*(a^2*x^2 - 2*a*x + 1)*
log(a*x - 1) - 8)/(a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2)
```

giac [A] time = 0.34, size = 52, normalized size = 0.74

$$-\frac{\log\left(|ax+1|\right)}{8\,a^4c^2}-\frac{7\,\log\left(|ax-1|\right)}{8\,a^4c^2}+\frac{5\,ax-4}{4\,(ax-1)^2a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a^4*c^2) - 7/8*log(abs(a*x - 1))/(a^4*c^2) + 1/4*(5
*a*x - 4)/((a*x - 1)^2*a^4*c^2)
```

maple [A] time = 0.04, size = 60, normalized size = 0.86

$$\frac{1}{4c^2a^4\left(ax-1\right)^2}+\frac{5}{4c^2a^4\left(ax-1\right)}-\frac{7\ln\left(ax-1\right)}{8c^2a^4}-\frac{\ln\left(ax+1\right)}{8a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4/c^2/a^4/(a*x-1)^2+5/4/c^2/a^4/(a*x-1)-7/8/c^2/a^4*ln(a*x-1)-1/8*ln(a*x+
1)/a^4/c^2
```

maxima [A] time = 0.32, size = 66, normalized size = 0.94

$$\frac{5\,ax-4}{4\left(a^6c^2x^2-2\,a^5c^2x+a^4c^2\right)}-\frac{\log\left(ax+1\right)}{8\,a^4c^2}-\frac{7\,\log\left(ax-1\right)}{8\,a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*(5*a*x - 4)/(a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2) - 1/8*log(a*x + 1)/(a
^4*c^2) - 7/8*log(a*x - 1)/(a^4*c^2)
```

mupad [B] time = 0.21, size = 65, normalized size = 0.93

$$\frac{\frac{5\,x}{4\,a^3}-\frac{1}{a^4}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}-\frac{7\,\ln\left(a\,x-1\right)}{8\,a^4\,c^2}-\frac{\ln\left(a\,x+1\right)}{8\,a^4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a*x + 1)^2)/((c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] ((5*x)/(4*a^3) - 1/a^4)/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) - (7*log(a*x - 1))/
(8*a^4*c^2) - log(a*x + 1)/(8*a^4*c^2)
```

sympy [A] time = 0.35, size = 63, normalized size = 0.90

$$-\frac{-5ax+4}{4a^6c^2x^2-8a^5c^2x+4a^4c^2}-\frac{\frac{7\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(-5*a*x + 4)/(4*a**6*c**2*x**2 - 8*a**5*c**2*x + 4*a**4*c**2) - (7*log(x -
 1/a)/8 + log(x + 1/a)/8)/(a**4*c**2)
```

3.1060 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{3}{4a^3c^2(1-ax)}+\frac{1}{4a^3c^2(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4a^3c^2}$$

```
[Out] 1/4/a^3/c^2/(-a*x+1)^2-3/4/a^3/c^2/(-a*x+1)+1/4*arctanh(a*x)/a^3/c^2
```

Rubi [A] time = 0.11, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6150, 88, 207}

$$-\frac{3}{4a^3c^2(1-ax)}+\frac{1}{4a^3c^2(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4a^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^3*c^2*(1 - a*x)^2) - 3/(4*a^3*c^2*(1 - a*x)) + ArcTanh[a*x]/(4*a^3*c
^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{x^2}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \left(-\frac{1}{2a^2(-1+ax)^3}-\frac{3}{4a^2(-1+ax)^2}-\frac{1}{4a^2(-1+a^2x^2)}\right)dx}{c^2}$$
$$= \frac{1}{4a^3c^2(1-ax)^2}-\frac{3}{4a^3c^2(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4a^2c^2}$$
$$= \frac{1}{4a^3c^2(1-ax)^2}-\frac{3}{4a^3c^2(1-ax)}+\frac{\tanh^{-1}(ax)}{4a^3c^2}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.69

$$\frac{3ax+(ax-1)^2\tanh^{-1}(ax)-2}{4a^3c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-2 + 3*a*x + (-1 + a*x)^2*ArcTanh[a*x])/(4*a^3*c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.58, size = 78, normalized size = 1.53

$$\frac{6\,ax+\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)-\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)-4}{8\left(a^5c^2x^2-2\,a^4c^2x+a^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] 1/8*(6*a*x + (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) - (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^5*c^2*x^2 - 2*a^4*c^2*x + a^3*c^2)
```

giac [A] time = 0.30, size = 52, normalized size = 1.02

$$\frac{\log\left(|ax+1|\right)}{8\,a^3c^2}-\frac{\log\left(|ax-1|\right)}{8\,a^3c^2}+\frac{3\,ax-2}{4\,(ax-1)^2a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(a*x + 1))/(a^3*c^2) - 1/8*log(abs(a*x - 1))/(a^3*c^2) + 1/4*(3*
a*x - 2)/((a*x - 1)^2*a^3*c^2)
```

maple [A] time = 0.03, size = 60, normalized size = 1.18

$$\frac{1}{4c^2a^3\left(ax-1\right)^2}+\frac{3}{4c^2a^3\left(ax-1\right)}-\frac{\ln\left(ax-1\right)}{8c^2a^3}+\frac{\ln\left(ax+1\right)}{8c^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4/c^2/a^3/(a*x-1)^2+3/4/c^2/a^3/(a*x-1)-1/8/c^2/a^3*ln(a*x-1)+1/8/c^2/a^3
*ln(a*x+1)
```

maxima [A] time = 0.32, size = 66, normalized size = 1.29

$$\frac{3\,ax-2}{4\left(a^5c^2x^2-2\,a^4c^2x+a^3c^2\right)}+\frac{\log\left(ax+1\right)}{8\,a^3c^2}-\frac{\log\left(ax-1\right)}{8\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*(3*a*x - 2)/(a^5*c^2*x^2 - 2*a^4*c^2*x + a^3*c^2) + 1/8*log(a*x + 1)/(a
^3*c^2) - 1/8*log(a*x - 1)/(a^3*c^2)
```

mupad [B] time = 0.07, size = 49, normalized size = 0.96

$$\frac{\frac{3\,x}{4\,a^2}-\frac{1}{2\,a^3}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}+\frac{\mathrm{atanh}\left(a\,x\right)}{4\,a^3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1)^2)/((c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] ((3*x)/(4*a^2) - 1/(2*a^3))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + atanh(a*x)/(4
*a^3*c^2)
```

sympy [A] time = 0.30, size = 61, normalized size = 1.20

$$-\frac{-3ax+2}{4a^5c^2x^2-8a^4c^2x+4a^3c^2}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(-3*a*x + 2)/(4*a**5*c**2*x**2 - 8*a**4*c**2*x + 4*a**3*c**2) - (log(x - 1
/a)/8 - log(x + 1/a)/8)/(a**3*c**2)
```

3.1061 $\int \frac{e^{2\tanh^{-1}(ax)}x}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{4a^2c^2(1-ax)}+\frac{1}{4a^2c^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4a^2c^2}$$

```
[Out] 1/4/a^2/c^2/(-a*x+1)^2-1/4/a^2/c^2/(-a*x+1)-1/4*arctanh(a*x)/a^2/c^2
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6150, 77, 207}

$$-\frac{1}{4a^2c^2(1-ax)}+\frac{1}{4a^2c^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a^2*c^2*(1 - a*x)^2) - 1/(4*a^2*c^2*(1 - a*x)) - ArcTanh[a*x]/(4*a^2*c
^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x}{(c-a^2cx^2)^2}\,dx &= \frac{\int \frac{x}{(1-ax)^3(1+ax)}\,dx}{c^2}\\
&= \frac{\int \left(-\frac{1}{2a(-1+ax)^3}-\frac{1}{4a(-1+ax)^2}+\frac{1}{4a(-1+a^2x^2)}\right)dx}{c^2}\\
&= \frac{1}{4a^2c^2(1-ax)^2}-\frac{1}{4a^2c^2(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4ac^2}\\
&= \frac{1}{4a^2c^2(1-ax)^2}-\frac{1}{4a^2c^2(1-ax)}-\frac{\tanh^{-1}(ax)}{4a^2c^2}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.67

$$\frac{ax-(ax-1)^2\tanh^{-1}(ax)}{4a^2c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a^2*c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.54, size = 77, normalized size = 1.51

$$\frac{2\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)}{8\left(a^4c^2x^2-2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1))/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

giac [A] time = 0.21, size = 47, normalized size = 0.92

$$-\frac{\log\left(|ax+1|\right)}{8\,a^2c^2}+\frac{\log\left(|ax-1|\right)}{8\,a^2c^2}+\frac{x}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a^2*c^2) + 1/8*log(abs(a*x - 1))/(a^2*c^2) + 1/4*x/
((a*x - 1)^2*a*c^2)
```

maple [A] time = 0.04, size = 60, normalized size = 1.18

$$\frac{1}{4c^2a^2\left(ax-1\right)^2}+\frac{1}{4c^2a^2\left(ax-1\right)}+\frac{\ln\left(ax-1\right)}{8c^2a^2}-\frac{\ln\left(ax+1\right)}{8c^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4/c^2/a^2/(a*x-1)^2+1/4/c^2/a^2/(a*x-1)+1/8/c^2/a^2*ln(a*x-1)-1/8/c^2/a^2
*ln(a*x+1)
```

maxima [A] time = 0.34, size = 59, normalized size = 1.16

$$\frac{x}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}-\frac{\log\left(ax+1\right)}{8\,a^2c^2}+\frac{\log\left(ax-1\right)}{8\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a^2*c^2) + 1/
8*log(a*x - 1)/(a^2*c^2)
```

mupad [B] time = 0.92, size = 42, normalized size = 0.82

$$\frac{x}{4\,a\,\left(a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2\right)}-\frac{\mathrm{atanh}\left(a\,x\right)}{4\,a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x + 1)^2)/((c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] x/(4*a*(c^2 + a^2*c^2*x^2 - 2*a*c^2*x)) - atanh(a*x)/(4*a^2*c^2)
```

sympy [A] time = 0.28, size = 53, normalized size = 1.04

$$\frac{x}{4a^3c^2x^2-8a^2c^2x+4ac^2}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] x/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) - (-log(x - 1/a)/8 + log(x
+ 1/a)/8)/(a**2*c**2)
```

3.1062 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$\frac{1}{4ac^2(1-ax)}+\frac{1}{4ac^2(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] 1/4/a/c^2/(-a*x+1)^2+1/4/a/c^2/(-a*x+1)+1/4*arctanh(a*x)/a/c^2
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{1}{4ac^2(1-ax)}+\frac{1}{4ac^2(1-ax)^2}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(4*a*c^2*(1 - a*x)^2) + 1/(4*a*c^2*(1 - a*x)) + ArcTanh[a*x]/(4*a*c^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^2}$$
$$= \frac{1}{4ac^2(1-ax)^2}+\frac{1}{4ac^2(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2}$$
$$= \frac{1}{4ac^2(1-ax)^2}+\frac{1}{4ac^2(1-ax)}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.69

$$\frac{-ax+(ax-1)^2\tanh^{-1}(ax)+2}{4ac^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2 - a*x + (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.51, size = 76, normalized size = 1.49

$$-\frac{2\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)-4}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.34, size = 51, normalized size = 1.00

$$\frac{\log\left(|ax+1|\right)}{8\,ac^2}-\frac{\log\left(|ax-1|\right)}{8\,ac^2}-\frac{ax-2}{4\left(ax-1\right)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(a*x + 1))/(a*c^2) - 1/8*log(abs(a*x - 1))/(a*c^2) - 1/4*(a*x -
2)/((a*x - 1)^2*a*c^2)
```

maple [A] time = 0.04, size = 60, normalized size = 1.18

$$\frac{1}{4c^2a\,(ax-1)^2}-\frac{1}{4c^2a\,(ax-1)}-\frac{\ln(ax-1)}{8c^2a}+\frac{\ln(ax+1)}{8a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/4/c^2/a/(a*x-1)^2-1/4/c^2/a/(a*x-1)-1/8/c^2/a*ln(a*x-1)+1/8*ln(a*x+1)/a/c
^2
```

maxima [A] time = 0.35, size = 63, normalized size = 1.24

$$-\frac{ax-2}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}+\frac{\log(ax+1)}{8\,ac^2}-\frac{\log(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + 1/8*log(a*x + 1)/(a*c^
2) - 1/8*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.06, size = 47, normalized size = 0.92

$$\frac{\operatorname{atanh}(a\,x)}{4\,a\,c^2}-\frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] atanh(a*x)/(4*a*c^2) - (x/4 - 1/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x)
```

sympy [A] time = 0.30, size = 56, normalized size = 1.10

$$-\frac{ax-2}{4a^3c^2x^2-8a^2c^2x+4ac^2}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(a*x - 2)/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) - (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**2)
```

3.1063 $\int \frac{e^{2\tanh^{-1}(ax)}}{x(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=64

$$\frac{3}{4c^2(1-ax)}+\frac{1}{4c^2(1-ax)^2}-\frac{7\log(1-ax)}{8c^2}-\frac{\log(ax+1)}{8c^2}+\frac{\log(x)}{c^2}$$

```
[Out] 1/4/c^2/(-a*x+1)^2+3/4/c^2/(-a*x+1)+ln(x)/c^2-7/8*ln(-a*x+1)/c^2-1/8*ln(a*x
+1)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 72}

$$\frac{3}{4c^2(1-ax)}+\frac{1}{4c^2(1-ax)^2}-\frac{7\log(1-ax)}{8c^2}-\frac{\log(ax+1)}{8c^2}+\frac{\log(x)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] 1/(4*c^2*(1 - a*x)^2) + 3/(4*c^2*(1 - a*x)) + Log[x]/c^2 - (7*Log[1 - a*x])
/(8*c^2) - Log[1 + a*x]/(8*c^2)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1}{x(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \left(\frac{1}{x} - \frac{a}{2(-1+ax)^3} + \frac{3a}{4(-1+ax)^2} - \frac{7a}{8(-1+ax)} - \frac{a}{8(1+ax)}\right)dx}{c^2}$$
$$= \frac{1}{4c^2(1-ax)^2} + \frac{3}{4c^2(1-ax)} + \frac{\log(x)}{c^2} - \frac{7\log(1-ax)}{8c^2} - \frac{\log(1+ax)}{8c^2}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.75

$$\frac{\frac{6}{1-ax} + \frac{2}{(ax-1)^2} - 7\log(1-ax) - \log(ax+1) + 8\log(x)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] (6/(1 - a*x) + 2/(-1 + a*x)^2 + 8*Log[x] - 7*Log[1 - a*x] - Log[1 + a*x])/(
8*c^2)
```

fricas [A] time = 0.56, size = 89, normalized size = 1.39

$$-\frac{6\,ax + \left(a^2x^2 - 2\,ax + 1\right)\log\left(ax+1\right) + 7\left(a^2x^2 - 2\,ax + 1\right)\log\left(ax-1\right) - 8\left(a^2x^2 - 2\,ax + 1\right)\log(x) - 8}{8\left(a^2c^2x^2 - 2\,ac^2x + c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(6*a*x + (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 7*(a^2*x^2 - 2*a*x + 1)*
log(a*x - 1) - 8*(a^2*x^2 - 2*a*x + 1)*log(x) - 8)/(a^2*c^2*x^2 - 2*a*c^2*x
 + c^2)
```

giac [A] time = 0.18, size = 50, normalized size = 0.78

$$-\frac{\log\left(|ax+1|\right)}{8\,c^2} - \frac{7\,\log\left(|ax-1|\right)}{8\,c^2} + \frac{\log\left(|x|\right)}{c^2} - \frac{3\,ax-4}{4\,(ax-1)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/c^2 - 7/8*log(abs(a*x - 1))/c^2 + log(abs(x))/c^2 -
1/4*(3*a*x - 4)/((a*x - 1)^2*c^2)
```

maple [A] time = 0.04, size = 54, normalized size = 0.84

$$\frac{\ln(x)}{c^2}+\frac{1}{4c^2\left(ax-1\right)^2}-\frac{3}{4c^2\left(ax-1\right)}-\frac{7\ln\left(ax-1\right)}{8c^2}-\frac{\ln\left(ax+1\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] ln(x)/c^2+1/4/c^2/(a*x-1)^2-3/4/c^2/(a*x-1)-7/8/c^2*ln(a*x-1)-1/8*ln(a*x+1)
/c^2
```

maxima [A] time = 0.35, size = 60, normalized size = 0.94

$$-\frac{3\,ax-4}{4\left(a^2c^2x^2-2\,ac^2x+c^2\right)}-\frac{\log\left(ax+1\right)}{8\,c^2}-\frac{7\,\log\left(ax-1\right)}{8\,c^2}+\frac{\log(x)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(3*a*x - 4)/(a^2*c^2*x^2 - 2*a*c^2*x + c^2) - 1/8*log(a*x + 1)/c^2 - 7
/8*log(a*x - 1)/c^2 + log(x)/c^2
```

mupad [B] time = 0.96, size = 60, normalized size = 0.94

$$\frac{\ln(x)}{c^2}-\frac{7\,\ln\left(a\,x-1\right)}{8\,c^2}-\frac{\ln\left(a\,x+1\right)}{8\,c^2}-\frac{\frac{3\,a\,x}{4}-1}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x*(c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] log(x)/c^2 - (7*log(a*x - 1))/(8*c^2) - log(a*x + 1)/(8*c^2) - ((3*a*x)/4 -
 1)/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x)
```

sympy [A] time = 0.48, size = 58, normalized size = 0.91

$$-\frac{3ax-4}{4a^2c^2x^2-8ac^2x+4c^2}-\frac{-\log\left(x\right)+\frac{7\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(3*a*x - 4)/(4*a**2*c**2*x**2 - 8*a*c**2*x + 4*c**2) - (-log(x) + 7*log(x
- 1/a)/8 + log(x + 1/a)/8)/c**2
```

3.1064 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=78

$$\frac{5a}{4c^2(1-ax)}+\frac{a}{4c^2(1-ax)^2}+\frac{2a\log(x)}{c^2}-\frac{17a\log(1-ax)}{8c^2}+\frac{a\log(ax+1)}{8c^2}-\frac{1}{c^2x}$$

```
[Out] -1/c^2/x+1/4*a/c^2/(-a*x+1)^2+5/4*a/c^2/(-a*x+1)+2*a*ln(x)/c^2-17/8*a*ln(-a
*x+1)/c^2+1/8*a*ln(a*x+1)/c^2
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{5a}{4c^2(1-ax)}+\frac{a}{4c^2(1-ax)^2}+\frac{2a\log(x)}{c^2}-\frac{17a\log(1-ax)}{8c^2}+\frac{a\log(ax+1)}{8c^2}-\frac{1}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(1/(c^2*x)) + a/(4*c^2*(1 - a*x)^2) + (5*a)/(4*c^2*(1 - a*x)) + (2*a*Log[x
])/c^2 - (17*a*Log[1 - a*x])/(8*c^2) + (a*Log[1 + a*x])/(8*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1}{x^2(1-ax)^3(1+ax)}\,dx}{c^2}$$

$$= \frac{\int \left(\frac{1}{x^2}+\frac{2a}{x}-\frac{a^2}{2(-1+ax)^3}+\frac{5a^2}{4(-1+ax)^2}-\frac{17a^2}{8(-1+ax)}+\frac{a^2}{8(1+ax)}\right)dx}{c^2}$$

$$= -\frac{1}{c^2x}+\frac{a}{4c^2(1-ax)^2}+\frac{5a}{4c^2(1-ax)}+\frac{2a\log(x)}{c^2}-\frac{17a\log(1-ax)}{8c^2}+\frac{a\log(1+ax)}{8c^2}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.73

$$\frac{\frac{10a}{1-ax}+\frac{2a}{(ax-1)^2}+16a\log(x)-17a\log(1-ax)+a\log(ax+1)-\frac{8}{x}}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-8/x + (10*a)/(1 - a*x) + (2*a)/(-1 + a*x)^2 + 16*a*Log[x] - 17*a*Log[1 -
a*x] + a*Log[1 + a*x])/(8*c^2)
```

fricas [A] time = 0.65, size = 120, normalized size = 1.54

$$-\frac{18\,a^2x^2-28\,ax-\left(a^3x^3-2\,a^2x^2+ax\right)\log\left(ax+1\right)+17\left(a^3x^3-2\,a^2x^2+ax\right)\log\left(ax-1\right)-16\left(a^3x^3-2\,a^2x^2+\right.}{8\left(a^2c^2x^3-2\,ac^2x^2+c^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/8*(18*a^2*x^2 - 28*a*x - (a^3*x^3 - 2*a^2*x^2 + a*x)*log(a*x + 1) + 17*(
a^3*x^3 - 2*a^2*x^2 + a*x)*log(a*x - 1) - 16*(a^3*x^3 - 2*a^2*x^2 + a*x)*lo
g(x) + 8)/(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x)
```

giac [A] time = 0.20, size = 65, normalized size = 0.83

$$\frac{a\log\left(|ax+1|\right)}{8\,c^2}-\frac{17\,a\log\left(|ax-1|\right)}{8\,c^2}+\frac{2\,a\log\left(|x|\right)}{c^2}-\frac{9\,a^2x^2-14\,ax+4}{4\,(ax-1)^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*a*log(abs(a*x + 1))/c^2 - 17/8*a*log(abs(a*x - 1))/c^2 + 2*a*log(abs(x)
)/c^2 - 1/4*(9*a^2*x^2 - 14*a*x + 4)/((a*x - 1)^2*c^2*x)
```

maple [A] time = 0.04, size = 68, normalized size = 0.87

$$-\frac{1}{c^2x}+\frac{2a\ln(x)}{c^2}+\frac{a}{4c^2\left(ax-1\right)^2}-\frac{5a}{4c^2\left(ax-1\right)}-\frac{17a\ln\left(ax-1\right)}{8c^2}+\frac{a\ln\left(ax+1\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/c^2/x+2*a*ln(x)/c^2+1/4/c^2*a/(a*x-1)^2-5/4/c^2*a/(a*x-1)-17/8/c^2*a*ln(
a*x-1)+1/8*a*ln(a*x+1)/c^2
```

maxima [A] time = 0.31, size = 76, normalized size = 0.97

$$-\frac{9\,a^2x^2-14\,ax+4}{4\left(a^2c^2x^3-2\,ac^2x^2+c^2x\right)}+\frac{a\log\left(ax+1\right)}{8\,c^2}-\frac{17\,a\log\left(ax-1\right)}{8\,c^2}+\frac{2\,a\log(x)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/4*(9*a^2*x^2 - 14*a*x + 4)/(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x) + 1/8*a*l
og(a*x + 1)/c^2 - 17/8*a*log(a*x - 1)/c^2 + 2*a*log(x)/c^2
```

mupad [B] time = 0.11, size = 76, normalized size = 0.97

$$\frac{2\,a\,\ln(x)}{c^2}-\frac{\frac{9\,a^2\,x^2}{4}-\frac{7\,a\,x}{2}+1}{a^2\,c^2\,x^3-2\,a\,c^2\,x^2+c^2\,x}-\frac{17\,a\,\ln\left(a\,x-1\right)}{8\,c^2}+\frac{a\,\ln\left(a\,x+1\right)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^2*(c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] (2*a*log(x))/c^2 - ((9*a^2*x^2)/4 - (7*a*x)/2 + 1)/(c^2*x - 2*a*c^2*x^2 + a
^2*c^2*x^3) - (17*a*log(a*x - 1))/(8*c^2) + (a*log(a*x + 1))/(8*c^2)
```

sympy [A] time = 0.58, size = 76, normalized size = 0.97

$$-\frac{9a^2x^2-14ax+4}{4a^2c^2x^3-8ac^2x^2+4c^2x}-\frac{-2a\log\left(x\right)+\frac{17a\log\left(x-\frac{1}{a}\right)}{8}-\frac{a\log\left(x+\frac{1}{a}\right)}{8}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(9*a**2*x**2 - 14*a*x + 4)/(4*a**2*c**2*x**3 - 8*a*c**2*x**2 + 4*c**2*x) -
 (-2*a*log(x) + 17*a*log(x - 1/a)/8 - a*log(x + 1/a)/8)/c**2
```

3.1065 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=99

$$\frac{7a^2}{4c^2(1-ax)}+\frac{a^2}{4c^2(1-ax)^2}+\frac{4a^2\log(x)}{c^2}-\frac{31a^2\log(1-ax)}{8c^2}-\frac{a^2\log(ax+1)}{8c^2}-\frac{2a}{c^2x}-\frac{1}{2c^2x^2}$$

```
[Out] -1/2/c^2/x^2-2*a/c^2/x+1/4*a^2/c^2/(-a*x+1)^2+7/4*a^2/c^2/(-a*x+1)+4*a^2*ln
(x)/c^2-31/8*a^2*ln(-a*x+1)/c^2-1/8*a^2*ln(a*x+1)/c^2
```

Rubi [A] time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{7a^2}{4c^2(1-ax)}+\frac{a^2}{4c^2(1-ax)^2}+\frac{4a^2\log(x)}{c^2}-\frac{31a^2\log(1-ax)}{8c^2}-\frac{a^2\log(ax+1)}{8c^2}-\frac{2a}{c^2x}-\frac{1}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^2),x]
```

```
[Out] -1/(2*c^2*x^2) - (2*a)/(c^2*x) + a^2/(4*c^2*(1 - a*x)^2) + (7*a^2)/(4*c^2*(
1 - a*x)) + (4*a^2*Log[x])/c^2 - (31*a^2*Log[1 - a*x])/(8*c^2) - (a^2*Log[1
 + a*x])/(8*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1}{x^3(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{4a^2}{x}-\frac{a^3}{2(-1+ax)^3}+\frac{7a^3}{4(-1+ax)^2}-\frac{31a^3}{8(-1+ax)}-\frac{a^3}{8(1+ax)}\right)dx}{c^2}$$
$$= -\frac{1}{2c^2x^2}-\frac{2a}{c^2x}+\frac{a^2}{4c^2(1-ax)^2}+\frac{7a^2}{4c^2(1-ax)}+\frac{4a^2\log(x)}{c^2}-\frac{31a^2\log(1-ax)}{8c^2}-\frac{a^2\log(1-}{8c^2}$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.73

$$-\frac{\frac{14a^2}{ax-1}-\frac{2a^2}{(ax-1)^2}-32a^2\log(x)+31a^2\log(1-ax)+a^2\log(ax+1)+\frac{16a}{x}+\frac{4}{x^2}}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^2),x]
```

```
[Out] -1/8*(4/x^2 + (16*a)/x - (2*a^2)/(-1 + a*x)^2 + (14*a^2)/(-1 + a*x) - 32*a^
2*Log[x] + 31*a^2*Log[1 - a*x] + a^2*Log[1 + a*x])/c^2
```

fricas [A] time = 0.67, size = 141, normalized size = 1.42

$$-\frac{30\,a^3x^3-44\,a^2x^2+8\,ax+\left(a^4x^4-2\,a^3x^3+a^2x^2\right)\log\left(ax+1\right)+31\left(a^4x^4-2\,a^3x^3+a^2x^2\right)\log\left(ax-1\right)-32\left(a^4x\right.}{8\left(a^2c^2x^4-2\,ac^2x^3+c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/8*(30*a^3*x^3 - 44*a^2*x^2 + 8*a*x + (a^4*x^4 - 2*a^3*x^3 + a^2*x^2)*log
(a*x + 1) + 31*(a^4*x^4 - 2*a^3*x^3 + a^2*x^2)*log(a*x - 1) - 32*(a^4*x^4 -
 2*a^3*x^3 + a^2*x^2)*log(x) + 4)/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*x^2)
```

giac [A] time = 1.09, size = 79, normalized size = 0.80

$$-\frac{a^2\log\left(|ax+1|\right)}{8\,c^2}-\frac{31\,a^2\log\left(|ax-1|\right)}{8\,c^2}+\frac{4\,a^2\log\left(|x|\right)}{c^2}-\frac{15\,a^3x^3-22\,a^2x^2+4\,ax+2}{4\,(ax-1)^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*a^2*log(abs(a*x + 1))/c^2 - 31/8*a^2*log(abs(a*x - 1))/c^2 + 4*a^2*log
(abs(x))/c^2 - 1/4*(15*a^3*x^3 - 22*a^2*x^2 + 4*a*x + 2)/((a*x - 1)^2*c^2*x
^2)
```

maple [A] time = 0.04, size = 87, normalized size = 0.88

$$-\frac{1}{2c^2x^2}-\frac{2a}{c^2x}+\frac{4a^2\ln(x)}{c^2}+\frac{a^2}{4c^2\left(ax-1\right)^2}-\frac{7a^2}{4c^2\left(ax-1\right)}-\frac{31a^2\ln\left(ax-1\right)}{8c^2}-\frac{a^2\ln\left(ax+1\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/c^2/x^2-2*a/c^2/x+4*a^2*ln(x)/c^2+1/4/c^2*a^2/(a*x-1)^2-7/4/c^2*a^2/(a
*x-1)-31/8/c^2*a^2*ln(a*x-1)-1/8*a^2*ln(a*x+1)/c^2
```

maxima [A] time = 0.31, size = 92, normalized size = 0.93

$$-\frac{a^2\log\left(ax+1\right)}{8\,c^2}-\frac{31\,a^2\log\left(ax-1\right)}{8\,c^2}+\frac{4\,a^2\log(x)}{c^2}-\frac{15\,a^3x^3-22\,a^2x^2+4\,ax+2}{4\left(a^2c^2x^4-2\,ac^2x^3+c^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -1/8*a^2*log(a*x + 1)/c^2 - 31/8*a^2*log(a*x - 1)/c^2 + 4*a^2*log(x)/c^2 -
1/4*(15*a^3*x^3 - 22*a^2*x^2 + 4*a*x + 2)/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*
x^2)
```

mupad [B] time = 0.12, size = 91, normalized size = 0.92

$$\frac{4\,a^2\,\ln(x)}{c^2}-\frac{\frac{15\,a^3\,x^3}{4}-\frac{11\,a^2\,x^2}{2}+a\,x+\frac{1}{2}}{a^2\,c^2\,x^4-2\,a\,c^2\,x^3+c^2\,x^2}-\frac{31\,a^2\,\ln\left(a\,x-1\right)}{8\,c^2}-\frac{a^2\,\ln\left(a\,x+1\right)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^3*(c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] (4*a^2*log(x))/c^2 - (a*x - (11*a^2*x^2)/2 + (15*a^3*x^3)/4 + 1/2)/(c^2*x^2
 - 2*a*c^2*x^3 + a^2*c^2*x^4) - (31*a^2*log(a*x - 1))/(8*c^2) - (a^2*log(a*
x + 1))/(8*c^2)
```

sympy [A] time = 0.61, size = 92, normalized size = 0.93

$$-\frac{15a^3x^3 - 22a^2x^2 + 4ax + 2}{4a^2c^2x^4 - 8ac^2x^3 + 4c^2x^2} - \frac{-4a^2\log(x) + \frac{31a^2\log\left(x-\frac{1}{a}\right)}{8} + \frac{a^2\log\left(x+\frac{1}{a}\right)}{8}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(15*a**3*x**3 - 22*a**2*x**2 + 4*a*x + 2)/(4*a**2*c**2*x**4 - 8*a*c**2*x**
3 + 4*c**2*x**2) - (-4*a**2*log(x) + 31*a**2*log(x - 1/a)/8 + a**2*log(x +
1/a)/8)/c**2
```

3.1066 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^4(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=110

$$\frac{9a^3}{4c^2(1-ax)}+\frac{a^3}{4c^2(1-ax)^2}+\frac{6a^3\log(x)}{c^2}-\frac{49a^3\log(1-ax)}{8c^2}+\frac{a^3\log(ax+1)}{8c^2}-\frac{4a^2}{c^2x}-\frac{a}{c^2x^2}-\frac{1}{3c^2x^3}$$

```
[Out] -1/3/c^2/x^3-a/c^2/x^2-4*a^2/c^2/x+1/4*a^3/c^2/(-a*x+1)^2+9/4*a^3/c^2/(-a*x
+1)+6*a^3*ln(x)/c^2-49/8*a^3*ln(-a*x+1)/c^2+1/8*a^3*ln(a*x+1)/c^2
```

Rubi [A] time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{9a^3}{4c^2(1-ax)}+\frac{a^3}{4c^2(1-ax)^2}-\frac{4a^2}{c^2x}+\frac{6a^3\log(x)}{c^2}-\frac{49a^3\log(1-ax)}{8c^2}+\frac{a^3\log(ax+1)}{8c^2}-\frac{a}{c^2x^2}-\frac{1}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^4*(c - a^2*c*x^2)^2),x]
```

```
[Out] -1/(3*c^2*x^3) - a/(c^2*x^2) - (4*a^2)/(c^2*x) + a^3/(4*c^2*(1 - a*x)^2) +
(9*a^3)/(4*c^2*(1 - a*x)) + (6*a^3*Log[x])/c^2 - (49*a^3*Log[1 - a*x])/(8*c
^2) + (a^3*Log[1 + a*x])/(8*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^4\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1}{x^4(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= \frac{\int \left(\frac{1}{x^4}+\frac{2a}{x^3}+\frac{4a^2}{x^2}+\frac{6a^3}{x}-\frac{a^4}{2(-1+ax)^3}+\frac{9a^4}{4(-1+ax)^2}-\frac{49a^4}{8(-1+ax)}+\frac{a^4}{8(1+ax)}\right)dx}{c^2}$$
$$= -\frac{1}{3c^2x^3}-\frac{a}{c^2x^2}-\frac{4a^2}{c^2x}+\frac{a^3}{4c^2(1-ax)^2}+\frac{9a^3}{4c^2(1-ax)}+\frac{6a^3\log(x)}{c^2}-\frac{49a^3\log(1-ax)}{8c^2}+\frac{a}{}$$

Mathematica [A] time = 0.09, size = 87, normalized size = 0.79

$$\frac{\frac{9a^3}{4-4ax}+\frac{a^3}{4(ax-1)^2}+6a^3\log(x)-\frac{49}{8}a^3\log(1-ax)+\frac{1}{8}a^3\log(ax+1)-\frac{4a^2}{x}-\frac{a}{x^2}-\frac{1}{3x^3}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^4*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-1/3*1/x^3 - a/x^2 - (4*a^2)/x + (9*a^3)/(4 - 4*a*x) + a^3/(4*(-1 + a*x)^2
) + 6*a^3*Log[x] - (49*a^3*Log[1 - a*x])/8 + (a^3*Log[1 + a*x])/8)/c^2
```

fricas [A] time = 0.63, size = 150, normalized size = 1.36

$$-\frac{150\,a^4x^4-228\,a^3x^3+56\,a^2x^2+8\,ax-3\left(a^5x^5-2\,a^4x^4+a^3x^3\right)\log\left(ax+1\right)+147\left(a^5x^5-2\,a^4x^4+a^3x^3\right)\log\left(a\right.}{24\left(a^2c^2x^5-2\,ac^2x^4+c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -1/24*(150*a^4*x^4 - 228*a^3*x^3 + 56*a^2*x^2 + 8*a*x - 3*(a^5*x^5 - 2*a^4*
x^4 + a^3*x^3)*log(a*x + 1) + 147*(a^5*x^5 - 2*a^4*x^4 + a^3*x^3)*log(a*x -
 1) - 144*(a^5*x^5 - 2*a^4*x^4 + a^3*x^3)*log(x) + 8)/(a^2*c^2*x^5 - 2*a*c^
2*x^4 + c^2*x^3)
```

giac [A] time = 0.16, size = 87, normalized size = 0.79

$$\frac{a^3\log\left(|ax+1|\right)}{8\,c^2}-\frac{49\,a^3\log\left(|ax-1|\right)}{8\,c^2}+\frac{6\,a^3\log\left(|x|\right)}{c^2}-\frac{75\,a^4x^4-114\,a^3x^3+28\,a^2x^2+4\,ax+4}{12\,(ax-1)^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*a^3*log(abs(a*x + 1))/c^2 - 49/8*a^3*log(abs(a*x - 1))/c^2 + 6*a^3*log(
abs(x))/c^2 - 1/12*(75*a^4*x^4 - 114*a^3*x^3 + 28*a^2*x^2 + 4*a*x + 4)/((a*
x - 1)^2*c^2*x^3)
```

maple [A] time = 0.04, size = 98, normalized size = 0.89

$$-\frac{1}{3c^2x^3}-\frac{a}{c^2x^2}-\frac{4a^2}{c^2x}+\frac{6a^3\ln(x)}{c^2}+\frac{a^3}{4c^2\left(ax-1\right)^2}-\frac{9a^3}{4c^2\left(ax-1\right)}-\frac{49a^3\ln\left(ax-1\right)}{8c^2}+\frac{a^3\ln\left(ax+1\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3/c^2/x^3-a/c^2/x^2-4*a^2/c^2/x+6*a^3*ln(x)/c^2+1/4/c^2*a^3/(a*x-1)^2-9/
4/c^2*a^3/(a*x-1)-49/8/c^2*a^3*ln(a*x-1)+1/8*a^3*ln(a*x+1)/c^2
```

maxima [A] time = 0.31, size = 100, normalized size = 0.91

$$\frac{a^3\log\left(ax+1\right)}{8\,c^2}-\frac{49\,a^3\log\left(ax-1\right)}{8\,c^2}+\frac{6\,a^3\log(x)}{c^2}-\frac{75\,a^4x^4-114\,a^3x^3+28\,a^2x^2+4\,ax+4}{12\left(a^2c^2x^5-2\,ac^2x^4+c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] 1/8*a^3*log(a*x + 1)/c^2 - 49/8*a^3*log(a*x - 1)/c^2 + 6*a^3*log(x)/c^2 - 1
/12*(75*a^4*x^4 - 114*a^3*x^3 + 28*a^2*x^2 + 4*a*x + 4)/(a^2*c^2*x^5 - 2*a*
c^2*x^4 + c^2*x^3)
```

mupad [B] time = 0.99, size = 100, normalized size = 0.91

$$\frac{6\,a^3\,\ln(x)}{c^2}-\frac{\frac{25\,a^4\,x^4}{4}-\frac{19\,a^3\,x^3}{2}+\frac{7\,a^2\,x^2}{3}+\frac{a\,x}{3}+\frac{1}{3}}{a^2\,c^2\,x^5-2\,a\,c^2\,x^4+c^2\,x^3}-\frac{49\,a^3\,\ln\left(a\,x-1\right)}{8\,c^2}+\frac{a^3\,\ln\left(a\,x+1\right)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^4*(c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] (6*a^3*log(x))/c^2 - ((a*x)/3 + (7*a^2*x^2)/3 - (19*a^3*x^3)/2 + (25*a^4*x^
4)/4 + 1/3)/(c^2*x^3 - 2*a*c^2*x^4 + a^2*c^2*x^5) - (49*a^3*log(a*x - 1))/(
8*c^2) + (a^3*log(a*x + 1))/(8*c^2)
```

sympy [A] time = 0.65, size = 100, normalized size = 0.91

$$-\frac{75a^4x^4-114a^3x^3+28a^2x^2+4ax+4}{12a^2c^2x^5-24ac^2x^4+12c^2x^3}-\frac{-6a^3\log(x)+\frac{49a^3\log\left(x-\frac{1}{a}\right)}{8}-\frac{a^3\log\left(x+\frac{1}{a}\right)}{8}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(75*a**4*x**4 - 114*a**3*x**3 + 28*a**2*x**2 + 4*a*x + 4)/(12*a**2*c**2*x*
*5 - 24*a*c**2*x**4 + 12*c**2*x**3) - (-6*a**3*log(x) + 49*a**3*log(x - 1/a
)/8 - a**3*log(x + 1/a)/8)/c**2
```

3.1067 $\int \frac{e^{2\tanh^{-1}(ax)}x^5}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=105

$$\frac{23}{16a^6c^3(1-ax)}+\frac{1}{16a^6c^3(ax+1)}-\frac{1}{2a^6c^3(1-ax)^2}+\frac{1}{12a^6c^3(1-ax)^3}+\frac{13\log(1-ax)}{16a^6c^3}+\frac{3\log(ax+1)}{16a^6c^3}$$

```
[Out] 1/12/a^6/c^3/(-a*x+1)^3-1/2/a^6/c^3/(-a*x+1)^2+23/16/a^6/c^3/(-a*x+1)+1/16/
a^6/c^3/(a*x+1)+13/16*ln(-a*x+1)/a^6/c^3+3/16*ln(a*x+1)/a^6/c^3
```

Rubi [A] time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{23}{16a^6c^3(1-ax)}+\frac{1}{16a^6c^3(ax+1)}-\frac{1}{2a^6c^3(1-ax)^2}+\frac{1}{12a^6c^3(1-ax)^3}+\frac{13\log(1-ax)}{16a^6c^3}+\frac{3\log(ax+1)}{16a^6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^5)/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a^6*c^3*(1 - a*x)^3) - 1/(2*a^6*c^3*(1 - a*x)^2) + 23/(16*a^6*c^3*(1
- a*x)) + 1/(16*a^6*c^3*(1 + a*x)) + (13*Log[1 - a*x])/(16*a^6*c^3) + (3*Lo
g[1 + a*x])/(16*a^6*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^5}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^5}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$= \frac{\int \left(\frac{1}{4a^5(-1+ax)^4}+\frac{1}{a^5(-1+ax)^3}+\frac{23}{16a^5(-1+ax)^2}+\frac{13}{16a^5(-1+ax)}-\frac{1}{16a^5(1+ax)^2}+\frac{3}{16a^5(1+ax)}\right)dx}{c^3}$$
$$= \frac{1}{12a^6c^3(1-ax)^3}-\frac{1}{2a^6c^3(1-ax)^2}+\frac{23}{16a^6c^3(1-ax)}+\frac{1}{16a^6c^3(1+ax)}+\frac{13\log(1-ax)}{16a^6c^3}+\frac{3}{}$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.83

$$\frac{-66a^3x^3+36a^2x^2+74ax+39(ax-1)^3(ax+1)\log(1-ax)+9(ax-1)^3(ax+1)\log(ax+1)-52}{48a^6c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^5)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-52 + 74*a*x + 36*a^2*x^2 - 66*a^3*x^3 + 39*(-1 + a*x)^3*(1 + a*x)*Log[1 -
 a*x] + 9*(-1 + a*x)^3*(1 + a*x)*Log[1 + a*x])/(48*a^6*c^3*(-1 + a*x)^3*(1
+ a*x))
```

fricas [A] time = 0.65, size = 123, normalized size = 1.17

$$-\frac{66\,a^3x^3-36\,a^2x^2-74\,ax-9\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax+1\right)-39\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax-1\right)}{48\left(a^{10}c^3x^4-2\,a^9c^3x^3+2\,a^7c^3x-a^6c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/48*(66*a^3*x^3 - 36*a^2*x^2 - 74*a*x - 9*(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)*log(a*x + 1) - 39*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 52)/(
a^10*c^3*x^4 - 2*a^9*c^3*x^3 + 2*a^7*c^3*x - a^6*c^3)
```

giac [A] time = 0.73, size = 75, normalized size = 0.71

$$\frac{3\,\log\left(|ax+1|\right)}{16\,a^6c^3}+\frac{13\,\log\left(|ax-1|\right)}{16\,a^6c^3}-\frac{33\,a^3x^3-18\,a^2x^2-37\,ax+26}{24\,(ax+1)(ax-1)^3a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(a*x + 1))/(a^6*c^3) + 13/16*log(abs(a*x - 1))/(a^6*c^3) - 1/24
*(33*a^3*x^3 - 18*a^2*x^2 - 37*a*x + 26)/((a*x + 1)*(a*x - 1)^3*a^6*c^3)
```

maple [A] time = 0.04, size = 90, normalized size = 0.86

$$-\frac{1}{2c^3a^6\left(ax-1\right)^2}-\frac{1}{12c^3a^6\left(ax-1\right)^3}-\frac{23}{16c^3a^6\left(ax-1\right)}+\frac{13\ln\left(ax-1\right)}{16c^3a^6}+\frac{1}{16a^6c^3\left(ax+1\right)}+\frac{3\ln\left(ax+1\right)}{16a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^5/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/2/c^3/a^6/(a*x-1)^2-1/12/c^3/a^6/(a*x-1)^3-23/16/c^3/a^6/(a*x-1)+13/16/c
^3/a^6*ln(a*x-1)+1/16/a^6/c^3/(a*x+1)+3/16*ln(a*x+1)/a^6/c^3
```

maxima [A] time = 0.32, size = 94, normalized size = 0.90

$$-\frac{33\,a^3x^3-18\,a^2x^2-37\,ax+26}{24\left(a^{10}c^3x^4-2\,a^9c^3x^3+2\,a^7c^3x-a^6c^3\right)}+\frac{3\,\log\left(ax+1\right)}{16\,a^6c^3}+\frac{13\,\log\left(ax-1\right)}{16\,a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^5/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/24*(33*a^3*x^3 - 18*a^2*x^2 - 37*a*x + 26)/(a^10*c^3*x^4 - 2*a^9*c^3*x^3
 + 2*a^7*c^3*x - a^6*c^3) + 3/16*log(a*x + 1)/(a^6*c^3) + 13/16*log(a*x - 1
)/(a^6*c^3)
```

mupad [B] time = 0.33, size = 94, normalized size = 0.90

$$\frac{13\,\ln\left(a\,x-1\right)}{16\,a^6\,c^3}-\frac{\frac{37\,x}{24\,a^5}-\frac{13}{12\,a^6}-\frac{11\,x^3}{8\,a^3}+\frac{3\,x^2}{4\,a^4}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}+\frac{3\,\ln\left(a\,x+1\right)}{16\,a^6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5*(a*x + 1)^2)/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] (13*log(a*x - 1))/(16*a^6*c^3) - ((37*x)/(24*a^5) - 13/(12*a^6) - (11*x^3)/
(8*a^3) + (3*x^2)/(4*a^4))/(c^3 + 2*a^3*c^3*x^3 - a^4*c^3*x^4 - 2*a*c^3*x)
+ (3*log(a*x + 1))/(16*a^6*c^3)
```

sympy [A] time = 0.53, size = 92, normalized size = 0.88

$$\frac{-33a^3x^3+18a^2x^2+37ax-26}{24a^{10}c^3x^4-48a^9c^3x^3+48a^7c^3x-24a^6c^3}+\frac{\frac{13\log\left(x-\frac{1}{a}\right)}{16}+\frac{3\log\left(x+\frac{1}{a}\right)}{16}}{a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**5/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-33*a**3*x**3 + 18*a**2*x**2 + 37*a*x - 26)/(24*a**10*c**3*x**4 - 48*a**9*
c**3*x**3 + 48*a**7*c**3*x - 24*a**6*c**3) + (13*log(x - 1/a)/16 + 3*log(x
+ 1/a)/16)/(a**6*c**3)
```

3.1068 $\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=86

$$\frac{11}{16a^5c^3(1-ax)}-\frac{1}{16a^5c^3(ax+1)}-\frac{3}{8a^5c^3(1-ax)^2}+\frac{1}{12a^5c^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{4a^5c^3}$$

```
[Out] 1/12/a^5/c^3/(-a*x+1)^3-3/8/a^5/c^3/(-a*x+1)^2+11/16/a^5/c^3/(-a*x+1)-1/16/
a^5/c^3/(a*x+1)-1/4*arctanh(a*x)/a^5/c^3
```

Rubi [A] time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6150, 88, 207}

$$\frac{11}{16a^5c^3(1-ax)}-\frac{1}{16a^5c^3(ax+1)}-\frac{3}{8a^5c^3(1-ax)^2}+\frac{1}{12a^5c^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{4a^5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a^5*c^3*(1 - a*x)^3) - 3/(8*a^5*c^3*(1 - a*x)^2) + 11/(16*a^5*c^3*(1
- a*x)) - 1/(16*a^5*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a^5*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^4}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$

$$= \frac{\int \left(\frac{1}{4a^4(-1+ax)^4}+\frac{3}{4a^4(-1+ax)^3}+\frac{11}{16a^4(-1+ax)^2}+\frac{1}{16a^4(1+ax)^2}+\frac{1}{4a^4\left(-1+a^2x^2\right)}\right)dx}{c^3}$$

$$= \frac{1}{12a^5c^3(1-ax)^3}-\frac{3}{8a^5c^3(1-ax)^2}+\frac{11}{16a^5c^3(1-ax)}-\frac{1}{16a^5c^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4a^4c^3}$$

$$= \frac{1}{12a^5c^3(1-ax)^3}-\frac{3}{8a^5c^3(1-ax)^2}+\frac{11}{16a^5c^3(1-ax)}-\frac{1}{16a^5c^3(1+ax)}-\frac{\tanh^{-1}(ax)}{4a^5c^3}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.74

$$\frac{-9a^3x^3+6a^2x^2+5ax-3(ax-1)^3(ax+1)\tanh^{-1}(ax)-4}{12a^5c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-4 + 5*a*x + 6*a^2*x^2 - 9*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x]
)/(12*a^5*c^3*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.55, size = 123, normalized size = 1.43

$$-\frac{18\,a^3x^3-12\,a^2x^2-10\,ax+3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax+1\right)-3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax-1\right)+}{24\left(a^9c^3x^4-2\,a^8c^3x^3+2\,a^6c^3x-a^5c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/24*(18*a^3*x^3 - 12*a^2*x^2 - 10*a*x + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)*log(a*x + 1) - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^
9*c^3*x^4 - 2*a^8*c^3*x^3 + 2*a^6*c^3*x - a^5*c^3)
```

giac [A] time = 0.19, size = 75, normalized size = 0.87

$$-\frac{\log\left(|ax+1|\right)}{8\,a^5c^3}+\frac{\log\left(|ax-1|\right)}{8\,a^5c^3}-\frac{9\,a^3x^3-6\,a^2x^2-5\,ax+4}{12\,(ax+1)(ax-1)^3a^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a^5*c^3) + 1/8*log(abs(a*x - 1))/(a^5*c^3) - 1/12*(
9*a^3*x^3 - 6*a^2*x^2 - 5*a*x + 4)/((a*x + 1)*(a*x - 1)^3*a^5*c^3)
```

maple [A] time = 0.04, size = 90, normalized size = 1.05

$$-\frac{1}{12c^3a^5\left(ax-1\right)^3}-\frac{3}{8c^3a^5\left(ax-1\right)^2}-\frac{11}{16c^3a^5\left(ax-1\right)}+\frac{\ln\left(ax-1\right)}{8c^3a^5}-\frac{1}{16a^5c^3\left(ax+1\right)}-\frac{\ln\left(ax+1\right)}{8c^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/12/c^3/a^5/(a*x-1)^3-3/8/c^3/a^5/(a*x-1)^2-11/16/c^3/a^5/(a*x-1)+1/8/c^3
/a^5*ln(a*x-1)-1/16/a^5/c^3/(a*x+1)-1/8/c^3/a^5*ln(a*x+1)
```

maxima [A] time = 0.33, size = 94, normalized size = 1.09

$$-\frac{9\,a^3x^3-6\,a^2x^2-5\,ax+4}{12\left(a^9c^3x^4-2\,a^8c^3x^3+2\,a^6c^3x-a^5c^3\right)}-\frac{\log\left(ax+1\right)}{8\,a^5c^3}+\frac{\log\left(ax-1\right)}{8\,a^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^4/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/12*(9*a^3*x^3 - 6*a^2*x^2 - 5*a*x + 4)/(a^9*c^3*x^4 - 2*a^8*c^3*x^3 + 2*
a^6*c^3*x - a^5*c^3) - 1/8*log(a*x + 1)/(a^5*c^3) + 1/8*log(a*x - 1)/(a^5*c
^3)
```

mupad [B] time = 0.07, size = 78, normalized size = 0.91

$$-\frac{\frac{5\,x}{12\,a^4}-\frac{1}{3\,a^5}-\frac{3\,x^3}{4\,a^2}+\frac{x^2}{2\,a^3}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}-\frac{\mathrm{atanh}\left(a\,x\right)}{4\,a^5\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(a*x + 1)^2)/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] - ((5*x)/(12*a^4) - 1/(3*a^5) - (3*x^3)/(4*a^2) + x^2/(2*a^3))/(c^3 + 2*a^3
*c^3*x^3 - a^4*c^3*x^4 - 2*a*c^3*x) - atanh(a*x)/(4*a^5*c^3)
```

sympy [A] time = 0.45, size = 88, normalized size = 1.02

$$\frac{-9a^3x^3+6a^2x^2+5ax-4}{12a^9c^3x^4-24a^8c^3x^3+24a^6c^3x-12a^5c^3}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**4/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-9*a**3*x**3 + 6*a**2*x**2 + 5*a*x - 4)/(12*a**9*c**3*x**4 - 24*a**8*c**3*
x**3 + 24*a**6*c**3*x - 12*a**5*c**3) + (log(x - 1/a)/8 - log(x + 1/a)/8)/(
a**5*c**3)
```

3.1069 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=86

$$\frac{3}{16a^4c^3(1-ax)}+\frac{1}{16a^4c^3(ax+1)}-\frac{1}{4a^4c^3(1-ax)^2}+\frac{1}{12a^4c^3(1-ax)^3}+\frac{\tanh^{-1}(ax)}{8a^4c^3}$$

```
[Out] 1/12/a^4/c^3/(-a*x+1)^3-1/4/a^4/c^3/(-a*x+1)^2+3/16/a^4/c^3/(-a*x+1)+1/16/a
^4/c^3/(a*x+1)+1/8*arctanh(a*x)/a^4/c^3
```

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6150, 88, 207}

$$\frac{3}{16a^4c^3(1-ax)}+\frac{1}{16a^4c^3(ax+1)}-\frac{1}{4a^4c^3(1-ax)^2}+\frac{1}{12a^4c^3(1-ax)^3}+\frac{\tanh^{-1}(ax)}{8a^4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a^4*c^3*(1 - a*x)^3) - 1/(4*a^4*c^3*(1 - a*x)^2) + 3/(16*a^4*c^3*(1 -
 a*x)) + 1/(16*a^4*c^3*(1 + a*x)) + ArcTanh[a*x]/(8*a^4*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^3}\,dx=\frac{\int \frac{x^3}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$=\frac{\int \left(\frac{1}{4a^3(-1+ax)^4}+\frac{1}{2a^3(-1+ax)^3}+\frac{3}{16a^3(-1+ax)^2}-\frac{1}{16a^3(1+ax)^2}-\frac{1}{8a^3\left(-1+a^2x^2\right)}\right)dx}{c^3}$$
$$=\frac{1}{12a^4c^3(1-ax)^3}-\frac{1}{4a^4c^3(1-ax)^2}+\frac{3}{16a^4c^3(1-ax)}+\frac{1}{16a^4c^3(1+ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{8a^3c^3}$$
$$=\frac{1}{12a^4c^3(1-ax)^3}-\frac{1}{4a^4c^3(1-ax)^2}+\frac{3}{16a^4c^3(1-ax)}+\frac{1}{16a^4c^3(1+ax)}+\frac{\tanh^{-1}(ax)}{8a^4c^3}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.74

$$\frac{-3a^3x^3-6a^2x^2+7ax+3(ax-1)^3(ax+1)\tanh^{-1}(ax)-2}{24a^4c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-2 + 7*a*x - 6*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x]
)/(24*a^4*c^3*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.61, size = 123, normalized size = 1.43

$$-\frac{6\,a^3x^3+12\,a^2x^2-14\,ax-3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax+1\right)+3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax-1\right)+}{48\left(a^8c^3x^4-2\,a^7c^3x^3+2\,a^5c^3x-a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/48*(6*a^3*x^3 + 12*a^2*x^2 - 14*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1
)*log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 4)/(a^8
*c^3*x^4 - 2*a^7*c^3*x^3 + 2*a^5*c^3*x - a^4*c^3)
```

giac [A] time = 0.19, size = 75, normalized size = 0.87

$$\frac{\log\left(|ax+1|\right)}{16\,a^4c^3}-\frac{\log\left(|ax-1|\right)}{16\,a^4c^3}-\frac{3\,a^3x^3+6\,a^2x^2-7\,ax+2}{24\,(ax+1)(ax-1)^3a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*log(abs(a*x + 1))/(a^4*c^3) - 1/16*log(abs(a*x - 1))/(a^4*c^3) - 1/24*
(3*a^3*x^3 + 6*a^2*x^2 - 7*a*x + 2)/((a*x + 1)*(a*x - 1)^3*a^4*c^3)
```

maple [A] time = 0.04, size = 90, normalized size = 1.05

$$-\frac{1}{12c^3a^4(ax-1)^3}-\frac{1}{4c^3a^4(ax-1)^2}-\frac{3}{16c^3a^4(ax-1)}-\frac{\ln(ax-1)}{16c^3a^4}+\frac{1}{16a^4c^3(ax+1)}+\frac{\ln(ax+1)}{16c^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/12/c^3/a^4/(a*x-1)^3-1/4/c^3/a^4/(a*x-1)^2-3/16/c^3/a^4/(a*x-1)-1/16/c^3
/a^4*ln(a*x-1)+1/16/a^4/c^3/(a*x+1)+1/16/c^3/a^4*ln(a*x+1)
```

maxima [A] time = 0.34, size = 94, normalized size = 1.09

$$-\frac{3a^3x^3+6a^2x^2-7ax+2}{24\left(a^8c^3x^4-2a^7c^3x^3+2a^5c^3x-a^4c^3\right)}+\frac{\log(ax+1)}{16a^4c^3}-\frac{\log(ax-1)}{16a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/24*(3*a^3*x^3 + 6*a^2*x^2 - 7*a*x + 2)/(a^8*c^3*x^4 - 2*a^7*c^3*x^3 + 2*
a^5*c^3*x - a^4*c^3) + 1/16*log(a*x + 1)/(a^4*c^3) - 1/16*log(a*x - 1)/(a^4
*c^3)
```

mupad [B] time = 0.93, size = 77, normalized size = 0.90

$$\frac{\frac{1}{12a^4}-\frac{7x}{24a^3}+\frac{x^3}{8a}+\frac{x^2}{4a^2}}{-a^4c^3x^4+2a^3c^3x^3-2ac^3x+c^3}+\frac{\operatorname{atanh}(ax)}{8a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a*x + 1)^2)/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] (1/(12*a^4) - (7*x)/(24*a^3) + x^3/(8*a) + x^2/(4*a^2))/(c^3 + 2*a^3*c^3*x^
3 - a^4*c^3*x^4 - 2*a*c^3*x) + atanh(a*x)/(8*a^4*c^3)
```

sympy [A] time = 0.43, size = 88, normalized size = 1.02

$$\frac{-3a^3x^3-6a^2x^2+7ax-2}{24a^8c^3x^4-48a^7c^3x^3+48a^5c^3x-24a^4c^3}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16}+\frac{\log\left(x+\frac{1}{a}\right)}{16}}{a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-3*a**3*x**3 - 6*a**2*x**2 + 7*a*x - 2)/(24*a**8*c**3*x**4 - 48*a**7*c**3*
x**3 + 48*a**5*c**3*x - 24*a**4*c**3) + (-log(x - 1/a)/16 + log(x + 1/a)/16
)/(a**4*c**3)
```

3.1070 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=31

$$-\frac{1-2ax}{6a^3c^3(1-ax)^3(ax+1)}$$

[Out] 1/6*(2*a*x-1)/a^3/c^3/(-a*x+1)^3/(a*x+1)

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 81}

$$-\frac{1-2ax}{6a^3c^3(1-ax)^3(ax+1)}$$

Antiderivative was successfully verified.

[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^3,x]

[Out] -(1 - 2*a*x)/(6*a^3*c^3*(1 - a*x)^3*(1 + a*x))

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^2}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$= -\frac{1-2ax}{6a^3c^3(1-ax)^3(1+ax)}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 0.97

$$\frac{1-2ax}{6a^3c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1 - 2*a*x)/(6*a^3*c^3*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.56, size = 49, normalized size = 1.58

$$-\frac{2\,ax-1}{6\left(a^7c^3x^4-2\,a^6c^3x^3+2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/6*(2*a*x - 1)/(a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^4*c^3*x - a^3*c^3)
```

giac [A] time = 0.19, size = 45, normalized size = 1.45

$$-\frac{1}{16\,(ax+1)a^3c^3}+\frac{3\,a^2x^2-12\,ax+5}{48\,(ax-1)^3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16/((a*x + 1)*a^3*c^3) + 1/48*(3*a^2*x^2 - 12*a*x + 5)/((a*x - 1)^3*a^3*
c^3)
```

maple [A] time = 0.03, size = 54, normalized size = 1.74

$$\frac{-\frac{1}{12a^3(ax-1)^3}-\frac{1}{8a^3(ax-1)^2}+\frac{1}{16a^3(ax-1)}-\frac{1}{16a^3(ax+1)}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(-1/12/a^3/(a*x-1)^3-1/8/a^3/(a*x-1)^2+1/16/a^3/(a*x-1)-1/16/a^3/(a*x
+1))
```

maxima [A] time = 0.32, size = 49, normalized size = 1.58

$$-\frac{2\,ax-1}{6\left(a^{7}c^{3}x^{4}-2\,a^{6}c^{3}x^{3}+2\,a^{4}c^{3}x-a^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/6*(2*a*x - 1)/(a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^4*c^3*x - a^3*c^3)
```

mupad [B] time = 0.07, size = 28, normalized size = 0.90

$$-\frac{2\,a\,x-1}{6\,a^{3}\,c^{3}\,{\left(a\,x-1\right)}^{3}\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1)^2)/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] -(2*a*x - 1)/(6*a^3*c^3*(a*x - 1)^3*(a*x + 1))
```

sympy [A] time = 0.36, size = 48, normalized size = 1.55

$$\frac{-2ax+1}{6a^{7}c^{3}x^{4}-12a^{6}c^{3}x^{3}+12a^{4}c^{3}x-6a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-2*a*x + 1)/(6*a**7*c**3*x**4 - 12*a**6*c**3*x**3 + 12*a**4*c**3*x - 6*a**
3*c**3)
```

3.1071 $\int \frac{e^{2\tanh^{-1}(ax)}x}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{16a^2c^3(1-ax)}+\frac{1}{16a^2c^3(ax+1)}+\frac{1}{12a^2c^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{8a^2c^3}$$

```
[Out] 1/12/a^2/c^3/(-a*x+1)^3-1/16/a^2/c^3/(-a*x+1)+1/16/a^2/c^3/(a*x+1)-1/8*arct
anh(a*x)/a^2/c^3
```

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6150, 77, 207}

$$-\frac{1}{16a^2c^3(1-ax)}+\frac{1}{16a^2c^3(ax+1)}+\frac{1}{12a^2c^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{8a^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a^2*c^3*(1 - a*x)^3) - 1/(16*a^2*c^3*(1 - a*x)) + 1/(16*a^2*c^3*(1 +
a*x)) - ArcTanh[a*x]/(8*a^2*c^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$= \frac{\int \left(\frac{1}{4a(-1+ax)^4} - \frac{1}{16a(-1+ax)^2} - \frac{1}{16a(1+ax)^2} + \frac{1}{8a\left(-1+a^2x^2\right)}\right)dx}{c^3}$$
$$= \frac{1}{12a^2c^3(1-ax)^3} - \frac{1}{16a^2c^3(1-ax)} + \frac{1}{16a^2c^3(1+ax)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{8ac^3}$$
$$= \frac{1}{12a^2c^3(1-ax)^3} - \frac{1}{16a^2c^3(1-ax)} + \frac{1}{16a^2c^3(1+ax)} - \frac{\tanh^{-1}(ax)}{8a^2c^3}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.88

$$\frac{-\frac{1}{16a^2(1-ax)} + \frac{1}{16a^2(ax+1)} + \frac{1}{12a^2(1-ax)^3} - \frac{\tanh^{-1}(ax)}{8a^2}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1/(12*a^2*(1 - a*x)^3) - 1/(16*a^2*(1 - a*x)) + 1/(16*a^2*(1 + a*x)) - Arc
Tanh[a*x]/(8*a^2))/c^3
```

fricas [B] time = 0.60, size = 123, normalized size = 1.81

$$\frac{6\,a^3x^3 - 12\,a^2x^2 + 2\,ax - 3\left(a^4x^4 - 2\,a^3x^3 + 2\,ax - 1\right)\log\left(ax+1\right) + 3\left(a^4x^4 - 2\,a^3x^3 + 2\,ax - 1\right)\log\left(ax-1\right) - 4}{48\left(a^6c^3x^4 - 2\,a^5c^3x^3 + 2\,a^3c^3x - a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) - 4)/(a^6*c
^3*x^4 - 2*a^5*c^3*x^3 + 2*a^3*c^3*x - a^2*c^3)
```

giac [A] time = 0.18, size = 74, normalized size = 1.09

$$-\frac{\log\left(|ax+1|\right)}{16\,a^{2}c^{3}}+\frac{\log\left(|ax-1|\right)}{16\,a^{2}c^{3}}+\frac{3\,a^{3}x^{3}-6\,a^{2}x^{2}+ax-2}{24\,(ax+1)(ax-1)^{3}a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*log(abs(a*x + 1))/(a^2*c^3) + 1/16*log(abs(a*x - 1))/(a^2*c^3) + 1/24
*(3*a^3*x^3 - 6*a^2*x^2 + a*x - 2)/((a*x + 1)*(a*x - 1)^3*a^2*c^3)
```

maple [A] time = 0.04, size = 75, normalized size = 1.10

$$-\frac{1}{12c^{3}a^{2}\left(ax-1\right)^{3}}+\frac{1}{16c^{3}a^{2}\left(ax-1\right)}+\frac{\ln\left(ax-1\right)}{16c^{3}a^{2}}+\frac{1}{16a^{2}c^{3}\left(ax+1\right)}-\frac{\ln\left(ax+1\right)}{16c^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/12/c^3/a^2/(a*x-1)^3+1/16/c^3/a^2/(a*x-1)+1/16/c^3/a^2*ln(a*x-1)+1/16/a^
2/c^3/(a*x+1)-1/16/c^3/a^2*ln(a*x+1)
```

maxima [A] time = 0.33, size = 93, normalized size = 1.37

$$\frac{3\,a^{3}x^{3}-6\,a^{2}x^{2}+ax-2}{24\left(a^{6}c^{3}x^{4}-2\,a^{5}c^{3}x^{3}+2\,a^{3}c^{3}x-a^{2}c^{3}\right)}-\frac{\log\left(ax+1\right)}{16\,a^{2}c^{3}}+\frac{\log\left(ax-1\right)}{16\,a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^3*x^3 - 6*a^2*x^2 + a*x - 2)/(a^6*c^3*x^4 - 2*a^5*c^3*x^3 + 2*a^3
*c^3*x - a^2*c^3) - 1/16*log(a*x + 1)/(a^2*c^3) + 1/16*log(a*x - 1)/(a^2*c^
3)
```

mupad [B] time = 0.92, size = 73, normalized size = 1.07

$$-\frac{\frac{x}{24\,a}+\frac{a\,x^{3}}{8}-\frac{1}{12\,a^{2}}-\frac{x^{2}}{4}}{-a^{4}\,c^{3}\,x^{4}+2\,a^{3}\,c^{3}\,x^{3}-2\,a\,c^{3}\,x+c^{3}}-\frac{\mathrm{atanh}\left(a\,x\right)}{8\,a^{2}\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x + 1)^2)/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] - (x/(24*a) + (a*x^3)/8 - 1/(12*a^2) - x^2/4)/(c^3 + 2*a^3*c^3*x^3 - a^4*c^
3*x^4 - 2*a*c^3*x) - atanh(a*x)/(8*a^2*c^3)
```

sympy [**A**] time = 0.45, size = 87, normalized size = 1.28

$$\frac{3a^3x^3 - 6a^2x^2 + ax - 2}{24a^6c^3x^4 - 48a^5c^3x^3 + 48a^3c^3x - 24a^2c^3} + \frac{\frac{\log\left(x - \frac{1}{a}\right)}{16} - \frac{\log\left(x + \frac{1}{a}\right)}{16}}{a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (3*a**3*x**3 - 6*a**2*x**2 + a*x - 2)/(24*a**6*c**3*x**4 - 48*a**5*c**3*x**
3 + 48*a**3*c**3*x - 24*a**2*c**3) + (log(x - 1/a)/16 - log(x + 1/a)/16)/(a
**2*c**3)
```

3.1072 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=86

$$\frac{3}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}+\frac{1}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] 1/12/a/c^3/(-a*x+1)^3+1/8/a/c^3/(-a*x+1)^2+3/16/a/c^3/(-a*x+1)-1/16/a/c^3/(
a*x+1)+1/4*arctanh(a*x)/a/c^3
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{3}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}+\frac{1}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a*c^3*(1 - a*x)^3) + 1/(8*a*c^3*(1 - a*x)^2) + 3/(16*a*c^3*(1 - a*x))
 - 1/(16*a*c^3*(1 + a*x)) + ArcTanh[a*x]/(4*a*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{1}{(1-ax)^4(1+ax)^2}\,dx}{c^3}\\
&= \frac{\int \left(\frac{1}{4(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^3}\\
&= \frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^2}+\frac{3}{16ac^3(1-ax)}-\frac{1}{16ac^3(1+ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}\\
&= \frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^2}+\frac{3}{16ac^3(1-ax)}-\frac{1}{16ac^3(1+ax)}+\frac{\tanh^{-1}(ax)}{4ac^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 0.73

$$-\frac{3a^3x^3-6a^2x^2+ax-3(ax-1)^3(ax+1)\tanh^{-1}(ax)+4}{12ac^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/12*(4 + a*x - 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a
*x])/(a*c^3*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.65, size = 121, normalized size = 1.41

$$-\frac{6\,a^3x^3-12\,a^2x^2+2\,ax-3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax+1\right)+3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax-1\right)+}{24\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)
*log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^5*
c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.21, size = 74, normalized size = 0.86

$$\frac{\log\left(|ax+1|\right)}{8\,ac^3}-\frac{\log\left(|ax-1|\right)}{8\,ac^3}-\frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(ax+1)(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(a*x + 1))/(a*c^3) - 1/8*log(abs(a*x - 1))/(a*c^3) - 1/12*(3*a^3
*x^3 - 6*a^2*x^2 + a*x + 4)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

maple [A] time = 0.04, size = 90, normalized size = 1.05

$$-\frac{1}{12c^3a\left(ax-1\right)^3}+\frac{1}{8c^3a\left(ax-1\right)^2}-\frac{3}{16c^3a\left(ax-1\right)}-\frac{\ln\left(ax-1\right)}{8a\,c^3}-\frac{1}{16a\,c^3\left(ax+1\right)}+\frac{\ln\left(ax+1\right)}{8a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/12/c^3/a/(a*x-1)^3+1/8/c^3/a/(a*x-1)^2-3/16/c^3/a/(a*x-1)-1/8/a/c^3*ln(a
*x-1)-1/16/a/c^3/(a*x+1)+1/8*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.32, size = 91, normalized size = 1.06

$$-\frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}+\frac{\log\left(ax+1\right)}{8\,ac^3}-\frac{\log\left(ax-1\right)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^3 - 6*a^2*x^2 + a*x + 4)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^
2*c^3*x - a*c^3) + 1/8*log(a*x + 1)/(a*c^3) - 1/8*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.07, size = 72, normalized size = 0.84

$$\frac{\frac{x}{12}-\frac{a\,x^2}{2}+\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}+\frac{\mathrm{atanh}\left(a\,x\right)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] (x/12 - (a*x^2)/2 + 1/(3*a) + (a^2*x^3)/4)/(c^3 + 2*a^3*c^3*x^3 - a^4*c^3*x
^4 - 2*a*c^3*x) + atanh(a*x)/(4*a*c^3)
```

sympy [A] time = 0.43, size = 83, normalized size = 0.97

$$\frac{-3a^3x^3+6a^2x^2-ax-4}{12a^5c^3x^4-24a^4c^3x^3+24a^2c^3x-12ac^3}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-3*a**3*x**3 + 6*a**2*x**2 - a*x - 4)/(12*a**5*c**3*x**4 - 24*a**4*c**3*x*
*3 + 24*a**2*c**3*x - 12*a*c**3) + (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c*
*3)
```

3.1073 $\int \frac{e^{2\tanh^{-1}(ax)}}{x(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=93

$$\frac{11}{16c^3(1-ax)}+\frac{1}{16c^3(ax+1)}+\frac{1}{4c^3(1-ax)^2}+\frac{1}{12c^3(1-ax)^3}-\frac{13\log(1-ax)}{16c^3}-\frac{3\log(ax+1)}{16c^3}+\frac{\log(x)}{c^3}$$

```
[Out] 1/12/c^3/(-a*x+1)^3+1/4/c^3/(-a*x+1)^2+11/16/c^3/(-a*x+1)+1/16/c^3/(a*x+1)+
ln(x)/c^3-13/16*ln(-a*x+1)/c^3-3/16*ln(a*x+1)/c^3
```

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{11}{16c^3(1-ax)}+\frac{1}{16c^3(ax+1)}+\frac{1}{4c^3(1-ax)^2}+\frac{1}{12c^3(1-ax)^3}-\frac{13\log(1-ax)}{16c^3}-\frac{3\log(ax+1)}{16c^3}+\frac{\log(x)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^3),x]
```

```
[Out] 1/(12*c^3*(1 - a*x)^3) + 1/(4*c^3*(1 - a*x)^2) + 11/(16*c^3*(1 - a*x)) + 1/
(16*c^3*(1 + a*x)) + Log[x]/c^3 - (13*Log[1 - a*x])/(16*c^3) - (3*Log[1 + a
*x])/(16*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1}{x(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$= \frac{\int \left(\frac{1}{x}+\frac{a}{4(-1+ax)^4}-\frac{a}{2(-1+ax)^3}+\frac{11a}{16(-1+ax)^2}-\frac{13a}{16(-1+ax)}-\frac{a}{16(1+ax)^2}-\frac{3a}{16(1+ax)}\right)dx}{c^3}$$
$$= \frac{1}{12c^3(1-ax)^3}+\frac{1}{4c^3(1-ax)^2}+\frac{11}{16c^3(1-ax)}+\frac{1}{16c^3(1+ax)}+\frac{\log(x)}{c^3}-\frac{13\log(1-ax)}{16c^3}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.71

$$\frac{\frac{33}{1-ax}+\frac{3}{ax+1}+\frac{12}{(ax-1)^2}-\frac{4}{(ax-1)^3}-39\log(1-ax)-9\log(ax+1)+48\log(x)}{48c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^3),x]
```

```
[Out] (33/(1 - a*x) - 4/(-1 + a*x)^3 + 12/(-1 + a*x)^2 + 3/(1 + a*x) + 48*Log[x]
- 39*Log[1 - a*x] - 9*Log[1 + a*x])/(48*c^3)
```

fricas [A] time = 0.79, size = 143, normalized size = 1.54

$$-\frac{30\,a^3x^3-36\,a^2x^2-38\,ax+9\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax+1\right)+39\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax-\right.}{48\left(a^4c^3x^4-2\,a^3c^3x^3+2\,ac^3x-c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(30*a^3*x^3 - 36*a^2*x^2 - 38*a*x + 9*(a^4*x^4 - 2*a^3*x^3 + 2*a*x -
1)*log(a*x + 1) + 39*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) - 48*(a
^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(x) + 52)/(a^4*c^3*x^4 - 2*a^3*c^3*x^3 +
 2*a*c^3*x - c^3)
```

giac [A] time = 0.36, size = 73, normalized size = 0.78

$$-\frac{3\,\log\left(|ax+1|\right)}{16\,c^3}-\frac{13\,\log\left(|ax-1|\right)}{16\,c^3}+\frac{\log\left(|x|\right)}{c^3}-\frac{15\,a^3x^3-18\,a^2x^2-19\,ax+26}{24\,(ax+1)(ax-1)^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -3/16*log(abs(a*x + 1))/c^3 - 13/16*log(abs(a*x - 1))/c^3 + log(abs(x))/c^3
 - 1/24*(15*a^3*x^3 - 18*a^2*x^2 - 19*a*x + 26)/((a*x + 1)*(a*x - 1)^3*c^3)
```

maple [A] time = 0.04, size = 78, normalized size = 0.84

$$\frac{\ln(x)}{c^3}-\frac{1}{12c^3\left(ax-1\right)^3}+\frac{1}{4c^3\left(ax-1\right)^2}-\frac{11}{16c^3\left(ax-1\right)}-\frac{13\ln\left(ax-1\right)}{16c^3}+\frac{1}{16c^3\left(ax+1\right)}-\frac{3\ln\left(ax+1\right)}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^3,x)
```

```
[Out] ln(x)/c^3-1/12/c^3/(a*x-1)^3+1/4/c^3/(a*x-1)^2-11/16/c^3/(a*x-1)-13/16/c^3*
ln(a*x-1)+1/16/c^3/(a*x+1)-3/16*ln(a*x+1)/c^3
```

maxima [A] time = 0.32, size = 89, normalized size = 0.96

$$-\frac{15\,a^3x^3-18\,a^2x^2-19\,ax+26}{24\left(a^4c^3x^4-2\,a^3c^3x^3+2\,ac^3x-c^3\right)}-\frac{3\,\log\left(ax+1\right)}{16\,c^3}-\frac{13\,\log\left(ax-1\right)}{16\,c^3}+\frac{\log(x)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*a^3*x^3 - 18*a^2*x^2 - 19*a*x + 26)/(a^4*c^3*x^4 - 2*a^3*c^3*x^3
+ 2*a*c^3*x - c^3) - 3/16*log(a*x + 1)/c^3 - 13/16*log(a*x - 1)/c^3 + log(x
)/c^3
```

mupad [B] time = 0.95, size = 88, normalized size = 0.95

$$\frac{\ln(x)}{c^3}-\frac{13\,\ln\left(a\,x-1\right)}{16\,c^3}-\frac{3\,\ln\left(a\,x+1\right)}{16\,c^3}-\frac{-\frac{5\,a^3\,x^3}{8}+\frac{3\,a^2\,x^2}{4}+\frac{19\,a\,x}{24}-\frac{13}{12}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x*(c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] log(x)/c^3 - (13*log(a*x - 1))/(16*c^3) - (3*log(a*x + 1))/(16*c^3) - ((19*
a*x)/24 + (3*a^2*x^2)/4 - (5*a^3*x^3)/8 - 13/12)/(c^3 + 2*a^3*c^3*x^3 - a^4
*c^3*x^4 - 2*a*c^3*x)
```

sympy [A] time = 0.70, size = 87, normalized size = 0.94

$$\frac{-15a^3x^3+18a^2x^2+19ax-26}{24a^4c^3x^4-48a^3c^3x^3+48ac^3x-24c^3}+\frac{\log\left(x\right)-\frac{13\log\left(x-\frac{1}{a}\right)}{16}-\frac{3\log\left(x+\frac{1}{a}\right)}{16}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-15*a**3*x**3 + 18*a**2*x**2 + 19*a*x - 26)/(24*a**4*c**3*x**4 - 48*a**3*c
**3*x**3 + 48*a*c**3*x - 24*c**3) + (log(x) - 13*log(x - 1/a)/16 - 3*log(x
+ 1/a)/16)/c**3
```

3.1074 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=109

$$\frac{23a}{16c^3(1-ax)}-\frac{a}{16c^3(ax+1)}+\frac{3a}{8c^3(1-ax)^2}+\frac{a}{12c^3(1-ax)^3}+\frac{2a\log(x)}{c^3}-\frac{9a\log(1-ax)}{4c^3}+\frac{a\log(ax+1)}{4c^3}-\frac{1}{c^3x}$$

```
[Out] -1/c^3/x+1/12*a/c^3/(-a*x+1)^3+3/8*a/c^3/(-a*x+1)^2+23/16*a/c^3/(-a*x+1)-1/
16*a/c^3/(a*x+1)+2*a*ln(x)/c^3-9/4*a*ln(-a*x+1)/c^3+1/4*a*ln(a*x+1)/c^3
```

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{23a}{16c^3(1-ax)}-\frac{a}{16c^3(ax+1)}+\frac{3a}{8c^3(1-ax)^2}+\frac{a}{12c^3(1-ax)^3}+\frac{2a\log(x)}{c^3}-\frac{9a\log(1-ax)}{4c^3}+\frac{a\log(ax+1)}{4c^3}-\frac{1}{c^3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^3),x]
```

```
[Out] -(1/(c^3*x)) + a/(12*c^3*(1 - a*x)^3) + (3*a)/(8*c^3*(1 - a*x)^2) + (23*a)/
(16*c^3*(1 - a*x)) - a/(16*c^3*(1 + a*x)) + (2*a*Log[x])/c^3 - (9*a*Log[1 -
 a*x])/(4*c^3) + (a*Log[1 + a*x])/(4*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^3}\,dx = \frac{\int \frac{1}{x^2(1-ax)^4(1+ax)^2}\,dx}{c^3}$$

$$= \frac{\int \left(\frac{1}{x^2}+\frac{2a}{x}+\frac{a^2}{4(-1+ax)^4}-\frac{3a^2}{4(-1+ax)^3}+\frac{23a^2}{16(-1+ax)^2}-\frac{9a^2}{4(-1+ax)}+\frac{a^2}{16(1+ax)^2}+\frac{a^2}{4(1+ax)}\right)dx}{c^3}$$

$$= -\frac{1}{c^3x}+\frac{a}{12c^3(1-ax)^3}+\frac{3a}{8c^3(1-ax)^2}+\frac{23a}{16c^3(1-ax)}-\frac{a}{16c^3(1+ax)}+\frac{2a\log(x)}{c^3}-\frac{9a\,l}{}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 0.72

$$\frac{\frac{69a}{1-ax}-\frac{3a}{ax+1}+\frac{18a}{(ax-1)^2}-\frac{4a}{(ax-1)^3}+96a\log(x)-108a\log(1-ax)+12a\log(ax+1)-\frac{48}{x}}{48c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-48/x + (69*a)/(1 - a*x) - (4*a)/(-1 + a*x)^3 + (18*a)/(-1 + a*x)^2 - (3*a
)/(1 + a*x) + 96*a*Log[x] - 108*a*Log[1 - a*x] + 12*a*Log[1 + a*x])/(48*c^3
)
```

fricas [A] time = 1.09, size = 175, normalized size = 1.61

$$-\frac{30\,a^4x^4-48\,a^3x^3-14\,a^2x^2+46\,ax-3\left(a^5x^5-2\,a^4x^4+2\,a^2x^2-ax\right)\log(ax+1)+27\left(a^5x^5-2\,a^4x^4+2\,a^2x^2\right.}{12\left(a^4c^3x^5-2\,a^3c^3x^4+2\,ac^3x^2-c^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/12*(30*a^4*x^4 - 48*a^3*x^3 - 14*a^2*x^2 + 46*a*x - 3*(a^5*x^5 - 2*a^4*x
^4 + 2*a^2*x^2 - a*x)*log(a*x + 1) + 27*(a^5*x^5 - 2*a^4*x^4 + 2*a^2*x^2 -
a*x)*log(a*x - 1) - 24*(a^5*x^5 - 2*a^4*x^4 + 2*a^2*x^2 - a*x)*log(x) - 12)
/(a^4*c^3*x^5 - 2*a^3*c^3*x^4 + 2*a*c^3*x^2 - c^3*x)
```

giac [A] time = 0.19, size = 88, normalized size = 0.81

$$\frac{a\log(|ax+1|)}{4\,c^3}-\frac{9\,a\log(|ax-1|)}{4\,c^3}+\frac{2\,a\log(|x|)}{c^3}-\frac{15\,a^4x^4-24\,a^3x^3-7\,a^2x^2+23\,ax-6}{6\,(ax+1)(ax-1)^3c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*a*log(abs(a*x + 1))/c^3 - 9/4*a*log(abs(a*x - 1))/c^3 + 2*a*log(abs(x))
/c^3 - 1/6*(15*a^4*x^4 - 24*a^3*x^3 - 7*a^2*x^2 + 23*a*x - 6)/((a*x + 1)*(a
*x - 1)^3*c^3*x)
```

maple [A] time = 0.04, size = 94, normalized size = 0.86

$$-\frac{1}{c^3x}+\frac{2a\ln(x)}{c^3}-\frac{a}{12c^3\left(ax-1\right)^3}+\frac{3a}{8c^3\left(ax-1\right)^2}-\frac{23a}{16c^3\left(ax-1\right)}-\frac{9a\ln\left(ax-1\right)}{4c^3}-\frac{a}{16c^3\left(ax+1\right)}+\frac{a\ln\left(ax+1\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/c^3/x+2*a*ln(x)/c^3-1/12/c^3*a/(a*x-1)^3+3/8/c^3*a/(a*x-1)^2-23/16/c^3*a
/(a*x-1)-9/4/c^3*a*ln(a*x-1)-1/16*a/c^3/(a*x+1)+1/4*a*ln(a*x+1)/c^3
```

maxima [A] time = 0.31, size = 104, normalized size = 0.95

$$-\frac{15\,a^4x^4-24\,a^3x^3-7\,a^2x^2+23\,ax-6}{6\left(a^4c^3x^5-2\,a^3c^3x^4+2\,ac^3x^2-c^3x\right)}+\frac{a\log\left(ax+1\right)}{4\,c^3}-\frac{9\,a\log\left(ax-1\right)}{4\,c^3}+\frac{2\,a\log(x)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/6*(15*a^4*x^4 - 24*a^3*x^3 - 7*a^2*x^2 + 23*a*x - 6)/(a^4*c^3*x^5 - 2*a^
3*c^3*x^4 + 2*a*c^3*x^2 - c^3*x) + 1/4*a*log(a*x + 1)/c^3 - 9/4*a*log(a*x -
 1)/c^3 + 2*a*log(x)/c^3
```

mupad [B] time = 0.99, size = 104, normalized size = 0.95

$$\frac{2\,a\,\ln(x)}{c^3}-\frac{-\frac{5\,a^4\,x^4}{2}+4\,a^3\,x^3+\frac{7\,a^2\,x^2}{6}-\frac{23\,a\,x}{6}+1}{-a^4\,c^3\,x^5+2\,a^3\,c^3\,x^4-2\,a\,c^3\,x^2+c^3\,x}-\frac{9\,a\,\ln\left(a\,x-1\right)}{4\,c^3}+\frac{a\,\ln\left(a\,x+1\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^2*(c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] (2*a*log(x))/c^3 - ((7*a^2*x^2)/6 - (23*a*x)/6 + 4*a^3*x^3 - (5*a^4*x^4)/2
+ 1)/(c^3*x - 2*a*c^3*x^2 + 2*a^3*c^3*x^4 - a^4*c^3*x^5) - (9*a*log(a*x - 1
))/(4*c^3) + (a*log(a*x + 1))/(4*c^3)
```

sympy [A] time = 0.74, size = 104, normalized size = 0.95

$$\frac{-15a^4x^4+24a^3x^3+7a^2x^2-23ax+6}{6a^4c^3x^5-12a^3c^3x^4+12ac^3x^2-6c^3x}+\frac{2a\log\left(x\right)-\frac{9a\log\left(x-\frac{1}{a}\right)}{4}+\frac{a\log\left(x+\frac{1}{a}\right)}{4}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-15*a**4*x**4 + 24*a**3*x**3 + 7*a**2*x**2 - 23*a*x + 6)/(6*a**4*c**3*x**5
 - 12*a**3*c**3*x**4 + 12*a*c**3*x**2 - 6*c**3*x) + (2*a*log(x) - 9*a*log(x
 - 1/a)/4 + a*log(x + 1/a)/4)/c**3
```

3.1075 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=134

$$\frac{39a^2}{16c^3(1-ax)}+\frac{a^2}{16c^3(ax+1)}+\frac{a^2}{2c^3(1-ax)^2}+\frac{a^2}{12c^3(1-ax)^3}+\frac{5a^2\log(x)}{c^3}-\frac{75a^2\log(1-ax)}{16c^3}-\frac{5a^2\log(ax+1)}{16c^3}-\frac{2a}{c^3x}-$$

```
[Out] -1/2/c^3/x^2-2*a/c^3/x+1/12*a^2/c^3/(-a*x+1)^3+1/2*a^2/c^3/(-a*x+1)^2+39/16
*a^2/c^3/(-a*x+1)+1/16*a^2/c^3/(a*x+1)+5*a^2*ln(x)/c^3-75/16*a^2*ln(-a*x+1)
/c^3-5/16*a^2*ln(a*x+1)/c^3
```

Rubi [A] time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$\frac{39a^2}{16c^3(1-ax)}+\frac{a^2}{16c^3(ax+1)}+\frac{a^2}{2c^3(1-ax)^2}+\frac{a^2}{12c^3(1-ax)^3}+\frac{5a^2\log(x)}{c^3}-\frac{75a^2\log(1-ax)}{16c^3}-\frac{5a^2\log(ax+1)}{16c^3}-\frac{2a}{c^3x}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/(2*c^3*x^2) - (2*a)/(c^3*x) + a^2/(12*c^3*(1 - a*x)^3) + a^2/(2*c^3*(1 -
 a*x)^2) + (39*a^2)/(16*c^3*(1 - a*x)) + a^2/(16*c^3*(1 + a*x)) + (5*a^2*Lo
g[x])/c^3 - (75*a^2*Log[1 - a*x])/(16*c^3) - (5*a^2*Log[1 + a*x])/(16*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^3}\,dx = \frac{\int \frac{1}{x^3(1-ax)^4(1+ax)^2}\,dx}{c^3}$$

$$= \frac{\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{5a^2}{x}+\frac{a^3}{4(-1+ax)^4}-\frac{a^3}{(-1+ax)^3}+\frac{39a^3}{16(-1+ax)^2}-\frac{75a^3}{16(-1+ax)}-\frac{a^3}{16(1+ax)^2}-\frac{5a^3}{16(1+ax)}\right)dx}{c^3}$$

$$= -\frac{1}{2c^3x^2}-\frac{2a}{c^3x}+\frac{a^2}{12c^3(1-ax)^3}+\frac{a^2}{2c^3(1-ax)^2}+\frac{39a^2}{16c^3(1-ax)}+\frac{a^2}{16c^3(1+ax)}+\frac{5a^2\log}{c^3}$$

Mathematica [A] time = 0.12, size = 98, normalized size = 0.73

$$\frac{\frac{117a^2}{1-ax}+\frac{3a^2}{ax+1}+\frac{24a^2}{(ax-1)^2}-\frac{4a^2}{(ax-1)^3}+240a^2\log(x)-225a^2\log(1-ax)-15a^2\log(ax+1)-\frac{96a}{x}-\frac{24}{x^2}}{48c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-24/x^2 - (96*a)/x + (117*a^2)/(1 - a*x) - (4*a^2)/(-1 + a*x)^3 + (24*a^2)
/(-1 + a*x)^2 + (3*a^2)/(1 + a*x) + 240*a^2*Log[x] - 225*a^2*Log[1 - a*x] -
 15*a^2*Log[1 + a*x])/(48*c^3)
```

fricas [A] time = 0.59, size = 197, normalized size = 1.47

$$-\frac{210\,a^5x^5-300\,a^4x^4-170\,a^3x^3+340\,a^2x^2-48\,ax+15\left(a^6x^6-2\,a^5x^5+2\,a^3x^3-a^2x^2\right)\log(ax+1)+225\left(a^6\right.}{48\left(a^4c^3x^6-2\,a^3c^3x^5+2\,ac^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/48*(210*a^5*x^5 - 300*a^4*x^4 - 170*a^3*x^3 + 340*a^2*x^2 - 48*a*x + 15*
(a^6*x^6 - 2*a^5*x^5 + 2*a^3*x^3 - a^2*x^2)*log(a*x + 1) + 225*(a^6*x^6 - 2
*a^5*x^5 + 2*a^3*x^3 - a^2*x^2)*log(a*x - 1) - 240*(a^6*x^6 - 2*a^5*x^5 + 2
*a^3*x^3 - a^2*x^2)*log(x) - 24)/(a^4*c^3*x^6 - 2*a^3*c^3*x^5 + 2*a*c^3*x^3
 - c^3*x^2)
```

giac [A] time = 0.19, size = 102, normalized size = 0.76

$$-\frac{5\,a^2\log(|ax+1|)}{16\,c^3}-\frac{75\,a^2\log(|ax-1|)}{16\,c^3}+\frac{5\,a^2\log(|x|)}{c^3}-\frac{105\,a^5x^5-150\,a^4x^4-85\,a^3x^3+170\,a^2x^2-24\,ax-12}{24\,(ax+1)(ax-1)^3c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -5/16*a^2*log(abs(a*x + 1))/c^3 - 75/16*a^2*log(abs(a*x - 1))/c^3 + 5*a^2*l
og(abs(x))/c^3 - 1/24*(105*a^5*x^5 - 150*a^4*x^4 - 85*a^3*x^3 + 170*a^2*x^2
 - 24*a*x - 12)/((a*x + 1)*(a*x - 1)^3*c^3*x^2)
```

maple [A] time = 0.04, size = 117, normalized size = 0.87

$$-\frac{1}{2c^3x^2}-\frac{2a}{c^3x}+\frac{5a^2\ln(x)}{c^3}-\frac{a^2}{12c^3\left(ax-1\right)^3}+\frac{a^2}{2c^3\left(ax-1\right)^2}-\frac{39a^2}{16c^3\left(ax-1\right)}-\frac{75a^2\ln\left(ax-1\right)}{16c^3}+\frac{a^2}{16c^3\left(ax+1\right)}-\frac{5a^2\ln\left(ax\right.}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/2/c^3/x^2-2*a/c^3/x+5*a^2*ln(x)/c^3-1/12/c^3*a^2/(a*x-1)^3+1/2/c^3*a^2/(
a*x-1)^2-39/16/c^3*a^2/(a*x-1)-75/16/c^3*a^2*ln(a*x-1)+1/16*a^2/c^3/(a*x+1)
-5/16*a^2*ln(a*x+1)/c^3
```

maxima [A] time = 0.33, size = 120, normalized size = 0.90

$$-\frac{105\,a^5x^5-150\,a^4x^4-85\,a^3x^3+170\,a^2x^2-24\,ax-12}{24\left(a^4c^3x^6-2\,a^3c^3x^5+2\,ac^3x^3-c^3x^2\right)}-\frac{5\,a^2\log\left(ax+1\right)}{16\,c^3}-\frac{75\,a^2\log\left(ax-1\right)}{16\,c^3}+\frac{5\,a^2\log(x)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/24*(105*a^5*x^5 - 150*a^4*x^4 - 85*a^3*x^3 + 170*a^2*x^2 - 24*a*x - 12)/
(a^4*c^3*x^6 - 2*a^3*c^3*x^5 + 2*a*c^3*x^3 - c^3*x^2) - 5/16*a^2*log(a*x +
1)/c^3 - 75/16*a^2*log(a*x - 1)/c^3 + 5*a^2*log(x)/c^3
```

mupad [B] time = 1.01, size = 119, normalized size = 0.89

$$\frac{5\,a^2\,\ln(x)}{c^3}-\frac{-\frac{35\,a^5\,x^5}{8}+\frac{25\,a^4\,x^4}{4}+\frac{85\,a^3\,x^3}{24}-\frac{85\,a^2\,x^2}{12}+a\,x+\frac{1}{2}}{-a^4\,c^3\,x^6+2\,a^3\,c^3\,x^5-2\,a\,c^3\,x^3+c^3\,x^2}-\frac{75\,a^2\,\ln\left(a\,x-1\right)}{16\,c^3}-\frac{5\,a^2\,\ln\left(a\,x+1\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^3*(c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] (5*a^2*log(x))/c^3 - (a*x - (85*a^2*x^2)/12 + (85*a^3*x^3)/24 + (25*a^4*x^4
)/4 - (35*a^5*x^5)/8 + 1/2)/(c^3*x^2 - 2*a*c^3*x^3 + 2*a^3*c^3*x^5 - a^4*c^
3*x^6) - (75*a^2*log(a*x - 1))/(16*c^3) - (5*a^2*log(a*x + 1))/(16*c^3)
```

sympy [A] time = 0.88, size = 121, normalized size = 0.90

$$\frac{-105a^5x^5+150a^4x^4+85a^3x^3-170a^2x^2+24ax+12}{24a^4c^3x^6-48a^3c^3x^5+48ac^3x^3-24c^3x^2}+\frac{5a^2\log(x)-\frac{75a^2\log\left(x-\frac{1}{a}\right)}{16}-\frac{5a^2\log\left(x+\frac{1}{a}\right)}{16}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-105*a**5*x**5 + 150*a**4*x**4 + 85*a**3*x**3 - 170*a**2*x**2 + 24*a*x + 1
2)/(24*a**4*c**3*x**6 - 48*a**3*c**3*x**5 + 48*a*c**3*x**3 - 24*c**3*x**2)
+ (5*a**2*log(x) - 75*a**2*log(x - 1/a)/16 - 5*a**2*log(x + 1/a)/16)/c**3
```

3.1076 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=121

$$\frac{5}{32ac^4(1-ax)}-\frac{5}{64ac^4(ax+1)}+\frac{3}{32ac^4(1-ax)^2}-\frac{1}{64ac^4(ax+1)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{32ac^4(1-ax)^4}+\frac{15\tanh^{-1}(ax)}{64ac^4}$$

```
[Out] 1/32/a/c^4/(-a*x+1)^4+1/16/a/c^4/(-a*x+1)^3+3/32/a/c^4/(-a*x+1)^2+5/32/a/c^
4/(-a*x+1)-1/64/a/c^4/(a*x+1)^2-5/64/a/c^4/(a*x+1)+15/64*arctanh(a*x)/a/c^4
```

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{5}{32ac^4(1-ax)}-\frac{5}{64ac^4(ax+1)}+\frac{3}{32ac^4(1-ax)^2}-\frac{1}{64ac^4(ax+1)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{32ac^4(1-ax)^4}+\frac{15\tanh^{-1}(ax)}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] 1/(32*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)^3) + 3/(32*a*c^4*(1 - a*x)
^2) + 5/(32*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 + a*x)^2) - 5/(64*a*c^4*(1 +
a*x)) + (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{1}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$

$$= \frac{\int \left(-\frac{1}{8(-1+ax)^5}+\frac{3}{16(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{5}{32(-1+ax)^2}+\frac{1}{32(1+ax)^3}+\frac{5}{64(1+ax)^2}-\frac{15}{64\left(-1+a^2x^2\right)}\right)dx}{c^4}$$

$$= \frac{1}{32ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{3}{32ac^4(1-ax)^2}+\frac{5}{32ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)^2}-$$

$$= \frac{1}{32ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{3}{32ac^4(1-ax)^2}+\frac{5}{32ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)^2}-$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.68

$$\frac{-15a^5x^5+30a^4x^4+10a^3x^3-50a^2x^2+17ax+15(ax-1)^4(ax+1)^2\tanh^{-1}(ax)+16}{64ac^4(ax-1)^4(ax+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (16 + 17*a*x - 50*a^2*x^2 + 10*a^3*x^3 + 30*a^4*x^4 - 15*a^5*x^5 + 15*(-1 +
 a*x)^4*(1 + a*x)^2*ArcTanh[a*x])/(64*a*c^4*(-1 + a*x)^4*(1 + a*x)^2)
```

fricas [B] time = 0.59, size = 217, normalized size = 1.79

$$-\frac{30\,a^5x^5-60\,a^4x^4-20\,a^3x^3+100\,a^2x^2-34\,ax-15\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\log\left(ax\right.}{128\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/128*(30*a^5*x^5 - 60*a^4*x^4 - 20*a^3*x^3 + 100*a^2*x^2 - 34*a*x - 15*(a
^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1
) + 15*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*lo
g(a*x - 1) - 32)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3
 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 1.12, size = 91, normalized size = 0.75

$$\frac{15 \log(|ax+1|)}{128\,ac^4} - \frac{15 \log(|ax-1|)}{128\,ac^4} - \frac{15\,a^5x^5 - 30\,a^4x^4 - 10\,a^3x^3 + 50\,a^2x^2 - 17\,ax - 16}{64\,(ax+1)^2(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 15/128*log(abs(a*x + 1))/(a*c^4) - 15/128*log(abs(a*x - 1))/(a*c^4) - 1/64*
(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/((a*x + 1
)^2*(a*x - 1)^4*a*c^4)
```

maple [A] time = 0.04, size = 120, normalized size = 0.99

$$\frac{1}{32c^4a\,(ax-1)^4} - \frac{1}{16c^4a\,(ax-1)^3} + \frac{3}{32c^4a\,(ax-1)^2} - \frac{5}{32c^4a\,(ax-1)} - \frac{15\ln(ax-1)}{128c^4a} - \frac{1}{64a\,c^4\,(ax+1)^2} - \frac{5}{64a\,c^4\,(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/32/c^4/a/(a*x-1)^4-1/16/c^4/a/(a*x-1)^3+3/32/c^4/a/(a*x-1)^2-5/32/c^4/a/(
a*x-1)-15/128/c^4/a*ln(a*x-1)-1/64/a/c^4/(a*x+1)^2-5/64/a/c^4/(a*x+1)+15/12
8*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.34, size = 140, normalized size = 1.16

$$-\frac{15\,a^5x^5 - 30\,a^4x^4 - 10\,a^3x^3 + 50\,a^2x^2 - 17\,ax - 16}{64\left(a^7c^4x^6 - 2\,a^6c^4x^5 - a^5c^4x^4 + 4\,a^4c^4x^3 - a^3c^4x^2 - 2\,a^2c^4x + ac^4\right)} + \frac{15 \log(ax+1)}{128\,ac^4} - \frac{15 \log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/64*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/(a^
7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a
^2*c^4*x + a*c^4) + 15/128*log(a*x + 1)/(a*c^4) - 15/128*log(a*x - 1)/(a*c^
4)
```

mupad [B] time = 0.14, size = 122, normalized size = 1.01

$$\frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4} - \frac{\frac{17\,x}{64} - \frac{25\,a\,x^2}{32} + \frac{1}{4\,a} + \frac{5\,a^2\,x^3}{32} + \frac{15\,a^3\,x^4}{32} - \frac{15\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6 + 2\,a^5\,c^4\,x^5 + a^4\,c^4\,x^4 - 4\,a^3\,c^4\,x^3 + a^2\,c^4\,x^2 + 2\,a\,c^4\,x - c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - a^2*c*x^2)^4*(a^2*x^2 - 1)),x)
```

```
[Out] (15*atanh(a*x))/(64*a*c^4) - ((17*x)/64 - (25*a*x^2)/32 + 1/(4*a) + (5*a^2*
x^3)/32 + (15*a^3*x^4)/32 - (15*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4
*x^3 + a^4*c^4*x^4 + 2*a^5*c^4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x)
```

sympy [A] time = 0.61, size = 143, normalized size = 1.18

$$-\frac{15a^5x^5 - 30a^4x^4 - 10a^3x^3 + 50a^2x^2 - 17ax - 16}{64a^7c^4x^6 - 128a^6c^4x^5 - 64a^5c^4x^4 + 256a^4c^4x^3 - 64a^3c^4x^2 - 128a^2c^4x + 64ac^4} - \frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128} - \frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] -(15*a**5*x**5 - 30*a**4*x**4 - 10*a**3*x**3 + 50*a**2*x**2 - 17*a*x - 16)/
(64*a**7*c**4*x**6 - 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 + 256*a**4*c**4
*x**3 - 64*a**3*c**4*x**2 - 128*a**2*c**4*x + 64*a*c**4) - (15*log(x - 1/a)
/128 - 15*log(x + 1/a)/128)/(a*c**4)
```

3.1077 $\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$-\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] 3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4-3/5*x^2*(-a^2*c*x^
2+c)^(1/2)/a^2-1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a-1/5*x^4*(-a^2*c*x^2+c)^(1/2)-
3/20*(5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

Rubi [A] time = 0.33, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1809, 833, 780, 217, 203}

$$-\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) - (x^3*Sqrt[c - a^2*c*x^2])/(2*a) - (x
^4*Sqrt[c - a^2*c*x^2])/5 - (3*(8 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) +
(3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = c\int \frac{x^3(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{\int \frac{x^3(-9a^2c-10a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2}$$
$$= -\frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x^2(30a^3c^2+36a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c}$$
$$= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{\int \frac{x(-72a^4c^3-90a^5c^3x)}{\sqrt{c-a^2cx^2}}\,dx}{60a^6c^2}$$
$$= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4} +$$
$$= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4} +$$
$$= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4} +$$

Mathematica [A] time = 0.14, size = 96, normalized size = 0.70

$$-\frac{15\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(a^2x^2-1)}\right) + (4a^4x^4+10a^3x^3+12a^2x^2+15ax+24)\sqrt{c-a^2cx^2}}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/20*(Sqrt[c - a^2*c*x^2]*(24 + 15*a*x + 12*a^2*x^2 + 10*a^3*x^3 + 4*a^4*x
^4) + 15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))]
)/a^4
```

fricas [A] time = 0.69, size = 184, normalized size = 1.34

$$\left[-\frac{2(4a^4x^4+10a^3x^3+12a^2x^2+15ax+24)\sqrt{-a^2cx^2+c} - 15\sqrt{-c}\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{40a^4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/40*(2*(4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-a^2*c*x
^2 + c) - 15*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c))/a^4, -1/20*((4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt
(-a^2*c*x^2 + c) + 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*
c*x^2 - c)))/a^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 210, normalized size = 1.53

$$\frac{x^2\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{5a^2c}+\frac{4\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{5c\,a^4}+\frac{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{2a^3c}-\frac{5x\sqrt{-a^2c\,x^2+c}}{4a^3}-\frac{5c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{4a^3\sqrt{a^2c}}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c+4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)+1/2/a^3*x
*(-a^2*c*x^2+c)^(3/2)/c-5/4/a^3*x*(-a^2*c*x^2+c)^(1/2)-5/4/a^3*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^4*(-(x-1/a)^2*a^2*c-2*
a*c*(x-1/a))^(1/2)+2/a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2
*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.60, size = 120, normalized size = 0.88

$$\frac{1}{20}a\left(\frac{4\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2}{a^3c}-\frac{25\sqrt{-a^2cx^2+c}\,x}{a^4}+\frac{10\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}{a^4c}+\frac{15\sqrt{c}\arcsin\left(ax\right)}{a^5}-\frac{40\sqrt{-a^2cx^2+c}}{a^5}+\frac{16}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/20*a*(4*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^3*c) - 25*sqrt(-a^2*c*x^2 + c)*x/a^
4 + 10*(-a^2*c*x^2 + c)^(3/2)*x/(a^4*c) + 15*sqrt(c)*arcsin(a*x)/a^5 - 40*s
qrt(-a^2*c*x^2 + c)/a^5 + 16*(-a^2*c*x^2 + c)^(3/2)/(a^5*c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3\sqrt{-a^2cx^2+c}}{ax-1}\,dx - \int \frac{ax^4\sqrt{-a^2cx^2+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**3*sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x**4*sqrt(
-a**2*c*x**2 + c)/(a*x - 1), x)
```

3.1078 $\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}+\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] 7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3-2/3*x^2*(-a^2*c*x^
2+c)^(1/2)/a-1/4*x^3*(-a^2*c*x^2+c)^(1/2)-1/24*(21*a*x+32)*(-a^2*c*x^2+c)^(
1/2)/a^3
```

Rubi [A] time = 0.29, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1809, 833, 780, 217, 203}

$$-\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}+\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) - (x^3*Sqrt[c - a^2*c*x^2])/4 - ((32 + 2
1*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) + (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(8*a^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = c\int \frac{x^2(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{\int \frac{x^2(-7a^2c-8a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{\int \frac{x(16a^3c^2+21a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} + \frac{(7c)\int \frac{1}{\sqrt{c-a^2cx^2}}}{8a^2}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} + \frac{(7c)\,\text{Subst}\Big(\int}{}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} + \frac{7\sqrt{c}\,\tan^{-1}\Big(}{8a^3}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 0.79

$$-\frac{21\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(a^2x^2-1)}\right) + (6a^3x^3+16a^2x^2+21ax+32)\sqrt{c-a^2cx^2}}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/24*(Sqrt[c - a^2*c*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt
[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/a^3
```

fricas [A] time = 0.74, size = 168, normalized size = 1.50

$$\left[-\frac{2\,(6\,a^3x^3+16\,a^2x^2+21\,ax+32)\sqrt{-a^2cx^2+c} - 21\,\sqrt{-c}\,\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{48\,a^3}, -\frac{(6\,a^3x^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/48*(2*(6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) - 21*
sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, -
1/24*((6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt
(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

giac [A] time = 0.19, size = 84, normalized size = 0.75

$$-\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,x+\frac{8}{a}\right)x+\frac{21}{a^2}\right)x+\frac{32}{a^3}\right)-\frac{7\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,a^2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x + 8/a)*x + 21/a^2)*x + 32/a^3) - 7/8*c*
log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

maple [B] time = 0.04, size = 186, normalized size = 1.66

$$\frac{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{4a^2c}-\frac{9x\sqrt{-a^2c\,x^2+c}}{8a^2}-\frac{9c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{8a^2\sqrt{a^2c}}+\frac{2\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{3a^3c}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{a^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c-9/8/a^2*x*(-a^2*c*x^2+c)^(1/2)-9/8/a^2*c/(
a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3/a^3*(-a^2*c*x
^2+c)^(3/2)/c-2/a^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+2/a^2*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.64, size = 96, normalized size = 0.86

$$-\frac{1}{24}\,a\left(\frac{27\sqrt{-a^2cx^2+c}\,x}{a^3}-\frac{6\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}{a^3c}-\frac{21\sqrt{c}\,\arcsin\left(ax\right)}{a^4}+\frac{48\sqrt{-a^2cx^2+c}}{a^4}-\frac{16\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{a^4c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/24*a*(27*sqrt(-a^2*c*x^2 + c)*x/a^3 - 6*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c)
 - 21*sqrt(c)*arcsin(a*x)/a^4 + 48*sqrt(-a^2*c*x^2 + c)/a^4 - 16*(-a^2*c*x^
2 + c)^(3/2)/(a^4*c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2\,\sqrt{c-a^2\,c\,x^2}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}\sqrt{-a^{2}cx^{2}+c}}{ax-1}\, dx - \int \frac{ax^{3}\sqrt{-a^{2}cx^{2}+c}}{ax-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**2*sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x**3*sqrt(
-a**2*c*x**2 + c)/(a*x - 1), x)
```

3.1079 $\int e^{2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2-1/3*x^2*(-a^2*c*x^2+c)
^(1/2)-1/3*(3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

Rubi [A] time = 0.18, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6151, 1809, 780, 217, 203}

$$-\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(x^2*Sqrt[c - a^2*c*x^2])/3 - ((5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) +
(Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= c\int \frac{x(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{\int \frac{x\left(-5a^2c-6a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\ &= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\ &= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\ &= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.10, size = 79, normalized size = 0.94

$$-\frac{\left(a^2x^2+3ax+5\right)\sqrt{c-a^2cx^2}+3\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/3*((5 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 3*Sqrt[c]*ArcTan[(a*x*Sqr
t[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/a^2
```

fricas [A] time = 0.84, size = 150, normalized size = 1.79

$$\left[-\frac{2\sqrt{-a^2cx^2+c}\left(a^2x^2+3\,ax+5\right)-3\sqrt{-c}\,\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{6\,a^2},-\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2+3\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x + 5) - 3*sqrt(-c)*log(2*a^2*
c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, -1/3*(sqrt(-a^2*c*x^
2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt
(c)*x/(a^2*c*x^2 - c)))/a^2]
```

giac [A] time = 0.23, size = 73, normalized size = 0.87

$$-\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x+\frac{3}{a}\right)x+\frac{5}{a^2}\right)-\frac{c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*sqrt(-a^2*c*x^2 + c)*((x + 3/a)*x + 5/a^2) - c*log(abs(-sqrt(-a^2*c)*x
 + sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

maple [B] time = 0.04, size = 164, normalized size = 1.95

$$\frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{3a^2c}-\frac{x\sqrt{-a^2c\,x^2+c}}{a}-\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{a\sqrt{a^2c}}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{a^2}+\frac{2c\arctan\left(\frac{\sqrt{a^2c}}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c}}\right.}{a\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c-x/a*(-a^2*c*x^2+c)^(1/2)-1/a*c/(a^2*c)^(1/2)
*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^2*(-(x-1/a)^2*a^2*c-2*a*c
```

```
*(x-1/a))^(1/2)+2/a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*
c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.52, size = 74, normalized size = 0.88

$$-\frac{1}{3}a\left(\frac{3\sqrt{-a^2cx^2+c}\,x}{a^2}-\frac{3\sqrt{c}\,\arcsin(ax)}{a^3}+\frac{6\sqrt{-a^2cx^2+c}}{a^3}-\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*a*(3*sqrt(-a^2*c*x^2 + c)*x/a^2 - 3*sqrt(c)*arcsin(a*x)/a^3 + 6*sqrt(-
a^2*c*x^2 + c)/a^3 - (-a^2*c*x^2 + c)^(3/2)/(a^3*c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x\sqrt{-a^2cx^2+c}}{ax-1}\,dx-\int\frac{ax^2\sqrt{-a^2cx^2+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x*sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x**2*sqrt(-a*
*2*c*x**2 + c)/(a*x - 1), x)
```

3.1080 $\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=86

$$-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] 3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a-3/2*(-a^2*c*x^2+c)^(
1/2)/a-1/2*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6141, 671, 641, 217, 203}

$$-\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) + (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
```

```
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= c\int \frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}(3c)\int \frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}(3c)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2}}\right.\\ &= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.88

$$-\frac{\sqrt{c-a^2cx^2}\left(\sqrt{1-a^2x^2}\,(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/2*(Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 -
a*x]/Sqrt[2]]))/(a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.70, size = 134, normalized size = 1.56

$$\left[-\frac{2\sqrt{-a^2cx^2+c}\,(ax+4)-3\sqrt{-c}\,\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{4\,a},-\frac{\sqrt{-a^2cx^2+c}\,(ax+4)+3\sqrt{c}\,\mathrm{arc}}{2\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) - 3*sqrt(-c)*log(2*a^2*c*x^2 + 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, -1/2*(sqrt(-a^2*c*x^2 + c)*(a*x +
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

giac [A] time = 0.23, size = 62, normalized size = 0.72

$$-\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right)-\frac{3\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) - 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(
-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

maple [A] time = 0.04, size = 134, normalized size = 1.56

$$-\frac{x\sqrt{-a^2c\,x^2+c}}{2}-\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{a}+\frac{2c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}\right)}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/2*x*(-a^2*c*x^2+c)^(1/2)-1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^
2*c*x^2+c)^(1/2))-2/a*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+2*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.46, size = 52, normalized size = 0.60

$$-\frac{1}{2}\,a\left(\frac{\sqrt{-a^2cx^2+c}\,x}{a}-\frac{3\sqrt{c}\,\arcsin\left(ax\right)}{a^2}+\frac{4\sqrt{-a^2cx^2+c}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*a*(sqrt(-a^2*c*x^2 + c)*x/a - 3*sqrt(c)*arcsin(a*x)/a^2 + 4*sqrt(-a^2*
c*x^2 + c)/a^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2cx^2+c}}{ax-1}\,dx - \int \frac{ax\sqrt{-a^2cx^2+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x*sqrt(-a**2*c*
x**2 + c)/(a*x - 1), x)
```

3.1081 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=78

$$-\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)-arctanh((-a^2*c*x^2+c)^(
1/2)/c^(1/2))*c^(1/2)-(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1809, 844, 217, 203, 266, 63, 208}

$$-\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
- Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= c\int \frac{(1+ax)^2}{x\sqrt{c-a^2cx^2}}\,dx \\
&= -\sqrt{c-a^2cx^2} - \frac{\int \frac{-a^2c-2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2} \\
&= -\sqrt{c-a^2cx^2} + c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx + (2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx \\
&= -\sqrt{c-a^2cx^2} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) + (2ac)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,d\right. \\
&= -\sqrt{c-a^2cx^2} + 2\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a^2} \\
&= -\sqrt{c-a^2cx^2} + 2\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 99, normalized size = 1.27

$$
-\sqrt{c-a^2cx^2} - \sqrt{c}\,\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - 2\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) + \sqrt{c}\,\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*
(-1 + a^2*x^2))] + Sqrt[c]*Log[x] - Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*
x^2]]
```

fricas [A] time = 0.66, size = 196, normalized size = 2.51

$$
\left[-2\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right) + \frac{1}{2}\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right) - \sqrt{-a^2cx^2+c}, -\sqrt{-c}\,\text{arct}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] [-2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*
sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) - sqrt
```

```
(-a^2*c*x^2 + c), -sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2
 - c)) + sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c
) - sqrt(-a^2*c*x^2 + c)]
```

giac [A] time = 0.19, size = 97, normalized size = 1.24

$$\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2\,a\sqrt{-c}\,\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\sqrt{-a^2cx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) + 2*
a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - sqrt(-
a^2*c*x^2 + c)
```

maple [A] time = 0.04, size = 128, normalized size = 1.64

$$\sqrt{-a^2c\,x^2+c}-\sqrt{c}\;\ln\left(\frac{2c+2\sqrt{c}\;\sqrt{-a^2c\,x^2+c}}{x}\right)-2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}+\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\;x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-2*(
-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1
/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-a^2cx^2+c}\,(ax+1)^2}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2/((a^2*x^2 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^2}{x\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2cx^2+c}}{ax^2-x}\,dx-\int \frac{ax\sqrt{-a^2cx^2+c}}{ax^2-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**2 - x), x) - Integral(a*x*sqrt(-a**2
*c*x**2 + c)/(a*x**2 - x), x)
```

3.1082 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)-2*a*arctanh((-a^2*c*x^2+
c)^(1/2)/c^(1/2))*c^(1/2)-(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.25, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1807, 844, 217, 203, 266, 63, 208}

$$-\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) + a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^
2]] - 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= c\int \frac{(1+ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{x} - \int \frac{-2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{x} + (2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{x} + (ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) + \left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+a^2cx
\right.\\
&= -\frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a}\\
&= -\frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - 2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 106, normalized size = 1.29

$$
-\frac{\sqrt{c-a^2cx^2}}{x} - 2a\sqrt{c}\,\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - a\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) + 2a\sqrt{c}\,\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) - a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt
[c]*(-1 + a^2*x^2))] + 2*a*Sqrt[c]*Log[x] - 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqr
t[c - a^2*c*x^2]]
```

fricas [A] time = 1.01, size = 212, normalized size = 2.59

$$
\left[-\frac{a\sqrt{c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right) - a\sqrt{c}\,x\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\,\sqrt{c}-2c}{x^2}\right) + \sqrt{-a^2cx^2+c}}{x}, -\frac{4a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-a^2
\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [-(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - a
*sqrt(c)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + s
qrt(-a^2*c*x^2 + c))/x, -1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sq
rt(-c)/(a^2*c*x^2 - c)) - a*sqrt(-c)*x*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2
+ c)*a*sqrt(-c)*x - c) + 2*sqrt(-a^2*c*x^2 + c))/x]
```

giac [A] time = 1.84, size = 133, normalized size = 1.62

$$\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{a^2\sqrt{-c}\,\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\frac{2\,a^2\sqrt{-c}\,c}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] 4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) +
a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + 2*a^
2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

maple [B] time = 0.05, size = 211, normalized size = 2.57

$$-2\sqrt{c}\;\ln\left(\frac{2c+2\sqrt{c}\;\sqrt{-a^2c\,x^2+c}}{x}\right)a+2\sqrt{-a^2c\,x^2+c}\;a-\frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}-a^2x\sqrt{-a^2c\,x^2+c}-\frac{a^2c\arctan\left(\frac{\sqrt{a^2c}\;x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] -2*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a+2*(-a^2*c*x^2+c)^(1
/2)*a-1/c/x*(-a^2*c*x^2+c)^(3/2)-a^2*x*(-a^2*c*x^2+c)^(1/2)-a^2*c/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*a*(-(x-1/a)^2*a^2*c-2*a
*c*(x-1/a))^(1/2)+2*a^2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*
a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2\sqrt{c}\int\frac{\sqrt{ax+1}\,\sqrt{-ax+1}}{a^2x^2-1}\,dx+a\sqrt{c}\,\log\left(\frac{\sqrt{-a^2cx^2+c}\,-\sqrt{c}}{\sqrt{-a^2cx^2+c}\,+\sqrt{c}}\right)-\frac{\sqrt{ax+1}\,\sqrt{-ax+1}\,\sqrt{c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] -a^2*sqrt(c)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^2*x^2 - 1), x) + a*s
qrt(c)*log((sqrt(-a^2*c*x^2 + c) - sqrt(c))/(sqrt(-a^2*c*x^2 + c) + sqrt(c)
)) - sqrt(a*x + 1)*sqrt(-a*x + 1)*sqrt(c)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^2}{x^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2cx^2+c}}{ax^3-x^2}\,dx-\int \frac{ax\sqrt{-a^2cx^2+c}}{ax^3-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**3 - x**2), x) - Integral(a*x*sqrt(-a
**2*c*x**2 + c)/(a*x**3 - x**2), x)
```

3.1083 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{3}{2}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-1/2*(-a^2*c*x^2+c)^(
1/2)/x^2-2*a*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.25, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1807, 807, 266, 63, 208}

$$-\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{3}{2}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2) - (2*a*Sqrt[c - a^2*c*x^2])/x - (3*a^2*Sqrt[c]
*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= c\int \frac{(1+ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{1}{2}\int \frac{-4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{1}{4}\left(3a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right) \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 79, normalized size = 1.01

$$-\frac{(4ax+1)\sqrt{c-a^2cx^2}}{2x^2}-\frac{3}{2}a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{3}{2}a^2\sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] -1/2*((1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 + (3*a^2*Sqrt[c]*Log[x])/2 - (3*
a^2*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

fricas [A] time = 1.08, size = 148, normalized size = 1.90

$$\left[\frac{3a^2\sqrt{c}x^2\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-2\sqrt{-a^2cx^2+c}(4ax+1)}{4x^2},-\frac{3a^2\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-a^2\cdots}}{2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x + 1))/x^2, -1/2*(3*a^2*sqrt(-c)*x
^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2
+ c)*(4*a*x + 1))/x^2]
```

giac [B] time = 0.19, size = 201, normalized size = 2.58

$$\frac{3a^2c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^3a^2c-4\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}\,c|a|+\left(\sqrt{\cdots}\right.}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="gia
c")
```

```
[Out] 3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c - 4*(sqrt(-a^2*c)*x - sq
rt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*x
^2 + c))*a^2*c^2 + 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))^2 - c)^2
```

maple [B] time = 0.04, size = 239, normalized size = 3.06

$$-\frac{3\sqrt{c}\ \ln\left(\frac{2c+2\sqrt{c}\ \sqrt{-a^2c\,x^2+c}}{x}\right)a^2}{2}+\frac{3\sqrt{-a^2c\,x^2+c}\ a^2}{2}-\frac{2a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}-2a^3x\sqrt{-a^2c\,x^2+c}-\frac{2a^3c\arctan\left(\frac{\sqrt{a^2c}}{\sqrt{-a^2c\,x}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -3/2*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^2+3/2*(-a^2*c*x^2
+c)^(1/2)*a^2-2*a/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^3*x*(-a^2*c*x^2+c)^(1/2)-2*a
^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*a^2*(-(x-
1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+2*a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2
)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))-1/2/c/x^2*(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-a^2cx^2+c}\,(ax+1)^2}{\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2/((a^2*x^2 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^2}{x^3\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-a^2cx^2+c}}{ax^4-x^3}\,dx-\int\frac{ax\sqrt{-a^2cx^2+c}}{ax^4-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**4 - x**3), x) - Integral(a*x*sqrt(-a
**2*c*x**2 + c)/(a*x**4 - x**3), x)
```

3.1084 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=101

$$-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\left(-\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-1/3*(-a^2*c*x^2+c)^(1/2)
/x^3-a*(-a^2*c*x^2+c)^(1/2)/x^2-5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.27, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1807, 835, 807, 266, 63, 208}

$$-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\left(-\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3) - (a*Sqrt[c - a^2*c*x^2])/x^2 - (5*a^2*Sqrt[c
 - a^2*c*x^2])/(3*x) - a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = c\int \frac{(1+ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{1}{3}\int \frac{-6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{\int \frac{10a^2c^2+6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + \left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + \frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\right.$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c}\right.$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.12, size = 82, normalized size = 0.81

$$a^3\sqrt{c}\log(x) - \frac{\left(5a^2x^2+3ax+1\right)\sqrt{c-a^2cx^2}}{3x^3} - a^3\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] -1/3*((1 + 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/x^3 + a^3*Sqrt[c]*Log[x]
 - a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

fricas [A] time = 0.57, size = 164, normalized size = 1.62

$$\left[\frac{3\,a^3\sqrt{c}\,x^3\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right) - 2\,\sqrt{-a^2cx^2+c}\left(5\,a^2x^2+3\,ax+1\right)}{6\,x^3}, -\frac{3\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-}}{a^2cx^2-c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3, -1/3*(3*a^
3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt
(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3]
```

giac [B] time = 0.83, size = 250, normalized size = 2.48

$$\frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\left(3\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^5a^3c-3\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}\,c|a|\cdots\right.}{3\left(\left(\sqrt{-a^2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="gia
c")
```

```
[Out] 2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c - 3*(sqrt(-a^2*c)*
x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) + 12*(sqrt(-a^2*c)*x - sq
rt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))*a^3*c^3 - 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqrt
(-a^2*c*x^2 + c))^2 - c)^3
```

maple [B] time = 0.05, size = 262, normalized size = 2.59

$$-\sqrt{c}\;\ln\left(\frac{2c+2\sqrt{c}\;\sqrt{-a^2c\,x^2+c}}{x}\right)a^3+\sqrt{-a^2c\,x^2+c}\;a^3-\frac{2a^2\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}-2a^4x\sqrt{-a^2c\,x^2+c}-\frac{2a^4c\arctan\left(\cdots\right)}{\sqrt{a^2c}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] -c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^3+(-a^2*c*x^2+c)^(1/2
)*a^3-2*a^2/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^4*x*(-a^2*c*x^2+c)^(1/2)-2*a^4*c/(
a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-1/3/c/x^3*(-a^2*c
*x^2+c)^(3/2)-2*a^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+2*a^4*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))-a/c/x^
2*(-a^2*c*x^2+c)^(3/2)
```

maxima [A] time = 0.47, size = 140, normalized size = 1.39

$$-\frac{\sqrt{ax+1}\,\sqrt{-ax+1}\,a^2\sqrt{c}}{x}+\frac{a^4c^{\frac{3}{2}}\log\left(\frac{\sqrt{-a^2cx^2+c}-\sqrt{c}}{\sqrt{-a^2cx^2+c}+\sqrt{c}}\right)-\frac{2\sqrt{-a^2cx^2+c}\,a^2c}{x^2}}{2\,ac}-\frac{\left(2\,a^2\sqrt{c}\,x^2+\sqrt{c}\right)\sqrt{ax+1}\,\sqrt{-ax+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="max
ima")
```

```
[Out] -sqrt(a*x + 1)*sqrt(-a*x + 1)*a^2*sqrt(c)/x + 1/2*(a^4*c^(3/2)*log((sqrt(-a
^2*c*x^2 + c) - sqrt(c))/(sqrt(-a^2*c*x^2 + c) + sqrt(c))) - 2*sqrt(-a^2*c*
x^2 + c)*a^2*c/x^2)/(a*c) - 1/3*(2*a^2*sqrt(c)*x^2 + sqrt(c))*sqrt(a*x + 1)
*sqrt(-a*x + 1)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(a\,x+1\right)}^2}{x^4\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^{2}cx^{2} + c}}{ax^{5} - x^{4}}\, dx - \int \frac{ax\sqrt{-a^{2}cx^{2} + c}}{ax^{5} - x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**5 - x**4), x) - Integral(a*x*sqrt(-a
**2*c*x**2 + c)/(a*x**5 - x**4), x)
```

3.1085 $\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7}{8}a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}$$

```
[Out] -7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-1/4*(-a^2*c*x^2+c)^(
1/2)/x^4-2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3-7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2-4/
3*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.31, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1807, 835, 807, 266, 63, 208}

$$-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{7}{8}a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) - (7*a^2*S
qrt[c - a^2*c*x^2])/(8*x^2) - (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) - (7*a^4*Sq
rt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = c\int \frac{(1+ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{1}{4}\int \frac{-8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{\int \frac{21a^2c^2+16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{\int \frac{-32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{1}{8}\left(7a^4c\right)\int$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{1}{16}\left(7a^4c\right)\text{Su}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{8}\left(7a^2\right)\text{Sub}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{7}{8}a^4\sqrt{c}\ \text{tanh}$$

Mathematica [A] time = 0.14, size = 95, normalized size = 0.73

$$\frac{7}{8}a^4\sqrt{c}\log(x) - \frac{7}{8}a^4\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) - \frac{\left(32a^3x^3+21a^2x^2+16ax+6\right)\sqrt{c-a^2cx^2}}{24x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] -1/24*(Sqrt[c - a^2*c*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^3*x^3))/x^4 + (7
*a^4*Sqrt[c]*Log[x])/8 - (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]
])/8
```

fricas [A] time = 0.68, size = 180, normalized size = 1.38

$$\left[\frac{21\,a^4\sqrt{c}\,x^4\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right) - 2\left(32\,a^3x^3+21\,a^2x^2+16\,ax+6\right)\sqrt{-a^2cx^2+c}}{48\,x^4}, -\frac{21\,a^4\sqrt{-c}\,x^4\,\text{arctan}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="fri
cas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, -1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2
*c*x^2 - c)) + (32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c))
/x^4]
```

giac [B] time = 0.20, size = 324, normalized size = 2.49

$$\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}-\frac{21\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2+96}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="gia
c")
```

```
[Out] 7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c
) - 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a^
2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 + 96*(sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2
 + c))^3*a^4*c^3 - 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(-
c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 + 32*a^3
*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

maple [B] time = 0.05, size = 287, normalized size = 2.21

$$-\frac{7\sqrt{c}\;\ln\left(\frac{2c+2\sqrt{c}\;\sqrt{-a^2c\,x^2+c}}{x}\right)a^4}{8}+\frac{7\sqrt{-a^2c\,x^2+c}\;a^4}{8}-\frac{2a^3\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}-2a^5x\sqrt{-a^2c\,x^2+c}-\frac{2a^5c\arctan\left(\frac{\sqrt{a^2}}{\sqrt{-a^2c}}\right)}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

```
[Out] -7/8*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^4+7/8*(-a^2*c*x^2
+c)^(1/2)*a^4-2*a^3/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^5*x*(-a^2*c*x^2+c)^(1/2)-2
*a^5*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3*a/c/x
^3*(-a^2*c*x^2+c)^(3/2)-2*a^4*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+2*a^5*
```

```
c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/
2))-9/8*a^2/c/x^2*(-a^2*c*x^2+c)^(3/2)-1/4/c/x^4*(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^2}{(a^2x^2-1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2/((a^2*x^2 - 1)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^2}{x^5\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2cx^2+c}}{ax^6-x^5}\,dx-\int \frac{ax\sqrt{-a^2cx^2+c}}{ax^6-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] -Integral(sqrt(-a**2*c*x**2 + c)/(a*x**6 - x**5), x) - Integral(a*x*sqrt(-a
**2*c*x**2 + c)/(a*x**6 - x**5), x)
```

3.1086 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=161

$$-\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}+\frac{c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^4}-\frac{(105ax+88)\left(c-a^2cx^2\right)^{3/2}}{420a^4}+$$

```
[Out] -11/35*x^2*(-a^2*c*x^2+c)^(3/2)/a^2-1/3*x^3*(-a^2*c*x^2+c)^(3/2)/a-1/7*x^4*
(-a^2*c*x^2+c)^(3/2)-1/420*(105*a*x+88)*(-a^2*c*x^2+c)^(3/2)/a^4+1/8*c^(3/2
)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a^4+1/8*c*x*(-a^2*c*x^2+c)^(1/2)
/a^3
```

Rubi [A] time = 0.34, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1809, 833, 780, 195, 217, 203}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^4}-\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}-\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a}-\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2}+\frac{cx\sqrt{c-a^2cx^2}}{8a^3}-\frac{(105ax+88)\left(c\ldots\right.}{420a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*x*Sqrt[c - a^2*c*x^2])/(8*a^3) - (11*x^2*(c - a^2*c*x^2)^(3/2))/(35*a^2)
 - (x^3*(c - a^2*c*x^2)^(3/2))/(3*a) - (x^4*(c - a^2*c*x^2)^(3/2))/7 - ((88
 + 105*a*x)*(c - a^2*c*x^2)^(3/2))/(420*a^4) + (c^(3/2)*ArcTan[(a*Sqrt[c]*x
)/Sqrt[c - a^2*c*x^2]])/(8*a^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^{3/2}\,dx = c\int x^3(1+ax)^2\sqrt{c-a^2cx^2}\,dx$$

$$= -\frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2} - \frac{\int x^3\left(-11a^2c-14a^3cx\right)\sqrt{c-a^2cx^2}\,dx}{7a^2}$$

$$= -\frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a} - \frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2} + \frac{\int x^2\left(42a^3c^2+66a^4c^2x\right)\sqrt{c-a^2c}}{42a^4c}$$

$$= -\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2} - \frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a} - \frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2} - \frac{\int x\left(-132a^4\right.}{}$$

$$= -\frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2} - \frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a} - \frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2} - \frac{(88+105ax}{42}$$

$$= \frac{cx\sqrt{c-a^2cx^2}}{8a^3} - \frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2} - \frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a} - \frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}$$

$$= \frac{cx\sqrt{c-a^2cx^2}}{8a^3} - \frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2} - \frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a} - \frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}$$

$$= \frac{cx\sqrt{c-a^2cx^2}}{8a^3} - \frac{11x^2\left(c-a^2cx^2\right)^{3/2}}{35a^2} - \frac{x^3\left(c-a^2cx^2\right)^{3/2}}{3a} - \frac{1}{7}x^4\left(c-a^2cx^2\right)^{3/2}$$

Mathematica [A] time = 0.14, size = 113, normalized size = 0.70

$$\frac{c\left(120a^6x^6+280a^5x^5+144a^4x^4-70a^3x^3-88a^2x^2-105ax-176\right)\sqrt{c-a^2cx^2}-105c^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{840a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-176 - 105*a*x - 88*a^2*x^2 - 70*a^3*x^3 + 144*a^4*
x^4 + 280*a^5*x^5 + 120*a^6*x^6) - 105*c^(3/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x
^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(840*a^4)
```

fricas [A] time = 0.74, size = 234, normalized size = 1.45

$$\left[\frac{105\sqrt{-c}\,c\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(120\,a^6cx^6+280\,a^5cx^5+144\,a^4cx^4-70\,a^3cx^3-88\,a^2\right.}{1680\,a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/1680*(105*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)
*x - c) + 2*(120*a^6*c*x^6 + 280*a^5*c*x^5 + 144*a^4*c*x^4 - 70*a^3*c*x^3 -
 88*a^2*c*x^2 - 105*a*c*x - 176*c)*sqrt(-a^2*c*x^2 + c))/a^4, -1/840*(105*c
^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - (120*a^6*
c*x^6 + 280*a^5*c*x^5 + 144*a^4*c*x^4 - 70*a^3*c*x^3 - 88*a^2*c*x^2 - 105*a
*c*x - 176*c)*sqrt(-a^2*c*x^2 + c))/a^4]
```

giac [A] time = 0.35, size = 117, normalized size = 0.73

$$\frac{1}{840}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(4\left(5\left(3a^2cx+7ac\right)x+18c\right)x-\frac{35c}{a}\right)x-\frac{44c}{a^2}\right)x-\frac{105c}{a^3}\right)x-\frac{176c}{a^4}\right)-\frac{c^2\log\left(\left|-\sqrt{-a^2c}\,x+\right.\right.}{8a^3\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] 1/840*sqrt(-a^2*c*x^2 + c)*((2*((4*(5*(3*a^2*c*x + 7*a*c)*x + 18*c)*x - 35*
c/a)*x - 44*c/a^2)*x - 105*c/a^3)*x - 176*c/a^4) - 1/8*c^2*log(abs(-sqrt(-a
^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^3*sqrt(-c)*abs(a))
```

maple [B] time = 0.05, size = 268, normalized size = 1.66

$$\frac{x^2\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{7a^2c}+\frac{16\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{35c\,a^4}+\frac{x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{3a^3c}-\frac{7x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{12a^3}-\frac{7cx\sqrt{-a^2c\,x^2+c}}{8a^3}-\frac{7c^2\arctan\left(\frac{\sqrt{a^2}}{\sqrt{-a^2c}}\right.}{8a^3\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/7*x^2*(-a^2*c*x^2+c)^(5/2)/a^2/c+16/35/c/a^4*(-a^2*c*x^2+c)^(5/2)+1/3/a^3
*x*(-a^2*c*x^2+c)^(5/2)/c-7/12/a^3*x*(-a^2*c*x^2+c)^(3/2)-7/8*c*x*(-a^2*c*x
^2+c)^(1/2)/a^3-7/8/a^3*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^
2+c)^(1/2))-2/3/a^4*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)+1/a^3*c*(-(x-1/a
)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+1/a^3*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/
2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.75, size = 213, normalized size = 1.32

$$\frac{1}{840}\,a\left(\frac{120\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2}{a^3c}-\frac{490\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}{a^4}+\frac{280\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}{a^4c}+\frac{840\sqrt{a^2cx^2-4\,acx+3\,c}\,cx}{a^4}-\frac{735}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/840*a*(120*(-a^2*c*x^2 + c)^(5/2)*x^2/(a^3*c) - 490*(-a^2*c*x^2 + c)^(3/2
)*x/a^4 + 280*(-a^2*c*x^2 + c)^(5/2)*x/(a^4*c) + 840*sqrt(a^2*c*x^2 - 4*a*c
*x + 3*c)*c*x/a^4 - 735*sqrt(-a^2*c*x^2 + c)*c*x/a^4 - 735*c^(3/2)*arcsin(a
*x)/a^5 - 560*(-a^2*c*x^2 + c)^(3/2)/a^5 + 384*(-a^2*c*x^2 + c)^(5/2)/(a^5*
c) - 1680*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c/a^5 + 840*c^3*arcsin(a*x - 2)/(
a^8*(-c/a^2)^(3/2)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\left(c-a^2\,c\,x^2\right)^{3/2}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^3*(c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [A] time = 18.31, size = 420, normalized size = 2.61

$$a^2c\left(\begin{cases}\frac{x^6\sqrt{-a^2cx^2+c}}{7}-\frac{x^4\sqrt{-a^2cx^2+c}}{35a^2}-\frac{4x^2\sqrt{-a^2cx^2+c}}{105a^4}-\frac{8\sqrt{-a^2cx^2+c}}{105a^6} & \text{for } a\neq 0\\ \frac{\sqrt{c}\,x^6}{6} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^7}{6\sqrt{a^2x^2-1}}-\frac{5i\sqrt{c}\,x^5}{24\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}}{48a^2\sqrt{a}} \\ -\frac{a^2\sqrt{c}\,x^7}{6\sqrt{-a^2x^2+1}}+\frac{5\sqrt{c}\,x^5}{24\sqrt{-a^2x^2+1}}+\frac{}{48}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 +
c)/(35*a**2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x*
*2 + c)/(105*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) + 2*a*c*Piecewise((I
*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2
*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(1
```

```
6*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2
) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*s
qrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c
)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5), True)) +
c*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 + c)/(1
5*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4/4, T
rue))
```

3.1087 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=136

$$-\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a}+\frac{3cx\sqrt{c-a^2cx^2}}{16a^2}-\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}+\frac{3c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a^3}-\frac{(45ax+32)\left(c-a^2cx^2\right)^{3/2}}{120a^3}$$

```
[Out] -2/5*x^2*(-a^2*c*x^2+c)^(3/2)/a-1/6*x^3*(-a^2*c*x^2+c)^(3/2)-1/120*(45*a*x+
32)*(-a^2*c*x^2+c)^(3/2)/a^3+3/16*c^(3/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)
^(1/2))/a^3+3/16*c*x*(-a^2*c*x^2+c)^(1/2)/a^2
```

Rubi [A] time = 0.31, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1809, 833, 780, 195, 217, 203}

$$\frac{3c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a^3}-\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2}-\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a}+\frac{3cx\sqrt{c-a^2cx^2}}{16a^2}-\frac{(45ax+32)\left(c-a^2cx^2\right)^{3/2}}{120a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (3*c*x*Sqrt[c - a^2*c*x^2])/(16*a^2) - (2*x^2*(c - a^2*c*x^2)^(3/2))/(5*a)
- (x^3*(c - a^2*c*x^2)^(3/2))/6 - ((32 + 45*a*x)*(c - a^2*c*x^2)^(3/2))/(12
0*a^3) + (3*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{3/2}\,dx = c\int x^2(1+ax)^2\sqrt{c-a^2cx^2}\,dx$$
$$= -\frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2} - \frac{\int x^2\left(-9a^2c-12a^3cx\right)\sqrt{c-a^2cx^2}\,dx}{6a^2}$$
$$= -\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a} - \frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2} + \frac{\int x\left(24a^3c^2+45a^4c^2x\right)\sqrt{c-a^2c}}{30a^4c}$$
$$= -\frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a} - \frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2} - \frac{(32+45ax)\left(c-a^2cx^2\right)^{3/2}}{120a^3} + \frac{(3c}{}$$
$$= \frac{3cx\sqrt{c-a^2cx^2}}{16a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a} - \frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2} - \frac{(32+45ax)\left(c-\right.}{120a^3}$$
$$= \frac{3cx\sqrt{c-a^2cx^2}}{16a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a} - \frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2} - \frac{(32+45ax)\left(c-\right.}{120a^3}$$
$$= \frac{3cx\sqrt{c-a^2cx^2}}{16a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{3/2}}{5a} - \frac{1}{6}x^3\left(c-a^2cx^2\right)^{3/2} - \frac{(32+45ax)\left(c-\right.}{120a^3}$$

Mathematica [A] time = 0.13, size = 105, normalized size = 0.77

$$\frac{c\left(40a^5x^5+96a^4x^4+50a^3x^3-32a^2x^2-45ax-64\right)\sqrt{c-a^2cx^2}-45c^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-64 - 45*a*x - 32*a^2*x^2 + 50*a^3*x^3 + 96*a^4*x^4
 + 40*a^5*x^5) - 45*c^(3/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 +
 a^2*x^2))])/(240*a^3)
```

fricas [A] time = 0.64, size = 216, normalized size = 1.59

$$\left[\frac{45\sqrt{-c}\,c\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(40\,a^5cx^5+96\,a^4cx^4+50\,a^3cx^3-32\,a^2cx^2-45\,acx-6\right.}{480\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/480*(45*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) + 2*(40*a^5*c*x^5 + 96*a^4*c*x^4 + 50*a^3*c*x^3 - 32*a^2*c*x^2 - 45*a
*c*x - 64*c)*sqrt(-a^2*c*x^2 + c))/a^3, -1/240*(45*c^(3/2)*arctan(sqrt(-a^2
*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - (40*a^5*c*x^5 + 96*a^4*c*x^4 + 5
0*a^3*c*x^3 - 32*a^2*c*x^2 - 45*a*c*x - 64*c)*sqrt(-a^2*c*x^2 + c))/a^3]
```

giac [A] time = 0.21, size = 107, normalized size = 0.79

$$\frac{1}{240}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(4\left(5a^2cx+12ac\right)x+25c\right)x-\frac{16c}{a}\right)x-\frac{45c}{a^2}\right)x-\frac{64c}{a^3}\right)-\frac{3c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+\right.\right.}{16a^2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] 1/240*sqrt(-a^2*c*x^2 + c)*((2*((4*(5*a^2*c*x + 12*a*c)*x + 25*c)*x - 16*c/
a)*x - 45*c/a^2)*x - 64*c/a^3) - 3/16*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a
^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

maple [B] time = 0.04, size = 244, normalized size = 1.79

$$\frac{x\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{6a^2c}-\frac{13x\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{24a^2}-\frac{13cx\sqrt{-a^2cx^2+c}}{16a^2}-\frac{13c^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{16a^2\sqrt{a^2c}}+\frac{2\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{5a^3c}-\frac{2\left(-\left(x-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/6*x*(-a^2*c*x^2+c)^(5/2)/a^2/c-13/24/a^2*x*(-a^2*c*x^2+c)^(3/2)-13/16*c*x
*(-a^2*c*x^2+c)^(1/2)/a^2-13/16/a^2*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*
x/(-a^2*c*x^2+c)^(1/2))+2/5/a^3*(-a^2*c*x^2+c)^(5/2)/c-2/3/a^3*(-(x-1/a)^2*
a^2*c-2*a*c*(x-1/a))^(3/2)+1/a^2*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x
+1/a^2*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-
1/a))^(1/2))
```

maxima [A] time = 0.65, size = 189, normalized size = 1.39

$$-\frac{1}{240}a\left(\frac{130\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}{a^3}-\frac{40\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}{a^3c}-\frac{240\sqrt{a^2cx^2-4acx+3c}\,cx}{a^3}+\frac{195\sqrt{-a^2cx^2+c}\,cx}{a^3}+\frac{195c^{\frac{3}{2}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/240*a*(130*(-a^2*c*x^2 + c)^(3/2)*x/a^3 - 40*(-a^2*c*x^2 + c)^(5/2)*x/(a
^3*c) - 240*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c*x/a^3 + 195*sqrt(-a^2*c*x^2 +
 c)*c*x/a^3 + 195*c^(3/2)*arcsin(a*x)/a^4 + 160*(-a^2*c*x^2 + c)^(3/2)/a^4
- 96*(-a^2*c*x^2 + c)^(5/2)/(a^4*c) + 480*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c
/a^4 - 240*c^3*arcsin(a*x - 2)/(a^7*(-c/a^2)^(3/2)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2\left(c-a^2\,c\,x^2\right)^{3/2}\,(a\,x+1)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^2*(c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 29.50, size = 515, normalized size = 3.79

$$a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^7}{6\sqrt{a^2x^2-1}}-\frac{5i\sqrt{c}\,x^5}{24\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,x^3}{48a^2\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{16a^4\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{16a^5} & \text{for } \left|a^2x^2\right|>1\\ -\frac{a^2\sqrt{c}\,x^7}{6\sqrt{-a^2x^2+1}}+\frac{5\sqrt{c}\,x^5}{24\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,x^3}{48a^2\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{16a^4\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{16a^5} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{x^4\sqrt{-a^2cx^2+c}}{5}\\ \frac{\sqrt{c}\,x^4}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)
*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1
)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a
**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5
*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*
x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(
16*a**5), True)) + 2*a*c*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sq
rt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a,
0)), (sqrt(c)*x**4/4, True)) + c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**
2*x**2 - 1)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a*
*2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1
), (-a**2*sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a
**2*x**2 + 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x
)/(8*a**3), True))
```

3.1088 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$\frac{c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{4a^2}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}+\frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{(15ax+14)\left(c-a^2cx^2\right)^{3/2}}{30a^2}$$

```
[Out] -1/5*x^2*(-a^2*c*x^2+c)^(3/2)-1/30*(15*a*x+14)*(-a^2*c*x^2+c)^(3/2)/a^2+1/4
*c^(3/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a^2+1/4*c*x*(-a^2*c*x^2+c
)^(1/2)/a
```

Rubi [A] time = 0.19, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6151, 1809, 780, 195, 217, 203}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{4a^2}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}+\frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{(15ax+14)\left(c-a^2cx^2\right)^{3/2}}{30a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*x*Sqrt[c - a^2*c*x^2])/(4*a) - (x^2*(c - a^2*c*x^2)^(3/2))/5 - ((14 + 15
*a*x)*(c - a^2*c*x^2)^(3/2))/(30*a^2) + (c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(4*a^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^{3/2}\,dx &= c\int x(1+ax)^2\sqrt{c-a^2cx^2}\,dx\\
&= -\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{\int x\left(-7a^2c-10a^3cx\right)\sqrt{c-a^2cx^2}\,dx}{5a^2}\\
&= -\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{(14+15ax)\left(c-a^2cx^2\right)^{3/2}}{30a^2}+\frac{c\int\sqrt{c-a^2cx^2}\,dx}{2a}\\
&= \frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{(14+15ax)\left(c-a^2cx^2\right)^{3/2}}{30a^2}+\frac{c^2\int\frac{1}{\sqrt{c-a^2}}}{4a}\\
&= \frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{(14+15ax)\left(c-a^2cx^2\right)^{3/2}}{30a^2}+\frac{c^2\,\mathrm{Subst}\left(\right.}{}\\
&= \frac{cx\sqrt{c-a^2cx^2}}{4a}-\frac{1}{5}x^2\left(c-a^2cx^2\right)^{3/2}-\frac{(14+15ax)\left(c-a^2cx^2\right)^{3/2}}{30a^2}+\frac{c^{3/2}\tan^{-1}}{4}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 97, normalized size = 0.87

$$\frac{c\left(12a^4x^4+30a^3x^3+16a^2x^2-15ax-28\right)\sqrt{c-a^2cx^2}-15c^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{60a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-28 - 15*a*x + 16*a^2*x^2 + 30*a^3*x^3 + 12*a^4*x^4
) - 15*c^(3/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/
(60*a^2)
```

fricas [A] time = 0.88, size = 198, normalized size = 1.78

$$\left[\frac{15\sqrt{-c}\,c\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(12\,a^4cx^4+30\,a^3cx^3+16\,a^2cx^2-15\,acx-28\,c\right)\sqrt{-a^2cx^2}}{120\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/120*(15*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) + 2*(12*a^4*c*x^4 + 30*a^3*c*x^3 + 16*a^2*c*x^2 - 15*a*c*x - 28*c)*sq
rt(-a^2*c*x^2 + c))/a^2, -1/60*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sq
rt(c)*x/(a^2*c*x^2 - c)) - (12*a^4*c*x^4 + 30*a^3*c*x^3 + 16*a^2*c*x^2 - 15
*a*c*x - 28*c)*sqrt(-a^2*c*x^2 + c))/a^2]
```

giac [A] time = 0.20, size = 98, normalized size = 0.88

$$\frac{1}{60}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\left(2\,a^2cx+5\,ac\right)x+8\,c\right)x-\frac{15\,c}{a}\right)x-\frac{28\,c}{a^2}\right)-\frac{c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{4\,a\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] 1/60*sqrt(-a^2*c*x^2 + c)*((2*(3*(2*a^2*c*x + 5*a*c)*x + 8*c)*x - 15*c/a)*x
 - 28*c/a^2) - 1/4*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a*
sqrt(-c)*abs(a))
```

maple [B] time = 0.04, size = 222, normalized size = 2.00

$$\frac{\left(-a^{2} c\,x^{2}+c\right)^{\frac{5}{2}}}{5 a^{2} c}-\frac{x \left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{2 a}-\frac{3 c x \sqrt{-a^{2} c\,x^{2}+c}}{4 a}-\frac{3 c^{2} \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c\,x^{2}+c}}\right)}{4 a \sqrt{a^{2} c}}-\frac{2 \left(-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2 a c \left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3 a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/5*(-a^2*c*x^2+c)^(5/2)/a^2/c-1/2*x/a*(-a^2*c*x^2+c)^(3/2)-3/4*c*x*(-a^2*c
*x^2+c)^(1/2)/a-3/4/a*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))-2/3/a^2*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)+1/a*c*(-(x-1/a)^2*
a^2*c-2*a*c*(x-1/a))^(1/2)*x+1/a*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(
-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.53, size = 167, normalized size = 1.50

$$-\frac{1}{60}\,a\left(\frac{30\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} x}{a^{2}}-\frac{60 \sqrt{a^{2} c x^{2}-4 \, a c x+3 \, c}\, c x}{a^{2}}+\frac{45 \sqrt{-a^{2} c x^{2}+c}\, c x}{a^{2}}+\frac{45 \, c^{\frac{3}{2}} \arcsin \left(a x\right)}{a^{3}}+\frac{40 \left(-a^{2} c x^{2}\right.}{a^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/60*a*(30*(-a^2*c*x^2 + c)^(3/2)*x/a^2 - 60*sqrt(a^2*c*x^2 - 4*a*c*x + 3*
c)*c*x/a^2 + 45*sqrt(-a^2*c*x^2 + c)*c*x/a^2 + 45*c^(3/2)*arcsin(a*x)/a^3 +
 40*(-a^2*c*x^2 + c)^(3/2)/a^3 - 12*(-a^2*c*x^2 + c)^(5/2)/(a^3*c) + 120*sq
rt(a^2*c*x^2 - 4*a*c*x + 3*c)*c/a^3 - 60*c^3*arcsin(a*x - 2)/(a^6*(-c/a^2)^
(3/2)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x*(c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [A] time = 16.12, size = 306, normalized size = 2.76

$$a^2c\left(\begin{cases}\frac{x^4\sqrt{-a^2cx^2+c}}{5}-\frac{x^2\sqrt{-a^2cx^2+c}}{15a^2}-\frac{2\sqrt{-a^2cx^2+c}}{15a^4} & \text{for } a\neq 0\\ \frac{\sqrt{c}\,x^4}{4} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}\,x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(a\\ -\frac{a^2\sqrt{c}\,x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\mathrm{as}}{8}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 +
c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4
/4, True)) + 2*a*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) -
 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x
**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt
(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1))
 - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), Tr
ue)) + c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c
*x**2 + c)**(3/2)/(3*a**2*c), True))
```

3.1089 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=107

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}+\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

```
[Out] -5/12*(-a^2*c*x^2+c)^(3/2)/a-1/4*(a*x+1)*(-a^2*c*x^2+c)^(3/2)/a+5/8*c^(3/2)
*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a+5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6141, 671, 641, 195, 217, 203}

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}+\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (5*c*x*Sqrt[c - a^2*c*x^2])/8 - (5*(c - a^2*c*x^2)^(3/2))/(12*a) - ((1 + a*
x)*(c - a^2*c*x^2)^(3/2))/(4*a) + (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
a^2*c*x^2]])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = c\int(1+ax)^2\sqrt{c-a^2cx^2}\,dx$$

$$= -\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{4}(5c)\int(1+ax)\sqrt{c-a^2cx^2}\,dx$$

$$= -\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{4}(5c)\int\sqrt{c-a^2cx^2}\,dx$$

$$= \frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{8}\left(5c^2\right)\int\frac{1}{\sqrt{c-a}}$$

$$= \frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{1}{8}\left(5c^2\right)\text{Subst}\left(\int\right.$$

$$= \frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}}{\sqrt{c-a^2}}\right.}{8a}$$

Mathematica [A] time = 0.09, size = 117, normalized size = 1.09

$$-\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(6a^4x^4+10a^3x^3-7a^2x^2-25ax+16\right)+30\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/24*(c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(16 - 25*a*x - 7*a^2*x^2 + 10*a
^3*x^3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*S
qrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.68, size = 180, normalized size = 1.68

$$\left[\frac{15\sqrt{-c}\,c\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(6a^3cx^3+16a^2cx^2+9acx-16c\right)\sqrt{-a^2cx^2+c}}{48a}, -\frac{15c^{\frac{3}{2}}}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) + 2*(6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c)
)/a, -1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
 c)) - (6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c))/
a]
```

giac [A] time = 0.76, size = 85, normalized size = 0.79

$$\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3a^2cx+8ac\right)x+9c\right)x-\frac{16c}{a}\right)-\frac{5c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{8\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x + 8*a*c)*x + 9*c)*x - 16*c/a) - 5/
8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

maple [B] time = 0.04, size = 186, normalized size = 1.74

$$-\frac{x\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{4}-\frac{3cx\sqrt{-a^{2}c\,x^{2}+c}}{8}-\frac{3c^{2}\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{8\sqrt{a^{2}c}}-\frac{2\left(-\left(x-\frac{1}{a}\right)^{2}a^{2}c-2ac\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3a}+c\sqrt{-\left(x-\frac{1}{a}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)-3/8*c*x*(-a^2*c*x^2+c)^(1/2)-3/8*c^2/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3/a*(-(x-1/a)^2*a^2*c-2*
a*c*(x-1/a))^(3/2)+c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+c^2/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.52, size = 146, normalized size = 1.36

$$-\frac{1}{24}\left(\frac{6\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}x}{a}-\frac{24\sqrt{a^{2}cx^{2}-4\,acx+3\,c}\,cx}{a}+\frac{9\sqrt{-a^{2}cx^{2}+c}\,cx}{a}+\frac{9\,c^{\frac{3}{2}}\arcsin\left(ax\right)}{a^{2}}+\frac{16\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}}{a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/24*(6*(-a^2*c*x^2 + c)^(3/2)*x/a - 24*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c*
x/a + 9*sqrt(-a^2*c*x^2 + c)*c*x/a + 9*c^(3/2)*arcsin(a*x)/a^2 + 16*(-a^2*c
*x^2 + c)^(3/2)/a^2 + 48*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c/a^2 - 24*c^3*arc
sin(a*x - 2)/(a^5*(-c/a^2)^(3/2)))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(c-a^{2}\,c\,x^{2}\right)^{3/2}\,\left(a\,x+1\right)^{2}}{a^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 18.34, size = 340, normalized size = 3.18

$$a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}\,x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c)
*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) -
I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4*
sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x
/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) + 2*a*c
*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 +
c)**(3/2)/(3*a**2*c), True)) + c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**
2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(
2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin
(a*x)/(2*a), True))
```

3.1090 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{3/2}}{x}\,dx$

Optimal. Leaf size=101

$$c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+c(ax+1)\sqrt{c-a^2cx^2}-\frac{1}{3}\left(c-a^2cx^2\right)^{3/2}$$

```
[Out] -1/3*(-a^2*c*x^2+c)^(3/2)+c^(3/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))-
c^(3/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))+c*(a*x+1)*(-a^2*c*x^2+c)^(1/2
)
```

Rubi [A] time = 0.29, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1809, 815, 844, 217, 203, 266, 63, 208}

$$c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+c(ax+1)\sqrt{c-a^2cx^2}-\frac{1}{3}\left(c-a^2cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x,x]
```

```
[Out] c*(1 + a*x)*Sqrt[c - a^2*c*x^2] - (c - a^2*c*x^2)^(3/2)/3 + c^(3/2)*ArcTan[
(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] - c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sq
rt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
```

```
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x}\,dx$$
$$= -\frac{1}{3}\left(c-a^2cx^2\right)^{3/2} - \frac{\int \frac{\left(-3a^2c-6a^3cx\right)\sqrt{c-a^2cx^2}}{x}\,dx}{3a^2}$$
$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + \frac{\int \frac{6a^4c^3+6a^5c^3x}{x\sqrt{c-a^2cx^2}}\,dx}{6a^4c}$$
$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + c^2\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx + \left(ac^2\right)\int \frac{}{\sqrt{c-}}$$
$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + \frac{1}{2}c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)$$
$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \frac{c\,\text{Subst}\left(\int\right.}{}$$
$$= c(1+ax)\sqrt{c-a^2cx^2} - \frac{1}{3}\left(c-a^2cx^2\right)^{3/2} + c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - c^{3/2}\tanh^{-1}$$

Mathematica [A] time = 0.11, size = 115, normalized size = 1.14

$$-c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+c^{3/2}\left(-\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)\right)+\frac{1}{3}c\left(a^2x^2+3ax+2\right)\sqrt{c-a^2cx^2}+c^{3/2}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x,x]
```

```
[Out] (c*(2 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2])/3 - c^(3/2)*ArcTan[(a*x*Sqrt[
c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))] + c^(3/2)*Log[x] - c^(3/2)*Log[c
+ Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

fricas [A] time = 0.69, size = 233, normalized size = 2.31

$$\left[-c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+\frac{1}{2}c^{\frac{3}{2}}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\frac{1}{3}\left(a^2cx^2+3acx+2c\right)\sqrt{-a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x,x, algorithm="frica
s")
```

```
[Out] [-c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*c^
(3/2)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + 1/3*(a
^2*c*x^2 + 3*a*c*x + 2*c)*sqrt(-a^2*c*x^2 + c), -sqrt(-c)*c*arctan(sqrt(-a^
2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 1/2*sqrt(-c)*c*log(2*a^2*c*x^2 + 2
*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) + 1/3*(a^2*c*x^2 + 3*a*c*x + 2*c)*s
qrt(-a^2*c*x^2 + c)]
```

giac [A] time = 1.06, size = 116, normalized size = 1.15

$$\frac{2\,c^2\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{a\sqrt{-c}\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(a^2cx+3\,ac\right)x+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) +
a*sqrt(-c)*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + 1/3*
sqrt(-a^2*c*x^2 + c)*((a^2*c*x + 3*a*c)*x + 2*c)
```

maple [B] time = 0.04, size = 179, normalized size = 1.77

$$\frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{3}-c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)+\sqrt{-a^2c\,x^2+c}\,c-\frac{2\left(-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3}+ac\sqrt{-\left(x-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x,x)
```

```
[Out] 1/3*(-a^2*c*x^2+c)^(3/2)-c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
+(-a^2*c*x^2+c)^(1/2)*c-2/3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)+a*c*(-(x
-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+a*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/
2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x,x, algorithm="maxim
a")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)^2}{x\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)), x)
```

sympy [C] time = 10.40, size = 267, normalized size = 2.64

$$a^2c\left(\begin{cases}0 & \text{for}\ c=0\\ \frac{\sqrt{c}x^2}{2} & \text{for}\ a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{ia^2\sqrt{c}x^3}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}x}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\ \mathrm{acosh}\,(ax)}{2a} & \text{for}\ \left|a^2x^2\right|>1\\ \frac{\sqrt{c}x\sqrt{-a^2x^2+1}}{2}+\frac{\sqrt{c}\ \mathrm{asin}\,(ax)}{2a} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}i\sqrt{c}\,\sqrt{a^2x^2-1} \\ \sqrt{c}\,\sqrt{-a^2x^2+1}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x,x)
```

```
[Out] a**2*c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x
**2 + c)**(3/2)/(3*a**2*c), True)) + 2*a*c*Piecewise((I*a**2*sqrt(c)*x**3/(
2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*ac
osh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sq
rt(c)*asin(a*x)/(2*a), True)) + c*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1)
- sqrt(c)*log(a*x) + sqrt(c)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Ab
s(a**2*x**2) > 1), (sqrt(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2
 - sqrt(c)*log(sqrt(-a**2*x**2 + 1) + 1), True))
```

3.1091 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2}-\frac{(c-a^2cx^2)^{3/2}}{x}$$

```
[Out] -(-a^2*c*x^2+c)^(3/2)/x-1/2*a*c^(3/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/
2))-2*a*c^(3/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))+1/2*a*c*(-a*x+4)*(-a^
2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2}-\frac{(c-a^2cx^2)^{3/2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^2,x]
```

```
[Out] (a*c*(4 - a*x)*Sqrt[c - a^2*c*x^2])/2 - (c - a^2*c*x^2)^(3/2)/x - (a*c^(3/2
)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/2 - 2*a*c^(3/2)*ArcTanh[Sqrt[c
 - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^2}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{x} - \int \frac{\left(-2ac+a^2cx\right)\sqrt{c-a^2cx^2}}{x}\,dx$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} + \frac{\int \frac{4a^3c^3-a^4c^3x}{x\sqrt{c-a^2cx^2}}\,dx}{2a^2c}$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} + \left(2ac^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \frac{1}{2}\left(a^2c\right.$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} + \left(ac^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x,\right.$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} - \frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \frac{(2c)\,\text{S}}{}$$
$$= \frac{1}{2}ac(4-ax)\sqrt{c-a^2cx^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{x} - \frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - 2ac^{3/}$$

Mathematica [A] time = 0.15, size = 124, normalized size = 1.11

$$-2ac^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{1}{2}ac^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\frac{c\left(a^2x^2+4ax-2\right)\sqrt{c-a^2cx^2}}{2x}+2ac^{3/2}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^2,x]
```

```
[Out] (c*(-2 + 4*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2])/(2*x) + (a*c^(3/2)*ArcTan[(a
*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/2 + 2*a*c^(3/2)*Log[x] -
 2*a*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

fricas [A] time = 0.70, size = 249, normalized size = 2.22

$$\left[\frac{ac^{\frac{3}{2}}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+2\,ac^{\frac{3}{2}}x\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+\left(a^2cx^2+4\,acx-2\,c\right)\sqrt{-a^2cx^2+c}}{2\,x},-\frac{8\,a\sqrt{}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [1/2*(a*c^(3/2)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c))
+ 2*a*c^(3/2)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2
) + (a^2*c*x^2 + 4*a*c*x - 2*c)*sqrt(-a^2*c*x^2 + c))/x, -1/4*(8*a*sqrt(-c)
*c*x*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x
*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - 2*(a^2*c*x^2
+ 4*a*c*x - 2*c)*sqrt(-a^2*c*x^2 + c))/x]
```

giac [A] time = 1.00, size = 165, normalized size = 1.47

$$\frac{4\,ac^2\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{2\,|a|}+\frac{2\,a^2\sqrt{-c}\,c^2}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^2,x, algorithm="gia
c")
```

```
[Out] 4*a*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 1/2*a^2*sqrt(-c)*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a
) + 2*a^2*sqrt(-c)*c^2/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs
(a)) + 1/2*sqrt(-a^2*c*x^2 + c)*(a^2*c*x + 4*a*c)
```

maple [B] time = 0.04, size = 286, normalized size = 2.55

$$\frac{2a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{3}-2a\,c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)+2a\sqrt{-a^2c\,x^2+c}\,c-\frac{\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{cx}-a^2x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}-\frac{3a\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^2,x)
```

```
[Out] 2/3*a*(-a^2*c*x^2+c)^(3/2)-2*a*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/
2))/x)+2*a*(-a^2*c*x^2+c)^(1/2)*c-1/c/x*(-a^2*c*x^2+c)^(5/2)-a^2*x*(-a^2*c*
x^2+c)^(3/2)-3/2*a^2*c*x*(-a^2*c*x^2+c)^(1/2)-3/2*a^2*c^2/(a^2*c)^(1/2)*arc
tan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3*a*(-(x-1/a)^2*a^2*c-2*a*c*(x-
1/a))^(3/2)+a^2*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+a^2*c^2/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^2,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,{\left(a\,x+1\right)}^2}{x^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)), x)
```

sympy [C] time = 19.26, size = 350, normalized size = 3.12

$$a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^3}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,x}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{2a} & \text{for } \left|a^2x^2\right|>1\\ \frac{\sqrt{c}\,x\sqrt{-a^2x^2+1}}{2}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}i\sqrt{c}\,\sqrt{a^2x^2-1}-\sqrt{c}\log(ax)+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}+ \\ \sqrt{c}\,\sqrt{-a^2x^2+1}+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}-\sqrt{c}\log\left(\sqrt{-a\right.\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**2,x)
```

```
[Out] a**2*c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x
/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1),
```

```
 (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True)) + 2*a*
c*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(a*x) + sqrt(c)*log
(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(c)*sqrt
(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a**2*x**2 +
 1) + 1), True)) + c*Piecewise((-I*a**2*sqrt(c)*x/sqrt(a**2*x**2 - 1) + I*a
*sqrt(c)*acosh(a*x) + I*sqrt(c)/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1
), (a**2*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - a*sqrt(c)*asin(a*x) - sqrt(c)/(x*
sqrt(-a**2*x**2 + 1)), True))
```

3.1092 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=121

$$-2a^2c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-\frac{1}{2}a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x}-\frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2}$$

```
[Out] -1/2*(-a^2*c*x^2+c)^(3/2)/x^2-2*a^2*c^(3/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+
c)^(1/2))-1/2*a^2*c^(3/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))-1/2*a*c*(-a
*x+4)*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.29, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 813, 844, 217, 203, 266, 63, 208}

$$-2a^2c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-\frac{1}{2}a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x}-\frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^3,x]
```

```
[Out] -(a*c*(4 - a*x)*Sqrt[c - a^2*c*x^2])/(2*x) - (c - a^2*c*x^2)^(3/2)/(2*x^2)
- 2*a^2*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] - (a^2*c^(3/2)*Ar
cTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^3}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^3}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} - \frac{1}{2}\int \frac{\left(-4ac-a^2cx\right)\sqrt{c-a^2cx^2}}{x^2}\,dx$$

$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} + \frac{1}{4}\int \frac{2a^2c^2-8a^3c^2x}{x\sqrt{c-a^2cx^2}}\,dx$$

$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} + \frac{1}{2}\left(a^2c^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \left(2a^3\right.$$

$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} + \frac{1}{4}\left(a^2c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx\right.$$

$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} - 2a^2c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \frac{1}{2}c\,\text{S}$$

$$= -\frac{ac(4-ax)\sqrt{c-a^2cx^2}}{2x} - \frac{\left(c-a^2cx^2\right)^{3/2}}{2x^2} - 2a^2c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \frac{1}{2}a^2c$$

Mathematica [A] time = 0.21, size = 129, normalized size = 1.07

$$\frac{1}{2}c\left(\frac{\left(2a^2x^2-4ax-1\right)\sqrt{c-a^2cx^2}}{x^2} - a^2\sqrt{c}\,\log\left(\sqrt{c}\,\sqrt{c-a^2cx^2}+c\right) + 4a^2\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) + a^2\sqrt{c}\,\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^3,x]
```

```
[Out] (c*(((-1 - 4*a*x + 2*a^2*x^2)*Sqrt[c - a^2*c*x^2])/x^2 + 4*a^2*Sqrt[c]*ArcT
an[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))] + a^2*Sqrt[c]*Log[x]
 - a^2*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]))/2
```

fricas [A] time = 0.66, size = 267, normalized size = 2.21

$$\left[\frac{8\,a^2c^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+a^2c^{\frac{3}{2}}x^2\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+2\left(2\,a^2cx^2-4\,acx-c\right)\sqrt{-a^2cx^2+c}}{4\,x^2},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/4*(8*a^2*c^(3/2)*x^2*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
- c)) + a^2*c^(3/2)*x^2*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*(2*a^2*c*x^2 - 4*a*c*x - c)*sqrt(-a^2*c*x^2 + c))/x^2, -1/2*(
a^2*sqrt(-c)*c*x^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) -
2*a^2*sqrt(-c)*c*x^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) - (2*a^2*c*x^2 - 4*a*c*x - c)*sqrt(-a^2*c*x^2 + c))/x^2]
```

giac [B] time = 0.23, size = 274, normalized size = 2.26

$$\frac{a^2c^2\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\,a^3\sqrt{-c}\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\sqrt{-a^2cx^2+c}\,a^2c-\frac{\left(\sqrt{-a^2c}\,x-\sqrt{-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^3,x, algorithm="gia
c")
```

```
[Out] a^2*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 2*a^3*sqrt(-c)*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a)
+ sqrt(-a^2*c*x^2 + c)*a^2*c - ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a
^2*c^2*abs(a) - 4*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(-c)*c^
2 + (sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^2*c^3*abs(a) + 4*a^3*sqrt(-c)
*c^3)/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^2*abs(a))
```

maple [B] time = 0.05, size = 316, normalized size = 2.61

$$\frac{a^2\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{6}-\frac{a^2c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{2}+\frac{a^2\sqrt{-a^2c\,x^2+c}\,c}{2}-\frac{2a\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{cx}-2a^3x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}-3a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^3,x)
```

```
[Out] 1/6*a^2*(-a^2*c*x^2+c)^(3/2)-1/2*a^2*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+
c)^(1/2))/x)+1/2*a^2*(-a^2*c*x^2+c)^(1/2)*c-2*a/c/x*(-a^2*c*x^2+c)^(5/2)-2*
a^3*x*(-a^2*c*x^2+c)^(3/2)-3*a^3*c*x*(-a^2*c*x^2+c)^(1/2)-3*a^3*c^2/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3*a^2*(-(x-1/a)^2*a^2
*c-2*a*c*(x-1/a))^(3/2)+a^3*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+a^3*
c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(
1/2))-1/2/c/x^2*(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^3,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,(a\,x+1)^2}{x^3\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)), x)
```

sympy [C] time = 8.22, size = 366, normalized size = 3.02

$$a^2c\left(\begin{cases} i\sqrt{c}\sqrt{a^2x^2-1}-\sqrt{c}\log(ax)+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}+i\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right) & \text{for } \left|a^2x^2\right|>1 \\ \sqrt{c}\sqrt{-a^2x^2+1}+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}-\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right)+2ac\left(\begin{cases} -\frac{ia^2\sqrt{c}\,x}{\sqrt{a^2x^2-1}}+ia\sqrt{c}\,\text{ac} \\ \frac{a^2\sqrt{c}\,x}{\sqrt{-a^2x^2+1}}-a\sqrt{c}\,\text{asin} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**3,x)

[Out] a**2*c*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(a*x) + sqrt(c
)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(c)
*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a**2*x
**2 + 1) + 1), True)) + 2*a*c*Piecewise((-I*a**2*sqrt(c)*x/sqrt(a**2*x**2 -
 1) + I*a*sqrt(c)*acosh(a*x) + I*sqrt(c)/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*
x**2) > 1), (a**2*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - a*sqrt(c)*asin(a*x) - sq
rt(c)/(x*sqrt(-a**2*x**2 + 1)), True)) + c*Piecewise((a**2*sqrt(c)*acosh(1/
(a*x))/2 + a*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(2*a*x**3*sqr
t(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**2*sqrt(c)*asin(1/(a*x
))/2 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))
```

3.1093 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=115

$$-\frac{ac(ax+1)\sqrt{c-a^2cx^2}}{x^2}-\frac{(c-a^2cx^2)^{3/2}}{3x^3}+a^3\left(-c^{3/2}\right)\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+a^3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -1/3*(-a^2*c*x^2+c)^(3/2)/x^3-a^3*c^(3/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)
^(1/2))+a^3*c^(3/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))-a*c*(a*x+1)*(-a^2
*c*x^2+c)^(1/2)/x^2
```

Rubi [A] time = 0.28, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 811, 844, 217, 203, 266, 63, 208}

$$a^3\left(-c^{3/2}\right)\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+a^3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(ax+1)\sqrt{c-a^2cx^2}}{x^2}-\frac{(c-a^2cx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^4,x]
```

```
[Out] -((a*c*(1 + a*x)*Sqrt[c - a^2*c*x^2])/x^2) - (c - a^2*c*x^2)^(3/2)/(3*x^3)
- a^3*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] + a^3*c^(3/2)*ArcTa
nh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^4}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^4}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - \frac{1}{3}\int \frac{\left(-6ac-3a^2cx\right)\sqrt{c-a^2cx^2}}{x^3}\,dx$$

$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} + \frac{\int \frac{-12a^3c^3-12a^4c^3x}{x\sqrt{c-a^2cx^2}}\,dx}{12c}$$

$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - \left(a^3c^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \left(a^4c^2\right)$$

$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - \frac{1}{2}\left(a^3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx\right.$$

$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - a^3c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + (ac)\,\text{S}$$

$$= -\frac{ac(1+ax)\sqrt{c-a^2cx^2}}{x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{3x^3} - a^3c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + a^3c^{3/2}$$

Mathematica [A] time = 0.15, size = 127, normalized size = 1.10

$$-a^3c^{3/2}\log(x)-\frac{c\left(2a^2x^2+3ax+1\right)\sqrt{c-a^2cx^2}}{3x^3}+a^3c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+a^3c^{3/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^4,x]
```

```
[Out] -1/3*(c*(1 + 3*a*x + 2*a^2*x^2)*Sqrt[c - a^2*c*x^2])/x^3 + a^3*c^(3/2)*ArcT
an[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))] - a^3*c^(3/2)*Log[x]
 + a^3*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

fricas [A] time = 0.69, size = 265, normalized size = 2.30

$$\left[\frac{6\,a^3c^{\frac{3}{2}}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+3\,a^3c^{\frac{3}{2}}x^3\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-2\left(2\,a^2cx^2+3\,acx+c\right)\sqrt{-a^2cx^2+c}}{6\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/6*(6*a^3*c^(3/2)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
- c)) + 3*a^3*c^(3/2)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - 2*(2*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a^2*c*x^2 + c))/x^3, 1/6*
(6*a^3*sqrt(-c)*c*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c))
 + 3*a^3*sqrt(-c)*c*x^3*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)
*x - c) - 2*(2*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a^2*c*x^2 + c))/x^3]
```

giac [B] time = 0.55, size = 259, normalized size = 2.25

$$-\frac{2\,a^3c^2\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^4\sqrt{-c}\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\frac{2\left(3\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^4,x, algorithm="gia
c")
```

```
[Out] -2*a^3*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) - a^4*sqrt(-c)*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a)
 - 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c^2*abs(a) + 6*(sqr
t(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^4*sqrt(-c)*c^3 - 3*(sqrt(-a^2*c)*x
- sqrt(-a^2*c*x^2 + c))*a^3*c^4*abs(a) - 2*a^4*sqrt(-c)*c^4)/(((sqrt(-a^2*c
)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^3*abs(a))
```

maple [B] time = 0.05, size = 339, normalized size = 2.95

$$-\frac{a^3\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{3}+a^3c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)-a^3\sqrt{-a^2c\,x^2+c}\,c-\frac{4a^2\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{3cx}-\frac{4a^4x\left(-a^2c\,x^2+c\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^4,x)
```

```
[Out] -1/3*a^3*(-a^2*c*x^2+c)^(3/2)+a^3*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^
(1/2))/x)-a^3*(-a^2*c*x^2+c)^(1/2)*c-4/3*a^2/c/x*(-a^2*c*x^2+c)^(5/2)-4/3*a
^4*x*(-a^2*c*x^2+c)^(3/2)-2*a^4*c*x*(-a^2*c*x^2+c)^(1/2)-2*a^4*c^2/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-1/3/c/x^3*(-a^2*c*x^2+c)
^(5/2)-2/3*a^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)+a^4*c*(-(x-1/a)^2*a^2
*c-2*a*c*(x-1/a))^(1/2)*x+a^4*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x
-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))-a/c/x^2*(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^{\frac{3}{2}}\int\frac{\sqrt{ax+1}\sqrt{-ax+1}}{x^2}\,dx-\frac{a^4c^{\frac{5}{2}}\log\left(\frac{\sqrt{-a^2cx^2+c}-\sqrt{c}}{\sqrt{-a^2cx^2+c}+\sqrt{c}}\right)+\frac{2\sqrt{-a^2cx^2+c}\,a^2c^2}{x^2}}{2\,ac}+\frac{\left(a^2c^{\frac{3}{2}}x^2-c^{\frac{3}{2}}\right)\sqrt{ax+1}\sqrt{-ax+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^4,x, algorithm="max
ima")
```

```
[Out] a^2*c^(3/2)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^2, x) - 1/2*(a^4*c^(5/
2)*log((sqrt(-a^2*c*x^2 + c) - sqrt(c))/(sqrt(-a^2*c*x^2 + c) + sqrt(c))) +
 2*sqrt(-a^2*c*x^2 + c)*a^2*c^2/x^2)/(a*c) + 1/3*(a^2*c^(3/2)*x^2 - c^(3/2)
)*sqrt(a*x + 1)*sqrt(-a*x + 1)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,{\left(a\,x+1\right)}^2}{x^4\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)), x)
```

sympy [C] time = 18.18, size = 359, normalized size = 3.12

$$a^2c\left(\begin{cases}-\frac{ia^2\sqrt{c}\,x}{\sqrt{a^2x^2-1}}+ia\sqrt{c}\,\operatorname{acosh}(ax)+\frac{i\sqrt{c}}{x\sqrt{a^2x^2-1}} & \text{for } \left|a^2x^2\right|>1\\ \frac{a^2\sqrt{c}\,x}{\sqrt{-a^2x^2+1}}-a\sqrt{c}\,\operatorname{asin}(ax)-\frac{\sqrt{c}}{x\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{a^2\sqrt{c}\,\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{a\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{\sqrt{c}}{2ax^3\sqrt{-1+\frac{1}{a^2x^2}}} \\ -\frac{ia^2\sqrt{c}\,\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**4,x)
```

```
[Out] a**2*c*Piecewise((-I*a**2*sqrt(c)*x/sqrt(a**2*x**2 - 1) + I*a*sqrt(c)*acosh
(a*x) + I*sqrt(c)/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**2*sqrt(
c)*x/sqrt(-a**2*x**2 + 1) - a*sqrt(c)*asin(a*x) - sqrt(c)/(x*sqrt(-a**2*x**
2 + 1)), True)) + 2*a*c*Piecewise((a**2*sqrt(c)*acosh(1/(a*x))/2 + a*sqrt(c
)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2
))), 1/Abs(a**2*x**2) > 1), (-I*a**2*sqrt(c)*asin(1/(a*x))/2 - I*a*sqrt(c)*
sqrt(1 - 1/(a**2*x**2))/(2*x), True)) + c*Piecewise((a**3*sqrt(c)*sqrt(-1 +
 1/(a**2*x**2))/3 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(3*x**2), 1/Abs(a**2
*x**2) > 1), (I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/3 - I*a*sqrt(c)*sqrt(1
 - 1/(a**2*x**2))/(3*x**2), True))
```

3.1094 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=106

$$-\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2}-\frac{(c-a^2cx^2)^{3/2}}{4x^4}-\frac{2a(c-a^2cx^2)^{3/2}}{3x^3}+\frac{5}{8}a^4c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -1/4*(-a^2*c*x^2+c)^(3/2)/x^4-2/3*a*(-a^2*c*x^2+c)^(3/2)/x^3+5/8*a^4*c^(3/2
)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))-5/8*a^2*c*(-a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A] time = 0.26, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1807, 807, 266, 47, 63, 208}

$$\frac{5}{8}a^4c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2}-\frac{2a(c-a^2cx^2)^{3/2}}{3x^3}-\frac{(c-a^2cx^2)^{3/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^5,x]
```

```
[Out] (-5*a^2*c*Sqrt[c - a^2*c*x^2])/(8*x^2) - (c - a^2*c*x^2)^(3/2)/(4*x^4) - (2
*a*(c - a^2*c*x^2)^(3/2))/(3*x^3) + (5*a^4*c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x
^2]/Sqrt[c]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^5}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4} - \frac{1}{4}\int \frac{\left(-8ac-5a^2cx\right)\sqrt{c-a^2cx^2}}{x^4}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3} + \frac{1}{4}\left(5a^2c\right)\int \frac{\sqrt{c-a^2cx^2}}{x^3}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3} + \frac{1}{8}\left(5a^2c\right)\text{Subst}\left(\int \frac{\sqrt{c-a^2cx}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3} - \frac{1}{16}\left(5a^4c^2\right)\text{Subst}\left(\int\right.$$
$$= -\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3} + \frac{1}{8}\left(5a^2c\right)\text{Subst}\left(\int\right.$$
$$= -\frac{5a^2c\sqrt{c-a^2cx^2}}{8x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{4x^4} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{3x^3} + \frac{5}{8}a^4c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\cdots}{\cdots}\right.$$

Mathematica [A] time = 0.15, size = 96, normalized size = 0.91

$$-\frac{5}{8}a^4c^{3/2}\log(x)+\frac{5}{8}a^4c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{c\left(16a^3x^3-9a^2x^2-16ax-6\right)\sqrt{c-a^2cx^2}}{24x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^5,x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-6 - 16*a*x - 9*a^2*x^2 + 16*a^3*x^3))/(24*x^4) - (
5*a^4*c^(3/2)*Log[x])/8 + (5*a^4*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2
]])/8
```

fricas [A] time = 0.60, size = 191, normalized size = 1.80

$$\left[\frac{15\,a^4c^{\frac{3}{2}}x^4\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\left(16\,a^3cx^3-9\,a^2cx^2-16\,acx-6\,c\right)\sqrt{-a^2cx^2+c}}{48\,x^4},\ \frac{15\,a^4\sqrt{-c}\,cx^4\,\text{arct}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^5,x, algorithm="fri
cas")
```

```
[Out] [1/48*(15*a^4*c^(3/2)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) + 2*(16*a^3*c*x^3 - 9*a^2*c*x^2 - 16*a*c*x - 6*c)*sqrt(-a^2*c*x
^2 + c))/x^4, 1/24*(15*a^4*sqrt(-c)*c*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(
-c)/(a^2*c*x^2 - c)) + (16*a^3*c*x^3 - 9*a^2*c*x^2 - 16*a*c*x - 6*c)*sqrt(-
a^2*c*x^2 + c))/x^4]
```

giac [B] time = 0.22, size = 371, normalized size = 3.50

$$-\frac{5\,a^4c^2\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}-\frac{9\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^7a^4c^2+48\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^6a^3\sqrt{-c}\,c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^5,x, algorithm="gia
c")
```

```
[Out] -5/4*a^4*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt
(-c) - 1/12*(9*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c^2 + 48*(sqrt
(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^6*a^3*sqrt(-c)*c^2*abs(a) - 33*(sqrt(-a^
2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^3 - 48*(sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^4*a^3*sqrt(-c)*c^3*abs(a) - 33*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2
 + c))^3*a^4*c^4 + 16*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(-c
)*c^4*abs(a) + 9*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^5 - 16*a^3*s
qrt(-c)*c^5*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

maple [B] time = 0.06, size = 364, normalized size = 3.43

$$-\frac{5a^4\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{24}+\frac{5a^4c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{8}-\frac{5a^4\sqrt{-a^2c\,x^2+c}\,c}{8}-\frac{2a^3\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{3cx}-\frac{2a^5x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^5,x)
```

```
[Out] -5/24*a^4*(-a^2*c*x^2+c)^(3/2)+5/8*a^4*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^
2+c)^(1/2))/x)-5/8*a^4*(-a^2*c*x^2+c)^(1/2)*c-2/3*a^3/c/x*(-a^2*c*x^2+c)^(5
/2)-2/3*a^5*x*(-a^2*c*x^2+c)^(3/2)-a^5*c*x*(-a^2*c*x^2+c)^(1/2)-a^5*c^2/(a^
2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3*a/c/x^3*(-a^2*c
```

```
*x^2+c)^(5/2)-2/3*a^4*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)+a^5*c*(-(x-1/a
)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+a^5*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)
*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))-7/8*a^2/c/x^2*(-a^2*c*x^2+c)^(5/
2)-1/4/c/x^4*(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^5,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)^2}{x^5\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)), x)
```

sympy [C] time = 14.70, size = 447, normalized size = 4.22

$$a^2c\left(\begin{cases}\frac{a^2\sqrt{c}\,\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{a\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{\sqrt{c}}{2ax^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for}\ \frac{1}{|a^2x^2|}>1\\ -\frac{ia^2\sqrt{c}\,\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{a^3\sqrt{c}\sqrt{-1+\frac{1}{a^2x^2}}}{3}-\frac{a\sqrt{c}\sqrt{-1+\frac{1}{a^2x^2}}}{3x^2} & \text{for}\ \frac{1}{|a^2 \cdots\\ \frac{ia^3\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{3}-\frac{ia\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} & \text{other}\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**5,x)
```

```
[Out] a**2*c*Piecewise((a**2*sqrt(c)*acosh(1/(a*x))/2 + a*sqrt(c)/(2*x*sqrt(-1 +
1/(a**2*x**2))) - sqrt(c)/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
```

```
**2) > 1), (-I*a**2*sqrt(c)*asin(1/(a*x))/2 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*
x**2))/(2*x), True)) + 2*a*c*Piecewise((a**3*sqrt(c)*sqrt(-1 + 1/(a**2*x**2
))/3 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1),
(I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/3 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*x*
*2))/(3*x**2), True)) + c*Piecewise((a**4*sqrt(c)*acosh(1/(a*x))/8 - a**3*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*a*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - sqrt(c)/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2)
> 1), (-I*a**4*sqrt(c)*asin(1/(a*x))/8 + I*a**3*sqrt(c)/(8*x*sqrt(1 - 1/(a*
*2*x**2))) - 3*I*a*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I*sqrt(c)/(4*
a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

3.1095 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=131

$$-\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5}-\frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4}-\frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3}+\frac{1}{4}a^5c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2}$$

```
[Out] -1/5*(-a^2*c*x^2+c)^(3/2)/x^5-1/2*a*(-a^2*c*x^2+c)^(3/2)/x^4-7/15*a^2*(-a^2
*c*x^2+c)^(3/2)/x^3+1/4*a^5*c^(3/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))-1
/4*a^3*c*(-a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A] time = 0.29, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{1}{4}a^5c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2}-\frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3}-\frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4}-\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^6,x]
```

```
[Out] -(a^3*c*Sqrt[c - a^2*c*x^2])/(4*x^2) - (c - a^2*c*x^2)^(3/2)/(5*x^5) - (a*(
c - a^2*c*x^2)^(3/2))/(2*x^4) - (7*a^2*(c - a^2*c*x^2)^(3/2))/(15*x^3) + (a
^5*c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^6}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^6}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{1}{5}\int \frac{\left(-10ac-7a^2cx\right)\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} + \frac{\int \frac{\left(28a^2c^2+10a^3c^2x\right)\sqrt{c-a^2cx^2}}{x^4}\,dx}{20c}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{2}\left(a^3c\right)\int \frac{\sqrt{c-a^2cx^2}}{x^3}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{4}\left(a^3c\right)\text{Subst}\left(\int \frac{\sqrt{}}{}\right.$$
$$= -\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} - \frac{1}{8}$$
$$= -\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{4}$$
$$= -\frac{a^3c\sqrt{c-a^2cx^2}}{4x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{2x^4} - \frac{7a^2\left(c-a^2cx^2\right)^{3/2}}{15x^3} + \frac{1}{4}a$$

Mathematica [A] time = 0.17, size = 104, normalized size = 0.79

$$-\frac{1}{4}a^5c^{3/2}\log(x)+\frac{1}{4}a^5c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{c\left(28a^4x^4+15a^3x^3-16a^2x^2-30ax-12\right)\sqrt{c-a^2cx^2}}{60x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^6,x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-12 - 30*a*x - 16*a^2*x^2 + 15*a^3*x^3 + 28*a^4*x^4
))/(60*x^5) - (a^5*c^(3/2)*Log[x])/4 + (a^5*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c
- a^2*c*x^2]])/4
```

fricas [A] time = 0.65, size = 209, normalized size = 1.60

$$\left[\frac{15\,a^5c^{\frac{3}{2}}x^5\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+2\left(28\,a^4cx^4+15\,a^3cx^3-16\,a^2cx^2-30\,acx-12\,c\right)\sqrt{-a^2cx^2+c}}{120\,x^5},\ \frac{15\,a^5\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^6,x, algorithm="fri
cas")
```

```
[Out] [1/120*(15*a^5*c^(3/2)*x^5*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
 - 2*c)/x^2) + 2*(28*a^4*c*x^4 + 15*a^3*c*x^3 - 16*a^2*c*x^2 - 30*a*c*x - 1
2*c)*sqrt(-a^2*c*x^2 + c))/x^5, 1/60*(15*a^5*sqrt(-c)*c*x^5*arctan(sqrt(-a^
2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (28*a^4*c*x^4 + 15*a^3*c*x^3 - 16*
a^2*c*x^2 - 30*a*c*x - 12*c)*sqrt(-a^2*c*x^2 + c))/x^5]
```

giac [B] time = 0.21, size = 414, normalized size = 3.16

$$-\frac{a^5c^2\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{2\sqrt{-c}}+\frac{15\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^9a^5c^2-60\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^8a^4\sqrt{-c}\,c^2|\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^6,x, algorithm="gia
c")
```

```
[Out] -1/2*a^5*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt
(-c) + 1/30*(15*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^9*a^5*c^2 - 60*(sqr
t(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^8*a^4*sqrt(-c)*c^2*abs(a) + 90*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^5*c^3 + 240*(sqrt(-a^2*c)*x - sqrt(-a^2
*c*x^2 + c))^6*a^4*sqrt(-c)*c^3*abs(a) - 40*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x
^2 + c))^4*a^4*sqrt(-c)*c^4*abs(a) - 90*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 +
 c))^3*a^5*c^5 + 80*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^4*sqrt(-c)*
c^5*abs(a) - 15*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^5*c^6 - 28*a^4*sq
rt(-c)*c^6*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^5
```

maple [B] time = 0.07, size = 388, normalized size = 2.96

$$-\frac{2a^2\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{3c\,x^3}-\frac{2a^4\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{3cx}-a^6cx\sqrt{-a^2c\,x^2+c}-\frac{a^6c^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+a^6c\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^6,x)
```

```
[Out] -2/3*a^2/c/x^3*(-a^2*c*x^2+c)^(5/2)-2/3*a^4/c/x*(-a^2*c*x^2+c)^(5/2)-a^6*c*
x*(-a^2*c*x^2+c)^(1/2)-a^6*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c
*x^2+c)^(1/2))+a^6*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+a^6*c^2/(a^2*
c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))-3/4
*a^3/c/x^2*(-a^2*c*x^2+c)^(5/2)-2/3*a^5*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3
/2)-1/12*a^5*(-a^2*c*x^2+c)^(3/2)-1/2*a/c/x^4*(-a^2*c*x^2+c)^(5/2)-1/5/c/x^
5*(-a^2*c*x^2+c)^(5/2)+1/4*a^5*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/
2))/x)-1/4*a^5*(-a^2*c*x^2+c)^(1/2)*c-2/3*a^6*x*(-a^2*c*x^2+c)^(3/2)
```

maxima [B] time = 0.46, size = 221, normalized size = 1.69

$$\frac{\left(a^{2}c^{\frac{3}{2}}x^{2}-c^{\frac{3}{2}}\right)\sqrt{ax+1}\,\sqrt{-ax+1}\,a^{2}}{3\,x^{3}}-\frac{a^{6}c^{\frac{5}{2}}\log\left(\frac{\sqrt{-a^{2}cx^{2}+c}-\sqrt{c}}{\sqrt{-a^{2}cx^{2}+c}+\sqrt{c}}\right)+\frac{2\left(\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}a^{6}c^{3}+\sqrt{-a^{2}cx^{2}+c}\,a^{6}c^{4}\right)}{\left(a^{2}cx^{2}-c\right)^{2}+2\left(a^{2}cx^{2}-c\right)c+c^{2}}}{8\,ac}+\frac{\left(2\,a^{4}c^{\frac{3}{2}}x^{4}+a^{2}c\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^6,x, algorithm="max
ima")
```

```
[Out] 1/3*(a^2*c^(3/2)*x^2 - c^(3/2))*sqrt(a*x + 1)*sqrt(-a*x + 1)*a^2/x^3 - 1/8*
(a^6*c^(5/2)*log((sqrt(-a^2*c*x^2 + c) - sqrt(c))/(sqrt(-a^2*c*x^2 + c) + s
qrt(c))) + 2*((-a^2*c*x^2 + c)^(3/2)*a^6*c^3 + sqrt(-a^2*c*x^2 + c)*a^6*c^4
)/((a^2*c*x^2 - c)^2 + 2*(a^2*c*x^2 - c)*c + c^2))/(a*c) + 1/15*(2*a^4*c^(3
/2)*x^4 + a^2*c^(3/2)*x^2 - 3*c^(3/2))*sqrt(a*x + 1)*sqrt(-a*x + 1)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)^2}{x^6\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^6*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^6*(a^2*x^2 - 1)), x)
```

sympy [C] time = 24.93, size = 484, normalized size = 3.69

$$a^2c\left(\begin{cases}\frac{a^3\sqrt{c}\sqrt{-1+\frac{1}{a^2x^2}}}{3}-\frac{a\sqrt{c}\sqrt{-1+\frac{1}{a^2x^2}}}{3x^2} & \text{for } \frac{1}{|a^2x^2|}>1\\ \frac{ia^3\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{3}-\frac{ia\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}\frac{a^4\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{a^3\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{3a\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{\sqrt{c}}{4ax^5\sqrt{-1}}\\ -\frac{ia^4\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}+\frac{ia^3\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3ia\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i\sqrt{c}}{4ax^5\sqrt{1-}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**6,x)
```

```
[Out] a**2*c*Piecewise((a**3*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(c)*sqrt(
-1 + 1/(a**2*x**2))/(3*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(c)*sqrt(1
 - 1/(a**2*x**2))/3 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))
+ 2*a*c*Piecewise((a**4*sqrt(c)*acosh(1/(a*x))/8 - a**3*sqrt(c)/(8*x*sqrt(-
1 + 1/(a**2*x**2))) + 3*a*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - sqrt(
c)/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**4*sqr
t(c)*asin(1/(a*x))/8 + I*a**3*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*I*a
*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I*sqrt(c)/(4*a*x**5*sqrt(1 - 1/
(a**2*x**2))), True)) + c*Piecewise((2*I*a**4*sqrt(c)*sqrt(a**2*x**2 - 1)/(
15*x) + I*a**2*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(c)*sqrt(a**2*
x**2 - 1)/(5*x**5), Abs(a**2*x**2) > 1), (2*a**4*sqrt(c)*sqrt(-a**2*x**2 +
1)/(15*x) + a**2*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3) - sqrt(c)*sqrt(-a**
2*x**2 + 1)/(5*x**5), True))
```

3.1096 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=156

$$-\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5}-\frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4}+\frac{3}{16}a^6c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2}-\frac{4a^3\left(c\right.}{}$$

```
[Out] -1/6*(-a^2*c*x^2+c)^(3/2)/x^6-2/5*a*(-a^2*c*x^2+c)^(3/2)/x^5-3/8*a^2*(-a^2*
c*x^2+c)^(3/2)/x^4-4/15*a^3*(-a^2*c*x^2+c)^(3/2)/x^3+3/16*a^6*c^(3/2)*arcta
nh((-a^2*c*x^2+c)^(1/2)/c^(1/2))-3/16*a^4*c*(-a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A] time = 0.33, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{3}{16}a^6c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2}-\frac{4a^3\left(c-a^2cx^2\right)^{3/2}}{15x^3}-\frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4}-\frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5}-\frac{\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^7,x]
```

```
[Out] (-3*a^4*c*Sqrt[c - a^2*c*x^2])/(16*x^2) - (c - a^2*c*x^2)^(3/2)/(6*x^6) - (
2*a*(c - a^2*c*x^2)^(3/2))/(5*x^5) - (3*a^2*(c - a^2*c*x^2)^(3/2))/(8*x^4)
- (4*a^3*(c - a^2*c*x^2)^(3/2))/(15*x^3) + (3*a^6*c^(3/2)*ArcTanh[Sqrt[c -
a^2*c*x^2]/Sqrt[c]])/16
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^7}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^7}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{1}{6}\int \frac{\left(-12ac-9a^2cx\right)\sqrt{c-a^2cx^2}}{x^6}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} + \frac{\int \frac{\left(45a^2c^2+24a^3c^2x\right)\sqrt{c-a^2cx^2}}{x^5}\,dx}{30c}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} - \frac{\int \frac{\left(-96a^3c^3-45a^4c^3x\right)\sqrt{c}}{x^4}}{120c^2}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} - \frac{4a^3\left(c-a^2cx^2\right)^{3/2}}{15x^3} -$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} - \frac{4a^3\left(c-a^2cx^2\right)^{3/2}}{15x^3} -$$
$$= -\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} -$$
$$= -\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} -$$
$$= -\frac{3a^4c\sqrt{c-a^2cx^2}}{16x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{6x^6} - \frac{2a\left(c-a^2cx^2\right)^{3/2}}{5x^5} - \frac{3a^2\left(c-a^2cx^2\right)^{3/2}}{8x^4} -$$

Mathematica [A] time = 0.24, size = 109, normalized size = 0.70

$$\frac{1}{240}c\left(-45a^6\sqrt{c}\,\log(x)+45a^6\sqrt{c}\,\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{\left(64a^5x^5+45a^4x^4+32a^3x^3-50a^2x^2-96ax-40\right.}{x^6}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^7,x]
```

```
[Out] (c*((Sqrt[c - a^2*c*x^2]*(-40 - 96*a*x - 50*a^2*x^2 + 32*a^3*x^3 + 45*a^4*x
^4 + 64*a^5*x^5))/x^6 - 45*a^6*Sqrt[c]*Log[x] + 45*a^6*Sqrt[c]*Log[c + Sqrt
[c]*Sqrt[c - a^2*c*x^2]]))/240
```

fricas [A] time = 0.79, size = 227, normalized size = 1.46

$$\left[\frac{45\,a^6c^{\frac{3}{2}}x^6\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+2\left(64\,a^5cx^5+45\,a^4cx^4+32\,a^3cx^3-50\,a^2cx^2-96\,acx-40\,c\right)\sqrt{-a^2cx^2}}{480\,x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^7,x, algorithm="fri
cas")
```

```
[Out] [1/480*(45*a^6*c^(3/2)*x^6*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
 - 2*c)/x^2) + 2*(64*a^5*c*x^5 + 45*a^4*c*x^4 + 32*a^3*c*x^3 - 50*a^2*c*x^2
 - 96*a*c*x - 40*c)*sqrt(-a^2*c*x^2 + c))/x^6, 1/240*(45*a^6*sqrt(-c)*c*x^6
*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (64*a^5*c*x^5 + 45
*a^4*c*x^4 + 32*a^3*c*x^3 - 50*a^2*c*x^2 - 96*a*c*x - 40*c)*sqrt(-a^2*c*x^2
 + c))/x^6]
```

giac [B] time = 0.57, size = 443, normalized size = 2.84

$$-\frac{3\,a^6c^2\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}}+\frac{45\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^{11}a^6c^2+65\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^9a^6c^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^7,x, algorithm="gia
c")
```

```
[Out] -3/8*a^6*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt
(-c) + 1/120*(45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^11*a^6*c^2 + 65*(s
qrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^9*a^6*c^3 + 960*(sqrt(-a^2*c)*x - sqr
t(-a^2*c*x^2 + c))^8*a^5*sqrt(-c)*c^3*abs(a) - 750*(sqrt(-a^2*c)*x - sqrt(-
a^2*c*x^2 + c))^7*a^6*c^4 - 640*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^6*a
^5*sqrt(-c)*c^4*abs(a) - 750*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^6*
c^5 + 65*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^6*c^6 - 384*(sqrt(-a^2
*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^5*sqrt(-c)*c^6*abs(a) + 45*(sqrt(-a^2*c)*
x - sqrt(-a^2*c*x^2 + c))*a^6*c^7 + 64*a^5*sqrt(-c)*c^7*abs(a))/((sqrt(-a^2
*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^6
```

maple [B] time = 0.08, size = 412, normalized size = 2.64

$$-\frac{a^{6} \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{16}+a^{7} c \sqrt{-\left(x -\frac{1}{a}\right)^{2} a^{2} c -2 a c \left(x -\frac{1}{a}\right)}\, x +\frac{a^{7} c^{2} \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-\left(x -\frac{1}{a}\right)^{2} a^{2} c -2 a c \left(x -\frac{1}{a}\right)}}\right)}{\sqrt{a^{2} c}}-\frac{2 a \left(-a^{2} c \,x^{2}+c \right)}{5 c \,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^7,x)
```

```
[Out] -1/16*a^6*(-a^2*c*x^2+c)^(3/2)+a^7*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)
*x+a^7*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-
1/a))^(1/2))-2/5*a/c/x^5*(-a^2*c*x^2+c)^(5/2)-2/3*a^5/c/x*(-a^2*c*x^2+c)^(5
/2)-a^7*c*x*(-a^2*c*x^2+c)^(1/2)-a^7*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)
*x/(-a^2*c*x^2+c)^(1/2))-13/24*a^2/c/x^4*(-a^2*c*x^2+c)^(5/2)-35/48*a^4/c/x
^2*(-a^2*c*x^2+c)^(5/2)-2/3*a^3/c/x^3*(-a^2*c*x^2+c)^(5/2)-1/6/c/x^6*(-a^2*
c*x^2+c)^(5/2)+3/16*a^6*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-
3/16*a^6*(-a^2*c*x^2+c)^(1/2)*c-2/3*a^7*x*(-a^2*c*x^2+c)^(3/2)-2/3*a^6*(-(x
-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} (a x+1)^{2}}{\left(a^{2} x^{2}-1\right) x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^7,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)^2}{x^7\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^7*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^7*(a^2*x^2 - 1)), x)
```

sympy [C] time = 15.99, size = 636, normalized size = 4.08

$$a^2c\left(\begin{cases} \frac{a^4\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8} - \frac{a^3\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}} + \frac{3a\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{\sqrt{c}}{4ax^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|} > 1 \\ -\frac{ia^4\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8} + \frac{ia^3\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3ia\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{i\sqrt{c}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise} \end{cases}\right) + 2ac\left(\begin{cases} \frac{2ia^4\sqrt{c}\sqrt{a^2x^2-1}}{15x} + \frac{ia^2\sqrt{c}\sqrt{a^2 \ldots}}{15x^3} \\ \frac{2a^4\sqrt{c}\sqrt{-a^2x^2+1}}{15x} + \frac{a^2\sqrt{c}\sqrt{- \ldots}}{15x \ldots} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**7,x)
```

```
[Out] a**2*c*Piecewise((a**4*sqrt(c)*acosh(1/(a*x))/8 - a**3*sqrt(c)/(8*x*sqrt(-1
 + 1/(a**2*x**2))) + 3*a*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c
)/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**4*sqrt
(c)*asin(1/(a*x))/8 + I*a**3*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*I*a*
sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I*sqrt(c)/(4*a*x**5*sqrt(1 - 1/(
a**2*x**2))), True)) + 2*a*c*Piecewise((2*I*a**4*sqrt(c)*sqrt(a**2*x**2 - 1
)/(15*x) + I*a**2*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(c)*sqrt(a*
*2*x**2 - 1)/(5*x**5), Abs(a**2*x**2) > 1), (2*a**4*sqrt(c)*sqrt(-a**2*x**2
 + 1)/(15*x) + a**2*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3) - sqrt(c)*sqrt(-
a**2*x**2 + 1)/(5*x**5), True)) + c*Piecewise((a**6*sqrt(c)*acosh(1/(a*x))/
16 - a**5*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*x**2))) + a**3*sqrt(c)/(48*x**3*s
qrt(-1 + 1/(a**2*x**2))) + 5*a*sqrt(c)/(24*x**5*sqrt(-1 + 1/(a**2*x**2))) -
 sqrt(c)/(6*a*x**7*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a*
*6*sqrt(c)*asin(1/(a*x))/16 + I*a**5*sqrt(c)/(16*x*sqrt(1 - 1/(a**2*x**2)))
 - I*a**3*sqrt(c)/(48*x**3*sqrt(1 - 1/(a**2*x**2))) - 5*I*a*sqrt(c)/(24*x**
5*sqrt(1 - 1/(a**2*x**2))) + I*sqrt(c)/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))),
True))
```

3.1097 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=181

$$-\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7}-\frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6}-\frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5}+\frac{1}{8}a^7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2}-\frac{22a^4\left(c-\right.}{10}$$

```
[Out] -1/7*(-a^2*c*x^2+c)^(3/2)/x^7-1/3*a*(-a^2*c*x^2+c)^(3/2)/x^6-11/35*a^2*(-a^
2*c*x^2+c)^(3/2)/x^5-1/4*a^3*(-a^2*c*x^2+c)^(3/2)/x^4-22/105*a^4*(-a^2*c*x^
2+c)^(3/2)/x^3+1/8*a^7*c^(3/2)*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))-1/8*a^
5*c*(-a^2*c*x^2+c)^(1/2)/x^2
```

Rubi [A] time = 0.37, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6151, 1807, 835, 807, 266, 47, 63, 208}

$$\frac{1}{8}a^7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2}-\frac{22a^4\left(c-a^2cx^2\right)^{3/2}}{105x^3}-\frac{a^3\left(c-a^2cx^2\right)^{3/2}}{4x^4}-\frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5}-\frac{a\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^8,x]
```

```
[Out] -(a^5*c*Sqrt[c - a^2*c*x^2])/(8*x^2) - (c - a^2*c*x^2)^(3/2)/(7*x^7) - (a*(
c - a^2*c*x^2)^(3/2))/(3*x^6) - (11*a^2*(c - a^2*c*x^2)^(3/2))/(35*x^5) - (
a^3*(c - a^2*c*x^2)^(3/2))/(4*x^4) - (22*a^4*(c - a^2*c*x^2)^(3/2))/(105*x^
3) + (a^7*c^(3/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
```

```
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}}{x^8}\,dx = c\int \frac{(1+ax)^2\sqrt{c-a^2cx^2}}{x^8}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{1}{7}\int \frac{\left(-14ac-11a^2cx\right)\sqrt{c-a^2cx^2}}{x^7}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} + \frac{\int \frac{\left(66a^2c^2+42a^3c^2x\right)\sqrt{c-a^2cx^2}}{x^6}\,dx}{42c}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{\int \frac{\left(-210a^3c^3-132a^4c^3x\right)}{x^5}}{210c^2}$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{a^3\left(c-a^2cx^2\right)^{3/2}}{4x^4} +$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{a^3\left(c-a^2cx^2\right)^{3/2}}{4x^4} -$$
$$= -\frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - \frac{a^3\left(c-a^2cx^2\right)^{3/2}}{4x^4} -$$
$$= -\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - $$
$$= -\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - $$
$$= -\frac{a^5c\sqrt{c-a^2cx^2}}{8x^2} - \frac{\left(c-a^2cx^2\right)^{3/2}}{7x^7} - \frac{a\left(c-a^2cx^2\right)^{3/2}}{3x^6} - \frac{11a^2\left(c-a^2cx^2\right)^{3/2}}{35x^5} - $$

Mathematica [A] time = 0.19, size = 120, normalized size = 0.66

$$-\frac{1}{8}a^7c^{3/2}\log(x)+\frac{1}{8}a^7c^{3/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{c\left(176a^6x^6+105a^5x^5+88a^4x^4+70a^3x^3-144a^2x^2-280ax\right.}{840x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2))/x^8,x]
```

[Out] (c*Sqrt[c - a^2*c*x^2]*(-120 - 280*a*x - 144*a^2*x^2 + 70*a^3*x^3 + 88*a^4*x^4 + 105*a^5*x^5 + 176*a^6*x^6))/(840*x^7) - (a^7*c^(3/2)*Log[x])/8 + (a^7*c^(3/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/8

fricas [A] time = 0.74, size = 245, normalized size = 1.35

$$\left[\frac{105\,a^{7}c^{\frac{3}{2}}x^{7}\log\left(-\frac{a^{2}cx^{2}-2\,\sqrt{-a^{2}cx^{2}+c}\,\sqrt{c}-2\,c}{x^{2}}\right)+2\left(176\,a^{6}cx^{6}+105\,a^{5}cx^{5}+88\,a^{4}cx^{4}+70\,a^{3}cx^{3}-144\,a^{2}cx^{2}-280\,acx\right.}{1680\,x^{7}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^8,x, algorithm="fricas")

[Out] [1/1680*(105*a^7*c^(3/2)*x^7*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + 2*(176*a^6*c*x^6 + 105*a^5*c*x^5 + 88*a^4*c*x^4 + 70*a^3*c*x^3 - 144*a^2*c*x^2 - 280*a*c*x - 120*c)*sqrt(-a^2*c*x^2 + c))/x^7, 1/840*(105*a^7*sqrt(-c)*c*x^7*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (176*a^6*c*x^6 + 105*a^5*c*x^5 + 88*a^4*c*x^4 + 70*a^3*c*x^3 - 144*a^2*c*x^2 - 280*a*c*x - 120*c)*sqrt(-a^2*c*x^2 + c))/x^7]

giac [B] time = 0.27, size = 529, normalized size = 2.92

$$-\frac{a^{7}c^{2}\arctan\left(-\frac{\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}+\frac{105\left(\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}\right)^{13}a^{7}c^{2}-700\left(\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}\right)^{11}a^{7}c^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^8,x, algorithm="giac")

[Out] -1/4*a^7*c^2*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) + 1/420*(105*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^13*a^7*c^2 - 700*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^11*a^7*c^3 + 1680*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^10*a^6*sqrt(-c)*c^3*abs(a) - 3395*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^9*a^7*c^4 - 7280*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^8*a^6*sqrt(-c)*c^4*abs(a) - 1120*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^6*a^6*sqrt(-c)*c^5*abs(a) + 3395*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^7*c^6 - 2016*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^4*a^6*sqrt(-c)*c^6*abs(a) + 700*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^7*c^7 + 1232*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^6*sqrt(-c)*c^7*abs(a) - 105*(sqrt(-

```
a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^7*c^8 - 176*a^6*sqrt(-c)*c^8*abs(a))/((s
qrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^7
```

maple [B] time = 0.10, size = 436, normalized size = 2.41

$$-\frac{a\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{3c\,x^6}-\frac{7a^3\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{12c\,x^4}-\frac{17a^5\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{24c\,x^2}-\frac{2a^8x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{3}-a^8cx\sqrt{-a^2c\,x^2+c}-\frac{a^8c^2\,\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^8,x)
```

```
[Out] -1/3*a/c/x^6*(-a^2*c*x^2+c)^(5/2)-7/12*a^3/c/x^4*(-a^2*c*x^2+c)^(5/2)-17/24
*a^5/c/x^2*(-a^2*c*x^2+c)^(5/2)-2/3*a^8*x*(-a^2*c*x^2+c)^(3/2)-a^8*c*x*(-a^
2*c*x^2+c)^(1/2)-a^8*c^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c
)^(1/2))-2/3*a^4/c/x^3*(-a^2*c*x^2+c)^(5/2)-16/35*a^2/c/x^5*(-a^2*c*x^2+c)^
(5/2)+1/8*a^7*c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-1/8*a^7*(-
a^2*c*x^2+c)^(1/2)*c+a^8*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+a^8*c^2
/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2
))-1/7/c/x^7*(-a^2*c*x^2+c)^(5/2)-2/3*a^6/c/x*(-a^2*c*x^2+c)^(5/2)-2/3*a^7*
(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)-1/24*a^7*(-a^2*c*x^2+c)^(3/2)
```

maxima [A] time = 0.45, size = 287, normalized size = 1.59

$$-\frac{3\,a^8c^{\frac{5}{2}}\log\left(\frac{\sqrt{-a^2cx^2+c}-\sqrt{c}}{\sqrt{-a^2cx^2+c}+\sqrt{c}}\right)-\frac{2\left(3\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^8c^3-8\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^8c^4-3\sqrt{-a^2cx^2+c}\,a^8c^5\right)}{\left(a^2cx^2-c\right)^3+3\left(a^2cx^2-c\right)^2c+3\left(a^2cx^2-c\right)c^2+c^3}}{48\,ac}+\frac{\left(2\,a^4c^{\frac{3}{2}}x^4+a^2c^{\frac{3}{2}}x^2-3\,c^{\frac{3}{2}}\right)\sqrt{ax}}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(3/2)/x^8,x, algorithm="max
ima")
```

```
[Out] -1/48*(3*a^8*c^(5/2)*log((sqrt(-a^2*c*x^2 + c) - sqrt(c))/(sqrt(-a^2*c*x^2
+ c) + sqrt(c))) - 2*(3*(-a^2*c*x^2 + c)^(5/2)*a^8*c^3 - 8*(-a^2*c*x^2 + c)
^(3/2)*a^8*c^4 - 3*sqrt(-a^2*c*x^2 + c)*a^8*c^5)/((a^2*c*x^2 - c)^3 + 3*(a^
2*c*x^2 - c)^2*c + 3*(a^2*c*x^2 - c)*c^2 + c^3))/(a*c) + 1/15*(2*a^4*c^(3/2
)*x^4 + a^2*c^(3/2)*x^2 - 3*c^(3/2))*sqrt(a*x + 1)*sqrt(-a*x + 1)*a^2/x^5 +
 1/105*(8*a^6*c^(3/2)*x^6 + 4*a^4*c^(3/2)*x^4 + 3*a^2*c^(3/2)*x^2 - 15*c^(3
/2))*sqrt(a*x + 1)*sqrt(-a*x + 1)/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)^2}{x^8\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^8*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(x^8*(a^2*x^2 - 1)), x)
```

sympy [C] time = 39.57, size = 660, normalized size = 3.65

$$a^2c\left(\begin{cases} \frac{2ia^4\sqrt{c}\,\sqrt{a^2x^2-1}}{15x}+\frac{ia^2\sqrt{c}\,\sqrt{a^2x^2-1}}{15x^3}-\frac{i\sqrt{c}\,\sqrt{a^2x^2-1}}{5x^5} & \text{for } \left|a^2x^2\right|>1 \\ \frac{2a^4\sqrt{c}\,\sqrt{-a^2x^2+1}}{15x}+\frac{a^2\sqrt{c}\,\sqrt{-a^2x^2+1}}{15x^3}-\frac{\sqrt{c}\,\sqrt{-a^2x^2+1}}{5x^5} & \text{otherwise} \end{cases}\right)+2ac\left(\begin{cases} \frac{a^6\sqrt{c}\,\operatorname{acosh}\left(\frac{1}{ax}\right)}{16}-\frac{a^5\sqrt{c}}{16x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{a^3 \cdots}{48x^3\sqrt{\cdots}} \\ -\frac{ia^6\sqrt{c}\,\operatorname{asin}\left(\frac{1}{ax}\right)}{16}+\frac{ia^5\sqrt{c}}{16x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{ia^3 \cdots}{48x^3\sqrt{\cdots}} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(3/2)/x**8,x)
```

```
[Out] a**2*c*Piecewise((2*I*a**4*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x) + I*a**2*sqrt
(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - I*sqrt(c)*sqrt(a**2*x**2 - 1)/(5*x**5),
 Abs(a**2*x**2) > 1), (2*a**4*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x) + a**2*sq
rt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3) - sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*x**5
), True)) + 2*a*c*Piecewise((a**6*sqrt(c)*acosh(1/(a*x))/16 - a**5*sqrt(c)/
(16*x*sqrt(-1 + 1/(a**2*x**2))) + a**3*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x
**2))) + 5*a*sqrt(c)/(24*x**5*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(6*a*x**7
*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**6*sqrt(c)*asin(1/
(a*x))/16 + I*a**5*sqrt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - I*a**3*sqrt(c)/
(48*x**3*sqrt(1 - 1/(a**2*x**2))) - 5*I*a*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2
*x**2))) + I*sqrt(c)/(6*a*x**7*sqrt(1 - 1/(a**2*x**2))), True)) + c*Piecewi
se((8*a**7*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/105 + 4*a**5*sqrt(c)*sqrt(-1 +
1/(a**2*x**2))/(105*x**2) + a**3*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(35*x**4)
 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(7*x**6), 1/Abs(a**2*x**2) > 1), (8*I
*a**7*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/105 + 4*I*a**5*sqrt(c)*sqrt(1 - 1/(a*
*2*x**2))/(105*x**2) + I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(35*x**4) - I
*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(7*x**6), True))
```

3.1098 $\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=187

$$-\frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2}-\frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2}-\frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a}+\frac{3c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{64a^4}-\frac{(315ax+208)\left(c-a^2cx^2\right)^{5/2}}{2520a^4}$$

```
[Out] 1/32*c*x*(-a^2*c*x^2+c)^(3/2)/a^3-13/63*x^2*(-a^2*c*x^2+c)^(5/2)/a^2-1/4*x^
3*(-a^2*c*x^2+c)^(5/2)/a-1/9*x^4*(-a^2*c*x^2+c)^(5/2)-1/2520*(315*a*x+208)*
(-a^2*c*x^2+c)^(5/2)/a^4+3/64*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/
2))/a^4+3/64*c^2*x*(-a^2*c*x^2+c)^(1/2)/a^3
```

Rubi [A] time = 0.37, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1809, 833, 780, 195, 217, 203}

$$\frac{3c^2x\sqrt{c-a^2cx^2}}{64a^3}+\frac{3c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{64a^4}-\frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2}-\frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a}-\frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2}+\frac{cx\left(c-a^2c\right.}{32a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (3*c^2*x*Sqrt[c - a^2*c*x^2])/(64*a^3) + (c*x*(c - a^2*c*x^2)^(3/2))/(32*a^
3) - (13*x^2*(c - a^2*c*x^2)^(5/2))/(63*a^2) - (x^3*(c - a^2*c*x^2)^(5/2))/
(4*a) - (x^4*(c - a^2*c*x^2)^(5/2))/9 - ((208 + 315*a*x)*(c - a^2*c*x^2)^(5
/2))/(2520*a^4) + (3*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(64
*a^4)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^{5/2}\,dx = c\int x^3(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$= -\frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2} - \frac{\int x^3\left(-13a^2c-18a^3cx\right)\left(c-a^2cx^2\right)^{3/2}\,dx}{9a^2}$$
$$= -\frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2} + \frac{\int x^2\left(54a^3c^2+104a^4c^2x\right)\left(c-a^2c\right.}{72a^4c}$$
$$= -\frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2} - \frac{\int x\left(-208a^4\right.}{}$$
$$= -\frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4\left(c-a^2cx^2\right)^{5/2} - \frac{(208+315a}{2}$$
$$= \frac{cx\left(c-a^2cx^2\right)^{3/2}}{32a^3} - \frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5/2}}{4a} - \frac{1}{9}x^4\left(c-a^2cx^2\right.$$
$$= \frac{3c^2x\sqrt{c-a^2cx^2}}{64a^3} + \frac{cx\left(c-a^2cx^2\right)^{3/2}}{32a^3} - \frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5}}{4a}$$
$$= \frac{3c^2x\sqrt{c-a^2cx^2}}{64a^3} + \frac{cx\left(c-a^2cx^2\right)^{3/2}}{32a^3} - \frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5}}{4a}$$
$$= \frac{3c^2x\sqrt{c-a^2cx^2}}{64a^3} + \frac{cx\left(c-a^2cx^2\right)^{3/2}}{32a^3} - \frac{13x^2\left(c-a^2cx^2\right)^{5/2}}{63a^2} - \frac{x^3\left(c-a^2cx^2\right)^{5}}{4a}$$

Mathematica [A] time = 0.22, size = 131, normalized size = 0.70

$$-\frac{c^2\left(945\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\left(2240a^8x^8+5040a^7x^7-320a^6x^6-7560a^5x^5-4416a^4x^4+630a^3x^3+832a^2\right.}{20160a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/20160*(c^2*(Sqrt[c - a^2*c*x^2]*(1664 + 945*a*x + 832*a^2*x^2 + 630*a^3*
x^3 - 4416*a^4*x^4 - 7560*a^5*x^5 - 320*a^6*x^6 + 5040*a^7*x^7 + 2240*a^8*x
^8) + 945*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))
]))/a^4
```

fricas [A] time = 0.77, size = 307, normalized size = 1.64

$$\left[\frac{945\sqrt{-c}\,c^2\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\left(2240\,a^8c^2x^8+5040\,a^7c^2x^7-320\,a^6c^2x^6-7560\,a^5c^2x\right.}{40320\,a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/40320*(945*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(
-c)*x - c) - 2*(2240*a^8*c^2*x^8 + 5040*a^7*c^2*x^7 - 320*a^6*c^2*x^6 - 756
0*a^5*c^2*x^5 - 4416*a^4*c^2*x^4 + 630*a^3*c^2*x^3 + 832*a^2*c^2*x^2 + 945*
a*c^2*x + 1664*c^2)*sqrt(-a^2*c*x^2 + c))/a^4, -1/20160*(945*c^(5/2)*arctan
(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (2240*a^8*c^2*x^8 + 50
40*a^7*c^2*x^7 - 320*a^6*c^2*x^6 - 7560*a^5*c^2*x^5 - 4416*a^4*c^2*x^4 + 63
0*a^3*c^2*x^3 + 832*a^2*c^2*x^2 + 945*a*c^2*x + 1664*c^2)*sqrt(-a^2*c*x^2 +
 c))/a^4]
```

giac [A] time = 0.39, size = 155, normalized size = 0.83

$$\frac{1}{20160}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(4\left(552\,c^2+5\left(189\,ac^2+2\left(4\,a^2c^2-7\left(4\,a^4c^2x+9\,a^3c^2\right)x\right)x\right)x\right)x-\frac{315\,c^2}{a}\right)x-\frac{416\,c^2}{a^2}\right)x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] 1/20160*sqrt(-a^2*c*x^2 + c)*((2*((4*(552*c^2 + 5*(189*a*c^2 + 2*(4*a^2*c^2
 - 7*(4*a^4*c^2*x + 9*a^3*c^2)*x)*x)*x)*x - 315*c^2/a)*x - 416*c^2/a^2)*x -
 945*c^2/a^3)*x - 1664*c^2/a^4) - 3/64*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(a^3*sqrt(-c)*abs(a))
```

maple [B] time = 0.05, size = 330, normalized size = 1.76

$$\frac{x^2\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{9a^2c}+\frac{20\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{63c\,a^4}+\frac{x\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{4a^3c}-\frac{3x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{8a^3}-\frac{15cx\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{32a^3}-\frac{45c^2x\sqrt{-a^2c\,x^2}}{64a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/9*x^2*(-a^2*c*x^2+c)^(7/2)/a^2/c+20/63/c/a^4*(-a^2*c*x^2+c)^(7/2)+1/4/a^3
*x*(-a^2*c*x^2+c)^(7/2)/c-3/8/a^3*x*(-a^2*c*x^2+c)^(5/2)-15/32*c*x*(-a^2*c*
x^2+c)^(3/2)/a^3-45/64*c^2*x*(-a^2*c*x^2+c)^(1/2)/a^3-45/64/a^3*c^3/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/5/a^4*(-(x-1/a)^2*a^2
*c-2*a*c*(x-1/a))^(5/2)+1/2/a^3*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x+
3/4/a^3*c^2*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+3/4/a^3*c^3/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.79, size = 239, normalized size = 1.28

$$\frac{1}{20160}\left(\frac{2240\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}x^{2}}{a^{3}c}-\frac{7560\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}x}{a^{4}}+\frac{5040\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}x}{a^{4}c}+\frac{630\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}cx}{a^{4}}+\frac{15120\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] 1/20160*(2240*(-a^2*c*x^2 + c)^(7/2)*x^2/(a^3*c) - 7560*(-a^2*c*x^2 + c)^(5
/2)*x/a^4 + 5040*(-a^2*c*x^2 + c)^(7/2)*x/(a^4*c) + 630*(-a^2*c*x^2 + c)^(3
/2)*c*x/a^4 + 15120*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^2*x/a^4 - 14175*sqrt(
-a^2*c*x^2 + c)*c^2*x/a^4 - 14175*c^(5/2)*arcsin(a*x)/a^5 - 8064*(-a^2*c*x^
2 + c)^(5/2)/a^5 + 6400*(-a^2*c*x^2 + c)^(7/2)/(a^5*c) - 30240*sqrt(a^2*c*x
^2 - 4*a*c*x + 3*c)*c^2/a^5 + 15120*c^4*arcsin(a*x - 2)/(a^8*(-c/a^2)^(3/2)
))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\left(c-a^2\,c\,x^2\right)^{5/2}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^3*(c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [A] time = 84.27, size = 763, normalized size = 4.08

$$-a^4c^2\left(\begin{cases}\frac{x^8\sqrt{-a^2cx^2+c}}{9}-\frac{x^6\sqrt{-a^2cx^2+c}}{63a^2}-\frac{2x^4\sqrt{-a^2cx^2+c}}{105a^4}-\frac{8x^2\sqrt{-a^2cx^2+c}}{315a^6}-\frac{16\sqrt{-a^2cx^2+c}}{315a^8} & \text{for } a\neq 0\\ \frac{\sqrt{c}\,x^8}{8} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^9}{8\sqrt{a^2x^2-1}}\\ -\frac{a^2\sqrt{c}\,x^9}{8\sqrt{-a^2x^2+}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**2*Piecewise((x**8*sqrt(-a**2*c*x**2 + c)/9 - x**6*sqrt(-a**2*c*x**
2 + c)/(63*a**2) - 2*x**4*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*x**2*sqrt(-
a**2*c*x**2 + c)/(315*a**6) - 16*sqrt(-a**2*c*x**2 + c)/(315*a**8), Ne(a, 0
)), (sqrt(c)*x**8/8, True)) - 2*a**3*c**2*Piecewise((I*a**2*sqrt(c)*x**9/(8
*sqrt(a**2*x**2 - 1)) - 7*I*sqrt(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(
c)*x**5/(192*a**2*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a*
*2*x**2 - 1)) + 5*I*sqrt(c)*x/(128*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*
acosh(a*x)/(128*a**7), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a*
*2*x**2 + 1)) + 7*sqrt(c)*x**7/(48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(19
2*a**2*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1
)) - 5*sqrt(c)*x/(128*a**6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128
*a**7), True)) + 2*a*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2
- 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2
*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(
c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-
a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(
48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) +
sqrt(c)*asin(a*x)/(16*a**5), True)) + c**2*Piecewise((x**4*sqrt(-a**2*c*x**
2 + c)/5 - x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)
/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4/4, True))
```

3.1099 $\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=162

$$\frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2}-\frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a}+\frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2}-\frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2}+\frac{11c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{128a^3}-\frac{(385ax$$

[Out] 11/192*c*x*(-a^2*c*x^2+c)^(3/2)/a^2-2/7*x^2*(-a^2*c*x^2+c)^(5/2)/a-1/8*x^3*(-a^2*c*x^2+c)^(5/2)-1/1680*(385*a*x+192)*(-a^2*c*x^2+c)^(5/2)/a^3+11/128*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a^3+11/128*c^2*x*(-a^2*c*x^2+c)^(1/2)/a^2

Rubi [A] time = 0.33, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1809, 833, 780, 195, 217, 203}

$$\frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2}+\frac{11c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{128a^3}-\frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2}-\frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a}+\frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2}-\frac{(385ax$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(5/2),x]

[Out] (11*c^2*x*Sqrt[c - a^2*c*x^2])/(128*a^2) + (11*c*x*(c - a^2*c*x^2)^(3/2))/(192*a^2) - (2*x^2*(c - a^2*c*x^2)^(5/2))/(7*a) - (x^3*(c - a^2*c*x^2)^(5/2))/8 - ((192 + 385*a*x)*(c - a^2*c*x^2)^(5/2))/(1680*a^3) + (11*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(128*a^3)

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{5/2}\,dx = c\int x^2(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$= -\frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} - \frac{\int x^2\left(-11a^2c-16a^3cx\right)\left(c-a^2cx^2\right)^{3/2}\,dx}{8a^2}$$
$$= -\frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} + \frac{\int x\left(32a^3c^2+77a^4c^2x\right)\left(c-a^2c\right.}{56a^4c}$$
$$= -\frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} - \frac{(192+385ax)\left(c-a^2cx^2\right)^{5/2}}{1680a^3} + \frac{(}{}$$
$$= \frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2cx^2\right)^{5/2} - \frac{(192+385a}{16}$$
$$= \frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2} + \frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2c\right.$$
$$= \frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2} + \frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2c\right.$$
$$= \frac{11c^2x\sqrt{c-a^2cx^2}}{128a^2} + \frac{11cx\left(c-a^2cx^2\right)^{3/2}}{192a^2} - \frac{2x^2\left(c-a^2cx^2\right)^{5/2}}{7a} - \frac{1}{8}x^3\left(c-a^2c\right.$$

Mathematica [A] time = 0.17, size = 123, normalized size = 0.76

$$-\frac{c^2\left(1155\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\left(1680a^7x^7+3840a^6x^6-280a^5x^5-6144a^4x^4-3710a^3x^3+768a^2x^2+1155a\right.\right.}{13440a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/13440*(c^2*(Sqrt[c - a^2*c*x^2]*(1536 + 1155*a*x + 768*a^2*x^2 - 3710*a^
3*x^3 - 6144*a^4*x^4 - 280*a^5*x^5 + 3840*a^6*x^6 + 1680*a^7*x^7) + 1155*Sq
rt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))]))/a^3
```

fricas [A] time = 0.88, size = 285, normalized size = 1.76

$$\left[\frac{1155\sqrt{-c}\,c^2\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\left(1680\,a^7c^2x^7+3840\,a^6c^2x^6-280\,a^5c^2x^5-6144\,a^4c\right.}{26880\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] [1/26880*(1155*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) - 2*(1680*a^7*c^2*x^7 + 3840*a^6*c^2*x^6 - 280*a^5*c^2*x^5 - 61
44*a^4*c^2*x^4 - 3710*a^3*c^2*x^3 + 768*a^2*c^2*x^2 + 1155*a*c^2*x + 1536*c
^2)*sqrt(-a^2*c*x^2 + c))/a^3, -1/13440*(1155*c^(5/2)*arctan(sqrt(-a^2*c*x^
2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (1680*a^7*c^2*x^7 + 3840*a^6*c^2*x^6
- 280*a^5*c^2*x^5 - 6144*a^4*c^2*x^4 - 3710*a^3*c^2*x^3 + 768*a^2*c^2*x^2 +
 1155*a*c^2*x + 1536*c^2)*sqrt(-a^2*c*x^2 + c))/a^3]
```

giac [A] time = 0.28, size = 143, normalized size = 0.88

$$\frac{1}{13440}\sqrt{-a^2cx^2+c}\left(\left(2\left(\left(1855\,c^2+4\left(768\,ac^2+5\left(7\,a^2c^2-6\left(7\,a^4c^2x+16\,a^3c^2\right)x\right)x\right)x\right)x-\frac{384\,c^2}{a}\right)x-\frac{1155\,c^2}{a^2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] 1/13440*sqrt(-a^2*c*x^2 + c)*((2*((1855*c^2 + 4*(768*a*c^2 + 5*(7*a^2*c^2 -
 6*(7*a^4*c^2*x + 16*a^3*c^2)*x)*x)*x)*x - 384*c^2/a)*x - 1155*c^2/a^2)*x -
 1536*c^2/a^3) - 11/128*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c))
)/(a^2*sqrt(-c)*abs(a))
```

maple [B] time = 0.04, size = 306, normalized size = 1.89

$$\frac{x\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{8a^2c}-\frac{17x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{48a^2}-\frac{85cx\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{192a^2}-\frac{85c^2x\sqrt{-a^2c\,x^2+c}}{128a^2}-\frac{85c^3\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{128a^2\sqrt{a^2c}}+\frac{2\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/8*x*(-a^2*c*x^2+c)^(7/2)/a^2/c-17/48/a^2*x*(-a^2*c*x^2+c)^(5/2)-85/192*c*
x*(-a^2*c*x^2+c)^(3/2)/a^2-85/128*c^2*x*(-a^2*c*x^2+c)^(1/2)/a^2-85/128/a^2
*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/7/a^3*(-a
^2*c*x^2+c)^(7/2)/c-2/5/a^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(5/2)+1/2/a^2*
c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x+3/4/a^2*c^2*(-(x-1/a)^2*a^2*c-2*
a*c*(x-1/a))^(1/2)*x+3/4/a^2*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-
1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.67, size = 215, normalized size = 1.33

$$-\frac{1}{13440}\,a\left(\frac{4760\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}{a^3}-\frac{1680\left(-a^2cx^2+c\right)^{\frac{7}{2}}x}{a^3c}-\frac{770\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx}{a^3}-\frac{10080\sqrt{a^2cx^2-4\,acx+3\,c}\,c^2}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] -1/13440*a*(4760*(-a^2*c*x^2 + c)^(5/2)*x/a^3 - 1680*(-a^2*c*x^2 + c)^(7/2)
*x/(a^3*c) - 770*(-a^2*c*x^2 + c)^(3/2)*c*x/a^3 - 10080*sqrt(a^2*c*x^2 - 4*
a*c*x + 3*c)*c^2*x/a^3 + 8925*sqrt(-a^2*c*x^2 + c)*c^2*x/a^3 + 8925*c^(5/2)
*arcsin(a*x)/a^4 + 5376*(-a^2*c*x^2 + c)^(5/2)/a^4 - 3840*(-a^2*c*x^2 + c)^
(7/2)/(a^4*c) + 20160*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^2/a^4 - 10080*c^4*a
rcsin(a*x - 2)/(a^7*(-c/a^2)^(3/2)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2\left(c-a^2\,c\,x^2\right)^{5/2}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^2*(c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 21.28, size = 687, normalized size = 4.24

$$-a^4c^2\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^9}{8\sqrt{a^2x^2-1}}-\frac{7i\sqrt{c}\,x^7}{48\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,x^5}{192a^2\sqrt{a^2x^2-1}}-\frac{5i\sqrt{c}\,x^3}{384a^4\sqrt{a^2x^2-1}}+\frac{5i\sqrt{c}\,x}{128a^6\sqrt{a^2x^2-1}}-\frac{5i\sqrt{c}\,\operatorname{acosh}(ax)}{128a^7} & \text{for } \left|a^2x^2\right|>1\\ -\frac{a^2\sqrt{c}\,x^9}{8\sqrt{-a^2x^2+1}}+\frac{7\sqrt{c}\,x^7}{48\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,x^5}{192a^2\sqrt{-a^2x^2+1}}+\frac{5\sqrt{c}\,x^3}{384a^4\sqrt{-a^2x^2+1}}-\frac{5\sqrt{c}\,x}{128a^6\sqrt{-a^2x^2+1}}+\frac{5\sqrt{c}\,\operatorname{asin}(ax)}{128a^7} & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqr
t(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**
2 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(
128*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2
*x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/
```

```
(48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) +
5*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt
(-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128*a**7), True)) - 2*a**3*c**2*Pi
ecewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a*
*2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(
105*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) + 2*a*c**2*Piecewise((x**4*sq
rt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a*
*2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4/4, True)) + c**2*Piecewi
se((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c)*x**3/(8*sqrt(
a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acos
h(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4*sqrt(-a**2*x**
2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(8*a**2*sqrt(
-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True))
```

3.1100 $\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^2}+\frac{c^2x\sqrt{c-a^2cx^2}}{8a}+\frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{(35ax+27)\left(c-a^2cx^2\right)^{5/2}}{105a^2}$$

```
[Out] 1/12*c*x*(-a^2*c*x^2+c)^(3/2)/a-1/7*x^2*(-a^2*c*x^2+c)^(5/2)-1/105*(35*a*x+
27)*(-a^2*c*x^2+c)^(5/2)/a^2+1/8*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^
(1/2))/a^2+1/8*c^2*x*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] time = 0.22, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6151, 1809, 780, 195, 217, 203}

$$\frac{c^2x\sqrt{c-a^2cx^2}}{8a}+\frac{c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^2}+\frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{(35ax+27)\left(c-a^2cx^2\right)^{5/2}}{105a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*x*Sqrt[c - a^2*c*x^2])/(8*a) + (c*x*(c - a^2*c*x^2)^(3/2))/(12*a) - (x
^2*(c - a^2*c*x^2)^(5/2))/7 - ((27 + 35*a*x)*(c - a^2*c*x^2)^(5/2))/(105*a^
2) + (c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(8*a^2)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^{5/2}\,dx=c\int x(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx$$

$$=-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{\int x\left(-9a^2c-14a^3cx\right)\left(c-a^2cx^2\right)^{3/2}\,dx}{7a^2}$$

$$=-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{(27+35ax)\left(c-a^2cx^2\right)^{5/2}}{105a^2}+\frac{c\int\left(c-a^2cx^2\right)^{3/2}\,dx}{3a}$$

$$=\frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{(27+35ax)\left(c-a^2cx^2\right)^{5/2}}{105a^2}+\frac{c^2\int\sqrt{c}$$

$$=\frac{c^2x\sqrt{c-a^2cx^2}}{8a}+\frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{(27+35ax)\left(c-a\right.}{105a^2}$$

$$=\frac{c^2x\sqrt{c-a^2cx^2}}{8a}+\frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{(27+35ax)\left(c-a\right.}{105a^2}$$

$$=\frac{c^2x\sqrt{c-a^2cx^2}}{8a}+\frac{cx\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{1}{7}x^2\left(c-a^2cx^2\right)^{5/2}-\frac{(27+35ax)\left(c-a\right.}{105a^2}$$

Mathematica [A] time = 0.15, size = 115, normalized size = 0.84

$$-\frac{c^2\left(105\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\left(120a^6x^6+280a^5x^5-24a^4x^4-490a^3x^3-312a^2x^2+105ax+216\right)\sqrt{c-a^2cx}\right.}{840a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/840*(c^2*(Sqrt[c - a^2*c*x^2]*(216 + 105*a*x - 312*a^2*x^2 - 490*a^3*x^3
 - 24*a^4*x^4 + 280*a^5*x^5 + 120*a^6*x^6) + 105*Sqrt[c]*ArcTan[(a*x*Sqrt[c
 - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))]))/a^2
```

fricas [A] time = 0.67, size = 263, normalized size = 1.92

$$\left[\frac{105\sqrt{-c}\,c^2\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\left(120\,a^6c^2x^6+280\,a^5c^2x^5-24\,a^4c^2x^4-490\,a^3c^2x^3-\right.}{1680\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/1680*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 2*(120*a^6*c^2*x^6 + 280*a^5*c^2*x^5 - 24*a^4*c^2*x^4 - 490*a^3
*c^2*x^3 - 312*a^2*c^2*x^2 + 105*a*c^2*x + 216*c^2)*sqrt(-a^2*c*x^2 + c))/a
^2, -1/840*(105*c^(5/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
- c)) + (120*a^6*c^2*x^6 + 280*a^5*c^2*x^5 - 24*a^4*c^2*x^4 - 490*a^3*c^2*x
^3 - 312*a^2*c^2*x^2 + 105*a*c^2*x + 216*c^2)*sqrt(-a^2*c*x^2 + c))/a^2]
```

giac [A] time = 0.89, size = 131, normalized size = 0.96

$$\frac{1}{840}\sqrt{-a^2cx^2+c}\left(\left(2\left(156\,c^2+\left(245\,ac^2+4\left(3\,a^2c^2-5\left(3\,a^4c^2x+7\,a^3c^2\right)x\right)x\right)x\right)x-\frac{105\,c^2}{a}\right)x-\frac{216\,c^2}{a^2}\right)-\frac{c^3\log\left(\left|\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

```
[Out] 1/840*sqrt(-a^2*c*x^2 + c)*((2*(156*c^2 + (245*a*c^2 + 4*(3*a^2*c^2 - 5*(3*
a^4*c^2*x + 7*a^3*c^2)*x)*x)*x)*x - 105*c^2/a)*x - 216*c^2/a^2) - 1/8*c^3*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

maple [B] time = 0.04, size = 284, normalized size = 2.07

$$\frac{\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{7a^2c}-\frac{x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{3a}-\frac{5cx\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{12a}-\frac{5c^2x\sqrt{-a^2c\,x^2+c}}{8a}-\frac{5c^3\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{8a\sqrt{a^2c}}-\frac{2\left(-\left(x-\frac{1}{a}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/7*(-a^2*c*x^2+c)^(7/2)/a^2/c-1/3*x/a*(-a^2*c*x^2+c)^(5/2)-5/12*c*x*(-a^2*
c*x^2+c)^(3/2)/a-5/8*c^2*x*(-a^2*c*x^2+c)^(1/2)/a-5/8/a*c^3/(a^2*c)^(1/2)*a
rctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/5/a^2*(-(x-1/a)^2*a^2*c-2*a*c
*(x-1/a))^(5/2)+1/2/a*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x+3/4/a*c^2*
(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+3/4/a*c^3/(a^2*c)^(1/2)*arctan((a^
2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.65, size = 193, normalized size = 1.41

$$-\frac{1}{840}\left(\frac{280\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}{a^2}-\frac{70\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx}{a^2}-\frac{630\sqrt{a^2cx^2-4\,acx+3\,c}\,c^2x}{a^2}+\frac{525\sqrt{-a^2cx^2+c}\,c^2x}{a^2}+\frac{525\,c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/840*(280*(-a^2*c*x^2 + c)^(5/2)*x/a^2 - 70*(-a^2*c*x^2 + c)^(3/2)*c*x/a^
2 - 630*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^2*x/a^2 + 525*sqrt(-a^2*c*x^2 + c
)*c^2*x/a^2 + 525*c^(5/2)*arcsin(a*x)/a^3 + 336*(-a^2*c*x^2 + c)^(5/2)/a^3
- 120*(-a^2*c*x^2 + c)^(7/2)/(a^3*c) + 1260*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)
*c^2/a^3 - 630*c^4*arcsin(a*x - 2)/(a^6*(-c/a^2)^(3/2)))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\left(c-a^2\,c\,x^2\right)^{5/2}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x*(c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [A] time = 70.64, size = 586, normalized size = 4.28

$$-a^4c^2\left(\begin{cases} \frac{x^6\sqrt{-a^2cx^2+c}}{7} - \frac{x^4\sqrt{-a^2cx^2+c}}{35a^2} - \frac{4x^2\sqrt{-a^2cx^2+c}}{105a^4} - \frac{8\sqrt{-a^2cx^2+c}}{105a^6} & \text{for } a \neq 0 \\ \frac{\sqrt{c}\,x^6}{6} & \text{otherwise} \end{cases}\right) - 2a^3c^2\left(\begin{cases} \frac{ia^2\sqrt{c}\,x^7}{6\sqrt{a^2x^2-1}} - \frac{5i\sqrt{c}\,x^5}{24\sqrt{a^2x^2-1}} - \cdots \\ -\frac{a^2\sqrt{c}\,x^7}{6\sqrt{-a^2x^2+1}} + \frac{5\sqrt{c}\,x^5}{24\sqrt{-a^2x^2+1}} \cdots \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**2*Piecewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**
2 + c)/(35*a**2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*
c*x**2 + c)/(105*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) - 2*a**3*c**2*Pi
ecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24
*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sq
rt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs
(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*
x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)
) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5),
 True)) + 2*a*c**2*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) -
 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x
```

```
**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt
(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1))
 - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), Tr
ue)) + c**2*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**
2*c*x**2 + c)**(3/2)/(3*a**2*c), True))
```

3.1101 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=130

$$\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}+\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

```
[Out] 7/24*c*x*(-a^2*c*x^2+c)^(3/2)-7/30*(-a^2*c*x^2+c)^(5/2)/a-1/6*(a*x+1)*(-a^2
*c*x^2+c)^(5/2)/a+7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a+7
/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6141, 671, 641, 195, 217, 203}

$$\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (7*c^2*x*Sqrt[c - a^2*c*x^2])/16 + (7*c*x*(c - a^2*c*x^2)^(3/2))/24 - (7*(c
 - a^2*c*x^2)^(5/2))/(30*a) - ((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) + (7*
c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx &= c\int (1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{1}{6}(7c)\int (1+ax)\left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{1}{6}(7c)\int \left(c-a^2cx^2\right)^{3/2}\,dx\\
&= \frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{1}{8}\left(7c^2\right)\int \sqrt{c
\\
&= \frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right.}{6a}\\
&= \frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right.}{6a}\\
&= \frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1+ax)\left(c-a^2cx^2\right.}{6a}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 135, normalized size = 1.04

$$\frac{c^2\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(40a^6x^6+56a^5x^5-106a^4x^4-182a^3x^3+57a^2x^2+231ax-96\right)-210\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-96 + 231*a*x + 57*a^2*x^2 - 182*a
^3*x^3 - 106*a^4*x^4 + 56*a^5*x^5 + 40*a^6*x^6) - 210*Sqrt[1 - a*x]*ArcSin[
Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.70, size = 241, normalized size = 1.85

$$\left[\frac{105\sqrt{-c}\,c^2\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\left(40\,a^5c^2x^5+96\,a^4c^2x^4-10\,a^3c^2x^3-192\,a^2c^2x^2-135\,ac^2x+96\,c^2\right)\sqrt{-a^2cx^2+c}}{480\,a},\ \ldots\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) - 2*(40*a^5*c^2*x^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^
2*x^2 - 135*a*c^2*x + 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, -1/240*(105*c^(5/2)*
arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5
+ 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^2*x^2 - 135*a*c^2*x + 96*c^2)
*sqrt(-a^2*c*x^2 + c))/a]
```

giac [A] time = 0.25, size = 116, normalized size = 0.89

$$-\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{16\,\sqrt{-c}\,|a|}+\frac{1}{240}\,\sqrt{-a^2cx^2+c}\left(\left(135\,c^2+2\left(96\,ac^2+\left(5\,a^2c^2-4\left(5\,a^4c^2x+12\,a^3c^2\right)x\right)x\right)x\right)x-\frac{96\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a)
) + 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 + 2*(96*a*c^2 + (5*a^2*c^2 - 4*(5*
a^4*c^2*x + 12*a^3*c^2)*x)*x)*x)*x - 96*c^2/a)
```

maple [B] time = 0.04, size = 242, normalized size = 1.86

$$-\frac{x\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}{6}-\frac{5cx\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{24}-\frac{5c^{2}x\sqrt{-a^{2}c\,x^{2}+c}}{16}-\frac{5c^{3}\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{16\sqrt{a^{2}c}}-\frac{2\left(-\left(x-\frac{1}{a}\right)^{2}a^{2}c-2ac\left(x-\right.\right.}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/6*x*(-a^2*c*x^2+c)^(5/2)-5/24*c*x*(-a^2*c*x^2+c)^(3/2)-5/16*c^2*x*(-a^2*
c*x^2+c)^(1/2)-5/16*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)
^(1/2))-2/5/a*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(5/2)+1/2*c*(-(x-1/a)^2*a^2*
c-2*a*c*(x-1/a))^(3/2)*x+3/4*c^2*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+3
/4*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a)
)^(1/2))
```

maxima [A] time = 0.55, size = 172, normalized size = 1.32

$$-\frac{1}{240}\left(\frac{40\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}x}{a}-\frac{70\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}cx}{a}-\frac{180\sqrt{a^{2}cx^{2}-4\,acx+3\,c}\,c^{2}x}{a}+\frac{75\sqrt{-a^{2}cx^{2}+c}\,c^{2}x}{a}+\frac{75\,c^{\frac{5}{2}}\,\mathrm{ar}}{\,}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/240*(40*(-a^2*c*x^2 + c)^(5/2)*x/a - 70*(-a^2*c*x^2 + c)^(3/2)*c*x/a - 1
80*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^2*x/a + 75*sqrt(-a^2*c*x^2 + c)*c^2*x/
a + 75*c^(5/2)*arcsin(a*x)/a^2 + 96*(-a^2*c*x^2 + c)^(5/2)/a^2 + 360*sqrt(a
^2*c*x^2 - 4*a*c*x + 3*c)*c^2/a^2 - 180*c^4*arcsin(a*x - 2)/(a^5*(-c/a^2)^(
3/2)))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(c-a^{2}\,c\,x^{2}\right)^{5/2}\,\left(a\,x+1\right)^{2}}{a^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 15.69, size = 478, normalized size = 3.68

$$-a^4c^2\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^7}{6\sqrt{a^2x^2-1}}-\frac{5i\sqrt{c}\,x^5}{24\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,x^3}{48a^2\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{16a^4\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{16a^5} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^7}{6\sqrt{-a^2x^2+1}}+\frac{5\sqrt{c}\,x^5}{24\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,x^3}{48a^2\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{16a^4\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{16a^5} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2c}}{5} \\ \frac{\sqrt{c}\,x^4}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqr
t(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
 - 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(
16*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
 + 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a
**2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*
x)/(16*a**5), True)) - 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
 - x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**
4), Ne(a, 0)), (sqrt(c)*x**4/4, True)) + 2*a*c**2*Piecewise((0, Eq(c, 0)),
(sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True
)) + c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c
)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) >
1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

3.1102 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=136

$$\frac{3}{4}c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{4}c^2(3ax+4)\sqrt{c-a^2cx^2}+\frac{1}{6}c(3ax+2)\left(c-a^2cx^2\right)^{3/2}-\frac{1}{5}\left(c-\right.$$

```
[Out] 1/6*c*(3*a*x+2)*(-a^2*c*x^2+c)^(3/2)-1/5*(-a^2*c*x^2+c)^(5/2)+3/4*c^(5/2)*a
rctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))-c^(5/2)*arctanh((-a^2*c*x^2+c)^(1/2
)/c^(1/2))+1/4*c^2*(3*a*x+4)*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1809, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{4}c^2(3ax+4)\sqrt{c-a^2cx^2}+\frac{3}{4}c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{6}c(3ax+2)\left(c-a^2cx^2\right)^{3/2}-\frac{1}{5}\left(c-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x,x]
```

```
[Out] (c^2*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/4 + (c*(2 + 3*a*x)*(c - a^2*c*x^2)^(3
/2))/6 - (c - a^2*c*x^2)^(5/2)/5 + (3*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/4 - c^(5/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
```

```
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x}\,dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x}\,dx$$

$$= -\frac{1}{5}\left(c-a^2cx^2\right)^{5/2} - \frac{\int \frac{\left(-5a^2c-10a^3cx\right)\left(c-a^2cx^2\right)^{3/2}}{x}\,dx}{5a^2}$$

$$= \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + \frac{\int \frac{\left(20a^4c^3+30a^5c^3x\right)\sqrt{c-a^2cx^2}}{x}\,dx}{20a^4c}$$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} - \int \frac{-4}{}$$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + c^3\int$$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + \frac{1}{2}c^3$$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + \frac{3}{4}c^{5/}$$

$$= \frac{1}{4}c^2(4+3ax)\sqrt{c-a^2cx^2} + \frac{1}{6}c(2+3ax)\left(c-a^2cx^2\right)^{3/2} - \frac{1}{5}\left(c-a^2cx^2\right)^{5/2} + \frac{3}{4}c^{5/}$$

Mathematica [A] time = 0.15, size = 136, normalized size = 1.00

$$-c^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-\frac{3}{4}c^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-\frac{1}{60}c^2\left(12a^4x^4+30a^3x^3-4a^2x^2-75ax-68\right)\sqrt{c-a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x,x]
```

```
[Out] -1/60*(c^2*Sqrt[c - a^2*c*x^2]*(-68 - 75*a*x - 4*a^2*x^2 + 30*a^3*x^3 + 12*
a^4*x^4)) - (3*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*
x^2))])/4 + c^(5/2)*Log[x] - c^(5/2)*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

fricas [A] time = 0.54, size = 295, normalized size = 2.17

$$\left[-\frac{3}{4}\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+\frac{1}{2}\,c^{\frac{5}{2}}\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}\,-2\,c}{x^2}\right)-\frac{1}{60}\left(12\,a^4c^2x^4+30\,a^3c^2x^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x,x, algorithm="frica
s")
```

```
[Out] [-3/4*c^(5/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/
2*c^(5/2)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) - 1/
60*(12*a^4*c^2*x^4 + 30*a^3*c^2*x^3 - 4*a^2*c^2*x^2 - 75*a*c^2*x - 68*c^2)*
sqrt(-a^2*c*x^2 + c), -sqrt(-c)*c^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a
^2*c*x^2 - c)) + 3/8*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*
a*sqrt(-c)*x - c) - 1/60*(12*a^4*c^2*x^4 + 30*a^3*c^2*x^3 - 4*a^2*c^2*x^2 -
 75*a*c^2*x - 68*c^2)*sqrt(-a^2*c*x^2 + c)]
```

giac [A] time = 0.70, size = 150, normalized size = 1.10

$$\frac{2\,c^3\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{3\,a\sqrt{-c}\,c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{4\,|a|}+\frac{1}{60}\,\sqrt{-a^2cx^2+c}\left(68\,c^2+\left(75\,ac^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x,x, algorithm="giac"
)
```

```
[Out] 2*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) +
3/4*a*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a)
+ 1/60*sqrt(-a^2*c*x^2 + c)*(68*c^2 + (75*a*c^2 + 2*(2*a^2*c^2 - 3*(2*a^4*c
^2*x + 5*a^3*c^2)*x)*x)*x)
```

maple [B] time = 0.04, size = 235, normalized size = 1.73

$$\frac{\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{5}+\frac{c\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{3}-c^{\frac{5}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)+\sqrt{-a^2c\,x^2+c}\,c^2-\frac{2\left(-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x,x)
```

```
[Out] 1/5*(-a^2*c*x^2+c)^(5/2)+1/3*c*(-a^2*c*x^2+c)^(3/2)-c^(5/2)*ln((2*c+2*c^(1/
2)*(-a^2*c*x^2+c)^(1/2))/x)+(-a^2*c*x^2+c)^(1/2)*c^2-2/5*(-(x-1/a)^2*a^2*c-
2*a*c*(x-1/a))^(5/2)+1/2*a*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x+3/4*a
*c^2*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+3/4*a*c^3/(a^2*c)^(1/2)*arcta
n((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x,x, algorithm="maxim
a")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}\,\left(a\,x+1\right)^2}{x\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x*(a^2*x^2 - 1)), x)
```

sympy [C] time = 33.54, size = 508, normalized size = 3.74

$$-a^4c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2cx^2+c}}{5}-\frac{x^2\sqrt{-a^2cx^2+c}}{15a^2}-\frac{2\sqrt{-a^2cx^2+c}}{15a^4} & \text{for } a\neq 0\\ \frac{\sqrt{c}\,x^4}{4} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}\,x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\mathrm{ac}}{8\ldots} \\ -\frac{a^2\sqrt{c}\,x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{8a^2\sqrt{-a^2x^2+1}}+\ldots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x,x)
```

```
[Out] -a**4*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**
2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*
x**4/4, True)) - 2*a**3*c**2*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x*
*2 - 1)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*s
```

```
qrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (
-a**2*sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*
x**2 + 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8
*a**3), True)) + 2*a*c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2
 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), A
bs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(
2*a), True)) + c**2*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(
a*x) + sqrt(c)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) >
 1), (sqrt(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log
(sqrt(-a**2*x**2 + 1) + 1), True))
```

3.1103 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=141

$$-\frac{9}{8}ac^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2ac^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2}+\frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/}$$

```
[Out] 1/12*a*c*(-9*a*x+8)*(-a^2*c*x^2+c)^(3/2)-(-a^2*c*x^2+c)^(5/2)/x-9/8*a*c^(5/
2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))-2*a*c^(5/2)*arctanh((-a^2*c*x^2
+c)^(1/2)/c^(1/2))+1/8*a*c^2*(-9*a*x+16)*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 815, 844, 217, 203, 266, 63, 208}

$$\frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2}-\frac{9}{8}ac^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2ac^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2}-$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^2,x]
```

```
[Out] (a*c^2*(16 - 9*a*x)*Sqrt[c - a^2*c*x^2])/8 + (a*c*(8 - 9*a*x)*(c - a^2*c*x^
2)^(3/2))/12 - (c - a^2*c*x^2)^(5/2)/x - (9*a*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/
Sqrt[c - a^2*c*x^2]])/8 - 2*a*c^(5/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^2}\,dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x^2}\,dx$$
$$= -\frac{\left(c-a^2cx^2\right)^{5/2}}{x} - \int \frac{\left(-2ac+3a^2cx\right)\left(c-a^2cx^2\right)^{3/2}}{x}\,dx$$
$$= \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} + \frac{\int \frac{\left(8a^3c^3-9a^4c^3x\right)\sqrt{c-a^2cx^2}}{x}\,dx}{4a^2c}$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} - \int$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} + (2$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} + (a$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} - \frac{9}{8}$$
$$= \frac{1}{8}ac^2(16-9ax)\sqrt{c-a^2cx^2} + \frac{1}{12}ac(8-9ax)\left(c-a^2cx^2\right)^{3/2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{x} - \frac{9}{8}$$

Mathematica [A] time = 0.24, size = 143, normalized size = 1.01

$$-2ac^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{9}{8}ac^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-\frac{c^2\left(6a^4x^4+16a^3x^3-3a^2x^2-64ax+24\right)\sqrt{c-a}}{24x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^2,x]
```

```
[Out] -1/24*(c^2*Sqrt[c - a^2*c*x^2]*(24 - 64*a*x - 3*a^2*x^2 + 16*a^3*x^3 + 6*a^
4*x^4))/x + (9*a*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^
2*x^2))])/8 + 2*a*c^(5/2)*Log[x] - 2*a*c^(5/2)*Log[c + Sqrt[c]*Sqrt[c - a^2
*c*x^2]]
```

fricas [A] time = 0.75, size = 313, normalized size = 2.22

$$\left[\frac{27\,ac^{\frac{5}{2}}x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+24\,ac^{\frac{5}{2}}x\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-\left(6\,a^4c^2x^4+16\,a^3c^2x^3-3\,a^2c^2x^2-6\right.}{24\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] [1/24*(27*a*c^(5/2)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
c)) + 24*a*c^(5/2)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c
)/x^2) - (6*a^4*c^2*x^4 + 16*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 64*a*c^2*x + 24*
c^2)*sqrt(-a^2*c*x^2 + c))/x, -1/48*(96*a*sqrt(-c)*c^2*x*arctan(sqrt(-a^2*c
*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - 27*a*sqrt(-c)*c^2*x*log(2*a^2*c*x^2 -
 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) + 2*(6*a^4*c^2*x^4 + 16*a^3*c^2*x
^3 - 3*a^2*c^2*x^2 - 64*a*c^2*x + 24*c^2)*sqrt(-a^2*c*x^2 + c))/x]
```

giac [A] time = 0.98, size = 195, normalized size = 1.38

$$\frac{4\,ac^3\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{9\,a^2\sqrt{-c}\,c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,|a|}+\frac{2\,a^2\sqrt{-c}\,c^3}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^2,x, algorithm="gia
c")
```

```
[Out] 4*a*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 9/8*a^2*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs
(a) + 2*a^2*sqrt(-c)*c^3/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*a
bs(a)) + 1/24*sqrt(-a^2*c*x^2 + c)*(64*a*c^2 + (3*a^2*c^2 - 2*(3*a^4*c^2*x
+ 8*a^3*c^2)*x)*x)
```

maple [B] time = 0.04, size = 367, normalized size = 2.60

$$\frac{2a\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}{5}+\frac{2ac\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{3}-2a\,c^{\frac{5}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^{2}c\,x^{2}+c}}{x}\right)+2a\sqrt{-a^{2}c\,x^{2}+c}\,c^{2}-\frac{\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}{cx}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^2,x)
```

```
[Out] 2/5*a*(-a^2*c*x^2+c)^(5/2)+2/3*a*c*(-a^2*c*x^2+c)^(3/2)-2*a*c^(5/2)*ln((2*c
+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2*a*(-a^2*c*x^2+c)^(1/2)*c^2-1/c/x*(-a^
2*c*x^2+c)^(7/2)-a^2*x*(-a^2*c*x^2+c)^(5/2)-5/4*a^2*c*x*(-a^2*c*x^2+c)^(3/2
)-15/8*a^2*c^2*x*(-a^2*c*x^2+c)^(1/2)-15/8*a^2*c^3/(a^2*c)^(1/2)*arctan((a^
2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/5*a*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(
5/2)+1/2*a^2*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x+3/4*a^2*c^2*(-(x-1/
a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+3/4*a^2*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^
(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}(ax+1)^{2}}{\left(a^{2}x^{2}-1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^2,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{{\left(c-a^{2}\,c\,x^{2}\right)}^{5/2}\,{\left(a\,x+1\right)}^{2}}{x^{2}\,\left(a^{2}\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x^2*(a^2*x^2 - 1)), x)
```

sympy [C] time = 69.47, size = 483, normalized size = 3.43

$$-a^4c^2\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}\,x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x**2,x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqr
t(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)
) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5
/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(
c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) - 2
*a**3*c**2*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2
*c*x**2 + c)**(3/2)/(3*a**2*c), True)) + 2*a*c**2*Piecewise((I*sqrt(c)*sqrt
(a**2*x**2 - 1) - sqrt(c)*log(a*x) + sqrt(c)*log(a**2*x**2)/2 + I*sqrt(c)*a
sin(1/(a*x)), Abs(a**2*x**2) > 1), (sqrt(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*
log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a**2*x**2 + 1) + 1), True)) + c**2*Pie
cewise((-I*a**2*sqrt(c)*x/sqrt(a**2*x**2 - 1) + I*a*sqrt(c)*acosh(a*x) + I*
sqrt(c)/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**2*sqrt(c)*x/sqrt(
-a**2*x**2 + 1) - a*sqrt(c)*asin(a*x) - sqrt(c)/(x*sqrt(-a**2*x**2 + 1)), T
rue))
```

3.1104 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=151

$$-3a^2c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\frac{1}{2}a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{1}{2}a^2c^2(6ax+1)\sqrt{c-a^2cx^2}-\frac{ac(ax+12)\left(c-a^2cx^2\right)^{3/2}}{6x}-\frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2}$$

```
[Out] -1/6*a*c*(a*x+12)*(-a^2*c*x^2+c)^(3/2)/x-1/2*(-a^2*c*x^2+c)^(5/2)/x^2-3*a^2
*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))+1/2*a^2*c^(5/2)*arctanh((
-a^2*c*x^2+c)^(1/2)/c^(1/2))-1/2*a^2*c^2*(6*a*x+1)*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6151, 1807, 813, 815, 844, 217, 203, 266, 63, 208}

$$-\frac{1}{2}a^2c^2(6ax+1)\sqrt{c-a^2cx^2}-3a^2c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\frac{1}{2}a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(ax+12)\left(c-a^2cx^2\right)^{3/2}}{6x}-\frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^3,x]
```

```
[Out] -(a^2*c^2*(1 + 6*a*x)*Sqrt[c - a^2*c*x^2])/2 - (a*c*(12 + a*x)*(c - a^2*c*x
^2)^(3/2))/(6*x) - (c - a^2*c*x^2)^(5/2)/(2*x^2) - 3*a^2*c^(5/2)*ArcTan[(a*
Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] + (a^2*c^(5/2)*ArcTanh[Sqrt[c - a^2*c*x^2]/
Sqrt[c]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^3}\,dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x^3}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - \frac{1}{2}\int \frac{\left(-4ac+a^2cx\right)\left(c-a^2cx^2\right)^{3/2}}{x^2}\,dx$$

$$= -\frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} + \frac{1}{4}\int \frac{\left(-2a^2c^2-24a^3c^2x\right)\sqrt{c-a}}{x}$$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - \int$$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - \frac{1}{2}($$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - \frac{1}{4}($$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - 3a^2$$

$$= -\frac{1}{2}a^2c^2(1+6ax)\sqrt{c-a^2cx^2} - \frac{ac(12+ax)\left(c-a^2cx^2\right)^{3/2}}{6x} - \frac{\left(c-a^2cx^2\right)^{5/2}}{2x^2} - 3a^2$$

Mathematica [A] time = 0.27, size = 151, normalized size = 1.00

$$\frac{1}{2}a^2c^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+3a^2c^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-\frac{1}{2}a^2c^{5/2}\log(x)-\frac{c^2\left(2a^4x^4+6a^3x^3-2a^2x^2+1\right.}{6x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^3,x]
```

```
[Out] -1/6*(c^2*Sqrt[c - a^2*c*x^2]*(3 + 12*a*x - 2*a^2*x^2 + 6*a^3*x^3 + 2*a^4*x
^4))/x^2 + 3*a^2*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^
2*x^2))] - (a^2*c^(5/2)*Log[x])/2 + (a^2*c^(5/2)*Log[c + Sqrt[c]*Sqrt[c - a
^2*c*x^2]])/2
```

fricas [A] time = 0.68, size = 329, normalized size = 2.18

$$\left[\frac{36\,a^2c^{\frac{5}{2}}x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+3\,a^2c^{\frac{5}{2}}x^2\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)-2\left(2\,a^4c^2x^4+6\,a^3c^2x^3-2\,a^2c^2x^2\right.}{12\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/12*(36*a^2*c^(5/2)*x^2*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^
2 - c)) + 3*a^2*c^(5/2)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c
) - 2*c)/x^2) - 2*(2*a^4*c^2*x^4 + 6*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + 12*a*c^2
*x + 3*c^2)*sqrt(-a^2*c*x^2 + c))/x^2, 1/6*(3*a^2*sqrt(-c)*c^2*x^2*arctan(s
qrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 9*a^2*sqrt(-c)*c^2*x^2*log(
2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - (2*a^4*c^2*x^4 + 6
*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + 12*a*c^2*x + 3*c^2)*sqrt(-a^2*c*x^2 + c))/x^
2]
```

giac [B] time = 0.99, size = 302, normalized size = 2.00

$$-\frac{a^2c^3\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{3\,a^3\sqrt{-c}\,c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\frac{1}{3}\sqrt{-a^2cx^2+c}\left(a^2c^2-\left(a^4c^2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^3,x, algorithm="gia
c")
```

```
[Out] -a^2*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 - 3*a^3*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(
a) + 1/3*sqrt(-a^2*c*x^2 + c)*(a^2*c^2 - (a^4*c^2*x + 3*a^3*c^2)*x) - ((sqr
t(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c^3*abs(a) - 4*(sqrt(-a^2*c)*x -
sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(-c)*c^3 + (sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))*a^2*c^4*abs(a) + 4*a^3*sqrt(-c)*c^4)/(((sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^2 - c)^2*abs(a))
```

maple [B] time = 0.05, size = 399, normalized size = 2.64

$$-\frac{a^{2}\left(-a^{2} c\,x^{2}+c\right)^{\frac{5}{2}}}{10}-\frac{a^{2} c\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{6}+\frac{a^{2} c^{\frac{5}{2}} \ln\left(\frac{2 c+2 \sqrt{c}\,\sqrt{-a^{2} c\,x^{2}+c}}{x}\right)}{2}-\frac{a^{2} \sqrt{-a^{2} c\,x^{2}+c}\;c^{2}}{2}-\frac{2 a\left(-a^{2} c\,x^{2}+c\right)^{\frac{7}{2}}}{c x}-2 a^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^3,x)
```

```
[Out] -1/10*a^2*(-a^2*c*x^2+c)^(5/2)-1/6*a^2*c*(-a^2*c*x^2+c)^(3/2)+1/2*a^2*c^(5/
2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-1/2*a^2*(-a^2*c*x^2+c)^(1/2)*
c^2-2*a/c/x*(-a^2*c*x^2+c)^(7/2)-2*a^3*x*(-a^2*c*x^2+c)^(5/2)-5/2*a^3*c*x*(
-a^2*c*x^2+c)^(3/2)-15/4*a^3*c^2*x*(-a^2*c*x^2+c)^(1/2)-15/4*a^3*c^3/(a^2*c
)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/5*a^2*(-(x-1/a)^2*a^
2*c-2*a*c*(x-1/a))^(5/2)+1/2*a^3*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x
+3/4*a^3*c^2*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+3/4*a^3*c^3/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))-1/2/c/x
^2*(-a^2*c*x^2+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} (a x+1)^{2}}{\left(a^{2} x^{2}-1\right) x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^3,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^{2}\,c\,x^{2}\right)^{5/2}\,(a\,x+1)^{2}}{x^{3}\,\left(a^{2}\,x^{2}-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x^3*(a^2*x^2 - 1)), x)
```

sympy [C] time = 118.63, size = 401, normalized size = 2.66

$$-a^4c^2\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^3}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,x}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{2a} & \text{for } \left|a^2x^2\right|>1\\ \frac{\sqrt{c}\,x\sqrt{-a^2x^2+1}}{2}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)+2ac^2\left(\begin{cases}-\frac{ia^2 \ldots}{\sqrt{a^2 \ldots}} \\ \frac{a^2\sqrt{\ldots}}{\sqrt{-a^2 \ldots}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x**3,x)
```

```
[Out] -a**4*c**2*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2
*c*x**2 + c)**(3/2)/(3*a**2*c), True)) - 2*a**3*c**2*Piecewise((I*a**2*sqrt
(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*
sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 +
 1)/2 + sqrt(c)*asin(a*x)/(2*a), True)) + 2*a*c**2*Piecewise((-I*a**2*sqrt(
c)*x/sqrt(a**2*x**2 - 1) + I*a*sqrt(c)*acosh(a*x) + I*sqrt(c)/(x*sqrt(a**2*
x**2 - 1)), Abs(a**2*x**2) > 1), (a**2*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - a*s
qrt(c)*asin(a*x) - sqrt(c)/(x*sqrt(-a**2*x**2 + 1)), True)) + c**2*Piecewis
e((a**2*sqrt(c)*acosh(1/(a*x))/2 + a*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2)))
 - sqrt(c)/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*
a**2*sqrt(c)*asin(1/(a*x))/2 - I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), T
rue))
```

3.1105 $\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=155

$$-\frac{a^2c^2(6ax+1)\sqrt{c-a^2cx^2}}{2x}-\frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2}-\frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3}-\frac{1}{2}a^3c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+3a^3c^{5/2}\tanh^{-1}$$

```
[Out] -1/6*a*c*(-a*x+6)*(-a^2*c*x^2+c)^(3/2)/x^2-1/3*(-a^2*c*x^2+c)^(5/2)/x^3-1/2
*a^3*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))+3*a^3*c^(5/2)*arctanh
((-a^2*c*x^2+c)^(1/2)/c^(1/2))-1/2*a^2*c^2*(6*a*x+1)*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.34, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6151, 1807, 813, 844, 217, 203, 266, 63, 208}

$$-\frac{a^2c^2(6ax+1)\sqrt{c-a^2cx^2}}{2x}-\frac{1}{2}a^3c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+3a^3c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^4,x]
```

```
[Out] -(a^2*c^2*(1 + 6*a*x)*Sqrt[c - a^2*c*x^2])/(2*x) - (a*c*(6 - a*x)*(c - a^2*
c*x^2)^(3/2))/(6*x^2) - (c - a^2*c*x^2)^(5/2)/(3*x^3) - (a^3*c^(5/2)*ArcTan
[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/2 + 3*a^3*c^(5/2)*ArcTanh[Sqrt[c - a^2
*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^4}\,dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x^4}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{3}\int \frac{\left(-6ac-a^2cx\right)\left(c-a^2cx^2\right)^{3/2}}{x^3}\,dx$$

$$= -\frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} + \frac{1}{8}\int \frac{\left(4a^2c^2-24a^3c^2x\right)\sqrt{c-a^2cx}}{x^2}$$

$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{16}\int$$

$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \left(3a^3c\right.$$

$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{2}\left(3a\right.$$

$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{2}a^3c^5$$

$$= -\frac{a^2c^2(1+6ax)\sqrt{c-a^2cx^2}}{2x} - \frac{ac(6-ax)\left(c-a^2cx^2\right)^{3/2}}{6x^2} - \frac{\left(c-a^2cx^2\right)^{5/2}}{3x^3} - \frac{1}{2}a^3c^5$$

Mathematica [A] time = 0.28, size = 149, normalized size = 0.96

$$-3a^3c^{5/2}\log(x)+3a^3c^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{1}{2}a^3c^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-\frac{c^2\left(3a^4x^4+12a^3x^3-2a^2x^2+\right.}{6x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^4,x]
```

```
[Out] -1/6*(c^2*Sqrt[c - a^2*c*x^2]*(2 + 6*a*x - 2*a^2*x^2 + 12*a^3*x^3 + 3*a^4*x
^4))/x^3 + (a^3*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2
```

```
*x^2))])/2 - 3*a^3*c^(5/2)*Log[x] + 3*a^3*c^(5/2)*Log[c + Sqrt[c]*Sqrt[c -
a^2*c*x^2]]
```

fricas [A] time = 0.73, size = 329, normalized size = 2.12

$$\left[\frac{3\,a^3c^{\frac{5}{2}}x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+9\,a^3c^{\frac{5}{2}}x^3\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-\left(3\,a^4c^2x^4+12\,a^3c^2x^3-2\,a^2c^2x^2+\right.}{6\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/6*(3*a^3*c^(5/2)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2
- c)) + 9*a^3*c^(5/2)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - (3*a^4*c^2*x^4 + 12*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + 6*a*c^2*x +
 2*c^2)*sqrt(-a^2*c*x^2 + c))/x^3, 1/12*(36*a^3*sqrt(-c)*c^2*x^3*arctan(sqr
t(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 3*a^3*sqrt(-c)*c^2*x^3*log(2*
a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - 2*(3*a^4*c^2*x^4 + 1
2*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + 6*a*c^2*x + 2*c^2)*sqrt(-a^2*c*x^2 + c))/x^
3]
```

giac [B] time = 0.23, size = 292, normalized size = 1.88

$$-\frac{6\,a^3c^3\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^4\sqrt{-c}\,c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{2\,|a|}-\frac{1}{2}\left(a^4c^2x+4\,a^3c^2\right)\sqrt{-a^2cx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^4,x, algorithm="gia
c")
```

```
[Out] -6*a^3*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) - 1/2*a^4*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/
abs(a) - 1/2*(a^4*c^2*x + 4*a^3*c^2)*sqrt(-a^2*c*x^2 + c) - 2/3*(3*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c^3*abs(a) + 3*(sqrt(-a^2*c)*x - sqrt
(-a^2*c*x^2 + c))^4*a^4*sqrt(-c)*c^3 - 3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2
+ c))*a^3*c^5*abs(a) + a^4*sqrt(-c)*c^5)/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^2 - c)^3*abs(a))
```

maple [B] time = 0.05, size = 423, normalized size = 2.73

$$-\frac{3a^3\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{5}-a^3c\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}+3a^3c^{\frac{5}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)-3a^3\sqrt{-a^2c\,x^2+c}\;c^2-\frac{2a^2\left(-a^2c\,x^2\right.}{3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^4,x)
```

```
[Out] -3/5*a^3*(-a^2*c*x^2+c)^(5/2)-a^3*c*(-a^2*c*x^2+c)^(3/2)+3*a^3*c^(5/2)*ln((
2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-3*a^3*(-a^2*c*x^2+c)^(1/2)*c^2-2/3*a
^2/c/x*(-a^2*c*x^2+c)^(7/2)-2/3*a^4*x*(-a^2*c*x^2+c)^(5/2)-5/6*a^4*c*x*(-a^
2*c*x^2+c)^(3/2)-5/4*a^4*c^2*x*(-a^2*c*x^2+c)^(1/2)-5/4*a^4*c^3/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-1/3/c/x^3*(-a^2*c*x^2+c)^(7
/2)-2/5*a^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(5/2)+1/2*a^4*c*(-(x-1/a)^2*a^
2*c-2*a*c*(x-1/a))^(3/2)*x+3/4*a^4*c^2*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/
2)*x+3/4*a^4*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a
*c*(x-1/a))^(1/2))-a/c/x^2*(-a^2*c*x^2+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^4,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\left(c-a^2\,c\,x^2\right)^{5/2}\,\left(a\,x+1\right)^2}{x^4\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x^4*(a^2*x^2 - 1)), x)
```

sympy [C] time = 26.91, size = 478, normalized size = 3.08

$$-a^4c^2\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^3}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,x}{2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{2a} & \text{for } |a^2x^2|>1\\ \frac{\sqrt{c}\,x\sqrt{-a^2x^2+1}}{2}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)-2a^3c^2\left(\begin{cases}i\sqrt{c}\sqrt{a^2x^2-1}-\sqrt{c}\log(ax)+\frac{\sqrt{c}\log\left(a^2x\right.}{2}\\ \sqrt{c}\sqrt{-a^2x^2+1}+\frac{\sqrt{c}\log\left(a^2x^2\right)}{2}-\sqrt{c}\log\left(\right.\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x**4,x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(
c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) >
 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True)) -
2*a**3*c**2*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(a*x) + s
qrt(c)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) > 1), (sq
rt(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a
**2*x**2 + 1) + 1), True)) + 2*a*c**2*Piecewise((a**2*sqrt(c)*acosh(1/(a*x)
)/2 + a*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(2*a*x**3*sqrt(-1
+ 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**2*sqrt(c)*asin(1/(a*x))/2
- I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True)) + c**2*Piecewise((a**3*
sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(3*
x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/3 - I
*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True))
```

3.1106 $\int \frac{e^{2\tanh^{-1}(ax)}(c-a^2cx^2)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=155

$$-\frac{(c-a^2cx^2)^{5/2}}{4x^4}-\frac{ac(9ax+16)(c-a^2cx^2)^{3/2}}{24x^3}+2a^4c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\frac{9}{8}a^4c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{a^3c^2(1\ldots}{}$$

```
[Out] -1/24*a*c*(9*a*x+16)*(-a^2*c*x^2+c)^(3/2)/x^3-1/4*(-a^2*c*x^2+c)^(5/2)/x^4+
2*a^4*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))+9/8*a^4*c^(5/2)*arct
anh((-a^2*c*x^2+c)^(1/2)/c^(1/2))+1/8*a^3*c^2*(-9*a*x+16)*(-a^2*c*x^2+c)^(1
/2)/x
```

Rubi [A] time = 0.33, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6151, 1807, 811, 813, 844, 217, 203, 266, 63, 208}

$$\frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x}+2a^4c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\frac{9}{8}a^4c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{ac(9ax+16)(c-a^2cx^2)^{\ldots}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2))/x^5,x]
```

```
[Out] (a^3*c^2*(16 - 9*a*x)*Sqrt[c - a^2*c*x^2])/(8*x) - (a*c*(16 + 9*a*x)*(c - a
^2*c*x^2)^(3/2))/(24*x^3) - (c - a^2*c*x^2)^(5/2)/(4*x^4) + 2*a^4*c^(5/2)*A
rcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] + (9*a^4*c^(5/2)*ArcTanh[Sqrt[c -
a^2*c*x^2]/Sqrt[c]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}}{x^5}\,dx = c\int \frac{(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}{x^5}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} - \frac{1}{4}\int \frac{\left(-8ac-3a^2cx\right)\left(c-a^2cx^2\right)^{3/2}}{x^4}\,dx$$

$$= -\frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} + \frac{\int \frac{\left(-32a^3c^3-18a^4c^3x\right)\sqrt{c-a^2cx^2}}{x^2}\,dx}{16c}$$

$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} - \int 3$$

$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} - \frac{1}{8}($$

$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} - \frac{1}{16}$$

$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} + 2a^4$$

$$= \frac{a^3c^2(16-9ax)\sqrt{c-a^2cx^2}}{8x} - \frac{ac(16+9ax)\left(c-a^2cx^2\right)^{3/2}}{24x^3} - \frac{\left(c-a^2cx^2\right)^{5/2}}{4x^4} + 2a^4$$

Mathematica [A] time = 0.27, size = 151, normalized size = 0.97

$$-\frac{9}{8}a^4c^{5/2}\log(x)+\frac{9}{8}a^4c^{5/2}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-2a^4c^{5/2}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-\frac{c^2\left(24a^4x^4-64a^3x^3-3a^2x^2\right.}{24x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x]))*(c - a^2*c*x^2)^(5/2))/x^5,x]
```

```
[Out] -1/24*(c^2*Sqrt[c - a^2*c*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 64*a^3*x^3 + 24*a^
4*x^4))/x^4 - 2*a^4*c^(5/2)*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 +
 a^2*x^2))] - (9*a^4*c^(5/2)*Log[x])/8 + (9*a^4*c^(5/2)*Log[c + Sqrt[c]*Sqr
t[c - a^2*c*x^2]])/8
```

fricas [A] time = 0.83, size = 329, normalized size = 2.12

$$\left[-\frac{96\,a^4c^{\frac{5}{2}}x^4\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)-27\,a^4c^{\frac{5}{2}}x^4\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\left(24\,a^4c^2x^4-64\,a^3c^2x^3-3\,a^2\right.}{48\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^5,x, algorithm="fri
cas")
```

```
[Out] [-1/48*(96*a^4*c^(5/2)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x
^2 - c)) - 27*a^4*c^(5/2)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt
(c) - 2*c)/x^2) + 2*(24*a^4*c^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a
*c^2*x + 6*c^2)*sqrt(-a^2*c*x^2 + c))/x^4, 1/24*(27*a^4*sqrt(-c)*c^2*x^4*ar
ctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 24*a^4*sqrt(-c)*c^2*x
^4*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) - (24*a^4*c^2
*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt(-a^2*c*x^2
 + c))/x^4]
```

giac [B] time = 0.37, size = 440, normalized size = 2.84

$$-\frac{9\,a^4c^3\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}+\frac{2\,a^5\sqrt{-c}\,c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\sqrt{-a^2cx^2+c}\,a^4c^2+\frac{3\left(\sqrt{-a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^5,x, algorithm="gia
c")
```

```
[Out] -9/4*a^4*c^3*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt
(-c) + 2*a^5*sqrt(-c)*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/
abs(a) - sqrt(-a^2*c*x^2 + c)*a^4*c^2 + 1/12*(3*(sqrt(-a^2*c)*x - sqrt(-a^2
*c*x^2 + c))^7*a^4*c^3*abs(a) - 96*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^
6*a^5*sqrt(-c)*c^3 + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^4*a
bs(a) + 192*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^4*a^5*sqrt(-c)*c^4 + 21
*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^4*c^5*abs(a) - 160*(sqrt(-a^2*
c)*x - sqrt(-a^2*c*x^2 + c))^2*a^5*sqrt(-c)*c^5 + 3*(sqrt(-a^2*c)*x - sqrt(
-a^2*c*x^2 + c))*a^4*c^6*abs(a) + 64*a^5*sqrt(-c)*c^6)/(((sqrt(-a^2*c)*x -
sqrt(-a^2*c*x^2 + c))^2 - c)^4*abs(a))
```

maple [B] time = 0.06, size = 447, normalized size = 2.88

$$\frac{2a^3\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{3cx}+\frac{5a^5cx\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{6}+\frac{5a^5c^2x\sqrt{-a^2c\,x^2+c}}{4}+\frac{5a^5c^3\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{4\sqrt{a^2c}}-\frac{2a\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{3c\,x^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^5,x)
```

```
[Out] 2/3*a^3/c/x*(-a^2*c*x^2+c)^(7/2)+5/6*a^5*c*x*(-a^2*c*x^2+c)^(3/2)+5/4*a^5*c
^2*x*(-a^2*c*x^2+c)^(1/2)+5/4*a^5*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/
(-a^2*c*x^2+c)^(1/2))-2/3*a/c/x^3*(-a^2*c*x^2+c)^(7/2)-2/5*a^4*(-(x-1/a)^2*
a^2*c-2*a*c*(x-1/a))^(5/2)+2/3*a^5*x*(-a^2*c*x^2+c)^(5/2)-5/8*a^2/c/x^2*(-a
^2*c*x^2+c)^(7/2)-1/4/c/x^4*(-a^2*c*x^2+c)^(7/2)-3/8*a^4*c*(-a^2*c*x^2+c)^(
3/2)+9/8*a^4*c^(5/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-9/8*a^4*(-a
^2*c*x^2+c)^(1/2)*c^2+1/2*a^5*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x+3/
4*a^5*c^2*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x+3/4*a^5*c^3/(a^2*c)^(1/2
)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))-9/40*a^4*(
-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2}{\left(a^2x^2-1\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(5/2)/x^5,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2/((a^2*x^2 - 1)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}\,\left(a\,x+1\right)^2}{x^5\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(x^5*(a^2*x^2 - 1)), x)
```

sympy [C] time = 65.52, size = 575, normalized size = 3.71

$$-a^4c^2\left(\begin{cases} i\sqrt{c}\,\sqrt{a^2x^2-1}-\sqrt{c}\,\log(ax)+\frac{\sqrt{c}\,\log\left(a^2x^2\right)}{2}+i\sqrt{c}\,\operatorname{asin}\left(\frac{1}{ax}\right) & \text{for } \left|a^2x^2\right|>1 \\ \sqrt{c}\,\sqrt{-a^2x^2+1}+\frac{\sqrt{c}\,\log\left(a^2x^2\right)}{2}-\sqrt{c}\,\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise} \end{cases}\right)-2a^3c^2\left(\begin{cases} -\frac{ia^2\sqrt{c}\,x}{\sqrt{a^2x^2-1}}+ia\sqrt{} \\ \frac{a^2\sqrt{c}\,x}{\sqrt{-a^2x^2+1}}-a\sqrt{c} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(5/2)/x**5,x)
```

```
[Out] -a**4*c**2*Piecewise((I*sqrt(c)*sqrt(a**2*x**2 - 1) - sqrt(c)*log(a*x) + sq
rt(c)*log(a**2*x**2)/2 + I*sqrt(c)*asin(1/(a*x)), Abs(a**2*x**2) > 1), (sqr
t(c)*sqrt(-a**2*x**2 + 1) + sqrt(c)*log(a**2*x**2)/2 - sqrt(c)*log(sqrt(-a*
*2*x**2 + 1) + 1), True)) - 2*a**3*c**2*Piecewise((-I*a**2*sqrt(c)*x/sqrt(a
**2*x**2 - 1) + I*a*sqrt(c)*acosh(a*x) + I*sqrt(c)/(x*sqrt(a**2*x**2 - 1)),
 Abs(a**2*x**2) > 1), (a**2*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - a*sqrt(c)*asin
(a*x) - sqrt(c)/(x*sqrt(-a**2*x**2 + 1)), True)) + 2*a*c**2*Piecewise((a**3
*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/3 - a*sqrt(c)*sqrt(-1 + 1/(a**2*x**2))/(3
*x**2), 1/Abs(a**2*x**2) > 1), (I*a**3*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/3 -
I*a*sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(3*x**2), True)) + c**2*Piecewise((a**4
*sqrt(c)*acosh(1/(a*x))/8 - a**3*sqrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3
*a*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - sqrt(c)/(4*a*x**5*sqrt(-1 +
1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-I*a**4*sqrt(c)*asin(1/(a*x))/8 +
I*a**3*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*I*a*sqrt(c)/(8*x**3*sqrt(1
 - 1/(a**2*x**2))) + I*sqrt(c)/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True))
```

3.1107 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=153

$$\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{128a}+\frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}-\cdots$$

```
[Out] 15/64*c^2*x*(-a^2*c*x^2+c)^(3/2)+3/16*c*x*(-a^2*c*x^2+c)^(5/2)-9/56*(-a^2*c
*x^2+c)^(7/2)/a-1/8*(a*x+1)*(-a^2*c*x^2+c)^(7/2)/a+45/128*c^(7/2)*arctan(a*
x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a+45/128*c^3*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6141, 671, 641, 195, 217, 203}

$$\frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{128a}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}-\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (45*c^3*x*Sqrt[c - a^2*c*x^2])/128 + (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 +
(3*c*x*(c - a^2*c*x^2)^(5/2))/16 - (9*(c - a^2*c*x^2)^(7/2))/(56*a) - ((1 +
 a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) + (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(128*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx = c\int (1+ax)^2\left(c-a^2cx^2\right)^{5/2}dx$$

$$= -\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a} + \frac{1}{8}(9c)\int (1+ax)\left(c-a^2cx^2\right)^{5/2}dx$$

$$= -\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a} - \frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a} + \frac{1}{8}(9c)\int \left(c-a^2cx^2\right)^{5/2}dx$$

$$= \frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2} - \frac{9\left(c-a^2cx^2\right)^{7/2}}{56a} - \frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a} + \frac{1}{16}\left(15c^2\right)\int ($$

$$= \frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2} + \frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2} - \frac{9\left(c-a^2cx^2\right)^{7/2}}{56a} - \frac{(1+ax)\left(c-a^2c\right.}{8a}$$

$$= \frac{45}{128}c^3x\sqrt{c-a^2cx^2} + \frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2} + \frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2} - \frac{9\left(c-a^2cx^2\right)}{56a}$$

$$= \frac{45}{128}c^3x\sqrt{c-a^2cx^2} + \frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2} + \frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2} - \frac{9\left(c-a^2cx^2\right)}{56a}$$

$$= \frac{45}{128}c^3x\sqrt{c-a^2cx^2} + \frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2} + \frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2} - \frac{9\left(c-a^2cx^2\right)}{56a}$$

Mathematica [A] time = 0.14, size = 151, normalized size = 0.99

$$-\frac{c^3\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(112a^8x^8+144a^7x^7-424a^6x^6-600a^5x^5+558a^4x^4+978a^3x^3-187a^2x^2-837ax+2\right.\right.}{896a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/896*(c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(256 - 837*a*x - 187*a^2*x^2
 + 978*a^3*x^3 + 558*a^4*x^4 - 600*a^5*x^5 - 424*a^6*x^6 + 144*a^7*x^7 + 11
2*a^8*x^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 -
a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.84, size = 286, normalized size = 1.87

$$\left[\frac{315\sqrt{-c}\,c^3\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(112\,a^7c^3x^7+256\,a^6c^3x^6-168\,a^5c^3x^5-768\,a^4c^3x^4-2\right.}{1792\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas"
)
```

```
[Out] [1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 2*(112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 - 768*a^
4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256*c^3)*sqrt
(-a^2*c*x^2 + c))/a, -1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt
(c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x
^5 - 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 25
6*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

giac [A] time = 0.34, size = 141, normalized size = 0.92

$$-\frac{45\,c^4\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{128\,\sqrt{-c}\,|a|}-\frac{1}{896}\,\sqrt{-a^2cx^2+c}\left(\frac{256\,c^3}{a}-\left(581\,c^3+2\left(384\,ac^3-\left(105\,a^2c^3+4\left(96\,a\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -45/128*c^4*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(
a)) - 1/896*sqrt(-a^2*c*x^2 + c)*(256*c^3/a - (581*c^3 + 2*(384*a*c^3 - (10
5*a^2*c^3 + 4*(96*a^3*c^3 + (21*a^4*c^3 - 2*(7*a^6*c^3*x + 16*a^5*c^3)*x)*x
)*x)*x)*x)*x)
```

maple [B] time = 0.04, size = 296, normalized size = 1.93

$$-\frac{x\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{8}-\frac{7cx\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{48}-\frac{35c^2x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{192}-\frac{35c^3x\sqrt{-a^2c\,x^2+c}}{128}-\frac{35c^4\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{128\sqrt{a^2c}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/8*x*(-a^2*c*x^2+c)^(7/2)-7/48*c*x*(-a^2*c*x^2+c)^(5/2)-35/192*c^2*x*(-a^
2*c*x^2+c)^(3/2)-35/128*c^3*x*(-a^2*c*x^2+c)^(1/2)-35/128*c^4/(a^2*c)^(1/2)
*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/7/a*(-(x-1/a)^2*a^2*c-2*a*c
*(x-1/a))^(7/2)+1/3*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(5/2)*x+5/12*c^2*(-(
x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x+5/8*c^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/
a))^(1/2)*x+5/8*c^4/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-
2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.57, size = 194, normalized size = 1.27

$$-\frac{1}{896}\left(\frac{112\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}x}{a}-\frac{168\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}cx}{a}-\frac{210\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}c^{2}x}{a}-\frac{560\sqrt{a^{2}cx^{2}-4\,acx+3\,c}\,c^{3}x}{a}+\frac{245}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima"
)
```

```
[Out] -1/896*(112*(-a^2*c*x^2 + c)^(7/2)*x/a - 168*(-a^2*c*x^2 + c)^(5/2)*c*x/a -
 210*(-a^2*c*x^2 + c)^(3/2)*c^2*x/a - 560*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c
^3*x/a + 245*sqrt(-a^2*c*x^2 + c)*c^3*x/a + 245*c^(7/2)*arcsin(a*x)/a^2 + 2
56*(-a^2*c*x^2 + c)^(7/2)/a^2 + 1120*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^3/a^
2 - 560*c^5*arcsin(a*x - 2)/(a^5*(-c/a^2)^(3/2)))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(c-a^2\,c\,x^2\right)^{7/2}\,{\left(a\,x+1\right)}^2}{a^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(7/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-((c - a^2*c*x^2)^(7/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 27.97, size = 1091, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] a**6*c**3*Piecewise((I*a**2*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqrt
(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**2
 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(1
28*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2*
x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/(
48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) + 5
*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt(
-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128*a**7), True)) + 2*a**5*c**3*Pie
cewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a**
2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(1
```

```
05*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) - a**4*c**3*Piecewise((I*a**2*
sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2*x**2
- 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(16*a**4
*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2) > 1)
, (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*sqrt(-a
**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(1
6*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5), True)) - 4*a**3
*c**3*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 + c
)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4/
4, True)) - a**2*c**3*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)
) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**
2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*s
qrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 +
1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3),
 True)) + 2*a*c**3*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)),
(-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)) + c**3*Piecewise((I*a**2*sqr
t(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I
*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2
+ 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

3.1108 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=137

$$\frac{x^2\sqrt{c-a^2cx^2}}{3a^2c}+\frac{11\sqrt{c-a^2cx^2}}{3a^4c}+\frac{(ax+1)^2}{a^4\sqrt{c-a^2cx^2}}-\frac{3\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^4\sqrt{c}}+\frac{x\sqrt{c-a^2cx^2}}{a^3c}$$

```
[Out] -3*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a^4/c^(1/2)+(a*x+1)^2/a^4/(-a^2
*c*x^2+c)^(1/2)+11/3*(-a^2*c*x^2+c)^(1/2)/a^4/c+x*(-a^2*c*x^2+c)^(1/2)/a^3/
c+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/c
```

Rubi [A] time = 0.36, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1635, 1815, 641, 217, 203}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3a^2c}+\frac{x\sqrt{c-a^2cx^2}}{a^3c}+\frac{11\sqrt{c-a^2cx^2}}{3a^4c}+\frac{(ax+1)^2}{a^4\sqrt{c-a^2cx^2}}-\frac{3\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (1 + a*x)^2/(a^4*Sqrt[c - a^2*c*x^2]) + (11*Sqrt[c - a^2*c*x^2])/(3*a^4*c)
+ (x*Sqrt[c - a^2*c*x^2])/(a^3*c) + (x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*c) - (
3*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a^4*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx = c\int \frac{x^3(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

$$= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} - \int \frac{(1+ax)\left(\frac{2}{a^3}+\frac{x}{a^2}+\frac{x^2}{a}\right)}{\sqrt{c-a^2cx^2}}\,dx$$

$$= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} + \frac{\int \frac{-\frac{6c}{a}-11cx-6acx^2}{\sqrt{c-a^2cx^2}}\,dx}{3a^2c}$$

$$= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{x\sqrt{c-a^2cx^2}}{a^3c} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} - \frac{\int \frac{18ac^2+22a^2c^2x}{\sqrt{c-a^2cx^2}}\,dx}{6a^4c^2}$$

$$= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{11\sqrt{c-a^2cx^2}}{3a^4c} + \frac{x\sqrt{c-a^2cx^2}}{a^3c} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} - \frac{3\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a^3}$$

$$= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{11\sqrt{c-a^2cx^2}}{3a^4c} + \frac{x\sqrt{c-a^2cx^2}}{a^3c} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x,\right.}{a^3}$$

$$= \frac{(1+ax)^2}{a^4\sqrt{c-a^2cx^2}} + \frac{11\sqrt{c-a^2cx^2}}{3a^4c} + \frac{x\sqrt{c-a^2cx^2}}{a^3c} + \frac{x^2\sqrt{c-a^2cx^2}}{3a^2c} - \frac{3\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^4\sqrt{c}}$$

Mathematica [A] time = 0.18, size = 97, normalized size = 0.71

$$\frac{9\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\frac{\left(a^3x^3+2a^2x^2+5ax-14\right)\sqrt{c-a^2cx^2}}{ax-1}}{3a^4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((Sqrt[c - a^2*c*x^2]*(-14 + 5*a*x + 2*a^2*x^2 + a^3*x^3))/(-1 + a*x) + 9*S
qrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^4*c
)
```

fricas [A] time = 0.62, size = 200, normalized size = 1.46

$$\left[-\frac{9\,(ax-1)\sqrt{-c}\,\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\left(a^3x^3+2\,a^2x^2+5\,ax-14\right)\sqrt{-a^2cx^2+c}}{6\left(a^5cx-a^4c\right)}, \frac{9\,(ax-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/6*(9*(a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) - 2*(a^3*x^3 + 2*a^2*x^2 + 5*a*x - 14)*sqrt(-a^2*c*x^2 + c))/(a
^5*c*x - a^4*c), 1/3*(9*(a*x - 1)*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqr
t(c)*x/(a^2*c*x^2 - c)) + (a^3*x^3 + 2*a^2*x^2 + 5*a*x - 14)*sqrt(-a^2*c*x^
2 + c))/(a^5*c*x - a^4*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] undef
```

maple [A] time = 0.04, size = 149, normalized size = 1.09

$$\frac{x^{2} \sqrt{-a^{2} c\, x^{2}+c}}{3 a^{2} c}+\frac{8 \sqrt{-a^{2} c\, x^{2}+c}}{3 a^{4} c}+\frac{x \sqrt{-a^{2} c\, x^{2}+c}}{a^{3} c}-\frac{3 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c\, x^{2}+c}}\right)}{a^{3} \sqrt{a^{2} c}}-\frac{2 \sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2 a c \left(x-\frac{1}{a}\right)}}{a^{5} c \left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/c+8/3*(-a^2*c*x^2+c)^(1/2)/a^4/c+x*(-a^2*c
*x^2+c)^(1/2)/a^3/c-3/a^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))-2/a^5/c/(x-1/a)*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)
```

maxima [A] time = 0.55, size = 113, normalized size = 0.82

$$-\frac{1}{3} a \left(\frac{6 \sqrt{-a^{2} c x^{2}+c}}{a^{6} c x-a^{5} c}-\frac{\sqrt{-a^{2} c x^{2}+c}\, x^{2}}{a^{3} c}-\frac{3 \sqrt{-a^{2} c x^{2}+c}\, x}{a^{4} c}+\frac{9 \arcsin \left(a x\right)}{a^{5} \sqrt{c}}-\frac{8 \sqrt{-a^{2} c x^{2}+c}}{a^{5} c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/3*a*(6*sqrt(-a^2*c*x^2 + c)/(a^6*c*x - a^5*c) - sqrt(-a^2*c*x^2 + c)*x^2
/(a^3*c) - 3*sqrt(-a^2*c*x^2 + c)*x/(a^4*c) + 9*arcsin(a*x)/(a^5*sqrt(c)) -
 8*sqrt(-a^2*c*x^2 + c)/(a^5*c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\,{\left(a\,x+1\right)}^2}{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a*x + 1)^2)/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(x^3*(a*x + 1)^2)/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{3}}{ax\sqrt{-a^{2}cx^{2}+c}-\sqrt{-a^{2}cx^{2}+c}}\,dx-\int \frac{ax^{4}}{ax\sqrt{-a^{2}cx^{2}+c}-\sqrt{-a^{2}cx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**3/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) -
Integral(a*x**4/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

3.1109 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=93

$$\frac{(ax+1)^2}{a^3\sqrt{c-a^2cx^2}}+\frac{(ax+6)\sqrt{c-a^2cx^2}}{2a^3c}-\frac{5\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a^3\sqrt{c}}$$

```
[Out] -5/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a^3/c^(1/2)+(a*x+1)^2/a^3/(-a
^2*c*x^2+c)^(1/2)+1/2*(a*x+6)*(-a^2*c*x^2+c)^(1/2)/a^3/c
```

Rubi [A] time = 0.25, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1635, 780, 217, 203}

$$\frac{(ax+1)^2}{a^3\sqrt{c-a^2cx^2}}+\frac{(ax+6)\sqrt{c-a^2cx^2}}{2a^3c}-\frac{5\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a^3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (1 + a*x)^2/(a^3*Sqrt[c - a^2*c*x^2]) + ((6 + a*x)*Sqrt[c - a^2*c*x^2])/(2*
a^3*c) - (5*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a^3*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{x^2(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{(1+ax)^2}{a^3\sqrt{c-a^2cx^2}} - \int \frac{\left(\frac{2}{a^2}+\frac{x}{a}\right)(1+ax)}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{(1+ax)^2}{a^3\sqrt{c-a^2cx^2}} + \frac{(6+ax)\sqrt{c-a^2cx^2}}{2a^3c} - \frac{5\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{2a^2}\\
&= \frac{(1+ax)^2}{a^3\sqrt{c-a^2cx^2}} + \frac{(6+ax)\sqrt{c-a^2cx^2}}{2a^3c} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)}{2a^2}\\
&= \frac{(1+ax)^2}{a^3\sqrt{c-a^2cx^2}} + \frac{(6+ax)\sqrt{c-a^2cx^2}}{2a^3c} - \frac{5\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a^3\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 1.01

$$\frac{\left(a^2x^2+3ax-8\right)\sqrt{c-a^2cx^2}+5\sqrt{c}\,(ax-1)\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{2a^3c(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((-8 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 5*Sqrt[c]*(-1 + a*x)*ArcTan[(
a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(2*a^3*c*(-1 + a*x))
```

fricas [A] time = 0.72, size = 184, normalized size = 1.98

$$\left[-\frac{5\,(ax-1)\sqrt{-c}\,\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\,\sqrt{-a^2cx^2+c}\,\left(a^2x^2+3\,ax-8\right)}{4\,\left(a^4cx-a^3c\right)},\ \frac{5\,(ax-1)\sqrt{c}\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/4*(5*(a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x - 8))/(a^4*c*x - a^3*
c), 1/2*(5*(a*x - 1)*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c
*x^2 - c)) + sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x - 8))/(a^4*c*x - a^3*c)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 126, normalized size = 1.35

$$\frac{x\sqrt{-a^2c\,x^2+c}}{2a^2c}-\frac{5\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2a^2\sqrt{a^2c}}+\frac{2\sqrt{-a^2c\,x^2+c}}{a^3c}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{a^4c\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x/a^2/c*(-a^2*c*x^2+c)^(1/2)-5/2/a^2/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)
*x/(-a^2*c*x^2+c)^(1/2))+2/a^3/c*(-a^2*c*x^2+c)^(1/2)-2/a^4/c/(x-1/a)*(-(x-
1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)
```

maxima [A] time = 0.50, size = 89, normalized size = 0.96

$$-\frac{1}{2}a\left(\frac{4\sqrt{-a^2cx^2+c}}{a^5cx-a^4c}-\frac{\sqrt{-a^2cx^2+c}\,x}{a^3c}+\frac{5\arcsin(ax)}{a^4\sqrt{c}}-\frac{4\sqrt{-a^2cx^2+c}}{a^4c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/2*a*(4*sqrt(-a^2*c*x^2 + c)/(a^5*c*x - a^4*c) - sqrt(-a^2*c*x^2 + c)*x/(
a^3*c) + 5*arcsin(a*x)/(a^4*sqrt(c)) - 4*sqrt(-a^2*c*x^2 + c)/(a^4*c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2\,(a\,x+1)^2}{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1)^2)/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(x^2*(a*x + 1)^2)/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx-\int \frac{ax^3}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**2/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) -
Integral(a*x**3/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

3.1110 $\int \frac{e^{2\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=84

$$\frac{(ax+1)^2}{a^2\sqrt{c-a^2cx^2}}+\frac{2\sqrt{c-a^2cx^2}}{a^2c}-\frac{2\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2\sqrt{c}}$$

```
[Out] -2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a^2/c^(1/2)+(a*x+1)^2/a^2/(-a^2
*c*x^2+c)^(1/2)+2*(-a^2*c*x^2+c)^(1/2)/a^2/c
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6151, 789, 641, 217, 203}

$$\frac{(ax+1)^2}{a^2\sqrt{c-a^2cx^2}}+\frac{2\sqrt{c-a^2cx^2}}{a^2c}-\frac{2\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (1 + a*x)^2/(a^2*Sqrt[c - a^2*c*x^2]) + (2*Sqrt[c - a^2*c*x^2])/(a^2*c) - (
2*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(a^2*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{x(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{(1+ax)^2}{a^2\sqrt{c-a^2cx^2}} - \frac{2\int \frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= \frac{(1+ax)^2}{a^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{c-a^2cx^2}}{a^2c} - \frac{2\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= \frac{(1+ax)^2}{a^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{c-a^2cx^2}}{a^2c} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\
&= \frac{(1+ax)^2}{a^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{c-a^2cx^2}}{a^2c} - \frac{2\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 78, normalized size = 0.93

$$\frac{\frac{(ax-3)\sqrt{c-a^2cx^2}}{ax-1}+2\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (((-3 + a*x)*Sqrt[c - a^2*c*x^2])/(-1 + a*x) + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c
 - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(a^2*c)
```

fricas [A] time = 0.79, size = 166, normalized size = 1.98

$$\left[-\frac{(ax-1)\sqrt{-c}\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-\sqrt{-a^2cx^2+c}\,(ax-3)}{a^3cx-a^2c}, \frac{2(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2}}{a^2cx}\right.}{a^3c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [-((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c) - sqrt(-a^2*c*x^2 + c)*(a*x - 3))/(a^3*c*x - a^2*c), (2*(a*x - 1)*sqr
t(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + sqrt(-a^2*c
*x^2 + c)*(a*x - 3))/(a^3*c*x - a^2*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] undef
```

maple [A] time = 0.04, size = 103, normalized size = 1.23

$$\frac{\sqrt{-a^2c\,x^2+c}}{a^2c}-\frac{2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{a\sqrt{a^2c}}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{a^3c\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a^2*c*x^2+c)^(1/2)/a^2/c-2/a/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c
*x^2+c)^(1/2))-2/a^3/c/(x-1/a)*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)
```

maxima [A] time = 0.46, size = 67, normalized size = 0.80

$$-a\left(\frac{2\sqrt{-a^2cx^2+c}}{a^4cx-a^3c}+\frac{2\arcsin(ax)}{a^3\sqrt{c}}-\frac{\sqrt{-a^2cx^2+c}}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -a*(2*sqrt(-a^2*c*x^2 + c)/(a^4*c*x - a^3*c) + 2*arcsin(a*x)/(a^3*sqrt(c))
- sqrt(-a^2*c*x^2 + c)/(a^3*c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,(a\,x+1)^2}{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x + 1)^2)/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(x*(a*x + 1)^2)/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx-\int \frac{ax^2}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) - Int
egral(a*x**2/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

3.1111 $\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=60

$$\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] -arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)+2*(a*x+1)/a/(-a^2*c*x^2
+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6141, 653, 217, 203}

$$\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*(1 + a*x))/(a*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/(a*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\ &= \frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}-\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= \frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}-\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\ &= \frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.37

$$\frac{2\sqrt{1-a^2x^2}\left(\sqrt{ax+1}+\sqrt{1-ax}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\,\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*(Sqrt[1 + a*x] + Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sq
rt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.67, size = 152, normalized size = 2.53

$$\left[-\frac{(ax-1)\sqrt{-c}\,\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+4\,\sqrt{-a^2cx^2+c}}{2\left(a^2cx-ac\right)},\ \frac{(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)-}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/2*((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x - a*c), ((a*x - 1)*sqrt(c)*arc
tan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2 +
 c))/(a^2*c*x - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.04, size = 80, normalized size = 1.33

$$-\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2c-2ac\left(x-\frac{1}{a}\right)}}{a^2c\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^2/c/(x-1/
a)*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)
```

maxima [A] time = 0.45, size = 45, normalized size = 0.75

$$-a\left(\frac{2\sqrt{-a^2cx^2+c}}{a^3cx-a^2c}+\frac{\arcsin\left(ax\right)}{a^2\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*(2*sqrt(-a^2*c*x^2 + c)/(a^3*c*x - a^2*c) + arcsin(a*x)/(a^2*sqrt(c)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{{\left(a\,x+1\right)}^2}{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(a*x + 1)^2/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx-\int \frac{1}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) - I
ntegral(1/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

3.1112 $\int \frac{e^{2\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+2*(a*x+1)/(-a^2*c*x^2+c)^(1/
2)
```

Rubi [A] time = 0.23, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1805, 266, 63, 208}

$$\frac{2(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*(1 + a*x))/Sqrt[c - a^2*c*x^2] - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/Sq
rt[c]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{(1+ax)^2}{x\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2(1+ax)}{\sqrt{c-a^2cx^2}}+\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2(1+ax)}{\sqrt{c-a^2cx^2}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= \frac{2(1+ax)}{\sqrt{c-a^2cx^2}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a^2c}\\
&= \frac{2(1+ax)}{\sqrt{c-a^2cx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 66, normalized size = 1.27

$$\frac{2\sqrt{c-a^2cx^2}}{c-acx}-\frac{\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{\sqrt{c}}+\frac{\log(x)}{\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[c - a^2*c*x^2])/(c - a*c*x) + Log[x]/Sqrt[c] - Log[c + Sqrt[c]*Sqrt
[c - a^2*c*x^2]]/Sqrt[c]
```

fricas [A] time = 0.92, size = 147, normalized size = 2.83

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-4\sqrt{-a^2cx^2+c}}{2(acx-c)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)+2\sqrt{-a^2cx^2}}{acx-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 4*sqrt(-a^2*c*x^2 + c))/(a*c*x - c), -((a*x - 1)*sqrt(-c)*arcta
n(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + 2*sqrt(-a^2*c*x^2 + c))/
(a*c*x - c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage2
```

maple [A] time = 0.04, size = 80, normalized size = 1.54

$$-\frac{\ln\left(\frac{2c+2\sqrt{c}\sqrt{-a^2c\,x^2+c}}{x}\right)}{\sqrt{c}}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{ac\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-2/a/c/(x-1/a)*(-(x-1/
a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)^2/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{(a\,x+1)^2}{x\,\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] -int((a*x + 1)^2/(x*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax^2\sqrt{-a^2cx^2+c}-x\sqrt{-a^2cx^2+c}}\,dx-\int \frac{1}{ax^2\sqrt{-a^2cx^2+c}-x\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x**2*sqrt(-a**2*c*x**2 + c) - x*sqrt(-a**2*c*x**2 + c)), x
) - Integral(1/(a*x**2*sqrt(-a**2*c*x**2 + c) - x*sqrt(-a**2*c*x**2 + c)),
x)
```

3.1113 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=77

$$\frac{2a(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{cx}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -2*a*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+2*a*(a*x+1)/(-a^2*c*x^2+
c)^(1/2)-(-a^2*c*x^2+c)^(1/2)/c/x
```

Rubi [A] time = 0.25, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1805, 807, 266, 63, 208}

$$\frac{2a(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{\sqrt{c-a^2cx^2}}{cx}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*a*(1 + a*x))/Sqrt[c - a^2*c*x^2] - Sqrt[c - a^2*c*x^2]/(c*x) - (2*a*ArcT
anh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{(1+ax)^2}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx \\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \int \frac{-1-2ax}{x^2\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{cx} + (2a)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{cx} + a\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) \\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{cx} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{ac} \\
&= \frac{2a(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{cx} - \frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 78, normalized size = 1.01

$$
\frac{(1-3ax)\sqrt{c-a^2cx^2}}{cx(ax-1)} - \frac{2a\log\left(\sqrt{c}\,\sqrt{c-a^2cx^2}+c\right)}{\sqrt{c}} + \frac{2a\log(x)}{\sqrt{c}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] ((1 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(c*x*(-1 + a*x)) + (2*a*Log[x])/Sqrt[c] -
 (2*a*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/Sqrt[c]
```

fricas [A] time = 0.70, size = 178, normalized size = 2.31

$$
\left[\frac{\left(a^2x^2-ax\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right) - \sqrt{-a^2cx^2+c}\,(3\,ax-1)}{acx^2-cx}, -\frac{2\left(a^2x^2-ax\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{\cdots}}{a^2cx^2-c}\right.}{acx^2-cx\cdots}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - sqrt(-a^2*c*x^2 + c)*(3*a*x - 1))/(a*c*x^2 - c*x), -(2*(a^2*x
^2 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) +
sqrt(-a^2*c*x^2 + c)*(3*a*x - 1))/(a*c*x^2 - c*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] undef
```

maple [A] time = 0.04, size = 99, normalized size = 1.29

$$-\frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{\sqrt{c}}-\frac{\sqrt{-a^2c\,x^2+c}}{cx}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{c\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -2*a/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-(-a^2*c*x^2+c)^(1/2
)/c/x-2/c/(x-1/a)*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{(ax+1)^2}{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{(a\,x+1)^2}{x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^2*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] -int((a*x + 1)^2/(x^2*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax^3\sqrt{-a^2cx^2+c} - x^2\sqrt{-a^2cx^2+c}}\,dx - \int \frac{1}{ax^3\sqrt{-a^2cx^2+c} - x^2\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x**3*sqrt(-a**2*c*x**2 + c) - x**2*sqrt(-a**2*c*x**2 + c))
, x) - Integral(1/(a*x**3*sqrt(-a**2*c*x**2 + c) - x**2*sqrt(-a**2*c*x**2 +
 c)), x)
```

3.1114 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=109

$$\frac{2a^2(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{2a\sqrt{c-a^2cx^2}}{cx}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}-\frac{5a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

```
[Out] -5/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+2*a^2*(a*x+1)/(-a^2*
c*x^2+c)^(1/2)-1/2*(-a^2*c*x^2+c)^(1/2)/c/x^2-2*a*(-a^2*c*x^2+c)^(1/2)/c/x
```

Rubi [A] time = 0.32, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1805, 1807, 807, 266, 63, 208}

$$\frac{2a^2(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{2a\sqrt{c-a^2cx^2}}{cx}-\frac{\sqrt{c-a^2cx^2}}{2cx^2}-\frac{5a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*a^2*(1 + a*x))/Sqrt[c - a^2*c*x^2] - Sqrt[c - a^2*c*x^2]/(2*c*x^2) - (2*
a*Sqrt[c - a^2*c*x^2])/(c*x) - (5*a^2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])
/(2*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx = c\int \frac{(1+ax)^2}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}} - \int \frac{-1-2ax-2a^2x^2}{x^3\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2cx^2} + \frac{\int \frac{4ac+5a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx}{2c}$$
$$= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2cx^2} - \frac{2a\sqrt{c-a^2cx^2}}{cx} + \frac{1}{2}\left(5a^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2cx^2} - \frac{2a\sqrt{c-a^2cx^2}}{cx} + \frac{1}{4}\left(5a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)$$
$$= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2cx^2} - \frac{2a\sqrt{c-a^2cx^2}}{cx} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{2c}$$
$$= \frac{2a^2(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2cx^2} - \frac{2a\sqrt{c-a^2cx^2}}{cx} - \frac{5a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.17, size = 94, normalized size = 0.86

$$\frac{\frac{\left(-8a^2x^2+3ax+1\right)\sqrt{c-a^2cx^2}}{x^2(ax-1)} - 5a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + 5a^2\sqrt{c}\log(x)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (((1 + 3*a*x - 8*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(x^2*(-1 + a*x)) + 5*a^2*Sqr
t[c]*Log[x] - 5*a^2*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/(2*c)
```

fricas [A] time = 0.65, size = 208, normalized size = 1.91

$$\left[\frac{5\left(a^3x^3-a^2x^2\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right) - 2\sqrt{-a^2cx^2+c}\left(8a^2x^2-3ax-1\right)}{4\left(acx^3-cx^2\right)}, -\frac{5\left(a^3x^3-a^2x^2\right)\sqrt{-c}\,\text{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(5*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c
)*sqrt(c) - 2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(8*a^2*x^2 - 3*a*x - 1))/(a*
c*x^3 - c*x^2), -1/2*(5*(a^3*x^3 - a^2*x^2)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2
 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2 + c)*(8*a^2*x^2 - 3*a*x -
 1))/(a*c*x^3 - c*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] undef
```

maple [A] time = 0.04, size = 124, normalized size = 1.14

$$-\frac{5a^2\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{2\sqrt{c}}-\frac{2a\sqrt{-a^2c\,x^2+c}}{cx}-\frac{2a\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{c\left(x-\frac{1}{a}\right)}-\frac{\sqrt{-a^2c\,x^2+c}}{2c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -5/2*a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-2*a*(-a^2*c*x^2
+c)^(1/2)/c/x-2*a/c/(x-1/a)*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)-1/2*(-a^
2*c*x^2+c)^(1/2)/c/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{(ax+1)^2}{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(a\,x+1)^2}{x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^3*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] -int((a*x + 1)^2/(x^3*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax^4\sqrt{-a^2cx^2+c}-x^3\sqrt{-a^2cx^2+c}}\,dx-\int \frac{1}{ax^4\sqrt{-a^2cx^2+c}-x^3\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x**4*sqrt(-a**2*c*x**2 + c) - x**3*sqrt(-a**2*c*x**2 + c))
, x) - Integral(1/(a*x**4*sqrt(-a**2*c*x**2 + c) - x**3*sqrt(-a**2*c*x**2 +
 c)), x)
```

3.1115 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^4\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=135

$$-\frac{8a^2\sqrt{c-a^2cx^2}}{3cx}-\frac{a\sqrt{c-a^2cx^2}}{cx^2}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}+\frac{2a^3(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -3*a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+2*a^3*(a*x+1)/(-a^2*c*
x^2+c)^(1/2)-1/3*(-a^2*c*x^2+c)^(1/2)/c/x^3-a*(-a^2*c*x^2+c)^(1/2)/c/x^2-8/
3*a^2*(-a^2*c*x^2+c)^(1/2)/c/x
```

Rubi [A] time = 0.41, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1805, 1807, 807, 266, 63, 208}

$$\frac{2a^3(ax+1)}{\sqrt{c-a^2cx^2}}-\frac{8a^2\sqrt{c-a^2cx^2}}{3cx}-\frac{a\sqrt{c-a^2cx^2}}{cx^2}-\frac{\sqrt{c-a^2cx^2}}{3cx^3}-\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^4*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*a^3*(1 + a*x))/Sqrt[c - a^2*c*x^2] - Sqrt[c - a^2*c*x^2]/(3*c*x^3) - (a*
Sqrt[c - a^2*c*x^2])/(c*x^2) - (8*a^2*Sqrt[c - a^2*c*x^2])/(3*c*x) - (3*a^3
*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^4\sqrt{c-a^2cx^2}}\,dx = c\int \frac{(1+ax)^2}{x^4\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \int \frac{-1-2ax-2a^2x^2-2a^3x^3}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} + \frac{\int \frac{6ac+8a^2cx+6a^3cx^2}{x^3\sqrt{c-a^2cx^2}}\,dx}{3c}$$
$$= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{\int \frac{-16a^2c^2-18a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c^2}$$
$$= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{8a^2\sqrt{c-a^2cx^2}}{3cx} + \left(3a^3\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{8a^2\sqrt{c-a^2cx^2}}{3cx} + \frac{1}{2}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-}}\right.$$
$$= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{8a^2\sqrt{c-a^2cx^2}}{3cx} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x\right.}{c}$$
$$= \frac{2a^3(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{3cx^3} - \frac{a\sqrt{c-a^2cx^2}}{cx^2} - \frac{8a^2\sqrt{c-a^2cx^2}}{3cx} - \frac{3a^3\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Mathematica [A] time = 0.17, size = 101, normalized size = 0.75

$$\frac{3a^3\log(x)}{\sqrt{c}} - \frac{3a^3\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{\sqrt{c}} + \frac{\left(-14a^3x^3+5a^2x^2+2ax+1\right)\sqrt{c-a^2cx^2}}{3cx^3(ax-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^4*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1 + 2*a*x + 5*a^2*x^2 - 14*a^3*x^3))/(3*c*x^3*(-1 + a
*x)) + (3*a^3*Log[x])/Sqrt[c] - (3*a^3*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/Sqrt[c]
```

fricas [A] time = 0.59, size = 224, normalized size = 1.66

$$\left[\frac{9\left(a^{4}x^{4}-a^{3}x^{3}\right)\sqrt{c}\,\log\left(-\frac{a^{2}cx^{2}+2\,\sqrt{-a^{2}cx^{2}+c}\,\sqrt{c}-2\,c}{x^{2}}\right)-2\left(14\,a^{3}x^{3}-5\,a^{2}x^{2}-2\,ax-1\right)\sqrt{-a^{2}cx^{2}+c}}{6\left(acx^{4}-cx^{3}\right)},-\frac{9\left(a^{4}x^{4}-a^{3}x^{3}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/6*(9*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c
)*sqrt(c) - 2*c)/x^2) - 2*(14*a^3*x^3 - 5*a^2*x^2 - 2*a*x - 1)*sqrt(-a^2*c*
x^2 + c))/(a*c*x^4 - c*x^3), -1/3*(9*(a^4*x^4 - a^3*x^3)*sqrt(-c)*arctan(sq
rt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + (14*a^3*x^3 - 5*a^2*x^2 - 2*
a*x - 1)*sqrt(-a^2*c*x^2 + c))/(a*c*x^4 - c*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] undef
```

maple [A] time = 0.05, size = 150, normalized size = 1.11

$$-\frac{3a^{3}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^{2}c\,x^{2}+c}}{x}\right)}{\sqrt{c}}-\frac{8a^{2}\sqrt{-a^{2}c\,x^{2}+c}}{3cx}-\frac{\sqrt{-a^{2}c\,x^{2}+c}}{3c\,x^{3}}-\frac{2a^{2}\sqrt{-\left(x-\frac{1}{a}\right)^{2}a^{2}c-2ac\left(x-\frac{1}{a}\right)}}{c\left(x-\frac{1}{a}\right)}-\frac{a\sqrt{-a^{2}c\,x^{2}+}}{c\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -3*a^3/c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-8/3*a^2*(-a^2*c*x
^2+c)^(1/2)/c/x-1/3*(-a^2*c*x^2+c)^(1/2)/c/x^3-2*a^2/c/(x-1/a)*(-(x-1/a)^2*
a^2*c-2*a*c*(x-1/a))^(1/2)-a*(-a^2*c*x^2+c)^(1/2)/c/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^{2}}{\sqrt{-a^{2}cx^{2}+c}\left(a^{2}x^{2}-1\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^4/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(a\,x+1)^2}{x^4\,\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^4*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] -int((a*x + 1)^2/(x^4*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax^5\sqrt{-a^2cx^2+c}-x^4\sqrt{-a^2cx^2+c}}\,dx-\int \frac{1}{ax^5\sqrt{-a^2cx^2+c}-x^4\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**4/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x**5*sqrt(-a**2*c*x**2 + c) - x**4*sqrt(-a**2*c*x**2 + c))
, x) - Integral(1/(a*x**5*sqrt(-a**2*c*x**2 + c) - x**4*sqrt(-a**2*c*x**2 +
 c)), x)
```

3.1116 $\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^4c^{3/2}}-\frac{\sqrt{c-a^2cx^2}}{a^4c^2}+\frac{(ax+1)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}}-\frac{8(ax+1)}{3a^4c\sqrt{c-a^2cx^2}}$$

```
[Out] 1/3*(a*x+1)^2/a^4/(-a^2*c*x^2+c)^(3/2)+2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^
(1/2))/a^4/c^(3/2)-8/3*(a*x+1)/a^4/c/(-a^2*c*x^2+c)^(1/2)-(-a^2*c*x^2+c)^(1
/2)/a^4/c^2
```

Rubi [A] time = 0.32, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1635, 641, 217, 203}

$$-\frac{\sqrt{c-a^2cx^2}}{a^4c^2}+\frac{2\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^4c^{3/2}}+\frac{(ax+1)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}}-\frac{8(ax+1)}{3a^4c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (1 + a*x)^2/(3*a^4*(c - a^2*c*x^2)^(3/2)) - (8*(1 + a*x))/(3*a^4*c*Sqrt[c -
 a^2*c*x^2]) - Sqrt[c - a^2*c*x^2]/(a^4*c^2) + (2*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(a^4*c^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = c\int \frac{x^3(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$

$$= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}} - \frac{1}{3}\int \frac{(1+ax)\left(\frac{2}{a^3}+\frac{3x}{a^2}+\frac{3x^2}{a}\right)}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

$$= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}} - \frac{8(1+ax)}{3a^4c\sqrt{c-a^2cx^2}} + \frac{\int \frac{\frac{6}{a^3}+\frac{3x}{a^2}}{\sqrt{c-a^2cx^2}}\,dx}{3c}$$

$$= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}} - \frac{8(1+ax)}{3a^4c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{a^4c^2} + \frac{2\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a^3c}$$

$$= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}} - \frac{8(1+ax)}{3a^4c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{a^4c^2} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)}{a^3c}$$

$$= \frac{(1+ax)^2}{3a^4\left(c-a^2cx^2\right)^{3/2}} - \frac{8(1+ax)}{3a^4c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{a^4c^2} + \frac{2\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^4c^{3/2}}$$

Mathematica [A] time = 0.17, size = 90, normalized size = 0.77

$$\frac{\frac{(-3a^2x^2+14ax-10)\sqrt{c-a^2cx^2}}{(ax-1)^2}-6\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(a^2x^2-1)}\right)}{3a^4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (((-10 + 14*a*x - 3*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(-1 + a*x)^2 - 6*Sqrt[c]*
ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^4*c^2)
```

fricas [A] time = 0.56, size = 229, normalized size = 1.96

$$\left[-\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+\sqrt{-a^2cx^2+c}\left(3\,a^2x^2-14\,ax+10\right)}{3\left(a^6c^2x^2-2\,a^5c^2x+a^4c^2\right)},-\frac{6\left(a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2
 + c)*a*sqrt(-c)*x - c) + sqrt(-a^2*c*x^2 + c)*(3*a^2*x^2 - 14*a*x + 10))/(
a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2), -1/3*(6*(a^2*x^2 - 2*a*x + 1)*sqrt(c)
*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2
 + c)*(3*a^2*x^2 - 14*a*x + 10))/(a^6*c^2*x^2 - 2*a^5*c^2*x + a^4*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 190, normalized size = 1.62

$$\frac{x^2}{a^2c\sqrt{-a^2c\,x^2+c}}-\frac{4}{c\,a^4\sqrt{-a^2c\,x^2+c}}-\frac{4x}{a^3c\sqrt{-a^2c\,x^2+c}}+\frac{2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{a^3c\sqrt{a^2c}}-\frac{2}{3a^5c\left(x-\frac{1}{a}\right)\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] x^2/a^2/c/(-a^2*c*x^2+c)^(1/2)-4/c/a^4/(-a^2*c*x^2+c)^(1/2)-4/a^3*x/c/(-a^2
*c*x^2+c)^(1/2)+2/a^3/c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)
^(1/2))-2/3/a^5/c/(x-1/a)/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+4/3/a^3/c/
(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x
```

maxima [B] time = 0.55, size = 260, normalized size = 2.22

$$\frac{1}{3}\left(\frac{a^3}{\sqrt{-a^2cx^2+c}\,a^9cx+\sqrt{-a^2cx^2+c}\,a^8c}-\frac{a^3}{\sqrt{-a^2cx^2+c}\,a^9cx-\sqrt{-a^2cx^2+c}\,a^8c}-\frac{a}{\sqrt{-a^2cx^2+c}\,a^7cx+\sqrt{-a^2cx^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/3*(a^3/(sqrt(-a^2*c*x^2 + c)*a^9*c*x + sqrt(-a^2*c*x^2 + c)*a^8*c) - a^3/
(sqrt(-a^2*c*x^2 + c)*a^9*c*x - sqrt(-a^2*c*x^2 + c)*a^8*c) - a/(sqrt(-a^2*
c*x^2 + c)*a^7*c*x + sqrt(-a^2*c*x^2 + c)*a^6*c) - a/(sqrt(-a^2*c*x^2 + c)*
a^7*c*x - sqrt(-a^2*c*x^2 + c)*a^6*c) + 3*x^2/(sqrt(-a^2*c*x^2 + c)*a^3*c)
- 8*x/(sqrt(-a^2*c*x^2 + c)*a^4*c) + 6*arcsin(a*x)/(a^5*c^(3/2)) - 12/(sqrt
(-a^2*c*x^2 + c)*a^5*c))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\,{\left(a\,x+1\right)}^2}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a*x + 1)^2)/((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(x^3*(a*x + 1)^2)/((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(x**3/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**
2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x
) - Integral(a*x**4/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt
(-a**2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c
)), x)
```

3.1117 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^3c^{3/2}}+\frac{(ax+1)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}}-\frac{5(ax+1)}{3a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] 1/3*(a*x+1)^2/a^3/(-a^2*c*x^2+c)^(3/2)+arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1
/2))/a^3/c^(3/2)-5/3*(a*x+1)/a^3/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1635, 778, 217, 203}

$$\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^3c^{3/2}}+\frac{(ax+1)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}}-\frac{5(ax+1)}{3a^3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (1 + a*x)^2/(3*a^3*(c - a^2*c*x^2)^(3/2)) - (5*(1 + a*x))/(3*a^3*c*Sqrt[c -
 a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]/(a^3*c^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= c\int \frac{x^2(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\\
&= \frac{(1+ax)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{3}\int \frac{\left(\frac{2}{a^2}+\frac{3x}{a}\right)(1+ax)}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{(1+ax)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}}-\frac{5(1+ax)}{3a^3c\sqrt{c-a^2cx^2}}+\frac{\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a^2c}\\
&= \frac{(1+ax)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}}-\frac{5(1+ax)}{3a^3c\sqrt{c-a^2cx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a^2c}\\
&= \frac{(1+ax)^2}{3a^3\left(c-a^2cx^2\right)^{3/2}}-\frac{5(1+ax)}{3a^3c\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^3c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 82, normalized size = 0.88

$$
\frac{\frac{(5ax-4)\sqrt{c-a^2cx^2}}{(ax-1)^2}-3\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{3a^3c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (((-4 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(-1 + a*x)^2 - 3*Sqrt[c]*ArcTan[(a*x*Sq
rt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^3*c^2)
```

fricas [A] time = 0.65, size = 215, normalized size = 2.31

$$\left[-\frac{3\left(a^{2}x^{2}-2\,ax+1\right)\sqrt{-c}\,\log\left(2\,a^{2}cx^{2}-2\,\sqrt{-a^{2}cx^{2}+c}\,a\sqrt{-c}\,x-c\right)-2\,\sqrt{-a^{2}cx^{2}+c}\,(5\,ax-4)}{6\left(a^{5}c^{2}x^{2}-2\,a^{4}c^{2}x+a^{3}c^{2}\right)},-\frac{3\left(a^{2}x^{2}-2\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [-1/6*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2
 + c)*a*sqrt(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c)*(5*a*x - 4))/(a^5*c^2*x^2
- 2*a^4*c^2*x + a^3*c^2), -1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*arctan(sqrt
(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^2 + c)*(5*a*x
 - 4))/(a^5*c^2*x^2 - 2*a^4*c^2*x + a^3*c^2)]
```

giac [A] time = 0.64, size = 44, normalized size = 0.47

$$-\frac{\log\left(\left|-\sqrt{-a^{2}c}\,x+\sqrt{-a^{2}cx^{2}+c}\right|\right)}{a^{2}\sqrt{-c}\,c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] -log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*c*abs(a))
```

maple [B] time = 0.04, size = 166, normalized size = 1.78

$$-\frac{3x}{a^{2}c\sqrt{-a^{2}c\,x^{2}+c}}+\frac{\arctan\left(\frac{\sqrt{a^{2}c}\;x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{a^{2}c\sqrt{a^{2}c}}-\frac{2}{a^{3}c\sqrt{-a^{2}c\,x^{2}+c}}-\frac{2}{3a^{4}c\left(x-\frac{1}{a}\right)\sqrt{-\left(x-\frac{1}{a}\right)^{2}a^{2}c-2ac\left(x-\frac{1}{a}\right)}}+\frac{}{3a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -3*x/a^2/c/(-a^2*c*x^2+c)^(1/2)+1/a^2/c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*
x/(-a^2*c*x^2+c)^(1/2))-2/a^3/c/(-a^2*c*x^2+c)^(1/2)-2/3/a^4/c/(x-1/a)/(-(x
-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+4/3/a^2/c/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a
))^(1/2)*x
```

maxima [B] time = 0.51, size = 230, normalized size = 2.47

$$\frac{1}{3}a\left(\frac{a}{\sqrt{-a^2cx^2+c}\,a^6cx+\sqrt{-a^2cx^2+c}\,a^5c}-\frac{a}{\sqrt{-a^2cx^2+c}\,a^6cx-\sqrt{-a^2cx^2+c}\,a^5c}-\frac{1}{\sqrt{-a^2cx^2+c}\,a^5cx+\sqrt{-a^2cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/3*a*(a/(sqrt(-a^2*c*x^2 + c)*a^6*c*x + sqrt(-a^2*c*x^2 + c)*a^5*c) - a/(s
qrt(-a^2*c*x^2 + c)*a^6*c*x - sqrt(-a^2*c*x^2 + c)*a^5*c) - 1/(sqrt(-a^2*c*
x^2 + c)*a^5*c*x + sqrt(-a^2*c*x^2 + c)*a^4*c) - 1/(sqrt(-a^2*c*x^2 + c)*a^
5*c*x - sqrt(-a^2*c*x^2 + c)*a^4*c) - 5*x/(sqrt(-a^2*c*x^2 + c)*a^3*c) + 3*
arcsin(a*x)/(a^4*c^(3/2)) - 6/(sqrt(-a^2*c*x^2 + c)*a^4*c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2\,{\left(a\,x+1\right)}^2}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1)^2)/((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(x^2*(a*x + 1)^2)/((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int\frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(x**2/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**
2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x
) - Integral(a*x**3/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt
(-a**2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c
)), x)
```

3.1118 $\int \frac{e^{2\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{(ax+1)^2}{3a^2\left(c-a^2cx^2\right)^{3/2}}-\frac{2(ax+1)}{3a^2c\sqrt{c-a^2cx^2}}$$

```
[Out] 1/3*(a*x+1)^2/a^2/(-a^2*c*x^2+c)^(3/2)-2/3*(a*x+1)/a^2/c/(-a^2*c*x^2+c)^(1/
2)
```

Rubi [A] time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6151, 789, 637}

$$\frac{(ax+1)^2}{3a^2\left(c-a^2cx^2\right)^{3/2}}-\frac{2(ax+1)}{3a^2c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (1 + a*x)^2/(3*a^2*(c - a^2*c*x^2)^(3/2)) - (2*(1 + a*x))/(3*a^2*c*Sqrt[c -
 a^2*c*x^2])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
```

```
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx = c\int \frac{x(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$
$$= \frac{(1+ax)^2}{3a^2\left(c-a^2cx^2\right)^{3/2}} - \frac{2\int \frac{1+ax}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{3a}$$
$$= \frac{(1+ax)^2}{3a^2\left(c-a^2cx^2\right)^{3/2}} - \frac{2(1+ax)}{3a^2c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 38, normalized size = 0.63

$$\frac{(2ax-1)\sqrt{c-a^2cx^2}}{3a^2c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
[Out] ((-1 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2*c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.68, size = 50, normalized size = 0.83

$$\frac{\sqrt{-a^2cx^2+c}\,(2\,ax-1)}{3\left(a^4c^2x^2-2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(2*a*x - 1)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

giac [B] time = 0.31, size = 117, normalized size = 1.95

$$-\frac{\left(ac+3\sqrt{-a^2c}\sqrt{c}\right)\mathrm{sgn}(x)}{3\left(a^3c^{\frac{5}{2}}-\sqrt{-a^2c}\,a^2c^2\right)} - \frac{2\left(a\sqrt{c}+3\sqrt{-a^2c+\frac{c}{x^2}}-\frac{3\sqrt{c}}{x}\right)}{3\left(a\sqrt{c}+\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^3\sqrt{c}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(a*c + 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^3*c^(5/2) - sqrt(-a^2*c)*a^2*
c^2) - 2/3*(a*sqrt(c) + 3*sqrt(-a^2*c + c/x^2) - 3*sqrt(c)/x)/((a*sqrt(c) +
 sqrt(-a^2*c + c/x^2) - sqrt(c)/x)^3*sqrt(c)*sgn(x))
```

maple [A] time = 0.03, size = 32, normalized size = 0.53

$$\frac{(2ax-1)(ax+1)^2}{3\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/3*(2*a*x-1)*(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)/a^2
```

maxima [B] time = 0.48, size = 218, normalized size = 3.63

$$\frac{1}{3}a\left(\frac{a}{\sqrt{-a^2cx^2+c}\,a^5cx+\sqrt{-a^2cx^2+c}\,a^4c}-\frac{a}{\sqrt{-a^2cx^2+c}\,a^5cx-\sqrt{-a^2cx^2+c}\,a^4c}-\frac{1}{\sqrt{-a^2cx^2+c}\,a^4cx+\sqrt{-a^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*a*(a/(sqrt(-a^2*c*x^2 + c)*a^5*c*x + sqrt(-a^2*c*x^2 + c)*a^4*c) - a/(s
qrt(-a^2*c*x^2 + c)*a^5*c*x - sqrt(-a^2*c*x^2 + c)*a^4*c) - 1/(sqrt(-a^2*c*
x^2 + c)*a^4*c*x + sqrt(-a^2*c*x^2 + c)*a^3*c) - 1/(sqrt(-a^2*c*x^2 + c)*a^
4*c*x - sqrt(-a^2*c*x^2 + c)*a^3*c) - 2*x/(sqrt(-a^2*c*x^2 + c)*a^2*c) - 3/
(sqrt(-a^2*c*x^2 + c)*a^3*c))
```

mupad [B] time = 1.02, size = 34, normalized size = 0.57

$$\frac{\sqrt{c-a^2\,c\,x^2}\;(2\,a\,x-1)}{3\,a^2\,c^2\,(a\,x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x + 1)^2)/((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(2*a*x - 1))/(3*a^2*c^2*(a*x - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(x/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**2*c
*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x) -
 Integral(a*x**2/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a
**2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)),
 x)
```

3.1119 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x}{3c\sqrt{c-a^2cx^2}}+\frac{2(ax+1)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 2/3*(a*x+1)/a/(-a^2*c*x^2+c)^(3/2)+1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6141, 653, 191}

$$\frac{x}{3c\sqrt{c-a^2cx^2}}+\frac{2(ax+1)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*(1 + a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$
$$= \frac{2(1+ax)}{3a\left(c-a^2cx^2\right)^{3/2}} + \frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= \frac{2(1+ax)}{3a\left(c-a^2cx^2\right)^{3/2}} + \frac{x}{3c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.24

$$-\frac{(ax-2)\sqrt{ax+1}\,\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/2}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/3*((-2 + a*x)*Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)^(3/2)*Sqrt
[c - a^2*c*x^2])
```

fricas [A] time = 0.79, size = 47, normalized size = 0.92

$$-\frac{\sqrt{-a^2cx^2+c}\,(ax-2)}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/3*sqrt(-a^2*c*x^2 + c)*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [B] time = 0.35, size = 148, normalized size = 2.90

$$-\frac{\left(ac-3\sqrt{-a^2c}\,\sqrt{c}\right)\mathrm{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}-\sqrt{-a^2c}\,ac^2\right)}+\frac{2\left(2\,a^2c+3\,a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}+\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^3c\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*c - 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) - sqrt(-a^2*c)*a*c^
2) + 2/3*(2*a^2*c + 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqr
t(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) + sqrt(-a^2*c + c/x^2) - sqrt
(c)/x)^3*c*sgn(x))
```

maple [A] time = 0.03, size = 31, normalized size = 0.61

$$-\frac{(ax-2)(ax+1)^2}{3\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(a*x-2)*(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)/a
```

maxima [B] time = 0.38, size = 196, normalized size = 3.84

$$\frac{1}{3}a\left(\frac{a}{\sqrt{-a^2cx^2+c}\,a^4cx+\sqrt{-a^2cx^2+c}\,a^3c}-\frac{a}{\sqrt{-a^2cx^2+c}\,a^4cx-\sqrt{-a^2cx^2+c}\,a^3c}-\frac{1}{\sqrt{-a^2cx^2+c}\,a^3cx+\sqrt{-a^2c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*a*(a/(sqrt(-a^2*c*x^2 + c)*a^4*c*x + sqrt(-a^2*c*x^2 + c)*a^3*c) - a/(s
qrt(-a^2*c*x^2 + c)*a^4*c*x - sqrt(-a^2*c*x^2 + c)*a^3*c) - 1/(sqrt(-a^2*c*
x^2 + c)*a^3*c*x + sqrt(-a^2*c*x^2 + c)*a^2*c) - 1/(sqrt(-a^2*c*x^2 + c)*a^
3*c*x - sqrt(-a^2*c*x^2 + c)*a^2*c) + x/(sqrt(-a^2*c*x^2 + c)*a*c))
```

mupad [B] time = 0.95, size = 33, normalized size = 0.65

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-2\right)}{3\,a\,c^2\,{\left(a\,x-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)),x)
```

```
[Out] -((c - a^2*c*x^2)^(1/2)*(a*x - 2))/(3*a*c^2*(a*x - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ax}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int\frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a*x/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**2
*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x)
 - Integral(1/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**2
*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x)
```

3.1120 $\int \frac{e^{2\tanh^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{4ax+3}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] 2/3*(a*x+1)/(-a^2*c*x^2+c)^(3/2)-arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(3
/2)+1/3*(4*a*x+3)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1805, 823, 12, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{4ax+3}{3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (2*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + (3 + 4*a*x)/(3*c*Sqrt[c - a^2*c*x
^2]) - ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = c\int \frac{(1+ax)^2}{x\left(c-a^2cx^2\right)^{5/2}}\,dx$$
$$= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} - \frac{1}{3}\int \frac{-3-4ax}{x\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{3+4ax}{3c\sqrt{c-a^2cx^2}} - \frac{\int -\frac{3a^2c^2}{x\sqrt{c-a^2cx^2}}\,dx}{3a^2c^3}$$
$$= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{3+4ax}{3c\sqrt{c-a^2cx^2}} + \frac{\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx}{c}$$
$$= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{3+4ax}{3c\sqrt{c-a^2cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)}{2c}$$
$$= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{3+4ax}{3c\sqrt{c-a^2cx^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a^2c^2}$$
$$= \frac{2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{3+4ax}{3c\sqrt{c-a^2cx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.12, size = 75, normalized size = 0.91

$$-\frac{\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{c^{3/2}} + \frac{(5-4ax)\sqrt{c-a^2cx^2}}{3c^2(ax-1)^2} + \frac{\log(x)}{c^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((5 - 4*a*x)*Sqrt[c - a^2*c*x^2])/(3*c^2*(-1 + a*x)^2) + Log[x]/c^(3/2) - L
og[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]/c^(3/2)
```

fricas [A] time = 0.51, size = 202, normalized size = 2.46

$$\left[\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-2\,\sqrt{-a^2cx^2+c}\,(4\,ax-5)}{6\left(a^2c^2x^2-2\,ac^2x+c^2\right)},-\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\arctan\right.}{3\left(a^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] [1/6*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 +
 c)*sqrt(c) - 2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 5))/(a^2*c^2*x^2
- 2*a*c^2*x + c^2), -1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(sqrt(-a^2
*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2 + c)*(4*a*x - 5))/(
a^2*c^2*x^2 - 2*a*c^2*x + c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.04, size = 152, normalized size = 1.85

$$\frac{1}{c\sqrt{-a^2c\,x^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{c^{\frac{3}{2}}}-\frac{2}{3ac\left(x-\frac{1}{a}\right)\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}-\frac{2\left(-2\left(x-\frac{1}{a}\right)a^2c-2ac\right.}{3a\,c^2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/c/(-a^2*c*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/
x)-2/3/a/c/(x-1/a)/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)-2/3/a/c^2*(-2*(x-
1/a)*a^2*c-2*a*c)/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{(-a^2cx^2+c)^{\frac{3}{2}}(a^2x^2-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)^2/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(a\,x+1)^2}{x\,(c-a^2\,c\,x^2)^{3/2}\,(a^2\,x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x*(c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)),x)
```

```
[Out] -int((a*x + 1)^2/(x*(c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{-a^3cx^4\sqrt{-a^2cx^2+c}+a^2cx^3\sqrt{-a^2cx^2+c}+acx^2\sqrt{-a^2cx^2+c}-cx\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^4\sqrt{-a^2cx^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a*x/(-a**3*c*x**4*sqrt(-a**2*c*x**2 + c) + a**2*c*x**3*sqrt(-a**2
*c*x**2 + c) + a*c*x**2*sqrt(-a**2*c*x**2 + c) - c*x*sqrt(-a**2*c*x**2 + c)
), x) - Integral(1/(-a**3*c*x**4*sqrt(-a**2*c*x**2 + c) + a**2*c*x**3*sqrt(
-a**2*c*x**2 + c) + a*c*x**2*sqrt(-a**2*c*x**2 + c) - c*x*sqrt(-a**2*c*x**2
 + c)), x)
```

3.1121 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{\sqrt{c-a^2cx^2}}{c^2x}+\frac{2a(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a(7ax+6)}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] 2/3*a*(a*x+1)/(-a^2*c*x^2+c)^(3/2)-2*a*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2)
)/c^(3/2)+1/3*a*(7*a*x+6)/c/(-a^2*c*x^2+c)^(1/2)-(-a^2*c*x^2+c)^(1/2)/c^2/x
```

Rubi [A] time = 0.34, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6151, 1805, 807, 266, 63, 208}

$$-\frac{\sqrt{c-a^2cx^2}}{c^2x}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2a(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}+\frac{a(7ax+6)}{3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (2*a*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + (a*(6 + 7*a*x))/(3*c*Sqrt[c - a
^2*c*x^2]) - Sqrt[c - a^2*c*x^2]/(c^2*x) - (2*a*ArcTanh[Sqrt[c - a^2*c*x^2]
/Sqrt[c]])/c^(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx = c\int \frac{(1+ax)^2}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx$$

$$= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} - \frac{1}{3}\int \frac{-3-6ax-4a^2x^2}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx$$

$$= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}} + \frac{\int \frac{3+6ax}{x^2\sqrt{c-a^2cx^2}}\,dx}{3c}$$

$$= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{c^2x} + \frac{(2a)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx}{c}$$

$$= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{c^2x} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)}{c}$$

$$= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{c^2x} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{ac^2}$$

$$= \frac{2a(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a(6+7ax)}{3c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{c^2x} - \frac{2a\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.14, size = 89, normalized size = 0.82

$$-\frac{2a\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{c^{3/2}} + \frac{\left(-10a^2x^2+14ax-3\right)\sqrt{c-a^2cx^2}}{3c^2x(ax-1)^2} + \frac{2a\log(x)}{c^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((-3 + 14*a*x - 10*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*c^2*x*(-1 + a*x)^2) + (
2*a*Log[x])/c^(3/2) - (2*a*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/c^(3/2)
```

fricas [A] time = 0.68, size = 238, normalized size = 2.20

$$\left[\frac{3\left(a^3x^3-2\,a^2x^2+ax\right)\sqrt{c}\,\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-\sqrt{-a^2cx^2+c}\left(10\,a^2x^2-14\,ax+3\right)}{3\left(a^2c^2x^3-2\,ac^2x^2+c^2x\right)},-\frac{6\left(a^3x^3-2\,a^2x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/3*(3*(a^3*x^3 - 2*a^2*x^2 + a*x)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c
*x^2 + c)*sqrt(c) - 2*c)/x^2) - sqrt(-a^2*c*x^2 + c)*(10*a^2*x^2 - 14*a*x +
 3))/(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x), -1/3*(6*(a^3*x^3 - 2*a^2*x^2 + a*
x)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a
^2*c*x^2 + c)*(10*a^2*x^2 - 14*a*x + 3))/(a^2*c^2*x^3 - 2*a*c^2*x^2 + c^2*x
)]
```

giac [A] time = 0.45, size = 100, normalized size = 0.93

$$-2\,a^2\sqrt{-c}\,c\left(\frac{2\,|a|\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{a^3c^3}-\frac{1}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)a^2c^2}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] -2*a^2*sqrt(-c)*c*(2*abs(a)*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))
/sqrt(-c))/(a^3*c^3) - 1/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*a
^2*c^2))*abs(a)
```

maple [A] time = 0.04, size = 178, normalized size = 1.65

$$\frac{2a}{c\sqrt{-a^2c\,x^2+c}}-\frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{c^{\frac{3}{2}}}-\frac{1}{cx\sqrt{-a^2c\,x^2+c}}+\frac{2a^2x}{c\sqrt{-a^2c\,x^2+c}}-\frac{2}{3c\left(x-\frac{1}{a}\right)\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

[Out] 2*a/c/(-a^2*c*x^2+c)^(1/2)-2*a/c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-1/c/x/(-a^2*c*x^2+c)^(1/2)+2*a^2/c*x/(-a^2*c*x^2+c)^(1/2)-2/3/c/(x-1/a)/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+4/3*a^2/c/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] -integrate((a*x + 1)^2/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(a\,x+1)^2}{x^2\,\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a*x + 1)^2/(x^2*(c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)),x)

[Out] -int((a*x + 1)^2/(x^2*(c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{-a^3cx^5\sqrt{-a^2cx^2+c}+a^2cx^4\sqrt{-a^2cx^2+c}+acx^3\sqrt{-a^2cx^2+c}-cx^2\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^5\sqrt{-a^2cx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**2/(-a**2*c*x**2+c)**(3/2),x)

[Out] -Integral(a*x/(-a**3*c*x**5*sqrt(-a**2*c*x**2 + c) + a**2*c*x**4*sqrt(-a**2*c*x**2 + c) + a*c*x**3*sqrt(-a**2*c*x**2 + c) - c*x**2*sqrt(-a**2*c*x**2 + c)), x) - Integral(1/(-a**3*c*x**5*sqrt(-a**2*c*x**2 + c) + a**2*c*x**4*sqrt(-a**2*c*x**2 + c) + a*c*x**3*sqrt(-a**2*c*x**2 + c) - c*x**2*sqrt(-a**2*c*x**2 + c)), x)

3.1122 $\int \frac{e^{2\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{7a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{2a\sqrt{c-a^2cx^2}}{c^2x}-\frac{\sqrt{c-a^2cx^2}}{2c^2x^2}+\frac{a^2(10ax+9)}{3c\sqrt{c-a^2cx^2}}+\frac{2a^2(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 2/3*a^2*(a*x+1)/(-a^2*c*x^2+c)^(3/2)-7/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c
^(1/2))/c^(3/2)+1/3*a^2*(10*a*x+9)/c/(-a^2*c*x^2+c)^(1/2)-1/2*(-a^2*c*x^2+c
)^(1/2)/c^2/x^2-2*a*(-a^2*c*x^2+c)^(1/2)/c^2/x
```

Rubi [A] time = 0.42, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6151, 1805, 1807, 807, 266, 63, 208}

$$-\frac{2a\sqrt{c-a^2cx^2}}{c^2x}-\frac{\sqrt{c-a^2cx^2}}{2c^2x^2}-\frac{7a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{a^2(10ax+9)}{3c\sqrt{c-a^2cx^2}}+\frac{2a^2(ax+1)}{3\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (2*a^2*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + (a^2*(9 + 10*a*x))/(3*c*Sqrt[
c - a^2*c*x^2]) - Sqrt[c - a^2*c*x^2]/(2*c^2*x^2) - (2*a*Sqrt[c - a^2*c*x^2
])/(c^2*x) - (7*a^2*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = c\int \frac{(1+ax)^2}{x^3\left(c-a^2cx^2\right)^{5/2}}\,dx$$

$$= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} - \frac{1}{3}\int \frac{-3-6ax-6a^2x^2-4a^3x^3}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx$$

$$= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}} + \frac{\int \frac{3+6ax+9a^2x^2}{x^3\sqrt{c-a^2cx^2}}\,dx}{3c}$$

$$= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2c^2x^2} - \frac{\int \frac{-12ac-21a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c^2}$$

$$= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2c^2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{c^2x} + \frac{\left(7a^2\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,d}{2c}$$

$$= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2c^2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{c^2x} + \frac{\left(7a^2\right)\text{Subst}\left(\int \frac{}{x\sqrt{c}}\right.}{4c}$$

$$= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2c^2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{c^2x} - \frac{7\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,d\right.}{2c}$$

$$= \frac{2a^2(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{a^2(9+10ax)}{3c\sqrt{c-a^2cx^2}} - \frac{\sqrt{c-a^2cx^2}}{2c^2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{c^2x} - \frac{7a^2\tanh^{-1}\left(\frac{\sqrt{c-a^2c}}{\sqrt{c}}\right.}{2c^{3/2}}$$

Mathematica [A] time = 0.15, size = 105, normalized size = 0.74

$$-\frac{7a^2\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)}{2c^{3/2}} + \frac{7a^2\log(x)}{2c^{3/2}} - \frac{\left(32a^3x^3-43a^2x^2+6ax+3\right)\sqrt{c-a^2cx^2}}{6c^2x^2(ax-1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/6*(Sqrt[c - a^2*c*x^2]*(3 + 6*a*x - 43*a^2*x^2 + 32*a^3*x^3))/(c^2*x^2*(
-1 + a*x)^2) + (7*a^2*Log[x])/(2*c^(3/2)) - (7*a^2*Log[c + Sqrt[c]*Sqrt[c -
 a^2*c*x^2]])/(2*c^(3/2))
```

fricas [A] time = 0.87, size = 266, normalized size = 1.87

$$\left[\frac{21\left(a^4x^4-2a^3x^3+a^2x^2\right)\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)-2\left(32a^3x^3-43a^2x^2+6ax+3\right)\sqrt{-a^2cx^2+c}}{12\left(a^2c^2x^4-2ac^2x^3+c^2x^2\right)}, -\frac{21\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] [1/12*(21*(a^4*x^4 - 2*a^3*x^3 + a^2*x^2)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(
-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) - 2*(32*a^3*x^3 - 43*a^2*x^2 + 6*a*x +
3)*sqrt(-a^2*c*x^2 + c))/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*x^2), -1/6*(21*(a
^4*x^4 - 2*a^3*x^3 + a^2*x^2)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)
/(a^2*c*x^2 - c)) + (32*a^3*x^3 - 43*a^2*x^2 + 6*a*x + 3)*sqrt(-a^2*c*x^2 +
 c))/(a^2*c^2*x^4 - 2*a*c^2*x^3 + c^2*x^2)]
```

giac [A] time = 0.41, size = 204, normalized size = 1.44

$$a^4c^2\left(\frac{7\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^3}-\frac{\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^3a-4\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2\sqrt{-c}\,|a|+\left(\sqrt{-\right.}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2a^3c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] a^4*c^2*(7*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/(a^2*s
qrt(-c)*c^3) - ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a - 4*(sqrt(-a^2*
c)*x - sqrt(-a^2*c*x^2 + c))^2*sqrt(-c)*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^
2*c*x^2 + c))*a*c + 4*sqrt(-c)*c*abs(a))/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^2 - c)^2*a^3*c^3))
```

maple [A] time = 0.05, size = 205, normalized size = 1.44

$$\frac{7a^2}{2c\sqrt{-a^2c\,x^2+c}}-\frac{7a^2\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{2c^{\frac{3}{2}}}-\frac{2a}{cx\sqrt{-a^2c\,x^2+c}}+\frac{4a^3x}{c\sqrt{-a^2c\,x^2+c}}-\frac{2a}{3c\left(x-\frac{1}{a}\right)\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 7/2*a^2/c/(-a^2*c*x^2+c)^(1/2)-7/2*a^2/c^(3/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^
2+c)^(1/2))/x)-2*a/c/x/(-a^2*c*x^2+c)^(1/2)+4*a^3/c*x/(-a^2*c*x^2+c)^(1/2)-
2/3*a/c/(x-1/a)/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)+4/3*a^3/c/(-(x-1/a)^
2*a^2*c-2*a*c*(x-1/a))^(1/2)*x-1/2/c/x^2/(-a^2*c*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(a\,x+1)^2}{x^3\,\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/(x^3*(c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)),x)
```

```
[Out] -int((a*x + 1)^2/(x^3*(c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{-a^3cx^6\sqrt{-a^2cx^2+c}+a^2cx^5\sqrt{-a^2cx^2+c}+acx^4\sqrt{-a^2cx^2+c}-cx^3\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^6\sqrt{-a^2cx^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a*x/(-a**3*c*x**6*sqrt(-a**2*c*x**2 + c) + a**2*c*x**5*sqrt(-a**2
*c*x**2 + c) + a*c*x**4*sqrt(-a**2*c*x**2 + c) - c*x**3*sqrt(-a**2*c*x**2 +
 c)), x) - Integral(1/(-a**3*c*x**6*sqrt(-a**2*c*x**2 + c) + a**2*c*x**5*sq
rt(-a**2*c*x**2 + c) + a*c*x**4*sqrt(-a**2*c*x**2 + c) - c*x**3*sqrt(-a**2*
c*x**2 + c)), x)
```

3.1123 $\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{2x}{5c^2\sqrt{c-a^2cx^2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2(ax+1)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] 2/5*(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)+1/5*x/c/(-a^2*c*x^2+c)^(3/2)+2/5*x/c^2/(
-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6141, 653, 192, 191}

$$\frac{2x}{5c^2\sqrt{c-a^2cx^2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2(ax+1)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (2*(1 + a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) + x/(5*c*(c - a^2*c*x^2)^(3/2)) +
 (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx\\ &= \frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}+\frac{3}{5}\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx\\ &= \frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{5c}\\ &= \frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2x}{5c^2\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.72

$$\frac{2a^3x^3-4a^2x^2+ax+2}{5ac^2(ax-1)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(5*a*c^2*(-1 + a*x)^2*Sqrt[c - a^2*c*x^2]
)
```

fricas [A] time = 0.77, size = 75, normalized size = 1.01

$$-\frac{\left(2\,a^3x^3-4\,a^2x^2+ax+2\right)\sqrt{-a^2cx^2+c}}{5\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 -
2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(-(a*x + 1)^2/((-a^2*c*x^2 + c)^(5/2)*(a^2*x^2 - 1)), x)
```

maple [A] time = 0.03, size = 47, normalized size = 0.64

$$\frac{\left(2x^3a^3-4a^2x^2+ax+2\right)(ax+1)^2}{5\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x)
[Out] 1/5*(2*a^3*x^3-4*a^2*x^2+a*x+2)*(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)/a
```

maxima [B] time = 0.40, size = 218, normalized size = 2.95

$$\frac{1}{5}a\left(\frac{a}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^4cx+\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^3c}-\frac{a}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^4cx-\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^3c}-\frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^3cx+\left(-a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima"
)
[Out] 1/5*a*(a/((-a^2*c*x^2 + c)^(3/2)*a^4*c*x + (-a^2*c*x^2 + c)^(3/2)*a^3*c) -
a/((-a^2*c*x^2 + c)^(3/2)*a^4*c*x - (-a^2*c*x^2 + c)^(3/2)*a^3*c) - 1/((-a^
2*c*x^2 + c)^(3/2)*a^3*c*x + (-a^2*c*x^2 + c)^(3/2)*a^2*c) - 1/((-a^2*c*x^2
 + c)^(3/2)*a^3*c*x - (-a^2*c*x^2 + c)^(3/2)*a^2*c) + 2*x/(sqrt(-a^2*c*x^2
+ c)*a*c^2) + x/((-a^2*c*x^2 + c)^(3/2)*a*c))
```

mupad [B] time = 1.06, size = 56, normalized size = 0.76

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2\right)}{5\,a\,c^3\,{\left(a\,x-1\right)}^3\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - a^2*c*x^2)^(5/2)*(a^2*x^2 - 1)),x)
```

```
[Out] -((c - a^2*c*x^2)^(1/2)*(a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2))/(5*a*c^3*(a*x -
1)^3*(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{a^5c^2x^5\sqrt{-a^2cx^2+c} - a^4c^2x^4\sqrt{-a^2cx^2+c} - 2a^3c^2x^3\sqrt{-a^2cx^2+c} + 2a^2c^2x^2\sqrt{-a^2cx^2+c} + ac^2x\sqrt{-a^2cx^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -Integral(a*x/(a**5*c**2*x**5*sqrt(-a**2*c*x**2 + c) - a**4*c**2*x**4*sqrt(
-a**2*c*x**2 + c) - 2*a**3*c**2*x**3*sqrt(-a**2*c*x**2 + c) + 2*a**2*c**2*x
**2*sqrt(-a**2*c*x**2 + c) + a*c**2*x*sqrt(-a**2*c*x**2 + c) - c**2*sqrt(-a
**2*c*x**2 + c)), x) - Integral(1/(a**5*c**2*x**5*sqrt(-a**2*c*x**2 + c) -
a**4*c**2*x**4*sqrt(-a**2*c*x**2 + c) - 2*a**3*c**2*x**3*sqrt(-a**2*c*x**2
+ c) + 2*a**2*c**2*x**2*sqrt(-a**2*c*x**2 + c) + a*c**2*x*sqrt(-a**2*c*x**2
 + c) - c**2*sqrt(-a**2*c*x**2 + c)), x)
```

3.1124 $\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$\frac{8x}{21c^3\sqrt{c-a^2cx^2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{2(ax+1)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] 2/7*(a*x+1)/a/(-a^2*c*x^2+c)^(7/2)+1/7*x/c/(-a^2*c*x^2+c)^(5/2)+4/21*x/c^2/
(-a^2*c*x^2+c)^(3/2)+8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6141, 653, 192, 191}

$$\frac{8x}{21c^3\sqrt{c-a^2cx^2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{2(ax+1)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (2*(1 + a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) + x/(7*c*(c - a^2*c*x^2)^(5/2)) +
 (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx &= c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{9/2}}\,dx\\ &= \frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}}+\frac{5}{7}\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx\\ &= \frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{7c}\\ &= \frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{21c^2}\\ &= \frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8x}{21c^3\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 96, normalized size = 0.99

$$-\frac{\sqrt{1-a^2x^2}\left(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6\right)}{21ac^3(1-ax)^{7/2}(ax+1)^{3/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/21*(Sqrt[1 - a^2*x^2]*(-6 - 9*a*x + 24*a^2*x^2 - 4*a^3*x^3 - 16*a^4*x^4
+ 8*a^5*x^5))/(a*c^3*(1 - a*x)^(7/2)*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.97, size = 124, normalized size = 1.28

$$-\frac{\left(8\,a^5x^5-16\,a^4x^4-4\,a^3x^3+24\,a^2x^2-9\,ax-6\right)\sqrt{-a^2cx^2+c}}{21\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas"
)

[Out] -1/21*(8*a^5*x^5 - 16*a^4*x^4 - 4*a^3*x^3 + 24*a^2*x^2 - 9*a*x - 6)*sqrt(-a
^2*c*x^2 + c)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 -
a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")

[Out] integrate(-(a*x + 1)^2/((-a^2*c*x^2 + c)^(7/2)*(a^2*x^2 - 1)), x)

maple [A] time = 0.03, size = 64, normalized size = 0.66

$$-\frac{\left(8x^5a^5-16x^4a^4-4x^3a^3+24a^2x^2-9ax-6\right)(ax+1)^2}{21\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x)

[Out] -1/21*(8*a^5*x^5-16*a^4*x^4-4*a^3*x^3+24*a^2*x^2-9*a*x-6)*(a*x+1)^2/(-a^2*c
*x^2+c)^(7/2)/a

maxima [B] time = 0.45, size = 242, normalized size = 2.49

$$\frac{1}{21}a\left(\frac{3a}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^4cx+\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^3c}-\frac{3a}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^4cx-\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^3c}-\frac{3}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^3cx+\left(-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima"
)

[Out] 1/21*a*(3*a/((-a^2*c*x^2 + c)^(5/2)*a^4*c*x + (-a^2*c*x^2 + c)^(5/2)*a^3*c)
 - 3*a/((-a^2*c*x^2 + c)^(5/2)*a^4*c*x - (-a^2*c*x^2 + c)^(5/2)*a^3*c) - 3/

```
((-a^2*c*x^2 + c)^(5/2)*a^3*c*x + (-a^2*c*x^2 + c)^(5/2)*a^2*c) - 3/((-a^2*
c*x^2 + c)^(5/2)*a^3*c*x - (-a^2*c*x^2 + c)^(5/2)*a^2*c) + 8*x/(sqrt(-a^2*c
*x^2 + c)*a*c^3) + 4*x/((-a^2*c*x^2 + c)^(3/2)*a*c^2) + 3*x/((-a^2*c*x^2 +
c)^(5/2)*a*c))
```

mupad [B] time = 1.10, size = 133, normalized size = 1.37

$$\frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,{\left(a\,x-1\right)}^4}-\frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,{\left(a\,x-1\right)}^3}+\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}+\frac{5}{28\,a\,c^4}\right)}{{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^2}-\frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x + 1)^2/((c - a^2*c*x^2)^(7/2)*(a^2*x^2 - 1)),x)
```

```
[Out] (c - a^2*c*x^2)^(1/2)/(28*a*c^4*(a*x - 1)^4) - (c - a^2*c*x^2)^(1/2)/(14*a*
c^4*(a*x - 1)^3) + ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) + 5/(28*a*c^4)))
/((a*x - 1)^2*(a*x + 1)^2) - (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x - 1)*
(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{-a^7c^3x^7\sqrt{-a^2cx^2+c}+a^6c^3x^6\sqrt{-a^2cx^2+c}+3a^5c^3x^5\sqrt{-a^2cx^2+c}-3a^4c^3x^4\sqrt{-a^2cx^2+c}-3a^3c^3x^3\sqrt{-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] -Integral(a*x/(-a**7*c**3*x**7*sqrt(-a**2*c*x**2 + c) + a**6*c**3*x**6*sqrt
(-a**2*c*x**2 + c) + 3*a**5*c**3*x**5*sqrt(-a**2*c*x**2 + c) - 3*a**4*c**3*
x**4*sqrt(-a**2*c*x**2 + c) - 3*a**3*c**3*x**3*sqrt(-a**2*c*x**2 + c) + 3*a
**2*c**3*x**2*sqrt(-a**2*c*x**2 + c) + a*c**3*x*sqrt(-a**2*c*x**2 + c) - c*
*3*sqrt(-a**2*c*x**2 + c)), x) - Integral(1/(-a**7*c**3*x**7*sqrt(-a**2*c*x
**2 + c) + a**6*c**3*x**6*sqrt(-a**2*c*x**2 + c) + 3*a**5*c**3*x**5*sqrt(-a
**2*c*x**2 + c) - 3*a**4*c**3*x**4*sqrt(-a**2*c*x**2 + c) - 3*a**3*c**3*x**
3*sqrt(-a**2*c*x**2 + c) + 3*a**2*c**3*x**2*sqrt(-a**2*c*x**2 + c) + a*c**3
*x*sqrt(-a**2*c*x**2 + c) - c**3*sqrt(-a**2*c*x**2 + c)), x)
```

3.1125 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=120

$$\frac{a^6c^3x^{m+7}}{m+7}+\frac{2a^5c^3x^{m+6}}{m+6}-\frac{a^4c^3x^{m+5}}{m+5}-\frac{4a^3c^3x^{m+4}}{m+4}-\frac{a^2c^3x^{m+3}}{m+3}+\frac{2ac^3x^{m+2}}{m+2}+\frac{c^3x^{m+1}}{m+1}$$

```
[Out] c^3*x^(1+m)/(1+m)+2*a*c^3*x^(2+m)/(2+m)-a^2*c^3*x^(3+m)/(3+m)-4*a^3*c^3*x^(
4+m)/(4+m)-a^4*c^3*x^(5+m)/(5+m)+2*a^5*c^3*x^(6+m)/(6+m)+a^6*c^3*x^(7+m)/(7
+m)
```

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 88}

$$-\frac{a^2c^3x^{m+3}}{m+3}-\frac{4a^3c^3x^{m+4}}{m+4}-\frac{a^4c^3x^{m+5}}{m+5}+\frac{2a^5c^3x^{m+6}}{m+6}+\frac{a^6c^3x^{m+7}}{m+7}+\frac{2ac^3x^{m+2}}{m+2}+\frac{c^3x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*x^(1 + m))/(1 + m) + (2*a*c^3*x^(2 + m))/(2 + m) - (a^2*c^3*x^(3 + m))
/(3 + m) - (4*a^3*c^3*x^(4 + m))/(4 + m) - (a^4*c^3*x^(5 + m))/(5 + m) + (2
*a^5*c^3*x^(6 + m))/(6 + m) + (a^6*c^3*x^(7 + m))/(7 + m)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^3\,dx = c^3\int x^m(1-ax)^2(1+ax)^4\,dx$$
$$= c^3\int\left(x^m+2ax^{1+m}-a^2x^{2+m}-4a^3x^{3+m}-a^4x^{4+m}+2a^5x^{5+m}+a^6x^{6+m}\right)dx$$
$$= \frac{c^3x^{1+m}}{1+m}+\frac{2ac^3x^{2+m}}{2+m}-\frac{a^2c^3x^{3+m}}{3+m}-\frac{4a^3c^3x^{4+m}}{4+m}-\frac{a^4c^3x^{5+m}}{5+m}+\frac{2a^5c^3x^{6+m}}{6+m}+$$

Mathematica [A] time = 0.07, size = 88, normalized size = 0.73

$$c^3x^{m+1}\left(\frac{a^6x^6}{m+7}+\frac{2a^5x^5}{m+6}-\frac{a^4x^4}{m+5}-\frac{4a^3x^3}{m+4}-\frac{a^2x^2}{m+3}+\frac{2ax}{m+2}+\frac{1}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^3,x]
```

```
[Out] c^3*x^(1 + m)*((1 + m)^(-1) + (2*a*x)/(2 + m) - (a^2*x^2)/(3 + m) - (4*a^3*
x^3)/(4 + m) - (a^4*x^4)/(5 + m) + (2*a^5*x^5)/(6 + m) + (a^6*x^6)/(7 + m))
```

fricas [B] time = 0.59, size = 540, normalized size = 4.50

$$\frac{\left(\left(a^6c^3m^6+21\,a^6c^3m^5+175\,a^6c^3m^4+735\,a^6c^3m^3+1624\,a^6c^3m^2+1764\,a^6c^3m+720\,a^6c^3\right)x^7+2\left(a^5c^3m^6+22\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] ((a^6*c^3*m^6 + 21*a^6*c^3*m^5 + 175*a^6*c^3*m^4 + 735*a^6*c^3*m^3 + 1624*a
^6*c^3*m^2 + 1764*a^6*c^3*m + 720*a^6*c^3)*x^7 + 2*(a^5*c^3*m^6 + 22*a^5*c^
3*m^5 + 190*a^5*c^3*m^4 + 820*a^5*c^3*m^3 + 1849*a^5*c^3*m^2 + 2038*a^5*c^3
*m + 840*a^5*c^3)*x^6 - (a^4*c^3*m^6 + 23*a^4*c^3*m^5 + 207*a^4*c^3*m^4 + 9
25*a^4*c^3*m^3 + 2144*a^4*c^3*m^2 + 2412*a^4*c^3*m + 1008*a^4*c^3)*x^5 - 4*
(a^3*c^3*m^6 + 24*a^3*c^3*m^5 + 226*a^3*c^3*m^4 + 1056*a^3*c^3*m^3 + 2545*a
^3*c^3*m^2 + 2952*a^3*c^3*m + 1260*a^3*c^3)*x^4 - (a^2*c^3*m^6 + 25*a^2*c^3
*m^5 + 247*a^2*c^3*m^4 + 1219*a^2*c^3*m^3 + 3112*a^2*c^3*m^2 + 3796*a^2*c^3
*m + 1680*a^2*c^3)*x^3 + 2*(a*c^3*m^6 + 26*a*c^3*m^5 + 270*a*c^3*m^4 + 1420
*a*c^3*m^3 + 3929*a*c^3*m^2 + 5274*a*c^3*m + 2520*a*c^3)*x^2 + (c^3*m^6 + 2
7*c^3*m^5 + 295*c^3*m^4 + 1665*c^3*m^3 + 5104*c^3*m^2 + 8028*c^3*m + 5040*c
^3)*x)*x^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 1306
8*m + 5040)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2-c\right)^3(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^3*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

maple [B] time = 0.03, size = 476, normalized size = 3.97

$$\frac{c^3x^{1+m}\left(a^6m^6x^6+21a^6m^5x^6+175a^6m^4x^6+2a^5m^6x^5+735a^6m^3x^6+44a^5m^5x^5+1624a^6m^2x^6+380a^5m^4x^5-a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*x^(1+m)*(a^6*m^6*x^6+21*a^6*m^5*x^6+175*a^6*m^4*x^6+2*a^5*m^6*x^5+735*a
^6*m^3*x^6+44*a^5*m^5*x^5+1624*a^6*m^2*x^6+380*a^5*m^4*x^5-a^4*m^6*x^4+1764
*a^6*m*x^6+1640*a^5*m^3*x^5-23*a^4*m^5*x^4+720*a^6*x^6+3698*a^5*m^2*x^5-207
*a^4*m^4*x^4-4*a^3*m^6*x^3+4076*a^5*m*x^5-925*a^4*m^3*x^4-96*a^3*m^5*x^3+16
80*a^5*x^5-2144*a^4*m^2*x^4-904*a^3*m^4*x^3-a^2*m^6*x^2-2412*a^4*m*x^4-4224
*a^3*m^3*x^3-25*a^2*m^5*x^2-1008*a^4*x^4-10180*a^3*m^2*x^3-247*a^2*m^4*x^2+
2*a*m^6*x-11808*a^3*m*x^3-1219*a^2*m^3*x^2+52*a*m^5*x-5040*a^3*x^3-3112*a^2
*m^2*x^2+540*a*m^4*x+m^6-3796*a^2*m*x^2+2840*a*m^3*x+27*m^5-1680*a^2*x^2+78
58*a*m^2*x+295*m^4+10548*a*m*x+1665*m^3+5040*a*x+5104*m^2+8028*m+5040)/(7+m
)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

maxima [B] time = 0.39, size = 304, normalized size = 2.53

$$\frac{\left(\left(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720\right)a^6c^3x^7+2\left(m^6+22\,m^5+190\,m^4+820\,m^3+1849\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*a^6*c^3*x^7 +
 2*(m^6 + 22*m^5 + 190*m^4 + 820*m^3 + 1849*m^2 + 2038*m + 840)*a^5*c^3*x^6
 - (m^6 + 23*m^5 + 207*m^4 + 925*m^3 + 2144*m^2 + 2412*m + 1008)*a^4*c^3*x^
5 - 4*(m^6 + 24*m^5 + 226*m^4 + 1056*m^3 + 2545*m^2 + 2952*m + 1260)*a^3*c^
3*x^4 - (m^6 + 25*m^5 + 247*m^4 + 1219*m^3 + 3112*m^2 + 3796*m + 1680)*a^2*
```

```
c^3*x^3 + 2*(m^6 + 26*m^5 + 270*m^4 + 1420*m^3 + 3929*m^2 + 5274*m + 2520)*
a*c^3*x^2 + (m^6 + 27*m^5 + 295*m^4 + 1665*m^3 + 5104*m^2 + 8028*m + 5040)*
c^3*x)*x^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 1306
8*m + 5040)
```

mupad [B] time = 1.23, size = 531, normalized size = 4.42

$$\frac{c^3\,x\,x^m\,\left(m^6+27\,m^5+295\,m^4+1665\,m^3+5104\,m^2+8028\,m+5040\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}+\frac{2\,a\,c^3\,x^m\,x^2\,\left(m^6+26\,m^5+270\,m^4\right.}{m^7+28\,m^6+322\,m^5+1960\,m^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(c - a^2*c*x^2)^3*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] (c^3*x*x^m*(8028*m + 5104*m^2 + 1665*m^3 + 295*m^4 + 27*m^5 + m^6 + 5040))/
(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
 + (2*a*c^3*x^m*x^2*(5274*m + 3929*m^2 + 1420*m^3 + 270*m^4 + 26*m^5 + m^6
+ 2520))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^
7 + 5040) + (a^6*c^3*x^m*x^7*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^
5 + m^6 + 720))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m
^6 + m^7 + 5040) + (2*a^5*c^3*x^m*x^6*(2038*m + 1849*m^2 + 820*m^3 + 190*m^
4 + 22*m^5 + m^6 + 840))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m
^5 + 28*m^6 + m^7 + 5040) - (a^4*c^3*x^m*x^5*(2412*m + 2144*m^2 + 925*m^3 +
 207*m^4 + 23*m^5 + m^6 + 1008))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4
 + 322*m^5 + 28*m^6 + m^7 + 5040) - (4*a^3*c^3*x^m*x^4*(2952*m + 2545*m^2 +
 1056*m^3 + 226*m^4 + 24*m^5 + m^6 + 1260))/(13068*m + 13132*m^2 + 6769*m^3
 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) - (a^2*c^3*x^m*x^3*(3796*m + 3
112*m^2 + 1219*m^3 + 247*m^4 + 25*m^5 + m^6 + 1680))/(13068*m + 13132*m^2 +
 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
```

sympy [A] time = 3.47, size = 3009, normalized size = 25.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**3,x)
```

```
[Out] Piecewise((a**6*c**3*log(x) - 2*a**5*c**3/x + a**4*c**3/(2*x**2) + 4*a**3*c
**3/(3*x**3) + a**2*c**3/(4*x**4) - 2*a*c**3/(5*x**5) - c**3/(6*x**6), Eq(m
, -7)), (a**6*c**3*x + 2*a**5*c**3*log(x) + a**4*c**3/x + 2*a**3*c**3/x**2
+ a**2*c**3/(3*x**3) - a*c**3/(2*x**4) - c**3/(5*x**5), Eq(m, -6)), (a**6*c
**3*x**2/2 + 2*a**5*c**3*x - a**4*c**3*log(x) + 4*a**3*c**3/x + a**2*c**3/(
2*x**2) - 2*a*c**3/(3*x**3) - c**3/(4*x**4), Eq(m, -5)), (a**6*c**3*x**3/3
+ a**5*c**3*x**2 - a**4*c**3*x - 4*a**3*c**3*log(x) + a**2*c**3/x - a*c**3/
x**2 - c**3/(3*x**3), Eq(m, -4)), (a**6*c**3*x**4/4 + 2*a**5*c**3*x**3/3 -
```

a**4*c**3*x**2/2 - 4*a**3*c**3*x - a**2*c**3*log(x) - 2*a*c**3/x - c**3/(2*
x**2), Eq(m, -3)), (a**6*c**3*x**5/5 + a**5*c**3*x**4/2 - a**4*c**3*x**3/3
- 2*a**3*c**3*x**2 - a**2*c**3*x + 2*a*c**3*log(x) - c**3/x, Eq(m, -2)), (a
6*c3*x**6/6 + 2*a**5*c**3*x**5/5 - a**4*c**3*x**4/4 - 4*a**3*c**3*x**3/
3 - a**2*c**3*x**2/2 + 2*a*c**3*x + c**3*log(x), Eq(m, -1)), (a**6*c**3*m**
6*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + 21*a**6*c**3*m**5*x**7*x**m/(m**7 + 28*m**6 + 322*m**5
 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 175*a**6*c**3*m**
4*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + 735*a**6*c**3*m**3*x**7*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1624*a**6*c**3*m
2*x7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m*
*2 + 13068*m + 5040) + 1764*a**6*c**3*m*x**7*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 720*a**6*c**3*x*
*7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1
3068*m + 5040) + 2*a**5*c**3*m**6*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 44*a**5*c**3*m**5*x**6
*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 130
68*m + 5040) + 380*a**5*c**3*m**4*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1640*a**5*c**3*m**3*x*
*6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1
3068*m + 5040) + 3698*a**5*c**3*m**2*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 4076*a**5*c**3*m*x*
*6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1
3068*m + 5040) + 1680*a**5*c**3*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960
*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - a**4*c**3*m**6*x**5*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) - 23*a**4*c**3*m**5*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**
4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - 207*a**4*c**3*m**4*x**5*x**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) - 925*a**4*c**3*m**3*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - 2144*a**4*c**3*m**2*x**5*x*
*m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*
m + 5040) - 2412*a**4*c**3*m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - 1008*a**4*c**3*x**5*x**m/(m
7 + 28*m6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) - 4*a**3*c**3*m**6*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) - 96*a**3*c**3*m**5*x**4*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) - 904*a**3*c**3*m**4*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) - 4224*a**3*c**3*m**3*x**4*x**m/(m
7 + 28*m6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) - 10180*a**3*c**3*m**2*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**
4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) - 11808*a**3*c**3*m*x**4*x**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +

```
 5040) - 5040*a**3*c**3*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) - a**2*c**3*m**6*x**3*x**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
- 25*a**2*c**3*m**5*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769
*m**3 + 13132*m**2 + 13068*m + 5040) - 247*a**2*c**3*m**4*x**3*x**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
- 1219*a**2*c**3*m**3*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 67
69*m**3 + 13132*m**2 + 13068*m + 5040) - 3112*a**2*c**3*m**2*x**3*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) - 3796*a**2*c**3*m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 67
69*m**3 + 13132*m**2 + 13068*m + 5040) - 1680*a**2*c**3*x**3*x**m/(m**7 + 2
8*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) +
2*a*c**3*m**6*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3
+ 13132*m**2 + 13068*m + 5040) + 52*a*c**3*m**5*x**2*x**m/(m**7 + 28*m**6 +
 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 540*a*c*
*3*m**4*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 1313
2*m**2 + 13068*m + 5040) + 2840*a*c**3*m**3*x**2*x**m/(m**7 + 28*m**6 + 322
*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 7858*a*c**3*
m**2*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m
**2 + 13068*m + 5040) + 10548*a*c**3*m*x**2*x**m/(m**7 + 28*m**6 + 322*m**5
 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5040*a*c**3*x**2*
x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1306
8*m + 5040) + c**3*m**6*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 676
9*m**3 + 13132*m**2 + 13068*m + 5040) + 27*c**3*m**5*x*x**m/(m**7 + 28*m**6
 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 295*c*
*3*m**4*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m
**2 + 13068*m + 5040) + 1665*c**3*m**3*x*x**m/(m**7 + 28*m**6 + 322*m**5 +
1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5104*c**3*m**2*x*x**
m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
 + 5040) + 8028*c**3*m*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769
*m**3 + 13132*m**2 + 13068*m + 5040) + 5040*c**3*x*x**m/(m**7 + 28*m**6 + 3
22*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040), True))
```

3.1126 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=67

$$-\frac{a^4c^2x^{m+5}}{m+5}-\frac{2a^3c^2x^{m+4}}{m+4}+\frac{2ac^2x^{m+2}}{m+2}+\frac{c^2x^{m+1}}{m+1}$$

```
[Out] c^2*x^(1+m)/(1+m)+2*a*c^2*x^(2+m)/(2+m)-2*a^3*c^2*x^(4+m)/(4+m)-a^4*c^2*x^(
5+m)/(5+m)
```

Rubi [A] time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 75}

$$-\frac{2a^3c^2x^{m+4}}{m+4}-\frac{a^4c^2x^{m+5}}{m+5}+\frac{2ac^2x^{m+2}}{m+2}+\frac{c^2x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^(1 + m))/(1 + m) + (2*a*c^2*x^(2 + m))/(2 + m) - (2*a^3*c^2*x^(4 + m
))/(4 + m) - (a^4*c^2*x^(5 + m))/(5 + m)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx = c^2\int x^m(1-ax)(1+ax)^3\,dx$$
$$= c^2\int\left(x^m+2ax^{1+m}-2a^3x^{3+m}-a^4x^{4+m}\right)dx$$
$$= \frac{c^2x^{1+m}}{1+m}+\frac{2ac^2x^{2+m}}{2+m}-\frac{2a^3c^2x^{4+m}}{4+m}-\frac{a^4c^2x^{5+m}}{5+m}$$

Mathematica [A] time = 0.11, size = 69, normalized size = 1.03

$$\frac{c^2x^{m+1}\left(2(m+3)\left(\frac{a^3x^3}{m+4}+\frac{3a^2x^2}{m+3}+\frac{3ax}{m+2}+\frac{1}{m+1}\right)-(ax+1)^4\right)}{m+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^(1 + m)*(-(1 + a*x)^4 + 2*(3 + m)*((1 + m)^(-1) + (3*a*x)/(2 + m) +
(3*a^2*x^2)/(3 + m) + (a^3*x^3)/(4 + m))))/(5 + m)
```

fricas [B] time = 0.68, size = 179, normalized size = 2.67

$$-\frac{\left(\left(a^4c^2m^3+7\,a^4c^2m^2+14\,a^4c^2m+8\,a^4c^2\right)x^5+2\left(a^3c^2m^3+8\,a^3c^2m^2+17\,a^3c^2m+10\,a^3c^2\right)x^4-2\left(ac^2m^3+10\right.}{m^4+12\,m^3+49\,m^2+78\,m+40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] -((a^4*c^2*m^3 + 7*a^4*c^2*m^2 + 14*a^4*c^2*m + 8*a^4*c^2)*x^5 + 2*(a^3*c^2
*m^3 + 8*a^3*c^2*m^2 + 17*a^3*c^2*m + 10*a^3*c^2)*x^4 - 2*(a*c^2*m^3 + 10*a
*c^2*m^2 + 29*a*c^2*m + 20*a*c^2)*x^2 - (c^2*m^3 + 11*c^2*m^2 + 38*c^2*m +
40*c^2)*x)*x^m/(m^4 + 12*m^3 + 49*m^2 + 78*m + 40)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2cx^2-c\right)^2(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^2*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

maple [B] time = 0.03, size = 146, normalized size = 2.18

$$-\frac{c^2x^{1+m}\left(a^4m^3x^4+7a^4m^2x^4+14a^4m\,x^4+2a^3m^3x^3+8x^4a^4+16a^3m^2x^3+34a^3m\,x^3+20x^3a^3-2a\,m^3x-20a\,m\right.}{(5+m)\,(4+m)\,(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^2,x)
```

```
[Out] -c^2*x^(1+m)*(a^4*m^3*x^4+7*a^4*m^2*x^4+14*a^4*m*x^4+2*a^3*m^3*x^3+8*a^4*x^
4+16*a^3*m^2*x^3+34*a^3*m*x^3+20*a^3*x^3-2*a*m^3*x-20*a*m^2*x-58*a*m*x-m^3-
40*a*x-11*m^2-38*m-40)/(5+m)/(4+m)/(2+m)/(1+m)
```

maxima [A] time = 0.37, size = 114, normalized size = 1.70

$$-\frac{\left(\left(m^3+7\,m^2+14\,m+8\right)a^4c^2x^5+2\left(m^3+8\,m^2+17\,m+10\right)a^3c^2x^4-2\left(m^3+10\,m^2+29\,m+20\right)ac^2x^2-\left(m^3\right.\right.}{m^4+12\,m^3+49\,m^2+78\,m+40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -((m^3 + 7*m^2 + 14*m + 8)*a^4*c^2*x^5 + 2*(m^3 + 8*m^2 + 17*m + 10)*a^3*c^
2*x^4 - 2*(m^3 + 10*m^2 + 29*m + 20)*a*c^2*x^2 - (m^3 + 11*m^2 + 38*m + 40)
*c^2*x)*x^m/(m^4 + 12*m^3 + 49*m^2 + 78*m + 40)
```

mupad [B] time = 1.02, size = 173, normalized size = 2.58

$$x^m\left(\frac{c^2\,x\,\left(m^3+11\,m^2+38\,m+40\right)}{m^4+12\,m^3+49\,m^2+78\,m+40}+\frac{2\,a\,c^2\,x^2\,\left(m^3+10\,m^2+29\,m+20\right)}{m^4+12\,m^3+49\,m^2+78\,m+40}-\frac{a^4\,c^2\,x^5\,\left(m^3+7\,m^2+14\,m+8\right)}{m^4+12\,m^3+49\,m^2+78\,m+40}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(c - a^2*c*x^2)^2*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] x^m*((c^2*x*(38*m + 11*m^2 + m^3 + 40))/(78*m + 49*m^2 + 12*m^3 + m^4 + 40)
 + (2*a*c^2*x^2*(29*m + 10*m^2 + m^3 + 20))/(78*m + 49*m^2 + 12*m^3 + m^4 +
 40) - (a^4*c^2*x^5*(14*m + 7*m^2 + m^3 + 8))/(78*m + 49*m^2 + 12*m^3 + m^4
 + 40) - (2*a^3*c^2*x^4*(17*m + 8*m^2 + m^3 + 10))/(78*m + 49*m^2 + 12*m^3
+ m^4 + 40))
```

sympy [A] time = 1.85, size = 706, normalized size = 10.54

$$\begin{cases} -a^4c^2\log(x) + \frac{2a^3c^2}{x} - \frac{2ac^2}{3x^3} - \frac{c^2}{4x^4} \\ -a^4c^2x - 2a^3c^2\log(x) - \frac{ac^2}{x^2} - \frac{c^2}{3x^3} \\ -\frac{a^4c^2x^3}{3} - a^3c^2x^2 + 2ac^2\log(x) - \frac{c^2}{x} \\ -\frac{a^4c^2x^4}{4} - \frac{2a^3c^2x^3}{3} + 2ac^2x + c^2\log(x) \\ -\frac{a^4c^2m^3x^5x^m}{m^4+12m^3+49m^2+78m+40} - \frac{7a^4c^2m^2x^5x^m}{m^4+12m^3+49m^2+78m+40} - \frac{14a^4c^2mx^5x^m}{m^4+12m^3+49m^2+78m+40} - \frac{8a^4c^2x^5x^m}{m^4+12m^3+49m^2+78m+40} - \frac{2a^3c^2m^3x^4x^m}{m^4+12m^3+49m^2+7} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((-a**4*c**2*log(x) + 2*a**3*c**2/x - 2*a*c**2/(3*x**3) - c**2/(4*
x**4), Eq(m, -5)), (-a**4*c**2*x - 2*a**3*c**2*log(x) - a*c**2/x**2 - c**2/
(3*x**3), Eq(m, -4)), (-a**4*c**2*x**3/3 - a**3*c**2*x**2 + 2*a*c**2*log(x)
 - c**2/x, Eq(m, -2)), (-a**4*c**2*x**4/4 - 2*a**3*c**2*x**3/3 + 2*a*c**2*x
 + c**2*log(x), Eq(m, -1)), (-a**4*c**2*m**3*x**5*x**m/(m**4 + 12*m**3 + 49
*m**2 + 78*m + 40) - 7*a**4*c**2*m**2*x**5*x**m/(m**4 + 12*m**3 + 49*m**2 +
 78*m + 40) - 14*a**4*c**2*m*x**5*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 4
0) - 8*a**4*c**2*x**5*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) - 2*a**3*
c**2*m**3*x**4*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) - 16*a**3*c**2*m
**2*x**4*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) - 34*a**3*c**2*m*x**4*
x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) - 20*a**3*c**2*x**4*x**m/(m**4
+ 12*m**3 + 49*m**2 + 78*m + 40) + 2*a*c**2*m**3*x**2*x**m/(m**4 + 12*m**3
+ 49*m**2 + 78*m + 40) + 20*a*c**2*m**2*x**2*x**m/(m**4 + 12*m**3 + 49*m**2
 + 78*m + 40) + 58*a*c**2*m*x**2*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40
) + 40*a*c**2*x**2*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) + c**2*m**3*
x*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40) + 11*c**2*m**2*x*x**m/(m**4 +
 12*m**3 + 49*m**2 + 78*m + 40) + 38*c**2*m*x*x**m/(m**4 + 12*m**3 + 49*m**
2 + 78*m + 40) + 40*c**2*x*x**m/(m**4 + 12*m**3 + 49*m**2 + 78*m + 40), Tru
e))
```

3.1127 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{a^2cx^{m+3}}{m+3}+\frac{2acx^{m+2}}{m+2}+\frac{cx^{m+1}}{m+1}$$

```
[Out] c*x^(1+m)/(1+m)+2*a*c*x^(2+m)/(2+m)+a^2*c*x^(3+m)/(3+m)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 43}

$$\frac{a^2cx^{m+3}}{m+3}+\frac{2acx^{m+2}}{m+2}+\frac{cx^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^(1 + m))/(1 + m) + (2*a*c*x^(2 + m))/(2 + m) + (a^2*c*x^(3 + m))/(3 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx&=c\int x^m(1+ax)^2\,dx\\&=c\int\left(x^m+2ax^{1+m}+a^2x^{2+m}\right)\,dx\\&=\frac{cx^{1+m}}{1+m}+\frac{2acx^{2+m}}{2+m}+\frac{a^2cx^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.81

$$cx^{m+1}\left(\frac{a^2x^2}{m+3}+\frac{2ax}{m+2}+\frac{1}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2),x]
[Out] c*x^(1 + m)*((1 + m)^(-1) + (2*a*x)/(2 + m) + (a^2*x^2)/(3 + m))
```

fricas [A] time = 0.55, size = 82, normalized size = 1.95

$$\frac{\left(\left(a^2cm^2+3\,a^2cm+2\,a^2c\right)x^3+2\left(acm^2+4\,acm+3\,ac\right)x^2+\left(cm^2+5\,cm+6\,c\right)x\right)x^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] ((a^2*c*m^2 + 3*a^2*c*m + 2*a^2*c)*x^3 + 2*(a*c*m^2 + 4*a*c*m + 3*a*c)*x^2
+ (c*m^2 + 5*c*m + 6*c)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2cx^2-c\right)(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate((a^2*c*x^2 - c)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

maple [A] time = 0.03, size = 74, normalized size = 1.76

$$\frac{c\,x^{1+m}\left(a^2m^2x^2+3a^2m\,x^2+2a^2x^2+2a\,m^2x+8amx+6ax+m^2+5m+6\right)}{(3+m)\,(2+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c),x)
[Out] c*x^(1+m)*(a^2*m^2*x^2+3*a^2*m*x^2+2*a^2*x^2+2*a*m^2*x+8*a*m*x+6*a*x+m^2+5*
m+6)/(3+m)/(2+m)/(1+m)
```

maxima [A] time = 0.38, size = 62, normalized size = 1.48

$$\frac{\left(\left(m^2+3\,m+2\right)a^2cx^3+2\left(m^2+4\,m+3\right)acx^2+\left(m^2+5\,m+6\right)cx\right)x^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] ((m^2 + 3*m + 2)*a^2*c*x^3 + 2*(m^2 + 4*m + 3)*a*c*x^2 + (m^2 + 5*m + 6)*c*
x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

mupad [B] time = 0.94, size = 92, normalized size = 2.19

$$x^m\left(\frac{c\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6}+\frac{2\,a\,c\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6}+\frac{a^2\,c\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(c - a^2*c*x^2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] x^m*((c*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (2*a*c*x^2*(4*m + m^2
 + 3))/(11*m + 6*m^2 + m^3 + 6) + (a^2*c*x^3*(3*m + m^2 + 2))/(11*m + 6*m^2
 + m^3 + 6))
```

sympy [A] time = 1.10, size = 299, normalized size = 7.12

$$\begin{cases} a^2c\log\left(x\right)-\frac{2ac}{x}-\frac{c}{2x^2} \\ a^2cx+2ac\log\left(x\right)-\frac{c}{x} \\ \frac{a^2cx^2}{2}+2acx+c\log\left(x\right) \\ \frac{a^2cm^2x^3x^m}{m^3+6m^2+11m+6}+\frac{3a^2cmx^3x^m}{m^3+6m^2+11m+6}+\frac{2a^2cx^3x^m}{m^3+6m^2+11m+6}+\frac{2acm^2x^2x^m}{m^3+6m^2+11m+6}+\frac{8acmx^2x^m}{m^3+6m^2+11m+6}+\frac{6acx^2x^m}{m^3+6m^2+11m+6}+\frac{cm^2xx^m}{m^3+6m^2+11m+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c),x)
```

```
[Out] Piecewise((a**2*c*log(x) - 2*a*c/x - c/(2*x**2), Eq(m, -3)), (a**2*c*x + 2*
a*c*log(x) - c/x, Eq(m, -2)), (a**2*c*x**2/2 + 2*a*c*x + c*log(x), Eq(m, -1
)), (a**2*c*m**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*a**2*c*m*x**3*x**
m/(m**3 + 6*m**2 + 11*m + 6) + 2*a**2*c*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6
) + 2*a*c*m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*c*m*x**2*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a*c*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + c*m*
*2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*c*m*x*x**m/(m**3 + 6*m**2 + 11*m +
 6) + 6*c*x*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

3.1128 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{c-a^2cx^2}\,dx$

Optimal. Leaf size=25

$$\frac{x^{m+1}\,{}_2F_1(2,m+1;m+2;ax)}{c(m+1)}$$

```
[Out] x^(1+m)*hypergeom([2, 1+m],[2+m],a*x)/c/(1+m)
```

Rubi [A] time = 0.08, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 64}

$$\frac{x^{m+1}\,{}_2F_1(2,m+1;m+2;ax)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, a*x])/(c*(1 + m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^m}{c-a^2cx^2}\,dx &= \frac{\int \frac{x^m}{(1-ax)^2}\,dx}{c}\\
&= \frac{x^{1+m}\,{}_2F_1(2,1+m;2+m;ax)}{c(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{x^{m+1}\,{}_2F_1(2,m+1;m+2;ax)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, a*x])/(c*(1 + m))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{a^2cx^2 - 2\,acx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m/(a^2*c*x^2 - 2*a*c*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2 x^m}{(a^2cx^2 - c)(a^2x^2 - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)*(a^2*x^2 - 1)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2\,x^m}{(-a^2x^2+1)\,(-a^2c\,x^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2 x^m}{(a^2cx^2 - c)(a^2x^2 - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)*(a^2*x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x^m\,{\left(a\,x+1\right)}^2}{\left(c-a^2\,c\,x^2\right)\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(a*x + 1)^2)/((c - a^2*c*x^2)*(a^2*x^2 - 1)),x)
[Out] -int((x^m*(a*x + 1)^2)/((c - a^2*c*x^2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{m}}{a^{2} x^{2} - 2 a x + 1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c),x)
[Out] Integral(x**m/(a**2*x**2 - 2*a*x + 1), x)/c
```

3.1129 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=113

$$\frac{(2m^2-4m+1)\,x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{8c^2(m+1)}+\frac{x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{8c^2(m+1)}+\frac{(2-m)x^{m+1}}{4c^2(1-ax)}+\frac{x^{m+1}}{4c^2(1-ax)^2}$$

```
[Out] 1/4*x^(1+m)/c^2/(-a*x+1)^2+1/4*(2-m)*x^(1+m)/c^2/(-a*x+1)+1/8*x^(1+m)*hyper
geom([1, 1+m],[2+m],-a*x)/c^2/(1+m)+1/8*(2*m^2-4*m+1)*x^(1+m)*hypergeom([1,
 1+m],[2+m],a*x)/c^2/(1+m)
```

Rubi [A] time = 0.18, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6150, 103, 151, 156, 64}

$$\frac{(2m^2-4m+1)\,x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{8c^2(m+1)}+\frac{x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{8c^2(m+1)}+\frac{(2-m)x^{m+1}}{4c^2(1-ax)}+\frac{x^{m+1}}{4c^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] x^(1 + m)/(4*c^2*(1 - a*x)^2) + ((2 - m)*x^(1 + m))/(4*c^2*(1 - a*x)) + (x^
(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(8*c^2*(1 + m)) + ((1 -
 4*m + 2*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(8*c^2*(1
+ m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{x^m}{(1-ax)^3(1+ax)}\,dx}{c^2}$$

$$= \frac{x^{1+m}}{4c^2(1-ax)^2} - \frac{\int \frac{x^m\left(-a(3-m)-a^2(1-m)x\right)}{(1-ax)^2(1+ax)}\,dx}{4ac^2}$$

$$= \frac{x^{1+m}}{4c^2(1-ax)^2} + \frac{(2-m)x^{1+m}}{4c^2(1-ax)} + \frac{\int \frac{x^m\left(2a^2(1-m)^2-2a^3(2-m)mx\right)}{(1-ax)(1+ax)}\,dx}{8a^2c^2}$$

$$= \frac{x^{1+m}}{4c^2(1-ax)^2} + \frac{(2-m)x^{1+m}}{4c^2(1-ax)} + \frac{\int \frac{x^m}{1+ax}\,dx}{8c^2} + \frac{\left(1-4m+2m^2\right)\int \frac{x^m}{1-ax}\,dx}{8c^2}$$

$$= \frac{x^{1+m}}{4c^2(1-ax)^2} + \frac{(2-m)x^{1+m}}{4c^2(1-ax)} + \frac{x^{1+m}\,{}_2F_1(1,1+m;2+m;-ax)}{8c^2(1+m)} + \frac{\left(1-4m+2m^2\right)x^{1+m}\,{}_2F}{8c^2(1-}$$

Mathematica [A] time = 0.07, size = 92, normalized size = 0.81

$$\frac{x^{m+1}\left(\left(2m^2-4m+1\right)(ax-1)^2\,{}_2F_1(1,m+1;m+2;ax)+(ax-1)^2\,{}_2F_1(1,m+1;m+2;-ax)+2(m+1)(m(ax-\right.}{8c^2(m+1)(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(2*(1 + m)*(3 - 2*a*x + m*(-1 + a*x)) + (-1 + a*x)^2*Hypergeomet
ric2F1[1, 1 + m, 2 + m, -(a*x)] + (1 - 4*m + 2*m^2)*(-1 + a*x)^2*Hypergeome
tric2F1[1, 1 + m, 2 + m, a*x]))/(8*c^2*(1 + m)*(-1 + a*x)^2)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^m}{a^4c^2x^4-2\,a^3c^2x^3+2\,ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="fricas"
)
```

```
[Out] integral(-x^m/(a^4*c^2*x^4 - 2*a^3*c^2*x^3 + 2*a*c^2*x - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2x^m}{\left(a^2cx^2-c\right)^2\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a*x + 1)^2*x^m/((a^2*c*x^2 - c)^2*(a^2*x^2 - 1)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2\,x^m}{\left(-a^2x^2+1\right)\left(-a^2c\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2 x^m}{\left(a^2cx^2-c\right)^2\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^2,x, algorithm="maxima"
)
```

```
[Out] -integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)^2*(a^2*x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^m\,(a\,x+1)^2}{\left(c-a^2\,c\,x^2\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(a*x + 1)^2)/((c - a^2*c*x^2)^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int((x^m*(a*x + 1)^2)/((c - a^2*c*x^2)^2*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^m}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -Integral(x**m/(a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1), x)/c**2
```

3.1130 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=203

$$\frac{(2-m)\left(2m^2-8m+3\right)x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{48c^3(m+1)}+\frac{(2-m)x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{16c^3(m+1)}-\frac{(2-m)(4-m)x^m}{24c^3(ax+1)}$$

```
[Out] -1/24*(2-m)*(4-m)*x^(1+m)/c^3/(a*x+1)+1/6*x^(1+m)/c^3/(-a*x+1)^3/(a*x+1)+1/
12*(4-m)*x^(1+m)/c^3/(-a*x+1)^2/(a*x+1)+1/24*(7-2*m)*(2-m)*x^(1+m)/c^3/(-a*
x+1)/(a*x+1)+1/16*(2-m)*x^(1+m)*hypergeom([1, 1+m],[2+m],-a*x)/c^3/(1+m)+1/
48*(2-m)*(2*m^2-8*m+3)*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)/c^3/(1+m)
```

Rubi [A] time = 0.35, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6150, 103, 151, 156, 64}

$$\frac{(2-m)\left(2m^2-8m+3\right)x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{48c^3(m+1)}+\frac{(2-m)x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{16c^3(m+1)}-\frac{(2-m)(4-m)x^m}{24c^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^3,x]
```

```
[Out] -((2 - m)*(4 - m)*x^(1 + m))/(24*c^3*(1 + a*x)) + x^(1 + m)/(6*c^3*(1 - a*x
)^3*(1 + a*x)) + ((4 - m)*x^(1 + m))/(12*c^3*(1 - a*x)^2*(1 + a*x)) + ((7 -
 2*m)*(2 - m)*x^(1 + m))/(24*c^3*(1 - a*x)*(1 + a*x)) + ((2 - m)*x^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(16*c^3*(1 + m)) + ((2 - m)*(3
- 8*m + 2*m^2)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(48*c^3*(
1 + m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^m}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$

$$= \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} - \frac{\int \frac{x^m\left(-a(5-m)-a^2(3-m)x\right)}{(1-ax)^3(1+ax)^2}\,dx}{6ac^3}$$

$$= \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{\int \frac{x^m\left(2a^2(2-m)(3-m)+2a^3(2-m)(4-m)x\right)}{(1-ax)^2(1+ax)^2}\,dx}{24a^2c^3}$$

$$= \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{(7-2m)(2-m)x^{1+m}}{24c^3(1-ax)(1+ax)} - \frac{\int \frac{x^m\left(2a^3(2-m)\left(1+7m\right.\right.}{(}}{}$$

$$= -\frac{(2-m)(4-m)x^{1+m}}{24c^3(1+ax)} + \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{(7-2m)(2-m)x}{24c^3(1-ax)(1+}$$

$$= -\frac{(2-m)(4-m)x^{1+m}}{24c^3(1+ax)} + \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{(7-2m)(2-m)x}{24c^3(1-ax)(1+}$$

$$= -\frac{(2-m)(4-m)x^{1+m}}{24c^3(1+ax)} + \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{(7-2m)(2-m)x}{24c^3(1-ax)(1+}$$

Mathematica [A] time = 0.16, size = 194, normalized size = 0.96

$$\frac{x^{m+1}\left(-2\left(m^2\left(-5a^3x^3+6a^2x^2+5ax-6\right)+m\left(2a^3x^3-3a^2x^2-6ax+11\right)+2\left(4a^3x^3-5a^2x^2-6ax+9\right)+m^3(a\right.\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(-2*(m^3*(-1 + a*x)^2*(1 + a*x) + m^2*(-6 + 5*a*x + 6*a^2*x^2 -
5*a^3*x^3) + m*(11 - 6*a*x - 3*a^2*x^2 + 2*a^3*x^3) + 2*(9 - 6*a*x - 5*a^2*
x^2 + 4*a^3*x^3)) - 3*(-2 + m)*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[1,
1 + m, 2 + m, -(a*x)] - (-6 + 19*m - 12*m^2 + 2*m^3)*(-1 + a*x)^3*(1 + a*x)
*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(48*c^3*(1 + m)*(-1 + a*x)^3*(1
+ a*x))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{a^6c^3x^6-2\,a^5c^3x^5-a^4c^3x^4+4\,a^3c^3x^3-a^2c^3x^2-2\,ac^3x+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] integral(x^m/(a^6*c^3*x^6 - 2*a^5*c^3*x^5 - a^4*c^3*x^4 + 4*a^3*c^3*x^3 - a
^2*c^3*x^2 - 2*a*c^3*x + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2 x^m}{\left(a^2cx^2-c\right)^3\left(a^2x^2-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)^3*(a^2*x^2 - 1)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2\, x^m}{\left(-a^2x^2+1\right)\left(-a^2c\, x^2+c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2 x^m}{\left(a^2cx^2-c\right)^3\left(a^2x^2-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] integrate((a*x + 1)^2*x^m/((a^2*c*x^2 - c)^3*(a^2*x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^m\,(a\,x+1)^2}{\left(c-a^2\,c\,x^2\right)^3\,\left(a^2\,x^2-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(a*x + 1)^2)/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] -int((x^m*(a*x + 1)^2)/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(x**m/(a**6*x**6 - 2*a**5*x**5 - a**4*x**4 + 4*a**3*x**3 - a**2*x**
2 - 2*a*x + 1), x)/c**3
```

3.1131 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=176

$$\frac{c^2(2m+7)x^{m+1}\sqrt{c-a^2cx^2}\ {}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+6)\sqrt{1-a^2x^2}}+\frac{2ac^2x^{m+2}\sqrt{c-a^2cx^2}\ {}_2F_1\left(-\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{1-a^2x^2}}-\frac{x^{m+1}\left(c-\right.}{m}$$

```
[Out] -x^(1+m)*(-a^2*c*x^2+c)^(5/2)/(6+m)+c^2*(7+2*m)*x^(1+m)*hypergeom([-3/2, 1/
2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*c*x^2+c)^(1/2)/(m^2+7*m+6)/(-a^2*x^2+1)
^(1/2)+2*a*c^2*x^(2+m)*hypergeom([-3/2, 1+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*c
*x^2+c)^(1/2)/(2+m)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1809, 808, 365, 364}

$$\frac{c^2(2m+7)x^{m+1}\sqrt{c-a^2cx^2}\ {}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+6)\sqrt{1-a^2x^2}}+\frac{2ac^2x^{m+2}\sqrt{c-a^2cx^2}\ {}_2F_1\left(-\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{1-a^2x^2}}-\frac{x^{m+1}\left(c-\right.}{m}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x^(1 + m)*(c - a^2*c*x^2)^(5/2))/(6 + m)) + (c^2*(7 + 2*m)*x^(1 + m)*Sqr
t[c - a^2*c*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((
1 + m)*(6 + m)*Sqrt[1 - a^2*x^2]) + (2*a*c^2*x^(2 + m)*Sqrt[c - a^2*c*x^2]*
Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[1 - a
^2*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{5/2}\,dx = c\int x^m(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx$$

$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{5/2}}{6+m} - \frac{\int x^m\left(-a^2c(7+2m)-2a^3c(6+m)x\right)\left(c-a^2cx^2\right)^{3/2}\,d}{a^2(6+m)}$$

$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{5/2}}{6+m} + (2ac)\int x^{1+m}\left(c-a^2cx^2\right)^{3/2}\,dx + \frac{(c(7+2m))\int x^m\left(c\right.}{6+r}$$

$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{5/2}}{6+m} + \frac{\left(2ac^2\sqrt{c-a^2cx^2}\right)\int x^{1+m}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}} + \frac{\left(c^2(7+\right.}{}$$

$$= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{5/2}}{6+m} + \frac{c^2(7+2m)x^{1+m}\sqrt{c-a^2cx^2}\;{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(6+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.22, size = 180, normalized size = 1.02

$$\frac{c^2x^{m+1}\sqrt{c-a^2cx^2}\left(\frac{2ax\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2}+1;\frac{m}{2}+2;a^2x^2\right)}{m+2}+\frac{{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{a^4x^4\,{}_2F_1\left(-\frac{1}{2},\frac{m+5}{2};\frac{m+7}{2};a^2x^2\right)}{m+5}-\frac{2a^3x^3\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2}+2;\frac{m}{2}+3;\right.}{m+4}\right.}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*x^(1 + m)*Sqrt[c - a^2*c*x^2]*((2*a*x*Hypergeometric2F1[-1/2, 1 + m/2,
 2 + m/2, a^2*x^2])/(2 + m) - (2*a^3*x^3*Hypergeometric2F1[-1/2, 2 + m/2, 3
 + m/2, a^2*x^2])/(4 + m) + Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a
^2*x^2]/(1 + m) - (a^4*x^4*Hypergeometric2F1[-1/2, (5 + m)/2, (7 + m)/2, a^
2*x^2])/(5 + m)))/Sqrt[1 - a^2*x^2]
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^4c^2x^4+2\,a^3c^2x^3-2\,ac^2x-c^2\right)\sqrt{-a^2cx^2+c}\,x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-(a^4*c^2*x^4 + 2*a^3*c^2*x^3 - 2*a*c^2*x - c^2)*sqrt(-a^2*c*x^2 +
 c)*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2\,x^m\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(5/2),x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^m\,\left(c-a^2\,c\,x^2\right)^{5/2}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^m*(c - a^2*c*x^2)^(5/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 23.16, size = 226, normalized size = 1.28

$$-\frac{a^4c^{\frac{5}{2}}x^5x^m\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}-\frac{a^3c^{\frac{5}{2}}x^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{\Gamma\left(\frac{m}{2}+3\right)}+\frac{ac^{\frac{5}{2}}x^2x^m\Gamma\left(\frac{m}{2}+1\right)}{\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -a**4*c**(5/2)*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7
/2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 7/2)) - a**3*c**(5/2)*x**
4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2 + 2), (m/2 + 3,), a**2*x**2*exp_pola
r(2*I*pi))/gamma(m/2 + 3) + a*c**(5/2)*x**2*x**m*gamma(m/2 + 1)*hyper((-1/2
, m/2 + 1), (m/2 + 2,), a**2*x**2*exp_polar(2*I*pi))/gamma(m/2 + 2) + c**(5
/2)*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), a**2*x**
2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3/2))
```

3.1132 $\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{c(2m+5)x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+4)\sqrt{1-a^2x^2}}+\frac{2acx^{m+2}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{1-a^2x^2}}-\frac{x^{m+1}\left(c-a\right.}{m+}$$

```
[Out] -x^(1+m)*(-a^2*c*x^2+c)^(3/2)/(4+m)+c*(5+2*m)*x^(1+m)*hypergeom([-1/2, 1/2+
1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*c*x^2+c)^(1/2)/(m^2+5*m+4)/(-a^2*x^2+1)^(
1/2)+2*a*c*x^(2+m)*hypergeom([-1/2, 1+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*c*x^2
+c)^(1/2)/(2+m)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1809, 808, 365, 364}

$$\frac{c(2m+5)x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+4)\sqrt{1-a^2x^2}}+\frac{2acx^{m+2}\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{1-a^2x^2}}-\frac{x^{m+1}\left(c-a\right.}{m+}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((x^(1 + m)*(c - a^2*c*x^2)^(3/2))/(4 + m)) + (c*(5 + 2*m)*x^(1 + m)*Sqrt[
c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1
+ m)*(4 + m)*Sqrt[1 - a^2*x^2]) + (2*a*c*x^(2 + m)*Sqrt[c - a^2*c*x^2]*Hype
rgeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[1 - a^2*x
^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^{3/2}\,dx &= c\int x^m(1+ax)^2\sqrt{c-a^2cx^2}\,dx\\
&= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{3/2}}{4+m}-\frac{\int x^m\left(-a^2c(5+2m)-2a^3c(4+m)x\right)\sqrt{c-a^2cx^2}\,dx}{a^2(4+m)}\\
&= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{3/2}}{4+m}+(2ac)\int x^{1+m}\sqrt{c-a^2cx^2}\,dx+\frac{(c(5+2m))\int x^m\sqrt{c-}\\
&= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{3/2}}{4+m}+\frac{\left(2ac\sqrt{c-a^2cx^2}\right)\int x^{1+m}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}+\frac{\left(c(5+2m)\right.}{}\\
&= -\frac{x^{1+m}\left(c-a^2cx^2\right)^{3/2}}{4+m}+\frac{c(5+2m)x^{1+m}\sqrt{c-a^2cx^2}\;{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(4+m)\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 158, normalized size = 0.92

$$\frac{cx^{m+1}\sqrt{c-a^2cx^2}\left(2a\left(m^2+4m+3\right)x\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2}+1;\frac{m}{2}+2;a^2x^2\right)+(m+2)\left(a^2(m+1)x^2\,{}_2F_1\left(-\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};a\right.\right.\right.}{(m+1)(m+2)(m+3)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*x^(1 + m)*Sqrt[c - a^2*c*x^2]*(2*a*(3 + 4*m + m^2)*x*Hypergeometric2F1[-
1/2, 1 + m/2, 2 + m/2, a^2*x^2] + (2 + m)*((3 + m)*Hypergeometric2F1[-1/2,
(1 + m)/2, (3 + m)/2, a^2*x^2] + a^2*(1 + m)*x^2*Hypergeometric2F1[-1/2, (3
 + m)/2, (5 + m)/2, a^2*x^2])))/((1 + m)*(2 + m)*(3 + m)*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2cx^2+2\,acx+c\right)\sqrt{-a^2cx^2+c}\,x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral((a^2*c*x^2 + 2*a*c*x + c)*sqrt(-a^2*c*x^2 + c)*x^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2\,x^m\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^m\,\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^m*(c - a^2*c*x^2)^(3/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 10.26, size = 172, normalized size = 1.00

$$\frac{a^2c^{\frac{3}{2}}x^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{ac^{\frac{3}{2}}x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|a^2x^2e^{2i\pi}\right)}{\Gamma\left(\frac{m}{2}+2\right)}+\frac{c^{\frac{3}{2}}xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1}{2\Gamma\left(\frac{m}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*x**3*x**m*gamma(m/2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/
2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 5/2)) + a*c**(3/2)*x**2*x*
*m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*exp_polar(2*
I*pi))/gamma(m/2 + 2) + c**(3/2)*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 +
 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(m/2 + 3/2))
```

3.1133 $\int e^{2\tanh^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}+\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

```
[Out] c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^2
+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)+2*a*c*x^(2+m)*hypergeom([1/2, 1+
1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)-x^(
1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

Rubi [A] time = 0.29, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1809, 808, 365, 364}

$$\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}+\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m)) + (c*(3 + 2*m)*x^(1 + m)*Sqrt[1
- a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*
(2 + m)*Sqrt[c - a^2*c*x^2]) + (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= c\int \frac{x^m(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{\int \frac{x^m\left(-a^2c(3+2m)-2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}\\
&= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+(2ac)\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx+\frac{(c(3+2m))\int \frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}\\
&= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{\left(2ac\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}+\frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)\int}{(2+m)\sqrt{c-a^2c}}\\
&= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}+\frac{2acx^2}{}
\end{aligned}$$

Mathematica [C] time = 0.23, size = 130, normalized size = 0.76

$$\frac{x^{m+1}\left(-\frac{\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{\sqrt{1-a^2x^2}}-\frac{2\sqrt{1-ax}\,\sqrt{-c(ax+1)}\,F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;ax,-ax\right)}{\sqrt{ax-1}\,\sqrt{ax+1}}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcTanh[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*((-2*Sqrt[1 - a*x]*Sqrt[-(c*(1 + a*x))]*AppellF1[1 + m, 1/2, -1/
2, 2 + m, a*x, -(a*x)])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (Sqrt[c - a^2*c*x^
2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2
]))/(1 + m)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,(ax+1)x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2\,x^m\sqrt{-a^2c\,x^2+c}}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^2x^m}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^2*x^m/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1),x)
```

```
[Out] int(-(x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^m\sqrt{-a^2cx^2+c}}{ax-1}\,dx-\int \frac{axx^m\sqrt{-a^2cx^2+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**m*sqrt(-a**2*c*x**2 + c)/(a*x - 1), x) - Integral(a*x*x**m*sqr
t(-a**2*c*x**2 + c)/(a*x - 1), x)
```

3.1134 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=169

$$-\frac{(2m+1)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)\sqrt{c-a^2cx^2}}-\frac{2a(m+1)\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{2(ax+1)}{\sqrt{c-a^2}}$$

```
[Out] 2*x^(1+m)*(a*x+1)/(-a^2*c*x^2+c)^(1/2)-(1+2*m)*x^(1+m)*hypergeom([1/2, 1/2+
1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(1+m)/(-a^2*c*x^2+c)^(1/2)-2
*a*(1+m)*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(
1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1806, 808, 365, 364}

$$-\frac{(2m+1)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)\sqrt{c-a^2cx^2}}-\frac{2a(m+1)\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{2(ax+1)}{\sqrt{c-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*x^(1 + m)*(1 + a*x))/Sqrt[c - a^2*c*x^2] - ((1 + 2*m)*x^(1 + m)*Sqrt[1 -
 a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*S
qrt[c - a^2*c*x^2]) - (2*a*(1 + m)*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] && !GtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{x^m(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\
&= \frac{2x^{1+m}(1+ax)}{\sqrt{c-a^2cx^2}} - \int \frac{x^m(1+2m+2a(1+m)x)}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2x^{1+m}(1+ax)}{\sqrt{c-a^2cx^2}} - (2a(1+m))\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx - (1+2m)\int \frac{x^m}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{2x^{1+m}(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{\left(2a(1+m)\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} - \frac{\left((1+2m)\sqrt{1-a^2x^2}\right)\int \frac{x^m}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{2x^{1+m}(1+ax)}{\sqrt{c-a^2cx^2}} - \frac{(1+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)\sqrt{c-a^2cx^2}} - \frac{2a(1+m)x^{2+m}\sqrt{1-a^2x^2}\ldots}{(2+m)\ldots}
\end{aligned}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.39

$$\frac{\sqrt{1-a^2x^2}\, x^{m+1} F_1\left(m+1;\frac{3}{2},-\frac{1}{2};m+2;ax,-ax\right)}{(m+1)\sqrt{ax-1}\,\sqrt{-c(ax+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 - a^2*x^2]*AppellF1[1 + m, 3/2, -1/2, 2 + m, a*x, -(a*x)]
)/((1 + m)*Sqrt[-1 + a*x]*Sqrt[-(c*(1 + a*x))])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\, x^m}{a^2cx^2-2\,acx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^m/(a^2*c*x^2 - 2*a*c*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2 x^m}{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)^2*x^m/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2\, x^m}{\left(-a^2x^2+1\right)\sqrt{-a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2 x^m}{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2*x^m/(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^m\,{\left(a\,x+1\right)}^2}{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(a*x + 1)^2)/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(x^m*(a*x + 1)^2)/((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^m}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx-\int \frac{axx^m}{ax\sqrt{-a^2cx^2+c}-\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(x**m/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x) -
Integral(a*x*x**m/(a*x*sqrt(-a**2*c*x**2 + c) - sqrt(-a**2*c*x**2 + c)), x)
```

3.1135 $\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=183

$$\frac{(1-2m)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{3c(m+1)\sqrt{c-a^2cx^2}}+\frac{2a(1-m)\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{3c(m+2)\sqrt{c-a^2cx^2}}+\frac{2(ax+1)x}{3\left(c-a^2cx\right.}$$

```
[Out] 2/3*x^(1+m)*(a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/3*(1-2*m)*x^(1+m)*hypergeom([3/2
, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/c/(1+m)/(-a^2*c*x^2+c)
^(1/2)+2/3*a*(1-m)*x^(2+m)*hypergeom([3/2, 1+1/2*m],[2+1/2*m],a^2*x^2)*(-a^
2*x^2+1)^(1/2)/c/(2+m)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6151, 1806, 808, 365, 364}

$$\frac{(1-2m)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{3c(m+1)\sqrt{c-a^2cx^2}}+\frac{2a(1-m)\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{3c(m+2)\sqrt{c-a^2cx^2}}+\frac{2(ax+1)x}{3\left(c-a^2cx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*x^(1 + m)*(1 + a*x))/(3*(c - a^2*c*x^2)^(3/2)) + ((1 - 2*m)*x^(1 + m)*Sq
rt[1 - a^2*x^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/(3*c
*(1 + m)*Sqrt[c - a^2*c*x^2]) + (2*a*(1 - m)*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hy
pergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(3*c*(2 + m)*Sqrt[c -
a^2*c*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1806

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, -Simp[((c*x)^(m + 1)*(f + g*x)*(a + b*x^2)^(p + 1))/(2*a*c*(p + 1)),
x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b
, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] && !GtQ[m, 0]
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^{3/2}}\,dx = c\int \frac{x^m(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$
$$= \frac{2x^{1+m}(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} - \frac{1}{3}\int \frac{x^m(-1+2m-2a(1-m)x)}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= \frac{2x^{1+m}(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{1}{3}(2a(1-m))\int \frac{x^{1+m}}{\left(c-a^2cx^2\right)^{3/2}}\,dx - \frac{1}{3}(-1+2m)\int \frac{x^m}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= \frac{2x^{1+m}(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2a(1-m)\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{3c\sqrt{c-a^2cx^2}} - \frac{\left((-1+2m)\sqrt{1-a^2x^2}\right)\int \frac{\cdots}{\left(1-a\cdots\right.}}{3c\sqrt{c-a^2cx^2}}$$
$$= \frac{2x^{1+m}(1+ax)}{3\left(c-a^2cx^2\right)^{3/2}} + \frac{(1-2m)x^{1+m}\sqrt{1-a^2x^2}\;{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{3c(1+m)\sqrt{c-a^2cx^2}} + \frac{2a(1-m)x^{2+m}\sqrt{1-\cdots}}{3c(2\cdots}$$

Mathematica [C] time = 0.25, size = 173, normalized size = 0.95

$$\frac{x^{m+1}\left(\sqrt{ax-1}\sqrt{ax+1}\sqrt{c-acx}\,F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-ax,ax\right)+(ax-1)\sqrt{-c(ax+1)}\left(F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+\cdots\right.\right.\right.}{8c^2(m+1)\sqrt{-(ax-1)^2}\sqrt{ax\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/
2, -1/2, 2 + m, -(a*x), a*x] + (-1 + a*x)*Sqrt[-(c*(1 + a*x))]*(AppellF1[1
+ m, 1/2, -1/2, 2 + m, a*x, -(a*x)] + 2*AppellF1[1 + m, 3/2, -1/2, 2 + m, a
*x, -(a*x)] + 4*AppellF1[1 + m, 5/2, -1/2, 2 + m, a*x, -(a*x)])))/(8*c^2*(1
 + m)*Sqrt[-(-1 + a*x)^2]*Sqrt[1 + a*x])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,x^m}{a^4c^2x^4-2\,a^3c^2x^3+2\,ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^m/(a^4*c^2*x^4 - 2*a^3*c^2*x^3 + 2*a*c^2*x
 - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2 x^m}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(-(a*x + 1)^2*x^m/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2 x^m}{\left(-a^2x^2+1\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2 x^m}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(a^2x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^m/(-a^2*c*x^2+c)^(3/2),x, algorithm="max
ima")
```

```
[Out] -integrate((a*x + 1)^2*x^m/((-a^2*c*x^2 + c)^(3/2)*(a^2*x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^m\,(a\,x+1)^2}{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(a*x + 1)^2)/((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)),x)
```

```
[Out] int(-(x^m*(a*x + 1)^2)/((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^m}{-a^3cx^3\sqrt{-a^2cx^2+c}+a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}-c\sqrt{-a^2cx^2+c}}\,dx-\int \frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**m/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(x**m/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sqrt(-a**
2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 + c)), x
) - Integral(a*x*x**m/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) + a**2*c*x**2*sq
rt(-a**2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) - c*sqrt(-a**2*c*x**2 +
 c)), x)
```

3.1136 $\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=55

$$-\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(1-ax)\right)}{ap}$$

```
[Out] -2^(1+p)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[1+p],-1/2*a*x+1/2)/a/p/((a*x
+1)^p)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6141, 678, 69}

$$-\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x
)/2])/(a*p*(1 + a*x)^p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
```

```
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$\begin{aligned}\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= c\int (1+ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\\ &= \left(c(1+ax)^{-p}(c-acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1+ax)^{1+p}(c-acx)^{-1+p}\,dx\\ &= -\frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1-ax)\right)}{ap}\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.24

$$-\frac{2^{p+1}(1-ax)^p\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(1 + p)*(1 - a*x)^p*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 +
 p, (1 - a*x)/2])/(a*p*(1 - a^2*x^2)^p))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(ax+1)\left(-a^2cx^2+c\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral(-(a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^2\left(-a^2cx^2+c\right)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate(-(a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 1), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2 \left(-a^2 c\, x^2+c\right)^p}{-a^2 x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x)
[Out] int((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^2\left(-a^2cx^2+c\right)^p}{a^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] -integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\left(c-a^2\, c\, x^2\right)^p\, (a\, x+1)^2}{a^2\, x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a^2*x^2 - 1),x)
[Out] int(-((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a^2*x^2 - 1), x)
```

sympy [C] time = 12.14, size = 653, normalized size = 11.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*(-a**2*c*x**2+c)**p,x)
[Out] -a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 + c**p*x**2*gamma(p)*gamm
a(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma
```

```
(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(
-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/
2 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a - 0**p*log(1/(a**2*x
**2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))
/a**2 + c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x
**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2
*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,
), 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) - Piecewise((0*
*p*log(a**2*x**2 - 1)/(2*a) - 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamm
a(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma
(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/
2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3
/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
 - 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p
), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*
p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 -
p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True
))
```

3.1137 $\int e^{3\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=136

$$\frac{23c\sin^{-1}(ax)}{16a^4}-\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2}-\frac{1}{6}acx^5\sqrt{1-a^2x^2}-\frac{3}{5}cx^4\sqrt{1-a^2x^2}-\frac{23cx^3\sqrt{1-a^2x^2}}{24a}-\frac{c(345ax+544)\sqrt{1-a^2x^2}}{240a^4}$$

```
[Out] 23/16*c*arcsin(a*x)/a^4-17/15*c*x^2*(-a^2*x^2+1)^(1/2)/a^2-23/24*c*x^3*(-a^
2*x^2+1)^(1/2)/a-3/5*c*x^4*(-a^2*x^2+1)^(1/2)-1/6*a*c*x^5*(-a^2*x^2+1)^(1/2
)-1/240*c*(345*a*x+544)*(-a^2*x^2+1)^(1/2)/a^4
```

Rubi [A] time = 0.28, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 1809, 833, 780, 216}

$$-\frac{1}{6}acx^5\sqrt{1-a^2x^2}-\frac{3}{5}cx^4\sqrt{1-a^2x^2}-\frac{23cx^3\sqrt{1-a^2x^2}}{24a}-\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2}-\frac{c(345ax+544)\sqrt{1-a^2x^2}}{240a^4}+\frac{23c\sin^{-1}(ax)}{16a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^3*(c - a^2*c*x^2),x]
```

```
[Out] (-17*c*x^2*Sqrt[1 - a^2*x^2])/(15*a^2) - (23*c*x^3*Sqrt[1 - a^2*x^2])/(24*a
) - (3*c*x^4*Sqrt[1 - a^2*x^2])/5 - (a*c*x^5*Sqrt[1 - a^2*x^2])/6 - (c*(544
 + 345*a*x)*Sqrt[1 - a^2*x^2])/(240*a^4) + (23*c*ArcSin[a*x])/(16*a^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
```

```
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)dx = c\int\frac{x^3(1+ax)^3}{\sqrt{1-a^2x^2}}dx$$

$$= -\frac{1}{6}acx^5\sqrt{1-a^2x^2} - \frac{c\int\frac{x^3\left(-6a^2-23a^3x-18a^4x^2\right)}{\sqrt{1-a^2x^2}}dx}{6a^2}$$

$$= -\frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} + \frac{c\int\frac{x^3\left(102a^4+115a^5x\right)}{\sqrt{1-a^2x^2}}dx}{30a^4}$$

$$= -\frac{23cx^3\sqrt{1-a^2x^2}}{24a} - \frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} - \frac{c\int\frac{x^2\left(-345a^5-408a^6x\right)}{\sqrt{1-a^2x^2}}d}{120a^6}$$

$$= -\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2} - \frac{23cx^3\sqrt{1-a^2x^2}}{24a} - \frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} + \frac{c}{\ }$$

$$= -\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2} - \frac{23cx^3\sqrt{1-a^2x^2}}{24a} - \frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} - \frac{c}{\ }$$

$$= -\frac{17cx^2\sqrt{1-a^2x^2}}{15a^2} - \frac{23cx^3\sqrt{1-a^2x^2}}{24a} - \frac{3}{5}cx^4\sqrt{1-a^2x^2} - \frac{1}{6}acx^5\sqrt{1-a^2x^2} - \frac{c}{\ }$$

Mathematica [A] time = 0.12, size = 70, normalized size = 0.51

$$\frac{345c\sin^{-1}(ax)-c\sqrt{1-a^2x^2}\left(40a^5x^5+144a^4x^4+230a^3x^3+272a^2x^2+345ax+544\right)}{240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^3*(c - a^2*c*x^2),x]
```

```
[Out] (-(c*Sqrt[1 - a^2*x^2]*(544 + 345*a*x + 272*a^2*x^2 + 230*a^3*x^3 + 144*a^4
*x^4 + 40*a^5*x^5)) + 345*c*ArcSin[a*x])/(240*a^4)
```

fricas [A] time = 0.59, size = 89, normalized size = 0.65

$$-\frac{690\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(40\,a^5cx^5+144\,a^4cx^4+230\,a^3cx^3+272\,a^2cx^2+345\,acx+544\,c\right)\sqrt{-a^2x^2+1}}{240\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c),x, algorithm="fri
cas")
```

```
[Out] -1/240*(690*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (40*a^5*c*x^5 + 144*
a^4*c*x^4 + 230*a^3*c*x^3 + 272*a^2*c*x^2 + 345*a*c*x + 544*c)*sqrt(-a^2*x^
2 + 1))/a^4
```

giac [A] time = 0.20, size = 78, normalized size = 0.57

$$-\frac{1}{240}\sqrt{-a^2x^2+1}\left(\left(2\left(\left(4\,(5\,acx+18\,c)x+\frac{115\,c}{a}\right)x+\frac{136\,c}{a^2}\right)x+\frac{345\,c}{a^3}\right)x+\frac{544\,c}{a^4}\right)+\frac{23\,c\arcsin\left(ax\right)\mathrm{sgn}(a)}{16\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c),x, algorithm="gia
c")
```

```
[Out] -1/240*sqrt(-a^2*x^2 + 1)*((2*((4*(5*a*c*x + 18*c)*x + 115*c/a)*x + 136*c/a
^2)*x + 345*c/a^3)*x + 544*c/a^4) + 23/16*c*arcsin(a*x)*sgn(a)/(a^3*abs(a))
```

maple [A] time = 0.09, size = 191, normalized size = 1.40

$$\frac{c\,a^3x^7}{6\sqrt{-a^2x^2+1}}+\frac{19ca\,x^5}{24\sqrt{-a^2x^2+1}}+\frac{23c\,x^3}{48a\sqrt{-a^2x^2+1}}-\frac{23cx}{16a^3\sqrt{-a^2x^2+1}}+\frac{23c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{16a^3\sqrt{a^2}}+\frac{3c\,a^2x^6}{5\sqrt{-a^2x^2+1}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c),x)
```

```
[Out] 1/6*c*a^3*x^7/(-a^2*x^2+1)^(1/2)+19/24*c*a*x^5/(-a^2*x^2+1)^(1/2)+23/48*c/a
*x^3/(-a^2*x^2+1)^(1/2)-23/16*c/a^3*x/(-a^2*x^2+1)^(1/2)+23/16*c/a^3/(a^2)^
(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3/5*c*a^2*x^6/(-a^2*x^2+1)^(
1/2)+8/15*c*x^4/(-a^2*x^2+1)^(1/2)+17/15*c/a^2*x^2/(-a^2*x^2+1)^(1/2)-34/15
*c/a^4/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.43, size = 169, normalized size = 1.24

$$\frac{a^3cx^7}{6\sqrt{-a^2x^2+1}}+\frac{3\,a^2cx^6}{5\sqrt{-a^2x^2+1}}+\frac{19\,acx^5}{24\sqrt{-a^2x^2+1}}+\frac{8\,cx^4}{15\sqrt{-a^2x^2+1}}+\frac{23\,cx^3}{48\sqrt{-a^2x^2+1}\,a}+\frac{17\,cx^2}{15\sqrt{-a^2x^2+1}\,a^2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c),x, algorithm="max
ima")
```

```
[Out] 1/6*a^3*c*x^7/sqrt(-a^2*x^2 + 1) + 3/5*a^2*c*x^6/sqrt(-a^2*x^2 + 1) + 19/24
*a*c*x^5/sqrt(-a^2*x^2 + 1) + 8/15*c*x^4/sqrt(-a^2*x^2 + 1) + 23/48*c*x^3/(
sqrt(-a^2*x^2 + 1)*a) + 17/15*c*x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 23/16*c*x/(s
```

```
qrt(-a^2*x^2 + 1)*a^3) + 23/16*c*arcsin(a*x)/a^4 - 34/15*c/(sqrt(-a^2*x^2 +
 1)*a^4)
```

mupad [B] time = 0.91, size = 140, normalized size = 1.03

$$\frac{23\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{16\,a^3\,\sqrt{-a^2}}-\frac{3\,c\,x^4\,\sqrt{1-a^2\,x^2}}{5}-\frac{23\,c\,x^3\,\sqrt{1-a^2\,x^2}}{24\,a}-\frac{17\,c\,x^2\,\sqrt{1-a^2\,x^2}}{15\,a^2}-\frac{23\,c\,x\,\sqrt{1-a^2\,x^2}}{16\,a^3}-\frac{a\,c\,x^5\,\sqrt{1-a^2\,x^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (23*c*asinh(x*(-a^2)^(1/2)))/(16*a^3*(-a^2)^(1/2)) - (3*c*x^4*(1 - a^2*x^2)
^(1/2))/5 - (23*c*x^3*(1 - a^2*x^2)^(1/2))/(24*a) - (17*c*x^2*(1 - a^2*x^2)
^(1/2))/(15*a^2) - (23*c*x*(1 - a^2*x^2)^(1/2))/(16*a^3) - (a*c*x^5*(1 - a^
2*x^2)^(1/2))/6 - (34*c*(1 - a^2*x^2)^(1/2))/(15*a^4)
```

sympy [A] time = 21.36, size = 483, normalized size = 3.55

$$a^3c\left(\begin{cases}-\frac{ix^7}{6\sqrt{a^2x^2-1}}-\frac{ix^5}{24a^2\sqrt{a^2x^2-1}}-\frac{5ix^3}{48a^4\sqrt{a^2x^2-1}}+\frac{5ix}{16a^6\sqrt{a^2x^2-1}}-\frac{5i\,\mathrm{acosh}\,(ax)}{16a^7} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^7}{6\sqrt{-a^2x^2+1}}+\frac{x^5}{24a^2\sqrt{-a^2x^2+1}}+\frac{5x^3}{48a^4\sqrt{-a^2x^2+1}}-\frac{5x}{16a^6\sqrt{-a^2x^2+1}}+\frac{5\,\mathrm{asin}\,(ax)}{16a^7} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2} - \\ \frac{x^6}{6}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(-a**2*c*x**2+c),x)
```

```
[Out] a**3*c*Piecewise((-I*x**7/(6*sqrt(a**2*x**2 - 1)) - I*x**5/(24*a**2*sqrt(a*
*2*x**2 - 1)) - 5*I*x**3/(48*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(16*a**6*sqr
t(a**2*x**2 - 1)) - 5*I*acosh(a*x)/(16*a**7), Abs(a**2*x**2) > 1), (x**7/(6
*sqrt(-a**2*x**2 + 1)) + x**5/(24*a**2*sqrt(-a**2*x**2 + 1)) + 5*x**3/(48*a
**4*sqrt(-a**2*x**2 + 1)) - 5*x/(16*a**6*sqrt(-a**2*x**2 + 1)) + 5*asin(a*x
)/(16*a**7), True)) + 3*a**2*c*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**
2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**
6), Ne(a, 0)), (x**6/6, True)) + 3*a*c*Piecewise((-I*x**5/(4*sqrt(a**2*x**2
 - 1)) - I*x**3/(8*a**2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2
 - 1)) - 3*I*acosh(a*x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*
x**2 + 1)) + x**3/(8*a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x*
*2 + 1)) + 3*asin(a*x)/(8*a**5), True)) + c*Piecewise((-x**2*sqrt(-a**2*x**
2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True
))
```

3.1138 $\int e^{3\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=111

$$\frac{13c\sin^{-1}(ax)}{8a^3}-\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{c(195ax+304)\sqrt{1-a^2x^2}}{120a^3}$$

```
[Out] 13/8*c*arcsin(a*x)/a^3-19/15*c*x^2*(-a^2*x^2+1)^(1/2)/a-3/4*c*x^3*(-a^2*x^2
+1)^(1/2)-1/5*a*c*x^4*(-a^2*x^2+1)^(1/2)-1/120*c*(195*a*x+304)*(-a^2*x^2+1)
^(1/2)/a^3
```

Rubi [A] time = 0.25, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6148, 1809, 833, 780, 216}

$$-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{c(195ax+304)\sqrt{1-a^2x^2}}{120a^3}+\frac{13c\sin^{-1}(ax)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2*(c - a^2*c*x^2),x]
```

```
[Out] (-19*c*x^2*Sqrt[1 - a^2*x^2])/(15*a) - (3*c*x^3*Sqrt[1 - a^2*x^2])/4 - (a*c
*x^4*Sqrt[1 - a^2*x^2])/5 - (c*(304 + 195*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3)
 + (13*c*ArcSin[a*x])/(8*a^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
```

```
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)dx &= c\int \frac{x^2(1+ax)^3}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{c\int \frac{x^2\left(-5a^2-19a^3x-15a^4x^2\right)}{\sqrt{1-a^2x^2}}\,dx}{5a^2}\\
&= -\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{1}{5}acx^4\sqrt{1-a^2x^2}+\frac{c\int \frac{x^2\left(65a^4+76a^5x\right)}{\sqrt{1-a^2x^2}}\,dx}{20a^4}\\
&= -\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{c\int \frac{x\left(-152a^5-195a^6x\right)}{\sqrt{1-a^2x^2}}\,dx}{60a^6}\\
&= -\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{c(304+195ax)\sqrt{1-}}{120a^3}\\
&= -\frac{19cx^2\sqrt{1-a^2x^2}}{15a}-\frac{3}{4}cx^3\sqrt{1-a^2x^2}-\frac{1}{5}acx^4\sqrt{1-a^2x^2}-\frac{c(304+195ax)\sqrt{1-}}{120a^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 62, normalized size = 0.56

$$\frac{195c\sin^{-1}(ax)-c\sqrt{1-a^2x^2}\left(24a^4x^4+90a^3x^3+152a^2x^2+195ax+304\right)}{120a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2*(c - a^2*c*x^2),x]
```

```
[Out] (-(c*Sqrt[1 - a^2*x^2]*(304 + 195*a*x + 152*a^2*x^2 + 90*a^3*x^3 + 24*a^4*x
^4)) + 195*c*ArcSin[a*x])/(120*a^3)
```

fricas [A] time = 0.61, size = 80, normalized size = 0.72

$$-\frac{390\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(24\,a^4cx^4+90\,a^3cx^3+152\,a^2cx^2+195\,acx+304\,c\right)\sqrt{-a^2x^2+1}}{120\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c),x, algorithm="fri
cas")
```

```
[Out] -1/120*(390*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c*x^4 + 90*a
^3*c*x^3 + 152*a^2*c*x^2 + 195*a*c*x + 304*c)*sqrt(-a^2*x^2 + 1))/a^3
```

giac [A] time = 0.23, size = 69, normalized size = 0.62

$$-\frac{1}{120}\sqrt{-a^2x^2+1}\left(\left(2\left(3\left(4\,acx+15\,c\right)x+\frac{76\,c}{a}\right)x+\frac{195\,c}{a^2}\right)x+\frac{304\,c}{a^3}\right)+\frac{13\,c\arcsin\left(ax\right)\operatorname{sgn}(a)}{8\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c),x, algorithm="gia
c")
```

```
[Out] -1/120*sqrt(-a^2*x^2 + 1)*((2*(3*(4*a*c*x + 15*c)*x + 76*c/a)*x + 195*c/a^2
)*x + 304*c/a^3) + 13/8*c*arcsin(a*x)*sgn(a)/(a^2*abs(a))
```

maple [A] time = 0.07, size = 170, normalized size = 1.53

$$\frac{c\,a^3x^6}{5\sqrt{-a^2x^2+1}}+\frac{16ca\,x^4}{15\sqrt{-a^2x^2+1}}+\frac{19c\,x^2}{15a\sqrt{-a^2x^2+1}}-\frac{38c}{15a^3\sqrt{-a^2x^2+1}}+\frac{3c\,a^2x^5}{4\sqrt{-a^2x^2+1}}+\frac{7c\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{13cx}{8a^2\sqrt{-a^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c),x)
```

```
[Out] 1/5*c*a^3*x^6/(-a^2*x^2+1)^(1/2)+16/15*c*a*x^4/(-a^2*x^2+1)^(1/2)+19/15*c/a
*x^2/(-a^2*x^2+1)^(1/2)-38/15*c/a^3/(-a^2*x^2+1)^(1/2)+3/4*c*a^2*x^5/(-a^2*
x^2+1)^(1/2)+7/8*c*x^3/(-a^2*x^2+1)^(1/2)-13/8*c*x/a^2/(-a^2*x^2+1)^(1/2)+1
3/8*c/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.42, size = 148, normalized size = 1.33

$$\frac{a^3cx^6}{5\sqrt{-a^2x^2+1}}+\frac{3\,a^2cx^5}{4\sqrt{-a^2x^2+1}}+\frac{16\,acx^4}{15\sqrt{-a^2x^2+1}}+\frac{7\,cx^3}{8\sqrt{-a^2x^2+1}}+\frac{19\,cx^2}{15\sqrt{-a^2x^2+1}\,a}-\frac{13\,cx}{8\sqrt{-a^2x^2+1}\,a^2}+\frac{13\,c\arcsin}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c),x, algorithm="max
ima")
```

```
[Out] 1/5*a^3*c*x^6/sqrt(-a^2*x^2 + 1) + 3/4*a^2*c*x^5/sqrt(-a^2*x^2 + 1) + 16/15
*a*c*x^4/sqrt(-a^2*x^2 + 1) + 7/8*c*x^3/sqrt(-a^2*x^2 + 1) + 19/15*c*x^2/(s
qrt(-a^2*x^2 + 1)*a) - 13/8*c*x/(sqrt(-a^2*x^2 + 1)*a^2) + 13/8*c*arcsin(a*
x)/a^3 - 38/15*c/(sqrt(-a^2*x^2 + 1)*a^3)
```

mupad [B] time = 0.03, size = 119, normalized size = 1.07

$$\frac{13\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a^2\,\sqrt{-a^2}}-\frac{3\,c\,x^3\,\sqrt{1-a^2\,x^2}}{4}-\frac{19\,c\,x^2\,\sqrt{1-a^2\,x^2}}{15\,a}-\frac{13\,c\,x\,\sqrt{1-a^2\,x^2}}{8\,a^2}-\frac{a\,c\,x^4\,\sqrt{1-a^2\,x^2}}{5}-\frac{38\,c\,\sqrt{1-a^2\,x^2}}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (13*c*asinh(x*(-a^2)^(1/2)))/(8*a^2*(-a^2)^(1/2)) - (3*c*x^3*(1 - a^2*x^2)^
(1/2))/4 - (19*c*x^2*(1 - a^2*x^2)^(1/2))/(15*a) - (13*c*x*(1 - a^2*x^2)^(1
/2))/(8*a^2) - (a*c*x^4*(1 - a^2*x^2)^(1/2))/5 - (38*c*(1 - a^2*x^2)^(1/2))
/(15*a^3)
```

sympy [C] time = 18.45, size = 371, normalized size = 3.34

$$a^3c\left(\begin{cases}-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^4}-\frac{8\sqrt{-a^2x^2+1}}{15a^6} & \text{for } a \neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\,\mathrm{acos}}{8a}\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\,\mathrm{as}}{}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(-a**2*c*x**2+c),x)
```

```
[Out] a**3*c*Piecewise((-x**4*sqrt(-a**2*x**2 + 1)/(5*a**2) - 4*x**2*sqrt(-a**2*x
**2 + 1)/(15*a**4) - 8*sqrt(-a**2*x**2 + 1)/(15*a**6), Ne(a, 0)), (x**6/6,
True)) + 3*a**2*c*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a*
*2*sqrt(a**2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*
x)/(8*a**5), Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*
a**2*sqrt(-a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x
)/(8*a**5), True)) + 3*a*c*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) -
 2*sqrt(-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + c*Piecewise(
(-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2)
> 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + as
in(a*x)/(2*a**3), True))
```

3.1139 $\int e^{3\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=120

$$-\frac{c\sqrt{1-a^2x^2}\,(ax+1)^3}{4a^2}-\frac{c\sqrt{1-a^2x^2}\,(ax+1)^2}{4a^2}-\frac{5c\sqrt{1-a^2x^2}\,(ax+1)}{8a^2}-\frac{15c\sqrt{1-a^2x^2}}{8a^2}+\frac{15c\sin^{-1}(ax)}{8a^2}$$

```
[Out] 15/8*c*arcsin(a*x)/a^2-15/8*c*(-a^2*x^2+1)^(1/2)/a^2-5/8*c*(a*x+1)*(-a^2*x^
2+1)^(1/2)/a^2-1/4*c*(a*x+1)^2*(-a^2*x^2+1)^(1/2)/a^2-1/4*c*(a*x+1)^3*(-a^2
*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6148, 795, 671, 641, 216}

$$-\frac{c\sqrt{1-a^2x^2}\,(ax+1)^3}{4a^2}-\frac{c\sqrt{1-a^2x^2}\,(ax+1)^2}{4a^2}-\frac{5c\sqrt{1-a^2x^2}\,(ax+1)}{8a^2}-\frac{15c\sqrt{1-a^2x^2}}{8a^2}+\frac{15c\sin^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x*(c - a^2*c*x^2),x]
```

```
[Out] (-15*c*Sqrt[1 - a^2*x^2])/(8*a^2) - (5*c*(1 + a*x)*Sqrt[1 - a^2*x^2])/(8*a^
2) - (c*(1 + a*x)^2*Sqrt[1 - a^2*x^2])/(4*a^2) - (c*(1 + a*x)^3*Sqrt[1 - a^
2*x^2])/(4*a^2) + (15*c*ArcSin[a*x])/(8*a^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)dx=c\int\frac{x(1+ax)^3}{\sqrt{1-a^2x^2}}dx$$
$$=-\frac{c(1+ax)^3\sqrt{1-a^2x^2}}{4a^2}+\frac{(3c)\int\frac{(1+ax)^3}{\sqrt{1-a^2x^2}}dx}{4a}$$
$$=-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{4a^2}-\frac{c(1+ax)^3\sqrt{1-a^2x^2}}{4a^2}+\frac{(5c)\int\frac{(1+ax)^2}{\sqrt{1-a^2x^2}}dx}{4a}$$
$$=-\frac{5c(1+ax)\sqrt{1-a^2x^2}}{8a^2}-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{4a^2}-\frac{c(1+ax)^3\sqrt{1-a^2x^2}}{4a^2}+\frac{(15c)}{}$$
$$=-\frac{15c\sqrt{1-a^2x^2}}{8a^2}-\frac{5c(1+ax)\sqrt{1-a^2x^2}}{8a^2}-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{4a^2}-\frac{c(1+ax)^3\sqrt{1}}{4a^2}$$
$$=-\frac{15c\sqrt{1-a^2x^2}}{8a^2}-\frac{5c(1+ax)\sqrt{1-a^2x^2}}{8a^2}-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{4a^2}-\frac{c(1+ax)^3\sqrt{1}}{4a^2}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.45

$$\frac{15c\sin^{-1}(ax)-c\sqrt{1-a^2x^2}\left(2a^3x^3+8a^2x^2+15ax+24\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x*(c - a^2*c*x^2),x]
```

```
[Out] (-(c*Sqrt[1 - a^2*x^2]*(24 + 15*a*x + 8*a^2*x^2 + 2*a^3*x^3)) + 15*c*ArcSin
[a*x])/(8*a^2)
```

fricas [A] time = 0.46, size = 71, normalized size = 0.59

$$-\frac{30\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^3cx^3+8\,a^2cx^2+15\,acx+24\,c\right)\sqrt{-a^2x^2+1}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] -1/8*(30*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^3*c*x^3 + 8*a^2*c*
x^2 + 15*a*c*x + 24*c)*sqrt(-a^2*x^2 + 1))/a^2
```

giac [A] time = 1.61, size = 58, normalized size = 0.48

$$-\frac{1}{8}\sqrt{-a^2x^2+1}\left(\left(2\,(acx+4\,c)x+\frac{15\,c}{a}\right)x+\frac{24\,c}{a^2}\right)+\frac{15\,c\arcsin\left(ax\right)\operatorname{sgn}(a)}{8\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] -1/8*sqrt(-a^2*x^2 + 1)*((2*(a*c*x + 4*c)*x + 15*c/a)*x + 24*c/a^2) + 15/8*
c*arcsin(a*x)*sgn(a)/(a*abs(a))
```

maple [A] time = 0.05, size = 148, normalized size = 1.23

$$\frac{c\,a^3x^5}{4\sqrt{-a^2x^2+1}}+\frac{13ca\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{15cx}{8a\sqrt{-a^2x^2+1}}+\frac{15c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a\sqrt{a^2}}+\frac{c\,a^2x^4}{\sqrt{-a^2x^2+1}}+\frac{2c\,x^2}{\sqrt{-a^2x^2+1}}-\frac{3c}{a^2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c),x)
```

```
[Out] 1/4*c*a^3*x^5/(-a^2*x^2+1)^(1/2)+13/8*c*a*x^3/(-a^2*x^2+1)^(1/2)-15/8*c/a*x
/(-a^2*x^2+1)^(1/2)+15/8*c/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^
(1/2))+c*a^2*x^4/(-a^2*x^2+1)^(1/2)+2*c*x^2/(-a^2*x^2+1)^(1/2)-3*c/a^2/(-a^
2*x^2+1)^(1/2)
```

maxima [A] time = 0.43, size = 126, normalized size = 1.05

$$\frac{a^3cx^5}{4\sqrt{-a^2x^2+1}}+\frac{a^2cx^4}{\sqrt{-a^2x^2+1}}+\frac{13\,acx^3}{8\sqrt{-a^2x^2+1}}+\frac{2\,cx^2}{\sqrt{-a^2x^2+1}}-\frac{15\,cx}{8\sqrt{-a^2x^2+1}\,a}+\frac{15\,c\arcsin\left(ax\right)}{8\,a^2}-\frac{3\,c}{\sqrt{-a^2x^2+1}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] 1/4*a^3*c*x^5/sqrt(-a^2*x^2 + 1) + a^2*c*x^4/sqrt(-a^2*x^2 + 1) + 13/8*a*c*
x^3/sqrt(-a^2*x^2 + 1) + 2*c*x^2/sqrt(-a^2*x^2 + 1) - 15/8*c*x/(sqrt(-a^2*x
^2 + 1)*a) + 15/8*c*arcsin(a*x)/a^2 - 3*c/(sqrt(-a^2*x^2 + 1)*a^2)
```

mupad [B] time = 0.03, size = 98, normalized size = 0.82

$$\frac{15\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,a\,\sqrt{-a^2}}-c\,x^2\,\sqrt{1-a^2\,x^2}-\frac{15\,c\,x\,\sqrt{1-a^2\,x^2}}{8\,a}-\frac{a\,c\,x^3\,\sqrt{1-a^2\,x^2}}{4}-\frac{3\,c\,\sqrt{1-a^2\,x^2}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (15*c*asinh(x*(-a^2)^(1/2)))/(8*a*(-a^2)^(1/2)) - c*x^2*(1 - a^2*x^2)^(1/2)
 - (15*c*x*(1 - a^2*x^2)^(1/2))/(8*a) - (a*c*x^3*(1 - a^2*x^2)^(1/2))/4 - (
3*c*(1 - a^2*x^2)^(1/2))/a^2
```

sympy [A] time = 16.33, size = 326, normalized size = 2.72

$$a^3c\left(\begin{cases}-\frac{ix^5}{4\sqrt{a^2x^2-1}}-\frac{ix^3}{8a^2\sqrt{a^2x^2-1}}+\frac{3ix}{8a^4\sqrt{a^2x^2-1}}-\frac{3i\,\mathrm{acosh}\,(ax)}{8a^5} & \text{for}\ \left|a^2x^2\right|>1\\ \frac{x^5}{4\sqrt{-a^2x^2+1}}+\frac{x^3}{8a^2\sqrt{-a^2x^2+1}}-\frac{3x}{8a^4\sqrt{-a^2x^2+1}}+\frac{3\,\mathrm{asin}\,(ax)}{8a^5} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{fo}\\ \frac{x^4}{4} & \text{ot}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(-a**2*c*x**2+c),x)
```

```
[Out] a**3*c*Piecewise((-I*x**5/(4*sqrt(a**2*x**2 - 1)) - I*x**3/(8*a**2*sqrt(a**
2*x**2 - 1)) + 3*I*x/(8*a**4*sqrt(a**2*x**2 - 1)) - 3*I*acosh(a*x)/(8*a**5)
, Abs(a**2*x**2) > 1), (x**5/(4*sqrt(-a**2*x**2 + 1)) + x**3/(8*a**2*sqrt(-
a**2*x**2 + 1)) - 3*x/(8*a**4*sqrt(-a**2*x**2 + 1)) + 3*asin(a*x)/(8*a**5),
 True)) + 3*a**2*c*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(
-a**2*x**2 + 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + 3*a*c*Piecewise((-I*
x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1)
, (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a
*x)/(2*a**3), True)) + c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2
 + 1)/a**2, True))
```

3.1140 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=91

$$-\frac{c\sqrt{1-a^2x^2}\,(ax+1)^2}{3a}-\frac{5c\sqrt{1-a^2x^2}\,(ax+1)}{6a}-\frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c\sin^{-1}(ax)}{2a}$$

```
[Out] 5/2*c*arcsin(a*x)/a-5/2*c*(-a^2*x^2+1)^(1/2)/a-5/6*c*(a*x+1)*(-a^2*x^2+1)^(
1/2)/a-1/3*c*(a*x+1)^2*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6138, 671, 641, 216}

$$-\frac{c\sqrt{1-a^2x^2}\,(ax+1)^2}{3a}-\frac{5c\sqrt{1-a^2x^2}\,(ax+1)}{6a}-\frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (-5*c*Sqrt[1 - a^2*x^2])/(2*a) - (5*c*(1 + a*x)*Sqrt[1 - a^2*x^2])/(6*a) -
(c*(1 + a*x)^2*Sqrt[1 - a^2*x^2])/(3*a) + (5*c*ArcSin[a*x])/(2*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx &= c\int \frac{(1+ax)^3}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{3}(5c)\int \frac{(1+ax)^2}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{5c(1+ax)\sqrt{1-a^2x^2}}{6a}-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}(5c)\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{5c\sqrt{1-a^2x^2}}{2a}-\frac{5c(1+ax)\sqrt{1-a^2x^2}}{6a}-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}(5c)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{5c\sqrt{1-a^2x^2}}{2a}-\frac{5c(1+ax)\sqrt{1-a^2x^2}}{6a}-\frac{c(1+ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5c\sin^{-1}(ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.63

$$-\frac{c\left(\sqrt{1-a^2x^2}\left(2a^2x^2+9ax+22\right)+30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -1/6*(c*(Sqrt[1 - a^2*x^2]*(22 + 9*a*x + 2*a^2*x^2) + 30*ArcSin[Sqrt[1 - a*
x]/Sqrt[2]]))/a
```

fricas [A] time = 0.63, size = 62, normalized size = 0.68

$$-\frac{30\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^2cx^2+9\,acx+22\,c\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] -1/6*(30*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^2*c*x^2 + 9*a*c*x
+ 22*c)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.29, size = 46, normalized size = 0.51

$$\frac{5\,c\arcsin\left(ax\right)\operatorname{sgn}(a)}{2\,|a|} - \frac{1}{6}\,\sqrt{-a^2x^2+1}\left((2\,acx+9\,c)x + \frac{22\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 5/2*c*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c*x + 9*c)*x
 + 22*c/a)
```

maple [A] time = 0.05, size = 125, normalized size = 1.37

$$\frac{c\,a^3x^4}{3\sqrt{-a^2x^2+1}}+\frac{10ca\,x^2}{3\sqrt{-a^2x^2+1}}-\frac{11c}{3a\sqrt{-a^2x^2+1}}+\frac{3c\,a^2x^3}{2\sqrt{-a^2x^2+1}}-\frac{3cx}{2\sqrt{-a^2x^2+1}}+\frac{5c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*c*a^3*x^4/(-a^2*x^2+1)^(1/2)+10/3*c*a*x^2/(-a^2*x^2+1)^(1/2)-11/3*c/a/(
-a^2*x^2+1)^(1/2)+3/2*c*a^2*x^3/(-a^2*x^2+1)^(1/2)-3/2*c*x/(-a^2*x^2+1)^(1/
2)+5/2*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.43, size = 106, normalized size = 1.16

$$\frac{a^3cx^4}{3\,\sqrt{-a^2x^2+1}}+\frac{3\,a^2cx^3}{2\,\sqrt{-a^2x^2+1}}+\frac{10\,acx^2}{3\,\sqrt{-a^2x^2+1}}-\frac{3\,cx}{2\,\sqrt{-a^2x^2+1}}+\frac{5\,c\arcsin\left(ax\right)}{2\,a}-\frac{11\,c}{3\,\sqrt{-a^2x^2+1}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] 1/3*a^3*c*x^4/sqrt(-a^2*x^2 + 1) + 3/2*a^2*c*x^3/sqrt(-a^2*x^2 + 1) + 10/3*
a*c*x^2/sqrt(-a^2*x^2 + 1) - 3/2*c*x/sqrt(-a^2*x^2 + 1) + 5/2*c*arcsin(a*x)
/a - 11/3*c/(sqrt(-a^2*x^2 + 1)*a)
```

mupad [B] time = 0.04, size = 74, normalized size = 0.81

$$\frac{5\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}} - \frac{3\,c\,x\,\sqrt{1-a^2\,x^2}}{2} - \frac{11\,c\,\sqrt{1-a^2\,x^2}}{3\,a} - \frac{a\,c\,x^2\,\sqrt{1-a^2\,x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (5*c*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2)) - (3*c*x*(1 - a^2*x^2)^(1/2))/
2 - (11*c*(1 - a^2*x^2)^(1/2))/(3*a) - (a*c*x^2*(1 - a^2*x^2)^(1/2))/3
```

sympy [A] time = 14.31, size = 218, normalized size = 2.40

$$a^3c\left(\begin{cases} -\frac{x^2\sqrt{-a^2x^2+1}}{3a^2} - \frac{2\sqrt{-a^2x^2+1}}{3a^4} & \text{for } a \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}\right) + 3a^2c\left(\begin{cases} -\frac{ix\sqrt{a^2x^2-1}}{2a^2} - \frac{i\operatorname{acosh}(ax)}{2a^3} & \text{for } |a^2x^2| > 1 \\ \frac{x^3}{2\sqrt{-a^2x^2+1}} - \frac{x}{2a^2\sqrt{-a^2x^2+1}} + \frac{\operatorname{asin}(ax)}{2a^3} & \text{otherwise} \end{cases}\right) + 3ac$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c),x)
```

```
[Out] a**3*c*Piecewise((-x**2*sqrt(-a**2*x**2 + 1)/(3*a**2) - 2*sqrt(-a**2*x**2 +
 1)/(3*a**4), Ne(a, 0)), (x**4/4, True)) + 3*a**2*c*Piecewise((-I*x*sqrt(a*
*2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3), Abs(a**2*x**2) > 1), (x**3/(
2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(2*a*
*3), True)) + 3*a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)
/a**2, True)) + c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (
sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0))
```

3.1141 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)}{x}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{2}acx\sqrt{1-a^2x^2}-3c\sqrt{1-a^2x^2}-c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{7}{2}c\sin^{-1}(ax)$$

```
[Out] 7/2*c*arcsin(a*x)-c*arctanh((-a^2*x^2+1)^(1/2))-3*c*(-a^2*x^2+1)^(1/2)-1/2*
a*c*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 1809, 844, 216, 266, 63, 208}

$$-\frac{1}{2}acx\sqrt{1-a^2x^2}-3c\sqrt{1-a^2x^2}-c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{7}{2}c\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x,x]
```

```
[Out] -3*c*Sqrt[1 - a^2*x^2] - (a*c*x*Sqrt[1 - a^2*x^2])/2 + (7*c*ArcSin[a*x])/2
- c*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x}\,dx = c\int \frac{(1+ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{1}{2}acx\sqrt{1-a^2x^2} - \frac{c\int \frac{-2a^2-7a^3x-6a^4x^2}{x\sqrt{1-a^2x^2}}\,dx}{2a^2}$$

$$= -3c\sqrt{1-a^2x^2} - \frac{1}{2}acx\sqrt{1-a^2x^2} + \frac{c\int \frac{2a^4+7a^5x}{x\sqrt{1-a^2x^2}}\,dx}{2a^4}$$

$$= -3c\sqrt{1-a^2x^2} - \frac{1}{2}acx\sqrt{1-a^2x^2} + c\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}(7ac)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$

$$= -3c\sqrt{1-a^2x^2} - \frac{1}{2}acx\sqrt{1-a^2x^2} + \frac{7}{2}c\sin^{-1}(ax) + \frac{1}{2}c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x,\right.$$

$$= -3c\sqrt{1-a^2x^2} - \frac{1}{2}acx\sqrt{1-a^2x^2} + \frac{7}{2}c\sin^{-1}(ax) - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2\right.}{a^2}$$

$$= -3c\sqrt{1-a^2x^2} - \frac{1}{2}acx\sqrt{1-a^2x^2} + \frac{7}{2}c\sin^{-1}(ax) - c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.74

$$-\frac{1}{2}c\left(\sqrt{1-a^2x^2}\,(ax+6) + 2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - 7\sin^{-1}(ax)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x,x]
```

```
[Out] -1/2*(c*((6 + a*x)*Sqrt[1 - a^2*x^2] - 7*ArcSin[a*x] + 2*ArcTanh[Sqrt[1 - a
^2*x^2]]))
```

fricas [A] time = 0.60, size = 69, normalized size = 1.05

$$-7\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}\,-1}{ax}\right) + c\log\left(\frac{\sqrt{-a^2x^2+1}\,-1}{x}\right) - \frac{1}{2}\sqrt{-a^2x^2+1}\,(acx+6\,c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x,x, algorithm="frica
s")
```

```
[Out] -7*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + c*log((sqrt(-a^2*x^2 + 1) - 1
)/x) - 1/2*sqrt(-a^2*x^2 + 1)*(a*c*x + 6*c)
```

giac [A] time = 0.21, size = 76, normalized size = 1.15

$$\frac{7\,ac\arcsin\left(ax\right)\mathrm{sgn}(a)}{2\,|a|}-\frac{ac\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\frac{1}{2}\,\sqrt{-a^2x^2+1}\,(acx+6\,c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x,x, algorithm="giac"
)
```

```
[Out] 7/2*a*c*arcsin(a*x)*sgn(a)/abs(a) - a*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*a
bs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/2*sqrt(-a^2*x^2 + 1)*(a*c*x + 6*c)
```

maple [B] time = 0.04, size = 121, normalized size = 1.83

$$\frac{c\,a^3x^3}{2\sqrt{-a^2x^2+1}}-\frac{cax}{2\sqrt{-a^2x^2+1}}+\frac{7ca\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}+\frac{3c\,a^2x^2}{\sqrt{-a^2x^2+1}}-\frac{3c}{\sqrt{-a^2x^2+1}}-c\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x,x)
```

```
[Out] 1/2*c*a^3*x^3/(-a^2*x^2+1)^(1/2)-1/2*c*a*x/(-a^2*x^2+1)^(1/2)+7/2*c*a/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+3*c*a^2*x^2/(-a^2*x^2+1)^(1
/2)-3*c/(-a^2*x^2+1)^(1/2)-c*arctanh(1/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.43, size = 111, normalized size = 1.68

$$\frac{a^3cx^3}{2\,\sqrt{-a^2x^2+1}}+\frac{3\,a^2cx^2}{\sqrt{-a^2x^2+1}}-\frac{acx}{2\,\sqrt{-a^2x^2+1}}+\frac{7}{2}\,c\arcsin\left(ax\right)-c\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{3\,c}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x,x, algorithm="maxim
a")
```

```
[Out] 1/2*a^3*c*x^3/sqrt(-a^2*x^2 + 1) + 3*a^2*c*x^2/sqrt(-a^2*x^2 + 1) - 1/2*a*c
*x/sqrt(-a^2*x^2 + 1) + 7/2*c*arcsin(a*x) - c*log(2*sqrt(-a^2*x^2 + 1)/abs(
x) + 2/abs(x)) - 3*c/sqrt(-a^2*x^2 + 1)
```

mupad [B] time = 0.91, size = 70, normalized size = 1.06

$$\frac{7\,a\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}-c\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)-\frac{a\,c\,x\,\sqrt{1-a^2\,x^2}}{2}-3\,c\,\sqrt{1-a^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (7*a*c*asinh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2)) - c*atanh((1 - a^2*x^2)^(1/2
)) - (a*c*x*(1 - a^2*x^2)^(1/2))/2 - 3*c*(1 - a^2*x^2)^(1/2)
```

sympy [C] time = 13.68, size = 197, normalized size = 2.98

$$a^3c\left(\begin{cases}-\frac{ix\sqrt{a^2x^2-1}}{2a^2}-\frac{i\,\mathrm{acosh}\,(ax)}{2a^3} & \text{for } \left|a^2x^2\right|>1\\ \frac{x^3}{2\sqrt{-a^2x^2+1}}-\frac{x}{2a^2\sqrt{-a^2x^2+1}}+\frac{\mathrm{asin}\,(ax)}{2a^3} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}\frac{x^2}{2} & \text{for } a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+3ac\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{asin}\left(x\right.\\ \sqrt{-\frac{1}{a^2}}\ \mathrm{asinh}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x,x)
```

```
[Out] a**3*c*Piecewise((-I*x*sqrt(a**2*x**2 - 1)/(2*a**2) - I*acosh(a*x)/(2*a**3)
, Abs(a**2*x**2) > 1), (x**3/(2*sqrt(-a**2*x**2 + 1)) - x/(2*a**2*sqrt(-a**
2*x**2 + 1)) + asin(a*x)/(2*a**3), True)) + 3*a**2*c*Piecewise((x**2/2, Eq(
a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True)) + 3*a*c*Piecewise((sqrt(a**(
-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**2)*asinh(x*sqrt(-a**2)), a*
*2 < 0)) + c*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(
a*x)), True))
```

3.1142 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)}{x^2}\,dx$

Optimal. Leaf size=66

$$-ac\sqrt{1-a^2x^2}-\frac{c\sqrt{1-a^2x^2}}{x}-3ac\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ac\sin^{-1}(ax)$$

```
[Out] 3*a*c*arcsin(a*x)-3*a*c*arctanh((-a^2*x^2+1)^(1/2))-a*c*(-a^2*x^2+1)^(1/2)-
c*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.19, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6148, 1807, 1809, 844, 216, 266, 63, 208}

$$-ac\sqrt{1-a^2x^2}-\frac{c\sqrt{1-a^2x^2}}{x}-3ac\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+3ac\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^2,x]
```

```
[Out] -(a*c*Sqrt[1 - a^2*x^2]) - (c*Sqrt[1 - a^2*x^2])/x + 3*a*c*ArcSin[a*x] - 3*
a*c*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^2}\,dx = c\int \frac{(1+ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{x} - c\int \frac{-3a-3a^2x-a^3x^2}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + \frac{c\int \frac{3a^3+3a^4x}{x\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + (3ac)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx + \left(3a^2c\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,d$$
$$= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + 3ac\sin^{-1}(ax) + \frac{1}{2}(3ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx\right.$$
$$= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + 3ac\sin^{-1}(ax) - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-\right.}{a}$$
$$= -ac\sqrt{1-a^2x^2} - \frac{c\sqrt{1-a^2x^2}}{x} + 3ac\sin^{-1}(ax) - 3ac\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.07, size = 52, normalized size = 0.79

$$c\left(-\frac{\sqrt{1-a^2x^2}\,(ax+1)}{x} - 3a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) + 3a\sin^{-1}(ax)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^2,x]
```

```
[Out] c*(-(((1 + a*x)*Sqrt[1 - a^2*x^2])/x) + 3*a*ArcSin[a*x] - 3*a*ArcTanh[Sqrt[
1 - a^2*x^2]])
```

fricas [A] time = 0.67, size = 80, normalized size = 1.21

$$-\frac{6\,acx\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - 3\,acx\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + acx + \sqrt{-a^2x^2+1}\,(acx+c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^2,x, algorithm="fri
cas")
```

```
[Out] -(6*a*c*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 3*a*c*x*log((sqrt(-a^2*x
^2 + 1) - 1)/x) + a*c*x + sqrt(-a^2*x^2 + 1)*(a*c*x + c))/x
```

giac [B] time = 0.23, size = 131, normalized size = 1.98

$$\frac{a^4cx}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}+\frac{3\,a^2c\arcsin\left(ax\right)\mathrm{sgn}(a)}{|a|}-\frac{3\,a^2c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}-\sqrt{-a^2x^2+1}\,ac-\frac{\left(\sqrt{-a^2x^2+1}\,|a\right.}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^2,x, algorithm="gia
c")
```

```
[Out] 1/2*a^4*c*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) + 3*a^2*c*arcsin(a*x)*
sgn(a)/abs(a) - 3*a^2*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^
2*abs(x)))/abs(a) - sqrt(-a^2*x^2 + 1)*a*c - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a)
 + a)*c/(x*abs(a))
```

maple [B] time = 0.05, size = 122, normalized size = 1.85

$$\frac{c\,a^3x^2}{\sqrt{-a^2x^2+1}}-\frac{ca}{\sqrt{-a^2x^2+1}}+\frac{c\,a^2x}{\sqrt{-a^2x^2+1}}+\frac{3c\,a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-3ca\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{c}{x\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^2,x)
```

```
[Out] c*a^3*x^2/(-a^2*x^2+1)^(1/2)-c*a/(-a^2*x^2+1)^(1/2)+c*a^2*x/(-a^2*x^2+1)^(1
/2)+3*c*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-3*c*a*arct
anh(1/(-a^2*x^2+1)^(1/2))-c/x/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.43, size = 111, normalized size = 1.68

$$\frac{a^3cx^2}{\sqrt{-a^2x^2+1}}+\frac{a^2cx}{\sqrt{-a^2x^2+1}}+3\,ac\arcsin\left(ax\right)-3\,ac\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{ac}{\sqrt{-a^2x^2+1}}-\frac{c}{\sqrt{-a^2x^2+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^2,x, algorithm="max
ima")
```

```
[Out] a^3*c*x^2/sqrt(-a^2*x^2 + 1) + a^2*c*x/sqrt(-a^2*x^2 + 1) + 3*a*c*arcsin(a*
x) - 3*a*c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - a*c/sqrt(-a^2*x^2
+ 1) - c/(sqrt(-a^2*x^2 + 1)*x)
```

mupad [B] time = 0.04, size = 79, normalized size = 1.20

$$\frac{3\,a^2\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-a\,c\,\sqrt{1-a^2\,x^2}-\frac{c\,\sqrt{1-a^2\,x^2}}{x}+a\,c\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,3\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] a*c*atan((1 - a^2*x^2)^(1/2)*1i)*3i - (c*(1 - a^2*x^2)^(1/2))/x - a*c*(1 -
a^2*x^2)^(1/2) + (3*a^2*c*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

sympy [C] time = 13.98, size = 150, normalized size = 2.27

$$a^3c\left(\begin{cases}\frac{x^2}{2} & \text{for}\ a^2=0\\ -\frac{\sqrt{-a^2x^2+1}}{a^2} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\ \mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+3ac\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**2,x)
```

```
[Out] a**3*c*Piecewise((x**2/2, Eq(a**2, 0)), (-sqrt(-a**2*x**2 + 1)/a**2, True))
 + 3*a**2*c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-
1/a**2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 3*a*c*Piecewise((-acosh(1/(a*x))
, 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + c*Piecewise((-I*sqrt(a*
*2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True))
```

3.1143 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)}{x^3}\,dx$

Optimal. Leaf size=76

$$-\frac{3ac\sqrt{1-a^2x^2}}{x}-\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{7}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^2c\sin^{-1}(ax)$$

```
[Out] a^2*c*arcsin(a*x)-7/2*a^2*c*arctanh((-a^2*x^2+1)^(1/2))-1/2*c*(-a^2*x^2+1)^
(1/2)/x^2-3*a*c*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.19, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 1807, 844, 216, 266, 63, 208}

$$-\frac{3ac\sqrt{1-a^2x^2}}{x}-\frac{c\sqrt{1-a^2x^2}}{2x^2}-\frac{7}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+a^2c\sin^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^3,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(2*x^2) - (3*a*c*Sqrt[1 - a^2*x^2])/x + a^2*c*ArcSin
[a*x] - (7*a^2*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^3}\,dx &= c\int \frac{(1+ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2} - \frac{1}{2}c\int \frac{-6a-7a^2x-2a^3x^2}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2} - \frac{3ac\sqrt{1-a^2x^2}}{x} + \frac{1}{2}c\int \frac{7a^2+2a^3x}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2} - \frac{3ac\sqrt{1-a^2x^2}}{x} + \frac{1}{2}\left(7a^2c\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx + \left(a^3c\right)\int \frac{1}{\sqrt{1-a^2x^2}} \\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2} - \frac{3ac\sqrt{1-a^2x^2}}{x} + a^2c\sin^{-1}(ax) + \frac{1}{4}\left(7a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}} \right. \\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2} - \frac{3ac\sqrt{1-a^2x^2}}{x} + a^2c\sin^{-1}(ax) - \frac{1}{2}(7c)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{\right. \\
&= -\frac{c\sqrt{1-a^2x^2}}{2x^2} - \frac{3ac\sqrt{1-a^2x^2}}{x} + a^2c\sin^{-1}(ax) - \frac{7}{2}a^2c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 60, normalized size = 0.79

$$
\frac{1}{2}c\left(-\frac{(6ax+1)\sqrt{1-a^2x^2}}{x^2} - 7a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) + 2a^2\sin^{-1}(ax)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^3,x]
```

```
[Out] (c*(-(((1 + 6*a*x)*Sqrt[1 - a^2*x^2])/x^2) + 2*a^2*ArcSin[a*x] - 7*a^2*ArcT
anh[Sqrt[1 - a^2*x^2]]))/2
```

fricas [A] time = 0.51, size = 85, normalized size = 1.12

$$
-\frac{4\,a^2cx^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - 7\,a^2cx^2\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) + \sqrt{-a^2x^2+1}\,(6\,acx+c)}{2\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^3,x, algorithm="fri
cas")
```

```
[Out] -1/2*(4*a^2*c*x^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 7*a^2*c*x^2*log(
(sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(6*a*c*x + c))/x^2
```

giac [B] time = 0.39, size = 179, normalized size = 2.36

$$\frac{a^3c\arcsin(ax)\,\mathrm{sgn}(a)}{|a|}+\frac{\left(a^3c+\frac{12\left(\sqrt{-a^2x^2+1}\,|a|+a\right)ac}{x}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}-\frac{7\,a^3c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{12\left(\sqrt{-a^2x^2+1}\,|a|+a\right)ac|a|}{x}+\cdots}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^3,x, algorithm="gia
c")
```

```
[Out] a^3*c*arcsin(a*x)*sgn(a)/abs(a) + 1/8*(a^3*c + 12*(sqrt(-a^2*x^2 + 1)*abs(a
) + a)*a*c/x)*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)) - 7/2*a^3*
c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/
8*(12*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a*c*abs(a)/x + (sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^2*c*abs(a)/(a*x^2))/a^2
```

maple [A] time = 0.05, size = 125, normalized size = 1.64

$$\frac{3c\,a^3x}{\sqrt{-a^2x^2+1}}+\frac{c\,a^3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}+\frac{c\,a^2}{2\sqrt{-a^2x^2+1}}-\frac{7c\,a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{3ca}{x\sqrt{-a^2x^2+1}}-\frac{c}{2x^2\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^3,x)
```

```
[Out] 3*c*a^3*x/(-a^2*x^2+1)^(1/2)+c*a^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x
^2+1)^(1/2))+1/2*c*a^2/(-a^2*x^2+1)^(1/2)-7/2*c*a^2*arctanh(1/(-a^2*x^2+1)^
(1/2))-3*c*a/x/(-a^2*x^2+1)^(1/2)-1/2*c/x^2/(-a^2*x^2+1)^(1/2)
```

maxima [A] time = 0.43, size = 116, normalized size = 1.53

$$\frac{3\,a^3cx}{\sqrt{-a^2x^2+1}}+a^2c\arcsin(ax)-\frac{7}{2}\,a^2c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{a^2c}{2\sqrt{-a^2x^2+1}}-\frac{3\,ac}{\sqrt{-a^2x^2+1}\,x}-\frac{c}{2\sqrt{-a^2x^2+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^3,x, algorithm="max
ima")
```

```
[Out] 3*a^3*c*x/sqrt(-a^2*x^2 + 1) + a^2*c*arcsin(a*x) - 7/2*a^2*c*log(2*sqrt(-a^
2*x^2 + 1)/abs(x) + 2/abs(x)) + 1/2*a^2*c/sqrt(-a^2*x^2 + 1) - 3*a*c/(sqrt(
-a^2*x^2 + 1)*x) - 1/2*c/(sqrt(-a^2*x^2 + 1)*x^2)
```

mupad [B] time = 0.05, size = 83, normalized size = 1.09

$$\frac{a^3\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}-\frac{3\,a\,c\,\sqrt{1-a^2\,x^2}}{x}-\frac{c\,\sqrt{1-a^2\,x^2}}{2\,x^2}+\frac{a^2\,c\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,7\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (a^2*c*atan((1 - a^2*x^2)^(1/2)*1i)*7i)/2 - (c*(1 - a^2*x^2)^(1/2))/(2*x^2)
 - (3*a*c*(1 - a^2*x^2)^(1/2))/x + (a^3*c*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/
2)
```

sympy [C] time = 10.01, size = 223, normalized size = 2.93

$$a^3c\left(\begin{cases}\sqrt{\frac{1}{a^2}}\ \mathrm{asin}\left(x\sqrt{a^2}\right) & \text{for}\ a^2>0\\ \sqrt{-\frac{1}{a^2}}\ \mathrm{asinh}\left(x\sqrt{-a^2}\right) & \text{for}\ a^2<0\end{cases}\right)+3a^2c\left(\begin{cases}-\mathrm{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\,\mathrm{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)+3ac\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ \left|a^2x^2\right| \\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**3,x)
```

```
[Out] a**3*c*Piecewise((sqrt(a**(-2))*asin(x*sqrt(a**2)), a**2 > 0), (sqrt(-1/a**
2)*asinh(x*sqrt(-a**2)), a**2 < 0)) + 3*a**2*c*Piecewise((-acosh(1/(a*x)),
1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), True)) + 3*a*c*Piecewise((-I*sqrt(
a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x**2 + 1)/x, True)) + c
*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Ab
s(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2
))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True))
```

3.1144 $\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^4}\,dx$

Optimal. Leaf size=94

$$-\frac{11a^2c\sqrt{1-a^2x^2}}{3x}-\frac{3ac\sqrt{1-a^2x^2}}{2x^2}-\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{5}{2}a^3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -5/2*a^3*c*arctanh((-a^2*x^2+1)^(1/2))-1/3*c*(-a^2*x^2+1)^(1/2)/x^3-3/2*a*c
*(-a^2*x^2+1)^(1/2)/x^2-11/3*a^2*c*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.20, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6148, 1807, 807, 266, 63, 208}

$$-\frac{11a^2c\sqrt{1-a^2x^2}}{3x}-\frac{3ac\sqrt{1-a^2x^2}}{2x^2}-\frac{c\sqrt{1-a^2x^2}}{3x^3}-\frac{5}{2}a^3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^4,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(3*x^3) - (3*a*c*Sqrt[1 - a^2*x^2])/(2*x^2) - (11*a^
2*c*Sqrt[1 - a^2*x^2])/(3*x) - (5*a^3*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^4}\,dx = c\int \frac{(1+ax)^3}{x^4\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3} - \frac{1}{3}c\int \frac{-9a-11a^2x-3a^3x^2}{x^3\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3} - \frac{3ac\sqrt{1-a^2x^2}}{2x^2} + \frac{1}{6}c\int \frac{22a^2+15a^3x}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3} - \frac{3ac\sqrt{1-a^2x^2}}{2x^2} - \frac{11a^2c\sqrt{1-a^2x^2}}{3x} + \frac{1}{2}\left(5a^3c\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3} - \frac{3ac\sqrt{1-a^2x^2}}{2x^2} - \frac{11a^2c\sqrt{1-a^2x^2}}{3x} + \frac{1}{4}\left(5a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a}}\right.$$
$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3} - \frac{3ac\sqrt{1-a^2x^2}}{2x^2} - \frac{11a^2c\sqrt{1-a^2x^2}}{3x} - \frac{1}{2}(5ac)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx\right.$$
$$= -\frac{c\sqrt{1-a^2x^2}}{3x^3} - \frac{3ac\sqrt{1-a^2x^2}}{2x^2} - \frac{11a^2c\sqrt{1-a^2x^2}}{3x} - \frac{5}{2}a^3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.64

$$-\frac{c\sqrt{1-a^2x^2}\left(22a^2x^2+9ax+2\right)}{6x^3} - \frac{5}{2}a^3c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^4,x]
```

```
[Out] -1/6*(c*Sqrt[1 - a^2*x^2]*(2 + 9*a*x + 22*a^2*x^2))/x^3 - (5*a^3*c*ArcTanh[
Sqrt[1 - a^2*x^2]])/2
```

fricas [A] time = 0.65, size = 66, normalized size = 0.70

$$\frac{15\,a^3cx^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(22\,a^2cx^2+9\,acx+2\,c\right)\sqrt{-a^2x^2+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/6*(15*a^3*c*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (22*a^2*c*x^2 + 9*a*c*x
 + 2*c)*sqrt(-a^2*x^2 + 1))/x^3
```

giac [B] time = 0.23, size = 218, normalized size = 2.32

$$\frac{\left(a^4c+\frac{9\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^2c}{x}+\frac{45\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2c}{x^2}\right)a^6x^3}{24\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3|a|}-\frac{5\,a^4c\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{45\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^4c}{x}+\frac{9\left(\sqrt{-a^2x\right.}}{}}{24\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^4,x, algorithm="gia
c")
```

```
[Out] 1/24*(a^4*c + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^2*c/x + 45*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^2*c/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*abs(a
)) - 5/2*a^4*c*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x))
)/abs(a) - 1/24*(45*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c/x + 9*(sqrt(-a^2*
x^2 + 1)*abs(a) + a)^2*a^2*c/x^2 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c/x^3)
/(a^2*abs(a))
```

maple [B] time = 0.04, size = 184, normalized size = 1.96

$$-c\left(\frac{a^3}{\sqrt{-a^2x^2+1}}+\frac{3x\,a^4}{\sqrt{-a^2x^2+1}}+2a^3\left(\frac{1}{\sqrt{-a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)-\frac{10a^2\left(-\frac{1}{x\sqrt{-a^2x^2+1}}+\frac{2a^2x}{\sqrt{-a^2x^2+1}}\right)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^4,x)
```

```
[Out] -c*(a^3/(-a^2*x^2+1)^(1/2)+3*x*a^4/(-a^2*x^2+1)^(1/2)+2*a^3*(1/(-a^2*x^2+1)
^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))-10/3*a^2*(-1/x/(-a^2*x^2+1)^(1/2)+2*a
^2*x/(-a^2*x^2+1)^(1/2))+1/3/x^3/(-a^2*x^2+1)^(1/2)-3*a*(-1/2/x^2/(-a^2*x^2
+1)^(1/2)+3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))))
```

maxima [A] time = 0.33, size = 128, normalized size = 1.36

$$\frac{11\,a^4cx}{3\sqrt{-a^2x^2+1}}-\frac{5}{2}\,a^3c\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{3\,a^3c}{2\sqrt{-a^2x^2+1}}-\frac{10\,a^2c}{3\sqrt{-a^2x^2+1}\,x}-\frac{3\,ac}{2\sqrt{-a^2x^2+1}\,x^2}-\frac{c}{3\sqrt{-a^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^4,x, algorithm="max
ima")
```

```
[Out] 11/3*a^4*c*x/sqrt(-a^2*x^2 + 1) - 5/2*a^3*c*log(2*sqrt(-a^2*x^2 + 1)/abs(x)
 + 2/abs(x)) + 3/2*a^3*c/sqrt(-a^2*x^2 + 1) - 10/3*a^2*c/(sqrt(-a^2*x^2 + 1
)*x) - 3/2*a*c/(sqrt(-a^2*x^2 + 1)*x^2) - 1/3*c/(sqrt(-a^2*x^2 + 1)*x^3)
```

mupad [B] time = 0.89, size = 82, normalized size = 0.87

$$-\frac{c\sqrt{1-a^2\,x^2}}{3\,x^3}-\frac{11\,a^2\,c\sqrt{1-a^2\,x^2}}{3\,x}-\frac{3\,a\,c\sqrt{1-a^2\,x^2}}{2\,x^2}+\frac{a^3\,c\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,5\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^3)/(x^4*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (a^3*c*atan((1 - a^2*x^2)^(1/2)*1i)*5i)/2 - (c*(1 - a^2*x^2)^(1/2))/(3*x^3)
 - (11*a^2*c*(1 - a^2*x^2)^(1/2))/(3*x) - (3*a*c*(1 - a^2*x^2)^(1/2))/(2*x^
2)
```

sympy [C] time = 28.08, size = 267, normalized size = 2.84

$$a^3c\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ |a^2x^2|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+3ac\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} \\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{}{2ax^3}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**4,x)
```

```
[Out] a**3*c*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)),
 True)) + 3*a**2*c*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1)
, (-sqrt(-a**2*x**2 + 1)/x, True)) + 3*a*c*Piecewise((-a**2*acosh(1/(a*x))/
2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1
/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**
2*x**2))), True)) + c*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sq
rt(a**2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 +
 1)/(3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True))
```

3.1145 $\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^5}\,dx$

Optimal. Leaf size=115

$$-\frac{15a^2c\sqrt{1-a^2x^2}}{8x^2}-\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{ac\sqrt{1-a^2x^2}}{x^3}-\frac{15}{8}a^4c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{3a^3c\sqrt{1-a^2x^2}}{x}$$

```
[Out] -15/8*a^4*c*arctanh((-a^2*x^2+1)^(1/2))-1/4*c*(-a^2*x^2+1)^(1/2)/x^4-a*c*(-
a^2*x^2+1)^(1/2)/x^3-15/8*a^2*c*(-a^2*x^2+1)^(1/2)/x^2-3*a^3*c*(-a^2*x^2+1)
^(1/2)/x
```

Rubi [A] time = 0.22, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 1807, 835, 807, 266, 63, 208}

$$-\frac{3a^3c\sqrt{1-a^2x^2}}{x}-\frac{15a^2c\sqrt{1-a^2x^2}}{8x^2}-\frac{ac\sqrt{1-a^2x^2}}{x^3}-\frac{c\sqrt{1-a^2x^2}}{4x^4}-\frac{15}{8}a^4c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^5,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(4*x^4) - (a*c*Sqrt[1 - a^2*x^2])/x^3 - (15*a^2*c*Sq
rt[1 - a^2*x^2])/(8*x^2) - (3*a^3*c*Sqrt[1 - a^2*x^2])/x - (15*a^4*c*ArcTan
h[Sqrt[1 - a^2*x^2]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^5}\,dx = c\int \frac{(1+ax)^3}{x^5\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4} - \frac{1}{4}c\int \frac{-12a-15a^2x-4a^3x^2}{x^4\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4} - \frac{ac\sqrt{1-a^2x^2}}{x^3} + \frac{1}{12}c\int \frac{45a^2+36a^3x}{x^3\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4} - \frac{ac\sqrt{1-a^2x^2}}{x^3} - \frac{15a^2c\sqrt{1-a^2x^2}}{8x^2} - \frac{1}{24}c\int \frac{-72a^3-45a^4x}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4} - \frac{ac\sqrt{1-a^2x^2}}{x^3} - \frac{15a^2c\sqrt{1-a^2x^2}}{8x^2} - \frac{3a^3c\sqrt{1-a^2x^2}}{x} + \frac{1}{8}\left(15a^4c\right)\int$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4} - \frac{ac\sqrt{1-a^2x^2}}{x^3} - \frac{15a^2c\sqrt{1-a^2x^2}}{8x^2} - \frac{3a^3c\sqrt{1-a^2x^2}}{x} + \frac{1}{16}\left(15a^4c\right)\mathrm{S}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4} - \frac{ac\sqrt{1-a^2x^2}}{x^3} - \frac{15a^2c\sqrt{1-a^2x^2}}{8x^2} - \frac{3a^3c\sqrt{1-a^2x^2}}{x} - \frac{1}{8}\left(15a^2c\right)\mathrm{Su}$$
$$= -\frac{c\sqrt{1-a^2x^2}}{4x^4} - \frac{ac\sqrt{1-a^2x^2}}{x^3} - \frac{15a^2c\sqrt{1-a^2x^2}}{8x^2} - \frac{3a^3c\sqrt{1-a^2x^2}}{x} - \frac{15}{8}a^4c\tanh$$

Mathematica [C] time = 0.12, size = 97, normalized size = 0.84

$$\frac{1}{2}ac\left(-\frac{\sqrt{1-a^2x^2}\left(6a^2x^2+3ax+2\right)}{x^3} - 2a^3\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-a^2x^2\right) - 3a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^5,x]
```

```
[Out] (a*c*(-((Sqrt[1 - a^2*x^2]*(2 + 3*a*x + 6*a^2*x^2))/x^3) - 3*a^3*ArcTanh[Sq
rt[1 - a^2*x^2]] - 2*a^3*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1
 - a^2*x^2]))/2
```

fricas [A] time = 0.55, size = 75, normalized size = 0.65

$$\frac{15\,a^4cx^4\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - \left(24\,a^3cx^3+15\,a^2cx^2+8\,acx+2\,c\right)\sqrt{-a^2x^2+1}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/8*(15*a^4*c*x^4*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (24*a^3*c*x^3 + 15*a^2*
c*x^2 + 8*a*c*x + 2*c)*sqrt(-a^2*x^2 + 1))/x^4
```

giac [B] time = 0.18, size = 280, normalized size = 2.43

$$\frac{\left(a^5c + \frac{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a^3c}{x} + \frac{32\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2ac}{x^2} + \frac{104\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3c}{ax^3}\right)a^8x^4}{64\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4|a|} - \frac{15\,a^5c\log\left(\frac{\left|-2\,\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{8\,|a|} - \frac{104\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/64*(a^5*c + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^3*c/x + 32*(sqrt(-a^2*x^2
 + 1)*abs(a) + a)^2*a*c/x^2 + 104*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*c/(a*x^
3))*a^8*x^4/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*abs(a)) - 15/8*a^5*c*log(1/2
*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/64*(104*(
sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^5*c*abs(a)/x + 32*(sqrt(-a^2*x^2 + 1)*abs(
a) + a)^2*a^3*c*abs(a)/x^2 + 8*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a*c*abs(a)
/x^3 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c*abs(a)/(a*x^4))/a^4
```

maple [B] time = 0.05, size = 231, normalized size = 2.01

$$-c\left(\frac{x\,a^5}{\sqrt{-a^2x^2+1}} + 3a^4\left(\frac{1}{\sqrt{-a^2x^2+1}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right) + 2a^3\left(-\frac{1}{x\sqrt{-a^2x^2+1}} + \frac{2a^2x}{\sqrt{-a^2x^2+1}}\right) - 3a\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^5,x)
```

```
[Out] -c*(x*a^5/(-a^2*x^2+1)^(1/2)+3*a^4*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^
2+1)^(1/2)))+2*a^3*(-1/x/(-a^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2))-3*a
*(-1/3/x^3/(-a^2*x^2+1)^(1/2)+4/3*a^2*(-1/x/(-a^2*x^2+1)^(1/2)+2*a^2*x/(-a^
2*x^2+1)^(1/2)))-13/4*a^2*(-1/2/x^2/(-a^2*x^2+1)^(1/2)+3/2*a^2*(1/(-a^2*x^2
+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))))+1/4/x^4/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.33, size = 149, normalized size = 1.30

$$\frac{3\,a^5cx}{\sqrt{-a^2x^2+1}}-\frac{15}{8}\,a^4c\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{15\,a^4c}{8\,\sqrt{-a^2x^2+1}}-\frac{2\,a^3c}{\sqrt{-a^2x^2+1}\,x}-\frac{13\,a^2c}{8\,\sqrt{-a^2x^2+1}\,x^2}-\frac{ac}{\sqrt{-a^2x^2+1}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^5,x, algorithm="max
ima")
```

```
[Out] 3*a^5*c*x/sqrt(-a^2*x^2 + 1) - 15/8*a^4*c*log(2*sqrt(-a^2*x^2 + 1)/abs(x) +
 2/abs(x)) + 15/8*a^4*c/sqrt(-a^2*x^2 + 1) - 2*a^3*c/(sqrt(-a^2*x^2 + 1)*x)
 - 13/8*a^2*c/(sqrt(-a^2*x^2 + 1)*x^2) - a*c/(sqrt(-a^2*x^2 + 1)*x^3) - 1/4
*c/(sqrt(-a^2*x^2 + 1)*x^4)
```

mupad [B] time = 0.88, size = 103, normalized size = 0.90

$$-\frac{c\,\sqrt{1-a^2\,x^2}}{4\,x^4}-\frac{15\,a^2\,c\,\sqrt{1-a^2\,x^2}}{8\,x^2}-\frac{3\,a^3\,c\,\sqrt{1-a^2\,x^2}}{x}-\frac{a\,c\,\sqrt{1-a^2\,x^2}}{x^3}+\frac{a^4\,c\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\,1\mathrm{i}\right)\,15\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^3)/(x^5*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (a^4*c*atan((1 - a^2*x^2)^(1/2)*1i)*15i)/8 - (c*(1 - a^2*x^2)^(1/2))/(4*x^4
) - (15*a^2*c*(1 - a^2*x^2)^(1/2))/(8*x^2) - (3*a^3*c*(1 - a^2*x^2)^(1/2))/
x - (a*c*(1 - a^2*x^2)^(1/2))/x^3
```

sympy [C] time = 12.00, size = 411, normalized size = 3.57

$$a^3c\left(\begin{cases}-\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ \left|a^2x^2\right|>1\\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{a^2\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for}\ \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\,\mathrm{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)+3ac\left(\begin{cases}-\frac{2ia^2\sqrt{a^2\cdots}}{3x\cdots} \\ -\frac{2a^2\sqrt{-\cdots}}{3\cdots}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**5,x)
```

```
[Out] a**3*c*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a*
*2*x**2 + 1)/x, True)) + 3*a**2*c*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqr
t(-1 + 1/(a**2*x**2))/(2*x), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2
```

```
 - I*a/(2*x*sqrt(1 - 1/(a**2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2)))
, True)) + 3*a*c*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a*
*2*x**2 - 1)/(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(
3*x) - sqrt(-a**2*x**2 + 1)/(3*x**3), True)) + c*Piecewise((-3*a**4*acosh(1
/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(
a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1)
, (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/
(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), Tr
ue))
```

3.1146 $\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^6}\,dx$

Optimal. Leaf size=144

$$-\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{13}{8}a^5c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{38a^4c\sqrt{1-a^2x^2}}{15x}-\frac{13a^3c\sqrt{1-a^2x^2}}{8x^2}$$

```
[Out] -13/8*a^5*c*arctanh((-a^2*x^2+1)^(1/2))-1/5*c*(-a^2*x^2+1)^(1/2)/x^5-3/4*a*
c*(-a^2*x^2+1)^(1/2)/x^4-19/15*a^2*c*(-a^2*x^2+1)^(1/2)/x^3-13/8*a^3*c*(-a^
2*x^2+1)^(1/2)/x^2-38/15*a^4*c*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.25, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6148, 1807, 835, 807, 266, 63, 208}

$$-\frac{38a^4c\sqrt{1-a^2x^2}}{15x}-\frac{13a^3c\sqrt{1-a^2x^2}}{8x^2}-\frac{19a^2c\sqrt{1-a^2x^2}}{15x^3}-\frac{3ac\sqrt{1-a^2x^2}}{4x^4}-\frac{c\sqrt{1-a^2x^2}}{5x^5}-\frac{13}{8}a^5c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^6,x]
```

```
[Out] -(c*Sqrt[1 - a^2*x^2])/(5*x^5) - (3*a*c*Sqrt[1 - a^2*x^2])/(4*x^4) - (19*a^
2*c*Sqrt[1 - a^2*x^2])/(15*x^3) - (13*a^3*c*Sqrt[1 - a^2*x^2])/(8*x^2) - (3
8*a^4*c*Sqrt[1 - a^2*x^2])/(15*x) - (13*a^5*c*ArcTanh[Sqrt[1 - a^2*x^2]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)}{x^6}\,dx = c\int \frac{(1+ax)^3}{x^6\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{c\sqrt{1-a^2x^2}}{5x^5} - \frac{1}{5}c\int \frac{-15a-19a^2x-5a^3x^2}{x^5\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{c\sqrt{1-a^2x^2}}{5x^5} - \frac{3ac\sqrt{1-a^2x^2}}{4x^4} + \frac{1}{20}c\int \frac{76a^2+65a^3x}{x^4\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{c\sqrt{1-a^2x^2}}{5x^5} - \frac{3ac\sqrt{1-a^2x^2}}{4x^4} - \frac{19a^2c\sqrt{1-a^2x^2}}{15x^3} - \frac{1}{60}c\int \frac{-195a^3-152a^4x}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{c\sqrt{1-a^2x^2}}{5x^5} - \frac{3ac\sqrt{1-a^2x^2}}{4x^4} - \frac{19a^2c\sqrt{1-a^2x^2}}{15x^3} - \frac{13a^3c\sqrt{1-a^2x^2}}{8x^2} + \frac{1}{120}c\int$$

$$= -\frac{c\sqrt{1-a^2x^2}}{5x^5} - \frac{3ac\sqrt{1-a^2x^2}}{4x^4} - \frac{19a^2c\sqrt{1-a^2x^2}}{15x^3} - \frac{13a^3c\sqrt{1-a^2x^2}}{8x^2} - \frac{38a^4c\sqrt{1}}{15}$$

$$= -\frac{c\sqrt{1-a^2x^2}}{5x^5} - \frac{3ac\sqrt{1-a^2x^2}}{4x^4} - \frac{19a^2c\sqrt{1-a^2x^2}}{15x^3} - \frac{13a^3c\sqrt{1-a^2x^2}}{8x^2} - \frac{38a^4c\sqrt{1}}{15}$$

$$= -\frac{c\sqrt{1-a^2x^2}}{5x^5} - \frac{3ac\sqrt{1-a^2x^2}}{4x^4} - \frac{19a^2c\sqrt{1-a^2x^2}}{15x^3} - \frac{13a^3c\sqrt{1-a^2x^2}}{8x^2} - \frac{38a^4c\sqrt{1}}{15}$$

$$= -\frac{c\sqrt{1-a^2x^2}}{5x^5} - \frac{3ac\sqrt{1-a^2x^2}}{4x^4} - \frac{19a^2c\sqrt{1-a^2x^2}}{15x^3} - \frac{13a^3c\sqrt{1-a^2x^2}}{8x^2} - \frac{38a^4c\sqrt{1}}{15}$$

Mathematica [C] time = 0.13, size = 110, normalized size = 0.76

$$-3a^5c\sqrt{1-a^2x^2}\ {}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-a^2x^2\right)-\frac{1}{2}a^5c\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{c\sqrt{1-a^2x^2}\left(76a^4x^4+15a^3x^3+38a^2x^2+6\right)}{30x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2))/x^6,x]
```

```
[Out] -1/30*(c*Sqrt[1 - a^2*x^2]*(6 + 38*a^2*x^2 + 15*a^3*x^3 + 76*a^4*x^4))/x^5
- (a^5*c*ArcTanh[Sqrt[1 - a^2*x^2]])/2 - 3*a^5*c*Sqrt[1 - a^2*x^2]*Hypergeo
metric2F1[1/2, 3, 3/2, 1 - a^2*x^2]
```

fricas [A] time = 0.64, size = 84, normalized size = 0.58

$$\frac{195\,a^5cx^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\left(304\,a^4cx^4+195\,a^3cx^3+152\,a^2cx^2+90\,acx+24\,c\right)\sqrt{-a^2x^2+1}}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^6,x, algorithm="fri
cas")
```

```
[Out] 1/120*(195*a^5*c*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) - (304*a^4*c*x^4 + 195
*a^3*c*x^3 + 152*a^2*c*x^2 + 90*a*c*x + 24*c)*sqrt(-a^2*x^2 + 1))/x^5
```

giac [B] time = 0.57, size = 332, normalized size = 2.31

$$\frac{\left(6\,a^{6}c+\frac{45\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)a^{4}c}{x}+\frac{170\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{2}a^{2}c}{x^{2}}+\frac{480\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{3}c}{x^{3}}+\frac{1380\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{4}c}{a^{2}x^{4}}\right)a^{10}x^{5}}{960\left(\sqrt{-a^{2}x^{2}+1}\,|a|+a\right)^{5}|a|}-\frac{13\,a^{6}c\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^6,x, algorithm="gia
c")
```

```
[Out] 1/960*(6*a^6*c + 45*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4*c/x + 170*(sqrt(-a^
2*x^2 + 1)*abs(a) + a)^2*a^2*c/x^2 + 480*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*
c/x^3 + 1380*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4*c/(a^2*x^4))*a^10*x^5/((sqrt
(-a^2*x^2 + 1)*abs(a) + a)^5*abs(a)) - 13/8*a^6*c*log(1/2*abs(-2*sqrt(-a^2*
x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/960*(1380*(sqrt(-a^2*x^2 +
1)*abs(a) + a)*a^8*c/x + 480*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^6*c/x^2 +
170*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3*a^4*c/x^3 + 45*(sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^4*a^2*c/x^4 + 6*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5*c/x^5)/(a^4*abs
(a))
```

maple [B] time = 0.05, size = 292, normalized size = 2.03

$$-c\left(a^{5}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)\right)+3a^{4}\left(-\frac{1}{x\sqrt{-a^{2}x^{2}+1}}+\frac{2a^{2}x}{\sqrt{-a^{2}x^{2}+1}}\right)-\frac{16a^{2}\left(-\frac{1}{3x^{3}\sqrt{-a^{2}x^{2}+1}}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^6,x)
```

```
[Out] -c*(a^5*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))+3*a^4*(-1/x/(-
a^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2))-16/5*a^2*(-1/3/x^3/(-a^2*x^2+1
```

```
)^(1/2)+4/3*a^2*(-1/x/(-a^2*x^2+1)^(1/2)+2*a^2*x/(-a^2*x^2+1)^(1/2)))+2*a^3
*(-1/2/x^2/(-a^2*x^2+1)^(1/2)+3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2
*x^2+1)^(1/2))))-3*a*(-1/4/x^4/(-a^2*x^2+1)^(1/2)+5/4*a^2*(-1/2/x^2/(-a^2*x
^2+1)^(1/2)+3/2*a^2*(1/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)))))+
1/5/x^5/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.33, size = 170, normalized size = 1.18

$$\frac{38\,a^6cx}{15\,\sqrt{-a^2x^2+1}}-\frac{13}{8}\,a^5c\log\left(\frac{2\,\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{13\,a^5c}{8\,\sqrt{-a^2x^2+1}}-\frac{19\,a^4c}{15\,\sqrt{-a^2x^2+1}\,x}-\frac{7\,a^3c}{8\,\sqrt{-a^2x^2+1}\,x^2}-\frac{16\,a}{15\,\sqrt{-a^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)/x^6,x, algorithm="max
ima")
```

```
[Out] 38/15*a^6*c*x/sqrt(-a^2*x^2 + 1) - 13/8*a^5*c*log(2*sqrt(-a^2*x^2 + 1)/abs(
x) + 2/abs(x)) + 13/8*a^5*c/sqrt(-a^2*x^2 + 1) - 19/15*a^4*c/(sqrt(-a^2*x^2
 + 1)*x) - 7/8*a^3*c/(sqrt(-a^2*x^2 + 1)*x^2) - 16/15*a^2*c/(sqrt(-a^2*x^2
+ 1)*x^3) - 3/4*a*c/(sqrt(-a^2*x^2 + 1)*x^4) - 1/5*c/(sqrt(-a^2*x^2 + 1)*x^
5)
```

mupad [B] time = 0.04, size = 124, normalized size = 0.86

$$-\frac{c\,\sqrt{1-a^2\,x^2}}{5\,x^5}-\frac{19\,a^2\,c\,\sqrt{1-a^2\,x^2}}{15\,x^3}-\frac{13\,a^3\,c\,\sqrt{1-a^2\,x^2}}{8\,x^2}-\frac{38\,a^4\,c\,\sqrt{1-a^2\,x^2}}{15\,x}-\frac{3\,a\,c\,\sqrt{1-a^2\,x^2}}{4\,x^4}+\frac{a^5\,c\,\mathrm{atan}\left(\sqrt{1-a^2\,x}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^3)/(x^6*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (a^5*c*atan((1 - a^2*x^2)^(1/2)*1i)*13i)/8 - (c*(1 - a^2*x^2)^(1/2))/(5*x^5
) - (19*a^2*c*(1 - a^2*x^2)^(1/2))/(15*x^3) - (13*a^3*c*(1 - a^2*x^2)^(1/2)
)/(8*x^2) - (38*a^4*c*(1 - a^2*x^2)^(1/2))/(15*x) - (3*a*c*(1 - a^2*x^2)^(1
/2))/(4*x^4)
```

sympy [C] time = 49.29, size = 518, normalized size = 3.60

$$a^3c\left(\begin{cases}-\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{-1+\frac{1}{a^2x^2}}}{2x} & \text{for}\ \frac{1}{|a^2x^2|}>1\\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2}-\frac{ia}{2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{i}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)+3a^2c\left(\begin{cases}-\frac{2ia^2\sqrt{a^2x^2-1}}{3x}-\frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for}\ \left|a^2x^2\right|>1\\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x}-\frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)/x**6,x)
```

```
[Out] a**3*c*Piecewise((-a**2*acosh(1/(a*x))/2 - a*sqrt(-1 + 1/(a**2*x**2))/(2*x)
, 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1/(a*x))/2 - I*a/(2*x*sqrt(1 - 1/(a**
2*x**2))) + I/(2*a*x**3*sqrt(1 - 1/(a**2*x**2))), True)) + 3*a**2*c*Piecewi
se((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(3*x**3), A
bs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a**2*x**2 +
 1)/(3*x**3), True)) + 3*a*c*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(
8*x*sqrt(-1 + 1/(a**2*x**2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*
a*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(
a*x))/8 - 3*I*a**3/(8*x*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(
a**2*x**2))) + I/(4*a*x**5*sqrt(1 - 1/(a**2*x**2))), True)) + c*Piecewise((
-8*a**5*sqrt(-1 + 1/(a**2*x**2))/15 - 4*a**3*sqrt(-1 + 1/(a**2*x**2))/(15*x
**2) - a*sqrt(-1 + 1/(a**2*x**2))/(5*x**4), 1/Abs(a**2*x**2) > 1), (-8*I*a*
*5*sqrt(1 - 1/(a**2*x**2))/15 - 4*I*a**3*sqrt(1 - 1/(a**2*x**2))/(15*x**2)
- I*a*sqrt(1 - 1/(a**2*x**2))/(5*x**4), True))
```

3.1147 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=121

$$-\frac{c^2(ax+1)^2\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{7c^2(ax+1)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\sin^{-1}(ax)}{8a}$$

```
[Out] -7/12*c^2*(-a^2*x^2+1)^(3/2)/a-7/20*c^2*(a*x+1)*(-a^2*x^2+1)^(3/2)/a-1/5*c^
2*(a*x+1)^2*(-a^2*x^2+1)^(3/2)/a+7/8*c^2*arcsin(a*x)/a+7/8*c^2*x*(-a^2*x^2+
1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6138, 671, 641, 195, 216}

$$-\frac{c^2(ax+1)^2\left(1-a^2x^2\right)^{3/2}}{5a}-\frac{7c^2(ax+1)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (7*c^2*x*Sqrt[1 - a^2*x^2])/8 - (7*c^2*(1 - a^2*x^2)^(3/2))/(12*a) - (7*c^2
*(1 + a*x)*(1 - a^2*x^2)^(3/2))/(20*a) - (c^2*(1 + a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^2*ArcSin[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=c^2\int(1+ax)^3\sqrt{1-a^2x^2}\,dx$$

$$=-\frac{c^2(1+ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{5}\left(7c^2\right)\int(1+ax)^2\sqrt{1-a^2x^2}\,dx$$

$$=-\frac{7c^2(1+ax)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{c^2(1+ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\left(7c^2\right)\int(1+ax)\sqrt{1-$$

$$=-\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}-\frac{7c^2(1+ax)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{c^2(1+ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{1}{4}\Big($$

$$=\frac{7}{8}c^2x\sqrt{1-a^2x^2}-\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}-\frac{7c^2(1+ax)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{c^2(1+ax)^2\left(1\right.}{5a}$$

$$=\frac{7}{8}c^2x\sqrt{1-a^2x^2}-\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}-\frac{7c^2(1+ax)\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{c^2(1+ax)^2\left(1\right.}{5a}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.62

$$\frac{c^2\left(\sqrt{1-a^2x^2}\left(24a^4x^4+90a^3x^3+112a^2x^2+15ax-136\right)-210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(Sqrt[1 - a^2*x^2]*(-136 + 15*a*x + 112*a^2*x^2 + 90*a^3*x^3 + 24*a^4*
x^4) - 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(120*a)
```

fricas [A] time = 0.83, size = 93, normalized size = 0.77

$$-\frac{210\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(24\,a^4c^2x^4+90\,a^3c^2x^3+112\,a^2c^2x^2+15\,ac^2x-136\,c^2\right)\sqrt{-a^2x^2+1}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] -1/120*(210*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (24*a^4*c^2*x^4 +
90*a^3*c^2*x^3 + 112*a^2*c^2*x^2 + 15*a*c^2*x - 136*c^2)*sqrt(-a^2*x^2 + 1)
)/a
```

giac [A] time = 0.23, size = 78, normalized size = 0.64

$$\frac{7\,c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{8\,|a|}+\frac{1}{120}\,\sqrt{-a^2x^2+1}\left(\left(15\,c^2+2\left(56\,ac^2+3\left(4\,a^3c^2x+15\,a^2c^2\right)x\right)x\right)x-\frac{136\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] 7/8*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/120*sqrt(-a^2*x^2 + 1)*((15*c^2 + 2*(
56*a*c^2 + 3*(4*a^3*c^2*x + 15*a^2*c^2)*x)*x)*x - 136*c^2/a)
```

maple [A] time = 0.07, size = 183, normalized size = 1.51

$$-\frac{c^2a^5x^6}{5\sqrt{-a^2x^2+1}}-\frac{11c^2a^3x^4}{15\sqrt{-a^2x^2+1}}+\frac{31c^2a\,x^2}{15\sqrt{-a^2x^2+1}}-\frac{17c^2}{15a\sqrt{-a^2x^2+1}}-\frac{3c^2a^4x^5}{4\sqrt{-a^2x^2+1}}+\frac{5c^2a^2x^3}{8\sqrt{-a^2x^2+1}}+\frac{c^2x}{8\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/5*c^2*a^5*x^6/(-a^2*x^2+1)^(1/2)-11/15*c^2*a^3*x^4/(-a^2*x^2+1)^(1/2)+31
/15*c^2*a*x^2/(-a^2*x^2+1)^(1/2)-17/15*c^2/a/(-a^2*x^2+1)^(1/2)-3/4*c^2*a^4
*x^5/(-a^2*x^2+1)^(1/2)+5/8*c^2*a^2*x^3/(-a^2*x^2+1)^(1/2)+1/8*c^2*x/(-a^2*
x^2+1)^(1/2)+7/8*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.43, size = 164, normalized size = 1.36

$$-\frac{a^5c^2x^6}{5\sqrt{-a^2x^2+1}}-\frac{3\,a^4c^2x^5}{4\sqrt{-a^2x^2+1}}-\frac{11\,a^3c^2x^4}{15\sqrt{-a^2x^2+1}}+\frac{5\,a^2c^2x^3}{8\sqrt{-a^2x^2+1}}+\frac{31\,ac^2x^2}{15\sqrt{-a^2x^2+1}}+\frac{c^2x}{8\sqrt{-a^2x^2+1}}+\frac{7\,c^2\arcsin(}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] -1/5*a^5*c^2*x^6/sqrt(-a^2*x^2 + 1) - 3/4*a^4*c^2*x^5/sqrt(-a^2*x^2 + 1) -
11/15*a^3*c^2*x^4/sqrt(-a^2*x^2 + 1) + 5/8*a^2*c^2*x^3/sqrt(-a^2*x^2 + 1) +
 31/15*a*c^2*x^2/sqrt(-a^2*x^2 + 1) + 1/8*c^2*x/sqrt(-a^2*x^2 + 1) + 7/8*c^
2*arcsin(a*x)/a - 17/15*c^2/(sqrt(-a^2*x^2 + 1)*a)
```

mupad [B] time = 0.03, size = 128, normalized size = 1.06

$$\frac{c^2\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{7\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}-\frac{17\,c^2\,\sqrt{1-a^2\,x^2}}{15\,a}+\frac{14\,a\,c^2\,x^2\,\sqrt{1-a^2\,x^2}}{15}+\frac{3\,a^2\,c^2\,x^3\,\sqrt{1-a^2\,x^2}}{4}+\frac{a^3\,c^2\,x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (c^2*x*(1 - a^2*x^2)^(1/2))/8 + (7*c^2*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(1/
2)) - (17*c^2*(1 - a^2*x^2)^(1/2))/(15*a) + (14*a*c^2*x^2*(1 - a^2*x^2)^(1/
2))/15 + (3*a^2*c^2*x^3*(1 - a^2*x^2)^(1/2))/4 + (a^3*c^2*x^4*(1 - a^2*x^2)
^(1/2))/5
```

sympy [C] time = 20.17, size = 340, normalized size = 2.81

$$a^3c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2x^2+1}}{5}-\frac{x^2\sqrt{-a^2x^2+1}}{15a^2}-\frac{2\sqrt{-a^2x^2+1}}{15a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+3a^2c^2\left(\begin{cases}\frac{ia^2x^5}{4\sqrt{a^2x^2-1}}-\frac{3ix^3}{8\sqrt{a^2x^2-1}}+\frac{ix}{8a^2\sqrt{a^2x^2-1}}-\frac{i\,\mathrm{acosh}(a}{8a^3}\\ -\frac{a^2x^5}{4\sqrt{-a^2x^2+1}}+\frac{3x^3}{8\sqrt{-a^2x^2+1}}-\frac{x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\mathrm{as}}{}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] a**3*c**2*Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1
)/(15*a**2) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True))
+ 3*a**2*c**2*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*
sqrt(a**2*x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a
**3), Abs(a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8
```

```
*sqrt(-a**2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**
3), True)) + 3*a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**
(3/2)/(3*a**2), True)) + c**2*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)
) - I*x/(2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1),
(x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

3.1148 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=143

$$-\frac{c^3(ax+1)^2\left(1-a^2x^2\right)^{5/2}}{7a}-\frac{3c^3(ax+1)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{9}{}$$

```
[Out] 3/8*c^3*x*(-a^2*x^2+1)^(3/2)-3/10*c^3*(-a^2*x^2+1)^(5/2)/a-3/14*c^3*(a*x+1)
*(-a^2*x^2+1)^(5/2)/a-1/7*c^3*(a*x+1)^2*(-a^2*x^2+1)^(5/2)/a+9/16*c^3*arcsi
n(a*x)/a+9/16*c^3*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6138, 671, 641, 195, 216}

$$-\frac{c^3(ax+1)^2\left(1-a^2x^2\right)^{5/2}}{7a}-\frac{3c^3(ax+1)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (9*c^3*x*Sqrt[1 - a^2*x^2])/16 + (3*c^3*x*(1 - a^2*x^2)^(3/2))/8 - (3*c^3*(
1 - a^2*x^2)^(5/2))/(10*a) - (3*c^3*(1 + a*x)*(1 - a^2*x^2)^(5/2))/(14*a) -
 (c^3*(1 + a*x)^2*(1 - a^2*x^2)^(5/2))/(7*a) + (9*c^3*ArcSin[a*x])/(16*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = c^3\int(1+ax)^3\left(1-a^2x^2\right)^{3/2}\,dx$$

$$= -\frac{c^3(1+ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{1}{7}\left(9c^3\right)\int(1+ax)^2\left(1-a^2x^2\right)^{3/2}\,dx$$

$$= -\frac{3c^3(1+ax)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{c^3(1+ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{1}{2}\left(3c^3\right)\int(1+ax)\left(1-a\right.$$

$$= -\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}-\frac{3c^3(1+ax)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{c^3(1+ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{1}{2}\left(3\right.$$

$$= \frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}-\frac{3c^3(1+ax)\left(1-a^2x^2\right)^{5/2}}{14a}-\frac{c^3(1+ax)^2\left(\right.}{7}$$

$$= \frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}-\frac{3c^3(1+ax)\left(1-a^2x^2\right.}{14a}$$

$$= \frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}-\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}-\frac{3c^3(1+ax)\left(1-a^2x^2\right.}{14a}$$

Mathematica [A] time = 0.12, size = 91, normalized size = 0.64

$$-\frac{c^3\left(\sqrt{1-a^2x^2}\left(80a^6x^6+280a^5x^5+208a^4x^4-350a^3x^3-656a^2x^2-245ax+368\right)+630\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/560*(c^3*(Sqrt[1 - a^2*x^2]*(368 - 245*a*x - 656*a^2*x^2 - 350*a^3*x^3 +
 208*a^4*x^4 + 280*a^5*x^5 + 80*a^6*x^6) + 630*ArcSin[Sqrt[1 - a*x]/Sqrt[2]
]))/a
```

fricas [A] time = 0.64, size = 114, normalized size = 0.80

$$-\frac{630\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(80\,a^6c^3x^6+280\,a^5c^3x^5+208\,a^4c^3x^4-350\,a^3c^3x^3-656\,a^2c^3x^2-245\,ac^3x+36\right.}{560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] -1/560*(630*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (80*a^6*c^3*x^6 +
280*a^5*c^3*x^5 + 208*a^4*c^3*x^4 - 350*a^3*c^3*x^3 - 656*a^2*c^3*x^2 - 245
*a*c^3*x + 368*c^3)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.22, size = 102, normalized size = 0.71

$$\frac{9\,c^3\arcsin\left(ax\right)\mathrm{sgn}(a)}{16\,|a|}-\frac{1}{560}\,\sqrt{-a^2x^2+1}\left(\frac{368\,c^3}{a}-\left(245\,c^3+2\left(328\,ac^3+\left(175\,a^2c^3-4\left(26\,a^3c^3+5\left(2\,a^5c^3x+\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] 9/16*c^3*arcsin(a*x)*sgn(a)/abs(a) - 1/560*sqrt(-a^2*x^2 + 1)*(368*c^3/a -
(245*c^3 + 2*(328*a*c^3 + (175*a^2*c^3 - 4*(26*a^3*c^3 + 5*(2*a^5*c^3*x + 7
*a^4*c^3)*x)*x)*x)*x)*x)
```

maple [A] time = 0.11, size = 229, normalized size = 1.60

$$\frac{c^3a^7x^8}{7\sqrt{-a^2x^2+1}}+\frac{8c^3a^5x^6}{35\sqrt{-a^2x^2+1}}-\frac{54c^3a^3x^4}{35\sqrt{-a^2x^2+1}}+\frac{64c^3a\,x^2}{35\sqrt{-a^2x^2+1}}-\frac{23c^3}{35a\sqrt{-a^2x^2+1}}+\frac{c^3a^6x^7}{2\sqrt{-a^2x^2+1}}-\frac{9c^3a^4x^5}{8\sqrt{-a^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/7*c^3*a^7*x^8/(-a^2*x^2+1)^(1/2)+8/35*c^3*a^5*x^6/(-a^2*x^2+1)^(1/2)-54/3
5*c^3*a^3*x^4/(-a^2*x^2+1)^(1/2)+64/35*c^3*a*x^2/(-a^2*x^2+1)^(1/2)-23/35*c
```

```
^3/a/(-a^2*x^2+1)^(1/2)+1/2*c^3*a^6*x^7/(-a^2*x^2+1)^(1/2)-9/8*c^3*a^4*x^5/
(-a^2*x^2+1)^(1/2)+3/16*c^3*a^2*x^3/(-a^2*x^2+1)^(1/2)+7/16*c^3*x/(-a^2*x^2
+1)^(1/2)+9/16*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.44, size = 210, normalized size = 1.47

$$\frac{a^7c^3x^8}{7\sqrt{-a^2x^2+1}}+\frac{a^6c^3x^7}{2\sqrt{-a^2x^2+1}}+\frac{8\,a^5c^3x^6}{35\sqrt{-a^2x^2+1}}-\frac{9\,a^4c^3x^5}{8\sqrt{-a^2x^2+1}}-\frac{54\,a^3c^3x^4}{35\sqrt{-a^2x^2+1}}+\frac{3\,a^2c^3x^3}{16\sqrt{-a^2x^2+1}}+\frac{64\,ac^3x^2}{35\sqrt{-a^2x^2+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] 1/7*a^7*c^3*x^8/sqrt(-a^2*x^2 + 1) + 1/2*a^6*c^3*x^7/sqrt(-a^2*x^2 + 1) + 8
/35*a^5*c^3*x^6/sqrt(-a^2*x^2 + 1) - 9/8*a^4*c^3*x^5/sqrt(-a^2*x^2 + 1) - 5
4/35*a^3*c^3*x^4/sqrt(-a^2*x^2 + 1) + 3/16*a^2*c^3*x^3/sqrt(-a^2*x^2 + 1) +
 64/35*a*c^3*x^2/sqrt(-a^2*x^2 + 1) + 7/16*c^3*x/sqrt(-a^2*x^2 + 1) + 9/16*
c^3*arcsin(a*x)/a - 23/35*c^3/(sqrt(-a^2*x^2 + 1)*a)
```

mupad [B] time = 0.05, size = 174, normalized size = 1.22

$$\frac{7\,c^3\,x\,\sqrt{1-a^2\,x^2}}{16}+\frac{9\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{16\,\sqrt{-a^2}}-\frac{23\,c^3\,\sqrt{1-a^2\,x^2}}{35\,a}+\frac{41\,a\,c^3\,x^2\,\sqrt{1-a^2\,x^2}}{35}+\frac{5\,a^2\,c^3\,x^3\,\sqrt{1-a^2\,x^2}}{8}-\frac{13\,a^3\,c^3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (7*c^3*x*(1 - a^2*x^2)^(1/2))/16 + (9*c^3*asinh(x*(-a^2)^(1/2)))/(16*(-a^2)
^(1/2)) - (23*c^3*(1 - a^2*x^2)^(1/2))/(35*a) + (41*a*c^3*x^2*(1 - a^2*x^2)
^(1/2))/35 + (5*a^2*c^3*x^3*(1 - a^2*x^2)^(1/2))/8 - (13*a^3*c^3*x^4*(1 - a
^2*x^2)^(1/2))/35 - (a^4*c^3*x^5*(1 - a^2*x^2)^(1/2))/2 - (a^5*c^3*x^6*(1 -
 a^2*x^2)^(1/2))/7
```

sympy [C] time = 25.47, size = 632, normalized size = 4.42

$$-a^5c^3\left(\begin{cases}\frac{x^6\sqrt{-a^2x^2+1}}{7}-\frac{x^4\sqrt{-a^2x^2+1}}{35a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{105a^4}-\frac{8\sqrt{-a^2x^2+1}}{105a^6} & \text{for}\ a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)-3a^4c^3\left(\begin{cases}\frac{ia^2x^7}{6\sqrt{a^2x^2-1}}-\frac{5ix^5}{24\sqrt{a^2x^2-1}}-\frac{i\cdots}{48a^2\sqrt{\cdots}}\\ -\frac{a^2x^7}{6\sqrt{-a^2x^2+1}}+\frac{5x^5}{24\sqrt{-a^2x^2+1}}+\cdots\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**5*c**3*Piecewise((x**6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 +
1)/(35*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 +
 1)/(105*a**6), Ne(a, 0)), (x**6/6, True)) - 3*a**4*c**3*Piecewise((I*a**2*
x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(
48*a**2*sqrt(a**2*x**2 - 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(
a*x)/(16*a**5), Abs(a**2*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) +
 5*x**5/(24*sqrt(-a**2*x**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x
/(16*a**4*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(16*a**5), True)) - 2*a**3*c**3
*Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**
2) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) + 2*a**2*
c**3*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2
*x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs
(a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a*
*2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True)
) + 3*a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*
a**2), True)) + c**3*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(
2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-
a**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

3.1149 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=165

$$-\frac{c^4(ax+1)^2\left(1-a^2x^2\right)^{7/2}}{9a}-\frac{11c^4(ax+1)\left(1-a^2x^2\right)^{7/2}}{72a}-\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{55}{192}c^4x\left(1-a^2x^2\right.$$

```
[Out] 55/192*c^4*x*(-a^2*x^2+1)^(3/2)+11/48*c^4*x*(-a^2*x^2+1)^(5/2)-11/56*c^4*(-
a^2*x^2+1)^(7/2)/a-11/72*c^4*(a*x+1)*(-a^2*x^2+1)^(7/2)/a-1/9*c^4*(a*x+1)^2
*(-a^2*x^2+1)^(7/2)/a+55/128*c^4*arcsin(a*x)/a+55/128*c^4*x*(-a^2*x^2+1)^(1
/2)
```

Rubi [A] time = 0.09, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6138, 671, 641, 195, 216}

$$-\frac{c^4(ax+1)^2\left(1-a^2x^2\right)^{7/2}}{9a}-\frac{11c^4(ax+1)\left(1-a^2x^2\right)^{7/2}}{72a}-\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{55}{192}c^4x\left(1-a^2x^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (55*c^4*x*Sqrt[1 - a^2*x^2])/128 + (55*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (11
*c^4*x*(1 - a^2*x^2)^(5/2))/48 - (11*c^4*(1 - a^2*x^2)^(7/2))/(56*a) - (11*
c^4*(1 + a*x)*(1 - a^2*x^2)^(7/2))/(72*a) - (c^4*(1 + a*x)^2*(1 - a^2*x^2)^
(7/2))/(9*a) + (55*c^4*ArcSin[a*x])/(128*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=c^4\int(1+ax)^3\left(1-a^2x^2\right)^{5/2}\,dx$$

$$=-\frac{c^4(1+ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{1}{9}\left(11c^4\right)\int(1+ax)^2\left(1-a^2x^2\right)^{5/2}\,dx$$

$$=-\frac{11c^4(1+ax)\left(1-a^2x^2\right)^{7/2}}{72a}-\frac{c^4(1+ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{1}{8}\left(11c^4\right)\int(1+ax)\left(1\right.$$

$$=-\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}-\frac{11c^4(1+ax)\left(1-a^2x^2\right)^{7/2}}{72a}-\frac{c^4(1+ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{1}{8}$$

$$=\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}-\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}-\frac{11c^4(1+ax)\left(1-a^2x^2\right)^{7/2}}{72a}-\frac{c^4(1+a}{}$$

$$=\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}-\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}-\frac{11c^4(1+ax)}{72}$$

$$=\frac{55}{128}c^4x\sqrt{1-a^2x^2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}-\frac{11c^4\left(1-a^2x\right.}{56a}$$

$$=\frac{55}{128}c^4x\sqrt{1-a^2x^2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}-\frac{11c^4\left(1-a^2x\right.}{56a}$$

Mathematica [A] time = 0.15, size = 107, normalized size = 0.65

$$\frac{c^4\left(\sqrt{1-a^2x^2}\left(896a^8x^8+3024a^7x^7+1024a^6x^6-7224a^5x^5-8448a^4x^4+3066a^3x^3+10240a^2x^2+4599ax-37\right.\right.}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(Sqrt[1 - a^2*x^2]*(-3712 + 4599*a*x + 10240*a^2*x^2 + 3066*a^3*x^3 -
8448*a^4*x^4 - 7224*a^5*x^5 + 1024*a^6*x^6 + 3024*a^7*x^7 + 896*a^8*x^8) -
6930*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(8064*a)
```

fricas [A] time = 0.65, size = 137, normalized size = 0.83

$$-\frac{6930\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(896\,a^8c^4x^8+3024\,a^7c^4x^7+1024\,a^6c^4x^6-7224\,a^5c^4x^5-8448\,a^4c^4x^4+3066\,a^3\right.}{8064\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="frica
s")
```

```
[Out] -1/8064*(6930*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (896*a^8*c^4*x^8
 + 3024*a^7*c^4*x^7 + 1024*a^6*c^4*x^6 - 7224*a^5*c^4*x^5 - 8448*a^4*c^4*x^
4 + 3066*a^3*c^4*x^3 + 10240*a^2*c^4*x^2 + 4599*a*c^4*x - 3712*c^4)*sqrt(-a
^2*x^2 + 1))/a
```

giac [A] time = 1.01, size = 125, normalized size = 0.76

$$\frac{55\,c^4\arcsin\left(ax\right)\operatorname{sgn}(a)}{128\,|a|}-\frac{1}{8064}\sqrt{-a^2x^2+1}\left(\frac{3712\,c^4}{a}-\left(4599\,c^4+2\left(5120\,ac^4+\left(1533\,a^2c^4-4\left(1056\,a^3c^4+\left(903\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac"
)
```

```
[Out] 55/128*c^4*arcsin(a*x)*sgn(a)/abs(a) - 1/8064*sqrt(-a^2*x^2 + 1)*(3712*c^4/
a - (4599*c^4 + 2*(5120*a*c^4 + (1533*a^2*c^4 - 4*(1056*a^3*c^4 + (903*a^4*
c^4 - 2*(64*a^5*c^4 + 7*(8*a^7*c^4*x + 27*a^6*c^4)*x)*x)*x)*x)*x)*x)*x)
```

maple [A] time = 0.25, size = 275, normalized size = 1.67

$$-\frac{c^4a^9x^{10}}{9\sqrt{-a^2x^2+1}}-\frac{c^4a^7x^8}{63\sqrt{-a^2x^2+1}}+\frac{74c^4a^5x^6}{63\sqrt{-a^2x^2+1}}-\frac{146c^4a^3x^4}{63\sqrt{-a^2x^2+1}}+\frac{109c^4a\,x^2}{63\sqrt{-a^2x^2+1}}-\frac{3c^4a^8x^9}{8\sqrt{-a^2x^2+1}}+\frac{61c^4a^6x^7}{48\sqrt{-a^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/9*c^4*a^9*x^10/(-a^2*x^2+1)^(1/2)-1/63*c^4*a^7*x^8/(-a^2*x^2+1)^(1/2)+74
/63*c^4*a^5*x^6/(-a^2*x^2+1)^(1/2)-146/63*c^4*a^3*x^4/(-a^2*x^2+1)^(1/2)+10
9/63*c^4*a*x^2/(-a^2*x^2+1)^(1/2)-3/8*c^4*a^8*x^9/(-a^2*x^2+1)^(1/2)+61/48*
c^4*a^6*x^7/(-a^2*x^2+1)^(1/2)-245/192*c^4*a^4*x^5/(-a^2*x^2+1)^(1/2)-73/38
4*c^4*a^2*x^3/(-a^2*x^2+1)^(1/2)+55/128*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*
x/(-a^2*x^2+1)^(1/2))+73/128*c^4*x/(-a^2*x^2+1)^(1/2)-29/63*c^4/a/(-a^2*x^2
+1)^(1/2)
```

maxima [A] time = 0.42, size = 256, normalized size = 1.55

$$-\frac{a^9c^4x^{10}}{9\sqrt{-a^2x^2+1}}-\frac{3\,a^8c^4x^9}{8\sqrt{-a^2x^2+1}}-\frac{a^7c^4x^8}{63\sqrt{-a^2x^2+1}}+\frac{61\,a^6c^4x^7}{48\sqrt{-a^2x^2+1}}+\frac{74\,a^5c^4x^6}{63\sqrt{-a^2x^2+1}}-\frac{245\,a^4c^4x^5}{192\sqrt{-a^2x^2+1}}-\frac{146\,a^3}{63\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxim
a")
```

```
[Out] -1/9*a^9*c^4*x^10/sqrt(-a^2*x^2 + 1) - 3/8*a^8*c^4*x^9/sqrt(-a^2*x^2 + 1) -
 1/63*a^7*c^4*x^8/sqrt(-a^2*x^2 + 1) + 61/48*a^6*c^4*x^7/sqrt(-a^2*x^2 + 1)
 + 74/63*a^5*c^4*x^6/sqrt(-a^2*x^2 + 1) - 245/192*a^4*c^4*x^5/sqrt(-a^2*x^2
 + 1) - 146/63*a^3*c^4*x^4/sqrt(-a^2*x^2 + 1) - 73/384*a^2*c^4*x^3/sqrt(-a^
2*x^2 + 1) + 109/63*a*c^4*x^2/sqrt(-a^2*x^2 + 1) + 73/128*c^4*x/sqrt(-a^2*x
^2 + 1) + 55/128*c^4*arcsin(a*x)/a - 29/63*c^4/(sqrt(-a^2*x^2 + 1)*a)
```

mupad [B] time = 0.93, size = 220, normalized size = 1.33

$$\frac{73\,c^4\,x\,\sqrt{1-a^2\,x^2}}{128}+\frac{55\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{128\,\sqrt{-a^2}}-\frac{29\,c^4\,\sqrt{1-a^2\,x^2}}{63\,a}+\frac{80\,a\,c^4\,x^2\,\sqrt{1-a^2\,x^2}}{63}+\frac{73\,a^2\,c^4\,x^3\,\sqrt{1-a^2\,x^2}}{192}-\frac{22}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (73*c^4*x*(1 - a^2*x^2)^(1/2))/128 + (55*c^4*asinh(x*(-a^2)^(1/2)))/(128*(-
a^2)^(1/2)) - (29*c^4*(1 - a^2*x^2)^(1/2))/(63*a) + (80*a*c^4*x^2*(1 - a^2*
x^2)^(1/2))/63 + (73*a^2*c^4*x^3*(1 - a^2*x^2)^(1/2))/192 - (22*a^3*c^4*x^4
*(1 - a^2*x^2)^(1/2))/21 - (43*a^4*c^4*x^5*(1 - a^2*x^2)^(1/2))/48 + (8*a^5
*c^4*x^6*(1 - a^2*x^2)^(1/2))/63 + (3*a^6*c^4*x^7*(1 - a^2*x^2)^(1/2))/8 +
(a^7*c^4*x^8*(1 - a^2*x^2)^(1/2))/9
```

sympy [C] time = 46.24, size = 996, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**7*c**4*Piecewise((x**8*sqrt(-a**2*x**2 + 1)/9 - x**6*sqrt(-a**2*x**2 + 1
)/(63*a**2) - 2*x**4*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*x**2*sqrt(-a**2*x*
*2 + 1)/(315*a**6) - 16*sqrt(-a**2*x**2 + 1)/(315*a**8), Ne(a, 0)), (x**8/8
, True)) + 3*a**6*c**4*Piecewise((I*a**2*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I
*x**7/(48*sqrt(a**2*x**2 - 1)) - I*x**5/(192*a**2*sqrt(a**2*x**2 - 1)) - 5*
I*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(128*a**6*sqrt(a**2*x**2 - 1)
) - 5*I*acosh(a*x)/(128*a**7), Abs(a**2*x**2) > 1), (-a**2*x**9/(8*sqrt(-a*
*2*x**2 + 1)) + 7*x**7/(48*sqrt(-a**2*x**2 + 1)) + x**5/(192*a**2*sqrt(-a**
2*x**2 + 1)) + 5*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(128*a**6*sqrt(
-a**2*x**2 + 1)) + 5*asin(a*x)/(128*a**7), True)) + a**5*c**4*Piecewise((x*
*6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 + 1)/(35*a**2) - 4*x**2*sq
rt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 + 1)/(105*a**6), Ne(a, 0)
), (x**6/6, True)) - 5*a**4*c**4*Piecewise((I*a**2*x**7/(6*sqrt(a**2*x**2 -
 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(48*a**2*sqrt(a**2*x**2 -
 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(16*a**5), Abs(a**2
*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*x**5/(24*sqrt(-a**2*x
**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x/(16*a**4*sqrt(-a**2*x**
2 + 1)) + asin(a*x)/(16*a**5), True)) - 5*a**3*c**4*Piecewise((x**4*sqrt(-a
**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**2) - 2*sqrt(-a**2*x**2 +
 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) + a**2*c**4*Piecewise((I*a**2*x**
5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*x**2 - 1)) + I*x/(8*a**2*
sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*x
**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**2*x**2 + 1)) - x/(8*a**2*
sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True)) + 3*a*c**4*Piecewise((x*
*2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a**2), True)) + c**4*Piece
wise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2*sqrt(a**2*x**2 - 1)) - I
*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a**2*x**2 + 1)/2 + asin(a*
x)/(2*a), True))
```

3.1150 $\int \frac{e^{3\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx$

Optimal. Leaf size=95

$$\frac{3\sin^{-1}(ax)}{a^3c}+\frac{(ax+1)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}}-\frac{2(ax+1)^2}{a^3c\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a^3c}$$

```
[Out] 1/3*(a*x+1)^3/a^3/c/(-a^2*x^2+1)^(3/2)+3*arcsin(a*x)/a^3/c-2*(a*x+1)^2/a^3/
c/(-a^2*x^2+1)^(1/2)-3*(-a^2*x^2+1)^(1/2)/a^3/c
```

Rubi [A] time = 0.19, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6148, 1635, 21, 669, 641, 216}

$$\frac{(ax+1)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}}-\frac{2(ax+1)^2}{a^3c\sqrt{1-a^2x^2}}-\frac{3\sqrt{1-a^2x^2}}{a^3c}+\frac{3\sin^{-1}(ax)}{a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)^3/(3*a^3*c*(1 - a^2*x^2)^(3/2)) - (2*(1 + a*x)^2)/(a^3*c*Sqrt[1 -
 a^2*x^2]) - (3*Sqrt[1 - a^2*x^2])/(a^3*c) + (3*ArcSin[a*x])/(a^3*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx = \frac{\int \frac{x^2(1+ax)^3}{(1-a^2x^2)^{5/2}}\,dx}{c}$$
$$= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{\left(\frac{3}{a^2}+\frac{3x}{a}\right)(1+ax)^2}{(1-a^2x^2)^{3/2}}\,dx}{3c}$$
$$= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{\int \frac{(1+ax)^3}{(1-a^2x^2)^{3/2}}\,dx}{a^2c}$$
$$= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^2}{a^3c\sqrt{1-a^2x^2}} + \frac{3\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx}{a^2c}$$
$$= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^2}{a^3c\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{a^3c} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2c}$$
$$= \frac{(1+ax)^3}{3a^3c\left(1-a^2x^2\right)^{3/2}} - \frac{2(1+ax)^2}{a^3c\sqrt{1-a^2x^2}} - \frac{3\sqrt{1-a^2x^2}}{a^3c} + \frac{3\sin^{-1}(ax)}{a^3c}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 0.82

$$\frac{3a^3x^3 - 16a^2x^2 + 9(ax-1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax) - 5ax + 14}{3a^3c(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (14 - 5*a*x - 16*a^2*x^2 + 3*a^3*x^3 + 9*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSi
n[a*x])/(3*a^3*c*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.62, size = 103, normalized size = 1.08

$$-\frac{14\,a^2x^2 - 28\,ax + 18\left(a^2x^2 - 2\,ax + 1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \left(3\,a^2x^2 - 19\,ax + 14\right)\sqrt{-a^2x^2+1} + 14}{3\left(a^5cx^2 - 2\,a^4cx + a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c),x, algorithm="fri
cas")
```

```
[Out] -1/3*(14*a^2*x^2 - 28*a*x + 18*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2
+ 1) - 1)/(a*x)) + (3*a^2*x^2 - 19*a*x + 14)*sqrt(-a^2*x^2 + 1) + 14)/(a^5*
c*x^2 - 2*a^4*c*x + a^3*c)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.04, size = 178, normalized size = 1.87

$$\frac{x^2}{ca\sqrt{-a^2x^2+1}}-\frac{6}{c\,a^3\sqrt{-a^2x^2+1}}-\frac{7x}{c\,a^2\sqrt{-a^2x^2+1}}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{c\,a^2\sqrt{a^2}}-\frac{4}{3c\,a^4\left(x-\frac{1}{a}\right)\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c),x)
```

```
[Out] 1/c/a*x^2/(-a^2*x^2+1)^(1/2)-6/c/a^3/(-a^2*x^2+1)^(1/2)-7/c*x/a^2/(-a^2*x^2
+1)^(1/2)+3/c/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))-4/3/
c/a^4/(x-1/a)/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+8/3/c/a^2/(-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)*x
```

maxima [B] time = 0.57, size = 364, normalized size = 3.83

$$\frac{1}{6}\left(\frac{a^3c^3}{\sqrt{-a^2x^2+1}\,a^8c^4x+\sqrt{-a^2x^2+1}\,a^7c^4}-\frac{a^3c^3}{\sqrt{-a^2x^2+1}\,a^8c^4x-\sqrt{-a^2x^2+1}\,a^7c^4}+\frac{3\,ac}{\sqrt{-a^2x^2+1}\,a^6c^2x+\sqrt{-a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c),x, algorithm="max
ima")
```

```
[Out] 1/6*(a^3*c^3/(sqrt(-a^2*x^2 + 1)*a^8*c^4*x + sqrt(-a^2*x^2 + 1)*a^7*c^4) -
a^3*c^3/(sqrt(-a^2*x^2 + 1)*a^8*c^4*x - sqrt(-a^2*x^2 + 1)*a^7*c^4) + 3*a*c
```

```
/(sqrt(-a^2*x^2 + 1)*a^6*c^2*x + sqrt(-a^2*x^2 + 1)*a^5*c^2) - 3*a*c/(sqrt(
-a^2*x^2 + 1)*a^6*c^2*x - sqrt(-a^2*x^2 + 1)*a^5*c^2) - 4*c/(sqrt(-a^2*x^2
+ 1)*a^5*c^2*x + sqrt(-a^2*x^2 + 1)*a^4*c^2) - 4*c/(sqrt(-a^2*x^2 + 1)*a^5*
c^2*x - sqrt(-a^2*x^2 + 1)*a^4*c^2) + 6*x^2/(sqrt(-a^2*x^2 + 1)*a^2*c) - 26
*x/(sqrt(-a^2*x^2 + 1)*a^3*c) + 18*arcsin(a*x)/(a^4*c) - 36/(sqrt(-a^2*x^2
+ 1)*a^4*c))*a
```

mupad [B] time = 0.94, size = 133, normalized size = 1.40

$$\frac{2\sqrt{1-a^2x^2}}{3\left(c\,a^5x^2-2\,c\,a^4x+c\,a^3\right)}+\frac{13\sqrt{1-a^2x^2}}{3\left(a\,c\sqrt{-a^2}-a^2\,c\,x\sqrt{-a^2}\right)\sqrt{-a^2}}-\frac{\sqrt{1-a^2x^2}}{a^3\,c}+\frac{3\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{a^2\,c\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1)^3)/((c - a^2*c*x^2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(3*(a^3*c + a^5*c*x^2 - 2*a^4*c*x)) + (13*(1 - a^2*
x^2)^(1/2))/(3*(a*c*(-a^2)^(1/2) - a^2*c*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (1
 - a^2*x^2)^(1/2)/(a^3*c) + (3*asinh(x*(-a^2)^(1/2)))/(a^2*c*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^2}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3ax^3}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^4}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{a^3x^5}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x**2/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**
2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a*x**3/(a**4*x**4*sqrt(-a**
2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
+ Integral(3*a**2*x**4/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-
a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**5/(a**4*x**4*
sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 +
 1)), x))/c
```

3.1151 $\int \frac{e^{3\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx$

Optimal. Leaf size=70

$$\frac{(ax+1)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}}-\frac{2(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{\sin^{-1}(ax)}{a^2c}$$

```
[Out] 1/3*(a*x+1)^3/a^2/c/(-a^2*x^2+1)^(3/2)+arcsin(a*x)/a^2/c-2*(a*x+1)/a^2/c/(-
a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6148, 789, 653, 216}

$$\frac{(ax+1)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}}-\frac{2(ax+1)}{a^2c\sqrt{1-a^2x^2}}+\frac{\sin^{-1}(ax)}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)^3/(3*a^2*c*(1 - a^2*x^2)^(3/2)) - (2*(1 + a*x))/(a^2*c*Sqrt[1 - a
^2*x^2]) + ArcSin[a*x]/(a^2*c)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx &= \frac{\int \frac{x(1+ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c}\\ &= \frac{(1+ax)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}}-\frac{\int \frac{(1+ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{ac}\\ &= \frac{(1+ax)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}}-\frac{2(1+ax)}{a^2c\sqrt{1-a^2x^2}}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{ac}\\ &= \frac{(1+ax)^3}{3a^2c\left(1-a^2x^2\right)^{3/2}}-\frac{2(1+ax)}{a^2c\sqrt{1-a^2x^2}}+\frac{\sin^{-1}(ax)}{a^2c}\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 1.00

$$\frac{-7a^2x^2+3(ax-1)\sqrt{1-a^2x^2}\,\sin^{-1}(ax)-2ax+5}{3a^2c(ax-1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] (5 - 2*a*x - 7*a^2*x^2 + 3*(-1 + a*x)*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(3*a^2
*c*(-1 + a*x)*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.66, size = 96, normalized size = 1.37

$$-\frac{5\,a^2x^2-10\,ax+6\left(a^2x^2-2\,ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}\,(7\,ax-5)+5}{3\left(a^4cx^2-2\,a^3cx+a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] -1/3*(5*a^2*x^2 - 10*a*x + 6*(a^2*x^2 - 2*a*x + 1)*arctan((sqrt(-a^2*x^2 +
1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*(7*a*x - 5) + 5)/(a^4*c*x^2 - 2*a^3*c*x
 + a^2*c)
```

giac [A] time = 0.35, size = 112, normalized size = 1.60

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{ac|a|}+\frac{2\left(\frac{12\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}-5\right)}{3\,ac\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] arcsin(a*x)*sgn(a)/(a*c*abs(a)) + 2/3*(12*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(
a^2*x) - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) - 5)/(a*c*((sqrt(-a^
2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a))
```

maple [B] time = 0.04, size = 155, normalized size = 2.21

$$-\frac{5x}{ca\sqrt{-a^2x^2+1}}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{ca\sqrt{a^2}}-\frac{3}{c\,a^2\sqrt{-a^2x^2+1}}-\frac{4}{3c\,a^3\left(x-\frac{1}{a}\right)\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}+\frac{}{3ca\sqrt{-a^2\left(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x/(-a^2*c*x^2+c),x)
```

```
[Out] -5/c*x/a/(-a^2*x^2+1)^(1/2)+1/c/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^
2+1)^(1/2))-3/c/a^2/(-a^2*x^2+1)^(1/2)-4/3/c/a^3/(x-1/a)/(-a^2*(x-1/a)^2-2*
a*(x-1/a))^(1/2)+8/3/c/a/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)*x
```

maxima [B] time = 0.52, size = 241, normalized size = 3.44

$$\frac{1}{3}a\left(\frac{2\,ac}{\sqrt{-a^2x^2+1}\,a^5c^2x+\sqrt{-a^2x^2+1}\,a^4c^2}-\frac{2\,ac}{\sqrt{-a^2x^2+1}\,a^5c^2x-\sqrt{-a^2x^2+1}\,a^4c^2}-\frac{2\,c}{\sqrt{-a^2x^2+1}\,a^4c^2x+\sqrt{-a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] 1/3*a*(2*a*c/(sqrt(-a^2*x^2 + 1)*a^5*c^2*x + sqrt(-a^2*x^2 + 1)*a^4*c^2) -
2*a*c/(sqrt(-a^2*x^2 + 1)*a^5*c^2*x - sqrt(-a^2*x^2 + 1)*a^4*c^2) - 2*c/(sq
rt(-a^2*x^2 + 1)*a^4*c^2*x + sqrt(-a^2*x^2 + 1)*a^3*c^2) - 2*c/(sqrt(-a^2*x
^2 + 1)*a^4*c^2*x - sqrt(-a^2*x^2 + 1)*a^3*c^2) - 7*x/(sqrt(-a^2*x^2 + 1)*a
^2*c) + 3*arcsin(a*x)/(a^3*c) - 9/(sqrt(-a^2*x^2 + 1)*a^3*c))
```

mupad [B] time = 0.95, size = 108, normalized size = 1.54

$$\frac{4}{3\,a^2\,c\,{\left(1-a^2\,x^2\right)}^{3/2}}-\frac{3}{a^2\,c\,\sqrt{1-a^2\,x^2}}-\frac{7\,x}{3\,a\,c\,\sqrt{1-a^2\,x^2}}+\frac{4\,x}{3\,a\,c\,{\left(1-a^2\,x^2\right)}^{3/2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}{a^3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1)^3)/((c - a^2*c*x^2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] 4/(3*a^2*c*(1 - a^2*x^2)^(3/2)) - 3/(a^2*c*(1 - a^2*x^2)^(1/2)) - (7*x)/(3*
a*c*(1 - a^2*x^2)^(1/2)) + (4*x)/(3*a*c*(1 - a^2*x^2)^(3/2)) - (asinh(x*(-a
^2)^(1/2))*(-a^2)^(1/2))/(a^3*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3ax^2}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^3}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 +
 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a*x**2/(a**4*x**4*sqrt(-a**2*x
**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + I
ntegral(3*a**2*x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**
2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(a**3*x**4/(a**4*x**4*sqr
t(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)
), x))/c
```

3.1152 $\int \frac{e^{3\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{3\tanh^{-1}(ax)}}{3ac}$$

```
[Out] 1/3*(a*x+1)^3/a/c/(-a^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6137}

$$\frac{e^{3\tanh^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(3*ArcTanh[a*x])/(3*a*c)
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{3\tanh^{-1}(ax)}}{3ac}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.61

$$\frac{(ax+1)^{3/2}}{3ac(1-ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)^(3/2)/(3*a*c*(1 - a*x)^(3/2))
```

fricas [A] time = 0.53, size = 54, normalized size = 3.00

$$\frac{a^2x^2 - 2\,ax + \sqrt{-a^2x^2 + 1}\,(ax + 1) + 1}{3\left(a^3cx^2 - 2\,a^2cx + ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] 1/3*(a^2*x^2 - 2*a*x + sqrt(-a^2*x^2 + 1)*(a*x + 1) + 1)/(a^3*c*x^2 - 2*a^2
*c*x + a*c)
```

giac [B] time = 0.34, size = 66, normalized size = 3.67

$$\frac{2\left(\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2} + 1\right)}{3\,c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x} - 1\right)^3 |a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 1)/(c*((sqrt(-a^2*x^2
+ 1)*abs(a) + a)/(a^2*x) - 1)^3*abs(a))
```

maple [A] time = 0.03, size = 28, normalized size = 1.56

$$\frac{(ax + 1)^3}{3ac\left(-a^2x^2 + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x)
```

```
[Out] 1/3*(a*x+1)^3/a/c/(-a^2*x^2+1)^(3/2)
```

maxima [B] time = 0.49, size = 329, normalized size = 18.28

$$\frac{1}{6}\left(\frac{a^3c^3}{\sqrt{-a^2x^2 + 1}\,a^6c^4x + \sqrt{-a^2x^2 + 1}\,a^5c^4} - \frac{a^3c^3}{\sqrt{-a^2x^2 + 1}\,a^6c^4x - \sqrt{-a^2x^2 + 1}\,a^5c^4} + \frac{3\,ac}{\sqrt{-a^2x^2 + 1}\,a^4c^2x + \sqrt{-a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] 1/6*(a^3*c^3/(sqrt(-a^2*x^2 + 1)*a^6*c^4*x + sqrt(-a^2*x^2 + 1)*a^5*c^4) -
a^3*c^3/(sqrt(-a^2*x^2 + 1)*a^6*c^4*x - sqrt(-a^2*x^2 + 1)*a^5*c^4) + 3*a*c
/(sqrt(-a^2*x^2 + 1)*a^4*c^2*x + sqrt(-a^2*x^2 + 1)*a^3*c^2) - 3*a*c/(sqrt(
-a^2*x^2 + 1)*a^4*c^2*x - sqrt(-a^2*x^2 + 1)*a^3*c^2) - 4*c/(sqrt(-a^2*x^2
+ 1)*a^3*c^2*x + sqrt(-a^2*x^2 + 1)*a^2*c^2) - 4*c/(sqrt(-a^2*x^2 + 1)*a^3*
c^2*x - sqrt(-a^2*x^2 + 1)*a^2*c^2) - 2*x/(sqrt(-a^2*x^2 + 1)*a*c) - 6/(sqr
t(-a^2*x^2 + 1)*a^2*c))*a
```

mupad [B] time = 0.06, size = 32, normalized size = 1.78

$$\frac{\sqrt{1-a^2\,x^2}\,\left(a\,x+1\right)}{3\,a\,c\,{\left(a\,x-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - a^2*c*x^2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(a*x + 1))/(3*a*c*(a*x - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{3ax}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^2}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{a^3x^3}{a^4x^4\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x}}$$
$$c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] (Integral(3*a*x/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x*
*2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x**2/(a**4*x**4*sqrt(
-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)),
 x) + Integral(a**3*x**3/(a**4*x**4*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt
(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**4*x**4*sqrt(-
a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)),
x))/c
```

3.1153 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)^2}+\frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3}$$

```
[Out] 1/5*(-a^2*x^2+1)^(1/2)/a/c^2/(-a*x+1)^3+2/15*(-a^2*x^2+1)^(1/2)/a/c^2/(-a*x
+1)^2+2/15*(-a^2*x^2+1)^(1/2)/a/c^2/(-a*x+1)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6138, 655, 659, 651}

$$\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)}+\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)^2}+\frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(5*a*c^2*(1 - a*x)^3) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*(
1 - a*x)^2) + (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*(1 - a*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
```

```
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^2}\\
&= \frac{\int \frac{1}{(1-ax)^3\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= \frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3}+\frac{2\int \frac{1}{(1-ax)^2\sqrt{1-a^2x^2}}\,dx}{5c^2}\\
&= \frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3}+\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)^2}+\frac{2\int \frac{1}{(1-ax)\sqrt{1-a^2x^2}}\,dx}{15c^2}\\
&= \frac{\sqrt{1-a^2x^2}}{5ac^2(1-ax)^3}+\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)^2}+\frac{2\sqrt{1-a^2x^2}}{15ac^2(1-ax)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.44

$$\frac{\sqrt{ax+1}\left(2a^2x^2-6ax+7\right)}{15ac^2(1-ax)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
[Out] (Sqrt[1 + a*x]*(7 - 6*a*x + 2*a^2*x^2))/(15*a*c^2*(1 - a*x)^(5/2))
```

fricas [A] time = 0.55, size = 91, normalized size = 0.94

$$\frac{7\,a^3x^3-21\,a^2x^2+21\,ax-\left(2\,a^2x^2-6\,ax+7\right)\sqrt{-a^2x^2+1}-7}{15\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] 1/15*(7*a^3*x^3 - 21*a^2*x^2 + 21*a*x - (2*a^2*x^2 - 6*a*x + 7)*sqrt(-a^2*x
^2 + 1) - 7)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [A] time = 0.28, size = 145, normalized size = 1.49

$$-\frac{2\left(\frac{20\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}-\frac{40\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{30\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}-\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}-7\right)}{15\,c^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] -2/15*(20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 40*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a^4*x^2) + 30*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) -
15*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 7)/(c^2*((sqrt(-a^2*x^2 +
1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))
```

maple [A] time = 0.03, size = 49, normalized size = 0.51

$$-\frac{\left(2a^2x^2-6ax+7\right)(ax+1)^2}{15\,(ax-1)\,c^2\left(-a^2x^2+1\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/15*(2*a^2*x^2-6*a*x+7)*(a*x+1)^2/(a*x-1)/c^2/(-a^2*x^2+1)^(3/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(a^2cx^2-c\right)^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((a^2*c*x^2 - c)^2*(-a^2*x^2 + 1)^(3/2)), x)
```

mupad [B] time = 0.91, size = 127, normalized size = 1.31

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{2\,a^3}{15\,c^2\,\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}-\frac{a^3}{5\,c^2\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^3}+\frac{2\,a^4}{15\,c^2\,{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)}^2\,\sqrt{-a^2}}\right)}{a^3\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((2*a^3)/(15*c^2*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)) -
a^3/(5*c^2*(x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)^3) + (2*a^4)/(15*c^2*(x*(-a^2)
^(1/2) - (-a^2)^(1/2)/a)^2*(-a^2)^(1/2))))/(a^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3ax}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^2}{-a^6x^6\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(3*a*x/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x
**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + I
ntegral(3*a**2*x**2/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a*
*2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
 + Integral(a**3*x**3/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-
a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)),
x) + Integral(1/(-a**6*x**6*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x
**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c*
*2
```

3.1154 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=119

$$\frac{8x}{35c^3\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}}+\frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}}$$

```
[Out] 8/35*x/c^3/(-a^2*x^2+1)^(1/2)+1/7/a/c^3/(-a*x+1)^3/(-a^2*x^2+1)^(1/2)+4/35/
a/c^3/(-a*x+1)^2/(-a^2*x^2+1)^(1/2)+4/35/a/c^3/(-a*x+1)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6138, 655, 659, 191}

$$\frac{8x}{35c^3\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}}+\frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8*x)/(35*c^3*Sqrt[1 - a^2*x^2]) + 1/(7*a*c^3*(1 - a*x)^3*Sqrt[1 - a^2*x^2]
) + 4/(35*a*c^3*(1 - a*x)^2*Sqrt[1 - a^2*x^2]) + 4/(35*a*c^3*(1 - a*x)*Sqrt
[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
```

```
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx=\frac{\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{9/2}}\,dx}{c^3}$$

$$=\frac{\int \frac{1}{(1-ax)^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$=\frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}}+\frac{4\int \frac{1}{(1-ax)^2\left(1-a^2x^2\right)^{3/2}}\,dx}{7c^3}$$

$$=\frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}}+\frac{12\int \frac{1}{(1-ax)\left(1-a^2x^2\right)^{3/2}}\,dx}{35c^3}$$

$$=\frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)\sqrt{1-a^2x^2}}+\frac{8\int \frac{1}{(1-a^2 \cdots}}{35 \cdots}$$

$$=\frac{8x}{35c^3\sqrt{1-a^2x^2}}+\frac{1}{7ac^3(1-ax)^3\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)^2\sqrt{1-a^2x^2}}+\frac{4}{35ac^3(1-ax)\sqrt{1-\cdots}}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.51

$$\frac{8a^4x^4-24a^3x^3+20a^2x^2+4ax-13}{35ac^3(ax-1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4)/(35*a*c^3*(-1 + a*x)^3*
Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.55, size = 144, normalized size = 1.21

$$\frac{13\,a^5x^5-39\,a^4x^4+26\,a^3x^3+26\,a^2x^2-39\,ax-\left(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13\right)\sqrt{-a^2x^2+1}+13}{35\left(a^6c^3x^5-3\,a^5c^3x^4+2\,a^4c^3x^3+2\,a^3c^3x^2-3\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] 1/35*(13*a^5*x^5 - 39*a^4*x^4 + 26*a^3*x^3 + 26*a^2*x^2 - 39*a*x - (8*a^4*x
^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt(-a^2*x^2 + 1) + 13)/(a^6*c^
3*x^5 - 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 - 3*a^2*c^3*x + a*c^3
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ax+1)^3}{\left(a^2cx^2-c\right)^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] integrate(-(a*x + 1)^3/((a^2*c*x^2 - c)^3*(-a^2*x^2 + 1)^(3/2)), x)
```

maple [A] time = 0.03, size = 63, normalized size = 0.53

$$-\frac{\left(8x^4a^4-24x^3a^3+20a^2x^2+4ax-13\right)(ax+1)}{35\,(ax-1)^2\,c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/35*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)*(a*x+1)/(a*x-1)^2/c^3/(-a^
2*x^2+1)^(3/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax+1)^3}{\left(a^2cx^2-c\right)^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] -integrate((a*x + 1)^3/((a^2*c*x^2 - c)^3*(-a^2*x^2 + 1)^(3/2)), x)
```

mupad [B] time = 1.20, size = 125, normalized size = 1.05

$$\frac{29\sqrt{1-a^2x^2}}{280\,a\,c^3\,(a\,x-1)^2}-\frac{13\sqrt{1-a^2x^2}}{140\,a\,c^3\,(a\,x-1)^3}+\frac{\sqrt{1-a^2x^2}}{14\,a\,c^3\,(a\,x-1)^4}-\frac{\sqrt{1-a^2x^2}\,\left(\frac{8\,x}{35\,c^3}+\frac{29}{280\,a\,c^3}\right)}{(a\,x-1)\,(a\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (29*(1 - a^2*x^2)^(1/2))/(280*a*c^3*(a*x - 1)^2) - (13*(1 - a^2*x^2)^(1/2))
/(140*a*c^3*(a*x - 1)^3) + (1 - a^2*x^2)^(1/2)/(14*a*c^3*(a*x - 1)^4) - ((1
 - a^2*x^2)^(1/2)*((8*x)/(35*c^3) + 29/(280*a*c^3)))/((a*x - 1)*(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3ax}{a^8x^8\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+1}-4a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int \frac{3a^2x^2}{a^8x^8\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (Integral(3*a*x/(a**8*x**8*sqrt(-a**2*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x*
*2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 +
1) + sqrt(-a**2*x**2 + 1)), x) + Integral(3*a**2*x**2/(a**8*x**8*sqrt(-a**2
*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x**2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2
 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + Inte
gral(a**3*x**3/(a**8*x**8*sqrt(-a**2*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x**
2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 + 1
) + sqrt(-a**2*x**2 + 1)), x) + Integral(1/(a**8*x**8*sqrt(-a**2*x**2 + 1)
- 4*a**6*x**6*sqrt(-a**2*x**2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a
**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**3
```

3.1155 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=141

$$\frac{16x}{63c^4\sqrt{1-a^2x^2}}+\frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}}+\frac{2}{21ac^4(1-ax)\left(1-a^2x^2\right)^{3/2}}+\frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}+\frac{1}{9ac^4(1-ax)^3\left(1\right.}$$

```
[Out] 8/63*x/c^4/(-a^2*x^2+1)^(3/2)+1/9/a/c^4/(-a*x+1)^3/(-a^2*x^2+1)^(3/2)+2/21/
a/c^4/(-a*x+1)^2/(-a^2*x^2+1)^(3/2)+2/21/a/c^4/(-a*x+1)/(-a^2*x^2+1)^(3/2)+
16/63*x/c^4/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6138, 655, 659, 192, 191}

$$\frac{16x}{63c^4\sqrt{1-a^2x^2}}+\frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}}+\frac{2}{21ac^4(1-ax)\left(1-a^2x^2\right)^{3/2}}+\frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}}+\frac{1}{9ac^4(1-ax)^3\left(1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (8*x)/(63*c^4*(1 - a^2*x^2)^(3/2)) + 1/(9*a*c^4*(1 - a*x)^3*(1 - a^2*x^2)^(
3/2)) + 2/(21*a*c^4*(1 - a*x)^2*(1 - a^2*x^2)^(3/2)) + 2/(21*a*c^4*(1 - a*x
)*(1 - a^2*x^2)^(3/2)) + (16*x)/(63*c^4*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
```

```
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6138

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
d, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && IGtQ[(n + 1)/2, 0] &&  !In
tegerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{(1+ax)^3}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^4}$$

$$= \frac{\int \frac{1}{(1-ax)^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^4}$$

$$= \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{(1-ax)^2\left(1-a^2x^2\right)^{5/2}}\,dx}{3c^4}$$

$$= \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{10\int \frac{1}{(1-ax)\left(1-a^2x^2\right)^{5/2}}\,dx}{21c^4}$$

$$= \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)\left(1-a^2x^2\right)^{3/2}} +$$

$$= \frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}} + \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{}{21ac^4(1}$$

$$= \frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}} + \frac{1}{9ac^4(1-ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{21ac^4(1-ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{}{21ac^4(1}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.53

$$\frac{16a^6x^6-48a^5x^5+24a^4x^4+56a^3x^3-66a^2x^2+6ax+19}{63ac^4(1-ax)^{9/2}(ax+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (19 + 6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a^4*x^4 - 48*a^5*x^5 + 16*a^6*x^
6)/(63*a*c^4*(1 - a*x)^(9/2)*(1 + a*x)^(3/2))
```

fricas [A] time = 0.60, size = 198, normalized size = 1.40

$$\frac{19\,a^7x^7-57\,a^6x^6+19\,a^5x^5+95\,a^4x^4-95\,a^3x^3-19\,a^2x^2+57\,ax-\left(16\,a^6x^6-48\,a^5x^5+24\,a^4x^4+56\,a^3x^3-66\right.}{63\left(a^8c^4x^7-3\,a^7c^4x^6+a^6c^4x^5+5\,a^5c^4x^4-5\,a^4c^4x^3-a^3c^4x^2+3\,a^2c^4x-ac^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="frica
s")
```

```
[Out] 1/63*(19*a^7*x^7 - 57*a^6*x^6 + 19*a^5*x^5 + 95*a^4*x^4 - 95*a^3*x^3 - 19*a
^2*x^2 + 57*a*x - (16*a^6*x^6 - 48*a^5*x^5 + 24*a^4*x^4 + 56*a^3*x^3 - 66*a
^2*x^2 + 6*a*x + 19)*sqrt(-a^2*x^2 + 1) - 19)/(a^8*c^4*x^7 - 3*a^7*c^4*x^6
+ a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 - a^3*c^4*x^2 + 3*a^2*c^4*x -
 a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(a^2cx^2-c\right)^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)^3/((a^2*c*x^2 - c)^4*(-a^2*x^2 + 1)^(3/2)), x)
```

maple [A] time = 0.03, size = 74, normalized size = 0.52

$$-\frac{16x^6a^6-48x^5a^5+24x^4a^4+56x^3a^3-66a^2x^2+6ax+19}{63\left(ax-1\right)^3c^4\left(-a^2x^2+1\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/63*(16*a^6*x^6-48*a^5*x^5+24*a^4*x^4+56*a^3*x^3-66*a^2*x^2+6*a*x+19)/(a*
x-1)^3/c^4/(-a^2*x^2+1)^(3/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(a^2cx^2-c\right)^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3/((a^2*c*x^2 - c)^4*(-a^2*x^2 + 1)^(3/2)), x)
```

mupad [B] time = 1.26, size = 156, normalized size = 1.11

$$\frac{13\sqrt{1-a^2x^2}}{252\,a\,c^4\,(a\,x-1)^4}-\frac{23\sqrt{1-a^2x^2}}{336\,a\,c^4\,(a\,x-1)^3}-\frac{\sqrt{1-a^2x^2}}{36\,a\,c^4\,(a\,x-1)^5}+\frac{\sqrt{1-a^2x^2}\,\left(\frac{197\,x}{1008\,c^4}+\frac{155}{1008\,a\,c^4}\right)}{(a\,x-1)^2\,(a\,x+1)^2}-\frac{16\,x\sqrt{1-a^2x^2}}{63\,c^4\,(a\,x-1)\,(a\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - a^2*c*x^2)^4*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (13*(1 - a^2*x^2)^(1/2))/(252*a*c^4*(a*x - 1)^4) - (23*(1 - a^2*x^2)^(1/2))
/(336*a*c^4*(a*x - 1)^3) - (1 - a^2*x^2)^(1/2)/(36*a*c^4*(a*x - 1)^5) + ((1
 - a^2*x^2)^(1/2)*((197*x)/(1008*c^4) + 155/(1008*a*c^4)))/((a*x - 1)^2*(a*
x + 1)^2) - (16*x*(1 - a^2*x^2)^(1/2))/(63*c^4*(a*x - 1)*(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{3ax}{-a^{10}x^{10}\sqrt{-a^2x^2+1}+5a^8x^8\sqrt{-a^2x^2+1}-10a^6x^6\sqrt{-a^2x^2+1}+10a^4x^4\sqrt{-a^2x^2+1}-5a^2x^2\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx+\int\frac{}{-a^{10}x^{10}\sqrt{-a^2x^2+1}+5a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (Integral(3*a*x/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a**2
*x**2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2*x*
*2 + 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) + In
tegral(3*a**2*x**2/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a
**2*x**2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2
*x**2 + 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x) +
 Integral(a**3*x**3/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-
a**2*x**2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**
2*x**2 + 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)
+ Integral(1/(-a**10*x**10*sqrt(-a**2*x**2 + 1) + 5*a**8*x**8*sqrt(-a**2*x*
*2 + 1) - 10*a**6*x**6*sqrt(-a**2*x**2 + 1) + 10*a**4*x**4*sqrt(-a**2*x**2
+ 1) - 5*a**2*x**2*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x))/c**4
```

3.1156 $\int e^{3\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=226

$$-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] -4*x*(-a^2*c*x^2+c)^(1/2)/a^3/(-a^2*x^2+1)^(1/2)-2*x^2*(-a^2*c*x^2+c)^(1/2)
/a^2/(-a^2*x^2+1)^(1/2)-4/3*x^3*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-3
/4*x^4*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-1/5*a*x^5*(-a^2*c*x^2+c)^(1/
2)/(-a^2*x^2+1)^(1/2)-4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^4/(-a^2*x^2+1)^(1
/2)
```

Rubi [A] time = 0.22, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - a^2*x^2]) - (2*x^2*Sqrt[c - a^2*c*
x^2])/(a^2*Sqrt[1 - a^2*x^2]) - (4*x^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a
^2*x^2]) - (3*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) - (a*x^5*Sqrt[
c - a^2*c*x^2])/(5*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x]
)/(a^4*Sqrt[1 - a^2*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}x^3\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^3(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^3}-\frac{4x}{a^2}-\frac{4x^2}{a}-3x^3-ax^4-\frac{4}{a^3(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{ax^5\sqrt{}}{5\sqrt{1}}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.35

$$-\frac{\sqrt{c-a^2cx^2}\left(ax\left(12a^4x^4+45a^3x^3+80a^2x^2+120ax+240\right)+240\log(1-ax)\right)}{60a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/60*(Sqrt[c - a^2*c*x^2]*(a*x*(240 + 120*a*x + 80*a^2*x^2 + 45*a^3*x^3 +
12*a^4*x^4) + 240*Log[1 - a*x]))/(a^4*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.67, size = 399, normalized size = 1.77

$$\left[\frac{120\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx+\left(a^4x^4-4a^3x^3+6a^2x^2-4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4-2a^3x^3+2ax-1}\right)+\left(12a^5x^5\right.}{60\left(a^6x^2-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/60*(120*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4
 - 4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a
*x - 1)) + (12*a^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x)*s
qrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a^4), -1/60*(240*(a^2*x^
2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2
*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (12*a
^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^3x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.04, size = 88, normalized size = 0.39

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(12x^5a^5+45x^4a^4+80x^3a^3+120a^2x^2+240ax+240\ln\left(ax-1\right)\right)}{60\left(a^2x^2-1\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/60*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(12*x^5*a^5+45*x^4*a^4+80*x^
3*a^3+120*a^2*x^2+240*a*x+240*ln(a*x-1))/(a^2*x^2-1)/a^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{c - a^2 c x^2} \, (a x + 1)^3}{\left(1 - a^2 x^2\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{-c (ax-1)(ax+1)} \, (ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x +
1))**(3/2), x)
```

3.1157 $\int e^{3\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=185

$$-\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] -4*x*(-a^2*c*x^2+c)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)-2*x^2*(-a^2*c*x^2+c)^(1/2)
/a/(-a^2*x^2+1)^(1/2)-x^3*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-1/4*a*x^4
*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/
a^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - a^2*x^2]) - (2*x^2*Sqrt[c - a^2*c*
x^2])/(a*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] -
 (a*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]
*Log[1 - a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^2}-\frac{4x}{a}-3x^2-ax^3-\frac{4}{a^2(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-}}{a}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.38

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(1-ax)}{a^3}-\frac{4x}{a^2}-\frac{ax^4}{4}-\frac{2x^2}{a}-x^3\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((-4*x)/a^2 - (2*x^2)/a - x^3 - (a*x^4)/4 - (4*Log[1 -
 a*x])/a^3))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.74, size = 381, normalized size = 2.06

$$\left[\frac{8\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx+\left(a^4x^4-4a^3x^3+6a^2x^2-4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4-2a^3x^3+2ax-1}\right)+\left(a^4x^4+4a^3\right.}{4\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/4*(8*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
- 1)) + (a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x)*sqrt(-a^2*c*x^2 + c)*sqr
t(-a^2*x^2 + 1))/(a^5*x^2 - a^3), -1/4*(16*(a^2*x^2 - 1)*sqrt(-c)*arctan(sq
rt(-a^2*c*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c
*x^4 - 2*a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (a^4*x^4 + 4*a^3*x^3 + 8*a^2*x
^2 + 16*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^5*x^2 - a^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 79, normalized size = 0.43

$$\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(x^{4} a^{4}+4 x^{3} a^{3}+8 a^{2} x^{2}+16 a x +16 \ln \left(a x -1\right)\right)}{4 \left(a^{2} x^{2}-1\right) a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(x^4*a^4+4*x^3*a^3+8*a^2*x^2+
16*a*x+16*ln(a*x-1))/(a^2*x^2-1)/a^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
[Out] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)^3}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x +
1))**(3/2), x)
```

3.1158 $\int e^{3\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=150

$$-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

```
[Out] -4*x*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-3/2*x^2*(-a^2*c*x^2+c)^(1/2)
/(-a^2*x^2+1)^(1/2)-1/3*a*x^3*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-4*ln(
-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 77}

$$-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[c - a^2*c*x^
2])/(2*Sqrt[1 - a^2*x^2]) - (a*x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2
]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a}-3x-ax^2-\frac{4}{a(-1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(1-ax)}{a^2}-\frac{ax^3}{3}-\frac{4x}{a}-\frac{3x^2}{2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((-4*x)/a - (3*x^2)/2 - (a*x^3)/3 - (4*Log[1 - a*x])/a
^2))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.80, size = 367, normalized size = 2.45

$$\left[\frac{12\left(a^2x^2-1\right)\sqrt{c}\,\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx+\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\left(2\,a^3x^3+9\right.}{6\left(a^4x^2-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(12*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
 - 1)) + (2*a^3*x^3 + 9*a^2*x^2 + 24*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1))/(a^4*x^2 - a^2), -1/6*(24*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c
*x^2 + c)*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*
a^3*c*x^3 - a^2*c*x^2 + 2*a*c*x)) - (2*a^3*x^3 + 9*a^2*x^2 + 24*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^4*x^2 - a^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^3x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3*x/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.04, size = 72, normalized size = 0.48

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(2x^3a^3+9a^2x^2+24ax+24\ln\left(ax-1\right)\right)}{6\left(a^2x^2-1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3+9*a^2*x^2+24*a*x+2
4*ln(a*x-1))/(a^2*x^2-1)/a^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)^3}{(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))
**(3/2), x)
```

3.1159 $\int e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=111

$$-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

```
[Out] -3*x*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-1/2*a*x^2*(-a^2*c*x^2+c)^(1/2)
/(-a^2*x^2+1)^(1/2)-4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-3*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] - (a*x^2*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a*Sqrt[1 - a^
2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(-3-ax+\frac{4}{1-ax}\right)dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.49

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a))/Sqrt[1 - a^2*
x^2]
```

fricas [A] time = 0.68, size = 345, normalized size = 3.11

$$\left[\frac{4\left(a^2x^2-1\right)\sqrt{c}\,\log\left(\frac{a^6cx^6-4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2+4\,acx+\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-2\,c}{a^4x^4-2\,a^3x^3+2\,ax-1}\right)+\sqrt{-a^2cx^2+}}{2\left(a^3x^2-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*(4*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 + 4*a*c*x + (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x
- 1)) + sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 6*a*x)*sqrt(-a^2*x^2 + 1))/(a^3*x^2
```

```
 - a), -1/2*(8*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2
- 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3 - a^2*c*x
^2 + 2*a*c*x)) - sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 6*a*x)*sqrt(-a^2*x^2 + 1))
/(a^3*x^2 - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.04, size = 63, normalized size = 0.57

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(a^2x^2+6ax+8\ln\left(ax-1\right)\right)}{2\left(a^2x^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a^2*x^2+6*a*x+8*ln(a*x-1))/(
a^2*x^2-1)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,\,(ax+1)^3}{(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**
(3/2), x)
```

3.1160 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=104

$$-\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -a*x*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a
^2*x^2+1)^(1/2)-4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 72}

$$-\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] -((a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]) + (Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2] - (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^2*x^
2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x(1-ax)}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.44

$$\frac{\sqrt{c-a^2cx^2}\,(-ax-4\log(1-ax)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
[Out] (Sqrt[c - a^2*c*x^2]*(-(a*x) + Log[x] - 4*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(ax+1)}{a^2x^3-2\,ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm=
"fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x + 1)/(a^2*x^3 - 2*a*x
^2 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

maple [A] time = 0.04, size = 55, normalized size = 0.53

$$\frac{(ax-\ln(x)+4\ln(ax-1))\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (a*x-ln(x)+4*ln(a*x-1))*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-
1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^3}{x\,{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,\,(ax+1)^3}{x\,(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.1161 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{3a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/x/(-a^2*x^2+1)^(1/2)+3*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/(
-a^2*x^2+1)^(1/2)-4*a*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{3a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) + (3*a*Sqrt[c - a^2*c*x^2]*Log
[x])/Sqrt[1 - a^2*x^2] - (4*a*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^
2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
```

```
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^2(1-ax)}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}+\frac{3a}{x}-\frac{4a^2}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-a^2x^2}}-\frac{4a\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.47

$$\frac{\sqrt{c-a^2cx^2}\left(3a\log(x)-4a\log(1-ax)-\frac{1}{x}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) + 3*a*Log[x] - 4*a*Log[1 - a*x]))/Sqrt[1 - a^
2*x^2]
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(ax+1)}{a^2x^4-2\,ax^3+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorith
m="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x + 1)/(a^2*x^4 - 2*a*x
^3 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

maple [A] time = 0.05, size = 60, normalized size = 0.56

$$\frac{(-3a\ln(x)x+4\ln(ax-1)\,xa+1)\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}}{\left(a^2x^2-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] (-3*a*ln(x)*x+4*ln(a*x-1)*x*a+1)*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/
(a^2*x^2-1)/x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorith
m="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^3}{x^2\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(ax-1)(ax+1)}\,(ax+1)^3}{x^2(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x +
 1))**(3/2)), x)
```

3.1162 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=149

$$-\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -1/2*(-a^2*c*x^2+c)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)-3*a*(-a^2*c*x^2+c)^(1/2)/x
/(-a^2*x^2+1)^(1/2)+4*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-4*a
^2*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - a^2*c*x^2])/
(x*Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2
] - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^3(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^3}+\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}} + \frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}} - \frac{4a^2\sqrt{c-a^2cx^2}\log(}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.42

$$\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(1-ax)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/2*1/x^2 - (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 - a
*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.73, size = 450, normalized size = 3.02

$$\left[\frac{4\left(a^4x^4-a^2x^2\right)\sqrt{c}\,\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3-6\,a^2+4\,a-1\right)x^4\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{2\left(a^2x^4-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorith
m="fricas")
```

```
[Out] [1/2*(4*(a^4*x^4 - a^2*x^2)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*
a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*
c*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 +
4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 -
2*a^3*x^5 + 2*a*x^3 - x^2)) - sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((6*a
 + 1)*x^2 - 6*a*x - 1))/(a^2*x^4 - x^2), -1/2*(8*(a^4*x^4 - a^2*x^2)*sqrt(-
c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a^2 - 2*a + 1)*x^2 -
 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c
*x^2 - 2*a*c*x + c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((6*a + 1)*x
^2 - 6*a*x - 1))/(a^2*x^4 - x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

maple [A] time = 0.04, size = 73, normalized size = 0.49

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(8a^2\ln(x)x^2-8\ln(ax-1)\,x^2a^2-6ax-1\right)}{2\left(a^2x^2-1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(8*a^2*ln(x)*x^2-8*ln(a*x-1)
*x^2*a^2-6*a*x-1)/(a^2*x^2-1)/x^2
```

maxima [A] time = 0.38, size = 160, normalized size = 1.07

$$-2\,(-1)^{-2\,a^2cx^2+2\,c}\,a^2\sqrt{c}\,\log\left(-2\,a^2c+\frac{2\,c}{x^2}\right)+\frac{1}{2}\,a^3\left(\frac{\sqrt{c}\,\log(ax+1)}{a}-\frac{\sqrt{c}\,\log(ax-1)}{a}\right)+\frac{a^2c}{2\,\sqrt{a^4cx^4-2\,a^2cx^2+c}}+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorith
m="maxima")
```

```
[Out] -2*(-1)^(-2*a^2*c*x^2 + 2*c)*a^2*sqrt(c)*log(-2*a^2*c + 2*c/x^2) + 1/2*a^3*
(sqrt(c)*log(a*x + 1)/a - sqrt(c)*log(a*x - 1)/a) + 1/2*a^2*c/sqrt(a^4*c*x^
4 - 2*a^2*c*x^2 + c) + 3/2*(a*sqrt(c)*log(a*x + 1) - a*sqrt(c)*log(a*x - 1)
 - 2*sqrt(c)/x)*a - 1/2*c/(sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^3}{x^3\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)^{3}}{x^{3}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x +
 1))**(3/2)), x)
```

3.1163 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=188

$$-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}+\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -1/3*(-a^2*c*x^2+c)^(1/2)/x^3/(-a^2*x^2+1)^(1/2)-3/2*a*(-a^2*c*x^2+c)^(1/2)
/x^2/(-a^2*x^2+1)^(1/2)-4*a^2*(-a^2*c*x^2+c)^(1/2)/x/(-a^2*x^2+1)^(1/2)+4*a
^3*ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-4*a^3*ln(-a*x+1)*(-a^2*c*x
^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}+\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[c - a^2*c*x^2])/
(2*x^2*Sqrt[1 - a^2*x^2]) - (4*a^2*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2
]) + (4*a^3*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^3*Sqrt[c -
 a^2*c*x^2]*Log[1 - a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^4}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^4(1-ax)}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^4}+\frac{3a}{x^3}+\frac{4a^2}{x^2}+\frac{4a^3}{x}-\frac{4a^4}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}-\frac{4}{}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 0.39

$$\frac{\sqrt{c-a^2cx^2}\left(4a^3\log(x)-4a^3\log(1-ax)-\frac{4a^2}{x}-\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/3*1/x^3 - (3*a)/(2*x^2) - (4*a^2)/x + 4*a^3*Log[x]
 - 4*a^3*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.03, size = 478, normalized size = 2.54

$$\left[\frac{12\left(a^5x^5-a^3x^3\right)\sqrt{c}\log\left(-\frac{4a^5cx^5-\left(2a^6-4a^5+6a^4-4a^3+a^2\right)cx^6-\left(4a^4+4a^3-6a^2+4a-1\right)cx^4+5a^2cx^2-4acx+\left(4a^3x^3-\left(4a^3-6a^2+4a-1\right)x\right.}{a^4x^6-2a^3x^5+2ax^3-x^2}\right.}{6\left(a^2x^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorith
m="fricas")
```

```
[Out] [1/6*(12*(a^5*x^5 - a^3*x^3)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6
*a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2
*c*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 +
 4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 -
 2*a^3*x^5 + 2*a*x^3 - x^2)) + sqrt(-a^2*c*x^2 + c)*(24*a^2*x^2 - (24*a^2 +
 9*a + 2)*x^3 + 9*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*x^5 - x^3), -1/6*(24*(a
^5*x^5 - a^3*x^3)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*
((2*a^2 - 2*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 - a^2)*c
*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) - sqrt(-a^2*c*x^2 + c)*(24*a^2
*x^2 - (24*a^2 + 9*a + 2)*x^3 + 9*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*x^5 - x
^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^4), x)
```

maple [A] time = 0.05, size = 81, normalized size = 0.43

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(24a^3\ln(x)x^3-24\ln\left(ax-1\right)x^3a^3-24a^2x^2-9ax-2\right)}{6\left(a^2x^2-1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] -1/6*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(24*a^3*ln(x)*x^3-24*ln(a*x-
1)*x^3*a^3-24*a^2*x^2-9*a*x-2)/(a^2*x^2-1)/x^3
```

maxima [A] time = 0.38, size = 216, normalized size = 1.15

$$-\frac{1}{2}\,(-1)^{-2\,a^2cx^2+2\,c}\,a^3\sqrt{c}\,\log\left(-2\,a^2c+\frac{2\,c}{x^2}\right)+\frac{1}{2}\,a^3\sqrt{c}\,\log\left(ax+1\right)-\frac{1}{2}\,a^3\sqrt{c}\,\log\left(ax-1\right)+\frac{3}{2}\left(a\sqrt{c}\,\log\left(ax+1\right)-a\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorith
m="maxima")
```

```
[Out] -1/2*(-1)^(-2*a^2*c*x^2 + 2*c)*a^3*sqrt(c)*log(-2*a^2*c + 2*c/x^2) + 1/2*a^
3*sqrt(c)*log(a*x + 1) - 1/2*a^3*sqrt(c)*log(a*x - 1) + 3/2*(a*sqrt(c)*log(
a*x + 1) - a*sqrt(c)*log(a*x - 1) - 2*sqrt(c)/x)*a^2 - 3/2*((-1)^(-2*a^2*c*
x^2 + 2*c)*a^2*sqrt(c)*log(-2*a^2*c + 2*c/x^2) - a^2*c/sqrt(a^4*c*x^4 - 2*a
^2*c*x^2 + c) + c/(sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*x^2))*a - 1/3*(3*a^2*s
qrt(c)*x^2 + sqrt(c))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^3}{x^4\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x^4*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x^4*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)^{3}}{x^{4}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x**4*(-(a*x - 1)*(a*x +
 1))**(3/2)), x)
```

3.1164 $\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=223

$$-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}+\frac{4a^4\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-a^2}}{x\sqrt{1-a^2}}$$

```
[Out] -1/4*(-a^2*c*x^2+c)^(1/2)/x^4/(-a^2*x^2+1)^(1/2)-a*(-a^2*c*x^2+c)^(1/2)/x^3
/(-a^2*x^2+1)^(1/2)-2*a^2*(-a^2*c*x^2+c)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)-4*a^3
*(-a^2*c*x^2+c)^(1/2)/x/(-a^2*x^2+1)^(1/2)+4*a^4*ln(x)*(-a^2*c*x^2+c)^(1/2)
/(-a^2*x^2+1)^(1/2)-4*a^4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2
)
```

Rubi [A] time = 0.22, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}+\frac{4a^4\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-a^2cx^2}\,\log(1-}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4*Sqrt[1 - a^2*x^2]) - (a*Sqrt[c - a^2*c*x^2])/(x
^3*Sqrt[1 - a^2*x^2]) - (2*a^2*Sqrt[c - a^2*c*x^2])/(x^2*Sqrt[1 - a^2*x^2])
 - (4*a^3*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - a^2*
c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])
/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^5}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^5(1-ax)}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^5}+\frac{3a}{x^4}+\frac{4a^2}{x^3}+\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{-1+ax}\right)dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}-\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{4a^4\sqrt{c-}}{\sqrt{1}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.35

$$\frac{\sqrt{c-a^2cx^2}\left(4a^4\log(x)-4a^4\log(1-ax)-\frac{4a^3}{x}-\frac{2a^2}{x^2}-\frac{a}{x^3}-\frac{1}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/4*1/x^4 - a/x^3 - (2*a^2)/x^2 - (4*a^3)/x + 4*a^4*
Log[x] - 4*a^4*Log[1 - a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.88, size = 504, normalized size = 2.26

$$\left[\frac{8\left(a^6x^6-a^4x^4\right)\sqrt{c}\,\log\left(-\frac{4\,a^5cx^5-\left(2\,a^6-4\,a^5+6\,a^4-4\,a^3+a^2\right)cx^6-\left(4\,a^4+4\,a^3-6\,a^2+4\,a-1\right)cx^4+5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3-6\,a^2+4\,a-1\right)x^4\right.}{a^4x^6-2\,a^3x^5+2\,ax^3-x^2}\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorith
m="fricas")
```

```
[Out] [1/4*(8*(a^6*x^6 - a^4*x^4)*sqrt(c)*log(-(4*a^5*c*x^5 - (2*a^6 - 4*a^5 + 6*
a^4 - 4*a^3 + a^2)*c*x^6 - (4*a^4 + 4*a^3 - 6*a^2 + 4*a - 1)*c*x^4 + 5*a^2*
c*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 - 6*a^2 + 4*a - 1)*x^4 - 6*a^2*x^2 +
4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) + c)/(a^4*x^6 -
2*a^3*x^5 + 2*a*x^3 - x^2)) + (16*a^3*x^3 - (16*a^3 + 8*a^2 + 4*a + 1)*x^4
+ 8*a^2*x^2 + 4*a*x + 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^2*x^6
- x^4), -1/4*(16*(a^6*x^6 - a^4*x^4)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1)*((2*a^2 - 2*a + 1)*x^2 - 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^
3 - (2*a^3 - a^2)*c*x^4 - (a^2 - 2*a + 1)*c*x^2 - 2*a*c*x + c)) - (16*a^3*x
^3 - (16*a^3 + 8*a^2 + 4*a + 1)*x^4 + 8*a^2*x^2 + 4*a*x + 1)*sqrt(-a^2*c*x^
2 + c)*sqrt(-a^2*x^2 + 1))/(a^2*x^6 - x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3/((-a^2*x^2 + 1)^(3/2)*x^5), x)
```

maple [A] time = 0.05, size = 89, normalized size = 0.40

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(16a^4\ln(x)x^4-16\ln(ax-1)\,x^4a^4-16x^3a^3-8a^2x^2-4ax-1\right)}{4\left(a^2x^2-1\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(16*a^4*ln(x)*x^4-16*ln(a*x-
1)*x^4*a^4-16*x^3*a^3-8*a^2*x^2-4*a*x-1)/(a^2*x^2-1)/x^4
```

maxima [A] time = 0.39, size = 306, normalized size = 1.37

$$-\frac{1}{2}\,(-1)^{-2\,a^2cx^2+2\,c}\,a^4\sqrt{c}\,\log\left(-2\,a^2c+\frac{2\,c}{x^2}\right)+\frac{a^4c}{2\,\sqrt{a^4cx^4-2\,a^2cx^2+c}}+\frac{1}{2}\left(a\sqrt{c}\,\log(ax+1)-a\sqrt{c}\,\log(ax-1)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorith
m="maxima")
```

```
[Out] -1/2*(-1)^(-2*a^2*c*x^2 + 2*c)*a^4*sqrt(c)*log(-2*a^2*c + 2*c/x^2) + 1/2*a^
4*c/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 1/2*(a*sqrt(c)*log(a*x + 1) - a*sqr
t(c)*log(a*x - 1) - 2*sqrt(c)/x)*a^3 - 3/2*((-1)^(-2*a^2*c*x^2 + 2*c)*a^2*s
qrt(c)*log(-2*a^2*c + 2*c/x^2) - a^2*c/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) +
c/(sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*x^2))*a^2 + 1/2*(3*a^3*sqrt(c)*log(a*x
 + 1) - 3*a^3*sqrt(c)*log(a*x - 1) - 2*(3*a^2*sqrt(c)*x^2 + sqrt(c))/x^3)*a
 - 1/4*a^2*c/(sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c)*x^2) - 1/4*c/(sqrt(a^4*c*x^
4 - 2*a^2*c*x^2 + c)*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^3}{x^5\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x^5*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(x^5*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{- c \left(a x - 1\right) \left(a x + 1\right)} \left(a x + 1\right)^{3}}{x^{5} \left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(x**5*(-(a*x - 1)*(a*x +
 1))**(3/2)), x)
```

3.1165 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=44

$$\frac{c(ax+1)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

```
[Out] 1/4*c*(a*x+1)^4*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 32}

$$\frac{c(ax+1)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*(1 + a*x)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(1+ax)^3\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{c(1+ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.32

$$\frac{c\left(\frac{a^3x^4}{4}+a^2x^3+\frac{3ax^2}{2}+x\right)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(x + (3*a*x^2)/2 + a^2*x^3 + (a^3*x^4)/4))/Sqrt[1 -
a^2*x^2]
```

fricas [A] time = 0.64, size = 67, normalized size = 1.52

$$-\frac{\left(a^3cx^4+4\,a^2cx^3+6\,acx^2+4\,cx\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{4\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/4*(a^3*c*x^4 + 4*a^2*c*x^3 + 6*a*c*x^2 + 4*c*x)*sqrt(-a^2*c*x^2 + c)*sqr
t(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.03, size = 50, normalized size = 1.14

$$\frac{x\left(x^{3}a^{3}+4a^{2}x^{2}+6ax+4\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{4\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*x*(a^3*x^3+4*a^2*x^2+6*a*x+4)*(-a^2*c*x^2+c)^(3/2)/(-a^2*x^2+1)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 1.05, size = 54, normalized size = 1.23

$$\frac{\sqrt{c-a^{2}\,c\,x^{2}}\,\left(\frac{c\,a^{3}\,x^{4}}{4}+c\,a^{2}\,x^{3}+\frac{3\,c\,a\,x^{2}}{2}+c\,x\right)}{\sqrt{1-a^{2}\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(3/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(c*x + a^2*c*x^3 + (a^3*c*x^4)/4 + (3*a*c*x^2)/2))/(
1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1)
)**(3/2), x)
```

3.1166 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=93

$$\frac{2c^2(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(ax+1)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

```
[Out] 2/5*c^2*(a*x+1)^5*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-1/6*c^2*(a*x+1)
^6*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{2c^2(ax+1)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(ax+1)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (2*c^2*(1 + a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) - (c^2*(1 +
 a*x)^6*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)(1+ax)^4\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int\left(2(1+ax)^4-(1+ax)^5\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{2c^2(1+ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(1+ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.56

$$-\frac{c^2(ax+1)^5(5ax-7)\sqrt{c-a^2cx^2}}{30a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/30*(c^2*(1 + a*x)^5*(-7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^
2])
```

fricas [A] time = 0.60, size = 98, normalized size = 1.05

$$\frac{\left(5\,a^5c^2x^6+18\,a^4c^2x^5+15\,a^3c^2x^4-20\,a^2c^2x^3-45\,ac^2x^2-30\,c^2x\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{30\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 + 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 - 45
*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.03, size = 81, normalized size = 0.87

$$\frac{x\left(5x^5a^5+18x^4a^4+15x^3a^3-20a^2x^2-45ax-30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{30\,(ax-1)\,(ax+1)\left(-a^2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5+18*a^4*x^4+15*a^3*x^3-20*a^2*x^2-45*a*x-30)*(-a^2*c*x^2+c
)^(5/2)/(a*x-1)/(a*x+1)/(-a^2*x^2+1)^(3/2)
```

maxima [B] time = 0.39, size = 237, normalized size = 2.55

$$-\frac{1}{3}\,a^2c^{\frac{5}{2}}x^3+\frac{1}{12}\left(\frac{2\,a^4c^3x^8}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}-\frac{5\,a^2c^3x^6}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}+\frac{3\,c^3x^4}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}\right)a^3+c^{\frac{5}{2}}x-\frac{1}{5}\left(3\,a^2c^{\frac{5}{2}}x^5-5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] -1/3*a^2*c^(5/2)*x^3 + 1/12*(2*a^4*c^3*x^8/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c
) - 5*a^2*c^3*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 3*c^3*x^4/sqrt(a^4*c*
x^4 - 2*a^2*c*x^2 + c))*a^3 + c^(5/2)*x - 1/5*(3*a^2*c^(5/2)*x^5 - 5*c^(5/2
)*x^3)*a^2 + 3/4*(a^4*c^3*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 3*a^2*c^3
*x^4/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 2*c^3/(sqrt(a^4*c*x^4 - 2*a^2*c*x^
2 + c)*a^2))*a
```

mupad [B] time = 1.08, size = 85, normalized size = 0.91

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(-\frac{a^5\,c^2\,x^6}{6}-\frac{3\,a^4\,c^2\,x^5}{5}-\frac{a^3\,c^2\,x^4}{2}+\frac{2\,a^2\,c^2\,x^3}{3}+\frac{3\,a\,c^2\,x^2}{2}+c^2\,x\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(5/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(c^2*x + (3*a*c^2*x^2)/2 + (2*a^2*c^2*x^3)/3 - (a^3*
c^2*x^4)/2 - (3*a^4*c^2*x^5)/5 - (a^5*c^2*x^6)/6))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1)(ax+1))^{\frac{5}{2}}(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1)
)**(3/2), x)
```

3.1167 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=139

$$\frac{c^3(ax+1)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}-\frac{4c^3(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{2c^3(ax+1)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] 2/3*c^3*(a*x+1)^6*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-4/7*c^3*(a*x+1)
^7*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+1/8*c^3*(a*x+1)^8*(-a^2*c*x^2+
c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c^3(ax+1)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}-\frac{4c^3(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{2c^3(ax+1)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (2*c^3*(1 + a*x)^6*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) - (4*c^3*(1
 + a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) + (c^3*(1 + a*x)^8*S
qrt[c - a^2*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx &= \frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int (1-ax)^2(1+ax)^5\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int \left(4(1+ax)^5-4(1+ax)^6+(1+ax)^7\right)dx}{\sqrt{1-a^2x^2}}\\
&= \frac{2c^3(1+ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{4c^3(1+ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{c^3(1+ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.43

$$\frac{c^3(ax+1)^6\left(21a^2x^2-54ax+37\right)\sqrt{c-a^2cx^2}}{168a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2),x]
[Out] (c^3*(1 + a*x)^6*(37 - 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(168*a*Sqr
t[1 - a^2*x^2])
```

fricas [A] time = 0.99, size = 120, normalized size = 0.86

$$-\frac{\left(21\,a^7c^3x^8+72\,a^6c^3x^7+28\,a^5c^3x^6-168\,a^4c^3x^5-210\,a^3c^3x^4+56\,a^2c^3x^3+252\,ac^3x^2+168\,c^3x\right)\sqrt{-a^2cx^2+c}}{168\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="f
ricas")
[Out] -1/168*(21*a^7*c^3*x^8 + 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 - 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 + 56*a^2*c^3*x^3 + 252*a*c^3*x^2 + 168*c^3*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.03, size = 97, normalized size = 0.70

$$\frac{x\left(21a^7x^7+72x^6a^6+28x^5a^5-168x^4a^4-210x^3a^3+56a^2x^2+252ax+168\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{168\,(ax-1)^2\,(ax+1)^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/168*x*(21*a^7*x^7+72*a^6*x^6+28*a^5*x^5-168*a^4*x^4-210*a^3*x^3+56*a^2*x^
2+252*a*x+168)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^2/(a*x+1)^2/(-a^2*x^2+1)^(3/2)
```

maxima [B] time = 0.43, size = 323, normalized size = 2.32

$$\frac{1}{5}\,a^4c^{\frac{7}{2}}x^5-\frac{2}{3}\,a^2c^{\frac{7}{2}}x^3+c^{\frac{7}{2}}x-\frac{1}{24}\left(\frac{3\,a^6c^4x^{10}}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}-\frac{11\,a^4c^4x^8}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}+\frac{14\,a^2c^4x^6}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}-\frac{6}{\sqrt{a^4cx^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="m
axima")
```

```
[Out] 1/5*a^4*c^(7/2)*x^5 - 2/3*a^2*c^(7/2)*x^3 + c^(7/2)*x - 1/24*(3*a^6*c^4*x^1
0/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 11*a^4*c^4*x^8/sqrt(a^4*c*x^4 - 2*a^2
*c*x^2 + c) + 14*a^2*c^4*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 6*c^4*x^4/
sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c))*a^3 + 1/35*(15*a^4*c^(7/2)*x^7 - 42*a^2*
c^(7/2)*x^5 + 35*c^(7/2)*x^3)*a^2 - 1/2*(a^6*c^4*x^8/sqrt(a^4*c*x^4 - 2*a^2
*c*x^2 + c) - 4*a^4*c^4*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 6*a^2*c^4*x
^4/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 3*c^4/(sqrt(a^4*c*x^4 - 2*a^2*c*x^2
+ c)*a^2))*a
```

mupad [B] time = 1.09, size = 107, normalized size = 0.77

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a^7\,c^3\,x^8}{8}+\frac{3\,a^6\,c^3\,x^7}{7}+\frac{a^5\,c^3\,x^6}{6}-a^4\,c^3\,x^5-\frac{5\,a^3\,c^3\,x^4}{4}+\frac{a^2\,c^3\,x^3}{3}+\frac{3\,a\,c^3\,x^2}{2}+c^3\,x\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(7/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(c^3*x + (3*a*c^3*x^2)/2 + (a^2*c^3*x^3)/3 - (5*a^3*
c^3*x^4)/4 - a^4*c^3*x^5 + (a^5*c^3*x^6)/6 + (3*a^6*c^3*x^7)/7 + (a^7*c^3*x
^8)/8))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1)
)**(3/2), x)
```

3.1168 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=185

$$-\frac{c^4(ax+1)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}+\frac{2c^4(ax+1)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{3c^4(ax+1)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}+\frac{8c^4(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}$$

```
[Out] 8/7*c^4*(a*x+1)^7*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-3/2*c^4*(a*x+1)
^8*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+2/3*c^4*(a*x+1)^9*(-a^2*c*x^2+
c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-1/10*c^4*(a*x+1)^10*(-a^2*c*x^2+c)^(1/2)/a/(-
a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{c^4(ax+1)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}+\frac{2c^4(ax+1)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{3c^4(ax+1)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}+\frac{8c^4(ax+1)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (8*c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (3*c^4*(1
 + a*x)^8*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - a^2*x^2]) + (2*c^4*(1 + a*x)^9
*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) - (c^4*(1 + a*x)^10*Sqrt[c -
a^2*c*x^2])/(10*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
```

```
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int(1-ax)^3(1+ax)^6\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int\left(8(1+ax)^6-12(1+ax)^7+6(1+ax)^8-(1+ax)^9\right)dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{8c^4(1+ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{3c^4(1+ax)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}+\frac{2c^4(1+ax)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.37

$$-\frac{c^4(ax+1)^7\left(21a^3x^3-77a^2x^2+98ax-44\right)\sqrt{c-a^2cx^2}}{210a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] -1/210*(c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(-44 + 98*a*x - 77*a^2*x^2 + 21
*a^3*x^3))/(a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.69, size = 120, normalized size = 0.65

$$\frac{\left(21\,a^9c^4x^{10}+70\,a^8c^4x^9-240\,a^6c^4x^7-210\,a^5c^4x^6+252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2-210\,c^4x\right)\sqrt{-a^2cx^2+}}{210\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="f
ricas")
```

```
[Out] 1/210*(21*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6
 + 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 - 210*c^4*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}(ax+1)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*(a*x + 1)^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [A] time = 0.03, size = 97, normalized size = 0.52

$$\frac{x\left(21a^9x^9+70x^8a^8-240x^6a^6-210x^5a^5+252x^4a^4+420x^3a^3-315ax-210\right)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}{210\,(ax-1)^3\,(ax+1)^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 1/210*x*(21*a^9*x^9+70*a^8*x^8-240*a^6*x^6-210*a^5*x^5+252*a^4*x^4+420*a^3*
x^3-315*a*x-210)*(-a^2*c*x^2+c)^(9/2)/(a*x-1)^3/(a*x+1)^3/(-a^2*x^2+1)^(3/2
)
```

maxima [B] time = 0.50, size = 409, normalized size = 2.21

$$-\frac{1}{7}\,a^6c^{\frac{9}{2}}x^7+\frac{3}{5}\,a^4c^{\frac{9}{2}}x^5-a^2c^{\frac{9}{2}}x^3+c^{\frac{9}{2}}x+\frac{1}{40}\left(\frac{4\,a^8c^5x^{12}}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}-\frac{19\,a^6c^5x^{10}}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}+\frac{35\,a^4c^5x^8}{\sqrt{a^4cx^4-2\,a^2cx^2+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="m
axima")
```

```
[Out] -1/7*a^6*c^(9/2)*x^7 + 3/5*a^4*c^(9/2)*x^5 - a^2*c^(9/2)*x^3 + c^(9/2)*x +
1/40*(4*a^8*c^5*x^12/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 19*a^6*c^5*x^10/sq
rt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 35*a^4*c^5*x^8/sqrt(a^4*c*x^4 - 2*a^2*c*x
^2 + c) - 30*a^2*c^5*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 10*c^5*x^4/sqr
t(a^4*c*x^4 - 2*a^2*c*x^2 + c))*a^3 - 1/105*(35*a^6*c^(9/2)*x^9 - 135*a^4*c
^(9/2)*x^7 + 189*a^2*c^(9/2)*x^5 - 105*c^(9/2)*x^3)*a^2 + 3/8*(a^8*c^5*x^10
/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 5*a^6*c^5*x^8/sqrt(a^4*c*x^4 - 2*a^2*c
*x^2 + c) + 10*a^4*c^5*x^6/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) - 10*a^2*c^5*x
```

```
^4/sqrt(a^4*c*x^4 - 2*a^2*c*x^2 + c) + 4*c^5/(sqrt(a^4*c*x^4 - 2*a^2*c*x^2
+ c)*a^2))*a
```

mupad [B] time = 1.09, size = 106, normalized size = 0.57

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(-\frac{a^9\,c^4\,x^{10}}{10}-\frac{a^8\,c^4\,x^9}{3}+\frac{8\,a^6\,c^4\,x^7}{7}+a^5\,c^4\,x^6-\frac{6\,a^4\,c^4\,x^5}{5}-2\,a^3\,c^4\,x^4+\frac{3\,a\,c^4\,x^2}{2}+c^4\,x\right)}{\sqrt{1-a^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(9/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(c^4*x + (3*a*c^4*x^2)/2 - 2*a^3*c^4*x^4 - (6*a^4*c^
4*x^5)/5 + a^5*c^4*x^6 + (8*a^6*c^4*x^7)/7 - (a^8*c^4*x^9)/3 - (a^9*c^4*x^1
0)/10))/(1 - a^2*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1)
)**(3/2), x)
```

3.1169 $\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{1-a^2x^2}}{a(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] 2*(-a^2*x^2+1)^(1/2)/a/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+ln(-a*x+1)*(-a^2*x^2+1
)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{2\sqrt{1-a^2x^2}}{a(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2
]*Log[1 - a*x])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1+ax}{(1-ax)^2}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{2\sqrt{1-a^2x^2}}{a(1-ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}\log(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.61

$$\frac{\sqrt{1-a^2x^2}\left(\frac{2}{1-ax}+\log(1-ax)\right)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2/(1 - a*x) + Log[1 - a*x]))/(a*Sqrt[c - a^2*c*x^2])
```

fricas [B] time = 0.61, size = 382, normalized size = 4.60

$$\left[\frac{4\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\,ax+\left(a^3x^3-a^2x^2-ax+1\right)\sqrt{c}\log\left(\frac{a^6cx^6-4a^5cx^5+5a^4cx^4-4a^2cx^2+4acx-\left(a^4x^4-4a^3x^3+6a^2x^2-\right.}{a^4x^4-2a^3x^3+2ax-1}\right.}{2\left(a^4cx^3-a^3cx^2-a^2cx+ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x + (a^3*x^3 - a^2*x^2 -
a*x + 1)*sqrt(c)*log((a^6*c*x^6 - 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 +
 4*a*c*x - (a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x)*sqrt(-a^2*c*x^2 + c)*s
qrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)))/(a^4*c
```

```
*x^3 - a^3*c*x^2 - a^2*c*x + a*c), (2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 +
1)*a*x + (a^3*x^3 - a^2*x^2 - a*x + 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)
*(a^2*x^2 - 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 - 2*a^3*c*x^3
 - a^2*c*x^2 + 2*a*c*x)))/(a^4*c*x^3 - a^3*c*x^2 - a^2*c*x + a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3/(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)), x)
```

maple [A] time = 0.04, size = 70, normalized size = 0.84

$$\frac{\left(-\ln(ax-1)xa+\ln(ax-1)+2\right)\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}}{\left(a^2x^2-1\right)ca\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-ln(a*x-1)*x*a+ln(a*x-1)+2)*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/(a^2
*x^2-1)/c/a/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3/(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,x+1)^3}{\sqrt{c-a^2\,c\,x^2}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x
 + 1))), x)
```

3.1170 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{1-a^2x^2}}{2ac(1-ax)^2\sqrt{c-a^2cx^2}}$$

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)/a/c/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 32}

$$\frac{\sqrt{1-a^2x^2}}{2ac(1-ax)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*c*(1 - a*x)^2*Sqrt[c - a^2*c*x^2])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^3}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}}{2ac(1-ax)^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.13

$$-\frac{\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{2ac^2(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[1 - a^2*x^2]*Sqrt[c - a^2*c*x^2])/(a*c^2*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.69, size = 72, normalized size = 1.53

$$\frac{\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\left(ax^2-2\,x\right)}{2\left(a^4c^2x^4-2\,a^3c^2x^3+2\,ac^2x-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x^2 - 2*x)/(a^4*c^2*x^4 - 2*
a^3*c^2*x^3 + 2*a*c^2*x - c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(3/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

maple [A] time = 0.03, size = 43, normalized size = 0.91

$$-\frac{(ax-1)(ax+1)^3}{2a\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(a*x-1)/a*(a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(3/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

mupad [B] time = 1.25, size = 74, normalized size = 1.57

$$\frac{\sqrt{c-a^2\,c\,x^2}}{2\,a^3\,c^2\,\left(\frac{\sqrt{1-a^2\,x^2}}{a^2}+x^2\,\sqrt{1-a^2\,x^2}-\frac{2\,x\,\sqrt{1-a^2\,x^2}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (c - a^2*c*x^2)^(1/2)/(2*a^3*c^2*((1 - a^2*x^2)^(1/2)/a^2 + x^2*(1 - a^2*x^
2)^(1/2) - (2*x*(1 - a^2*x^2)^(1/2))/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^3}{(-(ax-1)(ax+1))^{\frac{3}{2}}(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1
))**(3/2)), x)
```

3.1171 $\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{6ac^2(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

```
[Out] 1/6*(-a^2*x^2+1)^(1/2)/a/c^2/(-a*x+1)^3/(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^2+
1)^(1/2)/a/c^2/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^2+1)^(1/2)/a/c^2
/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+1/8*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/c^2/(-
a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{6ac^2(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(6*a*c^2*(1 - a*x)^3*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*
x^2]/(8*a*c^2*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a*c^2
*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^2
*Sqrt[c - a^2*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^4(1+ax)}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{1}{8(-1+ax)^2}-\frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{6ac^2(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-}}{8}$$

$$= \frac{\sqrt{1-a^2x^2}}{6ac^2(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-}}{8a}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 0.39

$$\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2+9ax+3(ax-1)^3\tanh^{-1}(ax)-10\right)}{24ac^2(ax-1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh[a*x]))
/(24*a*c^2*(-1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.78, size = 459, normalized size = 2.48

$$\left[\frac{3\left(a^5x^5-3\,a^4x^4+2\,a^3x^3+2\,a^2x^2-3\,ax+1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right)+4\left(\right.}{96\left(a^6c^3x^5-3\,a^5c^3x^4+2\,a^4c^3x^3+2\,a^3c^3x^2-3\,a^2c^3x+ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/96*(3*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*sqrt(c)*
log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1)) + 4*(10*a^3*x^3 - 27*a^2*x^2 + 21*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2
*x^2 + 1))/(a^6*c^3*x^5 - 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3 + 2*a^3*c^3*x^2 - 3
*a^2*c^3*x + a*c^3), 1/48*(3*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 -
 3*a*x + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqr
t(-c)*x/(a^4*c*x^4 - c)) + 2*(10*a^3*x^3 - 27*a^2*x^2 + 21*a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*c^3*x^5 - 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3
+ 2*a^3*c^3*x^2 - 3*a^2*c^3*x + a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(5/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

maple [A] time = 0.05, size = 159, normalized size = 0.86

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(3\ln\left(ax-1\right)x^3a^3-3a^3x^3\ln\left(ax+1\right)-9\ln\left(ax-1\right)x^2a^2+9\ln\left(ax+1\right)x^2a^2+6a^2\right.}{48\left(a^2x^2-1\right)c^3a\left(ax-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/48*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x^3*a^3-3*a^3*x
^3*ln(a*x+1)-9*ln(a*x-1)*x^2*a^2+9*ln(a*x+1)*x^2*a^2+6*a^2*x^2+9*ln(a*x-1)*
x*a-9*a*x*ln(a*x+1)-18*a*x-3*ln(a*x-1)+3*ln(a*x+1)+20)/(a^2*x^2-1)/c^3/a/(a
*x-1)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(5/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,x+1)^3}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\left(-c(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1
))**(5/2)), x)
```

3.1172 $\int \frac{e^{3\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{32ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{}{16a}$$

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)/a/c^3/(-a*x+1)^4/(-a^2*c*x^2+c)^(1/2)+1/12*(-a^2*x^
2+1)^(1/2)/a/c^3/(-a*x+1)^3/(-a^2*c*x^2+c)^(1/2)+3/32*(-a^2*x^2+1)^(1/2)/a/
c^3/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^2+1)^(1/2)/a/c^3/(-a*x+1)/(
-a^2*c*x^2+c)^(1/2)-1/32*(-a^2*x^2+1)^(1/2)/a/c^3/(a*x+1)/(-a^2*c*x^2+c)^(1
/2)+5/32*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{32ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(16*a*c^3*(1 - a*x)^4*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2
*x^2]/(12*a*c^3*(1 - a*x)^3*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2])/(3
2*a*c^3*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2*x^2]/(8*a*c^3*(1 -
a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(32*a*c^3*(1 + a*x)*Sqrt[c -
a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(32*a*c^3*Sqrt[c - a^2*c*x
^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^5(1+ax)^2}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4(-1+ax)^5}+\frac{1}{4(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{1}{8(-1+ax)^2}+\frac{1}{32(1+ax)^2}-\frac{5}{32\left(-1+a^2x^2\right)}\right)dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{16ac^3(1-ax)^4\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+$$

$$= \frac{\sqrt{1-a^2x^2}}{16ac^3(1-ax)^4\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+$$

Mathematica [A] time = 0.08, size = 101, normalized size = 0.36

$$\frac{\sqrt{1-a^2x^2}\left(-15a^4x^4+45a^3x^3-35a^2x^2-15ax+15(ax-1)^4(ax+1)\tanh^{-1}(ax)+32\right)}{96ac^3(ax-1)^4(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(32 - 15*a*x - 35*a^2*x^2 + 45*a^3*x^3 - 15*a^4*x^4 + 15
*(-1 + a*x)^4*(1 + a*x)*ArcTanh[a*x]))/(96*a*c^3*(-1 + a*x)^4*(1 + a*x)*Sqr
t[c - a^2*c*x^2])
```

fricas [A] time = 0.74, size = 565, normalized size = 2.03

$$\left[\frac{15\left(a^7x^7-3a^6x^6+a^5x^5+5a^4x^4-5a^3x^3-a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5a^4cx^4-5a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2}}{a^6x^6-3a^4x^4+3a^2x^2-1}\right.}{384\left(a^8c^4x^7-3a^7c^4x^6+a^6c^4x^5+5a^5c^4x^4-5a^4c^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="f
ricas")
```

```
[Out] [1/384*(15*(a^7*x^7 - 3*a^6*x^6 + a^5*x^5 + 5*a^4*x^4 - 5*a^3*x^3 - a^2*x^2
 + 3*a*x - 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*
x^3 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 -
3*a^4*x^4 + 3*a^2*x^2 - 1)) + 4*(32*a^5*x^5 - 81*a^4*x^4 + 19*a^3*x^3 + 99*
a^2*x^2 - 81*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^8*c^4*x^7 - 3
*a^7*c^4*x^6 + a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 - a^3*c^4*x^2 +
3*a^2*c^4*x - a*c^4), 1/192*(15*(a^7*x^7 - 3*a^6*x^6 + a^5*x^5 + 5*a^4*x^4
- 5*a^3*x^3 - a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*s
qrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 - c)) + 2*(32*a^5*x^5 - 81*a^4*x^
4 + 19*a^3*x^3 + 99*a^2*x^2 - 81*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 +
1))/(a^8*c^4*x^7 - 3*a^7*c^4*x^6 + a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^4*c^4*
x^3 - a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(7/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

maple [A] time = 0.05, size = 238, normalized size = 0.86

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(15\ln\left(ax-1\right)x^5a^5-15\ln\left(ax+1\right)x^5a^5-45\ln\left(ax-1\right)x^4a^4+45\ln\left(ax+1\right)x^4a^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/192*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x-1)*x^5*a^5-15*ln
(a*x+1)*x^5*a^5-45*ln(a*x-1)*x^4*a^4+45*ln(a*x+1)*x^4*a^4+30*x^4*a^4+30*ln(
a*x-1)*x^3*a^3-30*a^3*x^3*ln(a*x+1)-90*x^3*a^3+30*ln(a*x-1)*x^2*a^2-30*ln(a
*x+1)*x^2*a^2+70*a^2*x^2-45*ln(a*x-1)*x*a+45*a*x*ln(a*x+1)+30*a*x+15*ln(a*x
-1)-15*ln(a*x+1)-64)/(a^2*x^2-1)/c^4/a/(a*x-1)^4/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3/((-a^2*c*x^2 + c)^(7/2)*(-a^2*x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a\,x+1)^3}{\left(c-a^2\,c\,x^2\right)^{7/2}\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/((c - a^2*c*x^2)^(7/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int((a*x + 1)^3/((c - a^2*c*x^2)^(7/2)*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\left(-c(ax-1)(ax+1)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)**3/((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1
))**(7/2)), x)
```

3.1173 $\int e^{3\tanh^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=136

$$\frac{4x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;ax)}{(m+1)\sqrt{1-a^2x^2}}-\frac{3x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] -3*x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(1+m)/(-a^2*x^2+1)^(1/2)-a*x^(2+m)*(-a^2*c*
x^2+c)^(1/2)/(2+m)/(-a^2*x^2+1)^(1/2)+4*x^(1+m)*hypergeom([1, 1+m],[2+m],a*
x)*(-a^2*c*x^2+c)^(1/2)/(1+m)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6153, 6150, 88, 64}

$$\frac{4x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;ax)}{(m+1)\sqrt{1-a^2x^2}}-\frac{3x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-3*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) - (a*x^(2 +
m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) + (4*x^(1 + m)*Sqrt[c -
 a^2*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/((1 + m)*Sqrt[1 - a^2*
x^2])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}} \\
&= \frac{\sqrt{c-a^2cx^2}\int\frac{x^m(1+ax)^2}{1-ax}\,dx}{\sqrt{1-a^2x^2}} \\
&= \frac{\sqrt{c-a^2cx^2}\int\left(-3x^m-ax^{1+m}+\frac{4x^m}{1-ax}\right)dx}{\sqrt{1-a^2x^2}} \\
&= -\frac{3x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int\frac{x^m}{1-ax}\,dx}{\sqrt{1-a^2x^2}} \\
&= -\frac{3x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}+\frac{4x^{1+m}\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.54

$$
-\frac{x^{m+1}\sqrt{c-a^2cx^2}\left(-4(m+2)\,{}_2F_1(1,m+1;m+2;ax)+m(ax+3)+ax+6\right)}{(m+1)(m+2)\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((x^(1 + m)*Sqrt[c - a^2*c*x^2]*(6 + a*x + m*(3 + a*x) - 4*(2 + m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, a*x]))/((1 + m)*(2 + m)*Sqrt[1 - a^2*x^2]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(ax+1)x^m}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x + 1)*x^m/(a^2*x^2 - 2
*a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\,x^m\sqrt{-a^2c\,x^2+c}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)^3x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3*x^m/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}\,\left(ax+1\right)^3}{\left(-\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)**3/(-(a*x - 1)*(a*x +
1))**(3/2), x)
```

3.1174 $\int e^{3\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=251

$$\frac{(4m+2p+3)x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{m+1}{2},\frac{3}{2}-p;\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2p)}+\frac{a(4m+6p+5)x^{m+2}\left(1-a^2x^2\right)^{-p}\left(c-\right.}{(m+2)(m+}$$

```
[Out] (3+4*m+2*p)*x^(1+m)*(-a^2*c*x^2+c)^p*hypergeom([3/2-p, 1/2+1/2*m],[3/2+1/2*
m],a^2*x^2)/(1+m)/(m+2*p)/((-a^2*x^2+1)^p)+a*(5+4*m+6*p)*x^(2+m)*(-a^2*c*x^
2+c)^p*hypergeom([1+1/2*m, 3/2-p],[2+1/2*m],a^2*x^2)/(2+m)/(1+m+2*p)/((-a^2
*x^2+1)^p)-3*x^(1+m)*(-a^2*c*x^2+c)^p/(m+2*p)/(-a^2*x^2+1)^(1/2)-a*x^(2+m)*
(-a^2*c*x^2+c)^p/(1+m+2*p)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6153, 6148, 1809, 808, 364}

$$\frac{(4m+2p+3)x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{m+1}{2},\frac{3}{2}-p;\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2p)}+\frac{a(4m+6p+5)x^{m+2}\left(1-a^2x^2\right)^{-p}\left(c-\right.}{(m+2)(m+}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p,x]
```

```
[Out] (-3*x^(1 + m)*(c - a^2*c*x^2)^p)/((m + 2*p)*Sqrt[1 - a^2*x^2]) - (a*x^(2 +
m)*(c - a^2*c*x^2)^p)/((1 + m + 2*p)*Sqrt[1 - a^2*x^2]) + ((3 + 4*m + 2*p)*
x^(1 + m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(1 + m)/2, 3/2 - p, (3 + m)/2
, a^2*x^2])/((1 + m)*(m + 2*p)*(1 - a^2*x^2)^p) + (a*(5 + 4*m + 6*p)*x^(2 +
 m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(2 + m)/2, 3/2 - p, (4 + m)/2, a^2*
x^2])/((2 + m)*(1 + m + 2*p)*(1 - a^2*x^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
```

```
] &&  !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx$$
$$=-\frac{ax^{2+m}\left(c-a^2cx^2\right)^p}{(1+m+2p)\sqrt{1-a^2x^2}}-\frac{\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-\right.}{a^{2}}$$
$$=-\frac{3x^{1+m}\left(c-a^2cx^2\right)^p}{(m+2p)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\left(c-a^2cx^2\right)^p}{(1+m+2p)\sqrt{1-a^2x^2}}+\frac{\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)}{}$$
$$=-\frac{3x^{1+m}\left(c-a^2cx^2\right)^p}{(m+2p)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\left(c-a^2cx^2\right)^p}{(1+m+2p)\sqrt{1-a^2x^2}}+\frac{\left((3+4m+2p)\left(1-a^2x^2\right)^{-}\right.}{}$$
$$=-\frac{3x^{1+m}\left(c-a^2cx^2\right)^p}{(m+2p)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\left(c-a^2cx^2\right)^p}{(1+m+2p)\sqrt{1-a^2x^2}}+\frac{(3+4m+2p)x^{1+m}\left(1-a^2\right.}{}$$

Mathematica [A] time = 0.17, size = 186, normalized size = 0.74

$$x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{{}_2F_1\left(\frac{m+1}{2},\frac{3}{2}-p;\frac{m+3}{2};a^2x^2\right)}{m+1}+ax\left(\frac{3\,{}_2F_1\left(\frac{m+2}{2},\frac{3}{2}-p;\frac{m+4}{2};a^2x^2\right)}{m+2}+ax\left(\frac{3\,{}_2F_1\left(\frac{m+3}{2}\right.}{}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*(Hypergeometric2F1[(1 + m)/2, 3/2 - p, (3 + m)
/2, a^2*x^2]/(1 + m) + a*x*((3*Hypergeometric2F1[(2 + m)/2, 3/2 - p, (4 + m
)/2, a^2*x^2])/(2 + m) + a*x*((3*Hypergeometric2F1[(3 + m)/2, 3/2 - p, (5 +
 m)/2, a^2*x^2])/(3 + m) + (a*x*Hypergeometric2F1[(4 + m)/2, 3/2 - p, (6 +
m)/2, a^2*x^2])/(4 + m)))))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)\left(-a^2cx^2+c\right)^p x^m}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p*x^m/(a^2*x^2 - 2*a
*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{3} x^{m} \left(-a^{2} c\,x^{2}+c\right)^{p}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{3} \left(-a^{2} c x^{2}+c\right)^{p} x^{m}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^m*(-a^2*c*x^2+c)^p,x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x^m/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m \left(c-a^2\,c\,x^2\right)^p\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**m*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Timed out
```

3.1175 $\int e^{3\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=224

$$\frac{a(6p+17)x^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{3}{2}-p;\frac{7}{2};a^2x^2\right)}{10(p+2)}-\frac{ax^5\left(c-a^2cx^2\right)^p}{2(p+2)\sqrt{1-a^2x^2}}-\frac{3\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}+$$

```
[Out] -3*(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/a^4/(3+2*p)+1/10*a*(17+6*p)*x^5*(-a^
2*c*x^2+c)^p*hypergeom([5/2, 3/2-p],[7/2],a^2*x^2)/(2+p)/((-a^2*x^2+1)^p)+4
*(-a^2*c*x^2+c)^p/a^4/(1-2*p)/(-a^2*x^2+1)^(1/2)-1/2*a*x^5*(-a^2*c*x^2+c)^p
/(2+p)/(-a^2*x^2+1)^(1/2)+7*(-a^2*c*x^2+c)^p*(-a^2*x^2+1)^(1/2)/a^4/(1+2*p)
```

Rubi [A] time = 0.36, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6153, 6148, 1652, 446, 77, 459, 364}

$$\frac{a(6p+17)x^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{3}{2}-p;\frac{7}{2};a^2x^2\right)}{10(p+2)}-\frac{3\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}+\frac{7\sqrt{1-a^2x^2}\left(c-a^2c\right.}{a^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^p,x]
```

```
[Out] (4*(c - a^2*c*x^2)^p)/(a^4*(1 - 2*p)*Sqrt[1 - a^2*x^2]) - (a*x^5*(c - a^2*c
*x^2)^p)/(2*(2 + p)*Sqrt[1 - a^2*x^2]) + (7*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^
2)^p)/(a^4*(1 + 2*p)) - (3*(1 - a^2*x^2)^(3/2)*(c - a^2*c*x^2)^p)/(a^4*(3 +
 2*p)) + (a*(17 + 6*p)*x^5*(c - a^2*c*x^2)^p*Hypergeometric2F1[5/2, 3/2 - p
, 7/2, a^2*x^2])/(10*(2 + p)*(1 - a^2*x^2)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(1+3a^2x^2\right)dx+\left(\left(1-a^2x^2\right.\right.$$
$$=-\frac{ax^5\left(c-a^2cx^2\right)^p}{2(2+p)\sqrt{1-a^2x^2}}+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{3}{2}}\right.$$
$$=-\frac{ax^5\left(c-a^2cx^2\right)^p}{2(2+p)\sqrt{1-a^2x^2}}+\frac{a(17+6p)x^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{3}{2}-p;\right.}{10(2+p)}$$
$$=\frac{4\left(c-a^2cx^2\right)^p}{a^4(1-2p)\sqrt{1-a^2x^2}}-\frac{ax^5\left(c-a^2cx^2\right)^p}{2(2+p)\sqrt{1-a^2x^2}}+\frac{7\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(1+2p)}-\frac{3(1}{}$$

Mathematica [A] time = 0.20, size = 176, normalized size = 0.79

$$\frac{\left(c-a^2cx^2\right)^p\left(-\frac{105\left(1-a^2x^2\right)^{3/2}}{2p+3}+\frac{245\sqrt{1-a^2x^2}}{2p+1}+\frac{140}{(1-2p)\sqrt{1-a^2x^2}}+5a^7x^7\left(1-a^2x^2\right)^{-p}\,{}_2F_1\left(\frac{7}{2},\frac{3}{2}-p;\frac{9}{2};a^2x^2\right)+21a^5x^5\left(\right.\right.}{35a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^3*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(140/((1 - 2*p)*Sqrt[1 - a^2*x^2]) + (245*Sqrt[1 - a^2*x
^2])/(1 + 2*p) - (105*(1 - a^2*x^2)^(3/2))/(3 + 2*p) + (21*a^5*x^5*Hypergeo
metric2F1[5/2, 3/2 - p, 7/2, a^2*x^2])/(1 - a^2*x^2)^p + (5*a^7*x^7*Hyperge
ometric2F1[7/2, 3/2 - p, 9/2, a^2*x^2])/(1 - a^2*x^2)^p))/(35*a^4)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax^4+x^3\right)\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^4 + x^3)*sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a
*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 \left(-a^2cx^2+c\right)^p x^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x^3/(-a^2*x^2 + 1)^(3/2), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 x^3 \left(-a^2c\,x^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(a^2c^p\left(2\,p-1\right)x^2+2\,c^p\right)\left(-a^2x^2+1\right)^p}{\sqrt{-a^2x^2+1}\,\left(4\,p^2-1\right)a^4}-\int \frac{\left(a^3c^px^6+3\,a^2c^px^5+3\,ac^px^4\right)e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\left(a^2x^2-1\right)\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^3*(-a^2*c*x^2+c)^p,x, algorithm="m
axima")
```

```
[Out] -(a^2*c^p*(2*p - 1)*x^2 + 2*c^p)*(-a^2*x^2 + 1)^p/(sqrt(-a^2*x^2 + 1)*(4*p^
2 - 1)*a^4) - integrate((a^3*c^p*x^6 + 3*a^2*c^p*x^5 + 3*a*c^p*x^4)*e^(p*lo
g(a*x + 1) + p*log(-a*x + 1))/((a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^3*(c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(-c \left(ax-1\right) \left(ax+1\right)\right)^p \left(ax+1\right)^3}{\left(-\left(ax-1\right) \left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**3*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral(x**3*(-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1
))**(3/2), x)
```

3.1176 $\int e^{3\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=222

$$\frac{(2p+11)x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p{}_2F_1\left(\frac{3}{2},\frac{3}{2}-p;\frac{5}{2};a^2x^2\right)}{6(p+1)}-\frac{3x^3\left(c-a^2cx^2\right)^p}{2(p+1)\sqrt{1-a^2x^2}}-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}+5\sqrt{1}$$

```
[Out] -(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/a^3/(3+2*p)+1/6*(11+2*p)*x^3*(-a^2*c*x
^2+c)^p*hypergeom([3/2, 3/2-p],[5/2],a^2*x^2)/(1+p)/((-a^2*x^2+1)^p)+4*(-a^
2*c*x^2+c)^p/a^3/(1-2*p)/(-a^2*x^2+1)^(1/2)-3/2*x^3*(-a^2*c*x^2+c)^p/(1+p)/
(-a^2*x^2+1)^(1/2)+5*(-a^2*c*x^2+c)^p*(-a^2*x^2+1)^(1/2)/a^3/(1+2*p)
```

Rubi [A] time = 0.34, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6153, 6148, 1652, 459, 364, 446, 77}

$$\frac{(2p+11)x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p{}_2F_1\left(\frac{3}{2},\frac{3}{2}-p;\frac{5}{2};a^2x^2\right)}{6(p+1)}-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}+\frac{5\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^p,x]
```

```
[Out] (4*(c - a^2*c*x^2)^p)/(a^3*(1 - 2*p)*Sqrt[1 - a^2*x^2]) - (3*x^3*(c - a^2*c
*x^2)^p)/(2*(1 + p)*Sqrt[1 - a^2*x^2]) + (5*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^
2)^p)/(a^3*(1 + 2*p)) - ((1 - a^2*x^2)^(3/2)*(c - a^2*c*x^2)^p)/(a^3*(3 + 2
*p)) + ((11 + 2*p)*x^3*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/2, 3/2 - p, 5/
2, a^2*x^2])/(6*(1 + p)*(1 - a^2*x^2)^p)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(1+3a^2x^2\right)dx + \left(\left(1-a^2x^2\right)^{-}\right.$$

$$= -\frac{3x^3\left(c-a^2cx^2\right)^p}{2(1+p)\sqrt{1-a^2x^2}} + \frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{3}{2}+p}\right.$$

$$= -\frac{3x^3\left(c-a^2cx^2\right)^p}{2(1+p)\sqrt{1-a^2x^2}} + \frac{(11+2p)x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{3}{2}-p;\frac{5}{2};a\right.}{6(1+p)}$$

$$= \frac{4\left(c-a^2cx^2\right)^p}{a^3(1-2p)\sqrt{1-a^2x^2}} - \frac{3x^3\left(c-a^2cx^2\right)^p}{2(1+p)\sqrt{1-a^2x^2}} + \frac{5\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(1+2p)} - \frac{(1-}{}$$

Mathematica [A] time = 0.24, size = 179, normalized size = 0.81

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{3}{5}a^2x^5\,{}_2F_1\left(\frac{5}{2},\frac{3}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{3}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{\left(a^4x^4-4a^2p^2x^2\left(a^2x^2+\right.\right.}{a}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(((1 - a^2*x^2)^(-1/2 + p)*(-26 + 13*a^2*x^2 + a^4*x^4 -
 4*a^2*p^2*x^2*(3 + a^2*x^2) - 4*p*(3 + 5*a^2*x^2)))/(a^3*(-1 + 2*p)*(1 + 2
*p)*(3 + 2*p)) + (x^3*Hypergeometric2F1[3/2, 3/2 - p, 5/2, a^2*x^2])/3 + (3
*a^2*x^5*Hypergeometric2F1[5/2, 3/2 - p, 7/2, a^2*x^2])/5))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(ax^3+x^2\right)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x^3 + x^2)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a
*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 x^2\left(-a^2c\,x^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2*(-a^2*c*x^2+c)^p,x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x^2/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(c-a^2\,c\,x^2\right)^p\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(-c \left(ax-1\right) \left(ax+1\right)\right)^p \left(ax+1\right)^3}{\left(-\left(ax-1\right) \left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral(x**2*(-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1
))**(3/2), x)
```

3.1177 $\int e^{3\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=138

$$\frac{3\,2^{p+\frac{3}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(-2p^2-p+1\right)}-\frac{(ax+1)^3\left(c-a^2cx^2\right)^p}{2a^2(p+1)\sqrt{1-a^2x^2}}$$

```
[Out] 3*2^(3/2+p)*(-a*x+1)^(-1/2+p)*(-a^2*c*x^2+c)^p*hypergeom([-1/2+p, -3/2-p],[
1/2+p],-1/2*a*x+1/2)/a^2/(-2*p^2-p+1)/((-a^2*x^2+1)^p)-1/2*(a*x+1)^3*(-a^2*
c*x^2+c)^p/a^2/(1+p)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6153, 6148, 795, 676, 69}

$$\frac{3\,2^{p+\frac{3}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(-2p^2-p+1\right)}-\frac{(ax+1)^3\left(c-a^2cx^2\right)^p}{2a^2(p+1)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((1 + a*x)^3*(c - a^2*c*x^2)^p)/(2*a^2*(1 + p)*Sqrt[1 - a^2*x^2]) + (3*2^(
3/2 + p)*(1 - a*x)^(-1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-3/2 - p,
 -1/2 + p, 1/2 + p, (1 - a*x)/2])/(a^2*(1 - p - 2*p^2)*(1 - a^2*x^2)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 676

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(a^(p + 1)*d^(m - 1)*((d - e*x)/d)^(p + 1))/(a/d + (c*x)/e)^(p + 1), Int[(1
 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d, e, m}, x]
&& EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&
GtQ[a, 0] &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{3\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx$$

$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx$$

$$=-\frac{(1+ax)^3\left(c-a^2cx^2\right)^p}{2a^2(1+p)\sqrt{1-a^2x^2}}+\frac{\left(3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}}}{2a(1+p)}$$

$$=-\frac{(1+ax)^3\left(c-a^2cx^2\right)^p}{2a^2(1+p)\sqrt{1-a^2x^2}}+\frac{\left(3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{-\frac{3}{2}+p}(1+ax)^{\frac{3}{2}}}{2a(1+p)}$$

$$=-\frac{(1+ax)^3\left(c-a^2cx^2\right)^p}{2a^2(1+p)\sqrt{1-a^2x^2}}+\frac{3\,2^{\frac{3}{2}+p}(1-ax)^{-\frac{1}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{3}{2}\right.}{a^2(1-2p)(1+p)}$$

Mathematica [A] time = 0.38, size = 134, normalized size = 0.97

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(ax^3\,{}_2F_1\left(\frac{3}{2},\frac{3}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{\left(\frac{3-3a^2x^2}{2p+1}+\frac{4}{1-2p}\right)\left(1-a^2x^2\right)^{p-\frac{1}{2}}}{a^2}+\frac{1}{5}a^3x^5\,{}_2F_1\left(\frac{5}{2},\frac{3}{2}-p;\frac{7}{2}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcTanh[a*x])*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(((1 - a^2*x^2)^(-1/2 + p)*(4/(1 - 2*p) + (3 - 3*a^2*x^2
)/(1 + 2*p)))/a^2 + a*x^3*Hypergeometric2F1[3/2, 3/2 - p, 5/2, a^2*x^2] + (
a^3*x^5*Hypergeometric2F1[5/2, 3/2 - p, 7/2, a^2*x^2])/5))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(ax^2+x\right)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x^2 + x)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x
 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^3\left(-a^2cx^2+c\right)^px}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p*x/(-a^2*x^2 + 1)^(3/2), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^3\,x\left(-a^2c\,x^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(-a^{2}x^{2}+1\right)^{p}c^{p}}{\sqrt{-a^{2}x^{2}+1}\,a^{2}\left(2\,p-1\right)}-\int\frac{\left(a^{3}c^{p}x^{4}+3\,a^{2}c^{p}x^{3}+3\,ac^{p}x^{2}\right)e^{\left(p\log(ax+1)+p\log(-ax+1)\right)}}{\left(a^{2}x^{2}-1\right)\sqrt{ax+1}\,\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x*(-a^2*c*x^2+c)^p,x, algorithm="max
ima")
```

```
[Out] -(-a^2*x^2 + 1)^p*c^p/(sqrt(-a^2*x^2 + 1)*a^2*(2*p - 1)) - integrate((a^3*c
^p*x^4 + 3*a^2*c^p*x^3 + 3*a*c^p*x^2)*e^(p*log(a*x + 1) + p*log(-a*x + 1))/
((a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\left(c-a^{2}\,c\,x^{2}\right)^{p}\,\left(a\,x+1\right)^{3}}{\left(1-a^{2}\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{p}\left(ax+1\right)^{3}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral(x*(-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))*
*(3/2), x)
```

3.1178 $\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=85

$$\frac{2^{p+\frac{5}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{a(1-2p)}$$

```
[Out] 2^(5/2+p)*(-a*x+1)^(-1/2+p)*(-a^2*c*x^2+c)^p*hypergeom([-1/2+p, -3/2-p],[1/
2+p],-1/2*a*x+1/2)/a/(1-2*p)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 69}

$$\frac{2^{p+\frac{5}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{a(1-2p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(5/2 + p)*(1 - a*x)^(-1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-3/2
- p, -1/2 + p, 1/2 + p, (1 - a*x)/2])/(a*(1 - 2*p)*(1 - a^2*x^2)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{-\frac{3}{2}+p}(1+ax)^{\frac{3}{2}+p}\,dx\\ &= \frac{2^{\frac{5}{2}+p}(1-ax)^{-\frac{1}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{3}{2}-p,-\frac{1}{2}+p;\frac{1}{2}+p;\frac{1}{2}(1-ax)\right)}{a(1-2p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.98

$$\frac{2^{p+\frac{5}{2}}(1-ax)^{p-\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-\frac{3}{2},p-\frac{1}{2};p+\frac{1}{2};\frac{1}{2}(1-ax)\right)}{a-2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(5/2 + p)*(1 - a*x)^(-1/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-3/2
- p, -1/2 + p, 1/2 + p, (1 - a*x)/2])/((a - 2*a*p)*(1 - a^2*x^2)^p)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x +
 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac"
)
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/(-a^2*x^2 + 1)^(3/2), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{3} \left(-a^{2} c\,x^{2}+c\right)^{p}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{3} \left(-a^{2} c x^{2}+c\right)^{p}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxim
a")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,{\left(a\,x+1\right)}^3}{{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1)^3)/(1 - a^2*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{p} \left(a x + 1\right)^{3}}{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(-(a*x - 1)*(a*x + 1))**(
3/2), x)
```

3.1179 $\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx$

Optimal. Leaf size=193

$$\frac{a(6p+1)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)}{2p}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x\right.}{2p+1}$$

```
[Out] 1/2*a*(1+6*p)*x*(-a^2*c*x^2+c)^p*hypergeom([1/2, 3/2-p],[3/2],a^2*x^2)/p/((
-a^2*x^2+1)^p)+4*(-a^2*c*x^2+c)^p/(1-2*p)/(-a^2*x^2+1)^(1/2)-1/2*a*x*(-a^2*
c*x^2+c)^p/p/(-a^2*x^2+1)^(1/2)-(-a^2*c*x^2+c)^p*hypergeom([1, 1/2+p],[3/2+
p],-a^2*x^2+1)*(-a^2*x^2+1)^(1/2)/(1+2*p)
```

Rubi [A] time = 0.29, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6153, 6148, 1652, 446, 79, 65, 388, 245}

$$\frac{a(6p+1)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)}{2p}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x\right.}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x,x]
```

```
[Out] (4*(c - a^2*c*x^2)^p)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) - (a*x*(c - a^2*c*x^2)^
p)/(2*p*Sqrt[1 - a^2*x^2]) + (a*(1 + 6*p)*x*(c - a^2*c*x^2)^p*Hypergeometri
c2F1[1/2, 3/2 - p, 3/2, a^2*x^2])/(2*p*(1 - a^2*x^2)^p) - (Sqrt[1 - a^2*x^2
]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1
 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
```

```
mplerQ[p, 1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx$$

$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}}{x}\,dx$$

$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(1+3a^2x^2\right)}{x}\,dx+\left(\left(1-a^2x^2\right)^{-p}\left(c\right.\right.$$

$$=-\frac{ax\left(c-a^2cx^2\right)^p}{2p\sqrt{1-a^2x^2}}+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{3}{2}+p}\left(1+\right.}{x}\right.$$

$$=\frac{4\left(c-a^2cx^2\right)^p}{(1-2p)\sqrt{1-a^2x^2}}-\frac{ax\left(c-a^2cx^2\right)^p}{2p\sqrt{1-a^2x^2}}+\frac{a(1+6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F}{2p}$$

$$=\frac{4\left(c-a^2cx^2\right)^p}{(1-2p)\sqrt{1-a^2x^2}}-\frac{ax\left(c-a^2cx^2\right)^p}{2p\sqrt{1-a^2x^2}}+\frac{a(1+6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F}{2p}$$

Mathematica [A] time = 0.22, size = 159, normalized size = 0.82

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(-\frac{\left(1-a^2x^2\right)^{p-\frac{1}{2}}\,{}_2F_1\left(1,p-\frac{1}{2};p+\frac{1}{2};1-a^2x^2\right)}{2\left(p-\frac{1}{2}\right)}+3ax\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)+\frac{3\left(1-\right.}{1-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x,x]
```

```
[Out] ((c - a^2*c*x^2)^p*((3*(1 - a^2*x^2)^(-1/2 + p))/(1 - 2*p) + 3*a*x*Hypergeo
metric2F1[1/2, 3/2 - p, 3/2, a^2*x^2] - ((1 - a^2*x^2)^(-1/2 + p)*Hypergeom
etric2F1[1, -1/2 + p, 1/2 + p, 1 - a^2*x^2])/(2*(-1/2 + p)) + (a^3*x^3*Hype
rgeometric2F1[3/2, 3/2 - p, 5/2, a^2*x^2])/3))/(1 - a^2*x^2)^p
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)\left(-a^2cx^2+c\right)^p}{a^2x^3-2\,ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^3 - 2*a*x^2
 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="gia
c")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2c\,x^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x,x, algorithm="max
ima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,{\left(a\,x+1\right)}^3}{x\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1)^3)/(x*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{p} \left(a x + 1\right)^{3}}{x \left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**p/x,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))
**(3/2)), x)
```

3.1180 $\int \frac{e^{3\tanh^{-1}(ax)}(c-a^2cx^2)^p}{x^2}\,dx$

Optimal. Leaf size=187

$$a^2(5-2p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{3a\sqrt{1-a^2x^2}\,\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2\right.}{2p+1}$$

```
[Out] a^2*(5-2*p)*x*(-a^2*c*x^2+c)^p*hypergeom([1/2, 3/2-p],[3/2],a^2*x^2)/((-a^2
*x^2+1)^p)+4*a*(-a^2*c*x^2+c)^p/(1-2*p)/(-a^2*x^2+1)^(1/2)-(-a^2*c*x^2+c)^p
/x/(-a^2*x^2+1)^(1/2)-3*a*(-a^2*c*x^2+c)^p*hypergeom([1, 1/2+p],[3/2+p],-a^
2*x^2+1)*(-a^2*x^2+1)^(1/2)/(1+2*p)
```

Rubi [A] time = 0.33, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6153, 6148, 1807, 1652, 446, 79, 65, 12, 245}

$$a^2(5-2p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{3a\sqrt{1-a^2x^2}\,\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2\right.}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x^2,x]
```

```
[Out] (4*a*(c - a^2*c*x^2)^p)/((1 - 2*p)*Sqrt[1 - a^2*x^2]) - (c - a^2*c*x^2)^p/(
x*Sqrt[1 - a^2*x^2]) + (a^2*(5 - 2*p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1
[1/2, 3/2 - p, 3/2, a^2*x^2])/(1 - a^2*x^2)^p - (3*a*Sqrt[1 - a^2*x^2]*(c -
 a^2*c*x^2)^p*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1652

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] && !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^2}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}}{x^2}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} - \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-3a-a^2(5-2p)\right.}{x}$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int a^2(5-2p)\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\,dx - \left(\left(\right.\right.$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} - \frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{3}{2}+p}\left(-3a-a^3\right.}{x}\right.$$

$$= \frac{4a\left(c-a^2cx^2\right)^p}{(1-2p)\sqrt{1-a^2x^2}} - \frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + a^2(5-2p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2}\right.$$

$$= \frac{4a\left(c-a^2cx^2\right)^p}{(1-2p)\sqrt{1-a^2x^2}} - \frac{\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + a^2(5-2p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2}\right.$$

Mathematica [A] time = 0.26, size = 133, normalized size = 0.71

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(a\left(\frac{\left(1-a^2x^2\right)^{p-\frac{1}{2}}\left(3\,{}_2F_1\left(1,p-\frac{1}{2};p+\frac{1}{2};1-a^2x^2\right)+1\right)}{1-2p}+3ax\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)\right)-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x^2,x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(Hypergeometric2F1[-1/2, 3/2 - p, 1/2, a^2*x^2]/x) + a
*(3*a*x*Hypergeometric2F1[1/2, 3/2 - p, 3/2, a^2*x^2] + ((1 - a^2*x^2)^(-1/
2 + p)*(1 + 3*Hypergeometric2F1[1, -1/2 + p, 1/2 + p, 1 - a^2*x^2]))/(1 - 2
*p))))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)\left(-a^2cx^2+c\right)^p}{a^2x^4-2\,ax^3+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^4 - 2*a*x^3
 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2c\,x^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^2,x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,(a\,x+1)^3}{x^2\,\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1)^3)/(x^2*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1)\,(ax+1))^p\,(ax+1)^3}{x^2\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**p/x**2,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(x**2*(-(a*x - 1)*(a*x +
1))**(3/2)), x)
```

3.1181 $\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^3}\,dx$

Optimal. Leaf size=194

$$\frac{a^2(9-2p)\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p-\frac{1}{2};p+\frac{1}{2};1-a^2x^2\right)}{2(1-2p)\sqrt{1-a^2x^2}}-\frac{3a\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}}-\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}}+a^3(7-6p)x\left(1-a^2x^2\right)^{-p}$$

```
[Out] a^3*(7-6*p)*x*(-a^2*c*x^2+c)^p*hypergeom([1/2, 3/2-p],[3/2],a^2*x^2)/((-a^2
*x^2+1)^p)-1/2*(-a^2*c*x^2+c)^p/x^2/(-a^2*x^2+1)^(1/2)-3*a*(-a^2*c*x^2+c)^p
/x/(-a^2*x^2+1)^(1/2)+1/2*a^2*(9-2*p)*(-a^2*c*x^2+c)^p*hypergeom([1, -1/2+p
],[1/2+p],-a^2*x^2+1)/(1-2*p)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6153, 6148, 1807, 764, 266, 65, 245}

$$a^3(7-6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-p;\frac{3}{2};a^2x^2\right)+\frac{a^2(9-2p)\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p-\frac{1}{2};p+\frac{1}{2};1-a^2x^2\right)}{2(1-2p)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x^3,x]
```

```
[Out] -(c - a^2*c*x^2)^p/(2*x^2*Sqrt[1 - a^2*x^2]) - (3*a*(c - a^2*c*x^2)^p)/(x*S
qrt[1 - a^2*x^2]) + (a^3*(7 - 6*p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/
2, 3/2 - p, 3/2, a^2*x^2])/(1 - a^2*x^2)^p + (a^2*(9 - 2*p)*(c - a^2*c*x^2)
^p*Hypergeometric2F1[1, -1/2 + p, 1/2 + p, 1 - a^2*x^2])/(2*(1 - 2*p)*Sqrt[
1 - a^2*x^2])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6148

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a^2*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /
; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
0]) && IGtQ[(n + 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^3}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^3}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1+ax)^3\left(1-a^2x^2\right)^{-\frac{3}{2}+p}}{x^3}\,dx$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{3}{2}+p}\left(-6a-a^2(9\right.}{x^2}$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + \frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(a^2(9-2\right.}{}$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + \left(a^3(7-6p)\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \left(\right.$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + a^3(7-6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\ {}_2F_1\left(\right.$$

$$= -\frac{\left(c-a^2cx^2\right)^p}{2x^2\sqrt{1-a^2x^2}} - \frac{3a\left(c-a^2cx^2\right)^p}{x\sqrt{1-a^2x^2}} + a^3(7-6p)x\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\ {}_2F_1\left(\right.$$

Mathematica [A] time = 0.30, size = 154, normalized size = 0.79

$$a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(a\left(\frac{\left(1-a^2x^2\right)^{p-\frac{1}{2}}\left(3\,{}_2F_1\left(1,p-\frac{1}{2};p+\frac{1}{2};1-a^2x^2\right)+{}_2F_1\left(2,p-\frac{1}{2};p+\frac{1}{2};1-a^2x^2\right)\right)}{1-2p}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^p)/x^3,x]
```

```
[Out] (a*(c - a^2*c*x^2)^p*((-3*Hypergeometric2F1[-1/2, 3/2 - p, 1/2, a^2*x^2])/x
 + a*(a*x*Hypergeometric2F1[1/2, 3/2 - p, 3/2, a^2*x^2] + ((1 - a^2*x^2)^(-
1/2 + p)*(3*Hypergeometric2F1[1, -1/2 + p, 1/2 + p, 1 - a^2*x^2] + Hypergeo
metric2F1[2, -1/2 + p, 1/2 + p, 1 - a^2*x^2]))/(1 - 2*p))))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(ax+1)\left(-a^2cx^2+c\right)^p}{a^2x^5-2\,ax^4+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^5 - 2*a*x^4
 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="g
iac")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2c\,x^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x)
```

```
[Out] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3\left(-a^2cx^2+c\right)^p}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/x^3,x, algorithm="m
axima")
```

```
[Out] integrate((a*x + 1)^3*(-a^2*c*x^2 + c)^p/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,{\left(a\,x+1\right)}^3}{x^3\,{\left(1-a^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)),x)
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1)^3)/(x^3*(1 - a^2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{p}\left(ax+1\right)^{3}}{x^{3}\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*(-a**2*c*x**2+c)**p/x**3,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**3/(x**3*(-(a*x - 1)*(a*x +
1))**(3/2)), x)
```

3.1182 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$

Optimal. Leaf size=66

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

```
[Out] c^5*(a*x+1)^8/a-4/3*c^5*(a*x+1)^9/a+3/5*c^5*(a*x+1)^10/a-1/11*c^5*(a*x+1)^1
1/a
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (c^5*(1 + a*x)^8)/a - (4*c^5*(1 + a*x)^9)/(3*a) + (3*c^5*(1 + a*x)^10)/(5*a
) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx &= c^5\int(1-ax)^3(1+ax)^7\,dx\\ &= c^5\int\left(8(1+ax)^7-12(1+ax)^8+6(1+ax)^9-(1+ax)^{10}\right)dx\\ &= \frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.59

$$-\frac{c^5(ax+1)^8\left(15a^3x^3-54a^2x^2+67ax-29\right)}{165a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] -1/165*(c^5*(1 + a*x)^8*(-29 + 67*a*x - 54*a^2*x^2 + 15*a^3*x^3))/a
```

fricas [A] time = 0.45, size = 101, normalized size = 1.53

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

giac [A] time = 0.27, size = 101, normalized size = 1.53

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

maple [A] time = 0.02, size = 75, normalized size = 1.14

$$c^5\left(-\frac{1}{11}a^{10}x^{11}-\frac{2}{5}a^9x^{10}-\frac{1}{3}x^9a^8+a^7x^8+2x^7a^6-\frac{14}{5}a^4x^5-2x^4a^3+x^3a^2+2a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^5,x)
```

```
[Out] c^5*(-1/11*a^10*x^11-2/5*a^9*x^10-1/3*x^9*a^8+a^7*x^8+2*x^7*a^6-14/5*a^4*x^
5-2*x^4*a^3+x^3*a^2+2*a*x^2+x)
```

maxima [A] time = 0.35, size = 101, normalized size = 1.53

$$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

mupad [B] time = 0.93, size = 101, normalized size = 1.53

$$-\frac{a^{10}c^5x^{11}}{11}-\frac{2a^9c^5x^{10}}{5}-\frac{a^8c^5x^9}{3}+a^7c^5x^8+2a^6c^5x^7-\frac{14a^4c^5x^5}{5}-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^5*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] c^5*x + 2*a*c^5*x^2 + a^2*c^5*x^3 - 2*a^3*c^5*x^4 - (14*a^4*c^5*x^5)/5 + 2*
a^6*c^5*x^7 + a^7*c^5*x^8 - (a^8*c^5*x^9)/3 - (2*a^9*c^5*x^10)/5 - (a^10*c^
5*x^11)/11
```

sympy [B] time = 0.12, size = 109, normalized size = 1.65

$$-\frac{a^{10}c^5x^{11}}{11}-\frac{2a^9c^5x^{10}}{5}-\frac{a^8c^5x^9}{3}+a^7c^5x^8+2a^6c^5x^7-\frac{14a^4c^5x^5}{5}-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - 2*a**9*c**5*x**10/5 - a**8*c**5*x**9/3 + a**7*c**5*x
**8 + 2*a**6*c**5*x**7 - 14*a**4*c**5*x**5/5 - 2*a**3*c**5*x**4 + a**2*c**5
*x**3 + 2*a*c**5*x**2 + c**5*x
```

3.1183 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=52

$$\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

[Out] 4/7*c^4*(a*x+1)^7/a-1/2*c^4*(a*x+1)^8/a+1/9*c^4*(a*x+1)^9/a

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]

[Out] (4*c^4*(1 + a*x)^7)/(7*a) - (c^4*(1 + a*x)^8)/(2*a) + (c^4*(1 + a*x)^9)/(9*a)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx&=c^4\int(1-ax)^2(1+ax)^6\,dx\\&=c^4\int\left(4(1+ax)^6-4(1+ax)^7+(1+ax)^8\right)dx\\&=\frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.60

$$\frac{c^4(ax+1)^7\left(14a^2x^2-35ax+23\right)}{126a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^4,x]

[Out] (c^4*(1 + a*x)^7*(23 - 35*a*x + 14*a^2*x^2))/(126*a)

fricas [A] time = 0.65, size = 92, normalized size = 1.77

$$\frac{1}{9}a^8c^4x^9+\frac{1}{2}a^7c^4x^8+\frac{4}{7}a^6c^4x^7-\frac{2}{3}a^5c^4x^6-2a^4c^4x^5-a^3c^4x^4+\frac{4}{3}a^2c^4x^3+2ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="fricas")

[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x

giac [A] time = 0.35, size = 92, normalized size = 1.77

$$\frac{1}{9}a^8c^4x^9+\frac{1}{2}a^7c^4x^8+\frac{4}{7}a^6c^4x^7-\frac{2}{3}a^5c^4x^6-2a^4c^4x^5-a^3c^4x^4+\frac{4}{3}a^2c^4x^3+2ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="giac")

[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x

maple [A] time = 0.02, size = 69, normalized size = 1.33

$$c^4\left(\frac{1}{9}x^9a^8+\frac{1}{2}a^7x^8+\frac{4}{7}x^7a^6-\frac{2}{3}x^6a^5-2a^4x^5-x^4a^3+\frac{4}{3}x^3a^2+2ax^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^4,x)

[Out] c^4*(1/9*x^9*a^8+1/2*a^7*x^8+4/7*x^7*a^6-2/3*x^6*a^5-2*a^4*x^5-x^4*a^3+4/3*x^3*a^2+2*a*x^2+x)

maxima [A] time = 0.32, size = 92, normalized size = 1.77

$$\frac{1}{9}\,a^{8}c^{4}x^{9}+\frac{1}{2}\,a^{7}c^{4}x^{8}+\frac{4}{7}\,a^{6}c^{4}x^{7}-\frac{2}{3}\,a^{5}c^{4}x^{6}-2\,a^{4}c^{4}x^{5}-a^{3}c^{4}x^{4}+\frac{4}{3}\,a^{2}c^{4}x^{3}+2\,ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

mupad [B] time = 0.05, size = 92, normalized size = 1.77

$$\frac{a^8\,c^4\,x^9}{9}+\frac{a^7\,c^4\,x^8}{2}+\frac{4\,a^6\,c^4\,x^7}{7}-\frac{2\,a^5\,c^4\,x^6}{3}-2\,a^4\,c^4\,x^5-a^3\,c^4\,x^4+\frac{4\,a^2\,c^4\,x^3}{3}+2\,a\,c^4\,x^2+c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] c^4*x + 2*a*c^4*x^2 + (4*a^2*c^4*x^3)/3 - a^3*c^4*x^4 - 2*a^4*c^4*x^5 - (2*
a^5*c^4*x^6)/3 + (4*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/2 + (a^8*c^4*x^9)/9
```

sympy [B] time = 0.11, size = 100, normalized size = 1.92

$$\frac{a^{8}c^{4}x^{9}}{9}+\frac{a^{7}c^{4}x^{8}}{2}+\frac{4a^{6}c^{4}x^{7}}{7}-\frac{2a^{5}c^{4}x^{6}}{3}-2a^{4}c^{4}x^{5}-a^{3}c^{4}x^{4}+\frac{4a^{2}c^{4}x^{3}}{3}+2ac^{4}x^{2}+c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/2 + 4*a**6*c**4*x**7/7 - 2*a**5*c**4*x**6
/3 - 2*a**4*c**4*x**5 - a**3*c**4*x**4 + 4*a**2*c**4*x**3/3 + 2*a*c**4*x**2
 + c**4*x
```

3.1184 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=35

$$\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

```
[Out] 1/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/(7*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx&=c^3\int(1-ax)(1+ax)^5\,dx\\&=c^3\int\left(2(1+ax)^5-(1+ax)^6\right)dx\\&=\frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.66

$$-\frac{c^3(ax+1)^6(3ax-4)}{21a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/21*(c^3*(1 + a*x)^6*(-4 + 3*a*x))/a
```

fricas [A] time = 0.57, size = 59, normalized size = 1.69

$$-\frac{1}{7}a^6c^3x^7 - \frac{2}{3}a^5c^3x^6 - a^4c^3x^5 + \frac{5}{3}a^2c^3x^3 + 2ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

giac [A] time = 0.16, size = 59, normalized size = 1.69

$$-\frac{1}{7}a^6c^3x^7 - \frac{2}{3}a^5c^3x^6 - a^4c^3x^5 + \frac{5}{3}a^2c^3x^3 + 2ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

maple [A] time = 0.02, size = 45, normalized size = 1.29

$$c^3\left(-\frac{1}{7}x^7a^6 - \frac{2}{3}x^6a^5 - a^4x^5 + \frac{5}{3}x^3a^2 + 2a\,x^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(-1/7*x^7*a^6-2/3*x^6*a^5-a^4*x^5+5/3*x^3*a^2+2*a*x^2+x)
```

maxima [A] time = 0.33, size = 59, normalized size = 1.69

$$-\frac{1}{7}a^6c^3x^7 - \frac{2}{3}a^5c^3x^6 - a^4c^3x^5 + \frac{5}{3}a^2c^3x^3 + 2ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

mupad [B] time = 0.03, size = 59, normalized size = 1.69

$$-\frac{a^6\,c^3\,x^7}{7}-\frac{2\,a^5\,c^3\,x^6}{3}-a^4\,c^3\,x^5+\frac{5\,a^2\,c^3\,x^3}{3}+2\,a\,c^3\,x^2+c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
```

```
[Out] c^3*x + 2*a*c^3*x^2 + (5*a^2*c^3*x^3)/3 - a^4*c^3*x^5 - (2*a^5*c^3*x^6)/3 -
 (a^6*c^3*x^7)/7
```

sympy [B] time = 0.11, size = 63, normalized size = 1.80

$$-\frac{a^{6}c^{3}x^{7}}{7}-\frac{2a^{5}c^{3}x^{6}}{3}-a^{4}c^{3}x^{5}+\frac{5a^{2}c^{3}x^{3}}{3}+2ac^{3}x^{2}+c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**6*c**3*x**7/7 - 2*a**5*c**3*x**6/3 - a**4*c**3*x**5 + 5*a**2*c**3*x**3/
3 + 2*a*c**3*x**2 + c**3*x
```

3.1185 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(ax+1)^5}{5a}$$

```
[Out] 1/5*c^2*(a*x+1)^5/a
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 32}

$$\frac{c^2(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(1 + a*x)^5)/(5*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= c^2\int(1+ax)^4\,dx\\ &= \frac{c^2(1+ax)^5}{5a}\end{aligned}$$

Mathematica [B] time = 0.02, size = 37, normalized size = 2.18

$$c^2\left(\frac{a^4x^5}{5}+a^3x^4+2a^2x^3+2ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] c^2*(x + 2*a*x^2 + 2*a^2*x^3 + a^3*x^4 + (a^4*x^5)/5)
```

fricas [B] time = 0.59, size = 47, normalized size = 2.76

$$\frac{1}{5}\,a^4c^2x^5 + a^3c^2x^4 + 2\,a^2c^2x^3 + 2\,ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

giac [B] time = 0.16, size = 47, normalized size = 2.76

$$\frac{1}{5}\,a^4c^2x^5 + a^3c^2x^4 + 2\,a^2c^2x^3 + 2\,ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

maple [A] time = 0.02, size = 16, normalized size = 0.94

$$\frac{c^2\,(ax+1)^5}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/5*c^2*(a*x+1)^5/a
```

maxima [B] time = 0.32, size = 47, normalized size = 2.76

$$\frac{1}{5}\,a^4c^2x^5 + a^3c^2x^4 + 2\,a^2c^2x^3 + 2\,ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

mupad [B] time = 0.03, size = 47, normalized size = 2.76

$$\frac{a^4\,c^2\,x^5}{5}+a^3\,c^2\,x^4+2\,a^2\,c^2\,x^3+2\,a\,c^2\,x^2+c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] c^2*x + 2*a*c^2*x^2 + 2*a^2*c^2*x^3 + a^3*c^2*x^4 + (a^4*c^2*x^5)/5
```

sympy [B] time = 0.11, size = 48, normalized size = 2.82

$$\frac{a^4c^2x^5}{5}+a^3c^2x^4+2a^2c^2x^3+2ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**2,x)
[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4 + 2*a**2*c**2*x**3 + 2*a*c**2*x**2 + c**2
*x
```

3.1186 $\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=36

$$-\frac{1}{3}a^2cx^3-2acx^2-\frac{8c\log(1-ax)}{a}-7cx$$

```
[Out] -7*c*x-2*a*c*x^2-1/3*a^2*c*x^3-8*c*ln(-a*x+1)/a
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.28, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6140, 43}

$$-\frac{c(ax+1)^3}{3a}-\frac{c(ax+1)^2}{a}-\frac{8c\log(1-ax)}{a}-4cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -4*c*x - (c*(1 + a*x)^2)/a - (c*(1 + a*x)^3)/(3*a) - (8*c*Log[1 - a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= c\int\frac{(1+ax)^3}{1-ax}\,dx\\ &= c\int\left(-4+\frac{8}{1-ax}-2(1+ax)-(1+ax)^2\right)dx\\ &= -4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$-\frac{1}{3}a^2cx^3-2acx^2-\frac{8c\log(1-ax)}{a}-7cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2),x]
[Out] -7*c*x - 2*a*c*x^2 - (a^2*c*x^3)/3 - (8*c*Log[1 - a*x])/a
```

fricas [A] time = 0.63, size = 37, normalized size = 1.03

$$-\frac{a^3cx^3+6\,a^2cx^2+21\,acx+24\,c\log\left(ax-1\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -1/3*(a^3*c*x^3 + 6*a^2*c*x^2 + 21*a*c*x + 24*c*log(a*x - 1))/a
```

giac [A] time = 0.34, size = 44, normalized size = 1.22

$$-\frac{8\,c\log\left(|ax-1|\right)}{a}-\frac{a^5cx^3+6\,a^4cx^2+21\,a^3cx}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] -8*c*log(abs(a*x - 1))/a - 1/3*(a^5*c*x^3 + 6*a^4*c*x^2 + 21*a^3*c*x)/a^3
```

maple [A] time = 0.03, size = 34, normalized size = 0.94

$$-\frac{a^2c\,x^3}{3}-2ac\,x^2-7cx-\frac{8c\ln\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c),x)
[Out] -1/3*a^2*c*x^3-2*a*c*x^2-7*c*x-8*c/a*ln(a*x-1)
```

maxima [A] time = 0.32, size = 33, normalized size = 0.92

$$-\frac{1}{3}\,a^2cx^3-2\,acx^2-7\,cx-\frac{8\,c\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 - 2*a*c*x^2 - 7*c*x - 8*c*log(a*x - 1)/a
```

mupad [B] time = 0.05, size = 33, normalized size = 0.92

$$-7\,c\,x-\frac{a^2\,c\,x^3}{3}-\frac{8\,c\,\ln\left(a\,x-1\right)}{a}-2\,a\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] - 7*c*x - (a^2*c*x^3)/3 - (8*c*log(a*x - 1))/a - 2*a*c*x^2
```

sympy [A] time = 0.21, size = 36, normalized size = 1.00

$$-\frac{a^{2}cx^{3}}{3}-2acx^{2}-7cx-\frac{8c\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c),x)
[Out] -a**2*c*x**3/3 - 2*a*c*x**2 - 7*c*x - 8*c*log(a*x - 1)/a
```

3.1187 $\int \frac{e^{4\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=13

$$\frac{x}{c(1-ax)^2}$$

```
[Out] x/c/(-a*x+1)^2
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 34}

$$\frac{x}{c(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] x/(c*(1 - a*x)^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{e^{4\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx &= \frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c} \\ &= \frac{x}{c(1-ax)^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.92

$$\frac{(ax+1)^2}{4ac(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2),x]
[Out] (1 + a*x)^2/(4*a*c*(1 - a*x)^2)
```

fricas [A] time = 0.54, size = 19, normalized size = 1.46

$$\frac{x}{a^2cx^2 - 2\,acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

giac [A] time = 0.16, size = 12, normalized size = 0.92

$$\frac{x}{(ax-1)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] x/((a*x - 1)^2*c)
```

maple [B] time = 0.03, size = 28, normalized size = 2.15

$$\frac{\frac{1}{a(ax-1)} + \frac{1}{a(ax-1)^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c),x)
[Out] 1/c*(1/a/(a*x-1)+1/a/(a*x-1)^2)
```

maxima [A] time = 0.32, size = 19, normalized size = 1.46

$$\frac{x}{a^2cx^2 - 2\,acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

mupad [B] time = 0.92, size = 12, normalized size = 0.92

$$\frac{x}{c\,(a\,x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - a^2*c*x^2)*(a^2*x^2 - 1)^2),x)
[Out] x/(c*(a*x - 1)^2)
```

sympy [B] time = 0.24, size = 17, normalized size = 1.31

$$\frac{x}{a^2cx^2-2acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a**2*c*x**2+c),x)
[Out] x/(a**2*c*x**2 - 2*a*c*x + c)
```

3.1188 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3ac^2(1-ax)^3}$$

```
[Out] 1/3/a/c^2/(-a*x+1)^3
```

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 32}

$$\frac{1}{3ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(3*a*c^2*(1 - a*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{1}{(1-ax)^4}\,dx}{c^2}\\ &= \frac{1}{3ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 0.94

$$-\frac{1}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/3*1/(a*c^2*(-1 + a*x)^3)
```

fricas [B] time = 0.42, size = 41, normalized size = 2.28

$$-\frac{1}{3\left(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [A] time = 0.63, size = 15, normalized size = 0.83

$$-\frac{1}{3\,(ax-1)^3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/3/((a*x - 1)^3*a*c^2)
```

maple [A] time = 0.02, size = 16, normalized size = 0.89

$$-\frac{1}{3c^2a\,(ax-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3/c^2/a/(a*x-1)^3
```

maxima [B] time = 0.32, size = 41, normalized size = 2.28

$$-\frac{1}{3\left(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

mupad [B] time = 0.93, size = 40, normalized size = 2.22

$$\frac{1}{-3\,a^4\,c^2\,x^3+9\,a^3\,c^2\,x^2-9\,a^2\,c^2\,x+3\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - a^2*c*x^2)^2*(a^2*x^2 - 1)^2),x)
[Out] 1/(3*a*c^2 - 9*a^2*c^2*x + 9*a^3*c^2*x^2 - 3*a^4*c^2*x^3)
```

sympy [B] time = 0.29, size = 42, normalized size = 2.33

$$-\frac{1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a**2*c*x**2+c)**2,x)
[Out] -1/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3*a*c**2)
```

3.1189 $\int \frac{e^{4\tanh^{-1}(ax)}}{(c-a^2cx^2)^3} dx$

Optimal. Leaf size=87

$$\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

```
[Out] 1/8/a/c^3/(-a*x+1)^4+1/12/a/c^3/(-a*x+1)^3+1/16/a/c^3/(-a*x+1)^2+1/16/a/c^3
/(-a*x+1)+1/16*arctanh(a*x)/a/c^3
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(8*a*c^3*(1 - a*x)^4) + 1/(12*a*c^3*(1 - a*x)^3) + 1/(16*a*c^3*(1 - a*x)^
2) + 1/(16*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(16*a*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx &= \frac{\int \frac{1}{(1-ax)^5(1+ax)}\,dx}{c^3}\\
&= \frac{\int \left(-\frac{1}{2(-1+ax)^5}+\frac{1}{4(-1+ax)^4}-\frac{1}{8(-1+ax)^3}+\frac{1}{16(-1+ax)^2}-\frac{1}{16\left(-1+a^2x^2\right)}\right)dx}{c^3}\\
&= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{16c^3}\\
&= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{16ac^3}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.60

$$\frac{-3a^3x^3+12a^2x^2-19ax+3(ax-1)^4\tanh^{-1}(ax)+16}{48ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (16 - 19*a*x + 12*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^4*ArcTanh[a*x])/(48*a*
c^3*(-1 + a*x)^4)
```

fricas [B] time = 0.59, size = 147, normalized size = 1.69

$$-\frac{6\,a^3x^3-24\,a^2x^2+38\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax+1\right)+3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax-\right.}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(6*a^3*x^3 - 24*a^2*x^2 + 38*a*x - 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2
 - 4*a*x + 1)*log(a*x + 1) + 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1
)*log(a*x - 1) - 32)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c
^3*x + a*c^3)
```

giac [A] time = 0.19, size = 68, normalized size = 0.78

$$\frac{\log\left(|ax+1|\right)}{32\,ac^3}-\frac{\log\left(|ax-1|\right)}{32\,ac^3}-\frac{3\,a^3x^3-12\,a^2x^2+19\,ax-16}{48\,(ax-1)^4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*log(abs(a*x + 1))/(a*c^3) - 1/32*log(abs(a*x - 1))/(a*c^3) - 1/48*(3*a
^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/((a*x - 1)^4*a*c^3)
```

maple [A] time = 0.03, size = 90, normalized size = 1.03

$$\frac{1}{8c^3a\,(ax-1)^4}-\frac{1}{12c^3a\,(ax-1)^3}+\frac{1}{16c^3a\,(ax-1)^2}-\frac{1}{16c^3a\,(ax-1)}-\frac{\ln(ax-1)}{32a\,c^3}+\frac{\ln(ax+1)}{32a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/8/c^3/a/(a*x-1)^4-1/12/c^3/a/(a*x-1)^3+1/16/c^3/a/(a*x-1)^2-1/16/c^3/a/(a
*x-1)-1/32/a/c^3*ln(a*x-1)+1/32*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.32, size = 102, normalized size = 1.17

$$-\frac{3\,a^3x^3-12\,a^2x^2+19\,ax-16}{48\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}+\frac{\log(ax+1)}{32\,ac^3}-\frac{\log(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 +
 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + 1/32*log(a*x + 1)/(a*c^3) - 1/32*lo
g(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.10, size = 83, normalized size = 0.95

$$\frac{\operatorname{atanh}(a\,x)}{16\,a\,c^3}-\frac{\frac{19\,x}{48}-\frac{a\,x^2}{4}-\frac{1}{3\,a}+\frac{a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)^2),x)
```

```
[Out] atanh(a*x)/(16*a*c^3) - ((19*x)/48 - (a*x^2)/4 - 1/(3*a) + (a^2*x^3)/16)/(c
^3 + 6*a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x)
```

sympy [A] time = 0.60, size = 99, normalized size = 1.14

$$-\frac{3a^3x^3-12a^2x^2+19ax-16}{48a^5c^3x^4-192a^4c^3x^3+288a^3c^3x^2-192a^2c^3x+48ac^3}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(3*a**3*x**3 - 12*a**2*x**2 + 19*a*x - 16)/(48*a**5*c**3*x**4 - 192*a**4*c
**3*x**3 + 288*a**3*c**3*x**2 - 192*a**2*c**3*x + 48*a*c**3) - (log(x - 1/a
)/32 - log(x + 1/a)/32)/(a*c**3)
```

3.1190 $\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=122

$$\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{3\tanh^{-1}(ax)}{32ac^4}$$

```
[Out] 1/20/a/c^4/(-a*x+1)^5+1/16/a/c^4/(-a*x+1)^4+1/16/a/c^4/(-a*x+1)^3+1/16/a/c^
4/(-a*x+1)^2+5/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+3/32*arctanh(a*x)/a/c^4
```

Rubi [A] time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{3\tanh^{-1}(ax)}{32ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] 1/(20*a*c^4*(1 - a*x)^5) + 1/(16*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)
^3) + 1/(16*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 +
a*x)) + (3*ArcTanh[a*x])/(32*a*c^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{1}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$

$$= \frac{\int \left(\frac{1}{4(-1+ax)^6} - \frac{1}{4(-1+ax)^5} + \frac{3}{16(-1+ax)^4} - \frac{1}{8(-1+ax)^3} + \frac{5}{64(-1+ax)^2} + \frac{1}{64(1+ax)^2} - \frac{3}{32\left(-1+a^2x^2\right)}\right)dx}{c^4}$$

$$= \frac{1}{20ac^4(1-ax)^5} + \frac{1}{16ac^4(1-ax)^4} + \frac{1}{16ac^4(1-ax)^3} + \frac{1}{16ac^4(1-ax)^2} + \frac{5}{64ac^4(1-ax)} - \frac{}{64a}$$

$$= \frac{1}{20ac^4(1-ax)^5} + \frac{1}{16ac^4(1-ax)^4} + \frac{1}{16ac^4(1-ax)^3} + \frac{1}{16ac^4(1-ax)^2} + \frac{5}{64ac^4(1-ax)} - \frac{}{64a}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.66

$$\frac{-15a^5x^5+60a^4x^4-80a^3x^3+20a^2x^2+47ax+15(ax-1)^5(ax+1)\tanh^{-1}(ax)-48}{160ac^4(ax-1)^5(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (-48 + 47*a*x + 20*a^2*x^2 - 80*a^3*x^3 + 60*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)^5*(1 + a*x)*ArcTanh[a*x])/(160*a*c^4*(-1 + a*x)^5*(1 + a*x))
```

fricas [A] time = 0.75, size = 191, normalized size = 1.57

$$-\frac{30\,a^5x^5-120\,a^4x^4+160\,a^3x^3-40\,a^2x^2-94\,ax-15\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1\right)\log\left(ax+1\right)+}{320\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/320*(30*a^5*x^5 - 120*a^4*x^4 + 160*a^3*x^3 - 40*a^2*x^2 - 94*a*x - 15*(
a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 15*
(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) + 96
)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*
x - a*c^4)
```

giac [A] time = 0.18, size = 91, normalized size = 0.75

$$\frac{3\log(|ax+1|)}{64\,ac^4}-\frac{3\log(|ax-1|)}{64\,ac^4}-\frac{15\,a^5x^5-60\,a^4x^4+80\,a^3x^3-20\,a^2x^2-47\,ax+48}{160\,(ax+1)(ax-1)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 3/64*log(abs(a*x + 1))/(a*c^4) - 3/64*log(abs(a*x - 1))/(a*c^4) - 1/160*(15
*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/((a*x + 1)*(
a*x - 1)^5*a*c^4)
```

maple [A] time = 0.04, size = 120, normalized size = 0.98

$$-\frac{1}{20c^4a\,(ax-1)^5}+\frac{1}{16c^4a\,(ax-1)^4}-\frac{1}{16c^4a\,(ax-1)^3}+\frac{1}{16c^4a\,(ax-1)^2}-\frac{5}{64c^4a\,(ax-1)}-\frac{3\ln(ax-1)}{64c^4a}-\frac{1}{64a\,c^4\,(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/20/c^4/a/(a*x-1)^5+1/16/c^4/a/(a*x-1)^4-1/16/c^4/a/(a*x-1)^3+1/16/c^4/a/
(a*x-1)^2-5/64/c^4/a/(a*x-1)-3/64/c^4/a*ln(a*x-1)-1/64/a/c^4/(a*x+1)+3/64*l
n(a*x+1)/a/c^4
```

maxima [A] time = 0.33, size = 130, normalized size = 1.07

$$-\frac{15\,a^5x^5-60\,a^4x^4+80\,a^3x^3-20\,a^2x^2-47\,ax+48}{160\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4\right)}+\frac{3\log(ax+1)}{64\,ac^4}-\frac{3\log(ax-1)}{64\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/160*(15*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/(a
^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x -
a*c^4) + 3/64*log(a*x + 1)/(a*c^4) - 3/64*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.97, size = 111, normalized size = 0.91

$$\frac{3\,\mathrm{atanh}(a\,x)}{32\,a\,c^4}-\frac{\frac{47\,x}{160}+\frac{a\,x^2}{8}-\frac{3}{10\,a}-\frac{a^2\,x^3}{2}+\frac{3\,a^3\,x^4}{8}-\frac{3\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^4/((c - a^2*c*x^2)^4*(a^2*x^2 - 1)^2),x)
```

```
[Out] (3*atanh(a*x))/(32*a*c^4) - ((47*x)/160 + (a*x^2)/8 - 3/(10*a) - (a^2*x^3)/
2 + (3*a^3*x^4)/8 - (3*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 +
4*a^5*c^4*x^5 - a^6*c^4*x^6 - 4*a*c^4*x)
```

sympy **[A]** time = 0.61, size = 129, normalized size = 1.06

$$\frac{-15a^5x^5+60a^4x^4-80a^3x^3+20a^2x^2+47ax-48}{160a^7c^4x^6-640a^6c^4x^5+800a^5c^4x^4-800a^3c^4x^2+640a^2c^4x-160ac^4}+\frac{-\frac{3\log\left(x-\frac{1}{a}\right)}{64}+\frac{3\log\left(x+\frac{1}{a}\right)}{64}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (-15*a**5*x**5 + 60*a**4*x**4 - 80*a**3*x**3 + 20*a**2*x**2 + 47*a*x - 48)/
(160*a**7*c**4*x**6 - 640*a**6*c**4*x**5 + 800*a**5*c**4*x**4 - 800*a**3*c*
*4*x**2 + 640*a**2*c**4*x - 160*a*c**4) + (-3*log(x - 1/a)/64 + 3*log(x + 1
/a)/64)/(a*c**4)
```

3.1191 $\int e^{4\tanh^{-1}(ax)} \left(c-a^2cx^2\right)^p \, dx$

Optimal. Leaf size=63

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1} {}_2F_1\left(-p-2,p-1;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

```
[Out] 2^(2+p)*c*(a*x+1)^(1-p)*(-a^2*c*x^2+c)^(-1+p)*hypergeom([-1+p, -2-p],[p],-1
/2*a*x+1/2)/a/(1-p)
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6141, 678, 69}

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1} {}_2F_1\left(-p-2,p-1;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[-
2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
```

```
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rubi steps

$$\begin{aligned}\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= c^2\int(1+ax)^4\left(c-a^2cx^2\right)^{-2+p}\,dx\\ &= \left(c^2(1+ax)^{1-p}(c-acx)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\right)\int(1+ax)^{2+p}(c-acx)^{-2+p}\,dx\\ &= \frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\,{}_2F_1\left(-2-p,-1+p;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.14

$$-\frac{2^{p+2}(1-ax)^{p-1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-2,p-1;p;\frac{1}{2}(1-ax)\right)}{a(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
[Out] -((2^(2 + p)*(1 - a*x)^(-1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-2 - p,
 -1 + p, p, (1 - a*x)/2])/(a*(-1 + p)*(1 - a^2*x^2)^p))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^4\left(-a^2cx^2+c\right)^p}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)^4*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 1)^2, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4 \left(-a^2 c\,x^2 + c\right)^p}{\left(-a^2 x^2 + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x)
[Out] int((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^4 \left(-a^2 cx^2 + c\right)^p}{\left(a^2 x^2 - 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)^4*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c - a^2\,c\,x^2\right)^p\,(a\,x + 1)^4}{\left(a^2\,x^2 - 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1)^4)/(a^2*x^2 - 1)^2,x)
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1)^4)/(a^2*x^2 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1)\,(ax+1))^p\,(ax+1)^2}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

3.1192 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=127

$$\frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a}+\frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2}+\frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{35}{128}c^4x\sqrt{1-a^2x^2}+\frac{35c^4\sin^{-1}(ax)}{128a}$$

```
[Out] 35/192*c^4*x*(-a^2*x^2+1)^(3/2)+7/48*c^4*x*(-a^2*x^2+1)^(5/2)+1/8*c^4*x*(-a
^2*x^2+1)^(7/2)+1/9*c^4*(-a^2*x^2+1)^(9/2)/a+35/128*c^4*arcsin(a*x)/a+35/12
8*c^4*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 641, 195, 216}

$$\frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a}+\frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2}+\frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{35}{128}c^4x\sqrt{1-a^2x^2}+\frac{35c^4\sin^{-1}(ax)}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^ArcTanh[a*x],x]
```

```
[Out] (35*c^4*x*Sqrt[1 - a^2*x^2])/128 + (35*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (7*
c^4*x*(1 - a^2*x^2)^(5/2))/48 + (c^4*x*(1 - a^2*x^2)^(7/2))/8 + (c^4*(1 - a
^2*x^2)^(9/2))/(9*a) + (35*c^4*ArcSin[a*x])/(128*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4 dx = c^4\int (1-ax)\left(1-a^2x^2\right)^{7/2} dx$$
$$= \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + c^4\int \left(1-a^2x^2\right)^{7/2} dx$$
$$= \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} + \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{8}\left(7c^4\right)\int \left(1-a^2x^2\right)^{5/2} dx$$
$$= \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} + \frac{c^4\left(1-a^2x^2\right)^{9/2}}{9a} + \frac{1}{48}\left(35c^4\right)\int (1-$$
$$= \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2x^2\right)^{7/2} + \frac{c^4\left(1-a^2x^2\right)}{9a}$$
$$= \frac{35}{128}c^4x\sqrt{1-a^2x^2} + \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2\right.$$
$$= \frac{35}{128}c^4x\sqrt{1-a^2x^2} + \frac{35}{192}c^4x\left(1-a^2x^2\right)^{3/2} + \frac{7}{48}c^4x\left(1-a^2x^2\right)^{5/2} + \frac{1}{8}c^4x\left(1-a^2\right.$$

Mathematica [A] time = 0.15, size = 107, normalized size = 0.84

$$\frac{c^4\left(\sqrt{1-a^2x^2}\left(128a^8x^8-144a^7x^7-512a^6x^6+600a^5x^5+768a^4x^4-978a^3x^3-512a^2x^2+837ax+128\right)-630\right.}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^ArcTanh[a*x],x]
```

```
[Out] (c^4*(Sqrt[1 - a^2*x^2]*(128 + 837*a*x - 512*a^2*x^2 - 978*a^3*x^3 + 768*a^
4*x^4 + 600*a^5*x^5 - 512*a^6*x^6 - 144*a^7*x^7 + 128*a^8*x^8) - 630*ArcSin
[Sqrt[1 - a*x]/Sqrt[2]]))/(1152*a)
```

fricas [A] time = 0.66, size = 137, normalized size = 1.08

$$-\frac{630\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(128\,a^8c^4x^8-144\,a^7c^4x^7-512\,a^6c^4x^6+600\,a^5c^4x^5+768\,a^4c^4x^4-978\,a^3c^4x^3-5\right.}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/1152*(630*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (128*a^8*c^4*x^8
- 144*a^7*c^4*x^7 - 512*a^6*c^4*x^6 + 600*a^5*c^4*x^5 + 768*a^4*c^4*x^4 - 9
78*a^3*c^4*x^3 - 512*a^2*c^4*x^2 + 837*a*c^4*x + 128*c^4)*sqrt(-a^2*x^2 + 1
))/a
```

giac [A] time = 0.20, size = 124, normalized size = 0.98

$$\frac{35\,c^4\arcsin\left(ax\right)\mathrm{sgn}(a)}{128\,|a|}+\frac{1}{1152}\,\sqrt{-a^2x^2+1}\left(\frac{128\,c^4}{a}+\left(837\,c^4-2\left(256\,ac^4+\left(489\,a^2c^4-4\left(96\,a^3c^4+\left(75\,a^4c^4-2\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 35/128*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/1152*sqrt(-a^2*x^2 + 1)*(128*c^4/a
 + (837*c^4 - 2*(256*a*c^4 + (489*a^2*c^4 - 4*(96*a^3*c^4 + (75*a^4*c^4 - 2
*(32*a^5*c^4 - (8*a^7*c^4*x - 9*a^6*c^4)*x)*x)*x)*x)*x)*x)*x)
```

maple [A] time = 0.07, size = 201, normalized size = 1.58

$$-\frac{c^4a^5x^6\left(-a^2x^2+1\right)^{\frac{3}{2}}}{9}+\frac{c^4a^3x^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}-\frac{c^4a\,x^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3}+\frac{c^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}{9a}+\frac{c^4a^4x^5\left(-a^2x^2+1\right)^{\frac{3}{2}}}{8}-\frac{19c^4a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/9*c^4*a^5*x^6*(-a^2*x^2+1)^(3/2)+1/3*c^4*a^3*x^4*(-a^2*x^2+1)^(3/2)-1/3*
c^4*a*x^2*(-a^2*x^2+1)^(3/2)+1/9*c^4*(-a^2*x^2+1)^(3/2)/a+1/8*c^4*a^4*x^5*(
-a^2*x^2+1)^(3/2)-19/48*c^4*a^2*x^3*(-a^2*x^2+1)^(3/2)+29/64*c^4*x*(-a^2*x^
2+1)^(3/2)+35/128*c^4*x*(-a^2*x^2+1)^(1/2)+35/128*c^4/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.43, size = 182, normalized size = 1.43

$$-\frac{1}{9}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^5c^4x^6+\frac{1}{8}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4c^4x^5+\frac{1}{3}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^3c^4x^4-\frac{19}{48}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2c^4x^3-\frac{1}{3}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/9*(-a^2*x^2 + 1)^(3/2)*a^5*c^4*x^6 + 1/8*(-a^2*x^2 + 1)^(3/2)*a^4*c^4*x^
5 + 1/3*(-a^2*x^2 + 1)^(3/2)*a^3*c^4*x^4 - 19/48*(-a^2*x^2 + 1)^(3/2)*a^2*c
^4*x^3 - 1/3*(-a^2*x^2 + 1)^(3/2)*a*c^4*x^2 + 29/64*(-a^2*x^2 + 1)^(3/2)*c^
4*x + 35/128*sqrt(-a^2*x^2 + 1)*c^4*x + 1/9*(-a^2*x^2 + 1)^(3/2)*c^4/a + 35
/128*c^4*arcsin(a*x)/a
```

mupad [B] time = 0.93, size = 220, normalized size = 1.73

$$\frac{93\,c^4\,x\,\sqrt{1-a^2\,x^2}}{128}+\frac{35\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{128\,\sqrt{-a^2}}+\frac{c^4\,\sqrt{1-a^2\,x^2}}{9\,a}-\frac{4\,a\,c^4\,x^2\,\sqrt{1-a^2\,x^2}}{9}-\frac{163\,a^2\,c^4\,x^3\,\sqrt{1-a^2\,x^2}}{192}+\frac{2\,a^3\,c^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (93*c^4*x*(1 - a^2*x^2)^(1/2))/128 + (35*c^4*asinh(x*(-a^2)^(1/2)))/(128*(-
a^2)^(1/2)) + (c^4*(1 - a^2*x^2)^(1/2))/(9*a) - (4*a*c^4*x^2*(1 - a^2*x^2)^
(1/2))/9 - (163*a^2*c^4*x^3*(1 - a^2*x^2)^(1/2))/192 + (2*a^3*c^4*x^4*(1 -
a^2*x^2)^(1/2))/3 + (25*a^4*c^4*x^5*(1 - a^2*x^2)^(1/2))/48 - (4*a^5*c^4*x^
6*(1 - a^2*x^2)^(1/2))/9 - (a^6*c^4*x^7*(1 - a^2*x^2)^(1/2))/8 + (a^7*c^4*x
^8*(1 - a^2*x^2)^(1/2))/9
```

sympy [C] time = 20.60, size = 996, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] a**7*c**4*Piecewise((x**8*sqrt(-a**2*x**2 + 1)/9 - x**6*sqrt(-a**2*x**2 + 1
)/(63*a**2) - 2*x**4*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*x**2*sqrt(-a**2*x*
*2 + 1)/(315*a**6) - 16*sqrt(-a**2*x**2 + 1)/(315*a**8), Ne(a, 0)), (x**8/8
, True)) - a**6*c**4*Piecewise((I*a**2*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*x
**7/(48*sqrt(a**2*x**2 - 1)) - I*x**5/(192*a**2*sqrt(a**2*x**2 - 1)) - 5*I*
```

```
x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(128*a**6*sqrt(a**2*x**2 - 1))
- 5*I*acosh(a*x)/(128*a**7), Abs(a**2*x**2) > 1), (-a**2*x**9/(8*sqrt(-a**2
*x**2 + 1)) + 7*x**7/(48*sqrt(-a**2*x**2 + 1)) + x**5/(192*a**2*sqrt(-a**2*
x**2 + 1)) + 5*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(128*a**6*sqrt(-a
**2*x**2 + 1)) + 5*asin(a*x)/(128*a**7), True)) - 3*a**5*c**4*Piecewise((x*
*6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 + 1)/(35*a**2) - 4*x**2*sq
rt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 + 1)/(105*a**6), Ne(a, 0)
), (x**6/6, True)) + 3*a**4*c**4*Piecewise((I*a**2*x**7/(6*sqrt(a**2*x**2 -
 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(48*a**2*sqrt(a**2*x**2 -
 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(16*a**5), Abs(a**2
*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*x**5/(24*sqrt(-a**2*x
**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x/(16*a**4*sqrt(-a**2*x**
2 + 1)) + asin(a*x)/(16*a**5), True)) + 3*a**3*c**4*Piecewise((x**4*sqrt(-a
**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**2) - 2*sqrt(-a**2*x**2 +
 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) - 3*a**2*c**4*Piecewise((I*a**2*x
**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*x**2 - 1)) + I*x/(8*a**
2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2
*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**2*x**2 + 1)) - x/(8*a**
2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True)) - a*c**4*Piecewise((x*
*2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a**2), True)) + c**4*Piece
wise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2*sqrt(a**2*x**2 - 1)) - I
*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a**2*x**2 + 1)/2 + asin(a*
x)/(2*a), True))
```

3.1193 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=105

$$\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{16a}$$

```
[Out] 5/24*c^3*x*(-a^2*x^2+1)^(3/2)+1/6*c^3*x*(-a^2*x^2+1)^(5/2)+1/7*c^3*(-a^2*x^
2+1)^(7/2)/a+5/16*c^3*arcsin(a*x)/a+5/16*c^3*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 641, 195, 216}

$$\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5c^3\sin^{-1}(ax)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^ArcTanh[a*x],x]
```

```
[Out] (5*c^3*x*Sqrt[1 - a^2*x^2])/16 + (5*c^3*x*(1 - a^2*x^2)^(3/2))/24 + (c^3*x*
(1 - a^2*x^2)^(5/2))/6 + (c^3*(1 - a^2*x^2)^(7/2))/(7*a) + (5*c^3*ArcSin[a*
x])/(16*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= c^3\int(1-ax)\left(1-a^2x^2\right)^{5/2}\,dx\\
&= \frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+c^3\int\left(1-a^2x^2\right)^{5/2}\,dx\\
&= \frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{6}\left(5c^3\right)\int\left(1-a^2x^2\right)^{3/2}\,dx\\
&= \frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\frac{1}{8}\left(5c^3\right)\int\sqrt{1-a^2
\\
&= \frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+\\
&= \frac{5}{16}c^3x\sqrt{1-a^2x^2}+\frac{5}{24}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{1}{6}c^3x\left(1-a^2x^2\right)^{5/2}+\frac{c^3\left(1-a^2x^2\right)^{7/2}}{7a}+
\end{aligned}$$

Mathematica [A] time = 0.13, size = 91, normalized size = 0.87

$$-\frac{c^3\left(\sqrt{1-a^2x^2}\left(48a^6x^6-56a^5x^5-144a^4x^4+182a^3x^3+144a^2x^2-231ax-48\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{336a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^ArcTanh[a*x],x]
```

```
[Out] -1/336*(c^3*(Sqrt[1 - a^2*x^2]*(-48 - 231*a*x + 144*a^2*x^2 + 182*a^3*x^3 -
 144*a^4*x^4 - 56*a^5*x^5 + 48*a^6*x^6) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]
))/a
```

fricas [A] time = 0.72, size = 114, normalized size = 1.09

$$-\frac{210\,c^3\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(48\,a^6c^3x^6-56\,a^5c^3x^5-144\,a^4c^3x^4+182\,a^3c^3x^3+144\,a^2c^3x^2-231\,ac^3x-48\,c^3\right)}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)

[Out] -1/336*(210*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (48*a^6*c^3*x^6 - 56*a^5*c^3*x^5 - 144*a^4*c^3*x^4 + 182*a^3*c^3*x^3 + 144*a^2*c^3*x^2 - 231*a*c^3*x - 48*c^3)*sqrt(-a^2*x^2 + 1))/a

giac [A] time = 0.21, size = 101, normalized size = 0.96

$$\frac{5\,c^3 \arcsin\left(ax\right)\operatorname{sgn}(a)}{16\,|a|}+\frac{1}{336}\,\sqrt{-a^2x^2+1}\left(\frac{48\,c^3}{a}+\left(231\,c^3-2\left(72\,ac^3+\left(91\,a^2c^3-4\left(18\,a^3c^3-\left(6\,a^5c^3x-7\,a^4c^3\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] 5/16*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/336*sqrt(-a^2*x^2 + 1)*(48*c^3/a + (231*c^3 - 2*(72*a*c^3 + (91*a^2*c^3 - 4*(18*a^3*c^3 - (6*a^5*c^3*x - 7*a^4*c^3)*x)*x)*x)*x)*x)

maple [A] time = 0.05, size = 155, normalized size = 1.48

$$\frac{c^3a^3x^4\left(-a^2x^2+1\right)^{\frac{3}{2}}}{7}-\frac{2c^3a\,x^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{7}+\frac{c^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{7a}-\frac{c^3a^2x^3\left(-a^2x^2+1\right)^{\frac{3}{2}}}{6}+\frac{3c^3x\left(-a^2x^2+1\right)^{\frac{3}{2}}}{8}+\frac{5c^3x\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*c*x^2+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x)

[Out] 1/7*c^3*a^3*x^4*(-a^2*x^2+1)^(3/2)-2/7*c^3*a*x^2*(-a^2*x^2+1)^(3/2)+1/7*c^3*(-a^2*x^2+1)^(3/2)/a-1/6*c^3*a^2*x^3*(-a^2*x^2+1)^(3/2)+3/8*c^3*x*(-a^2*x^2+1)^(3/2)+5/16*c^3*x*(-a^2*x^2+1)^(1/2)+5/16*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))

maxima [A] time = 0.42, size = 136, normalized size = 1.30

$$\frac{1}{7}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^3c^3x^4-\frac{1}{6}\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2c^3x^3-\frac{2}{7}\left(-a^2x^2+1\right)^{\frac{3}{2}}ac^3x^2+\frac{3}{8}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^3x+\frac{5}{16}\sqrt{-a^2x^2+1}\,c^3x+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)

```
[Out] 1/7*(-a^2*x^2 + 1)^(3/2)*a^3*c^3*x^4 - 1/6*(-a^2*x^2 + 1)^(3/2)*a^2*c^3*x^3
 - 2/7*(-a^2*x^2 + 1)^(3/2)*a*c^3*x^2 + 3/8*(-a^2*x^2 + 1)^(3/2)*c^3*x + 5/
16*sqrt(-a^2*x^2 + 1)*c^3*x + 1/7*(-a^2*x^2 + 1)^(3/2)*c^3/a + 5/16*c^3*arc
sin(a*x)/a
```

mupad [B] time = 0.89, size = 174, normalized size = 1.66

$$\frac{11\,c^3\,x\,\sqrt{1-a^2\,x^2}}{16}+\frac{5\,c^3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{16\,\sqrt{-a^2}}+\frac{c^3\,\sqrt{1-a^2\,x^2}}{7\,a}-\frac{3\,a\,c^3\,x^2\,\sqrt{1-a^2\,x^2}}{7}-\frac{13\,a^2\,c^3\,x^3\,\sqrt{1-a^2\,x^2}}{24}+\frac{3\,a^3\,c^3\,x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (11*c^3*x*(1 - a^2*x^2)^(1/2))/16 + (5*c^3*asinh(x*(-a^2)^(1/2)))/(16*(-a^2
)^(1/2)) + (c^3*(1 - a^2*x^2)^(1/2))/(7*a) - (3*a*c^3*x^2*(1 - a^2*x^2)^(1/
2))/7 - (13*a^2*c^3*x^3*(1 - a^2*x^2)^(1/2))/24 + (3*a^3*c^3*x^4*(1 - a^2*x
^2)^(1/2))/7 + (a^4*c^3*x^5*(1 - a^2*x^2)^(1/2))/6 - (a^5*c^3*x^6*(1 - a^2*
x^2)^(1/2))/7
```

sympy [C] time = 11.91, size = 629, normalized size = 5.99

$$-a^5c^3\left(\begin{cases}\frac{x^6\sqrt{-a^2x^2+1}}{7}-\frac{x^4\sqrt{-a^2x^2+1}}{35a^2}-\frac{4x^2\sqrt{-a^2x^2+1}}{105a^4}-\frac{8\sqrt{-a^2x^2+1}}{105a^6} & \text{for } a\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)+a^4c^3\left(\begin{cases}\frac{ia^2x^7}{6\sqrt{a^2x^2-1}}-\frac{5ix^5}{24\sqrt{a^2x^2-1}}-\frac{ix^3}{48a^2\sqrt{a^2}} \\ -\frac{a^2x^7}{6\sqrt{-a^2x^2+1}}+\frac{5x^5}{24\sqrt{-a^2x^2+1}}+\frac{}{48}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -a**5*c**3*Piecewise((x**6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 +
1)/(35*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 +
 1)/(105*a**6), Ne(a, 0)), (x**6/6, True)) + a**4*c**3*Piecewise((I*a**2*x*
*7/(6*sqrt(a**2*x**2 - 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(48
*a**2*sqrt(a**2*x**2 - 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(a*
x)/(16*a**5), Abs(a**2*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5
*x**5/(24*sqrt(-a**2*x**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x/(
16*a**4*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(16*a**5), True)) + 2*a**3*c**3*P
iecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**2)
 - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) - 2*a**2*c*
*3*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*x
**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(a
**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**2
```

```
*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True))
- a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a**2
), True)) + c**3*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2*sq
rt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a**2
*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

3.1194 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=83

$$\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{3c^2\sin^{-1}(ax)}{8a}$$

```
[Out] 1/4*c^2*x*(-a^2*x^2+1)^(3/2)+1/5*c^2*(-a^2*x^2+1)^(5/2)/a+3/8*c^2*arcsin(a*
x)/a+3/8*c^2*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 641, 195, 216}

$$\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{3c^2\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^ArcTanh[a*x],x]
```

```
[Out] (3*c^2*x*Sqrt[1 - a^2*x^2])/8 + (c^2*x*(1 - a^2*x^2)^(3/2))/4 + (c^2*(1 - a
^2*x^2)^(5/2))/(5*a) + (3*c^2*ArcSin[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= c^2\int(1-ax)\left(1-a^2x^2\right)^{3/2}\,dx\\ &= \frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+c^2\int\left(1-a^2x^2\right)^{3/2}\,dx\\ &= \frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{4}\left(3c^2\right)\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{1}{8}\left(3c^2\right)\int\frac{1}{\sqrt{1-a^2x^2}}\\ &= \frac{3}{8}c^2x\sqrt{1-a^2x^2}+\frac{1}{4}c^2x\left(1-a^2x^2\right)^{3/2}+\frac{c^2\left(1-a^2x^2\right)^{5/2}}{5a}+\frac{3c^2\sin^{-1}(ax)}{8a}\end{aligned}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.90

$$\frac{c^2\left(\sqrt{1-a^2x^2}\left(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8\right)-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^ArcTanh[a*x],x]
```

```
[Out] (c^2*(Sqrt[1 - a^2*x^2]*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a^4*x^4)
- 30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(40*a)
```

fricas [A] time = 0.66, size = 93, normalized size = 1.12

$$-\frac{30\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(8\,a^4c^2x^4-10\,a^3c^2x^3-16\,a^2c^2x^2+25\,ac^2x+8\,c^2\right)\sqrt{-a^2x^2+1}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

[Out] -1/40*(30*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (8*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 16*a^2*c^2*x^2 + 25*a*c^2*x + 8*c^2)*sqrt(-a^2*x^2 + 1))/a

giac [A] time = 0.20, size = 78, normalized size = 0.94

$$\frac{3\,c^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{8\,|a|}+\frac{1}{40}\,\sqrt{-a^2x^2+1}\left(\left(25\,c^2-2\left(8\,ac^2-\left(4\,a^3c^2x-5\,a^2c^2\right)x\right)x\right)x+\frac{8\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] 3/8*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/40*sqrt(-a^2*x^2 + 1)*((25*c^2 - 2*(8*a*c^2 - (4*a^3*c^2*x - 5*a^2*c^2)*x)*x)*x + 8*c^2/a)

maple [A] time = 0.04, size = 109, normalized size = 1.31

$$-\frac{c^2a\,x^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{5}+\frac{c^2\left(-a^2x^2+1\right)^{\frac{3}{2}}}{5a}+\frac{c^2x\left(-a^2x^2+1\right)^{\frac{3}{2}}}{4}+\frac{3c^2x\sqrt{-a^2x^2+1}}{8}+\frac{3c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*c*x^2+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x)

[Out] -1/5*c^2*a*x^2*(-a^2*x^2+1)^(3/2)+1/5*c^2*(-a^2*x^2+1)^(3/2)/a+1/4*c^2*x*(-a^2*x^2+1)^(3/2)+3/8*c^2*x*(-a^2*x^2+1)^(1/2)+3/8*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))

maxima [A] time = 0.45, size = 90, normalized size = 1.08

$$-\frac{1}{5}\left(-a^2x^2+1\right)^{\frac{3}{2}}ac^2x^2+\frac{1}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2x+\frac{3}{8}\sqrt{-a^2x^2+1}\,c^2x+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{5\,a}+\frac{3\,c^2\arcsin\left(ax\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -1/5*(-a^2*x^2 + 1)^(3/2)*a*c^2*x^2 + 1/4*(-a^2*x^2 + 1)^(3/2)*c^2*x + 3/8*sqrt(-a^2*x^2 + 1)*c^2*x + 1/5*(-a^2*x^2 + 1)^(3/2)*c^2/a + 3/8*c^2*arcsin(a*x)/a

mupad [B] time = 0.04, size = 128, normalized size = 1.54

$$\frac{5\,c^2\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{3\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}+\frac{c^2\,\sqrt{1-a^2\,x^2}}{5\,a}-\frac{2\,a\,c^2\,x^2\,\sqrt{1-a^2\,x^2}}{5}-\frac{a^2\,c^2\,x^3\,\sqrt{1-a^2\,x^2}}{4}+\frac{a^3\,c^2\,x^4\,\sqrt{1-a^2\,x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (5*c^2*x*(1 - a^2*x^2)^(1/2))/8 + (3*c^2*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(
1/2)) + (c^2*(1 - a^2*x^2)^(1/2))/(5*a) - (2*a*c^2*x^2*(1 - a^2*x^2)^(1/2))
/5 - (a^2*c^2*x^3*(1 - a^2*x^2)^(1/2))/4 + (a^3*c^2*x^4*(1 - a^2*x^2)^(1/2)
)/5
```

sympy [C] time = 6.89, size = 337, normalized size = 4.06

$$a^3c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2x^2+1}}{5}-\frac{x^2\sqrt{-a^2x^2+1}}{15a^2}-\frac{2\sqrt{-a^2x^2+1}}{15a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)-a^2c^2\left(\begin{cases}\frac{ia^2x^5}{4\sqrt{a^2x^2-1}}-\frac{3ix^3}{8\sqrt{a^2x^2-1}}+\frac{ix}{8a^2\sqrt{a^2x^2-1}}-\frac{i\operatorname{acosh}(ax)}{8a^3}\\ -\frac{a^2x^5}{4\sqrt{-a^2x^2+1}}+\frac{3x^3}{8\sqrt{-a^2x^2+1}}-\frac{x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\operatorname{asin}}{8}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] a**3*c**2*Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1
)/(15*a**2) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True))
- a**2*c**2*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sq
rt(a**2*x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**
3), Abs(a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*s
qrt(-a**2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3)
, True)) - a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2
)/(3*a**2), True)) + c**2*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) -
I*x/(2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*s
qrt(-a**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

3.1195 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=55

$$\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

```
[Out] 1/3*c*(-a^2*x^2+1)^(3/2)/a+1/2*c*arcsin(a*x)/a+1/2*c*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6139, 641, 195, 216}

$$\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (c*x*Sqrt[1 - a^2*x^2])/2 + (c*(1 - a^2*x^2)^(3/2))/(3*a) + (c*ArcSin[a*x])
/(2*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx &= c\int(1-ax)\sqrt{1-a^2x^2}\,dx\\ &= \frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+c\int\sqrt{1-a^2x^2}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{1}{2}c\int\frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{1}{2}cx\sqrt{1-a^2x^2}+\frac{c\left(1-a^2x^2\right)^{3/2}}{3a}+\frac{c\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.09, size = 57, normalized size = 1.04

$$\frac{c\left(\left(-2a^2x^2+3ax+2\right)\sqrt{1-a^2x^2}-6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^ArcTanh[a*x],x]
```

```
[Out] (c*((2 + 3*a*x - 2*a^2*x^2)*Sqrt[1 - a^2*x^2] - 6*ArcSin[Sqrt[1 - a*x]/Sqrt
[2]]))/(6*a)
```

fricas [A] time = 0.79, size = 62, normalized size = 1.13

$$-\frac{6\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(2\,a^2cx^2-3\,acx-2\,c\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (2*a^2*c*x^2 - 3*a*c*x -
 2*c)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.67, size = 46, normalized size = 0.84

$$\frac{c \arcsin(ax)\operatorname{sgn}(a)}{2|a|} - \frac{1}{6}\sqrt{-a^2x^2+1}\left((2acx-3c)x - \frac{2c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*arcsin(a*x)*sgn(a)/abs(a) - 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c*x - 3*c)*x
 - 2*c/a)
```

maple [A] time = 0.03, size = 64, normalized size = 1.16

$$\frac{c\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3a} + \frac{cx\sqrt{-a^2x^2+1}}{2} + \frac{c\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/3*c*(-a^2*x^2+1)^(3/2)/a+1/2*c*x*(-a^2*x^2+1)^(1/2)+1/2*c/(a^2)^(1/2)*arc
tan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.41, size = 45, normalized size = 0.82

$$\frac{1}{2}\sqrt{-a^2x^2+1}\,cx + \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{3a} + \frac{c\arcsin(ax)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*c*x + 1/3*(-a^2*x^2 + 1)^(3/2)*c/a + 1/2*c*arcsin(a*
x)/a
```

mupad [B] time = 0.91, size = 80, normalized size = 1.45

$$\frac{\sqrt{1-a^2x^2}\left(\frac{cx\sqrt{-a^2}}{2} - \frac{ac}{3\sqrt{-a^2}} + \frac{a^3cx^2}{3\sqrt{-a^2}}\right)}{\sqrt{-a^2}} + \frac{c\,\mathrm{asinh}\left(x\sqrt{-a^2}\right)}{2\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((c*x*(-a^2)^(1/2))/2 - (a*c)/(3*(-a^2)^(1/2)) + (a^3*
c*x^2)/(3*(-a^2)^(1/2))))/(-a^2)^(1/2) + (c*asinh(x*(-a^2)^(1/2)))/(2*(-a^2
)^(1/2))
```

sympy [C] time = 4.44, size = 109, normalized size = 1.98

$$-ac\left(\begin{cases} \frac{x^2}{2} & \text{for } a^2 = 0 \\ -\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3a^2} & \text{otherwise} \end{cases}\right) + c\left(\begin{cases} \frac{ia^2x^3}{2\sqrt{a^2x^2-1}} - \frac{ix}{2\sqrt{a^2x^2-1}} - \frac{i\operatorname{acosh}(ax)}{2a} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{x\sqrt{-a^2x^2+1}}{2} + \frac{\operatorname{asin}(ax)}{2a} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -a*c*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a**2), T
rue)) + c*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2*sqrt(a**2
*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a**2*x**2 +
 1)/2 + asin(a*x)/(2*a), True))
```

3.1196 $\int \frac{e^{-\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=16

$$-\frac{e^{-\tanh^{-1}(ax)}}{ac}$$

```
[Out] -1/a/c/(a*x+1)*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6137}

$$-\frac{e^{-\tanh^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)),x]
```

```
[Out] -(1/(a*c*E^ArcTanh[a*x]))
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-\tanh^{-1}(ax)}}{ac}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.69

$$-\frac{\sqrt{1-ax}}{ac\sqrt{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)),x]
```

```
[Out] -(Sqrt[1 - a*x]/(a*c*Sqrt[1 + a*x]))
```

fricas [A] time = 0.59, size = 31, normalized size = 1.94

$$-\frac{ax+\sqrt{-a^2x^2+1}+1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas"
)
```

```
[Out] -(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*c*x + a*c)
```

giac [A] time = 0.22, size = 37, normalized size = 2.31

$$\frac{2}{c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/(c*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

maple [A] time = 0.03, size = 28, normalized size = 1.75

$$-\frac{\sqrt{-a^2x^2+1}}{ac\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c/(a*x+1)*(-a^2*x^2+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima"
)
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)*(a*x + 1)), x)
```

mupad [B] time = 0.05, size = 46, normalized size = 2.88

$$\frac{\sqrt{1-a^2\,x^2}}{c\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)*(a*x + 1)),x)
[Out] (1 - a^2*x^2)^(1/2)/(c*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c),x)
[Out] Integral(1/(a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c
```

3.1197 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=53

$$\frac{2x}{3c^2\sqrt{1-a^2x^2}}-\frac{1-ax}{3ac^2\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] 1/3*(a*x-1)/a/c^2/(-a^2*x^2+1)^(3/2)+2/3*x/c^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6139, 639, 191}

$$\frac{2x}{3c^2\sqrt{1-a^2x^2}}-\frac{1-ax}{3ac^2\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(1 - a*x)/(3*a*c^2*(1 - a^2*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{1-ax}{(1-a^2x^2)^{5/2}}\,dx}{c^2}$$
$$= -\frac{1-ax}{3ac^2(1-a^2x^2)^{3/2}} + \frac{2\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{3c^2}$$
$$= -\frac{1-ax}{3ac^2(1-a^2x^2)^{3/2}} + \frac{2x}{3c^2\sqrt{1-a^2x^2}}$$

Mathematica [**A**] time = 0.02, size = 43, normalized size = 0.81

$$\frac{2a^2x^2+2ax-1}{3ac^2\sqrt{1-ax}\,(ax+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-1 + 2*a*x + 2*a^2*x^2)/(3*a*c^2*Sqrt[1 - a*x]*(1 + a*x)^(3/2))
```

fricas [**A**] time = 0.60, size = 87, normalized size = 1.64

$$-\frac{a^3x^3+a^2x^2-ax+\left(2\,a^2x^2+2\,ax-1\right)\sqrt{-a^2x^2+1}-1}{3\left(a^4c^2x^3+a^3c^2x^2-a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="frica
s")
```

```
[Out] -1/3*(a^3*x^3 + a^2*x^2 - a*x + (2*a^2*x^2 + 2*a*x - 1)*sqrt(-a^2*x^2 + 1)
- 1)/(a^4*c^2*x^3 + a^3*c^2*x^2 - a^2*c^2*x - a*c^2)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^2*(a*x + 1)), x)
```

maple [A] time = 0.03, size = 42, normalized size = 0.79

$$\frac{2a^{2}x^{2}+2ax-1}{3\sqrt{-a^{2}x^{2}+1}\,\left(ax+1\right)a\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/3/(-a^2*x^2+1)^(1/2)*(2*a^2*x^2+2*a*x-1)/(a*x+1)/a/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}x^{2}+1}}{\left(a^{2}cx^{2}-c\right)^{2}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^2*(a*x + 1)), x)
```

mupad [B] time = 1.00, size = 48, normalized size = 0.91

$$-\frac{\sqrt{1-a^{2}\,x^{2}}\,\left(2\,a^{2}\,x^{2}+2\,a\,x-1\right)}{3\,a\,c^{2}\,\left(a\,x-1\right)\,{\left(a\,x+1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^2*(a*x + 1)),x)
```

```
[Out] -((1 - a^2*x^2)^(1/2)*(2*a*x + 2*a^2*x^2 - 1))/(3*a*c^2*(a*x - 1)*(a*x + 1)
^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{-a^{3}x^{3}\sqrt{-a^{2}x^{2}+1}-a^{2}x^{2}\sqrt{-a^{2}x^{2}+1}+ax\sqrt{-a^{2}x^{2}+1}+\sqrt{-a^{2}x^{2}+1}}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(1/(-a**3*x**3*sqrt(-a**2*x**2 + 1) - a**2*x**2*sqrt(-a**2*x**2 + 1
) + a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**2
```

3.1198 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=75

$$\frac{8x}{15c^3\sqrt{1-a^2x^2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{1-ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] 1/5*(a*x-1)/a/c^3/(-a^2*x^2+1)^(5/2)+4/15*x/c^3/(-a^2*x^2+1)^(3/2)+8/15*x/c
^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 639, 192, 191}

$$\frac{8x}{15c^3\sqrt{1-a^2x^2}}+\frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}}-\frac{1-ax}{5ac^3\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -(1 - a*x)/(5*a*c^3*(1 - a^2*x^2)^(5/2)) + (4*x)/(15*c^3*(1 - a^2*x^2)^(3/2
)) + (8*x)/(15*c^3*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3}$$

$$= -\frac{1-ax}{5ac^3\left(1-a^2x^2\right)^{5/2}} + \frac{4\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx}{5c^3}$$

$$= -\frac{1-ax}{5ac^3\left(1-a^2x^2\right)^{5/2}} + \frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{8\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{15c^3}$$

$$= -\frac{1-ax}{5ac^3\left(1-a^2x^2\right)^{5/2}} + \frac{4x}{15c^3\left(1-a^2x^2\right)^{3/2}} + \frac{8x}{15c^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.79

$$-\frac{8a^4x^4+8a^3x^3-12a^2x^2-12ax+3}{15ac^3(1-ax)^{3/2}(ax+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/15*(3 - 12*a*x - 12*a^2*x^2 + 8*a^3*x^3 + 8*a^4*x^4)/(a*c^3*(1 - a*x)^(3
/2)*(1 + a*x)^(5/2))
```

fricas [B] time = 0.84, size = 141, normalized size = 1.88

$$-\frac{3\,a^5x^5+3\,a^4x^4-6\,a^3x^3-6\,a^2x^2+3\,ax+\left(8\,a^4x^4+8\,a^3x^3-12\,a^2x^2-12\,ax+3\right)\sqrt{-a^2x^2+1}+3}{15\left(a^6c^3x^5+a^5c^3x^4-2\,a^4c^3x^3-2\,a^3c^3x^2+a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="frica
s")
```

```
[Out] -1/15*(3*a^5*x^5 + 3*a^4*x^4 - 6*a^3*x^3 - 6*a^2*x^2 + 3*a*x + (8*a^4*x^4 +
 8*a^3*x^3 - 12*a^2*x^2 - 12*a*x + 3)*sqrt(-a^2*x^2 + 1) + 3)/(a^6*c^3*x^5
+ a^5*c^3*x^4 - 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 + a^2*c^3*x + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^3(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^3*(a*x + 1)), x)
```

maple [A] time = 0.03, size = 58, normalized size = 0.77

$$-\frac{8x^4a^4+8x^3a^3-12a^2x^2-12ax+3}{15\left(-a^2x^2+1\right)^{\frac{3}{2}}(ax+1)\,c^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15/(-a^2*x^2+1)^(3/2)*(8*a^4*x^4+8*a^3*x^3-12*a^2*x^2-12*a*x+3)/(a*x+1)/
c^3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^3(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^3*(a*x + 1)), x)
```

mupad [B] time = 1.08, size = 64, normalized size = 0.85

$$-\frac{\sqrt{1-a^2\,x^2}\,\left(8\,a^4\,x^4+8\,a^3\,x^3-12\,a^2\,x^2-12\,a\,x+3\right)}{15\,a\,c^3\,{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^3*(a*x + 1)),x)
```

```
[Out] -((1 - a^2*x^2)^(1/2)*(8*a^3*x^3 - 12*a^2*x^2 - 12*a*x + 8*a^4*x^4 + 3))/(1
5*a*c^3*(a*x - 1)^2*(a*x + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^5x^5\sqrt{-a^2x^2+1}+a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}-2a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(1/(a**5*x**5*sqrt(-a**2*x**2 + 1) + a**4*x**4*sqrt(-a**2*x**2 + 1)
 - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) - 2*a**2*x**2*sqrt(-a**2*x**2 + 1) + a*
x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**3
```

3.1199 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=97

$$\frac{16x}{35c^4\sqrt{1-a^2x^2}}+\frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}}+\frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}}-\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}}$$

```
[Out] 1/7*(a*x-1)/a/c^4/(-a^2*x^2+1)^(7/2)+6/35*x/c^4/(-a^2*x^2+1)^(5/2)+8/35*x/c
^4/(-a^2*x^2+1)^(3/2)+16/35*x/c^4/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 639, 192, 191}

$$\frac{16x}{35c^4\sqrt{1-a^2x^2}}+\frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}}+\frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}}-\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] -(1 - a*x)/(7*a*c^4*(1 - a^2*x^2)^(7/2)) + (6*x)/(35*c^4*(1 - a^2*x^2)^(5/2
)) + (8*x)/(35*c^4*(1 - a^2*x^2)^(3/2)) + (16*x)/(35*c^4*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{9/2}}\,dx}{c^4}$$

$$= -\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6\int \frac{1}{\left(1-a^2x^2\right)^{7/2}}\,dx}{7c^4}$$

$$= -\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{24\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx}{35c^4}$$

$$= -\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}} + \frac{16\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{35c^4}$$

$$= -\frac{1-ax}{7ac^4\left(1-a^2x^2\right)^{7/2}} + \frac{6x}{35c^4\left(1-a^2x^2\right)^{5/2}} + \frac{8x}{35c^4\left(1-a^2x^2\right)^{3/2}} + \frac{16x}{35c^4\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 0.77

$$\frac{16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5}{35ac^4(1-ax)^{5/2}(ax+1)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] (-5 + 30*a*x + 30*a^2*x^2 - 40*a^3*x^3 - 40*a^4*x^4 + 16*a^5*x^5 + 16*a^6*x
^6)/(35*a*c^4*(1 - a*x)^(5/2)*(1 + a*x)^(7/2))
```

fricas [B] time = 0.59, size = 197, normalized size = 2.03

$$-\frac{5\,a^7x^7+5\,a^6x^6-15\,a^5x^5-15\,a^4x^4+15\,a^3x^3+15\,a^2x^2-5\,ax+\left(16\,a^6x^6+16\,a^5x^5-40\,a^4x^4-40\,a^3x^3+30\,a^2x\right.}{35\left(a^8c^4x^7+a^7c^4x^6-3\,a^6c^4x^5-3\,a^5c^4x^4+3\,a^4c^4x^3+3\,a^3c^4x^2-a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="frica
s")
```

```
[Out] -1/35*(5*a^7*x^7 + 5*a^6*x^6 - 15*a^5*x^5 - 15*a^4*x^4 + 15*a^3*x^3 + 15*a^
2*x^2 - 5*a*x + (16*a^6*x^6 + 16*a^5*x^5 - 40*a^4*x^4 - 40*a^3*x^3 + 30*a^2
*x^2 + 30*a*x - 5)*sqrt(-a^2*x^2 + 1) - 5)/(a^8*c^4*x^7 + a^7*c^4*x^6 - 3*a
^6*c^4*x^5 - 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 + 3*a^3*c^4*x^2 - a^2*c^4*x - a*
c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^4(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^4*(a*x + 1)), x)
```

maple [A] time = 0.03, size = 74, normalized size = 0.76

$$\frac{16x^6a^6+16x^5a^5-40x^4a^4-40x^3a^3+30a^2x^2+30ax-5}{35\left(-a^2x^2+1\right)^{\frac{5}{2}}(ax+1)\,c^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/35/(-a^2*x^2+1)^(5/2)*(16*a^6*x^6+16*a^5*x^5-40*a^4*x^4-40*a^3*x^3+30*a^2
*x^2+30*a*x-5)/(a*x+1)/c^4/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^4(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^4*(a*x + 1)), x)
```

mupad [B] time = 1.18, size = 145, normalized size = 1.49

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{8\,x}{35\,c^4}+\frac{1}{56\,a\,c^4}\right)}{{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^2}-\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{17\,x}{70\,c^4}-\frac{1}{7\,a\,c^4}\right)}{{\left(a\,x-1\right)}^3\,{\left(a\,x+1\right)}^3}-\frac{\sqrt{1-a^2\,x^2}}{56\,a\,c^4\,{\left(a\,x+1\right)}^4}-\frac{16\,x\,\sqrt{1-a^2\,x^2}}{35\,c^4\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^4*(a*x + 1)),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((8*x)/(35*c^4) + 1/(56*a*c^4)))/((a*x - 1)^2*(a*x + 1
)^2) - ((1 - a^2*x^2)^(1/2)*((17*x)/(70*c^4) - 1/(7*a*c^4)))/((a*x - 1)^3*(
a*x + 1)^3) - (1 - a^2*x^2)^(1/2)/(56*a*c^4*(a*x + 1)^4) - (16*x*(1 - a^2*x
^2)^(1/2))/(35*c^4*(a*x - 1)*(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{-a^7x^7\sqrt{-a^2x^2+1}-a^6x^6\sqrt{-a^2x^2+1}+3a^5x^5\sqrt{-a^2x^2+1}+3a^4x^4\sqrt{-a^2x^2+1}-3a^3x^3\sqrt{-a^2x^2+1}-3a^2x^2\sqrt{-a^2x^2+1}+ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Integral(1/(-a**7*x**7*sqrt(-a**2*x**2 + 1) - a**6*x**6*sqrt(-a**2*x**2 + 1
) + 3*a**5*x**5*sqrt(-a**2*x**2 + 1) + 3*a**4*x**4*sqrt(-a**2*x**2 + 1) - 3
*a**3*x**3*sqrt(-a**2*x**2 + 1) - 3*a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sq
rt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**4
```

3.1200 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^5} dx$

Optimal. Leaf size=119

$$\frac{128x}{315c^5\sqrt{1-a^2x^2}}+\frac{64x}{315c^5(1-a^2x^2)^{3/2}}+\frac{16x}{105c^5(1-a^2x^2)^{5/2}}+\frac{8x}{63c^5(1-a^2x^2)^{7/2}}-\frac{1-ax}{9ac^5(1-a^2x^2)^{9/2}}$$

```
[Out] 1/9*(a*x-1)/a/c^5/(-a^2*x^2+1)^(9/2)+8/63*x/c^5/(-a^2*x^2+1)^(7/2)+16/105*x
/c^5/(-a^2*x^2+1)^(5/2)+64/315*x/c^5/(-a^2*x^2+1)^(3/2)+128/315*x/c^5/(-a^2
*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 639, 192, 191}

$$\frac{128x}{315c^5\sqrt{1-a^2x^2}}+\frac{64x}{315c^5(1-a^2x^2)^{3/2}}+\frac{16x}{105c^5(1-a^2x^2)^{5/2}}+\frac{8x}{63c^5(1-a^2x^2)^{7/2}}-\frac{1-ax}{9ac^5(1-a^2x^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^5),x]
```

```
[Out] -(1 - a*x)/(9*a*c^5*(1 - a^2*x^2)^(9/2)) + (8*x)/(63*c^5*(1 - a^2*x^2)^(7/2
)) + (16*x)/(105*c^5*(1 - a^2*x^2)^(5/2)) + (64*x)/(315*c^5*(1 - a^2*x^2)^(
3/2)) + (128*x)/(315*c^5*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
```

```
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^5}\,dx = \frac{\int \frac{1-ax}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^5}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8\int \frac{1}{\left(1-a^2x^2\right)^{9/2}}\,dx}{9c^5}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16\int \frac{1}{\left(1-a^2x^2\right)^{7/2}}\,dx}{21c^5}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx}{105c^5}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{128}{}$$

$$= -\frac{1-ax}{9ac^5\left(1-a^2x^2\right)^{9/2}} + \frac{8x}{63c^5\left(1-a^2x^2\right)^{7/2}} + \frac{16x}{105c^5\left(1-a^2x^2\right)^{5/2}} + \frac{64x}{315c^5\left(1-a^2x^2\right)^{3/2}} + \frac{}{315c}$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.76

$$-\frac{128a^8x^8+128a^7x^7-448a^6x^6-448a^5x^5+560a^4x^4+560a^3x^3-280a^2x^2-280ax+35}{315ac^5(1-ax)^{7/2}(ax+1)^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^5),x]
```

```
[Out] -1/315*(35 - 280*a*x - 280*a^2*x^2 + 560*a^3*x^3 + 560*a^4*x^4 - 448*a^5*x^
5 - 448*a^6*x^6 + 128*a^7*x^7 + 128*a^8*x^8)/(a*c^5*(1 - a*x)^(7/2)*(1 + a*
x)^(9/2))
```

fricas [B] time = 0.75, size = 249, normalized size = 2.09

$$-\frac{35\,a^{9}x^{9}+35\,a^{8}x^{8}-140\,a^{7}x^{7}-140\,a^{6}x^{6}+210\,a^{5}x^{5}+210\,a^{4}x^{4}-140\,a^{3}x^{3}-140\,a^{2}x^{2}+35\,ax+\left(128\,a^{8}x^{8}+12\right.}{315\left(a^{10}c^{5}x^{9}+a^{9}c^{5}x^{8}-4\,a^{8}c^{5}x^{7}-4\,a^{7}c^{5}x^{6}+6\,a^{6}c^{5}x^{5}+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="frica
s")
```

```
[Out] -1/315*(35*a^9*x^9 + 35*a^8*x^8 - 140*a^7*x^7 - 140*a^6*x^6 + 210*a^5*x^5 +
 210*a^4*x^4 - 140*a^3*x^3 - 140*a^2*x^2 + 35*a*x + (128*a^8*x^8 + 128*a^7*
x^7 - 448*a^6*x^6 - 448*a^5*x^5 + 560*a^4*x^4 + 560*a^3*x^3 - 280*a^2*x^2 -
 280*a*x + 35)*sqrt(-a^2*x^2 + 1) + 35)/(a^10*c^5*x^9 + a^9*c^5*x^8 - 4*a^8
*c^5*x^7 - 4*a^7*c^5*x^6 + 6*a^6*c^5*x^5 + 6*a^5*c^5*x^4 - 4*a^4*c^5*x^3 -
4*a^3*c^5*x^2 + a^2*c^5*x + a*c^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{-a^{2}x^{2}+1}}{\left(a^{2}cx^{2}-c\right)^{5}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^5*(a*x + 1)), x)
```

maple [A] time = 0.03, size = 90, normalized size = 0.76

$$-\frac{128x^{8}a^{8}+128a^{7}x^{7}-448x^{6}a^{6}-448x^{5}a^{5}+560x^{4}a^{4}+560x^{3}a^{3}-280a^{2}x^{2}-280ax+35}{315\left(-a^{2}x^{2}+1\right)^{\frac{7}{2}}(ax+1)\,c^{5}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x)
```

```
[Out] -1/315/(-a^2*x^2+1)^(7/2)*(128*a^8*x^8+128*a^7*x^7-448*a^6*x^6-448*a^5*x^5+
560*a^4*x^4+560*a^3*x^3-280*a^2*x^2-280*a*x+35)/(a*x+1)/c^5/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2x^2+1}}{\left(a^2cx^2-c\right)^5(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^5,x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-a^2*x^2 + 1)/((a^2*c*x^2 - c)^5*(a*x + 1)), x)
```

mupad [B] time = 1.33, size = 177, normalized size = 1.49

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{53\,x}{252\,c^5}-\frac{5}{36\,a\,c^5}\right)}{{\left(a\,x-1\right)}^4\,{\left(a\,x+1\right)}^4}-\frac{\sqrt{1-a^2\,x^2}}{144\,a\,c^5\,{\left(a\,x+1\right)}^5}-\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{733\,x}{5040\,c^5}+\frac{5}{144\,a\,c^5}\right)}{{\left(a\,x-1\right)}^3\,{\left(a\,x+1\right)}^3}-\frac{128\,x\,\sqrt{1-a^2\,x^2}}{315\,c^5\,\left(a\,x-1\right)\,\left(a\,x+1\right)}+\frac{64}{315\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^5*(a*x + 1)),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((53*x)/(252*c^5) - 5/(36*a*c^5)))/((a*x - 1)^4*(a*x +
 1)^4) - (1 - a^2*x^2)^(1/2)/(144*a*c^5*(a*x + 1)^5) - ((1 - a^2*x^2)^(1/2)
*((733*x)/(5040*c^5) + 5/(144*a*c^5)))/((a*x - 1)^3*(a*x + 1)^3) - (128*x*(
1 - a^2*x^2)^(1/2))/(315*c^5*(a*x - 1)*(a*x + 1)) + (64*x*(1 - a^2*x^2)^(1/
2))/(315*c^5*(a*x - 1)^2*(a*x + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^9x^9\sqrt{-a^2x^2+1}+a^8x^8\sqrt{-a^2x^2+1}-4a^7x^7\sqrt{-a^2x^2+1}-4a^6x^6\sqrt{-a^2x^2+1}+6a^5x^5\sqrt{-a^2x^2+1}+6a^4x^4\sqrt{-a^2x^2+1}-4a^3x^3\sqrt{-a^2x^2+1}-4a^2x^2\sqrt{-a^2x^2+1}+a}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**5,x)
```

```
[Out] Integral(1/(a**9*x**9*sqrt(-a**2*x**2 + 1) + a**8*x**8*sqrt(-a**2*x**2 + 1)
 - 4*a**7*x**7*sqrt(-a**2*x**2 + 1) - 4*a**6*x**6*sqrt(-a**2*x**2 + 1) + 6*
a**5*x**5*sqrt(-a**2*x**2 + 1) + 6*a**4*x**4*sqrt(-a**2*x**2 + 1) - 4*a**3*
x**3*sqrt(-a**2*x**2 + 1) - 4*a**2*x**2*sqrt(-a**2*x**2 + 1) + a*x*sqrt(-a*
*2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**5
```

3.1201 $\int e^{-\tanh^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\, dx$

Optimal. Leaf size=83

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(1+m)/(-a^2*x^2+1)^(1/2)-a*x^(2+m)*(-a^2*c*x^2
+c)^(1/2)/(2+m)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 43}

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}-\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) - (a*x^(2 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\int e^{-\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\sqrt{c-a^2cx^2}\int x^m(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\sqrt{c-a^2cx^2}\int \left(x^m-ax^{1+m}\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}-\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.60

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}\left(\frac{1}{m+1}-\frac{ax}{m+2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] (x^(1 + m)*((1 + m)^(-1) - (a*x)/(2 + m))*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2
*x^2]
```

fricas [A] time = 0.71, size = 80, normalized size = 0.96

$$\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\left((am+a)x^2-(m+2)x\right)x^m}{\left(a^2m^2+3\,a^2m+2\,a^2\right)x^2-m^2-3\,m-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"fricas")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((a*m + a)*x^2 - (m + 2)*x)*x^m/((a
^2*m^2 + 3*a^2*m + 2*a^2)*x^2 - m^2 - 3*m - 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\,\sqrt{-a^2x^2 + 1}\,x^m}{ax + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^m/(a*x + 1), x)
```

maple [A] time = 0.03, size = 68, normalized size = 0.82

$$\frac{x^{1+m} (amx + ax - m - 2)\sqrt{-a^2c\,x^2 + c}\,\sqrt{-a^2x^2 + 1}}{(2 + m)(1 + m)(ax - 1)(ax + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+a*x-m-2)*(-a^2*c*x^2+c)^(1/2)*(-a^2*x^2+1)^(1/2)/(2+m)/(1+m)
/(a*x-1)/(a*x+1)
```

maxima [A] time = 0.37, size = 63, normalized size = 0.76

$$-\frac{\left(a\sqrt{c}\,(m + 1)x^2 - \sqrt{c}\,(m + 2)x\right)(ax + 1)(ax - 1)x^m}{\left(m^2 + 3\,m + 2\right)a^2x^2 - m^2 - 3\,m - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -(a*sqrt(c)*(m + 1)*x^2 - sqrt(c)*(m + 2)*x)*(a*x + 1)*(a*x - 1)*x^m/((m^2
+ 3*m + 2)*a^2*x^2 - m^2 - 3*m - 2)
```

mupad [B] time = 1.23, size = 52, normalized size = 0.63

$$\frac{x\,x^m\,\sqrt{c - a^2\,c\,x^2}\,\,(m - a\,x - a\,m\,x + 2)}{\sqrt{1 - a^2\,x^2}\,\,\left(m^2 + 3\,m + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] (x*x^m*(c - a^2*c*x^2)^(1/2)*(m - a*x - a*m*x + 2))/((1 - a^2*x^2)^(1/2)*(3
*m + m^2 + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-(ax-1)(ax+1)}\,\sqrt{-c(ax-1)(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x
+ 1), x)
```

3.1202 $\int e^{-\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}$$

```
[Out] 1/3*x^3*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-1/4*a*x^4*(-a^2*c*x^2+c)^(1
/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 43}

$$\frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] (x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2]) - (a*x^4*Sqrt[c - a^2*c*x^2
])/(4*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int x^2(1-ax)\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(x^2-ax^3\right)\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{x^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.57

$$
-\frac{x^3(3ax-4)\sqrt{c-a^2cx^2}}{12\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] -1/12*(x^3*(-4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.50, size = 50, normalized size = 0.68

$$
\frac{\sqrt{-a^2cx^2+c}\left(3\,ax^4-4\,x^3\right)\sqrt{-a^2x^2+1}}{12\left(a^2x^2-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 1/12*sqrt(-a^2*c*x^2 + c)*(3*a*x^4 - 4*x^3)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^2}{ax+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2/(a*x + 1), x)
```

maple [A] time = 0.03, size = 51, normalized size = 0.69

$$\frac{x^{3}\left(3ax-4\right)\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{-a^{2}x^{2}+1}}{12\left(ax-1\right)\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/12*x^3*(3*a*x-4)*(-a^2*c*x^2+c)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(a*x+1)
```

maxima [A] time = 0.35, size = 41, normalized size = 0.55

$$-\frac{\left(3\,a\sqrt{c}\,x^{4}-4\,\sqrt{c}\,x^{3}\right)(ax+1)(ax-1)}{12\left(a^{2}x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -1/12*(3*a*sqrt(c)*x^4 - 4*sqrt(c)*x^3)*(a*x + 1)*(a*x - 1)/(a^2*x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}\,\sqrt{c-a^{2}\,c\,x^{2}}\,\sqrt{1-a^{2}\,x^{2}}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x
+ 1), x)
```

3.1203 $\int e^{-\tanh^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

```
[Out] 1/2*x^2*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-1/3*a*x^3*(-a^2*c*x^2+c)^(1
/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 43}

$$\frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(2*Sqrt[1 - a^2*x^2]) - (a*x^3*Sqrt[c - a^2*c*x^2
])/(3*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(x-ax^2\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}} - \frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.57

$$-\frac{x^2(2ax-3)\sqrt{c-a^2cx^2}}{6\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^ArcTanh[a*x],x]
```

```
[Out] -1/6*(x^2*(-3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.74, size = 50, normalized size = 0.68

$$\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\left(2\,ax^3-3\,x^2\right)}{6\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(2*a*x^3 - 3*x^2)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x/(a*x + 1), x)
```

maple [A] time = 0.03, size = 51, normalized size = 0.69

$$\frac{x^{2} \left(2 a x -3\right) \sqrt{-a^{2} c \,x^{2}+c}\, \sqrt{-a^{2} x^{2}+1}}{6 \left(a x -1\right) \left(a x +1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/6*x^2*(2*a*x-3)*(-a^2*c*x^2+c)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(a*x+1)
```

maxima [A] time = 0.34, size = 41, normalized size = 0.55

$$-\frac{\left(2 \, a \sqrt{c} x^{3} - 3 \, \sqrt{c} x^{2}\right)(a x + 1)(a x - 1)}{6 \, \left(a^{2} x^{2} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/6*(2*a*sqrt(c)*x^3 - 3*sqrt(c)*x^2)*(a*x + 1)*(a*x - 1)/(a^2*x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{- \left(a x - 1\right) \left(a x + 1\right)} \sqrt{- c \left(a x - 1\right) \left(a x + 1\right)}}{a x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x + 1
), x)
```

3.1204 $\int e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

```
[Out] x*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-1/2*a*x^2*(-a^2*c*x^2+c)^(1/2)/(-
a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6143, 6140}

$$\frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcTanh[a*x],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] - (a*x^2*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - a^2*x^2])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int(1-ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}} - \frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.58

$$\frac{\left(x-\frac{ax^2}{2}\right)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcTanh[a*x],x]
```

```
[Out] ((x - (a*x^2)/2)*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.59, size = 47, normalized size = 0.68

$$\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\left(ax^2-2\,x\right)}{2\,\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x^2 - 2*x)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

maple [A] time = 0.02, size = 48, normalized size = 0.70

$$\frac{x\,(ax-2)\,\sqrt{-a^{2}c\,x^{2}+c}\;\sqrt{-a^{2}x^{2}+1}}{2\,(ax-1)\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(a*x+1)
```

maxima [A] time = 0.34, size = 48, normalized size = 0.70

$$-\frac{\left(a^{2}\sqrt{c}\,x^{2}-2\,a\sqrt{c}\,x+2\,\sqrt{c}\right)(ax+1)(ax-1)}{2\left(a^{3}x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/2*(a^2*sqrt(c)*x^2 - 2*a*sqrt(c)*x + 2*sqrt(c))*(a*x + 1)*(a*x - 1)/(a^3
*x^2 - a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^{2}\,c\,x^{2}}\;\sqrt{1-a^{2}\,x^{2}}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)\,(ax+1)}\,\sqrt{-c\,(ax-1)\,(ax+1)}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x + 1),
 x)
```

3.1205 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=66

$$\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

```
[Out] -a*x*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a
^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 43}

$$\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -((a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2]) + (Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{1-ax}{x}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-a+\frac{1}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.56

$$\frac{\sqrt{c-a^2cx^2}\,(\log(x)-ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcTanh[a*x]*x),x]
[Out] (Sqrt[c - a^2*c*x^2]*(-(a*x) + Log[x]))/Sqrt[1 - a^2*x^2]
```

fricas [B] time = 0.77, size = 261, normalized size = 3.95

$$\left[\frac{\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^2cx^6+a^2cx^2-cx^4-\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\left(x^4-1\right)\sqrt{c}-c}{a^2x^4-x^2}\right)+2\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(ax-a)}{2\left(a^2x^2-1\right)}, \frac{\left(a^2x^2-1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="f
ricas")
[Out] [1/2*((a^2*x^2 - 1)*sqrt(c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 - sqrt(-a^2*
c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^4 - 1)*sqrt(c) - c)/(a^2*x^4 - x^2)) + 2*s
qrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - a))/(a^2*x^2 - 1), ((a^2*x^2
```

```
- 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^2 + 1)*sqrt
(-c)/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1)*(a*x - a))/(a^2*x^2 - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/((a*x + 1)*x), x)
```

maple [A] time = 0.03, size = 47, normalized size = 0.71

$$\frac{(ax-\ln(x))\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (a*x-ln(x))*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/((a*x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\sqrt{-c(ax-1)(ax+1)}}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(x*(a*x +
1)), x)
```

3.1206 $\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/x/(-a^2*x^2+1)^(1/2)-a*ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a
^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 43}

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) - (a*Sqrt[c - a^2*c*x^2]*Log[x
])/Sqrt[1 - a^2*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
```

```
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{1-ax}{x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{a}{x}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}} - \frac{a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.59

$$\frac{\sqrt{c-a^2cx^2}\left(-a\log(x)-\frac{1}{x}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcTanh[a*x]*x^2),x]
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) - a*Log[x]))/Sqrt[1 - a^2*x^2]
```

fricas [B] time = 0.79, size = 263, normalized size = 3.81

$$\left[\frac{\left(a^3x^3-ax\right)\sqrt{c}\,\log\left(\frac{a^2cx^6+a^2cx^2-cx^4+\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\left(x^4-1\right)\sqrt{c}-c}{a^2x^4-x^2}\right)-2\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(x-1)}{2\left(a^2x^3-x\right)}, -\frac{\left(a^3x^3-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm=
"fricas")
[Out] [1/2*((a^3*x^3 - a*x)*sqrt(c)*log((a^2*c*x^6 + a^2*c*x^2 - c*x^4 + sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^4 - 1)*sqrt(c) - c)/(a^2*x^4 - x^2)) - 2
```

```
*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x - 1))/(a^2*x^3 - x), -((a^3*x^3
 - a*x)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(x^2 + 1)*s
qrt(-c)/(a^2*c*x^4 - (a^2 + 1)*c*x^2 + c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2
*x^2 + 1)*(x - 1))/(a^2*x^3 - x)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.04, size = 49, normalized size = 0.71

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-a^2x^2+1}\,\left(a\ln(x)x+1\right)}{\left(a^2x^2-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] (-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(a*ln(x)*x+1)/(a^2*x^2-1)/x
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/((a*x + 1)*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}{x^2\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(x^2*(a*x + 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(x^2*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\sqrt{-c(ax-1)(ax+1)}}{x^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*(a*x
 + 1)), x)
```

3.1207 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}-\frac{2c(1-ax)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] -2/3*c*(-a*x+1)^3*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+1/4*c*(-a*x+1)^
4*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}-\frac{2c(1-ax)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-2*c*(1 - a*x)^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (c*(1 - a*
x)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(1-ax)^2(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int\left(2(1-ax)^2-(1-ax)^3\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{2c(1-ax)^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.63

$$\frac{cx\left(3a^3x^3-4a^2x^2-6ax+12\right)\sqrt{c-a^2cx^2}}{12\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^ArcTanh[a*x],x]
```

```
[Out] (c*x*Sqrt[c - a^2*c*x^2]*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))/(12*Sqrt[1 -
 a^2*x^2])
```

fricas [A] time = 0.58, size = 68, normalized size = 0.75

$$-\frac{\left(3\,a^3cx^4-4\,a^2cx^3-6\,acx^2+12\,cx\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{12\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/12*(3*a^3*c*x^4 - 4*a^2*c*x^3 - 6*a*c*x^2 + 12*c*x)*sqrt(-a^2*c*x^2 + c)
*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

maple [A] time = 0.03, size = 65, normalized size = 0.71

$$\frac{x\left(3x^{3}a^{3}-4a^{2}x^{2}-6ax+12\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{-a^{2}x^{2}+1}}{12\left(ax+1\right)^{2}\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/12*x*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*(-a^2*c*x^2+c)^(3/2)*(-a^2*x^2+1)^(1/
2)/(a*x+1)^2/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\sqrt{-a^{2}x^{2}+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c-a^{2}\,c\,x^{2}\right)}^{3/2}\,\sqrt{1-a^{2}\,x^{2}}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)/(a*x +
1), x)
```

3.1208 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}+\frac{4c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(1-ax)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}$$

```
[Out] -c^2*(-a*x+1)^4*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+4/5*c^2*(-a*x+1)^
5*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-1/6*c^2*(-a*x+1)^6*(-a^2*c*x^2+
c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}+\frac{4c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(1-ax)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] -((c^2*(1 - a*x)^4*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2])) + (4*c^2*(1
- a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) - (c^2*(1 - a*x)^6*Sq
rt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)^3(1+ax)^2\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int\left(4(1-ax)^3-4(1-ax)^4+(1-ax)^5\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{c^2(1-ax)^4\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{4c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}-\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.43

$$-\frac{c^2(ax-1)^4\left(5a^2x^2+14ax+11\right)\sqrt{c-a^2cx^2}}{30a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^ArcTanh[a*x],x]
```

```
[Out] -1/30*(c^2*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a*S
qrt[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 98, normalized size = 0.70

$$\frac{\left(5\,a^5c^2x^6-6\,a^4c^2x^5-15\,a^3c^2x^4+20\,a^2c^2x^3+15\,ac^2x^2-30\,c^2x\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{30\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 - 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 + 15*
a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

maple [A] time = 0.03, size = 81, normalized size = 0.58

$$\frac{x\left(5x^5a^5-6x^4a^4-15x^3a^3+20a^2x^2+15ax-30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}{30\left(ax+1\right)^3\left(ax-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*(-a^2*c*x^2+c)
^(5/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^3/(a*x-1)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(5/2)/(a*x +
1), x)
```

3.1209 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=187

$$\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}-\frac{6c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{8c^3(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

```
[Out] -8/5*c^3*(-a*x+1)^5*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+2*c^3*(-a*x+1
)^6*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-6/7*c^3*(-a*x+1)^7*(-a^2*c*x^
2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+1/8*c^3*(-a*x+1)^8*(-a^2*c*x^2+c)^(1/2)/a/(
-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}-\frac{6c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{8c^3(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-8*c^3*(1 - a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) + (2*c^3*(
1 - a*x)^6*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (6*c^3*(1 - a*x)^7*
Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) + (c^3*(1 - a*x)^8*Sqrt[c - a^
2*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
```

```
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int(1-ax)^4(1+ax)^3\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int\left(8(1-ax)^4-12(1-ax)^5+6(1-ax)^6-(1-ax)^7\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{8c^3(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{6c^3(1-ax)^7\sqrt{c-a^2cx}}{7a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 0.36

$$\frac{c^3(ax-1)^5\left(35a^3x^3+135a^2x^2+185ax+93\right)\sqrt{c-a^2cx^2}}{280a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^ArcTanh[a*x],x]
```

```
[Out] (c^3*(-1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(93 + 185*a*x + 135*a^2*x^2 + 35*a^3*
x^3))/(280*a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 120, normalized size = 0.64

$$-\frac{\left(35\,a^7c^3x^8-40\,a^6c^3x^7-140\,a^5c^3x^6+168\,a^4c^3x^5+210\,a^3c^3x^4-280\,a^2c^3x^3-140\,ac^3x^2+280\,c^3x\right)\sqrt{-a^2cx^2+}}{280\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/280*(35*a^7*c^3*x^8 - 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 + 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 - 280*a^2*c^3*x^3 - 140*a*c^3*x^2 + 280*c^3*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

maple [A] time = 0.03, size = 97, normalized size = 0.52

$$\frac{x\left(35a^7x^7-40x^6a^6-140x^5a^5+168x^4a^4+210x^3a^3-280a^2x^2-140ax+280\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}}{280\left(ax+1\right)^4\left(ax-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/280*x*(35*a^7*x^7-40*a^6*x^6-140*a^5*x^5+168*a^4*x^4+210*a^3*x^3-280*a^2*
x^2-140*a*x+280)*(-a^2*c*x^2+c)^(7/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^4/(a*x-1)^
4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{7/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(7/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(7/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(7/2)/(a*x +
1), x)
```

3.1210 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=234

$$-\frac{c^4(1-ax)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}+\frac{8c^4(1-ax)^9\sqrt{c-a^2cx^2}}{9a\sqrt{1-a^2x^2}}-\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{32c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{8c^4(1-}{3a}$$

```
[Out] -8/3*c^4*(-a*x+1)^6*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+32/7*c^4*(-a*
x+1)^7*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-3*c^4*(-a*x+1)^8*(-a^2*c*x
^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+8/9*c^4*(-a*x+1)^9*(-a^2*c*x^2+c)^(1/2)/a/
(-a^2*x^2+1)^(1/2)-1/10*c^4*(-a*x+1)^10*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)
^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{c^4(1-ax)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}+\frac{8c^4(1-ax)^9\sqrt{c-a^2cx^2}}{9a\sqrt{1-a^2x^2}}-\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}+\frac{32c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{8c^4(1-}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] (-8*c^4*(1 - a*x)^6*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (32*c^4*
(1 - a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (3*c^4*(1 - a*x)
^8*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) + (8*c^4*(1 - a*x)^9*Sqrt[c -
 a^2*c*x^2])/(9*a*Sqrt[1 - a^2*x^2]) - (c^4*(1 - a*x)^10*Sqrt[c - a^2*c*x^2
])/(10*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}\,dx}{\sqrt{1-a^2x^2}}$$

$$=\frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int(1-ax)^5(1+ax)^4\,dx}{\sqrt{1-a^2x^2}}$$

$$=\frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int\left(16(1-ax)^5-32(1-ax)^6+24(1-ax)^7-8(1-ax)^8+(1-\right.}{\sqrt{1-a^2x^2}}$$

$$=-\frac{8c^4(1-ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{32c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{3c^4(1-ax)^8\sqrt{c-a^2c}}{a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.08, size = 76, normalized size = 0.32

$$-\frac{c^4(ax-1)^6\left(63a^4x^4+308a^3x^3+588a^2x^2+528ax+193\right)\sqrt{c-a^2cx^2}}{630a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^ArcTanh[a*x],x]
```

```
[Out] -1/630*(c^4*(-1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 + 528*a*x + 588*a^2*x^2 +
 308*a^3*x^3 + 63*a^4*x^4))/(a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.60, size = 142, normalized size = 0.61

$$\frac{\left(63\,a^9c^4x^{10}-70\,a^8c^4x^9-315\,a^7c^4x^8+360\,a^6c^4x^7+630\,a^5c^4x^6-756\,a^4c^4x^5-630\,a^3c^4x^4+840\,a^2c^4x^3+315\,a\right.}{630\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/630*(63*a^9*c^4*x^10 - 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 + 360*a^6*c^4*x^7
 + 630*a^5*c^4*x^6 - 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 + 840*a^2*c^4*x^3 +
315*a*c^4*x^2 - 630*c^4*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2
 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

maple [A] time = 0.03, size = 113, normalized size = 0.48

$$\frac{x\left(63a^9x^9-70x^8a^8-315a^7x^7+360x^6a^6+630x^5a^5-756x^4a^4-630x^3a^3+840a^2x^2+315ax-630\right)\left(-a^2c\,x^2+c\right.}{630\left(ax+1\right)^5\left(ax-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/630*x*(63*a^9*x^9-70*a^8*x^8-315*a^7*x^7+360*a^6*x^6+630*a^5*x^5-756*a^4*
x^4-630*a^3*x^3+840*a^2*x^2+315*a*x-630)*(-a^2*c*x^2+c)^(9/2)*(-a^2*x^2+1)^
(1/2)/(a*x+1)^5/(a*x-1)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt(-a^2*x^2 + 1)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{9/2}\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(9/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(9/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^{\frac{9}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(9/2)/(a*x + 1), x)
```

3.1211 $\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] ln(a*x+1)*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 31}

$$\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}
= \frac{\sqrt{1-a^2x^2}\int \frac{1}{1+ax}\,dx}{\sqrt{c-a^2cx^2}}
= \frac{\sqrt{1-a^2x^2}\log(1+ax)}{a\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Log[1 + a*x])/(a*Sqrt[c - a^2*c*x^2])
```

fricas [B] time = 0.60, size = 227, normalized size = 5.82

$$\left[\frac{\log\left(\frac{a^6cx^6+4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2-4\,acx-\left(a^4x^4+4\,a^3x^3+6\,a^2x^2+4\,ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-2\,c}{a^4x^4+2\,a^3x^3-2\,ax-1}\right)}{2\,a\sqrt{c}}, \frac{\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,(a^2x^2+\dots}{a^4cx^4+2\,a^3c\dots}\right.}{ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2 - 4*a*c*x - (
a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1))/(a*sqrt(c)), sqrt(-
c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqr
t(-c)/(a^4*c*x^4 + 2*a^3*c*x^3 - a^2*c*x^2 - 2*a*c*x))/(a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\sqrt{-a^2cx^2+c}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a^2*c*x^2 + c)*(a*x + 1)), x)
```

maple [A] time = 0.03, size = 40, normalized size = 1.03

$$\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\ \ln\left(ax+1\right)}{\sqrt{-a^{2}x^{2}+1}\ ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/(-a^2*x^2+1)^(1/2)/c/a*(-c*(a^2*x^2-1))^(1/2)*ln(a*x+1)
```

maxima [A] time = 0.33, size = 13, normalized size = 0.33

$$\frac{\log\left(ax+1\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="m
axima")
```

```
[Out] log(a*x + 1)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-a^2\,x^2}}{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\ \left(ax+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)
), x)
```

3.1212 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2ac(ax+1)\sqrt{c-a^2cx^2}}$$

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)/a/c/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+1/2*arctanh(a*x)*(
-a^2*x^2+1)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}\,\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{2ac(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a*c*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^
2]*ArcTanh[a*x])/(2*a*c*Sqrt[c - a^2*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)(1+ax)^2}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2(1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{c\sqrt{c-a^2cx^2}}\\
&= -\frac{\sqrt{1-a^2x^2}}{2ac(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\int \frac{1}{-1+a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}\\
&= -\frac{\sqrt{1-a^2x^2}}{2ac(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2ac\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.66

$$\frac{\sqrt{1-a^2x^2}\left(\frac{\tanh^{-1}(ax)}{2a}-\frac{1}{2a(ax+1)}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2)),x]
[Out] (Sqrt[1 - a^2*x^2]*(-1/2*1/(a*(1 + a*x)) + ArcTanh[a*x]/(2*a)))/(c*Sqrt[c -
 a^2*c*x^2])
```

fricas [A] time = 0.71, size = 343, normalized size = 3.81

$$\left[-\frac{4\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\,ax-\left(a^3x^3+a^2x^2-ax-1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5a^4cx^4-5a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+}}{a^6x^6-3a^4x^4+3a^2x^2-1}\right.}{8\left(a^4c^2x^3+a^3c^2x^2-a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [-1/8*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^3*x^3 + a^2*x^2 -
 a*x - 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3
+ a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^
4*x^4 + 3*a^2*x^2 - 1)))/(a^4*c^2*x^3 + a^3*c^2*x^2 - a^2*c^2*x - a*c^2), -
1/4*(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^3*x^3 + a^2*x^2 - a
*x - 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c
)*x/(a^4*c*x^4 - c)))/(a^4*c^2*x^3 + a^3*c^2*x^2 - a^2*c^2*x - a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)), x)
```

maple [A] time = 0.05, size = 88, normalized size = 0.98

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln(ax-1)\,xa-ax\ln(ax+1)+\ln(ax-1)-\ln(ax+1)+2\right)}{4\left(a^2x^2-1\right)c^2a\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a-a*x*ln(a*x+1)+
ln(a*x-1)-ln(a*x+1)+2)/(a^2*x^2-1)/c^2/a/(a*x+1)
```

maxima [A] time = 0.34, size = 50, normalized size = 0.56

$$-\frac{\sqrt{c}}{2\left(a^2c^2x+ac^2\right)}+\frac{\log(ax+1)}{4\,ac^{\frac{3}{2}}}-\frac{\log(ax-1)}{4\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="m
axima")
```

```
[Out] -1/2*sqrt(c)/(a^2*c^2*x + a*c^2) + 1/4*log(a*x + 1)/(a*c^(3/2)) - 1/4*log(a
*x - 1)/(a*c^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^(3/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^(3/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x +
 1)), x)
```

3.1213 $\int \frac{e^{-\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4ac^2(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

```
[Out] 1/8*(-a^2*x^2+1)^(1/2)/a/c^2/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/8*(-a^2*x^2+1)
^(1/2)/a/c^2/(a*x+1)^2/(-a^2*c*x^2+c)^(1/2)-1/4*(-a^2*x^2+1)^(1/2)/a/c^2/(a
*x+1)/(-a^2*c*x^2+c)^(1/2)+3/8*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/c^2/(-a^2*
c*x^2+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{4ac^2(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)^2\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(8*a*c^2*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^
2]/(8*a*c^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(4*a*c^2*(
1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^2
*Sqrt[c - a^2*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^2(1+ax)^3}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{8(-1+ax)^2}+\frac{1}{4(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}}{4ac^2(1+ax)\sqrt{c-a^2cx^2}} - \frac{\left(3\sqrt{1-\right.}{8}$$

$$= \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}}{4ac^2(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{1-}{8a}$$

Mathematica [A] time = 0.08, size = 83, normalized size = 0.45

$$\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2-3ax+3(ax-1)(ax+1)^2\tanh^{-1}(ax)+2\right)}{8a(ax-1)(acx+c)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2 - 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)*(1 + a*x)^2*ArcTan
h[a*x]))/(8*a*(-1 + a*x)*(c + a*c*x)^2*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.71, size = 451, normalized size = 2.46

$$\left[\frac{3\left(a^5x^5+a^4x^4-2\,a^3x^3-2\,a^2x^2+ax+1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right)-4\left(2\,a^3\right.}{32\left(a^6c^3x^5+a^5c^3x^4-2\,a^4c^3x^3-2\,a^3c^3x^2+a^2c^3x+ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="f
ricas")
```

```
[Out] [1/32*(3*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*sqrt(c)*log(
-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c*x^2
 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)
) - 4*(2*a^3*x^3 - a^2*x^2 - 5*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)
)/(a^6*c^3*x^5 + a^5*c^3*x^4 - 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 + a^2*c^3*x +
a*c^3), 1/16*(3*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*sqrt(
-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^
4 - c)) - 2*(2*a^3*x^3 - a^2*x^2 - 5*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1))/(a^6*c^3*x^5 + a^5*c^3*x^4 - 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 + a^2*c^
3*x + a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)), x)
```

maple [A] time = 0.05, size = 166, normalized size = 0.91

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(3\ln(ax-1)\,x^3a^3-3a^3x^3\ln(ax+1)+3\ln(ax-1)\,x^2a^2-3\ln(ax+1)\,x^2a^2+6a^2\right.}{16\left(a^2x^2-1\right)c^3a\,(ax-1)\,(ax+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x^3*a^3-3*a^3*x
^3*ln(a*x+1)+3*ln(a*x-1)*x^2*a^2-3*ln(a*x+1)*x^2*a^2+6*a^2*x^2-3*ln(a*x-1)*
x*a+3*a*x*ln(a*x+1)+6*a*x-3*ln(a*x-1)+3*ln(a*x+1)-4)/(a^2*x^2-1)/c^3/a/(a*x
-1)/(a*x+1)^2
```

maxima [A] time = 0.34, size = 83, normalized size = 0.45

$$-\frac{3 a^2x^2 + 3 ax - 2}{8 \left(a^4c^{\frac{5}{2}}x^3 + a^3c^{\frac{5}{2}}x^2 - a^2c^{\frac{5}{2}}x - ac^{\frac{5}{2}}\right)} + \frac{3 \log(ax+1)}{16 ac^{\frac{5}{2}}} - \frac{3 \log(ax-1)}{16 ac^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="m
axima")
```

```
[Out] -1/8*(3*a^2*x^2 + 3*a*x - 2)/(a^4*c^(5/2)*x^3 + a^3*c^(5/2)*x^2 - a^2*c^(5/
2)*x - a*c^(5/2)) + 3/16*log(a*x + 1)/(a*c^(5/2)) - 3/16*log(a*x - 1)/(a*c^
(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1 - a^2\,x^2}}{\left(c - a^2\,c\,x^2\right)^{5/2}\,(a\,x + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x +
 1)), x)
```

3.1214 $\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=276

$$\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{16ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{}{24a}$$

```
[Out] 1/32*(-a^2*x^2+1)^(1/2)/a/c^3/(-a*x+1)^2/(-a^2*c*x^2+c)^(1/2)+1/8*(-a^2*x^2
+1)^(1/2)/a/c^3/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/24*(-a^2*x^2+1)^(1/2)/a/c^3
/(a*x+1)^3/(-a^2*c*x^2+c)^(1/2)-3/32*(-a^2*x^2+1)^(1/2)/a/c^3/(a*x+1)^2/(-a
^2*c*x^2+c)^(1/2)-3/16*(-a^2*x^2+1)^(1/2)/a/c^3/(a*x+1)/(-a^2*c*x^2+c)^(1/2
)+5/16*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{16ac^3(ax+1)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(32*a*c^3*(1 - a*x)^2*Sqrt[c - a^2*c*x^2]) + Sqrt[1 - a^2
*x^2]/(8*a*c^3*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(24*a*c^3
*(1 + a*x)^3*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2])/(32*a*c^3*(1 + a*
x)^2*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2])/(16*a*c^3*(1 + a*x)*Sqrt[
c - a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a*c^3*Sqrt[c - a^2
*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^3(1+ax)^4}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{16(-1+ax)^3}+\frac{1}{8(-1+ax)^2}+\frac{1}{8(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{3}{16(1+ax)^2}-\frac{5}{16\left(-1+a^2x^2\right)}\right)dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{24ac^3(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{}{32a}$$

$$= \frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{24ac^3(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{}{32a}$$

Mathematica [A] time = 0.12, size = 101, normalized size = 0.37

$$\frac{\sqrt{1-a^2x^2}\left(-15a^4x^4-15a^3x^3+25a^2x^2+25ax+15(ax-1)^2(ax+1)^3\tanh^{-1}(ax)-8\right)}{48a(ax-1)^2(acx+c)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-8 + 25*a*x + 25*a^2*x^2 - 15*a^3*x^3 - 15*a^4*x^4 + 15
*(-1 + a*x)^2*(1 + a*x)^3*ArcTanh[a*x]))/(48*a*(-1 + a*x)^2*(c + a*c*x)^3*S
qrt[c - a^2*c*x^2])
```

fricas [A] time = 0.66, size = 565, normalized size = 2.05

$$\left[\frac{15\left(a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1\right)\sqrt{c}\log\left(-\frac{a^6cx^6+5a^4cx^4-5a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2}}{a^6x^6-3a^4x^4+3a^2x^2-1}\right.}{192\left(a^8c^4x^7+a^7c^4x^6-3a^6c^4x^5-3a^5c^4x^4+3a^4c^4x^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="f
ricas")
```

```
[Out] [1/192*(15*(a^7*x^7 + a^6*x^6 - 3*a^5*x^5 - 3*a^4*x^4 + 3*a^3*x^3 + 3*a^2*x
^2 - a*x - 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*
x^3 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 -
3*a^4*x^4 + 3*a^2*x^2 - 1)) - 4*(8*a^5*x^5 - 7*a^4*x^4 - 31*a^3*x^3 + 9*a^2
*x^2 + 33*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^8*c^4*x^7 + a^7*
c^4*x^6 - 3*a^6*c^4*x^5 - 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 + 3*a^3*c^4*x^2 - a
^2*c^4*x - a*c^4), 1/96*(15*(a^7*x^7 + a^6*x^6 - 3*a^5*x^5 - 3*a^4*x^4 + 3*
a^3*x^3 + 3*a^2*x^2 - a*x - 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(
-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 - c)) - 2*(8*a^5*x^5 - 7*a^4*x^4 - 31
*a^3*x^3 + 9*a^2*x^2 + 33*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^
8*c^4*x^7 + a^7*c^4*x^6 - 3*a^6*c^4*x^5 - 3*a^5*c^4*x^4 + 3*a^4*c^4*x^3 + 3
*a^3*c^4*x^2 - a^2*c^4*x - a*c^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)), x)
```

maple [A] time = 0.05, size = 238, normalized size = 0.86

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(15\ln\left(ax-1\right)x^5a^5-15\ln\left(ax+1\right)x^5a^5+15\ln\left(ax-1\right)x^4a^4-15\ln\left(ax+1\right)x^4a^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/96*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x-1)*x^5*a^5-15*ln(
a*x+1)*x^5*a^5+15*ln(a*x-1)*x^4*a^4-15*ln(a*x+1)*x^4*a^4+30*x^4*a^4-30*ln(a
*x-1)*x^3*a^3+30*a^3*x^3*ln(a*x+1)+30*x^3*a^3-30*ln(a*x-1)*x^2*a^2+30*ln(a*
x+1)*x^2*a^2-50*a^2*x^2+15*ln(a*x-1)*x*a-15*a*x*ln(a*x+1)-50*a*x+15*ln(a*x-
1)-15*ln(a*x+1)+16)/(a^2*x^2-1)/c^4/a/(a*x-1)^2/(a*x+1)^3
```

maxima [A] time = 0.35, size = 135, normalized size = 0.49

$$-\frac{15\,a^4\sqrt{c}\,x^4+15\,a^3\sqrt{c}\,x^3-25\,a^2\sqrt{c}\,x^2-25\,a\sqrt{c}\,x+8\,\sqrt{c}}{48\left(a^6c^4x^5+a^5c^4x^4-2\,a^4c^4x^3-2\,a^3c^4x^2+a^2c^4x+ac^4\right)}+\frac{5\,\log\left(ax+1\right)}{32\,ac^{\frac{7}{2}}}-\frac{5\,\log\left(ax-1\right)}{32\,ac^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="m
axima")
```

```
[Out] -1/48*(15*a^4*sqrt(c)*x^4 + 15*a^3*sqrt(c)*x^3 - 25*a^2*sqrt(c)*x^2 - 25*a*
sqrt(c)*x + 8*sqrt(c))/(a^6*c^4*x^5 + a^5*c^4*x^4 - 2*a^4*c^4*x^3 - 2*a^3*c
^4*x^2 + a^2*c^4*x + a*c^4) + 5/32*log(a*x + 1)/(a*c^(7/2)) - 5/32*log(a*x
- 1)/(a*c^(7/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{1-a^2\,x^2}}{{\left(c-a^2\,c\,x^2\right)}^{7/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x +
 1)), x)
```

3.1215 $\int e^{-\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=137

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}-p;\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax^{m+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p{}_2F_1\left(\frac{m+2}{2},\frac{1}{2}-p;\frac{m+4}{2}\right.}{m+2}$$

```
[Out] x^(1+m)*(-a^2*c*x^2+c)^p*hypergeom([1/2-p, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/
(1+m)/((-a^2*x^2+1)^p)-a*x^(2+m)*(-a^2*c*x^2+c)^p*hypergeom([1/2-p, 1+1/2*m
],[2+1/2*m],a^2*x^2)/(2+m)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6153, 6149, 808, 364}

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}-p;\frac{m+3}{2};a^2x^2\right)}{m+1}-\frac{ax^{m+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p{}_2F_1\left(\frac{m+2}{2},\frac{1}{2}-p;\frac{m+4}{2}\right.}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1/2 - p, (3 + m)/
2, a^2*x^2])/((1 + m)*(1 - a^2*x^2)^p) - (a*x^(2 + m)*(c - a^2*c*x^2)^p*Hyp
ergeometric2F1[(2 + m)/2, 1/2 - p, (4 + m)/2, a^2*x^2])/((2 + m)*(1 - a^2*x
^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
```

```
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}x^m\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx-\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2\right.\right.$$
$$=\frac{x^{1+m}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1+m}{2},\frac{1}{2}-p;\frac{3+m}{2};a^2x^2\right)}{1+m}-\frac{ax^{2+m}\left(1-a^2x\right.}{}$$

Mathematica [A] time = 0.06, size = 115, normalized size = 0.84

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{x^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}-p;\frac{m+1}{2}+1;a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_2F_1\left(\frac{m+2}{2},\frac{1}{2}-p;\frac{m+2}{2}+1;a^2x^2\right)}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]

[Out] ((c - a^2*c*x^2)^p*((x^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1/2 - p, 1 + (1
 + m)/2, a^2*x^2])/(1 + m) - (a*x^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1/2
- p, 1 + (2 + m)/2, a^2*x^2])/(2 + m)))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^px^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^m/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^m/(a*x + 1), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(-a^2c\,x^2+c\right)^p\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^m/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\left(c-a^2\,c\,x^2\right)^p\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\sqrt{-\left(ax-1\right)\left(ax+1\right)}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{p}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(a*x +
 1), x)
```

3.1216 $\int e^{-\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=85

$$-\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}$$

```
[Out] -(-a^2*x^2+1)^(1/2+p)/a^4/(1+2*p)+(-a^2*x^2+1)^(3/2+p)/a^4/(3+2*p)-1/5*a*x^
5*hypergeom([5/2, 1/2-p],[7/2],a^2*x^2)
```

Rubi [A] time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6149, 764, 266, 43, 364}

$$-\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}+\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^4*(1 + 2*p))) + (1 - a^2*x^2)^(3/2 + p)/(a^4*(
3 + 2*p)) - (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx &= \int x^3(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= -\left(a\int x^4\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\right)+\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= -\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\,\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{1}{2}+p}\,dx,x,x^2\right)\\
&= -\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\,\text{Subst}\left(\int\left(\frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{a^2}-\frac{\left(1-a^2x\right)^{\frac{1}{2}+p}}{a^2}\right)\right.\\
&= -\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^4(1+2p)}+\frac{\left(1-a^2x^2\right)^{\frac{3}{2}+p}}{a^4(3+2p)}-\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.91

$$-\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)-\frac{\left(a^2(2p+1)x^2+2\right)\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4\left(4p^2+8p+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]

[Out] -(((1 - a^2*x^2)^(1/2 + p)*(2 + a^2*(1 + 2*p)*x^2))/(a^4*(3 + 8*p + 4*p^2))
) - (a*x^5*Hypergeometric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/5
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^3}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^3/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^3/(a*x + 1), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-a^2x^2+1\right)^p \sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}} x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)*x^3/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(1-a^2\,x^2\right)^p\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
[Out] int((x^3*(1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(a*x + 1
), x)
```

3.1217 $\int e^{-\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3(2p+1)}-\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^3(2p+3)}$$

```
[Out] (-a^2*x^2+1)^(1/2+p)/a^3/(1+2*p)-(-a^2*x^2+1)^(3/2+p)/a^3/(3+2*p)+1/3*x^3*h
ypergeom([3/2, 1/2-p],[5/2],a^2*x^2)
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6149, 764, 364, 266, 43}

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3(2p+1)}-\frac{\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^3(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] (1 - a^2*x^2)^(1/2 + p)/(a^3*(1 + 2*p)) - (1 - a^2*x^2)^(3/2 + p)/(a^3*(3 +
 2*p)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] && !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx &= \int x^2(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= -\left(a\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\right)+\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{1}{2}a\,\text{Subst}\left(\int x\left(1-a^2x\right)^{-\frac{1}{2}+p}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{1}{2}a\,\text{Subst}\left(\int\left(\frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{a^2}-\frac{\left(1-a^2x\right)^{\frac{1}{2}+p}}{a^2}\right)\cdots\right.\\
&= \frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^3(1+2p)}-\frac{\left(1-a^2x^2\right)^{\frac{3}{2}+p}}{a^3(3+2p)}+\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.89

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{\left(a^2(2p+1)x^2+2\right)\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^3\left(4p^2+8p+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] ((1 - a^2*x^2)^(1/2 + p)*(2 + a^2*(1 + 2*p)*x^2))/(a^3*(3 + 8*p + 4*p^2)) +
 (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^2}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^2/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x^2/(a*x + 1), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-a^2x^2+1\right)^p\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}} x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)*x^2/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(1-a^2\,x^2\right)}^p\,\sqrt{1-a^2\,x^2}}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
[Out] int((x^2*(1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{-\left(ax-1\right)\left(ax+1\right)} \left(-\left(ax-1\right)\left(ax+1\right)\right)^{p}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(a*x + 1
), x)
```

3.1218 $\int e^{-\tanh^{-1}(ax)} x \left(1-a^2x^2\right)^p \, dx$

Optimal. Leaf size=58

$$-\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^2(2p+1)}$$

```
[Out] -(-a^2*x^2+1)^(1/2+p)/a^2/(1+2*p)-1/3*a*x^3*hypergeom([3/2, 1/2-p],[5/2],a^
2*x^2)
```

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6149, 764, 261, 364}

$$-\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -((1 - a^2*x^2)^(1/2 + p)/(a^2*(1 + 2*p))) - (a*x^3*Hypergeometric2F1[3/2,
1/2 - p, 5/2, a^2*x^2])/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx &= \int x(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= -\left(a\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\right)+\int x\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\ &= -\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}}{a^2(1+2p)}-\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.03

$$-\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{2a^2\left(p+\frac{1}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - a^2*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -1/2*(1 - a^2*x^2)^(1/2 + p)/(a^2*(1/2 + p)) - (a*x^3*Hypergeometric2F1[3/2
, 1/2 - p, 5/2, a^2*x^2])/3
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p*x/(a*x + 1), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a^2x^2+1\right)^p \sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}} x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)*x/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\left(1-a^2\,x^2\right)^p \sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x*(1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(-\left(ax-1\right)\left(ax+1\right)\right)^{p}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(a*x + 1),
x)
```

3.1219 $\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx$

Optimal. Leaf size=59

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{3}{2}}\,{}_2F_1\left(\frac{1}{2}-p,p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

```
[Out] -2^(1/2+p)*(-a*x+1)^(3/2+p)*hypergeom([1/2-p, 3/2+p],[5/2+p],-1/2*a*x+1/2)/
a/(3+2*p)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6140, 69}

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{3}{2}}\,{}_2F_1\left(\frac{1}{2}-p,p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^p/E^ArcTanh[a*x],x]
```

```
[Out] -((2^(1/2 + p)*(1 - a*x)^(3/2 + p)*Hypergeometric2F1[1/2 - p, 3/2 + p, 5/2
+ p, (1 - a*x)/2])/(a*(3 + 2*p)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx=\int(1-ax)^{\frac{1}{2}+p}(1+ax)^{-\frac{1}{2}+p}\,dx$$
$$=-\frac{2^{\frac{1}{2}+p}(1-ax)^{\frac{3}{2}+p}\,{}_2F_1\left(\frac{1}{2}-p,\frac{3}{2}+p;\frac{5}{2}+p;\frac{1}{2}(1-ax)\right)}{a(3+2p)}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.95

$$\frac{(ax-1)(2-2ax)^{p+\frac{1}{2}}\,{}_2F_1\left(\frac{1}{2}-p,p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^p/E^ArcTanh[a*x],x]
[Out] ((2 - 2*a*x)^(1/2 + p)*(-1 + a*x)*Hypergeometric2F1[1/2 - p, 3/2 + p, 5/2 +
 p, (1 - a*x)/2])/(a*(3 + 2*p))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x + 1), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2x^2+1\right)^p\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(1-a^2\,x^2\right)^p\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(a*x + 1), x)
```

3.1220 $\int \frac{e^{-\tanh^{-1}(ax)}(1-a^2x^2)^p}{x} dx$

Optimal. Leaf size=73

$$-\frac{(1-a^2x^2)^{p+\frac{1}{2}} {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)$$

```
[Out] -a*x*hypergeom([1/2, 1/2-p],[3/2],a^2*x^2)-(-a^2*x^2+1)^(1/2+p)*hypergeom([
1, 1/2+p],[3/2+p],-a^2*x^2+1)/(1+2*p)
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6149, 764, 266, 65, 245}

$$-\frac{(1-a^2x^2)^{p+\frac{1}{2}} {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^p/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -(a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2]) - ((1 - a^2*x^2)^(1/2
+ p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx &= \int \frac{(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx\\
&= -\left(a\int \left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\right)+\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx\\
&= -ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x}\,dx,x,x^2\right)\\
&= -ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{\frac{1}{2}+p}\,{}_2F_1\left(1,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.03

$$
-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^p/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -(a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2]) - ((1 - a^2*x^2)^(1/2
+ p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(2*(1/2 + p))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^2 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/((a*x + 1)*x), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^p\sqrt{-a^2x^2+1}}{(ax+1)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)/((a*x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^p\,\sqrt{1-a^2\,x^2}}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)),x)
[Out] int(((1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(x*(a*x + 1))
, x)
```

3.1221 $\int \frac{e^{-\tanh^{-1}(ax)}(1-a^2x^2)^p}{x^2}\,dx$

Optimal. Leaf size=74

$$\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\ {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

```
[Out] -hypergeom([-1/2, 1/2-p],[1/2],a^2*x^2)/x+a*(-a^2*x^2+1)^(1/2+p)*hypergeom(
[1, 1/2+p],[3/2+p],-a^2*x^2+1)/(1+2*p)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6149, 764, 364, 266, 65}

$$\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}\ {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^p/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) + (a*(1 - a^2*x^2)^(1/2
 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx &= \int \frac{(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx\\
&= -\left(a\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx\right)+\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx\\
&= -\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{\left(1-a^2x\right)^{-\frac{1}{2}+p}}{x}\,dx,x,x^2\right)\\
&= -\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}+\frac{a\left(1-a^2x^2\right)^{\frac{1}{2}+p}{}_2F_1\left(1,\frac{1}{2}+p;\frac{3}{2}+p;1-a^2x^2\right)}{1+2p}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.00

$$\frac{a\left(1-a^2x^2\right)^{p+\frac{1}{2}}{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^p/(E^ArcTanh[a*x]*x^2),x]
```

[Out] -(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) + (a*(1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{ax^3+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/(a*x^3 + x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2x^2+1\right)^p}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*x^2 + 1)^p/((a*x + 1)*x^2), x)

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^p\sqrt{-a^2x^2+1}}{(ax+1)\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)

[Out] int((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{p+\frac{1}{2}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((-a^2*x^2 + 1)^(p + 1/2)/((a*x + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-a^2\,x^2\right)^p\,\sqrt{1-a^2\,x^2}}{x^2\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(x^2*(a*x + 1)),x)
```

```
[Out] int(((1 - a^2*x^2)^p*(1 - a^2*x^2)^(1/2))/(x^2*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,(-(ax-1)(ax+1))^p}{x^2(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**p/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-(a*x - 1)*(a*x + 1))**p/(x**2*(a*x +
1)), x)
```

3.1222 $\int e^{-\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=134

$$-\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(2p+1)}$$

```
[Out] (-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/a^4/(3+2*p)-1/5*a*x^5*(-a^2*c*x^2+c)^p*
hypergeom([5/2, 1/2-p],[7/2],a^2*x^2)/((-a^2*x^2+1)^p)-(-a^2*c*x^2+c)^p*(-a
^2*x^2+1)^(1/2)/a^4/(1+2*p)
```

Rubi [A] time = 0.19, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6153, 6149, 764, 266, 43, 364}

$$-\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(2p+3)}-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^4*(1 + 2*p))) + ((1 - a^2*x^2)^(
3/2)*(c - a^2*c*x^2)^p)/(a^4*(3 + 2*p)) - (a*x^5*(c - a^2*c*x^2)^p*Hypergeo
metric2F1[5/2, 1/2 - p, 7/2, a^2*x^2])/(5*(1 - a^2*x^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{-\tanh^{-1}(ax)}x^3\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}x^3\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^3\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx-\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx\right.\right.$$
$$=-\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-\right.\right.$$
$$=-\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-\right.\right.$$
$$=-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^4(1+2p)}+\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^4(3+2p)}-\frac{1}{5}ax^5\left(1-a^2x^2\right)^{-p}\left(c-\right.$$

Mathematica [A] time = 0.08, size = 119, normalized size = 0.89

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{1}{2}\left(\frac{2\left(1-a^2x^2\right)^{p+\frac{3}{2}}}{a^4(2p+3)}-\frac{2\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{a^4(2p+1)}\right)-\frac{1}{5}ax^5\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-p;\frac{7}{2};a^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] ((c - a^2*c*x^2)^p*(((-2*(1 - a^2*x^2)^(1/2 + p))/(a^4*(1 + 2*p)) + (2*(1 -
 a^2*x^2)^(3/2 + p))/(a^4*(3 + 2*p)))/2 - (a*x^5*Hypergeometric2F1[5/2, 1/2
 - p, 7/2, a^2*x^2])/5))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^3}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^3/(a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-a^2c\,x^2+c\right)^p\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^3/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\left(c-a^2\,c\,x^2\right)^p\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x^3*(c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(ax-1)(ax+1)}\,\left(-c(ax-1)(ax+1)\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(a*x +
 1), x)
```

3.1223 $\int e^{-\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=133

$$\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}+\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(2p+1)}$$

```
[Out] -(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^p/a^3/(3+2*p)+1/3*x^3*(-a^2*c*x^2+c)^p*h
ypergeom([3/2, 1/2-p],[5/2],a^2*x^2)/((-a^2*x^2+1)^p)+(-a^2*c*x^2+c)^p*(-a^
2*x^2+1)^(1/2)/a^3/(1+2*p)
```

Rubi [A] time = 0.19, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6153, 6149, 764, 364, 266, 43}

$$\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(2p+3)}+\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^3*(1 + 2*p)) - ((1 - a^2*x^2)^(3/2
)*(c - a^2*c*x^2)^p)/(a^3*(3 + 2*p)) + (x^3*(c - a^2*c*x^2)^p*Hypergeometri
c2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}x^2\left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^2\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx-\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx\right.\right.\\
&= \frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-\right.\right.\\
&= \frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-\right.\right.\\
&= \frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^3(1+2p)}-\frac{\left(1-a^2x^2\right)^{3/2}\left(c-a^2cx^2\right)^p}{a^3(3+2p)}+\frac{1}{3}x^3\left(1-a^2x^2\right)^{-p}\left(c-a^2\right.
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 102, normalized size = 0.77

$$\frac{1}{3}\left(c-a^2cx^2\right)^p\left(x^3\left(1-a^2x^2\right)^{-p}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)+\frac{3\sqrt{1-a^2x^2}\,\left(a^2(2p+1)x^2+2\right)}{a^3\left(4p^2+8p+3\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] ((c - a^2*c*x^2)^p*((3*Sqrt[1 - a^2*x^2]*(2 + a^2*(1 + 2*p)*x^2))/(a^3*(3 +
 8*p + 4*p^2)) + (x^3*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(1 - a
^2*x^2)^p))/3
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^px^2}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^2/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^px^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^2/(a*x + 1), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(-a^2c\,x^2+c\right)^p\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x^2/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(c-a^2\,c\,x^2\right)^p\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(a*x +
 1), x)
```

3.1224 $\int e^{-\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=96

$$-\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(2p+1)}$$

```
[Out] -1/3*a*x^3*(-a^2*c*x^2+c)^p*hypergeom([3/2, 1/2-p],[5/2],a^2*x^2)/((-a^2*x^
2+1)^p)-(-a^2*c*x^2+c)^p*(-a^2*x^2+1)^(1/2)/a^2/(1+2*p)
```

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6153, 6149, 764, 261, 364}

$$-\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p)/(a^2*(1 + 2*p))) - (a*x^3*(c - a^2*
c*x^2)^p*Hypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/(3*(1 - a^2*x^2)^p)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x\left(1-a^2x^2\right)^{-\frac{1}{2}+p}\,dx-\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)\right.\\
&= -\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{a^2(1+2p)}-\frac{1}{3}ax^3\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x\right.
\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.92

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(-\frac{1}{3}ax^3\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-p;\frac{5}{2};a^2x^2\right)-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}}{2a^2\left(p+\frac{1}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c - a^2*c*x^2)^p)/E^ArcTanh[a*x],x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-1/2*(1 - a^2*x^2)^(1/2 + p)/(a^2*(1/2 + p)) - (a*x^3*H
ypergeometric2F1[3/2, 1/2 - p, 5/2, a^2*x^2])/3))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x/(a*x + 1), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a^2c\,x^2+c\right)^p\sqrt{-a^2x^2+1}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p*x/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(c-a^2\,c\,x^2\right)^p\,\sqrt{1-a^2\,x^2}}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(ax-1)(ax+1)}\,(-c(ax-1)(ax+1))^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(a*x + 1)
, x)
```

3.1225 $\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=86

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{3}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2}-p,p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

```
[Out] -2^(1/2+p)*(-a*x+1)^(3/2+p)*(-a^2*c*x^2+c)^p*hypergeom([1/2-p, 3/2+p],[5/2+
p],-1/2*a*x+1/2)/a/(3+2*p)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 69}

$$-\frac{2^{p+\frac{1}{2}}(1-ax)^{p+\frac{3}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2}-p,p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^ArcTanh[a*x],x]
```

```
[Out] -((2^(1/2 + p)*(1 - a*x)^(3/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2
- p, 3/2 + p, 5/2 + p, (1 - a*x)/2])/(a*(3 + 2*p)*(1 - a^2*x^2)^p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{\frac{1}{2}+p}(1+ax)^{-\frac{1}{2}+p}\,dx\\ &= -\frac{2^{\frac{1}{2}+p}(1-ax)^{\frac{3}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2}-p,\frac{3}{2}+p;\frac{5}{2}+p;\frac{1}{2}(1-ax)\right)}{a(3+2p)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.97

$$\frac{(ax-1)(2-2ax)^{p+\frac{1}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2}-p,p+\frac{3}{2};p+\frac{5}{2};\frac{1}{2}(1-ax)\right)}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^ArcTanh[a*x],x]
```

```
[Out] ((2 - 2*a*x)^(1/2 + p)*(-1 + a*x)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2 -
 p, 3/2 + p, 5/2 + p, (1 - a*x)/2])/(a*(3 + 2*p)*(1 - a^2*x^2)^p)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} c \,x^{2}+c \right)^{p} \sqrt{-a^{2} x^{2}+1}}{a x +1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2} x^{2} + 1} {\left(-a^{2} c x^{2} + c\right)}^{p}}{a x + 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^p\,\sqrt{1-a^2\,x^2}}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{- \left(a x - 1\right) \left(a x + 1\right)} \left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{p}}{a x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(a*x + 1),
x)
```

3.1226 $\int \frac{e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx$

Optimal. Leaf size=111

$$-ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\,\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

```
[Out] -a*x*(-a^2*c*x^2+c)^p*hypergeom([1/2, 1/2-p],[3/2],a^2*x^2)/((-a^2*x^2+1)^p
)-(-a^2*c*x^2+c)^p*hypergeom([1, 1/2+p],[3/2+p],-a^2*x^2+1)*(-a^2*x^2+1)^(1
/2)/(1+2*p)
```

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6153, 6149, 764, 266, 65, 245}

$$-ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\,\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/(E^ArcTanh[a*x]*x),x]
```

```
[Out] -((a*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2])/(1
- a^2*x^2)^p) - (Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1, 1
/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x}\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx$$

$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x}\,dx - \left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right.$$

$$= -ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)+\frac{1}{2}\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2\right.\right.$$

$$= -ax\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)-\frac{\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{}$$

Mathematica [A] time = 0.04, size = 103, normalized size = 0.93

$$\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\left(-\frac{\left(1-a^2x^2\right)^{p+\frac{1}{2}}\ {}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2\left(p+\frac{1}{2}\right)}-ax\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-p;\frac{3}{2};a^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/(E^ArcTanh[a*x]*x),x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(a*x*Hypergeometric2F1[1/2, 1/2 - p, 3/2, a^2*x^2]) -
((1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^2
])/(2*(1/2 + p))))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^2 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/((a*x + 1)*x), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2c\,x^2+c\right)^p\sqrt{-a^2x^2+1}}{(ax+1)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/((a*x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,\sqrt{1-a^2\,x^2}}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(x*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(-c\left(ax-1\right)\left(ax+1\right)\right)^p}{x\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(x*(a*x + 1
)), x)
```

3.1227 $\int \frac{e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=112

$$\frac{a\sqrt{1-a^2x^2}\,\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

```
[Out] -(-a^2*c*x^2+c)^p*hypergeom([-1/2, 1/2-p],[1/2],a^2*x^2)/x/((-a^2*x^2+1)^p)
+a*(-a^2*c*x^2+c)^p*hypergeom([1, 1/2+p],[3/2+p],-a^2*x^2+1)*(-a^2*x^2+1)^(
1/2)/(1+2*p)
```

Rubi [A] time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6153, 6149, 764, 364, 266, 65}

$$\frac{a\sqrt{1-a^2x^2}\,\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] -(((c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2])/(x*(1
- a^2*x^2)^p)) + (a*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^p*Hypergeometric2F1[1
, 1/2 + p, 3/2 + p, 1 - a^2*x^2])/(1 + 2*p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 6149

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^p, Int[(x^m*(1 - a^2*x^2)^(p + n/2))/(1 - a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c,
 0]) && ILtQ[(n - 1)/2, 0] &&  !IntegerQ[p - n/2]
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p}{x^2}\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{e^{-\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p}{x^2}\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{(1-ax)\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx\\ &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int \frac{\left(1-a^2x^2\right)^{-\frac{1}{2}+p}}{x^2}\,dx-\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right.\\ &= -\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}-\frac{1}{2}\left(a\left(1-a^2x^2\right)^{-p}\left(c-a^2c\right.\right.\\ &= -\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}+\frac{a\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^p}{1}\end{aligned}$$

Mathematica [A] time = 0.04, size = 102, normalized size = 0.91

$$(1-a^2x^2)^{-p}(c-a^2cx^2)^p\left(\frac{a(1-a^2x^2)^{p+\frac{1}{2}}\,{}_2F_1\left(1,p+\frac{1}{2};p+\frac{3}{2};1-a^2x^2\right)}{2p+1}-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-p;\frac{1}{2};a^2x^2\right)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/(E^ArcTanh[a*x]*x^2),x]
```

```
[Out] ((c - a^2*c*x^2)^p*(-(Hypergeometric2F1[-1/2, 1/2 - p, 1/2, a^2*x^2]/x) + (
a*(1 - a^2*x^2)^(1/2 + p)*Hypergeometric2F1[1, 1/2 + p, 3/2 + p, 1 - a^2*x^
2])/(1 + 2*p)))/(1 - a^2*x^2)^p
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,(-a^2cx^2+c)^p}{ax^3+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/(a*x^3 + x^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(-a^2c\,x^2+c)^p\sqrt{-a^2x^2+1}}{(ax+1)\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\left(-a^2cx^2+c\right)^p}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*(-a^2*c*x^2 + c)^p/((a*x + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,\sqrt{1-a^2\,x^2}}{x^2\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(x^2*(a*x + 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(1 - a^2*x^2)^(1/2))/(x^2*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(-c\left(ax-1\right)\left(ax+1\right)\right)^p}{x^2\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p/(x**2*(a*x
+ 1)), x)
```

3.1228 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=73

$$\frac{c^4(1-ax)^9}{9a}-\frac{3c^4(1-ax)^8}{4a}+\frac{12c^4(1-ax)^7}{7a}-\frac{4c^4(1-ax)^6}{3a}$$

```
[Out] -4/3*c^4*(-a*x+1)^6/a+12/7*c^4*(-a*x+1)^7/a-3/4*c^4*(-a*x+1)^8/a+1/9*c^4*(-a*x+1)^9/a
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^4(1-ax)^9}{9a}-\frac{3c^4(1-ax)^8}{4a}+\frac{12c^4(1-ax)^7}{7a}-\frac{4c^4(1-ax)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-4*c^4*(1 - a*x)^6)/(3*a) + (12*c^4*(1 - a*x)^7)/(7*a) - (3*c^4*(1 - a*x)^8)/(4*a) + (c^4*(1 - a*x)^9)/(9*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx&=c^4\int(1-ax)^5(1+ax)^3\,dx\\&=c^4\int\left(8(1-ax)^5-12(1-ax)^6+6(1-ax)^7-(1-ax)^8\right)dx\\&=-\frac{4c^4(1-ax)^6}{3a}+\frac{12c^4(1-ax)^7}{7a}-\frac{3c^4(1-ax)^8}{4a}+\frac{c^4(1-ax)^9}{9a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.53

$$-\frac{c^4(ax-1)^6\left(28a^3x^3+105a^2x^2+138ax+65\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(2*ArcTanh[a*x]),x]
[Out] -1/252*(c^4*(-1 + a*x)^6*(65 + 138*a*x + 105*a^2*x^2 + 28*a^3*x^3))/a
```

fricas [A] time = 0.53, size = 81, normalized size = 1.11

$$-\frac{1}{9}a^8c^4x^9+\frac{1}{4}a^7c^4x^8+\frac{2}{7}a^6c^4x^7-a^5c^4x^6+\frac{3}{2}a^3c^4x^4-\frac{2}{3}a^2c^4x^3-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 - a*c^4*x^2 + c^4*x
```

giac [A] time = 1.65, size = 78, normalized size = 1.07

$$-\frac{\left(28\,c^4-\frac{315\,c^4}{ax+1}+\frac{1440\,c^4}{(ax+1)^2}-\frac{3360\,c^4}{(ax+1)^3}+\frac{4032\,c^4}{(ax+1)^4}-\frac{2016\,c^4}{(ax+1)^5}\right)(ax+1)^9}{252\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] -1/252*(28*c^4 - 315*c^4/(a*x + 1) + 1440*c^4/(a*x + 1)^2 - 3360*c^4/(a*x +
 1)^3 + 4032*c^4/(a*x + 1)^4 - 2016*c^4/(a*x + 1)^5)*(a*x + 1)^9/a
```

maple [A] time = 0.03, size = 61, normalized size = 0.84

$$c^4\left(-\frac{1}{9}x^9a^8+\frac{1}{4}a^7x^8+\frac{2}{7}x^7a^6-x^6a^5+\frac{3}{2}x^4a^3-\frac{2}{3}x^3a^2-a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4/(a*x+1)^2*(-a^2*x^2+1),x)
[Out] c^4*(-1/9*x^9*a^8+1/4*a^7*x^8+2/7*x^7*a^6-x^6*a^5+3/2*x^4*a^3-2/3*x^3*a^2-a
*x^2+x)
```

maxima [A] time = 0.34, size = 81, normalized size = 1.11

$$-\frac{1}{9}a^8c^4x^9+\frac{1}{4}a^7c^4x^8+\frac{2}{7}a^6c^4x^7-a^5c^4x^6+\frac{3}{2}a^3c^4x^4-\frac{2}{3}a^2c^4x^3-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 + 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^
3*c^4*x^4 - 2/3*a^2*c^4*x^3 - a*c^4*x^2 + c^4*x
```

mupad [B] time = 0.05, size = 81, normalized size = 1.11

$$-\frac{a^8\,c^4\,x^9}{9}+\frac{a^7\,c^4\,x^8}{4}+\frac{2\,a^6\,c^4\,x^7}{7}-a^5\,c^4\,x^6+\frac{3\,a^3\,c^4\,x^4}{2}-\frac{2\,a^2\,c^4\,x^3}{3}-a\,c^4\,x^2+c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^4*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] c^4*x - a*c^4*x^2 - (2*a^2*c^4*x^3)/3 + (3*a^3*c^4*x^4)/2 - a^5*c^4*x^6 + (
2*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/4 - (a^8*c^4*x^9)/9
```

sympy [A] time = 0.09, size = 87, normalized size = 1.19

$$-\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{4}+\frac{2a^6c^4x^7}{7}-a^5c^4x^6+\frac{3a^3c^4x^4}{2}-\frac{2a^2c^4x^3}{3}-ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**8*c**4*x**9/9 + a**7*c**4*x**8/4 + 2*a**6*c**4*x**7/7 - a**5*c**4*x**6
+ 3*a**3*c**4*x**4/2 - 2*a**2*c**4*x**3/3 - a*c**4*x**2 + c**4*x
```

3.1229 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=55

$$-\frac{c^3(1-ax)^7}{7a}+\frac{2c^3(1-ax)^6}{3a}-\frac{4c^3(1-ax)^5}{5a}$$

```
[Out] -4/5*c^3*(-a*x+1)^5/a+2/3*c^3*(-a*x+1)^6/a-1/7*c^3*(-a*x+1)^7/a
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$-\frac{c^3(1-ax)^7}{7a}+\frac{2c^3(1-ax)^6}{3a}-\frac{4c^3(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-4*c^3*(1 - a*x)^5)/(5*a) + (2*c^3*(1 - a*x)^6)/(3*a) - (c^3*(1 - a*x)^7)/
(7*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx&=c^3\int(1-ax)^4(1+ax)^2\,dx\\&=c^3\int\left(4(1-ax)^4-4(1-ax)^5+(1-ax)^6\right)dx\\&=-\frac{4c^3(1-ax)^5}{5a}+\frac{2c^3(1-ax)^6}{3a}-\frac{c^3(1-ax)^7}{7a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.56

$$\frac{c^3(ax-1)^5\left(15a^2x^2+40ax+29\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(-1 + a*x)^5*(29 + 40*a*x + 15*a^2*x^2))/(105*a)
```

fricas [A] time = 0.59, size = 69, normalized size = 1.25

$$\frac{1}{7}a^6c^3x^7-\frac{1}{3}a^5c^3x^6-\frac{1}{5}a^4c^3x^5+a^3c^3x^4-\frac{1}{3}a^2c^3x^3-ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 - a*c^3*x^2 + c^3*x
```

giac [A] time = 0.17, size = 66, normalized size = 1.20

$$\frac{\left(15\,c^3-\frac{140\,c^3}{ax+1}+\frac{504\,c^3}{(ax+1)^2}-\frac{840\,c^3}{(ax+1)^3}+\frac{560\,c^3}{(ax+1)^4}\right)(ax+1)^7}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/105*(15*c^3 - 140*c^3/(a*x + 1) + 504*c^3/(a*x + 1)^2 - 840*c^3/(a*x + 1)
^3 + 560*c^3/(a*x + 1)^4)*(a*x + 1)^7/a
```

maple [A] time = 0.03, size = 52, normalized size = 0.95

$$c^3\left(\frac{1}{7}x^7a^6-\frac{1}{3}x^6a^5-\frac{1}{5}a^4x^5+x^4a^3-\frac{1}{3}x^3a^2-a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] c^3*(1/7*x^7*a^6-1/3*x^6*a^5-1/5*a^4*x^5+x^4*a^3-1/3*x^3*a^2-a*x^2+x)
```

maxima [A] time = 0.34, size = 69, normalized size = 1.25

$$\frac{1}{7}a^6c^3x^7 - \frac{1}{3}a^5c^3x^6 - \frac{1}{5}a^4c^3x^5 + a^3c^3x^4 - \frac{1}{3}a^2c^3x^3 - ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 - 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 - 1/3*a^2
*c^3*x^3 - a*c^3*x^2 + c^3*x
```

mupad [B] time = 0.04, size = 69, normalized size = 1.25

$$\frac{a^6\,c^3\,x^7}{7} - \frac{a^5\,c^3\,x^6}{3} - \frac{a^4\,c^3\,x^5}{5} + a^3\,c^3\,x^4 - \frac{a^2\,c^3\,x^3}{3} - a\,c^3\,x^2 + c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^3*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] c^3*x - a*c^3*x^2 - (a^2*c^3*x^3)/3 + a^3*c^3*x^4 - (a^4*c^3*x^5)/5 - (a^5*
c^3*x^6)/3 + (a^6*c^3*x^7)/7
```

sympy [A] time = 0.09, size = 70, normalized size = 1.27

$$\frac{a^6c^3x^7}{7} - \frac{a^5c^3x^6}{3} - \frac{a^4c^3x^5}{5} + a^3c^3x^4 - \frac{a^2c^3x^3}{3} - ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] a**6*c**3*x**7/7 - a**5*c**3*x**6/3 - a**4*c**3*x**5/5 + a**3*c**3*x**4 - a
**2*c**3*x**3/3 - a*c**3*x**2 + c**3*x
```

3.1230 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=37

$$\frac{c^2(1-ax)^5}{5a}-\frac{c^2(1-ax)^4}{2a}$$

```
[Out] -1/2*c^2*(-a*x+1)^4/a+1/5*c^2*(-a*x+1)^5/a
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 43}

$$\frac{c^2(1-ax)^5}{5a}-\frac{c^2(1-ax)^4}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c^2*(1 - a*x)^4)/(2*a) + (c^2*(1 - a*x)^5)/(5*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx&=c^2\int(1-ax)^3(1+ax)\,dx\\&=c^2\int\left(2(1-ax)^3-(1-ax)^4\right)\,dx\\&=-\frac{c^2(1-ax)^4}{2a}+\frac{c^2(1-ax)^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.86

$$c^2\left(-\frac{1}{5}a^4x^5+\frac{a^3x^4}{2}-ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(2*ArcTanh[a*x]),x]
[Out] c^2*(x - a*x^2 + (a^3*x^4)/2 - (a^4*x^5)/5)
```

fricas [A] time = 0.59, size = 37, normalized size = 1.00

$$-\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] -1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 + c^2*x
```

giac [A] time = 0.15, size = 54, normalized size = 1.46

$$-\frac{\left(2\,c^2-\frac{15\,c^2}{ax+1}+\frac{40\,c^2}{(ax+1)^2}-\frac{40\,c^2}{(ax+1)^3}\right)(ax+1)^5}{10\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] -1/10*(2*c^2 - 15*c^2/(a*x + 1) + 40*c^2/(a*x + 1)^2 - 40*c^2/(a*x + 1)^3)*
(a*x + 1)^5/a
```

maple [A] time = 0.02, size = 29, normalized size = 0.78

$$c^2\left(-\frac{1}{5}a^4x^5+\frac{1}{2}x^4a^3-a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/(a*x+1)^2*(-a^2*x^2+1),x)
[Out] c^2*(-1/5*a^4*x^5+1/2*x^4*a^3-a*x^2+x)
```

maxima [A] time = 0.34, size = 37, normalized size = 1.00

$$-\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 + c^2*x
```

mupad [B] time = 0.05, size = 37, normalized size = 1.00

$$-\frac{a^4\,c^2\,x^5}{5}+\frac{a^3\,c^2\,x^4}{2}-a\,c^2\,x^2+c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^2*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] c^2*x - a*c^2*x^2 + (a^3*c^2*x^4)/2 - (a^4*c^2*x^5)/5
```

sympy [A] time = 0.08, size = 36, normalized size = 0.97

$$-\frac{a^4c^2x^5}{5}+\frac{a^3c^2x^4}{2}-ac^2x^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**4*c**2*x**5/5 + a**3*c**2*x**4/2 - a*c**2*x**2 + c**2*x
```

3.1231 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=16

$$-\frac{c(1-ax)^3}{3a}$$

```
[Out] -1/3*c*(-a*x+1)^3/a
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6140, 32}

$$-\frac{c(1-ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(c*(1 - a*x)^3)/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= c\int(1-ax)^2\,dx\\ &= -\frac{c(1-ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.25

$$c\left(\frac{a^2x^3}{3}-ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)/E^(2*ArcTanh[a*x]),x]
[Out] c*(x - a*x^2 + (a^2*x^3)/3)
```

fricas [A] time = 0.67, size = 20, normalized size = 1.25

$$\frac{1}{3}a^2cx^3 - acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] 1/3*a^2*c*x^3 - a*c*x^2 + c*x
```

giac [B] time = 0.18, size = 34, normalized size = 2.12

$$\frac{(ax+1)^3\left(c - \frac{6c}{ax+1} + \frac{12c}{(ax+1)^2}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
[Out] 1/3*(a*x + 1)^3*(c - 6*c/(a*x + 1) + 12*c/(a*x + 1)^2)/a
```

maple [A] time = 0.02, size = 14, normalized size = 0.88

$$\frac{c(ax-1)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/(a*x+1)^2*(-a^2*x^2+1),x)
[Out] 1/3*c*(a*x-1)^3/a
```

maxima [A] time = 0.38, size = 20, normalized size = 1.25

$$\frac{1}{3}a^2cx^3 - acx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
[Out] 1/3*a^2*c*x^3 - a*c*x^2 + c*x
```

mupad [B] time = 0.04, size = 17, normalized size = 1.06

$$\frac{c\,x\,\left(a^2\,x^2-3\,a\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] (c*x*(a^2*x^2 - 3*a*x + 3))/3
```

sympy [A] time = 0.07, size = 19, normalized size = 1.19

$$\frac{a^2cx^3}{3}-acx^2+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] a**2*c*x**3/3 - a*c*x**2 + c*x
```

3.1232 $\int \frac{e^{-2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{ac(ax+1)}$$

[Out] -1/a/c/(a*x+1)

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 32}

$$-\frac{1}{ac(ax+1)}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)),x]

[Out] -(1/(a*c*(1 + a*x)))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx &= \frac{\int \frac{1}{(1+ax)^2}\,dx}{c}\\ &= -\frac{1}{ac(1+ax)}\end{aligned}$$

Mathematica [C] time = 0.02, size = 18, normalized size = 1.20

$$-\frac{e^{-2\tanh^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/2*1/(a*c*E^(2*ArcTanh[a*x]))
```

fricas [A] time = 0.54, size = 14, normalized size = 0.93

$$-\frac{1}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/(a^2*c*x + a*c)
```

giac [A] time = 0.17, size = 15, normalized size = 1.00

$$-\frac{1}{(ax+1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/((a*x + 1)*a*c)
```

maple [A] time = 0.02, size = 16, normalized size = 1.07

$$-\frac{1}{ac\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c/(a*x+1)
```

maxima [A] time = 0.36, size = 14, normalized size = 0.93

$$-\frac{1}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/(a^2*c*x + a*c)
```

mupad [B] time = 0.90, size = 13, normalized size = 0.87

$$-\frac{1}{a\,\left(c+a\,c\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a^2*c*x^2)*(a*x + 1)^2),x)
[Out] -1/(a*(c + a*c*x))
```

sympy [A] time = 0.12, size = 12, normalized size = 0.80

$$-\frac{1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c),x)
[Out] -1/(a**2*c*x + a*c)
```

3.1233 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4ac^2(ax+1)}-\frac{1}{4ac^2(ax+1)^2}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] -1/4/a/c^2/(a*x+1)^2-1/4/a/c^2/(a*x+1)+1/4*arctanh(a*x)/a/c^2
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$-\frac{1}{4ac^2(ax+1)}-\frac{1}{4ac^2(ax+1)^2}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] -1/(4*a*c^2*(1 + a*x)^2) - 1/(4*a*c^2*(1 + a*x)) + ArcTanh[a*x]/(4*a*c^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{1}{(1-ax)(1+ax)^3}\,dx}{c^2}$$
$$= \frac{\int \left(\frac{1}{2(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^2}$$
$$= -\frac{1}{4ac^2(1+ax)^2}-\frac{1}{4ac^2(1+ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2}$$
$$= -\frac{1}{4ac^2(1+ax)^2}-\frac{1}{4ac^2(1+ax)}+\frac{\tanh^{-1}(ax)}{4ac^2}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.67

$$\frac{-ax+(ax+1)^2\tanh^{-1}(ax)-2}{4a(acx+c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] (-2 - a*x + (1 + a*x)^2*ArcTanh[a*x])/(4*a*(c + a*c*x)^2)
```

fricas [A] time = 1.09, size = 76, normalized size = 1.55

$$-\frac{2\,ax-\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2+2\,ax+1\right)\log\left(ax-1\right)+4}{8\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*x - (a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 + 2*a*x + 1)*lo
g(a*x - 1) + 4)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.28, size = 55, normalized size = 1.12

$$-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^2}-\frac{\frac{ac^2}{ax+1}+\frac{ac^2}{(ax+1)^2}}{4\,a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(-2/(a*x + 1) + 1))/(a*c^2) - 1/4*(a*c^2/(a*x + 1) + a*c^2/(a*x
 + 1)^2)/(a^2*c^4)
```

maple [A] time = 0.03, size = 60, normalized size = 1.22

$$-\frac{\ln (ax-1)}{8c^2a}-\frac{1}{4a\,c^2\,(ax+1)^2}-\frac{1}{4a\,c^2\,(ax+1)}+\frac{\ln (ax+1)}{8a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/8/c^2/a*ln(a*x-1)-1/4/a/c^2/(a*x+1)^2-1/4/a/c^2/(a*x+1)+1/8*ln(a*x+1)/a/
c^2
```

maxima [A] time = 0.36, size = 63, normalized size = 1.29

$$-\frac{ax+2}{4\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}+\frac{\log (ax+1)}{8\,ac^2}-\frac{\log (ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) + 1/8*log(a*x + 1)/(a*c^
2) - 1/8*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.07, size = 47, normalized size = 0.96

$$\frac{\operatorname{atanh}(a\,x)}{4\,a\,c^2}-\frac{\frac{x}{4}+\frac{1}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a^2*c*x^2)^2*(a*x + 1)^2),x)
```

```
[Out] atanh(a*x)/(4*a*c^2) - (x/4 + 1/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x)
```

sympy [A] time = 0.28, size = 56, normalized size = 1.14

$$-\frac{ax+2}{4a^3c^2x^2+8a^2c^2x+4ac^2}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -(a*x + 2)/(4*a**3*c**2*x**2 + 8*a**2*c**2*x + 4*a*c**2) - (log(x - 1/a)/8
- log(x + 1/a)/8)/(a*c**2)
```

3.1234 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^3} dx$

Optimal. Leaf size=84

$$\frac{1}{16ac^3(1-ax)}-\frac{3}{16ac^3(ax+1)}-\frac{1}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] 1/16/a/c^3/(-a*x+1)-1/12/a/c^3/(a*x+1)^3-1/8/a/c^3/(a*x+1)^2-3/16/a/c^3/(a*
x+1)+1/4*arctanh(a*x)/a/c^3
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{1}{16ac^3(1-ax)}-\frac{3}{16ac^3(ax+1)}-\frac{1}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] 1/(16*a*c^3*(1 - a*x)) - 1/(12*a*c^3*(1 + a*x)^3) - 1/(8*a*c^3*(1 + a*x)^2)
 - 3/(16*a*c^3*(1 + a*x)) + ArcTanh[a*x]/(4*a*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{1}{(1-ax)^2(1+ax)^4}\,dx}{c^3}$$

$$= \frac{\int \left(\frac{1}{16(-1+ax)^2}+\frac{1}{4(1+ax)^4}+\frac{1}{4(1+ax)^3}+\frac{3}{16(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^3}$$

$$= \frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}-\frac{1}{8ac^3(1+ax)^2}-\frac{3}{16ac^3(1+ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}$$

$$= \frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}-\frac{1}{8ac^3(1+ax)^2}-\frac{3}{16ac^3(1+ax)}+\frac{\tanh^{-1}(ax)}{4ac^3}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.73

$$-\frac{3a^3x^3+6a^2x^2+ax-3(ax-1)(ax+1)^3\tanh^{-1}(ax)-4}{12a(ax-1)(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/12*(-4 + a*x + 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)*(1 + a*x)^3*ArcTanh[
a*x])/(a*(-1 + a*x)*(c + a*c*x)^3)
```

fricas [A] time = 0.62, size = 121, normalized size = 1.44

$$-\frac{6\,a^3x^3+12\,a^2x^2+2\,ax-3\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(ax+1\right)+3\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(ax-1\right)-8}{24\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^3*x^3 + 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)
*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x - 1) - 8)/(a^5*
c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.35, size = 97, normalized size = 1.15

$$-\frac{\log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{8\,ac^3}+\frac{1}{32\,ac^3\left(\frac{2}{ax+1}-1\right)}-\frac{\frac{9\,a^5c^6}{ax+1}+\frac{6\,a^5c^6}{(ax+1)^2}+\frac{4\,a^5c^6}{(ax+1)^3}}{48\,a^6c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(-2/(a*x + 1) + 1))/(a*c^3) + 1/32/(a*c^3*(2/(a*x + 1) - 1)) -
1/48*(9*a^5*c^6/(a*x + 1) + 6*a^5*c^6/(a*x + 1)^2 + 4*a^5*c^6/(a*x + 1)^3)/
(a^6*c^9)
```

maple [A] time = 0.04, size = 90, normalized size = 1.07

$$-\frac{1}{16c^3a(ax-1)}-\frac{\ln(ax-1)}{8ac^3}-\frac{1}{12ac^3(ax+1)^3}-\frac{1}{8ac^3(ax+1)^2}-\frac{3}{16ac^3(ax+1)}+\frac{\ln(ax+1)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/16/c^3/a/(a*x-1)-1/8/a/c^3*ln(a*x-1)-1/12/a/c^3/(a*x+1)^3-1/8/a/c^3/(a*x
+1)^2-3/16/a/c^3/(a*x+1)+1/8*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.36, size = 91, normalized size = 1.08

$$-\frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}+\frac{\log(ax+1)}{8\,ac^3}-\frac{\log(ax-1)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^3*x^3 + 6*a^2*x^2 + a*x - 4)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^
2*c^3*x - a*c^3) + 1/8*log(a*x + 1)/(a*c^3) - 1/8*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.09, size = 72, normalized size = 0.86

$$\frac{\frac{x}{12}+\frac{a\,x^2}{2}-\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3}+\frac{\mathrm{atanh}(a\,x)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a^2*c*x^2)^3*(a*x + 1)^2),x)
```

```
[Out] (x/12 + (a*x^2)/2 - 1/(3*a) + (a^2*x^3)/4)/(c^3 - 2*a^3*c^3*x^3 - a^4*c^3*x
^4 + 2*a*c^3*x) + atanh(a*x)/(4*a*c^3)
```

sympy [A] time = 0.41, size = 83, normalized size = 0.99

$$\frac{-3a^3x^3-6a^2x^2-ax+4}{12a^5c^3x^4+24a^4c^3x^3-24a^2c^3x-12ac^3}+\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-3*a**3*x**3 - 6*a**2*x**2 - a*x + 4)/(12*a**5*c**3*x**4 + 24*a**4*c**3*x*
*3 - 24*a**2*c**3*x - 12*a*c**3) + (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c*
*3)
```

3.1235 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=119

$$\frac{5}{64ac^4(1-ax)}-\frac{5}{32ac^4(ax+1)}+\frac{1}{64ac^4(1-ax)^2}-\frac{3}{32ac^4(ax+1)^2}-\frac{1}{16ac^4(ax+1)^3}-\frac{1}{32ac^4(ax+1)^4}+\frac{15\tanh^{-1}(ax)}{64ac^4}$$

```
[Out] 1/64/a/c^4/(-a*x+1)^2+5/64/a/c^4/(-a*x+1)-1/32/a/c^4/(a*x+1)^4-1/16/a/c^4/(
a*x+1)^3-3/32/a/c^4/(a*x+1)^2-5/32/a/c^4/(a*x+1)+15/64*arctanh(a*x)/a/c^4
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6140, 44, 207}

$$\frac{5}{64ac^4(1-ax)}-\frac{5}{32ac^4(ax+1)}+\frac{1}{64ac^4(1-ax)^2}-\frac{3}{32ac^4(ax+1)^2}-\frac{1}{16ac^4(ax+1)^3}-\frac{1}{32ac^4(ax+1)^4}+\frac{15\tanh^{-1}(ax)}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] 1/(64*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(32*a*c^4*(1 + a*x)^4
) - 1/(16*a*c^4*(1 + a*x)^3) - 3/(32*a*c^4*(1 + a*x)^2) - 5/(32*a*c^4*(1 +
a*x)) + (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{1}{(1-ax)^3(1+ax)^5}\,dx}{c^4}$$

$$= \frac{\int \left(-\frac{1}{32(-1+ax)^3}+\frac{5}{64(-1+ax)^2}+\frac{1}{8(1+ax)^5}+\frac{3}{16(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{5}{32(1+ax)^2}-\frac{15}{64\left(-1+a^2x^2\right)}\right)dx}{c^4}$$

$$= \frac{1}{64ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{32ac^4(1+ax)^4}-\frac{1}{16ac^4(1+ax)^3}-\frac{3}{32ac^4(1+ax)^2}-\frac{}{32a}$$

$$= \frac{1}{64ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{32ac^4(1+ax)^4}-\frac{1}{16ac^4(1+ax)^3}-\frac{3}{32ac^4(1+ax)^2}-\frac{}{32a}$$

Mathematica [A] time = 0.07, size = 80, normalized size = 0.67

$$\frac{-15a^5x^5-30a^4x^4+10a^3x^3+50a^2x^2+17ax+15(ax-1)^2(ax+1)^4\tanh^{-1}(ax)-16}{64a(ax-1)^2(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (-16 + 17*a*x + 50*a^2*x^2 + 10*a^3*x^3 - 30*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)^2*(1 + a*x)^4*ArcTanh[a*x])/(64*a*(-1 + a*x)^2*(c + a*c*x)^4)
```

fricas [B] time = 0.89, size = 217, normalized size = 1.82

$$-\frac{30\,a^5x^5+60\,a^4x^4-20\,a^3x^3-100\,a^2x^2-34\,ax-15\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\log\left(ax+\right.}{128\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/128*(30*a^5*x^5 + 60*a^4*x^4 - 20*a^3*x^3 - 100*a^2*x^2 - 34*a*x - 15*(a
^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1
) + 15*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*lo
g(a*x - 1) + 32)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3
 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.18, size = 122, normalized size = 1.03

$$-\frac{15 \log\left(\left|-\frac{2}{ax+1}+1\right|\right)}{128\,ac^4}+\frac{\frac{24}{ax+1}-11}{256\,ac^4\left(\frac{2}{ax+1}-1\right)^2}-\frac{\frac{5\,a^{11}c^{12}}{ax+1}+\frac{3\,a^{11}c^{12}}{(ax+1)^2}+\frac{2\,a^{11}c^{12}}{(ax+1)^3}+\frac{a^{11}c^{12}}{(ax+1)^4}}{32\,a^{12}c^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(-2/(a*x + 1) + 1))/(a*c^4) + 1/256*(24/(a*x + 1) - 11)/(a*c
^4*(2/(a*x + 1) - 1)^2) - 1/32*(5*a^11*c^12/(a*x + 1) + 3*a^11*c^12/(a*x +
1)^2 + 2*a^11*c^12/(a*x + 1)^3 + a^11*c^12/(a*x + 1)^4)/(a^12*c^16)
```

maple [A] time = 0.04, size = 120, normalized size = 1.01

$$\frac{1}{64c^4a\,(ax-1)^2}-\frac{5}{64c^4a\,(ax-1)}-\frac{15\ln(ax-1)}{128c^4a}-\frac{1}{32a\,c^4\,(ax+1)^4}-\frac{1}{16a\,c^4\,(ax+1)^3}-\frac{3}{32a\,c^4\,(ax+1)^2}-\frac{5}{32a\,c^4\,(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/64/c^4/a/(a*x-1)^2-5/64/c^4/a/(a*x-1)-15/128/c^4/a*ln(a*x-1)-1/32/a/c^4/(
a*x+1)^4-1/16/a/c^4/(a*x+1)^3-3/32/a/c^4/(a*x+1)^2-5/32/a/c^4/(a*x+1)+15/12
8*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.34, size = 140, normalized size = 1.18

$$-\frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}+\frac{15\,\log(ax+1)}{128\,ac^4}-\frac{15\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/64*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/(a^
7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a
^2*c^4*x + a*c^4) + 15/128*log(a*x + 1)/(a*c^4) - 15/128*log(a*x - 1)/(a*c^
4)
```

mupad [B] time = 0.99, size = 120, normalized size = 1.01

$$\frac{\frac{17\,x}{64}+\frac{25\,a\,x^2}{32}-\frac{1}{4\,a}+\frac{5\,a^2\,x^3}{32}-\frac{15\,a^3\,x^4}{32}-\frac{15\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4}+\frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a^2*c*x^2)^4*(a*x + 1)^2),x)
```

```
[Out] ((17*x)/64 + (25*a*x^2)/32 - 1/(4*a) + (5*a^2*x^3)/32 - (15*a^3*x^4)/32 - (
15*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^4*x^4 + 2*a^5*c^
4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) + (15*atanh(a*x))/(64*a*c^4)
```

sympy [A] time = 0.58, size = 143, normalized size = 1.20

$$-\frac{15a^5x^5+30a^4x^4-10a^3x^3-50a^2x^2-17ax+16}{64a^7c^4x^6+128a^6c^4x^5-64a^5c^4x^4-256a^4c^4x^3-64a^3c^4x^2+128a^2c^4x+64ac^4}-\frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128}-\frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] -(15*a**5*x**5 + 30*a**4*x**4 - 10*a**3*x**3 - 50*a**2*x**2 - 17*a*x + 16)/
(64*a**7*c**4*x**6 + 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 - 256*a**4*c**4
*x**3 - 64*a**3*c**4*x**2 + 128*a**2*c**4*x + 64*a*c**4) - (15*log(x - 1/a)
/128 - 15*log(x + 1/a)/128)/(a*c**4)
```

3.1236 $\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$-\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] -3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4-3/5*x^2*(-a^2*c*x
^2+c)^(1/2)/a^2+1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a-1/5*x^4*(-a^2*c*x^2+c)^(1/2)
-3/20*(-5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

Rubi [A] time = 0.33, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6152, 1809, 833, 780, 217, 203}

$$-\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) + (x^3*Sqrt[c - a^2*c*x^2])/(2*a) - (x
^4*Sqrt[c - a^2*c*x^2])/5 - (3*(8 - 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) -
(3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = c\int \frac{x^3(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{\int \frac{x^3(-9a^2c+10a^3cx)}{\sqrt{c-a^2cx^2}}dx}{5a^2}$$
$$= \frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x^2(-30a^3c^2+36a^4c^2x)}{\sqrt{c-a^2cx^2}}dx}{20a^4c}$$
$$= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{\int \frac{x(-72a^4c^3+90a^5c^3x)}{\sqrt{c-a^2cx^2}}dx}{60a^6c^2}$$
$$= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}$$
$$= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}$$
$$= -\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} - \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}$$

Mathematica [A] time = 0.15, size = 96, normalized size = 0.70

$$\frac{15\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(a^2x^2-1)}\right)+\left(-4a^4x^4+10a^3x^3-12a^2x^2+15ax-24\right)\sqrt{c-a^2cx^2}}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-24 + 15*a*x - 12*a^2*x^2 + 10*a^3*x^3 - 4*a^4*x^4) +
 15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20
*a^4)
```

fricas [A] time = 0.60, size = 184, normalized size = 1.34

$$\left[-\frac{2\left(4a^4x^4-10a^3x^3+12a^2x^2-15ax+24\right)\sqrt{-a^2cx^2+c}-15\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{40a^4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] [-1/40*(2*(4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-a^2*c*x
^2 + c) - 15*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
 - c))/a^4, -1/20*((4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt
(-a^2*c*x^2 + c) - 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*
c*x^2 - c)))/a^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 202, normalized size = 1.47

$$\frac{x^{2} \left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{5 a^{2} c}+\frac{4 \left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{5 c\,a^{4}}-\frac{x \left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{2 a^{3} c}+\frac{5 x \sqrt{-a^{2} c\,x^{2}+c}}{4 a^{3}}+\frac{5 c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c\,x^{2}+c}}\right)}{4 a^{3} \sqrt{a^{2} c}}-\frac{2 \sqrt{-\left(x+\frac{1}{a}\right)^{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c+4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)-1/2/a^3*x
*(-a^2*c*x^2+c)^(3/2)/c+5/4/a^3*x*(-a^2*c*x^2+c)^(1/2)+5/4/a^3*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^4*(-(x+1/a)^2*a^2*c+2*
a*c*(x+1/a))^(1/2)-2/a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2
*a^2*c+2*a*c*(x+1/a))^(1/2))
```

maxima [A] time = 0.44, size = 117, normalized size = 0.85

$$\frac{\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} x^{2}}{5\, a^{2} c}+\frac{5 \sqrt{-a^{2} c x^{2}+c}\, x}{4\, a^{3}}-\frac{\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} x}{2\, a^{3} c}-\frac{3 \sqrt{c}\, \arcsin \left(a x\right)}{4\, a^{4}}-\frac{2 \sqrt{-a^{2} c x^{2}+c}}{a^{4}}+\frac{4 \left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}}}{5\, a^{4} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] 1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) + 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 - 1
/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) - 3/4*sqrt(c)*arcsin(a*x)/a^4 - 2*sqrt(
-a^2*c*x^2 + c)/a^4 + 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x^3*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x^{3}\sqrt{-a^{2}cx^{2}+c}}{ax+1}\right)dx-\int \frac{ax^{4}\sqrt{-a^{2}cx^{2}+c}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**3*sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x**4*sqrt
(-a**2*c*x**2 + c)/(a*x + 1), x)
```

3.1237 $\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}+\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] 7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3+2/3*x^2*(-a^2*c*x^
2+c)^(1/2)/a-1/4*x^3*(-a^2*c*x^2+c)^(1/2)+1/24*(-21*a*x+32)*(-a^2*c*x^2+c)^
(1/2)/a^3
```

Rubi [A] time = 0.28, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6152, 1809, 833, 780, 217, 203}

$$-\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}+\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) - (x^3*Sqrt[c - a^2*c*x^2])/4 + ((32 - 21
*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) + (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(8*a^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = c\int \frac{x^2(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{\int \frac{x^2(-7a^2c+8a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{\int \frac{x(-16a^3c^2+21a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} + \frac{(7c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,d}{8a^2}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} + \frac{(7c)\,\mathrm{Subst}\left(\int \frac{}{1+}\right.}{}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} - \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} + \frac{7\sqrt{c}\tan^{-1}\left(\frac{a}{\sqrt{c-}}\right.}{8a^3}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.79

$$\frac{(-6a^3x^3+16a^2x^2-21ax+32)\sqrt{c-a^2cx^2} - 21\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}(a^2x^2-1)}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(32 - 21*a*x + 16*a^2*x^2 - 6*a^3*x^3) - 21*Sqrt[c]*Ar
cTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

fricas [A] time = 0.73, size = 168, normalized size = 1.50

$$\left[-\frac{2(6a^3x^3-16a^2x^2+21ax-32)\sqrt{-a^2cx^2+c} - 21\sqrt{-c}\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{48a^3}, -\frac{(6a^3x^3-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] [-1/48*(2*(6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) - 21*
sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, -
1/24*((6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt
(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

giac [B] time = 0.41, size = 221, normalized size = 1.97

$$-\frac{\left(336\,a^{5}\sqrt{c}\,\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)+\frac{75\,a^{5}\left(c-\frac{2\,c}{ax+1}\right)^{3}c\sqrt{-c+\frac{2\,c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)-83\,a^{5}\left(c-\frac{2\,c}{ax+1}\right)^{2}c^{2}\sqrt{-c+\frac{2\,c}{ax+1}}}{}\right.}{192\,a^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] -1/192*(336*a^5*sqrt(c)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x
 + 1))*sgn(a) + (75*a^5*(c - 2*c/(a*x + 1))^3*c*sqrt(-c + 2*c/(a*x + 1))*sg
n(1/(a*x + 1))*sgn(a) - 83*a^5*(c - 2*c/(a*x + 1))^2*c^2*sqrt(-c + 2*c/(a*x
 + 1))*sgn(1/(a*x + 1))*sgn(a) - 21*a^5*c^4*sqrt(-c + 2*c/(a*x + 1))*sgn(1/
(a*x + 1))*sgn(a) - 77*a^5*c^3*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*
sgn(a))*(a*x + 1)^4/c^4)*abs(a)/a^9
```

maple [A] time = 0.04, size = 178, normalized size = 1.59

$$\frac{x\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{4a^{2}c}-\frac{9x\sqrt{-a^{2}c\,x^{2}+c}}{8a^{2}}-\frac{9c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{8a^{2}\sqrt{a^{2}c}}-\frac{2\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{3a^{3}c}+\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}c+2ac\left(x+\frac{1}{a}\right)}}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c-9/8/a^2*x*(-a^2*c*x^2+c)^(1/2)-9/8/a^2*c/(
a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3/a^3*(-a^2*c*x
^2+c)^(3/2)/c+2/a^3*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2)+2/a^2*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2))
```

maxima [A] time = 0.43, size = 93, normalized size = 0.83

$$-\frac{9\sqrt{-a^{2}cx^{2}+c}\,x}{8\,a^{2}}+\frac{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}x}{4\,a^{2}c}+\frac{7\sqrt{c}\,\arcsin\left(ax\right)}{8\,a^{3}}+\frac{2\sqrt{-a^{2}cx^{2}+c}}{a^{3}}-\frac{2\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}}{3\,a^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] -9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 + 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) + 7/
8*sqrt(c)*arcsin(a*x)/a^3 + 2*sqrt(-a^2*c*x^2 + c)/a^3 - 2/3*(-a^2*c*x^2 +
c)^(3/2)/(a^3*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x^2*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(- \frac{x^{2} \sqrt{- a^{2} c x^{2} + c}}{a x + 1}\right)\, dx - \int \frac{a x^{3} \sqrt{- a^{2} c x^{2} + c}}{a x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**2*sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x**3*sqrt
(-a**2*c*x**2 + c)/(a*x + 1), x)
```

3.1238 $\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=85

$$-\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] -arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2-1/3*x^2*(-a^2*c*x^2+c
)^(1/2)-1/3*(-3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

Rubi [A] time = 0.18, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6152, 1809, 780, 217, 203}

$$-\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -(x^2*Sqrt[c - a^2*c*x^2])/3 - ((5 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) -
(Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= c\int\frac{x(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{\int\frac{x\left(-5a^2c+6a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\
&= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\int\frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\,\text{Subst}\left(\int\frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\
&= -\frac{1}{3}x^2\sqrt{c-a^2cx^2}-\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.94

$$\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-\left(a^2x^2-3ax+5\right)\sqrt{c-a^2cx^2}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (-((5 - 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2]) + 3*Sqrt[c]*ArcTan[(a*x*Sqrt[
c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

fricas [A] time = 0.61, size = 150, normalized size = 1.76

$$\left[-\frac{2\sqrt{-a^2cx^2+c}\left(a^2x^2-3\,ax+5\right)-3\sqrt{-c}\,\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{6\,a^2},-\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="frica
s")
```

```
[Out] [-1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(-c)*log(2*a^2*
c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, -1/3*(sqrt(-a^2*c*x^
2 + c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt
(c)*x/(a^2*c*x^2 - c)))/a^2]
```

giac [B] time = 0.41, size = 174, normalized size = 2.05

$$\frac{\left(24\,a^4\sqrt{c}\,\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)-\frac{\left(9\,a^4\left(c-\frac{2\,c}{ax+1}\right)^2c\sqrt{-c+\frac{2\,c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)+3\,a^4c^3\sqrt{-c+\frac{2\,c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a\right.}{c^3}\right.}{12\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac"
)
```

```
[Out] 1/12*(24*a^4*sqrt(c)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x +
1))*sgn(a) - (9*a^4*(c - 2*c/(a*x + 1))^2*c*sqrt(-c + 2*c/(a*x + 1))*sgn(1/
(a*x + 1))*sgn(a) + 3*a^4*c^3*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn
(a) + 8*a^4*c^2*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))*(a*x +
1)^3/c^3)*abs(a)/a^7
```

maple [B] time = 0.04, size = 154, normalized size = 1.81

$$\frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{3a^2c}+\frac{x\sqrt{-a^2c\,x^2+c}}{a}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{a\sqrt{a^2c}}-\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2ac\left(x+\frac{1}{a}\right)}}{a^2}-\frac{2c\arctan\left(\frac{\sqrt{}}{\sqrt{-\left(x+\frac{1}{a}\right)^2}}\right.}{a\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] 1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c+x/a*(-a^2*c*x^2+c)^(1/2)+1/a*c/(a^2*c)^(1/2)
*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^2*(-(x+1/a)^2*a^2*c+2*a*c
*(x+1/a))^(1/2)-2/a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*
c+2*a*c*(x+1/a))^(1/2))
```

maxima [A] time = 0.43, size = 70, normalized size = 0.82

$$\frac{\sqrt{-a^2cx^2+c}\,x}{a}-\frac{\sqrt{c}\,\arcsin\left(ax\right)}{a^2}-\frac{2\sqrt{-a^2cx^2+c}}{a^2}+\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxim
a")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*x/a - sqrt(c)*arcsin(a*x)/a^2 - 2*sqrt(-a^2*c*x^2 + c)
/a^2 + 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x\sqrt{-a^2cx^2+c}}{ax+1}\right)dx-\int\frac{ax^2\sqrt{-a^2cx^2+c}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x*sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x**2*sqrt(-a
**2*c*x**2 + c)/(a*x + 1), x)
```

3.1239 $\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=87

$$\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] 3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a+3/2*(-a^2*c*x^2+c)^(
1/2)/a+1/2*(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6142, 671, 641, 217, 203}

$$\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) + (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
```

```
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\ &= \frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}(3c)\int \frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx\\ &= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}(3c)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\ &= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 99, normalized size = 1.14

$$\frac{\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(a^2x^2-5ax+4\right)-6\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqrt[1 - a*x]
*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.63, size = 134, normalized size = 1.54

$$\left[-\frac{2\sqrt{-a^2cx^2+c}\,(ax-4)-3\sqrt{-c}\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{4a}, -\frac{\sqrt{-a^2cx^2+c}\,(ax-4)+3\sqrt{c}\text{ ar}}{2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) - 3*sqrt(-c)*log(2*a^2*c*x^2 + 2*sq
rt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, -1/2*(sqrt(-a^2*c*x^2 + c)*(a*x -
4) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

giac [A] time = 0.40, size = 126, normalized size = 1.45

$$-\frac{\left(12a^3\sqrt{c}\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{c}}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)-\frac{\left(3a^3c^2\sqrt{-c+\frac{2c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)+5a^3c\left(-c+\frac{2c}{ax+1}\right)^{\frac{3}{2}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)\right)(ax+1}{c^2}\right.}{4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*(12*a^3*sqrt(c)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x +
1))*sgn(a) - (3*a^3*c^2*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) +
5*a^3*c*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))*(a*x + 1)^2/c^2
)*abs(a)/a^5
```

maple [A] time = 0.04, size = 126, normalized size = 1.45

$$-\frac{x\sqrt{-a^2c\,x^2+c}}{2}-\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}+\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2ac\left(x+\frac{1}{a}\right)}}{a}+\frac{2c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2ac\left(x+\frac{1}{a}\right)}}\right)}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/2*x*(-a^2*c*x^2+c)^(1/2)-1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^
2*c*x^2+c)^(1/2))+2/a*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2)+2*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2))
```

maxima [A] time = 0.44, size = 47, normalized size = 0.54

$$-\frac{1}{2}\sqrt{-a^2cx^2+c}\,x+\frac{3\sqrt{c}\arcsin(ax)}{2a}+\frac{2\sqrt{-a^2cx^2+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*x + 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2
 + c)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{-a^2cx^2+c}}{ax+1}\right)dx-\int\frac{ax\sqrt{-a^2cx^2+c}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x*sqrt(-a**2*c
*x**2 + c)/(a*x + 1), x)
```

3.1240 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=78

$$-\sqrt{c-a^2cx^2}-2\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)-arctanh((-a^2*c*x^2+c)^
(1/2)/c^(1/2))*c^(1/2)-(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6152, 1809, 844, 217, 203, 266, 63, 208}

$$-\sqrt{c-a^2cx^2}-2\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
- Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = c\int \frac{(1-ax)^2}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= -\sqrt{c-a^2cx^2} - \frac{\int \frac{-a^2c+2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2}$$
$$= -\sqrt{c-a^2cx^2} + c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - (2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= -\sqrt{c-a^2cx^2} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) - (2ac)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\right.$$
$$= -\sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a^2}$$
$$= -\sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.10, size = 99, normalized size = 1.27

$$-\sqrt{c-a^2cx^2} - \sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + 2\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) + \sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*
(-1 + a^2*x^2))] + Sqrt[c]*Log[x] - Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*
x^2]]
```

fricas [A] time = 0.55, size = 196, normalized size = 2.51

$$\left[2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right) + \frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right) - \sqrt{-a^2cx^2+c}, -\sqrt{-c}\,\text{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="frica
s")
```

```
[Out] [2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*s
qrt(c)*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) - sqrt(
```

```
-a^2*c*x^2 + c), -sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2
- c)) + sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
 - sqrt(-a^2*c*x^2 + c)]
```

giac [A] time = 0.33, size = 124, normalized size = 1.59

$$-\left(\frac{(ax+1)\sqrt{-c+\frac{2c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{a^2}-\frac{2\,c\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{-c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{a^2\sqrt{-c}}-\frac{4\sqrt{c}\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{c}}\right)\mathrm{s}}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="giac"
)
```

```
[Out] -((a*x + 1)*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a)/a^2 - 2*c*arct
an(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))*sgn(a)/(a^2*sqrt(-c)
) - 4*sqrt(c)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x + 1))*sgn
(a)/a^2)*a*abs(a)
```

maple [A] time = 0.04, size = 120, normalized size = 1.54

$$\sqrt{-a^2c\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)-2\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2ac\left(x+\frac{1}{a}\right)}-\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2ac\left(x\right.}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x)
```

```
[Out] (-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)-2*(
-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2)-2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1
/2)*x/(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}{(ax+1)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x,x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}{x\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x*(a*x + 1)^2),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{-a^2cx^2+c}}{ax^2+x}\right)dx-\int \frac{ax\sqrt{-a^2cx^2+c}}{ax^2+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**2 + x), x) - Integral(a*x*sqrt(-a**
2*c*x**2 + c)/(a*x**2 + x), x)
```

3.1241 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+2*a*arctanh((-a^2*c*x^2+
c)^(1/2)/c^(1/2))*c^(1/2)-(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.25, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6152, 1807, 844, 217, 203, 266, 63, 208}

$$-\frac{\sqrt{c-a^2cx^2}}{x}+a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) + a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^
2]] + 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = c\int \frac{(1-ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x} - \int \frac{2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x} - (2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx + \left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x} - (ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) + \left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\right.$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + 2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.11, size = 106, normalized size = 1.29

$$-\frac{\sqrt{c-a^2cx^2}}{x} + 2a\sqrt{c}\,\log\left(\sqrt{c}\,\sqrt{c-a^2cx^2}+c\right) - a\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,\left(a^2x^2-1\right)}\right) - 2a\sqrt{c}\,\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/x) - a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt
[c]*(-1 + a^2*x^2))] - 2*a*Sqrt[c]*Log[x] + 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqr
t[c - a^2*c*x^2]]
```

fricas [A] time = 0.62, size = 211, normalized size = 2.57

$$\left[-\frac{a\sqrt{c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right) - a\sqrt{c}\,x\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right) + \sqrt{-a^2cx^2+c}}{x}, \frac{4\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}}{a^2cx^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="fri
cas")
```

```
[Out] [-(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - a
*sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + s
qrt(-a^2*c*x^2 + c))/x, 1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqr
t(-c)/(a^2*c*x^2 - c)) + a*sqrt(-c)*x*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 +
 c)*a*sqrt(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c))/x]
```

giac [B] time = 0.23, size = 168, normalized size = 2.05

$$-\left(\frac{4\,c\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{-c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{\sqrt{-c}}+2\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)-\frac{\left(\pi c+2\,\sqrt{-c}\,\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="gia
c")
```

```
[Out] -(4*c*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))*sgn(a)/sqr
t(-c) + 2*sqrt(c)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x + 1))
*sgn(a) - (pi*c + 2*sqrt(-c)*sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - c)*sgn(1/(a
*x + 1))*sgn(a)/sqrt(-c) + c*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(
a)/(c - c/(a*x + 1)))*abs(a)
```

maple [B] time = 0.04, size = 203, normalized size = 2.48

$$2\sqrt{c}\;\ln\left(\frac{2c+2\sqrt{c}\;\sqrt{-a^{2}c\,x^{2}+c}}{x}\right)a-2\sqrt{-a^{2}c\,x^{2}+c}\;a-\frac{\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{cx}-a^{2}x\sqrt{-a^{2}c\,x^{2}+c}-\frac{a^{2}c\arctan\left(\frac{\sqrt{a^{2}c}\;x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{\sqrt{a^{2}c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x)
```

```
[Out] 2*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a-2*(-a^2*c*x^2+c)^(1/
2)*a-1/c/x*(-a^2*c*x^2+c)^(3/2)-a^2*x*(-a^2*c*x^2+c)^(1/2)-a^2*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*a*(-(x+1/a)^2*a^2*c+2*a*
c*(x+1/a))^(1/2)+2*a^2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a
^2*c+2*a*c*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-a^{2}cx^{2}+c}\left(a^{2}x^{2}-1\right)}{\left(ax+1\right)^{2}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^2,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}{x^2\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x^2*(a*x + 1)^2),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x^2*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{-a^{2}cx^{2}+c}}{ax^{3}+x^{2}}\right)dx-\int \frac{ax\sqrt{-a^{2}cx^{2}+c}}{ax^{3}+x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**2,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**3 + x**2), x) - Integral(a*x*sqrt(-
a**2*c*x**2 + c)/(a*x**3 + x**2), x)
```

3.1242 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{3}{2}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-1/2*(-a^2*c*x^2+c)^(
1/2)/x^2+2*a*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.24, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6152, 1807, 807, 266, 63, 208}

$$\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{3}{2}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2) + (2*a*Sqrt[c - a^2*c*x^2])/x - (3*a^2*Sqrt[c]
*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= c\int \frac{(1-ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{1}{2}\int \frac{4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{1}{4}\left(3a^2c\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{3}{2}\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)\\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{3}{2}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 76, normalized size = 0.97

$$\frac{1}{2}\left(\frac{(4ax-1)\sqrt{c-a^2cx^2}}{x^2}-3a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+3a^2\sqrt{c}\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^3),x]
```

```
[Out] (((-1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 + 3*a^2*Sqrt[c]*Log[x] - 3*a^2*Sqrt
[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

fricas [A] time = 0.64, size = 149, normalized size = 1.91

$$\left[\frac{3\,a^2\sqrt{c}\,x^2\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}\,(4\,ax-1)}{4\,x^2},-\frac{3\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a}}{2\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="fri
cas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 1))/x^2, -1/2*(3*a^2*sqrt(-c)*x
^2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^2
+ c)*(4*a*x - 1))/x^2]
```

giac [B] time = 3.09, size = 152, normalized size = 1.95

$$\frac{1}{4}\left(\frac{12\,ac\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{-c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{\sqrt{-c}}-\frac{(3\,\pi ac-8\,ac)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{\sqrt{-c}}+\frac{3\,ac^2\sqrt{-c+\frac{2\,c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="gia
c")
```

```
[Out] 1/4*(12*a*c*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))*sgn(
a)/sqrt(-c) - (3*pi*a*c - 8*a*c)*sgn(1/(a*x + 1))*sgn(a)/sqrt(-c) + (3*a*c^
2*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) - 5*a*c*(-c + 2*c/(a*x +
 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))/(c - c/(a*x + 1))^2)*abs(a)
```

maple [B] time = 0.05, size = 231, normalized size = 2.96

$$-\frac{3\sqrt{c}\ \ln\left(\frac{2c+2\sqrt{c}\ \sqrt{-a^2c\,x^2+c}}{x}\right)a^2}{2}+\frac{3\sqrt{-a^2c\,x^2+c}\ a^2}{2}+\frac{2a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}+2a^3x\sqrt{-a^2c\,x^2+c}+\frac{2a^3c\arctan\left(\frac{\sqrt{a^2c}\ x}{\sqrt{-a^2c\,x^2}}\right)}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x)
```

```
[Out] -3/2*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^2+3/2*(-a^2*c*x^2
+c)^(1/2)*a^2+2*a/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^3*x*(-a^2*c*x^2+c)^(1/2)+2*a
^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-1/2/c/x^2*(
-a^2*c*x^2+c)^(3/2)-2*a^2*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2)-2*a^3*c/(a
^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}{(ax+1)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^3,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-a^2\,c\,x^2}\ \left(a^2\,x^2-1\right)}{x^3\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x^3*(a*x + 1)^2),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x^3*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{-a^2cx^2+c}}{ax^4+x^3}\right)dx-\int\frac{ax\sqrt{-a^2cx^2+c}}{ax^4+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**3,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**4 + x**3), x) - Integral(a*x*sqrt(-
a**2*c*x**2 + c)/(a*x**4 + x**3), x)
```

3.1243 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=99

$$-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-1/3*(-a^2*c*x^2+c)^(1/2)/
x^3+a*(-a^2*c*x^2+c)^(1/2)/x^2-5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.27, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6152, 1807, 835, 807, 266, 63, 208}

$$-\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3) + (a*Sqrt[c - a^2*c*x^2])/x^2 - (5*a^2*Sqrt[c
- a^2*c*x^2])/(3*x) + a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = c\int \frac{(1-ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{1}{3}\int \frac{6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{\int \frac{10a^2c^2-6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - \left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - \frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\right.$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c}\right.$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.15, size = 82, normalized size = 0.83

$$a^3\left(-\sqrt{c}\right)\log(x) + \frac{\left(-5a^2x^2+3ax-1\right)\sqrt{c-a^2cx^2}}{3x^3} + a^3\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^4),x]
```

```
[Out] ((-1 + 3*a*x - 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) - a^3*Sqrt[c]*Log[x]
 + a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

fricas [A] time = 0.59, size = 165, normalized size = 1.67

$$\left[\frac{3\,a^3\sqrt{c}\,x^3\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-2\sqrt{-a^2cx^2+c}\left(5\,a^2x^2-3\,ax+1\right)}{6\,x^3}, \frac{3\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="fri
cas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3, 1/3*(3*a^3
*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(
-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3]
```

giac [B] time = 0.39, size = 210, normalized size = 2.12

$$-\frac{1}{12}\left(\frac{24\,a^2c\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{-c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{\sqrt{-c}}-\frac{2\left(3\,\pi a^2c-10\,a^2c\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{\sqrt{-c}}+\frac{9\,a^2\left(c-\frac{2c}{ax+1}\right)^2c\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="gia
c")
```

```
[Out] -1/12*(24*a^2*c*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))*
sgn(a)/sqrt(-c) - 2*(3*pi*a^2*c - 10*a^2*c)*sgn(1/(a*x + 1))*sgn(a)/sqrt(-c
) + (9*a^2*(c - 2*c/(a*x + 1))^2*c*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1)
)*sgn(a) + 3*a^2*c^3*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) - 8*a
^2*c^2*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))/(c - c/(a*x + 1)
)^3)*abs(a)
```

maple [B] time = 0.05, size = 253, normalized size = 2.56

$$\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)a^3-\sqrt{-a^2c\,x^2+c}\,a^3-\frac{2a^2\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}-2a^4x\sqrt{-a^2c\,x^2+c}-\frac{2a^4c\arctan\left(\frac{\sqrt{}}{\sqrt{-}}\right)}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x)
```

```
[Out] c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^3-(-a^2*c*x^2+c)^(1/2)
*a^3-2*a^2/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^4*x*(-a^2*c*x^2+c)^(1/2)-2*a^4*c/(a
^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-1/3/c/x^3*(-a^2*c*
x^2+c)^(3/2)+a/c/x^2*(-a^2*c*x^2+c)^(3/2)+2*a^3*(-(x+1/a)^2*a^2*c+2*a*c*(x+
1/a))^(1/2)+2*a^4*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+
2*a*c*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}{\left(ax+1\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^4,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}{x^4\,{\left(a\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x^4*(a*x + 1)^2),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x^4*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(- \frac{\sqrt{- a^{2} c x^{2} + c}}{a x^{5} + x^{4}}\right)\, dx - \int \frac{a x \sqrt{- a^{2} c x^{2} + c}}{a x^{5} + x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**4,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**5 + x**4), x) - Integral(a*x*sqrt(-
a**2*c*x**2 + c)/(a*x**5 + x**4), x)
```

3.1244 $\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{7}{8}a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}$$

```
[Out] -7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-1/4*(-a^2*c*x^2+c)^(
1/2)/x^4+2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3-7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2+4/
3*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.31, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6152, 1807, 835, 807, 266, 63, 208}

$$\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{7}{8}a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) - (7*a^2*S
qrt[c - a^2*c*x^2])/(8*x^2) + (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) - (7*a^4*Sq
rt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = c\int \frac{(1-ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{1}{4}\int \frac{8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{\int \frac{21a^2c^2-16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{\int \frac{32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{1}{8}\left(7a^4c\right)$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{1}{16}\left(7a^4c\right.$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{8}\left(7a^2\right)\mathrm{S}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{7}{8}a^4\sqrt{c}\ \mathrm{ta}$$

Mathematica [A] time = 0.16, size = 95, normalized size = 0.73

$$\frac{7}{8}a^4\sqrt{c}\,\log(x) - \frac{7}{8}a^4\sqrt{c}\,\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right) + \frac{\left(32a^3x^3-21a^2x^2+16ax-6\right)\sqrt{c-a^2cx^2}}{24x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcTanh[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-6 + 16*a*x - 21*a^2*x^2 + 32*a^3*x^3))/(24*x^4) + (7
*a^4*Sqrt[c]*Log[x])/8 - (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]
])/8
```

fricas [A] time = 0.70, size = 181, normalized size = 1.39

$$\left[\frac{21\,a^4\sqrt{c}\,x^4\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+2\left(32\,a^3x^3-21\,a^2x^2+16\,ax-6\right)\sqrt{-a^2cx^2+c}}{48\,x^4}, -\frac{21\,a^4\sqrt{-c}\,x^4\,\mathrm{arctan}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="fri
cas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) + 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, -1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2
*c*x^2 - c)) - (32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c))
/x^4]
```

giac [B] time = 0.23, size = 258, normalized size = 1.98

$$\frac{1}{192}\left(\frac{336\,a^3c\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{-c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{\sqrt{-c}}-\frac{4\left(21\,\pi a^3c-64\,a^3c\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{\sqrt{-c}}+\frac{75\,a^3\left(c-\frac{2c}{ax+1}\right)^3c\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="gia
c")
```

```
[Out] 1/192*(336*a^3*c*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(-c))*sgn(1/(a*x + 1))
*sgn(a)/sqrt(-c) - 4*(21*pi*a^3*c - 64*a^3*c)*sgn(1/(a*x + 1))*sgn(a)/sqrt(
-c) + (75*a^3*(c - 2*c/(a*x + 1))^3*c*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x +
 1))*sgn(a) + 83*a^3*(c - 2*c/(a*x + 1))^2*c^2*sqrt(-c + 2*c/(a*x + 1))*sgn
(1/(a*x + 1))*sgn(a) + 21*a^3*c^4*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))
*sgn(a) - 77*a^3*c^3*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))/(c
 - c/(a*x + 1))^4)*abs(a)
```

maple [B] time = 0.05, size = 279, normalized size = 2.15

$$-\frac{7\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)a^4}{8}+\frac{7\sqrt{-a^2c\,x^2+c}\,a^4}{8}+\frac{2a^3\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}+2a^5x\sqrt{-a^2c\,x^2+c}+\frac{2a^5c\arctan\left(\frac{\sqrt{a^2c}\cdots}{\sqrt{-a^2c\,x\cdots}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x)
```

```
[Out] -7/8*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^4+7/8*(-a^2*c*x^2
+c)^(1/2)*a^4+2*a^3/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^5*x*(-a^2*c*x^2+c)^(1/2)+2
*a^5*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3*a/c/x
```

```
^3*(-a^2*c*x^2+c)^(3/2)-9/8*a^2/c/x^2*(-a^2*c*x^2+c)^(3/2)-1/4/c/x^4*(-a^2*
c*x^2+c)^(3/2)-2*a^4*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2)-2*a^5*c/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)}{(ax+1)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1)/x^5,x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)/((a*x + 1)^2*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}{x^5\,{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x^5*(a*x + 1)^2),x)
```

```
[Out] -int(((c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(x^5*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt{-a^2cx^2+c}}{ax^6+x^5}\right)dx-\int \frac{ax\sqrt{-a^2cx^2+c}}{ax^6+x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1)/x**5,x)
```

```
[Out] -Integral(-sqrt(-a**2*c*x**2 + c)/(a*x**6 + x**5), x) - Integral(a*x*sqrt(-
a**2*c*x**2 + c)/(a*x**6 + x**5), x)
```

3.1245 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=108

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}+\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

```
[Out] 5/12*(-a^2*c*x^2+c)^(3/2)/a+1/4*(-a*x+1)*(-a^2*c*x^2+c)^(3/2)/a+5/8*c^(3/2)
*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a+5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6142, 671, 641, 195, 217, 203}

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}+\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (5*c*x*Sqrt[c - a^2*c*x^2])/8 + (5*(c - a^2*c*x^2)^(3/2))/(12*a) + ((1 - a*
x)*(c - a^2*c*x^2)^(3/2))/(4*a) + (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
a^2*c*x^2]])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx = c\int(1-ax)^2\sqrt{c-a^2cx^2}\,dx$$

$$= \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} + \frac{1}{4}(5c)\int(1-ax)\sqrt{c-a^2cx^2}\,dx$$

$$= \frac{5\left(c-a^2cx^2\right)^{3/2}}{12a} + \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} + \frac{1}{4}(5c)\int\sqrt{c-a^2cx^2}\,dx$$

$$= \frac{5}{8}cx\sqrt{c-a^2cx^2} + \frac{5\left(c-a^2cx^2\right)^{3/2}}{12a} + \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} + \frac{1}{8}\left(5c^2\right)\int\frac{}{\sqrt{c}}$$

$$= \frac{5}{8}cx\sqrt{c-a^2cx^2} + \frac{5\left(c-a^2cx^2\right)^{3/2}}{12a} + \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} + \frac{1}{8}\left(5c^2\right)\text{Subst}$$

$$= \frac{5}{8}cx\sqrt{c-a^2cx^2} + \frac{5\left(c-a^2cx^2\right)^{3/2}}{12a} + \frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a} + \frac{5c^{3/2}\tan^{-1}\left(\right.}{8a}$$

Mathematica [A] time = 0.11, size = 117, normalized size = 1.08

$$-\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(6a^4x^4-22a^3x^3+25a^2x^2+7ax-16\right)+30\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/24*(c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-16 + 7*a*x + 25*a^2*x^2 - 22*
a^3*x^3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*
Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.80, size = 180, normalized size = 1.67

$$\left[\frac{15\sqrt{-c}\,c\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(6\,a^3cx^3-16\,a^2cx^2+9\,acx+16\,c\right)\sqrt{-a^2cx^2+c}}{48\,a}, -\frac{15\,c^{\frac{3}{2}}\,a\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) + 2*(6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c)
)/a, -1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
 c)) - (6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c))/
a]
```

giac [B] time = 0.28, size = 224, normalized size = 2.07

$$-\frac{\left(240\,a^5c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2\,c}{ax+1}}}{\sqrt{c}}\right)\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)-\frac{\left(15\,a^5\left(c-\frac{2\,c}{ax+1}\right)^3c^2\sqrt{-c+\frac{2\,c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)+73\,a^5\left(c-\frac{2\,c}{ax+1}\right)^2c^3\sqrt{-c+\frac{2\,c}{ax+1}}\,\mathrm{sg}\cdots\right.}{\cdots}\right.}{192\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/192*(240*a^5*c^(3/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a*x
 + 1))*sgn(a) - (15*a^5*(c - 2*c/(a*x + 1))^3*c^2*sqrt(-c + 2*c/(a*x + 1))*
```

```
sgn(1/(a*x + 1))*sgn(a) + 73*a^5*(c - 2*c/(a*x + 1))^2*c^3*sqrt(-c + 2*c/(a
*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 15*a^5*c^5*sqrt(-c + 2*c/(a*x + 1))*sgn(
1/(a*x + 1))*sgn(a) + 55*a^5*c^4*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1)
)*sgn(a))*(a*x + 1)^4/c^4)*abs(a)/a^7
```

maple [A] time = 0.04, size = 174, normalized size = 1.61

$$-\frac{x\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{4}-\frac{3cx\sqrt{-a^{2}c\,x^{2}+c}}{8}-\frac{3c^{2}\arctan\left(\frac{\sqrt{a^{2}c}\;x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{8\sqrt{a^{2}c}}+\frac{2\left(-\left(x+\frac{1}{a}\right)^{2}a^{2}c+2ac\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3a}+c\sqrt{-\left(x+\frac{1}{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)-3/8*c*x*(-a^2*c*x^2+c)^(1/2)-3/8*c^2/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3/a*(-(x+1/a)^2*a^2*c+2*
a*c*(x+1/a))^(3/2)+c*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2)*x+c^2/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2))
```

maxima [A] time = 0.47, size = 130, normalized size = 1.20

$$-\frac{1}{4}\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}x+\sqrt{a^{2}cx^{2}+4\,acx+3\,c}\,cx-\frac{3}{8}\,\sqrt{-a^{2}cx^{2}+c}\,cx-\frac{c^{3}\arcsin\left(ax+2\right)}{a\left(-c\right)^{\frac{3}{2}}}-\frac{3\,c^{\frac{3}{2}}\arcsin\left(ax\right)}{8\,a}+\frac{2\left(-a^{2}cx^{2}\right.}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] -1/4*(-a^2*c*x^2 + c)^(3/2)*x + sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c*x - 3/8*s
qrt(-a^2*c*x^2 + c)*c*x - c^3*arcsin(a*x + 2)/(a*(-c)^(3/2)) - 3/8*c^(3/2)*
arcsin(a*x)/a + 2/3*(-a^2*c*x^2 + c)^(3/2)/a + 2*sqrt(a^2*c*x^2 + 4*a*c*x +
 3*c)*c/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

[Out] -int(((c - a^2*c*x^2)^(3/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)

sympy [C] time = 6.78, size = 340, normalized size = 3.15

$$a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}\,x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)-2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0\\ -\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)+c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a**2*c*x**2+c)**(3/2)/(a*x+1)**2*(-a**2*x**2+1),x)

[Out] a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) - 2*a*c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)) + c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))

3.1246 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}+\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

```
[Out] 7/24*c*x*(-a^2*c*x^2+c)^(3/2)+7/30*(-a^2*c*x^2+c)^(5/2)/a+1/6*(-a*x+1)*(-a^
2*c*x^2+c)^(5/2)/a+7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a+
7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6142, 671, 641, 195, 217, 203}

$$\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (7*c^2*x*Sqrt[c - a^2*c*x^2])/16 + (7*c*x*(c - a^2*c*x^2)^(3/2))/24 + (7*(c
 - a^2*c*x^2)^(5/2))/(30*a) + ((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) + (7*
c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx = c\int(1-ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx$$

$$=\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{1}{6}(7c)\int(1-ax)\left(c-a^2cx^2\right)^{3/2}\,dx$$

$$=\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{1}{6}(7c)\int\left(c-a^2cx^2\right)^{3/2}\,dx$$

$$=\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{1}{8}\left(7c^2\right)\int\sqrt{}$$

$$=\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1-ax)\left(c-a^2cx^{}\right.}{6a}$$

$$=\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1-ax)\left(c-a^2cx^{}\right.}{6a}$$

$$=\frac{7}{16}c^2x\sqrt{c-a^2cx^2}+\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1-ax)\left(c-a^2cx^{}\right.}{6a}$$

Mathematica [A] time = 0.17, size = 135, normalized size = 1.03

$$\frac{c^2\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(40a^6x^6-136a^5x^5+86a^4x^4+202a^3x^3-327a^2x^2+39ax+96\right)-210\sqrt{1-ax}\sin^{-1}\left(\cdots\right.\right.}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(96 + 39*a*x - 327*a^2*x^2 + 202*a^
3*x^3 + 86*a^4*x^4 - 136*a^5*x^5 + 40*a^6*x^6) - 210*Sqrt[1 - a*x]*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.60, size = 241, normalized size = 1.84

$$\left[\frac{105\sqrt{-c}\,c^2\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\left(40\,a^5c^2x^5-96\,a^4c^2x^4-10\,a^3c^2x^3+192\,a^2c^2x^2-135\cdots\right.}{480\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) - 2*(40*a^5*c^2*x^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^
2*x^2 - 135*a*c^2*x - 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, -1/240*(105*c^(5/2)*
arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5
- 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^2*x^2 - 135*a*c^2*x - 96*c^2)
*sqrt(-a^2*c*x^2 + c))/a]
```

giac [B] time = 0.28, size = 320, normalized size = 2.44

$$-\frac{\left(6720\,a^7c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{c}}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)-\frac{\left(105\,a^7\left(c-\frac{2c}{ax+1}\right)^5c^3\sqrt{-c+\frac{2c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)-595\,a^7\left(c-\frac{2c}{ax+1}\right)^4c^4\sqrt{-c+\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/7680*(6720*a^7*c^(5/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1/(a
*x + 1))*sgn(a) - (105*a^7*(c - 2*c/(a*x + 1))^5*c^3*sqrt(-c + 2*c/(a*x + 1
```

```
))*sgn(1/(a*x + 1))*sgn(a) - 595*a^7*(c - 2*c/(a*x + 1))^4*c^4*sqrt(-c + 2*
c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) - 1686*a^7*(c - 2*c/(a*x + 1))^3*c^5*s
qrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 1386*a^7*(c - 2*c/(a*x +
1))^2*c^6*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 105*a^7*c^8*sq
rt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 595*a^7*c^7*(-c + 2*c/(a*x
 + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))*(a*x + 1)^6/c^6)*abs(a)/a^9
```

maple [B] time = 0.04, size = 226, normalized size = 1.73

$$-\frac{x\left(-a^{2} c\,x^{2}+c\right)^{\frac{5}{2}}}{6}-\frac{5 c x\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{24}-\frac{5 c^{2} x\sqrt{-a^{2} c\,x^{2}+c}}{16}-\frac{5 c^{3} \arctan\left(\frac{\sqrt{a^{2} c}\,x}{\sqrt{-a^{2} c\,x^{2}+c}}\right)}{16\sqrt{a^{2} c}}+\frac{2\left(-\left(x+\frac{1}{a}\right)^{2} a^{2} c+2 a c\left(x-\right.\right.}{5 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/6*x*(-a^2*c*x^2+c)^(5/2)-5/24*c*x*(-a^2*c*x^2+c)^(3/2)-5/16*c^2*x*(-a^2*
c*x^2+c)^(1/2)-5/16*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)
^(1/2))+2/5/a*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(5/2)+1/2*c*(-(x+1/a)^2*a^2*
c+2*a*c*(x+1/a))^(3/2)*x+3/4*c^2*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2)*x+3
/4*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a)
)^(1/2))
```

maxima [A] time = 0.42, size = 154, normalized size = 1.18

$$-\frac{1}{6}\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} x+\frac{7}{24}\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} c x+\frac{3}{4}\sqrt{a^{2} c x^{2}+4\,a c x+3\,c}\,c^{2} x-\frac{5}{16}\sqrt{-a^{2} c x^{2}+c}\,c^{2} x-\frac{3\,c^{4} \arcsin\left(a x+2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] -1/6*(-a^2*c*x^2 + c)^(5/2)*x + 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x + 3/4*sqrt(
a^2*c*x^2 + 4*a*c*x + 3*c)*c^2*x - 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x - 3/4*c^
4*arcsin(a*x + 2)/(a*(-c)^(3/2)) - 5/16*c^(5/2)*arcsin(a*x)/a + 2/5*(-a^2*c
*x^2 + c)^(5/2)/a + 3/2*sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c^2/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^{2}\,c\,x^{2}\right)^{5/2}\,\left(a^{2}\,x^{2}-1\right)}{\left(a\,x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(5/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - a^2*c*x^2)^(5/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [C] time = 10.27, size = 478, normalized size = 3.65

$$-a^4c^2\left(\begin{cases} \frac{ia^2\sqrt{c}\,x^7}{6\sqrt{a^2x^2-1}} - \frac{5i\sqrt{c}\,x^5}{24\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,x^3}{48a^2\sqrt{a^2x^2-1}} + \frac{i\sqrt{c}\,x}{16a^4\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{16a^5} & \text{for } |a^2x^2| > 1 \\ -\frac{a^2\sqrt{c}\,x^7}{6\sqrt{-a^2x^2+1}} + \frac{5\sqrt{c}\,x^5}{24\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,x^3}{48a^2\sqrt{-a^2x^2+1}} - \frac{\sqrt{c}\,x}{16a^4\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,\operatorname{asin}(ax)}{16a^5} & \text{otherwise} \end{cases}\right) + 2a^3c^2\left(\begin{cases} \frac{x^4\sqrt{-a^2c}}{5} \\ \frac{\sqrt{c}\,x^4}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqr
t(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
 - 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(
16*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
 + 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a
**2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*
x)/(16*a**5), True)) + 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
 - x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**
4), Ne(a, 0)), (sqrt(c)*x**4/4, True)) - 2*a*c**2*Piecewise((0, Eq(c, 0)),
(sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True
)) + c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c
)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) >
1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

3.1247 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=154

$$\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{128a}+\frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\cdots$$

```
[Out] 15/64*c^2*x*(-a^2*c*x^2+c)^(3/2)+3/16*c*x*(-a^2*c*x^2+c)^(5/2)+9/56*(-a^2*c
*x^2+c)^(7/2)/a+1/8*(-a*x+1)*(-a^2*c*x^2+c)^(7/2)/a+45/128*c^(7/2)*arctan(a
*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a+45/128*c^3*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6142, 671, 641, 195, 217, 203}

$$\frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{128a}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (45*c^3*x*Sqrt[c - a^2*c*x^2])/128 + (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 +
(3*c*x*(c - a^2*c*x^2)^(5/2))/16 + (9*(c - a^2*c*x^2)^(7/2))/(56*a) + ((1 -
 a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) + (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(128*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rubi steps

$$\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = c\int(1-ax)^2\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$= \frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{1}{8}(9c)\int(1-ax)\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$= \frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{1}{8}(9c)\int\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$= \frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1-ax)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{1}{16}\left(15c^2\right)\int$$
$$= \frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1-ax)\left(c-a\right.}{8a}$$
$$= \frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx\right.}{56a}$$
$$= \frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx\right.}{56a}$$
$$= \frac{45}{128}c^3x\sqrt{c-a^2cx^2}+\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}+\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx\right.}{56a}$$

Mathematica [A] time = 0.18, size = 151, normalized size = 0.98

$$-\frac{c^3\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(112a^8x^8-368a^7x^7+88a^6x^6+936a^5x^5-978a^4x^4-558a^3x^3+1349a^2x^2-325ax-25\right.\right.}{896a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -1/896*(c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-256 - 325*a*x + 1349*a^2*x
^2 - 558*a^3*x^3 - 978*a^4*x^4 + 936*a^5*x^5 + 88*a^6*x^6 - 368*a^7*x^7 + 1
12*a^8*x^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 -
 a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 1.35, size = 286, normalized size = 1.86

$$\left[\frac{315\sqrt{-c}\,c^3\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(112\,a^7c^3x^7-256\,a^6c^3x^6-168\,a^5c^3x^5+768\,a^4c^3x^4-2\right.}{1792\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas"
)
```

```
[Out] [1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 2*(112*a^7*c^3*x^7 - 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 + 768*a^
4*c^3*x^4 - 210*a^3*c^3*x^3 - 768*a^2*c^3*x^2 + 581*a*c^3*x + 256*c^3)*sqrt
(-a^2*c*x^2 + c))/a, -1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt
(c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 - 256*a^6*c^3*x^6 - 168*a^5*c^3*x
^5 + 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 - 768*a^2*c^3*x^2 + 581*a*c^3*x + 25
6*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

giac [B] time = 0.44, size = 416, normalized size = 2.70

$$-\frac{\left(80640\,a^9c^{\frac{7}{2}}\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{c}}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)-\frac{\left(315\,a^9\left(c-\frac{2c}{ax+1}\right)^7c^4\sqrt{-c+\frac{2c}{ax+1}}\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)-2415\,a^9\left(c-\frac{2c}{ax+1}\right)^6c^5\sqrt{-c\cdots}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/114688*(80640*a^9*c^(7/2)*arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))*sgn(1
/(a*x + 1))*sgn(a) - (315*a^9*(c - 2*c/(a*x + 1))^7*c^4*sqrt(-c + 2*c/(a*x
+ 1))*sgn(1/(a*x + 1))*sgn(a) - 2415*a^9*(c - 2*c/(a*x + 1))^6*c^5*sqrt(-c
+ 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 8043*a^9*(c - 2*c/(a*x + 1))^5*c
^6*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) + 17609*a^9*(c - 2*c/(a
*x + 1))^4*c^7*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a) - 15159*a^9
*(c - 2*c/(a*x + 1))^3*c^8*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a)
 + 8043*a^9*(c - 2*c/(a*x + 1))^2*c^9*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x +
 1))*sgn(a) + 315*a^9*c^11*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a)
 + 2415*a^9*c^10*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))*sgn(a))*(a*x +
 1)^8/c^8)*abs(a)/a^11
```

maple [B] time = 0.04, size = 276, normalized size = 1.79

$$-\frac{x\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{8}-\frac{7cx\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{48}-\frac{35c^2x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{192}-\frac{35c^3x\sqrt{-a^2c\,x^2+c}}{128}-\frac{35c^4\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{128\sqrt{a^2c}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] -1/8*x*(-a^2*c*x^2+c)^(7/2)-7/48*c*x*(-a^2*c*x^2+c)^(5/2)-35/192*c^2*x*(-a^
2*c*x^2+c)^(3/2)-35/128*c^3*x*(-a^2*c*x^2+c)^(1/2)-35/128*c^4/(a^2*c)^(1/2)
*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/7/a*(-(x+1/a)^2*a^2*c+2*a*c
*(x+1/a))^(7/2)+1/3*c*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(5/2)*x+5/12*c^2*(-(
x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(3/2)*x+5/8*c^3*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/
a))^(1/2)*x+5/8*c^4/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+
2*a*c*(x+1/a))^(1/2))
```

maxima [A] time = 0.42, size = 173, normalized size = 1.12

$$-\frac{1}{8}\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}x+\frac{3}{16}\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}cx+\frac{15}{64}\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}c^{2}x+\frac{5}{8}\sqrt{a^{2}cx^{2}+4\,acx+3\,c}\,c^{3}x-\frac{35}{128}\sqrt{-a^{2}cx^{2}+c}\,c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima"
)
```

```
[Out] -1/8*(-a^2*c*x^2 + c)^(7/2)*x + 3/16*(-a^2*c*x^2 + c)^(5/2)*c*x + 15/64*(-a
^2*c*x^2 + c)^(3/2)*c^2*x + 5/8*sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c^3*x - 35/
128*sqrt(-a^2*c*x^2 + c)*c^3*x - 5/8*c^5*arcsin(a*x + 2)/(a*(-c)^(3/2)) - 3
5/128*c^(7/2)*arcsin(a*x)/a + 2/7*(-a^2*c*x^2 + c)^(7/2)/a + 5/4*sqrt(a^2*c
*x^2 + 4*a*c*x + 3*c)*c^3/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c-a^{2}\,c\,x^{2}\right)^{7/2}\,\left(a^{2}\,x^{2}-1\right)}{\left(a\,x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^(7/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - a^2*c*x^2)^(7/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [C] time = 20.11, size = 1091, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] a**6*c**3*Piecewise((I*a**2*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqrt
(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**2
 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(1
28*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2*
x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/(
48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) + 5
*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt(
-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128*a**7), True)) - 2*a**5*c**3*Pie
cewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a**
2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(1
05*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) - a**4*c**3*Piecewise((I*a**2*
sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2*x**2
- 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(16*a**4
*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2) > 1)
, (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*sqrt(-a
**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(1
6*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5), True)) + 4*a**3
*c**3*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 + c
)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4/
4, True)) - a**2*c**3*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)
) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**
2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*s
qrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 +
1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3),
 True)) - 2*a*c**3*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)),
(-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)) + c**3*Piecewise((I*a**2*sqr
t(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I
*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2
+ 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

3.1248 $\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] -arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)-2*(-a*x+1)/a/(-a^2*c*x^
2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6142, 653, 217, 203}

$$-\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*(1 - a*x))/(a*Sqrt[c - a^2*c*x^2]) - ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*
c*x^2]]/(a*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\ &= -\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}-\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}-\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\ &= -\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}-\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 100, normalized size = 1.64

$$\frac{2\sqrt{1-a^2x^2}\left(\sqrt{ax+1}\,(ax-1)+\sqrt{1-ax}\,(ax+1)\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\,(ax+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*((-1 + a*x)*Sqrt[1 + a*x] + Sqrt[1 - a*x]*(1 + a*x)*Ar
cSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*(1 + a*x)*Sqrt[c - a^2*c*x^2
])
```

fricas [A] time = 0.74, size = 150, normalized size = 2.46

$$\left[-\frac{(ax+1)\sqrt{-c}\,\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+4\,\sqrt{-a^2cx^2+c}}{2\,\left(a^2cx+ac\right)},\ \frac{(ax+1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{a^2cx+ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="frica
s")
```

```
[Out] [-1/2*((a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) + 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x + a*c), ((a*x + 1)*sqrt(c)*arc
tan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2 +
 c))/(a^2*c*x + a*c)]
```

giac [B] time = 0.21, size = 107, normalized size = 1.75

$$-\frac{2\left(\frac{\left(c\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{-c}\sqrt{c}\right)\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)}{c^{\frac{3}{2}}}-\frac{\frac{\arctan\left(\frac{\sqrt{-c+\frac{2c}{ax+1}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{-c+\frac{2c}{ax+1}}}{c}}{\operatorname{sgn}\left(\frac{1}{ax+1}\right)\operatorname{sgn}(a)}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac"
)
```

```
[Out] -2*((c*arctan(sqrt(-c)/sqrt(c)) - sqrt(-c)*sqrt(c))*sgn(1/(a*x + 1))*sgn(a)
/c^(3/2) - (arctan(sqrt(-c + 2*c/(a*x + 1))/sqrt(c))/sqrt(c) - sqrt(-c + 2*
c/(a*x + 1))/c)/(sgn(1/(a*x + 1))*sgn(a)))/abs(a)
```

maple [A] time = 0.04, size = 74, normalized size = 1.21

$$-\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}-\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2c+2ac\left(x+\frac{1}{a}\right)}}{a^2c\left(x+\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a^2/c/(x+1/
a)*(-(x+1/a)^2*a^2*c+2*a*c*(x+1/a))^(1/2)
```

maxima [A] time = 0.45, size = 40, normalized size = 0.66

$$-\frac{2\sqrt{-a^2cx^2+c}}{a^2cx+ac}-\frac{\arcsin(ax)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x + a*c) - arcsin(a*x)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{a^2\,x^2-1}{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2),x)
```

```
[Out] -int((a^2*x^2 - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{ax\sqrt{-a^2cx^2+c}+\sqrt{-a^2cx^2+c}}\,dx-\int \left(-\frac{1}{ax\sqrt{-a^2cx^2+c}+\sqrt{-a^2cx^2+c}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] -Integral(a*x/(a*x*sqrt(-a**2*c*x**2 + c) + sqrt(-a**2*c*x**2 + c)), x) - I
ntegral(-1/(a*x*sqrt(-a**2*c*x**2 + c) + sqrt(-a**2*c*x**2 + c)), x)
```

3.1249 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{x}{3c\sqrt{c-a^2cx^2}}-\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -2/3*(-a*x+1)/a/(-a^2*c*x^2+c)^(3/2)+1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6142, 653, 191}

$$\frac{x}{3c\sqrt{c-a^2cx^2}}-\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (-2*(1 - a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) + x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$
$$= -\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}} + \frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= -\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}} + \frac{x}{3c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.21

$$-\frac{\sqrt{1-ax}\,(ax+2)\sqrt{1-a^2x^2}}{3ac(ax+1)^{3/2}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/3*(Sqrt[1 - a*x]*(2 + a*x)*Sqrt[1 - a^2*x^2])/(a*c*(1 + a*x)^(3/2)*Sqrt[
c - a^2*c*x^2])
```

fricas [A] time = 0.58, size = 47, normalized size = 0.90

$$-\frac{\sqrt{-a^2cx^2+c}\,(ax+2)}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/3*sqrt(-a^2*c*x^2 + c)*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.98, size = 82, normalized size = 1.58

$$\frac{\frac{2\sqrt{-c}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}{c^2} - \frac{3\,c\sqrt{-c+\frac{2\,c}{ax+1}}+\left(-c+\frac{2\,c}{ax+1}\right)^{\frac{3}{2}}}{c^3\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}}{6\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac"
)
```

```
[Out] 1/6*(2*sqrt(-c)*sgn(1/(a*x + 1))*sgn(a)/c^2 - (3*c*sqrt(-c + 2*c/(a*x + 1))
 + (-c + 2*c/(a*x + 1))^(3/2))/(c^3*sgn(1/(a*x + 1))*sgn(a)))/abs(a)
```

maple [A] time = 0.03, size = 31, normalized size = 0.60

$$-\frac{(ax-1)^2 (ax+2)}{3 \left(-a^2 c x^2+c\right)^{\frac{3}{2}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(a*x-1)^2*(a*x+2)/(-a^2*c*x^2+c)^(3/2)/a
```

maxima [A] time = 0.32, size = 60, normalized size = 1.15

$$\frac{x}{3\sqrt{-a^2cx^2+c}\,c}-\frac{2}{3\left(\sqrt{-a^2cx^2+c}\,a^2cx+\sqrt{-a^2cx^2+c}\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*x/(sqrt(-a^2*c*x^2 + c)*c) - 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x + sqrt(-
a^2*c*x^2 + c)*a*c)
```

mupad [B] time = 0.98, size = 33, normalized size = 0.63

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+2\right)}{3\,a\,c^2\,{\left(a\,x+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a^2*c*x^2)^(3/2)*(a*x + 1)^2),x)
```

```
[Out] -((c - a^2*c*x^2)^(1/2)*(a*x + 2))/(3*a*c^2*(a*x + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ax}{-a^3cx^3\sqrt{-a^2cx^2+c}-a^2cx^2\sqrt{-a^2cx^2+c}+acx\sqrt{-a^2cx^2+c}+c\sqrt{-a^2cx^2+c}}\,dx-\int\left(-\frac{}{-a^3cx^3\sqrt{-a^2cx^2+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -Integral(a*x/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) - a**2*c*x**2*sqrt(-a**2
*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) + c*sqrt(-a**2*c*x**2 + c)), x)
 - Integral(-1/(-a**3*c*x**3*sqrt(-a**2*c*x**2 + c) - a**2*c*x**2*sqrt(-a**
2*c*x**2 + c) + a*c*x*sqrt(-a**2*c*x**2 + c) + c*sqrt(-a**2*c*x**2 + c)), x
)
```

3.1250 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2x}{5c^2\sqrt{c-a^2cx^2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -2/5*(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)+1/5*x/c/(-a^2*c*x^2+c)^(3/2)+2/5*x/c^2
/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6142, 653, 192, 191}

$$\frac{2x}{5c^2\sqrt{c-a^2cx^2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (-2*(1 - a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) + x/(5*c*(c - a^2*c*x^2)^(3/2))
+ (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx\\ &= -\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}+\frac{3}{5}\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx\\ &= -\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{5c}\\ &= -\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}+\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2x}{5c^2\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 79, normalized size = 1.05

$$\frac{\sqrt{1-a^2x^2}\left(2a^3x^3+4a^2x^2+ax-2\right)}{5ac^2\sqrt{1-ax}\,(ax+1)^{5/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2 + a*x + 4*a^2*x^2 + 2*a^3*x^3))/(5*a*c^2*Sqrt[1 - a*
x]*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.83, size = 75, normalized size = 1.00

$$-\frac{\left(2\,a^3x^3+4\,a^2x^2+ax-2\right)\sqrt{-a^2cx^2+c}}{5\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/5*(2*a^3*x^3 + 4*a^2*x^2 + a*x - 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 +
2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

giac [B] time = 0.28, size = 220, normalized size = 2.93

$$\frac{a^3\left(\frac{5}{a^3c^2\sqrt{-c+\frac{2c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}-\frac{a^{12}\left(c-\frac{2c}{ax+1}\right)^2c^{20}\sqrt{-c+\frac{2c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)^4\mathrm{sgn}(a)^4+15\,a^{12}c^{22}\sqrt{-c+\frac{2c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)^4\mathrm{sgn}(a)^4+5\,a^{12}c^{21}\left(-c+\cdots\right.}{a^{15}c^{25}\mathrm{sgn}\left(\frac{1}{ax+1}\right)^5\mathrm{sgn}(a)^5}\cdots\right.}{40\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac"
)
```

```
[Out] 1/40*(a^3*(5/(a^3*c^2*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a)) - (
a^12*(c - 2*c/(a*x + 1))^2*c^20*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))^4
*sgn(a)^4 + 15*a^12*c^22*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))^4*sgn(a)
^4 + 5*a^12*c^21*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))^4*sgn(a)^4)/(a
^15*c^25*sgn(1/(a*x + 1))^5*sgn(a)^5)) - 16*sgn(1/(a*x + 1))*sgn(a)/(sqrt(-
c)*c^2))/abs(a)
```

maple [A] time = 0.03, size = 47, normalized size = 0.63

$$\frac{(ax-1)^2\left(2x^3a^3+4a^2x^2+ax-2\right)}{5\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/5*(a*x-1)^2*(2*a^3*x^3+4*a^2*x^2+a*x-2)/(-a^2*c*x^2+c)^(5/2)/a
```

maxima [A] time = 0.33, size = 79, normalized size = 1.05

$$-\frac{2}{5\left(\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^2cx+\left(-a^2cx^2+c\right)^{\frac{3}{2}}ac\right)}+\frac{2\,x}{5\sqrt{-a^2cx^2+c}\,c^2}+\frac{x}{5\left(-a^2cx^2+c\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x + (-a^2*c*x^2 + c)^(3/2)*a*c) + 2/5*x/
(sqrt(-a^2*c*x^2 + c)*c^2) + 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

mupad [B] time = 1.09, size = 56, normalized size = 0.75

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(2\,a^3\,x^3+4\,a^2\,x^2+a\,x-2\right)}{5\,a\,c^3\,\left(a\,x-1\right)\,{\left(a\,x+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)^2),x)
```

```
[Out] -((c - a^2*c*x^2)^(1/2)*(a*x + 4*a^2*x^2 + 2*a^3*x^3 - 2))/(5*a*c^3*(a*x -
1)*(a*x + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax}{a^5c^2x^5\sqrt{-a^2cx^2+c}+a^4c^2x^4\sqrt{-a^2cx^2+c}-2a^3c^2x^3\sqrt{-a^2cx^2+c}-2a^2c^2x^2\sqrt{-a^2cx^2+c}+ac^2x\sqrt{-a^2cx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] -Integral(a*x/(a**5*c**2*x**5*sqrt(-a**2*c*x**2 + c) + a**4*c**2*x**4*sqrt(
-a**2*c*x**2 + c) - 2*a**3*c**2*x**3*sqrt(-a**2*c*x**2 + c) - 2*a**2*c**2*x
**2*sqrt(-a**2*c*x**2 + c) + a*c**2*x*sqrt(-a**2*c*x**2 + c) + c**2*sqrt(-a
**2*c*x**2 + c)), x) - Integral(-1/(a**5*c**2*x**5*sqrt(-a**2*c*x**2 + c) +
 a**4*c**2*x**4*sqrt(-a**2*c*x**2 + c) - 2*a**3*c**2*x**3*sqrt(-a**2*c*x**2
 + c) - 2*a**2*c**2*x**2*sqrt(-a**2*c*x**2 + c) + a*c**2*x*sqrt(-a**2*c*x**
2 + c) + c**2*sqrt(-a**2*c*x**2 + c)), x)
```

3.1251 $\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=98

$$\frac{8x}{21c^3\sqrt{c-a^2cx^2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] -2/7*(-a*x+1)/a/(-a^2*c*x^2+c)^(7/2)+1/7*x/c/(-a^2*c*x^2+c)^(5/2)+4/21*x/c^
2/(-a^2*c*x^2+c)^(3/2)+8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6142, 653, 192, 191}

$$\frac{8x}{21c^3\sqrt{c-a^2cx^2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (-2*(1 - a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) + x/(7*c*(c - a^2*c*x^2)^(5/2))
+ (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx &= c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{9/2}}\,dx\\
&= -\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}+\frac{5}{7}\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx\\
&= -\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{7c}\\
&= -\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{21c^2}\\
&= -\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}+\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8x}{21c^3\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 96, normalized size = 0.98

$$-\frac{\sqrt{1-a^2x^2}\left(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6\right)}{21ac^3(1-ax)^{3/2}(ax+1)^{7/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2)),x]
[Out] -1/21*(Sqrt[1 - a^2*x^2]*(6 - 9*a*x - 24*a^2*x^2 - 4*a^3*x^3 + 16*a^4*x^4 +
 8*a^5*x^5))/(a*c^3*(1 - a*x)^(3/2)*(1 + a*x)^(7/2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.91, size = 124, normalized size = 1.27

$$-\frac{\left(8\,a^5x^5+16\,a^4x^4-4\,a^3x^3-24\,a^2x^2-9\,ax+6\right)\sqrt{-a^2cx^2+c}}{21\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="frica
s")
```

```
[Out] -1/21*(8*a^5*x^5 + 16*a^4*x^4 - 4*a^3*x^3 - 24*a^2*x^2 - 9*a*x + 6)*sqrt(-a
^2*c*x^2 + c)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 -
a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

giac [B] time = 0.62, size = 300, normalized size = 3.06

$$\frac{a^5\left(\frac{14\left(7c-\frac{15c}{ax+1}\right)}{a^5\left(c-\frac{2c}{ax+1}\right)c^3\sqrt{-c+\frac{2c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)\mathrm{sgn}(a)}+\frac{3a^{30}\left(c-\frac{2c}{ax+1}\right)^3c^{42}\sqrt{-c+\frac{2c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)^6\mathrm{sgn}(a)^6-21a^{30}\left(c-\frac{2c}{ax+1}\right)^2c^{43}\sqrt{-c+\frac{2c}{ax+1}}\,\mathrm{sgn}\left(\frac{1}{ax+1}\right)^6\mathrm{sgn}\cdots}{a^{35}c^{49}\mathrm{sgn}\left(\frac{1}{ax+1}\right)\cdots}\cdots\right)}{672\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac"
)
```

```
[Out] 1/672*(a^5*(14*(7*c - 15*c/(a*x + 1))/(a^5*(c - 2*c/(a*x + 1))*c^3*sqrt(-c
+ 2*c/(a*x + 1))*sgn(1/(a*x + 1))*sgn(a)) + (3*a^30*(c - 2*c/(a*x + 1))^3*c
^42*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))^6*sgn(a)^6 - 21*a^30*(c - 2*c
/(a*x + 1))^2*c^43*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))^6*sgn(a)^6 - 2
10*a^30*c^45*sqrt(-c + 2*c/(a*x + 1))*sgn(1/(a*x + 1))^6*sgn(a)^6 - 70*a^30
*c^44*(-c + 2*c/(a*x + 1))^(3/2)*sgn(1/(a*x + 1))^6*sgn(a)^6)/(a^35*c^49*sg
n(1/(a*x + 1))^7*sgn(a)^7)) - 256*sgn(1/(a*x + 1))*sgn(a)/(sqrt(-c)*c^3))/a
bs(a)
```

maple [A] time = 0.03, size = 64, normalized size = 0.65

$$-\frac{(ax-1)^2\left(8x^5a^5+16x^4a^4-4x^3a^3-24a^2x^2-9ax+6\right)}{21\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/21*(a*x-1)^2*(8*a^5*x^5+16*a^4*x^4-4*a^3*x^3-24*a^2*x^2-9*a*x+6)/(-a^2*c
*x^2+c)^(7/2)/a
```

maxima [A] time = 0.33, size = 98, normalized size = 1.00

$$-\frac{2}{7\left(\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^2cx+\left(-a^2cx^2+c\right)^{\frac{5}{2}}ac\right)}+\frac{8x}{21\sqrt{-a^2cx^2+c}\,c^3}+\frac{4x}{21\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^2}+\frac{x}{7\left(-a^2cx^2+c\right)^{\frac{5}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxim
a")
```

```
[Out] -2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x + (-a^2*c*x^2 + c)^(5/2)*a*c) + 8/21*x
/(sqrt(-a^2*c*x^2 + c)*c^3) + 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) + 1/7*x/(
(-a^2*c*x^2 + c)^(5/2)*c)
```

mupad [B] time = 1.12, size = 133, normalized size = 1.36

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}-\frac{5}{28\,a\,c^4}\right)}{{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^2}-\frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,{\left(a\,x+1\right)}^4}-\frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,{\left(a\,x+1\right)}^3}-\frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*x^2 - 1)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)^2),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) - 5/(28*a*c^4)))/((a*x - 1)^2*(a*x
+ 1)^2) - (c - a^2*c*x^2)^(1/2)/(28*a*c^4*(a*x + 1)^4) - (c - a^2*c*x^2)^(1
/2)/(14*a*c^4*(a*x + 1)^3) - (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x - 1)*
(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{ax}{-a^7c^3x^7\sqrt{-a^2cx^2+c}-a^6c^3x^6\sqrt{-a^2cx^2+c}+3a^5c^3x^5\sqrt{-a^2cx^2+c}+3a^4c^3x^4\sqrt{-a^2cx^2+c}-3a^3c^3x^3\sqrt{-a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] -Integral(a*x/(-a**7*c**3*x**7*sqrt(-a**2*c*x**2 + c) - a**6*c**3*x**6*sqrt
(-a**2*c*x**2 + c) + 3*a**5*c**3*x**5*sqrt(-a**2*c*x**2 + c) + 3*a**4*c**3*
x**4*sqrt(-a**2*c*x**2 + c) - 3*a**3*c**3*x**3*sqrt(-a**2*c*x**2 + c) - 3*a
**2*c**3*x**2*sqrt(-a**2*c*x**2 + c) + a*c**3*x*sqrt(-a**2*c*x**2 + c) + c*
*3*sqrt(-a**2*c*x**2 + c)), x) - Integral(-1/(-a**7*c**3*x**7*sqrt(-a**2*c*
x**2 + c) - a**6*c**3*x**6*sqrt(-a**2*c*x**2 + c) + 3*a**5*c**3*x**5*sqrt(-
a**2*c*x**2 + c) + 3*a**4*c**3*x**4*sqrt(-a**2*c*x**2 + c) - 3*a**3*c**3*x*
*3*sqrt(-a**2*c*x**2 + c) - 3*a**2*c**3*x**2*sqrt(-a**2*c*x**2 + c) + a*c**
3*x*sqrt(-a**2*c*x**2 + c) + c**3*sqrt(-a**2*c*x**2 + c)), x)
```

3.1252 $\int e^{-2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

```
[Out] c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^2
+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)-2*a*c*x^(2+m)*hypergeom([1/2, 1+
1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)-x^(
1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

Rubi [A] time = 0.28, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6152, 1809, 808, 365, 364}

$$\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}-\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] -((x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m)) + (c*(3 + 2*m)*x^(1 + m)*Sqrt[1
- a^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*
(2 + m)*Sqrt[c - a^2*c*x^2]) - (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= c\int \frac{x^m(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{\int \frac{x^m\left(-a^2c(3+2m)+2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}\\
&= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-(2ac)\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx+\frac{(c(3+2m))\int \frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}\\
&= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{\left(2ac\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}+\frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)}{(2+m)\sqrt{c-a}}\\
&= -\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}-\frac{2ac}{}
\end{aligned}$$

Mathematica [C] time = 0.20, size = 111, normalized size = 0.65

$$\frac{x^{m+1}\left(\frac{2\sqrt{c-acx}\,F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-ax,ax\right)}{\sqrt{1-ax}}-\frac{\sqrt{c-a^2cx^2}\;{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (x^(1 + m)*((2*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/2, -1/2, 2 + m, -(a*x), a*
x])/Sqrt[1 - a*x] - (Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2,
 (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]))/(1 + m)
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,(ax-1)x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-a^2c\,x^2+c}\;\left(-a^2x^2+1\right)}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-1\right)x^m}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 1)*x^m/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^m*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int((x^m*(c - a^2*c*x^2)^(1/2)*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x^m\sqrt{-a^2cx^2+c}}{ax+1}\right)dx-\int \frac{axx^m\sqrt{-a^2cx^2+c}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -Integral(-x**m*sqrt(-a**2*c*x**2 + c)/(a*x + 1), x) - Integral(a*x*x**m*sq
rt(-a**2*c*x**2 + c)/(a*x + 1), x)
```

3.1253 $\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=54

$$\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(ax+1)\right)}{ap}$$

```
[Out] 2^(1+p)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[1+p],1/2*a*x+1/2)/a/p/((-a*x+
1)^p)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6142, 678, 69}

$$\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(ax+1)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*ArcTanh[a*x]),x]
```

```
[Out] (2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*x)/
2])/(a*p*(1 - a*x)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
```

```
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= c\int(1-ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\\ &= \left(c(1-ax)^{-p}(c+acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{1+p}(c+acx)^{-1+p}\,dx\\ &= \frac{2^{1+p}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1+ax)\right)}{ap}\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.37

$$-\frac{2^{p-1}(1-ax)^{p+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1-p,p+2;p+3;\frac{1}{2}(1-ax)\right)}{a(p+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*ArcTanh[a*x]),x]
[Out] -((2^(-1 + p)*(1 - a*x)^(2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1 - p,
2 + p, 3 + p, (1 - a*x)/2])/(a*(2 + p)*(1 - a^2*x^2)^p))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{(ax-1)\left(-a^2cx^2+c\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="fricas")
[Out] integral(-(a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2x^2-1\right)\left(-a^2cx^2+c\right)^p}{(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} c \,x^{2}+c \right)^{p} \left(-a^{2} x^{2}+1\right)}{\left(a x +1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^{2} x^{2}-1\right) \left(-a^{2} c x^{2}+c\right)^{p}}{\left(a x+1\right)^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{{\left(c-a^2\,c\,x^2\right)}^p\,\left(a^2\,x^2-1\right)}{{\left(a\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - a^2*c*x^2)^p*(a^2*x^2 - 1))/(a*x + 1)^2,x)
```

```
[Out] -int(((c - a^2*c*x^2)^p*(a^2*x^2 - 1))/(a*x + 1)^2, x)
```

sympy [C] time = 10.90, size = 651, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)**2*(-a**2*x**2+1),x)
```

```
[Out] -a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1
/(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - c**p*x**2*gamma(p)*gamm
```

```
a(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma
(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(
-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/
2 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a + 0**p*log(1/(a**2*x
**2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))
/a**2 - c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x
**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2
*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,
), 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) + Piecewise((0*
*p*log(a**2*x**2 - 1)/(2*a) + 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamm
a(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma
(-p)*gamma(p + 1)) + a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/
2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3
/2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
 + 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p
), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) + a**(2*
p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 -
p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True
))
```

3.1254 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=167

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}$$

```
[Out] 55/192*c^4*x*(-a^2*x^2+1)^(3/2)+11/48*c^4*x*(-a^2*x^2+1)^(5/2)+11/56*c^4*(-
a^2*x^2+1)^(7/2)/a+11/72*c^4*(-a*x+1)*(-a^2*x^2+1)^(7/2)/a+1/9*c^4*(-a*x+1)
^2*(-a^2*x^2+1)^(7/2)/a+55/128*c^4*arcsin(a*x)/a+55/128*c^4*x*(-a^2*x^2+1)^
(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6139, 671, 641, 195, 216}

$$\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (55*c^4*x*Sqrt[1 - a^2*x^2])/128 + (55*c^4*x*(1 - a^2*x^2)^(3/2))/192 + (11
*c^4*x*(1 - a^2*x^2)^(5/2))/48 + (11*c^4*(1 - a^2*x^2)^(7/2))/(56*a) + (11*
c^4*(1 - a*x)*(1 - a^2*x^2)^(7/2))/(72*a) + (c^4*(1 - a*x)^2*(1 - a^2*x^2)^
(7/2))/(9*a) + (55*c^4*ArcSin[a*x])/(128*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = c^4\int (1-ax)^3\left(1-a^2x^2\right)^{5/2}\,dx$$

$$= \frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{1}{9}\left(11c^4\right)\int (1-ax)^2\left(1-a^2x^2\right)^{5/2}\,dx$$

$$= \frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{1}{8}\left(11c^4\right)\int (1-ax)\left(1\right.$$

$$= \frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{c^4(1-ax)^2\left(1-a^2x^2\right)^{7/2}}{9a}+\frac{1}{8}$$

$$= \frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11c^4(1-ax)\left(1-a^2x^2\right)^{7/2}}{72a}+\frac{c^4(1-}{}$$

$$= \frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{11c^4\left(1-a^2x^2\right)^{7/2}}{56a}+\frac{11c^4(1-ax)}{7}$$

$$= \frac{55}{128}c^4x\sqrt{1-a^2x^2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{11c^4\left(1-a^2\right.}{56a}$$

$$= \frac{55}{128}c^4x\sqrt{1-a^2x^2}+\frac{55}{192}c^4x\left(1-a^2x^2\right)^{3/2}+\frac{11}{48}c^4x\left(1-a^2x^2\right)^{5/2}+\frac{11c^4\left(1-a^2\right.}{56a}$$

Mathematica [A] time = 0.18, size = 107, normalized size = 0.64

$$-\frac{c^4\left(\sqrt{1-a^2x^2}\left(896a^8x^8-3024a^7x^7+1024a^6x^6+7224a^5x^5-8448a^4x^4-3066a^3x^3+10240a^2x^2-4599ax-3\right.\right.}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/8064*(c^4*(Sqrt[1 - a^2*x^2]*(-3712 - 4599*a*x + 10240*a^2*x^2 - 3066*a^
3*x^3 - 8448*a^4*x^4 + 7224*a^5*x^5 + 1024*a^6*x^6 - 3024*a^7*x^7 + 896*a^8
*x^8) + 6930*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/a
```

fricas [A] time = 0.58, size = 136, normalized size = 0.81

$$-\frac{6930\,c^4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(896\,a^8c^4x^8-3024\,a^7c^4x^7+1024\,a^6c^4x^6+7224\,a^5c^4x^5-8448\,a^4c^4x^4-3066\,a^3\right.}{8064\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/8064*(6930*c^4*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (896*a^8*c^4*x^8
 - 3024*a^7*c^4*x^7 + 1024*a^6*c^4*x^6 + 7224*a^5*c^4*x^5 - 8448*a^4*c^4*x^
4 - 3066*a^3*c^4*x^3 + 10240*a^2*c^4*x^2 - 4599*a*c^4*x - 3712*c^4)*sqrt(-a
^2*x^2 + 1))/a
```

giac [A] time = 0.32, size = 126, normalized size = 0.75

$$\frac{55\,c^4\arcsin\left(ax\right)\mathrm{sgn}(a)}{128\,|a|}+\frac{1}{8064}\,\sqrt{-a^2x^2+1}\left(\frac{3712\,c^4}{a}+\left(4599\,c^4-2\left(5120\,ac^4-\left(1533\,a^2c^4+4\left(1056\,a^3c^4-\left(903\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 55/128*c^4*arcsin(a*x)*sgn(a)/abs(a) + 1/8064*sqrt(-a^2*x^2 + 1)*(3712*c^4/
a + (4599*c^4 - 2*(5120*a*c^4 - (1533*a^2*c^4 + 4*(1056*a^3*c^4 - (903*a^4*
c^4 + 2*(64*a^5*c^4 + 7*(8*a^7*c^4*x - 27*a^6*c^4)*x)*x)*x)*x)*x)*x)*x)
```

maple [A] time = 0.05, size = 173, normalized size = 1.04

$$-\frac{c^4a^3x^4\left(-a^2x^2+1\right)^{\frac{5}{2}}}{9}-\frac{22c^4a\,x^2\left(-a^2x^2+1\right)^{\frac{5}{2}}}{63}+\frac{29c^4\left(-a^2x^2+1\right)^{\frac{5}{2}}}{63a}+\frac{3c^4a^2x^3\left(-a^2x^2+1\right)^{\frac{5}{2}}}{8}-\frac{7c^4x\left(-a^2x^2+1\right)^{\frac{5}{2}}}{48}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/9*c^4*a^3*x^4*(-a^2*x^2+1)^(5/2)-22/63*c^4*a*x^2*(-a^2*x^2+1)^(5/2)+29/6
3*c^4*(-a^2*x^2+1)^(5/2)/a+3/8*c^4*a^2*x^3*(-a^2*x^2+1)^(5/2)-7/48*c^4*x*(-
a^2*x^2+1)^(5/2)+55/192*c^4*x*(-a^2*x^2+1)^(3/2)+55/128*c^4*x*(-a^2*x^2+1)^
(1/2)+55/128*c^4/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.44, size = 154, normalized size = 0.92

$$-\frac{1}{9}\left(-a^2x^2+1\right)^{\frac{5}{2}}a^3c^4x^4+\frac{3}{8}\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2c^4x^3-\frac{22}{63}\left(-a^2x^2+1\right)^{\frac{5}{2}}ac^4x^2-\frac{7}{48}\left(-a^2x^2+1\right)^{\frac{5}{2}}c^4x+\frac{55}{192}\left(-a^2x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/9*(-a^2*x^2 + 1)^(5/2)*a^3*c^4*x^4 + 3/8*(-a^2*x^2 + 1)^(5/2)*a^2*c^4*x^
3 - 22/63*(-a^2*x^2 + 1)^(5/2)*a*c^4*x^2 - 7/48*(-a^2*x^2 + 1)^(5/2)*c^4*x
+ 55/192*(-a^2*x^2 + 1)^(3/2)*c^4*x + 29/63*(-a^2*x^2 + 1)^(5/2)*c^4/a + 55
/128*sqrt(-a^2*x^2 + 1)*c^4*x + 55/128*c^4*arcsin(a*x)/a
```

mupad [B] time = 0.07, size = 220, normalized size = 1.32

$$\frac{73\,c^4\,x\,\sqrt{1-a^2\,x^2}}{128}+\frac{55\,c^4\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{128\,\sqrt{-a^2}}+\frac{29\,c^4\,\sqrt{1-a^2\,x^2}}{63\,a}-\frac{80\,a\,c^4\,x^2\,\sqrt{1-a^2\,x^2}}{63}+\frac{73\,a^2\,c^4\,x^3\,\sqrt{1-a^2\,x^2}}{192}+\frac{22}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (73*c^4*x*(1 - a^2*x^2)^(1/2))/128 + (55*c^4*asinh(x*(-a^2)^(1/2)))/(128*(-
a^2)^(1/2)) + (29*c^4*(1 - a^2*x^2)^(1/2))/(63*a) - (80*a*c^4*x^2*(1 - a^2*
x^2)^(1/2))/63 + (73*a^2*c^4*x^3*(1 - a^2*x^2)^(1/2))/192 + (22*a^3*c^4*x^4
*(1 - a^2*x^2)^(1/2))/21 - (43*a^4*c^4*x^5*(1 - a^2*x^2)^(1/2))/48 - (8*a^5
*c^4*x^6*(1 - a^2*x^2)^(1/2))/63 + (3*a^6*c^4*x^7*(1 - a^2*x^2)^(1/2))/8 -
(a^7*c^4*x^8*(1 - a^2*x^2)^(1/2))/9
```

sympy [C] time = 20.38, size = 996, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -a**7*c**4*Piecewise((x**8*sqrt(-a**2*x**2 + 1)/9 - x**6*sqrt(-a**2*x**2 +
1)/(63*a**2) - 2*x**4*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*x**2*sqrt(-a**2*x
**2 + 1)/(315*a**6) - 16*sqrt(-a**2*x**2 + 1)/(315*a**8), Ne(a, 0)), (x**8/
8, True)) + 3*a**6*c**4*Piecewise((I*a**2*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*
I*x**7/(48*sqrt(a**2*x**2 - 1)) - I*x**5/(192*a**2*sqrt(a**2*x**2 - 1)) - 5
*I*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*x/(128*a**6*sqrt(a**2*x**2 - 1
)) - 5*I*acosh(a*x)/(128*a**7), Abs(a**2*x**2) > 1), (-a**2*x**9/(8*sqrt(-a
**2*x**2 + 1)) + 7*x**7/(48*sqrt(-a**2*x**2 + 1)) + x**5/(192*a**2*sqrt(-a*
*2*x**2 + 1)) + 5*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*x/(128*a**6*sqrt
(-a**2*x**2 + 1)) + 5*asin(a*x)/(128*a**7), True)) - a**5*c**4*Piecewise((x
**6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 + 1)/(35*a**2) - 4*x**2*s
qrt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 + 1)/(105*a**6), Ne(a, 0
)), (x**6/6, True)) - 5*a**4*c**4*Piecewise((I*a**2*x**7/(6*sqrt(a**2*x**2
- 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(48*a**2*sqrt(a**2*x**2
- 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(16*a**5), Abs(a**
2*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*x**5/(24*sqrt(-a**2*
x**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x/(16*a**4*sqrt(-a**2*x*
*2 + 1)) + asin(a*x)/(16*a**5), True)) + 5*a**3*c**4*Piecewise((x**4*sqrt(-
a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**2) - 2*sqrt(-a**2*x**2
+ 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) + a**2*c**4*Piecewise((I*a**2*x*
*5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*x**2 - 1)) + I*x/(8*a**2
*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*
x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**2*x**2 + 1)) - x/(8*a**2
*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True)) - 3*a*c**4*Piecewise((x
**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a**2), True)) + c**4*Piec
ewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2*sqrt(a**2*x**2 - 1)) -
I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a**2*x**2 + 1)/2 + asin(a
*x)/(2*a), True))
```

3.1255 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=145

$$\frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a}+\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{9c}{}$$

```
[Out] 3/8*c^3*x*(-a^2*x^2+1)^(3/2)+3/10*c^3*(-a^2*x^2+1)^(5/2)/a+3/14*c^3*(-a*x+1
)*(-a^2*x^2+1)^(5/2)/a+1/7*c^3*(-a*x+1)^2*(-a^2*x^2+1)^(5/2)/a+9/16*c^3*arc
sin(a*x)/a+9/16*c^3*x*(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6139, 671, 641, 195, 216}

$$\frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a}+\frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a}+\frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a}+\frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2}+\frac{9}{16}c^3x\sqrt{1-a^2x^2}+\frac{9c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (9*c^3*x*Sqrt[1 - a^2*x^2])/16 + (3*c^3*x*(1 - a^2*x^2)^(3/2))/8 + (3*c^3*(
1 - a^2*x^2)^(5/2))/(10*a) + (3*c^3*(1 - a*x)*(1 - a^2*x^2)^(5/2))/(14*a) +
 (c^3*(1 - a*x)^2*(1 - a^2*x^2)^(5/2))/(7*a) + (9*c^3*ArcSin[a*x])/(16*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = c^3\int(1-ax)^3\left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a} + \frac{1}{7}\left(9c^3\right)\int(1-ax)^2\left(1-a^2x^2\right)^{3/2}\,dx$$
$$= \frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a} + \frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a} + \frac{1}{2}\left(3c^3\right)\int(1-ax)\left(1-a\right.$$
$$= \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a} + \frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a} + \frac{c^3(1-ax)^2\left(1-a^2x^2\right)^{5/2}}{7a} + \frac{1}{2}\left(3c\right.$$
$$= \frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2} + \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a} + \frac{3c^3(1-ax)\left(1-a^2x^2\right)^{5/2}}{14a} + \frac{c^3(1-ax)^2}{7}$$
$$= \frac{9}{16}c^3x\sqrt{1-a^2x^2} + \frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2} + \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a} + \frac{3c^3(1-ax)\left(1-a^2x\right.}{14a}$$
$$= \frac{9}{16}c^3x\sqrt{1-a^2x^2} + \frac{3}{8}c^3x\left(1-a^2x^2\right)^{3/2} + \frac{3c^3\left(1-a^2x^2\right)^{5/2}}{10a} + \frac{3c^3(1-ax)\left(1-a^2x\right.}{14a}$$

Mathematica [A] time = 0.15, size = 91, normalized size = 0.63

$$\frac{c^3\left(\sqrt{1-a^2x^2}\left(80a^6x^6-280a^5x^5+208a^4x^4+350a^3x^3-656a^2x^2+245ax+368\right)-630\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^3*(Sqrt[1 - a^2*x^2]*(368 + 245*a*x - 656*a^2*x^2 + 350*a^3*x^3 + 208*a^
4*x^4 - 280*a^5*x^5 + 80*a^6*x^6) - 630*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(56
0*a)
```

fricas [A] time = 0.58, size = 115, normalized size = 0.79

$$-\frac{630\, c^{3} \arctan\left(\frac{\sqrt{-a^{2} x^{2}+1}-1}{a x}\right) - \left(80\, a^{6} c^{3} x^{6} - 280\, a^{5} c^{3} x^{5} + 208\, a^{4} c^{3} x^{4} + 350\, a^{3} c^{3} x^{3} - 656\, a^{2} c^{3} x^{2} + 245\, a c^{3} x + 36}{560\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/560*(630*c^3*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (80*a^6*c^3*x^6 -
280*a^5*c^3*x^5 + 208*a^4*c^3*x^4 + 350*a^3*c^3*x^3 - 656*a^2*c^3*x^2 + 245
*a*c^3*x + 368*c^3)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.20, size = 102, normalized size = 0.70

$$\frac{9\, c^{3} \arcsin\left(a x\right) \mathrm{sgn}\left(a\right)}{16\, {\left| a \right|}} + \frac{1}{560} \, \sqrt{-a^{2} x^{2} + 1} \left(\frac{368\, c^{3}}{a} + \left(245\, c^{3} - 2\, \left(328\, a c^{3} - \left(175\, a^{2} c^{3} + 4\, \left(26\, a^{3} c^{3} + 5\, \left(2\, a^{5} c^{3} x -\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 9/16*c^3*arcsin(a*x)*sgn(a)/abs(a) + 1/560*sqrt(-a^2*x^2 + 1)*(368*c^3/a +
(245*c^3 - 2*(328*a*c^3 - (175*a^2*c^3 + 4*(26*a^3*c^3 + 5*(2*a^5*c^3*x - 7
*a^4*c^3)*x)*x)*x)*x)*x)
```

maple [A] time = 0.04, size = 127, normalized size = 0.88

$$\frac{c^{3} a\, x^{2} \left(-a^{2} x^{2}+1\right)^{\frac{5}{2}}}{7}+\frac{23 c^{3} \left(-a^{2} x^{2}+1\right)^{\frac{5}{2}}}{35 a}-\frac{c^{3} x \left(-a^{2} x^{2}+1\right)^{\frac{5}{2}}}{2}+\frac{3 c^{3} x \left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{8}+\frac{9 c^{3} x \sqrt{-a^{2} x^{2}+1}}{16}+\frac{9 c^{3} \arctan}{16 \sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/7*c^3*a*x^2*(-a^2*x^2+1)^(5/2)+23/35*c^3*(-a^2*x^2+1)^(5/2)/a-1/2*c^3*x*(
-a^2*x^2+1)^(5/2)+3/8*c^3*x*(-a^2*x^2+1)^(3/2)+9/16*c^3*x*(-a^2*x^2+1)^(1/2
)+9/16*c^3/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 0.45, size = 108, normalized size = 0.74

$$\frac{1}{7}\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}ac^{3}x^{2}-\frac{1}{2}\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}c^{3}x+\frac{3}{8}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}c^{3}x+\frac{23\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}c^{3}}{35\,a}+\frac{9}{16}\sqrt{-a^{2}x^{2}+1}\,c^{3}x+\frac{9\,c^{3}\,\mathrm{arcsi}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/7*(-a^2*x^2 + 1)^(5/2)*a*c^3*x^2 - 1/2*(-a^2*x^2 + 1)^(5/2)*c^3*x + 3/8*(
-a^2*x^2 + 1)^(3/2)*c^3*x + 23/35*(-a^2*x^2 + 1)^(5/2)*c^3/a + 9/16*sqrt(-a
^2*x^2 + 1)*c^3*x + 9/16*c^3*arcsin(a*x)/a
```

mupad [B] time = 0.04, size = 174, normalized size = 1.20

$$\frac{7\,c^{3}\,x\,\sqrt{1-a^{2}\,x^{2}}}{16}+\frac{9\,c^{3}\,\mathrm{asinh}\left(x\,\sqrt{-a^{2}}\right)}{16\,\sqrt{-a^{2}}}+\frac{23\,c^{3}\,\sqrt{1-a^{2}\,x^{2}}}{35\,a}-\frac{41\,a\,c^{3}\,x^{2}\,\sqrt{1-a^{2}\,x^{2}}}{35}+\frac{5\,a^{2}\,c^{3}\,x^{3}\,\sqrt{1-a^{2}\,x^{2}}}{8}+\frac{13\,a^{3}\,c^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (7*c^3*x*(1 - a^2*x^2)^(1/2))/16 + (9*c^3*asinh(x*(-a^2)^(1/2)))/(16*(-a^2)
^(1/2)) + (23*c^3*(1 - a^2*x^2)^(1/2))/(35*a) - (41*a*c^3*x^2*(1 - a^2*x^2)
^(1/2))/35 + (5*a^2*c^3*x^3*(1 - a^2*x^2)^(1/2))/8 + (13*a^3*c^3*x^4*(1 - a
^2*x^2)^(1/2))/35 - (a^4*c^3*x^5*(1 - a^2*x^2)^(1/2))/2 + (a^5*c^3*x^6*(1 -
 a^2*x^2)^(1/2))/7
```

sympy [C] time = 13.78, size = 632, normalized size = 4.36

$$a^{5}c^{3}\left(\begin{cases}\frac{x^{6}\sqrt{-a^{2}x^{2}+1}}{7}-\frac{x^{4}\sqrt{-a^{2}x^{2}+1}}{35a^{2}}-\frac{4x^{2}\sqrt{-a^{2}x^{2}+1}}{105a^{4}}-\frac{8\sqrt{-a^{2}x^{2}+1}}{105a^{6}} & \text{for } a\neq 0\\ \frac{x^{6}}{6} & \text{otherwise}\end{cases}\right)-3a^{4}c^{3}\left(\begin{cases}\frac{ia^{2}x^{7}}{6\sqrt{a^{2}x^{2}-1}}-\frac{5ix^{5}}{24\sqrt{a^{2}x^{2}-1}}-\frac{ix^{3}}{48a^{2}\sqrt{a^{2}}} \\ -\frac{a^{2}x^{7}}{6\sqrt{-a^{2}x^{2}+1}}+\frac{5x^{5}}{24\sqrt{-a^{2}x^{2}+1}}+\frac{}{48a}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] a**5*c**3*Piecewise((x**6*sqrt(-a**2*x**2 + 1)/7 - x**4*sqrt(-a**2*x**2 + 1
)/(35*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)/(105*a**4) - 8*sqrt(-a**2*x**2 +
1)/(105*a**6), Ne(a, 0)), (x**6/6, True)) - 3*a**4*c**3*Piecewise((I*a**2*x
**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*x**5/(24*sqrt(a**2*x**2 - 1)) - I*x**3/(4
```

```
8*a**2*sqrt(a**2*x**2 - 1)) + I*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*acosh(a
*x)/(16*a**5), Abs(a**2*x**2) > 1), (-a**2*x**7/(6*sqrt(-a**2*x**2 + 1)) +
5*x**5/(24*sqrt(-a**2*x**2 + 1)) + x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - x/
(16*a**4*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(16*a**5), True)) + 2*a**3*c**3*
Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 + 1)/(15*a**2
) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True)) + 2*a**2*c
**3*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8*sqrt(a**2*
x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*a**3), Abs(
a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(8*sqrt(-a**
2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a**3), True))
 - 3*a*c**3*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)**(3/2)/(3*a
**2), True)) + c**3*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1)) - I*x/(2
*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (x*sqrt(-a
**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

3.1256 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=123

$$\frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\sin^{-1}(ax)}{8a}$$

```
[Out] 7/12*c^2*(-a^2*x^2+1)^(3/2)/a+7/20*c^2*(-a*x+1)*(-a^2*x^2+1)^(3/2)/a+1/5*c^
2*(-a*x+1)^2*(-a^2*x^2+1)^(3/2)/a+7/8*c^2*arcsin(a*x)/a+7/8*c^2*x*(-a^2*x^2
+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6139, 671, 641, 195, 216}

$$\frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a}+\frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{7}{8}c^2x\sqrt{1-a^2x^2}+\frac{7c^2\sin^{-1}(ax)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (7*c^2*x*Sqrt[1 - a^2*x^2])/8 + (7*c^2*(1 - a^2*x^2)^(3/2))/(12*a) + (7*c^2
*(1 - a*x)*(1 - a^2*x^2)^(3/2))/(20*a) + (c^2*(1 - a*x)^2*(1 - a^2*x^2)^(3/
2))/(5*a) + (7*c^2*ArcSin[a*x])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\int (1-ax)^3\sqrt{1-a^2x^2}\,dx$$

$$= \frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a} + \frac{1}{5}\left(7c^2\right)\int (1-ax)^2\sqrt{1-a^2x^2}\,dx$$

$$= \frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a} + \frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a} + \frac{1}{4}\left(7c^2\right)\int (1-ax)\sqrt{1-$$

$$= \frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a} + \frac{c^2(1-ax)^2\left(1-a^2x^2\right)^{3/2}}{5a} + \frac{1}{4}($$

$$= \frac{7}{8}c^2x\sqrt{1-a^2x^2} + \frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a} + \frac{c^2(1-ax)^2(}{5}$$

$$= \frac{7}{8}c^2x\sqrt{1-a^2x^2} + \frac{7c^2\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{7c^2(1-ax)\left(1-a^2x^2\right)^{3/2}}{20a} + \frac{c^2(1-ax)^2(}{5}$$

Mathematica [A] time = 0.12, size = 75, normalized size = 0.61

$$-\frac{c^2\left(\sqrt{1-a^2x^2}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)+210\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/120*(c^2*(Sqrt[1 - a^2*x^2]*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
24*a^4*x^4) + 210*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/a
```

fricas [A] time = 0.54, size = 92, normalized size = 0.75

$$-\frac{210\,c^2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\left(24\,a^4c^2x^4-90\,a^3c^2x^3+112\,a^2c^2x^2-15\,ac^2x-136\,c^2\right)\sqrt{-a^2x^2+1}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/120*(210*c^2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (24*a^4*c^2*x^4 -
90*a^3*c^2*x^3 + 112*a^2*c^2*x^2 - 15*a*c^2*x - 136*c^2)*sqrt(-a^2*x^2 + 1)
)/a
```

giac [A] time = 0.21, size = 78, normalized size = 0.63

$$\frac{7\,c^2\arcsin\left(ax\right)\mathrm{sgn}(a)}{8\,|a|}+\frac{1}{120}\,\sqrt{-a^2x^2+1}\left(\left(15\,c^2-2\left(56\,ac^2+3\left(4\,a^3c^2x-15\,a^2c^2\right)x\right)x\right)x+\frac{136\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] 7/8*c^2*arcsin(a*x)*sgn(a)/abs(a) + 1/120*sqrt(-a^2*x^2 + 1)*((15*c^2 - 2*(
56*a*c^2 + 3*(4*a^3*c^2*x - 15*a^2*c^2)*x)*x)*x + 136*c^2/a)
```

maple [A] time = 0.04, size = 189, normalized size = 1.54

$$-\frac{c^2\left(-a^2x^2+1\right)^{\frac{5}{2}}}{5a}-\frac{3c^2x\left(-a^2x^2+1\right)^{\frac{3}{2}}}{4}-\frac{9c^2x\sqrt{-a^2x^2+1}}{8}-\frac{9c^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8\sqrt{a^2}}+\frac{4c^2\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right.}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/5*c^2*(-a^2*x^2+1)^(5/2)/a-3/4*c^2*x*(-a^2*x^2+1)^(3/2)-9/8*c^2*x*(-a^2*
x^2+1)^(1/2)-9/8*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+4
/3*c^2/a*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(3/2)+2*c^2*(-a^2*(x+1/a)^2+2*a*(x+1/
a))^(1/2)*x+2*c^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1
/a))^(1/2))
```

maxima [C] time = 0.46, size = 147, normalized size = 1.20

$$-\frac{3}{4}\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2x-\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}c^2}{5a}+2\sqrt{a^2x^2+4ax+3}\,c^2x-\frac{9}{8}\sqrt{-a^2x^2+1}\,c^2x+\frac{4\left(-a^2x^2+1\right)^{\frac{3}{2}}c^2}{3a}-\frac{2ic^2\operatorname{arcsi}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -3/4*(-a^2*x^2 + 1)^(3/2)*c^2*x - 1/5*(-a^2*x^2 + 1)^(5/2)*c^2/a + 2*sqrt(a
^2*x^2 + 4*a*x + 3)*c^2*x - 9/8*sqrt(-a^2*x^2 + 1)*c^2*x + 4/3*(-a^2*x^2 +
1)^(3/2)*c^2/a - 2*I*c^2*arcsin(a*x + 2)/a - 9/8*c^2*arcsin(a*x)/a + 4*sqrt
(a^2*x^2 + 4*a*x + 3)*c^2/a
```

mupad [B] time = 0.04, size = 128, normalized size = 1.04

$$\frac{c^2\,x\,\sqrt{1-a^2\,x^2}}{8}+\frac{7\,c^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{8\,\sqrt{-a^2}}+\frac{17\,c^2\,\sqrt{1-a^2\,x^2}}{15\,a}-\frac{14\,a\,c^2\,x^2\,\sqrt{1-a^2\,x^2}}{15}+\frac{3\,a^2\,c^2\,x^3\,\sqrt{1-a^2\,x^2}}{4}-\frac{a^3\,c^2\,x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (c^2*x*(1 - a^2*x^2)^(1/2))/8 + (7*c^2*asinh(x*(-a^2)^(1/2)))/(8*(-a^2)^(1/
2)) + (17*c^2*(1 - a^2*x^2)^(1/2))/(15*a) - (14*a*c^2*x^2*(1 - a^2*x^2)^(1/
2))/15 + (3*a^2*c^2*x^3*(1 - a^2*x^2)^(1/2))/4 - (a^3*c^2*x^4*(1 - a^2*x^2)
^(1/2))/5
```

sympy [C] time = 8.10, size = 340, normalized size = 2.76

$$-a^3c^2\left(\begin{cases}\frac{x^4\sqrt{-a^2x^2+1}}{5}-\frac{x^2\sqrt{-a^2x^2+1}}{15a^2}-\frac{2\sqrt{-a^2x^2+1}}{15a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+3a^2c^2\left(\begin{cases}\frac{ia^2x^5}{4\sqrt{a^2x^2-1}}-\frac{3ix^3}{8\sqrt{a^2x^2-1}}+\frac{ix}{8a^2\sqrt{a^2x^2-1}}-\frac{i\operatorname{acosh}}{8a^3} \\ -\frac{a^2x^5}{4\sqrt{-a^2x^2+1}}+\frac{3x^3}{8\sqrt{-a^2x^2+1}}-\frac{x}{8a^2\sqrt{-a^2x^2+1}}+\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -a**3*c**2*Piecewise((x**4*sqrt(-a**2*x**2 + 1)/5 - x**2*sqrt(-a**2*x**2 +
1)/(15*a**2) - 2*sqrt(-a**2*x**2 + 1)/(15*a**4), Ne(a, 0)), (x**4/4, True))
 + 3*a**2*c**2*Piecewise((I*a**2*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*x**3/(8
*sqrt(a**2*x**2 - 1)) + I*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(8*
```

```
a**3), Abs(a**2*x**2) > 1), (-a**2*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*x**3/(
8*sqrt(-a**2*x**2 + 1)) - x/(8*a**2*sqrt(-a**2*x**2 + 1)) + asin(a*x)/(8*a*
*3), True)) - 3*a*c**2*Piecewise((x**2/2, Eq(a**2, 0)), (-(-a**2*x**2 + 1)*
*(3/2)/(3*a**2), True)) + c**2*Piecewise((I*a**2*x**3/(2*sqrt(a**2*x**2 - 1
)) - I*x/(2*sqrt(a**2*x**2 - 1)) - I*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1),
 (x*sqrt(-a**2*x**2 + 1)/2 + asin(a*x)/(2*a), True))
```

3.1257 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=93

$$\frac{c\sqrt{1-a^2x^2}\,(1-ax)^2}{3a}+\frac{5c\sqrt{1-a^2x^2}\,(1-ax)}{6a}+\frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c\sin^{-1}(ax)}{2a}$$

```
[Out] 5/2*c*arcsin(a*x)/a+5/2*c*(-a^2*x^2+1)^(1/2)/a+5/6*c*(-a*x+1)*(-a^2*x^2+1)^
(1/2)/a+1/3*c*(-a*x+1)^2*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6139, 671, 641, 216}

$$\frac{c\sqrt{1-a^2x^2}\,(1-ax)^2}{3a}+\frac{5c\sqrt{1-a^2x^2}\,(1-ax)}{6a}+\frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (5*c*Sqrt[1 - a^2*x^2])/(2*a) + (5*c*(1 - a*x)*Sqrt[1 - a^2*x^2])/(6*a) + (
c*(1 - a*x)^2*Sqrt[1 - a^2*x^2])/(3*a) + (5*c*ArcSin[a*x])/(2*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx &= c\int\frac{(1-ax)^3}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{c(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{3}(5c)\int\frac{(1-ax)^2}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{5c(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{c(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}(5c)\int\frac{1-ax}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{c(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{1}{2}(5c)\int\frac{1}{\sqrt{1-a^2}}\\ &= \frac{5c\sqrt{1-a^2x^2}}{2a}+\frac{5c(1-ax)\sqrt{1-a^2x^2}}{6a}+\frac{c(1-ax)^2\sqrt{1-a^2x^2}}{3a}+\frac{5c\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [A] time = 0.10, size = 70, normalized size = 0.75

$$\frac{c\left(\frac{\sqrt{ax+1}\left(-2a^3x^3+11a^2x^2-31ax+22\right)}{\sqrt{1-ax}}-30\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^(3*ArcTanh[a*x]),x]
[Out] (c*((Sqrt[1 + a*x]*(22 - 31*a*x + 11*a^2*x^2 - 2*a^3*x^3))/Sqrt[1 - a*x] -
30*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(6*a)
```

fricas [A] time = 0.61, size = 63, normalized size = 0.68

$$-\frac{30\,c\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(2\,a^2cx^2-9\,acx+22\,c\right)\sqrt{-a^2x^2+1}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(30*c*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (2*a^2*c*x^2 - 9*a*c*x
+ 22*c)*sqrt(-a^2*x^2 + 1))/a
```

giac [A] time = 0.27, size = 46, normalized size = 0.49

$$\frac{5\,c\arcsin\left(ax\right)\operatorname{sgn}(a)}{2\,|a|}+\frac{1}{6}\,\sqrt{-a^2x^2+1}\left((2\,acx-9\,c)x+\frac{22\,c}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 5/2*c*arcsin(a*x)*sgn(a)/abs(a) + 1/6*sqrt(-a^2*x^2 + 1)*((2*a*c*x - 9*c)*x
 + 22*c/a)
```

maple [A] time = 0.04, size = 133, normalized size = 1.43

$$\frac{2c\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a^3\left(x+\frac{1}{a}\right)^2}+\frac{5c\left(-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3a}+\frac{5c\sqrt{-a^2\left(x+\frac{1}{a}\right)^2+2a\left(x+\frac{1}{a}\right)}\,x}{2}+\frac{5c\operatorname{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 2*c/a^3/(x+1/a)^2*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(5/2)+5/3*c/a*(-a^2*(x+1/a)^
2+2*a*(x+1/a))^(3/2)+5/2*c*(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2)*x+5/2*c/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+1/a)^2+2*a*(x+1/a))^(1/2))
```

maxima [C] time = 0.50, size = 122, normalized size = 1.31

$$-\frac{1}{2}\sqrt{a^2x^2+4\,ax+3}\,cx+\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{a^2x+a}-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}c}{3\,a}+\frac{i\,c\arcsin\left(ax+2\right)}{2\,a}+\frac{3\,c\arcsin\left(ax\right)}{a}-\frac{\sqrt{a^2x^2+4\,ax+3}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*sqrt(a^2*x^2 + 4*a*x + 3)*c*x + (-a^2*x^2 + 1)^(3/2)*c/(a^2*x + a) - 1
/3*(-a^2*x^2 + 1)^(3/2)*c/a + 1/2*I*c*arcsin(a*x + 2)/a + 3*c*arcsin(a*x)/a
 - sqrt(a^2*x^2 + 4*a*x + 3)*c/a + 3*sqrt(-a^2*x^2 + 1)*c/a
```

mupad [B] time = 0.04, size = 74, normalized size = 0.80

$$\frac{11\,c\,\sqrt{1-a^2\,x^2}}{3\,a}-\frac{3\,c\,x\,\sqrt{1-a^2\,x^2}}{2}+\frac{5\,c\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,\sqrt{-a^2}}+\frac{a\,c\,x^2\,\sqrt{1-a^2\,x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] (11*c*(1 - a^2*x^2)^(1/2))/(3*a) - (3*c*x*(1 - a^2*x^2)^(1/2))/2 + (5*c*asi
nh(x*(-a^2)^(1/2)))/(2*(-a^2)^(1/2)) + (a*c*x^2*(1 - a^2*x^2)^(1/2))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx+\int \left(-\frac{2a^2x^2\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\right)dx+\int \frac{a^4x^4\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] c*(Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x)
+ Integral(-2*a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a
*x + 1), x) + Integral(a**4*x**4*sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x
**2 + 3*a*x + 1), x))
```

3.1258 $\int \frac{e^{-3\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$-\frac{e^{-3\tanh^{-1}(ax)}}{3ac}$$

```
[Out] -1/3/a/c/(a*x+1)^3*(-a^2*x^2+1)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6137}

$$-\frac{e^{-3\tanh^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(3*a*c*E^(3*ArcTanh[a*x]))
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-3\tanh^{-1}(ax)}}{3ac}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.61

$$-\frac{(1-ax)^{3/2}}{3ac(ax+1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/3*(1 - a*x)^(3/2)/(a*c*(1 + a*x)^(3/2))
```

fricas [B] time = 0.65, size = 55, normalized size = 3.06

$$-\frac{a^2x^2 + 2\,ax - \sqrt{-a^2x^2 + 1}\,(ax - 1) + 1}{3\,\left(a^3cx^2 + 2\,a^2cx + ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="frica
s")
```

```
[Out] -1/3*(a^2*x^2 + 2*a*x - sqrt(-a^2*x^2 + 1)*(a*x - 1) + 1)/(a^3*c*x^2 + 2*a^
2*c*x + a*c)
```

giac [B] time = 0.38, size = 66, normalized size = 3.67

$$\frac{2\left(\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+1\right)}{3\,c\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac"
)
```

```
[Out] 2/3*(3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 1)/(c*((sqrt(-a^2*x^2
+ 1)*abs(a) + a)/(a^2*x) + 1)^3*abs(a))
```

maple [A] time = 0.03, size = 28, normalized size = 1.56

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{3ac\,(ax+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/3/a/c/(a*x+1)^3*(-a^2*x^2+1)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxim
a")
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)*(a*x + 1)^3), x)
```

mupad [B] time = 0.96, size = 32, normalized size = 1.78

$$\frac{\sqrt{1-a^2\,x^2}\,\left(a\,x-1\right)}{3\,a\,c\,{\left(a\,x+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)*(a*x + 1)^3),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(a*x - 1))/(3*a*c*(a*x + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^3x^3+3a^2x^2+3ax+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] Integral(sqrt(-a**2*x**2 + 1)/(a**3*x**3 + 3*a**2*x**2 + 3*a*x + 1), x)/c
```

3.1259 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{2\sqrt{1-a^2x^2}}{15ac^2(ax+1)}-\frac{2\sqrt{1-a^2x^2}}{15ac^2(ax+1)^2}-\frac{\sqrt{1-a^2x^2}}{5ac^2(ax+1)^3}$$

```
[Out] -1/5*(-a^2*x^2+1)^(1/2)/a/c^2/(a*x+1)^3-2/15*(-a^2*x^2+1)^(1/2)/a/c^2/(a*x+
1)^2-2/15*(-a^2*x^2+1)^(1/2)/a/c^2/(a*x+1)
```

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 655, 659, 651}

$$-\frac{2\sqrt{1-a^2x^2}}{15ac^2(ax+1)}-\frac{2\sqrt{1-a^2x^2}}{15ac^2(ax+1)^2}-\frac{\sqrt{1-a^2x^2}}{5ac^2(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(5*a*c^2*(1 + a*x)^3) - (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*
(1 + a*x)^2) - (2*Sqrt[1 - a^2*x^2])/(15*a*c^2*(1 + a*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
```

```
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= \frac{\int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^2}\\
&= \frac{\int \frac{1}{(1+ax)^3\sqrt{1-a^2x^2}}\,dx}{c^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{5ac^2(1+ax)^3}+\frac{2\int \frac{1}{(1+ax)^2\sqrt{1-a^2x^2}}\,dx}{5c^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{5ac^2(1+ax)^3}-\frac{2\sqrt{1-a^2x^2}}{15ac^2(1+ax)^2}+\frac{2\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{15c^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{5ac^2(1+ax)^3}-\frac{2\sqrt{1-a^2x^2}}{15ac^2(1+ax)^2}-\frac{2\sqrt{1-a^2x^2}}{15ac^2(1+ax)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.46

$$-\frac{\sqrt{1-ax}\left(2a^2x^2+6ax+7\right)}{15ac^2(ax+1)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^2),x]
[Out] -1/15*(Sqrt[1 - a*x]*(7 + 6*a*x + 2*a^2*x^2))/(a*c^2*(1 + a*x)^(5/2))
```

fricas [A] time = 1.04, size = 89, normalized size = 0.95

$$-\frac{7\,a^3x^3+21\,a^2x^2+21\,ax+\left(2\,a^2x^2+6\,ax+7\right)\sqrt{-a^2x^2+1}+7}{15\left(a^4c^2x^3+3\,a^3c^2x^2+3\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fri
cas")
```

```
[Out] -1/15*(7*a^3*x^3 + 21*a^2*x^2 + 21*a*x + (2*a^2*x^2 + 6*a*x + 7)*sqrt(-a^2*
x^2 + 1) + 7)/(a^4*c^2*x^3 + 3*a^3*c^2*x^2 + 3*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.23, size = 145, normalized size = 1.54

$$\frac{2\left(\frac{20\left(\sqrt{-a^2x^2+1}\,|a|+a\right)}{a^2x}+\frac{40\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{a^4x^2}+\frac{30\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^3}{a^6x^3}+\frac{15\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^4}{a^8x^4}+7\right)}{15\,c^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)^5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="gia
c")
```

```
[Out] 2/15*(20*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 40*(sqrt(-a^2*x^2 + 1)*a
bs(a) + a)^2/(a^4*x^2) + 30*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) + 1
5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) + 7)/(c^2*((sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(a^2*x) + 1)^5*abs(a))
```

maple [A] time = 0.03, size = 42, normalized size = 0.45

$$-\frac{\sqrt{-a^2x^2+1}\;\left(2a^2x^2+6ax+7\right)}{15\,(ax+1)^3\,c^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/15*(-a^2*x^2+1)^(1/2)*(2*a^2*x^2+6*a*x+7)/(a*x+1)^3/c^2/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^2(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^2*(a*x + 1)^3), x)
```

mupad [B] time = 0.06, size = 125, normalized size = 1.33

$$-\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{a^3}{5\,c^2\,{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)}^3}-\frac{2\,a^3}{15\,c^2\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)}+\frac{2\,a^4}{15\,c^2\,{\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)}^2\,\sqrt{-a^2}}\right)}{a^3\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^2*(a*x + 1)^3),x)
```

```
[Out] -((1 - a^2*x^2)^(1/2)*(a^3/(5*c^2*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)^3) - (2
*a^3)/(15*c^2*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)) + (2*a^4)/(15*c^2*(x*(-a^2
)^(1/2) + (-a^2)^(1/2)/a)^2*(-a^2)^(1/2))))/(a^3*(-a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^3x^3\sqrt{-a^2x^2+1}+3a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(1/(a**3*x**3*sqrt(-a**2*x**2 + 1) + 3*a**2*x**2*sqrt(-a**2*x**2 +
1) + 3*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**2
```

3.1260 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=116

$$\frac{8x}{35c^3\sqrt{1-a^2x^2}}-\frac{4}{35ac^3(ax+1)\sqrt{1-a^2x^2}}-\frac{4}{35ac^3(ax+1)^2\sqrt{1-a^2x^2}}-\frac{1}{7ac^3(ax+1)^3\sqrt{1-a^2x^2}}$$

```
[Out] 8/35*x/c^3/(-a^2*x^2+1)^(1/2)-1/7/a/c^3/(a*x+1)^3/(-a^2*x^2+1)^(1/2)-4/35/a
/c^3/(a*x+1)^2/(-a^2*x^2+1)^(1/2)-4/35/a/c^3/(a*x+1)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6139, 655, 659, 191}

$$\frac{8x}{35c^3\sqrt{1-a^2x^2}}-\frac{4}{35ac^3(ax+1)\sqrt{1-a^2x^2}}-\frac{4}{35ac^3(ax+1)^2\sqrt{1-a^2x^2}}-\frac{1}{7ac^3(ax+1)^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (8*x)/(35*c^3*Sqrt[1 - a^2*x^2]) - 1/(7*a*c^3*(1 + a*x)^3*Sqrt[1 - a^2*x^2]
) - 4/(35*a*c^3*(1 + a*x)^2*Sqrt[1 - a^2*x^2]) - 4/(35*a*c^3*(1 + a*x)*Sqrt
[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
```

```
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{9/2}}\,dx}{c^3}$$

$$= \frac{\int \frac{1}{(1+ax)^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c^3}$$

$$= -\frac{1}{7ac^3(1+ax)^3\sqrt{1-a^2x^2}} + \frac{4\int \frac{1}{(1+ax)^2\left(1-a^2x^2\right)^{3/2}}\,dx}{7c^3}$$

$$= -\frac{1}{7ac^3(1+ax)^3\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)^2\sqrt{1-a^2x^2}} + \frac{12\int \frac{1}{(1+ax)\left(1-a^2x^2\right)^{3/2}}\,dx}{35c^3}$$

$$= -\frac{1}{7ac^3(1+ax)^3\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)^2\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)\sqrt{1-a^2x^2}} + \frac{8\int -}{}$$

$$= \frac{8x}{35c^3\sqrt{1-a^2x^2}} - \frac{1}{7ac^3(1+ax)^3\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)^2\sqrt{1-a^2x^2}} - \frac{4}{35ac^3(1+ax)\sqrt{1}}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.51

$$\frac{8a^4x^4+24a^3x^3+20a^2x^2-4ax-13}{35ac^3\sqrt{1-ax}\,(ax+1)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-13 - 4*a*x + 20*a^2*x^2 + 24*a^3*x^3 + 8*a^4*x^4)/(35*a*c^3*Sqrt[1 - a*x]
*(1 + a*x)^(7/2))
```

fricas [A] time = 0.57, size = 144, normalized size = 1.24

$$-\frac{13\,a^5x^5+39\,a^4x^4+26\,a^3x^3-26\,a^2x^2-39\,ax+\left(8\,a^4x^4+24\,a^3x^3+20\,a^2x^2-4\,ax-13\right)\sqrt{-a^2x^2+1}-13}{35\,\left(a^6c^3x^5+3\,a^5c^3x^4+2\,a^4c^3x^3-2\,a^3c^3x^2-3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fri
cas")
```

```
[Out] -1/35*(13*a^5*x^5 + 39*a^4*x^4 + 26*a^3*x^3 - 26*a^2*x^2 - 39*a*x + (8*a^4*
x^4 + 24*a^3*x^3 + 20*a^2*x^2 - 4*a*x - 13)*sqrt(-a^2*x^2 + 1) - 13)/(a^6*c
^3*x^5 + 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - 3*a^2*c^3*x - a*c^
3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^3(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="gia
c")
```

```
[Out] integrate(-(-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^3*(a*x + 1)^3), x)
```

maple [A] time = 0.03, size = 58, normalized size = 0.50

$$\frac{8x^4a^4+24x^3a^3+20a^2x^2-4ax-13}{35\sqrt{-a^2x^2+1}\,\left(ax+1\right)^3c^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/35/(-a^2*x^2+1)^(1/2)*(8*a^4*x^4+24*a^3*x^3+20*a^2*x^2-4*a*x-13)/(a*x+1)^
3/c^3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^3(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="max
ima")
```

```
[Out] -integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^3*(a*x + 1)^3), x)
```

mupad [B] time = 1.15, size = 125, normalized size = 1.08

$$-\frac{29\sqrt{1-a^2x^2}}{280\,a\,c^3\,(a\,x+1)^2}-\frac{13\sqrt{1-a^2x^2}}{140\,a\,c^3\,(a\,x+1)^3}-\frac{\sqrt{1-a^2x^2}}{14\,a\,c^3\,(a\,x+1)^4}-\frac{\sqrt{1-a^2x^2}\,\left(\frac{8\,x}{35\,c^3}-\frac{29}{280\,a\,c^3}\right)}{(a\,x-1)\,(a\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^3*(a*x + 1)^3),x)
```

```
[Out] - (29*(1 - a^2*x^2)^(1/2))/(280*a*c^3*(a*x + 1)^2) - (13*(1 - a^2*x^2)^(1/2
))/(140*a*c^3*(a*x + 1)^3) - (1 - a^2*x^2)^(1/2)/(14*a*c^3*(a*x + 1)^4) - (
(1 - a^2*x^2)^(1/2)*((8*x)/(35*c^3) - 29/(280*a*c^3)))/((a*x - 1)*(a*x + 1)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{-a^5x^5\sqrt{-a^2x^2+1}-3a^4x^4\sqrt{-a^2x^2+1}-2a^3x^3\sqrt{-a^2x^2+1}+2a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(1/(-a**5*x**5*sqrt(-a**2*x**2 + 1) - 3*a**4*x**4*sqrt(-a**2*x**2 +
 1) - 2*a**3*x**3*sqrt(-a**2*x**2 + 1) + 2*a**2*x**2*sqrt(-a**2*x**2 + 1) +
 3*a*x*sqrt(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**3
```

3.1261 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=138

$$\frac{16x}{63c^4\sqrt{1-a^2x^2}}+\frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}}-\frac{2}{21ac^4(ax+1)\left(1-a^2x^2\right)^{3/2}}-\frac{2}{21ac^4(ax+1)^2\left(1-a^2x^2\right)^{3/2}}-\frac{1}{9ac^4(ax+1)^3\left(1-\right.}$$

```
[Out] 8/63*x/c^4/(-a^2*x^2+1)^(3/2)-1/9/a/c^4/(a*x+1)^3/(-a^2*x^2+1)^(3/2)-2/21/a
/c^4/(a*x+1)^2/(-a^2*x^2+1)^(3/2)-2/21/a/c^4/(a*x+1)/(-a^2*x^2+1)^(3/2)+16/
63*x/c^4/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6139, 655, 659, 192, 191}

$$\frac{16x}{63c^4\sqrt{1-a^2x^2}}+\frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}}-\frac{2}{21ac^4(ax+1)\left(1-a^2x^2\right)^{3/2}}-\frac{2}{21ac^4(ax+1)^2\left(1-a^2x^2\right)^{3/2}}-\frac{1}{9ac^4(ax+1)^3\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (8*x)/(63*c^4*(1 - a^2*x^2)^(3/2)) - 1/(9*a*c^4*(1 + a*x)^3*(1 - a^2*x^2)^(
3/2)) - 2/(21*a*c^4*(1 + a*x)^2*(1 - a^2*x^2)^(3/2)) - 2/(21*a*c^4*(1 + a*x
)*(1 - a^2*x^2)^(3/2)) + (16*x)/(63*c^4*Sqrt[1 - a^2*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
```

```
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 6139

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^p, Int[(1 - a^2*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a, c, d
, p}, x] && EqQ[a^2*c + d, 0] && IntegerQ[p] && ILtQ[(n - 1)/2, 0] &&  !Int
egerQ[p - n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \frac{\int \frac{(1-ax)^3}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^4}$$

$$= \frac{\int \frac{1}{(1+ax)^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^4}$$

$$= -\frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{(1+ax)^2\left(1-a^2x^2\right)^{5/2}}\,dx}{3c^4}$$

$$= -\frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)^2\left(1-a^2x^2\right)^{3/2}} + \frac{10\int \frac{1}{(1+ax)\left(1-a^2x^2\right)^{5/2}}\,dx}{21c^4}$$

$$= -\frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)^2\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)\left(1-a^2x^2\right)^{3/2}} +$$

$$= \frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}} - \frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)^2\left(1-a^2x^2\right)^{3/2}} - \frac{}{21ac^4(1+}$$

$$= \frac{8x}{63c^4\left(1-a^2x^2\right)^{3/2}} - \frac{1}{9ac^4(1+ax)^3\left(1-a^2x^2\right)^{3/2}} - \frac{2}{21ac^4(1+ax)^2\left(1-a^2x^2\right)^{3/2}} - \frac{}{21ac^4(1+}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 0.54

$$-\frac{16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19}{63ac^4(1-ax)^{3/2}(ax+1)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] -1/63*(19 - 6*a*x - 66*a^2*x^2 - 56*a^3*x^3 + 24*a^4*x^4 + 48*a^5*x^5 + 16*
a^6*x^6)/(a*c^4*(1 - a*x)^(3/2)*(1 + a*x)^(9/2))
```

fricas [A] time = 0.65, size = 195, normalized size = 1.41

$$-\frac{19\,a^7x^7+57\,a^6x^6+19\,a^5x^5-95\,a^4x^4-95\,a^3x^3+19\,a^2x^2+57\,ax+\left(16\,a^6x^6+48\,a^5x^5+24\,a^4x^4-56\,a^3x^3-6\right.}{63\left(a^8c^4x^7+3\,a^7c^4x^6+a^6c^4x^5-5\,a^5c^4x^4-5\,a^4c^4x^3+a^3c^4x^2+3\,a^2c^4x+ac^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fri
cas")
```

```
[Out] -1/63*(19*a^7*x^7 + 57*a^6*x^6 + 19*a^5*x^5 - 95*a^4*x^4 - 95*a^3*x^3 + 19*
a^2*x^2 + 57*a*x + (16*a^6*x^6 + 48*a^5*x^5 + 24*a^4*x^4 - 56*a^3*x^3 - 66*
a^2*x^2 - 6*a*x + 19)*sqrt(-a^2*x^2 + 1) + 19)/(a^8*c^4*x^7 + 3*a^7*c^4*x^6
 + a^6*c^4*x^5 - 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 + a^3*c^4*x^2 + 3*a^2*c^4*x
+ a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^4(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="gia
c")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^4*(a*x + 1)^3), x)
```

maple [A] time = 0.03, size = 74, normalized size = 0.54

$$-\frac{16x^6a^6+48x^5a^5+24x^4a^4-56x^3a^3-66a^2x^2-6ax+19}{63\left(-a^2x^2+1\right)^{\frac{3}{2}}(ax+1)^3c^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/63/(-a^2*x^2+1)^(3/2)*(16*a^6*x^6+48*a^5*x^5+24*a^4*x^4-56*a^3*x^3-66*a^
2*x^2-6*a*x+19)/(a*x+1)^3/c^4/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^4(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="max
ima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a^2*c*x^2 - c)^4*(a*x + 1)^3), x)
```

mupad [B] time = 1.27, size = 156, normalized size = 1.13

$$\frac{\sqrt{1-a^2\,x^2}\,\left(\frac{197\,x}{1008\,c^4}-\frac{155}{1008\,a\,c^4}\right)}{{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^2}-\frac{13\,\sqrt{1-a^2\,x^2}}{252\,a\,c^4\,{\left(a\,x+1\right)}^4}-\frac{\sqrt{1-a^2\,x^2}}{36\,a\,c^4\,{\left(a\,x+1\right)}^5}-\frac{23\,\sqrt{1-a^2\,x^2}}{336\,a\,c^4\,{\left(a\,x+1\right)}^3}-\frac{16\,x\,\sqrt{1-a^2\,x^2}}{63\,c^4\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^4*(a*x + 1)^3),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*((197*x)/(1008*c^4) - 155/(1008*a*c^4)))/((a*x - 1)^2*
(a*x + 1)^2) - (13*(1 - a^2*x^2)^(1/2))/(252*a*c^4*(a*x + 1)^4) - (1 - a^2*
x^2)^(1/2)/(36*a*c^4*(a*x + 1)^5) - (23*(1 - a^2*x^2)^(1/2))/(336*a*c^4*(a*
x + 1)^3) - (16*x*(1 - a^2*x^2)^(1/2))/(63*c^4*(a*x - 1)*(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^7x^7\sqrt{-a^2x^2+1}+3a^6x^6\sqrt{-a^2x^2+1}+a^5x^5\sqrt{-a^2x^2+1}-5a^4x^4\sqrt{-a^2x^2+1}-5a^3x^3\sqrt{-a^2x^2+1}+a^2x^2\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+\sqrt{-a^2x^2+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Integral(1/(a**7*x**7*sqrt(-a**2*x**2 + 1) + 3*a**6*x**6*sqrt(-a**2*x**2 +
1) + a**5*x**5*sqrt(-a**2*x**2 + 1) - 5*a**4*x**4*sqrt(-a**2*x**2 + 1) - 5*
a**3*x**3*sqrt(-a**2*x**2 + 1) + a**2*x**2*sqrt(-a**2*x**2 + 1) + 3*a*x*sqr
t(-a**2*x**2 + 1) + sqrt(-a**2*x**2 + 1)), x)/c**4
```

3.1262 $\int e^{-3\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=225

$$-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^4\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] 4*x*(-a^2*c*x^2+c)^(1/2)/a^3/(-a^2*x^2+1)^(1/2)-2*x^2*(-a^2*c*x^2+c)^(1/2)/
a^2/(-a^2*x^2+1)^(1/2)+4/3*x^3*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-3/
4*x^4*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+1/5*a*x^5*(-a^2*c*x^2+c)^(1/2
)/(-a^2*x^2+1)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^4/(-a^2*x^2+1)^(1/2
)
```

Rubi [A] time = 0.22, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$\frac{ax^5\sqrt{c-a^2cx^2}}{5\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (4*x*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - a^2*x^2]) - (2*x^2*Sqrt[c - a^2*c*x
^2])/(a^2*Sqrt[1 - a^2*x^2]) + (4*x^3*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^
2*x^2]) - (3*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) + (a*x^5*Sqrt[c
 - a^2*c*x^2])/(5*Sqrt[1 - a^2*x^2]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/(a^4*Sqrt[1 - a^2*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}x^3\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^3(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^3}-\frac{4x}{a^2}+\frac{4x^2}{a}-3x^3+ax^4-\frac{4}{a^3(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{4x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-a^2x^2}}-\frac{2x^2\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{4x^3\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{ax^5\sqrt{c}}{5\sqrt{1-}}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 0.36

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(ax+1)}{a^4}+\frac{4x}{a^3}-\frac{2x^2}{a^2}+\frac{ax^5}{5}+\frac{4x^3}{3a}-\frac{3x^4}{4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^3 - (2*x^2)/a^2 + (4*x^3)/(3*a) - (3*x^4)/4 +
 (a*x^5)/5 - (4*Log[1 + a*x])/a^4))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.12, size = 399, normalized size = 1.77

$$\left[\frac{120\left(a^2x^2-1\right)\sqrt{c}\,\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx+\left(a^4x^4+4a^3x^3+6a^2x^2+4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\left(12a^5x^5-\right.}{60\left(a^6x^2-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/60*(120*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4
 - 4*a^2*c*x^2 - 4*a*c*x + (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a
*x - 1)) - (12*a^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x)*s
qrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a^4), -1/60*(240*(a^2*x^
2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2
*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 + 2*a^3*c*x^3 - a^2*c*x^2 - 2*a*c*x)) + (12*a
^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x)*sqrt(-a^2*c*x^2 +
 c)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\left(-a^2x^2 + 1\right)^{\frac{3}{2}} x^3}{(ax + 1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^3/(a*x + 1)^3, x)
```

maple [A] time = 0.04, size = 88, normalized size = 0.39

$$\frac{\sqrt{-c\left(a^2x^2 - 1\right)}\,\sqrt{-a^2x^2 + 1}\,\left(-12x^5a^5 + 45x^4a^4 - 80x^3a^3 + 120a^2x^2 - 240ax + 240\ln\left(ax + 1\right)\right)}{60\left(a^2x^2 - 1\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/60*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(-12*x^5*a^5+45*x^4*a^4-80*x
^3*a^3+120*a^2*x^2-240*a*x+240*ln(a*x+1))/(a^2*x^2-1)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\left(-a^2x^2 + 1\right)^{\frac{3}{2}} x^3}{(ax + 1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^3/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int((x^3*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(-\left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \sqrt{- c \left(a x - 1\right) \left(a x + 1\right)}}{\left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(a
*x + 1)**3, x)
```

3.1263 $\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=184

$$\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{a^3\sqrt{1-a^2x^2}}$$

```
[Out] -4*x*(-a^2*c*x^2+c)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)+2*x^2*(-a^2*c*x^2+c)^(1/2)
/a/(-a^2*x^2+1)^(1/2)-x^3*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+1/4*a*x^4
*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
^3/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-4*x*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - a^2*x^2]) + (2*x^2*Sqrt[c - a^2*c*
x^2])/(a*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] +
 (a*x^4*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c - a^2*c*x^2]
*Log[1 + a*x])/(a^3*Sqrt[1 - a^2*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{4x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-a^2x^2}}+\frac{2x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{ax^4\sqrt{c-a^2cx^2}}{4\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-}}{a}\end{aligned}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.38

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4\log(ax+1)}{a^3}-\frac{4x}{a^2}+\frac{ax^4}{4}+\frac{2x^2}{a}-x^3\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((-4*x)/a^2 + (2*x^2)/a - x^3 + (a*x^4)/4 + (4*Log[1 +
 a*x])/a^3))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.67, size = 383, normalized size = 2.08

$$\left[\frac{8\left(a^2x^2-1\right)\sqrt{c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx-\left(a^4x^4+4a^3x^3+6a^2x^2+4ax\right)\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\sqrt{c}-2c}{a^4x^4+2a^3x^3-2ax-1}\right)-\left(a^4x^4-4a^3x\right.}{4\left(a^5x^2-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/4*(8*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 - 4*a*c*x - (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x
- 1)) - (a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x)*sqrt(-a^2*c*x^2 + c)*sqr
t(-a^2*x^2 + 1))/(a^5*x^2 - a^3), 1/4*(16*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqr
t(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*
x^4 + 2*a^3*c*x^3 - a^2*c*x^2 - 2*a*c*x)) - (a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^
2 - 16*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^5*x^2 - a^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\left(-a^2x^2 + 1\right)^{\frac{3}{2}}x^2}{(ax + 1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^2/(a*x + 1)^3, x)
```

maple [A] time = 0.04, size = 79, normalized size = 0.43

$$-\frac{\sqrt{-c\left(a^2x^2 - 1\right)}\,\sqrt{-a^2x^2 + 1}\,\left(x^4a^4 - 4x^3a^3 + 8a^2x^2 - 16ax + 16\ln\left(ax + 1\right)\right)}{4\left(a^2x^2 - 1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/4*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(x^4*a^4-4*x^3*a^3+8*a^2*x^2
-16*a*x+16*ln(a*x+1))/(a^2*x^2-1)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\left(-a^2x^2 + 1\right)^{\frac{3}{2}}x^2}{(ax + 1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^2/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
[Out] int((x^2*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \sqrt{- c \left(a x - 1\right) \left(a x + 1\right)}}{\left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
[Out] Integral(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(a
*x + 1)**3, x)
```

3.1264 $\int e^{-3\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=149

$$-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2\sqrt{1-a^2x^2}}+\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}$$

```
[Out] 4*x*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-3/2*x^2*(-a^2*c*x^2+c)^(1/2)/
(-a^2*x^2+1)^(1/2)+1/3*a*x^3*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-4*ln(a
*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 77}

$$\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (4*x*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[c - a^2*c*x^2
])/(2*Sqrt[1 - a^2*x^2]) + (a*x^3*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - a^2*x^2]
) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{x(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{\sqrt{1-a^2x^2}}\\ &= \frac{4x\sqrt{c-a^2cx^2}}{a\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{ax^3\sqrt{c-a^2cx^2}}{3\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.42

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(ax+1)}{a^2}+\frac{ax^3}{3}+\frac{4x}{a}-\frac{3x^2}{2}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a - (3*x^2)/2 + (a*x^3)/3 - (4*Log[1 + a*x])/a^
2))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.78, size = 367, normalized size = 2.46

$$\left[\frac{12\left(a^2x^2-1\right)\sqrt{c}\,\log\left(\frac{a^6cx^6+4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2-4\,acx+\left(a^4x^4+4\,a^3x^3+6\,a^2x^2+4\,ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-2\,c}{a^4x^4+2\,a^3x^3-2\,ax-1}\right)-\left(2\,a^3x^3-9\,a\right.}{6\left(a^4x^2-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(12*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
 4*a^2*c*x^2 - 4*a*c*x + (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x
 - 1)) - (2*a^3*x^3 - 9*a^2*x^2 + 24*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^
2 + 1))/(a^4*x^2 - a^2), -1/6*(24*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c
*x^2 + c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 + 2*
a^3*c*x^3 - a^2*c*x^2 - 2*a*c*x)) + (2*a^3*x^3 - 9*a^2*x^2 + 24*a*x)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^4*x^2 - a^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x/(a*x + 1)^3, x)
```

maple [A] time = 0.04, size = 72, normalized size = 0.48

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-a^2x^2+1}\,\left(-2x^3a^3+9a^2x^2-24ax+24\ln\left(ax+1\right)\right)}{6\left(a^2x^2-1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/6*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(-2*x^3*a^3+9*a^2*x^2-24*a*x+
24*ln(a*x+1))/(a^2*x^2-1)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{c-a^2\,c\,x^2}\,\left(1-a^2\,x^2\right)^{3/2}}{(a\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x
+ 1)**3, x)
```

3.1265 $\int e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=110

$$\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{a\sqrt{1-a^2x^2}}$$

```
[Out] -3*x*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+1/2*a*x^2*(-a^2*c*x^2+c)^(1/2)
/(-a^2*x^2+1)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (a*x^2*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - a^2*x^2]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a*Sqrt[1 - a^
2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-3+ax+\frac{4}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{3x\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{ax^2\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.48

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{ax^2}{2}+\frac{4\log(ax+1)}{a}-3x\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-3*x + (a*x^2)/2 + (4*Log[1 + a*x])/a))/Sqrt[1 - a^2*
x^2]
```

fricas [A] time = 0.73, size = 347, normalized size = 3.15

$$\left[\frac{4\left(a^2x^2-1\right)\sqrt{c}\,\log\left(\frac{a^6cx^6+4\,a^5cx^5+5\,a^4cx^4-4\,a^2cx^2-4\,acx-\left(a^4x^4+4\,a^3x^3+6\,a^2x^2+4\,ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-2\,c}{a^4x^4+2\,a^3x^3-2\,ax-1}\right)-\sqrt{-a^2cx^2+c}}{2\left(a^3x^2-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] [1/2*(4*(a^2*x^2 - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 -
4*a^2*c*x^2 - 4*a*c*x - (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^2
*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x
- 1)) - sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 6*a*x)*sqrt(-a^2*x^2 + 1))/(a^3*x^2
```

```
 - a), 1/2*(8*(a^2*x^2 - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 +
 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 + 2*a^3*c*x^3 - a^2*c*x^
2 - 2*a*c*x)) - sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 6*a*x)*sqrt(-a^2*x^2 + 1))/
(a^3*x^2 - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

maple [A] time = 0.04, size = 63, normalized size = 0.57

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-a^2x^2+1}\,\left(a^2x^2-6ax+8\ln\left(ax+1\right)\right)}{2\left(a^2x^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(a^2*x^2-6*a*x+8*ln(a*x+1))/
(a^2*x^2-1)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(1-a^2\,x^2\right)^{3/2}}{(a\,x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(a*x +
1)**3, x)
```

3.1266 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=102

$$\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] a*x*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a^
2*x^2+1)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 72}

$$\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x),x]
```

```
[Out] (a*x*Sqrt[c - a^2*c*x^2])/Sqrt[1 - a^2*x^2] + (Sqrt[c - a^2*c*x^2]*Log[x])/
Sqrt[1 - a^2*x^2] - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x(1+ax)}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}\\ &= \frac{ax\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-a^2x^2}}-\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}\,(ax-4\log(ax+1)+\log(x))}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x),x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(ax-1)}{a^2x^3+2\,ax^2+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm=
"fricas")
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - 1)/(a^2*x^3 + 2*a*
x^2 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x), x)
```

maple [A] time = 0.04, size = 56, normalized size = 0.55

$$\frac{\left(-ax-\ln(x)+4\ln\left(ax+1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-a^2x^2+1}}{a^2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x)
```

```
[Out] (-a*x-ln(x)+4*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a^2*x^2
-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(1-a^2\,x^2\right)}^{3/2}}{x\,{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x*(a*x + 1)^3),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{x\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x*(a*x
 + 1)**3), x)
```

3.1267 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{3a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4a\sqrt{c-a^2cx^2}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/x/(-a^2*x^2+1)^(1/2)-3*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/(
-a^2*x^2+1)^(1/2)+4*a*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{3a\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4a\sqrt{c-a^2cx^2}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(x*Sqrt[1 - a^2*x^2])) - (3*a*Sqrt[c - a^2*c*x^2]*Log
[x])/Sqrt[1 - a^2*x^2] + (4*a*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^
2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
```

```
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^2(1+ax)}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{3a\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-a^2x^2}}+\frac{4a\sqrt{c-a^2cx^2}\,\log(1+ax)}{\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.47

$$\frac{\sqrt{c-a^2cx^2}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/Sqrt[1 - a^
2*x^2]
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(ax-1)}{a^2x^4+2\,ax^3+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorith
m="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - 1)/(a^2*x^4 + 2*a*
x^3 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^2), x)
```

maple [A] time = 0.04, size = 60, normalized size = 0.56

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-a^2x^2+1}\,\left(3a\ln(x)x-4ax\ln\left(ax+1\right)+1\right)}{\left(a^2x^2-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x)
```

```
[Out] (-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(3*a*ln(x)*x-4*a*x*ln(a*x+1)+1)/(
a^2*x^2-1)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^2,x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(1-a^2\,x^2\right)^{3/2}}{x^2\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x^2*(a*x + 1)^3),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x^2*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{x^{2}\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**2,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*(
a*x + 1)**3), x)
```

3.1268 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=148

$$\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] -1/2*(-a^2*c*x^2+c)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)+3*a*(-a^2*c*x^2+c)^(1/2)/x
/(-a^2*x^2+1)^(1/2)+4*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-4*a
^2*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*x^2*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2])/
(x*Sqrt[1 - a^2*x^2]) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2
] - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^3}\,dx}{\sqrt{1-a^2x^2}} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^3(1+ax)}\,dx}{\sqrt{1-a^2x^2}} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{\sqrt{1-a^2x^2}} \\
&= -\frac{\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{4a^2\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}\,\log(}{\sqrt{1-a^2x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.42

$$
\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{\sqrt{1-a^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 + a
*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.74, size = 447, normalized size = 3.02

$$
\left[\frac{4\left(a^4x^4-a^2x^2\right)\sqrt{c}\,\log\left(\frac{4\,a^5cx^5+\left(2\,a^6+4\,a^5+6\,a^4+4\,a^3+a^2\right)cx^6+\left(4\,a^4-4\,a^3-6\,a^2-4\,a-1\right)cx^4-5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3+6\,a^2+4\,a+1\right)x^4+6\right.}{a^4x^6+2\,a^3x^5-2\,ax^3-x^2}\right.}{2\left(a^2x^4-x^2\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorith
m="fricas")
```

```
[Out] [1/2*(4*(a^4*x^4 - a^2*x^2)*sqrt(c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*a
^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*c
*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 + 6*a^2 + 4*a + 1)*x^4 + 6*a^2*x^2 + 4
*a*x + 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^4*x^6 + 2
*a^3*x^5 - 2*a*x^3 - x^2)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((6*a
- 1)*x^2 - 6*a*x + 1))/(a^2*x^4 - x^2), 1/2*(8*(a^4*x^4 - a^2*x^2)*sqrt(-c)
*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((2*a^2 + 2*a + 1)*x^2 + 2
*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x
^2 - 2*a*c*x - c)) + sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*((6*a - 1)*x^2
 - 6*a*x + 1))/(a^2*x^4 - x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^3), x)
```

maple [A] time = 0.04, size = 73, normalized size = 0.49

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-a^2x^2+1}\,\left(8a^2\ln(x)x^2-8\ln\left(ax+1\right)x^2a^2+6ax-1\right)}{2\left(a^2x^2-1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x)
```

```
[Out] -1/2*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(8*a^2*ln(x)*x^2-8*ln(a*x+1)
*x^2*a^2+6*a*x-1)/(a^2*x^2-1)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^3,x, algorith
m="maxima")
```

[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(1-a^2\,x^2\right)}^{3/2}}{x^3\,{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x^3*(a*x + 1)^3),x)

[Out] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x^3*(a*x + 1)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{x^{3}\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**3,x)

[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**3*(
a*x + 1)**3), x)

3.1269 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=187

$$-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}-\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-a^2cx^2}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

```
[Out] -1/3*(-a^2*c*x^2+c)^(1/2)/x^3/(-a^2*x^2+1)^(1/2)+3/2*a*(-a^2*c*x^2+c)^(1/2)
/x^2/(-a^2*x^2+1)^(1/2)-4*a^2*(-a^2*c*x^2+c)^(1/2)/x/(-a^2*x^2+1)^(1/2)-4*a
^3*ln(x)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+4*a^3*ln(a*x+1)*(-a^2*c*x^
2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}-\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-a^2cx^2}\,\log(ax+1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*x^3*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2])/
(2*x^2*Sqrt[1 - a^2*x^2]) - (4*a^2*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2
]) - (4*a^3*Sqrt[c - a^2*c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] + (4*a^3*Sqrt[c -
 a^2*c*x^2]*Log[1 + a*x])/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^4}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^4(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3x^3\sqrt{1-a^2x^2}}+\frac{3a\sqrt{c-a^2cx^2}}{2x^2\sqrt{1-a^2x^2}}-\frac{4a^2\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{4a^3\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-a^2x^2}}+ 4$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.39

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^3\log(x)+4a^3\log(ax+1)-\frac{4a^2}{x}+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[x]
 + 4*a^3*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 0.78, size = 477, normalized size = 2.55

$$\left[\frac{12\left(a^5x^5-a^3x^3\right)\sqrt{c}\,\log\left(\frac{4a^5cx^5+\left(2a^6+4a^5+6a^4+4a^3+a^2\right)cx^6+\left(4a^4-4a^3-6a^2-4a-1\right)cx^4-5a^2cx^2-4acx-\left(4a^3x^3-\left(4a^3+6a^2+4a+1\right)x^4+6\right.}{a^4x^6+2a^3x^5-2ax^3-x^2}\right.}{6\left(a^2x^5-x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorith
m="fricas")
```

```
[Out] [1/6*(12*(a^5*x^5 - a^3*x^3)*sqrt(c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*
a^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*
c*x^2 - 4*a*c*x - (4*a^3*x^3 - (4*a^3 + 6*a^2 + 4*a + 1)*x^4 + 6*a^2*x^2 +
4*a*x + 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^4*x^6 +
2*a^3*x^5 - 2*a*x^3 - x^2)) + sqrt(-a^2*c*x^2 + c)*(24*a^2*x^2 - (24*a^2 -
9*a + 2)*x^3 - 9*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*x^5 - x^3), -1/6*(24*(a^
5*x^5 - a^3*x^3)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(
(2*a^2 + 2*a + 1)*x^2 + 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3 - (2*a^3 + a^2)*c*
x^4 + (a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) - sqrt(-a^2*c*x^2 + c)*(24*a^2*
x^2 - (24*a^2 - 9*a + 2)*x^3 - 9*a*x + 2)*sqrt(-a^2*x^2 + 1))/(a^2*x^5 - x^
3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^4), x)
```

maple [A] time = 0.05, size = 81, normalized size = 0.43

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-a^2x^2+1}\,\left(24a^3\ln(x)x^3-24a^3x^3\ln\left(ax+1\right)+24a^2x^2-9ax+2\right)}{6\left(a^2x^2-1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x)
```

```
[Out] 1/6*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(24*a^3*ln(x)*x^3-24*a^3*x^3*
ln(a*x+1)+24*a^2*x^2-9*a*x+2)/(a^2*x^2-1)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^4,x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(1-a^2\,x^2\right)^{3/2}}{x^4\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x^4*(a*x + 1)^3),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x^4*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{x^4\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**4,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**4*(
a*x + 1)**3), x)
```

3.1270 $\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=221

$$-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}+\frac{4a^4\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-a^2cx^2}\,\log(ax+1)}{\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-}}{x\sqrt{1-a}}$$

```
[Out] -1/4*(-a^2*c*x^2+c)^(1/2)/x^4/(-a^2*x^2+1)^(1/2)+a*(-a^2*c*x^2+c)^(1/2)/x^3
/(-a^2*x^2+1)^(1/2)-2*a^2*(-a^2*c*x^2+c)^(1/2)/x^2/(-a^2*x^2+1)^(1/2)+4*a^3
*(-a^2*c*x^2+c)^(1/2)/x/(-a^2*x^2+1)^(1/2)+4*a^4*ln(x)*(-a^2*c*x^2+c)^(1/2)
/(-a^2*x^2+1)^(1/2)-4*a^4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$\frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}+\frac{4a^4\log(x)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}-\frac{4a^4\sqrt{c-a^2cx^2}\,\log(ax}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*x^4*Sqrt[1 - a^2*x^2]) + (a*Sqrt[c - a^2*c*x^2])/(x
^3*Sqrt[1 - a^2*x^2]) - (2*a^2*Sqrt[c - a^2*c*x^2])/(x^2*Sqrt[1 - a^2*x^2])
 + (4*a^3*Sqrt[c - a^2*c*x^2])/(x*Sqrt[1 - a^2*x^2]) + (4*a^4*Sqrt[c - a^2*
c*x^2]*Log[x])/Sqrt[1 - a^2*x^2] - (4*a^4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/Sqrt[1 - a^2*x^2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^5}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^2}{x^5(1+ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4x^4\sqrt{1-a^2x^2}}+\frac{a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-a^2x^2}}-\frac{2a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-a^2x^2}}+\frac{4a^3\sqrt{c-a^2cx^2}}{x\sqrt{1-a^2x^2}}+\frac{4a^4\sqrt{c-}}{\sqrt{1}}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 0.35

$$\frac{\sqrt{c-a^2cx^2}\left(4a^4\log(x)-4a^4\log(ax+1)+\frac{4a^3}{x}-\frac{2a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcTanh[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^4*
Log[x] - 4*a^4*Log[1 + a*x]))/Sqrt[1 - a^2*x^2]
```

fricas [A] time = 1.02, size = 503, normalized size = 2.28

$$\left[\frac{8\left(a^6x^6-a^4x^4\right)\sqrt{c}\,\log\left(\frac{4\,a^5cx^5+\left(2\,a^6+4\,a^5+6\,a^4+4\,a^3+a^2\right)cx^6+\left(4\,a^4-4\,a^3-6\,a^2-4\,a-1\right)cx^4-5\,a^2cx^2-4\,acx+\left(4\,a^3x^3-\left(4\,a^3+6\,a^2+4\,a+1\right)x^4+6\right.}{a^4x^6+2\,a^3x^5-2\,ax^3-x^2}\right.}{4\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorith
m="fricas")
```

```
[Out] [1/4*(8*(a^6*x^6 - a^4*x^4)*sqrt(c)*log((4*a^5*c*x^5 + (2*a^6 + 4*a^5 + 6*a
^4 + 4*a^3 + a^2)*c*x^6 + (4*a^4 - 4*a^3 - 6*a^2 - 4*a - 1)*c*x^4 - 5*a^2*c
*x^2 - 4*a*c*x + (4*a^3*x^3 - (4*a^3 + 6*a^2 + 4*a + 1)*x^4 + 6*a^2*x^2 + 4
*a*x + 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^4*x^6 + 2
*a^3*x^5 - 2*a*x^3 - x^2)) - (16*a^3*x^3 - (16*a^3 - 8*a^2 + 4*a - 1)*x^4 -
 8*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^2*x^6 -
 x^4), 1/4*(16*(a^6*x^6 - a^4*x^4)*sqrt(-c)*arctan(-sqrt(-a^2*c*x^2 + c)*sq
rt(-a^2*x^2 + 1)*((2*a^2 + 2*a + 1)*x^2 + 2*a*x + 1)*sqrt(-c)/(2*a^3*c*x^3
- (2*a^3 + a^2)*c*x^4 + (a^2 + 2*a + 1)*c*x^2 - 2*a*c*x - c)) - (16*a^3*x^3
 - (16*a^3 - 8*a^2 + 4*a - 1)*x^4 - 8*a^2*x^2 + 4*a*x - 1)*sqrt(-a^2*c*x^2
+ c)*sqrt(-a^2*x^2 + 1))/(a^2*x^6 - x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^5), x)
```

maple [A] time = 0.05, size = 89, normalized size = 0.40

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-a^2x^2+1}\,\left(16a^4\ln(x)x^4-16\ln\left(ax+1\right)x^4a^4+16x^3a^3-8a^2x^2+4ax-1\right)}{4\left(a^2x^2-1\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x)
```

```
[Out] -1/4*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(16*a^4*ln(x)*x^4-16*ln(a*x+
1)*x^4*a^4+16*x^3*a^3-8*a^2*x^2+4*a*x-1)/(a^2*x^2-1)/x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/x^5,x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)/((a*x + 1)^3*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(1-a^2\,x^2\right)}^{3/2}}{x^5\,{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x^5*(a*x + 1)^3),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(x^5*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{x^{5}\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/x**5,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))/(x**5*(
a*x + 1)**3), x)
```

3.1271 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=189

$$\frac{c^4(1-ax)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}-\frac{2c^4(1-ax)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}-\frac{8c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}$$

```
[Out] -8/7*c^4*(-a*x+1)^7*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+3/2*c^4*(-a*x
+1)^8*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-2/3*c^4*(-a*x+1)^9*(-a^2*c*
x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+1/10*c^4*(-a*x+1)^10*(-a^2*c*x^2+c)^(1/2)
/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c^4(1-ax)^{10}\sqrt{c-a^2cx^2}}{10a\sqrt{1-a^2x^2}}-\frac{2c^4(1-ax)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}-\frac{8c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-8*c^4*(1 - a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) + (3*c^4*(
1 - a*x)^8*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - a^2*x^2]) - (2*c^4*(1 - a*x)^
9*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (c^4*(1 - a*x)^10*Sqrt[c -
 a^2*c*x^2])/(10*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
```

```
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{9/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int(1-ax)^6(1+ax)^3\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^4\sqrt{c-a^2cx^2}\right)\int\left(8(1-ax)^6-12(1-ax)^7+6(1-ax)^8-(1-ax)^9\right)dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{8c^4(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{3c^4(1-ax)^8\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}-\frac{2c^4(1-ax)^9\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.36

$$\frac{c^4(ax-1)^7\left(21a^3x^3+77a^2x^2+98ax+44\right)\sqrt{c-a^2cx^2}}{210a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^4*(-1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(44 + 98*a*x + 77*a^2*x^2 + 21*a^3*x^
3))/(210*a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.71, size = 120, normalized size = 0.63

$$-\frac{\left(21\,a^9c^4x^{10}-70\,a^8c^4x^9+240\,a^6c^4x^7-210\,a^5c^4x^6-252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2+210\,c^4x\right)\sqrt{-a^2cx^2+\cdots}}{210\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/210*(21*a^9*c^4*x^10 - 70*a^8*c^4*x^9 + 240*a^6*c^4*x^7 - 210*a^5*c^4*x^
6 - 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 + 210*c^4*x)*sqrt(-a^
2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

maple [A] time = 0.03, size = 97, normalized size = 0.51

$$\frac{x\left(21a^9x^9-70x^8a^8+240x^6a^6-210x^5a^5-252x^4a^4+420x^3a^3-315ax+210\right)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{210\,(ax+1)^6\,(ax-1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/210*x*(21*a^9*x^9-70*a^8*x^8+240*a^6*x^6-210*a^5*x^5-252*a^4*x^4+420*a^3*
x^3-315*a*x+210)*(-a^2*c*x^2+c)^(9/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^6/(a*x-1)^
6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{9/2}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(9/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - a^2*c*x^2)^(9/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

3.1272 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=142

$$-\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{4c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] -2/3*c^3*(-a*x+1)^6*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+4/7*c^3*(-a*x
+1)^7*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-1/8*c^3*(-a*x+1)^8*(-a^2*c*
x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{4c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^3*(1 - a*x)^6*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - a^2*x^2]) + (4*c^3*(
1 - a*x)^7*Sqrt[c - a^2*c*x^2])/(7*a*Sqrt[1 - a^2*x^2]) - (c^3*(1 - a*x)^8*
Sqrt[c - a^2*c*x^2])/(8*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{7/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int(1-ax)^5(1+ax)^2\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^3\sqrt{c-a^2cx^2}\right)\int\left(4(1-ax)^5-4(1-ax)^6+(1-ax)^7\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{2c^3(1-ax)^6\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{4c^3(1-ax)^7\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}-\frac{c^3(1-ax)^8\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.42

$$-\frac{c^3(ax-1)^6\left(21a^2x^2+54ax+37\right)\sqrt{c-a^2cx^2}}{168a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -1/168*(c^3*(-1 + a*x)^6*(37 + 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a
*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 1.10, size = 120, normalized size = 0.85

$$\frac{\left(21\,a^7c^3x^8-72\,a^6c^3x^7+28\,a^5c^3x^6+168\,a^4c^3x^5-210\,a^3c^3x^4-56\,a^2c^3x^3+252\,ac^3x^2-168\,c^3x\right)\sqrt{-a^2cx^2+c}\,\sqrt{-}}{168\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/168*(21*a^7*c^3*x^8 - 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 + 168*a^4*c^3*x^5 -
 210*a^3*c^3*x^4 - 56*a^2*c^3*x^3 + 252*a*c^3*x^2 - 168*c^3*x)*sqrt(-a^2*c*
x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} c x^{2}+c\right)^{\frac{7}{2}} \left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{\left(a x+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

maple [A] time = 0.03, size = 97, normalized size = 0.68

$$\frac{x \left(21 a^{7} x^{7}-72 x^{6} a^{6}+28 x^{5} a^{5}+168 x^{4} a^{4}-210 x^{3} a^{3}-56 a^{2} x^{2}+252 a x -168\right) \left(-a^{2} c \,x^{2}+c \right)^{\frac{7}{2}} \left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{168 \left(a x +1\right)^{5} \left(a x -1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/168*x*(21*a^7*x^7-72*a^6*x^6+28*a^5*x^5+168*a^4*x^4-210*a^3*x^3-56*a^2*x^
2+252*a*x-168)*(-a^2*c*x^2+c)^(7/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^5/(a*x-1)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} c x^{2}+c\right)^{\frac{7}{2}} \left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{\left(a x+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{7/2}\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(7/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - a^2*c*x^2)^(7/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}(-c(ax-1)(ax+1))^{\frac{7}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1))**(7/2)/(a*x
 + 1)**3, x)
```

3.1273 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{2c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

```
[Out] -2/5*c^2*(-a*x+1)^5*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)+1/6*c^2*(-a*x
+1)^6*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{2c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-2*c^2*(1 - a*x)^5*Sqrt[c - a^2*c*x^2])/(5*a*Sqrt[1 - a^2*x^2]) + (c^2*(1
- a*x)^6*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^2*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)^4(1+ax)\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int\left(2(1-ax)^4-(1-ax)^5\right)\,dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{2c^2(1-ax)^5\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{c^2(1-ax)^6\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.55

$$\frac{c^2(ax-1)^5(5ax+7)\sqrt{c-a^2cx^2}}{30a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c^2*(-1 + a*x)^5*(7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.70, size = 98, normalized size = 1.03

$$-\frac{\left(5\,a^5c^2x^6-18\,a^4c^2x^5+15\,a^3c^2x^4+20\,a^2c^2x^3-45\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{30\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -1/30*(5*a^5*c^2*x^6 - 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 - 4
5*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^2*x^2 -
1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

maple [A] time = 0.03, size = 81, normalized size = 0.85

$$\frac{x\left(5x^5a^5-18x^4a^4+15x^3a^3+20a^2x^2-45ax+30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{30\left(ax+1\right)^4\left(ax-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*(-a^2*c*x^2+c
)^(5/2)*(-a^2*x^2+1)^(3/2)/(a*x+1)^4/(a*x-1)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c-a^2\,c\,x^2\right)^{5/2}\,\left(1-a^2\,x^2\right)^{3/2}}{\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - a^2*c*x^2)^(5/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}{\left(ax+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1))**(5/2)/(a*x
 + 1)**3, x)
```

3.1274 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=45

$$-\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

[Out] -1/4*c*(-a*x+1)^4*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)

Rubi [A] time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 32}

$$-\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(c - a^2*c*x^2)^(3/2)/E^(3*ArcTanh[a*x]),x]

[Out] -(c*(1 - a*x)^4*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2])

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(1-ax)^3\,dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{c(1-ax)^4\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 1.29

$$\frac{c\left(-\frac{1}{4}a^3x^4+a^2x^3-\frac{3ax^2}{2}+x\right)\sqrt{c-a^2cx^2}}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(x - (3*a*x^2)/2 + a^2*x^3 - (a^3*x^4)/4))/Sqrt[1 -
a^2*x^2]
```

fricas [A] time = 0.65, size = 67, normalized size = 1.49

$$\frac{\left(a^3cx^4-4\,a^2cx^3+6\,acx^2-4\,cx\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{4\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/4*(a^3*c*x^4 - 4*a^2*c*x^3 + 6*a*c*x^2 - 4*c*x)*sqrt(-a^2*c*x^2 + c)*sqrt
(-a^2*x^2 + 1)/(a^2*x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*(-a^2*x^2 + 1)^(3/2)/(a*x + 1)^3, x)
```

maple [A] time = 0.03, size = 64, normalized size = 1.42

$$\frac{x\left(x^{3}a^{3}-4a^{2}x^{2}+6ax-4\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{4\left(ax-1\right)^{3}\left(ax+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] 1/4*x*(a^3*x^3-4*a^2*x^2+6*a*x-4)*(-a^2*c*x^2+c)^(3/2)*(-a^2*x^2+1)^(3/2)/(
a*x-1)^3/(a*x+1)^3
```

maxima [A] time = 0.35, size = 70, normalized size = 1.56

$$-\frac{\left(a^{4}c^{\frac{3}{2}}x^{4}-4\,a^{3}c^{\frac{3}{2}}x^{3}+6\,a^{2}c^{\frac{3}{2}}x^{2}-4\,ac^{\frac{3}{2}}x+4\,c^{\frac{3}{2}}\right)(ax+1)(ax-1)}{4\left(a^{3}x^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="m
axima")
```

```
[Out] -1/4*(a^4*c^(3/2)*x^4 - 4*a^3*c^(3/2)*x^3 + 6*a^2*c^(3/2)*x^2 - 4*a*c^(3/2)
*x + 4*c^(3/2))*(a*x + 1)*(a*x - 1)/(a^3*x^2 - a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c-a^{2}\,c\,x^{2}\right)^{3/2}\,\left(1-a^{2}\,x^{2}\right)^{3/2}}{\left(a\,x+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1))**(3/2)/(a*x
 + 1)**3, x)
```

3.1275 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2\sqrt{1-a^2x^2}}{a(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] -2*(-a^2*x^2+1)^(1/2)/a/(a*x+1)/(-a^2*c*x^2+c)^(1/2)-ln(a*x+1)*(-a^2*x^2+1)
^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 43}

$$-\frac{2\sqrt{1-a^2x^2}}{a(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(ax+1)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/(a*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^
2]*Log[1 + a*x])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{1-ax}{(1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{-1-ax}+\frac{2}{(1+ax)^2}\right)dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{1-a^2x^2}}{a(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1+ax)}{a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.66

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{2}{a(ax+1)}-\frac{\log(ax+1)}{a}\right)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2/(a*(1 + a*x)) - Log[1 + a*x]/a))/Sqrt[c - a^2*c*x^2]
```

fricas [B] time = 1.70, size = 381, normalized size = 4.65

$$\left[-\frac{4\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}\,ax-\left(a^3x^3+a^2x^2-ax-1\right)\sqrt{c}\log\left(\frac{a^6cx^6+4a^5cx^5+5a^4cx^4-4a^2cx^2-4acx+\left(a^4x^4+4a^3x^3+6a^2x^2+\right.}{a^4x^4+2a^3x^3-2ax-1}\right.}{2\left(a^4cx^3+a^3cx^2-a^2cx-ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/2*(4*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*x - (a^3*x^3 + a^2*x^2 -
 a*x - 1)*sqrt(c)*log((a^6*c*x^6 + 4*a^5*c*x^5 + 5*a^4*c*x^4 - 4*a^2*c*x^2
- 4*a*c*x + (a^4*x^4 + 4*a^3*x^3 + 6*a^2*x^2 + 4*a*x)*sqrt(-a^2*c*x^2 + c)*
sqrt(-a^2*x^2 + 1)*sqrt(c) - 2*c)/(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)))/(a^4*
```

```
c*x^3 + a^3*c*x^2 - a^2*c*x - a*c), -(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2
+ 1)*a*x + (a^3*x^3 + a^2*x^2 - a*x - 1)*sqrt(-c)*arctan(sqrt(-a^2*c*x^2 +
c)*(a^2*x^2 + 2*a*x + 2)*sqrt(-a^2*x^2 + 1)*sqrt(-c)/(a^4*c*x^4 + 2*a^3*c*x
^3 - a^2*c*x^2 - 2*a*c*x)))/(a^4*c*x^3 + a^3*c*x^2 - a^2*c*x - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}\,(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/(sqrt(-a^2*c*x^2 + c)*(a*x + 1)^3), x)
```

maple [A] time = 0.04, size = 69, normalized size = 0.84

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax\ln\left(ax+1\right)+\ln\left(ax+1\right)+2\right)}{\left(a^2x^2-1\right)c\left(ax+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] (-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)+ln(a*x+1)+2)/(a^2*
x^2-1)/c/(a*x+1)/a
```

maxima [A] time = 0.34, size = 33, normalized size = 0.40

$$-\frac{\log\left(ax+1\right)}{a\sqrt{c}}-\frac{2}{a^2\sqrt{c}\,x+a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm=
"maxima")
```

```
[Out] -log(a*x + 1)/(a*sqrt(c)) - 2/(a^2*sqrt(c)*x + a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-a^2\,x^2\right)}^{3/2}}{\sqrt{c-a^2\,c\,x^2}\,{\left(a\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x +
 1)**3), x)
```

3.1276 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{1-a^2x^2}}{2ac(ax+1)^2\sqrt{c-a^2cx^2}}$$

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)/a/c/(a*x+1)^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 32}

$$-\frac{\sqrt{1-a^2x^2}}{2ac(ax+1)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a*c*(1 + a*x)^2*Sqrt[c - a^2*c*x^2])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1+ax)^3}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}}{2ac(1+ax)^2\sqrt{c-a^2cx^2}}$$

Mathematica [**A**] time = 0.05, size = 53, normalized size = 1.15

$$\frac{\sqrt{1-a^2x^2}\,\sqrt{c-a^2cx^2}}{2ac^2(ax-1)(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*Sqrt[c - a^2*c*x^2])/(2*a*c^2*(-1 + a*x)*(1 + a*x)^3)
```

fricas [**A**] time = 0.56, size = 72, normalized size = 1.57

$$-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\left(ax^2+2\,x\right)}{2\,\left(a^4c^2x^4+2\,a^3c^2x^3-2\,ac^2x-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x^2 + 2*x)/(a^4*c^2*x^4 + 2
*a^3*c^2*x^3 - 2*a*c^2*x - c^2)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)^3), x)
```

maple [A] time = 0.03, size = 38, normalized size = 0.83

$$-\frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{2 \left(a x +1\right)^{2} a \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2/(a*x+1)^2/a*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

maxima [A] time = 0.43, size = 29, normalized size = 0.63

$$-\frac{1}{2 \left(a^{3} c^{\frac{3}{2}} x^{2}+2 \, a^{2} c^{\frac{3}{2}} x+a c^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -1/2/(a^3*c^(3/2)*x^2 + 2*a^2*c^(3/2)*x + a*c^(3/2))
```

mupad [B] time = 1.17, size = 58, normalized size = 1.26

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}{2\,a^5\,c^2\,\left(\frac{2\,x}{a^3}+\frac{1}{a^4}-x^4-\frac{2\,x^3}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^(3/2)*(a*x + 1)^3),x)
```

```
[Out] -((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2))/(2*a^5*c^2*((2*x)/a^3 + 1/a^4
- x^4 - (2*x^3)/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*
x + 1)**3), x)
```

3.1277 $\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{6ac^2(ax+1)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

```
[Out] -1/6*(-a^2*x^2+1)^(1/2)/a/c^2/(a*x+1)^3/(-a^2*c*x^2+c)^(1/2)-1/8*(-a^2*x^2+
1)^(1/2)/a/c^2/(a*x+1)^2/(-a^2*c*x^2+c)^(1/2)-1/8*(-a^2*x^2+1)^(1/2)/a/c^2/
(a*x+1)/(-a^2*c*x^2+c)^(1/2)+1/8*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/c^2/(-a^
2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{6ac^2(ax+1)^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8ac^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(6*a*c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2
*x^2]/(8*a*c^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^
2*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*a*c^
2*Sqrt[c - a^2*c*x^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx=\frac{\sqrt{1-a^2x^2}\int\frac{e^{-3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$=\frac{\sqrt{1-a^2x^2}\int\frac{1}{(1-ax)(1+ax)^4}\,dx}{c^2\sqrt{c-a^2cx^2}}$$
$$=\frac{\sqrt{1-a^2x^2}\int\left(\frac{1}{2(1+ax)^4}+\frac{1}{4(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{c^2\sqrt{c-a^2cx^2}}$$
$$=-\frac{\sqrt{1-a^2x^2}}{6ac^2(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-}}{8}$$
$$=-\frac{\sqrt{1-a^2x^2}}{6ac^2(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^2(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-}}{8a}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 0.40

$$\frac{\sqrt{1-a^2x^2}\left(-3a^2x^2-9ax+3(ax+1)^3\tanh^{-1}(ax)-10\right)}{24ac^2(ax+1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-10 - 9*a*x - 3*a^2*x^2 + 3*(1 + a*x)^3*ArcTanh[a*x]))/
(24*a*c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.65, size = 461, normalized size = 2.53

$$\left[\frac{3\left(a^5x^5+3\,a^4x^4+2\,a^3x^3-2\,a^2x^2-3\,ax-1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,\sqrt{c}-c}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right)-4}{96\left(a^6c^3x^5+3\,a^5c^3x^4+2\,a^4c^3x^3-2\,a^3c^3x^2-3\,a^2c^3x-ac^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"fricas")
```

```
[Out] [1/96*(3*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*sqrt(c)*
log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*x^3 + a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1)) - 4*(10*a^3*x^3 + 27*a^2*x^2 + 21*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2
*x^2 + 1))/(a^6*c^3*x^5 + 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^3*c^3*x^2 - 3
*a^2*c^3*x - a*c^3), 1/48*(3*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 -
 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*a*sqr
t(-c)*x/(a^4*c*x^4 - c)) - 2*(10*a^3*x^3 + 27*a^2*x^2 + 21*a*x)*sqrt(-a^2*c
*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^6*c^3*x^5 + 3*a^5*c^3*x^4 + 2*a^4*c^3*x^3
- 2*a^3*c^3*x^2 - 3*a^2*c^3*x - a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)^3), x)
```

maple [A] time = 0.05, size = 159, normalized size = 0.87

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(3\ln\left(ax-1\right)x^3a^3-3a^3x^3\ln\left(ax+1\right)+9\ln\left(ax-1\right)x^2a^2-9\ln\left(ax+1\right)x^2a^2+6a\right.}{48\left(a^2x^2-1\right)c^3a\left(ax+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/48*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x^3*a^3-3*a^3*x
^3*ln(a*x+1)+9*ln(a*x-1)*x^2*a^2-9*ln(a*x+1)*x^2*a^2+6*a^2*x^2+9*ln(a*x-1)*
x*a-9*a*x*ln(a*x+1)+18*a*x+3*ln(a*x-1)-3*ln(a*x+1)+20)/(a^2*x^2-1)/c^3/a/(a
*x+1)^3
```

maxima [A] time = 0.35, size = 93, normalized size = 0.51

$$-\frac{3\,a^2\sqrt{c}\,x^2+9\,a\sqrt{c}\,x+10\,\sqrt{c}}{24\left(a^4c^3x^3+3\,a^3c^3x^2+3\,a^2c^3x+ac^3\right)}+\frac{\log\left(ax+1\right)}{16\,ac^{\frac{5}{2}}}-\frac{\log\left(ax-1\right)}{16\,ac^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm=
"maxima")
```

```
[Out] -1/24*(3*a^2*sqrt(c)*x^2 + 9*a*sqrt(c)*x + 10*sqrt(c))/(a^4*c^3*x^3 + 3*a^3
*c^3*x^2 + 3*a^2*c^3*x + a*c^3) + 1/16*log(a*x + 1)/(a*c^(5/2)) - 1/16*log(
a*x - 1)/(a*c^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\left(ax+1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*
x + 1)**3), x)
```

3.1278 $\int \frac{e^{-3\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=275

$$\frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^3(ax+1)\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{12ac^3(ax+1)^3\sqrt{c-a^2cx^2}}-\frac{}{16}$$

```
[Out] 1/32*(-a^2*x^2+1)^(1/2)/a/c^3/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/16*(-a^2*x^2+
1)^(1/2)/a/c^3/(a*x+1)^4/(-a^2*c*x^2+c)^(1/2)-1/12*(-a^2*x^2+1)^(1/2)/a/c^3
/(a*x+1)^3/(-a^2*c*x^2+c)^(1/2)-3/32*(-a^2*x^2+1)^(1/2)/a/c^3/(a*x+1)^2/(-a
^2*c*x^2+c)^(1/2)-1/8*(-a^2*x^2+1)^(1/2)/a/c^3/(a*x+1)/(-a^2*c*x^2+c)^(1/2)
+5/32*arctanh(a*x)*(-a^2*x^2+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6143, 6140, 44, 207}

$$\frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{8ac^3(ax+1)\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}}{32ac^3(ax+1)^2\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{12ac^3(ax+1)^3\sqrt{c-a^2cx^2}}-\frac{}{16}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(32*a*c^3*(1 - a*x)*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x
^2]/(16*a*c^3*(1 + a*x)^4*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(12*a*c^
3*(1 + a*x)^3*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2])/(32*a*c^3*(1 + a
*x)^2*Sqrt[c - a^2*c*x^2]) - Sqrt[1 - a^2*x^2]/(8*a*c^3*(1 + a*x)*Sqrt[c -
a^2*c*x^2]) + (5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(32*a*c^3*Sqrt[c - a^2*c*x
^2])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^2(1+ax)^5}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{32(-1+ax)^2}+\frac{1}{4(1+ax)^5}+\frac{1}{4(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{32\left(-1+a^2x^2\right)}\right)dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{16ac^3(1+ax)^4\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{12ac^3(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{}{32a}$$

$$= \frac{\sqrt{1-a^2x^2}}{32ac^3(1-ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{16ac^3(1+ax)^4\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}}{12ac^3(1+ax)^3\sqrt{c-a^2cx^2}}-\frac{}{32a}$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.37

$$\frac{\sqrt{1-a^2x^2}\left(-15a^4x^4-45a^3x^3-35a^2x^2+15ax+15(ax-1)(ax+1)^4\tanh^{-1}(ax)+32\right)}{96ac^3(ax-1)(ax+1)^4\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(32 + 15*a*x - 35*a^2*x^2 - 45*a^3*x^3 - 15*a^4*x^4 + 15
*(-1 + a*x)*(1 + a*x)^4*ArcTanh[a*x]))/(96*a*c^3*(-1 + a*x)*(1 + a*x)^4*Sqr
t[c - a^2*c*x^2])
```

fricas [A] time = 0.83, size = 559, normalized size = 2.03

$$\left[\frac{15\left(a^7x^7+3\,a^6x^6+a^5x^5-5\,a^4x^4-5\,a^3x^3+a^2x^2+3\,ax+1\right)\sqrt{c}\,\log\left(-\frac{a^6cx^6+5\,a^4cx^4-5\,a^2cx^2-4\left(a^3x^3+ax\right)\sqrt{-a^2cx^2+c}\,\sqrt{-}}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}\right.}{384\left(a^8c^4x^7+3\,a^7c^4x^6+a^6c^4x^5-5\,a^5c^4x^4-5\,a^4c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm=
"fricas")
```

```
[Out] [1/384*(15*(a^7*x^7 + 3*a^6*x^6 + a^5*x^5 - 5*a^4*x^4 - 5*a^3*x^3 + a^2*x^2
 + 3*a*x + 1)*sqrt(c)*log(-(a^6*c*x^6 + 5*a^4*c*x^4 - 5*a^2*c*x^2 - 4*(a^3*
x^3 + a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*sqrt(c) - c)/(a^6*x^6 -
3*a^4*x^4 + 3*a^2*x^2 - 1)) - 4*(32*a^5*x^5 + 81*a^4*x^4 + 19*a^3*x^3 - 99*
a^2*x^2 - 81*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1))/(a^8*c^4*x^7 + 3
*a^7*c^4*x^6 + a^6*c^4*x^5 - 5*a^5*c^4*x^4 - 5*a^4*c^4*x^3 + a^3*c^4*x^2 +
3*a^2*c^4*x + a*c^4), 1/192*(15*(a^7*x^7 + 3*a^6*x^6 + a^5*x^5 - 5*a^4*x^4
- 5*a^3*x^3 + a^2*x^2 + 3*a*x + 1)*sqrt(-c)*arctan(2*sqrt(-a^2*c*x^2 + c)*s
qrt(-a^2*x^2 + 1)*a*sqrt(-c)*x/(a^4*c*x^4 - c)) - 2*(32*a^5*x^5 + 81*a^4*x^
4 + 19*a^3*x^3 - 99*a^2*x^2 - 81*a*x)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 +
1))/(a^8*c^4*x^7 + 3*a^7*c^4*x^6 + a^6*c^4*x^5 - 5*a^5*c^4*x^4 - 5*a^4*c^4*
x^3 + a^3*c^4*x^2 + 3*a^2*c^4*x + a*c^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)^3), x)
```

maple [A] time = 0.05, size = 238, normalized size = 0.87

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(15\ln\left(ax-1\right)x^5a^5-15\ln\left(ax+1\right)x^5a^5+45\ln\left(ax-1\right)x^4a^4-45\ln\left(ax+1\right)x^4a^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/192*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x-1)*x^5*a^5-15*ln
(a*x+1)*x^5*a^5+45*ln(a*x-1)*x^4*a^4-45*ln(a*x+1)*x^4*a^4+30*x^4*a^4+30*ln(
a*x-1)*x^3*a^3-30*a^3*x^3*ln(a*x+1)+90*x^3*a^3-30*ln(a*x-1)*x^2*a^2+30*ln(a
*x+1)*x^2*a^2+70*a^2*x^2-45*ln(a*x-1)*x*a+45*a*x*ln(a*x+1)-30*a*x-15*ln(a*x
-1)+15*ln(a*x+1)-64)/(a^2*x^2-1)/c^4/a/(a*x-1)/(a*x+1)^4
```

maxima [A] time = 0.34, size = 122, normalized size = 0.44

$$-\frac{15\,a^4x^4+45\,a^3x^3+35\,a^2x^2-15\,ax-32}{96\left(a^6c^{\frac{7}{2}}x^5+3\,a^5c^{\frac{7}{2}}x^4+2\,a^4c^{\frac{7}{2}}x^3-2\,a^3c^{\frac{7}{2}}x^2-3\,a^2c^{\frac{7}{2}}x-ac^{\frac{7}{2}}\right)}+\frac{5\,\log\left(ax+1\right)}{64\,ac^{\frac{7}{2}}}-\frac{5\,\log\left(ax-1\right)}{64\,ac^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm=
"maxima")
```

```
[Out] -1/96*(15*a^4*x^4 + 45*a^3*x^3 + 35*a^2*x^2 - 15*a*x - 32)/(a^6*c^(7/2)*x^5
 + 3*a^5*c^(7/2)*x^4 + 2*a^4*c^(7/2)*x^3 - 2*a^3*c^(7/2)*x^2 - 3*a^2*c^(7/2
)*x - a*c^(7/2)) + 5/64*log(a*x + 1)/(a*c^(7/2)) - 5/64*log(a*x - 1)/(a*c^(
7/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-a^2\,x^2\right)^{3/2}}{\left(c-a^2\,c\,x^2\right)^{7/2}\,\left(a\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)^3),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\left(ax+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*
x + 1)**3), x)
```

3.1279 $\int e^{-3\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=136

$$\frac{4x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;-ax)}{(m+1)\sqrt{1-a^2x^2}}-\frac{3x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

```
[Out] -3*x^(1+m)*(-a^2*c*x^2+c)^(1/2)/(1+m)/(-a^2*x^2+1)^(1/2)+a*x^(2+m)*(-a^2*c*
x^2+c)^(1/2)/(2+m)/(-a^2*x^2+1)^(1/2)+4*x^(1+m)*hypergeom([1, 1+m],[2+m],-a
*x)*(-a^2*c*x^2+c)^(1/2)/(1+m)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6153, 6150, 88, 64}

$$\frac{4x^{m+1}\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;-ax)}{(m+1)\sqrt{1-a^2x^2}}-\frac{3x^{m+1}\sqrt{c-a^2cx^2}}{(m+1)\sqrt{1-a^2x^2}}+\frac{ax^{m+2}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Sqrt[c - a^2*c*x^2])/((1 + m)*Sqrt[1 - a^2*x^2]) + (a*x^(2 +
m)*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - a^2*x^2]) + (4*x^(1 + m)*Sqrt[c -
 a^2*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/((1 + m)*Sqrt[1 - a
^2*x^2])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
```

```
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\tanh^{-1}(ax)}x^m\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^m(1-ax)^2}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-3x^m+ax^{1+m}+\frac{4x^m}{1+ax}\right)dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{3x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}+\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int \frac{x^m}{1+ax}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{3x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}}+\frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}}+\frac{4x^{1+m}\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2}{(1+m)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.54

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}\,(4(m+2)\,{}_2F_1(1,m+1;m+2;-ax)+m(ax-3)+ax-6)}{(m+1)(m+2)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcTanh[a*x]),x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2]*(-6 + a*x + m*(-3 + a*x) + 4*(2 + m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, -(a*x)]))/((1 + m)*(2 + m)*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,(ax-1)x^m}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^m/(a^2*x^2 +
2*a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-a^2c\,x^2+c}\,\left(-a^2x^2+1\right)^{\frac{3}{2}}}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,\left(-a^2x^2+1\right)^{\frac{3}{2}}x^m}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(-a^2*x^2 + 1)^(3/2)*x^m/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m \sqrt{c-a^2 c x^2} \left(1-a^2 x^2\right)^{3/2}}{(a x+1)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
[Out] int((x^m*(c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
[Out] Timed out
```

3.1280 $\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=86

$$-\frac{2^{p-\frac{1}{2}}(1-ax)^{p+\frac{5}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2}-p,p+\frac{5}{2};p+\frac{7}{2};\frac{1}{2}(1-ax)\right)}{a(2p+5)}$$

```
[Out] -2^(-1/2+p)*(-a*x+1)^(5/2+p)*(-a^2*c*x^2+c)^p*hypergeom([5/2+p, 3/2-p],[7/2
+p],-1/2*a*x+1/2)/a/(5+2*p)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 69}

$$-\frac{2^{p-\frac{1}{2}}(1-ax)^{p+\frac{5}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2}-p,p+\frac{5}{2};p+\frac{7}{2};\frac{1}{2}(1-ax)\right)}{a(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((2^(-1/2 + p)*(1 - a*x)^(5/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/2
 - p, 5/2 + p, 7/2 + p, (1 - a*x)/2])/(a*(5 + 2*p)*(1 - a^2*x^2)^p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-3\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{\frac{3}{2}+p}(1+ax)^{-\frac{3}{2}+p}\,dx$$
$$=-\frac{2^{-\frac{1}{2}+p}(1-ax)^{\frac{5}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2}-p,\frac{5}{2}+p;\frac{7}{2}+p;\frac{1}{2}(1-ax\right.}{a(5+2p)}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.00

$$-\frac{2^{p-\frac{3}{2}}(1-ax)^{p+\frac{5}{2}}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2}-p,p+\frac{5}{2};p+\frac{7}{2};\frac{1}{2}(1-ax)\right)}{a\left(p+\frac{5}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(3*ArcTanh[a*x]),x]
```

```
[Out] -((2^(-3/2 + p)*(1 - a*x)^(5/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/2
 - p, 5/2 + p, 7/2 + p, (1 - a*x)/2])/(a*(5/2 + p)*(1 - a^2*x^2)^p))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(ax-1)\left(-a^2cx^2+c\right)^p}{a^2x^2+2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*(a*x - 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 + 2*a*x
+ 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(-a^2cx^2+c\right)^p}{(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(-a^2*c*x^2 + c)^p/(a*x + 1)^3, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} c\,x^{2}+c\right)^{p} \left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}{\left(a x +1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}} \left(-a^{2} c x^{2}+c\right)^{p}}{\left(a x +1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/(a*x+1)^3*(-a^2*x^2+1)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*(-a^2*c*x^2 + c)^p/(a*x + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^p\,{\left(1-a^2\,x^2\right)}^{3/2}}{{\left(a\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3,x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(1 - a^2*x^2)^(3/2))/(a*x + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{p}}{\left(a x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/(a*x+1)**3*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*(-c*(a*x - 1)*(a*x + 1))**p/(a*x + 1
)**3, x)
```

3.1281 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=359

$$-\frac{(ax+1)^{11/4}(1-ax)^{13/4}}{6a}-\frac{11(ax+1)^{7/4}(1-ax)^{13/4}}{60a}-\frac{77(ax+1)^{3/4}(1-ax)^{13/4}}{480a}+\frac{77(ax+1)^{3/4}(1-ax)^{9/4}}{960a}+\frac{231(ax+}{}$$

```
[Out] 231/512*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a+231/1280*(-a*x+1)^(5/4)*(a*x+1)^(3/4
)/a+77/960*(-a*x+1)^(9/4)*(a*x+1)^(3/4)/a-77/480*(-a*x+1)^(13/4)*(a*x+1)^(3
/4)/a-11/60*(-a*x+1)^(13/4)*(a*x+1)^(7/4)/a-1/6*(-a*x+1)^(13/4)*(a*x+1)^(11
/4)/a-231/1024*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)-23
1/1024*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)+231/2048*ln
(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*2^(
1/2)-231/2048*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x
+1)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{11/4}(1-ax)^{13/4}}{6a}-\frac{11(ax+1)^{7/4}(1-ax)^{13/4}}{60a}-\frac{77(ax+1)^{3/4}(1-ax)^{13/4}}{480a}+\frac{77(ax+1)^{3/4}(1-ax)^{9/4}}{960a}+\frac{231(ax+}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (231*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(512*a) + (231*(1 - a*x)^(5/4)*(1 + a
*x)^(3/4))/(1280*a) + (77*(1 - a*x)^(9/4)*(1 + a*x)^(3/4))/(960*a) - (77*(1
 - a*x)^(13/4)*(1 + a*x)^(3/4))/(480*a) - (11*(1 - a*x)^(13/4)*(1 + a*x)^(7
/4))/(60*a) - ((1 - a*x)^(13/4)*(1 + a*x)^(11/4))/(6*a) + (231*ArcTan[1 - (
Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(512*Sqrt[2]*a) - (231*ArcTan[1
+ (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(512*Sqrt[2]*a) + (231*Log[1
+ Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])
/(1024*Sqrt[2]*a) - (231*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1
- a*x)^(1/4))/(1 + a*x)^(1/4)])/(1024*Sqrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}\,dx = \int (1-ax)^{9/4}(1+ax)^{11/4}\,dx$$
$$= -\frac{(1-ax)^{13/4}(1+ax)^{11/4}}{6a} + \frac{11}{12}\int (1-ax)^{9/4}(1+ax)^{7/4}\,dx$$
$$= -\frac{11(1-ax)^{13/4}(1+ax)^{7/4}}{60a} - \frac{(1-ax)^{13/4}(1+ax)^{11/4}}{6a} + \frac{77}{120}\int (1-ax)^{9/4}(1+ax$$
$$= -\frac{77(1-ax)^{13/4}(1+ax)^{3/4}}{480a} - \frac{11(1-ax)^{13/4}(1+ax)^{7/4}}{60a} - \frac{(1-ax)^{13/4}(1+ax)^{11/4}}{6a}$$
$$= \frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a} - \frac{77(1-ax)^{13/4}(1+ax)^{3/4}}{480a} - \frac{11(1-ax)^{13/4}(1+ax)^{7/4}}{60a}$$
$$= \frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a} + \frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a} - \frac{77(1-ax)^{13/4}(1+ax)^{3/4}}{480a}$$
$$= \frac{231\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{512a} + \frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a} + \frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}$$
$$= \frac{231\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{512a} + \frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a} + \frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}$$
$$= \frac{231\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{512a} + \frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a} + \frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}$$
$$= \frac{231\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{512a} + \frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a} + \frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}$$
$$= \frac{231\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{512a} + \frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a} + \frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}$$
$$= \frac{231\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{512a} + \frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a} + \frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}$$
$$= \frac{231\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{512a} + \frac{231(1-ax)^{5/4}(1+ax)^{3/4}}{1280a} + \frac{77(1-ax)^{9/4}(1+ax)^{3/4}}{960a}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.12

$$-\frac{16\ 2^{3/4}(1-ax)^{13/4}\,{}_2F_1\left(-\frac{11}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-ax)\right)}{13a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-16*2^(3/4)*(1 - a*x)^(13/4)*Hypergeometric2F1[-11/4, 13/4, 17/4, (1 - a*x
)/2])/(13*a)
```

fricas [B] time = 0.68, size = 557, normalized size = 1.55

$$-\frac{13860\,\sqrt{2}\,a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}\,a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x, algorith
m="fricas")
```

```
[Out] -1/30720*(13860*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^
4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^
2)*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a
*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 13860*sqrt(2)*a*
(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a
^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)
/(a*x - 1))*(a^(-4))^(1/4) + 1) + 3465*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2
)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x
 - a^2)*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 3465*sqrt(2)*a*(a^(
-4))^(1/4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*
(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)
) - 4*(1280*a^5*x^5 + 128*a^4*x^4 - 4144*a^3*x^3 - 520*a^2*x^2 + 5174*a*x +
 1547)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2x^2+1\right)^{\frac{5}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x, algorith
m="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(5/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,\left(-a^2x^2+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2x^2+1\right)^{\frac{5}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(5/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(1-a^2\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(5/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((1 - a^2*x^2)^(5/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Timed out
```

3.1282 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=307

$$-\frac{(ax+1)^{7/4}(1-ax)^{9/4}}{4a}-\frac{7(ax+1)^{3/4}(1-ax)^{9/4}}{24a}+\frac{7(ax+1)^{3/4}(1-ax)^{5/4}}{32a}+\frac{35(ax+1)^{3/4}\sqrt[4]{1-ax}}{64a}+\frac{35\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\right.}{128\sqrt{}}$$

```
[Out] 35/64*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a+7/32*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/a-7/
24*(-a*x+1)^(9/4)*(a*x+1)^(3/4)/a-1/4*(-a*x+1)^(9/4)*(a*x+1)^(7/4)/a-35/128
*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)-35/128*arctan(1+
(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)+35/256*ln(1-(-a*x+1)^(1/4)*
2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*2^(1/2)-35/256*ln(1+(
-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{7/4}(1-ax)^{9/4}}{4a}-\frac{7(ax+1)^{3/4}(1-ax)^{9/4}}{24a}+\frac{7(ax+1)^{3/4}(1-ax)^{5/4}}{32a}+\frac{35(ax+1)^{3/4}\sqrt[4]{1-ax}}{64a}+\frac{35\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\right.}{128\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (35*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(64*a) + (7*(1 - a*x)^(5/4)*(1 + a*x)^
(3/4))/(32*a) - (7*(1 - a*x)^(9/4)*(1 + a*x)^(3/4))/(24*a) - ((1 - a*x)^(9/
4)*(1 + a*x)^(7/4))/(4*a) + (35*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a
*x)^(1/4)])/(64*Sqrt[2]*a) - (35*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)])/(64*Sqrt[2]*a) + (35*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sq
rt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*Sqrt[2]*a) - (35*Log[1 + Sqrt
[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(128*
Sqrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx = \int (1-ax)^{5/4}(1+ax)^{7/4}\,dx$$

$$= -\frac{(1-ax)^{9/4}(1+ax)^{7/4}}{4a} + \frac{7}{8}\int (1-ax)^{5/4}(1+ax)^{3/4}\,dx$$

$$= -\frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a} - \frac{(1-ax)^{9/4}(1+ax)^{7/4}}{4a} + \frac{7}{16}\int \frac{(1-ax)^{5/4}}{\sqrt[4]{1+ax}}\,dx$$

$$= \frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a} - \frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a} - \frac{(1-ax)^{9/4}(1+ax)^{7/4}}{4a} + \frac{35}{64}$$

$$= \frac{35\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a} + \frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a} - \frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a} - (1$$

$$= \frac{35\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a} + \frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a} - \frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a} - (1$$

$$= \frac{35\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a} + \frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a} - \frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a} - (1$$

$$= \frac{35\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a} + \frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a} - \frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a} - (1$$

$$= \frac{35\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a} + \frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a} - \frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a} - (1$$

$$= \frac{35\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a} + \frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a} - \frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a} - (1$$

$$= \frac{35\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{64a} + \frac{7(1-ax)^{5/4}(1+ax)^{3/4}}{32a} - \frac{7(1-ax)^{9/4}(1+ax)^{3/4}}{24a} - (1$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.14

$$-\frac{8\,2^{3/4}(1-ax)^{9/4}\,{}_2F_1\left(-\frac{7}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-ax)\right)}{9a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-8*2^(3/4)*(1 - a*x)^(9/4)*Hypergeometric2F1[-7/4, 9/4, 13/4, (1 - a*x)/2]
)/(9*a)
```

fricas [B] time = 1.66, size = 541, normalized size = 1.76

$$-\frac{420\sqrt{2}\,a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}\,a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x, algorith
m="fricas")
```

```
[Out] -1/768*(420*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x
- a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*s
qrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqr
t(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 420*sqrt(2)*a*(a^(-4
))^(1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2
+ 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2
 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x
- 1))*(a^(-4))^(1/4) + 1) + 105*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*
x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)
*sqrt(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 105*sqrt(2)*a*(a^(-4))^(1/
4)*log(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))
^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) + 4*(4
8*a^3*x^3 + 8*a^2*x^2 - 118*a*x - 43)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^
2 + 1)/(a*x - 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,\left(-a^2x^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(1-a^2\,x^2\right)^{3/2}\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((1 - a^2*x^2)^(3/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))*(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

3.1283 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx$

Optimal. Leaf size=255

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{2a}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a}+\frac{3\,t}{}$$

```
[Out] 3/4*(-a*x+1)^(1/4)*(a*x+1)^(3/4)/a-1/2*(-a*x+1)^(5/4)*(a*x+1)^(3/4)/a-3/8*a
rctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)-3/8*arctan(1+(-a*x
+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)+3/16*ln(1-(-a*x+1)^(1/4)*2^(1/2)
/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*2^(1/2)-3/16*ln(1+(-a*x+1)^(
1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}}{2a}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}}{4a}+\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a}-\frac{3\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}\,a}+\frac{3\,t}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*Sqrt[1 - a^2*x^2],x]
```

```
[Out] (3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4))/(4*a) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/4
))/(2*a) + (3*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqr
t[2]*a) - (3*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt
[2]*a) + (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))
/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a) - (3*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] +
 (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx = \int \sqrt[4]{1-ax}\,(1+ax)^{3/4}\,dx$$

$$= -\frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} + \frac{3}{4}\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} + \frac{3}{8}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{2a}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{2a}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4a} - \cdots$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8a} \cdots$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} + \frac{3\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{8\sqrt{2}\,a} - \cdots$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}}{4a} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{4\sqrt{2}\,a} - \frac{3\tan^{-1}\cdots}{\cdots}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.16

$$-\frac{4\,2^{3/4}(1-ax)^{5/4}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-ax)\right)}{5a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-4*2^(3/4)*(1 - a*x)^(5/4)*Hypergeometric2F1[-3/4, 5/4, 9/4, (1 - a*x)/2])
/(5*a)
```

fricas [B] time = 0.99, size = 525, normalized size = 2.06

$$-\frac{12\sqrt{2}\,a\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^4}^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^4}^{\frac{1}{4}}-\sqrt{2}\,a\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\,\frac{1}{a^4}^{\frac{1}{4}}-1\right)+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x -
a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqr
t(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(
-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) + 12*sqrt(2)*a*(a^(-4))^
(1/4)*arctan(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1
)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 +
1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1
))*(a^(-4))^(1/4) + 1) + 3*sqrt(2)*a*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*x - a
^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt
(a^(-4)) - sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 3*sqrt(2)*a*(a^(-4))^(1/4)*log(
-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4)
- (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1)) - 4*sqrt(-a^2
*x^2 + 1)*(2*a*x + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2x^2+1}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{1-a^2\,x^2}\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\sqrt{-(ax-1)(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))*sqrt(-(a*x - 1)*(a*x + 1)), x
)
```

3.1284 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=193

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

```
[Out] 1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))
/a*2^(1/2)-1/2*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*
x+1)^(1/2))/a*2^(1/2)-arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(
1/2)-arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))/a*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6140, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - (Sqrt[2
]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a + Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(Sqrt[2]
*a) - Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)]/(Sqrt[2]*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\right.}{\sqrt{2}a}$$
$$= \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\right.}{a}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\log\left(\right.}{}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.21

$$-\frac{2\,2^{3/4}\sqrt[4]{1-ax}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-ax)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*2^(3/4)*(1 - a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - a*x)/2])/
a
```

fricas [B] time = 0.66, size = 473, normalized size = 2.45

$$-2\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\,a\sqrt{\frac{\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{-a^2x^2+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}\,a\sqrt{-\frac{\sqrt{-a}}{ax}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] -2*sqrt(2)*(a^(-4))^(1/4)*arctan(sqrt(2)*a*sqrt((sqrt(2)*(a^4*x - a^3)*sqrt
(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(a^(-4))
 - sqrt(-a^2*x^2 + 1))/(a*x - 1))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^
2*x^2 + 1)/(a*x - 1))*(a^(-4))^(1/4) - 1) - 2*sqrt(2)*(a^(-4))^(1/4)*arctan
(sqrt(2)*a*sqrt(-(sqrt(2)*(a^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))
*(a^(-4))^(3/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1
))*(a^(-4))^(1/4) - sqrt(2)*a*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^
(1/4) + 1) - 1/2*sqrt(2)*(a^(-4))^(1/4)*log((sqrt(2)*(a^4*x - a^3)*sqrt(-sq
rt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) + (a^3*x - a^2)*sqrt(a^(-4)) - s
qrt(-a^2*x^2 + 1))/(a*x - 1)) + 1/2*sqrt(2)*(a^(-4))^(1/4)*log(-(sqrt(2)*(a
^4*x - a^3)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))*(a^(-4))^(3/4) - (a^3*x - a
^2)*sqrt(a^(-4)) + sqrt(-a^2*x^2 + 1))/(a*x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/sqrt(-(a*x - 1)*(a*x + 1)), x
)
```

3.1285 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3a\sqrt{1-a^2x^2}}$$

```
[Out] -2/3*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-2*a*x+1)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6135}

$$-\frac{2(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 2*a*x))/(3*a*Sqrt[1 - a^2*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{3a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.86

$$\frac{2(2ax-1)}{3a(1-ax)^{3/4}\sqrt[4]{ax+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(3/2),x]
```

[Out] (2*(-1 + 2*a*x))/(3*a*(1 - a*x)^(3/4)*(1 + a*x)^(1/4))

fricas [A] time = 0.64, size = 56, normalized size = 1.51

$$-\frac{2\sqrt{-a^2x^2+1}\,(2\,ax-1)\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{3\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")

[Out] -2/3*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a^3*x^2 - a)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(3/2), x)

maple [A] time = 0.03, size = 54, normalized size = 1.46

$$-\frac{2\,(ax-1)\,(ax+1)\,(2ax-1)\,\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{3a\left(-a^2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(3/2),x)

[Out] -2/3*(a*x-1)*(a*x+1)*(2*a*x-1)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a/(-a^2*x^2+1)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(3/2), x)
```

mupad [B] time = 1.07, size = 69, normalized size = 1.86

$$-\frac{\left(\frac{2\,\sqrt{1-a^2\,x^2}}{3\,a^3}-\frac{4\,x\,\sqrt{1-a^2\,x^2}}{3\,a^2}\right)\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{\frac{1}{a^2}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] -(((2*(1 - a^2*x^2)^(1/2))/(3*a^3) - (4*x*(1 - a^2*x^2)^(1/2))/(3*a^2))*((a
*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(1/a^2 - x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/(-(a*x - 1)*(a*x + 1))**(3/2)
, x)
```

3.1286 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{2(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35a\left(1-a^2x^2\right)^{3/2}}-\frac{16(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35a\sqrt{1-a^2x^2}}$$

```
[Out] -2/35*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-6*a*x+1)/a/(-a^2*x^2+1)^(3/2)-16
/35*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-2*a*x+1)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6136, 6135}

$$-\frac{2(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35a\left(1-a^2x^2\right)^{3/2}}-\frac{16(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(35*a*(1 - a^2*x^2)^(3/2)) - (16*E^(Arc
Tanh[a*x]/2)*(1 - 2*a*x))/(35*a*Sqrt[1 - a^2*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{35a\left(1-a^2x^2\right)^{3/2}} + \frac{24}{35}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx$$
$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{35a\left(1-a^2x^2\right)^{3/2}} - \frac{16e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{35a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.64

$$-\frac{2\left(16a^3x^3-8a^2x^2-22ax+9\right)}{35a(1-ax)^{7/4}(ax+1)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-2*(9 - 22*a*x - 8*a^2*x^2 + 16*a^3*x^3))/(35*a*(1 - a*x)^(7/4)*(1 + a*x)^
(5/4))
```

fricas [A] time = 0.69, size = 78, normalized size = 1.04

$$-\frac{2\left(16\,a^3x^3-8\,a^2x^2-22\,ax+9\right)\sqrt{-a^2x^2+1}\,\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{35\left(a^5x^4-2\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(5/2),x, algorith
m="fricas")
```

```
[Out] -2/35*(16*a^3*x^3 - 8*a^2*x^2 - 22*a*x + 9)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-
a^2*x^2 + 1)/(a*x - 1))/(a^5*x^4 - 2*a^3*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(5/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(5/2), x)
```

maple [A] time = 0.03, size = 70, normalized size = 0.93

$$\frac{2\left(ax-1\right)\left(ax+1\right)\left(16x^{3}a^{3}-8a^{2}x^{2}-22ax+9\right)\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{35a\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(5/2),x)
```

```
[Out] 2/35*(a*x-1)*(a*x+1)*(16*a^3*x^3-8*a^2*x^2-22*a*x+9)*((a*x+1)/(-a^2*x^2+1)^
(1/2))^(1/2)/a/(-a^2*x^2+1)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(5/2), x)
```

mupad [B] time = 1.15, size = 115, normalized size = 1.53

$$-\frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,\left(\frac{18\,\sqrt{1-a^2\,x^2}}{35\,a^5}-\frac{44\,x\,\sqrt{1-a^2\,x^2}}{35\,a^4}+\frac{32\,x^3\,\sqrt{1-a^2\,x^2}}{35\,a^2}-\frac{16\,x^2\,\sqrt{1-a^2\,x^2}}{35\,a^3}\right)}{\frac{1}{a^4}+x^4-\frac{2\,x^2}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(1 - a^2*x^2)^(5/2),x)
```

```
[Out] -(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)*((18*(1 - a^2*x^2)^(1/2))/(35*a^5)
- (44*x*(1 - a^2*x^2)^(1/2))/(35*a^4) + (32*x^3*(1 - a^2*x^2)^(1/2))/(35*a^
2) - (16*x^2*(1 - a^2*x^2)^(1/2))/(35*a^3)))/(1/a^4 + x^4 - (2*x^2)/a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/(-(a*x - 1)*(a*x + 1))**(5/2)
, x)
```

3.1287 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{99a\left(1-a^2x^2\right)^{5/2}}-\frac{256(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693a\sqrt{1-a^2x^2}}-\frac{32(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693a\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -2/99*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-10*a*x+1)/a/(-a^2*x^2+1)^(5/2)-3
2/693*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-6*a*x+1)/a/(-a^2*x^2+1)^(3/2)-25
6/693*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-2*a*x+1)/a/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6136, 6135}

$$-\frac{2(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{99a\left(1-a^2x^2\right)^{5/2}}-\frac{256(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693a\sqrt{1-a^2x^2}}-\frac{32(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 10*a*x))/(99*a*(1 - a^2*x^2)^(5/2)) - (32*E^(Ar
cTanh[a*x]/2)*(1 - 6*a*x))/(693*a*(1 - a^2*x^2)^(3/2)) - (256*E^(ArcTanh[a*
x]/2)*(1 - 2*a*x))/(693*a*Sqrt[1 - a^2*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99a\left(1-a^2x^2\right)^{5/2}} + \frac{80}{99}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99a\left(1-a^2x^2\right)^{5/2}} - \frac{32e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{693a\left(1-a^2x^2\right)^{3/2}} + \frac{128}{231}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99a\left(1-a^2x^2\right)^{5/2}} - \frac{32e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{693a\left(1-a^2x^2\right)^{3/2}} - \frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{693a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.57

$$\frac{2\left(256a^5x^5-128a^4x^4-608a^3x^3+272a^2x^2+422ax-151\right)}{693a(1-ax)^{11/4}(ax+1)^{9/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (2*(-151 + 422*a*x + 272*a^2*x^2 - 608*a^3*x^3 - 128*a^4*x^4 + 256*a^5*x^5)
)/(693*a*(1 - a*x)^(11/4)*(1 + a*x)^(9/4))
```

fricas [A] time = 0.59, size = 104, normalized size = 0.93

$$-\frac{2\left(256\,a^5x^5-128\,a^4x^4-608\,a^3x^3+272\,a^2x^2+422\,ax-151\right)\sqrt{-a^2x^2+1}\sqrt{-\frac{\sqrt{-a^2x^2+1}}{ax-1}}}{693\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(7/2),x, algorith
m="fricas")
```

```
[Out] -2/693*(256*a^5*x^5 - 128*a^4*x^4 - 608*a^3*x^3 + 272*a^2*x^2 + 422*a*x - 1
51)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2*x^2 + 1)/(a*x - 1))/(a^7*x^6 - 3*a^5
*x^4 + 3*a^3*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(7/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(7/2), x)
```

maple [A] time = 0.03, size = 86, normalized size = 0.77

$$-\frac{2\left(ax-1\right)\left(ax+1\right)\left(256x^{5}a^{5}-128x^{4}a^{4}-608x^{3}a^{3}+272a^{2}x^{2}+422ax-151\right)\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{693a\left(-a^{2}x^{2}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(7/2),x)
```

```
[Out] -2/693*(a*x-1)*(a*x+1)*(256*a^5*x^5-128*a^4*x^4-608*a^3*x^3+272*a^2*x^2+422
*a*x-151)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a/(-a^2*x^2+1)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{\left(-a^{2}x^{2}+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(7/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(7/2), x)
```

mupad [B] time = 1.31, size = 165, normalized size = 1.47

$$-\frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,\left(\frac{302\,\sqrt{1-a^2\,x^2}}{693\,a^7}-\frac{844\,x\,\sqrt{1-a^2\,x^2}}{693\,a^6}-\frac{512\,x^5\,\sqrt{1-a^2\,x^2}}{693\,a^2}+\frac{256\,x^4\,\sqrt{1-a^2\,x^2}}{693\,a^3}+\frac{1216\,x^3\,\sqrt{1-a^2\,x^2}}{693\,a^4}-\frac{544\,x^2\,\sqrt{1-a^2\,x^2}}{693\,a^5}\right)}{\frac{1}{a^6}-x^6+\frac{3\,x^4}{a^2}-\frac{3\,x^2}{a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(1 - a^2*x^2)^(7/2),x)
```

```
[Out] -(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)*((302*(1 - a^2*x^2)^(1/2))/(693*a^7
) - (844*x*(1 - a^2*x^2)^(1/2))/(693*a^6) - (512*x^5*(1 - a^2*x^2)^(1/2))/(
```

```
693*a^2) + (256*x^4*(1 - a^2*x^2)^(1/2))/(693*a^3) + (1216*x^3*(1 - a^2*x^2
)^(1/2))/(693*a^4) - (544*x^2*(1 - a^2*x^2)^(1/2))/(693*a^5)))/(1/a^6 - x^6
 + (3*x^4)/a^2 - (3*x^2)/a^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(7/2),x)
[Out] Timed out
```

3.1288 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(1-a^2x^2)^{9/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{2(1-14ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{195a\left(1-a^2x^2\right)^{7/2}}-\frac{2048(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a\sqrt{1-a^2x^2}}-\frac{256(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a\left(1-a^2x^2\right)^{3/2}}-\frac{112(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -2/195*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-14*a*x+1)/a/(-a^2*x^2+1)^(7/2)-
112/6435*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-10*a*x+1)/a/(-a^2*x^2+1)^(5/2
)-256/6435*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-6*a*x+1)/a/(-a^2*x^2+1)^(3/
2)-2048/6435*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-2*a*x+1)/a/(-a^2*x^2+1)^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6136, 6135}

$$-\frac{2(1-14ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{195a\left(1-a^2x^2\right)^{7/2}}-\frac{2048(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a\sqrt{1-a^2x^2}}-\frac{256(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a\left(1-a^2x^2\right)^{3/2}}-\frac{112(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435a\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 14*a*x))/(195*a*(1 - a^2*x^2)^(7/2)) - (112*E^(
ArcTanh[a*x]/2)*(1 - 10*a*x))/(6435*a*(1 - a^2*x^2)^(5/2)) - (256*E^(ArcTan
h[a*x]/2)*(1 - 6*a*x))/(6435*a*(1 - a^2*x^2)^(3/2)) - (2048*E^(ArcTanh[a*x]
/2)*(1 - 2*a*x))/(6435*a*Sqrt[1 - a^2*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
```

```
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{9/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195a\left(1-a^2x^2\right)^{7/2}}+\frac{56}{65}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195a\left(1-a^2x^2\right)^{7/2}}-\frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435a\left(1-a^2x^2\right)^{5/2}}+\frac{896\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/2}}\,dx}{1287}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195a\left(1-a^2x^2\right)^{7/2}}-\frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435a\left(1-a^2x^2\right)^{5/2}}-\frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{6435a\left(1-a^2x^2\right)^{3/2}}+\frac{1024\int}{}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195a\left(1-a^2x^2\right)^{7/2}}-\frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435a\left(1-a^2x^2\right)^{5/2}}-\frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{6435a\left(1-a^2x^2\right)^{3/2}}-\frac{2048e^{\frac{1}{2}}}{643}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.54

$$-\frac{2\left(2048a^7x^7-1024a^6x^6-6912a^5x^5+3200a^4x^4+8240a^3x^3-3384a^2x^2-3838ax+1241\right)}{6435a(1-ax)^{15/4}(ax+1)^{13/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-2*(1241 - 3838*a*x - 3384*a^2*x^2 + 8240*a^3*x^3 + 3200*a^4*x^4 - 6912*a^
5*x^5 - 1024*a^6*x^6 + 2048*a^7*x^7))/(6435*a*(1 - a*x)^(15/4)*(1 + a*x)^(1
3/4))
```

fricas [A] time = 1.67, size = 126, normalized size = 0.85

$$-\frac{2\left(2048\,a^7x^7-1024\,a^6x^6-6912\,a^5x^5+3200\,a^4x^4+8240\,a^3x^3-3384\,a^2x^2-3838\,ax+1241\right)\sqrt{-a^2x^2+1}\,\sqrt{-}}{6435\left(a^9x^8-4\,a^7x^6+6\,a^5x^4-4\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(9/2),x, algorith
m="fricas")
```

```
[Out] -2/6435*(2048*a^7*x^7 - 1024*a^6*x^6 - 6912*a^5*x^5 + 3200*a^4*x^4 + 8240*a
^3*x^3 - 3384*a^2*x^2 - 3838*a*x + 1241)*sqrt(-a^2*x^2 + 1)*sqrt(-sqrt(-a^2
*x^2 + 1)/(a*x - 1))/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(9/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(9/2), x)
```

maple [A] time = 0.03, size = 102, normalized size = 0.68

$$\frac{2\left(ax-1\right)\left(ax+1\right)\left(2048a^7x^7-1024x^6a^6-6912x^5a^5+3200x^4a^4+8240x^3a^3-3384a^2x^2-3838ax+1241\right)\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{6435a\left(-a^2x^2+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(9/2),x)
```

```
[Out] 2/6435*(a*x-1)*(a*x+1)*(2048*a^7*x^7-1024*a^6*x^6-6912*a^5*x^5+3200*a^4*x^4
+8240*a^3*x^3-3384*a^2*x^2-3838*a*x+1241)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2
)/a/(-a^2*x^2+1)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2x^2+1\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*x^2+1)^(9/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*x^2 + 1)^(9/2), x)
```

mupad [B] time = 1.46, size = 211, normalized size = 1.42

$$-\frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,\left(\frac{2482\,\sqrt{1-a^2\,x^2}}{6435\,a^9}-\frac{7676\,x\,\sqrt{1-a^2\,x^2}}{6435\,a^8}+\frac{4096\,x^7\,\sqrt{1-a^2\,x^2}}{6435\,a^2}-\frac{2048\,x^6\,\sqrt{1-a^2\,x^2}}{6435\,a^3}-\frac{1536\,x^5\,\sqrt{1-a^2\,x^2}}{715\,a^4}+\frac{1280\,x^4\,\sqrt{1-a^2\,x^2}}{1287\,a^5}+\cdots\right.}{\frac{1}{a^8}+x^8-\frac{4\,x^6}{a^2}+\frac{6\,x^4}{a^4}-\frac{4\,x^2}{a^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(1 - a^2*x^2)^(9/2),x)
```

```
[Out] -(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)*((2482*(1 - a^2*x^2)^(1/2))/(6435*a
^9) - (7676*x*(1 - a^2*x^2)^(1/2))/(6435*a^8) + (4096*x^7*(1 - a^2*x^2)^(1/
2))/(6435*a^2) - (2048*x^6*(1 - a^2*x^2)^(1/2))/(6435*a^3) - (1536*x^5*(1 -
 a^2*x^2)^(1/2))/(715*a^4) + (1280*x^4*(1 - a^2*x^2)^(1/2))/(1287*a^5) + (3
296*x^3*(1 - a^2*x^2)^(1/2))/(1287*a^6) - (752*x^2*(1 - a^2*x^2)^(1/2))/(71
5*a^7)))/(1/a^8 + x^8 - (4*x^6)/a^2 + (6*x^4)/a^4 - (4*x^2)/a^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Timed out
```

3.1289 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=679

$$-\frac{c^2(ax+1)^{11/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{11c^2(ax+1)^{7/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{60a\sqrt{1-a^2x^2}}-\frac{77c^2(ax+1)^{3/4}(1-ax)^{13/4}\sqrt{c-a^2}}{480a\sqrt{1-a^2x^2}}$$

```
[Out] 231/512*c^2*(-a*x+1)^(1/4)*(a*x+1)^(3/4)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1
)^(1/2)+231/1280*c^2*(-a*x+1)^(5/4)*(a*x+1)^(3/4)*(-a^2*c*x^2+c)^(1/2)/a/(-
a^2*x^2+1)^(1/2)+77/960*c^2*(-a*x+1)^(9/4)*(a*x+1)^(3/4)*(-a^2*c*x^2+c)^(1/
2)/a/(-a^2*x^2+1)^(1/2)-77/480*c^2*(-a*x+1)^(13/4)*(a*x+1)^(3/4)*(-a^2*c*x^
2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-11/60*c^2*(-a*x+1)^(13/4)*(a*x+1)^(7/4)*(-a
^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-1/6*c^2*(-a*x+1)^(13/4)*(a*x+1)^(11/
4)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-231/1024*c^2*arctan(-1+(-a*x+1
)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*(-a^2*c*x^2+c)^(1/2)/a*2^(1/2)/(-a^2*x^2+1)^
(1/2)-231/1024*c^2*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*(-a^2*c*x
^2+c)^(1/2)/a*2^(1/2)/(-a^2*x^2+1)^(1/2)+231/2048*c^2*ln(1-(-a*x+1)^(1/4)*2
^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^2*c*x^2+c)^(1/2)/a*2
^(1/2)/(-a^2*x^2+1)^(1/2)-231/2048*c^2*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^
(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^2*c*x^2+c)^(1/2)/a*2^(1/2)/(-a^2*x^
2+1)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6143, 6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^2(ax+1)^{11/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}-\frac{11c^2(ax+1)^{7/4}(1-ax)^{13/4}\sqrt{c-a^2cx^2}}{60a\sqrt{1-a^2x^2}}-\frac{77c^2(ax+1)^{3/4}(1-ax)^{13/4}\sqrt{c-a^2}}{480a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (231*c^2*(1 - a*x)^(1/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(512*a*Sqrt[1
 - a^2*x^2]) + (231*c^2*(1 - a*x)^(5/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2]
)/(1280*a*Sqrt[1 - a^2*x^2]) + (77*c^2*(1 - a*x)^(9/4)*(1 + a*x)^(3/4)*Sqrt
[c - a^2*c*x^2])/(960*a*Sqrt[1 - a^2*x^2]) - (77*c^2*(1 - a*x)^(13/4)*(1 +
a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(480*a*Sqrt[1 - a^2*x^2]) - (11*c^2*(1 - a*
x)^(13/4)*(1 + a*x)^(7/4)*Sqrt[c - a^2*c*x^2])/(60*a*Sqrt[1 - a^2*x^2]) - (
c^2*(1 - a*x)^(13/4)*(1 + a*x)^(11/4)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - a^
2*x^2]) + (231*c^2*Sqrt[c - a^2*c*x^2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))
/(1 + a*x)^(1/4)])/(512*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) - (231*c^2*Sqrt[c - a^
```

```
2*c*x^2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(512*Sqrt[2
]*a*Sqrt[1 - a^2*x^2]) + (231*c^2*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]
/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(1024*Sqrt[2]*
a*Sqrt[1 - a^2*x^2]) - (231*c^2*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]/S
qrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(1024*Sqrt[2]*a*
Sqrt[1 - a^2*x^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{5/2}\,dx}{\sqrt{1-a^2x^2}}$$

$$=\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)^{9/4}(1+ax)^{11/4}\,dx}{\sqrt{1-a^2x^2}}$$

$$=-\frac{c^2(1-ax)^{13/4}(1+ax)^{11/4}\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}+\frac{\left(11c^2\sqrt{c-a^2cx^2}\right)\int(1-ax)^{9/4}(1+a}{12\sqrt{1-a^2x^2}}$$

$$=-\frac{11c^2(1-ax)^{13/4}(1+ax)^{7/4}\sqrt{c-a^2cx^2}}{60a\sqrt{1-a^2x^2}}-\frac{c^2(1-ax)^{13/4}(1+ax)^{11/4}\sqrt{c-a^2cx^2}}{6a\sqrt{1-a^2x^2}}$$

$$=-\frac{77c^2(1-ax)^{13/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{480a\sqrt{1-a^2x^2}}-\frac{11c^2(1-ax)^{13/4}(1+ax)^{7/4}\sqrt{c-a^2c}}{60a\sqrt{1-a^2x^2}}$$

$$=\frac{77c^2(1-ax)^{9/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{960a\sqrt{1-a^2x^2}}-\frac{77c^2(1-ax)^{13/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{480a\sqrt{1-a^2x^2}}$$

$$=\frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}}+\frac{77c^2(1-ax)^{9/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{960a\sqrt{1-a^2x^2}}$$

$$=\frac{231c^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}}+\frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}}$$

$$=\frac{231c^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}}+\frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}}$$

$$=\frac{231c^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}}+\frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}}$$

$$=\frac{231c^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}}+\frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}}$$

$$=\frac{231c^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}}+\frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}}$$

$$=\frac{231c^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}}+\frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}}$$

$$=\frac{231c^2\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{512a\sqrt{1-a^2x^2}}+\frac{231c^2(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{1280a\sqrt{1-a^2x^2}}$$

Mathematica [C] time = 0.04, size = 74, normalized size = 0.11

$$-\frac{16\,2^{3/4}c^2(1-ax)^{13/4}\sqrt{c-a^2cx^2}\;{}_2F_1\left(-\frac{11}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-ax)\right)}{13a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-16*2^(3/4)*c^2*(1 - a*x)^(13/4)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-11
/4, 13/4, 17/4, (1 - a*x)/2])/(13*a*Sqrt[1 - a^2*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algori
thm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algori
thm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(5/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*c*x**2+c)**(5/2),x)
[Out] Timed out
```

3.1290 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=547

$$-\frac{c(ax+1)^{7/4}(1-ax)^{9/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}-\frac{7c(ax+1)^{3/4}(1-ax)^{9/4}\sqrt{c-a^2cx^2}}{24a\sqrt{1-a^2x^2}}+\frac{7c(ax+1)^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}+\frac{35c}{}$$

```
[Out] 35/64*c*(-a*x+1)^(1/4)*(a*x+1)^(3/4)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1
/2)+7/32*c*(-a*x+1)^(5/4)*(a*x+1)^(3/4)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)
^(1/2)-7/24*c*(-a*x+1)^(9/4)*(a*x+1)^(3/4)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2
+1)^(1/2)-1/4*c*(-a*x+1)^(9/4)*(a*x+1)^(7/4)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x
^2+1)^(1/2)-35/128*c*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*(-a^2*
c*x^2+c)^(1/2)/a*2^(1/2)/(-a^2*x^2+1)^(1/2)-35/128*c*arctan(1+(-a*x+1)^(1/4
)*2^(1/2)/(a*x+1)^(1/4))*(-a^2*c*x^2+c)^(1/2)/a*2^(1/2)/(-a^2*x^2+1)^(1/2)+
35/256*c*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(
1/2))*(-a^2*c*x^2+c)^(1/2)/a*2^(1/2)/(-a^2*x^2+1)^(1/2)-35/256*c*ln(1+(-a*x
+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^2*c*x^2+c
)^(1/2)/a*2^(1/2)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6143, 6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c(ax+1)^{7/4}(1-ax)^{9/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}-\frac{7c(ax+1)^{3/4}(1-ax)^{9/4}\sqrt{c-a^2cx^2}}{24a\sqrt{1-a^2x^2}}+\frac{7c(ax+1)^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}+\frac{35c}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (35*c*(1 - a*x)^(1/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(64*a*Sqrt[1 - a
^2*x^2]) + (7*c*(1 - a*x)^(5/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(32*a*
Sqrt[1 - a^2*x^2]) - (7*c*(1 - a*x)^(9/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^
2])/(24*a*Sqrt[1 - a^2*x^2]) - (c*(1 - a*x)^(9/4)*(1 + a*x)^(7/4)*Sqrt[c -
a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x^2]) + (35*c*Sqrt[c - a^2*c*x^2]*ArcTan[1 -
(Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(64*Sqrt[2]*a*Sqrt[1 - a^2*x^2]
) - (35*c*Sqrt[c - a^2*c*x^2]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x
)^(1/4)])/(64*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) + (35*c*Sqrt[c - a^2*c*x^2]*Log[
1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)
])/(128*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) - (35*c*Sqrt[c - a^2*c*x^2]*Log[1 + Sq
rt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(12
8*Sqrt[2]*a*Sqrt[1 - a^2*x^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{\frac{1}{2}\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}}$$

$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(1-ax)^{5/4}(1+ax)^{7/4}\,dx}{\sqrt{1-a^2x^2}}$$

$$=-\frac{c(1-ax)^{9/4}(1+ax)^{7/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}+\frac{\left(7c\sqrt{c-a^2cx^2}\right)\int(1-ax)^{5/4}(1+ax)^{3/4}}{8\sqrt{1-a^2x^2}}$$

$$=-\frac{7c(1-ax)^{9/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{24a\sqrt{1-a^2x^2}}-\frac{c(1-ax)^{9/4}(1+ax)^{7/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}+$$

$$=\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-\frac{7c(1-ax)^{9/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{24a\sqrt{1-a^2x^2}}-$$

$$=\frac{35c\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-$$

$$=\frac{35c\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-$$

$$=\frac{35c\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-$$

$$=\frac{35c\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-$$

$$=\frac{35c\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-$$

$$=\frac{35c\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-$$

$$=\frac{35c\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{64a\sqrt{1-a^2x^2}}+\frac{7c(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{32a\sqrt{1-a^2x^2}}-$$

Mathematica [C] time = 0.03, size = 72, normalized size = 0.13

$$-\frac{8\, 2^{3/4} c (1-a x)^{9/4} \sqrt{c-a^2 c x^2}\, {}_2F_1\left(-\frac{7}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2} (1-a x)\right)}{9 a \sqrt{1-a^2 x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-8*2^(3/4)*c*(1 - a*x)^(9/4)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-7/4, 9
/4, 13/4, (1 - a*x)/2])/(9*a*Sqrt[1 - a^2*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2 c x^2 + c\right)^{\frac{3}{2}} \sqrt{\frac{a x + 1}{\sqrt{-a^2 x^2 + 1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algori
thm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a x + 1}{\sqrt{-a^2 x^2 + 1}}}\, \left(-a^2 c\, x^2 + c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c-a^2\,c\,x^2\right)^{3/2}\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(3/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.1291 $\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=429

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}+\frac{3\sqrt{c-a^2cx^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a\sqrt{1-a^2x^2}}-$$

```
[Out] 3/4*(-a*x+1)^(1/4)*(a*x+1)^(3/4)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-
1/2*(-a*x+1)^(5/4)*(a*x+1)^(3/4)*(-a^2*c*x^2+c)^(1/2)/a/(-a^2*x^2+1)^(1/2)-
3/8*arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*(-a^2*c*x^2+c)^(1/2)/a*
2^(1/2)/(-a^2*x^2+1)^(1/2)-3/8*arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4
))*(-a^2*c*x^2+c)^(1/2)/a*2^(1/2)/(-a^2*x^2+1)^(1/2)+3/16*ln(1-(-a*x+1)^(1/
4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^2*c*x^2+c)^(1/2)
/a*2^(1/2)/(-a^2*x^2+1)^(1/2)-3/16*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4
)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^2*c*x^2+c)^(1/2)/a*2^(1/2)/(-a^2*x^2+1)
^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6143, 6140, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ax+1)^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}}+\frac{3(ax+1)^{3/4}\sqrt[4]{1-ax}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}}+\frac{3\sqrt{c-a^2cx^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{8\sqrt{2}a\sqrt{1-a^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*(1 - a*x)^(1/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - a^2*x
^2]) - ((1 - a*x)^(5/4)*(1 + a*x)^(3/4)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 -
a^2*x^2]) + (3*Sqrt[c - a^2*c*x^2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1
+ a*x)^(1/4)])/(4*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) - (3*Sqrt[c - a^2*c*x^2]*Arc
Tan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(4*Sqrt[2]*a*Sqrt[1 - a
^2*x^2]) + (3*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sq
rt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a*Sqrt[1 - a^2*x^2]) -
(3*Sqrt[c - a^2*c*x^2]*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 -
a*x)^(1/4))/(1 + a*x)^(1/4)])/(8*Sqrt[2]*a*Sqrt[1 - a^2*x^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\frac{1}{2}\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \sqrt[4]{1-ax}\,(1+ax)^{3/4}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{c-a^2cx^2}\right)\int \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\,dx}{4\sqrt{1-a^2x^2}}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{c-a^2\right.}{}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{c-a^2\right.}{}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{c-a^2\right.}{}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{c-a^2\right.}{}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{c-a^2\right.}{}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} + \frac{3\sqrt{c-a^2c}}{}$$

$$= \frac{3\sqrt[4]{1-ax}\,(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} + \frac{3\sqrt{c-a^2c}}{4}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.17

$$-\frac{4\,2^{3/4}(1-ax)^{5/4}\sqrt{c-a^2cx^2}\;{}_2F_1\left(-\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-ax)\right)}{5a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-4*2^(3/4)*(1 - a*x)^(5/4)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-3/4, 5/4
, 9/4, (1 - a*x)/2])/(5*a*Sqrt[1 - a^2*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c}\sqrt{\frac{ax + 1}{\sqrt{-a^2x^2 + 1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax + 1}{\sqrt{-a^2x^2 + 1}}}\, \sqrt{-a^2c\,x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c}\sqrt{\frac{ax + 1}{\sqrt{-a^2x^2 + 1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)),
 x)
```

3.1292 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt{1-a^2x^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}}+\frac{\sqrt{2}\sqrt{1-a^2x^2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{ax}}\right.}{a\sqrt{c-a^2cx^2}}$$

```
[Out] 1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))
*(-a^2*x^2+1)^(1/2)/a*2^(1/2)/(-a^2*c*x^2+c)^(1/2)-1/2*ln(1+(-a*x+1)^(1/4)*
2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2))*(-a^2*x^2+1)^(1/2)/a*2^
(1/2)/(-a^2*c*x^2+c)^(1/2)-arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*
2^(1/2)*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)-arctan(1+(-a*x+1)^(1/4)*2
^(1/2)/(a*x+1)^(1/4))*2^(1/2)*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6143, 6140, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{1-a^2x^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a\sqrt{c-a^2cx^2}}+\frac{\sqrt{2}\sqrt{1-a^2x^2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{ax}}\right.}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[2]*Sqrt[1 - a^2*x^2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(
1/4)])/(a*Sqrt[c - a^2*c*x^2]) - (Sqrt[2]*Sqrt[1 - a^2*x^2]*ArcTan[1 + (Sqr
t[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/(a*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 -
 a^2*x^2]*Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(
1 + a*x)^(1/4)])/(Sqrt[2]*a*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1
 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]
)/(Sqrt[2]*a*Sqrt[c - a^2*c*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{1}{(1-ax)^{3/4}\sqrt[4]{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= -\frac{\left(4\sqrt{1-a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a\sqrt{c-a^2cx^2}}$$
$$= -\frac{\left(4\sqrt{1-a^2x^2}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}}$$
$$= -\frac{\left(2\sqrt{1-a^2x^2}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} - \frac{\left(2\sqrt{1-a^2x^2}\right)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}}$$
$$= -\frac{\sqrt{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.}{a\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\,\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\,\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a\sqrt{c-a^2cx^2}} - \frac{(\sqrt{2}\,\sqrt{}}{}$$
$$= \frac{\sqrt{2}\sqrt{1-a^2x^2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} - \frac{\sqrt{2}\sqrt{1-a^2x^2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}\,\text{lo}}{\sqrt{}}$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.22

$$-\frac{2\,2^{3/4}\sqrt[4]{1-ax}\,\sqrt{1-a^2x^2}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-ax)\right)}{a\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*2^(3/4)*(1 - a*x)^(1/4)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/4, 1/4, 5
/4, (1 - a*x)/2])/(a*Sqrt[c - a^2*c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/sqrt(-a^2*c*x^2 + c), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algori
thm="maxima")
```

[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/sqrt(-a^2*c*x^2 + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(1/2),x)

[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)

[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)

3.1293 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3ac\sqrt{c-a^2cx^2}}$$

[Out] -2/3*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-2*a*x+1)/a/c/(-a^2*c*x^2+c)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6135}

$$-\frac{2(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3ac\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(3/2),x]

[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 2*a*x))/(3*a*c*Sqrt[c - a^2*c*x^2])

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{3ac\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.56

$$\frac{2(2ax-1)\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/4}\sqrt[4]{ax+1}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*(-1 + 2*a*x)*Sqrt[1 - a^2*x^2])/(3*a*c*(1 - a*x)^(3/4)*(1 + a*x)^(1/4)*S
qrt[c - a^2*c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.03, size = 55, normalized size = 1.34

$$-\frac{2\left(ax-1\right)\left(ax+1\right)\left(2ax-1\right)\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{3a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -2/3*(a*x-1)*(a*x+1)*(2*a*x-1)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a/(-a^2*c
*x^2+c)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 1.26, size = 49, normalized size = 1.20

$$\frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,\left(\frac{4\,x}{3\,c}-\frac{2}{3\,a\,c}\right)}{\sqrt{c-a^2\,c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] (((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)*((4*x)/(3*c) - 2/(3*a*c)))/(c - a^2*
c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt((a*x + 1)/sqrt(-a**2*x**2 + 1))/(-c*(a*x - 1)*(a*x + 1))**(3/
2), x)
```

3.1294 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{16(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35ac^2\sqrt{c-a^2cx^2}}-\frac{2(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35ac\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -2/35*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-6*a*x+1)/a/c/(-a^2*c*x^2+c)^(3/2
)-16/35*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-2*a*x+1)/a/c^2/(-a^2*c*x^2+c)^
(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6136, 6135}

$$-\frac{16(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35ac^2\sqrt{c-a^2cx^2}}-\frac{2(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{35ac\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(35*a*c*(c - a^2*c*x^2)^(3/2)) - (16*E^
(ArcTanh[a*x]/2)*(1 - 2*a*x))/(35*a*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] && !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{35ac\left(c-a^2cx^2\right)^{3/2}} + \frac{24\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{35c}$$
$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{35ac\left(c-a^2cx^2\right)^{3/2}} - \frac{16e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{35ac^2\sqrt{c-a^2cx^2}}$$

Mathematica [A]   time = 0.04, size = 80, normalized size = 0.96

$$-\frac{2\sqrt{1-a^2x^2}\left(16a^3x^3-8a^2x^2-22ax+9\right)}{35ac^2(1-ax)^{7/4}(ax+1)^{5/4}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*(9 - 22*a*x - 8*a^2*x^2 + 16*a^3*x^3))/(35*a*c^2*(1 -
 a*x)^(7/4)*(1 + a*x)^(5/4)*Sqrt[c - a^2*c*x^2])
```

fricas [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.03, size = 71, normalized size = 0.86

$$\frac{2\left(ax-1\right)\left(ax+1\right)\left(16x^{3}a^{3}-8a^{2}x^{2}-22ax+9\right)\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{35a\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 2/35*(a*x-1)*(a*x+1)*(16*a^3*x^3-8*a^2*x^2-22*a*x+9)*((a*x+1)/(-a^2*x^2+1)^
(1/2))^(1/2)/a/(-a^2*c*x^2+c)^(5/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 1.33, size = 97, normalized size = 1.17

$$-\frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,\left(\frac{18}{35\,a^3\,c^2}+\frac{32\,x^3}{35\,c^2}-\frac{44\,x}{35\,a^2\,c^2}-\frac{16\,x^2}{35\,a\,c^2}\right)}{\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)*(18/(35*a^3*c^2) + (32*x^3)/(35*c^2
) - (44*x)/(35*a^2*c^2) - (16*x^2)/(35*a*c^2)))/((c - a^2*c*x^2)^(1/2)/a^2
- x^2*(c - a^2*c*x^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
[Out] Timed out
```

3.1295 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{256(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693ac^3\sqrt{c-a^2cx^2}}-\frac{32(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693ac^2\left(c-a^2cx^2\right)^{3/2}}-\frac{2(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{99ac\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -2/99*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-10*a*x+1)/a/c/(-a^2*c*x^2+c)^(5/
2)-32/693*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-6*a*x+1)/a/c^2/(-a^2*c*x^2+c
)^(3/2)-256/693*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-2*a*x+1)/a/c^3/(-a^2*c
*x^2+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6136, 6135}

$$-\frac{256(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693ac^3\sqrt{c-a^2cx^2}}-\frac{32(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{693ac^2\left(c-a^2cx^2\right)^{3/2}}-\frac{2(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{99ac\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 10*a*x))/(99*a*c*(c - a^2*c*x^2)^(5/2)) - (32*E
^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(693*a*c^2*(c - a^2*c*x^2)^(3/2)) - (256*E^(
ArcTanh[a*x]/2)*(1 - 2*a*x))/(693*a*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99ac\left(c-a^2cx^2\right)^{5/2}}+\frac{80\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{99c}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99ac\left(c-a^2cx^2\right)^{5/2}}-\frac{32e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{693ac^2\left(c-a^2cx^2\right)^{3/2}}+\frac{128\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{231c^2}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{99ac\left(c-a^2cx^2\right)^{5/2}}-\frac{32e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{693ac^2\left(c-a^2cx^2\right)^{3/2}}-\frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-2ax)}{693ac^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.77

$$\frac{2\sqrt{1-a^2x^2}\left(256a^5x^5-128a^4x^4-608a^3x^3+272a^2x^2+422ax-151\right)}{693ac^3(1-ax)^{11/4}(ax+1)^{9/4}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*(-151 + 422*a*x + 272*a^2*x^2 - 608*a^3*x^3 - 128*a^4*
x^4 + 256*a^5*x^5))/(693*a*c^3*(1 - a*x)^(11/4)*(1 + a*x)^(9/4)*Sqrt[c - a^
2*c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algori
thm="giac")

[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(7/2), x)

maple [A] time = 0.03, size = 87, normalized size = 0.70

$$-\frac{2\left(ax-1\right)\left(ax+1\right)\left(256x^{5}a^{5}-128x^{4}a^{4}-608x^{3}a^{3}+272a^{2}x^{2}+422ax-151\right)\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{693a\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(7/2),x)

[Out] -2/693*(a*x-1)*(a*x+1)*(256*a^5*x^5-128*a^4*x^4-608*a^3*x^3+272*a^2*x^2+422
*a*x-151)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a/(-a^2*c*x^2+c)^(7/2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algori
thm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.

mupad [B] time = 1.42, size = 139, normalized size = 1.12

$$-\frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,\left(\frac{302}{693\,a^5\,c^3}-\frac{512\,x^5}{693\,c^3}-\frac{844\,x}{693\,a^4\,c^3}+\frac{256\,x^4}{693\,a\,c^3}+\frac{1216\,x^3}{693\,a^2\,c^3}-\frac{544\,x^2}{693\,a^3\,c^3}\right)}{\frac{\sqrt{c-a^2\,c\,x^2}}{a^4}+x^4\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x^2\,\sqrt{c-a^2\,c\,x^2}}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(7/2),x)

[Out] -(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)*(302/(693*a^5*c^3) - (512*x^5)/(693
*c^3) - (844*x)/(693*a^4*c^3) + (256*x^4)/(693*a*c^3) + (1216*x^3)/(693*a^2
*c^3) - (544*x^2)/(693*a^3*c^3)))/((c - a^2*c*x^2)^(1/2)/a^4 + x^4*(c - a^2
*c*x^2)^(1/2) - (2*x^2*(c - a^2*c*x^2)^(1/2))/a^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
[Out] Timed out
```

3.1296 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2048(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^4\sqrt{c-a^2cx^2}}-\frac{256(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^3\left(c-a^2cx^2\right)^{3/2}}-\frac{112(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^2\left(c-a^2cx^2\right)^{5/2}}-\frac{2(1-14ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{195ac\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] -2/195*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-14*a*x+1)/a/c/(-a^2*c*x^2+c)^(7
/2)-112/6435*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-10*a*x+1)/a/c^2/(-a^2*c*x
^2+c)^(5/2)-256/6435*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-6*a*x+1)/a/c^3/(-
a^2*c*x^2+c)^(3/2)-2048/6435*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-2*a*x+1)/
a/c^4/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6136, 6135}

$$-\frac{2048(1-2ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^4\sqrt{c-a^2cx^2}}-\frac{256(1-6ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^3\left(c-a^2cx^2\right)^{3/2}}-\frac{112(1-10ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{6435ac^2\left(c-a^2cx^2\right)^{5/2}}-\frac{2(1-14ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{195ac\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-2*E^(ArcTanh[a*x]/2)*(1 - 14*a*x))/(195*a*c*(c - a^2*c*x^2)^(7/2)) - (112
*E^(ArcTanh[a*x]/2)*(1 - 10*a*x))/(6435*a*c^2*(c - a^2*c*x^2)^(5/2)) - (256
*E^(ArcTanh[a*x]/2)*(1 - 6*a*x))/(6435*a*c^3*(c - a^2*c*x^2)^(3/2)) - (2048
*E^(ArcTanh[a*x]/2)*(1 - 2*a*x))/(6435*a*c^4*Sqrt[c - a^2*c*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
```

```
&& EqQ[a^2*c + d, 0] && LtQ[p, -1] && !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195ac\left(c-a^2cx^2\right)^{7/2}} + \frac{56\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx}{65c}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195ac\left(c-a^2cx^2\right)^{7/2}} - \frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435ac^2\left(c-a^2cx^2\right)^{5/2}} + \frac{896\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{1287c^2}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195ac\left(c-a^2cx^2\right)^{7/2}} - \frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435ac^2\left(c-a^2cx^2\right)^{5/2}} - \frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{6435ac^3\left(c-a^2cx^2\right)^{3/2}} + \frac{1024\int}{2}$$

$$= -\frac{2e^{\frac{1}{2}\tanh^{-1}(ax)}(1-14ax)}{195ac\left(c-a^2cx^2\right)^{7/2}} - \frac{112e^{\frac{1}{2}\tanh^{-1}(ax)}(1-10ax)}{6435ac^2\left(c-a^2cx^2\right)^{5/2}} - \frac{256e^{\frac{1}{2}\tanh^{-1}(ax)}(1-6ax)}{6435ac^3\left(c-a^2cx^2\right)^{3/2}} - \frac{2048e^{\frac{1}{2}}}{6435}$$

Mathematica [A] time = 0.06, size = 112, normalized size = 0.68

$$-\frac{2\sqrt{1-a^2x^2}\left(2048a^7x^7-1024a^6x^6-6912a^5x^5+3200a^4x^4+8240a^3x^3-3384a^2x^2-3838ax+1241\right)}{6435ac^4(1-ax)^{15/4}(ax+1)^{13/4}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*(1241 - 3838*a*x - 3384*a^2*x^2 + 8240*a^3*x^3 + 3200
*a^4*x^4 - 6912*a^5*x^5 - 1024*a^6*x^6 + 2048*a^7*x^7))/(6435*a*c^4*(1 - a*
x)^(15/4)*(1 + a*x)^(13/4)*Sqrt[c - a^2*c*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/2), x)
```

maple [A] time = 0.03, size = 103, normalized size = 0.62

$$\frac{2\,(ax-1)\,(ax+1)\left(2048a^7x^7-1024x^6a^6-6912x^5a^5+3200x^4a^4+8240x^3a^3-3384a^2x^2-3838ax+1241\right)\sqrt{}}{6435a\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 2/6435*(a*x-1)*(a*x+1)*(2048*a^7*x^7-1024*a^6*x^6-6912*a^5*x^5+3200*a^4*x^4
+8240*a^3*x^3-3384*a^2*x^2-3838*a*x+1241)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2
)/a/(-a^2*c*x^2+c)^(9/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 1.49, size = 183, normalized size = 1.11

$$-\frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}\,\left(\frac{2482}{6435\,a^7\,c^4}+\frac{4096\,x^7}{6435\,c^4}-\frac{7676\,x}{6435\,a^6\,c^4}-\frac{2048\,x^6}{6435\,a\,c^4}-\frac{1536\,x^5}{715\,a^2\,c^4}+\frac{1280\,x^4}{1287\,a^3\,c^4}+\frac{3296\,x^3}{1287\,a^4\,c^4}-\frac{752\,x^2}{715\,a^5\,c^4}\right)}{\frac{\sqrt{c-a^2\,c\,x^2}}{a^6}-x^6\,\sqrt{c-a^2\,c\,x^2}+\frac{3\,x^4\,\sqrt{c-a^2\,c\,x^2}}{a^2}-\frac{3\,x^2\,\sqrt{c-a^2\,c\,x^2}}{a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(9/2),x)
```

```
[Out] -(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)*(2482/(6435*a^7*c^4) + (4096*x^7)/(
6435*c^4) - (7676*x)/(6435*a^6*c^4) - (2048*x^6)/(6435*a*c^4) - (1536*x^5)/
(715*a^2*c^4) + (1280*x^4)/(1287*a^3*c^4) + (3296*x^3)/(1287*a^4*c^4) - (75
2*x^2)/(715*a^5*c^4)))/((c - a^2*c*x^2)^(1/2)/a^6 - x^6*(c - a^2*c*x^2)^(1/
2) + (3*x^4*(c - a^2*c*x^2)^(1/2))/a^2 - (3*x^2*(c - a^2*c*x^2)^(1/2))/a^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

3.1297 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/4}}\,dx$

Optimal. Leaf size=201

$$-\frac{2\sqrt[4]{1-a^2x^2}\,(1-ax)^{3/2}}{3a^4c\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt[4]{1-a^2x^2}\,\sqrt{1-ax}}{a^4c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,a^4c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] -2/3*(-a*x+1)^(3/2)*(-a^2*x^2+1)^(1/4)/a^4/c/(-a^2*c*x^2+c)^(1/4)+1/2*(-a^2
*x^2+1)^(1/4)*arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))/a^4/c/(-a^2*c*x^2+c)^(1/4
)*2^(1/2)+(-a^2*x^2+1)^(1/4)/a^4/c/(-a^2*c*x^2+c)^(1/4)/(-a*x+1)^(1/2)+2*(-
a^2*x^2+1)^(1/4)*(-a*x+1)^(1/2)/a^4/c/(-a^2*c*x^2+c)^(1/4)
```

Rubi [A] time = 0.30, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6153, 6150, 87, 43, 783, 78, 63, 207}

$$-\frac{2\sqrt[4]{1-a^2x^2}\,(1-ax)^{3/2}}{3a^4c\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt[4]{1-a^2x^2}\,\sqrt{1-ax}}{a^4c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,a^4c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x^3)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] (1 - a^2*x^2)^(1/4)/(a^4*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) + (2*Sqrt[1
 - a*x]*(1 - a^2*x^2)^(1/4))/(a^4*c*(c - a^2*c*x^2)^(1/4)) - (2*(1 - a*x)^(
3/2)*(1 - a^2*x^2)^(1/4))/(3*a^4*c*(c - a^2*c*x^2)^(1/4)) + ((1 - a^2*x^2)^
(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a^4*c*(c - a^2*c*x^2)^(1/4))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
```

tegerQ[n/2]

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/4}}\,dx = \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}\int \frac{x^3}{(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}\int \left(-\frac{x}{a^2\sqrt{1-ax}}-\frac{x}{a^2\sqrt{1-ax}\left(-1+a^2x^2\right)}\right)dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= -\frac{\sqrt[4]{1-a^2x^2}\int \frac{x}{\sqrt{1-ax}}\,dx}{a^2c\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}\int \frac{x}{\sqrt{1-ax}\left(-1+a^2x^2\right)}\,dx}{a^2c\sqrt[4]{c-a^2cx^2}}$$

$$= -\frac{\sqrt[4]{1-a^2x^2}\int \frac{x}{(-1-ax)(1-ax)^{3/2}}\,dx}{a^2c\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}\int \left(\frac{1}{a\sqrt{1-ax}}-\frac{\sqrt{1-ax}}{a}\right)dx}{a^2c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}} + \frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^4c\sqrt[4]{c-a^2cx^2}} - \frac{2(1-ax)^{3/2}\sqrt[4]{1-a^2x^2}}{3a^4c\sqrt[4]{c-a^2cx^2}} + \frac{\sqrt[4]{1-a^2x^2}\int}{2a^3c\sqrt[4]{c}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}} + \frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^4c\sqrt[4]{c-a^2cx^2}} - \frac{2(1-ax)^{3/2}\sqrt[4]{1-a^2x^2}}{3a^4c\sqrt[4]{c-a^2cx^2}} - \frac{\sqrt[4]{1-a^2x^2}\ \mathrm{Su}}{}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{a^4c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}} + \frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^4c\sqrt[4]{c-a^2cx^2}} - \frac{2(1-ax)^{3/2}\sqrt[4]{1-a^2x^2}}{3a^4c\sqrt[4]{c-a^2cx^2}} + \frac{\sqrt[4]{1-a^2x^2}\ \mathrm{ta}}{\sqrt{2}\,a^4c\sqrt[4]{}}$$

Mathematica [C] time = 0.06, size = 84, normalized size = 0.42

$$-\frac{\sqrt[4]{1-a^2x^2}\left(2\left(a^2x^2+ax-5\right)+3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1-ax)\right)\right)}{3a^4c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x^3)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] -1/3*((1 - a^2*x^2)^(1/4)*(2*(-5 + a*x + a^2*x^2) + 3*Hypergeometric2F1[-1/
2, 1, 1/2, (1 - a*x)/2]))/(a^4*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}\, x^{3}}{\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="maxima")
```

```
[Out] integrate(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{{\left(c-a^2\,c\,x^2\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(5/4),x)
```

```
[Out] int((x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(5/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**3/(-a**2*c*x**2+c)**(5/
4),x)
```

```
[Out] Timed out
```

3.1298 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/4}}\,dx$

Optimal. Leaf size=153

$$\frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,a^3c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] -1/2*(-a^2*x^2+1)^(1/4)*arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))/a^3/c/(-a^2*c*x
^2+c)^(1/4)*2^(1/2)+(-a^2*x^2+1)^(1/4)/a^3/c/(-a^2*c*x^2+c)^(1/4)/(-a*x+1)^
(1/2)+2*(-a^2*x^2+1)^(1/4)*(-a*x+1)^(1/2)/a^3/c/(-a^2*c*x^2+c)^(1/4)
```

Rubi [A] time = 0.26, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {6153, 6150, 87, 627, 51, 63, 207}

$$\frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,a^3c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x^2)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] (1 - a^2*x^2)^(1/4)/(a^3*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) + (2*Sqrt[1
 - a*x]*(1 - a^2*x^2)^(1/4))/(a^3*c*(c - a^2*c*x^2)^(1/4)) - ((1 - a^2*x^2)
^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a^3*c*(c - a^2*c*x^2)^(1/4)
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/4}}\,dx &= \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}} \\
&= \frac{\sqrt[4]{1-a^2x^2}\int \frac{x^2}{(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}} \\
&= \frac{\sqrt[4]{1-a^2x^2}\int \left(-\frac{1}{a^2\sqrt{1-ax}}-\frac{1}{a^2\sqrt{1-ax}\left(-1+a^2x^2\right)}\right)dx}{c\sqrt[4]{c-a^2cx^2}} \\
&= \frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{\sqrt{1-ax}\left(-1+a^2x^2\right)}\,dx}{a^2c\sqrt[4]{c-a^2cx^2}} \\
&= \frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{(-1-ax)(1-ax)^{3/2}}\,dx}{a^2c\sqrt[4]{c-a^2cx^2}} \\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{(-1-ax)\sqrt{1-ax}}\,dx}{2a^2c\sqrt[4]{c-a^2cx^2}} \\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx,x,\sqrt{1-ax}\right)}{a^3c\sqrt[4]{c-a^2cx^2}} \\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^3c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}+\frac{2\sqrt{1-ax}\,\sqrt[4]{1-a^2x^2}}{a^3c\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,a^3c\sqrt[4]{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 70, normalized size = 0.46

$$
\frac{\sqrt[4]{1-a^2x^2}\left({}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1-ax)\right)-2ax+2\right)}{a^3c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x^2)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] ((1 - a^2*x^2)^(1/4)*(2 - 2*a*x + Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)
/2]))/(a^3*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="giac")
```

```
[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\, x^2}{\left(-a^2c\, x^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="maxima")
```

```
[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{{\left(c-a^2\,c\,x^2\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(5/4),x)
```

```
[Out] int((x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(5/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**2/(-a**2*c*x**2+c)**(5/
4),x)
```

```
[Out] Timed out
```

3.1299 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{5/4}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,a^2c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] 1/2*(-a^2*x^2+1)^(1/4)*arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))/a^2/c/(-a^2*c*x^
2+c)^(1/4)*2^(1/2)+(-a^2*x^2+1)^(1/4)/a^2/c/(-a^2*c*x^2+c)^(1/4)/(-a*x+1)^(
1/2)
```

Rubi [A] time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 78, 63, 206}

$$\frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,a^2c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] (1 - a^2*x^2)^(1/4)/(a^2*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) + ((1 - a^2
*x^2)^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a^2*c*(c - a^2*c*x^2)^
(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/4}}\,dx &= \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}\int \frac{x}{(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx}{2ac\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1-ax}\right)}{a^2c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{a^2c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,a^2c\sqrt[4]{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.70

$$\frac{\sqrt[4]{1-a^2x^2}\left(\frac{1}{a^2\sqrt{1-ax}}+\frac{\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,a^2}\right)}{c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] ((1 - a^2*x^2)^(1/4)*(1/(a^2*Sqrt[1 - a*x]) + ArcTanh[Sqrt[1 - a*x]/Sqrt[2]
]/(Sqrt[2]*a^2)))/(c*(c - a^2*c*x^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="giac")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,x}{\left(-a^2c\,x^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="maxima")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{{\left(c-a^2\,c\,x^2\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(5/4),x)
```

```
[Out] int((x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(5/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x/(-a**2*c*x**2+c)**(5/4),
x)
```

```
[Out] Timed out
```

3.1300 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/4}}\,dx$

Optimal. Leaf size=106

$$\frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,ac\sqrt[4]{c-a^2cx^2}}$$

```
[Out] -1/2*(-a^2*x^2+1)^(1/4)*arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))/a/c/(-a^2*c*x^2
+c)^(1/4)*2^(1/2)+(-a^2*x^2+1)^(1/4)/a/c/(-a^2*c*x^2+c)^(1/4)/(-a*x+1)^(1/2
)
```

Rubi [A] time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6143, 6140, 51, 63, 206}

$$\frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,ac\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] (1 - a^2*x^2)^(1/4)/(a*c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - ((1 - a^2*x
^2)^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*a*c*(c - a^2*c*x^2)^(1/4
))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/4}}\,dx &= \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{\sqrt{1-ax}\,(1+ax)}\,dx}{2c\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1-ax}\right)}{ac\sqrt[4]{c-a^2cx^2}}\\
&= \frac{\sqrt[4]{1-a^2x^2}}{ac\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,ac\sqrt[4]{c-a^2cx^2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.60

$$\frac{\sqrt[4]{1-a^2x^2}\ {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1-ax)\right)}{ac\sqrt{1-ax}\ \sqrt[4]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(5/4),x]
```

```
[Out] ((1 - a^2*x^2)^(1/4)*Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2])/(a*c*Sqr
t[1 - a*x]*(c - a^2*c*x^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2c\,x^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(5/4),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{{\left(c-a^2\,c\,x^2\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(5/4),x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(5/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(5/4),x)
```

```
[Out] Timed out
```

3.1301 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{2\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] -2*(-a^2*x^2+1)^(1/4)*arctanh((-a*x+1)^(1/2))/c/(-a^2*c*x^2+c)^(1/4)+1/2*(-
a^2*x^2+1)^(1/4)*arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))/c/(-a^2*c*x^2+c)^(1/4)
*2^(1/2)+(-a^2*x^2+1)^(1/4)/c/(-a^2*c*x^2+c)^(1/4)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {6153, 6150, 85, 156, 63, 208, 206}

$$\frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{2\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(5/4)),x]
```

```
[Out] (1 - a^2*x^2)^(1/4)/(c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - (2*(1 - a^2*x
^2)^(1/4)*ArcTanh[Sqrt[1 - a*x]])/(c*(c - a^2*c*x^2)^(1/4)) + ((1 - a^2*x^2
)^(1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*c*(c - a^2*c*x^2)^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/4}}\,dx = \frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{x(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} + \frac{\sqrt[4]{1-a^2x^2}\int \frac{2a+a^2x}{x\sqrt{1-ax}(1+ax)}\,dx}{2ac\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} + \frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{x\sqrt{1-ax}}\,dx}{c\sqrt[4]{c-a^2cx^2}} - \frac{\left(a\sqrt[4]{1-a^2x^2}\right)\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx}{2c\sqrt[4]{c-a^2cx^2}}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} + \frac{\sqrt[4]{1-a^2x^2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}} - \frac{\left(2\sqrt[4]{1-a^2x^2}\right)\text{Sub}}{ac}$$

$$= \frac{\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}} - \frac{2\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}} + \frac{\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,c\sqrt[4]{c-a^2cx^2}}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.55

$$-\frac{\sqrt[4]{1-a^2x^2}\left({}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1-ax)\right)-2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-ax\right)\right)}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(5/4)),x]
```

```
[Out] -((((1 - a^2*x^2)^(1/4)*(Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2] - 2*Hy
pergeometric2F1[-1/2, 1, 1/2, 1 - a*x]))/(c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(
1/4)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(5/4)*x), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x\left(-a^2c\,x^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(5/4),x, algo
rithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(5/4)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{x\,{\left(c-a^2\,c\,x^2\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(x*(c - a^2*c*x^2)^(5/4)),x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(x*(c - a^2*c*x^2)^(5/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x/(-a**2*c*x**2+c)**(5/4),
x)
```

```
[Out] Timed out
```

3.1302 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=196

$$\frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,c\sqrt[4]{c-a^2cx^2}}$$

```
[Out] -a*(-a^2*x^2+1)^(1/4)*arctanh((-a*x+1)^(1/2))/c/(-a^2*c*x^2+c)^(1/4)-1/2*a*
(-a^2*x^2+1)^(1/4)*arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))/c/(-a^2*c*x^2+c)^(1/
4)*2^(1/2)+2*a*(-a^2*x^2+1)^(1/4)/c/(-a^2*c*x^2+c)^(1/4)/(-a*x+1)^(1/2)-(-a
^2*x^2+1)^(1/4)/c/x/(-a^2*c*x^2+c)^(1/4)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6153, 6150, 103, 152, 156, 63, 208, 206}

$$\frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\,\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1-a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{\sqrt{2}\,c\sqrt[4]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(x^2*(c - a^2*c*x^2)^(5/4)),x]
```

```
[Out] (2*a*(1 - a^2*x^2)^(1/4))/(c*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - (1 - a^
2*x^2)^(1/4)/(c*x*Sqrt[1 - a*x]*(c - a^2*c*x^2)^(1/4)) - (a*(1 - a^2*x^2)^(
1/4)*ArcTanh[Sqrt[1 - a*x]])/(c*(c - a^2*c*x^2)^(1/4)) - (a*(1 - a^2*x^2)^(
1/4)*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/(Sqrt[2]*c*(c - a^2*c*x^2)^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
```

```
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/4}}\,dx=\frac{\sqrt[4]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{5/4}}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$=\frac{\sqrt[4]{1-a^2x^2}\int \frac{1}{x^2(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$=-\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\int \frac{-\frac{a}{2}-\frac{3a^2x}{2}}{x(1-ax)^{3/2}(1+ax)}\,dx}{c\sqrt[4]{c-a^2cx^2}}$$

$$=\frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\sqrt[4]{1-a^2x^2}\int \frac{\frac{a^2}{2}+a^3x}{x\sqrt{1-ax}(1+ax)}\,dx}{ac\sqrt[4]{c-a^2cx^2}}$$

$$=\frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}+\frac{\left(a\sqrt[4]{1-a^2x^2}\right)\int \frac{1}{x\sqrt{1-ax}}\,dx}{2c\sqrt[4]{c-a^2cx^2}}+\frac{\left(a^2\sqrt[4]{1-\right.}{}$$

$$=\frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\right.}{c\sqrt[4]{c-a^2cx^2}}$$

$$=\frac{2a\sqrt[4]{1-a^2x^2}}{c\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{\sqrt[4]{1-a^2x^2}}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1-a^2x^2}\tanh^{-1}\left(\sqrt{1-ax}\right)}{c\sqrt[4]{c-a^2cx^2}}-\frac{a\sqrt[4]{1}}{}$$

Mathematica [C] time = 0.04, size = 86, normalized size = 0.44

$$\frac{\sqrt[4]{1-a^2x^2}\left(ax\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1-ax)\right)+ax\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-ax\right)-1\right)}{cx\sqrt{1-ax}\sqrt[4]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(x^2*(c - a^2*c*x^2)^(5/4)),x]
```

```
[Out] ((1 - a^2*x^2)^(1/4)*(-1 + a*x*Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]
 + a*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 - a*x]))/(c*x*Sqrt[1 - a*x]*(c - a
^2*c*x^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(5/4)*x^2),
x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x^2\left(-a^2c\,x^2+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/4),x, al
gorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(5/4)*x^2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{x^2\,{\left(c-a^2\,c\,x^2\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(x^2*(c - a^2*c*x^2)^(5/4)),x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(x^2*(c - a^2*c*x^2)^(5/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x**2/(-a**2*c*x**2+c)**(5/
4),x)
```

```
[Out] Timed out
```

3.1303 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{9/8}}\,dx$

Optimal. Leaf size=200

$$-\frac{4x^2\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}+\frac{64\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)}{105a^4c\sqrt[8]{c-a^2cx^2}}+\frac{8(6-ax)\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{21a^4c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

```
[Out] -4/7*x^2*(a*x+1)^(1/8)*(-a^2*x^2+1)^(1/8)/a^2/c/(-a*x+1)^(3/8)/(-a^2*c*x^2+
c)^(1/8)+8/21*(-a*x+6)*(a*x+1)^(1/8)*(-a^2*x^2+1)^(1/8)/a^4/c/(-a*x+1)^(3/8
)/(-a^2*c*x^2+c)^(1/8)+64/105*2^(1/8)*(-a*x+1)^(5/8)*(-a^2*x^2+1)^(1/8)*hyp
ergeom([5/8, 7/8],[13/8],-1/2*a*x+1/2)/a^4/c/(-a^2*c*x^2+c)^(1/8)
```

Rubi [A] time = 0.25, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {6153, 6150, 100, 146, 69}

$$\frac{64\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)}{105a^4c\sqrt[8]{c-a^2cx^2}}-\frac{4x^2\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}+\frac{8(6-ax)\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{21a^4c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x^3)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (-4*x^2*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(7*a^2*c*(1 - a*x)^(3/8)*(c -
a^2*c*x^2)^(1/8)) + (8*(6 - a*x)*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(21*a
^4*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8)) + (64*2^(1/8)*(1 - a*x)^(5/8)*(
1 - a^2*x^2)^(1/8)*Hypergeometric2F1[5/8, 7/8, 13/8, (1 - a*x)/2])/(105*a^4
*c*(c - a^2*c*x^2)^(1/8))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{9/8}}\,dx = \frac{\sqrt[8]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{9/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}$$

$$= \frac{\sqrt[8]{1-a^2x^2}\int \frac{x^3}{(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}$$

$$= -\frac{4x^2\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}} - \frac{\left(4\sqrt[8]{1-a^2x^2}\right)\int \frac{x\left(-2-\frac{ax}{2}\right)}{(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{7a^2c\sqrt[8]{c-a^2cx^2}}$$

$$= -\frac{4x^2\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}} + \frac{8(6-ax)\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{21a^4c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}} - \frac{\left(16\sqrt[8]{1-a^2x^2}\right)\int \frac{1}{(1-ax)^{3/8}(}}{21a^3c\sqrt[8]{c-a^2cx^2}}$$

$$= -\frac{4x^2\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{7a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}} + \frac{8(6-ax)\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{21a^4c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}} + \frac{64\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x}}{105a^4c\sqrt[8]{c}}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.54

$$-\frac{4\sqrt[8]{1-a^2x^2}\left(5\sqrt[8]{ax+1}\left(3a^2x^2+2ax-12\right)+16\sqrt[8]{2}(ax-1)\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)\right)}{105a^4c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x^3)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (-4*(1 - a^2*x^2)^(1/8)*(5*(1 + a*x)^(1/8)*(-12 + 2*a*x + 3*a^2*x^2) + 16*2
^(1/8)*(-1 + a*x)*Hypergeometric2F1[5/8, 7/8, 13/8, (1 - a*x)/2]))/(105*a^4
*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
-1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}\, x^{3}}{\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{\left(-a^{2}cx^{2}+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^3/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="maxima")
```

```
[Out] integrate(x^3*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3}\sqrt{\frac{a\,x+1}{\sqrt{1-a^{2}\,x^{2}}}}}{\left(c-a^{2}\,c\,x^{2}\right)^{9/8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(9/8),x)
```

```
[Out] int((x^3*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(9/8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**3/(-a**2*c*x**2+c)**(9/
8),x)
```

```
[Out] Timed out
```

3.1304 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{9/8}}\,dx$

Optimal. Leaf size=41

$$\frac{4(2-ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3a^3c\sqrt[8]{c-a^2cx^2}}$$

```
[Out] 4/3*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*(-a*x+2)/a^3/c/(-a^2*c*x^2+c)^(1/8)
```

Rubi [A] time = 0.12, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {6146}

$$\frac{4(2-ax)e^{\frac{1}{2}\tanh^{-1}(ax)}}{3a^3c\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x^2)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (4*E^(ArcTanh[a*x]/2)*(2 - a*x))/(3*a^3*c*(c - a^2*c*x^2)^(1/8))
```

Rule 6146

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[((1 - a*n*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*d*n*(n^
2 - 1)), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && EqQ[n^2 + 2*(
p + 1), 0] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{9/8}}\,dx=\frac{4e^{\frac{1}{2}\tanh^{-1}(ax)}(2-ax)}{3a^3c\sqrt[8]{c-a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.54

$$-\frac{4(ax-2)\sqrt[8]{ax+1}\,\sqrt[8]{1-a^2x^2}}{3a^3c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x^2)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (-4*(-2 + a*x)*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(3*a^3*c*(1 - a*x)^(3/8
)*(c - a^2*c*x^2)^(1/8))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="giac")
```

```
[Out] integrate(x^2*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

maple [A] time = 0.03, size = 54, normalized size = 1.32

$$\frac{4\left(ax-1\right)\left(ax+1\right)\left(ax-2\right)\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{3a^3\left(-a^2c\,x^2+c\right)^{\frac{9}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] 4/3*(a*x-1)*(a*x+1)*(a*x-2)*((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/a^3/(-a^2*c*
x^2+c)^(9/8)
```

maxima [A] time = 1.06, size = 28, normalized size = 0.68

$$-\frac{4\left(ax+1\right)^{\frac{1}{8}}\left(ax-2\right)}{3\left(-ax+1\right)^{\frac{3}{8}}a^3c^{\frac{9}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x^2/(-a^2*c*x^2+c)^(9/8),x, al
gorithm="maxima")
```

```
[Out] -4/3*(a*x + 1)^(1/8)*(a*x - 2)/((-a*x + 1)^(3/8)*a^3*c^(9/8))
```

mupad [B] time = 1.26, size = 52, normalized size = 1.27

$$\frac{\left(\frac{8}{3\,a^3\,c}-\frac{4\,x}{3\,a^2\,c}\right)\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{{\left(c-a^2\,c\,x^2\right)}^{1/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(9/8),x)
```

```
[Out] ((8/(3*a^3*c) - (4*x)/(3*a^2*c))*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c
- a^2*c*x^2)^(1/8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x**2/(-a**2*c*x**2+c)**(9/
8),x)
```

```
[Out] Timed out
```

3.1305 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{9/8}}\,dx$

Optimal. Leaf size=133

$$\frac{8\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)}{15a^2c\sqrt[8]{c-a^2cx^2}}+\frac{4\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{3a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

```
[Out] 4/3*(a*x+1)^(1/8)*(-a^2*x^2+1)^(1/8)/a^2/c/(-a*x+1)^(3/8)/(-a^2*c*x^2+c)^(1
/8)+8/15*2^(1/8)*(-a*x+1)^(5/8)*(-a^2*x^2+1)^(1/8)*hypergeom([5/8, 7/8],[13
/8],-1/2*a*x+1/2)/a^2/c/(-a^2*c*x^2+c)^(1/8)
```

Rubi [A] time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6153, 6150, 78, 69}

$$\frac{8\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)}{15a^2c\sqrt[8]{c-a^2cx^2}}+\frac{4\sqrt[8]{ax+1}\sqrt[8]{1-a^2x^2}}{3a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(ArcTanh[a*x]/2)*x)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (4*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8))/(3*a^2*c*(1 - a*x)^(3/8)*(c - a^2*c
*x^2)^(1/8)) + (8*2^(1/8)*(1 - a*x)^(5/8)*(1 - a^2*x^2)^(1/8)*Hypergeometri
c2F1[5/8, 7/8, 13/8, (1 - a*x)/2])/(15*a^2*c*(c - a^2*c*x^2)^(1/8))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{9/8}}\,dx &= \frac{\sqrt[8]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}x}{\left(1-a^2x^2\right)^{9/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}\\
&= \frac{\sqrt[8]{1-a^2x^2}\int \frac{x}{(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}\\
&= \frac{4\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{3a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}-\frac{\left(2\sqrt[8]{1-a^2x^2}\right)\int \frac{1}{(1-ax)^{3/8}(1+ax)^{7/8}}\,dx}{3ac\sqrt[8]{c-a^2cx^2}}\\
&= \frac{4\sqrt[8]{1+ax}\sqrt[8]{1-a^2x^2}}{3a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}+\frac{8\sqrt[8]{2}(1-ax)^{5/8}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)}{15a^2c\sqrt[8]{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 0.70

$$-\frac{4\sqrt[8]{1-a^2x^2}\left(2\sqrt[8]{2}(ax-1)\,{}_2F_1\left(\frac{5}{8},\frac{7}{8};\frac{13}{8};\frac{1}{2}(1-ax)\right)-5\sqrt[8]{ax+1}\right)}{15a^2c(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(ArcTanh[a*x]/2)*x)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (-4*(1 - a^2*x^2)^(1/8)*(-5*(1 + a*x)^(1/8) + 2*2^(1/8)*(-1 + a*x)*Hypergeo
metric2F1[5/8, 7/8, 13/8, (1 - a*x)/2]))/(15*a^2*c*(1 - a*x)^(3/8)*(c - a^2
*c*x^2)^(1/8))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="giac")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}\,x}{\left(-a^2c\,x^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)*x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="maxima")
```

```
[Out] integrate(x*sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{{\left(c-a^2\,c\,x^2\right)}^{9/8}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(9/8),x)
```

```
[Out] int((x*((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2))/(c - a^2*c*x^2)^(9/8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)*x/(-a**2*c*x**2+c)**(9/8),
x)
```

```
[Out] Timed out
```

3.1306 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/8}}\,dx$

Optimal. Leaf size=74

$$\frac{4\sqrt[8]{2}\,\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(-\frac{3}{8},\frac{7}{8};\frac{5}{8};\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

```
[Out] 4/3*2^(1/8)*(-a^2*x^2+1)^(1/8)*hypergeom([-3/8, 7/8],[5/8],-1/2*a*x+1/2)/a/
c/(-a*x+1)^(3/8)/(-a^2*c*x^2+c)^(1/8)
```

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6143, 6140, 69}

$$\frac{4\sqrt[8]{2}\,\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(-\frac{3}{8},\frac{7}{8};\frac{5}{8};\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (4*2^(1/8)*(1 - a^2*x^2)^(1/8)*Hypergeometric2F1[-3/8, 7/8, 5/8, (1 - a*x)/
2])/(3*a*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
```

```
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/8}}\,dx=\frac{\sqrt[8]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^{9/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}$$
$$=\frac{\sqrt[8]{1-a^2x^2}\int \frac{1}{(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}$$
$$=\frac{4\sqrt[8]{2}\sqrt[8]{1-a^2x^2}\,{}_2F_1\left(-\frac{3}{8},\frac{7}{8};\frac{5}{8};\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 0.93

$$\frac{4\sqrt[8]{2-2a^2x^2}\,{}_2F_1\left(-\frac{3}{8},\frac{7}{8};\frac{5}{8};\frac{1}{2}(1-ax)\right)}{3ac(1-ax)^{3/8}\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(c - a^2*c*x^2)^(9/8),x]
```

```
[Out] (4*(2 - 2*a^2*x^2)^(1/8)*Hypergeometric2F1[-3/8, 7/8, 5/8, (1 - a*x)/2])/(3
*a*c*(1 - a*x)^(3/8)*(c - a^2*c*x^2)^(1/8))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x, algori
thm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^{2}x^{2}+1}}}}{\left(-a^{2}cx^{2}+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/(-a^2*c*x^2+c)^(9/8),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/(-a^2*c*x^2 + c)^(9/8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{{\left(c-a^2\,c\,x^2\right)}^{9/8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(9/8),x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(c - a^2*c*x^2)^(9/8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/(-a**2*c*x**2+c)**(9/8),x)
[Out] Timed out
```

3.1307 $\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x(c-a^2cx^2)^{9/8}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\,2^{5/8}\sqrt[8]{ax+1}\,\sqrt[8]{1-a^2x^2}\,F_1\left(\frac{1}{8};\frac{11}{8},1;\frac{9}{8};\frac{1}{2}(ax+1),ax+1\right)}{c\sqrt[8]{c-a^2cx^2}}$$

```
[Out] -2*2^(5/8)*(a*x+1)^(1/8)*(-a^2*x^2+1)^(1/8)*AppellF1(1/8,11/8,1,9/8,1/2*a*x
+1/2,a*x+1)/c/(-a^2*c*x^2+c)^(1/8)
```

Rubi [A] time = 0.22, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6153, 6150, 136}

$$-\frac{2\,2^{5/8}\sqrt[8]{ax+1}\,\sqrt[8]{1-a^2x^2}\,F_1\left(\frac{1}{8};\frac{11}{8},1;\frac{9}{8};\frac{1}{2}(ax+1),ax+1\right)}{c\sqrt[8]{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(9/8)),x]
```

```
[Out] (-2*2^(5/8)*(1 + a*x)^(1/8)*(1 - a^2*x^2)^(1/8)*AppellF1[1/8, 11/8, 1, 9/8,
 (1 + a*x)/2, 1 + a*x])/(c*(c - a^2*c*x^2)^(1/8))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{9/8}}\,dx &= \frac{\sqrt[8]{1-a^2x^2}\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{9/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}\\ &= \frac{\sqrt[8]{1-a^2x^2}\int \frac{1}{x(1-ax)^{11/8}(1+ax)^{7/8}}\,dx}{c\sqrt[8]{c-a^2cx^2}}\\ &= -\frac{2\,2^{5/8}\sqrt[8]{1+ax}\,\sqrt[8]{1-a^2x^2}\,F_1\left(\frac{1}{8};\frac{11}{8},1;\frac{9}{8};\frac{1}{2}(1+ax),1+ax\right)}{c\sqrt[8]{c-a^2cx^2}}\end{aligned}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{e^{\frac{1}{2}\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{9/8}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(9/8)),x]
[Out] Integrate[E^(ArcTanh[a*x]/2)/(x*(c - a^2*c*x^2)^(9/8)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="giac")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(9/8)*x), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{x\left(-a^2c\,x^2+c\right)^{\frac{9}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x)
```

```
[Out] int(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax+1}{\sqrt{-a^2x^2+1}}}}{\left(-a^2cx^2+c\right)^{\frac{9}{8}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a^2*x^2+1)^(1/2))^(1/2)/x/(-a^2*c*x^2+c)^(9/8),x, algo
rithm="maxima")
```

```
[Out] integrate(sqrt((a*x + 1)/sqrt(-a^2*x^2 + 1))/((-a^2*c*x^2 + c)^(9/8)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x+1}{\sqrt{1-a^2\,x^2}}}}{x\,{\left(c-a^2\,c\,x^2\right)}^{9/8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(x*(c - a^2*c*x^2)^(9/8)),x)
```

```
[Out] int(((a*x + 1)/(1 - a^2*x^2)^(1/2))^(1/2)/(x*(c - a^2*c*x^2)^(9/8)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x+1)/(-a**2*x**2+1)**(1/2))**(1/2)/x/(-a**2*c*x**2+c)**(9/8),
x)
```

```
[Out] Timed out
```

3.1308 $\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=68

$$-\frac{c2^{\frac{n}{2}+2}(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-1,2-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

```
[Out] -2^(2+1/2*n)*c*(-a*x+1)^(2-1/2*n)*hypergeom([-1-1/2*n, 2-1/2*n],[3-1/2*n],-
1/2*a*x+1/2)/a/(4-n)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6140, 69}

$$-\frac{c2^{\frac{n}{2}+2}(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-1,2-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -((2^(2 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[-1 - n/2, 2 - n/2, 3
 - n/2, (1 - a*x)/2])/(a*(4 - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = c\int (1-ax)^{1-\frac{n}{2}}(1+ax)^{1+\frac{n}{2}}\,dx$$
$$= -\frac{2^{2+\frac{n}{2}}c(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(-1-\frac{n}{2},2-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(4-n)}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.96

$$\frac{c2^{\frac{n}{2}+2}(1-ax)^{2-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-1,2-\frac{n}{2};3-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(n-4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (2^(2 + n/2)*c*(1 - a*x)^(2 - n/2)*Hypergeometric2F1[-1 - n/2, 2 - n/2, 3 -
 n/2, (1 - a*x)/2])/(a*(-4 + n))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}\left(-a^2c\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x)
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(a^2cx^2-c\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -integrate((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{atanh}(a\,x)}\,\left(c-a^2\,c\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a^2*c*x^2),x)
[Out] int(exp(n*atanh(a*x))*(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int a^2x^2e^{n\,\mathrm{atanh}\,(ax)}\,dx+\int\left(-e^{n\,\mathrm{atanh}\,(ax)}\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c),x)
[Out] -c*(Integral(a**2*x**2*exp(n*atanh(a*x)), x) + Integral(-exp(n*atanh(a*x)),
 x))
```

3.1309 $\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=70

$$-\frac{c^2 2^{\frac{n}{2}+3}(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-2,3-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

```
[Out] -2^(3+1/2*n)*c^2*(-a*x+1)^(3-1/2*n)*hypergeom([3-1/2*n, -2-1/2*n],[4-1/2*n]
,-1/2*a*x+1/2)/a/(6-n)
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 69}

$$-\frac{c^2 2^{\frac{n}{2}+3}(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-2,3-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] -((2^(3 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[-2 - n/2, 3 - n/2,
 4 - n/2, (1 - a*x)/2])/(a*(6 - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx = c^2\int (1-ax)^{2-\frac{n}{2}}(1+ax)^{2+\frac{n}{2}}\,dx$$
$$= -\frac{2^{3+\frac{n}{2}}c^2(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(-2-\frac{n}{2},3-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(6-n)}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 0.96

$$\frac{c^2 2^{\frac{n}{2}+3}(1-ax)^{3-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-2,3-\frac{n}{2};4-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(n-6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (2^(3 + n/2)*c^2*(1 - a*x)^(3 - n/2)*Hypergeometric2F1[-2 - n/2, 3 - n/2, 4
 - n/2, (1 - a*x)/2])/(a*(-6 + n))
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2-c\right)^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} \left(-a^{2} c \,x^{2}+c \right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x)
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} c x^{2} - c\right)^{2} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2} \, n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{e}}^{n\,\mathrm{atanh}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^2,x)
[Out] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{2} \left(\int \left(- 2 a^{2} x^{2} e^{n \operatorname{atanh}{\left(a x \right)}}\right)\, dx + \int a^{4} x^{4} e^{n \operatorname{atanh}{\left(a x \right)}}\, dx + \int e^{n \operatorname{atanh}{\left(a x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**2,x)
[Out] c**2*(Integral(-2*a**2*x**2*exp(n*atanh(a*x)), x) + Integral(a**4*x**4*exp(
n*atanh(a*x)), x) + Integral(exp(n*atanh(a*x)), x))
```

3.1310 $\int e^{n \tanh^{-1}(ax)} (c - a^2 c x^2)^3 \, dx$

Optimal. Leaf size=70

$$-\frac{c^3 2^{\frac{n}{2}+4} (1-ax)^{4-\frac{n}{2}} {}_2F_1\left(-\frac{n}{2}-3, 4-\frac{n}{2}; 5-\frac{n}{2}; \frac{1}{2}(1-ax)\right)}{a(8-n)}$$

```
[Out] -2^(4+1/2*n)*c^3*(-a*x+1)^(4-1/2*n)*hypergeom([4-1/2*n, -3-1/2*n],[5-1/2*n]
,-1/2*a*x+1/2)/a/(8-n)
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6140, 69}

$$-\frac{c^3 2^{\frac{n}{2}+4} (1-ax)^{4-\frac{n}{2}} {}_2F_1\left(-\frac{n}{2}-3, 4-\frac{n}{2}; 5-\frac{n}{2}; \frac{1}{2}(1-ax)\right)}{a(8-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -((2^(4 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[-3 - n/2, 4 - n/2,
 5 - n/2, (1 - a*x)/2])/(a*(8 - n)))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=c^3\int(1-ax)^{3-\frac{n}{2}}(1+ax)^{3+\frac{n}{2}}\,dx$$
$$=-\frac{2^{4+\frac{n}{2}}c^3(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(-3-\frac{n}{2},4-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(8-n)}$$

Mathematica [**A**] time = 0.02, size = 67, normalized size = 0.96

$$\frac{c^3 2^{\frac{n}{2}+4}(1-ax)^{4-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-3,4-\frac{n}{2};5-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a(n-8)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (2^(4 + n/2)*c^3*(1 - a*x)^(4 - n/2)*Hypergeometric2F1[-3 - n/2, 4 - n/2, 5
 - n/2, (1 - a*x)/2])/(a*(-8 + n))
```

fricas [**F**] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*((a*x + 1)/(a
*x - 1))^(1/2*n), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int-\left(a^2cx^2-c\right)^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n \,\mathrm{arctanh}(a x)} \left(-a^{2} c \,x^{2}+c \right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(a^{2} c x^{2} - c\right)^{3} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2} \, n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{e}}^{n\,\mathrm{atanh}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^3,x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- c^{3} \left(\int 3 a^{2} x^{2} e^{n \operatorname{atanh}{\left(a x \right)}}\, dx + \int \left(- 3 a^{4} x^{4} e^{n \operatorname{atanh}{\left(a x \right)}}\right)\, dx + \int a^{6} x^{6} e^{n \operatorname{atanh}{\left(a x \right)}}\, dx + \int \left(- e^{n \operatorname{atanh}{\left(a x \right)}}\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2*exp(n*atanh(a*x)), x) + Integral(-3*a**4*x**4*e
xp(n*atanh(a*x)), x) + Integral(a**6*x**6*exp(n*atanh(a*x)), x) + Integral(
-exp(n*atanh(a*x)), x))
```

3.1311 $\int \frac{e^{n \tanh^{-1}(ax)} x^4}{c-a^2 c x^2} \, dx$

Optimal. Leaf size=209

$$\frac{2^{\frac{n}{2}-1} n \left(n^2+8\right) (1-ax)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{3a^5 c(2-n)}+\frac{(ax+1)^{n/2} \left(-a\left(n^2+6\right) nx+n^3+n^2+8n+6\right) (1}{6a^5 cn}$$

```
[Out] -1/6*n*x^2*(a*x+1)^(1/2*n)/a^3/c/((-a*x+1)^(1/2*n))-1/3*x^3*(a*x+1)^(1/2*n)
/a^2/c/((-a*x+1)^(1/2*n))+1/6*(a*x+1)^(1/2*n)*(6+8*n+n^2+n^3-a*n*(n^2+6)*x)
/a^5/c/n/((-a*x+1)^(1/2*n))+1/3*2^(-1+1/2*n)*n*(n^2+8)*(-a*x+1)^(1-1/2*n)*h
ypergeom([1-1/2*n, 1-1/2*n],[2-1/2*n],-1/2*a*x+1/2)/a^5/c/(2-n)
```

Rubi [A] time = 0.26, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6150, 100, 153, 143, 69}

$$\frac{2^{\frac{n}{2}-1} n \left(n^2+8\right) (1-ax)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{3a^5 c(2-n)}+\frac{(ax+1)^{n/2} \left(-a\left(n^2+6\right) nx+n^3+n^2+8n+6\right) (1}{6a^5 cn}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] -(n*x^2*(1 + a*x)^(n/2))/(6*a^3*c*(1 - a*x)^(n/2)) - (x^3*(1 + a*x)^(n/2))/
(3*a^2*c*(1 - a*x)^(n/2)) + ((1 + a*x)^(n/2)*(6 + 8*n + n^2 + n^3 - a*n*(6
+ n^2)*x))/(6*a^5*c*n*(1 - a*x)^(n/2)) + (2^(-1 + n/2)*n*(8 + n^2)*(1 - a*x
)^(1 - n/2)*Hypergeometric2F1[1 - n/2, 1 - n/2, 2 - n/2, (1 - a*x)/2])/(3*a
^5*c*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^4}{c-a^2cx^2}\,dx = \frac{\int x^4(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}$$
$$= -\frac{x^3(1-ax)^{-n/2}(1+ax)^{n/2}}{3a^2c} - \frac{\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-3-anx)\,dx}{3a^2c}$$
$$= -\frac{nx^2(1-ax)^{-n/2}(1+ax)^{n/2}}{6a^3c} - \frac{x^3(1-ax)^{-n/2}(1+ax)^{n/2}}{3a^2c} + \frac{\int x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\left(2a\right.}{6a^4c}$$
$$= -\frac{nx^2(1-ax)^{-n/2}(1+ax)^{n/2}}{6a^3c} - \frac{x^3(1-ax)^{-n/2}(1+ax)^{n/2}}{3a^2c} + \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(6+8n+\right.}{6a^5cn}$$
$$= -\frac{nx^2(1-ax)^{-n/2}(1+ax)^{n/2}}{6a^3c} - \frac{x^3(1-ax)^{-n/2}(1+ax)^{n/2}}{3a^2c} + \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(6+8n+\right.}{6a^5cn}$$

Mathematica [A] time = 0.23, size = 172, normalized size = 0.82

$$\frac{(1-ax)^{-n/2}\left(2^{\frac{n}{2}+1}n\left(n^2-2n+2\right)(ax-1)\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)-(n-2)\left((ax+1)^{n/2}\left(2n\left(a^3x^3+2ax\right.\right.\right.\right.}{6a^5c(n-2)n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2),x]
```

```
[Out] (2^(1 + n/2)*n*(2 - 2*n + n^2)*(-1 + a*x)*Hypergeometric2F1[1 - n/2, -1/2*n
, 2 - n/2, (1 - a*x)/2] - (-2 + n)*((1 + a*x)^(n/2)*(-6 + (n + a*n*x)^2 + 2
*n*(3 + 2*a*x + a^3*x^3)) - 2^(2 + n/2)*n*(3 + n)*Hypergeometric2F1[-1/2*n,
 -1/2*n, 1 - n/2, (1 - a*x)/2]))/(6*a^5*c*(-2 + n)*n*(1 - a*x)^(n/2))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^{4} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^{2}cx^{2}-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)} x^{4}}{-a^{2}c\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x)
[Out] int(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{4} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^{2}cx^{2}-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{4}\,e^{n \operatorname{atanh}(a\,x)}}{c-a^{2}\,c\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*exp(n*atanh(a*x)))/(c - a^2*c*x^2),x)
[Out] int((x^4*exp(n*atanh(a*x)))/(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^4 e^{n \operatorname{atanh}(ax)}}{a^2x^2-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**4/(-a**2*c*x**2+c),x)
[Out] -Integral(x**4*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

3.1312 $\int \frac{e^{n \tanh^{-1}(ax)} x^3}{c-a^2cx^2} \, dx$

Optimal. Leaf size=160

$$\frac{2^{\frac{n}{2}-1}\left(n^2+2\right)(1-ax)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^4c(2-n)}+\frac{(ax+1)^{n/2}\left(-an^2x+n^2+n+2\right)(1-ax)^{-n/2}}{2a^4cn}-\frac{x^2(}{}$$

```
[Out] -1/2*x^2*(a*x+1)^(1/2*n)/a^2/c/((-a*x+1)^(1/2*n))+1/2*(a*x+1)^(1/2*n)*(-a*n
^2*x+n^2+n+2)/a^4/c/n/((-a*x+1)^(1/2*n))+2^(-1+1/2*n)*(n^2+2)*(-a*x+1)^(1-1
/2*n)*hypergeom([1-1/2*n, 1-1/2*n],[2-1/2*n],-1/2*a*x+1/2)/a^4/c/(2-n)
```

Rubi [A] time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6150, 100, 143, 69}

$$\frac{2^{\frac{n}{2}-1}\left(n^2+2\right)(1-ax)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^4c(2-n)}+\frac{(ax+1)^{n/2}\left(-an^2x+n^2+n+2\right)(1-ax)^{-n/2}}{2a^4cn}-\frac{x^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] -(x^2*(1 + a*x)^(n/2))/(2*a^2*c*(1 - a*x)^(n/2)) + ((1 + a*x)^(n/2)*(2 + n
+ n^2 - a*n^2*x))/(2*a^4*c*n*(1 - a*x)^(n/2)) + (2^(-1 + n/2)*(2 + n^2)*(1
- a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, 1 - n/2, 2 - n/2, (1 - a*x)/2])
/(a^4*c*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^3}{c-a^2cx^2}\,dx = \frac{\int x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}$$

$$= -\frac{x^2(1-ax)^{-n/2}(1+ax)^{n/2}}{2a^2c} - \frac{\int x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-2-anx)\,dx}{2a^2c}$$

$$= -\frac{x^2(1-ax)^{-n/2}(1+ax)^{n/2}}{2a^2c} + \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(2+n+n^2-an^2x\right)}{2a^4cn} - \frac{\left(2+n^2\right)\int(1-a}{}$$

$$= -\frac{x^2(1-ax)^{-n/2}(1+ax)^{n/2}}{2a^2c} + \frac{(1-ax)^{-n/2}(1+ax)^{n/2}\left(2+n+n^2-an^2x\right)}{2a^4cn} + \frac{2^{-1+\frac{n}{2}}\left(2+n^2\right)(}{}$$

Mathematica [A] time = 0.07, size = 120, normalized size = 0.75

$$\frac{(1-ax)^{-n/2}\left(2^{n/2}n\left(n^2+2\right)(ax-1)\,{}_2F_1\left(1-\frac{n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)-(n-2)(ax+1)^{n/2}\left(n\left(a^2x^2-1\right)+n^2(ax\right.\right.}{2a^4c(n-2)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2),x]
```

```
[Out] (-((-2 + n)*(1 + a*x)^(n/2)*(-2 + n^2*(-1 + a*x) + n*(-1 + a^2*x^2))) + 2^(
n/2)*n*(2 + n^2)*(-1 + a*x)*Hypergeometric2F1[1 - n/2, 1 - n/2, 2 - n/2, (1
 - a*x)/2])/(2*a^4*c*(-2 + n)*n*(1 - a*x)^(n/2))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}x^3}{-a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*atanh(a*x)))/(c - a^2*c*x^2),x)
```

```
[Out] int((x^3*exp(n*atanh(a*x)))/(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^3 e^{n\,\mathrm{atanh}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(x**3*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

3.1313 $\int \frac{e^{n \tanh^{-1}(ax)} x^2}{c-a^2 c x^2} \, dx$

Optimal. Leaf size=86

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^3c(2-n)}+\frac{e^{n\tanh^{-1}(ax)}}{a^3cn}$$

```
[Out] exp(n*arctanh(a*x))/a^3/c/n+2^(1+1/2*n)*(-a*x+1)^(1-1/2*n)*hypergeom([-1/2*
n, 1-1/2*n],[2-1/2*n],-1/2*a*x+1/2)/a^3/c/(2-n)
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.48, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6150, 90, 79, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2} {}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^3c}-\frac{x(ax+1)^{n/2}(1-ax)^{-n/2}}{a^2c}+\frac{(1-n)(ax+1)^{n/2}(1-ax)^{-n/2}}{a^3cn}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] ((1 - n)*(1 + a*x)^(n/2))/(a^3*c*n*(1 - a*x)^(n/2)) - (x*(1 + a*x)^(n/2))/(
a^2*c*(1 - a*x)^(n/2)) + (2^(1 + n/2)*Hypergeometric2F1[-n/2, -n/2, 1 - n/2
, (1 - a*x)/2])/(a^3*c*(1 - a*x)^(n/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}x^2}{c-a^2cx^2}\,dx &= \frac{\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}\\
&= -\frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2c} - \frac{\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-1-anx)\,dx}{a^2c}\\
&= \frac{(1-n)(1-ax)^{-n/2}(1+ax)^{n/2}}{a^3cn} - \frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2c} + \frac{n\int (1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{a^2c}\\
&= \frac{(1-n)(1-ax)^{-n/2}(1+ax)^{n/2}}{a^3cn} - \frac{x(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2c} + \frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\right.}{a^3c}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 82, normalized size = 0.95

$$\frac{(1-ax)^{-n/2}\left(2^{\frac{n}{2}+1}n\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)-(ax+1)^{n/2}(anx+n-1)\right)}{a^3cn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2),x]
```

```
[Out] (-((1 + a*x)^(n/2)*(-1 + n + a*n*x)) + 2^(1 + n/2)*n*Hypergeometric2F1[-1/2
*n, -1/2*n, 1 - n/2, (1 - a*x)/2])/(a^3*c*n*(1 - a*x)^(n/2))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(-x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}x^2}{-a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x)
[Out] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*atanh(a*x)))/(c - a^2*c*x^2),x)
```

```
[Out] int((x^2*exp(n*atanh(a*x)))/(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2 e^{n\,\mathrm{atanh}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(x**2*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

3.1314 $\int \frac{e^{n\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx$

Optimal. Leaf size=94

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2cn}-\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{a^2cn}$$

```
[Out] -(a*x+1)^(1/2*n)/a^2/c/n/((-a*x+1)^(1/2*n))+2^(1+1/2*n)*hypergeom([-1/2*n,
-1/2*n],[1-1/2*n],-1/2*a*x+1/2)/a^2/c/n/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6150, 79, 69}

$$\frac{2^{\frac{n}{2}+1}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2cn}-\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{a^2cn}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] -((1 + a*x)^(n/2)/(a^2*c*n*(1 - a*x)^(n/2))) + (2^(1 + n/2)*Hypergeometric2
F1[-n/2, -n/2, 1 - n/2, (1 - a*x)/2])/(a^2*c*n*(1 - a*x)^(n/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}x}{c-a^2cx^2}\,dx &= \frac{\int x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}\\ &= -\frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2cn}+\frac{\int(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}\,dx}{ac}\\ &= -\frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{a^2cn}+\frac{2^{1+\frac{n}{2}}(1-ax)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^2cn}\end{aligned}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.79

$$\frac{(1-ax)^{-n/2}\left(2^{\frac{n}{2}+1}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{1}{2}(1-ax)\right)-(ax+1)^{n/2}\right)}{a^2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2),x]
```

```
[Out] (-(1 + a*x)^(n/2) + 2^(1 + n/2)*Hypergeometric2F1[-1/2*n, -1/2*n, 1 - n/2,
(1 - a*x)/2])/(a^2*c*n*(1 - a*x)^(n/2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}x}{-a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x)
[Out] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,e^{n\,\mathrm{atanh}(a\,x)}}{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*atanh(a*x)))/(c - a^2*c*x^2),x)
[Out] int((x*exp(n*atanh(a*x)))/(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{xe^{n\,\mathrm{atanh}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c),x)
[Out] -Integral(x*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c
```

3.1315 $\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{n\tanh^{-1}(ax)}}{acn}$$

[Out] exp(n*arctanh(a*x))/a/c/n

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6137}

$$\frac{e^{n\tanh^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2),x]

[Out] E^(n*ArcTanh[a*x])/(a*c*n)

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{n\tanh^{-1}(ax)}}{acn}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.83

$$\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{acn}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2),x]

[Out] (1 + a*x)^(n/2)/(a*c*n*(1 - a*x)^(n/2))

fricas [A] time = 0.82, size = 27, normalized size = 1.50

$$\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] ((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

maple [A] time = 0.03, size = 18, normalized size = 1.00

$$\frac{e^{n \operatorname{arctanh}(ax)}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c),x)
[Out] exp(n*arctanh(a*x))/a/c/n
```

maxima [A] time = 0.33, size = 30, normalized size = 1.67

$$\frac{e^{\left(\frac{1}{2}n\log(ax+1)-\frac{1}{2}n\log(ax-1)\right)}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] e^(1/2*n*log(a*x + 1) - 1/2*n*log(a*x - 1))/(a*c*n)
```

mupad [B] time = 1.11, size = 31, normalized size = 1.72

$$\frac{(a\,x+1)^{n/2}}{a\,c\,n\,(1-a\,x)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a^2*c*x^2),x)
```

```
[Out] (a*x + 1)^(n/2)/(a*c*n*(1 - a*x)^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty}x & \text{for}\ c=0 \wedge n=0 \\ \tilde{\infty}\int e^{n\,\text{atanh}\,(ax)}\,dx & \text{for}\ c=0 \\ -\frac{\log\left(x-\frac{1}{a}\right)}{2ac}+\frac{\log\left(x+\frac{1}{a}\right)}{2ac} & \text{for}\ n=0 \\ \frac{e^{n\,\text{atanh}\,(ax)}}{acn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c),x)
```

```
[Out] Piecewise((zoo*x, Eq(c, 0) & Eq(n, 0)), (zoo*Integral(exp(n*atanh(a*x)), x)
, Eq(c, 0)), (-log(x - 1/a)/(2*a*c) + log(x + 1/a)/(2*a*c), Eq(n, 0)), (exp
(n*atanh(a*x))/(a*c*n), True))
```

3.1316 $\int \frac{e^{n\tanh^{-1}(ax)}}{x(c-a^2cx^2)}\,dx$

Optimal. Leaf size=90

$$\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{cn}-\frac{2(1-ax)^{-n/2}(ax+1)^{n/2}\,{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{ax+1}{1-ax}\right)}{cn}$$

```
[Out] (a*x+1)^(1/2*n)/c/n/((-a*x+1)^(1/2*n))-2*(a*x+1)^(1/2*n)*hypergeom([1, 1/2*
n],[1+1/2*n],(a*x+1)/(-a*x+1))/c/n/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6150, 96, 131}

$$\frac{(1-ax)^{-n/2}(ax+1)^{n/2}}{cn}-\frac{2(1-ax)^{1-\frac{n}{2}}(ax+1)^{\frac{n-2}{2}}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{c(2-n)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)),x]
```

```
[Out] (1 + a*x)^(n/2)/(c*n*(1 - a*x)^(n/2)) - (2*(1 - a*x)^(1 - n/2)*(1 + a*x)^((
-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)])/(c*
(2 - n))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)}\,dx = \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}}{x}\,dx}{c}$$

$$= \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} + \frac{\int \frac{(1-ax)^{-n/2}(1+ax)^{-1+\frac{n}{2}}}{x}\,dx}{c}$$

$$= \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} - \frac{2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{1+ax}\right)}{c(2-n)}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 0.94

$$\frac{(1-ax)^{-n/2}(ax+1)^{\frac{n}{2}-1}\left((n-2)(ax+1)-2n(ax-1)\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)\right)}{c(n-2)n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)),x]
```

```
[Out] ((1 + a*x)^(-1 + n/2)*((-2 + n)*(1 + a*x) - 2*n*(-1 + a*x)*Hypergeometric2F
1[1, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)]))/(c*(-2 + n)*n*(1 - a*x)^(n/2)
)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^3-cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^3 - c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)*x), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x\left(-a^2c\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x)
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x\,\left(c-a^2\,c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)),x)
```

```
[Out] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{a^2x^3-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c),x)
[Out] -Integral(exp(n*atanh(a*x))/(a**2*x**3 - x), x)/c
```

3.1317 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)}\,dx$

Optimal. Leaf size=123

$$-\frac{2a(ax+1)^{n/2}(1-ax)^{-n/2}\,{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{ax+1}{1-ax}\right)}{c}+\frac{a(n+1)(ax+1)^{n/2}(1-ax)^{-n/2}}{cn}-\frac{(ax+1)^{n/2}(1-ax)^{-n/2}}{cx}$$

```
[Out] a*(1+n)*(a*x+1)^(1/2*n)/c/n/((-a*x+1)^(1/2*n))-(a*x+1)^(1/2*n)/c/x/((-a*x+1
)^(1/2*n))-2*a*(a*x+1)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n],(a*x+1)/(-a*x
+1))/c/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6150, 129, 155, 12, 131}

$$-\frac{2an(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{c(2-n)}+\frac{a(n+1)(ax+1)^{n/2}(1-ax)^{-n/2}}{cn}-\frac{(ax+1)^{n/2}(1-ax)^{-n/2}}{cx}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] (a*(1 + n)*(1 + a*x)^(n/2))/(c*n*(1 - a*x)^(n/2)) - (1 + a*x)^(n/2)/(c*x*(1
 - a*x)^(n/2)) - (2*a*n*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeo
metric2F1[1, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)])/(c*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)}\,dx = \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}}{x^2}\,dx}{c}$$

$$= -\frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cx} - \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}(-an-a^2x)}{x}\,dx}{c}$$

$$= \frac{a(1+n)(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} - \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cx} + \frac{\int \frac{a^2n^2(1-ax)^{-n/2}(1+ax)^{-1+\frac{n}{2}}}{x}\,dx}{acn}$$

$$= \frac{a(1+n)(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} - \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cx} + \frac{(an)\int \frac{(1-ax)^{-n/2}(1+ax)^{-1+\frac{n}{2}}}{x}\,dx}{c}$$

$$= \frac{a(1+n)(1-ax)^{-n/2}(1+ax)^{n/2}}{cn} - \frac{(1-ax)^{-n/2}(1+ax)^{n/2}}{cx} - \frac{2an(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\ 2}{c(2-n}$$

Mathematica [A] time = 0.06, size = 103, normalized size = 0.84

$$\frac{(1-ax)^{-n/2}(ax+1)^{\frac{n}{2}-1}\left((n-2)(ax+1)(n(ax-1)+ax)-2an^2x(ax-1)\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)\right)}{c(n-2)nx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)),x]
```

```
[Out] ((1 + a*x)^(-1 + n/2)*((-2 + n)*(1 + a*x)*(a*x + n*(-1 + a*x)) - 2*a*n^2*x*
(-1 + a*x)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)]))/(c
*(-2 + n)*n*x*(1 - a*x)^(n/2))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^4-cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^4 - c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)*x^2), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\operatorname{arctanh}(ax)}}{x^2\left(-a^2c\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x)
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\operatorname{atanh}(a\,x)}}{x^2\,\left(c-a^2\,c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)),x)
[Out] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{e^{n \operatorname{atanh}(ax)}}{a^2x^4-x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(exp(n*atanh(a*x))/(a**2*x**4 - x**2), x)/c
```

3.1318 $\int \frac{e^{n\tanh^{-1}(ax)}x^4}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=376

$$-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(\frac{2-n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^5c^2(2-n)}-\frac{(n+3)\left(2-n^2\right)(ax+1)^{n/2}(1-ax)^{-\frac{n}{2}-1}}{a^5c^2\left(4-n^2\right)}-\frac{(n+3)\left(2-n^2\right)(}{a^5c^2n}$$

```
[Out] (1-n)*(3+n)*(-a*x+1)^(-1-1/2*n)*(a*x+1)^(-1+1/2*n)/a^5/c^2/(2-n)+(3+n)*x*(-
a*x+1)^(-1-1/2*n)*(a*x+1)^(-1+1/2*n)/a^4/c^2-x^3*(-a*x+1)^(-1-1/2*n)*(a*x+1
)^(-1+1/2*n)/a^2/c^2+(-a*x+1)^(1-1/2*n)*(a*x+1)^(-1+1/2*n)/a^5/c^2/(2-n)-(a
*x+1)^(-1+1/2*n)/a^5/c^2/((-a*x+1)^(1/2*n))-(3+n)*(-n^2+2)*(-a*x+1)^(-1-1/2
*n)*(a*x+1)^(1/2*n)/a^5/c^2/(-n^2+4)-(3+n)*(-n^2+2)*(a*x+1)^(1/2*n)/a^5/c^2
/n/(-n^2+4)/((-a*x+1)^(1/2*n))-2^(1/2*n)*n*(-a*x+1)^(1-1/2*n)*hypergeom([1-
1/2*n, 1-1/2*n],[2-1/2*n],-1/2*a*x+1/2)/a^5/c^2/(2-n)
```

Rubi [A] time = 0.39, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6150, 100, 159, 89, 79, 69, 90, 45, 37}

$$-\frac{2^{n/2}n(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(\frac{2-n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^5c^2(2-n)}-\frac{(n+3)\left(2-n^2\right)(ax+1)^{n/2}(1-ax)^{-\frac{n}{2}-1}}{a^5c^2\left(4-n^2\right)}-\frac{(n+3)\left(2-n^2\right)(}{a^5c^2n}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] ((1 - n)*(3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^5*c^2*(2 -
 n)) + ((3 + n)*x*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^4*c^2) -
(x^3*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^2*c^2) + ((1 - a*x)^(1
 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^5*c^2*(2 - n)) - (1 + a*x)^((-2 + n)/2)/
(a^5*c^2*(1 - a*x)^(n/2)) - ((3 + n)*(2 - n^2)*(1 - a*x)^(-1 - n/2)*(1 + a*
x)^(n/2))/(a^5*c^2*(4 - n^2)) - ((3 + n)*(2 - n^2)*(1 + a*x)^(n/2))/(a^5*c^
2*n*(4 - n^2)*(1 - a*x)^(n/2)) - (2^(n/2)*n*(1 - a*x)^(1 - n/2)*Hypergeomet
ric2F1[(2 - n)/2, 1 - n/2, 2 - n/2, (1 - a*x)/2])/(a^5*c^2*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int x^4(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2}$$

$$= -\frac{x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^2c^2} - \frac{\int x^2(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}(-3-anx)\,dx}{a^2c^2}$$

$$= -\frac{x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^2c^2} - \frac{n\int x^2(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{a^2c^2} + \frac{(3+n)\int x^2(1-ax}{a}$$

$$= \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2} - \frac{x^3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^2c^2} - \frac{(1-ax)^{-n/2}(1+ax}{a^5c^2}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^5c^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2} - \frac{x^3(1-}{}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^5c^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2} - \frac{x^3(1-}{}$$

$$= \frac{(1-n)(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^5c^2(2-n)} + \frac{(3+n)x(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2} - \frac{x^3(1-}{}$$

Mathematica [A] time = 0.17, size = 178, normalized size = 0.47

$$-\frac{(1-ax)^{-\frac{n}{2}-1}\left((ax+1)^{n/2}\left(n^2\left(1-2a^2x^2\right)+6a^2x^2+n\left(-4a^3x^3+4a^2x^2+6ax-4\right)+n^3(ax-1)^2(ax+1)-6\right)-2\right.}{a^5c^2(n-2)n(n+2)(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^4)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*((1 + a*x)^(n/2)*(-6 + 6*a^2*x^2 + n^3*(-1 + a*x)^2
*(1 + a*x) + n^2*(1 - 2*a^2*x^2) + n*(-4 + 6*a*x + 4*a^2*x^2 - 4*a^3*x^3))
- 2^(n/2)*n^2*(2 + n)*(-1 + a*x)^2*(1 + a*x)*Hypergeometric2F1[1 - n/2, 1 -
 n/2, 2 - n/2, (1 - a*x)/2]))/(a^5*c^2*(-2 + n)*n*(2 + n)*(1 + a*x)))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2a^2c^2x^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)} x^4}{\left(-a^2c\,x^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^4/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,e^{n\,\mathrm{atanh}(a\,x)}}{\left(c - a^2\,c\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^2,x)
```

```
[Out] int((x^4*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^4 e^{n \operatorname{atanh}(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**4/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**4*exp(n*atanh(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

3.1319 $\int \frac{e^{n\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=311

$$\frac{2^{\frac{n}{2}+2}(1-ax)^{-\frac{n}{2}-1}\,{}_2F_1\left(-\frac{n}{2}-1,-\frac{n}{2}-1;-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^4c^2(n+2)}+\frac{2(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{a^4c^2n\left(4-n^2\right)}-\frac{(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}-1}}{a^4c^2(n+2)}+\frac{3(ax}{}$$

```
[Out] -(-a*x+1)^(-1-1/2*n)*(a*x+1)^(-1+1/2*n)/a^4/c^2/(2+n)+2*(-a*x+1)^(1-1/2*n)*
(a*x+1)^(-1+1/2*n)/a^4/c^2/n/(-n^2+4)-2*(a*x+1)^(-1+1/2*n)/a^4/c^2/n/(2+n)/
((-a*x+1)^(1/2*n))+3*(-a*x+1)^(-1-1/2*n)*(a*x+1)^(1/2*n)/a^4/c^2/(2+n)+3*(a
*x+1)^(1/2*n)/a^4/c^2/n/(2+n)/((-a*x+1)^(1/2*n))-3*(-a*x+1)^(-1-1/2*n)*(a*x
+1)^(1+1/2*n)/a^4/c^2/(2+n)+2^(2+1/2*n)*(-a*x+1)^(-1-1/2*n)*hypergeom([-1-1
/2*n, -1-1/2*n],[-1/2*n],-1/2*a*x+1/2)/a^4/c^2/(2+n)
```

Rubi [A] time = 0.25, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6150, 128, 45, 37, 69}

$$\frac{2^{\frac{n}{2}+2}(1-ax)^{-\frac{n}{2}-1}\,{}_2F_1\left(-\frac{n}{2}-1,-\frac{n}{2}-1;-\frac{n}{2};\frac{1}{2}(1-ax)\right)}{a^4c^2(n+2)}+\frac{2(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{a^4c^2n\left(4-n^2\right)}-\frac{(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}-1}}{a^4c^2(n+2)}+\frac{3(ax}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^4*c^2*(2 + n))) + (2*(1
- a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2))/(a^4*c^2*n*(4 - n^2)) - (2*(1 + a*
x)^((-2 + n)/2))/(a^4*c^2*n*(2 + n)*(1 - a*x)^(n/2)) + (3*(1 - a*x)^(-1 - n
/2)*(1 + a*x)^(n/2))/(a^4*c^2*(2 + n)) + (3*(1 + a*x)^(n/2))/(a^4*c^2*n*(2
+ n)*(1 - a*x)^(n/2)) - (3*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((2 + n)/2))/(a^4
*c^2*(2 + n)) + (2^(2 + n/2)*(1 - a*x)^(-1 - n/2)*Hypergeometric2F1[-1 - n/
2, -1 - n/2, -n/2, (1 - a*x)/2])/(a^4*c^2*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 128

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[
m, 0] && ILtQ[n, 0]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int x^3(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2}$$

$$= \frac{\int\left(-\frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}}{a^3}+\frac{3(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}}{a^3}+\frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{1+\frac{n}{2}}}{a^3}-\frac{3(1-ax)^{-2-\frac{n}{2}}(1+ax)^{n/2}}{a^3}\right) d}{c^2}$$

$$= -\frac{\int(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{a^3c^2}+\frac{\int(1-ax)^{-2-\frac{n}{2}}(1+ax)^{1+\frac{n}{2}}\,dx}{a^3c^2}+\frac{3\int(1-ax)^{-2-\frac{n}{2}}(1+ax}{a^3c^2}$$

$$= -\frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2(2+n)}+\frac{3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}}{a^4c^2(2+n)}-\frac{3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{a^4c^2(2+n)}+$$

$$= -\frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2(2+n)}-\frac{2(1-ax)^{-n/2}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2n(2+n)}+\frac{3(1-ax)^{-1-\frac{n}{2}}(1+ax)^{n/2}}{a^4c^2(2+n)}+$$

$$= -\frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2(2+n)}+\frac{2(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{a^4c^2n\left(4-n^2\right)}-\frac{2(1-ax)^{-n/2}(1+ax)^{\frac{1}{2}(-2+n}}{a^4c^2n(2+n)}$$

Mathematica [A] time = 5.40, size = 145, normalized size = 0.47

$$-\frac{e^{n\tanh^{-1}(ax)}\left(-\left(n^2-4\right)\left(a^2x^2-1\right){}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;-e^{2\tanh^{-1}(ax)}\right)+(n-2)n\left(a^2x^2-1\right)e^{2\tanh^{-1}(ax)}{}_2F_1\left(1,\frac{n}{2}+1;\right.\right.}{a^4c^2(n-2)n(n+2)\left(a^2x^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(n*(-1 + a*n*x - a^2*x^2) + E^(2*ArcTanh[a*x])*(-2 +
n)*n*(-1 + a^2*x^2)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcTanh[a*
x])] - (-4 + n^2)*(-1 + a^2*x^2)*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*A
rcTanh[a*x])]))/(a^4*c^2*(-2 + n)*n*(2 + n)*(-1 + a^2*x^2)))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)} x^3}{\left(-a^2c\,x^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,e^{n\,\mathrm{atanh}(a\,x)}}{\left(c - a^2\,c\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^2,x)
```

```
[Out] int((x^3*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{3}e^{n\,\mathrm{atanh}\,(ax)}}{a^{4}x^{4}-2a^{2}x^{2}+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(x**3*exp(n*atanh(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

3.1320 $\int \frac{e^{n\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{\left(2-n^2\right)e^{n\tanh^{-1}(ax)}}{a^3c^2n\left(4-n^2\right)}-\frac{(n-2ax)e^{n\tanh^{-1}(ax)}}{a^3c^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

```
[Out] -exp(n*arctanh(a*x))*(-n^2+2)/a^3/c^2/n/(-n^2+4)-exp(n*arctanh(a*x))*(-2*a*
x+n)/a^3/c^2/(-n^2+4)/(-a^2*x^2+1)
```

Rubi [A] time = 0.13, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6147, 6137}

$$-\frac{(n-2ax)e^{n\tanh^{-1}(ax)}}{a^3c^2\left(4-n^2\right)\left(1-a^2x^2\right)}-\frac{\left(2-n^2\right)e^{n\tanh^{-1}(ax)}}{a^3c^2n\left(4-n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(2 - n^2))/(a^3*c^2*n*(4 - n^2))) - (E^(n*ArcTanh[a*x
])*(n - 2*a*x))/(a^3*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6147

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> -Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a
*d*(n^2 - 4*(p + 1)^2)), x] + Dist[(n^2 + 2*(p + 1))/(d*(n^2 - 4*(p + 1)^2)
), Int[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}
, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] && !IntegerQ[n] && NeQ[n^2 - 4*(p
+ 1)^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^2}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-2ax)}{a^3c^2\left(4-n^2\right)\left(1-a^2x^2\right)} - \frac{\left(2-n^2\right)\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx}{a^2c\left(4-n^2\right)}$$
$$= -\frac{e^{n\tanh^{-1}(ax)}\left(2-n^2\right)}{a^3c^2n\left(4-n^2\right)} - \frac{e^{n\tanh^{-1}(ax)}(n-2ax)}{a^3c^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.06, size = 65, normalized size = 0.82

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(-a^2\left(n^2-2\right)x^2+2anx-2\right)}{a^3c^2n\left(n^2-4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(-2 + 2*a*n*x - a^2*(-2 + n^2)
*x^2))/(a^3*c^2*n*(-4 + n^2)))
```

fricas [A] time = 0.47, size = 91, normalized size = 1.15

$$-\frac{\left(2\,anx-\left(a^2n^2-2\,a^2\right)x^2-2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^3c^2n^3-4\,a^3c^2n-\left(a^5c^2n^3-4\,a^5c^2n\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*n*x - (a^2*n^2 - 2*a^2)*x^2 - 2)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c
^2*n^3 - 4*a^3*c^2*n - (a^5*c^2*n^3 - 4*a^5*c^2*n)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

maple [A] time = 0.03, size = 62, normalized size = 0.78

$$-\frac{e^{n \operatorname{arctanh}(ax)} \left(a^2 n^2 x^2 - 2a^2 x^2 - 2nax + 2\right)}{\left(a^2 x^2 - 1\right) c^2 a^3 n \left(n^2 - 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -exp(n*arctanh(a*x))*(a^2*n^2*x^2-2*a^2*x^2-2*a*n*x+2)/(a^2*x^2-1)/c^2/a^3/
n/(n^2-4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2 c x^2 - c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

mupad [B] time = 1.17, size = 93, normalized size = 1.18

$$\frac{(a\,x+1)^{n/2} \left(\frac{2}{a^5\,c^2\,n\,\left(n^2-4\right)} - \frac{2\,x}{a^4\,c^2\,\left(n^2-4\right)} + \frac{x^2\,\left(n^2-2\right)}{a^3\,c^2\,n\,\left(n^2-4\right)}\right)}{\left(\frac{1}{a^2} - x^2\right)\,(1 - a\,x)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^2,x)
```

```
[Out] ((a*x + 1)^(n/2)*(2/(a^5*c^2*n*(n^2 - 4)) - (2*x)/(a^4*c^2*(n^2 - 4)) + (x^
2*(n^2 - 2))/(a^3*c^2*n*(n^2 - 4))))/((1/a^2 - x^2)*(1 - a*x)^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty}x^3e^{-\infty n} \\ \tilde{\infty}x^3e^{\infty n} \\ \tilde{\infty}\int x^2e^{n\,\mathrm{atanh}\,(ax)}\,dx \\ -\frac{a^2x^2\,\mathrm{atanh}\,(ax)}{4a^5c^2x^2e^{2\,\mathrm{atanh}\,(ax)}-4a^3c^2e^{2\,\mathrm{atanh}\,(ax)}}-\frac{2ax\,\mathrm{atanh}\,(ax)}{4a^5c^2x^2e^{2\,\mathrm{atanh}\,(ax)}-4a^3c^2e^{2\,\mathrm{atanh}\,(ax)}}-\frac{3ax}{4a^5c^2x^2e^{2\,\mathrm{atanh}\,(ax)}-4a^3c^2e^{2\,\mathrm{atanh}\,(ax)}}-\frac{\mathrm{atanh}\,(a}{4a^5c^2x^2e^{2\,\mathrm{atanh}\,(ax)}-4} \\ \frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{4a^5c^2x^2-4a^3c^2}-\frac{a^2x^2\log\left(x+\frac{1}{a}\right)}{4a^5c^2x^2-4a^3c^2}-\frac{2ax}{4a^5c^2x^2-4a^3c^2}-\frac{\log\left(x-\frac{1}{a}\right)}{4a^5c^2x^2-4a^3c^2}+\frac{\log\left(x+\frac{1}{a}\right)}{4a^5c^2x^2-4a^3c^2} \\ \frac{\int\frac{x^2e^{2\,\mathrm{atanh}\,(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2} \\ -\frac{a^2n^2x^2e^{n\,\mathrm{atanh}\,(ax)}}{a^5c^2n^3x^2-4a^5c^2nx^2-a^3c^2n^3+4a^3c^2n}+\frac{2a^2x^2e^{n\,\mathrm{atanh}\,(ax)}}{a^5c^2n^3x^2-4a^5c^2nx^2-a^3c^2n^3+4a^3c^2n}+\frac{2anxe^{n\,\mathrm{atanh}\,(ax)}}{a^5c^2n^3x^2-4a^5c^2nx^2-a^3c^2n^3+4a^3c^2n}-\frac{2e^{n\,\mathrm{atanh}\,(}}{a^5c^2n^3x^2-4a^5c^2nx^2-} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((zoo*x**3*exp(-oo*n), Eq(a, -1/x)), (zoo*x**3*exp(oo*n), Eq(a, 1/
x)), (zoo*Integral(x**2*exp(n*atanh(a*x)), x), Eq(c, 0)), (-a**2*x**2*atanh
(a*x)/(4*a**5*c**2*x**2*exp(2*atanh(a*x)) - 4*a**3*c**2*exp(2*atanh(a*x)))
- 2*a*x*atanh(a*x)/(4*a**5*c**2*x**2*exp(2*atanh(a*x)) - 4*a**3*c**2*exp(2*
atanh(a*x))) - 3*a*x/(4*a**5*c**2*x**2*exp(2*atanh(a*x)) - 4*a**3*c**2*exp(
2*atanh(a*x))) - atanh(a*x)/(4*a**5*c**2*x**2*exp(2*atanh(a*x)) - 4*a**3*c*
*2*exp(2*atanh(a*x))) - 2/(4*a**5*c**2*x**2*exp(2*atanh(a*x)) - 4*a**3*c**2
*exp(2*atanh(a*x))), Eq(n, -2)), (a**2*x**2*log(x - 1/a)/(4*a**5*c**2*x**2
- 4*a**3*c**2) - a**2*x**2*log(x + 1/a)/(4*a**5*c**2*x**2 - 4*a**3*c**2) -
2*a*x/(4*a**5*c**2*x**2 - 4*a**3*c**2) - log(x - 1/a)/(4*a**5*c**2*x**2 - 4
*a**3*c**2) + log(x + 1/a)/(4*a**5*c**2*x**2 - 4*a**3*c**2), Eq(n, 0)), (In
tegral(x**2*exp(2*atanh(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2, Eq(n,
 2)), (-a**2*n**2*x**2*exp(n*atanh(a*x))/(a**5*c**2*n**3*x**2 - 4*a**5*c**2
*n*x**2 - a**3*c**2*n**3 + 4*a**3*c**2*n) + 2*a**2*x**2*exp(n*atanh(a*x))/(
a**5*c**2*n**3*x**2 - 4*a**5*c**2*n*x**2 - a**3*c**2*n**3 + 4*a**3*c**2*n)
+ 2*a*n*x*exp(n*atanh(a*x))/(a**5*c**2*n**3*x**2 - 4*a**5*c**2*n*x**2 - a**
3*c**2*n**3 + 4*a**3*c**2*n) - 2*exp(n*atanh(a*x))/(a**5*c**2*n**3*x**2 - 4
*a**5*c**2*n*x**2 - a**3*c**2*n**3 + 4*a**3*c**2*n), True))
```

3.1321 $\int \frac{e^{n\tanh^{-1}(ax)}x}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=69

$$\frac{(2-anx)e^{n\tanh^{-1}(ax)}}{a^2c^2\left(4-n^2\right)\left(1-a^2x^2\right)}-\frac{e^{n\tanh^{-1}(ax)}}{a^2c^2\left(4-n^2\right)}$$

```
[Out] -exp(n*arctanh(a*x))/a^2/c^2/(-n^2+4)+exp(n*arctanh(a*x))*(-a*n*x+2)/a^2/c^
2/(-n^2+4)/(-a^2*x^2+1)
```

Rubi [A] time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6145, 6136, 6137}

$$\frac{n(n-2ax)e^{n\tanh^{-1}(ax)}}{2a^2c^2\left(4-n^2\right)\left(1-a^2x^2\right)}-\frac{e^{n\tanh^{-1}(ax)}}{a^2c^2\left(4-n^2\right)}+\frac{e^{n\tanh^{-1}(ax)}}{2a^2c^2\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(E^(n*ArcTanh[a*x])/(a^2*c^2*(4 - n^2))) + E^(n*ArcTanh[a*x])/(2*a^2*c^2*(
1 - a^2*x^2)) + (E^(n*ArcTanh[a*x])*n*(n - 2*a*x))/(2*a^2*c^2*(4 - n^2)*(1
- a^2*x^2))
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6145

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[((c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(2*d*(p + 1)), x] - Dist[
(a*c*n)/(2*d*(p + 1)), Int[(c + d*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && I
ntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^2}\,dx = \frac{e^{n\tanh^{-1}(ax)}}{2a^2c^2\left(1-a^2x^2\right)} - \frac{n\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{2a}$$

$$= \frac{e^{n\tanh^{-1}(ax)}}{2a^2c^2\left(1-a^2x^2\right)} + \frac{e^{n\tanh^{-1}(ax)}n(n-2ax)}{2a^2c^2\left(4-n^2\right)\left(1-a^2x^2\right)} - \frac{n\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx}{ac\left(4-n^2\right)}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}}{a^2c^2\left(4-n^2\right)} + \frac{e^{n\tanh^{-1}(ax)}}{2a^2c^2\left(1-a^2x^2\right)} + \frac{e^{n\tanh^{-1}(ax)}n(n-2ax)}{2a^2c^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.81

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(a^2x^2-anx+1\right)}{a^2c^2\left(n^2-4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(1 - a*n*x + a^2*x^2))/(a^2*c^
2*(-4 + n^2)))
```

fricas [A] time = 0.46, size = 78, normalized size = 1.13

$$-\frac{\left(a^2x^2-anx+1\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2n^2-4\,a^2c^2-\left(a^4c^2n^2-4\,a^4c^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

[Out] -(a^2*x^2 - a*n*x + 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*n^2 - 4*a^2*c
^2 - (a^4*c^2*n^2 - 4*a^4*c^2)*x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^2,x, algorithm="giac")

[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)

maple [A] time = 0.03, size = 47, normalized size = 0.68

$$\frac{e^{n\,\mathrm{arctanh}(ax)}\left(a^2x^2-nax+1\right)}{\left(a^2x^2-1\right)c^2a^2\left(n^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^2,x)

[Out] exp(n*arctanh(a*x))*(a^2*x^2-a*n*x+1)/(a^2*x^2-1)/c^2/a^2/(n^2-4)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)

mupad [B] time = 1.42, size = 46, normalized size = 0.67

$$\frac{e^{n\,\mathrm{atanh}(a\,x)}\,\left(a^2\,x^2-n\,a\,x+1\right)}{a^2\,c^2\,\left(n^2-4\right)\,\left(a^2\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^2,x)
```

```
[Out] (exp(n*atanh(a*x))*(a^2*x^2 - a*n*x + 1))/(a^2*c^2*(n^2 - 4)*(a^2*x^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty} x^2 e^{-\infty n} \\ \tilde{\infty} x^2 e^{\infty n} \\ \tilde{\infty} \int x e^{n \operatorname{atanh}(ax)}\, dx \\ -\frac{a^2x^2 \operatorname{atanh}(ax)}{4a^4c^2x^2e^{2\operatorname{atanh}(ax)} - 4a^2c^2e^{2\operatorname{atanh}(ax)}} - \frac{2ax \operatorname{atanh}(ax)}{4a^4c^2x^2e^{2\operatorname{atanh}(ax)} - 4a^2c^2e^{2\operatorname{atanh}(ax)}} + \frac{ax}{4a^4c^2x^2e^{2\operatorname{atanh}(ax)} - 4a^2c^2e^{2\operatorname{atanh}(ax)}} - \frac{\operatorname{atanh}(a}{4a^4c^2x^2e^{2\operatorname{atanh}(ax)} - 4} \\ \frac{\int \frac{xe^{2\operatorname{atanh}(ax)}}{a^4x^4 - 2a^2x^2 + 1}\, dx}{c^2} \\ \frac{a^2x^2e^{n\operatorname{atanh}(ax)}}{a^4c^2n^2x^2 - 4a^4c^2x^2 - a^2c^2n^2 + 4a^2c^2} - \frac{anxe^{n\operatorname{atanh}(ax)}}{a^4c^2n^2x^2 - 4a^4c^2x^2 - a^2c^2n^2 + 4a^2c^2} + \frac{e^{n\operatorname{atanh}(ax)}}{a^4c^2n^2x^2 - 4a^4c^2x^2 - a^2c^2n^2 + 4a^2c^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((zoo*x**2*exp(-oo*n), Eq(a, -1/x)), (zoo*x**2*exp(oo*n), Eq(a, 1/
x)), (zoo*Integral(x*exp(n*atanh(a*x)), x), Eq(c, 0)), (-a**2*x**2*atanh(a*
x)/(4*a**4*c**2*x**2*exp(2*atanh(a*x)) - 4*a**2*c**2*exp(2*atanh(a*x))) - 2
*a*x*atanh(a*x)/(4*a**4*c**2*x**2*exp(2*atanh(a*x)) - 4*a**2*c**2*exp(2*ata
nh(a*x))) + a*x/(4*a**4*c**2*x**2*exp(2*atanh(a*x)) - 4*a**2*c**2*exp(2*ata
nh(a*x))) - atanh(a*x)/(4*a**4*c**2*x**2*exp(2*atanh(a*x)) - 4*a**2*c**2*ex
p(2*atanh(a*x))), Eq(n, -2)), (Integral(x*exp(2*atanh(a*x))/(a**4*x**4 - 2*
a**2*x**2 + 1), x)/c**2, Eq(n, 2)), (a**2*x**2*exp(n*atanh(a*x))/(a**4*c**2
*n**2*x**2 - 4*a**4*c**2*x**2 - a**2*c**2*n**2 + 4*a**2*c**2) - a*n*x*exp(n
*atanh(a*x))/(a**4*c**2*n**2*x**2 - 4*a**4*c**2*x**2 - a**2*c**2*n**2 + 4*a
**2*c**2) + exp(n*atanh(a*x))/(a**4*c**2*n**2*x**2 - 4*a**4*c**2*x**2 - a**
2*c**2*n**2 + 4*a**2*c**2), True))
```

3.1322 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=72

$$\frac{2e^{n\tanh^{-1}(ax)}}{ac^2n\left(4-n^2\right)}-\frac{(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

```
[Out] 2*exp(n*arctanh(a*x))/a/c^2/n/(-n^2+4)-exp(n*arctanh(a*x))*(-2*a*x+n)/a/c^2
/(-n^2+4)/(-a^2*x^2+1)
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6136, 6137}

$$\frac{2e^{n\tanh^{-1}(ax)}}{ac^2n\left(4-n^2\right)}-\frac{(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^(n*ArcTanh[a*x]))/(a*c^2*n*(4 - n^2)) - (E^(n*ArcTanh[a*x])*(n - 2*a*x
))/(a*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-2ax)}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)} + \frac{2\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx}{c\left(4-n^2\right)}$$
$$= \frac{2e^{n\tanh^{-1}(ax)}}{ac^2n\left(4-n^2\right)} - \frac{e^{n\tanh^{-1}(ax)}(n-2ax)}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 0.94

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(-2a^2x^2+2anx-n^2+2\right)}{ac^2(n-2)n(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(2 - n^2 + 2*a*n*x - 2*a^2*x^2
))/(a*c^2*(-2 + n)*n*(2 + n)))
```

fricas [A] time = 0.46, size = 79, normalized size = 1.10

$$\frac{\left(2\,a^2x^2-2\,anx+n^2-2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^2n^3-4\,ac^2n-\left(a^3c^2n^3-4\,a^3c^2n\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*x^2 - 2*a*n*x + n^2 - 2)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^3 -
4*a*c^2*n - (a^3*c^2*n^3 - 4*a^3*c^2*n)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

maple [A] time = 0.03, size = 55, normalized size = 0.76

$$-\frac{e^{n\,\mathrm{arctanh}(ax)}\left(2a^{2}x^{2}-2nax+n^{2}-2\right)}{\left(a^{2}x^{2}-1\right)c^{2}an\left(n^{2}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -exp(n*arctanh(a*x))*(2*a^2*x^2-2*a*n*x+n^2-2)/(a^2*x^2-1)/c^2/a/n/(n^2-4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^{2}cx^{2}-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

mupad [B] time = 1.21, size = 93, normalized size = 1.29

$$\frac{(a\,x+1)^{n/2}\left(\frac{2\,x^{2}}{a\,c^{2}\,n\left(n^{2}-4\right)}-\frac{2\,x}{a^{2}\,c^{2}\left(n^{2}-4\right)}+\frac{n^{2}-2}{a^{3}\,c^{2}\,n\left(n^{2}-4\right)}\right)}{\left(\frac{1}{a^{2}}-x^{2}\right)(1-a\,x)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a^2*c*x^2)^2,x)
```

```
[Out] ((a*x + 1)^(n/2)*((2*x^2)/(a*c^2*n*(n^2 - 4)) - (2*x)/(a^2*c^2*(n^2 - 4)) +
 (n^2 - 2)/(a^3*c^2*n*(n^2 - 4))))/((1/a^2 - x^2)*(1 - a*x)^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty} x e^{-\infty n} \\ \tilde{\infty} x e^{\infty n} \\ \tilde{\infty} \int e^{n \operatorname{atanh}(ax)}\, dx \\ -\frac{a^2x^2 \operatorname{atanh}(ax)}{4a^3c^2x^2e^{2\operatorname{atanh}(ax)}-4ac^2e^{2\operatorname{atanh}(ax)}} - \frac{2ax \operatorname{atanh}(ax)}{4a^3c^2x^2e^{2\operatorname{atanh}(ax)}-4ac^2e^{2\operatorname{atanh}(ax)}} + \frac{ax}{4a^3c^2x^2e^{2\operatorname{atanh}(ax)}-4ac^2e^{2\operatorname{atanh}(ax)}} - \frac{\operatorname{atanh}(ax)}{4a^3c^2x^2e^{2\operatorname{atanh}(ax)}-4ac^2} \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} + \frac{a^2x^2\log\left(x+\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} - \frac{2ax}{4a^3c^2x^2-4ac^2} + \frac{\log\left(x-\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} - \frac{\log\left(x+\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} \\ \frac{\int \frac{e^{2\operatorname{atanh}(ax)}}{a^4x^4-2a^2x^2+1}\, dx}{c^2} \\ -\frac{2a^2x^2e^{n\operatorname{atanh}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} + \frac{2anxe^{n\operatorname{atanh}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} - \frac{n^2e^{n\operatorname{atanh}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} + \frac{2e^{n\operatorname{atanh}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((zoo*x*exp(-oo*n), Eq(a, -1/x)), (zoo*x*exp(oo*n), Eq(a, 1/x)), (
zoo*Integral(exp(n*atanh(a*x)), x), Eq(c, 0)), (-a**2*x**2*atanh(a*x)/(4*a*
*3*c**2*x**2*exp(2*atanh(a*x)) - 4*a*c**2*exp(2*atanh(a*x))) - 2*a*x*atanh(
a*x)/(4*a**3*c**2*x**2*exp(2*atanh(a*x)) - 4*a*c**2*exp(2*atanh(a*x))) + a*
x/(4*a**3*c**2*x**2*exp(2*atanh(a*x)) - 4*a*c**2*exp(2*atanh(a*x))) - atanh
(a*x)/(4*a**3*c**2*x**2*exp(2*atanh(a*x)) - 4*a*c**2*exp(2*atanh(a*x))) + 2
/(4*a**3*c**2*x**2*exp(2*atanh(a*x)) - 4*a*c**2*exp(2*atanh(a*x))), Eq(n, -
2)), (-a**2*x**2*log(x - 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) + a**2*x**2*log
(x + 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) - 2*a*x/(4*a**3*c**2*x**2 - 4*a*c**
2) + log(x - 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) - log(x + 1/a)/(4*a**3*c**2
*x**2 - 4*a*c**2), Eq(n, 0)), (Integral(exp(2*atanh(a*x))/(a**4*x**4 - 2*a*
*2*x**2 + 1), x)/c**2, Eq(n, 2)), (-2*a**2*x**2*exp(n*atanh(a*x))/(a**3*c**
2*n**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n) + 2*a*n*x*exp(
n*atanh(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a
*c**2*n) - n**2*exp(n*atanh(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2
 - a*c**2*n**3 + 4*a*c**2*n) + 2*exp(n*atanh(a*x))/(a**3*c**2*n**3*x**2 - 4
*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n), True))
```

3.1323 $\int \frac{e^{n\tanh^{-1}(ax)}}{x(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=190

$$-\frac{2(ax+1)^{n/2}(1-ax)^{-n/2}\,{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{ax+1}{1-ax}\right)}{c^2n}-\frac{\left(-n^2-n+4\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{c^2n\left(4-n^2\right)}+\frac{(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}-1}}{c^2(n+2)}+$$

```
[Out] (-a*x+1)^(-1-1/2*n)*(a*x+1)^(-1+1/2*n)/c^2/(2+n)-(-n^2-n+4)*(-a*x+1)^(1-1/2
*n)*(a*x+1)^(-1+1/2*n)/c^2/n/(-n^2+4)+(4+n)*(a*x+1)^(-1+1/2*n)/c^2/n/(2+n)/
((-a*x+1)^(1/2*n))-2*(a*x+1)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n],(a*x+1)
/(-a*x+1))/c^2/n/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.20, antiderivative size = 200, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6150, 129, 155, 12, 131}

$$-\frac{2(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{c^2(2-n)}-\frac{\left(-n^2-n+4\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{c^2n\left(4-n^2\right)}+\frac{(ax+1)^{\frac{n-2}{2}}(1-ax)^{-\frac{n}{2}}}{c^2(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] ((1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*(2 + n)) - ((4 - n - n^2
)*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*n*(4 - n^2)) + ((4 + n)*
(1 + a*x)^((-2 + n)/2))/(c^2*n*(2 + n)*(1 - a*x)^(n/2)) - (2*(1 - a*x)^(1 -
 n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (1 - a*
x)/(1 + a*x)])/(c^2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
```

```
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}}{x}\,dx}{c^2}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\left(-a(2+n)-2a^2x\right)}{x}\,dx}{ac^2(2+n)}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} + \frac{(4+n)(1-ax)^{-n/2}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n(2+n)} + \frac{\int \frac{(1-ax)^{-n/2}(1+ax)^{-2+\frac{n}{2}}\left(a^2 n\right.}{x}}{a^2c^2n(2-}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{\left(4-n-n^2\right)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n\left(4-n^2\right)} + \frac{(4+n)(1-ax)^{-n}}{c^2n(2}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{\left(4-n-n^2\right)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n\left(4-n^2\right)} + \frac{(4+n)(1-ax)^{-n}}{c^2n(2}$$

$$= \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{\left(4-n-n^2\right)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n\left(4-n^2\right)} + \frac{(4+n)(1-ax)^{-n}}{c^2n(2}$$

Mathematica [A] time = 0.07, size = 121, normalized size = 0.64

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(n^2\left(a^2x^2-ax-1\right)+a^2nx^2-4a^2x^2-2(n+2)n(ax-1)^2\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)+n\right.}{c^2n\left(n^2-4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(4 + n - 4*a^2*x^2 + a^2*n*x^2
 + n^2*(-1 - a*x + a^2*x^2) - 2*n*(2 + n)*(-1 + a*x)^2*Hypergeometric2F1[1,
 1 - n/2, 2 - n/2, (1 - a*x)/(1 + a*x)]))/(c^2*n*(-4 + n^2)))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^5-2\,a^2c^2x^3+c^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^5 - 2*a^2*c^2*x^3 + c^2*x
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)^2*x), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x\left(-a^2c\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x\left(c-a^2\,c\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)^2),x)
[Out] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^{n \operatorname{atanh}(ax)}}{a^4x^5-2a^2x^3+x}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c)**2,x)
[Out] Integral(exp(n*atanh(a*x))/(a**4*x**5 - 2*a**2*x**3 + x), x)/c**2
```

3.1324 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=239

$$-\frac{2a(ax+1)^{n/2}(1-ax)^{-n/2}\,{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{ax+1}{1-ax}\right)}{c^2}+\frac{a\left(n^2+4n+6\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{-n/2}}{c^2n(n+2)}-\frac{a\left(-n^3-n^2+4n+6\right)}{c^2n\left(4\right.}$$

```
[Out] a*(3+n)*(-a*x+1)^(-1-1/2*n)*(a*x+1)^(-1+1/2*n)/c^2/(2+n)-(-a*x+1)^(-1-1/2*n
)*(a*x+1)^(-1+1/2*n)/c^2/x-a*(-n^3-n^2+4*n+6)*(-a*x+1)^(1-1/2*n)*(a*x+1)^(-
1+1/2*n)/c^2/n/(-n^2+4)+a*(n^2+4*n+6)*(a*x+1)^(-1+1/2*n)/c^2/n/(2+n)/((-a*x
+1)^(1/2*n))-2*a*(a*x+1)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n],(a*x+1)/(-a
*x+1))/c^2/((-a*x+1)^(1/2*n))
```

Rubi [A] time = 0.25, antiderivative size = 253, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6150, 129, 155, 12, 131}

$$-\frac{2an(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}\,{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{c^2(2-n)}-\frac{a\left(-n^3-n^2+4n+6\right)(ax+1)^{\frac{n-2}{2}}(1-ax)^{1-\frac{n}{2}}}{c^2n\left(4-n^2\right)}+\frac{a\left(n^2+4\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] (a*(3 + n)*(1 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*(2 + n)) - ((1
 - a*x)^(-1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*x) - (a*(6 + 4*n - n^2 - n^
3)*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2))/(c^2*n*(4 - n^2)) + (a*(6 +
4*n + n^2)*(1 + a*x)^((-2 + n)/2))/(c^2*n*(2 + n)*(1 - a*x)^(n/2)) - (2*a*n
*(1 - a*x)^(1 - n/2)*(1 + a*x)^((-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2
 - n/2, (1 - a*x)/(1 + a*x)])/(c^2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^2}\,dx = \frac{\int \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}}{x^2}\,dx}{c^2}$$

$$= -\frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} - \frac{\int \frac{(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\left(-an-3a^2x\right)}{x}\,dx}{c^2}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} + \frac{\int \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-2}}{}}{ac}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} + \frac{a\left(6+4n+n^2\right)(1}{c^2}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} - \frac{a\left(6+4n-n^2-n\right.}{c}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} - \frac{a\left(6+4n-n^2-n\right.}{c}$$

$$= \frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} - \frac{a\left(6+4n-n^2-n\right.}{c}$$

Mathematica [A] time = 0.10, size = 163, normalized size = 0.68

$$-\frac{(1-ax)^{-\frac{n}{2}-1}(ax+1)^{\frac{n}{2}-1}\left(-6a^3x^3+an^2x\left(a^2x^2-2\right)+n\left(-4a^3x^3+6a^2x^2+4ax-4\right)-2a(n+2)n^2x(ax-1)^2\,{}_2\right.}{c^2(n-2)n(n+2)x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^2),x]
```

```
[Out] -(((1 - a*x)^(-1 - n/2)*(1 + a*x)^(-1 + n/2)*(6*a*x - 6*a^3*x^3 + n^3*(-1 +
 a*x)^2*(1 + a*x) + a*n^2*x*(-2 + a^2*x^2) + n*(-4 + 4*a*x + 6*a^2*x^2 - 4*
a^3*x^3) - 2*a*n^2*(2 + n)*x*(-1 + a*x)^2*Hypergeometric2F1[1, 1 - n/2, 2 -
 n/2, (1 - a*x)/(1 + a*x)]))/(c^2*(-2 + n)*n*(2 + n)*x))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^6-2a^2c^2x^4+c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^6 - 2*a^2*c^2*x^4 + c^2*x
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)^2*x^2), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x^2\left(-a^2c\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x)
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((a^2*c*x^2 - c)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{x^2\,{\left(c-a^2\,c\,x^2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)^2),x)
[Out] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{e^{n \operatorname{atanh}{\left(ax \right)}}}{a^{4} x^{6} - 2 a^{2} x^{4} + x^{2}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c)**2,x)
[Out] Integral(exp(n*atanh(a*x))/(a**4*x**6 - 2*a**2*x**4 + x**2), x)/c**2
```

3.1325 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(n-4ax)e^{n\tanh^{-1}(ax)}}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}+\frac{24e^{n\tanh^{-1}(ax)}}{ac^3n\left(n^4-20n^2+64\right)}$$

```
[Out] 24*exp(n*arctanh(a*x))/a/c^3/n/(n^4-20*n^2+64)-exp(n*arctanh(a*x))*(-4*a*x+
n)/a/c^3/(-n^2+16)/(-a^2*x^2+1)^2-12*exp(n*arctanh(a*x))*(-2*a*x+n)/a/c^3/(
n^4-20*n^2+64)/(-a^2*x^2+1)
```

Rubi [A] time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6136, 6137}

$$-\frac{(n-4ax)e^{n\tanh^{-1}(ax)}}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}+\frac{24e^{n\tanh^{-1}(ax)}}{ac^3n\left(n^4-20n^2+64\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (24*E^(n*ArcTanh[a*x]))/(a*c^3*n*(64 - 20*n^2 + n^4)) - (E^(n*ArcTanh[a*x])
*(n - 4*a*x))/(a*c^3*(16 - n^2)*(1 - a^2*x^2)^2) - (12*E^(n*ArcTanh[a*x])*(
n - 2*a*x))/(a*c^3*(4 - n^2)*(16 - n^2)*(1 - a^2*x^2))
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} + \frac{12\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{c(16-n^2)}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} - \frac{12e^{n\tanh^{-1}(ax)}(n-2ax)}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)} + \frac{24\int \frac{e^{n\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx}{c^2(64-20n^2+n^4)}$$

$$= \frac{24e^{n\tanh^{-1}(ax)}}{ac^3n(64-20n^2+n^4)} - \frac{e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} - \frac{12e^{n\tanh^{-1}(ax)}(n-2ax)}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.87

$$\frac{(1-ax)^{-\frac{n}{2}-2}(ax+1)^{\frac{n}{2}-2}\left(4n^2\left(3a^2x^2-4\right)-8anx\left(3a^2x^2-5\right)+24\left(a^2x^2-1\right)^2-4an^3x+n^4\right)}{ac^3(n-4)(n-2)n(n+2)(n+4)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] ((1 - a*x)^(-2 - n/2)*(1 + a*x)^(-2 + n/2)*(n^4 - 4*a*n^3*x + 24*(-1 + a^2*
x^2)^2 - 8*a*n*x*(-5 + 3*a^2*x^2) + 4*n^2*(-4 + 3*a^2*x^2)))/(a*c^3*(-4 + n
)*(-2 + n)*n*(2 + n)*(4 + n))
```

fricas [A] time = 0.55, size = 174, normalized size = 1.37

$$\frac{\left(24\,a^4x^4-24\,a^3nx^3+n^4+12\left(a^2n^2-4\,a^2\right)x^2-16\,n^2-4\left(an^3-10\,an\right)x+24\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^5-20\,ac^3n^3+64\,ac^3n+\left(a^5c^3n^5-20\,a^5c^3n^3+64\,a^5c^3n\right)x^4-2\left(a^3c^3n^5-20\,a^3c^3n^3+64\,a^3c^3n\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (24*a^4*x^4 - 24*a^3*n*x^3 + n^4 + 12*(a^2*n^2 - 4*a^2)*x^2 - 16*n^2 - 4*(a
*n^3 - 10*a*n)*x + 24)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^5 - 20*a*c^3*
n^3 + 64*a*c^3*n + (a^5*c^3*n^5 - 20*a^5*c^3*n^3 + 64*a^5*c^3*n)*x^4 - 2*(a
^3*c^3*n^5 - 20*a^3*c^3*n^3 + 64*a^3*c^3*n)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

maple [A] time = 0.03, size = 101, normalized size = 0.80

$$\frac{\left(24x^4a^4-24x^3a^3n+12a^2n^2x^2-4a\,n^3x-48a^2x^2+n^4+40nax-16n^2+24\right)\mathrm{e}^{n\,\mathrm{arctanh}(ax)}}{\left(a^2x^2-1\right)^2c^3a\left(n^2-16\right)\left(n^2-4\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^3,x)
[Out] (24*a^4*x^4-24*a^3*n*x^3+12*a^2*n^2*x^2-4*a*n^3*x-48*a^2*x^2+n^4+40*a*n*x-1
6*n^2+24)*exp(n*arctanh(a*x))/(a^2*x^2-1)^2/c^3/a/(n^2-16)/(n^2-4)/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

mupad [B] time = 1.39, size = 179, normalized size = 1.41

$$\frac{(a\,x+1)^{n/2}\left(\frac{24\,x^4}{a\,c^3\,n\,\left(n^4-20\,n^2+64\right)}-\frac{4\,x\,\left(n^2-10\right)}{a^4\,c^3\,\left(n^4-20\,n^2+64\right)}-\frac{24\,x^3}{a^2\,c^3\,\left(n^4-20\,n^2+64\right)}+\frac{n^4-16\,n^2+24}{a^5\,c^3\,n\,\left(n^4-20\,n^2+64\right)}+\frac{x^2\,\left(12\,n^2-48\right)}{a^3\,c^3\,n\,\left(n^4-20\,n^2+64\right)}\right)}{(1-a\,x)^{n/2}\,\left(\frac{1}{a^4}+x^4-\frac{2\,x^2}{a^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a^2*c*x^2)^3,x)
```

```
[Out] ((a*x + 1)^(n/2)*((24*x^4)/(a*c^3*n*(n^4 - 20*n^2 + 64)) - (4*x*(n^2 - 10))
/(a^4*c^3*(n^4 - 20*n^2 + 64)) - (24*x^3)/(a^2*c^3*(n^4 - 20*n^2 + 64)) + (
n^4 - 16*n^2 + 24)/(a^5*c^3*n*(n^4 - 20*n^2 + 64)) + (x^2*(12*n^2 - 48))/(a
^3*c^3*n*(n^4 - 20*n^2 + 64))))/((1 - a*x)^(n/2)*(1/a^4 + x^4 - (2*x^2)/a^2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.1326 $\int \frac{e^{n\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=197

$$-\frac{(n-6ax)e^{n\tanh^{-1}(ax)}}{ac^4(36-n^2)(1-a^2x^2)^3}-\frac{360(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^4(4-n^2)(16-n^2)(36-n^2)(1-a^2x^2)}-\frac{30(n-4ax)e^{n\tanh^{-1}(ax)}}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2}+\frac{}{ac^4}$$

```
[Out] 720*exp(n*arctanh(a*x))/a/c^4/n/(-n^2+36)/(n^4-20*n^2+64)-exp(n*arctanh(a*x
))*(-6*a*x+n)/a/c^4/(-n^2+36)/(-a^2*x^2+1)^3-30*exp(n*arctanh(a*x))*(-4*a*x
+n)/a/c^4/(n^4-52*n^2+576)/(-a^2*x^2+1)^2-360*exp(n*arctanh(a*x))*(-2*a*x+n
)/a/c^4/(-n^2+36)/(n^4-20*n^2+64)/(-a^2*x^2+1)
```

Rubi [A] time = 0.19, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6136, 6137}

$$-\frac{(n-6ax)e^{n\tanh^{-1}(ax)}}{ac^4(36-n^2)(1-a^2x^2)^3}-\frac{360(n-2ax)e^{n\tanh^{-1}(ax)}}{ac^4(4-n^2)(16-n^2)(36-n^2)(1-a^2x^2)}-\frac{30(n-4ax)e^{n\tanh^{-1}(ax)}}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2}+\frac{}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (720*E^(n*ArcTanh[a*x]))/(a*c^4*n*(36 - n^2)*(64 - 20*n^2 + n^4)) - (E^(n*A
rcTanh[a*x])*(n - 6*a*x))/(a*c^4*(36 - n^2)*(1 - a^2*x^2)^3) - (30*E^(n*Arc
Tanh[a*x])*(n - 4*a*x))/(a*c^4*(16 - n^2)*(36 - n^2)*(1 - a^2*x^2)^2) - (36
0*E^(n*ArcTanh[a*x])*(n - 2*a*x))/(a*c^4*(4 - n^2)*(16 - n^2)*(36 - n^2)*(1
 - a^2*x^2))
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6137

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcTanh[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
```

```
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}+\frac{30\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{c\left(36-n^2\right)}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{30e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}+\frac{360\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{c^2\left(576-52n^2+n^4\right)}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{30e^{n\tanh^{-1}(ax)}(n-4ax)}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}-\frac{360e^{n\tanh^{-1}(ax}}{ac^4\left(4-n^2\right)\left(576-52\right.}$$

$$= \frac{720e^{n\tanh^{-1}(ax)}}{ac^4n\left(4-n^2\right)\left(576-52n^2+n^4\right)}-\frac{e^{n\tanh^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{30e^{n\tanh^{-1}(ax)}(n-4a}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-\right.}$$

Mathematica [A] time = 0.13, size = 170, normalized size = 0.86

$$-\frac{(1-ax)^{-\frac{n}{2}-3}(ax+1)^{\frac{n}{2}-3}\left(n^4\left(50-30a^2x^2\right)+120an^3x\left(a^2x^2-2\right)-720\left(a^2x^2-1\right)^3-8n^2\left(45a^4x^4-105a^2x^2+\right.\right.}{ac^4(n-6)(n-4)(n-2)n(n+2)(n+4)(n+6)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -((((1 - a*x)^(-3 - n/2)*(1 + a*x)^(-3 + n/2)*(-n^6 + 6*a*n^5*x + n^4*(50 -
30*a^2*x^2) + 120*a*n^3*x*(-2 + a^2*x^2) - 720*(-1 + a^2*x^2)^3 + 48*a*n*x*
(33 - 40*a^2*x^2 + 15*a^4*x^4) - 8*n^2*(68 - 105*a^2*x^2 + 45*a^4*x^4)))/(a
*c^4*(-6 + n)*(-4 + n)*(-2 + n)*n*(2 + n)*(4 + n)*(6 + n)))
```

fricas [A] time = 0.72, size = 309, normalized size = 1.57

$$\frac{\left(720\,a^6x^6-720\,a^5nx^5+n^6+360\left(a^4n^2-6\,a^4\right)x^4-50\,n^4-120\left(a^3n^3-16\,a^3n\right)x^3+30\left(a^2n^4\right.\right.}{ac^4n^7-56\,ac^4n^5+784\,ac^4n^3-\left(a^7c^4n^7-56\,a^7c^4n^5+784\,a^7c^4n^3-2304\,a^7c^4n\right)x^6-2304\,ac^4n+3\left(a^5c^4n^7-5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] (720*a^6*x^6 - 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 - 6*a^4)*x^4 - 50*n^4 - 1
20*(a^3*n^3 - 16*a^3*n)*x^3 + 30*(a^2*n^4 - 28*a^2*n^2 + 72*a^2)*x^2 + 544*
n^2 - 6*(a*n^5 - 40*a*n^3 + 264*a*n)*x - 720)*((a*x + 1)/(a*x - 1))^(1/2*n)
/(a*c^4*n^7 - 56*a*c^4*n^5 + 784*a*c^4*n^3 - (a^7*c^4*n^7 - 56*a^7*c^4*n^5
+ 784*a^7*c^4*n^3 - 2304*a^7*c^4*n)*x^6 - 2304*a*c^4*n + 3*(a^5*c^4*n^7 - 5
6*a^5*c^4*n^5 + 784*a^5*c^4*n^3 - 2304*a^5*c^4*n)*x^4 - 3*(a^3*c^4*n^7 - 56
*a^3*c^4*n^5 + 784*a^3*c^4*n^3 - 2304*a^3*c^4*n)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

maple [A] time = 0.03, size = 167, normalized size = 0.85

$$-\frac{\left(720x^6a^6-720a^5x^5n+360a^4n^2x^4-120a^3n^3x^3-2160x^4a^4+30a^2n^4x^2+1920x^3a^3n-6a\,n^5x-840a^2n^2x^2+n\right.}{\left(a^2x^2-1\right)^3c^4an\left(n^6-56n^4+784n^2-23\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -(720*a^6*x^6-720*a^5*n*x^5+360*a^4*n^2*x^4-120*a^3*n^3*x^3-2160*a^4*x^4+30
*a^2*n^4*x^2+1920*a^3*n*x^3-6*a*n^5*x-840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160*
a^2*x^2-50*n^4-1584*a*n*x+544*n^2-720)*exp(n*arctanh(a*x))/(a^2*x^2-1)^3/c^
4/a/n/(n^6-56*n^4+784*n^2-2304)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

mupad [B] time = 1.37, size = 301, normalized size = 1.53

$$\frac{(a\,x+1)^{n/2}\left(\frac{n^6-50\,n^4+544\,n^2-720}{a^7\,c^4\,n\,\left(n^6-56\,n^4+784\,n^2-2304\right)}-\frac{720\,x^5}{a^2\,c^4\,\left(n^6-56\,n^4+784\,n^2-2304\right)}-\frac{x^3\,\left(120\,n^2-1920\right)}{a^4\,c^4\,\left(n^6-56\,n^4+784\,n^2-2304\right)}+\frac{720\,x^6}{a\,c^4\,n\,\left(n^6-56\,n^4+784\,n^2\right.}}{(1-a\,x)^{n/2}\left(\frac{1}{a^6}-x^6+\frac{3\,x^4}{a^2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a^2*c*x^2)^4,x)
```

```
[Out] ((a*x + 1)^(n/2)*((544*n^2 - 50*n^4 + n^6 - 720)/(a^7*c^4*n*(784*n^2 - 56*n
^4 + n^6 - 2304)) - (720*x^5)/(a^2*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) - (
x^3*(120*n^2 - 1920))/(a^4*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) + (720*x^6)
/(a*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) - (6*x*(n^4 - 40*n^2 + 264))/(a^
6*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^2*(30*n^4 - 840*n^2 + 2160))/(a
^5*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^4*(360*n^2 - 2160))/(a^3*c^4
*n*(784*n^2 - 56*n^4 + n^6 - 2304))))/((1 - a*x)^(n/2)*(1/a^6 - x^6 + (3*x^
4)/a^2 - (3*x^2)/a^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

3.1327 $\int e^{n\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=256

$$-\frac{x^2\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n+3}{2}}(1-ax)^{\frac{3-n}{2}}}{5a^2\sqrt{1-a^2x^2}}-\frac{2^{\frac{n-1}{2}}n\left(n^2+11\right)\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{15a^4(3-n)\sqrt{1-a^2x^2}}$$

```
[Out] -1/5*x^2*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(3/2+1/2*n)*(-a^2*c*x^2+c)^(1/2)/a^2/
(-a^2*x^2+1)^(1/2)-1/60*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(3/2+1/2*n)*(3*a*n*x+n
^2+8)*(-a^2*c*x^2+c)^(1/2)/a^4/(-a^2*x^2+1)^(1/2)-1/15*2^(-1/2+1/2*n)*n*(n^
2+11)*(-a*x+1)^(3/2-1/2*n)*hypergeom([3/2-1/2*n, -1/2-1/2*n],[5/2-1/2*n],-1
/2*a*x+1/2)*(-a^2*c*x^2+c)^(1/2)/a^4/(3-n)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 100, 147, 69}

$$-\frac{2^{\frac{n-1}{2}}n\left(n^2+11\right)\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{15a^4(3-n)\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n+3}{2}}\left(3anx+n^2\right.}{60a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(x^2*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2])/(5*a
^2*Sqrt[1 - a^2*x^2]) - ((1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*(8 + n
^2 + 3*a*n*x)*Sqrt[c - a^2*c*x^2])/(60*a^4*Sqrt[1 - a^2*x^2]) - (2^((-1 + n
)/2)*n*(11 + n^2)*(1 - a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2
F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(15*a^4*(3 - n)*Sqrt[1 -
 a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{n\tanh^{-1}(ax)}x^3\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^3(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{x^2(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{5a^2\sqrt{1-a^2x^2}} - \frac{\sqrt{c-a^2cx^2}\int x(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{5a^2\sqrt{1-a^2x^2}}$$
$$= -\frac{x^2(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{5a^2\sqrt{1-a^2x^2}} - \frac{(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\left(8+n^2+3anx\right)}{60a^4\sqrt{1-a^2x^2}}$$
$$= -\frac{x^2(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{5a^2\sqrt{1-a^2x^2}} - \frac{(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\left(8+n^2+3anx\right)}{60a^4\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.56, size = 236, normalized size = 0.92

$$\frac{\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3}{2}-\frac{n}{2}}\left(a^2(n-3)x^2(ax+1)^{\frac{n+3}{2}} - 2^{\frac{n+7}{2}}n\,{}_2F_1\left(\frac{1}{2}(-n-5),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right) + 2^{\frac{n+7}{2}}(n-1)\,{}_2F_1\left(\frac{1}{2}(\right.\right.}{5a^4(3-n)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(3/2 - n/2)*Sqrt[c - a^2*c*x^2]*(a^2*(-3 + n)*x^2*(1 + a*x)^((3
+ n)/2) - 2^((7 + n)/2)*n*Hypergeometric2F1[(-5 - n)/2, (3 - n)/2, (5 - n)/
2, (1 - a*x)/2] + 2^((7 + n)/2)*(-1 + n)*Hypergeometric2F1[(-3 - n)/2, (3 -
 n)/2, (5 - n)/2, (1 - a*x)/2] - 2^((3 + n)/2)*(-2 + n)*Hypergeometric2F1[(
-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2]))/(5*a^4*(3 - n)*Sqrt[1 - a^2
*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n \,\mathrm{arctanh}(a x)} x^{3} \sqrt{-a^{2} c \,x^{2}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2} c x^{2}+c}\, x^{3} \left(\frac{a x+1}{a x-1}\right)^{\frac{1}{2} \, n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\mathrm{e}}^{n\,\mathrm{atanh}\left(a\,x\right)}\,\sqrt{c-a^2\,c\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(x^3*exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{-c\,(ax-1)\,(ax+1)}\,e^{n\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x)), x)
```

3.1328 $\int e^{n\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=244

$$-\frac{x\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n+3}{2}}(1-ax)^{\frac{3-n}{2}}}{4a^2\sqrt{1-a^2x^2}}-\frac{2^{\frac{n-1}{2}}\left(n^2+3\right)\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{3a^3(3-n)\sqrt{1-a^2x^2}}-$$

```
[Out] -1/12*n*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(3/2+1/2*n)*(-a^2*c*x^2+c)^(1/2)/a^3/(
-a^2*x^2+1)^(1/2)-1/4*x*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(3/2+1/2*n)*(-a^2*c*x^
2+c)^(1/2)/a^2/(-a^2*x^2+1)^(1/2)-1/3*2^(-1/2+1/2*n)*(n^2+3)*(-a*x+1)^(3/2-
1/2*n)*hypergeom([3/2-1/2*n, -1/2-1/2*n],[5/2-1/2*n],-1/2*a*x+1/2)*(-a^2*c*
x^2+c)^(1/2)/a^3/(3-n)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 90, 80, 69}

$$-\frac{2^{\frac{n-1}{2}}\left(n^2+3\right)\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{3a^3(3-n)\sqrt{1-a^2x^2}}-\frac{n\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n+3}{2}}(1-ax)^{\frac{3-n}{2}}}{12a^3\sqrt{1-a^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2])/(12*a^
3*Sqrt[1 - a^2*x^2]) - (x*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[
c - a^2*c*x^2])/(4*a^2*Sqrt[1 - a^2*x^2]) - (2^((-1 + n)/2)*(3 + n^2)*(1 -
a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[(-1 - n)/2, (3 - n)/
2, (5 - n)/2, (1 - a*x)/2])/(3*a^3*(3 - n)*Sqrt[1 - a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{n\tanh^{-1}(ax)}x^2\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^2(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{x(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{4a^2\sqrt{1-a^2x^2}} - \frac{\sqrt{c-a^2cx^2}\int(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}(\cdots}{4a^2\sqrt{1-a^2x^2}}$$
$$= -\frac{n(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{12a^3\sqrt{1-a^2x^2}} - \frac{x(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{4a^2\sqrt{1-a^2x^2}} + \cdots$$
$$= -\frac{n(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{12a^3\sqrt{1-a^2x^2}} - \frac{x(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{4a^2\sqrt{1-a^2x^2}} - \cdots$$

Mathematica [A] time = 0.21, size = 135, normalized size = 0.55

$$\frac{\sqrt{c-a^2cx^2}(1-ax)^{\frac{3}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}\left(n^2+3\right){}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)-(n-3)(ax+1)^{\frac{n+3}{2}}(3ax+n)\right)}{12a^3(n-3)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(3/2 - n/2)*Sqrt[c - a^2*c*x^2]*(-((-3 + n)*(1 + a*x)^((3 + n)/2
)*(n + 3*a*x)) + 2^((3 + n)/2)*(3 + n^2)*Hypergeometric2F1[(-1 - n)/2, (3 -
 n)/2, (5 - n)/2, (1 - a*x)/2]))/(12*a^3*(-3 + n)*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2cx^2+c}\,x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c}\, x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)} x^2 \sqrt{-a^2c\,x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c}\, x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\, e^{n\,\mathrm{atanh}(a\,x)}\, \sqrt{c - a^2\,c\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2),x)
[Out] int(x^2*exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{-c\,(ax-1)\,(ax+1)}\, e^{n\,\mathrm{atanh}\,(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x)), x)
```

3.1329 $\int e^{n\tanh^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=173

$$-\frac{2^{\frac{n+3}{2}} n\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{3a^2(3-n)\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n+3}{2}}(1-ax)^{\frac{3-n}{2}}}{3a^2\sqrt{1-a^2x^2}}$$

```
[Out] -1/3*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(3/2+1/2*n)*(-a^2*c*x^2+c)^(1/2)/a^2/(-a^
2*x^2+1)^(1/2)-1/3*2^(3/2+1/2*n)*n*(-a*x+1)^(3/2-1/2*n)*hypergeom([3/2-1/2*
n, -1/2-1/2*n],[5/2-1/2*n],-1/2*a*x+1/2)*(-a^2*c*x^2+c)^(1/2)/a^2/(3-n)/(-a
^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6153, 6150, 80, 69}

$$-\frac{2^{\frac{n+3}{2}} n\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{3a^2(3-n)\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n+3}{2}}(1-ax)^{\frac{3-n}{2}}}{3a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((1 - a*x)^((3 - n)/2)*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2])/(3*a^2*S
qrt[1 - a^2*x^2]) - (2^((3 + n)/2)*n*(1 - a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x
^2]*Hypergeometric2F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(3*a^
2*(3 - n)*Sqrt[1 - a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{n\tanh^{-1}(ax)}x\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\sqrt{c-a^2cx^2}\int x(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(n\sqrt{c-a^2cx^2}\right)\int(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{3a\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-a^2x^2}} - \frac{2^{\frac{3+n}{2}}n(1-ax)^{\frac{3-n}{2}}\sqrt{c-a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-1-\right.}{3a^2(3-n)\sqrt{1-a^2x}}$$

Mathematica [A] time = 0.14, size = 125, normalized size = 0.72

$$\frac{\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}n\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)-(n-3)(ax+1)^{\frac{n+3}{2}}\right)}{3a^2(n-3)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(3/2 - n/2)*Sqrt[c - a^2*c*x^2]*(-((-3 + n)*(1 + a*x)^((3 + n)/2
)) + 2^((3 + n)/2)*n*Hypergeometric2F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1
 - a*x)/2]))/(3*a^2*(-3 + n)*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2cx^2+c}\,x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}x\sqrt{-a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a^2cx^2+c}\,x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,e^{n\,\mathrm{atanh}(a\,x)}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2),x)
[Out] int(x*exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-c\,(ax-1)\,(ax+1)}\,e^{n\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x)), x)
```

3.1330 $\int e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=104

$$-\frac{2^{\frac{n+3}{2}}\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a(3-n)\sqrt{1-a^2x^2}}$$

```
[Out] -2^(3/2+1/2*n)*(-a*x+1)^(3/2-1/2*n)*hypergeom([3/2-1/2*n, -1/2-1/2*n],[5/2-
1/2*n],-1/2*a*x+1/2)*(-a^2*c*x^2+c)^(1/2)/a/(3-n)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 69}

$$-\frac{2^{\frac{n+3}{2}}\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a(3-n)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((2^((3 + n)/2)*(1 - a*x)^((3 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F
1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(a*(3 - n)*Sqrt[1 - a^2*x
^2]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{n\tanh^{-1}(ax)}\sqrt{1-a^2x^2}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}\int (1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{2^{\frac{3+n}{2}}(1-ax)^{\frac{3-n}{2}}\sqrt{c-a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a(3-n)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.97

$$\frac{2^{\frac{n+3}{2}}\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3}{2}-\frac{n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a(n-3)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2^((3 + n)/2)*(1 - a*x)^(3/2 - n/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[
(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(a*(-3 + n)*Sqrt[1 - a^2*x^
2])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arctanh}(ax)} \sqrt{-a^2 c\, x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2 c x^2 + c} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{atanh}(a\, x)} \sqrt{c - a^2\, c\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\,(ax-1)\,(ax+1)}\, e^{n \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x)), x)
```

3.1331 $\int \frac{e^{n \tanh^{-1}(ax)} \sqrt{c-a^2cx^2}}{x} \, dx$

Optimal. Leaf size=269

$$\frac{2^{\frac{n+1}{2}} n\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{\left(n^2-4n+3\right)\sqrt{1-a^2x^2}}+\frac{2\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\right.}{(1-n)\sqrt{1-a^2x^2}}$$

```
[Out] -(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*c*x^2+c)^(1/2)/(1-n)/(-a^2
*x^2+1)^(1/2)+2*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*hypergeom([1, -1/
2+1/2*n],[1/2+1/2*n],(a*x+1)/(-a*x+1))*(-a^2*c*x^2+c)^(1/2)/(1-n)/(-a^2*x^2
+1)^(1/2)+2^(1/2+1/2*n)*n*(-a*x+1)^(3/2-1/2*n)*hypergeom([3/2-1/2*n, 1/2-1/
2*n],[5/2-1/2*n],-1/2*a*x+1/2)*(-a^2*c*x^2+c)^(1/2)/(n^2-4*n+3)/(-a^2*x^2+1
)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 299, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 105, 69, 131}

$$\frac{2\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1}{2}(-n-1)}\,{}_2F_1\left(1,\frac{1}{2}(-n-1);\frac{1-n}{2};\frac{1-ax}{ax+1}\right)}{(n+1)\sqrt{1-a^2x^2}}-\frac{2^{\frac{n+3}{2}}\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{1}{2}(-n-1)}\,{}_2F_1\left(\frac{1}{2}(-n-\right.}{(n+1)\sqrt{1-a^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (2*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[c - a^2*c*x^2]*Hyperge
ometric2F1[1, (-1 - n)/2, (1 - n)/2, (1 - a*x)/(1 + a*x)])/((1 + n)*Sqrt[1
- a^2*x^2]) - (2^((3 + n)/2)*(1 - a*x)^((-1 - n)/2)*Sqrt[c - a^2*c*x^2]*Hyp
ergeometric2F1[(-1 - n)/2, (-1 - n)/2, (1 - n)/2, (1 - a*x)/2])/((1 + n)*Sq
rt[1 - a^2*x^2]) + (2^((3 + n)/2)*(1 - a*x)^((1 - n)/2)*Sqrt[c - a^2*c*x^2]
*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/((1 - n)
*Sqrt[1 - a^2*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{c-a^2cx^2}\right)\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{\sqrt{1-a^2x^2}}$$

$$= \frac{2^{\frac{3+n}{2}}(1-ax)^{\frac{1-n}{2}}\sqrt{c-a^2cx^2}\ {}_2F_1\left(\frac{1}{2}(-1-n),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{(1-n)\sqrt{1-a^2x^2}} + \frac{\sqrt{c-a^2cx^2}\int}{\sqrt{}}$$

$$= \frac{2(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1+n}{2}}\sqrt{c-a^2cx^2}\ {}_2F_1\left(1,\frac{1}{2}(-1-n);\frac{1-n}{2};\frac{1-ax}{1+ax}\right)}{(1+n)\sqrt{1-a^2x^2}} - \frac{2^{\frac{3+n}{2}}(1-a}{}$$

Mathematica [A] time = 0.17, size = 207, normalized size = 0.77

$$\frac{2\sqrt{c-a^2cx^2}(1-ax)^{\frac{1}{2}(-n-1)}\left((n-1)(ax+1)^{\frac{n+1}{2}}\ {}_2F_1\left(1,-\frac{n}{2}-\frac{1}{2};\frac{1}{2}-\frac{n}{2};\frac{1-ax}{ax+1}\right)+2^{\frac{n+1}{2}}\left((n+1)(ax-1)\ {}_2F_1\left(-\frac{n}{2}-\frac{1}{2},\right.\right.\right.}{\left(n^2-1\right)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (2*(1 - a*x)^((-1 - n)/2)*Sqrt[c - a^2*c*x^2]*((-1 + n)*(1 + a*x)^((1 + n)/
2)*Hypergeometric2F1[1, -1/2 - n/2, 1/2 - n/2, (1 - a*x)/(1 + a*x)] + 2^((1
 + n)/2)*(-((-1 + n)*Hypergeometric2F1[-1/2 - n/2, -1/2 - n/2, 1/2 - n/2, 1
/2 - (a*x)/2]) + (1 + n)*(-1 + a*x)*Hypergeometric2F1[-1/2 - n/2, 1/2 - n/2
, 3/2 - n/2, 1/2 - (a*x)/2])))/((-1 + n^2)*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)} \sqrt{-a^{2} c\, x^{2}+c}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2} c x^{2}+c} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n \operatorname{atanh}(a\,x)}\, \sqrt{c-a^2\,c\,x^2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2))/x,x)
```

```
[Out] int((exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,e^{n\,\mathrm{atanh}\,(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x))/x, x)
```

3.1332 $\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=268

$$-\frac{2an\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{1-a^2x^2}}+\frac{a2^{\frac{n+1}{2}}\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(\right.}{(1-n)\sqrt{1-a^2x^2}}$$

```
[Out] -(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(-a^2*c*x^2+c)^(1/2)/x/(-a^2*x^2+
1)^(1/2)-2*a*n*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*hypergeom([1, 1/2-
1/2*n],[3/2-1/2*n],(-a*x+1)/(a*x+1))*(-a^2*c*x^2+c)^(1/2)/(1-n)/(-a^2*x^2+1
)^(1/2)+2^(1/2+1/2*n)*a*(-a*x+1)^(1/2-1/2*n)*hypergeom([1/2-1/2*n, 1/2-1/2*
n],[3/2-1/2*n],-1/2*a*x+1/2)*(-a^2*c*x^2+c)^(1/2)/(1-n)/(-a^2*x^2+1)^(1/2)
```

Rubi [C] time = 0.23, antiderivative size = 97, normalized size of antiderivative = 0.36, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 136}

$$\frac{a2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-a^2cx^2}\,(ax+1)^{\frac{n+3}{2}}F_1\left(\frac{n+3}{2};\frac{n-1}{2},2;\frac{n+5}{2};\frac{1}{2}(ax+1),ax+1\right)}{(n+3)\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (2^(3/2 - n/2)*a*(1 + a*x)^((3 + n)/2)*Sqrt[c - a^2*c*x^2]*AppellF1[(3 + n)
/2, (-1 + n)/2, 2, (5 + n)/2, (1 + a*x)/2, 1 + a*x])/((3 + n)*Sqrt[1 - a^2*
x^2])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{n\tanh^{-1}(ax)}\sqrt{1-a^2x^2}}{x^2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(1-ax)^{\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{2^{\frac{3}{2}-\frac{n}{2}}a(1+ax)^{\frac{3+n}{2}}\sqrt{c-a^2cx^2}\,F_1\left(\frac{3+n}{2};\frac{1}{2}(-1+n),2;\frac{5+n}{2};\frac{1}{2}(1+ax),1+ax\right)}{(3+n)\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.47, size = 138, normalized size = 0.51

$$-\frac{c\sqrt{1-a^2x^2}\,e^{n\tanh^{-1}(ax)}\left((n+1)\sqrt{1-a^2x^2}+2axe^{\tanh^{-1}(ax)}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-e^{2\tanh^{-1}(ax)}\right)+2anxe^{\tanh^{-1}(ax)}\,{}_2F_1\left(\right.\right.}{(n+1)x\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -((c*E^(n*ArcTanh[a*x])*Sqrt[1 - a^2*x^2]*((1 + n)*Sqrt[1 - a^2*x^2] + 2*a*
E^ArcTanh[a*x]*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E^(2*ArcTanh[a
*x])] + 2*a*E^ArcTanh[a*x]*n*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E
^(2*ArcTanh[a*x])]))/((1 + n)*x*Sqrt[c - a^2*c*x^2]))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)} \sqrt{-a^2 c\, x^2 + c}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2 c x^2 + c} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n \operatorname{atanh}(a\,x)} \sqrt{c - a^2\,c\,x^2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2))/x^2,x)
```

```
[Out] int((exp(n*atanh(a*x))*(c - a^2*c*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,e^{n\,\operatorname{atanh}\,(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*atanh(a*x))/x**2, x)
```

3.1333 $\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=105

$$-\frac{c2^{\frac{n+5}{2}}\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{5-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-3),\frac{5-n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{a(5-n)\sqrt{1-a^2x^2}}$$

```
[Out] -2^(5/2+1/2*n)*c*(-a*x+1)^(5/2-1/2*n)*hypergeom([5/2-1/2*n, -3/2-1/2*n],[7/
2-1/2*n],-1/2*a*x+1/2)*(-a^2*c*x^2+c)^(1/2)/a/(5-n)/(-a^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 69}

$$-\frac{c2^{\frac{n+5}{2}}\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{5-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-3),\frac{5-n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{a(5-n)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((2^((5 + n)/2)*c*(1 - a*x)^((5 - n)/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric
2F1[(-3 - n)/2, (5 - n)/2, (7 - n)/2, (1 - a*x)/2])/(a*(5 - n)*Sqrt[1 - a^2
*x^2]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
```

```
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int e^{n\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{3/2}\,dx}{\sqrt{1-a^2x^2}}$$
$$=\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(1-ax)^{\frac{3}{2}-\frac{n}{2}}(1+ax)^{\frac{3}{2}+\frac{n}{2}}\,dx}{\sqrt{1-a^2x^2}}$$
$$=-\frac{2^{\frac{5+n}{2}}c(1-ax)^{\frac{5-n}{2}}\sqrt{c-a^2cx^2}\,{}_2F_1\left(\frac{1}{2}(-3-n),\frac{5-n}{2};\frac{7-n}{2};\frac{1}{2}(1-ax)\right)}{a(5-n)\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.09, size = 102, normalized size = 0.97

$$\frac{c2^{\frac{n+5}{2}}\sqrt{c-a^2cx^2}\,(1-ax)^{\frac{5}{2}-\frac{n}{2}}\,{}_2F_1\left(-\frac{n}{2}-\frac{3}{2},\frac{5}{2}-\frac{n}{2};\frac{7}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{a(n-5)\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (2^((5 + n)/2)*c*(1 - a*x)^(5/2 - n/2)*Sqrt[c - a^2*c*x^2]*Hypergeometric2F
1[-3/2 - n/2, 5/2 - n/2, 7/2 - n/2, 1/2 - (a*x)/2])/(a*(-5 + n)*Sqrt[1 - a^
2*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n
), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n \,\mathrm{arctanh}(a x)} \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(-a^{2} c x^{2} + c\right)}^{\frac{3}{2}} \left(\frac{a x + 1}{a x - 1}\right)^{\frac{1}{2} \, n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{e}}^{n\,\mathrm{atanh}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} e^{n \operatorname{atanh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*exp(n*atanh(a*x)), x)
```

3.1334 $\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=275

$$-\frac{x^2\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{3a^2\sqrt{c-a^2cx^2}}-\frac{2^{\frac{n-1}{2}}n\left(n^2+5\right)\sqrt{1-a^2x^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{3a^4(1-n)(3-n)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a}}{}$$

```
[Out] -1/3*x^2*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/a^2/(-
a^2*c*x^2+c)^(1/2)-1/6*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(4+n+n^2+a*
(1-n)*n*x)*(-a^2*x^2+1)^(1/2)/a^4/(1-n)/(-a^2*c*x^2+c)^(1/2)-1/3*2^(-1/2+1/
2*n)*n*(n^2+5)*(-a*x+1)^(3/2-1/2*n)*hypergeom([3/2-1/2*n, 1/2-1/2*n],[5/2-1
/2*n],-1/2*a*x+1/2)*(-a^2*x^2+1)^(1/2)/a^4/(n^2-4*n+3)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 100, 146, 69}

$$-\frac{2^{\frac{n-1}{2}}n\left(n^2+5\right)\sqrt{1-a^2x^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{3a^4(1-n)(3-n)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}\left(a(1-n)nx+n^2+n\right.}{6a^4(1-n)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(x^2*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(3*a^2
*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*(4 + n
 + n^2 + a*(1 - n)*n*x)*Sqrt[1 - a^2*x^2])/(6*a^4*(1 - n)*Sqrt[c - a^2*c*x^
2]) - (2^((-1 + n)/2)*n*(5 + n^2)*(1 - a*x)^((3 - n)/2)*Sqrt[1 - a^2*x^2]*H
ypergeometric2F1[(1 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(3*a^4*(1 -
 n)*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int x^3(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= -\frac{x^2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int x(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}(-2-anx)}{3a^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{x^2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\left(4+n+n^2+a(1-n)nx\right)\sqrt{}}{6a^4(1-n)\sqrt{c-a^2cx^2}}$$

$$= -\frac{x^2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{3a^2\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\left(4+n+n^2+a(1-n)nx\right)\sqrt{}}{6a^4(1-n)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.29, size = 187, normalized size = 0.68

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}-\frac{n}{2}}\left(2^{\frac{n}{2}+1}n\left(n^2+5\right)(ax-1)\,{}_2F_1\left(\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2};\frac{5}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)-\sqrt{2}\,(n-3)(ax+1)^{\frac{n+1}{2}}\left(n\left(2a^2x\right.\right.\right.}{6\sqrt{2}\,a^4(n-3)(n-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-(Sqrt[2]*(-3 + n)*(1 + a*x)^((1
+ n)/2)*(n^2*(-1 + a*x) - 2*(2 + a^2*x^2) + n*(-1 - a*x + 2*a^2*x^2))) + 2^
(1 + n/2)*n*(5 + n^2)*(-1 + a*x)*Hypergeometric2F1[1/2 - n/2, 3/2 - n/2, 5/
2 - n/2, 1/2 - (a*x)/2]))/(6*Sqrt[2]*a^4*(-3 + n)*(-1 + n)*Sqrt[c - a^2*c*x
^2])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2
 - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)} x^3}{\sqrt{-a^2 c\, x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{\sqrt{-a^2cx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\, e^{n \operatorname{atanh}(a\, x)}}{\sqrt{c - a^2\, c\, x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^3*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{n \operatorname{atanh}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.1335 $\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=253

$$-\frac{x\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{2a^2\sqrt{c-a^2cx^2}}-\frac{2^{\frac{n+1}{2}}\left(n^2+1\right)\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^3\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{(\cdots}{\cdots}$$

```
[Out] 1/2*(1-n)*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/a^3/(
1+n)/(-a^2*c*x^2+c)^(1/2)-1/2*x*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(-
a^2*x^2+1)^(1/2)/a^2/(-a^2*c*x^2+c)^(1/2)-2^(1/2+1/2*n)*(n^2+1)*(-a*x+1)^(1
/2-1/2*n)*hypergeom([1/2-1/2*n, -1/2-1/2*n],[3/2-1/2*n],-1/2*a*x+1/2)*(-a^2
*x^2+1)^(1/2)/a^3/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 90, 79, 69}

$$-\frac{2^{\frac{n+1}{2}}\left(n^2+1\right)\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^3\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{x\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{2a^2\sqrt{c-a^2cx^2}}+\frac{(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - n)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*
a^3*(1 + n)*Sqrt[c - a^2*c*x^2]) - (x*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 +
 n)/2)*Sqrt[1 - a^2*x^2])/(2*a^2*Sqrt[c - a^2*c*x^2]) - (2^((1 + n)/2)*(1 +
 n^2)*(1 - a*x)^((1 - n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[(-1 - n)/2,
 (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/(a^3*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int x^2(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= -\frac{x(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}(-1-anx)\,d}{2a^2\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^3(1+n)\sqrt{c-a^2cx^2}} - \frac{x(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}} + \frac{\left(\left(1+n^2\right)\right.}{}$$

$$= \frac{(1-n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^3(1+n)\sqrt{c-a^2cx^2}} - \frac{x(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2a^2\sqrt{c-a^2cx^2}} - \frac{2^{\frac{1+n}{2}}\left(1+\right.}{}$$

Mathematica [A] time = 0.18, size = 141, normalized size = 0.56

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}\left(n^2+1\right){}_2F_1\left(-\frac{n}{2}-\frac{1}{2},\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)-(n-1)(ax+1)^{\frac{n+1}{2}}(anx+ax+n-1)\right)}{2a^3\left(n^2-1\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)*(1 + a*x)^((1 + n)/2)*
(-1 + n + a*x + a*n*x)) + 2^((3 + n)/2)*(1 + n^2)*Hypergeometric2F1[-1/2 -
n/2, 1/2 - n/2, 3/2 - n/2, 1/2 - (a*x)/2]))/(2*a^3*(-1 + n^2)*Sqrt[c - a^2*
c*x^2])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2
 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^{2}cx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{{\rm e}^{n\,\mathrm{arctanh}\left(ax\right)}x^{2}}{\sqrt{-a^{2}c\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^{2}cx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2}\,{\rm e}^{n\,\mathrm{atanh}\left(a\,x\right)}}{\sqrt{c-a^{2}\,c\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int((x^2*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 e^{n \operatorname{atanh}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.1336 $\int \frac{e^{n\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2^{\frac{n+3}{2}}n\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{a^2(n+1)\sqrt{c-a^2cx^2}}$$

```
[Out] -(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/a^2/(1+n)/(-a^
2*c*x^2+c)^(1/2)-2^(3/2+1/2*n)*n*(-a*x+1)^(1/2-1/2*n)*hypergeom([1/2-1/2*n,
 -1/2-1/2*n],[3/2-1/2*n],-1/2*a*x+1/2)*(-a^2*x^2+1)^(1/2)/a^2/(-n^2+1)/(-a^
2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6153, 6150, 79, 69}

$$-\frac{2^{\frac{n+3}{2}}n\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^2\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{a^2(n+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^2*(1 +
 n)*Sqrt[c - a^2*c*x^2])) - (2^((3 + n)/2)*n*(1 - a*x)^((1 - n)/2)*Sqrt[1 -
 a^2*x^2]*Hypergeometric2F1[(-1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])
/(a^2*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
```

```
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}x}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int x(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{a^2(1+n)\sqrt{c-a^2cx^2}}+\frac{\left(n\sqrt{1-a^2x^2}\right)\int(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{\frac{1+n}{2}}\,dx}{a(1+n)\sqrt{c-a^2cx^2}}\\ &= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{a^2(1+n)\sqrt{c-a^2cx^2}}-\frac{2^{\frac{3+n}{2}}n(1-ax)^{\frac{1-n}{2}}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{1-n}{2};\frac{3-n}{2};\right.}{a^2\left(1-n^2\right)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 124, normalized size = 0.70

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}\left(2^{\frac{n+3}{2}}n\,{}_2F_1\left(-\frac{n}{2}-\frac{1}{2},\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)-(n-1)(ax+1)^{\frac{n+1}{2}}\right)}{a^2\left(n^2-1\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)*(1 + a*x)^((1 + n)/2))
 + 2^((3 + n)/2)*n*Hypergeometric2F1[-1/2 - n/2, 1/2 - n/2, 3/2 - n/2, 1/2
- (a*x)/2]))/(a^2*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 -
 c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}x}{\sqrt{-a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(1/2),x)
[Out] int((x*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{xe^{n\,\mathrm{atanh}\,(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x*exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.1337 $\int \frac{e^{n \tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}} dx$

Optimal. Leaf size=104

$$-\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] -2^(1/2+1/2*n)*(-a*x+1)^(1/2-1/2*n)*hypergeom([1/2-1/2*n, 1/2-1/2*n],[3/2-1
/2*n],-1/2*a*x+1/2)*(-a^2*x^2+1)^(1/2)/a/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6143, 6140, 69}

$$-\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a(1-n)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -((2^((1 + n)/2)*(1 - a*x)^((1 - n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[
(1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/(a*(1 - n)*Sqrt[c - a^2*c*x^
2]))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
```

```
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= -\frac{2^{\frac{1+n}{2}}(1-ax)^{\frac{1-n}{2}}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a(1-n)\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 101, normalized size = 0.97

$$\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}-\frac{n}{2}}\,{}_2F_1\left(\frac{1}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{a(n-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2^((1 + n)/2)*(1 - a*x)^(1/2 - n/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1/
2 - n/2, 1/2 - n/2, 3/2 - n/2, 1/2 - (a*x)/2])/(a*(-1 + n)*Sqrt[c - a^2*c*x
^2])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{\sqrt{-a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a^2*c*x^2)^(1/2),x)
[Out] int(exp(n*atanh(a*x))/(c - a^2*c*x^2)^(1/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.1338 $\int \frac{e^{n \tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,(ax+1)^{\frac{n-1}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] -2*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*hypergeom([1, 1/2-1/2*n],[3/2-
1/2*n],(-a*x+1)/(a*x+1))*(-a^2*x^2+1)^(1/2)/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 131}

$$-\frac{2\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,(ax+1)^{\frac{n-1}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeo
metric2F1[1, (1 - n)/2, (3 - n)/2, (1 - a*x)/(1 + a*x)])/((1 - n)*Sqrt[c -
a^2*c*x^2])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\tanh^{-1}(ax)}}{x\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{1+ax}\right)}{(1-n)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 99, normalized size = 0.98

$$\frac{2\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}-\frac{n}{2}}(ax+1)^{\frac{n-1}{2}}\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\frac{1-ax}{ax+1}\right)}{(n-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*(1 - a*x)^(1/2 - n/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeom
etric2F1[1, 1/2 - n/2, 3/2 - n/2, (1 - a*x)/(1 + a*x)])/((-1 + n)*Sqrt[c -
a^2*c*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^3-cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^3 - c
*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x\sqrt{-a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)^(1/2)),x)
[Out] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{x\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*atanh(a*x))/(x*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

3.1339 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{2an\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{x\sqrt{c-a^2cx^2}}$$

```
[Out] -(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/x/(-a^2*c*x^2+
c)^(1/2)-2*a*n*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*hypergeom([1, 1/2-
1/2*n],[3/2-1/2*n],(-a*x+1)/(a*x+1))*(-a^2*x^2+1)^(1/2)/(1-n)/(-a^2*c*x^2+c
)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6153, 6150, 96, 131}

$$-\frac{2an\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{x\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] -(((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(x*Sqrt[c
 - a^2*c*x^2])) - (2*a*n*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[
1 - a^2*x^2]*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (1 - a*x)/(1 + a*x)
])/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
```

```
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx}{\sqrt{c-a^2cx^2}}\\
&= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}+\frac{\left(an\sqrt{1-a^2x^2}\right)\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx}{\sqrt{c-a^2cx^2}}\\
&= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{x\sqrt{c-a^2cx^2}}-\frac{2an(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}\,{}_2F_1\left(1,\frac{1-n}{2}\right.}{(1-n)\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 117, normalized size = 0.70

$$\frac{\sqrt{1-a^2x^2}(1-ax)^{\frac{1}{2}-\frac{n}{2}}(ax+1)^{\frac{n-1}{2}}\left(2anx\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\frac{1-ax}{ax+1}\right)-(n-1)(ax+1)\right)}{(n-1)x\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)
*(1 + a*x)) + 2*a*n*x*Hypergeometric2F1[1, 1/2 - n/2, 3/2 - n/2, (1 - a*x)/
(1 + a*x)]))/((-1 + n)*x*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^4-cx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^4 - c
*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x^2), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)}}{x^2\sqrt{-a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x^2\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

3.1340 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=242

$$-\frac{a^2(n^2+1)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}-\frac{an\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{2x\sqrt{c-a^2cx^2}}-\sqrt{1-\ldots}$$

```
[Out] -1/2*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/x^2/(-a^2*
c*x^2+c)^(1/2)-1/2*a*n*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(1/2+1/2*n)*(-a^2*x^2+1
)^(1/2)/x/(-a^2*c*x^2+c)^(1/2)-a^2*(n^2+1)*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1
/2+1/2*n)*hypergeom([1, 1/2-1/2*n],[3/2-1/2*n],(-a*x+1)/(a*x+1))*(-a^2*x^2+
1)^(1/2)/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 151, 12, 131}

$$-\frac{a^2(n^2+1)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{1-ax}{ax+1}\right)}{(1-n)\sqrt{c-a^2cx^2}}-\frac{an\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n+1}{2}}(1-ax)^{\frac{1-n}{2}}}{2x\sqrt{c-a^2cx^2}}-\sqrt{1-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] -((1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*x^2*Sqr
t[c - a^2*c*x^2]) - (a*n*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((1 + n)/2)*Sqrt[1
 - a^2*x^2])/(2*x*Sqrt[c - a^2*c*x^2]) - (a^2*(1 + n^2)*(1 - a*x)^((1 - n)/
2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (1 - n)/2,
 (3 - n)/2, (1 - a*x)/(1 + a*x)])/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}}{x^3}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\left(-an-a^2x\right)}{x^2}\,dx}{2\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}\int \ldots}{\ldots}$$

$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x\sqrt{c-a^2cx^2}} + \frac{\left(a^2\left(1+n^2\right)\sqrt{\ldots}\right.}{\ldots}$$

$$= -\frac{(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1+n}{2}}\sqrt{1-a^2x^2}}{2x\sqrt{c-a^2cx^2}} - \frac{a^2\left(1+n^2\right)(1\ldots}{\ldots}$$

Mathematica [A] time = 0.14, size = 134, normalized size = 0.55

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}-\frac{n}{2}}(ax+1)^{\frac{n-1}{2}}\left(2a^2\left(n^2+1\right)x^2\,{}_2F_1\left(1,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\frac{1-ax}{ax+1}\right)-(n-1)(ax+1)(anx+1)\right)}{2(n-1)x^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^3*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] ((1 - a*x)^(1/2 - n/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-1 + n)
*(1 + a*x)*(1 + a*n*x)) + 2*a^2*(1 + n^2)*x^2*Hypergeometric2F1[1, 1/2 - n/
2, 3/2 - n/2, (1 - a*x)/(1 + a*x)]))/(2*(-1 + n)*x^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^5-cx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^5 - c
*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x^3), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x^3\sqrt{-a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(sqrt(-a^2*c*x^2 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x^3\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x^3*(c - a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(exp(n*atanh(a*x))/(x^3*(c - a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{x^3 \sqrt{-c (ax-1) (ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

3.1341 $\int \frac{e^{n\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{x^2\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{a^2c\sqrt{c-a^2cx^2}}-\frac{2^{\frac{n-1}{2}}n\sqrt{1-a^2x^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{3-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a^4c(3-n)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}}{}$$

```
[Out] -x^2*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/a^2/c/(-
a^2*c*x^2+c)^(1/2)+(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(2+2*n+n^2-a*
n*(3+2*n)*x)*(-a^2*x^2+1)^(1/2)/a^4/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)-2^(-1/2
+1/2*n)*n*(-a*x+1)^(3/2-1/2*n)*hypergeom([3/2-1/2*n, 3/2-1/2*n],[5/2-1/2*n]
,-1/2*a*x+1/2)*(-a^2*x^2+1)^(1/2)/a^4/c/(3-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6153, 6150, 100, 145, 69}

$$-\frac{2^{\frac{n-1}{2}}n\sqrt{1-a^2x^2}\,(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(\frac{3-n}{2},\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-ax)\right)}{a^4c(3-n)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}\left(-a(2n+3)nx+n^2+2n+2\right.}{a^4c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((x^2*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^
2*c*Sqrt[c - a^2*c*x^2])) + ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*
(2 + 2*n + n^2 - a*n*(3 + 2*n)*x)*Sqrt[1 - a^2*x^2])/(a^4*c*(1 - n^2)*Sqrt[
c - a^2*c*x^2]) - (2^((-1 + n)/2)*n*(1 - a*x)^((3 - n)/2)*Sqrt[1 - a^2*x^2]
*Hypergeometric2F1[(3 - n)/2, (3 - n)/2, (5 - n)/2, (1 - a*x)/2])/(a^4*c*(3
 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int x^3(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{x^2(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{a^2c\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int x(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}(\cdots}{a^2c\sqrt{c-a^2cx^2}}$$

$$= -\frac{x^2(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{a^2c\sqrt{c-a^2cx^2}} + \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(2+2n+n\cdots\right.}{a^4c\left(1-n^2\right)\sqrt{c-a\cdots}}$$

$$= -\frac{x^2(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{a^2c\sqrt{c-a^2cx^2}} + \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\left(2+2n+n\cdots\right.}{a^4c\left(1-n^2\right)\sqrt{c-a\cdots}}$$

Mathematica [A] time = 0.29, size = 186, normalized size = 0.69

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{-\frac{n}{2}-\frac{1}{2}}\left(-4a^4x^2(ax+1)^{\frac{n-1}{2}} + \frac{a^2 2^{\frac{n+3}{2}} n(ax-1)^2\,{}_2F_1\left(\frac{3}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2};\frac{5}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)}{n-3} + \frac{4a^2\left(n^2(2ax-1)+n(3ax-2)-2\right)(ax+1)^{\frac{n\cdots}{}}}{n^2-1}\cdots\right.}{4a^6c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^(-1/2 - n/2)*Sqrt[1 - a^2*x^2]*(-4*a^4*x^2*(1 + a*x)^((-1 + n)/2
) + (4*a^2*(1 + a*x)^((-1 + n)/2)*(-2 + n^2*(-1 + 2*a*x) + n*(-2 + 3*a*x)))
/(-1 + n^2) + (2^((3 + n)/2)*a^2*n*(-1 + a*x)^2*Hypergeometric2F1[3/2 - n/2
, 3/2 - n/2, 5/2 - n/2, 1/2 - (a*x)/2])/(-3 + n)))/(4*a^6*c*Sqrt[c - a^2*c*
x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)} x^3}{\left(-a^2 c\, x^2 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{\left(-a^2 c x^2 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\, e^{n \operatorname{atanh}(a\, x)}}{\left(c - a^2\, c\, x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^3*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{n \operatorname{atanh}(ax)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3*exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.1342 $\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^3c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\tanh^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arctanh(a*x))*(-a*x+n)/a^3/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)+2^(1/2+1/
2*n)*(-a*x+1)^(1/2-1/2*n)*hypergeom([1/2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],-1/2
*a*x+1/2)*(-a^2*x^2+1)^(1/2)/a^3/c/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6147, 6143, 6140, 69}

$$\frac{2^{\frac{n+1}{2}}\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^3c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\tanh^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) + (
2^((1 + n)/2)*(1 - a*x)^((1 - n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[(1
- n)/2, (1 - n)/2, (3 - n)/2, (1 - a*x)/2])/(a^3*c*(1 - n)*Sqrt[c - a^2*c*x
^2])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6147

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> -Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a
*d*(n^2 - 4*(p + 1)^2)), x] + Dist[(n^2 + 2*(p + 1))/(d*(n^2 - 4*(p + 1)^2)
), Int[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}
, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p
+ 1)^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx}{a^2c}\\
&= -\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{\sqrt{1-a^2x^2}}\,dx}{a^2c\sqrt{c-a^2cx^2}}\\
&= -\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\int (1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{1}{2}+\frac{n}{2}}\,dx}{a^2c\sqrt{c-a^2cx^2}}\\
&= -\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{2^{\frac{1+n}{2}}(1-ax)^{\frac{1-n}{2}}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-ax)\right)}{a^3c(1-n)\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 155, normalized size = 1.01

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{-\frac{n}{2}-\frac{1}{2}}\left(2^{\frac{n+1}{2}}(n+1)(ax-1)\sqrt{ax+1}\,{}_2F_1\left(\frac{1}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};\frac{1}{2}-\frac{ax}{2}\right)+(n-ax)(ax+1)^{n/2}\right)}{a^3c(n-1)(n+1)\sqrt{ax+1}\,\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^(-1/2 - n/2)*Sqrt[1 - a^2*x^2]*((n - a*x)*(1 + a*x)^(n/2) + 2^((
1 + n)/2)*(1 + n)*(-1 + a*x)*Sqrt[1 + a*x]*Hypergeometric2F1[1/2 - n/2, 1/2
 - n/2, 3/2 - n/2, 1/2 - (a*x)/2]))/(a^3*c*(-1 + n)*(1 + n)*Sqrt[1 + a*x]*S
qrt[c - a^2*c*x^2])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{arctanh}(ax)}x^2}{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^2*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 e^{n\,\mathrm{atanh}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.1343 $\int \frac{e^{n\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{(1-anx)e^{n\tanh^{-1}(ax)}}{a^2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] exp(n*arctanh(a*x))*(-a*n*x+1)/a^2/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6144}

$$\frac{(1-anx)e^{n\tanh^{-1}(ax)}}{a^2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcTanh[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6144

```
Int[(E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[((1 - a*n*x)*E^(n*ArcTanh[a*x]))/(d*(n^2 - 1)*Sqrt[c + d*x^2]),
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx=\frac{e^{n\tanh^{-1}(ax)}(1-anx)}{a^2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 81, normalized size = 1.76

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{-\frac{n}{2}-\frac{1}{2}}(ax+1)^{\frac{n-1}{2}}(anx-1)}{a^2c(n-1)(n+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^(-1/2 - n/2)*(1 + a*x)^((-1 + n)/2)*(-1 + a*n*x)*Sqrt[1 - a^2*x^
2])/(a^2*c*(-1 + n)*(1 + n)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.59, size = 82, normalized size = 1.78

$$\frac{\sqrt{-a^2cx^2+c}\,(anx-1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2c^2n^2-a^2c^2-\left(a^4c^2n^2-a^4c^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*(a*n*x - 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*n^2
 - a^2*c^2 - (a^4*c^2*n^2 - a^4*c^2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.03, size = 49, normalized size = 1.07

$$-\frac{(ax-1)\,(ax+1)\,(nax-1)\,\mathrm{e}^{n\,\mathrm{arctanh}(ax)}}{a^2\left(n^2-1\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(a*n*x-1)*exp(n*arctanh(a*x))/a^2/(n^2-1)/(-a^2*c*x^2+c)^(
3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [B] time = 1.08, size = 68, normalized size = 1.48

$$-\frac{e^{\frac{n\,\ln(a\,x+1)}{2}-\frac{n\,\ln(1-a\,x)}{2}}\,\left(\frac{1}{a^2\,c\,\left(n^2-1\right)}-\frac{n\,x}{a\,c\,\left(n^2-1\right)}\right)}{\sqrt{c-a^2\,c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] -(exp((n*log(a*x + 1))/2 - (n*log(1 - a*x))/2)*(1/(a^2*c*(n^2 - 1)) - (n*x)
/(a*c*(n^2 - 1))))/(c - a^2*c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{xe^{n\,\mathrm{atanh}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.1344 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{(n-ax)e^{n\tanh^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arctanh(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6135}

$$-\frac{(n-ax)e^{n\tanh^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{e^{n\tanh^{-1}(ax)}(n-ax)}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 81, normalized size = 1.76

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-1)}(n-ax)(ax+1)^{\frac{n-1}{2}}}{ac(n-1)(n+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(n - a*x)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])
/(a*c*(-1 + n)*(1 + n)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.41, size = 80, normalized size = 1.74

$$-\frac{\sqrt{-a^2cx^2+c}\,(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-\left(a^3c^2n^2-a^3c^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 -
a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.03, size = 49, normalized size = 1.07

$$\frac{(ax-1)(ax+1)(ax-n)\,\mathrm{e}^{n\,\mathrm{arctanh}(ax)}}{\left(n^2-1\right)a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(a*x-n)*exp(n*arctanh(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [B] time = 1.10, size = 65, normalized size = 1.41

$$-\frac{e^{\frac{n\,\ln(a\,x+1)}{2}-\frac{n\,\ln(1-a\,x)}{2}}\left(\frac{x}{c\,\left(n^2-1\right)}-\frac{n}{a\,c\,\left(n^2-1\right)}\right)}{\sqrt{c-a^2\,c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] -(exp((n*log(a*x + 1))/2 - (n*log(1 - a*x))/2)*(x/(c*(n^2 - 1)) - n/(a*c*(n
^2 - 1))))/(c - a^2*c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.1345 $\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=243

$$\frac{2\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{ax+1}{1-ax}\right)}{c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n+2)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,(a\cdots}{c(n+\cdots}$$

```
[Out] (-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c/(1+n)/(-a^2
*c*x^2+c)^(1/2)-(2+n)*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1
)^(1/2)/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)+2*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/
2+1/2*n)*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a*x+1)/(-a*x+1))*(-a^2*x^2+
1)^(1/2)/c/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 247, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 155, 12, 131}

$$-\frac{2\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c(3-n)\sqrt{c-a^2cx^2}}-\frac{(n+2)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,(\cdots}{c(n\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(c*(1 + n
)*Sqrt[c - a^2*c*x^2]) - ((2 + n)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)
/2)*Sqrt[1 - a^2*x^2])/(c*(1 - n^2)*Sqrt[c - a^2*c*x^2]) - (2*(1 - a*x)^((3
 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 -
 n)/2, (5 - n)/2, (1 - a*x)/(1 + a*x)])/(c*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}}{x}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{1}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\left(-a(1+n)-a^2x\right)}{x}\,d}{ac(1+n)\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(2+n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}} +$$

$$= \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(2+n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}} +$$

$$= \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(2+n)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}} -$$

Mathematica [A] time = 0.16, size = 149, normalized size = 0.61

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-1)}(ax+1)^{\frac{n-3}{2}}\left(2\left(n^2-1\right)(ax-1)^2\,{}_2F_1\left(1,\frac{3}{2}-\frac{n}{2};\frac{5}{2}-\frac{n}{2};\frac{1-ax}{ax+1}\right)-(n-3)(ax+1)(n(ax-2)+\right.}{c(n-3)(n-1)(n+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-3 + n
)*(1 + a*x)*(-1 + 2*a*x + n*(-2 + a*x))) + 2*(-1 + n^2)*(-1 + a*x)^2*Hyperg
eometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1 - a*x)/(1 + a*x)]))/(c*(-3 + n)*(-1
 + n)*(1 + n)*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^5-2\,a^2c^2x^3+c^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^5 -
2*a^2*c^2*x^3 + c^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x\,\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)^(3/2)),x)
[Out] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(n*atanh(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.1346 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=321

$$\frac{2an\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{ax+1}{1-ax}\right)}{c(1-n)\sqrt{c-a^2cx^2}}-\frac{a\left(n^2+2n+2\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+$$

```
[Out] a*(2+n)*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c/(1+
n)/(-a^2*c*x^2+c)^(1/2)-(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^
2+1)^(1/2)/c/x/(-a^2*c*x^2+c)^(1/2)-a*(n^2+2*n+2)*(-a*x+1)^(1/2-1/2*n)*(a*x
+1)^(-1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)+2*a*n*(
-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*hypergeom([1, -1/2+1/2*n],[1/2+1/2
*n],(a*x+1)/(-a*x+1))*(-a^2*x^2+1)^(1/2)/c/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 325, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 155, 12, 131}

$$-\frac{2an\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c(3-n)\sqrt{c-a^2cx^2}}-\frac{a\left(n^2+2n+2\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}}{c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a*(2 + n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])
/(c*(1 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 +
 n)/2)*Sqrt[1 - a^2*x^2])/(c*x*Sqrt[c - a^2*c*x^2]) - (a*(2 + 2*n + n^2)*(1
 - a*x)^((1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(c*(1 - n^2)*
Sqrt[c - a^2*c*x^2]) - (2*a*n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*
Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (1 - a*x)/(1 +
 a*x)])/(c*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}}{x^2}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\left(-an-2a^2x\right.}{x}}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-}}{cx\sqrt{c-a^2cx^2}}$$

$$= \frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-}}{cx\sqrt{c-a^2cx^2}}$$

$$= \frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-}}{cx\sqrt{c-a^2cx^2}}$$

$$= \frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-}}{cx\sqrt{c-a^2cx^2}}$$

<u>Mathematica</u> [A] time = 0.20, size = 173, normalized size = 0.54

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-1)}(ax+1)^{\frac{n-3}{2}}\left(2an\left(n^2-1\right)x(ax-1)^2\,{}_2F_1\left(1,\frac{3}{2}-\frac{n}{2};\frac{5}{2}-\frac{n}{2};\frac{1-ax}{ax+1}\right)-(n-3)(ax+1)\left(2a^2x^2\right.\right.}{c(n-3)(n-1)(n+1)x\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-3 + n
)*(1 + a*x)*(-1 + 2*a^2*x^2 + n^2*(-1 + a*x)^2 + a*n*x*(-3 + 2*a*x))) + 2*a
*n*(-1 + n^2)*x*(-1 + a*x)^2*Hypergeometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1
- a*x)/(1 + a*x)]))/(c*(-3 + n)*(-1 + n)*(1 + n)*x*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^6-2\,a^2c^2x^4+c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^6 -
2*a^2*c^2*x^4 + c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x^2), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int\frac{e^{n\,\mathrm{arctanh}(ax)}}{x^2\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}}{x^2\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}{\left(ax \right)}}}{x^{2} \left(- c \left(ax - 1\right) \left(ax + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(x**2*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.1347 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=417

$$\frac{a^2\left(n^2+3\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{ax+1}{1-ax}\right)}{c(1-n)\sqrt{c-a^2cx^2}}+\frac{a^2\left(n^2+2n+3\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-a}{2c(n+1)\sqrt{c-a^2cx^2}}$$

```
[Out] 1/2*a^2*(n^2+2*n+3)*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1)
^(1/2)/c/(1+n)/(-a^2*c*x^2+c)^(1/2)-1/2*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2
+1/2*n)*(-a^2*x^2+1)^(1/2)/c/x^2/(-a^2*c*x^2+c)^(1/2)-1/2*a*n*(-a*x+1)^(-1/
2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c/x/(-a^2*c*x^2+c)^(1/2)-1
/2*a^2*(n^3+2*n^2+5*n+6)*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^
2+1)^(1/2)/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)+a^2*(n^2+3)*(-a*x+1)^(1/2-1/2*n)
*(a*x+1)^(-1/2+1/2*n)*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a*x+1)/(-a*x+1
))*(-a^2*x^2+1)^(1/2)/c/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 422, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6153, 6150, 129, 151, 155, 12, 131}

$$-\frac{a^2\left(n^2+3\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c(3-n)\sqrt{c-a^2cx^2}}+\frac{a^2\left(n^2+2n+3\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-}{2c(n+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^2*(3 + 2*n + n^2)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 -
 a^2*x^2])/(2*c*(1 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-1 - n)/2)*(1 +
 a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c*x^2*Sqrt[c - a^2*c*x^2]) - (a*n*
(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c*x*Sqr
t[c - a^2*c*x^2]) - (a^2*(6 + 5*n + 2*n^2 + n^3)*(1 - a*x)^((1 - n)/2)*(1 +
 a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]) -
 (a^2*(3 + n^2)*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x
^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (1 - a*x)/(1 + a*x)])/(c*(3
- n)*Sqrt[c - a^2*c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
```

```
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}}{x^3}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{3}{2}+\frac{n}{2}}\left(-an-3a^2x\right)}{x^2}}{2c\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x}}{2cx\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(3+2n+n^2\right)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-}}{2cx^2\sqrt{c-a^2c}}$$

$$= \frac{a^2\left(3+2n+n^2\right)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-}}{2cx^2\sqrt{c-a^2c}}$$

$$= \frac{a^2\left(3+2n+n^2\right)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-}}{2cx^2\sqrt{c-a^2c}}$$

$$= \frac{a^2\left(3+2n+n^2\right)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-}}{2cx^2\sqrt{c-a^2c}}$$

Mathematica [A] time = 0.25, size = 219, normalized size = 0.53

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-1)}(ax+1)^{\frac{n-3}{2}}\left(2a^2\left(n^4+2n^2-3\right)x^2(ax-1)^2\,{}_2F_1\left(1,\frac{3}{2}-\frac{n}{2};\frac{5}{2}-\frac{n}{2};\frac{1-ax}{ax+1}\right)-(n-3)(ax+1)\right.}{2c(n-3)(n-1)(n+1)x^2\sqrt{c-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] ((1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-((-3 + n
)*(1 + a*x)*(-1 - 3*a^2*x^2 + 6*a^3*x^3 + a*n^3*x*(-1 + a*x)^2 + n^2*(-1 +
```

```
a*x)^2*(1 + 2*a*x) + a*n*x*(-1 - 6*a*x + 5*a^2*x^2))) + 2*a^2*(-3 + 2*n^2 +
 n^4)*x^2*(-1 + a*x)^2*Hypergeometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1 - a*x)
/(1 + a*x)]))/(2*c*(-3 + n)*(-1 + n)*(1 + n)*x^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^7-2a^2c^2x^5+c^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^7 -
2*a^2*c^2*x^5 + c^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x^3), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x^3\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x^3\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x^3*(c - a^2*c*x^2)^(3/2)),x)
```

```
[Out] int(exp(n*atanh(a*x))/(x^3*(c - a^2*c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}{\left(ax \right)}}}{x^{3} \left(- c \left(ax - 1\right) \left(ax + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(x**3*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.1348 $\int \frac{e^{n\tanh^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=407

$$\frac{x^3\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1}{2}(-n-3)}}{ac^2(n+3)\sqrt{c-a^2cx^2}}-\frac{3(2-n)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}}+\frac{3\left(-n^2+2n+1\right)\sqrt{1-a^2x^2}}{a^4c^2(3-n)(n+1)}$$

```
[Out] x^3*(-a*x+1)^(-3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/2)/a/c^2/(3+
n)/(-a^2*c*x^2+c)^(1/2)-3*(2-n)*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*
(-a^2*x^2+1)^(1/2)/a^4/c^2/(-n^2+9)/(-a^2*c*x^2+c)^(1/2)-3*x*(-a*x+1)^(-1/2
-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/2)/a^3/c^2/(3+n)/(-a^2*c*x^2+c
)^(1/2)+3*(-n^2+2*n+1)*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2
+1)^(1/2)/a^4/c^2/(-n^3-n^2+9*n+9)/(-a^2*c*x^2+c)^(1/2)-3*(-n^2+2*n+1)*(-a*
x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+1/2*n)*(-a^2*x^2+1)^(1/2)/a^4/c^2/(n^4-10*n^
2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6153, 6150, 94, 90, 79, 45, 37}

$$-\frac{3(2-n)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}}+\frac{3\left(-n^2+2n+1\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1}{2}(-n-1)}}{a^4c^2(3-n)(n+1)(n+3)\sqrt{c-a^2cx^2}}-\frac{3\left(-n^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (x^3*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(a*c^
2*(3 + n)*Sqrt[c - a^2*c*x^2]) - (3*(2 - n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x
)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(a^4*c^2*(9 - n^2)*Sqrt[c - a^2*c*x^2]) -
 (3*x*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(a^3
*c^2*(3 + n)*Sqrt[c - a^2*c*x^2]) + (3*(1 + 2*n - n^2)*(1 - a*x)^((-1 - n)/
2)*(1 + a*x)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^4*c^2*(3 - n)*(1 + n)*(3 +
n)*Sqrt[c - a^2*c*x^2]) - (3*(1 + 2*n - n^2)*(1 - a*x)^((1 - n)/2)*(1 + a*x
)^((-1 + n)/2)*Sqrt[1 - a^2*x^2])/(a^4*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*
c*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
```

```
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}x^3}{\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int x^3(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{\left(3\sqrt{1-a^2x^2}\right)\int x^2(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}}{ac^2(3+n)\sqrt{c-a^2cx^2}}$$

$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{3x(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{a^3c^2(3+n)\sqrt{c-a^2cx^2}} -$$

$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{3(2-n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}}$$

$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{3(2-n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}}$$

$$= \frac{x^3(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{ac^2(3+n)\sqrt{c-a^2cx^2}} - \frac{3(2-n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a}}{a^4c^2\left(9-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.22, size = 112, normalized size = 0.28

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(-a^3n\left(n^2-7\right)x^3+3a^2\left(n^2-3\right)x^2-6anx+6\right)}{a^4c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(6 - 6*a
*n*x + 3*a^2*(-3 + n^2)*x^2 - a^3*n*(-7 + n^2)*x^3))/(a^4*c^2*(9 - 10*n^2 +
 n^4)*Sqrt[c - a^2*c*x^2]))
```

fricas [A] time = 0.56, size = 175, normalized size = 0.43

$$\frac{\sqrt{-a^2cx^2+c}\left(\left(a^3n^3-7\,a^3n\right)x^3+6\,anx-3\left(a^2n^2-3\,a^2\right)x^2-6\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3+\left(a^8c^3n^4-10\,a^8c^3n^2+9\,a^8c^3\right)x^4-2\left(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*((a^3*n^3 - 7*a^3*n)*x^3 + 6*a*n*x - 3*(a^2*n^2 - 3*a^
2)*x^2 - 6)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^3*n^4 - 10*a^4*c^3*n^2 + 9
*a^4*c^3 + (a^8*c^3*n^4 - 10*a^8*c^3*n^2 + 9*a^8*c^3)*x^4 - 2*(a^6*c^3*n^4
- 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 93, normalized size = 0.23

$$-\frac{(ax-1)\,(ax+1)\left(a^3n^3x^3-7x^3a^3n-3a^2n^2x^2+9a^2x^2+6nax-6\right){\rm e}^{n\,\mathrm{arctanh}(ax)}}{a^4\left(n^4-10n^2+9\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(a^3*n^3*x^3-7*a^3*n*x^3-3*a^2*n^2*x^2+9*a^2*x^2+6*a*n*x-6
)*exp(n*arctanh(a*x))/a^4/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 1.28, size = 162, normalized size = 0.40

$$-\frac{(a\,x+1)^{n/2}\,\left(\frac{6}{a^6\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{6\,n\,x}{a^5\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{x^2\,\left(3\,n^2-9\right)}{a^4\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{n\,x^3\,\left(n^2-7\right)}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{(1-a\,x)^{n/2}\,\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -((a*x + 1)^(n/2)*(6/(a^6*c^2*(n^4 - 10*n^2 + 9)) - (6*n*x)/(a^5*c^2*(n^4 -
 10*n^2 + 9)) + (x^2*(3*n^2 - 9))/(a^4*c^2*(n^4 - 10*n^2 + 9)) - (n*x^3*(n^
2 - 7))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/((1 - a*x)^(n/2)*((c - a^2*c*x^2)^(1
/2)/a^2 - x^2*(c - a^2*c*x^2)^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{n\,\mathrm{atanh}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.1349 $\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{(3-n^2)(n-ax)e^{n\tanh^{-1}(ax)}}{a^3c^2(n^4-10n^2+9)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\tanh^{-1}(ax)}}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}$$

```
[Out] -exp(n*arctanh(a*x))*(-3*a*x+n)/a^3/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+exp(n*a
rctanh(a*x))*(-n^2+3)*(-a*x+n)/a^3/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6147, 6135}

$$\frac{(3-n^2)(n-ax)e^{n\tanh^{-1}(ax)}}{a^3c^2(n^4-10n^2+9)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\tanh^{-1}(ax)}}{a^3c(9-n^2)(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
 + (E^(n*ArcTanh[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(9 - 10*n^2 + n^4)*Sqr
t[c - a^2*c*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6147

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> -Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a
*d*(n^2 - 4*(p + 1)^2)), x] + Dist[(n^2 + 2*(p + 1))/(d*(n^2 - 4*(p + 1)^2)
), Int[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}
, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p
+ 1)^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-3ax)}{a^3c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}} - \frac{\left(3-n^2\right)\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{a^2c\left(9-n^2\right)}$$
$$= -\frac{e^{n\tanh^{-1}(ax)}(n-3ax)}{a^3c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}} + \frac{e^{n\tanh^{-1}(ax)}\left(3-n^2\right)(n-ax)}{a^3c^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.22, size = 125, normalized size = 1.23

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(-3a^3x^3-a^2n^3x^2+an^2x\left(a^2x^2+2\right)+n\left(3a^2x^2-2\right)\right)}{a^3c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-(a^2*n
^3*x^2) - 3*a^3*x^3 + a*n^2*x*(2 + a^2*x^2) + n*(-2 + 3*a^2*x^2)))/(a^3*c^2
*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2]))
```

fricas [A] time = 0.50, size = 180, normalized size = 1.76

$$-\frac{\sqrt{-a^2cx^2+c}\left(2\,an^2x+\left(a^3n^2-3\,a^3\right)x^3-\left(a^2n^3-3\,a^2n\right)x^2-2\,n\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3+\left(a^7c^3n^4-10\,a^7c^3n^2+9\,a^7c^3\right)x^4-2\left(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(2*a*n^2*x + (a^3*n^2 - 3*a^3)*x^3 - (a^2*n^3 - 3*a^2
*n)*x^2 - 2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*n^4 - 10*a^3*c^3*n^2
+ 9*a^3*c^3 + (a^7*c^3*n^4 - 10*a^7*c^3*n^2 + 9*a^7*c^3)*x^4 - 2*(a^5*c^3*n
^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.03, size = 96, normalized size = 0.94

$$\frac{(ax-1)(ax+1)\left(a^{3}n^{2}x^{3}-a^{2}n^{3}x^{2}-3x^{3}a^{3}+3n\,x^{2}a^{2}+2n^{2}xa-2n\right)\mathrm{e}^{n\,\mathrm{arctanh}(ax)}}{\left(n^{4}-10n^{2}+9\right)a^{3}\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(a^3*n^2*x^3-a^2*n^3*x^2-3*a^3*x^3+3*a^2*n*x^2+2*a*n^2*x-2*
n)*exp(n*arctanh(a*x))/(n^4-10*n^2+9)/a^3/(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 1.23, size = 162, normalized size = 1.59

$$\frac{\mathrm{e}^{\frac{n\,\ln(a\,x+1)}{2}-\frac{n\,\ln(1-a\,x)}{2}}\left(\frac{2\,n}{a^{5}\,c^{2}\left(n^{4}-10\,n^{2}+9\right)}-\frac{x^{3}\left(n^{2}-3\right)}{a^{2}\,c^{2}\left(n^{4}-10\,n^{2}+9\right)}-\frac{2\,n^{2}\,x}{a^{4}\,c^{2}\left(n^{4}-10\,n^{2}+9\right)}+\frac{n\,x^{2}\left(n^{2}-3\right)}{a^{3}\,c^{2}\left(n^{4}-10\,n^{2}+9\right)}\right)}{\frac{\sqrt{c-a^{2}\,c\,x^{2}}}{a^{2}}-x^{2}\,\sqrt{c-a^{2}\,c\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] (exp((n*log(a*x + 1))/2 - (n*log(1 - a*x))/2)*((2*n)/(a^5*c^2*(n^4 - 10*n^2
 + 9)) - (x^3*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n^2*x)/(a^4*c^2*
(n^4 - 10*n^2 + 9)) + (n*x^2*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/((c
- a^2*c*x^2)^(1/2)/a^2 - x^2*(c - a^2*c*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 e^{n \operatorname{atanh}(ax)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.1350 $\int \frac{e^{n\tanh^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{2n(n-ax)e^{n\tanh^{-1}(ax)}}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}+\frac{n(n-3ax)e^{n\tanh^{-1}(ax)}}{3a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{e^{n\tanh^{-1}(ax)}}{3a^2c\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/3*exp(n*arctanh(a*x))/a^2/c/(-a^2*c*x^2+c)^(3/2)+1/3*exp(n*arctanh(a*x))*
n*(-3*a*x+n)/a^2/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+2*exp(n*arctanh(a*x))*n*(-
a*x+n)/a^2/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6145, 6136, 6135}

$$\frac{2n(n-ax)e^{n\tanh^{-1}(ax)}}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}+\frac{n(n-3ax)e^{n\tanh^{-1}(ax)}}{3a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{e^{n\tanh^{-1}(ax)}}{3a^2c\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] E^(n*ArcTanh[a*x])/(3*a^2*c*(c - a^2*c*x^2)^(3/2)) + (E^(n*ArcTanh[a*x])*n*
(n - 3*a*x))/(3*a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) + (2*E^(n*ArcTanh[a*
x])*n*(n - a*x))/(a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rule 6145

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[((c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(2*d*(p + 1)), x] - Dist[
(a*c*n)/(2*d*(p + 1)), Int[(c + d*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && I
ntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{e^{n\tanh^{-1}(ax)}}{3a^2c\left(c-a^2cx^2\right)^{3/2}} - \frac{n\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{3a}$$

$$= \frac{e^{n\tanh^{-1}(ax)}}{3a^2c\left(c-a^2cx^2\right)^{3/2}} + \frac{e^{n\tanh^{-1}(ax)}n(n-3ax)}{3a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}} - \frac{(2n)\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{ac\left(9-n^2\right)}$$

$$= \frac{e^{n\tanh^{-1}(ax)}}{3a^2c\left(c-a^2cx^2\right)^{3/2}} + \frac{e^{n\tanh^{-1}(ax)}n(n-3ax)}{3a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}} + \frac{2e^{n\tanh^{-1}(ax)}n(n-ax)}{a^2c^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.19, size = 114, normalized size = 0.86

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(-n^2\left(2a^2x^2+1\right)+anx\left(2a^2x^2-3\right)+an^3x+3\right)}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(3 + a*n^3
*x + a*n*x*(-3 + 2*a^2*x^2) - n^2*(1 + 2*a^2*x^2)))/(a^2*c^2*(9 - 10*n^2 +
n^4)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.86, size = 171, normalized size = 1.29

$$\frac{\left(2\,a^3nx^3-2\,a^2n^2x^2-n^2+\left(an^3-3\,an\right)x+3\right)\sqrt{-a^2cx^2+c}\,\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2c^3n^4-10\,a^2c^3n^2+9\,a^2c^3+\left(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3\right)x^4-2\left(a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] (2*a^3*n*x^3 - 2*a^2*n^2*x^2 - n^2 + (a*n^3 - 3*a*n)*x + 3)*sqrt(-a^2*c*x^2
 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^3*n^4 - 10*a^2*c^3*n^2 + 9*a^2*c
^3 + (a^6*c^3*n^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^4 - 2*(a^4*c^3*n^4 - 10*a
^4*c^3*n^2 + 9*a^4*c^3)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.03, size = 86, normalized size = 0.65

$$-\frac{(ax-1)(ax+1)\left(2x^3a^3n-2a^2n^2x^2+a\,n^3x-3nax-n^2+3\right)\mathrm{e}^{n\,\mathrm{arctanh}(ax)}}{a^2\left(n^4-10n^2+9\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(2*a^3*n*x^3-2*a^2*n^2*x^2+a*n^3*x-3*a*n*x-n^2+3)*exp(n*ar
ctanh(a*x))/a^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 1.19, size = 163, normalized size = 1.23

$$-\frac{e^{\frac{n\ln(a\,x+1)}{2}-\frac{n\ln(1-a\,x)}{2}}\left(\frac{n^2-3}{a^4\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{2\,n^2\,x^2}{a^2\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{2\,n\,x^3}{a\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{n\,x\,\left(n^2-3\right)}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -(exp((n*log(a*x + 1))/2 - (n*log(1 - a*x))/2)*((n^2 - 3)/(a^4*c^2*(n^4 - 1
0*n^2 + 9)) + (2*n^2*x^2)/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n*x^3)/(a*c^2*(
n^4 - 10*n^2 + 9)) - (n*x*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/((c - a
^2*c*x^2)^(1/2)/a^2 - x^2*(c - a^2*c*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{xe^{n\,\mathrm{atanh}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.1351 $\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{6(n-ax)e^{n\tanh^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\tanh^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -exp(n*arctanh(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*a
rctanh(a*x))*(-a*x+n)/a/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6136, 6135}

$$-\frac{6(n-ax)e^{n\tanh^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\tanh^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcTanh[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}} + \frac{6\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{c\left(9-n^2\right)}$$
$$= -\frac{e^{n\tanh^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}} - \frac{6e^{n\tanh^{-1}(ax)}(n-ax)}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.16, size = 123, normalized size = 1.21

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(6a^3x^3+n\left(7-6a^2x^2\right)+3an^2x-9ax-n^3\right)}{ac^2(n-3)(n-1)(n+1)(n+3)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(-n^3 -
9*a*x + 3*a*n^2*x + 6*a^3*x^3 + n*(7 - 6*a^2*x^2)))/(a*c^2*(-3 + n)*(-1 + n
)*(1 + n)*(3 + n)*Sqrt[c - a^2*c*x^2]))
```

fricas [A] time = 0.52, size = 165, normalized size = 1.62

$$-\frac{\left(6\,a^3x^3-6\,a^2nx^2-n^3+3\left(an^2-3\,a\right)x+7\,n\right)\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^4-10\,ac^3n^2+\left(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3\right)x^4+9\,ac^3-2\left(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.03, size = 84, normalized size = 0.82

$$\frac{(ax-1)(ax+1)\left(6x^{3}a^{3}-6n\,x^{2}a^{2}+3n^{2}xa-n^{3}-9ax+7n\right)\mathrm{e}^{n\,\mathrm{arctanh}(ax)}}{a\left(n^{4}-10n^{2}+9\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcta
nh(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 1.21, size = 160, normalized size = 1.57

$$-\frac{\mathrm{e}^{\frac{n\,\ln(a\,x+1)}{2}-\frac{n\,\ln(1-a\,x)}{2}}\left(\frac{6\,x^{3}}{c^{2}\left(n^{4}-10\,n^{2}+9\right)}+\frac{7\,n-n^{3}}{a^{3}\,c^{2}\left(n^{4}-10\,n^{2}+9\right)}+\frac{3\,x\left(n^{2}-3\right)}{a^{2}\,c^{2}\left(n^{4}-10\,n^{2}+9\right)}-\frac{6\,n\,x^{2}}{a\,c^{2}\left(n^{4}-10\,n^{2}+9\right)}\right)}{\frac{\sqrt{c-a^{2}\,c\,x^{2}}}{a^{2}}-x^{2}\,\sqrt{c-a^{2}\,c\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -(exp((n*log(a*x + 1))/2 - (n*log(1 - a*x))/2)*((6*x^3)/(c^2*(n^4 - 10*n^2
+ 9)) + (7*n - n^3)/(a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2
*(n^4 - 10*n^2 + 9)) - (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/((c - a^2*c*x
^2)^(1/2)/a^2 - x^2*(c - a^2*c*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}(ax)}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(exp(n*atanh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.1352 $\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=417

$$\frac{2\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{ax+1}{1-ax}\right)}{c^2(1-n)\sqrt{c-a^2cx^2}}-\frac{\left(n^2+6n+15\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1-n}{2}}}{c^2(n+3)\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{(-n\cdots}{\cdots}$$

```
[Out] (-a*x+1)^(-3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c^2/(3+n)/(-a
^2*c*x^2+c)^(1/2)+(6+n)*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^
2+1)^(1/2)/c^2/(n^2+4*n+3)/(-a^2*c*x^2+c)^(1/2)-(n^2+6*n+15)*(-a*x+1)^(1/2-
1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c^2/(-n^3-3*n^2+n+3)/(-a^2*c
*x^2+c)^(1/2)+(-n^3-2*n^2+7*n+18)*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)
*(-a^2*x^2+1)^(1/2)/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)+2*(-a*x+1)^(1/2
-1/2*n)*(a*x+1)^(-1/2+1/2*n)*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a*x+1)/
(-a*x+1))*(-a^2*x^2+1)^(1/2)/c^2/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 421, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 155, 12, 131}

$$-\frac{2\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c^2(3-n)\sqrt{c-a^2cx^2}}-\frac{\left(n^2+6n+15\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1-n}{2}}}{c^2(n+3)\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{(-\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(3 +
 n)*Sqrt[c - a^2*c*x^2]) + ((6 + n)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 +
 n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(1 + n)*(3 + n)*Sqrt[c - a^2*c*x^2]) - ((15
+ 6*n + n^2)*(1 - a*x)^((1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]
)/(c^2*(3 + n)*(1 - n^2)*Sqrt[c - a^2*c*x^2]) + ((18 + 7*n - 2*n^2 - n^3)*(
1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(9 - 10
*n^2 + n^4)*Sqrt[c - a^2*c*x^2]) - (2*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3
+ n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (1 - a
*x)/(1 + a*x)])/(c^2*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}}{x}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{3}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\left(-a(3+n)-3a^2x\right.}{x}}{ac^2(3+n)\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}}$$

$$= \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} + \frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.30, size = 222, normalized size = 0.53

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(an^2x\left(-2a^2x^2+3ax+2\right)-\left(n^3\left(a^3x^3-2a^2x^2+2\right)\right)+n\left(7a^3x^3-18a^2x^2\right.\right.}{c^2\left(n^4-10n^2+9\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(a*n^2*x
*(2 + 3*a*x - 2*a^2*x^2) - n^3*(2 - 2*a^2*x^2 + a^3*x^3) + 3*(2 - 6*a*x - 3
*a^2*x^2 + 6*a^3*x^3) + n*(18 - 6*a*x - 18*a^2*x^2 + 7*a^3*x^3) + 2*(-3 - n
 + 3*n^2 + n^3)*(-1 + a*x)^3*Hypergeometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1
- a*x)/(1 + a*x)]))/(c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2]))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^6c^3x^7-3\,a^4c^3x^5+3\,a^2c^3x^3-c^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^7 -
 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 - c^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arctanh}(ax)}}{x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x\,\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)^(5/2)),x)
[Out] int(exp(n*atanh(a*x))/(x*(c - a^2*c*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{atanh}\,(ax)}}{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(exp(n*atanh(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(5/2)), x)
```

3.1353 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=507

$$\frac{2an\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{ax+1}{1-ax}\right)}{c^2(1-n)\sqrt{c-a^2cx^2}}+\frac{a\left(n^2+6n+12\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1}{2}(-n-1}}{c^2(n+1)(n+3)\sqrt{c-a^2cx^2}}$$

```
[Out] a*(4+n)*(-a*x+1)^(-3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c^2/(
3+n)/(-a^2*c*x^2+c)^(1/2)-(-a*x+1)^(-3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*
x^2+1)^(1/2)/c^2/x/(-a^2*c*x^2+c)^(1/2)+a*(n^2+6*n+12)*(-a*x+1)^(-1/2-1/2*n
)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c^2/(n^2+4*n+3)/(-a^2*c*x^2+c)^(1
/2)-a*(n^3+6*n^2+15*n+24)*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x
^2+1)^(1/2)/c^2/(-n^3-3*n^2+n+3)/(-a^2*c*x^2+c)^(1/2)+a*(-n^4-2*n^3+7*n^2+1
8*n+24)*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c^2/(n
^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)+2*a*n*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-1/2+
1/2*n)*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a*x+1)/(-a*x+1))*(-a^2*x^2+1)
^(1/2)/c^2/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 511, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6153, 6150, 129, 155, 12, 131}

$$-\frac{2an\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c^2(3-n)\sqrt{c-a^2cx^2}}+\frac{a\left(n^2+6n+12\right)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{1}{2}(-n}}{c^2(n+1)(n+3)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a*(4 + n)*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])
/(c^2*(3 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3
 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*x*Sqrt[c - a^2*c*x^2]) + (a*(12 + 6*n + n^
2)*(1 - a*x)^((-1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(1
 + n)*(3 + n)*Sqrt[c - a^2*c*x^2]) - (a*(24 + 15*n + 6*n^2 + n^3)*(1 - a*x)
^((1 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(3 + n)*(1 - n^
2)*Sqrt[c - a^2*c*x^2]) + (a*(24 + 18*n + 7*n^2 - 2*n^3 - n^4)*(1 - a*x)^((
3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(c^2*(9 - 10*n^2 + n^4)
*Sqrt[c - a^2*c*x^2]) - (2*a*n*(1 - a*x)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)
*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (1 - a*x)/(1
+ a*x)])/(c^2*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^2\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}}{x^2}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\left(-an-4a^2x\right)}{x}\,d}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a}}{c^2x\sqrt{c-a^2cx^2}}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a}}{c^2x\sqrt{c-a^2cx^2}}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a}}{c^2x\sqrt{c-a^2cx^2}}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a}}{c^2x\sqrt{c-a^2cx^2}}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a}}{c^2x\sqrt{c-a^2cx^2}}$$

$$= \frac{a(4+n)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a}}{c^2x\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.36, size = 268, normalized size = 0.53

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(24a^4x^4-36a^2x^2+an^3x\left(-2a^3x^3+3a^2x^2+2ax-4\right)+anx\left(18a^3x^3-33\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*(9 - 36*
a^2*x^2 + 24*a^4*x^4 - n^4*(-1 + a*x)^3*(1 + a*x) + a*n^3*x*(-4 + 2*a*x + 3
*a^2*x^2 - 2*a^3*x^3) + a*n*x*(34 - 18*a*x - 33*a^2*x^2 + 18*a^3*x^3) + n^2
*(-10 + 18*a*x + 6*a^2*x^2 - 18*a^3*x^3 + 7*a^4*x^4) + 2*a*n*(-3 - n + 3*n^
2 + n^3)*x*(-1 + a*x)^3*Hypergeometric2F1[1, 3/2 - n/2, 5/2 - n/2, (1 - a*x
)/(1 + a*x)]))/(c^2*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*x*Sqrt[c - a^2*c*x^2]
))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^6c^3x^8-3\,a^4c^3x^6+3\,a^2c^3x^4-c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^8 -
 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 - c^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x^2), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)}}{x^{2} \left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{atanh}(a\,x)}}{x^2\,{\left(c-a^2\,c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(exp(n*atanh(a*x))/(x^2*(c - a^2*c*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{atanh}{\left(a x \right)}}}{x^{2} \left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(exp(n*atanh(a*x))/(x**2*(-c*(a*x - 1)*(a*x + 1))**(5/2)), x)
```

3.1354 $\int \frac{e^{n\tanh^{-1}(ax)}}{x^3(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=623

$$\frac{a^2(n^2+5)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-1}{2}}(1-ax)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{ax+1}{1-ax}\right)}{c^2(1-n)\sqrt{c-a^2cx^2}}+\frac{a^2(n^2+4n+5)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-$$

```
[Out] 1/2*a^2*(n^2+4*n+5)*(-a*x+1)^(-3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)
^(1/2)/c^2/(3+n)/(-a^2*c*x^2+c)^(1/2)-1/2*(-a*x+1)^(-3/2-1/2*n)*(a*x+1)^(-3
/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c^2/x^2/(-a^2*c*x^2+c)^(1/2)-1/2*a*n*(-a*x+1)^
(-3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c^2/x/(-a^2*c*x^2+c)^(
1/2)+1/2*a^2*(n^3+6*n^2+17*n+30)*(-a*x+1)^(-1/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)
*(-a^2*x^2+1)^(1/2)/c^2/(n^2+4*n+3)/(-a^2*c*x^2+c)^(1/2)-1/2*a^2*(n^4+6*n^3
+20*n^2+54*n+75)*(-a*x+1)^(1/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/
2)/c^2/(-n^3-3*n^2+n+3)/(-a^2*c*x^2+c)^(1/2)+1/2*a^2*(-n^5-2*n^4+2*n^3+8*n^
2+59*n+90)*(-a*x+1)^(3/2-1/2*n)*(a*x+1)^(-3/2+1/2*n)*(-a^2*x^2+1)^(1/2)/c^2
/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)+a^2*(n^2+5)*(-a*x+1)^(1/2-1/2*n)*(a*x+
1)^(-1/2+1/2*n)*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a*x+1)/(-a*x+1))*(-a
^2*x^2+1)^(1/2)/c^2/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 628, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6153, 6150, 129, 151, 155, 12, 131}

$$-\frac{a^2(n^2+5)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-ax)^{\frac{3-n}{2}}\,{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{1-ax}{ax+1}\right)}{c^2(3-n)\sqrt{c-a^2cx^2}}+\frac{a^2(n^2+4n+5)\sqrt{1-a^2x^2}\,(ax+1)^{\frac{n-3}{2}}(1-$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^2*(5 + 4*n + n^2)*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 -
 a^2*x^2])/(2*c^2*(3 + n)*Sqrt[c - a^2*c*x^2]) - ((1 - a*x)^((-3 - n)/2)*(1
 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*x^2*Sqrt[c - a^2*c*x^2]) - (
a*n*(1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2
*x*Sqrt[c - a^2*c*x^2]) + (a^2*(30 + 17*n + 6*n^2 + n^3)*(1 - a*x)^((-1 - n
)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*(1 + n)*(3 + n)*Sqrt[
c - a^2*c*x^2]) - (a^2*(75 + 54*n + 20*n^2 + 6*n^3 + n^4)*(1 - a*x)^((1 - n
)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*(3 + n)*(1 - n^2)*Sqr
t[c - a^2*c*x^2]) + (a^2*(90 + 59*n + 8*n^2 + 2*n^3 - 2*n^4 - n^5)*(1 - a*x
```

```
)^((3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2])/(2*c^2*(9 - 10*n^2
+ n^4)*Sqrt[c - a^2*c*x^2]) - (a^2*(5 + n^2)*(1 - a*x)^((3 - n)/2)*(1 + a*x
)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2,
 (1 - a*x)/(1 + a*x)])/(c^2*(3 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{n\tanh^{-1}(ax)}}{x^3\left(1-a^2x^2\right)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}}{x^3}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\int \frac{(1-ax)^{-\frac{5}{2}-\frac{n}{2}}(1+ax)^{-\frac{5}{2}+\frac{n}{2}}\left(-an-5a^2x\right)}{x^2}$$

$$= -\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}} - \frac{an(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2x\sqrt{c-a^2cx^2}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+}}{2c^2x^2\sqrt{c-a^2cx}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+}}{2c^2x^2\sqrt{c-a^2cx}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+}}{2c^2x^2\sqrt{c-a^2cx}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+}}{2c^2x^2\sqrt{c-a^2cx}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+}}{2c^2x^2\sqrt{c-a^2cx}}$$

$$= \frac{a^2\left(5+4n+n^2\right)(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{2c^2(3+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+}}{2c^2x^2\sqrt{c-a^2cx}}$$

Mathematica [A] time = 0.83, size = 267, normalized size = 0.43

$$\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-3)}(ax+1)^{\frac{n-3}{2}}\left(-\frac{a^2(1-ax)\left(-(ax-1)^2\left(2\left(n^6-5n^4-41n^2+45\right){}_2F_1\left(1,\frac{3}{2}-\frac{n}{2};\frac{5}{2}-\frac{n}{2};\frac{1-ax}{ax+1}\right)-n^6+n^5+8n^4+2n^3+35n^2-87n\right.\right.}{(n-3)^2(n-1)(n+1)}\right.}{2c^2\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(x^3*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] ((1 - a*x)^((-3 - n)/2)*(1 + a*x)^((-3 + n)/2)*Sqrt[1 - a^2*x^2]*((a^2*(5 +
 4*n + n^2))/(3 + n) - x^(-2) - (a*n)/x - (a^2*(1 - a*x)*(-((-3 + n)^2*(-1
+ n)*(30 + 17*n + 6*n^2 + n^3)) + (-3 + n)^2*(75 + 54*n + 20*n^2 + 6*n^3 +
n^4)*(-1 + a*x) - (-1 + a*x)^2*(-270 - 87*n + 35*n^2 + 2*n^3 + 8*n^4 + n^5
- n^6 + 2*(45 - 41*n^2 - 5*n^4 + n^6)*Hypergeometric2F1[1, 3/2 - n/2, 5/2 -
 n/2, (1 - a*x)/(1 + a*x)])))/((-3 + n)^2*(-1 + n)*(1 + n)*(3 + n))))/(2*c^
2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^6c^3x^9-3\,a^4c^3x^7+3\,a^2c^3x^5-c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^9 -
 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 - c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x^3), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)}}{x^{3} \left(-a^{2} c\, x^{2}+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n \operatorname{atanh}(a\, x)}}{x^{3} \left(c-a^{2}\, c\, x^{2}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(x^3*(c - a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(exp(n*atanh(a*x))/(x^3*(c - a^2*c*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.1355 $\int \frac{e^{n \tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{120(n-ax)e^{n\tanh^{-1}(ax)}}{ac^3(1-n^2)(9-n^2)(25-n^2)\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\tanh^{-1}(ax)}}{ac^2(9-n^2)(25-n^2)(c-a^2cx^2)^{3/2}}-\frac{(n-5ax)e^{n\tanh^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}$$

```
[Out] -exp(n*arctanh(a*x))*(-5*a*x+n)/a/c/(-n^2+25)/(-a^2*c*x^2+c)^(5/2)-20*exp(n
*arctanh(a*x))*(-3*a*x+n)/a/c^2/(n^4-34*n^2+225)/(-a^2*c*x^2+c)^(3/2)-120*e
xp(n*arctanh(a*x))*(-a*x+n)/a/c^3/(-n^2+25)/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(
1/2)
```

Rubi [A] time = 0.19, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6136, 6135}

$$-\frac{120(n-ax)e^{n\tanh^{-1}(ax)}}{ac^3(1-n^2)(9-n^2)(25-n^2)\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\tanh^{-1}(ax)}}{ac^2(9-n^2)(25-n^2)(c-a^2cx^2)^{3/2}}-\frac{(n-5ax)e^{n\tanh^{-1}(ax)}}{ac(25-n^2)(c-a^2cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -((E^(n*ArcTanh[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
- (20*E^(n*ArcTanh[a*x])*(n - 3*a*x))/(a*c^2*(9 - n^2)*(25 - n^2)*(c - a^2*
c*x^2)^(3/2)) - (120*E^(n*ArcTanh[a*x])*(n - a*x))/(a*c^3*(1 - n^2)*(9 - n^
2)*(25 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6135

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcTanh[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6136

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*c*(n^2
- 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)), I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
 && EqQ[a^2*c + d, 0] && LtQ[p, -1] &&  !IntegerQ[n] && NeQ[n^2 - 4*(p + 1)
^2, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{e^{n\tanh^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}} + \frac{20\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{c\left(25-n^2\right)}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}} - \frac{20e^{n\tanh^{-1}(ax)}(n-3ax)}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}} + \frac{120\int \frac{e^{n\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{c^2\left(9-n^2\right)\left(25-n^2\right.}$$

$$= -\frac{e^{n\tanh^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}} - \frac{20e^{n\tanh^{-1}(ax)}(n-3ax)}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}} - \frac{120e^{n\tanh^{-}}}{ac^3\left(1-n^2\right)\left(9-n^2\right)}$$

Mathematica [A] time = 0.26, size = 182, normalized size = 1.10

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)^{\frac{1}{2}(-n-5)}(ax+1)^{\frac{n-5}{2}}\left(n^3\left(30-20a^2x^2\right)+10an^2x\left(6a^2x^2-11\right)+n\left(-120a^4x^4+260a^2x^2-149\right)\right.}{ac^3(n-5)(n-3)(n-1)(n+1)(n+3)(n+5)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(((1 - a*x)^((-5 - n)/2)*(1 + a*x)^((-5 + n)/2)*Sqrt[1 - a^2*x^2]*(-n^5 +
5*a*n^4*x + n^3*(30 - 20*a^2*x^2) + 10*a*n^2*x*(-11 + 6*a^2*x^2) + n*(-149
+ 260*a^2*x^2 - 120*a^4*x^4) + 15*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)))/(a*c^
3*(-5 + n)*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(5 + n)*Sqrt[c - a^2*c*x^2]))
```

fricas [A] time = 0.48, size = 291, normalized size = 1.75

$$-\frac{\left(120\,a^5x^5-120\,a^4nx^4-n^5+60\left(a^3n^2-5\,a^3\right)x^3+30\,n^3-20\left(a^2n^3-13\,a^2n\right)x^2+5\right.}{ac^4n^6-35\,ac^4n^4+259\,ac^4n^2-\left(a^7c^4n^6-35\,a^7c^4n^4+259\,a^7c^4n^2-225\,a^7c^4\right)x^6-225\,ac^4+3\left(a^5c^4n^6-35\,a^5c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -(120*a^5*x^5 - 120*a^4*n*x^4 - n^5 + 60*(a^3*n^2 - 5*a^3)*x^3 + 30*n^3 - 2
0*(a^2*n^3 - 13*a^2*n)*x^2 + 5*(a*n^4 - 22*a*n^2 + 45*a)*x - 149*n)*sqrt(-a
^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^6 - 35*a*c^4*n^4 + 259
*a*c^4*n^2 - (a^7*c^4*n^6 - 35*a^7*c^4*n^4 + 259*a^7*c^4*n^2 - 225*a^7*c^4)
```

```
*x^6 - 225*a*c^4 + 3*(a^5*c^4*n^6 - 35*a^5*c^4*n^4 + 259*a^5*c^4*n^2 - 225*
a^5*c^4)*x^4 - 3*(a^3*c^4*n^6 - 35*a^3*c^4*n^4 + 259*a^3*c^4*n^2 - 225*a^3*
c^4)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

maple [A] time = 0.04, size = 140, normalized size = 0.84

$$\frac{(ax-1)\,(ax+1)\left(120x^5a^5-120n\,a^4x^4+60a^3n^2x^3-20a^2n^3x^2-300x^3a^3+5a\,n^4x+260n\,x^2a^2-n^5-110n^2xa\right.}{a\left(n^6-35n^4+259n^2-225\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(120*a^5*x^5-120*a^4*n*x^4+60*a^3*n^2*x^3-20*a^2*n^3*x^2-30
0*a^3*x^3+5*a*n^4*x+260*a^2*n*x^2-n^5-110*a*n^2*x+30*n^3+225*a*x-149*n)*exp
(n*arctanh(a*x))/a/(n^6-35*n^4+259*n^2-225)/(-a^2*c*x^2+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

mupad [B] time = 1.46, size = 276, normalized size = 1.66

$$-\frac{(a\,x+1)^{n/2}\,\left(\frac{120\,x^5}{c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}-\frac{120\,n\,x^4}{a\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}+\frac{x^3\,\left(60\,n^2-300\right)}{a^2\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}-\frac{n\,\left(n^4-30\,n^2+149\right)}{a^5\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}+\right.}{(1-a\,x)^{n/2}\,\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^4}+x^4\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x^2\,\sqrt{c-a^2\,c\,x^2}}{a^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] -((a*x + 1)^(n/2)*((120*x^5)/(c^3*(259*n^2 - 35*n^4 + n^6 - 225)) - (120*n*
x^4)/(a*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) + (x^3*(60*n^2 - 300))/(a^2*c^3
*(259*n^2 - 35*n^4 + n^6 - 225)) - (n*(n^4 - 30*n^2 + 149))/(a^5*c^3*(259*n
^2 - 35*n^4 + n^6 - 225)) + (5*x*(n^4 - 22*n^2 + 45))/(a^4*c^3*(259*n^2 - 3
5*n^4 + n^6 - 225)) - (20*n*x^2*(n^2 - 13))/(a^3*c^3*(259*n^2 - 35*n^4 + n^
6 - 225))))/((1 - a*x)^(n/2)*((c - a^2*c*x^2)^(1/2)/a^4 + x^4*(c - a^2*c*x^
2)^(1/2) - (2*x^2*(c - a^2*c*x^2)^(1/2))/a^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.1356 $\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=42

$$\frac{c^2x^{m+1}F_1\left(m+1;\frac{n-4}{2},-\frac{n}{2}-2;m+2;ax,-ax\right)}{m+1}$$

```
[Out] c^2*x^(1+m)*AppellF1(1+m,-2+1/2*n,-2-1/2*n,2+m,a*x,-a*x)/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 133}

$$\frac{c^2x^{m+1}F_1\left(m+1;\frac{n-4}{2},-\frac{n}{2}-2;m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*x^(1 + m)*AppellF1[1 + m, (-4 + n)/2, -2 - n/2, 2 + m, a*x, -(a*x)])/(
1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx=c^2\int x^m(1-ax)^{2-\frac{n}{2}}(1+ax)^{2+\frac{n}{2}}\,dx$$
$$=\frac{c^2x^{1+m}F_1\left(1+m;\frac{1}{2}(-4+n),-2-\frac{n}{2};2+m;ax,-ax\right)}{1+m}$$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2,x]
[Out] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^2, x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*x^m*((a*x + 1)/(a*x - 1))^(1/2
*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2-c\right)^2x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate((a^2*c*x^2 - c)^2*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)}x^m\left(-a^2c\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2-c\right)^2 x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*atanh(a*x))*(c - a^2*c*x^2)^2,x)
```

```
[Out] int(x^m*exp(n*atanh(a*x))*(c - a^2*c*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m*(-a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.1357 $\int e^{n \tanh^{-1}(ax)} x^m \left(c - a^2 c x^2\right) dx$

Optimal. Leaf size=40

$$\frac{cx^{m+1}F_1\left(m+1;\frac{n-2}{2},-\frac{n}{2}-1;m+2;ax,-ax\right)}{m+1}$$

```
[Out] c*x^(1+m)*AppellF1(1+m,-1+1/2*n,-1-1/2*n,2+m,a*x,-a*x)/(1+m)
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6150, 133}

$$\frac{cx^{m+1}F_1\left(m+1;\frac{n-2}{2},-\frac{n}{2}-1;m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2),x]
```

```
[Out] (c*x^(1 + m)*AppellF1[1 + m, (-2 + n)/2, -1 - n/2, 2 + m, a*x, -(a*x)])/(1
+ m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n \tanh^{-1}(ax)} x^m \left(c - a^2 c x^2\right) dx &= c \int x^m (1-ax)^{1-\frac{n}{2}} (1+ax)^{1+\frac{n}{2}} dx \\ &= \frac{cx^{1+m}F_1\left(1+m;\frac{1}{2}(-2+n),-1-\frac{n}{2};2+m;ax,-ax\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.32, size = 0, normalized size = 0.00

$$\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2),x]
[Out] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2), x]
```

fricas [F] time = 1.98, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2cx^2-c\right)x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] integral(-(a^2*c*x^2 - c)*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2cx^2-c\right)x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] integrate(-(a^2*c*x^2 - c)*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arctanh}(ax)}x^m\left(-a^2c\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x)
[Out] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(a^2cx^2-c\right)x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,\mathrm{e}^{n\,\mathrm{atanh}(a\,x)}\,\left(c-a^2\,c\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*atanh(a*x))*(c - a^2*c*x^2),x)
```

```
[Out] int(x^m*exp(n*atanh(a*x))*(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(-x^m e^{n\,\mathrm{atanh}\,(ax)}\right)dx+\int a^2x^2x^me^{n\,\mathrm{atanh}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(-x**m*exp(n*atanh(a*x)), x) + Integral(a**2*x**2*x**m*exp(n*at
anh(a*x)), x))
```

3.1358 $\int \frac{e^{n\tanh^{-1}(ax)}x^m}{c-a^2cx^2}\,dx$

Optimal. Leaf size=42

$$\frac{x^{m+1}F_1\left(m+1;\frac{n+2}{2},1-\frac{n}{2};m+2;ax,-ax\right)}{c(m+1)}$$

```
[Out] x^(1+m)*AppellF1(1+m,1+1/2*n,1-1/2*n,2+m,a*x,-a*x)/c/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{n+2}{2},1-\frac{n}{2};m+2;ax,-ax\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (x^(1 + m)*AppellF1[1 + m, (2 + n)/2, 1 - n/2, 2 + m, a*x, -(a*x)])/(c*(1 +
 m))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^m}{c-a^2cx^2}\,dx = \frac{\int x^m(1-ax)^{-1-\frac{n}{2}}(1+ax)^{-1+\frac{n}{2}}\,dx}{c}$$
$$= \frac{x^{1+m}F_1\left(1+m;\frac{2+n}{2},1-\frac{n}{2};2+m;ax,-ax\right)}{c(1+m)}$$

Mathematica [B] time = 0.24, size = 106, normalized size = 2.52

$$\frac{x^m\left(e^{-2\tanh^{-1}(ax)}-1\right)^m\left(e^{-2\tanh^{-1}(ax)}+1\right)^m\left(-e^{-4\tanh^{-1}(ax)}\left(e^{2\tanh^{-1}(ax)}-1\right)^2\right)^{-m}e^{n\tanh^{-1}(ax)}F_1\left(-\frac{n}{2};m,-m;1-\frac{n}{2};-\right.}{acn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2),x]
```

```
[Out] (E^(n*ArcTanh[a*x])*(-1 + E^(-2*ArcTanh[a*x]))^m*(1 + E^(-2*ArcTanh[a*x]))^
m*x^m*AppellF1[-1/2*n, m, -m, 1 - n/2, -E^(-2*ArcTanh[a*x]), E^(-2*ArcTanh[
a*x])])/(a*c*(-((-1 + E^(2*ArcTanh[a*x]))^2/E^(4*ArcTanh[a*x])))^m*n)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x, algorithm="giac")
```

[Out] integrate(-x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)} x^m}{-a^2 c x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x)

[Out] int(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2 c x^2 - c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c),x, algorithm="maxima")

[Out] -integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m e^{n \operatorname{atanh}(a x)}}{c - a^2 c x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^m*exp(n*atanh(a*x)))/(c - a^2*c*x^2),x)

[Out] int((x^m*exp(n*atanh(a*x)))/(c - a^2*c*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^m e^{n \operatorname{atanh}(ax)}}{a^2 x^2 - 1} \, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*atanh(a*x))*x**m/(-a**2*c*x**2+c),x)

[Out] -Integral(x**m*exp(n*atanh(a*x))/(a**2*x**2 - 1), x)/c

3.1359 $\int \frac{e^{n\tanh^{-1}(ax)}x^m}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=42

$$\frac{x^{m+1}F_1\left(m+1;\frac{n+4}{2},2-\frac{n}{2};m+2;ax,-ax\right)}{c^2(m+1)}$$

[Out] x^(1+m)*AppellF1(1+m,2+1/2*n,2-1/2*n,2+m,a*x,-a*x)/c^2/(1+m)

Rubi [A] time = 0.10, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 133}

$$\frac{x^{m+1}F_1\left(m+1;\frac{n+4}{2},2-\frac{n}{2};m+2;ax,-ax\right)}{c^2(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2,x]

[Out] (x^(1 + m)*AppellF1[1 + m, (4 + n)/2, 2 - n/2, 2 + m, a*x, -(a*x)])/(c^2*(1 + m))

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^2}\,dx = \frac{\int x^m(1-ax)^{-2-\frac{n}{2}}(1+ax)^{-2+\frac{n}{2}}\,dx}{c^2}$$
$$= \frac{x^{1+m}F_1\left(1+m;\frac{4+n}{2},2-\frac{n}{2};2+m;ax,-ax\right)}{c^2(1+m)}$$

Mathematica [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{e^{n\tanh^{-1}(ax)}x^m}{\left(c-a^2cx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2,x]
```

```
[Out] Integrate[(E^(n*ArcTanh[a*x])*x^m)/(c - a^2*c*x^2)^2, x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arctanh}(ax)} x^{m}}{\left(-a^{2} c\, x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x)
[Out] int(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\, n}}{\left(a^{2} c x^{2}-c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m}\, e^{n \operatorname{atanh}(a\, x)}}{\left(c-a^{2}\, c\, x^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^2,x)
[Out] int((x^m*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{m} e^{n \operatorname{atanh}{\left(ax \right)}}}{a^{4} x^{4} - 2 a^{2} x^{2} + 1}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m/(-a**2*c*x**2+c)**2,x)
[Out] Integral(x**m*exp(n*atanh(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

3.1360 $\int e^{n\tanh^{-1}(ax)}x^m\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=70

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^pF_1\left(m+1;\frac{1}{2}(n-2p),-\frac{n}{2}-p;m+2;ax,-ax\right)}{m+1}$$

```
[Out] x^(1+m)*(-a^2*c*x^2+c)^p*AppellF1(1+m,1/2*n-p,-1/2*n-p,2+m,a*x,-a*x)/(1+m)/
((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.16, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 133}

$$\frac{x^{m+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^pF_1\left(m+1;\frac{1}{2}(n-2p),-\frac{n}{2}-p;m+2;ax,-ax\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(c - a^2*c*x^2)^p*AppellF1[1 + m, (n - 2*p)/2, -n/2 - p, 2 + m,
a*x, -(a*x)])/((1 + m)*(1 - a^2*x^2)^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
```

tegerQ[n/2]

Rubi steps

$$\begin{aligned}
\int e^{n\tanh^{-1}(ax)} x^m \left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{n\tanh^{-1}(ax)} x^m \left(1-a^2x^2\right)^p\,dx\\
&= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x^m(1-ax)^{-\frac{n}{2}+p}(1+ax)^{\frac{n}{2}+p}\,dx\\
&= \frac{x^{1+m}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p F_1\left(1+m;\frac{1}{2}(n-2p),-\frac{n}{2}-p;2+m;ax,-ax\right)}{1+m}
\end{aligned}$$

Mathematica [F] time = 0.58, size = 0, normalized size = 0.00

$$\int e^{n\tanh^{-1}(ax)} x^m \left(c-a^2cx^2\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p,x]
[Out] Integrate[E^(n*ArcTanh[a*x])*x^m*(c - a^2*c*x^2)^p, x]
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-a^2cx^2+c\right)^p x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((-a^2*c*x^2 + c)^p*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^p x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)} x^m \left(-a^2 c\, x^2 + c\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x)
[Out] int(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2 + c\right)^p x^m \left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^m*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p*x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\, e^{n\,\mathrm{atanh}(a\,x)} \left(c - a^2\, c\, x^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*atanh(a*x))*(c - a^2*c*x^2)^p,x)
[Out] int(x^m*exp(n*atanh(a*x))*(c - a^2*c*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(-c\left(ax-1\right)\left(ax+1\right)\right)^p e^{n\,\mathrm{atanh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**m*(-a**2*c*x**2+c)**p,x)
[Out] Integral(x**m*(-c*(a*x - 1)*(a*x + 1))**p*exp(n*atanh(a*x)), x)
```

3.1361 $\int e^{n \tanh^{-1}(ax)} x \left(c - a^2 c x^2\right)^p \, dx$

Optimal. Leaf size=177

$$-\frac{n 2^{\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}{}_2F_1\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{a^2(p+1)(-n+2p+2)}-\frac{\left(1-a^2x^2\right)^{-p}\left(c\right.}{}$$

```
[Out] -1/2*(-a*x+1)^(1-1/2*n+p)*(a*x+1)^(1+1/2*n+p)*(-a^2*c*x^2+c)^p/a^2/(1+p)/((
-a^2*x^2+1)^p)-2^(1/2*n+p)*n*(-a*x+1)^(1-1/2*n+p)*(-a^2*c*x^2+c)^p*hypergeo
m([-1/2*n-p, 1-1/2*n+p],[2-1/2*n+p],-1/2*a*x+1/2)/a^2/(1+p)/(2-n+2*p)/((-a^
2*x^2+1)^p)
```

Rubi [A] time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6153, 6150, 80, 69}

$$-\frac{n 2^{\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}{}_2F_1\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{a^2(p+1)(-n+2p+2)}-\frac{\left(1-a^2x^2\right)^{-p}\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((1 - a*x)^(1 - n/2 + p)*(1 + a*x)^(1 + n/2 + p)*(c - a^2*c*x^2)^p)/(2*a^2
*(1 + p)*(1 - a^2*x^2)^p) - (2^(n/2 + p)*n*(1 - a*x)^(1 - n/2 + p)*(c - a^2
*c*x^2)^p*Hypergeometric2F1[-n/2 - p, 1 - n/2 + p, 2 - n/2 + p, (1 - a*x)/2
])/(a^2*(1 + p)*(2 - n + 2*p)*(1 - a^2*x^2)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{n\tanh^{-1}(ax)}x\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int x(1-ax)^{-\frac{n}{2}+p}(1+ax)^{\frac{n}{2}+p}\,dx$$
$$=-\frac{(1-ax)^{1-\frac{n}{2}+p}(1+ax)^{1+\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{2a^2(1+p)}+\frac{\left(n\left(1-a^2x^2\right)^{-p}\left(c-\right.\right.}{}$$
$$=-\frac{(1-ax)^{1-\frac{n}{2}+p}(1+ax)^{1+\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{2a^2(1+p)}-\frac{2^{\frac{n}{2}+p}n(1-ax)^{1-\frac{n}{2}+p}}{}$$

Mathematica [A] time = 0.12, size = 136, normalized size = 0.77

$$-\frac{\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}\left(n2^{\frac{n}{2}+p+1}{}_2F_1\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)-(n-2(p+\right.}{2a^2(p+1)(-n+2p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x*(c - a^2*c*x^2)^p,x]
```

```
[Out] -1/2*((1 - a*x)^(1 - n/2 + p)*(c - a^2*c*x^2)^p*(-((n - 2*(1 + p))*(1 + a*x
)^(1 + n/2 + p)) + 2^(1 + n/2 + p)*n*Hypergeometric2F1[-1/2*n - p, 1 - n/2
+ p, 2 - n/2 + p, (1 - a*x)/2]))/(a^2*(1 + p)*(2 - n + 2*p)*(1 - a^2*x^2)^p
)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-a^2cx^2+c\right)^p x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((-a^2*c*x^2 + c)^p*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^p x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arctanh}(ax)} x\left(-a^2c\,x^2+c\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x)
[Out] int(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^p x\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p*x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,e^{n\,\mathrm{atanh}(a\,x)}\left(c-a^2\,c\,x^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(n*atanh(a*x))*(c - a^2*c*x^2)^p,x)
```

```
[Out] int(x*exp(n*atanh(a*x))*(c - a^2*c*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{p}e^{n\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral(x*(-c*(a*x - 1)*(a*x + 1))**p*exp(n*atanh(a*x)), x)
```

3.1362 $\int e^{n \tanh^{-1}(ax)} \left(c - a^2cx^2\right)^p \, dx$

Optimal. Leaf size=103

$$-\frac{2^{\frac{n}{2}+p+1} \left(1-a^2x^2\right)^{-p} \left(c-a^2cx^2\right)^p (1-ax)^{-\frac{n}{2}+p+1} {}_2F_1\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{a(-n+2p+2)}$$

```
[Out] -2^(1+1/2*n+p)*(-a*x+1)^(1-1/2*n+p)*(-a^2*c*x^2+c)^p*hypergeom([-1/2*n-p, 1
-1/2*n+p],[2-1/2*n+p],-1/2*a*x+1/2)/a/(2-n+2*p)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6143, 6140, 69}

$$-\frac{2^{\frac{n}{2}+p+1} \left(1-a^2x^2\right)^{-p} \left(c-a^2cx^2\right)^p (1-ax)^{-\frac{n}{2}+p+1} {}_2F_1\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{a(-n+2p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(1 + n/2 + p)*(1 - a*x)^(1 - n/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric
2F1[-n/2 - p, 1 - n/2 + p, 2 - n/2 + p, (1 - a*x)/2])/(a*(2 - n + 2*p)*(1 -
 a^2*x^2)^p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
```

```
EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{n\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{-\frac{n}{2}+p}(1+ax)^{\frac{n}{2}+p}\,dx$$
$$=-\frac{2^{1+\frac{n}{2}+p}(1-ax)^{1-\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{n}{2}-p,1-\frac{n}{2}+p;2-\frac{n}{2}+\right.}{a(2-n+2p)}$$

Mathematica [A] time = 0.04, size = 102, normalized size = 0.99

$$-\frac{2^{\frac{n}{2}+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p(1-ax)^{-\frac{n}{2}+p+1}\,{}_2F_1\left(-\frac{n}{2}-p,-\frac{n}{2}+p+1;-\frac{n}{2}+p+2;\frac{1}{2}(1-ax)\right)}{a\left(-\frac{n}{2}+p+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((2^(n/2 + p)*(1 - a*x)^(1 - n/2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[
-1/2*n - p, 1 - n/2 + p, 2 - n/2 + p, (1 - a*x)/2])/(a*(1 - n/2 + p)*(1 - a
^2*x^2)^p))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n \,\mathrm{arctanh}(a x)} \left(-a^{2} c \,x^{2}+c \right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x)
[Out] int(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c x^{2}+c\right)^{p} \left(\frac{a x+1}{a x-1}\right)^{\frac{1}{2} \, n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{e}}^{n\,\mathrm{atanh}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^p,x)
[Out] int(exp(n*atanh(a*x))*(c - a^2*c*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{p} e^{n \operatorname{atanh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*exp(n*atanh(a*x)), x)
```

3.1363 $\int e^{2(1+p)\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{-p}\,dx$

Optimal. Leaf size=41

$$\frac{(1-ax)^{1-2p}}{a(1-2p)}+\frac{(1-ax)^{-2p}}{ap}$$

```
[Out] (-a*x+1)^(1-2*p)/a/(1-2*p)+1/a/p/((-a*x+1)^(2*p))
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6140, 43}

$$\frac{(1-ax)^{1-2p}}{a(1-2p)}+\frac{(1-ax)^{-2p}}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(1 + p)*ArcTanh[a*x])/(1 - a^2*x^2)^p,x]
```

```
[Out] (1 - a*x)^(1 - 2*p)/(a*(1 - 2*p)) + 1/(a*p*(1 - a*x)^(2*p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2(1+p)\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{-p}\,dx &= \int (1-ax)^{-1-2p}(1+ax)\,dx\\ &= \int \left(2(1-ax)^{-1-2p}-(1-ax)^{-2p}\right)dx\\ &= \frac{(1-ax)^{1-2p}}{a(1-2p)}+\frac{(1-ax)^{-2p}}{ap}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.76

$$\frac{(1-ax)^{-2p}(apx+p-1)}{ap(2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(1 + p)*ArcTanh[a*x])/(1 - a^2*x^2)^p,x]
```

```
[Out] (-1 + p + a*p*x)/(a*p*(-1 + 2*p)*(1 - a*x)^(2*p))
```

fricas [A] time = 0.53, size = 80, normalized size = 1.95

$$-\frac{\left(a^2px^2-ax-p+1\right)\left(\frac{ax+1}{ax-1}\right)^{p+1}}{\left(2\,ap^2-ap+\left(2\,a^2p^2-a^2p\right)x\right)\left(-a^2x^2+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*x^2+1)^p),x, algorithm="fricas")
```

```
[Out] -(a^2*p*x^2 - a*x - p + 1)*((a*x + 1)/(a*x - 1))^(p + 1)/((2*a*p^2 - a*p +
(2*a^2*p^2 - a^2*p)*x)*(-a^2*x^2 + 1)^p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax+1}{ax-1}\right)^{p+1}}{\left(-a^2x^2+1\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*x^2+1)^p),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(p + 1)/(-a^2*x^2 + 1)^p, x)
```

maple [A] time = 0.03, size = 59, normalized size = 1.44

$$-\frac{(ax-1)\left(apx+p-1\right)\mathrm{e}^{2\left(1+p\right)\operatorname{arctanh}(ax)}\left(-a^2x^2+1\right)^{-p}}{ap\left(2p-1\right)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*(1+p)*arctanh(a*x))/((-a^2*x^2+1)^p),x)
```

```
[Out] -(a*x-1)*(a*p*x+p-1)*exp(2*(1+p)*arctanh(a*x))/a/p/(2*p-1)/(a*x+1)/((-a^2*x
^2+1)^p)
```

maxima [A] time = 0.34, size = 40, normalized size = 0.98

$$-\frac{apx+p-1}{\left(2\,(-1)^{p}\,p^{2}-(-1)^{p}\,p\right)(ax-1)^{2\,p}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*x^2+1)^p),x, algorithm="maxima")
```

```
[Out] -(a*p*x + p - 1)/((2*(-1)^p*p^2 - (-1)^p*p)*(a*x - 1)^(2*p)*a)
```

mupad [B] time = 1.12, size = 84, normalized size = 2.05

$$\frac{p\,(a\,x+1)^{p}-(a\,x+1)^{p}+a\,p\,x\,(a\,x+1)^{p}}{2\,a\,p^{2}\,\left(1-a^{2}\,x^{2}\right)^{p}\,(1-a\,x)^{p}-a\,p\,\left(1-a^{2}\,x^{2}\right)^{p}\,(1-a\,x)^{p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atanh(a*x)*(p + 1))/(1 - a^2*x^2)^p,x)
```

```
[Out] (p*(a*x + 1)^p - (a*x + 1)^p + a*p*x*(a*x + 1)^p)/(2*a*p^2*(1 - a^2*x^2)^p*
(1 - a*x)^p - a*p*(1 - a^2*x^2)^p*(1 - a*x)^p)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*atanh(a*x))/((-a**2*x**2+1)**p),x)
```

```
[Out] Timed out
```

3.1364 $\int e^{2(1+p)\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{-p}\,dx$

Optimal. Leaf size=95

$$\frac{\left(1-a^2x^2\right)^p(1-ax)^{1-2p}\left(c-a^2cx^2\right)^{-p}}{a(1-2p)}+\frac{\left(1-a^2x^2\right)^p(1-ax)^{-2p}\left(c-a^2cx^2\right)^{-p}}{ap}$$

```
[Out] (-a*x+1)^(1-2*p)*(-a^2*x^2+1)^p/a/(1-2*p)/((-a^2*c*x^2+c)^p)+(-a^2*x^2+1)^p
/a/p/((-a*x+1)^(2*p))/((-a^2*c*x^2+c)^p)
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6143, 6140, 43}

$$\frac{\left(1-a^2x^2\right)^p(1-ax)^{1-2p}\left(c-a^2cx^2\right)^{-p}}{a(1-2p)}+\frac{\left(1-a^2x^2\right)^p(1-ax)^{-2p}\left(c-a^2cx^2\right)^{-p}}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*(1 + p)*ArcTanh[a*x])/(c - a^2*c*x^2)^p,x]
```

```
[Out] ((1 - a*x)^(1 - 2*p)*(1 - a^2*x^2)^p)/(a*(1 - 2*p)*(c - a^2*c*x^2)^p) + (1
- a^2*x^2)^p/(a*p*(1 - a*x)^(2*p)*(c - a^2*c*x^2)^p)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{2(1+p)\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{-p}\,dx &= \left(\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}\right)\int e^{2(1+p)\tanh^{-1}(ax)}\left(1-a^2x^2\right)^{-p}\,dx\\ &= \left(\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}\right)\int (1-ax)^{-1-2p}(1+ax)\,dx\\ &= \left(\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}\right)\int \left(2(1-ax)^{-1-2p}-(1-ax)^{-2p}\right)\,dx\\ &= \frac{(1-ax)^{1-2p}\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}}{a(1-2p)}+\frac{(1-ax)^{-2p}\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right.}{ap}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.61

$$\frac{(1-ax)^{-2p}(apx+p-1)\left(1-a^2x^2\right)^p\left(c-a^2cx^2\right)^{-p}}{ap(2p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*(1 + p)*ArcTanh[a*x])/(c - a^2*c*x^2)^p,x]
```

```
[Out] ((-1 + p + a*p*x)*(1 - a^2*x^2)^p)/(a*p*(-1 + 2*p)*(1 - a*x)^(2*p)*(c - a^2
*c*x^2)^p)
```

fricas [A] time = 0.48, size = 81, normalized size = 0.85

$$-\frac{\left(a^2px^2-ax-p+1\right)\left(\frac{ax+1}{ax-1}\right)^{p+1}}{\left(2\,ap^2-ap+\left(2\,a^2p^2-a^2p\right)x\right)\left(-a^2cx^2+c\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*c*x^2+c)^p),x, algorithm="fricas
")
```

```
[Out] -(a^2*p*x^2 - a*x - p + 1)*((a*x + 1)/(a*x - 1))^(p + 1)/((2*a*p^2 - a*p +
(2*a^2*p^2 - a^2*p)*x)*(-a^2*c*x^2 + c)^p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax+1}{ax-1}\right)^{p+1}}{\left(-a^2cx^2+c\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*c*x^2+c)^p),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(p + 1)/(-a^2*c*x^2 + c)^p, x)
```

maple [A] time = 0.03, size = 60, normalized size = 0.63

$$-\frac{(ax-1)\left(apx+p-1\right)\mathrm{e}^{2\left(1+p\right)\operatorname{arctanh}(ax)}\left(-a^2c\,x^2+c\right)^{-p}}{ap\left(2p-1\right)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*(1+p)*arctanh(a*x))/((-a^2*c*x^2+c)^p),x)
```

```
[Out] -(a*x-1)*(a*p*x+p-1)*exp(2*(1+p)*arctanh(a*x))/a/p/(2*p-1)/(a*x+1)/((-a^2*c
*x^2+c)^p)
```

maxima [A] time = 0.36, size = 41, normalized size = 0.43

$$-\frac{apx+p-1}{\left(2\,p^2-p\right)(ax-1)^{2\,p}a\,(-c)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*arctanh(a*x))/((-a^2*c*x^2+c)^p),x, algorithm="maxima
")
```

```
[Out] -(a*p*x + p - 1)/((2*p^2 - p)*(a*x - 1)^(2*p)*a*(-c)^p)
```

mupad [B] time = 1.01, size = 86, normalized size = 0.91

$$-\frac{p\,(a\,x+1)^p-(a\,x+1)^p+a\,p\,x\,(a\,x+1)^p}{a\,p\,\left(c-a^2\,c\,x^2\right)^p\,(1-a\,x)^p-2\,a\,p^2\,\left(c-a^2\,c\,x^2\right)^p\,(1-a\,x)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*atanh(a*x)*(p + 1))/(c - a^2*c*x^2)^p,x)
```

```
[Out] -(p*(a*x + 1)^p - (a*x + 1)^p + a*p*x*(a*x + 1)^p)/(a*p*(c - a^2*c*x^2)^p*(
1 - a*x)^p - 2*a*p^2*(c - a^2*c*x^2)^p*(1 - a*x)^p)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*(1+p)*atanh(a*x))/((-a**2*c*x**2+c)**p),x)
```

```
[Out] Timed out
```

3.1365 $\int e^{2p\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=49

$$\frac{(ax+1)^{2p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(2p+1)}$$

[Out] (a*x+1)^(1+2*p)*(-a^2*c*x^2+c)^p/a/(1+2*p)/((-a^2*x^2+1)^p)

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6143, 6140, 32}

$$\frac{(ax+1)^{2p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(2p+1)}$$

Antiderivative was successfully verified.

[In] Int[E^(2*p*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]

[Out] ((1 + a*x)^(1 + 2*p)*(c - a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 - a^2*x^2)^p)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{2p\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{2p\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx$$
$$=\left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1+ax)^{2p}\,dx$$
$$=\frac{(1+ax)^{1+2p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(1+2p)}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.73

$$\frac{(ax+1)\left(c-a^2cx^2\right)^p e^{2p\tanh^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcTanh[a*x])*(c - a^2*c*x^2)^p,x]
[Out] (E^(2*p*ArcTanh[a*x])*(1 + a*x)*(c - a^2*c*x^2)^p)/(a + 2*a*p)
```

fricas [A] time = 0.46, size = 42, normalized size = 0.86

$$\frac{(ax+1)\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^p}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] (a*x + 1)*(-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p/(2*a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^p\left(\frac{ax+1}{ax-1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p, x)
```

maple [A] time = 0.03, size = 38, normalized size = 0.78

$$\frac{(ax+1)\,\mathrm{e}^{2p\,\mathrm{arctanh}(ax)}\left(-a^2c\,x^2+c\right)^p}{\left(1+2p\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p,x)
```

```
[Out] (a*x+1)/(1+2*p)/a*exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p
```

maxima [A] time = 0.34, size = 34, normalized size = 0.69

$$\frac{\left(a\,(-c)^p\,x+(-c)^p\right)(ax+1)^{2\,p}}{a\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arctanh(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] (a*(-c)^p*x + (-c)^p)*(a*x + 1)^(2*p)/(a*(2*p + 1))
```

mupad [B] time = 1.06, size = 43, normalized size = 0.88

$$\frac{{\left(c-a^2\,c\,x^2\right)}^p\,{\left(a\,x+1\right)}^{p+1}}{a\,\left(2\,p+1\right)\,{\left(1-a\,x\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*atanh(a*x))*(c - a^2*c*x^2)^p,x)
```

```
[Out] ((c - a^2*c*x^2)^p*(a*x + 1)^(p + 1))/(a*(2*p + 1)*(1 - a*x)^p)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x}{\sqrt{c}} & \text{for}\: a = 0 \wedge p = -\frac{1}{2} \\ c^{p} x & \text{for}\: a = 0 \\ \int \frac{e^{- \operatorname{atanh}{\left(a x \right)}}}{\sqrt{- c \left(a x - 1\right) \left(a x + 1\right)}}\, dx & \text{for}\: p = - \frac{1}{2} \\ \frac{a x \left(- a^{2} c x^{2} + c\right)^{p} e^{2 p \operatorname{atanh}{\left(a x \right)}}}{2 a p + a} + \frac{\left(- a^{2} c x^{2} + c\right)^{p} e^{2 p \operatorname{atanh}{\left(a x \right)}}}{2 a p + a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*atanh(a*x))*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Piecewise((x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x, Eq(a, 0)), (Integra
l(exp(-atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p, -1/2)), (a*x*(-a
**2*c*x**2 + c)**p*exp(2*p*atanh(a*x))/(2*a*p + a) + (-a**2*c*x**2 + c)**p*
exp(2*p*atanh(a*x))/(2*a*p + a), True))
```

3.1366 $\int e^{-2p\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=51

$$-\frac{(1-ax)^{2p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(2p+1)}$$

```
[Out] -(-a*x+1)^(1+2*p)*(-a^2*c*x^2+c)^p/a/(1+2*p)/((-a^2*x^2+1)^p)
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6143, 6140, 32}

$$-\frac{(1-ax)^{2p+1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*p*ArcTanh[a*x]),x]
```

```
[Out] -(((1 - a*x)^(1 + 2*p)*(c - a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 - a^2*x^2)^p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6143

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPart[p], Int[
(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] &&
EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-2p\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int e^{-2p\tanh^{-1}(ax)}\left(1-a^2x^2\right)^p\,dx$$
$$= \left(\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{2p}\,dx$$
$$= -\frac{(1-ax)^{1+2p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p}{a(1+2p)}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.71

$$\frac{(ax-1)\left(c-a^2cx^2\right)^p e^{-2p\tanh^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*p*ArcTanh[a*x]),x]
[Out] ((-1 + a*x)*(c - a^2*c*x^2)^p)/(E^(2*p*ArcTanh[a*x])*(a + 2*a*p))
```

fricas [A] time = 0.54, size = 44, normalized size = 0.86

$$\frac{(ax-1)\left(-a^2cx^2+c\right)^p}{\left(2\,ap+a\right)\left(\frac{ax+1}{ax-1}\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="fricas")
[Out] (a*x - 1)*(-a^2*c*x^2 + c)^p/((2*a*p + a)*((a*x + 1)/(a*x - 1))^p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax+1}{ax-1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

maple [A] time = 0.03, size = 40, normalized size = 0.78

$$\frac{(ax-1)\left(-a^{2}c\,x^{2}+c\right)^{p} \mathrm{e}^{-2p\,\mathrm{arctanh}(ax)}}{\left(1+2p\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x)),x)
[Out] (a*x-1)/(1+2*p)/a*(-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x))
```

maxima [A] time = 0.35, size = 36, normalized size = 0.71

$$\frac{\left(a\,(-c)^{p}\,x-(-c)^{p}\right)(ax-1)^{2\,p}}{a\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arctanh(a*x)),x, algorithm="maxima")
[Out] (a*(-c)^p*x - (-c)^p)*(a*x - 1)^(2*p)/(a*(2*p + 1))
```

mupad [B] time = 1.02, size = 44, normalized size = 0.86

$$-\frac{\left(c-a^{2}\,c\,x^{2}\right)^{p}\,(1-a\,x)^{p+1}}{a\,\left(2\,p+1\right)\,(a\,x+1)^{p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*p*atanh(a*x))*(c - a^2*c*x^2)^p,x)
[Out] -((c - a^2*c*x^2)^p*(1 - a*x)^(p + 1))/(a*(2*p + 1)*(a*x + 1)^p)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x}{\sqrt{c}} & \text{for}\: a = 0 \wedge p = -\frac{1}{2} \\ c^{p} x & \text{for}\: a = 0 \\ \int \frac{e^{\operatorname{atanh}{(ax)}}}{\sqrt{-c(ax-1)(ax+1)}}\, dx & \text{for}\: p = -\frac{1}{2} \\ \frac{ax\left(-a^{2}cx^{2}+c\right)^{p}}{2ape^{2p\operatorname{atanh}{(ax)}}+ae^{2p\operatorname{atanh}{(ax)}}} - \frac{\left(-a^{2}cx^{2}+c\right)^{p}}{2ape^{2p\operatorname{atanh}{(ax)}}+ae^{2p\operatorname{atanh}{(ax)}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/exp(2*p*atanh(a*x)),x)
```

```
[Out] Piecewise((x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x, Eq(a, 0)), (Integra
l(exp(atanh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p, -1/2)), (a*x*(-a*
*2*c*x**2 + c)**p/(2*a*p*exp(2*p*atanh(a*x)) + a*exp(2*p*atanh(a*x))) - (-a
**2*c*x**2 + c)**p/(2*a*p*exp(2*p*atanh(a*x)) + a*exp(2*p*atanh(a*x))), Tru
e))
```

3.1367 $\int e^{n\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx$

Optimal. Leaf size=53

$$\frac{(1-anx)\left(c-a^2cx^2\right)^{-\frac{n^2}{2}}e^{n\tanh^{-1}(ax)}}{a^3cn\left(1-n^2\right)}$$

```
[Out] exp(n*arctanh(a*x))*(-a*n*x+1)/a^3/c/n/(-n^2+1)/((-a^2*c*x^2+c)^(1/2*n^2))
```

Rubi [A] time = 0.11, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {6146}

$$\frac{(1-anx)\left(c-a^2cx^2\right)^{-\frac{n^2}{2}}e^{n\tanh^{-1}(ax)}}{a^3cn\left(1-n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(-1 - n^2/2),x]
```

```
[Out] (E^(n*ArcTanh[a*x])*(1 - a*n*x))/(a^3*c*n*(1 - n^2)*(c - a^2*c*x^2)^(n^2/2)
)
```

Rule 6146

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[((1 - a*n*x)*(c + d*x^2)^(p + 1)*E^(n*ArcTanh[a*x]))/(a*d*n*(n^
2 - 1)), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && EqQ[n^2 + 2*(
p + 1), 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{n\tanh^{-1}(ax)}x^2\left(c-a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\frac{e^{n\tanh^{-1}(ax)}(1-anx)\left(c-a^2cx^2\right)^{-\frac{n^2}{2}}}{a^3cn\left(1-n^2\right)}$$

Mathematica [A] time = 0.09, size = 92, normalized size = 1.74

$$\frac{(1-ax)^{-\frac{1}{2}n(n+1)}(ax+1)^{-\frac{1}{2}(n-1)n}(anx-1)\left(1-a^2x^2\right)^{\frac{n^2}{2}}\left(c-a^2cx^2\right)^{-\frac{n^2}{2}}}{a^3c(n-1)n(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcTanh[a*x])*x^2*(c - a^2*c*x^2)^(-1 - n^2/2),x]
```

```
[Out] ((-1 + a*n*x)*(1 - a^2*x^2)^(n^2/2))/(a^3*c*(-1 + n)*n*(1 + n)*(1 - a*x)^((
n*(1 + n))/2)*(1 + a*x)^(((-1 + n)*n)/2)*(c - a^2*c*x^2)^(n^2/2))
```

fricas [A] time = 0.48, size = 77, normalized size = 1.45

$$-\frac{\left(a^{3}nx^{3}-a^{2}x^{2}-anx+1\right)\left(-a^{2}cx^{2}+c\right)^{-\frac{1}{2}n^{2}-1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^{3}n^{3}-a^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"fricas")
```

```
[Out] -(a^3*n*x^3 - a^2*x^2 - a*n*x + 1)*(-a^2*c*x^2 + c)^(-1/2*n^2 - 1)*((a*x +
1)/(a*x - 1))^(1/2*n)/(a^3*n^3 - a^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^{2}cx^{2}+c\right)^{-\frac{1}{2}n^{2}-1}x^{2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(-1/2*n^2 - 1)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)
, x)
```

maple [A] time = 0.03, size = 58, normalized size = 1.09

$$-\frac{(ax-1)\,(ax+1)\,(nax-1)\,\mathrm{e}^{n\,\mathrm{arctanh}(ax)}\left(-a^{2}c\,x^{2}+c\right)^{-1-\frac{n^{2}}{2}}}{a^{3}n\left(n^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(-1-1/2*n^2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(a*n*x-1)*exp(n*arctanh(a*x))*(-a^2*c*x^2+c)^(-1-1/2*n^2)/
a^3/n/(n^2-1)
```

maxima [A] time = 0.36, size = 75, normalized size = 1.42

$$\frac{(anx-1)e^{\left(-\frac{1}{2}n^2\log(ax+1)-\frac{1}{2}n^2\log(ax-1)+\frac{1}{2}n\log(ax+1)-\frac{1}{2}n\log(ax-1)\right)}}{\left(n^3-n\right)a^3(-c)^{\frac{1}{2}n^2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arctanh(a*x))*x^2*(-a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm=
"maxima")
```

```
[Out] (a*n*x - 1)*e^(-1/2*n^2*log(a*x + 1) - 1/2*n^2*log(a*x - 1) + 1/2*n*log(a*x
 + 1) - 1/2*n*log(a*x - 1))/((n^3 - n)*a^3*(-c)^(1/2*n^2)*c)
```

mupad [B] time = 1.09, size = 101, normalized size = 1.91

$$\frac{e^{\frac{n\ln(a\,x+1)}{2}-\frac{n\ln(1-a\,x)}{2}}-a\,n\,x\,e^{\frac{n\ln(a\,x+1)}{2}-\frac{n\ln(1-a\,x)}{2}}}{a^3\,c\,n\left(c-a^2\,c\,x^2\right)^{\frac{n^2}{2}}-a^3\,c\,n^3\left(c-a^2\,c\,x^2\right)^{\frac{n^2}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*atanh(a*x)))/(c - a^2*c*x^2)^(n^2/2 + 1),x)
```

```
[Out] (exp((n*log(a*x + 1))/2 - (n*log(1 - a*x))/2) - a*n*x*exp((n*log(a*x + 1))/
2 - (n*log(1 - a*x))/2))/(a^3*c*n*(c - a^2*c*x^2)^(n^2/2) - a^3*c*n^3*(c -
a^2*c*x^2)^(n^2/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*atanh(a*x))*x**2*(-a**2*c*x**2+c)**(-1-1/2*n**2),x)
```

```
[Out] Timed out
```

3.1368 $\int \frac{e^{6\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{19}}\,dx$

Optimal. Leaf size=31

$$-\frac{1-6ax}{210a^3c^{19}(1-ax)^{21}(ax+1)^{15}}$$

[Out] 1/210*(6*a*x-1)/a^3/c^19/(-a*x+1)^21/(a*x+1)^15

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 81}

$$-\frac{1-6ax}{210a^3c^{19}(1-ax)^{21}(ax+1)^{15}}$$

Antiderivative was successfully verified.

[In] Int[(E^(6*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^19,x]

[Out] -(1 - 6*a*x)/(210*a^3*c^19*(1 - a*x)^21*(1 + a*x)^15)

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{6\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{19}}\,dx=\frac{\int \frac{x^2}{(1-ax)^{22}(1+ax)^{16}}\,dx}{c^{19}}$$
$$=-\frac{1-6ax}{210a^3c^{19}(1-ax)^{21}(1+ax)^{15}}$$

Mathematica [A] time = 1.27, size = 30, normalized size = 0.97

$$\frac{1-6ax}{210a^3c^{19}(ax-1)^{21}(ax+1)^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(6*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^19,x]
```

```
[Out] (1 - 6*a*x)/(210*a^3*c^19*(-1 + a*x)^21*(1 + a*x)^15)
```

fricas [B] time = 1.48, size = 379, normalized size = 12.23

$$-\frac{}{210\left(a^{39}c^{19}x^{36}-6\,a^{38}c^{19}x^{35}+70\,a^{36}c^{19}x^{33}-105\,a^{35}c^{19}x^{32}-336\,a^{34}c^{19}x^{31}+896\,a^{33}c^{19}x^{30}+720\,a^{32}c^{19}x^{29}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^6/(-a^2*x^2+1)^3*x^2/(-a^2*c*x^2+c)^19,x, algorithm="fric
as")
```

```
[Out] -1/210*(6*a*x - 1)/(a^39*c^19*x^36 - 6*a^38*c^19*x^35 + 70*a^36*c^19*x^33 -
 105*a^35*c^19*x^32 - 336*a^34*c^19*x^31 + 896*a^33*c^19*x^30 + 720*a^32*c^
19*x^29 - 3900*a^31*c^19*x^28 + 280*a^30*c^19*x^27 + 10752*a^29*c^19*x^26 -
 6552*a^28*c^19*x^25 - 20020*a^27*c^19*x^24 + 21840*a^26*c^19*x^23 + 24960*
a^25*c^19*x^22 - 43472*a^24*c^19*x^21 - 18018*a^23*c^19*x^20 + 60060*a^22*c
^19*x^19 - 60060*a^20*c^19*x^17 + 18018*a^19*c^19*x^16 + 43472*a^18*c^19*x^
15 - 24960*a^17*c^19*x^14 - 21840*a^16*c^19*x^13 + 20020*a^15*c^19*x^12 + 6
552*a^14*c^19*x^11 - 10752*a^13*c^19*x^10 - 280*a^12*c^19*x^9 + 3900*a^11*c
^19*x^8 - 720*a^10*c^19*x^7 - 896*a^9*c^19*x^6 + 336*a^8*c^19*x^5 + 105*a^7
*c^19*x^4 - 70*a^6*c^19*x^3 + 6*a^4*c^19*x - a^3*c^19)
```

giac [B] time = 0.25, size = 299, normalized size = 9.65

$$-\frac{358229025\,a^{14}x^{14}+5340869100\,a^{13}x^{13}+37114698075\,a^{12}x^{12}+159416118225\,a^{11}x^{11}+473088806190\,a^{10}x^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^6/(-a^2*x^2+1)^3*x^2/(-a^2*c*x^2+c)^19,x, algorithm="giac
")
```

```
[Out] -1/901943132160*(358229025*a^14*x^14 + 5340869100*a^13*x^13 + 37114698075*a
^12*x^12 + 159416118225*a^11*x^11 + 473088806190*a^10*x^10 + 1026819468675*
a^9*x^9 + 1682288472150*a^8*x^8 + 2115551402250*a^7*x^7 + 2054435046125*a^6
*x^6 + 1535397250002*a^5*x^5 + 870854759775*a^4*x^4 + 364307533205*a^3*x^3
+ 106553746740*a^2*x^2 + 19571887695*a*x + 1710785408)/((a*x + 1)^15*a^3*c^
19) + 1/901943132160*(358229025*a^20*x^20 - 7555375800*a^19*x^19 + 75901131
600*a^18*x^18 - 483051354975*a^17*x^17 + 2184946607340*a^16*x^16 - 74692054
50840*a^15*x^15 + 20031221295000*a^14*x^14 - 43177004037300*a^13*x^13 + 760
13078916950*a^12*x^12 - 110448380006328*a^11*x^11 + 133277726128008*a^10*x^
10 - 133908931763530*a^9*x^9 + 111933156213900*a^8*x^8 - 77492989590120*a^7
*x^7 + 44041557267624*a^6*x^6 - 20244576347604*a^5*x^5 + 7349182966545*a^4*
x^4 - 2026362494800*a^3*x^3 + 396520754280*a^2*x^2 - 48177926223*a*x + 2584
181888)/((a*x - 1)^21*a^3*c^19)
```

maple [B] time = 0.06, size = 426, normalized size = 13.74

$$-\frac{3991995}{8589934592a^3(ax-1)^4}+\frac{1964315}{4294967296a^3(ax-1)^3}-\frac{930465}{2147483648a^3(ax-1)^2}-\frac{1}{65536a^3(ax-1)^{19}}-\frac{858429}{4294967296a^3(ax+1)^5}-\frac{2211105}{8589934592a^3(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^6/(-a^2*x^2+1)^3*x^2/(-a^2*c*x^2+c)^19,x)
```

```
[Out] 1/c^19*(-3991995/8589934592/a^3/(a*x-1)^4+1964315/4294967296/a^3/(a*x-1)^3-
930465/2147483648/a^3/(a*x-1)^2-1/65536/a^3/(a*x-1)^19-858429/4294967296/a^
3/(a*x+1)^5-2211105/8589934592/a^3/(a*x+1)^4-1344005/4294967296/a^3/(a*x+1)
^3-1550775/4294967296/a^3/(a*x+1)^2-13/16777216/a^3/(a*x+1)^13-1/62914560/a
^3/(a*x+1)^15-9/58720256/a^3/(a*x+1)^14-275/100663296/a^3/(a*x+1)^12-253/33
554432/a^3/(a*x+1)^11-5819/335544320/a^3/(a*x+1)^10-13915/402653184/a^3/(a*
x+1)^9-16445/268435456/a^3/(a*x+1)^8-740025/7516192768/a^3/(a*x+1)^7-312455
/2147483648/a^3/(a*x+1)^6-1/1376256/a^3/(a*x-1)^21+3/655360/a^3/(a*x-1)^20+
7/196608/a^3/(a*x-1)^18-17/262144/a^3/(a*x-1)^17+51/524288/a^3/(a*x-1)^16-3
23/2621440/a^3/(a*x-1)^15+969/7340032/a^3/(a*x-1)^14-969/8388608/a^3/(a*x-1
)^13+3553/50331648/a^3/(a*x-1)^12-7429/83886080/a^3/(a*x-1)^10+37145/201326
592/a^3/(a*x-1)^9-37145/134217728/a^3/(a*x-1)^8+334305/939524096/a^3/(a*x-1
)^7-111435/268435456/a^3/(a*x-1)^6+1938969/4294967296/a^3/(a*x-1)^5+3411705
/8589934592/a^3/(a*x-1)-3411705/8589934592/a^3/(a*x+1))
```

maxima [B] time = 0.57, size = 379, normalized size = 12.23

$$-\frac{}{210\left(a^{39}c^{19}x^{36}-6a^{38}c^{19}x^{35}+70a^{36}c^{19}x^{33}-105a^{35}c^{19}x^{32}-336a^{34}c^{19}x^{31}+896a^{33}c^{19}x^{30}+720a^{32}c^{19}x^{29}-39\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^6/(-a^2*x^2+1)^3*x^2/(-a^2*c*x^2+c)^19,x, algorithm="maxi
ma")
```

```
[Out] -1/210*(6*a*x - 1)/(a^39*c^19*x^36 - 6*a^38*c^19*x^35 + 70*a^36*c^19*x^33 -
 105*a^35*c^19*x^32 - 336*a^34*c^19*x^31 + 896*a^33*c^19*x^30 + 720*a^32*c^
19*x^29 - 3900*a^31*c^19*x^28 + 280*a^30*c^19*x^27 + 10752*a^29*c^19*x^26 -
 6552*a^28*c^19*x^25 - 20020*a^27*c^19*x^24 + 21840*a^26*c^19*x^23 + 24960*
a^25*c^19*x^22 - 43472*a^24*c^19*x^21 - 18018*a^23*c^19*x^20 + 60060*a^22*c
^19*x^19 - 60060*a^20*c^19*x^17 + 18018*a^19*c^19*x^16 + 43472*a^18*c^19*x^
15 - 24960*a^17*c^19*x^14 - 21840*a^16*c^19*x^13 + 20020*a^15*c^19*x^12 + 6
552*a^14*c^19*x^11 - 10752*a^13*c^19*x^10 - 280*a^12*c^19*x^9 + 3900*a^11*c
^19*x^8 - 720*a^10*c^19*x^7 - 896*a^9*c^19*x^6 + 336*a^8*c^19*x^5 + 105*a^7
*c^19*x^4 - 70*a^6*c^19*x^3 + 6*a^4*c^19*x - a^3*c^19)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1)^6)/((c - a^2*c*x^2)^19*(a^2*x^2 - 1)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [B] time = 3.15, size = 405, normalized size = 13.06

$$\overline{210a^{39}c^{19}x^{36} - 1260a^{38}c^{19}x^{35} + 14700a^{36}c^{19}x^{33} - 22050a^{35}c^{19}x^{32} - 70560a^{34}c^{19}x^{31} + 188160a^{33}c^{19}x^{30} + 15120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**6/(-a**2*x**2+1)**3*x**2/(-a**2*c*x**2+c)**19,x)
```

```
[Out] (-6*a*x + 1)/(210*a**39*c**19*x**36 - 1260*a**38*c**19*x**35 + 14700*a**36*
c**19*x**33 - 22050*a**35*c**19*x**32 - 70560*a**34*c**19*x**31 + 188160*a*
*33*c**19*x**30 + 151200*a**32*c**19*x**29 - 819000*a**31*c**19*x**28 + 588
00*a**30*c**19*x**27 + 2257920*a**29*c**19*x**26 - 1375920*a**28*c**19*x**2
5 - 4204200*a**27*c**19*x**24 + 4586400*a**26*c**19*x**23 + 5241600*a**25*c
**19*x**22 - 9129120*a**24*c**19*x**21 - 3783780*a**23*c**19*x**20 + 126126
00*a**22*c**19*x**19 - 12612600*a**20*c**19*x**17 + 3783780*a**19*c**19*x**
16 + 9129120*a**18*c**19*x**15 - 5241600*a**17*c**19*x**14 - 4586400*a**16*
c**19*x**13 + 4204200*a**15*c**19*x**12 + 1375920*a**14*c**19*x**11 - 22579
20*a**13*c**19*x**10 - 58800*a**12*c**19*x**9 + 819000*a**11*c**19*x**8 - 1
51200*a**10*c**19*x**7 - 188160*a**9*c**19*x**6 + 70560*a**8*c**19*x**5 + 2
2050*a**7*c**19*x**4 - 14700*a**6*c**19*x**3 + 1260*a**4*c**19*x - 210*a**3
*c**19)
```

3.1369 $\int \frac{e^{4\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^9}\,dx$

Optimal. Leaf size=31

$$-\frac{1-4ax}{60a^3c^9(1-ax)^{10}(ax+1)^6}$$

```
[Out] 1/60*(4*a*x-1)/a^3/c^9/(-a*x+1)^10/(a*x+1)^6
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 81}

$$-\frac{1-4ax}{60a^3c^9(1-ax)^{10}(ax+1)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(4*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^9,x]
```

```
[Out] -(1 - 4*a*x)/(60*a^3*c^9*(1 - a*x)^10*(1 + a*x)^6)
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{4\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^9}\,dx = \frac{\int \frac{x^2}{(1-ax)^{11}(1+ax)^7}\,dx}{c^9}$$
$$= -\frac{1-4ax}{60a^3c^9(1-ax)^{10}(1+ax)^6}$$

Mathematica [A] time = 0.25, size = 30, normalized size = 0.97

$$\frac{4ax-1}{60a^3c^9(ax-1)^{10}(ax+1)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(4*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^9,x]
```

```
[Out] (-1 + 4*a*x)/(60*a^3*c^9*(-1 + a*x)^10*(1 + a*x)^6)
```

fricas [B] time = 0.47, size = 169, normalized size = 5.45

$$\frac{4\,ax-1}{60\,\left(a^{19}c^9x^{16}-4\,a^{18}c^9x^{15}+20\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}-36\,a^{14}c^9x^{11}+64\,a^{13}c^9x^{10}+20\,a^{12}c^9x^9-90\,a^{11}c^9x^8+20$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2/(-a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] 1/60*(4*a*x - 1)/(a^19*c^9*x^16 - 4*a^18*c^9*x^15 + 20*a^16*c^9*x^13 - 20*a
^15*c^9*x^12 - 36*a^14*c^9*x^11 + 64*a^13*c^9*x^10 + 20*a^12*c^9*x^9 - 90*a
^11*c^9*x^8 + 20*a^10*c^9*x^7 + 64*a^9*c^9*x^6 - 36*a^8*c^9*x^5 - 20*a^7*c^
9*x^4 + 20*a^6*c^9*x^3 - 4*a^4*c^9*x + a^3*c^9)
```

giac [B] time = 0.19, size = 139, normalized size = 4.48

$$-\frac{2145\,a^5x^5+12540\,a^4x^4+30030\,a^3x^3+37080\,a^2x^2+23841\,ax+6476}{983040\,(ax+1)^6a^3c^9}+\frac{2145\,a^9x^9-21780\,a^8x^8+99660\,a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2/(-a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 + 12540*a^4*x^4 + 30030*a^3*x^3 + 37080*a^2*x^2 + 2
3841*a*x + 6476)/((a*x + 1)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 - 21780*a^8
```

```
*x^8 + 99660*a^7*x^7 - 270480*a^6*x^6 + 481446*a^5*x^5 - 584920*a^4*x^4 + 4
86220*a^3*x^3 - 265680*a^2*x^2 + 84065*a*x - 9908)/((a*x - 1)^10*a^3*c^9)
```

maple [B] time = 0.04, size = 186, normalized size = 6.00

$$\frac{\frac{1}{1280a^3(ax-1)^{10}} - \frac{1}{768a^3(ax-1)^9} - \frac{7}{6144a^3(ax-1)^6} + \frac{21}{10240a^3(ax-1)^5} - \frac{21}{8192a^3(ax-1)^4} + \frac{11}{4096a^3(ax-1)^3} - \frac{165}{65536a^3(ax-1)^2} + \frac{143}{65536a^3(ax-}}{c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^4/(-a^2*x^2+1)^2*x^2/(-a^2*c*x^2+c)^9,x)
```

```
[Out] 1/c^9*(1/1280/a^3/(a*x-1)^10-1/768/a^3/(a*x-1)^9-7/6144/a^3/(a*x-1)^6+21/10
240/a^3/(a*x-1)^5-21/8192/a^3/(a*x-1)^4+11/4096/a^3/(a*x-1)^3-165/65536/a^3
/(a*x-1)^2+143/65536/a^3/(a*x-1)+1/1024/a^3/(a*x-1)^8-1/12288/a^3/(a*x+1)^6
-7/20480/a^3/(a*x+1)^5-11/8192/a^3/(a*x+1)^3-121/65536/a^3/(a*x+1)^2-143/65
536/a^3/(a*x+1)-13/16384/a^3/(a*x+1)^4)
```

maxima [B] time = 0.36, size = 169, normalized size = 5.45

$$\frac{4\,ax-1}{60\left(a^{19}c^9x^{16} - 4\,a^{18}c^9x^{15} + 20\,a^{16}c^9x^{13} - 20\,a^{15}c^9x^{12} - 36\,a^{14}c^9x^{11} + 64\,a^{13}c^9x^{10} + 20\,a^{12}c^9x^9 - 90\,a^{11}c^9x^8 + 20\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^4/(-a^2*x^2+1)^2*x^2/(-a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] 1/60*(4*a*x - 1)/(a^19*c^9*x^16 - 4*a^18*c^9*x^15 + 20*a^16*c^9*x^13 - 20*a
^15*c^9*x^12 - 36*a^14*c^9*x^11 + 64*a^13*c^9*x^10 + 20*a^12*c^9*x^9 - 90*a
^11*c^9*x^8 + 20*a^10*c^9*x^7 + 64*a^9*c^9*x^6 - 36*a^8*c^9*x^5 - 20*a^7*c^
9*x^4 + 20*a^6*c^9*x^3 - 4*a^4*c^9*x + a^3*c^9)
```

mupad [B] time = 1.51, size = 28, normalized size = 0.90

$$\frac{4\,a\,x-1}{60\,a^3\,c^9\,{\left(a\,x-1\right)}^{10}\,{\left(a\,x+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1)^4)/((c - a^2*c*x^2)^9*(a^2*x^2 - 1)^2),x)
```

```
[Out] (4*a*x - 1)/(60*a^3*c^9*(a*x - 1)^10*(a*x + 1)^6)
```

sympy [B] time = 1.22, size = 180, normalized size = 5.81

$$-\frac{-4ax+1}{60a^{19}c^9x^{16} - 240a^{18}c^9x^{15} + 1200a^{16}c^9x^{13} - 1200a^{15}c^9x^{12} - 2160a^{14}c^9x^{11} + 3840a^{13}c^9x^{10} + 1200a^{12}c^9x^9 - 5400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**4/(-a**2*x**2+1)**2*x**2/(-a**2*c*x**2+c)**9,x)
```

```
[Out] -(-4*a*x + 1)/(60*a**19*c**9*x**16 - 240*a**18*c**9*x**15 + 1200*a**16*c**9
*x**13 - 1200*a**15*c**9*x**12 - 2160*a**14*c**9*x**11 + 3840*a**13*c**9*x*
*10 + 1200*a**12*c**9*x**9 - 5400*a**11*c**9*x**8 + 1200*a**10*c**9*x**7 +
3840*a**9*c**9*x**6 - 2160*a**8*c**9*x**5 - 1200*a**7*c**9*x**4 + 1200*a**6
*c**9*x**3 - 240*a**4*c**9*x + 60*a**3*c**9)
```

3.1370 $\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=31

$$-\frac{1-2ax}{6a^3c^3(1-ax)^3(ax+1)}$$

```
[Out] 1/6*(2*a*x-1)/a^3/c^3/(-a*x+1)^3/(a*x+1)
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 81}

$$-\frac{1-2ax}{6a^3c^3(1-ax)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] -(1 - 2*a*x)/(6*a^3*c^3*(1 - a*x)^3*(1 + a*x))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{2\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^3}\,dx = \frac{\int \frac{x^2}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$

$$= -\frac{1-2ax}{6a^3c^3(1-ax)^3(1+ax)}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 0.97

$$\frac{1-2ax}{6a^3c^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^3,x]
```

```
[Out] (1 - 2*a*x)/(6*a^3*c^3*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.65, size = 49, normalized size = 1.58

$$-\frac{2\,ax-1}{6\left(a^7c^3x^4-2\,a^6c^3x^3+2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/6*(2*a*x - 1)/(a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^4*c^3*x - a^3*c^3)
```

giac [A] time = 0.22, size = 45, normalized size = 1.45

$$-\frac{1}{16\,(ax+1)a^3c^3}+\frac{3\,a^2x^2-12\,ax+5}{48\,(ax-1)^3\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16/((a*x + 1)*a^3*c^3) + 1/48*(3*a^2*x^2 - 12*a*x + 5)/((a*x - 1)^3*a^3*
c^3)
```

maple [A] time = 0.03, size = 54, normalized size = 1.74

$$\frac{-\frac{1}{12a^3(ax-1)^3}-\frac{1}{8a^3(ax-1)^2}+\frac{1}{16a^3(ax-1)}-\frac{1}{16a^3(ax+1)}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(-1/12/a^3/(a*x-1)^3-1/8/a^3/(a*x-1)^2+1/16/a^3/(a*x-1)-1/16/a^3/(a*x
+1))
```

maxima [A] time = 0.32, size = 49, normalized size = 1.58

$$-\frac{2\,ax-1}{6\left(a^{7}c^{3}x^{4}-2\,a^{6}c^{3}x^{3}+2\,a^{4}c^{3}x-a^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(-a^2*x^2+1)*x^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/6*(2*a*x - 1)/(a^7*c^3*x^4 - 2*a^6*c^3*x^3 + 2*a^4*c^3*x - a^3*c^3)
```

mupad [B] time = 0.10, size = 28, normalized size = 0.90

$$-\frac{2\,a\,x-1}{6\,a^{3}\,c^{3}\,{\left(a\,x-1\right)}^{3}\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x + 1)^2)/((c - a^2*c*x^2)^3*(a^2*x^2 - 1)),x)
```

```
[Out] -(2*a*x - 1)/(6*a^3*c^3*(a*x - 1)^3*(a*x + 1))
```

sympy [A] time = 0.35, size = 48, normalized size = 1.55

$$\frac{-2ax+1}{6a^{7}c^{3}x^{4}-12a^{6}c^{3}x^{3}+12a^{4}c^{3}x-6a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(-a**2*x**2+1)*x**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-2*a*x + 1)/(6*a**7*c**3*x**4 - 12*a**6*c**3*x**3 + 12*a**4*c**3*x - 6*a**
3*c**3)
```

3.1371 $\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=31

$$\frac{2ax+1}{6a^3c^3(1-ax)(ax+1)^3}$$

```
[Out] 1/6*(2*a*x+1)/a^3/c^3/(-a*x+1)/(a*x+1)^3
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 81}

$$\frac{2ax+1}{6a^3c^3(1-ax)(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (1 + 2*a*x)/(6*a^3*c^3*(1 - a*x)*(1 + a*x)^3)
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^3}\,dx = \frac{\int \frac{x^2}{(1-ax)^2(1+ax)^4}\,dx}{c^3}$$
$$= \frac{1+2ax}{6a^3c^3(1-ax)(1+ax)^3}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 0.97

$$-\frac{2ax+1}{6a^3c^3(ax-1)(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(2*ArcTanh[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/6*(1 + 2*a*x)/(a^3*c^3*(-1 + a*x)*(1 + a*x)^3)
```

fricas [A] time = 0.61, size = 49, normalized size = 1.58

$$-\frac{2\,ax+1}{6\left(a^7c^3x^4+2\,a^6c^3x^3-2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas"
)
```

```
[Out] -1/6*(2*a*x + 1)/(a^7*c^3*x^4 + 2*a^6*c^3*x^3 - 2*a^4*c^3*x - a^3*c^3)
```

giac [B] time = 0.17, size = 76, normalized size = 2.45

$$\frac{1}{32\,a^3c^3\left(\frac{2}{ax+1}-1\right)}+\frac{\frac{3\,a^3c^6}{ax+1}+\frac{6\,a^3c^6}{(ax+1)^2}-\frac{4\,a^3c^6}{(ax+1)^3}}{48\,a^6c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32/(a^3*c^3*(2/(a*x + 1) - 1)) + 1/48*(3*a^3*c^6/(a*x + 1) + 6*a^3*c^6/(a
*x + 1)^2 - 4*a^3*c^6/(a*x + 1)^3)/(a^6*c^9)
```

maple [A] time = 0.04, size = 54, normalized size = 1.74

$$\frac{-\frac{1}{16a^3(ax-1)}-\frac{1}{12a^3(ax+1)^3}+\frac{1}{8a^3(ax+1)^2}+\frac{1}{16a^3(ax+1)}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/c^3*(-1/16/a^3/(a*x-1)-1/12/a^3/(a*x+1)^3+1/8/a^3/(a*x+1)^2+1/16/a^3/(a*x
+1))
```

maxima [A] time = 0.33, size = 49, normalized size = 1.58

$$-\frac{2\,ax+1}{6\left(a^7c^3x^4+2\,a^6c^3x^3-2\,a^4c^3x-a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^2*(-a^2*x^2+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima"
)
```

```
[Out] -1/6*(2*a*x + 1)/(a^7*c^3*x^4 + 2*a^6*c^3*x^3 - 2*a^4*c^3*x - a^3*c^3)
```

mupad [B] time = 0.11, size = 28, normalized size = 0.90

$$-\frac{2\,a\,x+1}{6\,a^3\,c^3\,\left(a\,x-1\right)\,{\left(a\,x+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a^2*x^2 - 1))/((c - a^2*c*x^2)^3*(a*x + 1)^2),x)
```

```
[Out] -(2*a*x + 1)/(6*a^3*c^3*(a*x - 1)*(a*x + 1)^3)
```

sympy [A] time = 0.36, size = 49, normalized size = 1.58

$$\frac{-2ax-1}{6a^7c^3x^4+12a^6c^3x^3-12a^4c^3x-6a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**2*(-a**2*x**2+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] (-2*a*x - 1)/(6*a**7*c**3*x**4 + 12*a**6*c**3*x**3 - 12*a**4*c**3*x - 6*a**
3*c**3)
```

3.1372 $\int \frac{e^{-4\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^9}\,dx$

Optimal. Leaf size=31

$$\frac{4ax+1}{60a^3c^9(1-ax)^6(ax+1)^{10}}$$

```
[Out] 1/60*(4*a*x+1)/a^3/c^9/(-a*x+1)^6/(a*x+1)^10
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6150, 81}

$$\frac{4ax+1}{60a^3c^9(1-ax)^6(ax+1)^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^9),x]
```

```
[Out] (1 + 4*a*x)/(60*a^3*c^9*(1 - a*x)^6*(1 + a*x)^10)
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-4\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^9}\,dx = \frac{\int \frac{x^2}{(1-ax)^7(1+ax)^{11}}\,dx}{c^9}$$
$$= \frac{1+4ax}{60a^3c^9(1-ax)^6(1+ax)^{10}}$$

Mathematica [A] time = 0.21, size = 30, normalized size = 0.97

$$\frac{4ax+1}{60a^3c^9(ax-1)^6(ax+1)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(4*ArcTanh[a*x])*(c - a^2*c*x^2)^9),x]
```

```
[Out] (1 + 4*a*x)/(60*a^3*c^9*(-1 + a*x)^6*(1 + a*x)^10)
```

fricas [B] time = 0.63, size = 169, normalized size = 5.45

$$\frac{4\,ax+1}{60\,\left(a^{19}c^9x^{16}+4\,a^{18}c^9x^{15}-20\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}+36\,a^{14}c^9x^{11}+64\,a^{13}c^9x^{10}-20\,a^{12}c^9x^9-90\,a^{11}c^9x^8-20\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^4*(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^9,x, algorithm="frica
s")
```

```
[Out] 1/60*(4*a*x + 1)/(a^19*c^9*x^16 + 4*a^18*c^9*x^15 - 20*a^16*c^9*x^13 - 20*a
^15*c^9*x^12 + 36*a^14*c^9*x^11 + 64*a^13*c^9*x^10 - 20*a^12*c^9*x^9 - 90*a
^11*c^9*x^8 - 20*a^10*c^9*x^7 + 64*a^9*c^9*x^6 + 36*a^8*c^9*x^5 - 20*a^7*c^
9*x^4 - 20*a^6*c^9*x^3 + 4*a^4*c^9*x + a^3*c^9)
```

giac [B] time = 0.20, size = 139, normalized size = 4.48

$$-\frac{2145\,a^5x^5-12540\,a^4x^4+30030\,a^3x^3-37080\,a^2x^2+23841\,ax-6476}{983040\,(ax-1)^6a^3c^9}+\frac{2145\,a^9x^9+21780\,a^8x^8+99660\,a^7x^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^4*(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^9,x, algorithm="giac"
)
```

```
[Out] -1/983040*(2145*a^5*x^5 - 12540*a^4*x^4 + 30030*a^3*x^3 - 37080*a^2*x^2 + 2
3841*a*x - 6476)/((a*x - 1)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 + 21780*a^8
```

```
*x^8 + 99660*a^7*x^7 + 270480*a^6*x^6 + 481446*a^5*x^5 + 584920*a^4*x^4 + 4
86220*a^3*x^3 + 265680*a^2*x^2 + 84065*a*x + 9908)/((a*x + 1)^10*a^3*c^9)
```

maple [B] time = 0.04, size = 186, normalized size = 6.00

$$\frac{\frac{1}{12288a^3(ax-1)^6}-\frac{7}{20480a^3(ax-1)^5}-\frac{11}{8192a^3(ax-1)^3}+\frac{121}{65536a^3(ax-1)^2}-\frac{143}{65536a^3(ax-1)}+\frac{13}{16384a^3(ax-1)^4}-\frac{1}{1280a^3(ax+1)^{10}}-\frac{1}{768a^3(ax+}}{c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^4*(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^9,x)
```

```
[Out] 1/c^9*(1/12288/a^3/(a*x-1)^6-7/20480/a^3/(a*x-1)^5-11/8192/a^3/(a*x-1)^3+12
1/65536/a^3/(a*x-1)^2-143/65536/a^3/(a*x-1)+13/16384/a^3/(a*x-1)^4-1/1280/a
^3/(a*x+1)^10-1/768/a^3/(a*x+1)^9+7/6144/a^3/(a*x+1)^6+21/10240/a^3/(a*x+1)
^5+21/8192/a^3/(a*x+1)^4+11/4096/a^3/(a*x+1)^3+165/65536/a^3/(a*x+1)^2+143/
65536/a^3/(a*x+1)-1/1024/a^3/(a*x+1)^8)
```

maxima [B] time = 0.36, size = 169, normalized size = 5.45

$$\frac{4\,ax+1}{60\,\left(a^{19}c^9x^{16}+4\,a^{18}c^9x^{15}-20\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}+36\,a^{14}c^9x^{11}+64\,a^{13}c^9x^{10}-20\,a^{12}c^9x^9-90\,a^{11}c^9x^8-20\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^4*(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^9,x, algorithm="maxim
a")
```

```
[Out] 1/60*(4*a*x + 1)/(a^19*c^9*x^16 + 4*a^18*c^9*x^15 - 20*a^16*c^9*x^13 - 20*a
^15*c^9*x^12 + 36*a^14*c^9*x^11 + 64*a^13*c^9*x^10 - 20*a^12*c^9*x^9 - 90*a
^11*c^9*x^8 - 20*a^10*c^9*x^7 + 64*a^9*c^9*x^6 + 36*a^8*c^9*x^5 - 20*a^7*c^
9*x^4 - 20*a^6*c^9*x^3 + 4*a^4*c^9*x + a^3*c^9)
```

mupad [B] time = 2.54, size = 28, normalized size = 0.90

$$\frac{4\,a\,x+1}{60\,a^3\,c^9\,{\left(a\,x-1\right)}^6\,{\left(a\,x+1\right)}^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 - 1)^2)/((c - a^2*c*x^2)^9*(a*x + 1)^4),x)
```

```
[Out] (4*a*x + 1)/(60*a^3*c^9*(a*x - 1)^6*(a*x + 1)^10)
```

sympy [B] time = 1.21, size = 182, normalized size = 5.87

$$-\frac{-4ax-1}{60a^{19}c^9x^{16}+240a^{18}c^9x^{15}-1200a^{16}c^9x^{13}-1200a^{15}c^9x^{12}+2160a^{14}c^9x^{11}+3840a^{13}c^9x^{10}-1200a^{12}c^9x^9-5400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**4*(-a**2*x**2+1)**2/(-a**2*c*x**2+c)**9,x)
```

```
[Out] -(-4*a*x - 1)/(60*a**19*c**9*x**16 + 240*a**18*c**9*x**15 - 1200*a**16*c**9
*x**13 - 1200*a**15*c**9*x**12 + 2160*a**14*c**9*x**11 + 3840*a**13*c**9*x*
*10 - 1200*a**12*c**9*x**9 - 5400*a**11*c**9*x**8 - 1200*a**10*c**9*x**7 +
3840*a**9*c**9*x**6 + 2160*a**8*c**9*x**5 - 1200*a**7*c**9*x**4 - 1200*a**6
*c**9*x**3 + 240*a**4*c**9*x + 60*a**3*c**9)
```

3.1373 $\int \frac{e^{5\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{27/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{(1-5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{15}(ax+1)^{10}\sqrt{c-a^2cx^2}}$$

```
[Out] -1/120*(-5*a*x+1)*(-a^2*x^2+1)^(1/2)/a^3/c^13/(-a*x+1)^15/(a*x+1)^10/(-a^2*
c*x^2+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 81}

$$-\frac{(1-5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{15}(ax+1)^{10}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(5*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(27/2),x]
```

```
[Out] -((1 - 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(1 - a*x)^15*(1 + a*x)^10*Sq
rt[c - a^2*c*x^2])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{5\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{27/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{5\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{27/2}}\,dx}{c^{13}\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^{16}(1+ax)^{11}}\,dx}{c^{13}\sqrt{c-a^2cx^2}}$$

$$= -\frac{(1-5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{15}(1+ax)^{10}\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.59, size = 59, normalized size = 0.98

$$\frac{(1-5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(ax-1)^{15}(ax+1)^{10}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(5*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(27/2),x]
```

```
[Out] ((1 - 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(-1 + a*x)^15*(1 + a*x)^10*Sq
rt[c - a^2*c*x^2])
```

fricas [B] time = 0.98, size = 496, normalized size = 8.27

$$-\frac{\left(a^{22}x^{25}-5\,a^{21}x^{24}+40\,a^{19}x^{22}-50\,a^{18}x^{21}-126\,a\right.}{120\left(a^{27}c^{14}x^{27}-5\,a^{26}c^{14}x^{26}-a^{25}c^{14}x^{25}+45\,a^{24}c^{14}x^{24}-50\,a^{23}c^{14}x^{23}-166\,a^{22}c^{14}x^{22}+330\,a^{21}c^{14}x^{21}+286\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^5/(-a^2*x^2+1)^(5/2)*x^2/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] -1/120*(a^22*x^25 - 5*a^21*x^24 + 40*a^19*x^22 - 50*a^18*x^21 - 126*a^17*x^
20 + 280*a^16*x^19 + 160*a^15*x^18 - 765*a^14*x^17 + 105*a^13*x^16 + 1248*a
```

```
^12*x^15 - 720*a^11*x^14 - 1260*a^10*x^13 + 1260*a^9*x^12 + 720*a^8*x^11 -
1248*a^7*x^10 - 105*a^6*x^9 + 765*a^5*x^8 - 160*a^4*x^7 - 280*a^3*x^6 + 126
*a^2*x^5 + 50*a*x^4 - 40*x^3)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^27
*c^14*x^27 - 5*a^26*c^14*x^26 - a^25*c^14*x^25 + 45*a^24*c^14*x^24 - 50*a^2
3*c^14*x^23 - 166*a^22*c^14*x^22 + 330*a^21*c^14*x^21 + 286*a^20*c^14*x^20
- 1045*a^19*c^14*x^19 - 55*a^18*c^14*x^18 + 2013*a^17*c^14*x^17 - 825*a^16*
c^14*x^16 - 2508*a^15*c^14*x^15 + 1980*a^14*c^14*x^14 + 1980*a^13*c^14*x^13
 - 2508*a^12*c^14*x^12 - 825*a^11*c^14*x^11 + 2013*a^10*c^14*x^10 - 55*a^9*
c^14*x^9 - 1045*a^8*c^14*x^8 + 286*a^7*c^14*x^7 + 330*a^6*c^14*x^6 - 166*a^
5*c^14*x^5 - 50*a^4*c^14*x^4 + 45*a^3*c^14*x^3 - a^2*c^14*x^2 - 5*a*c^14*x
+ c^14)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^5 x^2}{\left(-a^2cx^2+c\right)^{\frac{27}{2}}\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^5/(-a^2*x^2+1)^(5/2)*x^2/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*x + 1)^5*x^2/((-a^2*c*x^2 + c)^(27/2)*(-a^2*x^2 + 1)^(5/2)), x
)
```

maple [A] time = 0.04, size = 49, normalized size = 0.82

$$-\frac{(ax-1)(ax+1)^6(5ax-1)}{120a^3\left(-a^2x^2+1\right)^{\frac{5}{2}}\left(-a^2c\,x^2+c\right)^{\frac{27}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^5/(-a^2*x^2+1)^(5/2)*x^2/(-a^2*c*x^2+c)^(27/2),x)
```

```
[Out] -1/120*(a*x-1)*(a*x+1)^6*(5*a*x-1)/a^3/(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(2
7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^5 x^2}{\left(-a^2cx^2+c\right)^{\frac{27}{2}}\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^5/(-a^2*x^2+1)^(5/2)*x^2/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
```

```
[Out] integrate((a*x + 1)^5*x^2/((-a^2*c*x^2 + c)^(27/2)*(-a^2*x^2 + 1)^(5/2)), x
)
```

mupad [B] time = 2.40, size = 583, normalized size = 9.72

$$-\frac{}{120\,a^3\,c^{14}\,\sqrt{1-a^2\,x^2}-600\,a^4\,c^{14}\,x\,\sqrt{1-a^2\,x^2}+4800\,a^6\,c^{14}\,x^3\,\sqrt{1-a^2\,x^2}-6000\,a^7\,c^{14}\,x^4\,\sqrt{1-a^2\,x^2}-15120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1)^5)/((c - a^2*c*x^2)^(27/2)*(1 - a^2*x^2)^(5/2)),x)
```

```
[Out] -((c - a^2*c*x^2)^(1/2) - 5*a*x*(c - a^2*c*x^2)^(1/2))/(120*a^3*c^14*(1 - a
^2*x^2)^(1/2) - 600*a^4*c^14*x*(1 - a^2*x^2)^(1/2) + 4800*a^6*c^14*x^3*(1 -
 a^2*x^2)^(1/2) - 6000*a^7*c^14*x^4*(1 - a^2*x^2)^(1/2) - 15120*a^8*c^14*x^
5*(1 - a^2*x^2)^(1/2) + 33600*a^9*c^14*x^6*(1 - a^2*x^2)^(1/2) + 19200*a^10
*c^14*x^7*(1 - a^2*x^2)^(1/2) - 91800*a^11*c^14*x^8*(1 - a^2*x^2)^(1/2) + 1
2600*a^12*c^14*x^9*(1 - a^2*x^2)^(1/2) + 149760*a^13*c^14*x^10*(1 - a^2*x^2
)^(1/2) - 86400*a^14*c^14*x^11*(1 - a^2*x^2)^(1/2) - 151200*a^15*c^14*x^12*
(1 - a^2*x^2)^(1/2) + 151200*a^16*c^14*x^13*(1 - a^2*x^2)^(1/2) + 86400*a^1
7*c^14*x^14*(1 - a^2*x^2)^(1/2) - 149760*a^18*c^14*x^15*(1 - a^2*x^2)^(1/2)
 - 12600*a^19*c^14*x^16*(1 - a^2*x^2)^(1/2) + 91800*a^20*c^14*x^17*(1 - a^2
*x^2)^(1/2) - 19200*a^21*c^14*x^18*(1 - a^2*x^2)^(1/2) - 33600*a^22*c^14*x^
19*(1 - a^2*x^2)^(1/2) + 15120*a^23*c^14*x^20*(1 - a^2*x^2)^(1/2) + 6000*a^
24*c^14*x^21*(1 - a^2*x^2)^(1/2) - 4800*a^25*c^14*x^22*(1 - a^2*x^2)^(1/2)
+ 600*a^27*c^14*x^24*(1 - a^2*x^2)^(1/2) - 120*a^28*c^14*x^25*(1 - a^2*x^2)
^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**5/(-a**2*x**2+1)**(5/2)*x**2/(-a**2*c*x**2+c)**(27/2),x)
```

```
[Out] Timed out
```

3.1374 $\int \frac{e^{3\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{11/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{(1-3ax)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^6(ax+1)^3\sqrt{c-a^2cx^2}}$$

```
[Out] -1/24*(-3*a*x+1)*(-a^2*x^2+1)^(1/2)/a^3/c^5/(-a*x+1)^6/(a*x+1)^3/(-a^2*c*x^
2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 81}

$$-\frac{(1-3ax)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^6(ax+1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(11/2),x]
```

```
[Out] -((1 - 3*a*x)*Sqrt[1 - a^2*x^2])/(24*a^3*c^5*(1 - a*x)^6*(1 + a*x)^3*Sqrt[c
 - a^2*c*x^2])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{11/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{3\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^5\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^7(1+ax)^4}\,dx}{c^5\sqrt{c-a^2cx^2}}$$
$$= -\frac{(1-3ax)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^6(1+ax)^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.12, size = 59, normalized size = 0.98

$$\frac{(3ax-1)\sqrt{1-a^2x^2}}{24a^3c^5(ax-1)^6(ax+1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcTanh[a*x])*x^2)/(c - a^2*c*x^2)^(11/2),x]
```

```
[Out] ((-1 + 3*a*x)*Sqrt[1 - a^2*x^2])/(24*a^3*c^5*(-1 + a*x)^6*(1 + a*x)^3*Sqrt[
c - a^2*c*x^2])
```

fricas [B] time = 0.54, size = 193, normalized size = 3.22

$$-\frac{\left(a^6x^9-3\,a^5x^8+8\,a^3x^6-6\,a^2x^5-6\,ax^4+8\,x^3\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{24\left(a^{11}c^6x^{11}-3\,a^{10}c^6x^{10}-a^9c^6x^9+11\,a^8c^6x^8-6\,a^7c^6x^7-14\,a^6c^6x^6+14\,a^5c^6x^5+6\,a^4c^6x^4-11\,a^3c^6x^3+a^2c^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] -1/24*(a^6*x^9 - 3*a^5*x^8 + 8*a^3*x^6 - 6*a^2*x^5 - 6*a*x^4 + 8*x^3)*sqrt(
-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^11*c^6*x^11 - 3*a^10*c^6*x^10 - a^9*c
```

```
^6*x^9 + 11*a^8*c^6*x^8 - 6*a^7*c^6*x^7 - 14*a^6*c^6*x^6 + 14*a^5*c^6*x^5 +
 6*a^4*c^6*x^4 - 11*a^3*c^6*x^3 + a^2*c^6*x^2 + 3*a*c^6*x - c^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 x^2}{\left(-a^2cx^2+c\right)^{\frac{11}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
```

```
[Out] integrate((a*x + 1)^3*x^2/((-a^2*c*x^2 + c)^(11/2)*(-a^2*x^2 + 1)^(3/2)), x
)
```

maple [A] time = 0.03, size = 49, normalized size = 0.82

$$-\frac{(ax-1)(ax+1)^4(3ax-1)}{24a^3\left(-a^2x^2+1\right)^{\frac{3}{2}}\left(-a^2c\,x^2+c\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c)^(11/2),x)
```

```
[Out] -1/24*(a*x-1)*(a*x+1)^4*(3*a*x-1)/a^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3 x^2}{\left(-a^2cx^2+c\right)^{\frac{11}{2}}\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/(-a^2*x^2+1)^(3/2)*x^2/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
```

```
[Out] integrate((a*x + 1)^3*x^2/((-a^2*c*x^2 + c)^(11/2)*(-a^2*x^2 + 1)^(3/2)), x
)
```

mupad [B] time = 1.58, size = 186, normalized size = 3.10

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{1}{24\,a^{12}\,c^6}-\frac{x}{8\,a^{11}\,c^6}\right)}{\frac{\sqrt{1-a^2\,x^2}}{a^9}+x^9\,\sqrt{1-a^2\,x^2}-\frac{3\,x\,\sqrt{1-a^2\,x^2}}{a^8}-\frac{3\,x^8\,\sqrt{1-a^2\,x^2}}{a}+\frac{8\,x^6\,\sqrt{1-a^2\,x^2}}{a^3}-\frac{6\,x^5\,\sqrt{1-a^2\,x^2}}{a^4}-\frac{6\,x^4\,\sqrt{1-a^2\,x^2}}{a^5}+\frac{8\,x^3\,\sqrt{1-a^2\,x^2}}{a^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1)^3)/((c - a^2*c*x^2)^(11/2)*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] -((c - a^2*c*x^2)^(1/2)*(1/(24*a^12*c^6) - x/(8*a^11*c^6)))/((1 - a^2*x^2)^
(1/2)/a^9 + x^9*(1 - a^2*x^2)^(1/2) - (3*x*(1 - a^2*x^2)^(1/2))/a^8 - (3*x^
8*(1 - a^2*x^2)^(1/2))/a + (8*x^6*(1 - a^2*x^2)^(1/2))/a^3 - (6*x^5*(1 - a^
2*x^2)^(1/2))/a^4 - (6*x^4*(1 - a^2*x^2)^(1/2))/a^5 + (8*x^3*(1 - a^2*x^2)^
(1/2))/a^6)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/(-a**2*x**2+1)**(3/2)*x**2/(-a**2*c*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

3.1375 $\int \frac{e^{\tanh^{-1}(ax)} x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)/a^3/c/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+3/4*ln(-a*x+1)*(
-a^2*x^2+1)^(1/2)/a^3/c/(-a^2*c*x^2+c)^(1/2)+1/4*ln(a*x+1)*(-a^2*x^2+1)^(1/
2)/a^3/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6153, 6150, 88}

$$\frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a^3*c*(1 - a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[1 - a^2
*x^2]*Log[1 - a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - a^2*x^2]*Log[
1 + a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^2(1+ax)}\,dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}\int \left(\frac{1}{2a^2(-1+ax)^2}+\frac{3}{4a^2(-1+ax)}+\frac{1}{4a^2(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-a^2x^2}}{2a^3c(1-ax)\sqrt{c-a^2cx^2}}+\frac{3\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\,\log(1+ax)}{4a^3c\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 76, normalized size = 0.55

$$\frac{\sqrt{1-a^2x^2}\left(\frac{1}{2a^3(1-ax)}+\frac{3\log(1-ax)}{4a^3}+\frac{\log(ax+1)}{4a^3}\right)}{c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcTanh[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1/(2*a^3*(1 - a*x)) + (3*Log[1 - a*x])/(4*a^3) + Log[1
 + a*x]/(4*a^3)))/(c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^2}{a^5c^2x^5-a^4c^2x^4-2\,a^3c^2x^3+2\,a^2c^2x^2+ac^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2/(a^5*c^2*x^5 - a^4*c^
2*x^4 - 2*a^3*c^2*x^3 + 2*a^2*c^2*x^2 + a*c^2*x - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

maple [A] time = 0.05, size = 90, normalized size = 0.65

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(3\ln\left(ax-1\right)xa+ax\ln\left(ax+1\right)-3\ln\left(ax-1\right)-\ln\left(ax+1\right)-2\right)}{4\left(a^2x^2-1\right)c^2a^3\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x*a+a*x*ln(a*x+
1)-3*ln(a*x-1)-ln(a*x+1)-2)/(a^2*x^2-1)/c^2/a^3/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a^2*x^2+1)^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt(-a^2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a\,x+1\right)}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((x^2*(a*x + 1))/((c - a^2*c*x^2)^(3/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,(ax+1)}{\sqrt{-(ax-1)\,(ax+1)}\,\,(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(-a**2*x**2+1)**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*(a*x + 1)/(sqrt(-(a*x - 1)*(a*x + 1))*(-c*(a*x - 1)*(a*x + 1)
)**(3/2)), x)
```

3.1376 $\int \frac{e^{-\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{1-a^2x^2}}{2a^3c(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)/a^3/c/(a*x+1)/(-a^2*c*x^2+c)^(1/2)-1/4*ln(-a*x+1)*(
-a^2*x^2+1)^(1/2)/a^3/c/(-a^2*c*x^2+c)^(1/2)-3/4*ln(a*x+1)*(-a^2*x^2+1)^(1/
2)/a^3/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 88}

$$-\frac{\sqrt{1-a^2x^2}}{2a^3c(ax+1)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\,\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\,\log(ax+1)}{4a^3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a^3*c*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*
x^2]*Log[1 - a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*Log
[1 + a*x])/(4*a^3*c*Sqrt[c - a^2*c*x^2])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1-a^2x^2}\int \frac{e^{-\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)(1+ax)^2}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{\sqrt{1-a^2x^2}\int \left(-\frac{1}{4a^2(-1+ax)}+\frac{1}{2a^2(1+ax)^2}-\frac{3}{4a^2(1+ax)}\right)dx}{c\sqrt{c-a^2cx^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}}{2a^3c(1+ax)\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log(1-ax)}{4a^3c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\log(1+ax)}{4a^3c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 0.52

$$-\frac{\sqrt{1-a^2x^2}\,((ax+1)\log(1-ax)+3(ax+1)\log(ax+1)+2)}{4a^3(acx+c)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^ArcTanh[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/4*(Sqrt[1 - a^2*x^2]*(2 + (1 + a*x)*Log[1 - a*x] + 3*(1 + a*x)*Log[1 + a
*x]))/(a^3*(c + a*c*x)*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}\,x^2}{a^5c^2x^5+a^4c^2x^4-2\,a^3c^2x^3-2\,a^2c^2x^2+ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)*x^2/(a^5*c^2*x^5 + a^4*c^2
*x^4 - 2*a^3*c^2*x^3 - 2*a^2*c^2*x^2 + a*c^2*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2x^2+1}\,x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2/((-a^2*c*x^2 + c)^(3/2)*(a*x + 1)), x)
```

maple [A] time = 0.05, size = 88, normalized size = 0.64

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln(ax-1)\,xa+3ax\ln(ax+1)+\ln(ax-1)+3\ln(ax+1)+2\right)}{4\left(a^2x^2-1\right)c^2a^3\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a+3*a*x*ln(a*x+1
)+ln(a*x-1)+3*ln(a*x+1)+2)/(a^2*x^2-1)/c^2/a^3/(a*x+1)
```

maxima [A] time = 0.35, size = 52, normalized size = 0.38

$$-\frac{\sqrt{c}}{2\left(a^4c^2x+a^3c^2\right)}-\frac{3\,\log(ax+1)}{4\,a^3c^{\frac{3}{2}}}-\frac{\log(ax-1)}{4\,a^3c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)*(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -1/2*sqrt(c)/(a^4*c^2*x + a^3*c^2) - 3/4*log(a*x + 1)/(a^3*c^(3/2)) - 1/4*l
og(a*x - 1)/(a^3*c^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{1-a^2\,x^2}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^(1/2))/((c - a^2*c*x^2)^(3/2)*(a*x + 1)),x)
```

```
[Out] int((x^2*(1 - a^2*x^2)^(1/2))/((c - a^2*c*x^2)^(3/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(ax-1)(ax+1)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(
a*x + 1)), x)
```

3.1377 $\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{(c-a^2cx^2)^{11/2}}\,dx$

Optimal. Leaf size=60

$$\frac{(3ax+1)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^3(ax+1)^6\sqrt{c-a^2cx^2}}$$

```
[Out] 1/24*(3*a*x+1)*(-a^2*x^2+1)^(1/2)/a^3/c^5/(-a*x+1)^3/(a*x+1)^6/(-a^2*c*x^2+
c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 81}

$$\frac{(3ax+1)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^3(ax+1)^6\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(11/2)),x]
```

```
[Out] ((1 + 3*a*x)*Sqrt[1 - a^2*x^2])/(24*a^3*c^5*(1 - a*x)^3*(1 + a*x)^6*Sqrt[c
- a^2*c*x^2])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
```

```
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) &&  !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{11/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{-3\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{11/2}}\,dx}{c^5\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^4(1+ax)^7}\,dx}{c^5\sqrt{c-a^2cx^2}}\\ &= \frac{(1+3ax)\sqrt{1-a^2x^2}}{24a^3c^5(1-ax)^3(1+ax)^6\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 59, normalized size = 0.98

$$-\frac{(3ax+1)\sqrt{1-a^2x^2}}{24a^3c^5(ax-1)^3(ax+1)^6\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(3*ArcTanh[a*x])*(c - a^2*c*x^2)^(11/2)),x]
```

```
[Out] -1/24*((1 + 3*a*x)*Sqrt[1 - a^2*x^2])/(a^3*c^5*(-1 + a*x)^3*(1 + a*x)^6*Sqr
t[c - a^2*c*x^2])
```

fricas [B] time = 0.95, size = 192, normalized size = 3.20

$$\frac{\left(a^6x^9+3\,a^5x^8-8\,a^3x^6-6\,a^2x^5+6\,ax^4+8\,x^3\right)\sqrt{-a^2cx^2+c}\,\sqrt{-a^2x^2+1}}{24\left(a^{11}c^6x^{11}+3\,a^{10}c^6x^{10}-a^9c^6x^9-11\,a^8c^6x^8-6\,a^7c^6x^7+14\,a^6c^6x^6+14\,a^5c^6x^5-6\,a^4c^6x^4-11\,a^3c^6x^3-a^2c^6x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="fricas")
```

```
[Out] 1/24*(a^6*x^9 + 3*a^5*x^8 - 8*a^3*x^6 - 6*a^2*x^5 + 6*a*x^4 + 8*x^3)*sqrt(-
a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^11*c^6*x^11 + 3*a^10*c^6*x^10 - a^9*c^
```

```
6*x^9 - 11*a^8*c^6*x^8 - 6*a^7*c^6*x^7 + 14*a^6*c^6*x^6 + 14*a^5*c^6*x^5 -
6*a^4*c^6*x^4 - 11*a^3*c^6*x^3 - a^2*c^6*x^2 + 3*a*c^6*x + c^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}{\left(-a^2cx^2+c\right)^{\frac{11}{2}}(ax+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^2/((-a^2*c*x^2 + c)^(11/2)*(a*x + 1)^3), x
)
```

maple [A] time = 0.03, size = 49, normalized size = 0.82

$$-\frac{(ax-1)(3ax+1)\left(-a^2x^2+1\right)^{\frac{3}{2}}}{24\,(ax+1)^2\,a^3\left(-a^2c\,x^2+c\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11/2),x)
```

```
[Out] -1/24*(a*x-1)*(3*a*x+1)*(-a^2*x^2+1)^(3/2)/(a*x+1)^2/a^3/(-a^2*c*x^2+c)^(11
/2)
```

maxima [A] time = 0.36, size = 93, normalized size = 1.55

$$-\frac{3\,ax+1}{24\left(a^{12}c^{\frac{11}{2}}x^9+3\,a^{11}c^{\frac{11}{2}}x^8-8\,a^9c^{\frac{11}{2}}x^6-6\,a^8c^{\frac{11}{2}}x^5+6\,a^7c^{\frac{11}{2}}x^4+8\,a^6c^{\frac{11}{2}}x^3-3\,a^4c^{\frac{11}{2}}x-a^3c^{\frac{11}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^3*(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(11/2),x, algorit
hm="maxima")
```

```
[Out] -1/24*(3*a*x + 1)/(a^12*c^(11/2)*x^9 + 3*a^11*c^(11/2)*x^8 - 8*a^9*c^(11/2)
*x^6 - 6*a^8*c^(11/2)*x^5 + 6*a^7*c^(11/2)*x^4 + 8*a^6*c^(11/2)*x^3 - 3*a^4
*c^(11/2)*x - a^3*c^(11/2))
```

mupad [B] time = 1.54, size = 187, normalized size = 3.12

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}+3\,a\,x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{1-a^2\,x^2}}{24\,a^{14}\,c^6\,x^{11}+72\,a^{13}\,c^6\,x^{10}-24\,a^{12}\,c^6\,x^9-264\,a^{11}\,c^6\,x^8-144\,a^{10}\,c^6\,x^7+336\,a^9\,c^6\,x^6+336\,a^8\,c^6\,x^5-144\,a^7\,c^6\,x^4 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^(3/2))/((c - a^2*c*x^2)^(11/2)*(a*x + 1)^3),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2) + 3*a*x*(c - a^2*c*x^2)^(1/2)*(1
 - a^2*x^2)^(1/2))/(24*a^3*c^6 + 72*a^4*c^6*x - 24*a^5*c^6*x^2 - 264*a^6*c^
6*x^3 - 144*a^7*c^6*x^4 + 336*a^8*c^6*x^5 + 336*a^9*c^6*x^6 - 144*a^10*c^6*
x^7 - 264*a^11*c^6*x^8 - 24*a^12*c^6*x^9 + 72*a^13*c^6*x^10 + 24*a^14*c^6*x
^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**3*(-a**2*x**2+1)**(3/2)/(-a**2*c*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

3.1378 $\int \frac{e^{-5\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{27/2}}\,dx$

Optimal. Leaf size=60

$$\frac{(5ax+1)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{10}(ax+1)^{15}\sqrt{c-a^2cx^2}}$$

```
[Out] 1/120*(5*a*x+1)*(-a^2*x^2+1)^(1/2)/a^3/c^13/(-a*x+1)^10/(a*x+1)^15/(-a^2*c*
x^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6153, 6150, 81}

$$\frac{(5ax+1)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{10}(ax+1)^{15}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(E^(5*ArcTanh[a*x])*(c - a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((1 + 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(1 - a*x)^10*(1 + a*x)^15*Sqr
t[c - a^2*c*x^2])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6153

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_
Symbol] :> Dist[(c^IntPart[p]*(c + d*x^2)^FracPart[p])/(1 - a^2*x^2)^FracPa
rt[p], Int[x^m*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d
, m, n, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && !In
tegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-5\tanh^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{27/2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{e^{-5\tanh^{-1}(ax)}x^2}{\left(1-a^2x^2\right)^{27/2}}\,dx}{c^{13}\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{1-a^2x^2}\int \frac{x^2}{(1-ax)^{11}(1+ax)^{16}}\,dx}{c^{13}\sqrt{c-a^2cx^2}}\\ &= \frac{(1+5ax)\sqrt{1-a^2x^2}}{120a^3c^{13}(1-ax)^{10}(1+ax)^{15}\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.52, size = 59, normalized size = 0.98

$$\frac{(5ax+1)\sqrt{1-a^2x^2}}{120a^3c^{13}(ax-1)^{10}(ax+1)^{15}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(E^(5*ArcTanh[a*x])*(c - a^2*c*x^2)^(27/2)),x]
```

```
[Out] ((1 + 5*a*x)*Sqrt[1 - a^2*x^2])/(120*a^3*c^13*(-1 + a*x)^10*(1 + a*x)^15*Sq
rt[c - a^2*c*x^2])
```

fricas [B] time = 0.95, size = 497, normalized size = 8.28

$$\frac{\left(a^{22}x^{25}+5\,a^{21}x^{24}-40\,a^{19}x^{22}-50\,a^{18}x^{21}+126\,a^{1}\right.}{120\left(a^{27}c^{14}x^{27}+5\,a^{26}c^{14}x^{26}-a^{25}c^{14}x^{25}-45\,a^{24}c^{14}x^{24}-50\,a^{23}c^{14}x^{23}+166\,a^{22}c^{14}x^{22}+330\,a^{21}c^{14}x^{21}-286\,a^{20}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^5*(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="fricas")
```

```
[Out] 1/120*(a^22*x^25 + 5*a^21*x^24 - 40*a^19*x^22 - 50*a^18*x^21 + 126*a^17*x^2
0 + 280*a^16*x^19 - 160*a^15*x^18 - 765*a^14*x^17 - 105*a^13*x^16 + 1248*a^
```

```
12*x^15 + 720*a^11*x^14 - 1260*a^10*x^13 - 1260*a^9*x^12 + 720*a^8*x^11 + 1
248*a^7*x^10 - 105*a^6*x^9 - 765*a^5*x^8 - 160*a^4*x^7 + 280*a^3*x^6 + 126*
a^2*x^5 - 50*a*x^4 - 40*x^3)*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/(a^27*
c^14*x^27 + 5*a^26*c^14*x^26 - a^25*c^14*x^25 - 45*a^24*c^14*x^24 - 50*a^23
*c^14*x^23 + 166*a^22*c^14*x^22 + 330*a^21*c^14*x^21 - 286*a^20*c^14*x^20 -
 1045*a^19*c^14*x^19 + 55*a^18*c^14*x^18 + 2013*a^17*c^14*x^17 + 825*a^16*c
^14*x^16 - 2508*a^15*c^14*x^15 - 1980*a^14*c^14*x^14 + 1980*a^13*c^14*x^13
+ 2508*a^12*c^14*x^12 - 825*a^11*c^14*x^11 - 2013*a^10*c^14*x^10 - 55*a^9*c
^14*x^9 + 1045*a^8*c^14*x^8 + 286*a^7*c^14*x^7 - 330*a^6*c^14*x^6 - 166*a^5
*c^14*x^5 + 50*a^4*c^14*x^4 + 45*a^3*c^14*x^3 + a^2*c^14*x^2 - 5*a*c^14*x -
 c^14)
```

giac [B] time = 5.60, size = 489, normalized size = 8.15

$$\frac{2451570\sqrt{c}\left(\frac{2}{ax+1}-1\right)^{9}+1514205\sqrt{c}\left(\frac{2}{ax+1}-1\right)^{8}+769120\sqrt{c}\left(\frac{2}{ax+1}-1\right)^{7}+318780\sqrt{c}\left(\frac{2}{ax+1}-1\right)^{6}+106260\sqrt{}}{2013265920\,a^{3}c^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^5*(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="giac")
```

```
[Out] 1/2013265920*(2451570*sqrt(c)*(2/(a*x + 1) - 1)^9 + 1514205*sqrt(c)*(2/(a*x
 + 1) - 1)^8 + 769120*sqrt(c)*(2/(a*x + 1) - 1)^7 + 318780*sqrt(c)*(2/(a*x
+ 1) - 1)^6 + 106260*sqrt(c)*(2/(a*x + 1) - 1)^5 + 27830*sqrt(c)*(2/(a*x +
1) - 1)^4 + 5520*sqrt(c)*(2/(a*x + 1) - 1)^3 + 780*sqrt(c)*(2/(a*x + 1) - 1
)^2 + 70*sqrt(c)*(2/(a*x + 1) - 1) + 3*sqrt(c))/(a^3*c^14*(2/(a*x + 1) - 1)
^10) - 1/2013265920*(2*a^42*c^(393/2)*(2/(a*x + 1) - 1)^15 + 45*a^42*c^(393
/2)*(2/(a*x + 1) - 1)^14 + 480*a^42*c^(393/2)*(2/(a*x + 1) - 1)^13 + 3220*a
^42*c^(393/2)*(2/(a*x + 1) - 1)^12 + 15180*a^42*c^(393/2)*(2/(a*x + 1) - 1)
^11 + 53130*a^42*c^(393/2)*(2/(a*x + 1) - 1)^10 + 141680*a^42*c^(393/2)*(2/
(a*x + 1) - 1)^9 + 288420*a^42*c^(393/2)*(2/(a*x + 1) - 1)^8 + 432630*a^42*
c^(393/2)*(2/(a*x + 1) - 1)^7 + 408595*a^42*c^(393/2)*(2/(a*x + 1) - 1)^6 -
 891480*a^42*c^(393/2)*(2/(a*x + 1) - 1)^4 - 2080120*a^42*c^(393/2)*(2/(a*x
 + 1) - 1)^3 - 3120180*a^42*c^(393/2)*(2/(a*x + 1) - 1)^2 - 3565920*a^42*c^
(393/2)*(2/(a*x + 1) - 1))/(a^45*c^210)
```

maple [A] time = 0.04, size = 49, normalized size = 0.82

$$-\frac{(ax-1)\,(5ax+1)\left(-a^{2}x^{2}+1\right)^{\frac{5}{2}}}{120\,(ax+1)^{4}\,a^{3}\left(-a^{2}c\,x^{2}+c\right)^{\frac{27}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)^5*(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(27/2),x)
```

```
[Out] -1/120*(a*x-1)*(5*a*x+1)*(-a^2*x^2+1)^(5/2)/(a*x+1)^4/a^3/(-a^2*c*x^2+c)^(2
7/2)
```

maxima [B] time = 0.96, size = 273, normalized size = 4.55

$$\frac{}{120\left(a^{28}c^{14}x^{25}+5\,a^{27}c^{14}x^{24}-40\,a^{25}c^{14}x^{22}-50\,a^{24}c^{14}x^{21}+126\,a^{23}c^{14}x^{20}+280\,a^{22}c^{14}x^{19}-160\,a^{21}c^{14}x^{18}-765\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)^5*(-a^2*x^2+1)^(5/2)/(-a^2*c*x^2+c)^(27/2),x, algorit
hm="maxima")
```

```
[Out] 1/120*(5*a*sqrt(c)*x + sqrt(c))/(a^28*c^14*x^25 + 5*a^27*c^14*x^24 - 40*a^2
5*c^14*x^22 - 50*a^24*c^14*x^21 + 126*a^23*c^14*x^20 + 280*a^22*c^14*x^19 -
 160*a^21*c^14*x^18 - 765*a^20*c^14*x^17 - 105*a^19*c^14*x^16 + 1248*a^18*c
^14*x^15 + 720*a^17*c^14*x^14 - 1260*a^16*c^14*x^13 - 1260*a^15*c^14*x^12 +
 720*a^14*c^14*x^11 + 1248*a^13*c^14*x^10 - 105*a^12*c^14*x^9 - 765*a^11*c^
14*x^8 - 160*a^10*c^14*x^7 + 280*a^9*c^14*x^6 + 126*a^8*c^14*x^5 - 50*a^7*c
^14*x^4 - 40*a^6*c^14*x^3 + 5*a^4*c^14*x + a^3*c^14)
```

mupad [B] time = 2.22, size = 363, normalized size = 6.05

$$\frac{}{-120\,a^{30}\,c^{14}\,x^{27}-600\,a^{29}\,c^{14}\,x^{26}+120\,a^{28}\,c^{14}\,x^{25}+5400\,a^{27}\,c^{14}\,x^{24}+6000\,a^{26}\,c^{14}\,x^{23}-19920\,a^{25}\,c^{14}\,x^{22}-396}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^(5/2))/((c - a^2*c*x^2)^(27/2)*(a*x + 1)^5),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(1 - a^2*x^2)^(1/2) + 5*a*x*(c - a^2*c*x^2)^(1/2)*(1
 - a^2*x^2)^(1/2))/(120*a^3*c^14 + 600*a^4*c^14*x - 120*a^5*c^14*x^2 - 5400
*a^6*c^14*x^3 - 6000*a^7*c^14*x^4 + 19920*a^8*c^14*x^5 + 39600*a^9*c^14*x^6
 - 34320*a^10*c^14*x^7 - 125400*a^11*c^14*x^8 + 6600*a^12*c^14*x^9 + 241560
*a^13*c^14*x^10 + 99000*a^14*c^14*x^11 - 300960*a^15*c^14*x^12 - 237600*a^1
6*c^14*x^13 + 237600*a^17*c^14*x^14 + 300960*a^18*c^14*x^15 - 99000*a^19*c^
14*x^16 - 241560*a^20*c^14*x^17 - 6600*a^21*c^14*x^18 + 125400*a^22*c^14*x^
19 + 34320*a^23*c^14*x^20 - 39600*a^24*c^14*x^21 - 19920*a^25*c^14*x^22 + 6
000*a^26*c^14*x^23 + 5400*a^27*c^14*x^24 + 120*a^28*c^14*x^25 - 600*a^29*c^
14*x^26 - 120*a^30*c^14*x^27)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)**5*(-a**2*x**2+1)**(5/2)/(-a**2*c*x**2+c)**(27/2),x)
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
   as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```